AN ABSTRACT OF THE THESIS OF

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Several computer codes based on one and two-group diffusion theory models were developed for SHUFFLE. The programs were developed to calculate power distributions in a two-dimensional quarter core geometry of a pressurized power reactor. The various coarse-mesh numerical computations for the power calculations yield the following:

the Borresen's scheme applied to the modified onegroup power calculation came up with an improved power distribution,

the modified Borresen's method yielded a more accurate power calculations than the Borresen's scheme,

the face dependent discontinuity factor method have a better prediction of the power distribution than the node averaged discontinuity factor method,

Both the face dependent discontinuity factor method and the modified Borresen's methods for the two-group model have quite attractive features.

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TABLE OF CONTENTS

1.	Page
2.	FORMULATION AND DERIVATION OF THE COARSE-MESH DIFFUSION EQUATIONS
3.	BORRESEN'S METHODS OF NODAL AVERAGING
4.	THE DISCONTINUITY FACTOR METHOD24 4.1 Derivation of The Nodal Equations24
5.	RESULTS
6. 7.	CONCLUSION

LIST OF FIGURES

<u>Figure</u>	Page
2.1	Five-point difference scheme for the (1/2-group) models5
2.2	A representation of the boundary condition10
3.1	A planar diagram showing the notations used for nodes and interfaces14
3.2	A two region slab representing two adjacent half nodes in the x-direction18
4.1	A representation of the node(i,j) in relation to its neighboring nodes28
5.1	Oconee quarter core geometry with fuel type loading
5.2	Quarter core configuration for actual core and rectangular geometry with boundary conditions37

LIST OF TABLES

<u>Tabl</u>	<u>e</u> <u>Page</u>
5.1	Material properties of fuel and water for the Benchmark problems40
5.2	Design Data For The Oconee II Nuclear Power Plant41
5.3	Fast Flux Face-Dependent Discontinuity Factors42
5.4	Thermal Flux Face-dependent Discontinuity Factors44
5.5	Fast Flux Node Average Discontinuity Factors46
5.6	Thermal Flux Node Average Discontinuity Factors47
5.7	Assembly Power Densities for the 2-D Benchmark Problem (Borresen's Method Using Modified One- Group Diffusion Theory Model)48
5.8	Assembly Power Densities for the 2-D Benchmark Problem (Modified Borresen's Method Using Modified One-Group Diffusion Theory Model)49
5.9	Assembly Power Densities for the 2-D Benchmark Problem (Borresen's Method Using Two-Group Diffusion Theory Model)50
5.10	Assembly Power Densities for the 2-D Benchmark Problem (Modified Borresen's Method Using Two- Group Diffusion Theory Model)51
5.11	Assembly Power Densities for the 2-D Benchmark Problem (Face Dependent Discontinuity factor Method Using Two-Group Diffusion Theory Model)52
5.12	Assembly Power Densities for the 2-D Benchmark Problem (Node Averaged Discontinuity factor Method Using Two-Group Diffusion Theory Model)53
5.13	Summary of Results for the Benchmark Problem54

A Comparative Study of Nodal Coarse-Mesh Methods for Pressurized Water Reactors

1.0 INTRODUCTION

The two major concerns in the study of nuclear reactor core analysis are (1) calculating flux distribution and (2) determining criticality. Neutron diffusion theory is by far the most widely used method of treatment. Fine-mesh finite difference diffusion theory calculations have been relied on heavily by reactor designers and analysts, but when one considers the two and three dimensional geometries it becomes expensive.

In this study various coarse-mesh methods were examined. A common theme that binds all the various forms of the nodal and coarse mesh methods is that they can be reduced to a finite difference equations. This means that they can be solved by existing finite difference codes with some form of modifications. The SHUFFLE computer code developed by Stout[5] was modified. The major issue concerning the use of nodal methods is to find a simple but accurate relationship between the average flux in a node and the current on the node surfaces.

Borresen's method[2] of averaging the fluxes to obtain

the nodal fluxes was used, together with a modified scheme[3,4]. The node averaged fluxes were calculated by the use of finite difference coefficients that used the fluxes and diffusion coefficients of neighboring nodes. The modified Borresen's method[3,4] introduces nonlinearity to the flux calculations.

The Discontinuity factor method[6,7,8,9,13]; a method based on the continuity of current across nodal boundaries was also studied. Both a face-dependent and a node-averaged discontinuity factors were calculated, and used to generate the nodal fluxes and power distribution. The method allows for the discontinuity of the fluxes at the node boundaries but maintains the continuity of the currents. The results obtained closely agrees with the 2DB calculations[10].

Chapter 2 discussed the general formulation of the diffusion equations and the development of the finite difference scheme together with the boundary conditions used. In chapter 3 the Borresen's and the modified Borresen's methods were discussed. The discontinuity factor methods were considered in chapter 4. The results and conclusions were addressed in chapters 5 and 6 respectively.

2.0 FORMULATION AND DERIVATION OF THE COARSE-MESH DIFFUSION EQUATIONS

Satisfactory calculation of power distribution of a reactor core model is necessary in any reactor analysis. In this chapter, formulation of modified 1-group and the 2-group diffusion theory is presented. The Borresen Scheme and a modified form of the Borresen method was used to obtain node-averaged fluxes. These fluxes were then used in the power calculations. Initially, point dependent fluxes were obtained from the five-point difference scheme and it was from these that the node averaged fluxes were calculated.

2.1 Two-Group Diffusion Equations

The 2-group diffusion equations in x,y geometry are

$$-D_{1}\nabla^{2}\Phi_{1}(x,y) + \Sigma_{R_{1}}(x,y)\Phi_{1}(x,y) = S_{1}(x,y)$$
(2-1)

$$-D_2 \nabla^2 \Phi_2(x, y) + \Sigma_{a_2}(x, y) \Phi_2(x, y) = \Sigma_{sl_1}(x, y) \Phi_1(x, y)$$
(2-2)

where

$$s_{1}(x,y) = \frac{1}{k} \left[v \Sigma_{f_{1}}(x,y) \Phi_{1}(x,y) + v \Sigma_{f_{2}}(x,y) \Phi_{2}(x,y) \right],$$

$$\begin{split} \Phi \left(x,y \right) &= \text{Fast flux at } x,y \ , \\ \Phi \left(x,y \right) &= \text{Thermal flux at } x,y \ , \\ D_1 &= \text{Fast diffusion coefficient }, \\ D_2 &= \text{Thermal diffusion coefficient }, \\ \nu \Sigma_{f_1} \left(x,y \right) &= \text{fast fission source cross-section at } x,y \ , \\ \nu \Sigma_{f_2} \left(x,y \right) &= \text{thermal fission source cross-section at } x,y \ , \\ \Sigma_{sl} \left(x,y \right) &= \text{group transfer scattering} \\ &= \text{cross-section at } x,y \ , \text{ and} \\ \Sigma_{R_1} \left(x,y \right) &= \text{removal cross-section at } x,y \ . \end{split}$$

For a large thermal reactor, we can simplify the 2-group diffusion equations by neglecting the thermal leakage. The diffusion equation for the thermal group, Eq(2-2), becomes

$$\Phi_2 = \frac{\Sigma_{sl_1} \Phi_1}{\Sigma_{a_2}} \tag{2-3}$$

In solving the equations the reactor core was divided into a number of identical nodes. A mesh grid was constructed with mesh points located at the center of each node and difference equations were developed for the diffusion equations. Figure 2-1 shows a five-point difference scheme representation, where the center mesh point (i,j) can be represented by the node numbered 0.



Figure 2.1 Five-point difference scheme for the (1/2-group) models.

Integrating over the volume (unit) from $(x - \Delta x/2)$ to $(x + \Delta x/2)$ and from $(y - \Delta y/2)$ to $(y + \Delta y/2)$ yields the spatial difference equation associated with each point. Using Green's theorem[14], the volume integral of the leakage term transformed into a surface integral

$$\int D\nabla^2 \, \mathbf{\Phi} \, dV = \int D \, \nabla \mathbf{\Phi} \cdot dA \tag{2-4}$$

Equations (2-1) and (2-2) become:

$$-\int D\nabla \Phi_1 \cdot dA + \int \Sigma_{R_1} \Phi_1 dV = \int \frac{1}{k_{off}} \left(\nu \Sigma_{f_1} \Phi_1 + \nu \Sigma_{f_2} \Phi_2 \right) dV , \qquad (2-5)$$

and

$$-\int D\nabla \Phi_2 \cdot dA + \int \Sigma_{a_2} \Phi_2 dV = \int \Sigma_{sl_1} \Phi_1 dV \qquad (2-6)$$

Substituting for the thermal flux using the expression in eq(2-3) into eq(2-5) results in the following:

$$-\int D\nabla \Phi_1 \cdot dA + \int \Sigma_{R_1} \Phi_1 dV = \int \frac{1}{k_{off}} (\mathbf{v} \Sigma_{f_1} + \mathbf{v} \Sigma_{f_2}) \Phi_1 dV \quad , \qquad (2-7)$$

