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The adverse economic conditions resulting from a downward trend in egg prices and increasing costs of production have centered the interest of commercial egg producers on efforts to make more efficient use of the most important factor of production, i.e. the laying bird. Attention has been given to the economic feasibility of replacement policies involving the use of extended laying cycles and force molting programs in commercial egg production. Budget analyses of data available from force molting trials have indicated that such policies could provide alternatives to traditional (annual replacement) policies. This thesis was an attempt to incorporate all available information into a single study aimed at providing firstly, a more complete description of the producer's set of alternative actions with the inclusion of several force molting policies and secondly, information as to the producer's optimum action under conditions of certainty and uncertainty given this set of actions.

The analysis under certainty was concerned with the programming of replacement alternatives assuming price, cost and production levels were known. A simple economic model was developed. An enumerative procedure was used to define the producer's set of alternative actions over an assumed two year planning period. The optimum choice of an action and the ranking of actions was studied using a wide range of price, cost and production variable levels. The criterion of maximizing net income was used. The ranking procedure was used to delimit a small set of undominated actions. These undominated actions were subject to a sensitivity analysis.

The major results of the certainty analysis were:

1. With a mean egg price over the planning period greater or equal to 32.0 cents per dozen, and the cost of a replacement pullet $1.50 or less, an annual replacement policy was optimum. The sensitivity analysis showed that for some price-cost conditions an increase of 0.2 cents per pound in the cost of layers ration resulted in the optimum action changing from an annual replacement policy to one requiring a force molting program.

2. With mean egg prices of less than 25.5 cents per dozen, unless the producer could purchase replacements for less than $1.40 a force molted action was optimum.

3. With mean egg prices between 25.0 and 29.0 cents per dozen a replacement cost of \$1.20-\$1.30 favored annual replacement. As this cost was increased a force molted action became optimum at progressively lower total variable cost levels.

4. The in-lay period was shown to have a significant effect on the choice of action due to its influence on the form of the egg price cycle over the planning period.

The analysis of the replacement problem under uncertainty began with a discussion of the choice criterion of maximizing expected utility. The sources of producer's net income variability were defined. Estimates of the moments of the distribution of net income for each of the actions considered were obtained. These distributions were determined to be normal. A mean-variance (E-V) frontier was constructed for a series of 13 in-lay periods. The criterion of maximizing expected utility was illustrated using three theoretical utility functions which reflected risk preference, aversion and indifference.

The results of the uncertainty analysis showed that the choice between non-force molted and force molted actions was a function of the producer's attitude towards risk. Producer's utility was maximized by choosing: (1) a force molted action with three molts over the planning period for risk preference, (2) one of the non-force molted actions with replacement of the first flock after either 14, 13

or 12 months of production, depending on the in-lay period, for risk aversion, (3) a force molted action with two force molts over the planning period for risk indifference.

It was concluded that force molting policies can provide alternatives to traditional annual (non-force molted) replacement policies, but recommendations for their use in commercial egg production must be based on consideration of producers' attitudes towards risk.

A Study of the Economics of Force Molting in Commercial Egg Production Under Conditions of Certainty and Uncertainty

by

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A STUDY OF THE ECONOMICS OF FORCE MOLTING IN COMMERCIAL EGG PRODUCTION UNDER CONDITIONS OF CERTAINTY AND UNCERTAINTY

CHAPTER I

INTRODUCTION

Historical Background

The adverse trends in egg prices and production costs which have occurred since 1950 have resulted in marked changes in the commercial egg producing industry. A decline in the per capita consumption, combined with a relatively fixed supply of eggs, has resulted in a steady downward trend in egg prices (U.S.D.A. Agricultural Statistics 1950-1968). As a consequence of increasing quantities of broiler meat on the market, the salvage value for a culled laying hen decreased from a level of 23 cents per pound in 1950 to seven cents per pound in 1968 (U.S.D.A., 1950-1968). This decline has caused a significant increase in the cost of livestock depreciation. Increased costs for other inputs such as labor and equipment have also added to what might be termed a cost-price squeeze in the industry.

Faced with these adverse trends egg producers have had to find ways of increasing the efficiency of resource utilization in their operations in order to survive. Efforts to counteract these trends in the industry have resulted in considerable structural changes in the industry since 1950,and attempts to make more efficient use of the most costly production input, i.e. the laying bird.

The structural changes have required considerable long term capital investment and have involved a changeover from extensive to intensive (controlled environment) housing, increased physical size and scale of operation, and moves toward vertically integrated operations with complete control of input and output flows. These integrated operations may include ownership of the breeding farms, hatcheries, rearing and laying facilities, feed mill, and processing plants for eggs and culled birds.

The most important influence on efficient use of the laying birds has been the steadily increasing cost of livestock depreciation. As a result, producers have questioned the continued use of 'traditional' annual replacement policies and have considered the feasibility of extending the laying life of their birds beyond the end of the first year (cycle) of production.[1] The use of extended laying cycles of in

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[1] This has been made possible by the move from extensive to intensive housing. Under extensive conditions the pattern of egg production and the onset of a natural molt (rest period) was governed by daylength. The move to intensive housing gave the producer greater control over production and in the absence of declining daylength birds could be kept in-lay for extended periods without molting.

some instances up to two years in length, proved to be moderately successful in as far as they did result in increased total output per bird, and hence a reduction in the cost of livestock depreciation per unit of output (Clayton, 1963; Richardson, 1961).

The advantage of the lower depreciation cost was countered however by a number of serious economic disadvantages caused firstly, by poor egg quality over the second year resulting in a high percentage of B grade and reject eggs, and secondly, by high feed/ eggs conversion ratios. Both these factors become increasingly important after the end of the first year. There was also the problem of high levels of mortality sustained by birds in their second year of production. Poor egg quality caused the greatest loss in egg revenue to the producer, and as retail outlets came to demand eggs of high quality from their suppliers it resulted in an increasing problem to the producer.

It has been known for some time that a natural molt was followed initially by favourable increases in egg quality, egg size and hen-month egg production. Experimental laying trials have been conducted to investigate the influence of force molting[2] on these

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[2] Force molting is a procedure conducted under controlled environment conditions. It involves subjecting birds to unnatural stress, such as, starvation and removal of water for a short period (Bell, 1965a; Parlour, 1966a; Hansen, 1966; Marble, 1963), or the administering of an antipituitary drug which inhibits secretion of oestrogenic hormones (Bearse, 1967; Blount _et_ _al_., 1964;

factors of production (Bell, 1965a; B.O.C.M., 1965; Clayton and Sykes, 1966; Fuge, 1962; Hansen, 1960, 1966; Hyre, 1966; Len et al., 1964; Marble, 1963; Morrison and Aho, 1964; Noles, 1966; Parlour, 1966a; Snyder and Orr, 1960). The results of these trials show: (1) that a controlled molt can be induced using several different methods e.g. using drugs (Bearse, 1967; Blount et al., 1964, Sykes, 1964) or by the imposition of unnatural stress by the removal of food and water for a short period (Bell, 1965a; Hansen, 1966; Marble, 1963; Parlour, 1966), and (2) that an enforced rest period in the laying cycle did result in a significant improvement in egg quality, egg size and hen-month egg production. The results from these trials have also indicated that replacement policies using extended laying cycles which incorporate a force molting program might provide effective economic alternatives to traditional annual replacement policies.

The Problem

The main intent of the study from which this thesis is written is to examine the economic feasibility of using force molting programs in commercial egg laying operations. This involves the

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B.O.C.M., 1965; Sykes, 1964). This induced 'rest' period lasts from 4-8 weeks. See Himeno and Tanabe (1957) for a discussion of the mechanism of molting in the hen.

programming of production and those decisions pertaining to the periodic replacement of the laying bird when the use of a force molting program is included in the producer's set of alternative actions.

It will be assumed that problems in the industry related to optimum size, scale, and rate of growth, can be separated from those actually involving the programming of production. Thus, assuming that the price-cost relationships are such that a producer would be willing and able to stay in production, the principle decision to be examined is the replacement policy to be subscribed to by the producer.

Thus, given that the producer has one or more laying houses and other resource endowments, the problem becomes one of determining what laying flock replacement policy the producer could follow in order to maximize net revenue, minimize costs, or to optimize some other such decision criterion which would define his action as being the best possible. This problem involves questions concerning: (1) the optimum timing of replacements, (2) the optimum length of the laying cycle, and (3) the feasibility of using a force molting program.

The problem of determining optimal replacement policies for egg laying enterprises has received some attention in the literature but little emphasis has been placed on the importance of force molting programs and how their inclusion might alter the nature of the replacement problem and its solution.[3] This omission has been mainly due to the fact that the results from force molting trials have only recently become available for economic analysis. The work that has been published on the importance of force molting programs on the planning of flock replacements has been somewhat limited in nature in that it has failed to emphasize the magnitude and complexity of the decision making problem when extended laying cycles are included in the producer's set of alternative actions. The replacement problem can be treated as:

1. A decision making problem under certainty, where it is assumed that the consequence of each action is known, i.e. each action invariably leads to a specific single valued outcome.

2. A decision making problem under uncertainty, where the decision problem requires a choice when it is uncertain as to what the possible outcomes of an action or the respective likelihood of these outcomes might be.

Most of the published research on the replacement of poultry flocks can be classified under certainty. It was the intent of this study to complement the work in this area and to contribute to an understanding of the replacement problem under conditions of uncertainty.

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[3] This literature is reviewed in Chapter II.

Objectives and Outline of the Thesis

The Decision Making Problem Under Certainty

The first objective of this study was to develop an analysis of the replacement problem assuming that levels of costs, prices and production parameters were known with certainty.

Chapter II of this thesis discusses the methodological procedures which have been used to study the replacement problem under certainty.

Chapter III is concerned with the development of a relatively simple economic model designed and directed towards approximating the firms optimum flock replacement and output policies assuming the criterion of maximizing net returns. The optimum replacement policy is studied for a wide range of price, cost and production conditions.

Chapter IV lists and discusses the sources of experimental data used in the analysis.

Chapter V is concerned with the results of a preliminary analysis of the producer's alternative actions in the set A. This is followed by a sensitivity analysis of these actions using a large number of price, cost and production variable combinations. It was anticipated that these results would indicate a fairly small set of actions $\mathscr{a}$ which would be studied under conditions of uncertainty. Of prime importance in the stages of the certainty analysis was the determination of those conditions which resulted in the choice of a force molted action as the producer's optimum action.

The Decision Making Problem Under Uncertainty

The objectives of this part of the study was to use the set of actions $\mathcal{A}$ to seek solutions to the replacement problem under conditions of uncertainty.

Chapter VI discusses the choice criterion of maximizing producer's expected utility. The sources of producers' income variability are defined and the procedures for determining the stochastic nature of these variables are discussed. A simulation method for deriving the distribution of net income for each action in $\mathcal{A}$ over the planning period for 13 in-lay periods is given. A method for combining estimates of the moments of the distribution of net income and information on the form of the utility function to produce the expected utility from each of the producer's alternative actions is given in Chapter VII. The results of the uncertainty analysis are discussed in this chapter.

Chapter VIII summarizes the results and conclusions of this thesis.

It should be emphasized here that in all stages of the analysis interest will be focused on the nature of the optimum policies indicated rather than the precise numerical values that satisfy the maximizing criterion function.

By incorporating the results from the force molting trials that have been completed to date it was hoped that the fulfillment of specified objectives would firstly, provide valuable information on the future usefulness of force molting programs in commercial egg production, secondly, serve to implement the decision making processes of those producers actively involved in the field and finally, would provide guidelines for future research work.

CHAPTER II

METHODOLOGICAL REVIEW OF THE REPLACEMENT PROBLEM UNDER CONDITIONS OF CERTAINTY

The decision making problem is usually separated into two categories. First, it is partitioned according to whether the decision is made by an individual or group, and second, according to whether the decision occurs under conditions of certainty, risk, or uncertainty (Luce and Raiffa, 1957). We will be concerned here with individual decision making under certainty[4] where it is assumed that each action is known and leads invariably to a specific outcome. The theory of decision making under certainty (also described as the theory of riskless choice) embraces the concept of the economic man[5] (Edwards, 1954), who is assumed to be: (1) completely informed, i.e. he knows not only what courses of action are open to him but also what the outcome of any action will be, (2) infinitely sensitive, i.e. the alternatives available to him are continuous infinitely divisible functions. This assumption leads rather conveniently

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[4] In cases of certainty the possible events in the state space θ are known or at least are assumed to be known, and accordingly θ contains but one element and the distinction between action and strategy vanishes (Tedford, 1964b. p. 10).

[5] This concept of economic man has been central to the development of economic theory and perhaps explains why few models and results deduced from this theory could be classified as falling outside the certainty class.

to functions which are easily integrable and differentiable, and (3) rational, i.e. he is able to weakly order the states into which he can get, and he makes his choice so as to maximize something e.g. profit or utility.

A problem under certainty may therefore be regarded as simply one of choosing between alternatives on the basis of known situations given that the assumptions listed under (1), (2) and (3) above hold for the decision maker.

Past studies, which are reviewed and discussed in this chapter, have used models or structural relationships based upon the programming approach to production where emphasis has been placed on those factors the decision maker can control and change in order to maximize (or minimize) some index in the face of price, cost, or environmental change. These models have involved exact single valued specifications of these relationships.

The programming methods applied in quantitative economics (such as linear and dynamic programming) were developed primarily to solve problems set within closed systems where the objects of interest were inanimate. In the area of replacement theory they first found application in problems involving the replacement of equipment, machinery and capital goods (Alchain, 1958; Bellman, 1955; Bellman and Dreyfus, 1962; Burt, 1963; Dean, 1961; Hadley, 1963). These methods were later incorporated into the area of quantitative economics and have since found wide application to problems of optimization, resource allocation, and replacement of livestock and equipment in agricultural operations (Baum, 1959; Billingham and Wheeler, 1962; Burt, 1968; Eidman, Carter, and Dean, 1968; Halter and White, 1962; Jenkins and Halter, 1963; Low and Brookhouse, 1967; McKee, Dean and Gervais, 1958; Pouliquen, 1966; White, 1959). The notion of maximization employed by these methods is mathematically useful in that it makes it possible for the theory to analytically specify a unique point or subset of points among those available to the decision maker. In general these methods provide an optimal policy (policies) rather than precise numerical values that furnish the maximum or minimum of the criterion function (Bellman and Dreyfus, 1962, p. 13). The reason for this lies in the large errors that invariably accompany specification of the variables and parameters in the model. So long as it can be assumed that these errors are uniformly distributed over the set of alternatives then they are unlikely to affect the nature of the optimal policies, but they will certainly affect the maximum or minimum value of the criterion function.

Review of Literature on the Methodological Procedures Used to Determine Optimum Replacement Policies in Commercial Egg Production

The analytic replacement models that have been proposed in the literature have been concerned with the problem of reinvestment in the laying enterprise rather than with the problem of net investment. There is a clear distinction here between replacement and output expansion decisions. Replacement decisions are regarded as being of the reinvestment type where investment is necessary to replace a unit of capital stock (e.g. the laying bird) in order to maintain, rather than expand, productive capacity (Smith, 1961).

In general an economic replacement policy may be defined as one that optimally spaces the purchases of the firm's capital stock. Such a policy may be determined by finding that period of economic service life for the capital stock which maximizes the present value of the net earnings profile for the present and all future replaceable units of stock in the renewal chain which extends as far into the future as the firms planning horizon (Tedford, 1964a). However, the models in the literature have varied according to the relationship presumed to exist between the length of life for each flock of birds t_i in the renewal chain, and the length of the planning horizon N relevant to the enterprise. If a flock's economic length of life is t_i, i.e. each $t_i = N$, then only a single investment decision need be made. If however, $t_i \neq N$, then an optimum policy must be determined and incorporated into the analysis (Tedford, 1964b, p. 17). Thus, the economic life of a unit of replaceable capital stock cannot be determined in isolation from the specification of the time span of the planning horizon.

The literature on optimum replacement policies in commercial egg production falls into two main categories depending on the programming method used. The first concerns the application of dynamic programming to solving the replacement problem. The second deals with other methods such as linear programming, enumerative procedures and simple budgeting.

Dynamic Programming Methods

The use of dynamic programming involves the application of Bellmans' "principle of optimality" which states that:

> an optimum policy has the property that whatever the initial state and decision are the remaining decisions must constitute an optimum policy with regard to the state resulting from the first decision (1962, p. 15).

Thus, given that we have some limited quantity of resource x to be allocated over an N stage process such that

$$x_1 + x_2 + . . . + x_N = x ; x \geq 0$$

the "principle" says that having chosen some initial x_N we do not have to examine all the policies involving that choice of x_N but rather those policies that are optimal for N-1 stage processes given a quantity $x-x_N$ of the resource. In this way the process remains essentially additive rather than multiplicative.

We wish to maximize the function

$$R(x_1, x_2, \ldots, x_N) = g_1(x_1)+g_2(x_2)+ \ldots +g_N(x_N)$$

over the region $x_i \geq 0; \sum_{i=1}^{N} x_i = x.$

The allocations are made one at a time. First a quantity of resource is assigned to the Nth activity, then to the N-1th and so on in a "backwards" allocation process.

The maximum of $R_1(x_1 x_2, \ldots, x_N)$ is a function of x and N;

$$f_N(x) = \max_{\{x_i\}} R_1(x_1, x_2, \ldots, x_N).$$

The function $f_N(x)$ is then the optimal return of a quantity of resources x to N activities.

It is clear that

$$f_N(0) = 0, \quad N = 1, 2, \ldots$$

provided that $g_i(0) = 0$ for each i.

Also that:

$$f_1(x) = g_1(x) \text{ for } x \geq 0 \qquad 2.1$$

The initial allocation of x_N to the Nth activity results in a total return

$$g_N(x_N) = f_{N-1}(x-x_N)$$

and the optimal choice of x_N is one which maximizes this function.

The basic equation is:

$$f_N(x) = \max_{0 \leq x_N \leq x} [g_N(x_N) + f_{N-1}(x-x_N)], \quad N = 2, 3, \ldots,; x \geq 0, \qquad 2.2$$

with $f_1(x)$ determined from 2.1 above.

The application of the "principle of optimality" to the replacement of laying birds involves a multistage recurrent replacement decision process over some planning horizon N. For the sake of simplicity we will consider for the moment only those processes where the decision at each stage involves the choice between two alternative actions, i.e. either to replace the presently held bird with another of a different age, or to continue with the presently held bird. The basic differences between the models proposed in the literature concern: (1) the length of the planning horizon N, (2) the bird's economic length of life (production period), and (3) the age at which birds are allowed to enter the replacement chain.

Length of the Planning Period N

The length of planning period (planning horizon, or enterprise life) assumed in the studies to date have varied from one (Low and Brookhouse, 1967) to ten years (Halter and White, 1962; White, 1959; Low and Brookhouse, 1967). The planning horizon should be clearly distinguished from the terms production period and enterprise period which were defined by Halter and White as follows:

> A production period is the length of time over which the product is accumulated and numbered here over the twelve months of the year whereas each enterprise period is of the same length as the production period but numbered over the life of the enterprise (1962, p. 6).

The choice of length of planning horizon determines the length of time over which the decision process is to be considered. This choice is a function of an evaluation as to what the "life of the enterprise" should be, and also of the fact that the programming method itself dictates that the recursive process must be continued over a long enough period of time so that the choice of $g_N(x_N)$ (the initial allocation of x_N to the Nth activity) does not affect the optimum policy. A ten year planning horizon has been the choice for most analyses. Low and Brookhouse used a one year planning horizon and concluded:

> The feature that is unrealistic about the example worked out above [assuming a 12 month planning horizon] is the length of the planning period. Changing the end date changes the problem. It almost certainly changes the solution in detail if not in emphasis. The shorter the planning horizon the more pronounced the effect of a change in end date (My insert) (1967, p. 344).

This conclusion,that with such a short planning horizon the choice of allocation to the first stage affects the solution, could have been predicted from the theory of dynamic programming and

Bellman's "principle of optimality" outlined previously.

The Production Period

The choice of the production period determines the upper constraint on the age at which a bird is allowed to remain in the renewal chain. Thus, if a production period of 12 months is assumed, then an optimum replacement policy is determined subject to the constraint that when a bird reaches 12 months of age it must be culled and the producer has no alternative but to replace it. It represents an evaluation of the economic laying life of the bird. This constraint has been specified as 12 months (Halter and White, 1962), 16 months (Low and Brookhouse, 1967), and 30 months (Pouliquen, 1966) in the studies completed to date.

The Age of Entry of Birds into the Renewal Chain

The assumptions made concerning the age at which a bird enters the replacement chain has had an important effect on the optimum policies resulting from the studies completed to date. These differences can best be illustrated by comparing the assumptions made by Halter and White (1962) with those of Low and Brookhouse (1967).

In their analysis of the replacement problem, Halter and White assumed that at each stage of the decision process two courses of action could be taken; either, the presently held bird could be kept for another time period, or it could be replaced with a bird of a different age, i.e. if the presently held bird was of age j months she could feasibly be replaced with a bird of any age i, $i = 1, 2, \ldots 13$, $i \neq j$.

In their later study Low and Brookhouse, although assuming that only two courses of action could be taken at each decision point as in Halter and White's study, included the important constraint that the presently held bird of age j months could be replaced only with a point-of-lay pullet of age $i = 1$.

That these different assumptions had an important effect on the resulting optimum policy can be appreciated by comparing the conclusions from the two studies. The following statements summarize these conclusions. Halter and White stated:

> The replacement policy reads as follows: At the beginning of the enterprise, January 1, 1948, hens of age 3(7 months old) are placed in cages. On January 31, 1948, these hens will be of age 4(8 months old)...the policy says to replace with hens of age 4 in February.... At the end of February, hens of age 5(9 months old) are on hand and at this time the policy dictates replacement with age 2 birds.... Thus until October 1 the same group of hens is kept. At this time however, the hens of age 9 are replaced with hens of age 2 (1962, p. 16).

Whereas Low and Brookhouse stated:

> The planning period was taken to be either one or ten years. When the shorter period is used the selection problem becomes one of selecting a date for the annual replacement of the flock.... An examination of the physical properties of the

> optimum policies for the different situations shows that there are two common characteristics. Firstly...where the ten year planning horizon is assumed, flocks are always kept in-lay for more than a year.... (1967, p. 346).

The distinction between these two sets of conclusions centers on the assumptions made regarding the valuation of birds entering and leaving the replacement chain. Thus, Halter and White's assumption that a bird of age j could be sold and replaced with a bird of any other age i, $i \neq j$, depended on the additional assumption that the purchase price of the replacement hen (or equally her sale price) was a function of her salvage value plus the value of her egg laying potential. By assuming that the laying bird was an asset that could not be meaningfully valued after reaching point-of-lay by estimating the price it could bring on the open market[6], Low and Brookhouse restricted the birds allowed to enter the replacement chain to point-of-lay pullets only.

Critique

It is interesting to note that the conclusions reached as to the optimal laying cycles for commercial egg layers when dynamic programming methods are used, have differed considerably; there has

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[6] Cocks and Murray (1966) maintain that this is because the market is prepared to buy birds culled from the laying flock for their carcass value only.

been no concensus of opinion to date. This has been mainly due to the fact that the models proposed are operational only within the bounds of the restrictive assumptions made in the analysis as has been shown in the preceding discussion. The results attained have depended on the assumptions about the length of the planning horizon, the production period, and the age at which a bird enters and leaves the renewal chain.

In addition to the differences in these three assumptions, the models in the literature have used different input specifications concerning egg prices, cost levels, and production coefficients. Because these input specifications are variables and not fixed constants over time the results of these models hold only for the set of parameter values used in the analysis.[7]

In general too little attention has been focused on the sensitivity analysis of the replacement policies to imposed changes in costs, prices, and input coefficients, and too much attention has centered on the determination of the "optimum policy" using a single set of parameter values. Notable exceptions to this criticism are found to some extent in the work of Low and Brookhouse (1967), and to a much greater degree in Pouliquen's study (1966).

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[7] It may be advisable in practical planning to recalculate the optimum policy at six or twelve month intervals changing as necessary the estimates of the important parameters as recommended by Low and Brookhouse (1967, p. 350).

These criticisms should not deter from the fact that these models have considered some of the important elements of the replacement decision problem and in this respect may be of more value than the answers they have yielded.

The only dynamic programming study completed to date which has incorporated the results of some of the recent force molting trials is the work of Pouliquen (1966). This analysis of the replacement problem used data resulting from the Wye College trials (see page 65). In this work Pouliquen argued for the use of a detailed (sensitivity) analysis of the replacement problem over a relatively short planning horizon of 30 to 40 months because of the high degree of uncertainty associated with the distant future. With such a short planning horizon, Pouliquen noted that a simple enumeration of all the alternative policies over the planning horizon would have been simpler than using dynamic programming, and would have achieved essentially the same results. The reason for the decision to use a dynamic programming model seemed to center on its potential use in more complex models of the replacement procedure rather than on its usefulness in solving the particular problems posed by Pouliquen's model.[8] The position will be taken in Chapter III of this thesis that

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[8] It is not clear whether Pouliquen was aware of the limitations of the dynamic programming method before model construction, or after trying to define the model parameters. As will be discussed in Chapter III, the choice of a relatively short planning horizon for the

the application of the dynamic programming method to more extensive replacement problems may be limited by the very nature of their complexity. Thus, for some models of the replacement problem (in particular the model posed in Chapter III of this thesis), it may be necessary to either, restrict the planning period to a maximum of two to four years in order to use a dynamic programming method, or revert to a simple enumerative procedure for determining the possible alternative actions over the planning horizon, and solve the replacement problem using some other method besides dynamic programming.

Other Programming Methods

Linear Programming

Linear programming involves the analysis of problems of a linear function of variables which is to be maximized (or minimized) when these variables are subject to a number of constraints in the form of linear inequalities (Dorfmann, Samuelson, and Solow, 1958).

There have been two studies which have used the linear programming method to find a solution to the optimum replacement

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more complex replacement problems may not be a matter of some rationale regarding the uncertainty associated with the distant future as Pouliquen argued, but rather as a constraint placed upon the planning horizon by the very complexity of the problem itself.

problem (Billingham and Wheeler, 1962; McKee *et al.*, 1958). It was unfortunate that the published accounts of these studies failed to present information on the model used, in the form of the linear functions or the constraints which formed the specification of the problem. The only information given to the reader by Billingham and Wheeler was that: "the solution was calculated by linear programming methods for alternatives involving the 12 possible months of hatch and 18 different length of lay ranging from 7 to 24 months" (1958, p. 123).

It was not clear from this statement, nor from the text of these papers, *why* the authors felt that linear programming was the best method for determining the optimum combination of the 12 months of hatch and 18 lengths of lay. Because of this oversight it was not possible to comment on the suitability of this method for solving the replacement problem as stated by Billingham and Wheeler. McKee, and Billingham and Wheeler, have not hesitated to negate much of the significance of their work in their conclusions, and to point out the severe limitations this method places on the eventual solution. One of the most obvious limitations is that the results indicate that in order to maximize the objective function the producer must cage birds of many different ages in the same laying house. This can lead to obvious problems as stated by Billingham and Wheeler:

> These solutions depend upon the assumption that the new birds can be introduced at anytime without affecting the performance prospects of either old or new birds.... This assumption can hardly be expected to be extended on a monthly basis to offset mortality losses. Disease control measures, lighting schedules, the social hierarchy of the flock, procurement difficulties and various other problems would be involved (1958, p. 134).

McKee, and Billingham and Wheeler also pointed out that differences in seasonal egg price relationships, could affect the results materially, as could changes in other assumed parameter levels.

Enumerative Procedures

In the critique of the use of dynamic programming methods it was noted that for those problems where it was feasible to hypothesize a relatively short planning period of say two or three years duration, it would have been possible to determine the optimum replacement policy using a simple enumerative procedure to define the producers' alternative actions over the planning period. An example of this enumerative procedure was given in a recent paper by Noles (1967). In this paper the replacement problem was stated as one of finding the optimum combination of flocks which could be held in production for any one of seven laying season lengths (varying from 48 to 72 weeks). It was assumed that a flock could be housed on any one of 13 housing dates during the year. The method Noles used to find this optimum combination was based on the assumption that:

> One could maximize net returns over time if one could determine the sequence of flocks with the highest average net income [per 28 day period] where the sequence of flocks constituted a cycle of flock replacements (My insert) (1967, p. 56).

Noles's statement of the method used to find this optimum combination served to illustrate two points of crucial importance. The first concerns the need to state specifically the length of the planning period over which this optimum combination is to be determined prior to solution of the problem, and the second concerns the manner in which the magnitude of this problem expands as the length of the planning period increases.

The result of not recognizing the importance of the first point is suitably illustrated by Noles's approach. Thus, concerning the method used to solve the optimizing problem he stated:

> The first step in the determination of the optimum policy was the arbitrary selection of five consecutive flocks for the initial planning period. With five flocks the planning period extended from 5.0 to 7.3 years since flocks could be kept in-lay from 48 to 72 weeks (My emphasis) (1967, p. 12).

The point to notice here is that the selection of the "initial planning period" as "five consecutive flocks" implies a logical contradiction in terms. A planning period is either 5.0 or it is 7.3 years long. These two events are mutually exclusive.[9] Since all the

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[9] The economic life of a unit of replaceable capital stock, e.g. the laying bird, cannot be determined in isolation from the specification of the time span of the planning period (Tedford, 1964b, p. 17).

possible combinations of "five consecutive flocks" did cover a large range of different time spans it was invalid for Noles to assume that these combinations were even comparable. In order to illustrate the fallacies in Noles's methodology let us assume that his maximizing criterion holds, and find one case which rejects the procedure. Let us consider the simple case below; with just three lengths of laying cycle and a "two flock planning period".

Let P_j = egg price in cents per dozen in month j, j = 1, 2, . . .

