A MULTI-STAGE STOCHASTIC DECISION MODEL FOR DETERMINING OPTIMAL REPLACEMENT OF DAIRY COWS

by

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A THESIS

submitted to

OREGON STATE UNIVERSITY

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

August 1962

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Date thesis is presented July 18, 1962

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ACKNOWLEDGEMENTS

The writer wishes to express appreciation to the many individuals that have made a contribution to this study. To my wife, Carol, I express my sincere thanks for her encouragement, understanding, and patience. To my major professor, Dr. Albert N. Halter, I extend appreciation for his helpful attitude, advise, and guidance during the many hours spent on this study and during the writer's graduate program.

A special appreciation is extended to those who furnished the majority of the data, Mr. Don Anderson, Dairy Specialist, Oregon State University and Mr. Dexter N. Putnam, Dairy Specialist, Pennsylvania State University.

Appreciation is extended to the members of the Departments of Agricultural Economics, Statistics, and Mathematics for suggestions made concerning the research work and for their contribution to the writer's training.

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A MULTI-STAGE STOCHASTIC DECISION MODEL FOR DETERMINING OPTIMAL REPLACEMENT OF DAIRY COWS

CHAPTER I

THE REPLACEMENT DECISION MAKING PROBLEM

The basic problem of replacement is concerned with what type of remedial action and when the remedial action should be taken in an enterprise with respect to productive units for which diminishing productivity occurs over time. Replacement theory is designed to determine the remedial action and the point in time at which the productive unit should be restored to its original or a more productive position. The decision at any point in time as to whether or not remedial action will be taken and the type of remedial action to be taken is based upon a criterion of optimality. The criterion of optimality merely specifies what is to be maximized or minimized over the life span of the enterprise. An enterprise is made up of more than one productive unit and/or is in operation for more than one time period.

When the decision to replace a productive unit is made at time t_o, a decision is also made regarding the time at which the unit used for replacement will be replaced. This sequence or set of decisions is called a policy. A policy specifies the age of the unit to be replaced and the age of the replacement for the life span of the enterprise. Obviously, there are many such policies. Of the set of all policies the policy which determines the actions that attain the criterion of optimality is called the optimal policy.

Single and Multi-stage Decision Making

The character of the problem or the nature of the enterprise specifies a number of decision points. These decision points exist or can be specified for any kind of productive units and are such that they follow a time sequence in a continually operating enterprise. In a continually operating enterprise the decision at any time t_o is a function of decisions made at preceeding and subsequent decision points. Thus in the replacement problem, the decision to replace a productive unit at any time t_o will depend upon the past sequence of replacements as well as the subsequent sequence.

In single stage decision making the criterion of optimality can be attained by considering a decision at time t_o independent of the decisions for the preceeding and subsequent enterprise periods. In multi-stage decision making the criterion of optimality can be attained only by considering all decision points simultaneously.

Replacement decision making problems of continually operating enterprises are contained in the set of multi-stage decision making problems. Thus, replacement problems can be solved as a multi-stage decision making problem, i.e., by considering all replacements for the life span of the enterprise simultaneously.

Examples of Multi-stage Decision Making

Cleland White, in a study at the University of Kentucky in 1959, demonstrated the use of multi-stage decision making in determining an optimal policy for the replacement of caged laying hens (16). Multi-stage decision making was used by White because the past decision determines the age of hen on hand during the present enterprise period thus influencing the net returns during that period. White also observed that the subsequent decisions affect the net return that can be obtained from replacing the present animal. This is true because of variation in production of hens of different ages and also because prices and costs will vary through time. Each of these factors will have an affect upon the present decision. Thus, White used multi-stage decision making in determining an optimal policy for replacement of caged laying hens in a continually operating enterprise.

A comparison of multi-stage and single stage decision making for an abstract replacement problem is shown in the following example.

Let there exist an enterprise which has three enterprise periods, three decision points, and two possible actions at each decision point. Let one of these actions represent the keeping of the present productive unit and the other action represent the replacement of the present productive unit by a new productive unit. Also, let the enterprise be terminated at the end of the third enterprise period. Suppose the net returns from a new production unit is 15 in the first and second enterprise periods; due to an increase in the cost of the unit at the third decision point. the net return is 14 in the third enterprise period. Further suppose that the net returns for units of age one, two, three and four are 16, 10, 8, and 6 respectively. If the age of the present unit is two then the problem is: what is the optimal replacement policy to follow for the life span of the enterprise?

The restrictions of the problem can be presented in the following diagram where t = number of the enterprise period, a_t = decision point, R_i = replacement with a new unit in the i^{th} enterprise period, K_i^j = the keeping of a unit of age j in the i^{th} enterprise period, $[R_i]$ = net return from the replacement in the

i^{th} enterprise period, and $[K_i^j]$ = net return from the present unit of age j in the i^{th} enterprise period.

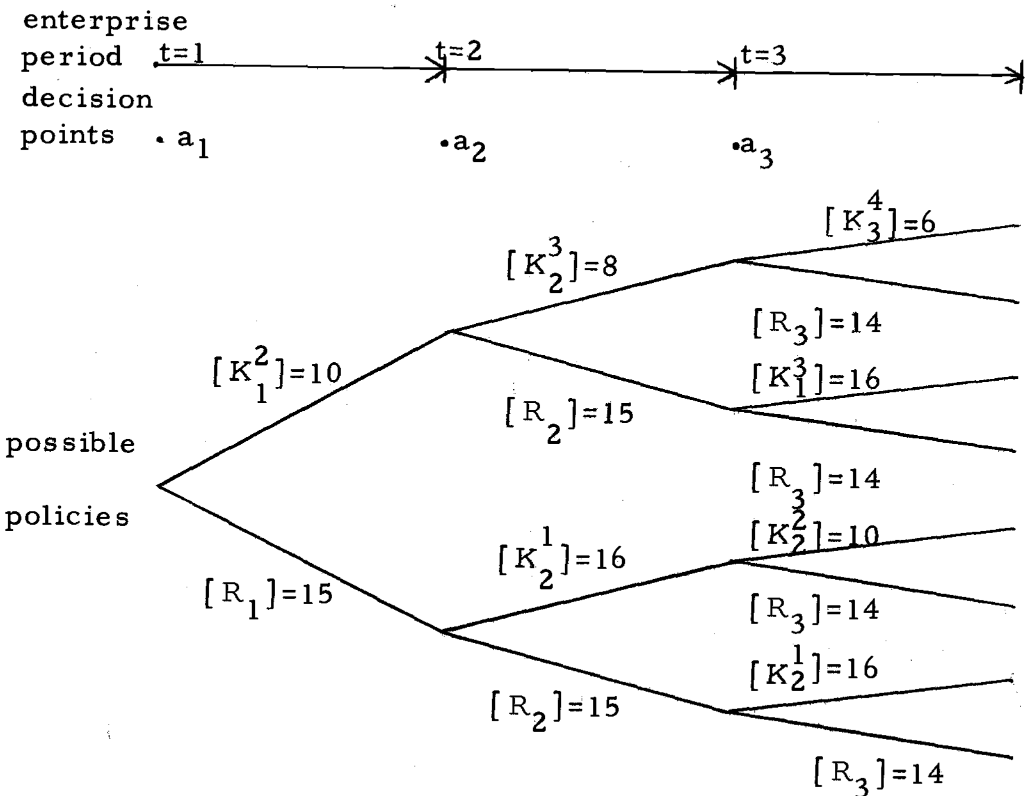

Figure 1. Diagram of abstract replacement problem.

A single stage decision making replacement policy for the example would be obtained by looking at each decision point independent of the other decision points. At decision point a_1, the unit of age two can be kept, K_1^2, with a net return of 10 or the present unit can be replaced, R_1, with a net return of 15. The most profitable single stage decision is to replace the present unit and obtain the return of 15. At decision point a_2, the present unit is the unit used as a replacement of a_1. This unit can be kept, K_2^1, with a return of 16 or replaced, R_2, with a return of 15. At a_2 the most profitable single stage decision is K_2^1 with a return of 16. At decision point a_3, the present unit can be kept, K_3^2, with a net return of 10 or replaced, R_3, with a return of 14. The most profitable action is to replace the present unit and obtain a net return of 14. Combining the decisions made at a_1, a_2 and a_3, single stage decision making would generate the replacement policy: replace the unit at a_1, keep the unit at a_2, and replace the unit at a_3. The net return from this policy is 45.

In a small example such as this a multi-stage replacement policy can be obtained by enumerating all possible replacement policies. The policy which yields the maximum net return will be the optimal sequence of decisions. The eight possible replacement policies and the associated net return of each are $[K_1^2, K_2^3, K_3^4] = 24$, $[K_1^2, K_2^3, R_3] = 32$, $[K_1^2, R_2, K_3^1] = 41$, $[K_1^2, R_2, R_3] = 39$, $[R_1, K_2^1, K_3^2] = 41$, $[R_1, K_2^1, R_3] = 45$, $[R_1, R_2, K_3^1] = 46$, and $[R_1, R_2, R_3] = 44$.

The policy R_1, R_2, K_3^1 yields the maximum net returns. Therefore the optimal sequence of decisions is to replace at a_1, replace at a_2, and keep the present unit at a_3. The optimal replacement policy arrived at by use of multi-stage decision making is a policy which yields higher net returns to the enterprise than the policy recommended by the use of single stage decision making. Hence, the pitfalls of single stage decision making and the necessity for the use of multi-stage decision making in obtaining an optimal replacement policy for a continually operating enterprise are apparent.

Dairy Cow Replacement Problem

The dairy cow, as other biological productive units, loses efficiency over time. As the present animal loses productive efficiency the enterprise can be restored by replacing her with an animal in a more productive lactation. The decision concerning whether the animal is kept or replaced at any decision point in the life span of the enterprise is dependent upon the criterion of optimality, the net return from the present animal, and the net returns from possible replacement animals.

