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Waldo George Magnuson, Jr. for the Electrical Engineer
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Abstract approved

The design of a preamplifier for use with a piezoelectric transducer is generally approached from the point of view that an extremely high input impedance is required to obtain low frequency response. However, piezoelectric transducers are essentially charge generators and this fact may be utilized in designing preamplifiers to be used with them. In this thesis the concept of a charge amplifier is discussed and a practical expression is derived in terms of frequency response, input impedance, amplifier gain, and feedback capacitor value. It is shown that modest input impedances may be used and low frequency response maintained, using this concept. A practical amplifier design is illustrated and its performance is described.

A CHARGE SENSITIVE AMPLIFIER FOR USE WITH PIEZOELECTRIC TRANSDUCERS

by

WALDO GEORGE MAGNUSON JR

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APPROVED:

Professor of Electrical Engineering

Head of Department Electrical Engineering

Chairman of School Graduate Committee

Dean of Graduate School

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TABLE OF CONTENTS

LIST OF FIGURES

A CHARGE SENSITIVE AMPLIFIER FOR USE WITH PIEZOELECTRIC TRANSDUCERS

INTRODUCTION

The measurement of acceleration in the development and evaluation of weapon systems is generally complicated by the requirements placed on the instrument system. In particular, when one of the requirements of acceleration measurement is to record the high frequency response due to shock impacts as well as the ability to reproduce the longer duration acceleration experienced by the system, the design and implementing of the instrument system is not easily achieved.

The high-speed water entry of aircraft-launched or rocket-thrown torpedoes serves as one example where a knowledge of both the transient and steady-state acceleration history is highly desirable (2, p. 3). Such a knowledge is quite valuable if not necessary in designing shock absorbing mountings for delicate control relays, electronic programing and guidance panels, and for the attaining of high reliability in the release of parachute stablizers at water entry. In applications such as these an acceleration time-history extending to as long as fifty milliseconds coupled with a knowledge of transient behavior is often a requirement of the acceleration measuring instrumentation. The resulting acceleration system would require then a flat frequency response from 0.1 to 10,000 cps.

Piezoelectric accelerometers constructed of quartz crystals provide the most suitable transducer to measure the above described acceleration time histories because of their reduced pyroelectric effect and their high natural resonant frequency. However piezoelectric accelerometers are characterized by the fact that they are charge generators, that is their simplified equivalent circuit consists of a charge generator in parallel with a capacitor. The voltage at the output terminals of the transducer is equal to the generated charge divided by the transducer's capacity. In order to maintain a low frequency response the general procedure is to match the transducer to a very high (100 to 1,000 megohms) impedance circuit.

Using the fact that piezoelectric accelerometers are charge generators it is possible to use much lower input impedances without sacrificing low frequency response by utilizing a charge sensitive preamplifier. This is particularly valuable when the preamplifier must experience the same acceleration as the transducer as in remote telemetry systems.

Whereas vacuum tubes are very difficult if not impossible to render insensitive to system acceleration, transistors produce negligible output. The lower impedances associated with transistors requires a different treatment of the transducer output in order to maintain low frequency response. In this report such a preamplifier is described. Its output voltage is proportional to the input charge and these preamplifiers are particularly suited to use with transistors.

THE CHARGE SENSITIVE AMPLIFIER

The charge sensitive amplifier can be most easily described by first considering it to have an input impedance high enough to be considered infinite. In practice this might be realized with a cathode follower vacuum tube circuit or similar high impedance vacuum tube circuit, or a very high impedance field effect transistor circuit employing bootstrapping techniques. Values in excess of 100 megohms might be adequate for particular piezoelectric transducers at some specified low frequency response. With the assumption that the input impedance is high enough so as to be able to be considered infinite, the charge sensitive amplifier block diagram reduces to that in Figure la (3, p. 20). The amplifier has an internal gain of -A. The capacitor C_f constitutes the feedback path and C the capacity of the accelerometer, the cable and wiring capacity, and the capacity associated with the amplifier input. The accelerometer is represented by the charge source Q.