PERIOD $_j$	1	2	3	4	5	6	. . .
EGG PRICE P_j	2	5	4	2	1	6	. . .

Let Q_i = number of dozen eggs laid by a hypothetical flock with a maximum life of three months, i = 1, 2, 3. The eggs laid by this flock over its life are shown below:

AGE$_i$	1	2	3
Q_i	3	2	4

Using Noles's method let us "arbitrarily" select a planning period of "two consecutive flocks." This implies that the planning period as stated, is a variable and has a range of from two to six months in length (two flocks kept for one month each to two flocks kept for three months each.)

Let the set C be all possible (3^2) combinations of ages over the planning period such that:

$$C = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

Calculations of total revenue and mean net revenue per period for each of the above combinations produced the results shown in Table I.

Table I. Total revenue and mean net revenue figures for all combinations over a 'two flock' planning period.

Combination	1,1	1,2	1,3	2,1	2,2	2,3	3,1	3,2	3,3
Total Length	2	3	4	3	4	5	4	5	6
Total Revenue	21	29	37	28	32	36	38	40	64
Mean Revenue/Period	10.50	9.67	9.25	9.33	8.00	7.20	9.50	8.00	10.67

Using the criterion of maximizing the average return per month for "two consecutive flocks" we would select the combination (3, 3) from Table I, giving an average return of 10.67 per month, over a total planning period of six months.

Any statement to the effect that this combination represents an optimum policy for the "initial planning period" would be incorrect. The combinations listed in the table are not comparable since they cover different planning periods. The choice of the combination (3, 3) logically implies that the planning period extends over six months.

The question that must be posed here is, does this combination represent the optimum over this planning period? The answer is no; this can readily be seen by considering some other combination which also extends over six months.

Let us consider the combination (1, 1, 1, 3). The total revenue from this policy is 65 with a mean per month of 10.83. Since 10.83 exceeds 10.67 the policy (1, 1, 1, 3) is better than (3, 3) over a six month planning period.

The fallacy in Noles's criterion lies in his assumption that the optimum policy determines the length of the planning period. This obviously is incorrect, since the object of any optimizing problem is to determine the best action or series of actions which can be taken subject to the constraint that only a limited quantity of resources are available to allocate over the length of the planning period. For as Bellman and Dreyfus stated: "The maximizing problem arises from the fact that we have only a limited quantity of resources available" (My emphasis) (1962, p. 5).

Thus, the definition of the constraints must come logically prior to the optimum solution, or in other words, the definition of the planning period (over which these limited resources are to be allocated) must precede and not follow the solution to the optimizing problem.

That the selection of "five consecutive flocks" for the initial "planning period" was not a matter of "arbitrary selection" but simply a matter of computational convenience can be seen from the following example.

Noles lists seven possible flock lengths to be combined over the "five flock planning period". Thus, there are 7^5 or approximately 16,800 different combinations to be analyzed. If, however, the planning period is extended to say " a seven flock planning period", there are now 7^7 or approximately 860,000 combinations to be considered. Thus, just this simple "arbitrary" reduction of the planning period from a seven to a "five flock planning period" has reduced the size of the problem by 98%. It should be obvious from the above that the magnitude of the enumerative problem expands exponentially as the planning period increases. This fact is of course one of the major reasons why dynamic programming is a more efficient programming method than an enumerative procedure for this type of replacement problem. The application of Bellman's "principle of optimality" shows that the optimizing procedure is essentially additive rather than multiplicative, so the computational effort required to maximize some criterion over say 10^{10} alternatives is now only twice that required to maximize over 10^5 alternatives, rather than 10^5 times if the enumerative method was used on the same problem.

Simple Budgeting

Most of the work on the economics of force molting policies, except for the recent study of Pouliquen (1966), can be included under this heading. Prior to Pouliquen's work the most thorough study of force molting had been completed by the author in a masters thesis project (Parlour, 1966a). In this thesis a simple comparative procedure was used in attempting to answer the question as to whether a producer should force molt birds at thirteen months of age and continue for a second cycle of production, or whether it would be more economical to replace with a new flock of replacement pullets at this time. These were the only alternative actions considered.

Major emphasis was placed on the influence of egg prices and the cost of the replacement pullet on the final solution. Questions concerning the best way to force molt birds, i.e. either by a conventional or drug induced method, and the influence of breed differences (medium-heavy versus light) on performance over the two cycles, were also considered. This work indicated the need for more trials to investigate the effects of force molting at different periods, e.g. at 6, 12 and 18 months, or at 9 and 18 months (an investigation which was later to be completed at Washington Agricultural Experiment Station). It was the author's feeling that any concrete recommendations on the usefulness of force molting should be kept in abeyance until such a time as more comprehensive force molting trials had been completed, and these could be subject to more sophisticated analytical procedures. These summarize the reasons for wishing to further this work in a Ph.D. thesis.

Other simple budgeting studies which have been reported in the literature have involved financial comparisons between experimental flocks using the actual costs, prices and production figures pertaining to the experiments (Bell, 1965a; Bragg, 1968b; Cooper, 1964; Cox, 1966; Fuge, 1962; Hansen, 1966; Hyre, 1966; Len et al., 1964; Marble, 1963; Morrison and Aho, 1964; Parlour, 1966; Simon, 1966). These simple budgeting studies have been successful in identifying a problem which centers on the decision as to whether a producer should use a force molting procedure in his replacement policy, or continue to follow the "traditional" policy of annual replacement. The concensus of opinion from these studies has been that force molting can play an important role in the producer's replacement plans, and they have served to identify most of the relevant factors which could influence this role.

Summary and Conclusions

This chapter has presented the methodological procedures used in attempting to solve the problem of determining optimum replacement policies in commercial egg production. It has served to emphasize the critical nature of the assumptions underlying decision making under certainty when applied to the problem of poultry flock replacement.

It was shown that when the dynamic programming method was used, the differences in the solutions to the problem posed were due primarily to the assumptions regarding the length of the planning period, length of the production period, and the age of entry of birds into the renewal chain. It was hypothesized that for more complex replacement problems it may be necessary to look for a simpler programming method rather than attempting to fit the dynamic programming method to the problem. More will be said on this subject in the next chapter. The discussion of the application of linear programming was inconclusive, but the results from these applications have been so impractical as to warrant asking the question as to whether these applications were not a case of fitting the problem to the method rather than vice versa.

The consideration of an enumerative procedure emphasized the importance of making economic comparisons of alternative policies over the same time period. It was also shown that for most replacement problems where both dynamic programming and enumerative methods can be used, the former method is preferable because of its computational convenience.

Finally, the simple budgeting methods which have been used to analyze most of the results from the force molting experiments have indicated the need to consider force molting programs as an integral part of the producers set of alternative actions. They have also identified many of the important factors likely to affect such a consideration.

This methodological review leads into the discussion of models and model building which begins Chapter III. This chapter will also include a statement of the model used in the certainty analysis, and a discussion of the methodological problems met in attaining the objectives previously defined in Chapter I.

CHAPTER III

THE CONSTRUCTION OF A DETERMINISTIC MODEL AND THE CHOICE OF A PROGRAMMING METHOD

The discussion in Chapter II was directed towards a précis of the different methods that have been employed in searching for solutions to a class of replacement problems. The problems of model formulation and specification which should logically precede questions concerning the method used to solve the problem were mentioned briefly in this discussion. In most of the works cited the impression has been that the model and the programming method have been regarded as synonymous. For some methods, e.g. dynamic and linear programming, it is easy to understand how the problem has been specified in such a way that it readily fits a standard programming method, thus minimizing the need for model specification.

It may be the case however that the replacement problem will not readily adapt to one of the standard programming methods. This will not become apparent until a model of the system under study has been specified. Because the study behind this thesis was concerned with the analysis of such a problem, it was felt that it would be advantageous to begin this chapter with some comments on models and model building. This discussion leads into a statement of a model describing the replacement procedure when force molting is included in the producers' set of alternative actions. This is followed by consideration of the methodological problems to be met in trying to use this model to attain the objectives specified in Chapter I. This chapter concludes with a section devoted to the specifications of the variables and parameters used in the analysis, and a review of the problem objectives.

Models and Model Building

A model, by definition, is a substitute for, or a scaled down version of, some real system (or equipment) which is the ultimate object of the analyst's concern. Its value as such arises from understanding of obscure behavior characteristics more effectively than could be achieved by observing the real system (Baumol, 1966; Forrester, 1964; Naylor, 1967). As it is a substitute problem it should be sufficiently simple and orderly to be amenable to study. Ideally, it should increase our understanding of the real world as well as provide a useful guide to judgement and intuitive decisions.

Baumol (1966) lists three types of models: analytic, predictive and descriptive. Economic models have often been judged by their ability to predict the specific state of some system at some future time and many have customarily failed to pass this test. Forrester (1964) maintains that the value of an economic model need not be measured solely by its ability to predict a specific outcome in the future. Indeed there are sound reasons for believing that this measure is unnecessarily restrictive. For as Schoeffler stated:

> Modern research in logic and in related fields has shown that the extreme view of prediction, and the derivative artificialities in economic analysis are all unnecessary for the useful and successful practice of economics (1955, p. 121).

The analytical, and to a lesser extent the descriptive functions of a model, were emphasized by Forrester who stressed the importance of studying the behavioral characteristics of the model in order to increase our understanding of real world phenomena. The mechanism of the model must represent the mechanism of actuality, and in this respect one of the most important uses of a model could be to explore system behavior outside the normal and historical range of operation. This behavior will be outside the region of any data that could have been produced in the past, data which must necessarily place limiting conditions on the information provided by the model. If our objective is to clarify our thinking about the system a model can be useful if it represents only what we believe to be the nature of the system under study. Thus we could rely less on statistics and formal data, and make better use of our vast store of descriptive information in formulating the model.

The assumptions which embody the model builders subjective evaluation of the important features of the real system under study, must be made before any data collection. The routine clerical collection of numerical data is unlikely to expose any new concepts or previously unknown but significant variables. Because many of the relationships assumed in model formulation may be in the form of hypotheses, it is essential that the data be collected at a stage when it can validly be used to test such hypotheses. This stage is after model formulation and not before. It will also be seen that it is only after the model and related hypotheses have been postulated that we are aware of what data are needed.

The absolute worth of a model can be no greater than the worth of its objectives. Usefulness can only be related to a clear statement of purpose. Thus, the goals set the framework for what a model must do, but too often validity and significance are decided outside the context of model purpose. The danger here is that the quantitative procedure may take on an aura of authenticity by and of its own right, and the underlying assumptions based on judgement or merely faith may have been completely forgotten.

The attitude that the validity of a model should be judged by its suitability for a particular purpose, and that it is sound and dependable if it accomplishes what is expected of it, will be adopted in this thesis. Validity as an abstract concept divorced from purpose will be regarded as having no useful meaning. The position that the accuracy of a model depends upon how accurate a solution is required and how decisions change as the model is modified, will also be adopted.

Statement of a Model of the Flock Replacement Procedure Including Force Molting Programs

A model of the replacement procedure when force molting is included in the producer's set of alternative actions, can be represented visually in a decision tree diagram. Such a diagram, with the inclusion of five force molting programs, is shown in Figure 1. A description of these force molting programs and the relevant states of production for birds of different ages is given below:

Let s_{jk} = a state or production for a bird of age j months[10] subjected to the kth force molting procedure. The set of all states is denoted:

$$S = \{s_{jk}\} \quad , \; j = 0, 1, 2, \ldots, 26, \; k = 0, 1, \ldots, 5$$

where $j = 0$ designates a four week clean-out period,

$j = 1, 2, \ldots, 26$ designates the age in months, and

$k = 0$ designates a bird that has not been force molted,

$k = 1$ designates a bird that has been force molted once at age nine months,

$k = 2$ designates a bird that has been force molted once at age

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[10] All future references to "a month" will refer to a 28 day period, except where otherwise specified.

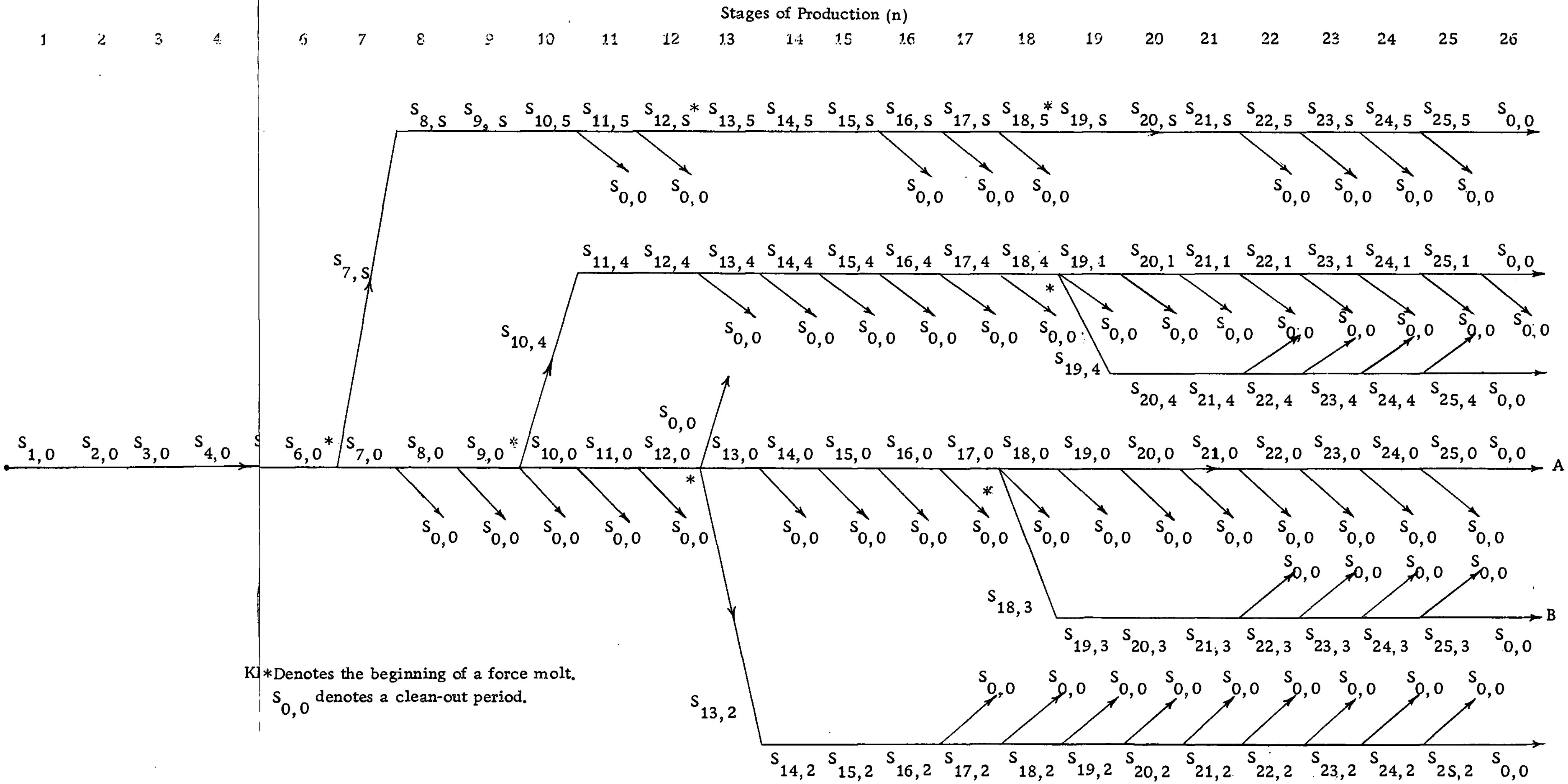

Figure 1. Decisitree diagram showing the sequence of states of production for five force molted and one non-force molted alternves over a 26 month planning period.

13 months,

k = 3 designates a bird that has been force molted once at age 17 months,

k = 4 designates a bird that has been force molted twice at age nine and 19 months,

k = 5 designates a bird that has been force molted three times at age six, 13, and 19 months.

The decision tree diagram (model) was based upon the following important assumptions and constraints:

1. The enterprise was of the simplest possible form, i.e. a single laying unit.

2. The producer followed an all-in, all-out replacement policy for this laying unit.

3. Only point-of-lay birds were allowed to enter the replacement chain.

4. Birds were not kept in-lay for longer than two years.

5. Birds would not be replaced until they were at least seven months old, nor would they be replaced until at least two months following the onset of a force molt.

6. The producer would allow a minimum disease break of four weeks every two years.

A verbal description of the model shown diagrammatically in Figure 1 should help in understanding the model and the problems encountered in the search for a programming method to solve the flock replacement problem.

Referring to Figure 1 the following points should be noted:

1. All possible stages[11] and states of production are shown for a 26 month planning period only. The s_{jk} states are listed along the six horizontal pathways.[12] Each of these six pathways indicate a replacement policy which involves keeping a bird (flock) in-lay for a 26 month period (25 months of lay plus one month clean-out period). Over this 26 month period the flock could theoretically be subjected to one of the five force molting procedures listed above, or it could be kept in-lay for the 26 months without a force molt. Examples of these two pathways are shown in Figure 1 along paths B and A respectively.

2. Besides these six horizontal pathways, the figure shows the stages at which the producer could theoretically make a choice (decision) as to what course of action he should pursue in subsequent stages of the planning period. These branches indicate decision points. In most cases the decision calls for a simple replace/continue choice (indicated by $\begin{smallmatrix}\longrightarrow \\ \searrow s_{0,0}\end{smallmatrix}$), while at some a three way choice,

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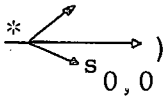
[11] The stages being defined as the four weekly periods into which the planning period is divided.

[12] A pathway being defined as: any sequence of states of production which begin with state $s_{1,0}$ and extends over the length of the planning period, subject to the assumptions and constraints listed above.

i.e. replace/continue/force molt is called for (indicated by ). If all the possible combinations of the stages of production had been listed over the 26 stages shown in the diagram, then it should be apparent that there would be many more alternative pathways which extend over the length of the planning period besides the six shown in the figure. An example of such an alternative pathway might be

$$s_{1,0}\; s_{2,0} \cdots s_{7,0}\; s_{0,0}\; s_{1,0}\; s_{2,0} \cdots s_{7,0}\; s_{0,0}\; s_{1,0}$$

$$s_{2,0} \cdots s_{9,0}\; s_{0,0}.$$

Such a pathway satisfies the constraints of the model, and indicates a replacement policy over the 26 month period which calls for the first flock to be held for seven months, the second for seven months, and the third for nine months.

All the alternative pathways are far too numerous to include in the decision tree diagram for even such a short planning period as 26 months. It should also be apparent that as the planning period is lengthened the number of alternative paths expands exponentially.

Thus, with these two points in mind, the replacement problem can be stated as one of determining the optimal combination or mapping of the s_{jk} states in S over the N stage process such that $n = 1, 2, \ldots, N$ where N is the length of the planning period, given some optimizing criterion such as maximizing net revenue, and subject to the assumptions and constraints listed previously.

The Choice of a Programming Method

The description of the model shown in Figure 1 established that the replacement problem with the inclusion of a set of force molting procedures is far more complex than any similar type problem that has been proposed to date. This can be appreciated by considering that all previous models with one exception have dealt only with the problem of mapping together states of production along a single pathway, e.g. path A in Figure 1. Halter and White (1962), and Low and Brookhouse (1967) have demonstrated that this problem can be solved without difficulty using dynamic programming methods. The replacement problem as posed by Pouliquen (1966), included the possibility of introducing a force molt at 13 months of age and he was therefore concerned with the mapping of states along two pathways, e.g. A and B in Figure 1.

The review of the literature on optimum policies in commercial egg production showed that dynamic programming methods have found wide application in this area of research. Thus it seemed in order, at least at the outset, to consider the possibility of using an approach based upon this programming method. However, further examination of the replacement model and the resulting methodological problems when the possibility of a large number of force molting programs were included in the producer's set of possible actions, led to a rejection of this method as computationally infeasible. The reasons for this decision follow from the discussion below.

The Relative Merits and Demerits of the Dynamic Programming Method

In order to fully appreciate the significance of the methodological problems involved when dynamic programming was proposed as a possible programming method, one must revert to consideration of the optimizing procedure this method uses in a multistage decision process. The main stages in this procedure were detailed in Chapter II in the section on dynamic programming. The exposition below is intended to complement this discussion and at the same time to relate to the particular replacement decision problem on hand.

Let us consider the nth and n-1th stages in some N stage decision process. Let s_{jk} be a state in the set S of all possible states such that:

$$S = \{s_{jk}\} \; ; \; j = 1, 2, \ldots, 26, \; k = 0, 1, 2, \ldots, 5.$$

Let $f_n(s_{jk})$ be the net revenue accruing to an optimum policy from stages n through N, beginning at the nth stage in state s_{jk}. Then it follows that:

$$f_{n-1}(s_{jk}) = \max_{a_i \in A(S)} \left[r_{n-1}(s_{jk}, a_i) + f_n(T_n(s_{jk}, a_i)) \right]$$

where:

1. $A(S)$ is the set of all possible actions a_i that can be followed in passing from the nth to the n-1th stage or production.

2. $r_{n-1}(s_{jk}, a_i)$ is the net revenue for the n-1th stage as a result of being in state s_{jk} after taking action a_i such that $a_i \in A(S)$.

3. $T_n(s_{jk}, a_i)$ is a decision rule which defines what state s_{jk} the process will be in at stage n, given that it was in some state s_{jk} in stage n-1 and action a_i was taken.

In any multistage decision problem, specification problems center on determination of $f_n(s_{jk})$ for each stage n and each state in S, in defining $f_n(T_n(s_{jk}, a_i))$ and in specifying $A(S)$ prior to beginning the search for the optimum policy over the N stage process. This problem can be illustrated in reference to the decision tree diagram shown in Figure 1. Let us consider the problem of specifying $T_n(s_{jk}, a_i)$. In order to completely specify these decision rules each alternative pathway would have to be traced backwards in time from the initial starting point in stage N. It should be noted that at each of the decision points on the tree a new series of pathways begins, so that these essentially increase at almost an exponential rate over time. The number of possible pathways, and hence $T_n(s_{jk}, a_i)$, over the relatively short planning period of 26 months, would be considerable. For a longer time period, of say 10 or 20 years, the specification problem would be enormous. It will be appreciated that the same problems would prevent specification of A(S) as defined above.

Thus, for the problem of finding the optimal combination of states in S over an N stage horizon for all states and stages shown in Figure 1, the estimation of these parameters would be extremely complex, even if it were to be assumed that the state variables (s_{jk}) could be defined with sufficient accuracy.

In the dynamic programming models proposed to date these specification problems have been minimized by firstly, considering the multistage decision process for only the single path of states as shown in A of Figure 1 (Low and Brookhouse, 1967; Halter and White, 1962; White, 1958), and secondly, reducing N by restricting the length of the planning horizon (Pouliquen, 1966).

The complexity of the decision rules $f_n(T_n(s_{jk}, a_i))$ and the problems associated with calculating $f_n(s_{jk})$ for all $s_{jk} \in S$ and all n, $n = 1, 2, \ldots, N$, and defining the set S with sufficient accuracy for the replacement problem with the inclusion of five possible force molting procedures, led to the conclusion that an analysis based upon dynamic programming would not be methodologically appropriate. That is not to say that this method could not have been used, but there is a point at which one must balance the additional cost and time needed to overcome the above mentioned difficulties against the alternative of developing a simpler analytical approach. In making this decision one must consider the expected end results, and also judge this decision in the light of firstly, the objectives of the analysis and secondly, the additional accuracy which is likely to be forthcoming in comparison to that which might be gained from a simpler approach to the problem.

It was the author's opinion that the objectives of this thesis specified at the end of Chapter I could not be adequately satisfied using an analysis based upon the dynamic programming method, and even if this had not been the case a search for a simpler method would have been necessitated by the computational difficulties discussed above.

The Choice of an Enumerative Method to Attain Specified Objectives Using a Deterministic Model

The result of the preceding discussion on the dynamic programming method led to the decision to reject this method and to use instead a simple enumerative procedure to compare the producer's alternative actions over some specified planning period. As was mentioned previously, for a planning period of any appreciable length, the problem of the enumeration of all possible alternatives becomes extremely laborious. In order to simplify this problem the following two assumptions were made.

1. The producer's planning period extended over a two year (26 months) period. This planning period was regarded as the longest period of time over which he would make a replacement decision. This appeared to be a realistic assumption both from the standpoint of the uncertainty involved in egg production and the conceivable length of egg laying cycles for birds of present genetic stock.

2. The state of the enterprise would be the same at the end as at the beginning of the planning period. This ensured that all alternative actions were compared under identical conditions as well as over the same length of time. In this analysis this state was set at $s_{0,0}$ or a clean-out period.

With the inclusion of these additional assumptions it was possible to enumerate all possible actions the producer could theoretically follow over the two year planning period. These actions were included in the set A.

Listing of All Actions in the Set A

Definition: Let an action a_i be a series of states of production where the state of production prevailing in any one of the 26 stages of the planning period is defined by s_{jk}. Let this action begin at the end of a clean-out period in state $s_{1,0}$ and end at the conclusion of a clean-out period in state $s_{0,0}$. This series being subject to the assumptions and constraints listed above.

Thus, the series of states listed below would be examples of an action in A over the 26 stage planning period:

$$s_{1,0}\ s_{2,0}\ \cdots\ s_{25,0}\ s_{0,0}. \qquad 3.1$$

This series defines the laying cycle over the planning period for a bird which is kept in-lay for 25 months without a force molt with a one month clean-out period ($s_{0,0}$) at the end of the planning period. Another example is:

$$s_{1,0}\ s_{2,0}\ \cdots\ s_{12,0}\ s_{13,2}\ s_{14,2}\ \cdots\ s_{25,2}\ s_{0,0} \qquad 3.2$$

This series of states defines the laying cycle over the planning period for a bird which is force molted at 13 months of age and kept in-lay for a further 12 months of production with a one month clean-out period at the end of the planning period.

With the aid of the decision tree diagram in Figure 1, and the definition of an action, it was possible to make a complete listing of the states over all stages of production for each action in A. States of production prevailing in each stage of the planning period for these actions are given in Tables II and III. For the sake of clarity it was decided to differentiate between those actions which included a force molting program during the planning period, and those which

Table II. The set A^o. Tabulated j, k indices for the s^a_{jk} states of production comprising each non-force molted action over the planning period.

a_i	Stages of Production (n)																									
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	1,0	2,0...																...18,0	1,0	2,0...					... 7,0	0,0
2	1,0	2,0...															... 17,0	1,0	2,0...						... 8,0	0,0
3	1,0	2,0...														... 16,0	1,0	2,0...							... 9,0	0,0
4	1,0	2,0...													... 15,0	1,0	2,0...								...10,0	0,0
5	1,0	2,0...												... 14,0	1,0	2,0...									...11,0	0,0
6	1,0	2,0...											...13,0	1,0	2,0...										...12,0	0,0
7	1,0	2,0...										... 12,0	1,0	2,0...											...13,0	0,0
8	1,0	2,0...									... 11,0	1,0	2,0...												...14,0	0,0
9	1,0	2,0...								... 10,0	1,0	2,0...													...15,0	0,0
10	1,0	2,0...							...9,0	1,0	2,0...														...16,0	0,0
11	1,0	2,0...						...8,0	1,0	2,0...															...17,0	0,0
12	1,0	2,0...					...7,0	1,0	2,0...																...18,0	0,0
13	1,0	2,0...									...11,0	1,0	2,0...					... 7,0	1,0	2,0...					... 7,0	0,0
14	1,0	2,0...								...10,0	1,0	2,0...					... 7,0	1,0	2,0...						... 8,0	0,0
15	1,0	2,0...							...9,0	1,0	2,0...					...7,0	1,0	2,0...							... 9,0	0,0
16	1,0	2,0...						...8,0	1,0	2,0...					... 7,0	1,0	2,0...								...10,0	0,0
17	1,0	2,0...					...7,0	1,0	2,0...					... 7,0	1,0	2,0...									...11,0	0,0
18	1,0	2,0...								...10,0	1,0	2,0...						...8,0	1,0	2,0...					... 7,0	0,0
19	1,0	2,0...							...9,0	1,0	2,0...						...8,0	1,0	2,0...						... 8,0	0,0
20	1,0	2,0...							...9,0	1,0	2,0...							...9,0	1,0	2,0...					... 7,0	0,0
21	1,0	2,0...						...8,0	1,0	2,0...						...8,0	1,0	2,0...							... 9,0	0,0
22	1,0	2,0...						...8,0	1,0	2,0...							...9,0	1,0	2,0...						... 8,0	0,0
23	1,0	2,0...						...8,0	1,0	2,0...								... 10,0	1,0	2,0...					... 7,0	0,0
24	1,0	2,0...					...7,0	1,0	2,0...						...8,0	1,0	2,0...								...10,0	0,0
25	1,0	2,0...					...7,0	1,0	2,0...							...9,0	1,0	2,0...							... 9,0	0,0
26	1,0	2,0...					...7,0	1,0	2,0...								... 10,0	1,0	2,0...						... 8,0	0,0
27	1,0	2,0...					...7,0	1,0	2,0...									...11,0	1,0	2,0...					... 7,0	0,0

[a] Where j represents the age of the bird, and k represents the force molting treatment.

