Auxiliary material

Calculation of the dynamic capillary pressure at wetting front

The basic assumption is that water flows down at the wetting front pore by pore. Mass balance consideration allows us to calculate the velocity, V, though the pore neck.

$$V = \frac{Q}{\pi {r_n}^2} \quad \text{(A1)}$$

where Q (cm^3/min) is the flux in the finger and r_n (cm) is the radius of the pore neck. The capillary number (Ca) is can be found for this velocity V as:

$$Ca = \frac{V\mu}{\sigma} \quad \text{(A2)}$$

where μ is the viscosity of water (0.00089 Pa.s) and σ is the surface tension of water (0.07197 N/m).

The Hoffman-Jiang equation in Baver et al (2013, this issue) allows us the find the dynamic contact angle for a zero static contact angle for any capillary number as

$$\frac{1-\cos\theta_{d,0}}{2} = \tanh(4.96\,\mathrm{Ca}^{.702}) \quad \text{(A3)}$$

Solving for $\theta_{d,0}$ gives:

$$\theta_{d,0} = \arccos\,(1 - 2\ \tanh(4.96\,\mathrm{Ca}^{.702})) \quad \text{(A4)}$$

According to Baver et al (2013, this issue) the dynamic contact angle of non-zero static contact angles can be found as:

$$\theta_d = \frac{\theta_{d,0} - \theta_s}{(1 - \frac{\theta_s}{\pi})} \quad \text{(A5)}$$

Substituting Eq. A5 into Eq. A4:

$$\theta_d = \theta_s + (1 - \frac{\theta_s}{\pi})\mathrm{arccos}\,(1 - 2 * \tanh(4.96\,\mathrm{Ca}^{.702}) \quad \text{(A6)}$$

Then θ_d is can be used to calculate the dynamic pressure at the wetting front, h, (cm), with the Laplace equation as:.

$$h = \frac{2\sigma cos\theta}{\rho g r_n} \quad \text{(A7)}$$

where σ is the surface tension of water (0.07197 N/m), ρ is the density of water (1000 kg/m^3), and r_n is the pore neck size.

In the DiCarlo (2007) experiments a high initial moisture content will increase the effective flux through the pore. We adjusted the velocity in an ad hoc manner as:

$$V = \frac{Q\left(1+\frac{m_i}{m_s}\right)}{\pi r_n{}^2} \quad \text{(A8)}$$

where m_i is the initial moisture content and m_s is the saturated moisture content.

Table A1: Finger properties for calculating fluxes in finger in Bauters et al (1998).

percent of water repellent grains	velocity of fastest finger (cm/min), solved by slope, Fig 4`	finger width (cm) from Table 4`	Calculated flux cm/min
0.0	0.58	45	0.40
3.1	1.94	2.1	1.14
5.0	2.59	2.8	2.03
5.7	2.87	2.7	2.17
9.0	5.24	3.5*	5.14

*Fingers split

` Fig 4 and Table 4 refer to Bauters et al. (1998)