The dairy cow replacement problem is a multi-stage decision making problem which consists of T decision points with the decision at any point in the enterprise dependent upon subsequent and preceding decisions similar to the hen replacement problem. Because the productivity of a present animal of a given lactation will either increase of decrease during the next lactation, the returns for the subsequent enterprise periods will be affected.

The enterprise period in the dairy cow replacement problem is defined to be the time from the beginning of a lactation to the beginning of the next. Hence for dairy cows it corresponds to essentially a year.[1] In addition, price of the product or prices of the inputs will not remain constant throughout the life span of the enterprise. Therefore to solve for an optimal replacement policy for a continually operating dairy enterprise it is necessary to use multi-stage decision making. The criterion of optimality which was used to determine the replacement policy for dairy cows is the maximization of net return over the life span of the enterprise.

[1]The enterprise period used by White in the caged layer replacement problem was a month (15, p. 1538). A year was used in the dairy cow replacement problem because of the longer productive life of dairy cows. However, it should be noted that this length of the enterprise period is arbitrary.

Stochastic Nature of the Dairy Cow

As with many other productive units, the dairy cow can fail at any point during a given lactation. The failure of a dairy cow is defined as the removal of the animal from the enterprise for sickness, physical injury or death, i.e., the recovery rate of a failure for dairy purposes is considered to be near zero. If the animal is a failure and she did not die then she can be sold for beef. An animal is considered as succeeding if she did not fail during the given lactation.

The likelihood of finding a replacement of a given lactation depends upon the number of animals available. Since not all calves are heifers, not all heifers become productive and some heifers that become productive in the first lactation fail in subsequent lactations, the likelihood of finding a replacement varies between lactations. The frequency function of a population of dairy cows by lactation in Iowa is shown in Table 1.

Later in Chapter III this frequency function will be interpreted as the likelihood of finding a cow in given lactation.

Table 1. Frequency Function of All Cows by Lactation in Iowa Cow-Testing Associations for Years 1927-28 and 1930-36.[a]

Lactation[b]	Frequency Function
1	.2432
2	.1758
3	.1412
4	.1148
5	.0969
6	.0732
7	.0569
8	.0369
9	.0258
10	.0142
11	.0105
12 and above	.0106

[a] Calculated from (7, p. 1026).

[b] A cow of lactation "1" is considered to be an animal of at least two years of age and not over three years of age. Thus a "1" is an animal ready to begin or in her first lactation; a "2" is an animal ready to begin or in her second lactation, etc. Henceforth, reference to only the number of the lactation will be made.

Procedure of Analysis

The major objective of this study was to determine optimal replacement policies for dairy cows in a continually operating dairy enterprise under various price and cost situations.[1] Instrumental objectives were the estimation of the various stochastic properties of the dairy cow and the isolation of the important economic factors which affect the optimal replacement policy.

The methodology for the study was based upon the concept of stochastic multi-stage decision making as presented by Richard Bellman (3, p. 61-79). Bellman and his associates at Rand Corporation have centered their research work in the area of mathematical decision making which includes multi-stage decision problems.

Also, the work by White at the University of Kentucky has been used in specifying the methodology to be used. Although White noted the importance of the stochastic nature of biological production units (16, 49-50), he concerned himself with obtaining a deterministic optimal replacement policy for caged laying hens. This study is an extension of White's work in that consideration

[1] This study is part of an Oregon Agricultural Experiment Station Project entitled "Economic Replacement Policies in Continually Operating Animal Enterprises", Project No. 478.

is given to the stochastic properties of the productive unit. These properties and their measurement are presented in Chapter III.

To obtain the optimal replacement policy for the continually operating dairy enterprise stochastic multi-stage programming was used.[1] One of the advantages of using stochastic multi-stage programming is that the elements of chance of a given event associated with the problem can be incorporated into the decision making model. The model is precisely stated in the form of a recursive equation and is presented in Chapter II.

Because the decision making model is based upon a recursive equation, the programming of the replacement problem for a digital computer is facilitated. The optimal replacement policies were obtained by programming the IBM 1620 with Fortran language. The program statements used in programming the recursive equation for the computer are presented in Appendix I.

The numerical estimation of the economic and stochastic components of the replacement model are presented in Chapter III.

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[1]Multi-stage programming, called dynamic programming by Bellman, is used to identify the mathematical tool used in solution of multi-stage decision processes. The term, multi-stage programming, is used in this study to allow the reader to avoid the confusion which has been created by the current usage of dynamic programming in agricultural economics. In agricultural economics the term dynamics programming is used to imply a general problem area rather than a specific mathematical tool.

Chapter IV is devoted to the presentation and discussion of the optimal policies derived under various price assumptions. The implications and limitations of the study are discussed in Chapter V.

CHAPTER II

THE REPLACEMENT DECISION MAKING MODEL

The problem of replacement of dairy cows in a continually operating enterprise was discussed briefly in Chapter I in the context of multi-stage decision making. It was indicated that a decision model can be specified which can be solved for an optimal replacement policy. The development of a decision model and the method of solution are presented in this chapter.

Economic Factors Involved in Making a Dairy Cow Replacement Decision

Economic factors which are considered in the decision relative to the replacement of the present animal are: (1) the market value of the present animal, (2) the market value of the possible replacement animal, (3) the nuisance cost associated with replacing the present animal, hereafter called transaction costs, (4) the net market value of the production from the present animal, (5) the net market value of the production from the possible replacements, (6) the maximum net return that can be obtained in subsequent enterprise periods if the present animal is retained, and (7) the maximum net return that can be obtained in subsequent enterprise periods from the replacement of the present animal.

The market value of the present animal and possible replacements is a function of the current production level, the number of lactations remaining for the animal, the production in subsequent lactations, the price of beef, and the supply and demand for dairy cows. The transaction costs entail commission charges, transportation costs, and the value of the time and effort involved in making the replacement. The net market value of production for the present and replacement animals is determined by considering the market value of production minus the associated costs of production. The maximum net return in subsequent enterprise periods is made up of the market value of the animals, the transaction costs, and the market value of the production.

Stochastic Factors Invovled in Making a Dairy Cow Replacement Decision

In any enterprise period the present animal may be afflicted with mastitis, brucellosis, ketosis, milk fever, and other diseases which will cause the production of the animal to diminish to a non-profitable level. This animal is essentially a failure. Other events which can cause an animal to become a failure are sterility, and accidents resulting in physical injury. In fact the animal may just die. Of course, all of these events can happen to the replacement as well. The possibility of these events occurring gives to an individual animal a stochastic property of failure. The stochastic property of an animal succeeding may be considered as the possibility that failure does not occur.

In any enterprise period it may be taken as a certainty that any kind and quality of replacement is available. However, when one considers the replacement of an entire herd or a portion thereof, it is less likely that the same kind and quality of animals for replacement will be available. For example, (1) the owner of a herd of 25 animals will not have 25 raised replacements of the same age on hand during any enterprise period and (2) the owner of a herd of 25 animals will have a difficult and expensive task if he tries to find 25 cows of a given lactation and quality. However, this doesn't imply that it is an impossibility; it merely implies that there is some likelihood associated with the finding of a replacement. This is the second stochastic factor which influences the dairy cow replacement decision.

Thus, the stochastic factors which influence the dairy cow replacement decision are (1) the likelihood of the animal failing, (2) the likelihood of the animal succeeding, and (3) the likelihood of finding a cow in a given lactation.

Making the Dairy Cow Replacement Decision

The comparison of the net returns from the present animal with those from the possible replacements constitute the replacement decision. This comparison at any decision point a_t must satisfy the criterion of optimality, i.e., the decision made at a_t must be the one which makes the sequence of replacement decisions yield the optimal policy. Similarly, in making the net returns comparison at a_{t+1} it is necessary for the decision to satisfy the criterion of optimality. Thus over the entire life of the enterprise each decision must satisfy the criterion of optimality.

Development of the Deterministic Replacement Model

The decision concerning whether or not the dairy cow is to be replaced at decision point a_t is a function of the net returns during the present enterprise period and the maximum net returns that can be obtained in subsequent time periods. Let NR_j equal the net returns of possible replacement animals in the present enterprise period and $\overline{NR}_j$ equal the maximum net returns that can be obtained in subsequent enterprise periods if an animal of lactation j is used as the replacement.[1] Let $\pi_{t,i}$ equal the maximum net return to

[1] If the present animal is considered as a replacement for itself then j represents the lactation of all possible replacements. Hence, the net return comparison between the present animal and the possible replacements is achieved.

the enterprise for the t^{th} and subsequent enterprise periods from a policy which has a present animal of lactation i, then it follows that

$$\pi_{t,i} = \underset{j=1}{\overset{J}{\text{Max}}} [NR_j + \overline{NR}_j]$$

where j = the lactation of the possible replacements. Now, NR_j equals the market value of the present animal minus the market value of the replacement minus the cost of making the transaction plus the net market value of the production of the replacement during the present enterprise period (again i can equal j). This can be demonstrated by considering a present animal of lactation five in the first enterprise period. If animals of lactation j = 1, 2, ... , 12 are considered as possible replacements then $\pi_{1,5}$ can be determined. Let $pv_{1,5}$ = market value of the present animal of lactation five in enterprise period one, $rc_{1,j}$ = market value of the replacement animal of lactation j in enterprise period one, $r_{1,j}$ = net market value of the production from the replacement animal of lactation j in the first enterprise period and $\overline{NR}_j$ be as previously defined. It now follows that the maximum net return to the enterprise for the present and subsequent enterprise periods is

$$\pi_{1,5} = \text{Max} \begin{bmatrix} pv_{1,5} - rc_{1,1} - \Delta_1 + r_{1,1} + \overline{NR}_1 \\ pv_{1,5} - rc_{1,2} - \Delta_2 + r_{1,2} + \overline{NR}_2 \\ \vdots \\ pv_{1,5} - rc_{1,5} - \Delta_5 + r_{1,5} + \overline{NR}_5 \\ \vdots \\ pv_{1,5} - rc_{1,12} - \Delta_{12} + r_{1,12} + \overline{NR}_{1,12} \end{bmatrix} .$$

When $j = 5$ then $\Delta_5 = 0$ since the present animal would be replaced by itself and hence no transaction cost would be incurred. The above set of equations can be rewritten as

$$\pi_{1,5} = pv_{1,5} + \underset{j=1}{\overset{12}{\text{Max}}} \left[-rc_{1,j} - \Delta_j + r_{1,j} + \overline{NR}_j \right] \quad \text{and}$$

can be generalized to

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{\text{Max}}} \left[- rc_{t,j} - \Delta_j + r_{t,j} + \overline{NR}_j \right] \quad \text{where}$$

t = enterprise period ($t = 1, 2, \ldots, T$),

i = lactation of present animal ($i = 1, 2, \ldots, J$),

j = lactation of replacement animal ($j = 1, 2, \ldots, J$),

and $\Delta_j = 0$ when $i = j$.