Table III. The set A^f. Tabulated j, k indices for the s_{jk}[a] states of production comprising each of the force molted actions over the planning period.

a_i	Stages of Production (n)																									
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
28	1,0	2,0...				6,0	7,5	8,5...																	...25,5	0,0
29	1,0	2,0...				...6,0	7,5	8,5...									...17,5	1,0	2,0...						... 8,0	0,0
30	1,0	2,0...				...6,0	7,5	8,5...								...16,5	1,0	2,0...							... 9,0	0,0
31	1,0	2,0...				...6,0	7,5	8,5...							...15,5	1,0	2,0...								...10,0	0,0
32	1,0	2,0...				...6,0	7,5	8,5...			...11,5	1,0	2,0...												...14,0	0,0
33	1,0	2,0...				...6,0	7,5	8,5...			...11,5	1,0	2,0...					...7,0	1,0	2,0...					... 7,0	0,0
34	1,0	2,0...				...6,0	7,5	8,5.....		10,5	1,0	2,0...													...15,0	0,0
35	1,0	2,0...				...6,0	7,5	8,5.....		10,5	1,0	2,0...					...7,0	1,0	2,0...						... 8,0	0,0
36	1,0	2,0...				...6,0	7,5	8,5.....		10,5	1,0	2,0...						...8,0	1,0	2,0...					... 7,0	0,0
37	1,0	2,0...							...9,0	10,4	11,4...														...25,4	0,0
38	1,0	2,0...							...9,0	10,4	11,4...						...17,4	1,0	2,0...						... 8,0	0,0
39	1,0	2,0...							...9,0	10,4	11,4...					...16,4	1,0	2,0...							... 9,0	0,0
40	1,0	2,0...							...9,0	10,4	11,4...				...15,4	1,0	2,0...								...10,0	0,0
41	1,0	2,0...							...9,0	10,4	11,4...			...14,4	1,0	2,0...									...11,0	0,0
42	1,0	2,0...							...9,0	10,4	11,4......		13,4	1,0	2,0...										...12,0	0,0
43	1,0	2,0...										...12,0	13,2	14,2...											...25,0	0,0
44	1,0	2,0...										...12,0	13,2	14,2...				...18,2	1,0	2,0...					... 7,0	0,0
45	1,0	2,0...										...12,0	13,2	14,2...			...17,2	1,0	2,0...						... 8,0	0,0
46	1,0	2,0...										...12,0	13,2	14,2......		16,2	1,0	2,0...							... 9,0	0,0
47	1,0	2,0...							...9,0	10,1	11,1...														...25,1	0,0
48	1,0	2,0...							...9,0	10,1	11,1...							...18,1	1,0	2,0...					... 7,0	0,0
49	1,0	2,0...							...9,0	10,1	11,1...						...17,1	1,0	2,0...						... 8,0	0,0
50	1,0	2,0...							...9,0	10,1	11,1...					...16,1	1,0	2,0...							... 9,0	0,0
51	1,0	2,0...							...9,0	10,1	11,1...				...15,1	1,0	2,0...								...10,0	0,0
52	1,0	2,0...							...9,0	10,1	11,1...			...14,1	1,0	2,0...									...11,0	0,0
53	1,0	2,0...							...9,0	10,1	11,1......		13,1	1,0	2,0...										...12,0	0,0
54	1,0	2,0...															...17,0	18,3	19,3...						...25,3	0,0

[a] Where j represents the age of the bird, and k represents the force molting treatment.

did not. The actions in the former category were included in the set A^f, and those in the latter in set A^o. Thus, the series 3.1 above would be included in the set A^o, and the series 3.2 in the set A^f.

From the actions listed in Table II it will be seen that:

$$A^o = \{a_i\} , \quad i = 1, 2, \ldots, 27. \qquad 3.3$$

and from Table III that

$$A^f = \{a_i\} , \quad i = 28, 29, \ldots, 54 \qquad 3.4$$

Thus, $A = A^o \cup A^f = \{a_i\} , \quad i = 1, 2, \ldots, 54.$

Thus, over the two year planning period the producer could theoretically follow any one of the 54 actions listed in Tables II and III.

In order to make a comparison among the alternative actions on an economic basis, costs and returns must be calculated for each alternative. The following sections define the variables and parameters used in determining the economic benefits from each alternative.

Cost and Revenue Equations for the ith Action in A Over the Planning Period N

Let π_i = the net revenue (income) accruing to the producer as a result of taking action a_i over N.

TR_i = total revenue from egg sales.

LD_i = livestock depreciation (the difference between the cost of the point-of-lay pullet and the salvage value of the culled bird).

TCF_i = total feed cost.

FC_i = fixed costs.

Production Variables

Let l_i = the number of laying flock replacements made over the length of the planning period.

Let q_{gni} = the number of dozen eggs laid per hen-month in the gth grade in the nth stage of production;

where:

g = 1 designates small AA grade eggs

g = 2 designates medium AA grade eggs

g = 3 designates large B grade eggs

g = 4 designates large A grade eggs

g = 5 designates large AA grade eggs

g = 6 designates extra large AA grade eggs

g = 7 designates jumbo AA grade eggs

g = 8 designates reject and commercial grade eggs.

Let Q_{ni} = total hen-month egg production in the nth stage of production.[13]

$$= \sum_{g=1}^{8} q_{gni}$$

and $q_{gni} = Q_{ni} \cdot GD_{gni}$

where GD_{gni} = percentage of total hen-month egg production laid in the gth grade in the nth stage of production.[14]

Mortality

Let M_i^1 = percentage flock mortality over the first year of the planning period.

Let M_i^2 = percentage flock mortality over the second year of the planning period.

Let p_{ni} = the accumulative flock mortality up to and including the nth stage of production

$$= (V)m_{ni}$$

where:

V = initial flock size, i.e. number of birds in state $s_{1,0}$.

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[13] Note that: $Q_{ni} = f(Q_{jk}) = f(s_{jk})$, where Q_{jk} = hen-month egg production in the s_{jk}th state of production.

[14] Note that: $GD_{gni} = f(GD_{gjk}) = f(s_{jk})$, where GD_{gjk} = percentage of total hen-month egg production laid in the gth grade in the s_{jk}th state of production.

m_{ni} = accumulative percentage flock mortality up to and including the nth stage of production.[15]

Production Function

The production function is:

$$Q_{ni} = f(Y_{ni}, X_{1ni}, X_{2ni} \mid X_3, X_4, \ldots, X_n) \qquad 3.5$$

where:

Y_{ni} = hen-month feed consumption in the nth stage of production.[16]

X_{1ni} = age of the bird in months (=j).

X_{2ni} = a dummy variable to account for the effects of the force molting treatment on egg production.

$X_3, X_4, \ldots, X_n$ = fixed parameters which include breed characteristics, climatic conditions, and management factors such as housing, lighting and degree of environmental control.

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[15] Note that: $m_{ni} = f(m_{jk}) = f(s_{jk})$, where m_{jk} = the percentage flock mortality over the s_{jk}th state of production.

[16] Note that: $Y_{ni} = f(Y_{jk})$, the hen-month feed consumption in the s_{jk}th state of production.

Production Parameters

Let W = the weight of a culled bird.

V = initial flock size, i.e. number of birds in state $s_{1,0}$.

Output Prices

Let P_{gt} = producer prices for eggs in the gth grade in the t'th month of the year, $t = 1, 2, \ldots, 13$.

P^c = salvage price per pound liveweight for culled birds.

Input Prices

Let C^r = cost of a point-of-lay pullet.

C^f = cost of layer ration per pound.

Other Costs[17]

Let C^c = per bird cost of cleaning out the laying unit at the end of the planning period.

C^t = per bird cost of transferring a point-of-lay pullet

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[17] Other costs such as labor, equipment depreciation and interest charges were regarded as fixed in the short-run and were not considered.

into the laying unit.

Cost and Revenue Equations

With these defintions the following cost and revenue equations can be derived:

$$\pi_i = TR_i - (LD_i + TCF_i + FC_i).$$

where:

$$TR_i = \sum_{n=1}^{N} (V - p_{ni})(q_{gni} \cdot P_{gt}) \qquad 3.6$$

$$LD_i = l_i \cdot V(C^r - (1 - (M_i^1 + M_i^2)/100)\, P^c \cdot W) \qquad 3.7$$

$$TCF_i = \sum_{n=1}^{N} (V - p_{ni})(Y_{ni} \cdot P^f) \qquad 3.8$$

$$FC_i = V(l_i \cdot C^t + C^c) \qquad 3.9$$

Thus, it follows that the net revenue equation in terms of the variables and parameters defined above is:

$$\pi_i = \sum_{n=1}^{N} (V - p_{ni})(q_{gni} \cdot P_{gt} - Y_{ni} \cdot P^f) -$$

$$l_i \cdot V[(C^r - (1 - (M_i^1 + M_i^2)/100)P^c \cdot W) + C^t + C^c/l_i] \qquad 3.10$$

$i = 1, 2, \ldots, 54$

$n = 1, 2, \ldots, 26$

$g = 1, 2, \ldots, 7$

$t = 1, 2, \ldots, 13.$

Review of the Objectives to be Attained Using the Deterministic Model

At this stage having successfully defined the replacement problem with the inclusion of force molted alternatives within manageable limits it may be well to restate the objectives of the analysis in terms of the relationships defined in the preceding sections. These objectives were:

1. To compare the net revenue (π_i) from each action in A over N. This comparison was carried out under a wide range of egg prices (P_{gt}), feed cost (C^f), replacement cost (C^r), cull price (P^c) and production (q_{gni}) conditions. The procedure was to rank the actions in A on the basis of π_i in order to determine those actions which consistently produced the highest levels of π_i, i.e. those which "dominated"[18] other actions in A under all price, cost and production conditions. The undominated actions producing the highest levels of π_i formed the set A' where $A' \subset A$.

2. To subject those actions in A' to a sensitivity analysis using a large number of price, cost and production conditions with the object of studying the relative ranking of actions in A' (especially the relationship between actions in A^o, and $A^{f'}$) and the sensitivity

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[18] A definition of the term "dominance" as used in this thesis will be given at the beginning of Chapter V.

of the optimum actions to small changes in prices and costs. Of particular interest was the influence of the in-lay period on these factors.

The set of actions A' will be used in the analysis under conditions of uncertainty in Chapter VI. The results of the certainty analysis presented in Chapter V, will provide useful information which will be incorporated into the later stochastic analysis. Thus, these two stages in the analysis, the deterministic and stochastic stages, should be regarded as part of a continuum in the analysis of the replacement problem rather than as separate distinct entities.

CHAPTER IV

INPUT DATA FOR THE MODEL AND THE METHODOLOGICAL PROCEDURE FOR ANALYSIS OF THE REPLACEMENT/FORCE MOLT MODEL

The last section of Chapter III defined the components of the evaluation procedure as: production variables, input and output prices, and other costs which were needed to arrive at estimates of net revenue (π_i) for each action in A over the two year planning period. To carry out this evaluation procedure required specific information. The compilation of this information from several data sources is the subject of this chapter. The first section outlines the experimental force molting trials which provided most of the production data. The following section lists and discusses the data on egg production, egg size, egg quality, feed consumption, mortality, and final body weight collected from these trials. Subsequent sections consider the collection of data on the input and output price levels, with particular attention being given to the levels of these prices to be used in the analysis. The chapter ends with a review of the procedures used in the analysis.

Production Data

The hen-month egg production data (Q_{jk}), the monthly distribution of eggs by grade (GD_{gjk}), and monthly feed consumption figures

(Y_{jk}) for each of the s_{jk} states of production used in the analysis are listed in Table I of Appendix A.

It was not possible to obtain all the necessary data from one single source. However, every effort was made to ensure that data were compiled from essentially homogeneous sources. From the outline of the trials given below it will be seen that in most cases the same force molting method (conventional) was used. The differences in production due to the influence of breed differences were minimized by considering only data from those trials which used a light hybrid (Leghorn type) bird. It has been shown that there are significant differences between the response of light and heavy hybrids to the same force molting treatment (Len _et_ _al_., 1964; Marble, 1963; Parlour, 1966).

Only data from those experiments conducted under intensive (controlled) environment housing conditions over the laying cycle were considered. Differences in production due to the influence of age at point-of-lay were minimized by considering only those flocks housed in the laying quarters at 21-23 weeks of age. Differences in production due to management and husbandry practices could not be quantified but it was felt that these differences would be marginal because of the controlled conditions under which the experiments were conducted.

Sources of Information

The production data were collected from the four primary sources listed below:

1. Washington State Agricultural Experiment Station trials
2. Cornell University Agricultural Experiment Station trials
3. Wye College (University of London) trials
4. Skylane Farms Oregon

In addition to the original production data obtained from these trials, useful information was supplied by secondary sources reported in the literature (Bell, 1964; Bragg, 1968; Cooper, 1964; Hansen, 1966; Len *et al.*, 1964; Marble, 1963; Mehner and Torges, 1967; Morrison and Aho, 1964).

Washington State Agricultural Experiment Station Trials, 1964-1966

These trials, conducted under the direction of Reed Hansen, were designed primarily to determine the effect of a number of force molting procedures on egg production and egg quality. A conventional force molting method was used requiring a reduction in lighting followed by complete removal of feed and water for 48 hours. This was followed by a three week period of reduced feeding in order to keep the birds on an enforced rest.

The following force molting procedures were used over a two year laying cycle:

1. Control--no force molting over the two year period.
2. Six month molts--birds were force molted three times, at six, 13 and 19 months of age.
3. Nine month molts--birds were force molted twice at nine and 19 months of age.
4. Thirteen month molt--birds were force molted once at 13 months of age.

Three replicate flocks of 50 birds housed at point-of-lay were used for each of the above procedures. Complete weekly production records on these trials were available for analysis.[19] Some additional information was also obtained from the published reports of the trials (Hansen, 1966).

Cornell University Agricultural Experiment Station Trials, 1959-1961

These trials, directed by D. Marble, covered two years of egg production for 11 different breed combinations at two consecutive New York random sample tests. A total of 44 flocks of 50 birds housed per flock were used in the trials. All the experimental flocks were

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[19] R. Hansen and G. Bearse of the Agricultural Experiment Station, were most cooperative in allowing access to the original records of these trials, and in providing much valuable information in numerous personal communications.

force molted at 17 months of age using the same procedure as for the Washington trials discussed above. Complete original production records were furnished by Cornell upon request.[20] These trials were reported in the literature by Marble (1963). Additional information, not available from inspection of the records, was obtained from this report.

Wye College (University of London) Trials, 1962-1964

These trials, under the direction of A. H. Sykes, also covered a two year laying period. Three treatments and two different breeds of birds were used. These treatments were:

1. Control--no force molting over the two year period.
2. Thirteen month molt using a conventional force molting procedure.
3. Thirteen month molt using a drug molting procedure (ICI 33828).[21]

Two breeds, a light and a medium-light hybrid, were used giving a

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[20] Professor Bruckner of Cornell University was kind enough to provide the complete records from these trials.

[21] A drug developed by Imperial Chemical Industries of Great Britain, as a result of work in the area of the control of female ovulation using the "pill". In chickens it acts upon the pituitary gland to prevent ovulation. Information on the effectiveness of this drug and its possible further use in the poultry industry can be found in the literature (Bearse, 1967; Blount _et al._, 1964; B.O.C.M, 1965; Shipston, 1965; Sykes, 1964).

total of six breed by treatment groups. There was no replication within these six groups.

These trials have been reported in the literature (Clayton and Sykes, 1966; Parlour, 1966b) and in the author's unpublished master's thesis (Parlour, 1966a).

Other Sources

Information pertaining to flocks force molted at nine months of age and kept for an extended laying cycle was supplied by B. Franken of Skylane Farms, Oregon, a large commercial egg laying enterprise.[22]

Hen-Month Egg Production

One of the major determinants of the monthly distribution of net revenue over the planning period for an action in the set A is the pattern of monthly egg production. This pattern is a function of the age of the bird and the force molting treatment to which the bird is subjected. The egg production patterns for the five force molting procedures, $k = 1, 2, \ldots, 5$, are shown in Figures 2 through 6 respectively. The dependent variable is percent hen-month egg production.

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[22] The data from this source became available only after completion of the preliminary analysis. Those actions which required this data before estimates of π_i could be obtained were however included later in the sensitivity analysis.

These percentages were calculated on the basis of 100 percent production being equal to one egg per day over the 28 day month, or 2.33 dozen per hen-month. Thus 50 percent production would represent 1.16 dozen eggs per hen-month, i.e. 1.16/2.33 x 100 = 50%. The egg production figures (Q_{jk}) relevant to the s_{jk} states of production are shown in Table I of Appendix A.

The decline in the percentage hen-month production with age is well illustrated by Figure 4. All the figures show the manner in which force molting affected egg production for the duration of the

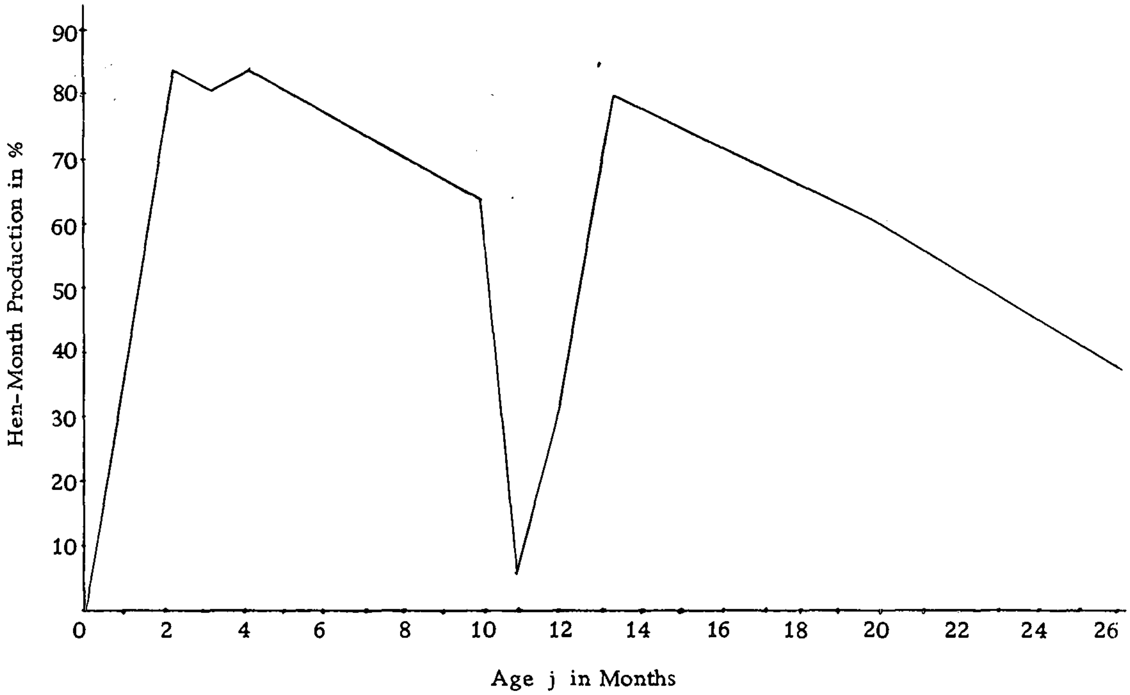

Figure 2. Hen-month egg production percentages for a flock force molted at 9 months of age (k=1).

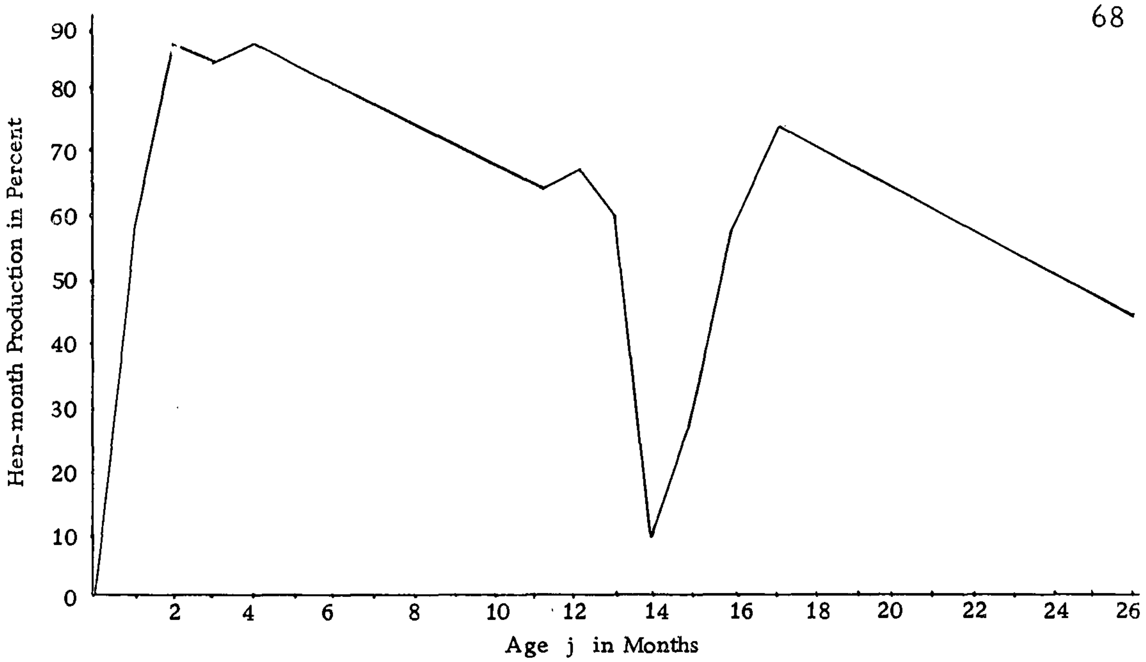

Figure 3. Hen-month egg production percentages for a flock force molted at 13 months of age. k = 2

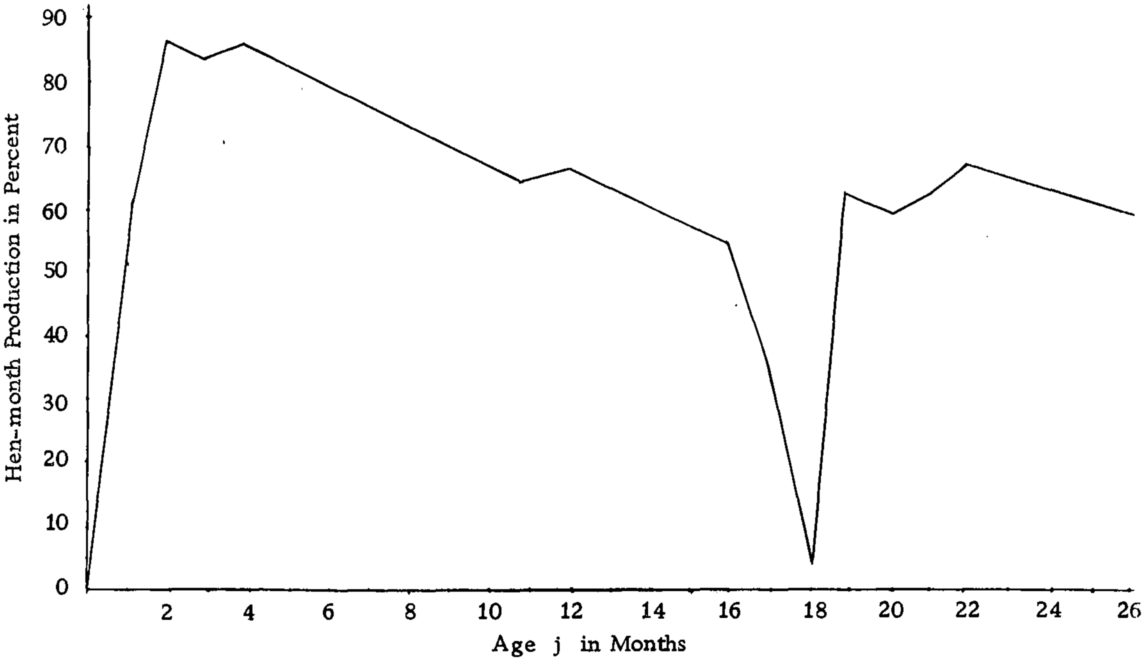

Figure 4. Hen-month egg production percentages for a flock force molted at 17 months of age. k = 3

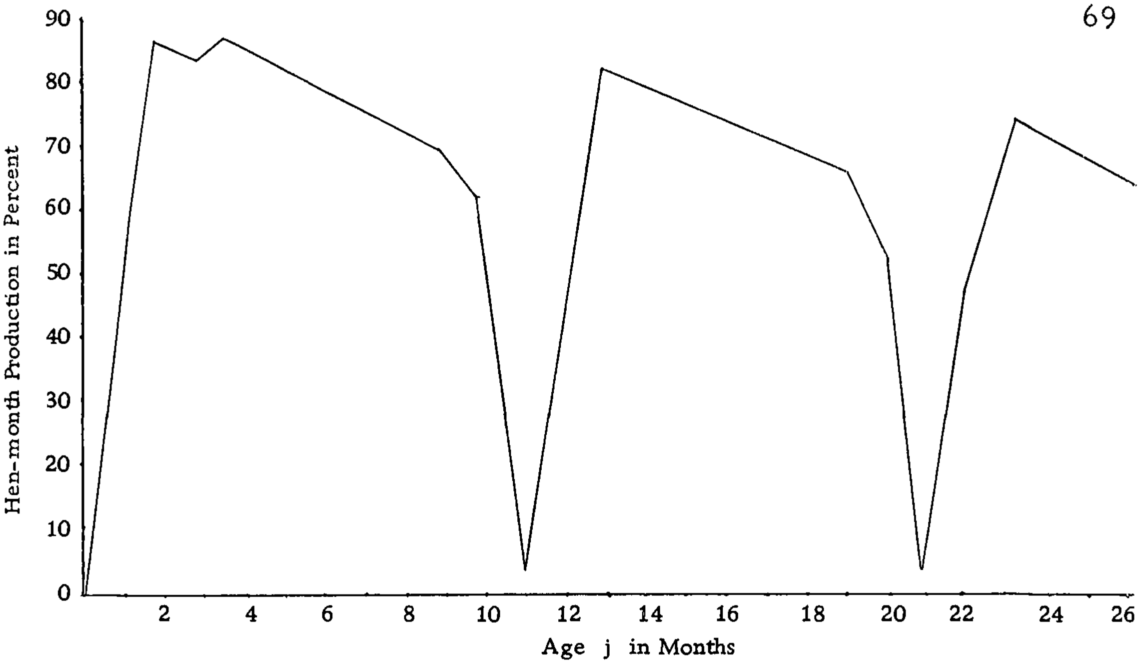

Figure 5. Hen-month egg production percentages for a flock force molted at 9 and 19 months of age. k = 4

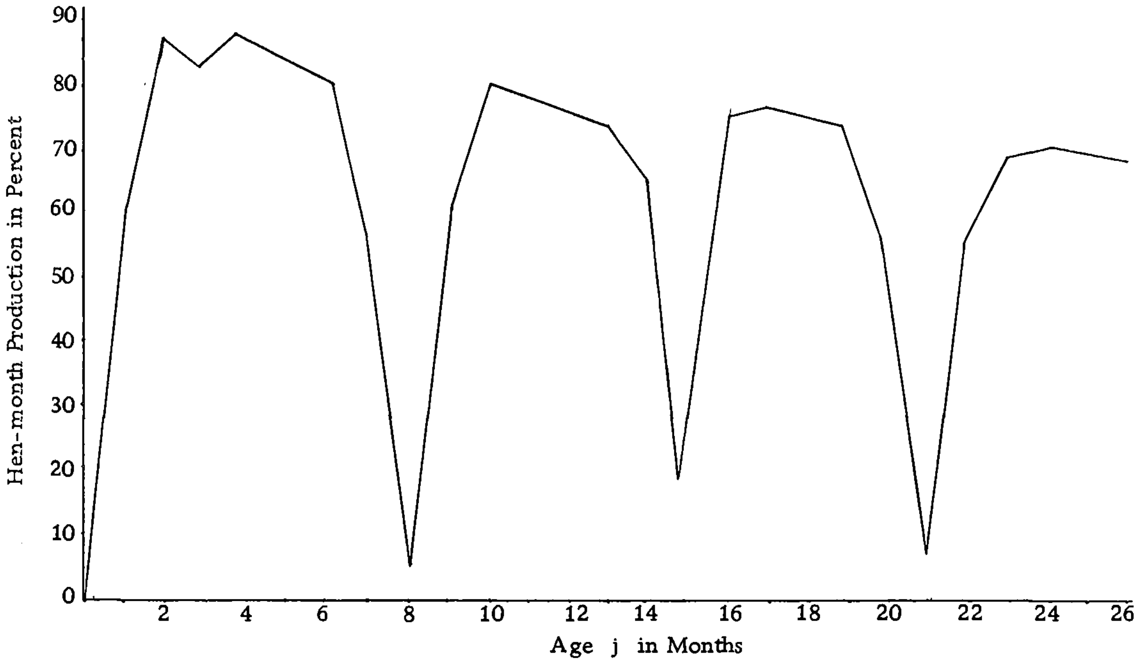

Figure 6. Hen-month egg production percentages for a flock force molted at 6, 13 and 19 months of age. k = 5

molt and how post-molt production figures increase above pre-molt levels. Figures 2 through 6 also show that force molting resulted in a halt in the decline in hen-month egg production (at least temporarily) and caused a subsequent return to a level that approximated the level observed four to five months prior to the onset of the molt.

The effects of frequency of force molting over the two year period, and the age at which the first force molt was applied, on the total number of months in the planning period when hen-month egg production was above 70 percent, are summarized in Table IV. These results show that as the frequency of force molting was increased from one to three molts over the planning period, the number of months when production was above 70 percent increased significantly. Where a single force molt was applied, the earlier the molt the greater the number of months in the planning period when hen-month egg production exceeded 70 percent.

Table IV. Total number of months above 70 percent hen-month egg production for five force molting procedures.

k	Force Molting Procedure	Number of Months Above 70 Percent Production
1	1 Force molt at 9 months of age	13
2	1 Force molt at 13 months of age	9
3	1 Force molt at 17 months of age	9
4	2 Force molts at 9 and 19 months of age	18
5	3 Force molts at 6, 13 and 19 months of age	17

Egg Size Distribution

A second determinant of the distribution of monthly net revenue is the distribution of egg size by age and force molting procedure. Figures 7 and 8 show how this egg size distribution changed with age for a flock force molted at 17 months of age, and for one force molted at 6, 13 and 19 months of age, respectively.[23] These figures show that during the first six months of production small and

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[23] The data used to derive these figures are shown in Table I of Appendix A.