The node interface boundary condition correspond to the continuity of current density across the interface. Consider the case where the nodes i and it's adjacent node i+1 separated by a distance, h, and equidistant from a mesh line separating the two regions. The continuity of the neutron current density of the node interface gives the following:

$$-D_{i} \frac{\Phi_{i+\frac{1}{2}} - \Phi_{i}}{h/2} = -D_{i+1} \frac{\Phi_{i+1} - \Phi_{i+\frac{1}{2}}}{h/2}$$
(2-8)

 Φ_i , Φ_{i+} , and Φ_{i+1} are the fluxes at the mid-point of node i, the interface, and the mid-point of node i+1, respectively. The flux at the node interface is given as:

$$\Phi_{i+\nu_{2}} = \frac{D_{i}\Phi_{i} + D_{i+1}\Phi_{i+1}}{D_{i} + D_{i+1}} , \qquad (2-9)$$

Using the above expression the flux gradient can be written as:

$$\nabla \Phi_{i+1/2} = \left(\frac{D_{i+1}}{D_i + D_{i+1}}\right) \left(\frac{\Phi_{i+1} - \Phi_i}{h/2}\right)$$
(2-10)

The five-point difference scheme applied to equations (2-1) and (2-2) would result in the following:

$$\sum_{k=1}^{4} \frac{\overline{D}_{1k} A_k}{h_k} (\Phi_{1k} - \Phi_{10}) - \Sigma_{R_{10}} \Phi_{10} V_0 + S_{10} V_0 = 0$$
 (2-11)

where

$$\begin{split} V_0 &= \text{volume associated with mesh 0}, \\ \Phi_{1k} &= \text{fast flux of node k (at mesh point k)}, \\ h_k &= \text{distance between mesh point k and mesh point 0}, \\ A_k &= \text{area of the boundary separating mesh point k} \\ &= \text{and mesh point 0}, \text{ and} \end{split}$$

$$\overline{D}_{1k} = \frac{D_{10}D_{1k}(\Delta R_0 + \Delta R_k)}{D_{10}\Delta R_k + D_{1k}\Delta R_0}$$
(2-12)

is the effective diffusion coefficient and R represents either x or y, depending on whether k is 1,2 or 3,4, respectively.

Solving for Φ_{10} we get:

$$\Phi_{10} = \frac{S_{10}V_0 + \sum_{k=1}^{4} C_{1k}\Phi_{1k}}{C_{15}}$$
(2-13)

where

$$C_{1k} = \frac{\overline{D}_{1k}A_k}{h_k}$$

$$C_{15} = \Sigma_{R_{10}} V_0 + \sum_{k=1}^4 C_{1k}$$

2.2 Thermal Group Calculations

2.2.1 1.5-Group Approximation

From the assumption leading to Eq(2-3), the thermal flux can easily be determined from the fast flux calculation:

$$\Phi_2 = \frac{\Sigma_{sl_1} \Phi_1}{\Sigma_{a_2}}$$

2.2.2 2-Group Approximation

For the full 2-group case, the integration of Eq(2-2) yields:

$$\sum_{k=1}^{4} \frac{\overline{D}_{2k} A_k}{h_k} (\Phi_{2k} - \Phi_{20}) - \Sigma_{a_2} \Phi_{20} V_0 + \Sigma_{sl_{10}} \Phi_{10} V_0 = 0$$
(2-14)

The thermal flux becomes:

$$\Phi_{20} = \frac{\Sigma_{sl_{10}} V_0 \Phi_{10} + \sum_{k=1}^{4} C_{2k} \Phi_{2k}}{C_{25}}$$
(2-15)

where

Inner iterations were performed on the fast and

$$C_{25} = \Sigma_{a_2} V_0 + \sum_{k=1}^4 C_{2k}$$

$$C_{2k} = \frac{\overline{D}_{2k}A_k}{h_k}$$

thermal fluxes until the desired convergence criterion was met. When the inner iteration converges, the eigenvalue was corrected and the new eigenvalue was then used for another iteration (outer),

$$K_{eff}^{(n)} = K_{eff}^{(n-1)} \underline{(\text{total fission})^n}_{(\text{total fission})^{n-1}}$$

An over-relaxation technique was used for faster convergence of the fluxes. To calculate the power produced in each node, node-averaged fluxes are needed. The averaging techniques used are discussed in the next chapter.

2.3 Boundary Conditions

The center line for a quarter core of the Oconee Nuclear Reactor[12] bisects the fuel assemblies(nodes) in both the x and y direction. Thus the boundary condition applied to the two (inner-most) sides was the zero flux gradient due to symmetry. This is accomplished by setting the coefficients multiplying this term at these points to zero. The other two sides were the reflector boundary and the flux can be assumed to be zero at some extrapolated distance from the boundary surface. For this boundary, the coefficient becomes:

$$\frac{\overline{D}_{gk}A_k}{h_k} = \frac{\overline{D}_{gk}A_k}{.5\delta R_{I_n} + .71\lambda_{tr}} ,$$

where $R_{\rm In}$ is the last mesh interval, and the denominator represents the extrapolated distance.



Figure 2.2 A representation of the boundary condition

3.0 BORRESEN METHODS OF NODE-AVERAGING

Now that the fast and thermal point-wise fluxes have been determined, nodal power can be calculated using the cross-section data supplied. The point fluxes, when used in the power calculation, will yield a significantly large error. This is due to the fluxes in the neighboring bundles. Therefore, a node-averaged flux was used in the determination of nodal power.

In this chapter, methods of averaging the point fluxes to obtain node-averaged fluxes were discussed. First, the use of the Borresen method was discussed, followed by the modified form of the Borresen scheme.

3.1 Derivation of the Equations

The fast flux diffusion equation is:

$$-\nabla \cdot D_1 \nabla \Phi_1 = S_1 \cdot \Phi_1 \tag{3-1}$$

where

$$S_{1} = \frac{1}{k_{eff}} (\mathbf{v} \Sigma_{f_{1}} + \mathbf{v} \Sigma_{f_{2}} \cdot \frac{\Phi_{2}}{\Phi_{1}}) - \Sigma_{a_{1}} - \Sigma_{s_{12}}$$
(3-2)

A volume integration is performed over Eq(3-1),

$$-\int_{B} D_{1} \left(\nabla \boldsymbol{\Phi}_{1} \cdot \boldsymbol{n} \right) dB = \int_{V} S_{1} \boldsymbol{\Phi}_{1} dV \quad , \qquad (3-3)$$

where

B = nodal surface area, v = nodal volume, and n = nodal surface unit vector.

The right hand side of Eq(3-3) can be written as:

$$\int_{V} S_{1} \Phi_{1} dV = \overline{S}_{1} \cdot \overline{\Phi}_{1} V \quad , \qquad (3-4)$$

$$S_{1} = \frac{1}{k_{eff}} \left(\nu \Sigma_{f_{1}} + \nu \Sigma_{f_{2}} \frac{\Sigma_{s_{12}}}{\Sigma_{a_{2}}} F \right) - \Sigma_{a_{1}} - \Sigma_{s_{12}}$$
(3-5)

$$F = \frac{\overline{\Phi}_2}{\Phi_{2,as}} \tag{3-6}$$

with

$$\overline{\Phi}_{2} = \frac{1}{V} \int_{V} \Phi_{2} dV \qquad (3-7)$$

$$\overline{\Phi}_{2,as} = \frac{\Sigma_{s_{12}}}{\Sigma_{a_2}} \Phi_1 \tag{3-8}$$

$$\overline{\Phi}_1 = \frac{1}{V} \int_V \Phi_1 dV \quad , \tag{3-9}$$

where F; the actual to asymptotic node average flux ratio, is initially taken as unity.

A five-point finite difference scheme was formulated for the left hand side of Eq(3-3), with mesh points at the node centers. The continuity of current and flux densities across the interface $(i+\frac{1}{2})$, see figure (3.1), resulted in the expression below

$$-D_{i}\frac{\Phi_{i+1/2}-\Phi_{i}}{h/2} = -D_{i+1}\frac{\Phi_{i+1}-\Phi_{i+1/2}}{h/2}$$
(3-10)

from the definition of the net fast group current density

$$J_{i+1/2} = -D_i \frac{\Phi_{i+1} - \Phi_i}{h/2} , \qquad (3-11)$$

where

$$D_i$$
 = fast diffusion coefficient for node i,
 Φ_i = mid-point flux value for node i,
 $\Phi_{i+\frac{1}{2}}$ = flux value on interface between node i and
i+1, and

h = mesh width.