Now consider $\overline{NR}_j$, the maximum net return that can be obtained in subsequent enterprise periods. If an animal of lactation j is used as a replacement, it will be in lactation j + 1 in the t + 1[st] enterprise period. At the beginning of the t + 1[st] enterprise period the animal shall be eligible for replacement. Thus, $\overline{NR}_j$ is the net returns that can be obtained in the t + 1[st] enterprise period by replacing with an animal of lactation j' plus the maximum net return that can be obtained in enterprise periods subsequent to t + 1, i.e., stated mathematically, $\overline{NR}_j = \underset{j'=1}{\overset{J}{\text{Max}}} \left[NR_{j'} + \overline{NR}_{j'} \right]$.

But $\pi_{t+1,\ j+1} = \underset{j'=1}{\overset{J}{\text{Max}}} \left[NR_{j'} + \overline{NR}_{j'} \right]$ and hence $\overline{NR}_j = \pi_{t+1,j+1}$.

The deterministic replacement equation now can be rewritten as

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{\text{Max}}} \left[-rc_{t,j} + r_{t,j} - \Delta_j + \pi_{t+1,j+1} \right].$$

Development of the Stochastic Replacement Model

The preceding equation presents the replacement model under non-stochastic conditions. To modify the above equation for stochastic conditions one needs to incorporate the likelihood of failure, success and acquisition. These stochastic factors are entered into the model by finding the expected value of the net return from the replacement, $r_{t,j}$.[1] The expected value of $r_{t,j}$ depends upon the likelihood of finding an animal of lactation j, the likelihood of success of an animal of lactation j, and the likelihood of failure of an animal of lactation j. If we let P_j = likelihood of finding an animal of lactation j, q_j = the likelihood of success of an animal of lactation j, and p_j = likelihood of failure of an animal of lactation j, then the expected value of $r_{t,j}$ is

$$E(r_{t,j}) = P_j(q_j \text{ (value of success}_{t,j}) + p_j \text{(value of failure}_{t,j})).$$

If the lactation of the present animal is the same as the replacement being considered then the likelihood of finding the present animal is one ($P_j = 1$). The replacement model now becomes

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{\text{Max}}} \left[-rc_{t,j} + P_j(q_j \cdot VS_{t,j} + p_j VF_{t,j}) - \Delta_j + \pi_{t+1,j+1} \right]$$

where

$0 \leq P_j < 1$ except when i = j then $P_j = 1$,

$0 \leq q_j \leq 1$,

$0 \leq p_j \leq 1$,

$q_j + p_j = 1$,

$VS_{t,j}$ = value of success of an animal of lactation j in enterprise period t,

[1]Expected values are discussed in Chapter III.

$VF_{t,j}$ = value of failure of an animal of lactation j in enterprise period t,

and the other symbols as previously defined.

Solution of the Replacement Equation

One method of solution of a multi-stage replacement problem is the computation of the returns from all possible policies. This method was used to obtain the optimal policy for the abstract example presented in Chapter I. However, if cows of 12 different lactations are considered as replacements and the life span of the enterprise is 12 then there would be approximately 8.9×10^{12} possible replacement policies. Clearly, even with the fastest computers the problem of enumeration, storage, and comparison of possible policies becomes relatively impossible. If the possible policies and the associated net returns are calculated at the rate of 10 per second or 36,000 per hour, then 2.47×10^{8} hours of continuous computer operation would be necessary to obtain the optimal replacement policy. Fortunately, another method of solution of the replacement equation is available. The method of solution which will yield the same results as all possible combinations is based upon the mathematical concept of recursion relations. A recursion relation is such that any term of a sequence after a specified term can be obtained as a function of the preceeding terms. Thus, an equation which expresses a recursion relation can be solved for the sequence of terms by specifying some initial term.

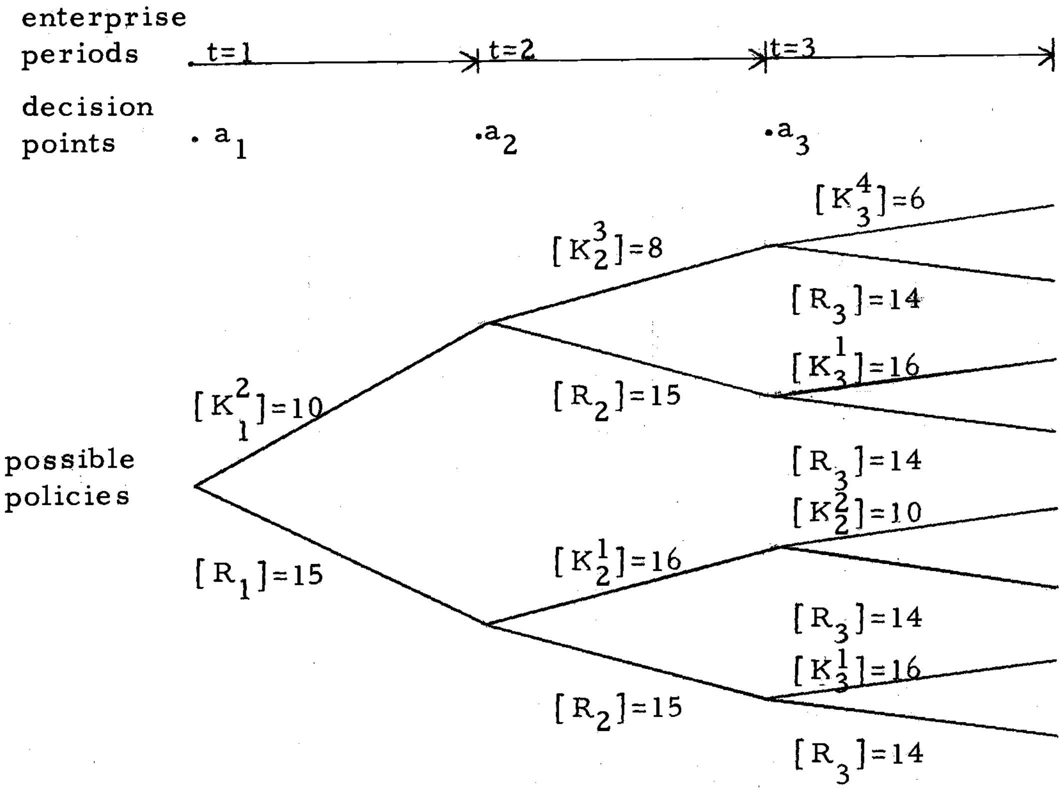

Figure 2. Diagram of abstract replacement program.

This method can be demonstrated by considering the abstract example presented in Chapter I. To solve the example in this manner the problem is divided into three one dimensional problems and the solution is initialized at the third enterprise period. The three problems are as follows:

Problem 1

If K_2^3 was the unit used in the second enterprise period, then follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_3^4] = 6 \\ [R_3] = 14 \end{bmatrix} = [R_3] = 14.$$

If R_2 was the unit used in the second enterprise period, then follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_3^1] = 16 \\ [R_3] = 14 \end{bmatrix} = [K_3^1] = 16.$$

If K_2^1 was the unit used in the second enterprise period, then follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_3^2] = 10 \\ [R_3] = 14 \end{bmatrix} = [R_3] = 14.$$

Problem 2

If K_1^2 was the unit used in the first enterprise period, then follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_2^3, R_3] = 22 \\ [R_2, K_3^1] = 31 \end{bmatrix} = [R_2, K_3^1] = 31.$$

If R_1 was the unit used in the first enterprise period, then follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_2^1, R_3] = 30 \\ [R_2, K_3^1] = 31 \end{bmatrix} = [R_2, K_3^1] = 31.$$

The policies R_3 and K_3^1 were found to return the maximum in the first problem.

Problem 3

For the present unit at a_1 follow the policy which attains

$$\text{Max} \begin{bmatrix} [K_1^2, R_2, K_3^1] = 41 \\ \\ [R_1, R_2, K_3^1] = 46 \end{bmatrix} = [R_1, R_2, K_3^1] = 46.$$

The policy R_2, K_3^1 was found to return the maximum in the second problem.

The optimal policy R_1, R_2, K_3^1 is the same sequence of decisions that was determined by enumerating all possible policies in Chapter I. This sequence of decisions which is the optimal policy was obtained by organizing the problem in such a manner that a recursive approach could be used. This method of solution is called multi-stage programming.

The replacement equation is a recursive equation and its solution can be found if an initial position is specified and the enterprise periods are relabeled. The enterprise periods are relabeled from the specified initial position, i.e., instead of indexing the enterprise periods as $t = 1, 2, 3, \ldots, T$ they are now indexed as $t = T, T-1, T-2, \ldots, 3, 2, 1$. The stochastic replacement equation can now be written as

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{\text{Max}}} \left[P_j(q_j VS_{t,j} + p_j VF_{t,j}) - rc_{t,j} - \Delta_j + \pi_{t-1,j+1} \right].$$

The solution can be initialized by specifying, $\pi_{t-1,j+1}$, at $\pi_{o,j+1}$ where $t = 1$, the end of the enterprise. In the dairy cow replacement problem $\pi_{0,j+1}$ is the market value of an animal of lactation $j + 1$, since the most profitable alternative at the end of the enterprise is to sell the animal.