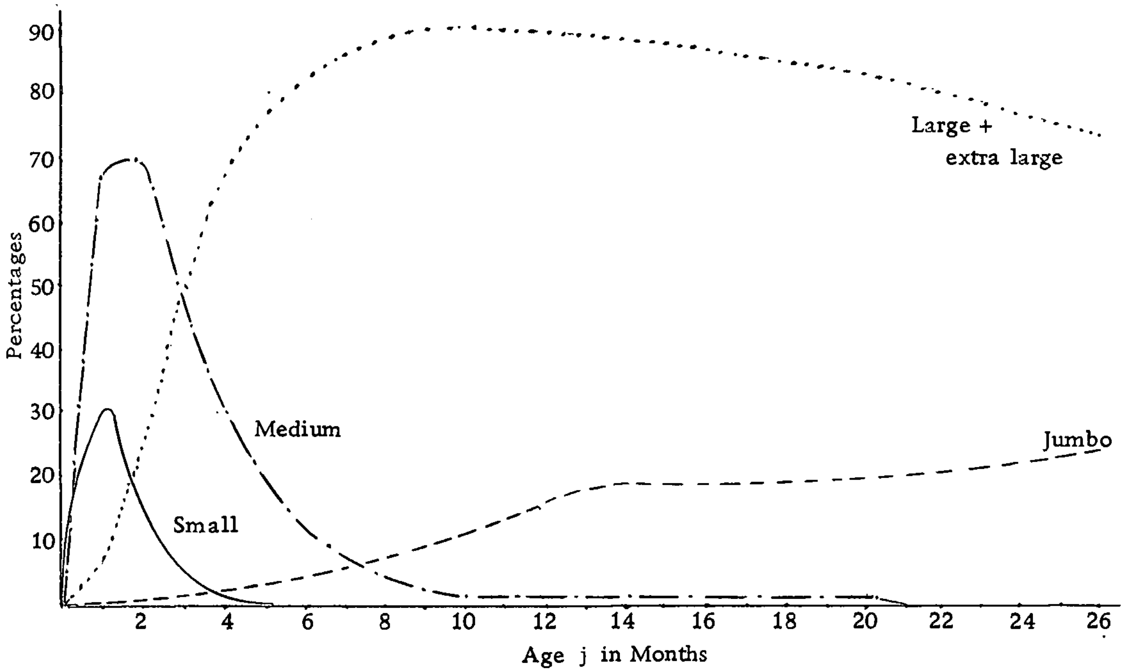

Figure 7. Egg size distribution for a flock force molted once at 17 months of age. k=3

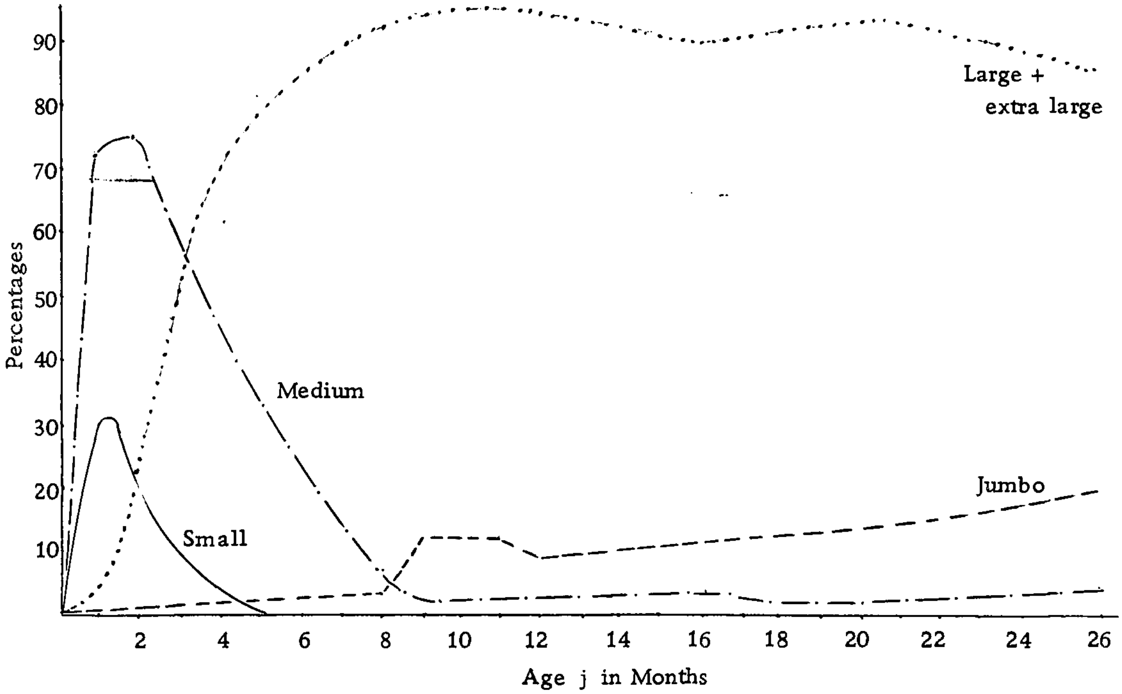

Figure 8. Egg size distribution for a flock force molted three times at 6, 13 and 19 months of age. k=5

medium size eggs predominated, after which time 90-98 percent of all eggs laid fell into the large-jumbo size categories. It was interesting to note from Figure 8 that neither, the first molt at six months of age, nor subsequent molts at nine and 19 months had any significant effects on the monthly egg size distribution.

Egg Quality

Whether a producer sells his egg output on a private retail market, to a co-operative, or to a wholesale buyer, he will obtain a quality price premium for AA grade eggs over A or B grade eggs. As will be seen when egg prices are considered, these price premiums can be quite large, with the result that low quality eggs can lead to significant losses in potential egg revenue to the producer. Thus, another determinant of the monthly distribution of net revenue is the difference in egg quality between flocks of different ages and force molting procedure.

As an example of how age and force molting procedure affect egg quality, Figure 9 shows the incidence of B grade eggs for four flocks--three subjected to force molting procedures and the fourth in continuous production over a two year period.

It was apparent from Figure 9 that one of the major problems incurred when keeping birds in-lay for extended periods without a

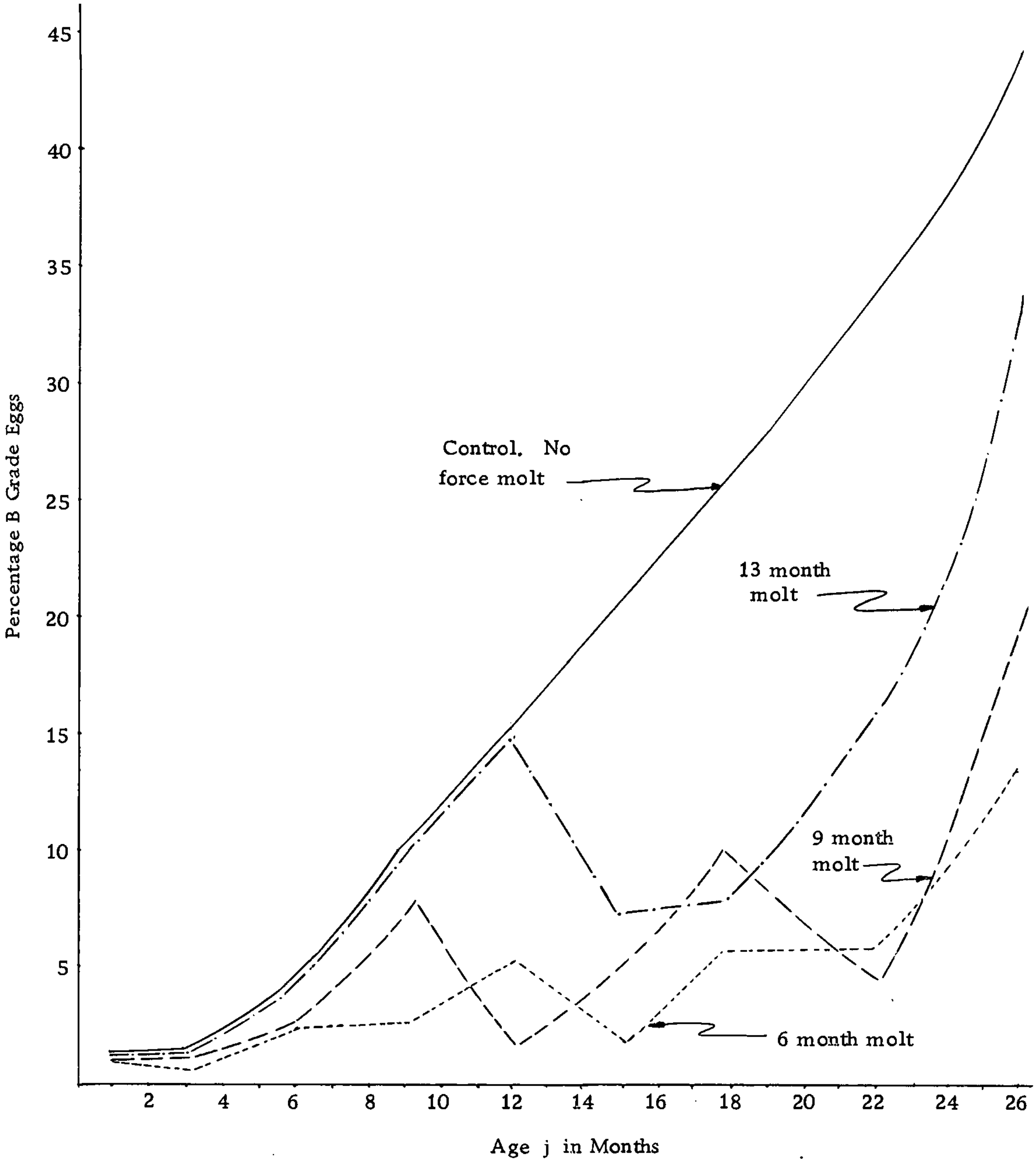

Figure 9. The incidence of B grade (including commercial and reject) eggs for three force molted and one non-force molted flocks over a two-year period.[a] ([a] Source: Washington Agricultural Experiment Station data Hansen, 1966).

force molt concerns the increasing preponderance of B grade and reject eggs with increasing age. This trend is well illustrated in Figure 9 by the upward trend in low quality eggs for those flocks kept for the two year period without a force molt. Also, the relationship between the frequency of force molting and the incidence of low quality eggs is apparent from the figure. The monthly distributions of egg quality as reflected in the grade distributions (GD_{gjk}) are given in Table I of Appendix A.

Because of the direct effect of egg quality on the producer's egg revenue, it was essential that estimates of the quality distribution of market grade eggs for each stage of production in the model be obtained. The data collected failed to provide these estimates directly, but enough information was provided in the literature to enable indirect estimates. The changes in egg quality, according to age and force molting procedures, were given by Marble (1963) and Hansen (1966). These trends are shown in Tables V and VI. Table VI shows measurements of egg quality over the two year period for the flocks in the Washington trials, in Haugh Units. Assuming that 80 Haugh Units are required in 24 hour old eggs to ensure AA quality, it will be seen that eggs from the hens rested every six or nine months maintained AA quality almost to the end of the two years. Also, it can be seen from this table that the flocks force molted at 13 months ($k = 2$), and control flocks ($k = 0$)

Table V. Albumen quality at periodic intervals during pullet and hen years.[a]

Age j or Month of Production	Albumen Quality			
	Grade AA	Grade A	Grade B	Grade C
4	80.1	19.3	0.6	0.0
7	61.2	35.1	3.7	0.0
10	42.1	47.9	8.0	2.0
13	44.6	40.8	12.4	2.2
14	38.6	40.8	14.4	6.2
15	38.7	49.1	9.1	3.1
18[b]	58.2	39.7	2.1	0.0
19	67.0	31.8	1.2	0.0
20	47.4	46.3	6.3	0.0
21	41.6	49.6	8.5	0.0

[a] Source: Marble (1963, Table 15, p. 25)

[b] Flock Force Molted.

Table VI. Albumen quality as Haugh units.[a]

Age j or Month of Production	Treatment			
	Control (k = 0)	6 Months (k = 5)	9 Months (k = 4)	13 Months (k = 2)
1	95.1	94.8	95.2	95.3
3	89.8	89.7	90.1	89.2
6	85.5	85.9[b]	85.3	84.7
9	80.8	85.9	81.6	80.1
12	77.7	82.9	85.1	76.7
15	73.0	85.1	81.9	81.7
18	69.5	80.7	79.0	80.3
22	67.6	81.0	81.0	76.8
25	68.0	77.4	78.2	74.3

[a] Source: Hansen (1966, Table 4, p. 5)

[b] ═══ Denotes Force Molt (Rest) Between Observations

presented an egg quality problem over the second year. Table II of Appendix A provides additional information used in estimating the monthly breakdown of egg production into market grades.

The trends in egg and shell quality following force molting have been reported in the literature. Mehner and Torges (1967) reported that egg breaking strength as well as albumen quality improved in direct ratio (correlation) to the level shown prior to the molt; the lower this level the greater the post-molt improvement. Improvements in egg and shell quality following force molting were also recorded by Berg and Bearse (1967), Len,*et al.*, (1964), Snyder and Orr (1960), Hansen (1960, 1966), Parlour (1966), and Hyre, *et al.*, (1966). In most cases it was noted that force molting had the effect of increasing egg and shell quality to the levels shown five to seven months prior to the molt, but that this improvement was a function of the length of the enforced rest.

Direct estimates of the monthly egg quality breakdown were obtained from Skylane farms for birds force molted at nine months of age.

The estimated figures for the egg quality breakdown are shown in Table I of Appendix A, and are listed as the percentage of total eggs laid per hen-month falling in each of the seven market grades of eggs (GD_{gjk}). It should be noted that the large egg category was the only one which was subdivided into quality divisions, e.g. AA, A and B. This is not to imply that all eggs laid in the other egg size categories were AA eggs, only that having no information on the subdivisions of these other size categories this was a necessary assumption.

Feed Consumption

The figures for feed consumption (Y_{jk}) in pounds per hen-month are shown in Table I of Appendix A. This table also shows the weight of feed consumed per dozen eggs laid as a measure of feed conversion efficiency. The decrease in feed conversion efficiency with age, and the subsequent improvement in efficiency following a force molt was indicated by the results from these trials and can be seen from the tabulated results. These results were supported by other experimental results reported by Len, et al. (1964), Bell (1965a), and Parlour (1966a).

Mortality

The results from these trials have shown that flock mortality is subject to extreme variation depending on severity of the force molting method, the general health of the flock prior to a force molt, breed of bird used, overall husbandry and management conditions, and whether or not a culling program is used prior to force molting. Thus, it was not possible to obtain a consistent set of figures to be used in the model. The figures shown in Table VII are based upon the actual results obtained from the experimental sources. Because of the extreme variability of these figures, the actual mortality figures used in the analysis were modified to take account of other mortality estimates from secondary sources of information. Thus, flock mortality estimates obtained from commercial flocks in the field indicate that mortality, at least over the first year of production, is distributed lognormally with a mean of approximately 14 percent.

Table VII. Flock mortality over first and second years of production for the experimental force molted flocks.

Source	Force Molting at	Strain of Bird	Percent Mortality First Year	Second Year
Wye College Trials	13 months	Light Hybrid	17.0	26.0
		Medium-Heavy Hybrid	9.0	14.0
Washington State, Agricultural Experiment Station Trials	No molt	Light Hybrid	2.0	8.0
	13 months	Light Hybrid	6.0	15.0
	9 and 19 months	Light Hybrid	6.0	7.0
	6, 13 and 19 months	Light Hybrid	5.0	7.0
Cornell University, Agricultural Experiment Station Trials		Light Hybrid	6.6	12.8
California Random Sample Tests[a]	No molt	Mixed Light Hybrid and Light-Heavy	10.0	6.5
	17 months	Crosses	10.0	6.0

[a] Reported by Len, et al., (1964).

Thus, the experimental results obtained from the Washington trials, especially the mortality figures for the control flocks over the first year of production, would be in the extreme left-hand tail of this mortality distribution. On the basis of information from non-experimental sources (reported in Chapter VI), the mortality percentages from these trials were adjusted so as to approximate commercial conditions, while at the same time maintaining the relative levels of mortality shown in Table VII for the different force molting treatments. These adjusted mortality figures are given in Table VIII.

Table VIII. The adjusted flock mortality percentages used in the analysis.

No. of Force Molts Over a Two Year Period	Force Molted At	Percentage Mortality First Year	Percentage Mortality Second Year
1	17 Months	12.0	13.0
1	13 Months	12.0	11.0
1	9 Months	10.0	13.0
2	9 and 19 Months	10.0	10.0
3	6, 13 and 19 Months	9.0	10.0
No Molt	-	12.0	14.0

Final Body Weight

The final body weight (W) of the light hybrids used in the Washington trials was 4.95 pounds (with a range of 4.88 to 4.98 for the four treatment groups). The birds used in the Wye College trials were weighed out at a mean of 5.00 pounds. The birds used by B. Franken of Skylane Farms, showed little increase in weight over the two year period. Mean weight at the end of the first year was 4.00 pounds and 4.06 pounds at the end of the second. This prevention of increased body weight over the second year was reportedly due to the weekly checks made by Franken, and his use of the feed ration protein level to control body weight close to 4.00 pounds. Such a procedure was not reported in the experimental flocks, and this no doubt accounts for the tendency for these birds to gain weight in the second year. It seems likely that if a closer check had been kept to prevent weight gains some increase in feed conversion efficiency would have resulted. Unfortunately it was not possible to allow for these weight gains by making appropriate adjustments in the feed conversion efficiency figures in the experimental results.

On the basis of the above observations, birds were assumed to weigh 4.00 pounds up to the end of the first year and 5.00 pounds up to the end of the second year of production. The exception was made for those stages of production which used data from Franken's flocks over the second year. Birds in these stages were assumed to weigh 4.00 pounds for the two year period.

Output and Input Prices

Output Prices

Egg Prices

In obtaining estimates for egg prices (P_{gt}) to be used in the model, two conditions had to be satisfied. Firstly, they had to be monthly (or if possible weekly) prices paid to producers at the farm in cents per dozen eggs. Secondly, the prices had to reflect both size and quality differentials. The price quotations from the major egg markets such as Los Angeles, Chicago and New York, failed to satisfy both these conditions. Some of the smaller market quotations did give prices paid at the farm but failed to give the necessary quality breakdown. The source of egg prices finally selected was a large local producers' co-operative[24] which was able to supply the required weekly price information for the ten year period, 1958-1967. These egg prices for all market grades are given in Table I of Appendix B. The relationship between the mean annual egg price

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[24] In deference to the expressed wishes of the manager of this co-operative the name of the co-operative is not mentioned in this thesis.

from this source as compared to the National averages as reported by the U.S.D.A., can be seen from Figure 10.

Figure 11 illustrates the seasonal fluctuations in mean monthly egg prices over the ten year period, 1958-1967. These fluctuations, as the graph shows, were periodic (every 13 months). Low prices predominated in the summer months with peak prices occurring in the later autumn and winter months. These price fluctuations reflected the seasonal supply and demand situation in the market (Gerra, 1959). Figure 10 taken in conjunction with Figure 11 shows

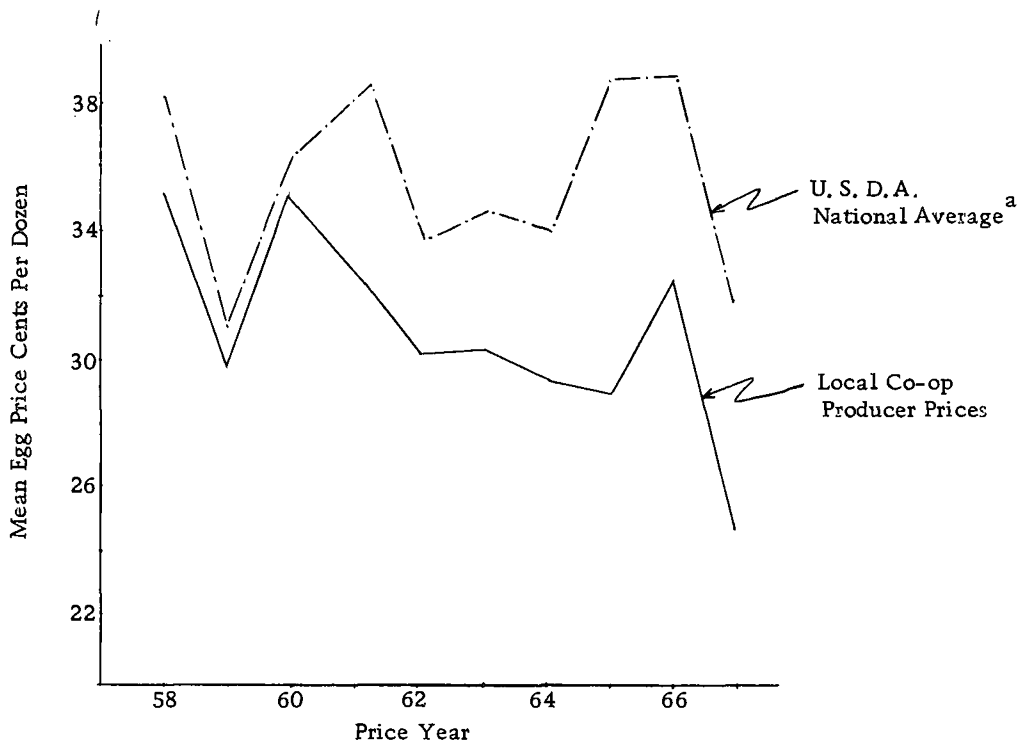

Figure 10. Mean annual egg prices 1958-1967. U.S.D.A. national averages compared to local co-op producer prices. ([a] Source: U.S.D.A. Agricultural Statistics, 1958-1967)

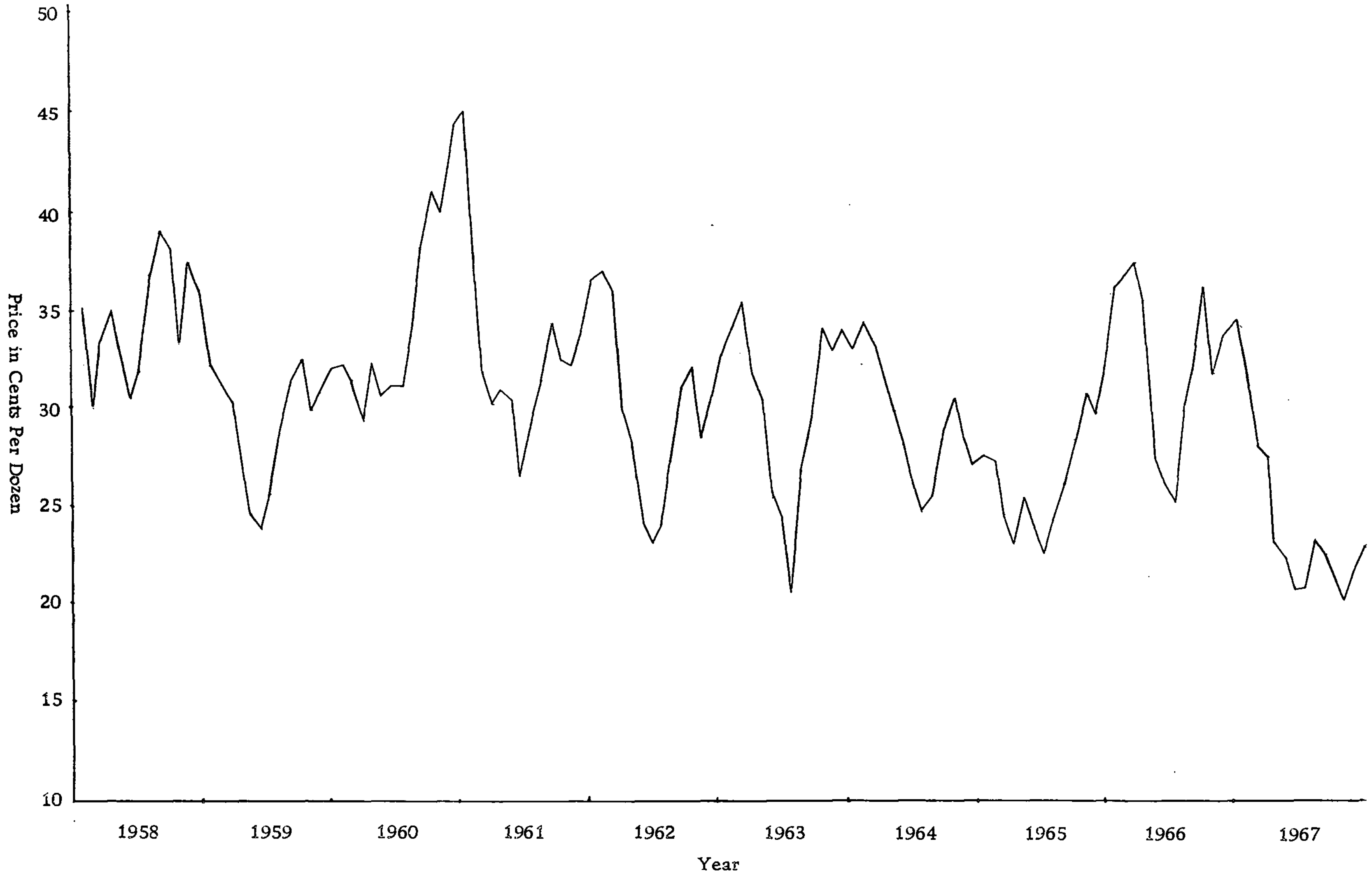

Figure 11. Producer egg prices, monthly means, 1958-1967.

the general downward trend in mean egg prices over the ten year period.[25]

The ten years prices listed in Table I of Appendix B were used to derive five historical price structures, each of two years duration. The price structures listed in Table IX were those used in the preliminary analysis of the replacement problem. These price structures numbered from one through five were listed according to the mean egg price (P^p) prevailing over the two year planning period in approximately a descending order of magnitude. It should be noted that the seven egg price structures listed in Table IX did not include the addition of price premiums for extra large and jumbo size eggs. Thus, all eggs falling in the large AA-jumbo AA grades were priced at the large AA egg price. In the analysis estimates of net revenue were obtained using these egg price structures. In addition, these estimates were obtained using the same egg price structures, but with the inclusion of price premiums for extra large AA eggs and jumbo AA eggs of two cents and four cents per dozen respectively, over the large AA egg price.

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[25] A trend analysis was carried out on these ten year egg prices using the method suggested by H. B. Mann, and reported by Tintner (1965, p. 212). The analysis yielded an r value of -0.4667, which indicated a significant negative trend over the ten year period.

Table IX. Egg price structures used in the preliminary analysis.

Price Structure	Price Years 1	Price Years 2	Historical or Hypothetical	Mean Egg Prices in Cents/Doz./per Month[a] In-Lay period 1 Yr. 1	Yr. 2	P^p [d]	In-Lay Period 7 Yr. 1	Yr. 2	P^p
1	1960	1961	Historical[b]	32.6	28.5	30.8	30.2	29.9	30.8
2	1958	1959	"	30.0	26.3	28.2	28.5	27.8	28.2
3	1962	1963	"	26.7	27.3	27.0	24.7	29.3	27.0
4	1964	1965	"	25.7	24.4	25.1	24.4	25.7	25.1
5	1966	1967	"	29.6	20.9	25.3	28.0	22.5	25.3
6	PEXT 1	PEXT 2	Hypothetical[c]	23.0	22.5	22.8	21.5	24.0	22.8
7	PEXT 3	PEXT 4	"	20.5	18.3	19.4	19.5	19.3	19.4

[a] prices do not include the addition of price premiums for extra large AA or jumbo AA grade eggs.

[b] from observed egg prices.

[c] extrapolation of the downward trend in egg prices, 1958-1967.

[d] P^p = mean egg price per month over the two year planning period.

Cull Prices

Table III of Appendix A shows the monthly salvage prices paid to producers for culled birds over the ten year period, 1958-1967. These national figures were compiled by the U.S.D.A. The tabulated mean annual figures show a marked downward trend over the ten year period. The monthly figures show that salvage prices have not fluctuated significantly from month to month over any single year. It was therefore assumed that cull prices were fixed for the length of the planning period. Two price levels of six and twelve cents per pound liveweight were used in the preliminary analysis.

Input Prices

Feed Cost

The cost of feed (C^f) amounts to some 50-65 percent of the producer's total costs depending on the price paid per ton. A wide range of costs are given in the literature depending on whether the producer mills his own feed or whether he buys from a retail supplier. Most of the budget studies conducted on the results of the force molting trials have used a feed price comparable to the cost of obtaining feed from a retail supplier. Thus, Hansen (1966) used a cost of $82.87 per ton, Hyre, et al. (1966) $70.00 per ton, and Morrison and Aho (1964) a cost of $72.00 per ton. U.S.D.A.

quotations for the period 1960-1968 (see Table IV of Appendix A) show that the cost of layer ration purchased from a retail supplier has maintained a level of approximately \$86.00 per ton. However, on-farm mixing and milling of rations can result in a considerable cost savings which may amount to as much as 25-30 percent, depending on the percentage protein content of the ration. Cost studies of commercial enterprises emphasize just how variable feed cost can be. In order to account for this variability, three feed costs, 1.5, 3.0 and 5.0 cents per pound (\$30, \$60 and \$100 per ton) were used in the preliminary analysis. The values of 1.5 and 5.0 cents represented feed costs which have not yet been observed in the field; it was hoped that their use in the analysis might provide valuable information on the effects of such extreme feed cost levels on the optimum and near optimum actions.