From above, the flux at the interface becomes

$$\Phi_{i+\frac{1}{2}} = \frac{D_i \Phi_i + D_{i+1} \Phi_{i+1}}{D_i + D_{i+1}} , \qquad (3-12)$$

and

$$J_{i+1/2} = \frac{2D_i D_{i+1}}{h(D_i + D_{i+1})} \left(\Phi_i - \Phi_{i+1} \right)$$
(3-12)

Similarly or the interface between node i-1 and node i:

$$J_{i-\nu_{2}} = -\frac{2D_{i}D_{i-1}}{h(D_{i} + D_{i-1})} \left(\Phi_{i} - \Phi_{i-1}\right)$$
(3-14)

(i-1,j)	← (i-½,j) (i,j)	← (i+½,j) (i+1,j)
	← h →	

Figure 3.1 A planar diagram showing the notations used for nodes and interfaces

The left hand side of Eq(3-3) can be expressed as :

$$-\int_{B} D(\nabla \Phi \cdot n) dB \approx J_{i+\nu_2} - J_{i-\nu_2}$$
(3-15)

A two-dimensional finite-difference form of Eq(3-3) with an approximation of

$$\frac{D_i D_j}{D_i + D_j} \approx 1/2 \sqrt{D_i D_j}$$

The above approximation is used to eliminate the use of the coefficient on the left hand side of the equation, which would otherwise occupy computer memory during the computation, with minimal error. Expressing the node-averaged fast (and thermal) flux as a weighted average of the mid-point flux in that node and the fluxes on the four adjacent interfaces,

$$\overline{\Phi}_{i} = b \Phi_{i} + 2C \sum_{4j} \Phi_{j}^{i}$$
(3-16)

where

$$b = \frac{3a}{3a+2(1-a)}$$
(3-17)

$$C = 1 - \frac{a}{4[3a+2(1-a)]}$$
(3-18)

and

$$\Phi_j^1$$
 = the flux on the interface between node i and
node j,and

3.2 Modified Borresen's Method

Borresen's coarse-mesh 1.5 (two)-group diffusion theory scheme is an efficient tool for power distribution calculations. It's major drawback is in cases where large thermal gradients occur. To overcome this, Kim et al. [4] presented a modification which was made with regard to the thermal group. In this technique the nodal interface thermal group was determined analytically, and using this a node-averaged thermal flux was obtained. A discussion of the modification follow below.

3.3 Mathematical Formulation of the Modified Borresen's Scheme

The interpolation formula for the node-averaged group flux Φ_i for a three-dimensional case is

$$\overline{\Phi}_{gi} = b_g \Phi_{gi} + 2C_g \left(\sum_{4j} \Phi_{gi}^j + w R \sum_{2j} \Phi_{gi}^j \right)$$
(3-19)

with the nodal interface group flux given by

$$\Phi_{gi}^{j} = \frac{D_{gi}\Phi_{gi} + D_{gj}\Phi_{gj}}{D_{gi} + D_{gj}}$$
(3-20)

Only a two-dimensional analysis was being considered in this study. Therefore, the R in Eq(3-19) was set to zero. Thus, the effect of the w in Eq(3-19) was not there. The f is a parameter that was not present in the original formulation by Borresen, but was added by Kim and Levine [3,4]. It gives different weights to the nodal interface group fluxes, depending on whether they are at vertical or horizontal surfaces. Thus, for the two-dimensional analysis, Eq(3-19) is exactly the same as Eq(3-16).

In the original Borresen scheme, an approximation similar to Eq(2-3) was used:

$$\Phi_{2i} = \Sigma_{sl_i} \frac{\overline{\Phi}_{1_i}}{\Sigma_{a_{2_i}}}$$
(3-21)

But in the modified scheme the node-averaged thermal flux was computed using an interpolation formula with separate weight factors for the thermal groups:

$$\overline{\Phi}_{2i} = b_{2i} \Phi_{2i} + 2C_{2i} \sum_{j=1}^{4} \Phi_{2i}^{j}$$
(3-22)

where

$$b_{2i} = [1 - 2 \tanh(hK_i/2) / hK_i]^2 , \qquad (3-23)$$

$$C_{2i} = (1 - b_{2i})/2 \tag{3-24}$$

$$K_i = \sqrt{\frac{\Sigma_{at_i}}{D_{t_i}}} \tag{3-25}$$

How the expression given above for equation (3-22) is obtained will be shown in the following section.

3.4 Solution to the 1-D Diffusion theory equation

Consider a one-dimensional thermal group diffusion equation for a two-region slab representing two adjacent half-nodes in the x-direction, figure(3.2):

$$-D_{2i}\frac{d^2}{dx^2}\Phi_{2i} + \Sigma_{a_{2i}}\Phi_{2i} = S_i \qquad \text{for } -\frac{h}{2} \le x \le 0 \qquad (3-26)$$

and

$$-D_{2j}\frac{d^2}{dx^2}\Phi_{2j} + \Sigma_{a_{2j}}\Phi_{2j} = S_j \qquad \text{for } 0 \le x \le \frac{h}{2} \qquad (3-27)$$

with boundary conditions

$$\Phi_{2i}(-\frac{h}{2}) = \Phi_{2i} = (\Sigma_{s12_i} / \Sigma_{a_{2i}}) \overline{\Phi}_{1i}$$
(3-28)

and

$$\Phi_{2j}(-\frac{h}{2}) = \Phi_{2j} = (\Sigma_{s12j} / \Sigma_{a_{2j}}) \overline{\Phi}_{1j}$$
(3-29)

and the interface continuity conditions

$$\Phi_{2i}(0) = \Phi_{2j}(0) = \Phi_{2i}^{j}$$
(3-30)

and

$$-D_{2i}\Phi_{2i}(0) = D_{2j}\Phi_{2j}(0) \tag{3-31}$$



Figure 3.2 A two region slab representing two adjacent half nodes in the x-direction

Solving equations (3-26) and (3-27) assuming a constant source, resulted in a solution of the form

$$\Phi_{2i} = \Phi_{2i} + A_1 \cosh(x/L_i) + B_1 \sinh(x/L_i)$$
(3-32)

and

$$\Phi_{2j} = \Phi_{2j} + A_2 \cosh(x/L_j) + B_2 \sinh(x/L_j)$$
(3-33)

Applying the boundary conditions

$$\Phi_{2i}(h/2) = \Phi_{2i} + A_1 \cosh(h/2L_i) + B_1 \sinh(h/2L_i)$$
(3-34)

$$\Phi_{2j}(-h/2) = \Phi_{2j} + A_2 \cosh(-h/2L_j) + B_2 \sinh(-h/2L_j) \quad (3-35)$$

$$\Phi_{2i}(0) = \Phi_{2i}^{j} = \Phi_{2i}^{j} + A_{1}$$
(3-36)

$$\Phi_{2j}(0) = \Phi_{2j}^{j} = \Phi_{2j}^{j} + A_{2}$$
(3-37)

and

$$-D_{2i}\frac{B_1}{L_i} = -D_{2j}\frac{B_2}{L_j}$$
(3-38)

from equation (3-38)

$$B_2 = B_1 \frac{D_{2i}L_j}{D_{2j}L_i}$$
(3-39)

and substituting for B_2 in equation (3-35) and rearranging the equations (3-34) and (3-35), the two equations thus become

$$A_{2} + B_{1} \frac{D_{2i}L_{j}}{D_{2j}L_{i}} \tanh(h/2L_{j}) = 0$$
(3-40)

and

$$A_{1} - B_{1} \tanh(h/2L_{j}) = 0$$
 (3-41)

From equations (3-36), (3-37), (3-40) and (3-41) we can obtain all the unknowns. Thus

$$A_{1} = \frac{\frac{D_{2j}}{L_{j}} (\Phi_{2j} - \Phi_{2i}) \tanh(h/2L_{i})}{DM}$$
(3-42)

$$A_{2} = \frac{\frac{D_{2i}}{L_{i}} (\Phi_{2i} - \Phi_{2j}) \tanh(h/2L_{j})}{DM}$$
(3-43)

$$B_{1} = \frac{\frac{D_{2j}}{L_{j}} (\Phi_{2j} - \Phi_{2i})}{DM}$$
(3-44)

and

$$B_{2} = \frac{\frac{D_{2i}}{L_{i}} (\Phi_{2j} - \Phi_{2i})}{DM}$$
(3-45)

where

$$DM = \frac{D_{2i}}{L_i} \tanh(h/2L_j) + \frac{D_{2j}}{L_j} \tanh(h/2L_i)$$

and from equation (3-36),

$$\Phi_{2i}^{j} = \frac{\frac{D_{2i}K_{i}}{T_{i}} \Phi_{2i} + \frac{D_{2j}K_{j}}{T_{j}} \Phi_{2j}}{\frac{D_{2i}K_{i}}{T_{i}} + \frac{D_{2j}K_{j}}{T_{j}}}$$
(3-46)

where

$$K_{I} = \left[\Sigma_{a_{2I}} / D_{2i} \right]^{\frac{1}{2}}$$

and

Similar expression can be obtained for the y-directed surfaces of the node i.