The decision making model will be used to obtain optimal policies which are presented in Chapter IV after the economic and stochastic components of the model are discussed and, using illustrative data, estimated in the next chapter.

CHAPTER III

ESTIMATION OF THE COMPONENTS OF THE REPLACEMENT EQUATION

Optimal replacement policies for dairy cows are dependent upon the value of the various components of the equation as presented in Chapter II. For the optimal policies to be determined each component of the model must be specified.

In order for the equation to be solved the limit of j is specified as well as the initial position. For this study j varies from 1 to 12, i. e., cows of 12 different lactations are considered. Initial positions are always considered to be at the termination of the enterprise. This chapter is concerned with the estimation of the stochastic and economic components of the replacement equation.

Stochastic Properties of the Dairy Cow Replacement Problem

The stochastic properties of the dairy cow replacement problem are the likelihood of finding an animal of a given lactation, the likelihood of that animal failing, and the likelihood of that animal succeeding. An estimate of the likelihood of a given event can be obtained from a sample of observations. The estimate of the likelihood of an event is called the probability of the event, i.e., the likelihood of an event is based upon the population and the probability of an event is the sample estimate of the likelihood value. This section is concerned with the calculation of the probabilities associated with the stochastic properties of the replacement problem.

Probability of Finding a Dairy Cow of a Specified Age, P_j

Suppose that within a specified environment the possible states of nature are E_1, E_2, ... , E_n and the environment periodically changes one of the states to another in a random fashion. The possible states of nature, E_i, are denoted as the simple events and the action which induces a possible change from one state of nature to another is denoted as a trial. The probability of a simple event occurring is defined as the ratio of the number of occurrences of a given event divided by the total number of trials.

The sample space contains the set of all possible events. The first property of the sample space is that each conceivable outcome of a trial or experiment is represented by one and only one point in the corresponding sample space. The second property of the sample space is that each point in the sample space has associated with it a non-negative number called the probability of the corresponding simple event.

These concepts can be demonstrated by considering an urn which contains a number of red, green and white balls. The sample space is [Red Ball, White Ball, Green Ball], i.e., the simple events are observing a red ball, a white ball or a green ball on any given draw. Now suppose that a ball is drawn at random, a trial, the color is observed and recorded, the ball is replaced, and another ball is drawn until 100 balls have been drawn. Suppose the frequency of occurrence is [White, 50; Red, 30; Green, 20]. Then it is said that the probability of the event, a white ball, is $Pr[W] = \frac{\text{Number of occurrences of W}}{\text{Number of draws}} = \frac{50}{100} = \frac{1}{2}$. The $Pr[R] = 0.3$ and $Pr[G] = 0.2$ are obtained in a similar manner.

A discrete random variable is a real valued function defined over the events or outcomes of a trial. A random variable for the preceding example can be denoted as x_1 = white, x_2 = red, and x_3 = green. The probabilities associated with the random variables are $Pr[x_1] = Pr[W]$, $Pr[x_2] = Pr[R]$, and $Pr[x_3] = Pr[G]$.

In the preceding example the $Pr[x_1]$ $Pr[x_2]$, and $Pr[x_3]$ were obtained by drawing balls at random from an urn. Similarly, the finding of a cow of a given lactation to be used as a replacement can be considered as a random event. Thus a random variable can be defined as the possible lactation of the replacement, i.e., $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, ... , $x_{12} = 12$. The probability that can be associated with the value of the random variable x_j can be obtained by the same procedure as in the urn example, i.e., the number of cows of lactation j that are available divided by the total number of cows. The data of Cannon and Hansen (7, p. 1026) were used to obtain the probability of finding an animal of a specified lactation. These are shown in Table 2.

Probability of Failure and Success, p_j and q_j, of Dairy Cows

In order to consider the probability of failure or success of a dairy cow in a given lactation the previously discussed concepts on probability are extended to the consideration of conditional probability. Assume that the balls in the urn in addition to being three different colors are each numbered with either a 1, 2, or 3. Let the random variable x_j represent the color of the ball and the random variable y_i represent the number on the ball. Now, the probability of observing y_i given that x_j has been observed is the conditional probability,

$$Pr[y_i | x_j] = Pr[y_i x_j]/Pr[x_j].$$

Suppose the experiment of drawing a ball at random, recording the color of the ball and the number on the ball, and replacing the ball is repeated 100 times then consider that the frequency of the 100 draws are as presented in Table 3.

Table 2. The Number of Cows Available and the Probability, P_j, of a Cow in a Given Lactation

x_j Lactation	Number of Cows Available[a]	$Pr[x_j]$	P_j
1	31,447	31,447/129,320	.2432
2	22,735	22,735/129,320	.1758
3	18,258	18,258/129,320	.1412
4	14,840	14,840/129,320	.1148
5	12,535	12,535/129,320	.0969
6	9,471	9,471/129,320	.0732
7	7,361	7,361/129,320	.0569
8	4,776	4,776/129,320	.0369
9	3,338	3,338/129,320	.0258
10	1,829	1,829/129,320	.0142
11	1,362	1,362/129,320	.0105
12 and over	1,368	1,368/129,320	.0106
Total	129,320	1.0000	1.0000

[a] Calculated from (7, p. 1026).

Table 3. Frequency of Occurrence of Drawing a Ball from an Urn Containing Balls with Two Characteristics

Color of the Ball, x_i	Number on the Ball, y_j			Total
	$y_1 = 1$	$y_2 = 2$	$y_3 = 3$	
x_1 = white	20	15	15	50
x_2 = red	10	15	5	30
x_3 = green	10	5	5	20
Total	40	35	25	100

The probability of observing y_1 given that x_1 has been observed is

$$\Pr[y_1|x_1] = \Pr[y_1x_1]/\Pr[x_1] = \frac{20}{100} \Big/ \frac{50}{100} = \frac{2}{5}$$

where

$$\Pr[y_1x_1] \doteq \frac{\text{number of occurrences of } y_1x_1}{\text{total number of trials}} = \frac{20}{100} = \frac{1}{5}$$

and

$$\Pr[x_1] = \frac{\text{number of occurrences of } x_1}{\text{total number of trials}} = \frac{50}{100} = \frac{1}{2} .$$

Thus the probability of observing the number "1" given that a white is drawn is equal to $2/5$. Similarly the rest of the conditional probabilities can be obtained.

Suppose the balls in the urn take on one more characteristic, that is a "hat" covers some of the numbers. Now let z_1 represent the presence of a "hat" and z_2 the absence of a "hat", then the conditional probability of obtaining a "hat" given the color of the ball and the number on the ball is

$$Pr[z_k | x_i y_j] = Pr[z_k x_i y_j] / Pr[x_i y_j].$$

Suppose the previous experiment is performed again and the frequency of the three characteristics are recorded as in Table 4. The conditional probabilities related to any one of the possible outcomes can be obtained.

For example,

$$Pr[z_1 | x_1 y_1] = Pr[z_1 x_1 y_1] / Pr[x_1 y_1] = \frac{10}{100} \cdot \frac{100}{40} = \frac{1}{4}$$

or

$$Pr[x_1 | z_2 y_2] = Pr[x_1 z_2 y_2] / Pr[z_2 y_2] = \frac{10}{100} \cdot \frac{100}{20} = \frac{1}{2}.$$

To visualize the conditional probability of a dairy cow failing let x_i which was the color of the ball, represent the lactation of the cow where i = 1, 2, ... , 12; y_j which was number on the ball, represent the butterfat production level of the cow where j = 1, 2, 3; z_1, which was the presence of the hat over the number, represent the failure of the cow; and z_2, which was the absence of a hat, represent the success of the cow. Just as in the urn

Table 4. Frequency of Occurrence of Drawing a Ball from an Urn Containing Balls with Three Characteristics

Color of the Ball x_j	Number on the Ball, y_i: $y_1=1$		$y_2=2$		$y_3=3$		Total
	z_1 =Hat	z_2 = No Hat	z_1=Hat	z_2=No Hat	z_1=Hat	z_2=No Hat	
x_1 = white	10	5	5	10	10	10	50
x_2 = red	10	5	5	5	5	0	30
x_3 = green	5	5	5	5	0	0	20
Subtotal	25	15	15	20	15	10	
Total	40		35		25		100

example where the probability of observing a hat was conditioned upon the occurrence of the color and number on the ball, so is the probability of failure of a given animal conditional upon the butterfat level and the lactation of the cow. These conditional probabilities are shown in Table 5.

The conditional probabilities shown in Table 5 were not obtained as easily as those in the urn example. In fact the data used in this study in the estimation of the probability of failure and success were nearly impossible to obtain. The Dairy Herd Improvement Association's program in most states keeps statistics on cows removed from the herd according to whether the animal was sold for dairy, sold for beef, or died.[1] It was necessary to find data that would allow the number of failures, i.e., those cows removed because of disease, physical injury, or accident, to be specified. It was found that the Pennsylvania DHIA program kept adequate records on IBM cards to allow sorting of actual failures from removals for dairy purposes and low production.

Through the cooperation of Mr. Dexter N. Putnam, Dairy Specialist, Pennsylvania State University, over 10,000 cards containing the information concerning removal during 1960 were

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[1] Dairy Herd Improvement Associations were contacted in Idaho, Utah, Arizona, Oregon, Washington, Kansas, Iowa, New York and Pennsylvania.