Cost of the Replacement Pullet

The cost of the replacement pullet (C^r) depends on whether the producer rears his own birds or whether he buys from an outside supplier. If he rears his own replacements the cost may be as low as \$1.20 for a point-of-lay bird. The upper range on this cost is about \$1.70 per bird for purchased pullets (Stratton, 1964, 1965a, 1965b).

The budget studies of the force molting trials completed to date have assumed a fairly high cost for the replacement pullet. Marble (1963) used a cost of $1.75 for a 22 week old pullet. Hansen (1966) used a cost of $1.60 for a 20 week old pullet. Hyre, *et al.* (1966) a cost of $1.50, Morrison and Aho (1964) a cost of $2.00. Bell (1966) used a cost of $1.80 for a purchased pullet and $1.40 for a home reared pullet. Commercial cost studies also show that there is an appreciable range in price that the producer pays for this factor of production (Bell, 1965b; Kelsey and Sheppard, 1966; Niles and Williams, 1962; Stratton, 1964, 1965a, 1965b). In order to investigate the effects of varying the replacement cost, three cost levels of $1.00, $1.50 and $2.00 per pullet were used in the preliminary analysis.

Other Costs

The transfer cost (C^t), the cost of transferring the point-of-lay pullet from the rearing to the laying unit, was set at one cent per bird housed. The clean-out cost (C^c), the cost of cleaning and disinfecting the laying unit at the end of the planning period, was set at two cents per bird housed.

It was assumed that miscellaneous costs such as wages, equipment, depreciation, vaccines, medication, repairs, taxes, and utilities were fixed in the short-run (e.g. over the length of the planning period) and would not vary as a result of the particular replacement policy the producer may choose to follow. The only cost which is likely to be regarded as variable in the short run is labor, in that it may be necessary to employ some temporary labor for the short period when houses are cleaned-out and refilled with point-of-lay pullets. These extra charges have been accounted for by including transfer and clean-out costs in the net revenue equation, 3.10.

Calculation Procedure

The previous sections have discussed the levels of production variables, input and output costs, and other costs used in the analysis. The values of net revenue (π_i) were calculated for each action in A using equation 3.10 for the following levels of prices and costs:

1. Five egg price structures each of two years duration. Estimates of π_i were obtained assuming: (1) price premiums (PP) on extra large AA and jumbo AA grade eggs, and (2) no price premiums (NP) on these grade eggs.

2. Two levels of cull prices (P^c); six and 12 cents per pound liveweight for culled birds.

3. Three levels of feed cost (C^f); 1.5, 3.0 and 5.0 cents per pound for layers ration.

4. Three levels of replacement cost (C^r); \$1.00, \$1.50 and \$2.00 for a point-of-lay pullet.

In addition to the price and cost levels listed in 1, 2, 3 and 4 above, it was felt that it would be advantageous to introduce a number of hypothetical levels for egg prices and egg quality into the analysis in order to study their effects both on the optimum action in A, and on the relative ranking of these actions over the two year planning period.

Hypothetical Egg Price Structures

In addition to the five egg price structures based upon the observed time series data, four hypothetical series (designated PEXT1, ..., PEXT4 in the text) were also used. These prices are listed in Table I of Appendix B. A trend analysis of the ten years annual means for the historical price data indicated a significant downward trend in egg prices over this period.[26] The hypothetical price structures were used in order to investigate the effect of an extrapolation of this downward trend. The mean annual prices pertaining to each of these four price years are given in Table IX. The inclusion of these prices resulted in two additional price structures (each of two years duration) giving a total of seven price structures to be used in the analysis.

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[26] Ibid., op. cit.

Hypothetical Egg Quality Distributions

In addition to using the egg quality distributions obtained from the experimental data (listed in Table I of Appendix A), it was decided to investigate the effect of increasing egg quality for those stages of production involving birds greater than 13 months old. This had the effect of increasing the comparative advantage of those actions in which birds were kept in-lay for extended laying periods. The object here was to see whether by increasing egg quality the relative ranking of actions in A^f and A^o would alter significantly. These increases in egg quality were confined to the large egg size category. The hypothetical distributions used were:

Ho^1: The original monthly egg quality distributions for large eggs (as shown in Table I of Appendix B) but with the number of large eggs increased by ten percent and the numbers of eggs in both the large A and large B grades decreased by five percent each.

Thus, suppose that the original quality distribution for large eggs in a particular month was:

Grade	B	A	AA	Total
No. of dozen eggs	10%	20%	70%	100%
Percentage Large eggs	10%	20%	70%	100%

Increasing the number of large eggs by ten percent and decreasing the numbers of large A and large B by five percent would result in the revised distribution:

Grade	B	A	AA	Total
No. of dozen eggs	5%	15%	80%	100%
Percentage Large eggs	5%	15%	80%	100%

Ho^2: As for Ho^1, but with the percentage of large AA eggs increased by 20 percent.

In both Ho^1 and Ho^2, only the distribution of large eggs between the quality grades AA, A and B was altered. The total number of eggs allocated to the large egg size category was not changed, as can be seen from the above examples.

The In-Lay Period

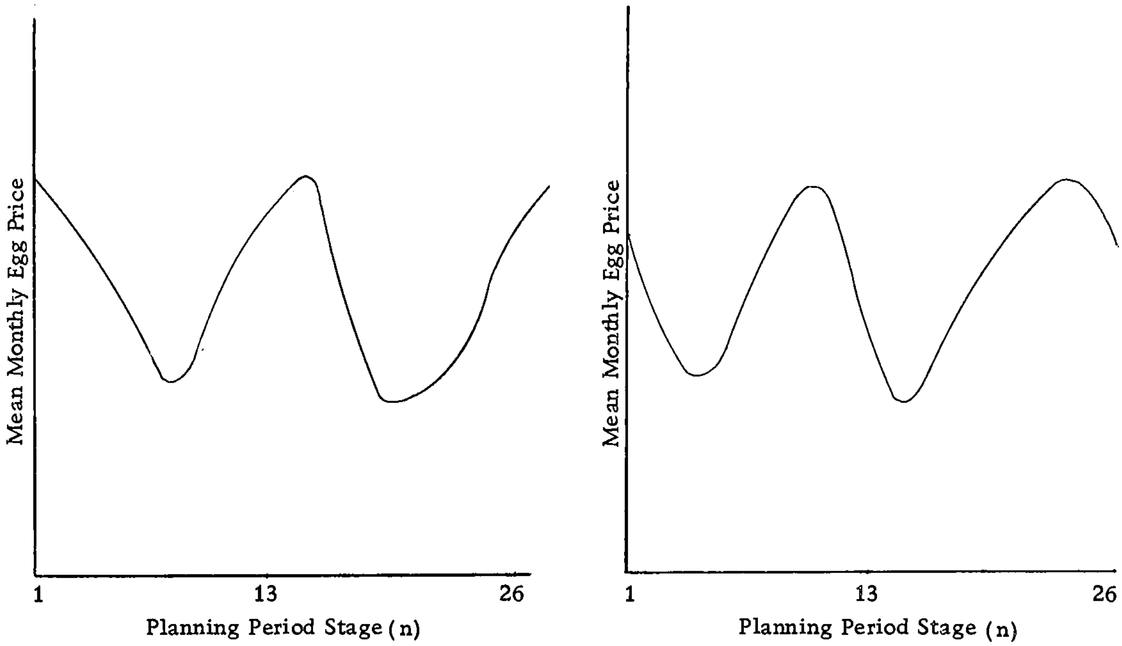
The in-lay period refers to the time of the year when a bird (flock) begins egg production. This period usually coincides with the transfer of the birds from rearing to laying units when the birds are between 20 and 24 weeks of age. The choice of the in-lay period is assumed to be under direct producer control. The annual periodic fluctuations in egg prices as shown in Figure 11, combined with the changing nature of egg production (in terms of monthly output, grade and quality distributions) with increasing age and force molting treatment (as shown in Figures 7 and 8), cause the choice of in-lay period to have a significant influence on the total revenue from egg sales over the planning period.

The effects of shifting the in-lay period on the form of the egg price cycle over the planning period are shown in Figure 12. This figure shows how the monthly egg income earned by a flock can be expected to change in relation to the period of the year when the flock is brought into lay.

In order to demonstrate the effects of altering the in-lay period on the optimum actions in A and also on the relative ranking of actions in A, it was proposed that two in-lay periods would be used in the analysis. These were set at period 1 (January 1) and period 7 (July 15). These in-lay periods were designated INLAY 1 and INLAY 7 in the text.

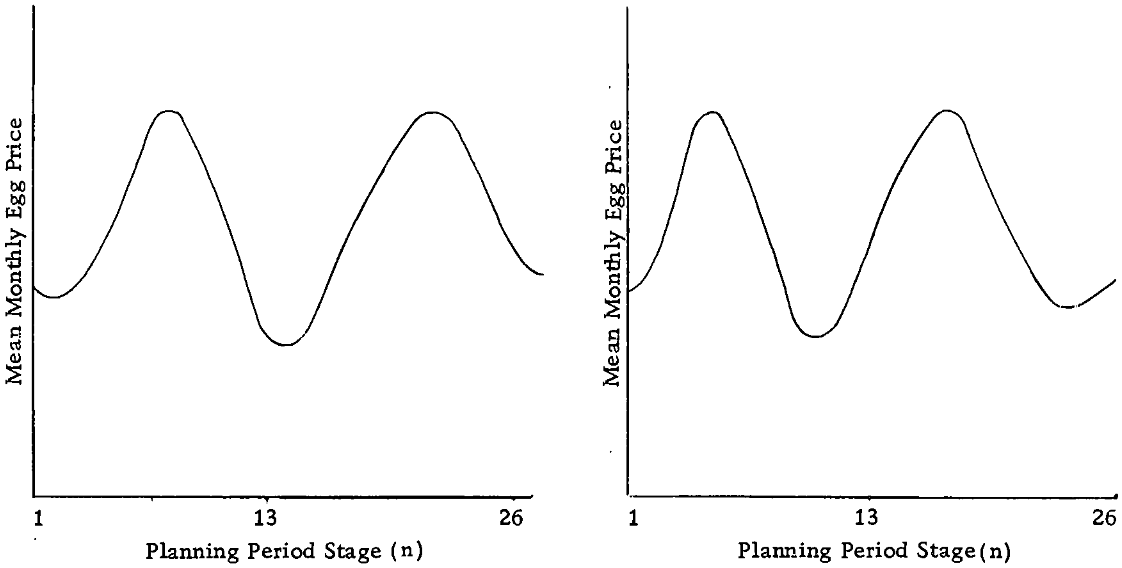
Summary and Conclusions

This chapter has discussed the collection of production data from a number of experimental force molting trials and several additional sources. The levels of costs and prices to be used in the first (or preliminary) stage of the deterministic analysis were specified. In addition, several hypothetical egg price structures and egg quality distributions were included in the analysis. Two in-lay periods were specified.



Graph 1. In-lay period 1.

Graph 2. In-lay period 4.



Graph 3. In-lay period 7.

Graph 4. In-lay period 10.

Figure 12. A series of four graphs showing how the egg price cycle over the planning period changes in relation to the in-lay period.

The levels of production variables, costs and prices were used to produce 630 price, cost and production variable combinations; each one reflecting a different set of economic conditions. These combinations are given in the next chapter. Values of net revenue (π_i) for each action in A were obtained for each of these combinations using the equation given in 3.10. The ranking procedures, and the determination of both the optimum, and "undominated" actions in A are discussed in the next chapter.

CHAPTER V

THE RESULTS OF THE PRELIMINARY AND SENSITIVITY ANALYSES USING THE DETERMINISTIC MODEL

The Preliminary Analysis

The levels of the production variables, input and output prices, and in-lay periods, specified in Chapter IV, produced a total of 630 variable combinations of possible economic conditions, under which estimates of the net income (π_i) for each action in A were calculated (using equation 3.10). These values of π_i were then used to rank the 54 actions in A in descending order of magnitude for each of these 630 combinations.

The rankings of the 54 actions were then studied for each combination in order to determine those actions which consistently appeared among the top 20 actions in these rankings. It was found that 15 actions consistently appeared among these top 20 actions, i.e. these actions consistently produced π_i values which were larger than those actions resulting from the "lower" 34 actions. These 15 actions were termed the "dominant" actions, and were included in the set A'. The remaining five actions included in A' were regarded as only "marginally dominant" in that for the majority of the combinations studied they appeared among the top 20 actions, but for some combinations they fell below the 20th position. In no case was an action which fell below 25th position for more than one of the 630 rankings studied, included in A'.

Actions Included in the Set A'

The 20 actions in A selected on the criterion of being "undominated" were included in the set A'. These actions are listed below:

$$A' = A^{o'} \cup A^{f'}$$

where

$$A^{o'} = \{a_i\} \; ; \; i = 4, 5, 6, 7, 8, 9$$

$$A^{f'} = \{a_i\} \; ; \; i = 28, 30, 31, 32, 37, 39, 40, 41, 42, 43, 45, 46, 47, 54.$$

Table X shows the j, k indices for the s_{jk} states which comprised each of these actions in A' over the 26 stages of production. It should be noted that not all the state indices over the 26 stages of production for any action were actually written in the table. This was in order to avoid confusion. The indices included in Table X were written so as to accentuate those stages of production when the action underwent an important change. Thus, let us consider the tabulated indices for action a_4 which is listed in Table X as:

Table X. Tabulated j, k indices for the s_{jk} states of production for the actions in A'[a].

Action[b] $\frac{a_i}{i}$	Stages of Production (n) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
4	1,0	2,0 ...													...15,0	1,0	2,0...								...10,0	0,0
5	1,0	2,0 ...												...14,0	1,0	2,0...									...11,0	0,0
6	1,0	2,0 ...											...13,0	1,0	2,0...										...12,0	0,0
7	1,0	2,0 ...										...12,0	1,0	2,0...											...13,0	0,0
8	1,0	2,0 ...									...11,0	1,0	2,0...												...14,0	0,0
9	1,0	2,0 ...								...10,0	1,0	2,0...													...15,0	0,0
28	1,0	2,0 ...				...6,0	7,5	8,5...																	...25,5	0,0
30	1,0	2,0 ...				...6,0	7,5	8,5...								...16,5	1,0	2,0...							... 9,0	0,0
31	1,0	2,0 ...				...6,0	7,5	8,5...							...15,5	1,0	2,0...								...10,0	0,0
32	1,0	2,0 ...				...6,0	7,5	8,5...			...11,5	1,0	2,0...												...14,0	0,0
37	1,0	2,0 ...							...9,0	10,4	11,4...														...25,4	0,0
39	1,0	2,0 ...							...9,0	10,4	11,4...					...16,4	1,0	2,0...							... 9,0	0,0
40	1,0	2,0 ...							...9,0	10,4	11,4...				...15,4	1,0	2,0...								...10,0	0,0
41	1,0	2,0 ...							...9,0	10,4	11,4...			...14,4...											...11,0	0,0
42	1,0	2,0 ...							...9,0	10,4	11,4......		13,4...												...12,0	0,0
43	1,0	2,0 ...										...12,0	13,2...												...25,2	0,0
45	1,0	2,0 ...										...12,0	13,2...				...17,2	1,0	2,0...						... 8,0	0,0
46	1,0	2,0 ...										...12,0	13,2...			...16,2	1,0	2,0...							... 9,0	0,0
47	1,0	2,0 ...							...9,0	10,1	11,1...														...25,1	0,0
54	1,0	2,0 ...															...17,0	18,3	19,3...						...25,3	0,0

[a] The first index j represents the age of the bird on hand in the nth stage of production. The second index k represents the force molting program the flock is subjected to over the planning period (See Table XI for a description of these treatments).

[b] Note that: all a_i, i = 1, 2, ..., 27 are non-force molted actions, and
all a_i, i = 28, 29, ..., 54 are force molted actions.

Stage	1	2 . . .	15	16	17 . . .	25	26
j, k indices	1, 0	2, 0 . . .	15, 0	1, 0	2, 0. . .	10, 0	0, 0

It was listed in this way to accentuate the fact that at the end of the 15th stage of production (15, 0) the flock was culled, and replaced at the beginning of stage 16 with a replacement flock (1, 0). At the end of stage 25 this flock was ten months old (10, 0) at which time it was culled, and the 26th stage of production spent in cleaning out the laying unit (0, 0).

In those actions where a flock underwent a first force molt, this was indicated by the change in the k subscript. Thus, consider the action a_{28}:

Stage	1	2 . . .	6	7	8 . . .	25	26
State	1, 0	2, 0. . .	6, 0	7, 5	8, 5. . .	25, 5	0, 0

This action underwent the first force molt at the end of the sixth stage of production when the flock was at the end of its sixth month of production (6, 0). The force molt was indicated by a change in "k" subscripts from "0" to "5", i.e. 6, 0 to 7, 5. The change to "5" indicated that this first force molt was the first of three possible force molts over the two year period (see the listing of force molted actions in Table III).

It should be noted for all actions that the "j" subscripts are continuous except where the series is interrupted by the introduction of a replacement flock (in which case "j" again reverts to "1" as in $s_{1,0}$).

Table XI gives a description of each of the actions in A' to help in understanding Table X. Table XI shows the total months of production for the first and second replacement flock over the planning period and the age at which the flocks in $A^{f'}$ underwent a force molting treatment. The actions in A' are listed in the same order in both Tables X and XI.

The 20 actions in the set A' had several important common characteristics which deserve mention. Firstly, it was noticed that none of these actions required more than two replacement flocks, even though several of the actions in A included this possibility. Secondly, none of the actions showed a flock being culled before ten months of age even though the possibility of culling at seven months was included. There were four notable exceptions to this general rule. As can be seen from Tables X and XI, four of the force molted actions did show culling prior to the tenth month of production, as in the 25th stage of production when culling was mandatory. These four actions were limited in significance because they belonged to that set of five actions previously defined as being "marginally dominant". Thirdly, all of the force molting treatments, in one way or another, were included in A' as indicated by the appearance of the whole range

Table XI. Description of the actions included in the set A'.

$\frac{a_i}{i}$	Number of months for which the first flock is in-lay	Stage of production when second flock housed	Number of months for which the second flock is in-lay	Flock force molted at
4	15	16	10	a
5	14	15	11	a
6	13	14	12	a
7	12	13	13	a
8	11	12	14	a
9	10	11	15	a
28	25	-	-	6, 13, and 19 months
30	16	17	9	6 and 13 months
31	15	16	10	6 and 13 months
32	11	12	14	6 months
37	25	-	-	9 and 19 months
39	16	17	9	9 months
40	15	16	10	9 months
41	14	15	11	9 months
42	13	14	12	9 months
43	25	-	-	13 months
45	17	18	8	13 months
46	16	17	9	13 months
47[b]	25	—	—	9 months
54	25	–	–	17 months

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[a] No force molting over the two-year planning period.

[b] This action was not included in the analysis until after completion of the preliminary analysis and therefore did not appear in the set A.

of "k" subscripts in Table X.

Of the 20 actions included in A', only a few actually appeared as optimum actions (i.e. with the greatest π_i value) under all the 630 variable combinations studied. These optimum actions will be discussed in the following section.

The Optimum Actions in A

Table XII shows the optimum action in A for each of the 630 variable combinations studied. The table was set up in the following manner. The 18 levels of the sum of livestock depreciation[27] and feed cost were listed across the top of the table so as to produce a complete spectrum of increasing total costs in passing from left to right across the table. The horizontal rows of the table were divided according to the prevailing egg prices (which were functions of: (1) the in-lay period, (2) the egg price structure used, and (3) the presence or absence of price premiums), and the hypothetical egg quality distribution for large eggs, Ho^1 and Ho^2.

The division of the horizontal rows into five distinct blocks of seven rows each makes for easier inspection of the results. Consider now just one of these five blocks. The figures listed within this block

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[27] The difference between the replacement cost of the point-of-lay pullets, and the cull value of the flock at the end of the planning period.

Table XII. Tabulated 'i' indices for the optimum actions in A for all variable combinations studied. Preliminary analysis results.

Block	PP or NP^a Ho^2 or Ho^{2b}	Egg Price Structure $(P^p)^c$	In-Lay Period	Feed Cost Per Pound (C^f)																	
				1.5¢						3.0¢						5.0¢					
				Replacement Cost Per Hen-Housed (C^r)																	
				1.00		1.50		2.00		1.00		1.50		2.00		1.00		1.50		2.00	
				Cull Price Per Pound Liveweight (P^c)																	
				12	6	12	6	12	6	12	6	12	6	12	6	12	6	12	6	12	6
1	NP	1	1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37
		2		6	6	6	6	6	37	6	6	6	6	6	37	6	6	6	6	6	37
		3		7	7	7	7	37	37	7	7	7	7	37	37	7	7	7	37	37	37
		4		6	6	6	6	37	37	6	6	6	37	37	37	6	6	6	28	28	37
		5		7	7	7	7	37	37	7	7	7	37	37	37	7	7	7	37	37	37
		6		7	7	7	7	37	37	7	7	7	37	37	37	7	7	37	37	37	37
		7		6	7	7	37	37	37	7	6	6	37	37	37	6	6	*	*	*	*
2	NP	1	7	7	7	7	7	7	37	7	7	7	7	37	37	7	7	7	37	37	37
		2		7	7	7	7	7	37	7	7	7	7	37	37	7	7	7	37	37	37
		3		6	6	6	6	6	37	7	7	7	7	7	37	7	7	7	7	37	37
		4		6	6	6	6	6	37	6	6	6	6	37	37	6	6	6	37	37	37
		5		7	6	7	6	37	37	7	6	6	7	37	37	7	7	7	37	37	37
		6		5	5	5	5	37	37	5	5	5	37	37	37	5	5	28	28	28	37
		7		7	7	7	37	37	37	7	7	7	37	37	37	39	*	*	*	*	*
3	PP	1	1	7	7	7	7	7	7	7	7	7	7	7	37	7	7	7	7	28	37
		2		6	6	6	6	6	37	6	6	6	6	37	37	6	6	6	37	37	37
		3		7	7	7	7	7	37	7	7	7	7	37	37	7	7	7	37	37	37
		4		6	6	6	6	37	37	6	6	6	37	37	37	6	6	28	37	37	37
		5		7	7	7	7	37	37	7	7	7	37	37	37	7	7	37	37	37	37
		6		7	7	7	37	37	37	7	7	7	37	37	37	7	7	37	37	37	37
		7		7	7	37	37	37	37	7	7	37	37	37	37	7	37	37	*	*	*
4	Ho^1 NP	1	1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37
		2		6	7	6	7	7	37	6	6	6	7	6	37	7	7	6	7	37	37
		3		7	7	7	7	37	37	7	7	7	28	28	28	7	7	28	28	28	37
		4		6	6	6	6	37	37	6	6	6	28	28	28	6	6	28	28	28	37
		5		7	7	7	7	37	37	7	7	7	7	37	37	7	7	7	37	37	37
		6		7	7	7	7	37	37	7	7	7	37	37	37	7	7	37	37	37	37
		7		6	7	6	37	37	37	7	6	7	37	37	37	6	7	7	*	*	*
5	Ho^2 NP	1	1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	28	37
		2		7	7	7	7	7	37	7	7	7	7	37	37	7	7	6	37	37	37
		3		7	7	7	7	7	37	7	7	7	7	37	37	7	7	7	37	37	37
		4		6	6	6	6	37	37	6	6	6	37	37	37	6	6	28	28	28	37
		5		7	7	7	7	37	37	7	7	7	37	37	37	7	7	7	37	37	37
		6		7	7	7	37	37	37	7	7	7	37	37	37	7	7	37	37	37	37
		7		7	7	7	37	37	37	7	7	37	37	37	37	7	7	37	*	*	*

* Denotes $\pi_i \leq \$0.00$.

[a] PP indicates that the egg price structures include price premiums for extra large AA and jumbo AA grade eggs. NP indicates egg price structures without price premiums.

[b] Ho^1: original large egg grade distribution + 10% increase in large AA eggs.
Ho^2: original large egg grade distribution + 20% increase in large AA eggs.

[c] See Table IX for a list of these egg price structures.

could be read in the following three different ways:

1. Across each row, showing the changes in optimum policy as costs of production increase for a given set of egg prices and quality distribution for large eggs.

2. Down each column showing the changes in the optimum action for a given level of production costs over the range of decreasing egg prices (in passing from egg price structure one to seven in the table).

3. Diagonally across the table showing the changes in the optimum action as both egg prices and production costs are varied simultaneously.

Discussion of the Results

Before discussing the optimum actions shown in Table XII some general remarks on these results will be made.

Firstly, of the 20 actions included in the set A', only six actions appear as optimum actions in Table XII. These are:

$$a_5, a_6, a_7, a_{28}, a_{37}, a_{39}$$[28]

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[28] Where: a_5 represents a non-force molted action. First flock kept in-lay for 14 months and the second for 11 months.

a_6 represents a non-force molted action. First flock kept in-lay for 13 months and the second for 12 months.

a_7 represents a non-force molted action. First flock kept in-lay for 12 months and the second for 13 months.

Of these six actions, a_5, a_6, a_7, a_{28}, and a_{37} were optimum for the majority of the 630 variable combinations studied. It should be noted that a_6 and a_7 were non-force molted actions while a_{28} and a_{37} were force molted actions involving three and two molts respectively over the two year planning period.

Secondly, neither a_{43} (the force molted action with one molt at 13 months of age) nor a_{54} (the force molted action with one molt at 17 months of age) appeared as optimum actions in Table XII. Thirdly, in general, when the cost of replacement (C^r) was \$1.00 per bird (or less) the producers optimum action was to replace his flock every year (annual replacement). At the increased cost of \$1.50 the influence of the cull price became the important factor. At the lower cull price of six cents per pound the choice of all optimum actions became sensitive to the effect of feed cost and egg price conditions. At the highest cost of replacement (i.e. C^r = \$2.00) the levels of cull price, feed cost, and egg prices must all be considered in order to determine the optimum action.

More specifically, the results in Table XII indicate that:

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a_{28} represents a force molted action. The flock being subsubjected to three force molts; at 6, 13 and 19 months of age.

a_{37} represents a force molted action. The flock being subjected to two force molts; at 9 and 19 months of age.

a_{39} represents a force molted action. The first flock kept in-lay for 16 months with a force molt at 9 months of age. Second flock kept in-lay for 9 months.

1. If one regards it as unlikely that mean egg prices over a two year period will in the future surpass 29.0 cents per dozen, then one can direct attention to the optimum actions under price structures 4, 5, 6 and 7.

2. If one further assumes that cull prices will remain at about six cents per pound liveweight then one can state that: (1) for a replacement cost of $1.00 per bird an annual replacement policy (action a_6 or a_7) can be followed, (2) for a replacement cost of $1.50 per bird and a feed cost of $30 per ton (which is well below any attainable level for today's producers), then an annual policy can still be followed. For feed costs of $60 per ton (or greater) a force molted action with either two molts (a_{28}), or three molts (a_{37}), can be followed over the planning period, (3) a force molted action can always be followed if the replacement cost is $2.00 or greater. These results are summarized more concisely in Table XIII.

In determining the optimum action for a given set of price, cost and production conditions, Table XIII indicates whether a non-force molted action (NFA) or a force molted action (FMA) would be optimum over the two year planning period. The precise nature of this action can be seen by referring back to Table XII. Let us consider a set of economic conditions which might reflect those pertaining to the industry today. Suppose the replacement cost is fixed at $1.50 per hen-housed, the cull price at six cents per pound liveweight,

Table XIII. A summary of the effects of the variable combinations on the optimum actions in A .[a]

	Replacement Cost Per Hen Housed					
	C^r = \$1.00	C^r = \$1.50		C^r = \$2.00		
Cull Price (P^c) 12 cents Per Pound Liveweight	NFA Optimal for All Combinations Feed Costs, Egg Prices and Other Production Conditions.	NFA Optimal for All Combinations of Feed Costs, Egg Prices, and Other Production Conditions.		C^f = 1.5 cents	P^p = 1, 2, (3, 4) P^p = (3, 4), 5, . . . , 7	NFA FMA
				C^f = 3.0 cents	P^p = 1, (2) P^p = (2), 3, 4, . . . , 7	NFA FMA
				C^f = 5.0 cents	P^p = (1) P^p = (1), 2, . . . , 7	NFA FMA
Cull Price (P^c) 6 cents Per Pound Liveweight		C^f = 1.5 cents NFA Optimal for all Combinations of Egg Prices and Production Conditions		C^f = 1.5 cents	P^p = 1 P^p = 2, 3, . . . , 7	NFA FMA
		C^f = 3.0 cents	P^p = 1, 2, 3. NFA P^p = 4, 5, 6, 7. FMA	C^f = 3.0 cents	P^p = (1) P^p = (1), 2, . . . , 7	NFA FMA
		C^f = 5.0 cents	P^p = 1, (2).[b] NFA P^p = (2), 3, . . . , 7. FMA	C^f = 5.0 cents	FMA for all P^p = 1, 2, . . . , 7	

[a] The following explanation of abbreviations should aid in understanding the table.

NFA = a non-force molted action (e. g., a_6 or a_7).
FMA = a force molted action (e. g., a_{28} or a_{37}).
C^f = cost of feed in cents per pound.
P^p = egg price structure (see Table XV).

[b] P^p = 1, (2) indicates that under price structure 2 there was no clear cut distinction between the non-force molted and the force molted policies.

and the feed cost at $60 per ton. Reference to Table XIII immediately shows that if the expected mean egg price over the next two years is between 27.0 and 35.0 cents per dozen ($P^p = 1, 2$ or 3) then an annual replacement policy is optimum, e. g. a_6 or a_7. If the expected mean price is less than 27.0 cents per dozen then a force molted action is optimum, e. g. a_{28} or a_{37}. Reference to Table XII will indicate the optimum action that can be followed for these two egg price conditions.