The feedback capacitor C_f may be replaced by an equivalent shunting capacitor to ground of value $C_f(1 + A)$ as shown in Figure lb. This can easily be shown by writing the expression for the current in C_f. This current is

$$I_{C_f} = C_f \frac{d(E_{in} - E_{out})}{dt}$$

however

$$E_{in} - E_{out} = E_{in}(1 + A)$$

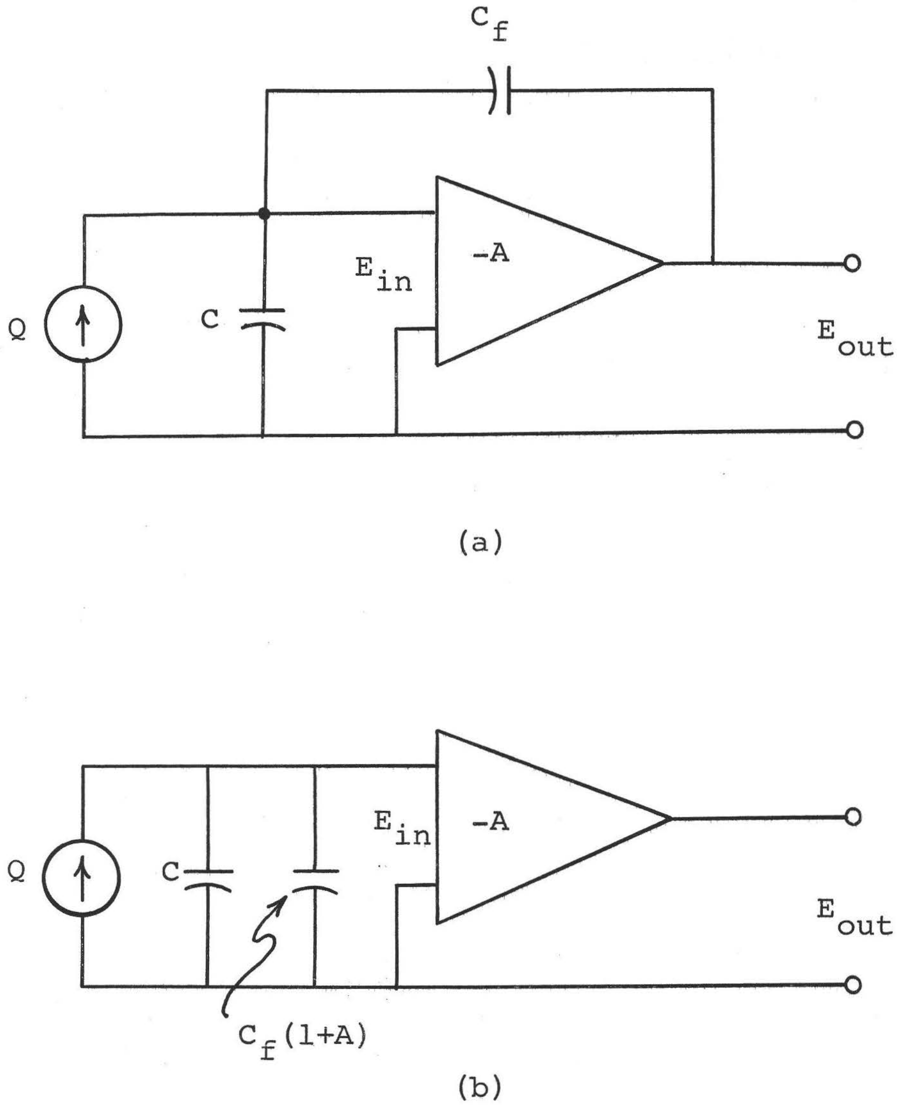

Figure 1.

Representation of charge sensitive amplifier in block diagram form.

which gives

$$I_{C_f} = C_f \frac{d\, E_{in}\, (1 + A)}{dt}$$

$$= (1 + A)\, C_f \frac{d\, E_{in}}{dt} \quad .$$

Obviously nothing would be changed at the input if a capacitor of value $(1 + A)\ C_f$ were shunted to ground since the same current would flow.

The voltage developed by a fixed amount of charge Q is $V = Q/C$ thus the voltage at the input of Figure 1b is given by

$$E_{in} = \frac{Q}{C + C_f\, (1 + A)} \quad .$$

If $C_f\ (1 + A)$ is much larger than C then E_{in} is approximately

$$E_{in} \cong \frac{Q}{C_f\, A}$$

and since

$$E_{out} = -A\, E_{in}$$

$$E_{out} \cong \frac{Q}{C_f} \quad .$$

With the absence of the amplifier the accelerometer output is given by Q/C and with the amplifier present the output is $-\,Q/C_f$. The contribution of the amplifier is then the ratio of $-\,C/C_f$. This ratio may be made quite large in practical circuits.

CHARGE AMPLIFIER CIRCUIT ANALYSIS

In the design of a charge sensitive amplifier using transistors, the presence of input impedance modifies the above results. The following section will analyze a charge sensitive amplifier including the shunting effect of finite resistance at the input of the amplifier. It will be shown that the amplifier output is given approximately by $-\,S/C_f$ where S is the sensitivity of the accelerometer. Design restrictions will be found that relate the amplifier input impedance, amplifier gain, feedback capacitor, and the lowest frequency of interest.