Table 5. Probabilities of Failure and Success of Dairy Cows Given Butterfat Level and Lactation[a]

Lactation	Butterfat Level 1 Less than 350 pounds		Butterfat Level 2 350 to 450 pounds		Butterfat Level 3 Above 450 pounds	
	Probability of Failure	Probability of Success	Probability of Failure	Probability of Success	Probability of Failure	Probability of Success
1	.0438	.9562	.0543	.9457	.0674	.9326
2	.0662	.9338	.0755	.9245	.0835	.9165
3	.0825	.9175	.0937	.9063	.1222	.8778
4	.0927	.9073	.1196	.8804	.1438	.8562
5	.1027	.8973	.1350	.8650	.1678	.8322
6	.1393	.8607	.1557	.8443	.1751	.8249
7	.1322	.8678	.1576	.8424	.1813	.8187
8	.1821	.8179	.1662	.8338	.1947	.8053
9	.1614	.8386	.1358	.8642	.1336	.8664
10	.1189	.8811	.1578	.8422	.1527	.8473
11	.1243	.8757	.1245	.8755	.0927	.9073
12 and above	.0452	.9548	.0847	.9153	.0509	.9491

[a]Calculated from data in Appendix Table 4.

obtained. Also, cards containing the average herd size and herd butterfat production were obtained. Failures were classified according to the lactation of failure, herd size from which the animal failed, and butterfat level of the herd from which the animal failed. In comparing the proportion of failures in these data with year end summaries it was found that cows which failed before 90 days had not been included. It was then learned from Putnam that the records of animals failing before they had completed the first 90 days of their lactation were not retained on IBM cards. To determine a correction factor for this situation, the September, 1960 to August, 1961 Monthly Reports for 87 herds in Centre County, Pennsylvania were obtained from Putnam. The failures in these herds were then analyzed using the same classification as was used on other removals. This sample was then used to adjust the original set of data so as to include those animals which had failed before 90 days.

To determine whether or not failure by lactation, herd size, and butterfat levels were independent several Chi square contingency tests were run. It was concluded that failure by lactation was independent of herd size, since the calculated value of Chi square was 34.67 compared to 47.37 at the five percentage point of the Chi square distribution with 33 degrees of freedom. This result led to the conclusion that the probability of failure of an animal in a given lactation is not conditional ipon the size of the herd. Because this was a cross sectional analysis, managerial ability is already reflected in the size of the herd. Thus, no dependence between the proportion of failures by lactation and herd size could be expected. However, this does not say that if the same level of managerial ability was used on different herd sizes that the same results would be observed.

The failure of the animal by lactation and the butterfat level of the herd were concluded to be dependent, since the calculated value of Chi square was 119.02 compared to 73.29 at the five percentage point of the Chi square distribution with 55 degrees of freedom. This result led to the conclusion that the failure of an animal in a given lactation was conditional upon the butterfat level of the herd.

Estimation of Economic Compenents

The economic components of the dairy cow replacement model which must be specified numerically are the market value of the present and possible replacement animals, the expected net return of each of the possible replacement animals, transaction costs, and the initial position. For this study the market value of the present and replacement animals, $pv_{t,i}$ and $rc_{t,j}$ is considered to be the same for two animals of the same lactation and butterfat level. The expected net return of the replacement animal is

$$P_j(q_j VS_{t,j} + p_j VF_{t,j}).$$

Transaction cost, Δ_j, is equal to zero when the animal being considered for replacement is of the same lactation as the present animal. The initial position, $\pi_{o,j+1}$, is specified as the market value of the animal at the termination of the enterprise.

Market Value of the Animal, $pv_{t,i}$ and $rc_{t,j}$

The market value of the present animal and possible replacement animals in a given enterprise period represents a value for beef plus a value associated with expected returns from diary production. The market value of an animal for this study was estimated using data obtained from a questionnaire mailed to a sample of DHIA herd owners in Oregon.[1] Data were collected on 326 transactions of cows bought or sold for dairy purposes by Oregon DHIA herd owners during 1961. Data collected on each transaction were: (1) the number of lactations the animal had

[1] Of the 313 questionnaires mailed, 188 were returned for a response of 60 percent.

completed when purchased or sold, (2) the price paid or received for the animal, (3) the sale charges incurred in the transaction, (4) the number of miles the animal was transported, and (5) the previous production of the animal or the expected production of the animal if no previous records were available. From these data an equation for estimating dairy cow market value was derived.

Least squares estimates were determined for the parameters of the equation:

$$rc_{t,j} = pv_{t,j} = B_0 + B_1 j + B_2 j^2 + B_3(k) + B_4(abl_j) + B_5(abbf_k) + e_{t,j}$$

where

$rc_{t,j}$ = market value of the replacement of lactation j,

$pv_{t,j}$ = market value of the present animal of lactation j(since $rc_{t,j} = pv_{t,j}$ when i = j),

j = lactation of the animal (1, 2, 3, ... , 12),

$$k = \text{butterfat level} = \begin{bmatrix} 1 \text{ if butterfat production} < 350 \text{ pounds} \\ 2 \text{ if } 350 \leq \text{butterfat production} \leq 450 \text{ pounds} \\ 3 \text{ if butterfat production} > 450 \text{ pounds} \end{bmatrix}$$

ablj = index of the number of cows available by lactation,[1] and

$abbf_k$ = index of the number of cows available by butterfat level.[2]

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[1] The index is based upon the previous data of Cannon and Hansen and is presented in Appendix Table 1.

[2] The index is based upon 9576 records of 305 day lactations of Oregon DHIA dairy cows. This index is presented in Appendix Table 2.

The regression equation was

$$rc_{t,j} = pv_{t,j} = 249.14 + 0.92(j) + 0.50(j^2) + 17.59(k) - 4.76(abl_j) - 60.4(abbf_k).$$

The standard errors of the estimate of the parameters were

$$\sigma_{B1} = 9.3, \ \sigma_{B2} = 1.02, \ \sigma_{B3} = 13.7, \ \sigma_{B4} = 8.0 \text{ and } \sigma_{B5} = 50.3.$$

The estimated market values of animals by lactation and butterfat level as derived by the preceding equation are presented in Table 6.

Expected Net Return, $P_j(q_j VS_{t,j} + p_j VF_{t,j})$

The expected value of an outcome of an experiment or game is simply the sum of the returns from the various outcomes times the probabilities associated with the outcomes. In the urn example if a game is established such that there is a payoff R_2 associated with observing a white ball with a "hat" and a different payoff R_1 associated with observing a white ball without a "hat" and no payoff otherwise, then the expected return from a draw is

$$E(\text{Return}) = Pr[W]\,(Pr[\text{Hat}\,|W]R_2 + Pr[\text{No Hat}\,|W]R_1) + (1 - Pr[\bar{W}])0.$$

Table 6. Market Value of Animals by Lactation and Butterfat Level Used in this Study

Lactation	Estimated Market Value		
	Butterfat Level 1 (dollars)	Butterfat Level 2 (dollars)	Butterfat Level 3 (dollars)
1	227.09	220.56	248.16
2	227.70	221.17	248.76
3	229.51	222.98	250.58
4	232.06	225.54	253.13
5	235.65	229.13	256.72
6	238.24	231.71	259.30
7	241.16	234.64	262.23
8	238.62	232.09	259.69
9	234.62	228.09	255.68
10[a]	207.97	201.44	229.03
11	191.37	184.84	212.44
12 and above	208.83	202.30	229.89

[a]The data from the questionnaire did not include any transactions on animals above lactation nine. The market value for animals of lactations 10, 11, and 12 is an extrapolation of the data.

If $R_1 = 10$, $R_2 = 2$, $Pr[W] = .5$, $Pr[Hat | W] = .5$ and $Pr[No\ Hat | W] = .5$, then $E(Return) = .5((.5)(2) + (.5)(10)) = 3.0$. It should be noted that the expected return of the game is not equal to the return on any given draw; rather if the game is repeated many times the average return is the expected return. The expected return from a replacement animal can be obtained in a similar fashion if the following associations are made:

1. The appearance of a hat is associated with the failure of an animal.
2. The appearance of no hat is associated with the success of an animal.
3. The appearance of a white ball is associated with the finding of an animal of a given lactation.
4. The payoff R_1 is associated with net returns if the animal succeeds.
5. The payoff R_2 is associated with the net returns if the animal fails.
6. The number of plays of the game is associated with the number of enterprise periods.

Thus the expected net return of the replacement animal of lactation j can be expressed as

E (net returns) $= P_j(q_j VS_{t,j} + p_j VF_{t,j})$. As in the preceeding example it is very likely that the actual net returns for any given animal and enterprise period will not equal the expected net returns; however, as noted for repeated trials in the urn example, the average net return for a continually operating enterprise is equal to the expected net return.

Net Return from Success, $VS_{t,j}$

The estimated net return for an animal of lactation j if she succeeds was obtained by subtracting certain costs of production from the market value of the production. Costs and returns which are constant to both the present animal and possible replacements were not considered in the net return estimation, since their inclusion would not change the replacement policy. Costs and returns considered to be constant to animals of all lactations and butterfat levels are labor, barn and facilities charge, value of waste product, value of the calf, and breeding fees.

The value of the production of an animal of a given lactation can be obtained by multiplying the pounds of butterfat times the price of the product. The pounds of butterfat varies by lactation. The relationship between the number of the cow's lactation and production was obtained from data presented by Brody (6. p. 691). A quadratic equation was fitted to Brody's data to obtain an index of how production varies by lactation. This equation is plotted in

Figure 3. Suppose the index is to be obtained for a cow in the third lactation, then a "3" is substituted into the equation and solved. In the third lactation the index of production is 1.23, i.e., an animal in the third lactation will produce 123 percent of its first lactation. Thus, the animal which produced 300 pounds of butterfat in its first lactation will produce 369 pounds in its third lactation.