The Effect of Shifting the In-Lay Period

Two in-lay periods were used in the analysis. The results shown in blocks 1 and 2 of Table XII demonstrated the effects of shifting the in-lay period from period 1 to period 7. The effects were:

1. To increase the number of variable combinations for which a force molting action was optimum. This can be seen by reading across the first horizontal row in each of blocks 1 and 2 and comparing the frequency with which the force molting action a_{37} appears.

2. To decrease the levels of net revenue (π_i) accruing to each action. This decrease being caused by the change in the form of the egg price cycle as a result of shifting the in-lay period (this change is shown in Graphs 1 and 4 of Figure 12). This effect is shown in Table XII by the increase in the incidence of variable combinations

that resulted in negative net revenues.

3. Shifting the in-lay period from period 1 to 7 resulted in a significant change in the form of the egg price cycle and increased the comparative advantage of the force molted actions, but also decreased the net revenue accruing to each action.

The Effect of the Price Premiums

In order to compare the effects of adding price premiums for extra large AA and jumbo AA eggs to the egg price structures shown in Table XII the results shown in blocks 1 and 3 of the table were compared. The addition of price premiums increased the comparative advantage of the force molted actions a_{28} and a_{37} which was not a totally unexpected result. The effect however was not as marked as might be expected from a comparison of the percentage of all eggs laid in the extra large AA and jumbo grades for these force molted actions and the non-force molted actions a_6 and a_7 (see the GD_{gjk} values for g = 6 and 7, in Table I, Appendix A).

The Effect of Changing the Egg Quality Distributions for Large Grade Eggs

The results of introducing the two additional egg quality distributions for large eggs, Ho^1 and Ho^2, were best seen by comparing the optimum actions in blocks 1, 4 and 5 in Table XII.

Ho^1: A comparison of blocks 1 and 4 (original distribution versus Ho^1) showed no increase in the frequency with which force molted actions appeared as optimum actions in passing from block 1 to 4. The change in quality distribution had no effect on the non-force molted actions (as was to be expected) but it did change several of the optimum force molted actions from a_{37} in block 1 to a_{28} in block 4, i.e. it decreased the advantage of two force molts a_{37} over the planning period as compared to three force molts a_{28}. This change can be seen by comparing the optimum actions under egg price structure 4 in blocks 1 and 4.

Ho^2: A comparison of blocks 1 and 5 indicated a marginal increase in the frequency of a force molted action as the optimum action (comparing the optimum actions under egg price structures 1 and 2 in blocks 1 and 5).

The Relative Ranking of Actions in A

One of the stated objectives of this analysis was to study the relative ranking of the actions in A in order to determine those actions which would be included in the set A'. One of the outcomes of this ranking procedure was that it was possible to inspect those actions which produced π_i values which were close to those produced by the optimum actions. The results of this procedure led to two conclusions. Firstly, that for many of the 630 combinations studied the top three actions produced π_i values which were very close to each other--the spread between the optimum and the third from optimum π_i values being as little as \$3, and for most combinations, only \$1 to \$2 separating the top two π_i values. Secondly, that a sensitivity analysis was needed on the actions in A'. This conclusion followed from an inspection of the top five actions in A for each of the 630 variable combinations studied. By examining these actions it was possible to trace the movement of a particular action from say the fifth ranked position to the optimum position. Of particular interest was the movement of the force molted actions in relation to the non-force molted actions. These relative movements can best be seen by considering the actions listed in Table XIV. This table shows just the top five actions in A under price structure 1 and 5 in block 1 of Table XII.

With reference to Table XIV consider the movement of action a_{37} for the cases denoted $\boxed{37}$ and ㊲ . The table shows that under price structure 1 with a feed cost of 5.0 cents per pound, a replacement cost of \$2.00 per hen-housed and a cull price of 12 cents per pound, that action a_{37} ($\boxed{37}$) was the fourth ranked action. A decrease in the cull price to six cents per pound caused this action to become optimum. It was apparent from this that the relative rankings of the actions a_6, a_7, and a_{37} were extremely sensitive to small changes in the cull price level. A similar large

Table XIV. The top five actions in A. No price premiums. In-lay period 1. Price structures 1 and 5 only.

Egg Price Structure P^p	Mean Egg Price	Feed Cost Per Pound (C^f)																	
		1.5¢						3.0¢						5.0¢					
		Replacement Cost Per Hen-Housed (C^r)																	
		1.00		1.50		2.00		1.00		1.50		2.00		1.00		1.50		2.00	
		Cull Price Per Pound Liveweight (P^c)																	
		12	6	12	6	12	6	12	6	12	6	12	6	12	6	12	6	12	6
1	30.8	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	$\boxed{37}$
		6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	28
		5	5	5	5	5	5	5	5	5	5	5	37	5	5	5	5	28	7
		4	4	4	4	4	37	4	4	4	4	4	28	4	4	4	4	$\boxed{37}$	6
		32	32	32	32	37	4	32	32	32	32	37	5	32	32	32	28	5	5
5	25.2	7	7	7	7	㊲	37	7	7	7	7	37	37	7	7	7	37	37	37
		6	6	6	6	7	28	6	6	6	37	28	28	6	6	6	28	28	28
		5	5	5	5	6	7	5	5	5	6	7	43	5	5	37	7	7	43
		4	4	4	㊲	28	6	4	4	37	28	6	7	4	37	28	6	6	54
		32	32	37	4	5	43	32	32	4	5	5	6	32	4	5	5	5	7

change in the ranking of a_{37} was indicated in Table XIV by the movement of ㊲ . This followed as the result of a 50 cent increase in the replacement cost.

The point emphasized by these two examples is that in many cases the results of the preliminary analysis showed that the relative ranking of the actions changed so abruptly that it was impossible to gauge the sensitivity of these actions to small changes in the levels of the variables used in the analysis.

In order to complete the analysis under conditions of certainty it was decided to conduct a sensitivity analysis on the actions in A' by increasing the number of levels of replacement costs, feed costs, and cull prices while at the same time narrowing the range in costs and prices covered by these variables. At the same time it was felt that it would be informative to increase the number of in-lay periods studied from two to four. The analysis was conducted using the egg grade distributions (GD_{gjk}) shown in Table I of Appendix A.

The main object of the sensitivity analysis was to determine the changes in the optimum action for small changes in the levels of production costs and prices. In particular, interest was focused on the relationship between the force molted and non-force molted actions in the set A'.

The Sensitivity Analysis

The methodological procedure was the same as that used in the preliminary analysis. The same ranking procedure was used. The differences centered on the price and cost levels used in this analysis. These levels are discussed below under the appropriate headings.

Egg Price Structure

It was assumed that all egg price structures would include egg price premiums of two cents for extra large AA and four cents per dozen for the price of jumbo AA grade eggs over the price of large AA grade eggs. The seven egg price structures used in the sensitivity analysis are listed in Table XV. It will be seen by comparing the prices shown in Tables IX and XV that these differ from those used in the preliminary analysis. Table XV shows that the two years prices used for each price structure had mean egg prices over the first and second years which were close to each other, e.g. the mean egg price over years one and two of the planning period for price structure 1A were 35.45 and 34.83 cents per dozen for in-lay period 1. By reordering the historical price data it was possible to obtain a smooth downward trend in egg prices as can be seen from Table XV. This table also shows the effect of changing the in-lay

Table XV. Egg price structures used in the sensitivity analysis.[a]

Price Structure	Price Years		Historical or Hypothetical Price Structure	Mean Egg Price in Cents Per Dozen In-Lay Period 1			4			7			10		
	1	2		Year 1	Year 2	P^p[d]	Year 1	Year 2	P^p	Year 1	Year 2	P^p	Year 1	Year 2	P^p
1A	1960	1958	Historical[b]	35.45	34.83	35.14	36.10	34.18	35.14	36.42	33.86	35.14	36.63	33.65	35.14
2A	1966	1961	Historical	32.37	32.07	32.22	31.88	32.56	32.22	28.83	35.61	32.22	32.12	32.32	32.22
3A	1963	1962	Historical	29.72	29.70	29.71	30.18	29.24	29.71	29.90	29.52	29.71	30.22	29.20	29.71
4A	1959	1964	Historical	29.70	28.84	29.27	30.22	28.32	29.27	30.66	27.88	29.27	29.63	28.91	29.27
5A	1965	1967	Historical	27.22	23.72	25.47	28.09	22.85	25.47	27.71	23.23	25.47	26.81	24.13	25.47
6A	Pext1	Pext2	Hypothetical[c]	26.11	25.92	26.01	25.86	26.16	26.01	26.04	25.98	26.01	25.59	26.43	26.01
7A	Pext3	Pext4	Hypothetical	23.48	21.26	22.37	22.50	22.24	22.37	22.36	22.38	22.37	22.00	22.74	22.37

[a] Prices include the addition of price premiums for extra large AA and jumbo AA grade eggs (See Table I, Appendix A.)

[b] From observed egg prices.

[c] Extrapolation of the downward trend in egg prices, 1958-1967.

[d] P^p = mean egg price per month over the two-year planning period.

period on the mean egg prices over the first and second years of the planning period. The manner in which the in-lay period affects the form of the egg price cycle was shown in Figure 12.

Feed Costs

The upper and lower feed cost levels used in the preliminary analysis were outside any levels as yet observed in the field. Their use in the analysis did accentuate the effects of increasing feed cost on the resulting optimum policy. However, in the sensitivity analysis it was felt that it would be more informative to narrow the total range of these costs, while at the same time increasing the number of levels within this range. Consequently, six levels of feed costs, 2.4, 2.6, 2.8, 3.0, 3.2, and 3.4 cents per pound were used in the sensitivity analysis.

The upper level of 3.4 cents was the cost level that a producer buying ready mixed feed in bulk from an outside supplier would be expected to incur. The lower cost level of 2.4 cents was close to the lowest a producer might be expected to achieve with bulk buying of grain and protein supplements, combined with the use of on-farm milling and mixing facilities.

Cull Prices

In addition to the two levels of six and twelve cents per pound used in the preliminary analysis, a third level of ten cents per pound was included in the sensitivity analysis.

Replacement Costs

Six replacement cost levels were used. These were set at 1.2, 1.3, 1.4, 1.5, 1.6, and 1.7 dollars for a point-of-lay pullet of 22 weeks of age. This range was wide enough to cover almost all levels observed in the field and the cost increments were small enough to allow the effect of a ten cent increase in this cost on the optimum action to be detected.

In-Lay Periods

The results from the preliminary analysis showed how changing the in-lay period altered the optimum action and the relative ranking of actions for a given set of conditions. In order to demonstrate these changes more fully the number of in-lay periods was increased from two to four. These were set at periods 1, 4, 7, and 10. The mean egg price prevailing over the planning period for each of these in-lay periods are given in Table XV.

The combination of seven egg price structures, six feed cost, three cull price, and six replacement cost levels, yielded a total of 756 sets of economic conditions for each of the four in-lay periods considered.

Using the same procedure and computer program as for the preliminary analysis, net revnue (π_i) values were estimated (using equation 3.10) for each of the actions in A' for each of the above combinations of prices, costs and in-lay periods. These actions were then ranked according to the π_i values in descending order of magnitude. The results of this analytical procedure are given in the next section.

The Results of the Sensitivity Analysis

The optimum actions in the set of actions A' under the complete range of prices and costs listed above for each of the four in-lay periods considered, are shown in Table XVI. The following description of the layout of this table should aid in an understanding of the main conclusions to be deduced from this analysis.

Table XVI is divided into four main sections according to the in-lay period used. Within each section the possible combinations of cull prices and feed costs were listed in such a way as to provide a spectrum of increasing costs in passing from left to right across a particular section. The rows within each section were divided, firstly according to price structure and secondly, according to the replacement cost (C^r) used. The egg price structures were listed according to the mean egg price (P^p) prevailing over the planning period in approximately descending order of magnitude from 1A to

Table XVI. Tabulated 'i' indices for the optimum actions in A', four in-lay periods, and various cost and price conditions.

Price Structure (P^p)[a]	Replacement Cost (C^r) Per Hen Housed $	In-Lay Period 1																		In-Lay Period 4																	
		Cull Price in Cents Per Pound Liveweight (P^c)																																			
		12.0¢						10.0¢						6.0¢						12.0¢						10.0¢						6.0¢					
		Feed Cost in Cents Per Pound (C^f)[b]																																			
		1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1A	1.2	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.3	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
2A	1.2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.5	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37
3A	1.2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.5	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	54
	1.6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	37	7	7	7	7	7	7	7	7	7	7	7	7	54	54	54	54	54	54
	1.7	7	7	7	7	7	7	7	7	7	7	37	37	37	37	37	37	37	37	7	7	7	7	7	7	54	54	54	54	54	54	54	54	54	54	54	54
4A	1.2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	1.5	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	37	37	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	54	54	54
	1.6	7	7	7	7	7	7	7	7	7	7	7	7	37	37	37	37	37	37	7	7	7	7	7	7	7	7	7	54	54	54	54	54	54	54	54	54
	1.7	7	7	7	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	7	7	7	7	7	7	54	54	54	54	54	54	54	54	54	54	54	54
5A	1.2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	37	37
	1.3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	7	7	7	7	7	7	7	7	37	37	37	37	37	37	37	37	37	37
	1.4	7	7	7	7	7	7	7	7	7	7	37	37	37	37	37	37	37	37	7	7	7	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37
	1.5	7	7	7	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.6	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
6A	1.2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	37	7	7	7	7	28	28	7	7	7	28	28	28	7	7	7	28	28	28
	1.3	7	7	7	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	7	7	7	7	28	28	37	37	37	28	28	28	37	37	37	37	37	37
	1.4	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	7	7	7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.5	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.6	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
7A	1.2	6	6	6	6	6	28	6	6	6	6	6	28	37	37	37	37	37	37	7	7	28	28	28	28	37	37	37	37	37	37	37	37	37	37	37	37
	1.3	6	6	6	6	6	28	37	37	37	37	37	37	37	37	37	37	37	37	7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.4	6	6	6	6	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.5	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.6	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
	1.7	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37

Table XVI (Cont'd.)

Price Structure (P^p)[a]	Replacement Cost (C^r) Per Hen-Housed $	In-Lay Period 7 — Cull Price in Cents Per Pound Liveweight (P^c): 12.0¢ 10.0¢ 6.0¢ — Feed Cost in Cents Per Pound (C^f)[b]: 1 2 3 4 5 6 1 2 3 4 5 6 1 2 3 4 5 6	In-Lay Period 10 — Cull Price in Cents Per Pound Liveweight (P^c): 12.0¢ 10.0¢ 6.0¢ — Feed Cost in Cents Per Pound (C^f)[b]: 1 2 3 4 5 6 1 2 3 4 5 6 1 2 3 4 5 6
1A	1.2		
	1.3		
	1.4		
	1.5	7	5 37
	1.6		37
	1.7	37	37
2A	1.2		
	1.3		
	1.4		
	1.5	7	5
	1.6		
	1.7	37	37
3A	1.2		
	1.3		
	1.4	6	5 37
	1.5		37
	1.6	37	37
	1.7	37	37
4A	1.2		37
	1.3	6	37
	1.4		5 37
	1.5	37	37
	1.6	37	37
	1.7	37	37
5A	1.2	37	5 37
	1.3	7 37	37
	1.4	37	37
	1.5	37	37
	1.6	37	37
	1.7	37	37
6A	1.2	37	
	1.3	5 37	6 37
	1.4	37	37
	1.5	37	37
	1.6	37	37
	1.7	37	37
7A	1.2	28 37	54
	1.3	7 37	54
	1.4	37	4 54
	1.5	37	54
	1.6	37	54
	1.7	37	54

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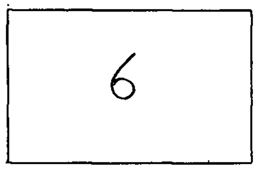
[a] See Table XV for the levels of P^p for each of the in-lay periods.

[b] Where 1, 2, 3, 4, 5, and 6 designate feed cost levels of 2.4, 2.6, 2.8, 3.0, 3.2 and 3.4 cents per pound respectively.

7A. The replacement costs were listed in ascending order of magnitude from $1.20 to $1.70 per hen-housed. Thus, considering the possible combinations of cull prices, feed costs, replacement costs and price structures, these reflect an increasing total variable cost level and a decreasing egg price level in passing from the top left hand corner to the lower right hand corner of a given section of the table.

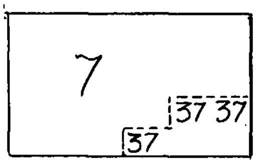
The designation of the optimum action used in the table was as follows:

Consider the actions listed for in-lay period one. It will be seen from the table that for price structure 1A that only one figure, "6" appears in the center of the table thus:



This indicates that the optimum action for all price/cost combinations in this particular block was a_6 (an annual replacement policy).

Consider now the same in-lay period but the block pertaining to price structure 3A which looks like:



This layout indicates that for all price/cost combinations to the left of the dotted line action a_7 (an annual replacement policy) was the optimum action. To the right of this line the optimum actions for the combinations take on the first number listed for the particular row of the table. In this case shown above, action a_{37} (a flock force molted at six, 13 and 19 months of age) was optimal for all combinations to the right of the dotted line.

In-Lay Period 1

In-lay period 1 takes the first stage of production for each action in A' as commencing on January 1 for each of the egg price structures used. The planning period in this case covered a two year period from January 1 of the first price year to December 31 of the second price year. The approximate form of the egg price cycle that might be expected to prevail over this period was shown in Graph 1 of Figure 12. With this in-lay period egg prices would be expected to reach low points (troughs) at stages n equal to 7 and 20 of the planning period, with high points (peaks) at n equal to 1, 13 and 26.

With the mean egg price (P^p) over the planning period equal to or greater than 32.0 cents per dozen the optimum action was a non-force molted action (a_6 or a_7). As P^p was decreased from 32.0 to 22.0 cents per dozen actions a_6 and a_7 were replaced by force molting actions a_{28} and a_{37}, at progressively lower total variable cost levels.[29] The historical price structures used, i.e. 1A to 5A, showed that for the lowest replacement cost of \$1.20 per bird the optimum actions always involved a non-force molted action regardless of feed cost or cull price level. With a further lowering of the mean egg price to 22.0 cents per dozen then actions a_{28} and a_{37} did become optimum only at the upper end of the total variable cost range.

The effect of increasing total variable costs on the optimum action can best be seen by following the changes that occurred for any given price structure, e.g. 4A. Thus, with P^p equal to 29.01 cents per dozen and a replacement cost greater or equal to \$1.40 per bird, a non-force molted action (a_7) was always optimum. An increase of ten cents in the replacement cost to \$1.50 per bird resulted in the force molted action a_{37} becoming optimum when the cull price was set at six cents per pound and feed cost greater or equal to 3.0 cents per pound. A further ten cents increase in the replacement cost to \$1.60 per bird further increased the range of cull prices and feed costs over which the force molted action a_{37} was optimum. Similar series of changes were observed for each of the price structures listed.

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[29] a_{28} is the force molted action with molts at 6, 13 and 19 months. a_{37} is the force molted action with molts at 9 and 19 months.

The hypothetical price structures 6A and 7A served to further emphasize that low egg prices favored the force molting actions, a_{28} and a_{37}. At the lowest egg price levels used in the analysis it can be seen that the force molted actions a_{28} and a_{37} almost completely dominated the non-force molted action, a_6.

In-Lay Period 4

The shift in the in-lay period resulted in changing the form of the egg price cycle from that shown in Graph 1 to that shown in Graph 2 of Figure 12. With this in-lay period the egg price cycle would be expected to reach troughs at stages n equal to 5 and 18 of the planning period, with peaks at n equal to 10 and 23. The results shown in Table XVI showed that the price/cost combinations that resulted in a force molting action becoming optimum were very similar to those for in-lay period 1. The major change was in the appearance of action a_{54} as the optimum action[30] under price structures 3A and 4A. The shift in in-lay period, also resulted in a force molting action becoming optimum at somewhat lower total cost levels for any given egg price structure, i.e. the shift of in-lay period from 1 to 4 favored the force molted actions. This change became more

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[30] a_{54} is a force molted action with one molt at 17 months of age.

prominent at the lower egg price levels. This can be seen by comparing the optimum actions under structures 4A and 5A, for these two in-lay periods.

In-Lay Period 7

The form of the expected egg price cycle for this in-lay period is shown in Graph 3 of Figure 12. This egg price cycle has troughs at stages n equal to 2 and 15 of the planning period with peaks at n equal to 7 and 20. The results shown in Table XVI did not show any significant differences between the optimum actions under in-lay periods 7 and 1. There were, however, some differences between the optimal actions under in-lay periods 7 and 4. For the higher egg price structures, e.g. 1A and 2A, force molted actions appeared as optimum actions with greater frequency under in-lay period 7 than under either 1 or 4. At the lower price levels, e.g. 3A through 7A, there were no obvious differences between the frequency with which force molted actions appeared as optimum when the actions in Table XVI under in-lay periods 1, 4 and 7 were compared.

In-Lay Period 10

The form of the expected egg price cycle for this in-lay period is shown in Graph 4 of Figure 12. This egg price cycle has troughs at stages n equal to 10 and 23 of the planning period, and peaks at n equal to 5 and 18. With this in-lay period the optimal non-force molted action under most of the egg price structures used was a_5 as shown in Table XVI. Action a_5 indicated that the first flock should be kept in-lay for 14 months and the second for only 11 months. This result represented a slight change from the annual replacement policies (a_6 and a_7) indicated as optimum for in-lay periods 1, 4 and 7.

Those actions appearing as optimum in Table XVI under in-lay period 10, were the same as those indicated for the other three in-lay periods used in the analysis. It was noted that under the price structure 1A (P^p equal to 35.14 cents per dozen) that a force molted policy, a_{37}, was optimum for a larger range of price/cost combinations than for any of the other in-lay periods. This did not hold for the other six price structures. In fact, the results showed that under price structures 3A, 4A and 5A, in-lay period 10 favored a force molted action (a_{37}), more so than any of the other three in-lay periods, whereas for structures 6A and 7A it was the least favorable.

Summary and Conclusions

The results of the deterministic analysis demonstrate how important it is to consider the question of the economic feasibility of force molting policies as alternatives to say an annual replacement policy in the light of the complete range of feed costs, replacement costs, cull prices and egg prices that are likely to occur. Previous work in this area has shown how erroneous conclusions drawn for analyses conducted using single valued cost and price parameters can be.

The general conclusions from this analysis were as follows:

1. With the mean egg price over the planning period at greater than 32.0 cents per dozen and the cost of a replacement pullet at $1.50, an annual replacement policy was the optimum action, regardless of the in-lay period. With a higher replacement cost of $1.60 and $1.70 then attention is centered on the levels of feed costs and cull prices. Thus, as the results show, a 0.2 cent increase in the cost of a pound of layers ration resulted in the optimum policy changing from an annual replacement policy to one which required a force molting program.

2. At the lower egg price levels (less than 25.5 cents per dozen) unless the producer is able to either rear or buy replacements for less than $1.40 per bird a force molting action a_{28}, a_{37} or a_{54} would be the optimum action.

3. With egg prices at the intermediate levels of 25.0 to 29.0 cents per dozen, a low replacement cost of $1.20 to $1.30 per bird in the main tended to favor a non-force molted action (a_5, a_6, or

a_7). As the replacement cost increased however, then a force molted action (a_{28}, a_{37} or a_{54}) became optimum at progressively lower total variable cost levels.

4. The hypothesis, proposed earlier, that altering the in-lay period would change the relative ranking of the actions in A' was supported to a certain extent by these results. That such changes did result in some differences in those price/cost combinations for which a force molted action became optimal, was demonstrated. These changes were no doubt due to the manner in which altering the in-lay period changed the form of the egg price cycle over the planning period. Because of the significant differences between the egg production patterns for force molted and non-force molted actions, over the planning period, these changes in the egg price cycle did have different effects on both the monthly pattern of egg income and hence net revenue for the force molted and non-force molted actions. These differences will be shown in a later section on the analysis of the replacement problem under uncertainty.

It is interesting to note that variations in the in-lay period had little effect on those actions in A' which appeared as optimum actions. Thus, of the 20 actions included in A' at the outset of the sensitivity analysis, only four non-force molted action a_4, a_5, a_6 and a_7, and three force molted actions a_{28}, a_{37} and a_{54} appeared as optimum actions.[31] A description of these actions can be found in Table XI (and footnote 28, page 105).

5. The results from the preliminary and sensitivity analyses showed that of the large number of theoretical actions included in the set A, only a limited number of these need to be considered in the final section of this thesis which attempts to look at the problem of choosing between the producer's alternatives under conditions of uncertainty. The seven actions listed in 4 above were included in the set of actions to be considered under conditions of uncertainty. In addition to these, four other actions, a_{32}, a_{43}, a_{46} and a_{47} (see Table XI for a description of these actions) which were consistently very close to appearing as optimum actions, were included in the set $\mathcal{A}$ of eleven actions to be considered under conditions of uncertainty.

6. The ranking procedure used both in the preliminary and the sensitivity analyses showed that for all combinations of variables studied, the top five actions resulted in net revenues which were quite close to each other. Without any information on the degree of variability of net revenue for these actions it was not possible to state categorically that the producer's "best" action would be confined

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[31] These actions are, as might be expected, the same as those which appeared as optimum in the preliminary analysis except for action a_{47} which was added after the preliminary analysis had been completed.

to the optimum actions shown in Table XVI. The methodological procedure for choosing between the top actions will be given in the next chapter.

CHAPTER VI

REFORMULATION OF THE REPLACEMENT PROBLEM UNDER CONDITIONS OF UNCERTAINTY

The results of the analysis given in the previous chapters provide information on the relative ranking of the replacement actions given that the levels of pertinent variables and parameters are known with certainty. The ranking procedure used in the analysis showed that of the original 54 actions included in the set A only seven actions actually appeared as optimum actions for the range of variable and parameter values studied. The ranking procedure also provided information on near optimum actions, i. e. those which, although never attaining the optimum position, were consistently close to being optimum. These optimum and near optimum actions, discussed at the end of the last chapter, were included in the set $\mathcal{A}$ of actions to be considered under conditions of uncertainty. The set was defined as

$$\mathcal{A} = \{a_4, a_5, a_6, a_7, a_{28}, a_{32}, a_{37}, a_{43}, a_{46}, a_{47}, a_{54}\}$$

A description of these actions can be found in Table XI, page 102.

Thus, assuming that one can restrict the field of choice to those actions contained in set $\mathcal{A}$, the replacement problem under conditions of uncertainty can be expressed as one of choosing between these actions given that the expected levels of costs, prices and production variables which define the state of nature prevailing over the length of the planning period are not known with certainty.

The Replacement Problem Under Uncertainty Using the Criterion of Maximizing Expected Utility

One way in which a choice among risky actions can be effected is on the basis of information about the moments of the distribution of net income (X) for each action in $\mathcal{A}$ over the state space Θ, where the space Θ represents all possible states of nature θ which might prevail over the planning period.[32] The procedures for deciding among the alternative actions when information on the moments of these distributions are known have been detailed in the literature on decision making under uncertainty. In particular, Halter and Dean (1969) gave the methodology for the general case where no prior information is available on the form of the distribution of net income (defined as $f(X)$) over the states of nature, and Markowitz (1967) gave the details for the specific case when it is assumed that this distribution is normal.

Information about the moments of the distribution of $f(X)$ can be incorporated into the decision making problem via the producer's

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[32] Where the state set Θ is defined in terms of the price, cost and production variable conditions prevailing over the planning period.

utility function when the choice criterion is one of maximizing expected utility. The manner in which this is achieved has been detailed by Halter and Dean (1969) for the general case mentioned previously. This procedure can be summarized as follows:

Let U(X) denote the utility of some action a where the random variable X represents the continuous outcomes from the action a over the state space Θ. Then expected utility for an action a is given by the equation

$$U(a) = EU(X) = U[E(X)] + \frac{1}{2}\sigma^2 \frac{d^2U[E(X)]}{dX^2} + \frac{1}{3!} g_1 \frac{d^3U[E(X)]}{dX^3} + \frac{1}{4!} g_2 \frac{d^4U[E(X)]}{dX^4} + \dots \qquad 6.1$$

where: the expectation of the constant E(X) = E(X)

the expectation of the constant [X - E(X)] = 0

the expectation of the constant $[X - E(X)]^2$ = σ^2 i.e. the variance of the distribution of X.

the expectation of $[X - E(X)]^3$ = g_1 i.e. the skewness of the distribution of X.

the expectation of $[X - E(X)]^4$ = g_2 i.e. the kurtosis of the distribution of X.

The equation 6.1 above gives the expected utility U(a) for any probability distribution of net income, f(X) over the state space Θ for an action a in terms of: (1) the moments of the distribution f(X), i. e. the mean (E(X)), variance (σ^2), skewness (g_1) and kurtosis (g_2), and (2) the first four derivatives of the utility function.

The number of terms used to calculate U(a) for an action depends firstly, on the number of moments that describe the distribution of X, and secondly the number of derivatives which can be taken from the utility function. Thus, if f(X) is normally distributed only the first two moments of the distribution need be used. Also, if the utility function is described by a quadratic function only the first two derivatives of the function will be non-zero and U(a) will be calculated using only these two terms.

It will be appreciated that if prior information shows f(X) to be normally distributed then the estimation of U(a) from equation 6. 1 is simplified considerably. For the particular replacement problem studied in this thesis no such prior information was available. Because of this, it was necessary to estimate the form of the distribution $f(X_i)$[33] for each of the eleven actions in $\mathcal{A}$ over the state space Θ from available empirical data. This estimation was carried out using a computer simulation procedure. The following sections outline the approach to the estimation of the distribution of net income for each of the actions in $\mathcal{A}$. The next section will be concerned

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[33] The distribution $f(X_i)$ has the 'i' subscript to indicate its dependence on the a_ith action in $\mathcal{A}$.

with consideration of the sources of producers' income variability which must be considered in the estimation procedure. Discussion of the utility function and its incorporation into the decision problem will be delayed until Chapter VII.