The input resistance of the amplifier will be removed from the amplifier and considered as part of the external circuit. The charge generated by the accelerometer will be assumed to be of the form

$$Q = CV = S \sin \omega t \quad .$$

Referring to Figure 2 the currents I_1 and I_2 are written as

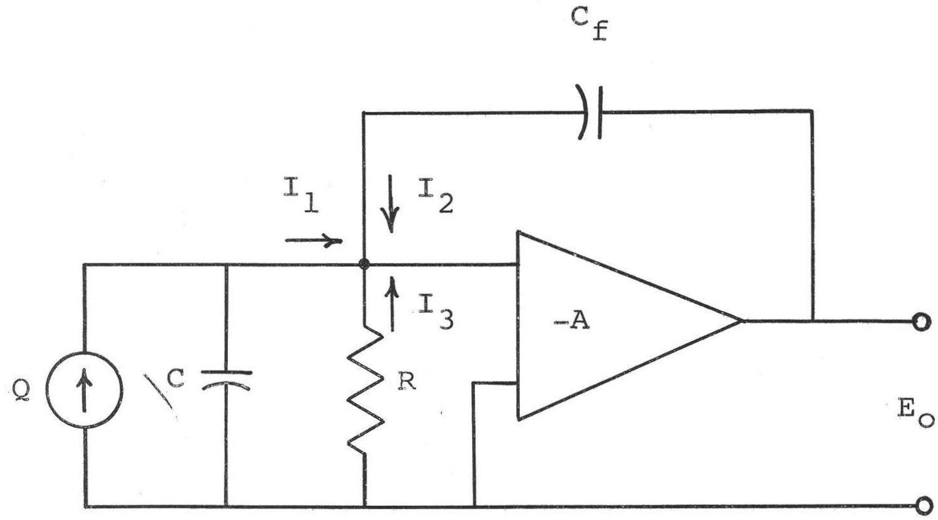

Figure 2.

Block diagram of charge sensitive amplifier including effects of input resistance.

$$I_1 = C\,\frac{dE}{dt} = C\,\frac{d\,\frac{Q}{C}}{dt} = \frac{dQ}{dt}$$

$$I_2 = C_f\,\frac{d(E_o - E_i)}{dt} \cong C_f\,\frac{dE_o}{dt} = -\,ARC_f\,\frac{dI_3}{dt}\,.$$

Then using Kirchhoff's current law to sum currents

$$I_1 + I_2 + I_3 = 0$$

or

$$ARC_f\,\frac{dI_3}{dt} + I_3 = \frac{dQ}{dt}\,.$$

If Q is differentiated and substituted into the above equation, the equation becomes, after rearranging

$$\frac{dI_3}{dt} + \frac{1}{ARC_f}\,I_3 = \frac{\omega S}{ARC_f}\cos\omega t\,.$$

This is a linear first-order differential equation whose solution (see Appendix) is

$$I_3\,(t) = \frac{\omega\,S}{\sqrt{1-(\omega ARC_f)^2}}\cos\,(\omega t - \Phi) - \frac{\omega\,S}{1-(\omega ARC_f)^2}\,\epsilon^{-\frac{t}{ARC_f}}\,.$$

When $t \gg o$ the second term becomes very small (see Appendix) and the above equation reduces to

$$I_3 = \frac{\omega\,S}{\sqrt{1-(\omega\,ARC_f)^2}}\cos\,(\omega t - \Phi)\,.$$

The output voltage is given by

$$Eo = - RAI_3$$

$$= \frac{\omega SAR}{\sqrt{1-(\omega ARC_f)^2}} \cos(\omega t - \Phi) .$$

The magnitude of Eo is

$$|Eo| = \frac{\omega SAR}{\sqrt{1-(\omega ARC_f)^2}} .$$

When $\omega ARC_f \gg 1$ the above equation reduces to the final result

$$|Eo| = \frac{S}{C_f} .$$

Eo is then simply the sensitivity of the accelerometer divided by the value of the feedback capacitor.

DESIGN PROCEDURE

Using the expressions derived in the last section a typical design example will now be investigated. The two expressions to be used are the equations

$$\omega ARC_f \gg 1$$

and

$$|Eo| = \frac{S}{C_f} .$$

For this example a quartz piezoelectric accelerometer manufactured by Massa Laboratories will be considered. Their type M-191 accelerometer has a sensitivity of S = 4 micro-micro coulomb/g and a capacity of C = 110 pf. The sensitivity is expressed in equivalent g's where g is the acceleration of gravity.