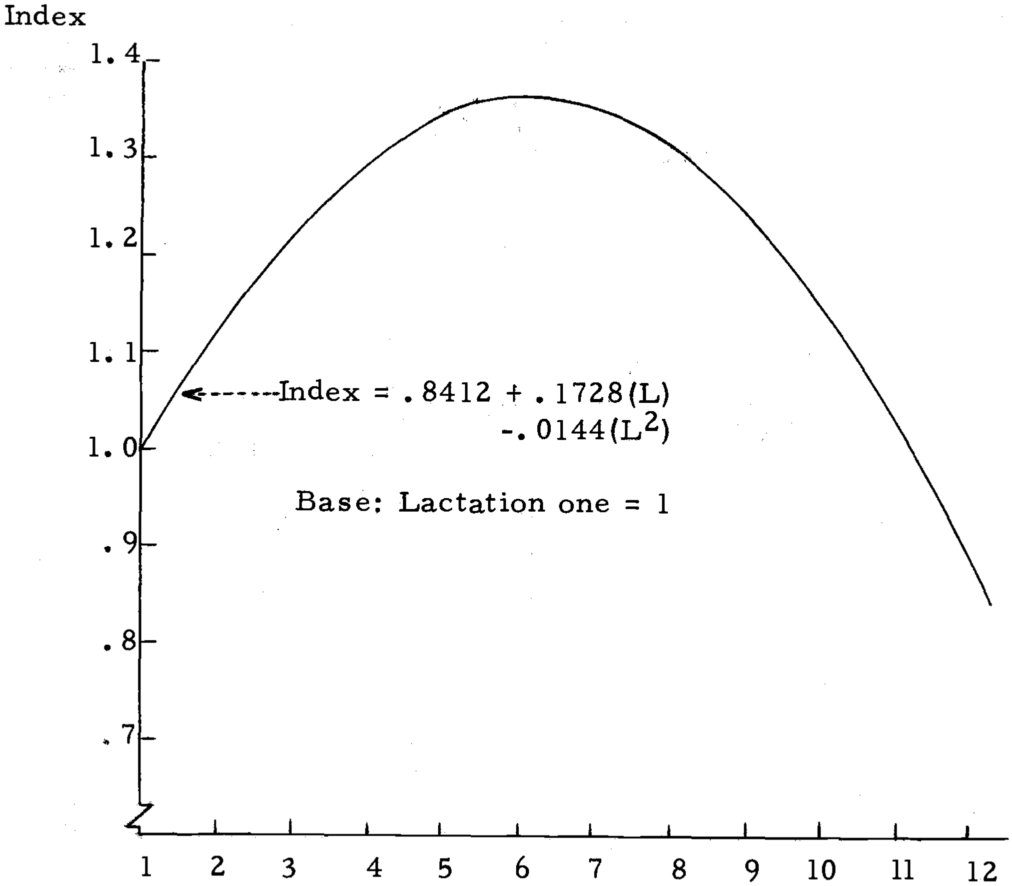

Figure 3. Index of butterfat production by lactation.

If the price of butterfat is \$1.35 per pound, the base level of production is 300 pounds, and the animal is in the third lactation, then the value of production is 481.15 dollars.

Costs of forage and concentrates fed were estimated. The forage costs were estimated to be \$11.00 per month[1] times a weight index to take into consideration that an animal of lactation one in general has not reached her full growth. The weight index used was calculated from a sample of 100 records from the Pennsylvania State DHIA Centre County Monthly Reports and is presented in Appendix Table 3. For example, an animal in its sixth lactation has a weight index of 1. 312; therefore, the forage costs would be estimated at (\$11.00) (12) (1.312) = \$173.18 for an enterprise period of twelve months.

The cost of concentrates were estimated by fitting a linear equation with cost of concentrates dependent upon the amount of butterfat produced. The data for estimation of the equation was obtained from the Pennsylvania State DHIA Monthly Reports. The equation for concentrate costs is

$$cc = 43.256 + (.3125)\,(\text{pounds of butterfat})\,.$$ [2]

[1] The figure of \$11.00 per month is used to estimate cost of forage for the Monthly Report to the herd owners of Pennsylvania DHIA. It is used in this study since it was readily available.

[2] The above relationships are included in the computer program as shown in Appendix I; therefore no specific calculations of a net return is shown here.

Net Return from Failure, $VF_{t,j}$

If an animal fails she can either be sold for beef or she will be removed from the herd because of death. Of those animals failing in Pennsylvania in 1960, ten percent were deaths and the remaining 90 percent were sold for beef. Thus, the probability that an animal will be sold for beef given failure is .90. The expected return from a failure is

$$VF_{t,j} = Pr[beef](\text{value of beef}) + Pr[death](\text{value of death})$$

$$= .90\ (\text{value of beef}) + .10\ (\text{value of death}) .$$

The value of death is assumed to be near zero and the value of beef is estimated by multiplying the weight of the animal in its first lactation times the previously indicated weight index times the price per pound. Thus, if the animal's base weight in its first lactation is 1200 pounds, the price is \$14.00 per hundred, and the index is 1.312, then the value of beef is \$220.42 and the expected return from the failure is (.90)(\$220.42) + (.10)(0) = \$198.38. The expected net return of the failure is obtained by subtracting the market value of the animal, $rc_{t,j}$ from the failure's expected return. If the price paid for the animal in the example was \$238.24, then

$$VF_{t,j} = 198.38 - 238.24 = -39.86,$$

i.e., a loss would be incurred if the animal fails.

Transactions Costs, Δ_j

The cost of making the transaction from the present animal to its replacement includes commission charges, transportation, etc. For this study, these were obtained from the mail questionnaire data. The average mileage per transaction was 32 miles and the average commission charge was $2.26. If 12 cents per mile were charged, then the transportation costs would be $3.84 and the average transportation plus commission cost is $6.10. The amount of labor used for the transaction would vary and would be of different value to different individuals; hence, for purposes of analysis the transaction cost was set at $10.00 allowing $3.90 for the value of the labor involved.[1]

Initial Position, $\pi_{o,j+1}$

As shown previously, some initial position must be specified in order to solve the stochastic multi-stage replacement

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[1]Data were not available to allow a rough approximation to be obtained of the costs incurred in searching for an animal of a given lactation. The relative difficulty of finding cows in specified lactations is reflected by P_j.

equation. The initial position is assumed to be the termination of the enterprise. Thus, $\pi_{o, j+1}$ is the sale price of the animal of lactation j + 1, i.e., the market value. Initial positions by lactation and butterfat level as used in this study are presented in Table 7. For purposes of computation $\pi_{o, 13}$ is assumed to be equal to $\pi_{o, 12}$.

Summary

This chapter has shown how numerical values were provided for the components of the stochastic multi-stage replacement equation. The equation was programmed for the IBM 1620 and the various numerical estimates as given in this chapter were used to yield optimal replacement policies for dairy cows in a continually operating enterprise. The results of this analysis are presented in the following chapter.

Table 7. Initial Positions, $\pi_{o,j+1}$, by Lactation and Butterfat Levels[a]

Lactation j	$\pi_{o,j+1}$ Butterfat Level 1	Butterfat Level 2	Butterfat Level 3
1	227.70	221.17	248.76
2	229.51	222.98	250.58
3	232.06	225.54	253.13
4	235.65	229.13	256.72
5	238.24	231.71	259.30
6	241.16	234.64	262.23
7	238.62	232.09	259.69
8	234.62	228.09	255.68
9	207.97	201.44	229.03
10	191.37	184.84	212.44
11	208.83	202.30	229.89
12	208.83	202.30	229.89

[a]Calculated by using the equation on page 41.

CHAPTER IV

OPTIMAL DAIRY COW REPLACEMENT POLICIES

Optimal replacement policies for dairy cows in a continually operating enterprise were obtained by the use of the stochastic recurrence equation discussed in Chapter II and the numerical values of the components of the equation presented in Chapter III. Replacement policies were determined for animals of three butterfat levels and under various price conditions. The prices for the year 1961 were used as base prices and the effects of variations in feed prices, dairy cow prices, canner and cutter prices, and milk prices were observed. Also, the actual prices for the 1950-1961 period were used to determine what the optimal policy for animals in each butterfat level would have been during that particular sequence of enterprise periods. It is assumed that the probability of finding a cow in a given lactation, the probability of failure, and the probability of success are constant between cows throughout the length of the enterprise. Also, the production relationship between pounds of butterfat and lactation is assumed constant between cows and over time. This chapter presents the optimal replacement policies under these various conditions.

Effects of Price Variations Upon Optimal Policies

Optimal replacement policies were determined under different price conditions. The prices used were 1961 averages and various combinations of increases and decreases. The optimal policies for butterfat level one, production less than 350 pounds, are presented in Table 8. The optimal policies for the second butterfat level, production between 350 and 450 pounds, are presented in Table 9. The optimal policies of the third butterfat level, over 450 pounds, are presented in Table 10.

The effect of the price variations are as follows:

1. With dairy cow prices, feed prices, canner and cutter prices, and milk prices constant at the 1961 level a policy of 1, 2, 3, 4, 5, 6 is obtained for each butterfat level.[1]

2. With dairy cow prices, feed prices, canner and cutter prices, and milk prices each at 120 percent of the 1961 level no change in the optimal policy is observed.

3. With dairy cow prices, feed prices, canner and cutter prices, and milk prices each at 80 percent of the 1961 level no change in the optimal policy occurred.

4. With dairy cow prices, feed prices, and canner and cutter prices at 1961 levels and milk prices at 120 percent of the 1961 level, the replacement cycle lengthens.

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[1] The policy is read: obtain an animal of lactation one, i.e., ready to begin her first lactation, keep her until she has completed her sixth lactation and then replace her with an animal of lactation one.

Table 8. Optimal Replacement Policies for Dairy Cows in Butterfat Level I Under Various Price Conditions.