To determine the thermal group weight factor, b_2 , consider the one-dimensional diffusion equation for node i.

$$-D_{2i}\frac{d^2}{dx^2}\Phi_{2i} + \Sigma_{a_{2i}}\Phi_{2i} = S_i \qquad \text{for } -\frac{h}{2} \le x \le +\frac{h}{2} \qquad (3-47)$$

$$S_{i} = \Sigma_{sl_{i}} \overline{\Phi}_{1i} = \Sigma_{a_{2i}} \Phi_{2i}$$
(3-48)

and

$$\Phi_{2i}(-h/2) = \Phi_{2i}^{ls}$$
$$\Phi_{2i}(-h/2) = \Phi_{2i}^{rs}$$

The node-averaged thermal flux is

$$\overline{\Phi}_{2i} = \frac{1}{h} \int_{-h/2}^{h/2} \Phi_{2i}(x) \, dx \tag{3-49}$$

Substituting for Φ_{2i} , using a similar solution as performed above,

$$\overline{\Phi}_{2i} = \frac{1}{h} \left[\Phi_{2i} + A_1 \cosh(x/L_i) + B_1 \sinh(x/L_i) \right]$$
(3-50)

where

$$A_{1} = \frac{\Phi_{2i}^{ls} + \Phi_{2i}^{rs} - 2\Phi_{2i}}{2\cosh(h/2L_{i})}$$
(3-51)

$$B_{1} = \frac{\Phi_{2i}^{ls} + \Phi_{2i}^{rs}}{2\sinh(h/2L_{i})}$$
(3-52)

Thus,

$$\overline{\Phi}_{2i} = a_{2i} \Phi_{2i} + \frac{1 - a_{2i}}{2} \left(\Phi_{2i}^{ls} + \Phi_{2i}^{rs} \right)$$
(3-53)

where

 Φ_{21}^{1s} = thermal flux at the left surface of node i Φ_{2i}^{rs} = thermal flux at the right surface of node i, and

$$a_{2i} = 1 - 2 \tanh(hK_i/2) / hK_i$$
 (3-54)

Thus the b_{2i} is an approximation of the product of a_{2i} along the x and y directions. Thus,

$$b_{2i} = [1 - 2 \tanh(hK_i/2) / hK_i]^2$$

The treatment of the boundary conditions is the same as the

one discussed in chapter 2. (section 2.3).

3.5 Power Calculation

The power distribution was obtained from the node averaged fast and thermal flux distributions. The node average fast and thermal fluxes were obtained by the averaging techniques discussed from the node-centered fast and thermal flux distributions. Thus

 $P = v \Sigma_{f_1} \overline{\Phi}_1 + v \Sigma_{f_2} \overline{\Phi}_2$

4.0 THE DISCONTINUITY FACTOR METHOD

The discontinuity factor method was developed for power reactor analyses in which "discontinuity factors" were used in each of the node interfaces. The face dependent discontinuity factor and the node averaged discontinuity factor methods are discussed in this chapter. The conventional diffusion theory model is the fundamental basis of the discontinuity factor method with the exception of introducing "discontinuity factors" in order to predict accurately the node-averaged reaction rates, provided that the discontinuity factors are obtained from a reference solution.

4.1 Derivation of the Nodal Equations

Consider the multi-group diffusion theory equation,

$$\nabla \cdot \vec{J}_{g}(\vec{r}) + \Sigma_{rg}(\vec{r}) \Phi_{g}(\vec{r}) = \sum_{g'=1}^{G} \left[\frac{1}{\lambda} M_{gg'}(\vec{r}) + \Sigma_{gg'}(\vec{r}) \right] \Phi_{g'}(\vec{r}) \quad (4-1)$$

where

$$\begin{split} \vec{J}_g(\vec{r}) &= \int_{\Delta E_g} \vec{J}(\vec{r}, E) \, dE \quad , \\ \Phi_g(\vec{r}) &= \int_{\Delta E_g} \Phi(\vec{r}, E) \, dE \quad , \end{split}$$

$$\begin{split} \Sigma_{rg}(\vec{r}) \, \Phi_g(\vec{r}) &= \int_{\Delta E_g} \Sigma_r(\vec{r}, E) \, \Phi(\vec{r}, E) \, dE \quad , \\ M_{gg}, (\vec{r}) \, \Phi_{g'}(\vec{r}) \\ &= \int_{\Delta E_g} dE \!\!\int_{\Delta E_{g'}} dE' \sum_j \chi^j(E) \, \mathbf{v} \Sigma_f^{\ j}(\vec{r}, E') \, \Phi(\vec{r}, E') \end{split}$$

$$\Sigma_{gg'}(\vec{r}) \Phi_{g'}(\vec{r}) = \int_{\Delta E_g} dE \int_{\Delta E_{g'}} dE \cdot \Sigma_s(\vec{r}, E' \to E) \Phi(\vec{r}, E')$$

$$(4-2)$$

Subscript g or g' refer to energy groups, superscript j identify particular isotopes, $\Sigma_{gg'}(\vec{r})$ is the macroscopic cross section for removal of neutrons from group g' to group g, $\Sigma_{rg}(\vec{r})$ is the macroscopic cross section for removal of neutrons from group g (includes absorption), $\chi(E)$ is the probability that fission neutrons will be born with energies between E and E+AE.

By the use of the net current we can derive a formally exact finite-difference equation. The reactor was partitioned into a large number of nodes of the size

$$V_{ij} = h_x^i h_y^j$$
 ,

where h can vary in size.

With the location of nodes labeled by two numbers (i,j) ; figure(4.1), Eq(4-1) was integrated over the volume (area) of any one node,

$$\frac{1}{V_{ij}} \sum_{n=1}^{6} \int_{s_n} \vec{J}_g(\vec{r}) \cdot \vec{n}_{s_n} ds + \overline{\Sigma}_g^{ij} \overline{\Phi}_{gg'}^{ij}$$

$$= \sum_{g'=1}^{G} \frac{1}{\lambda} M_{gg'}^{ij} \Phi^{ij} + \sum_{g'\neq} \overline{\Sigma}_{gg'}^{ij} \overline{\Phi}_{g'}^{ij}$$
(4-3)

where

$$\begin{split} V_{ij}\overline{\Phi}_{g}^{ij} &= \int_{V} \Phi_{g} dV , \\ V_{ij}\overline{\Sigma}_{rg}^{ij}, \overline{\Phi}_{g}^{ij} &= \int_{V} [\Sigma_{rg} - \Sigma_{gg},] \Phi_{g} dV , \\ V_{ij}\overline{M}_{gg}^{ij}, \overline{\Phi}_{g'}^{ij} &= \int_{V} M_{gg}, \Phi_{g}, dV , \\ V_{ij}\overline{\Sigma}_{gg}^{ij}, \overline{\Phi}_{g'}^{ij} &= \int_{V} \Sigma_{gg}, \Phi_{g}, dV , \end{split}$$

$$(4-4)$$

The formulation of the above integration was based on three dimensions, in which $h_z^{\ k}$ was considered as unity and the z-component was suppressed.

In order to obtain formally exact nodal equations of the finite-difference form for $\overline{\Phi}^{ij}$ we shall relate $(\int J \cdot n \, ds)$ to $\overline{\Phi}^{ij}$ in exactly the same way as we would if deriving the finite-difference equations from the diffusion equation Eq(4-1). Two approximate expressions can be written for the face-integrated current between node (i,j) and node (i+1,j),

$$\int dy J_{gx}(x_{i+l_{x}}, y) \approx -\overline{D}_{g}^{i+1,j} \frac{h_{y}^{j} \overline{\Phi}_{g}^{i+1,j} - \int dy \Phi_{g}(x_{i+l_{x}}, y)}{h_{x}^{i+1}/2}$$

$$\approx -\overline{D}_{g}^{ij} \frac{\int dy \Phi_{g}(x_{i+l_{x}}, y) - h_{y}^{j} \overline{\Phi}_{g}^{ij}}{h_{x}^{i+1}/2} \qquad (4-5)$$

The approximations were based on the validity of Fick's law ($J_{gu}(\vec{r}) = -D_{gu}(\vec{r}) (\partial/\partial u) \Phi_{g}(\vec{r})$) on the interface between the two nodes. The reason for the two expressions is that one can approach the interface from either of the two directions.

$$J_{gx}(x_{i+1/2}, y) = -\overline{D}_{g}^{i+1,j} \frac{\partial \Phi_{g}(x_{i+1}, y)}{\partial x}$$
$$= -\overline{D}_{g}^{ij} \frac{\partial \Phi_{g}(x_{i}^{*}, y)}{\partial x} \qquad (4-6)$$

By dividing the first expression in Eq(4-5) by $\overline{D}^{i+1,j}$, and the second by $\overline{D}^{i,j}$, rearranging and adding the two equations will result in

$$\frac{1}{h_{y}^{j}} \int dy J_{gx}(x_{i+\nu_{2}}, y) \approx - \left[\frac{h_{x}^{i}}{2\overline{D}_{g}^{i,j}} + \frac{h_{x}^{i+1}}{2\overline{D}_{g}^{i+1,j}}\right]^{-1} (\overline{\Phi}_{g}^{i+1,j} - \overline{\Phi}_{g}^{i,j}) \qquad (4-7a)$$



Figure 4.1 A representation of the node (i,j) in relation to its neighboring nodes.