In the typical applications cited in the introduction the following example will be chosen as a design example. A typical acceleration time history at the propeller shaft on the Mk 43 Mod 3 torpedo is indicated in Figure 3 (2, p. 16). The acceleration initially rises to a peak structural response value between 75 and 200 g in the first few milliseconds. The average acceleration then decays to a value of approximately 24 g at 15 ms following water entry, and gradually reduces to a level of about 14 g after 55 ms.

The design requirements may be stated as the following question. What will be the requirements for the amplifier input impedance and amplifier gain for specified values of feedback capacity and low frequency time constant in order to provide a given output sensitivity?

For example let us say the output requirement is 0.2

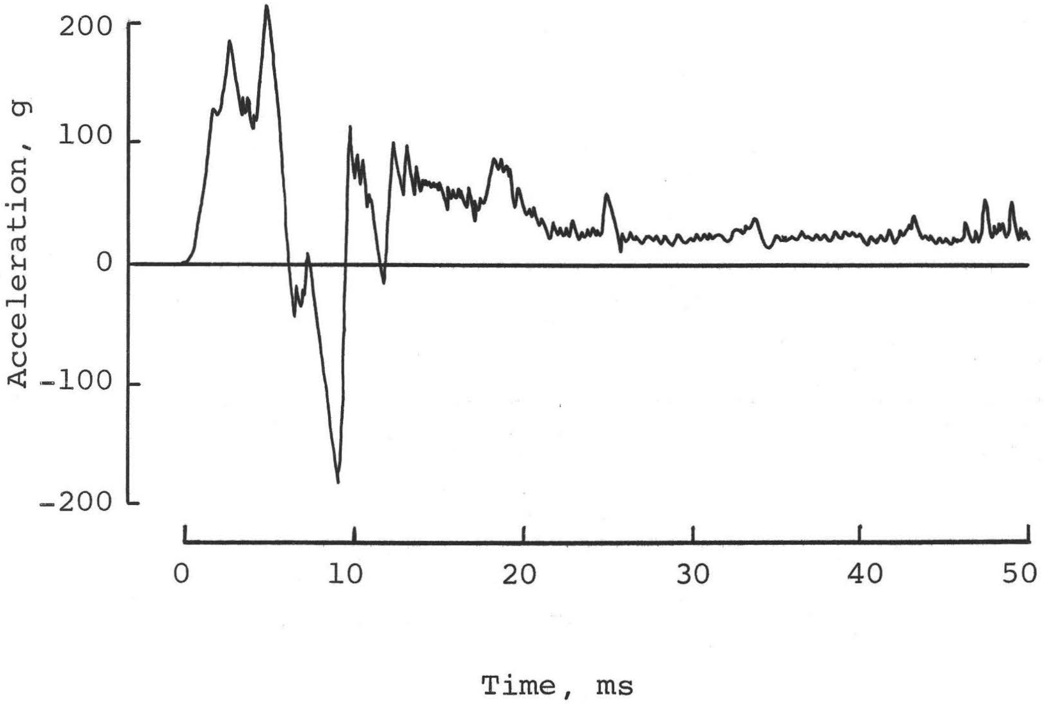

Figure 3.

Acceleration time history at the propeller shaft of the Mk 43 Mod 3 torpedo at water entry.

mv per g of acceleration. This would be an appropriate level at the acceleration levels designed for (1000 g peak) to drive the Tele-Dynamics, Inc., transistorized, high sensitivity, voltage-controlled subcarrier oscillator type TID-1251B. This VCO has an input sensitivity of 250 millivolts to produce full deviation as defined by IRIG standards. Then with the Massa M-191 accelerometer this requires the feedback capacitor value to be

$$C_f = \frac{S}{|Eo|} = \frac{4 \text{ pq/g}}{0.2 \text{ mv/g}}$$

$$= 20 \times 10^{-9} \text{ f} = 0.02 \, \mu\text{f} \quad .$$

Furthermore consider that we are interested in recording the acceleration time history for a duration of 50 ms with less than five percent error. An expression for the low frequency cutoff point may be derived by considering a square wave applied to the input and observing the amount of droop or tilt at the output (4, p. 33). Expressing the tolerable error as a percentage tile P in the waveform, where P is defined by

$$P = \frac{E - E'}{\frac{E - E'}{2}} \times 100 \quad .$$

E is the initial amplitude of the applied square wave at the output of the amplifier and E' is the amplitude of the output after a time T. When RC/T $\gg$ 1 which is true for the case under consideration, P reduces to the following expression