Cow Prices	Feed Prices	Canner and Cutter Prices	Milk Prices	Replacement Policy
1950-1961	1950-1961	1950-1961	1950-1961	5, 6, 7, 1, 2, 3, 4, 5, 1, 2, 3, 4
1961	1961	1961	1961	1, 2, 3, 4, 5, 6
120% 1961	120% 1961	120% 1961	120% 1961	1, 2, 3, 4, 5, 6
80% 1961	80% 1961	80% 1961	80% 1961	1, 2, 3, 4, 5, 6
1961	1961	1961	120% 1961	1, 2, 3, 4, 5, 6, 1 or 7[a]
1961	1961	1961	80% 1961	1, 2, 3, 4, 5, 1 or 6
1961	120% 1961	1961	1961	1, 2, 3, 4, 5, 1 or 6
1961	80% 1961	1961	1961	1, 2, 3, 4, 5, 6, 7
120% 1961	1961	120% 1961	1961	1, 2, 3, 4, 5, 6
80% 1961	1961	80% 1961	1961	1, 2, 3, 4, 5, 6

[a] From the standpoint of net returns it makes no difference if the animal is kept or replaced.

Table 9. Optimal Replacement Policies for Dairy Cows in Butterfat Level II Under Various Price Conditions.

Cow Prices	Feed Prices	Canner and Cutter Prices	Milk Prices	Replacement Policy
1950-1961	1950-1961	1950-1961	1950-1961	2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 7, 8
1961	1961	1961	1961	1, 2, 3, 4, 5, 6
120% 1961	120% 1961	120% 1961	120% 1961	1, 2, 3, 4, 5, 6
80% 1961	80% 1961	80% 1961	80% 1961	1, 2, 3, 4, 5, 6
1961	1961	1961	120% 1961	1, 2, 3, 4, 5, 6, 7
1961	1961	1961	80% 1961	1, 2, 3, 4, 5, 1 or 6
1961	120% 1961	1961	1961	1, 2, 3, 4, 5, 1 or 6
1961	80% 1961	1961	1961	1, 2, 3, 4, 5, 6, 1 or 7
120% 1961	1961	120% 1961	1961	1, 2, 3, 4, 5, 6
80% 1961	1961	80% 1961	1961	1, 2, 3, 4, 5, 6

Table 10. Optimal Replacement Policies for Dairy Cows in Butterfat Level III Under Various Price Conditions.

Cow Prices	Feed Prices	Canner and Cutter Prices	Milk Prices	Replacement Policy
1950-1961	1950-1961	1950-1961	1950-1961	2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 7, 8
1961	1961	1961	1961	1, 2, 3, 4, 5, 6
120% 1961	120% 1961	120% 1961	120% 1961	1, 2, 3, 4, 5, 6
80% 1961	80% 1961	80% 1961	80% 1961	1, 2, 3, 4, 5, 6
1961	1961	1961	120% 1961	1, 2, 3, 4, 5, 6, 1 or 7
1961	1961	1961	80% 1961	1, 2, 3, 4, 5, 1 or 6
1961	120% 1961	1961	1961	1, 2, 3, 4, 5, 1 or 6
1961	80% 1961	1961	1961	1, 2, 3, 4, 5, 6, 1 or 7
120% 1961	1961	120% 1961	1961	1, 2, 3, 4, 5, 6
80% 1961	1961	80% 1961	1961	1, 2, 3, 4, 5, 6

5. With dairy cow prices, feed prices, and canner and cutter prices at 1961 levels and milk prices at 80 percent of the 1961 level the replacement cycle shortens.

6. With dairy cow prices, milk prices and canner and cutter prices at 1961 levels and feed prices at 120 percent of 1961 levels the replacement cycle shortens.

7. With dairy cow prices, milk prices, and canner and cutter prices at 1961 levels and feed prices at 80 percent of 1961 levels the replacement cycle lengthens.

8. With feed prices and milk prices at 1961 levels and an increase or decrease of 20 percent in the 1961 prices of dairy cows and canners and cutters no change in the optimal policy occurred.

It should be noted that an increase in milk prices tended to lengthen the replacement cycle while a decrease tended to shorten the cycle. An increase in feed prices resulted in a decrease in the length of the replacement cycle and a decrease resulted in a longer cycle. Since the prices of dairy cows and canners and cutters generally move together (See Table 11), it should be noted that when these prices increase or decrease no change is observed in the length of the replacement cycle.

Optimal Replacement Policies for 1950-1961

It is quite evident that when only one variable changes the replacement policy stabilizes and repeats. However, under actual conditions, milk prices may change in one direction while feed prices and canner and cutter prices may change in an opposite direction.

Likewise, milk prices and dairy cow prices need not move in the same direction. To demonstrate that the replacement cycles may not be of the same length under actual conditions the replacement equation was solved using prices from the 12 year period 1950 through 1961 inclusive. Price indexes of dairy cows, feed, canners and cutters, and milk for 1950-1961 are shown in Table 11. The indexes were determined using 1961 prices as a base.

The optimal replacement policies for the period 1950-1961 were determined for each of the three butterfat levels. The optimal policy for each butterfat level was (2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 7, 8). This policy is read as follows: Begin with a cow of lactation two in 1951, keep her from 1951 through 1954, then in 1955 replace her with a cow in its first lactation and keep her through 1961 when more data would be necessary to determine the next decision. The replacement animal is assumed to be in the same butterfat level as the present animal when they are in the same lactation.

Discussion of Replacement Policies

It is of interest and also highly significant that in all cases of the hypothetical price variations and the actual prices for the twelve year period of 1950-1961 that the replacement animal was always an animal of lactation one, i. e., an animal ready to begin her first

TABLE 11

Price Indexes of Dairy Cows, Feed, Canners and Cutters, and Milk, 1950-1961.

Year	Price of Dairy Cow Index[a]	Feed Cost Index[b]	Canner and Cutter Index[a]	Milk Price Index[c]
1961	1.000	1.000	1.000	1.000
1960	.995	1.000	.983	.976
1959	1.040	.996	1.131	.950
1958	.940	1.003	1.150	.932
1957	.740	1.042	.839	.968
1956	.680	1.042	.695	.976
1955	.650	1.069	.695	.941
1954	.660	1.142	.668	.929
1953	.790	1.183	.742	1.021
1952	1.080	1.301	1.170	1.180
1951	1.103	1.214	1.455	1.171
1950	.884	1.062	1.146	.988

[a] Calculated from (13, p. 14).

[b] Calculated from (14, p. 14).

[c] Calculated from (14, p. 7).

lactation. Results from the mail questionnaire indicated that 80.6 percent of the Oregon DHIA herd owners sampled did not buy any replacements in 1961, i.e., they raised the replacements used in 1961 and hence implies these herd owners used replacements of lactation one. Actual culling rates observed for Oregon DHIA herds for 1954 to 1961 are shown in Table 12. The culling rate is the percentage of cows removed each year for not only failures as previously defined but also low production.

Table 12. Culling Rate by Years for Oregon DHIA Herds[a].

Year	Culling Rates
	Percentage
1954	20
1955	19
1956	22
1957	26
1958	24
1959	25
1960	27
1961	26

[a] Calculated from (9).

The average culling rate for these eight years was 23 percent. The optimal policy for these eight years as determined by the replacement model was (1, 2, 3, 4, 5, 6, 7, 8) implying a culling rate of 12 percent. Hence, herd owners were replacing animals twice as fast as the price relationships over the eight year period would indicate. This is not saying the herd owners are irrational nor that the model is illogical. It is likely that if herd owners in 1954 had had price data for enterprise periods following 1954, they would have lengthened the replacement cycle of their herds thereby decreasing the number of removals. That is, hindsight is better than foresight. However, this discrepancy could also mean that herd owners are production maximizers and an animal would be removed as a low producer before she becomes less profitable than her replacement. Production maximizers are those who may over emphasize the improvement of herd averages at the cost of profit.

Assuming 1961 price levels and relationships had been anticipated correctly by Oregon DHIA herd owners and had expected these to prevail in the future, the optimal replacement policy would have been (1, 2, 3, 4, 5, 6). This implies approximately a 17 percent culling rate. However, a culling rate of 26 percent was shown in Table 12 for 1961. While the discrepancy here is not as great as when the 1954 to 1961 period is considered, this much discrepancy implies that it may be profitable to put this model to practical use. To do this the data gathering procedure now being used by dairy management advisors would need to be modified only slightly to obtain data that would be of value in providing information to be used to make decision making more profitable.

CHAPTER V

SUMMARY AND CONCLUSIONS

This thesis has attempted to organize the replacement decision process in such a manner that a variety of factors which influence the decision can be considered. Replacement problems have the fundamental property that decisions at one point in time are dependent upon decisions at other points in time. The dairy cow replacement decision process has this fundamental property. In addition, the dairy cow replacement problem involves consideration of both economic and stochastic components. These are: market value of the animal, net return from the success of an animal, net return from the failure of the animal, transaction costs, probability of finding a cow of a given lactation, and probability of success and failure. The simultaneous consideration of the economic components, the stochastic components, and the fundamental property gave rise to the development of a stochastic multi-stage decision making model.

This model was expressed in the form of a mathematical equation. It was shown that the equation could be solved as a recurrence relation by renumbering the enterprise periods and specifying an initial position. The solution was made obtainable by programming the IBM 1620 to solve the replacement equation.

Utilizing data (1) obtained from Pennsylvania DHIA records and Iowa cow-testing associations concerning the stochastic factors, (2) presented by Brody concerning production variation by lactation, and (3) obtained from Oregon DHIA herd owners on prices paid and received for productive dairy cows, the equation was solved for various milk, feed, and beef price assumptions. It was assumed that the data obtained from Pennsylvania, Iowa, and Brody applied to dairy cows in Oregon. This does not say that it would be useless to attempt to obtain these data for dairy cows in Oregon; rather the data used were the best available. Since price data which are relevant to Oregon DHIA herd owners were used, a comparison of optimal policies obtained from the solution of the replacement equation and actual practice was made. This comparison showed that the DHIA herd owners in Oregon may be replacing their cows on too short of a cycle from the standpoint of the maximization of profits.