Similarly, the expressions for the currents across the other three faces of the node (i,j) are:

$$\frac{1}{h_{y}^{j}} \int dy J_{gx}(x_{i-\nu_{z}}, y) \approx -\left[\frac{h_{x}^{i}}{2\overline{D}_{g}^{i,j}} + \frac{h_{x}^{i-1}}{2\overline{D}_{g}^{i-1,j}}\right]^{-1} (\overline{\Phi}_{g}^{i-1,j} - \overline{\Phi}_{g}^{ij}) \quad (4-7b)$$

$$\frac{1}{h_x^{i}} \int dy J_{gx}(x, y_{i+v_2}) \approx - \left[\frac{h_y^{j}}{2\overline{D}_g^{i,j}} + \frac{h_y^{j+1}}{2\overline{D}_g^{i,j+1}} \right]^{-1} (\overline{\Phi}_g^{i,j+1} - \overline{\Phi}_g^{ij}) \quad (4-7c)$$

$$\frac{1}{h_x^{i}} \int dy J_{gx}(x, y_{j-\nu_2}) \approx - \left[\frac{h_y^{j}}{2\overline{D}_g^{i,j}} + \frac{h_y^{j-1}}{2\overline{D}_g^{i,j-1}} \right]^{-1} (\overline{\Phi}_g^{i,j-1} - \overline{\Phi}_g^{ij}) \quad (4-7d)$$

Equations (4-7a), (4-7b), (4-7c) and (4-7d) yield one of the standard set of finite-difference equations. But the approximation made was a poor one for a large h. To obtain a somewhat formally exact set of finite-difference equations, we can alter Eq(4-5) and force it be exact. Eq(4-5) can be used to define the values of $\overline{D}^{i+1,j}$ and $\overline{D}^{i,j}$ associated with the two faces of boundary separating node (i,j) from node (i+1,j). If the face integrated fluxes, currents and volume average fluxes are known from a reference transport theory calculations, the node average \overline{D} 's could be obtained by solving eq(4-5). Thus, the surface fluxes of the two expressions in eq(4-5) are divided by the "discontinuity factors", Eq(4-5) then becomes:

$$\int dy \int J_{gx}(x_{i+\frac{1}{2}}, y)$$

$$= -\overline{D}_{g}^{i+1,j} \frac{h_{y}^{j} \overline{\Phi}_{g}^{i+1,j} - \frac{1}{f_{gx}^{i+1,j}} \int dy \Phi_{g}(x_{i+\frac{1}{2}}, y)}{h_{x}^{i+1}/2}$$

$$= -\overline{D}_{g}^{ij} \frac{\frac{1}{f_{gx}^{ij}}}{h_{x}^{j+1}/2} (4-8)$$

Comparing eq(4-8) to eq(4-5) shows that the finite difference diffusion theory expression for the integrated surface current can be forced to be correct by allowing the integrated surface flux to be discontinuous across the surface. Thus a fictitious flux-integral was defined as:

$$\int dy \Phi_g^{hom} \left(X_{i+1}^{\pm}, y \right)$$

by the relationship

$$\int dy \Phi_{g}^{hom} (X_{i+1}^{+}, y) \equiv \frac{1}{f_{gx^{-}}^{i+1, j}} \int dy \Phi_{g} (x_{i+1}, y)$$

$$\int dy \Phi_{g}^{hom} (X_{i+1}^{-}, y) \equiv \frac{1}{f_{gx^{+}}^{i+1, j}} \int dy \Phi_{g} (x_{i+1}, y)$$

$$(4-9)$$

Since the net currents' expressions are dependent upon the interfaces surrounding a node, the f's are face-dependent discontinuity factors. The four discontinuity factors in node (i,j) are obtained as:

$$f_{gx^{+}}^{i,j} = \frac{\int dy \Phi_{g}(x_{i+1}, y)}{\int dy \Phi_{g}^{hom}(x_{i+1}^{-}, y)}$$

$$f_{gx^{-}}^{i,j} = \frac{\int dy \Phi_{g}(x_{i}, y)}{\int dy \Phi_{g}^{hom}(x_{i}^{+}, y)}$$

$$f_{gy^{+}}^{i,j} = \frac{\int dy \Phi_{g}(x, y_{j+1})}{\int dy \Phi_{g}^{hom}(x, y_{j+1}^{-})}$$

$$f_{gy^{-}}^{i,j} = \frac{\int dy \Phi_{g}(x, y_{j})}{\int dy \Phi_{g}^{hom}(x, y_{j}^{+})}$$
(4-10)

The discontinuity factors account for the fact that we are assuming Fick's law with the value \bar{D}_g to be correct and for the fact that we are making the finite-difference approximation. The f's have no physical significance.

By eliminating the face-integrated flux from Eq(4-8), this result, together with its analogous expressions for the three other surfaces, was substituted into Eq(4-3). We then obtain the following:

$$\begin{split} & -\frac{1}{h_x^i} \left[\frac{h_x^i f_{gx^+}^{ij}}{2\overline{D}_g^{i,j}} + \frac{h_x^{i+1} f_{gx^-}^{i+1,j}}{2\overline{D}_g^{i+1,j}} \right]^{-1} \\ & \left[f_{gx^-}^{i+1,j} \overline{\Phi}_g^{i+1,j} - f_{gx^+}^{ij} \overline{\Phi}_g^{i,j} \right] , \\ & -\frac{1}{h_y^i} \left[\frac{h_y^i f_{gy^+}^{ij}}{2\overline{D}_g^{i,j}} + \frac{h_y^{i+1} f_{gy^-}^{i,j+1}}{2\overline{D}_g^{i,j+1}} \right]^{-1} \\ & \left[f_{gy^-}^{i,j+1} \overline{\Phi}_g^{i,j+1} - f_{gy^+}^{ij} \overline{\Phi}_g^{i,j} \right] , \\ & -\frac{1}{h_x^i} \left[\frac{h_x^{i-1} f_{gx^+}^{i-1,j}}{2\overline{D}_g^{i-1,j}} + \frac{h_x^i f_{gx^-}^{i,j}}{2\overline{D}_g^{ij}} \right]^{-1} \\ & \left[f_{gx^+}^{i-1,j} \overline{\Phi}_g^{i-1,j} - f_{gx^-}^{ij} \overline{\Phi}_g^{i,j} \right] , \\ & -\frac{1}{h_y^j} \left[\frac{h_y^{j-1} f_{gy^+}^{i,j-1}}{2\overline{D}_g^{i,j-1}} + \frac{h_y^j f_{gy^-}^{i,j}}{2\overline{D}_g^{i,j}} \right]^{-1} \\ & \left[f_{gy^+}^{i,j-1} \overline{\Phi}_g^{i,j-1} - f_{gy^-}^{ij} \overline{\Phi}_g^{i,j} \right] , \end{split}$$

 $\overline{\Sigma}_{g}^{i,j}\overline{\Phi}_{g}^{i,j}$,

$$= \sum_{g'=1}^{G} \frac{1}{\lambda} \overline{M}_{gg'}^{ij} \overline{\Phi}^{ij} + \sum_{g'\neq g} \overline{\Sigma}_{gg'}^{ij} \overline{\Phi}_{g'}^{ij} ,$$

$$g = 1, 2 \dots G$$

$$i = 1, 2 \dots I - 1$$

$$j = 1, 2 \dots J - 1 \qquad (4 - 11)$$

A less cumbersome method of using the discontinuity factors is by the use of a node-averaged discontinuity factor, \overline{f} . The four Discontinuity factors as shown in eq(4-10) for a given energy group in a given node are replaced by their average value. Since fuel cells are rotationally symmetric and embedded in a lattice of similar cells, the assumption seemed reasonable. This makes it easier to program than using face dependent discontinuity factors; instead of using four f's per node, only one is used. The total leakage out of a node is preserved. By using Eq(4-5) for all the four faces, the result is:

$$\int dy J_{gx}(x_{i+\frac{1}{2}}, y) - \int dy J_{gx}(x_{i}, y)$$

$$+ \int dx J_{gy}(x, y_{i+\frac{1}{2}}) - \int dx J_{gy}(x, y_{i})$$

$$= -\frac{2\overline{D}_{g}^{i,j}}{\overline{f}_{g}^{i,j}} \left[\frac{1}{h_{x}^{i}} \left[dy \Phi_{g}(x_{i+\frac{1}{2}}, y) - \int dy \Phi_{g}(x_{i-\frac{1}{2}}, y) \right] \right]$$

$$\left[-\frac{1}{h_{y}^{j}} \left[\int dx \Phi_{g}(x, y_{j-\frac{1}{2}}) - \int dx \Phi_{g}(x, y_{j-\frac{1}{2}}) \right] \right]$$

$$(4-12)$$

 \bar{f}^{ij} can be taken as a simple arithmetic average of the four surface dependent discontinuity factors. Using \bar{f} , eq(4-10) becomes:

$$-\frac{1}{h_x^{i}} \left[\frac{h_x^{i}}{2\overline{D}_g^{*i,j}} + \frac{h_x^{i+1}}{2\overline{D}_g^{*i+1,j}} \right]^{-1} \left[\overline{\Phi}_g^{*i+1,j} - \overline{\Phi}_g^{*i,j} \right] ,$$

$$-\frac{1}{h_y^{j}} \left[\frac{h_y^{j}}{2\overline{D}_g^{*i,j}} + \frac{h_y^{j+1}}{2\overline{D}_g^{*i,j+1}} \right]^{-1} \left[\overline{\Phi}_g^{*i,j+1} - \overline{\Phi}_g^{*i,j} \right] ,$$