$$P = \frac{100\ T}{RC} \quad .$$

Since the low frequency - 3db point is given by f = $1/2\ \pi\ RC$, P reduces to

$$P = 100\ \pi\ \frac{f_1}{f}$$

in which $f = \frac{1}{2\ T}$ is the frequency of the applied square wave. This leads to a low-frequency cutoff in the present example of

$$f_1 = \frac{fP}{100\ \pi} = \frac{(10)\,(5)}{100\ \pi} = 0.159 \text{ cps}$$

or

$$\omega = 2\ \pi\ f = 1 \quad .$$

Choosing an input impedance of 10^7 ohms the requirement that

$$\omega\ ARC_f \gg 1$$

yields the following necessary amplifier gain

$$\omega\ ARC_f = 100$$

or

$$A = \frac{100}{\omega RC_f} = \frac{100}{(1)(10^7)(20 \times 10^{-9})} = 200 \quad .$$

In summary, the results for the design example are

$$S = 4 \text{ pq/g}$$

$$C = 0.02 \ \mu \text{f}$$

$$R = 10 \text{ megohms}$$

$$A = -200$$

Preamplifier Sensitivity = 0.2 mv/g

Preamplifier Passband = 0.16 to 10,000 cps.

DESIGN EXAMPLE

In the previous section design specifications were derived and in this section a particular circuit design using low noise silicon planar transistor will be presented.

Transistor amplifiers are usually considered to have relatively low input impedances. In the amplifier under consideration the design criteria for the first stage are (1) high input impedance, (2) low noise figure, and (3) good thermal stability. By employing feedback techniques the loading effect of the base and collector circuits may be greatly reduced. In particular, when the use of positive feedback is employed to reduce the shunting of the base biasing network the input impedance will be limited only by the series emitter resistance and the shunting effect of the collector resistance.

Consider the circuit in Figure 4. The use of the feedback resistor R_f improves the dc operating point stability but has the disadvantage of reducing the input impedance due to negative feedback (1, p. 61). However, the unity gain amplifier provides a positive feedback path which both reduces the loading effects of R_f and the collector resistance. With this feedback path the effective ac resistance of R_f becomes

$$(R_f)ac = R_f/(1-A_v)$$

where A_v is the voltage gain of the unity gain amplifier, emitter follower, and is given a low frequency by

$$A_v = 1/(1-r_e/R_e) \quad .$$

This is approximately equal in the present example to

$$A_v \cong 1/(1-50/22 \times 10^3)$$

$$\cong 0.99 \quad .$$

The collector potential is forced to follow the input signal and thus any collector to base resistance is

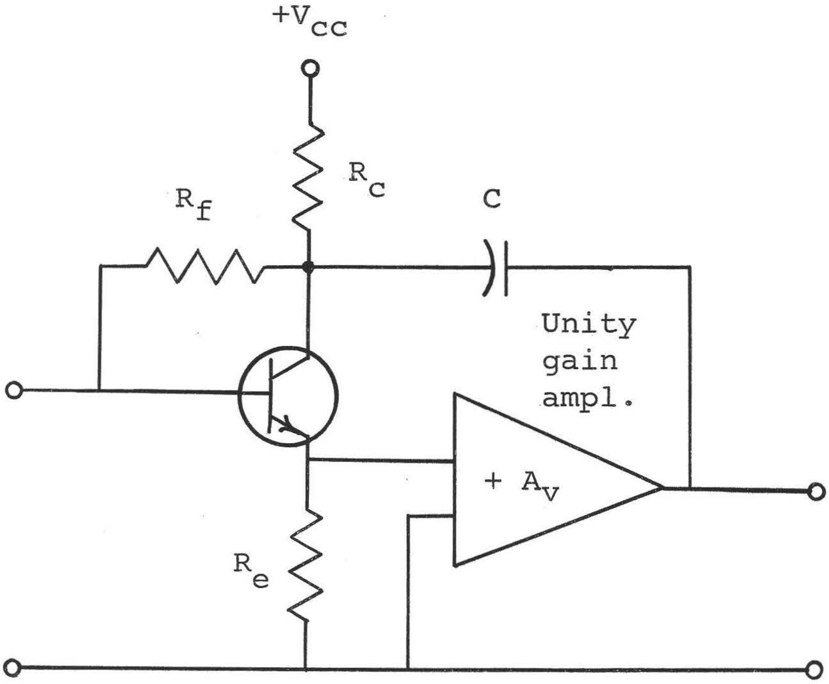

Figure 4.