Unique to this analysis was the definition of a failure, the use of this definition in classifying cow removals, and the subsequent determination of the probabilities of failure and success. Another unique aspect of the analysis was the inclusion of the fact that it is unlikely that an entire herd of dairy cows can be replaced by cows of the same lactation. This was accomplished by interpreting the population distribution of dairy cows as the probability of finding a cow of a given lactation.

In pointing out some of the limitations of the analysis, bold among them would be the probability associated with the event that a heifer would be of a given production level when it is to be used as a replacement. This probability could be considered as conditional upon the dam's production record and the production record of the sire's offspring. The task of obtaining these probabilities remains for another thesis. Also, left to another thesis is describing the decision process, developing a model, and obtaining the solution of the optimal herd composition problem. A hint to this problem is:

$$\pi_{t;i_1, i_2, \ldots, i_n} = p v_{t;i_1, i_2, \ldots, i_n}$$

$$+ \max_{\substack{j_1 = 1, J \\ j_2 = j_1, J \\ j_n = j_n - 1, J}} \left\{ \begin{bmatrix} r_{t,j_1} - rc_{t,j_1} - \Delta_{j_1} \\ + r_{t,j_2} - rc_{t,j_2} - \Delta_{j_2} \\ \cdot \\ \cdot \\ \cdot \\ + r_{t,j_n} - rc_{t,j_n} - \Delta_{j_n} \end{bmatrix} + \pi_{t-1;j_1+1, j_2+1, \ldots; j_n+1} \right\}$$

where N is the size of the herd and the other notation is the same as used previously. The equation describing the herd composition problem is deterministic, but could be modified to be stochastic by incorporating the stochastic factors of the individual dairy cow replacement problem.

Extensions of the Model

The use of multi-stage decision models in determining optimal actions offers great potential for research workers. The theoretical development of models for multi-stage decision problems is considerably advanced beyond the application to actual problems. The multi-stage approach offers the researcher the opportunity to study problems in which decisions are dependent upon one another through time.

An area apart from dairy in which the model offers great potential is in the area of range management. For example, the decision to reseed a given range at any given time is dependent upon decisions made at other decision points. The use of a multi-stage decision model would make feasible the study of reseeding policies for many enterprise periods. Factors which could be considered would be cattle prices, probability of obtaining a sufficient stand for different levels of grazing, different seeding and spraying actions and the prices of inputs, i.e., seed, labor, etc.

Another application of the approach in agriculture is the replacement of fruit trees in an orchard. Unfortunately, there exists no organization which has kept individual fruit tree records to provide data to make the problem easily solvable. This does not say that the problem cannot be solved, it merely says that the necessary data is not readily accessible. A problem similar to the replacement of fruit trees is the decision to cut and replace timber producing trees. Interesting in this problem would be the relationship between thickness of stand and rate of cut.

To persons working in extension, the multi-stage replacement equation offers even greater possibility. Perhaps of most promise in dairy cow replacement is the obtaining of "shadow prices", i.e., the range of dairy cow prices over which the replacement decision would not change. This information can be obtained by solving the equation for optimal policies under various levels of replacement heifer prices. This information would answer the question of how much can the herd owner afford to pay for replacement animals (or pay for raising replacements) before the length of the replacement cycle changes.

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APPENDICES

APPENDIX I

Multi-stage Replacement Program for IBM 1620

The following Fortran notation is used:

PI(J+I) = $\pi_{t-1, j+1}$

P(J)= p_j

I-P(J)= q_j

APV(J)= $\underset{j=1}{\overset{12}{\text{Max}}}\left[\quad\quad\right]$

SPI(I)= $\pi_{t, i}$

AL(J)= Index of number of cows available by age
AB(K)=Index of number of cows available by butterfat level
TB(J)=Value of butterfat production of animal in lactation j
TC(J)=Cost of production
SP(J)= P_j
TEMP(J)= $rc_{t,j}$ = $pv_{t,j}$
FC(J)=Weight index
E=Transaction cost
N=length of enterprise
BF=Butterfat production of animal in first lactation
IB=Butterfat level of the herd
VF= $VF_{t,j}$
R= $VS_{t,j}$
ZA=Price level of dairy cows
ZB=Price level of feed
ZC=Price level of canners and cutters
ZD=Price level of milk

```
      DIMENSION PI(13), P(12), APV(12), SPI(12), AL(12), AB(3)
      DIMENSION TB(12), TC(12), SP(12), TEMP(13), FC(12)
   50 DO 1 I=1, 12
    1 READ 201, P(I)
      DO 3 I=1, 12
    3 READ 201, SP(I)
      DO 6 I=1, 12
    6 READ 201, FC(I)
      READ 202, E
      READ 203, N
      READ 202, BF
      READ 203, IB
      BFC=IB
      DO 2 I=1, 12
    2 READ 202, AL(I)
      DO 4 I=1, 3
    4 READ 202, AB(I)
   30 DO 32 I=1, 12
      X=I
   32 TEMP(I)=249. 14 +. 92*X+. 5*(X**2)+17. 5864*BFC-4. 75*AL(I)
                 -60. 42*AB(IB)
      TEMP(13)=TEMP(12)
   36 DO 5 I=1, 13
    5 PI (I)=TEMP(I)
      DO 40 I=1, 12
      X=I
      TA=84. 17+17. 28*X-1. 44*(X**2)
      TB(I)=1. 35*TA*BF*. 01
   40 TC (I)=164. 256+.3125*BF*TA*.01
      DO 28 LT=1, N
      IF (SENSE SWITCH 2)71, 72
   72 PRINT 100, LT
   71 PUNCH 203, LT
      READ 204, ZA, ZB, ZC, ZD
      DO 26 J=1, 12
      DO 22 I=1, 12
      VF=156. *ZC*FC(1)-TEMP(I)*ZA
      VBF=ZD*TB(I)
      R=VBF-TC(I)*ZB*FC(I)
      RC=ZA*TEMP(I)
      IF (I-J)21, 20, 21
   20 APV(I)=(1. -P(I))*R+P(I)*VF-RC+PI(I+1)
```

```
      GO TO 22
   21 APV(I)=SP(I)*((1.-P(I))*R+P(I)*VF)-RC-E+PI(I+1)
   22 CONTINUE
      OPT=-9999.
      DO 25 I=1, 12
      IF (APV(I)-OPT)25, 25, 23
   23 OPT=APV(I)
   25 CONTINUE
      PV=TEMP(J)*ZA
      SPI(J)=PV+OPT
      DO 26 I=1, 12
      IF(APV(I)-OPT)26, 12, 26
   12 IF(SENSE SWITCH 2)74, 75
   75 PRINT 205, J, SPI(J), I
   74 PUNCH 205, J, SPI(J), I
   26 CONTINUE
      DO 27 J=1, 12
   27 PI(J)=SPI(J)
   28 CONTINUE
      GO TO 50
  201 FORMAT (F6.4)
  202 FORMAT (F5.2)
  203 FORMAT (I4)
  204 FORMAT (F4.3, F4.3, F4.3, F4.3)
  205 FORMAT (I4, F18.2, I10)
  100 FORMAT (4H LT , I4)
      END
```

Appendix Table 1. Index of the Number of Cows Available by Lactation Used in Regression Equation for Dairy Cow Prices

Lactation	Number of Cows [a]	Index [b]
1	31, 447	1.000
2	22, 735	1.383
3	18, 258	1.723
4	14, 840	2.119
5	12, 535	2.509
6	9, 471	3.321
7	7, 361	4.272
8	4, 776	6.588
9	3, 338	9.422
10	1, 829	17.241
11	1, 362	23.152
12 and over	1, 368	22.100

[a] Obtained from: (7, p. 1026).

[b] Index is determined by dividing the number of cows in each lactation into the number of cows available in the first lactation.

*****

Appendix Table 2. Index of the Number of Cows Available by Butterfat Level as Obtained From Analysis of 9, 576 Oregon Cows Completing a 305 Day Lactation in 1961 Used in Regression Equation for Dairy Cow Prices

Butterfat Level	Number of Cows	Index [a]
Less than 350 pounds	2, 363	0.6010
350 to 450 pounds	3, 932	1.0000
Over 450 pounds	3, 281	0.8344
TOTAL	9, 576	

[a] Index is determined using the number of cows in the second butterfat level as a base.

Appendix Table 3. Index of the Weight of Dairy Cows by Lactation Used in this Study

Lactation	Number of Cows in Sample[a]	Average Weight (Pounds)	Index[b]
1	25	1120	1.000
2	18	1210	1.080
3	14	1370	1.223
4	13	1400	1.250
5	11	1450	1.286
6 and above	19	1470	1.312

[a] Cows used in the sample were selected randomly from the Centre County, Pennsylvania, DHIA Monthly Reports.

[b] Index was calculated by dividing each weight by the average weight of animals in the first lactation.

Appendix Table 4. Number of Cows Failing and Total Number of Cows by Lactation and Butterfat Level as Obtained From Analysis of Pennsylvania DHIA Data

	Number of Cows Failing by Butterfat Level			Number of Cows by Butterfat Level [a]		
Lactation	Less than 350 lbs. butterfat	350-450 lbs. of butterfat	Over 450 lbs. of butterfat	Less than 350 lbs. butterfat	350-450 lbs. of butterfat	Over 450 lbs. of butterfat
1	171	949	556	39, 058	17, 485	8, 251
2	187	955	498	2, 826	12, 645	5, 967
3	187	951	585	2, 268	10, 149	4, 789
4	171	987	560	1, 844	8, 250	3, 893
5	160	941	552	1, 558	6, 970	3, 289
6	164	820	435	1, 177	5, 265	2, 484
7	121	645	350	915	4, 092	1, 931
8	108	441	244	593	2, 654	1, 253
9	67	252	117	415	1, 856	876
10	27	160	73	227	1, 014	478
11	21	94	33	169	755	356
12 and over	8	67	19	177	791	373

[a] The number of cows by butterfat level was obtained for the Pennsylvania data by sorting herd cards by butterfat levels, summing the number cows in each level, and applying the Iowa age distribution.