$$- \frac{1}{h_x^i} \left[\frac{h_x^{i-1}}{2\overline{D}_g^{*i-1,j}} + \frac{h_x^i}{2\overline{D}_g^{*i,j}} \right]^{-1} \left[\overline{\Phi}_g^{*i-1,j} - \overline{\Phi}_g^{*i,j} \right] , \\ - \frac{1}{h_y^j} \left[\frac{h_y^{j-1}}{2\overline{D}_g^{*i,j-1}} + \frac{h_y^j}{2\overline{D}_g^{*i,j}} \right]^{-1} \left[\overline{\Phi}_g^{*i,j-1} - \overline{\Phi}_g^{*i,j} \right] ,$$

$$+ \overline{\Sigma}_{rg'}^{*ij} \overline{\Phi}_{g'}^{*ij}$$

$$= \sum_{g'=1}^{G} \frac{1}{\lambda} \overline{M}_{gg'}^{*ij} \overline{\Phi}^{*ij} + \sum_{g'\neq g} \overline{\Sigma}_{gg'}^{*ij} \overline{\Phi}_{g'}^{*ij} \qquad (4-13)$$

where

$$\begin{split} \overline{\Phi}_{g}^{*ij} &= \overline{\Phi}_{g}^{i,j} \overline{f}_{g}^{i,j} , \\ \overline{D}_{g}^{*i,j} &= \overline{D}_{g}^{i,j} / \overline{f}_{g}^{i,j} \\ \overline{\Sigma}_{g}^{*ij} &= \overline{\Sigma}_{g}^{i,j} / \overline{f}_{g}^{i,j} , \\ \overline{\Sigma}_{gg'}^{*ij} &= \overline{N}_{gg'}^{ij} / \overline{f}_{g}^{i,j} , \\ \overline{\Sigma}_{gg'}^{*ij} &= \overline{\Sigma}_{gg'}^{ij} / \overline{f}_{g}^{i,j} , \end{split}$$

$$(4-14)$$

The boundary conditions employed are similar to the treatment made in Chapter 2.

5.0 RESULTS

A comparison of the results obtained for the various methods studied in this thesis was made with the benchmark test case calculations. The benchmark calculations were performed with 2DB code[10] using data from Oconee Nuclear Reactor[12]. A quarter core of mesh size 21.811 cm was used, which is the fuel assembly size of Oconee Nuclear Reactor core. The fuel loading by type and the core geometry are shown in figures (5.1) and (5.2). The design parameters of the Oconee nuclear reactor is shown in table (5.1). The material properties for the fuel and moderator are shown in table (5.2).

5.1 Discontinuity Factor Calculations

The face-dependent and the node-averaged discontinuity factors were obtained from the 2DB code calculations. A listing of the fast flux and thermal flux face-dependent discontinuity factors are shown in tables(5.3) and (5.4), and those of the fast and thermal node-averaged discontinuity factors are shown in tables (5.4) and (5.5).

		r		·	· · · · · · · · · · · · · · · · · · ·		r	1 — Č
2	1	2	1	3	1	2	3	
1	2	1	2	1	2	1	3	
2	1	3	1	2	1	2	3	
1	2	1	2	1	2	3		
3	1	2	1	2	3	3		
1	2	1	2	3	3			
2	1	2	3	3				
3	3	3						

∣ €

Region	Fuel Type (w/o U-235)	
1	1.5	
2	2.0	
3	3.0	

Figure 5.1 Oconee quarter core geometry with fuel type loading

									-
								4	
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								4	
							4	4	
							4		
							4	4	
						4	4	4	
			· · · · ·						
					4	4	4	4	-
			4	4	4	4	4	4	
4	4	4	4	4	4	4	4	4	
				l	l	l	l	l	J

Α

D

Side	Boundary Condition	Region	Material
A	$(\partial/\partial y)\Phi_{g} = 0$	4	Reflecto (Water)
В	$(\partial/\partial y)\Phi_{g} = 0$		
С	$\Phi_{g} = 0$		
D	$\Phi_{g} = 0$		

Figure 5.2 Quarter core configuration for actual core and rectangular geometry with boundary conditions

5.2 Comparison of the various Methods

The results for all of the various methods implemented were compared with the 2DB benchmark calculations. Table (5.13) shows the percentage relative errors of all the methods studied to the benchmark results. The relative percent error of K_{eff} , mean power density and maximum power density are computed.

In the determination of both the K_{eff} and power distribution the 2-group Borresen's and the modified 2-group Borresen's methods have identical relative percent errors. Of the two methods, the two group Borresen's method has the least relative percent error for the power distribution. The two methods have the same K_{eff} , since the modification only affects the power calculations.

There is a significant improvement in the results from the modified one-group Borresen's method (A in table(5.13)) to the 2-group methods(C and D in table(5.13)). The relative errors are lower for the 2-group calculations. But there is no significant difference between the modified one-group modified Borresen's method (B in table(5.13)) and the twogroup calculations.

The power distribution of the six methods(A,B,C,D,E,F) are shown together with the benchmark values in figures (5.7) to (5.12).

The results of the discontinuity factor methods closely agree with the benchmark results. Between the two discontinuity factor methods, the face dependent discontinuity factor method have lower relative percent errors (K_{eff} , mean and maximum) than the node-averaged discontinuity factor method. The face-dependent discontinuity factor method has the lowest mean and maximum percent error of all the methods studied.

A fundamental disadvantage of the discontinuity factor method is the requirement that a reference reactor solution has to be known, and from it the discontinuity factors are then determined. It would be worthwhile to develop a method of determining the discontinuity factors without a reference calculation, such as CASMO color sets.

Table 5.1 Material properties of fuel and water for the Benchmark problems

Region	Group	D	Σ_{a}	$v\Sigma_{f}$	Σ_{s12}
1	1	1.47011	.0079912	.0042851	.0174688
	2	0.389794	.0504723	.0721688	
2	1	1 46264	0082909	0049451	0168676
2	2	0.389884	.0602743	.0933065	.0100070
3	1	1.4557	.0088974	.0062028	.0159719
	2	0.38934	.078408	.13230	
4	1	1.561	.000434	0.0	.03002
	2	.31128	.008818	0.0	

Table 5.2 Design Data for the Oconee II Nuclear Power Plant

Rated thermal output 2568 Mw Vessel coolant inlet temperature 554 F Vessel coolant outlet temperature 604.7 \mathbf{F} Core outlet temperature 605.5 F Core operating pressure 2185 psig Total reactor coolant flow rate 13.13×10^7 lb/hr Average heat flux 171,470 Btu/hr-ft² ft² Core flow area 49.19 Average core fuel temperature 1540 F Total number fuel assemblies 177 Number of fuel rods per assembly 208 Fuel rod pitch 0.568 in Fuel assembly pitch 8.587 in Active fuel length 144 in Assembly lattice 8 x 8

[a]	b]	.e	5
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Table 5.3 Fast Flux Face-Dependent Discontinuity Factors

Ι	J	TOP	BOTTOM	\mathbf{LEFT}	RIGHT
1	1	1.004	1.017	1.004	1.017
1	2	0.949	0.952	0.993	0.981
1	3	1.057	1.055	1.003	1.016
1	4	0.944	0.899	0.997	0.985
1	5	1.179	1.178	1.017	1.050
1	6	0.896	0.936	0.999	0.986
1	7	1.040	0.962	1.008	1.020
1	8	1.199	3.257	1.004	1.004
1	9	0.537	0.192	1.004	1.004
2	1	0.993	0.981	0.949	0.952
2	2	1.047	1.051	1.047	1.051
2	3	0.936	0.894	0.966	0.966
2	4	1.045	1.046	1.052	1.052
2	5	0.893	0.935	0.967	0.970
2	6	1.048	1.041	1.056	1.052
2	7	0.949	0.945	0.948	0.916
2	8	1.005	1.013	1.270	3.207
2	9	1.018	0.000	0.536	0.101
3	1	1.003	1.016	1.057	1.055
3	2	0.966	0.966	0.936	0.894
3	3	1.173	1.175	1.173	1.175
3	4	0.962	0.964	0.896	0.941
3	5	1.043	1.046	1.053	1.059
3	6	0.937	0.935	0.957	0.962
3	/	1.042	0.951	1.08/	0.996
3	8	1.058	3.132	1.122	2.316
3	9	1.081	0.000	0.521	0.100
4 1	1 2	0.997	0.985	0.944	0.899
4	2	1.052	1.052	1.045	1.046
4	3	1 050	1 040	0.962	1 040
ч Л	4 5	1.050	0 944	1.050	1.049
<u>т</u>	5	1 045	0.944	1 078	0.955
Ā	7	1 082	1 034	1 170	2 924
4	8	0 611	0 807	0 581	0 181
4	9	0.765	0.000	0.551	0.401
5	1	1 017	1 050	1 179	1 178
5	2	0 967	0 970	0 893	0 935
5	3	1.053	1.059	1 043	1 046
5	4	0.949	0.953	0.944	0.944
5	5	1.063	0.968	1.063	0.968
5	6	1.131	1.080	1.140	1.086
5	7	1.080	3,065	1.037	2.340
5	8	1.023	1.288	0.523	0.209
5	9	0.824	0.000	0.572	0,100