Feedback technique for bootstrapping collector and base resistances.

multiplied by about 100 times.

Lowering the input impedance by negative feedback through R_f and then raising it by positive feedback through the unity gain amplifier is, in general, not the best method for attaining high input impedances. However the advantages of increased bias stability and economy of components coupled with the fact that an extremely high input impedance is not needed, justify it in the present case.

Referring to the input amplifier of Figure 4, the input impedance may be written as

$$R_{in} \cong \left[\beta_1 (R_{e_1} \parallel \beta_2 R_{e_2})\right] \parallel \left[R_f/(1-A_1A_2)\right]$$

where the subscripts refer to the first and second transistors and the symbol $\parallel$ means "in parallel with." Calculation of R_{in} for the values in Fig. 5 and the measured β values give an input impedance of 14 megohms. Measured values of input impedance versus frequency indicated an input impedance of 11.8 megohms that was constant to 3,000 cps. Above 3,000 cps the impedance decreased smoothly to 4 megohms at 10,000 cps.

Following the high impedance unity gain stage is a

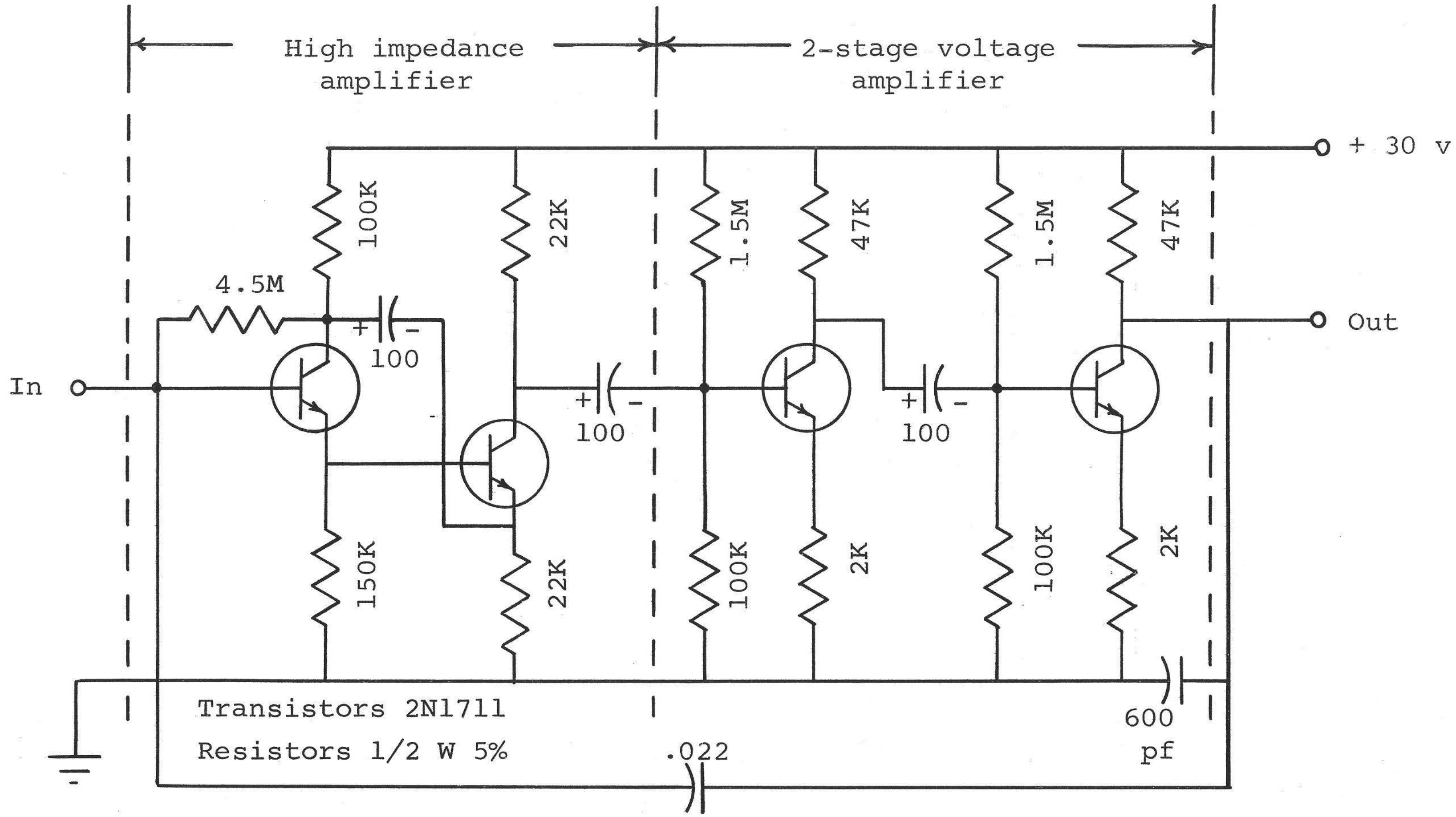

Figure 5.