The Sources of Producers Income Variability

In order to arrive at estimates of the moments of the distribution of net income over the states of nature that might prevail over the planning period for a particular action in $\mathcal{A}$, it was necessary to first define the sources of income variability. By defining these variables, the decision making problem is placed in a more realistic context in that emphasis is placed on decision making in an ex ante context, i. e. where in planning his replacement policy the producer is planning for a future time period where uncertainties regarding costs, prices and levels of production influence his decision and directly affect his expected net income level.

The major sources of income variability facing the commercial poultry producer are:

1. The future levels of input and output prices where variation in these levels are due to forces external to the producer's sphere of control. Of these prices, the fluctuations in egg prices have by far the greatest influence on the variability of producer income. The form and magnitude of the fluctuations which have occurred over the

past ten years in egg prices were shown in Figure 11, page 84

2. The expected levels of egg production performance. Evaluation of the available technical information suggests that many of the biological, environmental, and physical factors which underlie and influence egg production have not been quantified. It would seem that the level of egg production is subject to wide variation, and also the relationship that exists between egg production, feed consumption and the other variable inputs is also subject to variation that cannot be described by an exact function, i. e. the production function may be stochastic in nature. Observations of commercial producers actual practices also suggest that little direct control is exercised over the short run variable inputs such as feed and water. The accepted practice is to follow an ad lib feeding program so that the precise functional relationship between the amount of feed consumed and the level of output is difficult to ascertain.

3. The distribution of egg size is a source of uncertainty rarely mentioned in the literature, but which nevertheless must be considered of prime importance. Because of existing price differentials, the variation in the distribution of eggs between each of the five market sizes (small, medium, large, extra large, and jumbo) can result in appreciable variation in total revenue from egg sales. The reason for neglecting the egg size distribution in previous studies is difficult to understand considering the large price differentials that

exist between eggs in the small and large egg size categories.

4. As can be seen from Figure 9, egg quality does not maintain a constant level over the life cycle of the bird. Throughout this cycle egg quality is subject to gradual deterioration, resulting in increasing losses in potential egg revenue to the producer. Expected values for the percentage of eggs laid in each grade over the life cycle of a bird have been measured (see Table I, Appendix A), but no information is available on the degree of variability of egg quality. The influence of environmental, managerial and various stress factors on the distribution of egg quality have not been measured. Because of the high price premiums paid on the market for eggs in the A and AA categories, lack of knowledge regarding the degree of variability of the distribution of total egg production between C, B, A and AA quality grades adds to the uncertainty problem facing the producer.

5. Flock mortality is a variable which the producer can control to a limited extent under controlled environment conditions, but which is still subject to random effects beyond his control. As will be seen in the next section, even under intensive conditions variations in the levels of flock mortality still exist.

These then are the major factors affecting income variability facing the commercial egg producer, and consideration of these factors emphasize the fact that in an ex ante sense the producer must make a choice between alternative actions without perfect knowledge about the state of nature likely to prevail over the planning period. The producer's decision making process is made even more difficult by the lack of reliable information on these sources of uncertainty.

Estimation of the Stochastic Nature of Variables Affecting Expected Net Income Levels

The variables which affect the levels of the producer's expected net income as a result of following one of the eleven actions in $\mathcal{A}$ can also be regarded as describing the continuous range of states of nature contained in the state space Θ_i.[34] Since egg prices vary seasonally and egg production varies with the age of the hen, a convenient aggregate variable to represent the time dimension is a four week period (a month, or stage of production). Also, since available price, cost and production data are usually collected on a monthly basis, it is consistent to regard the state space Θ_i as being composed of a series of 26 subspaces Θ_{ni}, $n = 1, 2, \ldots, 26$. Thus, it was possible to estimate the stochastic nature of the relevant variables on a monthly basis and to use this information to derive the estimates of the monthly distribution of net income $(f(X_{ni}))$. The distribution of net income over the 26 month planning period could

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[34] The state space Θ_i is subscripted to indicate that it is in fact a function of which of the 'i' actions, $i = 1, 2, \ldots, 11$ in $\mathcal{A}$ is being considered.

then be derived on the basis of these monthly estimates. Therefore, in the following sections, the stochastic nature of the relevant price, cost, and production variables will be described in terms of monthly estimates.

Production Variables

Egg Production

Estimates of the expected value of hen-month egg production ($\overline{Q}_{ni}$) and the variance of Q_{ni} (var (Q_{ni})) for each stage of production n for each action in $\mathcal{A}$ were required. Theoretically the best method for deriving these estimates would have involved a prior estimation of the stochastic nature of the production function:

$$Q_{ni} = f(Y_{ni}, X_{1ni}, X_{2ni}, X_{3ni} \mid X_4, X_5, \ldots, X_n)^{35}$$

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[35] Note that: $Q_{ni} = f(Q_{jk})$, where Q_{jk} = the hen-month egg production in the s_{jk}th state of production

and $Y_{ni} = f(Y_{jk})$, where Y_{jk} = the hen-month feed consumption in the s_{jk}th state of production.

Thus, the subscripts 'n' and 'i' are used to indicate the stage of production n for the action i whereas the subscripts 'j' and 'k' are used to indicate the age of the bird j and the force molting treatment k. In the following sections it will often be more convenient to use variables subscripted by stage (n) and action (i), but the functional relationships indicated above should be remembered.

as defined in equation 3.5. However, such an estimation could not be made because of the lack of sufficient data. Many of the biological, environmental and physical factors which affect egg production have not been quantified. Because the present state of information did not allow for a direct estimation of $\overline{Q}_{ni}$ and var (Q_{ni}) it was decided to use an indirect estimation procedure. The lack of sufficient treatment replication in the Washington and Wye College trials resulted in estimates of $\overline{Q}_{jk}$ for many of the s_{jk} states of production based on only one to three treatment replicates. Without secondary sources of information on the $\overline{Q}_{jk}$ values, these trial estimates listed in Table I of Appendix A had to be used in the analysis. Obviously with so few degrees of freedom it would have been impossible to obtain significant estimates of var (Q_{jk}) from the trial data. It would have been interesting to have subjected the hypothesis that var (Q_{jk}) was a constant and not a function of age (j) or force molting treatment (k) to statistical test. Without sufficient experimental data such a procedure could not be followed. Instead, in order to obtain estimates of var (Q_{jk}) it was necessary to use a data source that provided sufficient treatment replication for these estimates to be made for at least some of the s_{jk} states of production. This data was supplied by the Cornell Agricultural Experiment Station trials. It was necessary to assume that estimates made from this data source could be applied to explain the variability in monthly egg production for those states of production for which estimates could not be derived.

The procedure used was to calculate monthly variance estimates of egg production on a per bird basis from the Cornell University data. Eleven treatment replicates were available from these trials over two years of egg production on which to base these estimates. Prior to making these estimates the following hypotheses were proposed:

$$\begin{aligned}
Ho^1:\ \mathrm{Var}(Q_{jk}) &= \mathrm{Var}(Q_{ik}) \\
Ha^1:\ \mathrm{Var}(Q_{jk}) &\neq \mathrm{Var}(Q_{ik}) \\
k &= 0 \\
i &\neq j \\
i &= 1, 2, \ldots, 13 \\
j &= 1, 2, \ldots, 13
\end{aligned}$$

$$\begin{aligned}
Ho^2:\ \mathrm{Var}(Q_{jk}) &= \mathrm{Var}(Q_{ik}) \\
Ha^2:\ \mathrm{Var}(Q_{jk}) &\neq \mathrm{Var}(Q_{ik}) \\
k &= 3 \\
i &\neq j \\
i &= 14, 15, \ldots, 26 \\
j &= 14, 15, \ldots, 26
\end{aligned}$$

The first hypothesis was that all the variances of monthly egg production over the stages of the first year of production ($j = 1, 2, \ldots, 13$) were equal. The second was that all the variances over the stages of the second year of production (j = 14, 15, . . . , 26) were also equal. The Bartlett test of variances[36] was used to test these hypotheses. The test lead to a failure to reject either Ho^1 or Ho^2 at the 5 percent significance level, i.e. there were no significant differences between the monthly variability of egg production over either the first or second years of production at this significance level.

Following on from this result, pooled estimates of the monthly variance of hen-month egg production were estimated for the first year (S_1^2), the second year (S_2^2) and the combined first and second years (S_{1+2}^2) using the procedure given by Bartlett. These estimates were:

$$S_1^2 = 0.14322$$

$$S_2^2 = 0.14655$$

A pooled variance estimate of hen-month egg production for the 26 months of production produced an estimate of

$$S_{1+2}^2 = 0.15010 = \sigma_q^2$$

It was assumed, on the basis of these results, that the variance

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[36] See Donald. A. S. Fraser. 1964. Statistics: an introduction. 3d. ed. New York, John Wiley and Sons. p. 256.

of monthly per bird egg production for each of the s_{jk} states of production (and hence for each of the N stages of production for each action in $\mathcal{A}$), was a constant equal to 0.15010 and that the distribution of egg production for each monthly stage of production (n) could be represented as

$$Q_{ni} \sim N(\bar{Q}_{ni}, 0.15010) \qquad 6.2$$

Egg Size Distribution

The degree of variability of the distribution of hen-month egg production between the five egg sizes (small, medium, large, extra large and jumbo) could not be estimated, even by indirect methods. Data needed to make such estimates were not available from any of the experimental trial sources. A thorough study of the literature indicated that this is one source of egg production variability which has yet to be subjected to empirical measurement. It was possible to obtain estimates of the expected values for the percentage of the total hen-month egg production in each of the five egg sizes. For most stages of production these were based upon only a few observations and should therefore be regarded as only approximations. The estimation of these mean percentages were discussed in Chapter III. The monthly egg size percentages (GD_{gjk}) used in the analysis are listed in Table I of Appendix A.

Egg Quality Distribution

This was the second of the important sources of uncertainty discussed previously on which no experimental data were available. Obtaining estimates of the quality distribution between the different sizes of eggs was extremely difficult because of the lack of sufficient measurements on the flocks in the experimental force molting trials. Very little attention has been paid to the variability of egg quality as a potential source of income variability. One would imagine however that in the light of the premiums that the producer is able to obtain for high quality (A and AA grade) eggs, that the collection of information on this parameter would have some economic justification.

The present state of information allowed only estimates of the mean percentage of total eggs laid per bird in each quality grade for each state of production to be obtained. These figures for GD_{gjk} are listed in Table I of Appendix A.

Feed Consumption

Theoretically the best method for including the variability of the hen-month feed consumption into the analysis would have been via the production function. However, in the discussion of the distribution of monthly egg production, it was stated that the form of the production function could not be estimated due to the fact that many of the factors which underlie this function have yet to be quantified.

Because it was not possible to approach this problem directly through the production function, an indirect method was used to incorporate the variability in hen-month feed consumption into the uncertainty analysis.

The method used involved expressing hen-month feed consumption as a function of percentage hen-month egg production, age of the bird, and the force molting procedure used. Thus:

$$Y_{jk} = f(X_{1jk}, X_{2jk}, X_{3jk}, \ldots, X_{7jk} \mid X_8, X_9, \ldots, X_n)$$

where: Y_{jk} = hen-month feed consumption in pounds

X_{1jk} = percent hen-month egg production (see Table I, Appendix A)

X_{2jk} = age of the bird in months

X_{3jk} = 1 prior to the first force molt
0 otherwise

X_{4jk} = 1 for those stages of production prior to the second and after the first force molt
0 otherwise

X_{5jk} = 1 for those stages of production prior to the third and after the second force molt
0 otherwise

X_{6jk} = 1 for those stages of production after the third force molt
0 otherwise

X_{7jk} = 1 for the first stage of production after a force molt

0 otherwise

$X_8, X_9, \ldots, X_n$ = fixed environmental factors such as lighting, housing, and the general level of husbandry.

On the basis of the above functional relationship the following model was proposed:

$$Y_{jk} = \beta_0 + \beta_1 X_{1jk} + \beta_2 X_{2jk} + \ldots + \beta_7 X_{7jk} + S_Y$$

assuming 1. $Y_{jkm} \neq Y_{jkn}$, $m \neq n$, i.e. Y_{jk} values are uncorrelated,

and 2. $S_Y \sim N(0, \sigma_Y^2)$.

A stepwise regression analysis of this model using the data collected from the force molting trials (listed in Table I of Appendix A) produced the following estimated relationship:

$$\hat{Y}_{jk} = 6.27280 + 0.01555X_{1jk} - 0.01902X_{2jk} - 0.52251X_{3jk} + 0.19453X_{5jk} + 0.73363X_{7jk} \qquad 6.3$$

$$R^2 = 0.5779$$

$$\hat{\sigma}_Y^2 = 0.141689 \text{ (estimates } \sigma_Y^2\text{)}$$

$$S_Y \sim N(0, 0.14169)$$

The procedure for incorporating this relationship into the estimation of net income variability will be given in a later section.

Flock Mortality

The effect of the variability of flock mortality could theoretically have been included in the analysis using one of the following methods:

1. By obtaining estimates of the mean and variance of monthly flock mortality for each of the s_{jk} states of production directly from the experimental data. Insufficient data were available from experimental sources to allow this procedure to be followed.

2. By obtaining the mean and variance of the distributions of annual flock mortality for each of the actions in $\mathcal{A}$. The monthly estimates could then be obtained by dividing the expected value for annual flock mortality by 13[37] (assuming a constant variance for each month of production). The experimental data allowed for estimates of the means of these annual distributions but not of the variances. The variances were obtained from data collected from 80 commercial flocks in California (Bell, 1964, 1965a, 1965b; Stratton, 1964, 1965a, 1965b). A histogram of the California data showing the distribution of

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[37] A classical least squares regression analysis of the mortality figures from the Cornell trials showed that there was a significant positive linear relationship between the age of the flock and the percentage accumulative mortality. This relationship held for both first and second years of production for the trial flocks. This analysis is given in Appendix B.

this mortality was constructed. This is shown in Figure 13. The data used to derive this figure are given in Table II of Appendix B. This histogram shows that the mortality distribution had a significant positive skew. Eidman, Carter and Dean (1968) described a similar mortality distribution for turkey flocks. They used a lognormal function to describe this distribution. Visual inspection of Figure 13 indicated that this distribution might also provide a good fit to describe mortality among poultry flocks.

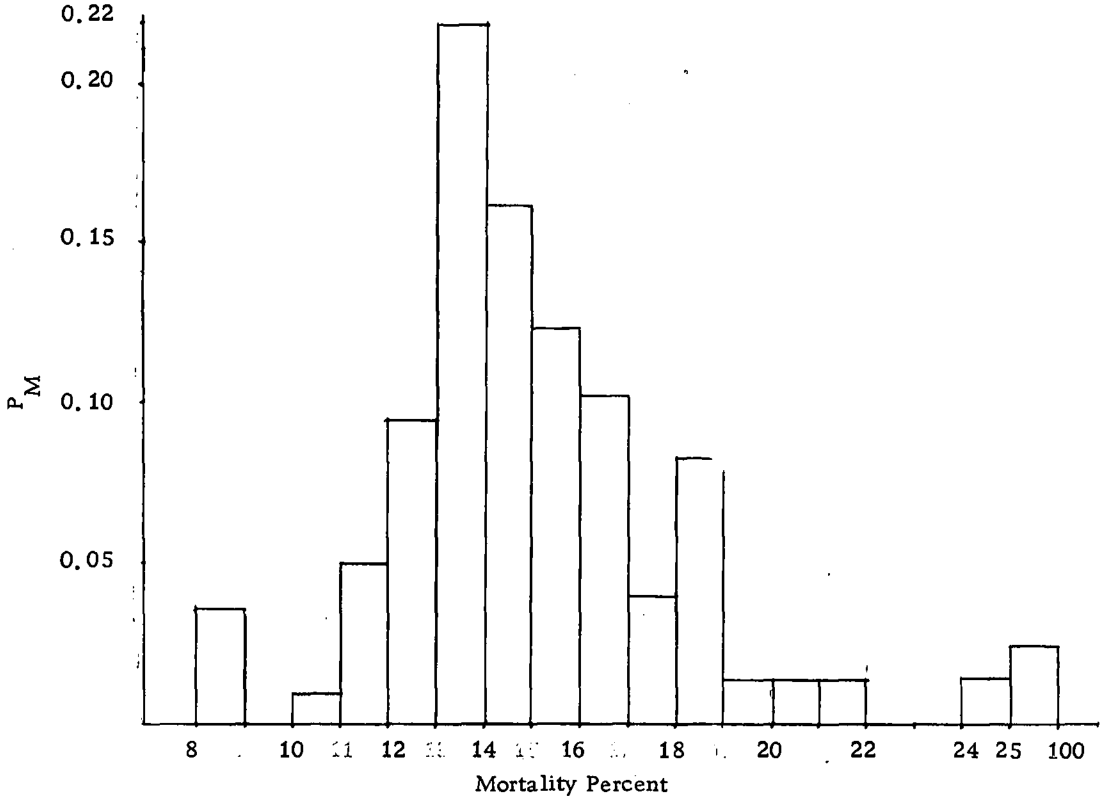

Figure 13. A histogram of the accumulative mortality for 80 California flocks over the first year of production.

The lognormal distribution permitted the fitting of the normal curve to the logarithmic transformations of the observations, thus making the function relatively easy to handle in the analysis. The following analysis is after Aitchison and Brown (1957, p. 37-54).

Let M = a positive variate $(0 < m \leq \infty)$ be the annual percentage mortality for a flock of birds.

Then if $Y = \log M \sim N(\mu_M, \sigma^2_M)$, then $M \sim \Lambda(\mu_M, \sigma^2_M)$ and the distribution of M is completely specified by μ_M and σ^2_M (Note that m is a positive variate such that $P_M = \Pr(0 \leq m \leq M)$).

A plot of log M against P_M on probability paper provided a quick check as to whether the mortality distribution shown in Figure 13 might feasibly be regarded as lognormally distributed. A plot of log M against P_M (figures shown in Table II of Appendix B) resulted in a positive linear relationship between log M and P_M indicating a lognormal fit such that:

$$M \sim \Lambda(\mu_M, \sigma^2_M)$$

A subsequent χ^2 goodness of fit test showed that the hypothesis $M \sim \Lambda(\mu_M, \sigma^2_M)$, could not be rejected at the 5 percent significance level. Estimates of μ_M and σ^2_M were obtained from the plot of log M against P_M. These estimates were:

$$\overline{M} = 14.462$$

and $$S_M^2 = 1.375.$$

Estimates of μ_M and σ_M^2 were also calculated using the maximum likelihood method given by Aitchison and Brown. This method yielded the estimates

$$\overline{M} = 14.765$$

and $$S_M^2 = 1.471$$

which were close to those obtained from the plot of log M against P_M.

It followed from these estimates that

$$M \sim \Lambda(14.765,\ 1.471)$$

and $$Y = \log M \sim N(3.8607,\ 0.3804) \qquad 6.4$$

Unfortunately the value of $\overline{M}$ estimated above could not realistically be applied to all actions in $\mathcal{a}$ over the two year planning period. For, as was shown in Table VII, the force molting procedure had a significant effect on the expected mortality level over the planning period. In order to account for these differences it was assumed that the variance estimate $S_M^2 = 1.471$ given above would hold for all actions in $\mathcal{a}$ over both first and second years of the planning period[38] and that the differences in flock mortality between the various actions would be reflected simply in the mortality percentages listed in Table VIII. These values were estimated on the basis of results from the force molting experiments as described in Chapter IV.

Egg Prices

The seasonal and cyclical fluctuations in producer egg prices have been the major source of income variability in the commercial poultry industry. The form of these fluctuations was shown in Figure 11. This figure showed the movement of the mean monthly egg price over a ten year period. It was apparent from the figure that over the past ten years, egg prices have shown cycles of annual periodicity. Although this periodicity has been repeated from year to year, those months when egg prices have been at their highest (peaks) and lowest (troughs) have varied somewhat between years.

It will be seen from the monthly grade prices listed in Table I of Appendix B that yet another facet of the egg price cycle has been the extent of price differentials between the different grades of eggs

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[38] This assumption was based on the testable hypothesis that the variance of the monthly mortality distribution was a constant, i. e. it was independent of force molting procedure and age of the bird.

at different periods of the egg price cycle. Of special interest in accounting for egg price variability are the price differentials for the lower five egg grades, i. e. for small AA through large B. In general the large A--large AA differential has remained fairly constant at about 1.5 cents per dozen, as was noted in Chapter IV. Visual inspection of the ten years price data listed in Appendix B indicated that the grade differentials between the lower five grades of eggs have maintained a fairly constant relationship to each other over time, i. e. these differentials have been smallest during the period of peak prices (in the autumn and winter months) and largest during the periods of trough prices (spring and summer months). The following procedure was used to incorporate both the monthly fluctuations in the egg price cycle and the differential relationship between the various egg grades within a particular month, into the description of the state spaces $\ominus_{ni}$:

1. It was decided to use the monthly prices of small AA grade eggs (P_{1t}) as <u>indicator prices</u> upon which to base price estimates of medium AA, large B and large A grade eggs. Using the ten years price data for small AA eggs listed in Appendix B, a series of 13 discrete frequency distributions were derived--one for each month of the price year. These distributions were based on ten observations per month. These frequency distributions were then used to derive accumulative probability distributions for small AA grade eggs.

Thus, for a particular month of the year (t) the accumulative distribution for small AA grade eggs ($F(P_{1t})$) could be represented as follows:

$$F(P_{1t}) = \begin{cases} 0.0, & P_{1t} < 17 \\ 0.2, & 17 \leq P_{1t} < 21 \\ 0.4, & 21 \leq P_{1t} < 23 \\ 0.7, & 23 \leq P_{1t} < 26 \\ 1.0, & 26 \leq P_{1t} \end{cases}$$

2. The monthly small AA grade price was used here as an indicator price from which other differential grade prices could be estimated from the following equations:

$$P_{2t} = \beta_{1,0} + \beta_{2,1} P_{1t} + S_2$$

$$P_{3t} = \beta_{2,0} + \beta_{3,1} P_{1t} + \beta_{3,2} P_{2t} + S_3$$

$$P_{4t} = \beta_{3,0} + \beta_{4,1} P_{1t} + \beta_{4,2} P_{2t} + \beta_{4,3} P_{3t} + S_4$$

where:

P_{2t} = the price of medium AA grade eggs in cents per dozen

P_{3t} = the price of large B grade eggs in cents per dozen

P_{4t} = the price of large A grade eggs in cents per dozen

S_2, S_3 and S_4 are disturbance terms.

The values of the parameters of these models were estimated using classical least squares regression procedures under the assumptions that:

$$S_2 \sim N(0, \sigma^2_{P_1}),$$

$$S_3 \sim N(0, \sigma^2_{P_2}),$$

and $$S_4 \sim N(0, \sigma^2_{P_3}).$$

The regression analysis produced the following estimated relationships:

$$\hat{P}_{2t} = 10.34027 + 0.92837\ P_{1t} \qquad (R^2 = 0.7776) \qquad 6.5$$

$$\hat{\sigma}^2_{P_2} = 6.3044, \text{ hence } S_2 \sim N(0,\ 6.3044)$$

$$\hat{P}_{3t} = 5.16122 + 0.10409\ P_{1t} + 0.64742\ P_{2t} \qquad 6.6$$

$$(R^2 = 0.7269)$$

$$\hat{\sigma}^2_{P_3} = 6.2655, \text{ hence } S_3 \sim N(0,\ 6.2655)$$

$$\hat{P}_{4t} = 8.16419 - 0.31381\ P_{1t} + 0.78122\ P_{2t} \qquad 6.7$$

$$+ 0.40368\ P_{3t} \qquad (R^2 = 0.8718)$$

$$\hat{\sigma}^2_{P_4} = 3.3142 \text{ hence } S_4 \sim N(0,\ 3.3142).$$

Thus, given an initial value for P_{1t} in a particular month t (stage of production n), it was possible to generate successive estimates of P_{2t}, P_{3t} and P_{4t} for that month on the basis of the relationships in 6.5, 6.6 and 6.7 above. Estimates for the prices of large AA (P_{5t}), extra large AA (P_{6t}) and jumbo AA (P_{7t}) grade eggs were then obtained by adding a constant differential to the estimated price for P_{4t} such that:

$$P_{5t} = P_{4t} + 1.5 \qquad 6.8$$

$$P_{6t} = P_{4t} + 3.5 \qquad 6.9$$

$$P_{7t} = P_{4t} + 5.5. \qquad 6.10$$

The procedure for incorporating the relationships estimated in 6.5 through 6.10 above into the uncertainty analysis, and the estimation of producers net income variability, will be detailed later in this chapter.

Summary

This section has outlined the procedures used to estimate the distributions of the variables describing the states of nature Θ_{ni}. Indirect methods for estimating some of these distributions were discussed. Some important distributions could not be obtained because of lack of sufficient information.

Before embarking upon a description of the procedure used to estimate distribution of net income $f(X_i)$ for each action over the states of nature Θ_i some of the salient points which have been noted in the various parts of the preceding discussion need to be emphasized.

It should be obvious that, because of the lack of good experimental information, the derivation of indirect estimates for the production variables merely sufficed to demonstrate estimation methods, rather than provide the reliable information on the stochastic nature of these variables as was required for the uncertainty analysis. The result of these indirect methods was that general estimates were obtained which did not allow differences between the variables describing the states of nature for the various actions to be detected, especially possible differences between force molted and non-force molted actions.

In view of these points then, the contents of this and following sections of the chapter can best be regarded as describing methodological procedures which could be followed in obtaining better estimates of $f(X_i)$ at some later period when sufficient experimental information becomes available. However, the approximations used in the analysis will serve to show how, given good estimates at the outset, these might be used to define the producer's best course of action under conditions of uncertainty.

The Derivation of the Moments of the Distribution of Net Income Over the States of Nature for Each Action in $\mathcal{A}$

A necessary condition for obtaining the information needed to effect a choice between the producer's alternative actions was the estimation of the distribution of net income $(f(X_i))$ over the state space Θ_i for each of the eleven actions in $\mathcal{A}$. As was stated previously, it was proposed to approach this problem by first determining the monthly distributions of net income $(f(X_{ni}))$ over the state spaces Θ_{ni} for each action, and then to use these estimates to arrive at the distribution of $f(X_i)$. Theoretically such a procedure requires prior specification of the distribution of the states of nature over the state set Θ_{ni} for each month (stage) of production, for each of the eleven actions in $\mathcal{A}$. However, any attempts to specify these distributions analytically would have involved extreme difficulties (remembering that these distributions are continuous or discrete functions of the six variables discussed in the previous section). The analytical problem was further complicated when one considers the introduction of the in-lay period as a parameter whose particular value alters the form of the state space Θ_{ni} due to its effect on the form of the egg price cycle over the planning period.

Fortunately, it was not necessary to attempt an analytical solution to this complex estimation problem since a simulation[39]

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[39] A simulation of a system or an organism is the operation of a model or simulator which is a representation of the system or organism. The model is amenable to manipulations which would be impossible or too expensive or unpractical to perform on the entity it portrays. The operation of the model can be studied and, from it, properties concerning the behavior of the actual system or subsystem can be inferred (Shubik, 1960, p. 909).

procedure provided a method whereby estimates of the distribution $f(X_{ni})$ could be obtained without first having to specify Θ_{ni}. The simulation procedure used briefly was as follows: For a particular action a_i and a given in-lay period, 100 sets of 'observations' of the variables such as monthly egg production, feed consumption, mortality and egg prices were generated using the estimated distributions and relationships described previously in equations 6.2 - 6.10. These monthly 'observations' were then used to calculate 100 estimates of expected monthly net income $(E(X_{ni}))$. The estimates were then used to derive the distribution of $f(X_{ni})$ and to calculate the moments of this distribution. This procedure was repeated for each of the 26 months of the planning period. The monthly estimates of $f(X_{ni})$ $n = 1, 2, \ldots, 26$, were then used to estimate the moments of the distribution of $f(X_i)$. This procedure was then repeated for each of the eleven actions in $\mathcal{A}$ and for the thirteen in-lay periods considered.

During the course of this procedure it was possible to hypothesize the form of the distribution of $f(X_{ni})$, and to subject this hypothesis to a statistical test. The hypothesis was that:

$$f(X_{ni}) \sim N(\mu_{X_{ni}}, \sigma^2_{X_{ni}}), \quad n = 1, 2, \ldots, 26 \qquad 6.11$$

$$i = 1, 2, \ldots, 11$$

i. e. that expected monthly net income was distributed normally with an expected value equal to $\mu_{X_{ni}}$ and a variance of $\sigma^2_{X_{ni}}$. The statistical test used in the simulation procedure is given in Appendix C.

With this outline of the proposed simulation procedure, the next chapter presents the results of the simulation runs. A detailed description of the simulation routine along with a flow diagram can be found in Appendix C.

Summary

The first section of this chapter discussed the problem of choosing between the alternative actions in $\mathcal{A}$ under conditions of uncertainty. The method for effecting such a choice, using the criterion of maximizing expected utility, was proposed. It was seen that in order to derive the expected utility from a particular action required information on (1) the distribution of net income accruing to the producer as a result of following that action, and (2) the form of the producer's utility function. This chapter has been concerned with problems associated with obtaining information on (1) above.

The next chapter will present the results of the simulation runs, and will show how these results along with information on the form of the producer's utility function can be combined to effect a choice between the producer's alternative actions under conditions of uncertainty.