6	1	0.999	0.986	0.896	0.936
6	2	1.056	1.052	1.048	1.041
6	3	0.957	0.962	0.937	0.935
6	4	1.078	0.985	1.045	0.954
6	5	1.140	1.086	1.131	1.080
6	6	1.055	2.553	1.055	2.553
6	7	0.608	0.693	0.589	0.570
6	8	0.757	0.430	0.669	0.273
6	9	0.892	0.000	0.584	0.100
7	1	1.008	1.020	1.040	0.962
7	2	0.948	0.916	0.949	0.945
7	3	1.087	0.996	1.042	0.951
7	4	1.170	2.823	1.082	1.034
7	5	1.037	2.340	1.080	3.065
7	6	0.589	0.570	0.608	0.693
7	7	0.703	0.362	0.703	0.362
7	8	0.698	0.442	0.657	0.366
7	9	0.741	0.002	0.641	0.100
8	1	1.004	1.004	1.199	3.257
8	2	1.270	3.207	1.005	1.013
8	3	1.122	2.316	1.058	3.132
8	4	0.581	0.481	0.611	0.807
8	5	0.523	0.209	1.023	1.288
8	6	0.669	0.273	0.757	0.430
8	7	0.657	0.366	0.698	0.442
8	8	0.681	0.395	0.681	0.395
8	9	0.701	0.013	0.663	0.100
9	1	1.004	1.004	0.537	0.192
9	2	0.536	0.101	1.018	0.000
9	3	0.521	0.100	1.081	0.000
9	4	0.652	0.100	0.765	0.000
9	5	0.572	0.100	0.824	0.000
9	6	0.584	0.100	0.892	0.000
9	7	0.641	0.100	0.741	0.002
9	8	0.663	0.100	0.701	0.013
9	9	0.682	5.019	0.682	5.018

Table 5.4 Thermal Flux Face-Dependent Discontinuity Factors

I	J	TOP	BOTTOM	LEFT	RIGHT
1	1	0.989	0.909	0.989	0.909
1	2	1.328	1.334	1.008	1.106
1	3	0.801	0.799	0.988	0.907
1	4	1.344	3.327	1.010	1.103
1	5	0.591	0.595	0.979	0.782
1	6	3.452	1.326	1.012	1.104
1	7	0.800	1.999	0.992	0.909
Ţ	8	0.662	0.289	1.003	0.999
1 2	9	0.533	0.471	1.004	1.004
2	1 2	1.008	1.106	1.328	1.334
2	2	0.790	0.807	0.796	0.807
2	7	1.320	3.309	1.329	1.330
2	-7 5	3 287	1 3 2 6	0.808	0.810
2	6	0 806	1.320	T.335	T.33T
2	7	1.348	1,337	1 360	3 997
2	8	1.022	1.031	0 565	0 288
2	9	1.020	0.000	0.533	0.200
3	1	0.988	0.907	0.801	0.799
3	2	1.329	1.330	1.328	3,309
3	3	0.591	0.592	0.591	0.592
3	4	1.322	1.324	3.365	1.336
3	5	0.793	0.794	0.797	0.803
3	6	1.307	1.348	1.352	1.395
3	7	0.796	1.554	0.802	1.580
3	8	0.939	0.289	0.727	0.296
3	9	1.051	0.000	0.577	0.105
4	1	1.010	1.103	1.344	3.327
4	2	0.808	0.810	0.804	0.805
4	3	3.365	1.336	1.322	1.324
4	4	0.807	0.797	0.807	0.797
4	5	1.320	1.370	1.333	1.385
4	6 7	0.794	1.902	0.805	1.951
4 1	0	0.742	1.050	0.673	0.289
ч Л	o Q	1 146	1.059	0.589	0.831
7 5	9 1	1.140	0.000	0.849	0.101
5	2	1 335	0.702	0.591	0.595
5	3	0.797	1.331	J.207 0 703	1.326
5	4	1.333	1.385	1 320	1 370
5	5	0.801	1.932	0.801	1 932
5	6	0.700	0.963	0.701	0.957
5	7	0.960	0.293	1,102	0.295
5	8	0.991	0.893	0.598	0.674
5	9	0.884	0.000	0.668	0.100

6	1	1.012	1.104	3.452	1.326
6	2	0.809	0.802	0.806	0.793
6	3	1.352	1.395	1.307	1.348
6	4	0.805	1.951	0.794	1.902
6	5	0.701	0.957	0.700	0.963
6	6	1.086	0.295	1.086	0.295
6	7	0.632	0.916	0.628	0.863
6	8	1.109	0.725	0.890	0.480
6	9	0.911	0.000	0.655	0.100
7	1	0.992	0.909	0.800	1.999
7	2	1.360	3.887	1.348	1.337
7	3	0.802	1.580	0.796	1.554
7	4	0.673	0.289	0.742	1.050
7	5	1.102	0.295	0.960	0.293
7	6	0.628	0.863	0.632	0.916
7	7	0.965	0.612	0.965	0.612
7	8	0.769	0.524	0.724	0.438
7	9	0.775	0.001	0.674	0.100
8	1	1.003	0.999	0.662	0.289
8	2	0.565	0.288	1.022	1.031
8	3	0.727	0.296	0.939	0.289
8	4	0.589	0.831	0.700	1.059
8	5	0.598	0.674	0.991	0.893
8	6	0.890	0.480	1.109	0.725
8	7	0.724	0.438	0.769	0.524
8	8	0.713	0.438	0.713	0.438
8	9	0.720	0.007	0.685	0.100
9	1	1.004	1.004	0.533	0.471
9	2	0.533	0.100	1.020	0.000
9	3	0.577	0.100	1.051	0.000
9	4	0.849	0.100	1.146	0.000
9	5	0.668	0.100	0.884	0.000
9	6	0.655	0.100	0.911	0.000
9	7	0.674	0.100	0.775	0.001
9	8	0.685	0.100	0.720	0.007
9	9	0.696	0.123	0.696	0.123

J	= 1	2	3	4	5	6	7	8	9
I									
1	1.011	0.969	1.033	0.956	1.106	0.954	1.007	1.616	0.684
2	0.969	1.049	0.941	1.049	0.941	1.050	0.940	1.624	0.414
3	1.033	0.941	1.174	0.941	1.050	0.948	1.019	1.907	0.426
4	0.956	1.049	0.941	1.050	0.947	1.016	1.527	0.620	0.379
5	1.106	0.941	1.050	0.947	1.015	1.109	1.881	0.761	0.374
6	0.954	1.050	0.948	1.015	1.109	1.804	0.615	0.532	0.394
7	1.007	0.940	1.019	1.527	1.881	0.615	0.532	0.541	0.371
8	1.616	1.624	1.907	0.624	0.761	0.532	0.541	0.538	0.369
9	0.684	0.414	0.426	0.379	0.374	0.394	0.371	0.369	2.850

Table 5.5 Fast Flux Node Average Discontinuity Factors

J	= 1	2	3	4	5	6	7	8	9
I									
1	0.949	1.194	0.874	1.696	0.737	1.723	1.175	0.738	0.753
2	1.194	0.802	1.824	0.807	1.820	0.802	1.983	0.726	0.415
3	0.874	1.824	0.592	1.837	0.797	1.351	1.183	0.563	0.433
4	1.696	0.807	1.837	0.802	1.352	1.363	0.689	0.795	0.524
5	0.737	1.820	0.797	1.352	1.366	0.830	0.662	0.789	0.413
6	1.723	0.802	1.351	1.363	0.830	0.691	0.760	0.801	0.416
7	1.175	1.983	1.183	0.689	0.662	0.760	0.789	0.614	0.388
8	0.738	0.726	0.563	0.795	0.789	0.801	0.614	0.576	0.378
9	0.753	0.415	0.433	0.524	0.413	0.416	0.388	0.378	0.410

Table 5.6 Thermal Flux Node Average Discontinuity Factors

Table 5.7 Assembly Power Densities for the 2-D Benchmark Problem (Borresen's Method Using Modified One-Group Diffusion Theory Model)

I	J=	1	2	3	4	5	6	7	8
1		1.118 1.180	1.023 1.088	1.212 1.219	1.044 1.117	1.406 1.358	0.875 0.925	0.836 0.822	0.711 0.676
2		1.023 1.088	1.216 1.197	1.066 1.163	1.218 1.195	0.980 1.063	1.004 0.973	0.709 0.740	0.687 0.633
3		1.212 1.219	1.066 1.163	1.528 1.497	1.042 1.133	1.122 1.115	0.869 0.912	0.814 0.811	0.636 0.505
4		1.044 1.117	1.218 1.195	1.042 1.133	1.188 1.173	0.993 1.059	1.071 1.105	0.995 0.888	
5		1.406 1.358	0.980 1.063	1.122 1.115	0.993 1.059	1.187 1.252	1.267 1.295	0.869 0.750	
6		0.875 0.925	1.004 0.973	0.869 0.912	1.071 1.105	1.267 1.295	1.045 0.898		
7		0.836 0.822	0.709 0.740	0.814 0.811	0.995 0.888	0.869 0.750			
8		0.711 0.676	0.687 0.633	0.636 0.505	> 2 > I	2DB BENG PRESENT	CHMARK I METHOD	RESULTS	
						Maar			