Circuit schematic of charge sensitive amplifier in design example.

two stage RC coupled voltage amplifier. Bias stability has been sacrificed to some extent in the design in order to increase the gain. The open loop gain of the amplifier is 48 db (250) and varies less than one half db between 0.15 and 3,000 cps. At 0.1 cps the gain is down 6 db and at 10,000 cps the gain is also - 6 db. Above 10,000 cps the gain decreases uniformly at - 6 db per octave.

Collector to emitter voltages have been kept below 5 volts and emitter currents below 500 μa for low noise and low drift performance. The open loop noise level is 7.4 mv referred to the input and is mostly $\frac{1}{f}$ or flicker noise. This noise corresponds to approximately 0.2 equivalent g's for the Massa M-191 accelerometer. The 600 pf capacitor at the output suppresses high frequency oscillations and causes the high frequency roll off beginning at approximately 10,000 cps.

Sensitivity of the amplifier was 0.18 mv/g which agrees very well with the calculated value of 0.182 mv/g using the measured amplifier characteristics and the equations $|Eo| = S/C_f$ and $\omega ARC = 100$.

CONCLUSIONS

This thesis has described a technique for designing a preamplifier to be used with piezoelectric transducers. The fact that piezoelectric transducers are charge generators has been used to derive practical design expressions in terms of frequency response, input impedance, amplifier gain, feedback capacitor value and output voltage sensitivity in terms of input charge.

A design procedure is presented and the performance characteristics of an amplifier design example are found to be in concurrence with the prediction of the design expressions.

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APPENDIX

APPENDIX

The appendix will present the derivation of the solution for the current as a function of time for the charge sensitive amplifier circuit analysis. The differential equation to be solved is

$$\frac{dI_3}{dt} + \frac{1}{ARC_f} I_3 = \frac{\omega S}{ARC_f} \cos \omega t \quad .$$

The corresponding Laplace transform equation is

$$sI_3(s) + \frac{1}{ARC_f} I_3(s) - \frac{\omega S}{ARC_f} \frac{s}{s^2 + \omega^2}$$

and making the substitutions

$$I_3(s) = I(s), \alpha = \frac{1}{ARC_f}, \text{ and } \beta = \frac{\omega S}{ARC_f}$$

the equation becomes

$$sI(s) + \alpha I(s) = \beta \frac{s}{s^2 + \omega^2}$$

$$I(s)(s + \alpha) = \frac{\beta s}{s^2 + \omega^2}$$

$$I(s) = \frac{\beta s}{(s^2 + \omega^2)(s + \alpha)} \quad .$$

Then expanding by partial fractions

$$\frac{\beta \, s}{(s^2 + \omega^2)(s + \alpha)} = \frac{Ks + M}{s^2 + \omega^2} + \frac{N}{s + \alpha}$$

and multiplying out and solving for K, M, and N yields

$$\beta \, s = Ks^2 + Ks\,\alpha + Ms + M\,\alpha + Ns^2 + N\,\omega^2 \,.$$

Next equating like coefficients and solving

$$K + N = 0 \quad \text{or } K = -N$$

$$K\,\alpha + M = \beta$$

$$M\,\alpha + N\omega^2 = 0 \quad \text{or } M = -\frac{N\omega^2}{\alpha}$$

$$-N\,\alpha - \frac{N\omega^2}{\alpha} = \beta$$

$$N = -\frac{\beta}{\alpha + \dfrac{\omega^2}{\alpha}} = -\frac{\alpha\beta}{\alpha^2 + \omega^2}$$

$$M = \frac{\beta^2}{\alpha^2 + \omega^2} \text{ and } K = \frac{\alpha\beta}{\alpha^2 + \omega^2} \,.$$

