NOTES

On the Depth of the Nocturnal Boundary Layer

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ABSTRACT

The depth of the nocturnal boundary layer, modeled by diagnostic functions of surface fluxes, is only weakly related to "observed" depths estimated from observed profiles of either wind or temperature as has been shown in previous studies. This is partly due to influences of nonstationarity and large errors in the estimate of the small surface fluxes. However, the weak relationship between the modeled and profile-derived depths also is due to the inability of the profile-derived depths to represent the actual depth of the turbulence. The diagnostic models perform significantly better when tested against an improved estimate of the actual depth of the turbulence as computed from profiles of the Richardson number.

Yu (1978) showed that several simple diagnostic and time-dependent models all fail to explain more than 25% of the variance of the nocturnal boundary-layer depth when the depth is estimated from profiles of temperature or wind. These models presumably attempt to predict the depth of the turbulence and not the deeper inversion layer since the models are based on surface turbulent fluxes. Since Yu's study, the time-dependent models are being replaced by slightly more sophisticated versions (Smeda, 1979) which often are posed in terms of a Richardson number (Manins and Sawford, 1979; Mahrt 1981; Nieuwstadt and Tennekes, 1981). However, the original diagnostic models continue to enjoy use as a rough estimate of the boundary-layer depth or as a scaling depth.

Yu's results suggest that the diagnostic models could be virtually useless if most of the unexplained 75% of the variance is due to nonstationarity or other inadequacies of the model. On the other hand, some of the unexplained variance could be due to errors in the deduced surface fluxes and due to the inability of profile-derived depths to represent the actual depth of the turbulence. For example, the height of the low-level wind maximum h_u may reflect the influences of baroclinity and the history of the wind more than the instantaneous distribution of turbulence especially in the frequent case where the wind maximum occurs in a region of weak shear. As a further example, the depth of the cooling h_θ or surface inversion h_i may be strongly influenced by clear-air radiational cooling (André and Mahrt, 1982; Garratt and Brost, 1982). In this note, we will attempt to show that the depth of significant turbulence appears to be considerably thinner than the various profile-derived depths used by Yu (1978) and Melgarejo and Deardorff (1975) and that the diagnostic models perform considerably better when tested against a suitable estimate of the turbulence.

Here we will estimate the depth of the turbulence as the thickness of the layer below which the Richardson number is less than a critical value. The Richardson number Ri is computed over 50 m layers from the same Wangara temperature and wind profiles (Clarke *et al.*, 1971) analyzed by Yu. In particular,

$$\mathrm{Ri} = \frac{g}{\theta_0}\frac{\Delta\theta\ 50\ \mathrm{m}}{(\Delta u^2 + \Delta v^2)},$$

where $\Delta\theta$, Δu and Δv are the changes of potential temperature and wind components across 50 m layers. For the surface conditions, we use zero flow speed and the surface air temperature reported for the radiosonde.

The relationship between the turbulence and the occurrence of a critical Richardson number may be rather complicated although such a concept is widely used in studies of stratified turbulent flows. Fortunately, the Richardson number was normally observed to be small and relatively constant with height in a layer near the ground and then capped by a sharp increase of the Richardson number (André and Mahrt, 1981). Consequently, the estimated depth of the turbulence is not very sensitive to the numerical

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TABLE 1. Nocturnal boundary-layer depths.

Day	Hour (L)	h_{Ri} (m)	h_θ (m)	h_u (m)	h_i (m)
1	0600	33	190	280	200
	1800	39	200	220	150
	2100	45	220	320	250
	2400	38	355	210	200
4	0300	44	445	260	300
	0600	145	455	305	300
6	1800	76	125	120	100
	2100	30	200	120	150
7	0300	46	300	140	250
	0600	45	235	125	250
	1800	29	180	270	200
	2100	30	320	330	200
	2400	34	295	170	150
10	2400	30	210	200	150
11	2400	115	290	160	150
12	0600	60	265	120	300
	2100	60	465	190	200
	2400	60	275	110	250
13	0300	39	160	350	500
	0600	37	360	280	400
	1800	32	230	300	150
	2100	34	195	220	200
	2400	43	205	330	250
14	0300	28	115	270	150
	0600	138	310	210	350
	2100	31	180	160	200
	2400	45	320	320	300
16	2400	25	150	140	150
18	2400	63	155	150	50
19	0300	36	200	120	200
	0600	34	205	170	250
25	2100	33	250	115	150
26	0600	28	220	200	200
	2100	53	250	280	150
30	2100	118	200	500	200
	2400	138	340	350	300
31	0300	169	320	165	250
	0600	67	290	205	250
	2100	40	195	160	200
	2400	40	110	210	150
32	0300	36	390	145	200
	0600	46	380	170	300
	2400	30	150	195	150
33	0300	27	305	105	300
	1800	52	245	200	50
	2100	77	185	105	100
	2400	47	400	200	150
34	0300	58	450	215	400
	0600	50	650	105	500
	1800	74	250	245	100
	2100	67	225	190	200
35	0600	242	395	300	400
39	0300	200	375	165	350
	2400	7	200	100	200
40	0300	6	125	100	150
42	2100	28	215	100	250
	2400	32	205	150	200
43	0300	76	225	150	250
44	0300	250	330	450	
	0600	499	550	520	

choice of "cutoff" or critical value. Here we choose 0.5 for the cutoff value and use linear interpolation of the Richardson number between layer mid-levels to compute the implied depth of the turbulence. This inferred depth of the turbulence h_{Ri} is listed in Table 1 with the values reported in Yu for the other depth estimates.

As a diagnostic model of the depth of the nocturnal boundary layer, we use only Cu_*/f, where C is a nondimensional constant whose value does not influence the variance analysis to be presented below. The other existing diagnostic models need not be separately considered since they involve functions of u_*/f and the Monin-Obukhov length L and since u_*/f and L are very highly correlated (Mahrt and Heald, 1979; Venkatram, 1980). That is, the correlations between the profile-derived depths and model depth u_*/f (Table 2) would be nearly the same if we replaced u_*/f with L or $(u_*L/f)^{1/2}$.

Table 2 shows that nearly 50% of the variance of the depth of the inferred turbulence is explained by the diagnostic model u_*/f. This diagnostic model explains less than 25% of the other profile-derived depths as previously demonstrated by Yu (1978). Similar conclusions result from values of u_* provided by Hicks (1976) (Table 2). Scatter plots indicate that the correlation between the profile-depth estimates and the diagnostic model is weakest for small u_* when surface fluxes are most difficult to mea-

TABLE 2. Correlations between diagnostic models u_*/f and various profile-defined depths. Values of u_*/f are from Melgarejo and Deardorff (1975) [MD] and Hicks (1976) [H].

	MD	H
h_u	0.42	0.46
h_θ	0.42	0.42
h_i	0.23	0.32
h_{Ri}	0.86	0.76

sure. Using values of surface fluxes reported by Melgarejo and Deardorff and computing averages or least-square fits, we find that the implied depth of the turbulence is roughly 0.06 u_*/f, 0.6 $(u_*L/f)^{1/2}$ or 6 L.

In conclusion, simple diagnostic models of the depth of the nocturnal boundary layer perform considerably better when tested with a suitable estimate of the depth of the turbulence instead of previous profile-derived estimates. While nonstationarity is undoubtedly important, diagnostic expressions for the depth seem to have some use as a rough estimate when simplicity is required. The recent time-dependent models cited above should statistically perform better since they contain two adjustable parameters.

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