Mean error(%) = 4.72Max. error(%) = -20.5CPU Time (s) = 14.4 Table 5.8 Assembly Power Densities for the 2-D Benchmark Problem (Modified Borresen's Method Using Modified One-Group Diffusion Theory Model)

J= I	1	2	3	4	5	6	7	8
1	1.118 1.101	1.023 1.057	1.212 1.223	1.044 1.078	1.406 1.385	0.875 0.891	0.836 0.814	0.711 0.695
2	1.023 1.057	1.216 1.220	1.066 1.108	1.218 1.215	0.980 1.011	1.004 0.989	0.709 0.707	0.687 0.659
3	1.212 1.223	1.066 1.108	1.528 1.554	1.042 1.081	1.122 1.131	0.869 0.881	0.814 0.794	0.636 0.577
4	1.044 1.078	1.218 1.215	1.042 1.081	1.188 1.190	0.993 1.022	1.071 1.078	0.995 0.940	
5	1.406 1.385	0.980 1.011	1.122 1.131	0.993 1.022	1.187 1.220	1.267 1.275	0.869 0.828	
6	0.875 0.891	1.004 0.989	0.869 0.881	1.071 1.078	1.267 1.275	1.045 1.001		
7	0.836 0.814	0.709 0.707	0.814 0.794	0.995 0.940	0.869 0.828			
8	0.711 0.695	0.687 0.659	0.636 0.577	> 2 >]	2DB PRESENT	METHOD		

Mean error(%) = 2.07Max. error(%) = -9.28CPU Time (s) = 15.1

Table 5.9	Assembly Power Densities for the 2-D Benchmark
	Problem (Borresen's Method Using Two-Group
	Diffusion Theory Model)

J= I	1	2	3	4	5	6	7	8
1	1.118 1.033	1.023 0.978	1.212 1.184	1.044 0.996	1.406 1.420	0.875 0.842	0.836 0.822	0.711 0.750
2	1.023 0.978	1.216 1.171	1.066 1.021	1.218 1.175	0.980 0.948	1.004 0.982	0.709 0.689	0.687 0.730
3	1.212 1.184	1.066 1.021	1.528 1.583	1.042 1.003	1.122 1.113	0.869 0.844	0.814 0.823	0.636 0.646
4	1.044 0.996	1.218 1.175	1.042 1.003	1.188 1.166	0.993 0.967	1.071 1.088	0.995 1.035	
5	1.406 1.420	0.980 0.948	1.122 1.113	0.993 0.967	1.187 1.216	1.267 1.383	0.869 0.916	
6	0.875 0.842	1.004 0.982	0.869 0.844	1.071 1.088	1.267 1.383	1.045 1.088		
7	0.836 0.822	0.709 0.689	0.814 0.823	0.995 1.035	0.869 0.916			
8	0.711 0.750	0.687 0.730	0.636 0.646	> : > :	2DB PRESENT	METHOD		
					Moan	orror (× _ ~	0.1

Mean error(%)	=	2.81
<pre>Max. error(%)</pre>	=	9.16
CPU Time (s)	=	47.8

Table 5.10 Assembly Power Densities for the 2-D Benchmark Problem (Modified Borresen's Method Using Two-Group Diffusion Theory Model)

J= I	1	2	3	4	5	6	7	8
1	1.118 1.028	1.023 0.976	1.212 1.179	1.044 0.995	1.406 1.412	0.875 0.841	0.836 0.820	0.711 0.749
2	1.023 0.976	1.216 1.179	1.066 1.018	1.218 1.175	0.980 0.943	1.004 0.983	0.709 0.688	0.687 0.730
3	1.212 1.179	1.066 1.018	1.528 1.580	1.042 1.002	1.122 1.113	0.869 0.844	0.814 0.824	0.636 0.647
4	1.044 0.995	1.218 1.175	1.042 1.002	1.188 1.166	0.993 0.968	1.071 1.091	0.995 1.038	
5	1.406 1.412	0.980 0.943	1.122 1.113	0.993 0.968	1.187 1.220	1.267 1.388	0.869 0.920	
6	0.875 0.841	1.004 0.983	0.869 0.844	1.071 1.091	1.267 1.388	1.045 1.093		
7	0.836 0.820	0.709 0.688	0.814 0.824	0.995 1.038	0.869 0.920			
8	0.711 0.749	0.687 0.730	0.636 0.647	> ;	2DB PRESENT	METHOD		

Mean error(%)	=	2.92
Max. error(%)	=	9.55
CPU Time (s)	=	56.3

Table	5.11	Assembly Power Densities for the 2-D Benchmark
		Problem (Face Dependent Discontinuity factor
		Method Using Two-Group Diffusion Theory Model)

J= I	1	2	3	4	5	6	7	8
1	1.118 1.145	1.023 1.025	1.212 1.187	1.044 1.051	1.406 1.356	0.875 0.872	0.836 0.861	0.711 0.715
2	1.023 1.025	1.216 1.180	1.066 1.090	1.218 1.235	0.980 0.995	1.004 0.996	0.709 0.711	0.687 0.700
3	1.212 1.187	1.066 1.090	1.528 1.487	1.042 1.064	1.122 1.098	0.869 0.885	0.814 0.808	0.636 0.637
4	1.044 1.051	1.218 1.235	1.042 1.064	1.188 1.150	0.993 0.991	1.071 1.073	0.995 0.979	
5	1.406 1.356	0.980 0.995	1.122 1.098	0.993 0.991	1.187 1.207	1.267 1.294	0.869 0.861	
6	0.875 0.872	1.004 0.996	0.869 0.885	1.071 1.073	1.267 1.294	1.045 1.036		
7	0.836 0.861	0.709 0.711	0.814 0.808	0.995 0.979	0.869 0.863			
8	0.711 0.715	0.687 0.700	0.636 0.637	> >	2DB PRESENT	METHOD		

<pre>Mean error(%)</pre>	=	1.40
<pre>Max. error(%)</pre>	=	3.88
CPU Time (s)	=	24.2

Table 5.12 Assembly Power Densities for the 2-D Benchmark Problem (Node Averaged Discontinuity factor Method Using Two-Group Diffusion Theory Model)

	1	2	3	4	5	6	7	8
1	1.118 1.160	1.023 1.024	1.212 1.241	1.044 1.022	1.406 1.430	0.875 0.851	0.836 0.834	0.711 0.660
2	1.023 1.024	1.216 1.281	1.066 1.026	1.218 1.305	0.980 0.939	1.004 1.056	0.709 0.677	0.687 0.627
3	1.212 1.241	1.066 1.026	1.528 1.676	1.042 1.001	1.122 1.183	0.869 0.859	0.814 0.822	0.636 0.596
4	1.044 1.022	1.218 1.305	1.042 1.001	1.188 1.244	0.993 1.000	1.071 1.057	0.995 0.952	
5	1.406 1.430	0.980 0.939	1.121 1.183	0.993 1.000	1.187 1.176	1.267 1.344	0.869 0.819	
6	0.875 0.851	1.004 1.056	0.869 0.859	1.071 1.057	1.267 1.344	1.045 0.993		
7	0.836 0.834	0.709 0.677	0.814 0.822	0.995 0.952	0.869 0.819			
8	0.711 0.660	0.687 0.627	0.636 0.596	> 2 >]	2DB PRESENT	METHOD		

Mean error(%) = 3.14 Max. error(%) = 9.69 CPU Time (s) = 22.3

(%)	A	В	с	D	E	F	G
Error in K _{eff}	.03	.03	10	10	.03	.03	0.0
Mean Error	4.72	2.07	2.81	2.92	1.40	3.14	0.0
Max Error	-20.5	-9.28	9.16	9.55	3.88	9.69	0.0

47.8

56.3

24.2

22.3

6360.

Table 5.13 Summary of Results for the Benchmark Problem

Reference Eigenvalue: 1.2008

CPU Time (sec)

Mean Error: Mean Relative Assembly Power Difference Max. Error: Maximum Relative Assembly Power Difference

A => Modified one-Group Borresen's Method

14.4

B => Modified One-Group Modified Borresen's Method

15.1

- C => Two-Group Borresen's Method
- D => Two-Group Modified Borresen's Method
- E => Face-Dependent Discontinuity Factor Method
- F => Node-Averaged Discontinuity Factor Method
- G => 2DB Reference Calculation

6.0 CONCLUSION

As stated in Chapter 1, the objective of this thesis was to modify the one-group and two-group diffusion theory code (SHUFFLE), and compare the various methods to a benchmark (2DB) calculations.

The following conclusions were determined: when the discontinuity factors were obtained from the reference solutions, the results of the discontinuity factor methods closely resemble that of the reference solution. In both the one-group and two-group calculations, the Borresen's and the modified Borresen's have very nearly identical results. Although the modified Borresen's method has an overall lower mean power and maximum relative percent errors.

In the power distribution results of the discontinuity factor methods, the face-dependent discontinuity factor method has a lower percent relative errors than the nodeaverege discontinuity factor method.

The two-group modified Borresen's method appear to the most attractive. An aspect that needs to be studied in the future would be a means of determining the discontinuity factors without the use of the reference solution.

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