Then

$$I(s) = \frac{\alpha\beta \quad s}{(\alpha^2 + \omega^2)(s^2 + \omega^2)} + \frac{\beta\,\omega^2}{(\alpha^2 + \omega^2)} \frac{1}{(s^2 + \omega^2)}$$

$$- \frac{\alpha\beta}{(\alpha^2 - \omega^2)} \frac{1}{(s + \alpha)}$$

taking the inverse yields

$$I(t) = \frac{\alpha\beta}{\alpha^2 + \omega^2} \cos \omega t + \frac{\beta\ \omega}{(\alpha^2 + \omega^2)} \sin \omega t$$

$$- \frac{\alpha\beta}{(\alpha^2 + \omega^2)} \epsilon^{-\alpha t} .$$

Returning to the substitutions made for I_3 , α and β $I_3(t)$ becomes

$$I_3(t) = \frac{\omega\ S}{(ARC_f)^2} \frac{1}{(\frac{1}{ARC_f})^2 + \omega^2} \cos \omega t$$

$$+ \frac{\omega^2\ S}{ARC_f \left[(\frac{1}{ARC})^2 - \omega^2\right]} \sin \omega t$$

$$- \frac{\omega\ S}{(ARC_f)^2} \frac{1}{(\frac{1}{ARC_f})^2 - \omega^2} \epsilon^{-\frac{t}{ARC_f}}$$

$$= \frac{\omega\ S}{1+(\omega\, ARC_f)^2} \cos \omega t + \frac{\omega^2\ S\ ARC_f}{1+(\omega\, ARC_f)^2} \sin \omega t - \frac{\omega\ S}{1+(\omega\, ARC_f)^2}$$

$$\epsilon^{-\frac{t}{ARC_f}}$$

$$I_3(t) = \omega S \left[\frac{\cos \omega t}{1+(\omega ARC_f)^2} + \frac{\omega ARC_f}{1+(\omega ARC_f)^2} \sin \omega t \right]$$

$$- \frac{\omega S}{1+(\omega ARC_f)^2} \in^{-\frac{t}{ARC_f}} .$$

Making the substitutions

$$\tan \Phi = \omega ARC_f$$

$$\cos \Phi = \frac{1}{\sqrt{1+(\omega ARC_f)^2}}$$

$$\sin \Phi = \frac{ARC_f}{\sqrt{1+(\omega ARC_f)^2}}$$

$$I_3(t) = \omega S \frac{\cos \omega t \cos \Phi + \sin \omega t \sin \Phi}{\sqrt{1+(\omega ARC_f)^2}}$$

$$- \frac{S}{1+(\omega ARC_f)^2} \in^{-\frac{t}{ARC_f}}$$

and by using the trignometric identity

$$\cos x \cos y + \sin x \sin y = \cos (x - y)$$

the solution for $I_3(t)$ becomes

$$I_3(t) = \frac{S}{\sqrt{1-(\omega ARC_f)^2}} \cos(\omega t - \Phi) - \frac{S}{\sqrt{1-(\omega ARC_f)^2}}$$

$$\epsilon^{-\frac{t}{ARC_f}}$$

which is the desired result. This equation reduces, when $t \gg 0$, to

$$I_3(t) = \frac{S}{\sqrt{1-(\omega ARC_f)^2}} \cos(\omega t - \Phi) \quad .$$

This will be verified by numerically evaluating the two terms of the equation using the typical values obtained from the design example and then comparing their values.

For

$$A = -200$$

$$R = 10^7 \text{ ohms}$$

$$C = 0.02\ \mu f$$

and a radian frequency of $\omega = 2\pi f = 10{,}000$ cps, which corresponds to the upper cutoff frequency and worst case, the first term may be evaluated as follows

Ist Term of $I_3(t)$

$$\frac{S}{\sqrt{1-(\omega ARC_f)^2}} \cos(\omega t - \Phi) =$$

$$\frac{S \cos(\omega t - \Phi)}{\sqrt{1-\left[(2\pi)(10^4)(-200)(10^7)(20\text{x}10^{-9})\right]^2}}$$

$$\cong 10^{-6} \omega S \cos(\omega t - \Phi)$$

and for the steady state condition may be replaced by

$$= 10^{-6} \omega S$$

2nd Term of $I_3(t)$

$$\frac{\omega S}{1-(\omega ARC_f)^2} \epsilon^{-\frac{t}{ARC_f}}$$

$$= \frac{\omega S \epsilon^{-\frac{t}{ARC_f}}}{1-\left[(2\pi)(10^4)(-200)(10^7)(20\text{x}10^{-9})\right]^2}$$

$$\cong 10^{-12} \omega S \epsilon^{-\frac{t}{ARC_f}}$$

letting $\epsilon^{-\frac{t}{ARC_f}}$ be a maximum the above reduces to

$$= 10^{-12} \omega S .$$

By comparing the two terms it is easily seen that the second is smaller by approximately 10^6 times and may be neglected without affecting the results.