CHAPTER VII

RESULTS OF THE UNCERTAINTY ANALYSIS

Moments of the Distribution of Net Income for Each Action in $\mathcal{A}$

The simulation procedure outlined in the previous chapter produced estimates of the moments of the distribution of net income $f(X_i)$ for each of the eleven actions in $\mathcal{A}$ for each of the thirteen in-lay periods. The simulation procedure incorporated a subroutine to test the hypothesis that each of the monthly distributions of net revenue $f(X_{ni})$, $n = 1, 2, \ldots, 26$, was normally distributed. The test was based on the chi squared statistic (Hogg and Craig, 1967, p. 303). This hypothesis failed to be rejected at the five percent significance level for all $f(X_{ni})$. On the basis of this result the moments of the $f(X_i)$ distributions were estimated using a simple summation procedure. Thus, if $E(X_{ni})$ and $\text{var}(X_{ni})$ were the expected value and variance of the distribution $f(X_{ni})$, i.e. $f(X_{ni}) \sim N(E(X_{ni}), \text{var}(X_{ni}))$, then the expected value and variance of $f(X_i)$ were estimated from the relationships:

$$E(X_i) = \sum_{n=1}^{N} E(X_{ni})$$

and $$\operatorname{var}(X_i) = \sum_{n=1}^{N} \operatorname{var}(X_{ni}), \quad n = 1, 2, \ldots, 26 \quad i = 1, 2, \quad , 11$$

The values for the expected values $E(X_i)$, and the standard errors $(\operatorname{var}(X_i))^{1/2}$, of the net income distributions $f(X_i)$ obtained from the simulation procedure, are shown in Table XVII. The tabulated results are discussed in the following sections.

Expected Net Income Levels

The results given in Table XVII show that the non-force molted actions a_4, a_5, a_6 and a_7 produced lower net income levels than the force molted actions a_{28}, a_{32}, a_{37}, a_{39}, a_{43}, a_{47} and a_{54} for all of the in-lay periods considered. The levels of $E(X_i)$ for the four non-force molted actions were close to each other; no more than \$20 separating these levels in any in-lay period. The income levels for actions a_5 and a_6 were consistently higher than for actions a_4 and a_7. The levels of $E(X_i)$ for the seven force molted actions were more widely dispersed than for the non-force molted actions; the difference between the highest (a_{37}) and lowest (a_{39}) values was about \$130 for most in-lay periods.

Of the eleven actions in $\mathcal{A}$, a_{37} produced the highest net income for all the in-lay periods, while actions a_{32} and a_{39} produced the lowest net incomes for periods 9, 10 and 11, and periods

Table XVII. Means and standard errors of the net income distributions for actions in the set $\mathcal{A}$.

Action a_i	Mean and Standard Error of f(Xi)	In-Lay Period												
		1	2	3	4	5	6	7	8	9	10	11	12	13
a_4	$E(X_4)$	510	500	497	496	498	510	508	515	520	522	528	524	519
	$S.E(X_4)$	326	324	321	360	384	417	431	434	452	461	437	400	367
a_5	$E(X_5)$	519	508	501	499	496	500	509	516	519	523	527	530	525
	$S.E(X_5)$	321	318	307	324	354	404	431	439	444	463	469	447	378
a_6	$E(X_6)$	520	509	503	499	499	500	507	516	521	524	527	530	528
	$S.E(X_6)$	329	323	308	332	355	418	438	437	447	474	473	454	403
a_7	$E(X_7)$	514	504	497	494	490	493	498	505	512	513	517	522	522
	$S.E(X_7)$	354	304	291	293	317	372	426	442	452	461	478	472	425
a_{28}	$E(X_{28})$	605	595	593	592	592	584	581	589	596	602	607	612	611
	$S.E(X_{28})$	737	723	724	735	730	719	700	699	727	754	769	772	758
a_{32}	$E(X_{32})$	531	517	513	510	505	501	500	506	513	517	533	533	537
	$S.E(X_{32})$	568	529	494	524	527	536	542	557	591	597	608	625	613
a_{37}	$E(X_{37})$	624	611	602	598	599	596	601	610	619	624	626	629	630
	$S.E(X_{37})$	686	670	641	636	642	657	671	679	702	722	723	720	709
a_{39}	$E(X_{39})$	490	475	469	466	469	475	483	491	519	522	519	517	515
	$S.E(X_{39})$	589	571	550	550	551	575	589	602	604	606	589	589	582
a_{43}	$E(X_{43})$	562	555	553	552	554	557	563	569	572	575	576	575	569
	$S.E(X_{43})$	506	489	475	487	499	537	553	561	577	599	598	584	548
a_{47}	$E(X_{47})$	579	569	557	551	550	557	563	572	581	584	587	589	588
	$S.E(X_{47})$	632	605	557	538	539	577	604	628	656	671	685	687	660
a_{54}	$E(X_{54})$	577	572	571	569	572	572	576	581	581	578	579	583	585
	$S.E(X_{54})$	522	513	508	520	523	550	563	559	570	580	569	560	542

Notes: Tabulated figures were derived assuming a flock size of 100 birds housed at point-of-lay. A description of the actions in this table can be found in Table XI.

1-8, 12 and 13 respectively.

The Variance of Net Income

The most important information obtained from the simulation procedure concerned the differences in income variability between non-force molted and force molted actions. The results in Table XVII show that force molted actions were subject to significantly higher variability than the non-force molted actions. This result could be explained in part by the severe fluctuations in egg production and feed consumption over the planning period for the force molted actions as compared to the relative stability of these variables for the non-force molted actions (see Figures 2-6). If this were the case one would expect the action with the highest frequency of force molting (e. g. a_{28}--three molts over the planning period) to have the highest net income variability. As can be seen from Table XVII this was in fact the case. This argument was further supported by noting that there was a direct correlation between frequency of force molting and the level of income variability. Thus, the actions a_{28}, a_{37} and a_{43} with, three, two and one molt respectively over the planning period resulted in income variances such that

$$\mathrm{var}(X_{28}) > \mathrm{var}(X_{37}) > \mathrm{var}(X_{43})$$

The In-Lay Period

The results in Table XVII show that for the non-force molted actions a_4, a_5 and a_6, expected net income can be maximized by ensuring that the first stage of production for these actions begins at in-lay period 10, 11 or 12. For action a_7, and the force molted actions a_{28}, a_{32}, a_{37}, a_{39}, a_{43}, a_{47} and a_{54} the best in-lay periods are 11, 12 and 13. Thus, for most of the actions the optimum in-lay date lies between the end of August and the beginning of January of the first year of the planning period.

The following sections show how the results given in Table XVII together with information on the form of the producer's utility function can be used to effect a choice between the actions in $\mathcal{A}$ under conditions of uncertainty.

The Mean-Variance Efficiency Frontier

A useful initial theoretical framework for choosing between the actions in $\mathcal{A}$ is given by the mean-variance (E-V) efficiency framework. This method defines a boundary (frontier) as one providing the minimum variance (or standard error) of net income for each level of expected net income. In decision making terms, any action not lying on the efficiency frontier is dominated by those that do. Thus, because the distribution of net income for each of the actions in $\mathcal{A}$

was determined to be normal (i.e., specified by only two moments), it was possible to use this method as a convenient way of initially reducing the producer's field of choice to those actions lying on the efficiency frontier.

Figures 14 and 15 show two such frontiers derived for in-lay periods 1 and 9 respectively, from the results given in Table XVII. Table XVIII lists the actions lying on the frontier for the thirteen in-lay periods considered. Table XVIII shows that: (1) of the eleven actions in $\mathcal{A}$ only actions a_{32}, a_{39} and a_{47} were completely dominated, i.e. they did not appear on the efficiency frontier for any of the in-lay periods, and (2) the actions lying on the frontier changed according to the in-lay period.

The Criterion of Maximizing Expected Utility

Maximizing expected utility will select an action along the E-V frontier for any producer. Since this study was not intended to derive utility functions for individual producers, the procedure of applying the criterion of maximizing expected utility will be illustrated with theoretical utility functions.

The approach used was to derive three theoretical utility functions which reflected risk preference, indifference and aversion for money gains. The object was to demonstrate how the choice of the optimum action, i.e. that which maximizes the producer's

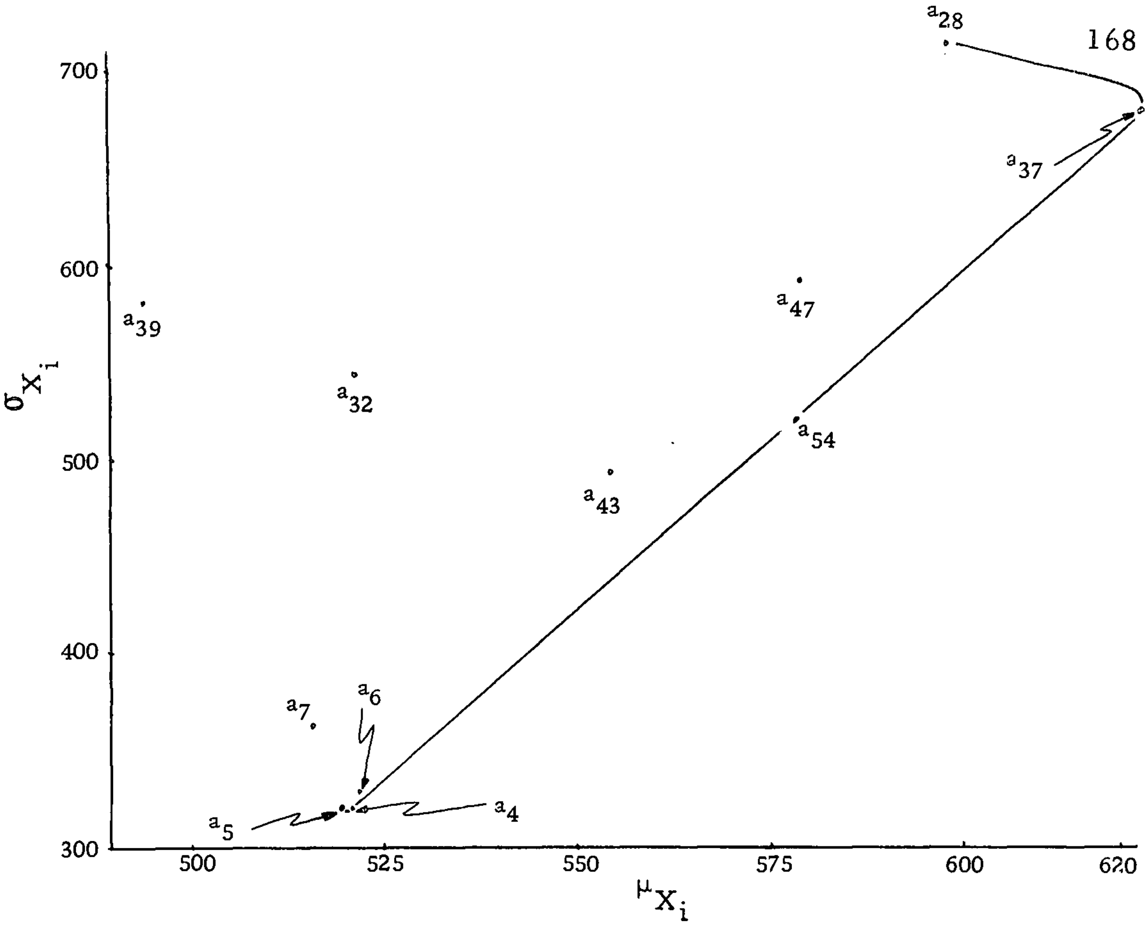


Figure 14. Action on the mean-variance efficiency frontier. In-lay period 1.

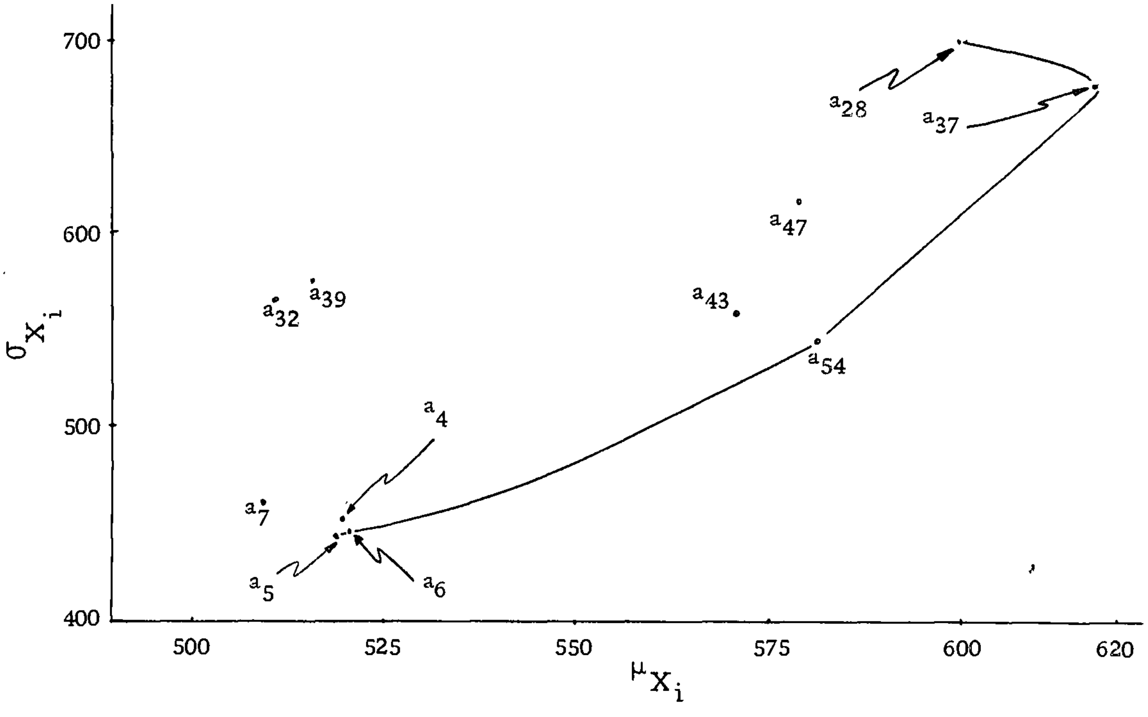


Figure 15. Actions on the mean-variance efficiency frontier. In lay-period 9.

Table XVIII. Actions lying on the mean-variance frontier for each of the thirteen in-lay periods.

	In-lay Period												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Actions Lying on the E-V Frontier	a_4	a_5	a_5	a_5	a_7	a_4	a_4	a_4	a_4	a_4	a_4	a_4	a_4
	a_5	a_7	a_6	a_7	a_{28}	a_7	a_5	a_5	a_5	a_5	a_5	a_{28}	a_5
	a_6	a_{28}	a_7	a_{28}	a_{37}	a_{28}	a_7	a_6	a_6	a_7	a_6	a_{37}	a_6
	a_{28}	a_{37}	a_{28}	a_{37}	a_{54}	a_{37}	a_{28}	a_7	a_7	a_{28}	a_{28}	a_{54}	a_{28}
	a_{37}	a_{54}	a_{37}	a_{54}		a_{43}	a_{37}	a_{28}	a_{28}	a_{37}	a_{37}		a_{37}
	a_{54}		a_{54}			a_{54}	a_{43}	a_{37}	a_{37}	a_{54}			a_{54}
							a_{54}	a_{54}	a_{54}				

NOTE: Key to actions in the table.

a_4 = non-force molted action. First flock kept in-lay for 15 months, and the second for 10 months.

a_5 = non-force molted action. First flock kept in-lay for 14 months, and the second for 11 months.

a_6 = non-force molted action. First flock kept in-lay for 13 months, and the second for 12 months.

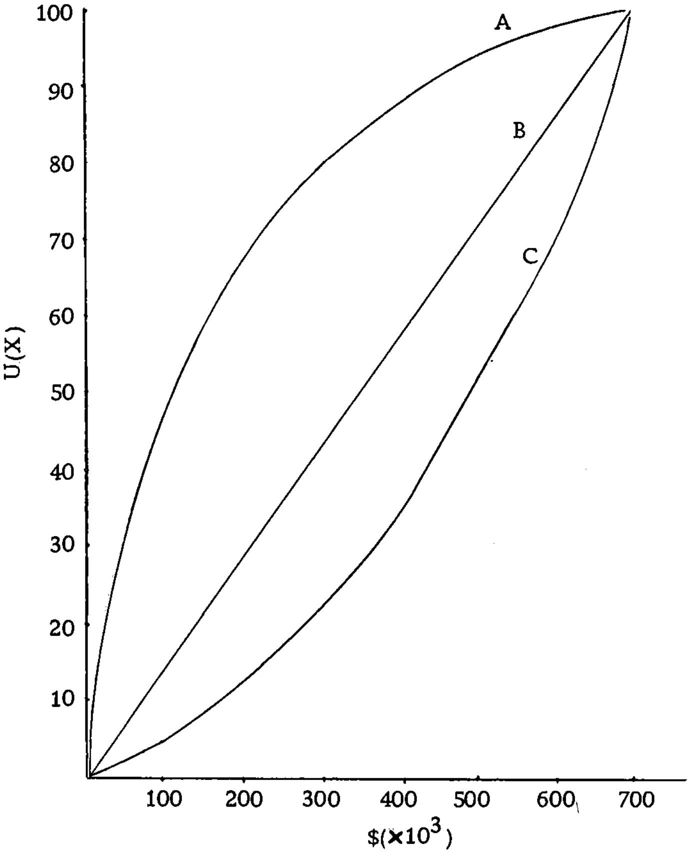
a_7 = non-force molted action. First flock kept in-lay for 12 months, and the second for 13 months.

a_{28} = force molted action. Three molts; at the 6, 13 and 19th month of the two year planning period.

a_{37} = force molted action. Two molts; at the 9 and 19th month of the two year planning period.

a_{43} = force molted action. One molt; at the 13th month of the two year planning period.

a_{54} = force molted action. One molt; at the 17th month of the two year planning period.



KEY: A $U(X) = 0.296X - 0.0002095X^2$, $0 \leq X \leq 700$
Risk aversion for money gains

B $U(X) = 0.143X$
Risk indifference

C $U(X) = 0.015X + 0.000182X^2$, $0 \leq X \leq 700$
Risk preference for money gains

Figure 16. Three derived utility functions.

$$U(a_i) = U[E(X_i)] + \frac{1}{2} \operatorname{var}(X_i) \frac{d^2 U[E(X_i)]}{dX^2} \qquad 7.4$$

as given by equation 6.1. The formula gives the expected utility for any action when the parameters of the utility function are inserted. For the three utility functions given above, the formula provides the following three statements for utility in terms of the mean and variance of net income:

$$U(a_i) = 0.015[E(X_i)] + 0.000182 [E(X_i)]^2 + 0.000182 [E(X_i)] \operatorname{var}(X_i) \qquad \text{From 7.1} \qquad 7.5$$

$$U(a_i) = 0.296 [E(X_i)] + 0.0002095 [E(X_i)]^2 - 0.0002095 [E(X_i)] \operatorname{var}(X_i) \qquad \text{From 7.2} \qquad 7.6$$

$$U(a_i) = 0.143 [E(X_i)] \qquad \text{From 7.3} \qquad 7.7$$

Using equations 7.5, 7.6 and 7.7 and the moments of the net income distributions given in Table XVII, the expected utility from each of the actions lying on the efficiency frontier for each in-lay period were calculated. The actions that maximize and minimize the producer's expected utility assuming risk preference, risk aversion and risk indifference are given in Table XIX.

Table XIX. Actions[a] producing the maximum and minimum expected utility for conditions of of risk preference, aversion and indifference.

	Maximum or Minimum	In-lay Period												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Risk Preference[b]	Max	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}
	Min	a_4	a_7	a_7	a_7	a_7	a_7	a_7	a_7	a_7	a_7	a_4	a_4	a_4
Risk Aversion[c]	Max	a_5	a_7	a_7	a_7	a_7	a_7	a_5	a_4	a_5	a_4	a_4	a_4	a_4
	Min	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}	a_{28}
Risk Indifference[d]	Max	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}	a_{37}
	Min	a_{39}	a_{39}	a_{39}	a_{39}	a_{39}	a_{39}	a_{39}	a_{39}	a_7	a_7	a_7	a_{39}	a_{39}

[a] A key to these actions was given in the previous table.

[b] Risk preference: $U(X) = 0.015X + 0.000182X^2$, $0 \le X \le 700$

[c] Risk aversion: $U(X) = 0.296X - 0.0002095X^2$, $0 \le X \le 700$

[d] Risk indifference: $U(X) = 0.143X$, $0 \le X \le 700$

Risk Preference

The results in Table XIX indicate that if the producer has a risk preference he can follow the force molted action a_{28} (three molts over the two year planning period) to maximize his expected utility.[40] A non-force molted action (either a_4 or a_7--depending on the in-lay period) minimized the producer's expected utility.

Risk Aversion

The results in Table XIX show that if the producer has a risk aversion he can follow one of the non-force molted actions a_4, a_5 or a_7 to maximize his expected utility; the particular action chosen being the function of the in-lay period. Thus, where the planning period commences at any one of the in-lay periods 2 through 6 he can follow action a_7 (annual replacement), and for the other in-lay periods he can follow either a_4 or a_5, to maximize his expected utility.[41] The force molted action a_{28} minimized the producer's

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[40] The results showed that action a_{37} (two molts over the two year planning period) was the producer's second best alternative.

[41] Where a_4 = a non-force molted action which calls for the first flock to be kept for <u>15</u> months and then replaced with a second flock which is then kept for <u>10</u> months, and a_5 = a non-force molted action which calls for the first flock to be kept for 14 months and then replaced with a second flock which is then kept in-lay for 11 months.

expected utility.

Risk Indifference

If the producer is indifferent to risk, then his expected utility is simply a function of the expected net income from an action as shown in equation 7.7. In this case, the producer can maximize his expected utility by following action a_{37} (two molts over the two year planning period).

Summary

This chapter has given the derived estimates for the expected value and variance of net income for each action in $\mathcal{A}$ for a range of in-lay periods. The choice of an action from this set was separated into two stages. Firstly, a mean-variance efficiency frontier was constructed for each in-lay period and the choice of action was then centered on those actions lying on this frontier. Secondly, three utility functions were derived for the case where the producer was assumed to have a laying unit capacity of 100,000 birds. A method for affecting the final choice of action using the criterion of maximizing expected utility was given.

It was shown for the particular utility functions used in this study that if the producer had a risk preference he could maximize his utility by following the force molted action a_{28}. If he was indifferent to risks, a_{37} maximized his utility. Where the producer had a risk aversion one of the non-force molted actions, a_4, a_5 or a_7 could be chosen; the particular choice of action being dictated by the in-lay period.

These results, their significance and possible influence on further research work will be discussed in the concluding chapter of this thesis.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

The adverse economic conditions resulting from a downward trend in egg prices and increasing costs of production have resulted in what might be termed a cost-price squeeze in the commercial egg producing industry. These adverse conditions have resulted particularly in increasing costs of livestock depreciation. In an effort to make more efficient use of the most important factor of production, i.e. the laying bird, attention has been focused on the economic feasibility of replacement policies involving the use of extended laying cycles and force molting programs. These policies being considered as alternatives to the annual replacement policies traditionally followed by producers in the industry.

The literature on the subject of programming of production in commercial egg laying operations has concentrated mainly on the determination of optimum replacement policies without inclusion of force molted alternatives. Those few studies which have made such an inclusion, have largely been in the form of specific reports or analyses on a particular force molting trial, and have therefore been fairly narrow in their scope. With few exceptions, the work in these areas has concentrated on the problem of determining optimum flock replacement policies under conditions of certainty. Standard programming methods such as linear and dynamic programming and budgeting methods have been applied to obtain diverse solutions to this problem. The replacement problem facing the commercial producer under conditions of uncertainty has received some attention in the literature, but here again no consideration has been given to the possible inclusion of force molting policies in the producer's set of alternatives.

The failure of past work to consider the importance of force molting policies is due mainly to the fact that information on the possible economic significance of such policies has only recently become available; thus, the most significant experiments, i. e. the Washington State Experiment Station trials and the Wye College trials, were not completed until mid-1966.

The analyses of data from these and other force molting trials have provided some information on the economics of force molting and the role of extended laying cycles in commercial egg production. No attempt has been made to incorporate all available information into a single study aimed at providing firstly, a more complete description of the producer's set of alternative replacement policies (actions) with the inclusion of several force molting policies and secondly, information as to the producer's best choice of action, assuming both conditions of certainty <u>and</u> uncertainty, given this set of alternatives.

This thesis attempted to obtain solutions to some of the problems mentioned above with the following objectives in mind:

1. To develop a methodological procedure for studying the replacement problem under certainty when the set A of producer's alternative actions included several force molted actions. The choice criterion was assumed to be maximizing net revenue. The problem was formulated initially as a decision problem under certainty to illustrate the relevant factors and interrelationships, and to study the choice of optimum action for a wide range of price, cost and production conditions. Emphasis was placed on determining the economic conditions which might result in the introduction of a force molting program into the producer's optimum replacement policy.

2. To study the replacement problem under conditions of uncertainty in order to account for the stochastic nature of price, cost and production variables in the decision making process. The choice criterion was maximizing expected utility. Attempts were made to estimate the moments of the distributions of such variables as egg prices, egg production, feed consumption and flock mortality. These estimates, together with information on the form of the producer's utility function, were then used to show how the producer's choice of optimum action could be effected under conditions of uncertainty.

The certainty analysis was discussed in the context of programming methods. The methods of dynamic and linear programming and simple budgeting were considered. Emphasis was placed in the discussion of dynamic programming as a possible programming method. It was concluded that specification problems prevented the use of dynamic programming in the analysis, and that the use of a simple enumerative procedure would overcome these problems and at the same time allow the study objectives to be attained. Using this enumerative procedure, 54 alternative actions, spanning an assumed two year planning period, were defined. Using a simple economic model, the net revenues accruing to each action in A were estimated for a wide range of price, cost and production conditions. A ranking procedure was then used to define a subset A' of undominated actions, where dominance was defined to be a function of the frequency with which an action in A appeared among the top 20 actions for the whole range of economic conditions. The actions in A' were subject to a sensitivity analysis using restricted price and cost ranges but with a large number of levels within these narrow ranges. Some attention was given to the influence of the in-lay period on the optimum action.

The results of the certainty analysis showed:

1. With a mean egg price over the planning period greater or equal to 32.0 cents per dozen and the cost of a replacement pullet at \$1.50 or less, an annual replacement policy was optimum. With replacement costs of \$1.60-\$1.70 then the levels of feed costs and cull prices became critical in deciding between force molted and

non-force molted actions. The analysis showed that a 0.2 cent per pound increase in the cost of layers ration resulted in the optimum policy changing from an annual replacement policy to one requiring a force molting program.

2. With mean egg prices of less than 25.5 cents per dozen, unless the producer can purchase replacements for less than \$1.40, a force molted action with either 3, 2, or 1 force molts over the two year planning period would be optimum; the choice of action being a function of the in-lay period.

3. With mean egg prices at a level of 25.0-29.0 cents per dozen, a low replacement cost of \$1.20-\$1.30 favored annual replacement. As replacement cost increased however a force molted action became optimum at progressively lower total variable cost levels.

4. The in-lay period was shown to have a significant influence on the choice of action due to its effect on the form of the egg price cycle facing the producer over the planning period.

5. Of the original 54 actions in A only eleven, four non-force molted and seven force molted, actions were classified as undominated. These actions were included in the set $\mathcal{A}$ to be studied under conditions of uncertainty.

6. The ranking procedure used in the certainty analysis showed that for all price, cost and production conditions studied the *top five* actions produced net revenue figures which were close to each other.

Without any information on the degree of variability of net income for each of these actions it was not possible to state categorically that the producer's best action would be confined to the optimum action.

The uncertainty analysis began with a definition of the replacement problem as one of choosing between the alternative actions in $\mathcal{A}$ given that the expected levels of costs, prices and production variables which define the state of nature prevailing over the planning period were not known with certainty. This was followed by a discussion of the choice criterion of maximizing expected utility. The derivation of expected utility in terms of (1) the moments of the distributions of net income for an action, and (2) the producer's utility function, was given.

In order to derive estimates for (1) above, the sources of producers' income variability were first defined. These were: (1) egg prices, (2) egg production, (3) egg size distribution, (4) egg quality distribution, (5) feed consumption, and (6) flock mortality. The estimation of the stochastic nature of these variables was discussed. The estimation procedures used were not entirely successful for the following reasons:

1. The force molting trials, which were the main sources of experimental data had, with few exceptions, been poorly designed. Lack of sufficient treatment replication precluded variance estimates for such production variables as egg production, feed consumption

and flock mortality.

2. Where measurements on such variables as egg size and egg quality (which affect market grades) had been conducted these were measured at infrequent intervals over the duration of the force molting trials for too few treatment replicates. In those cases where measurements had been taken the data were not reported in a form readily available for economic analysis.

These deficiencies were due mainly to the fact that the force molting trials were designed primarily to provide poultry scientists and physiologists, and not economists, with experimental data. This was unfortunate, as greater consideration of the possible economic importance of these trials might have resulted in better designed experiments that could have provided information of value to both physiologists and economists.

The lack of sufficient reliable information resulted in several important constraints on the estimation procedures. However, it was felt that by proceeding with the analysis that the foundation for further work based upon better information could be laid. Because of this, more emphasis was placed on methodological procedure, although it was hoped that the results of the analysis would provide some guide as to the producer's optimum replacement policies under conditions of uncertainty.

The estimates of the distributional form of important variables were used to derive the moments of the distribution of net income $f(X_i)$ for each action in $\mathcal{A}$ over the planning period N. This was achieved using a simulation procedure which estimated the monthly distributions of net income $f(X_{ni})$ for each action using generated monthly net income 'observations'. The moments of these monthly distributions were then used to derive the moments of the distribution $f(X_i)$ for each action in $\mathcal{A}$. This procedure was repeated for a series of 13 in-lay periods.

The hypothesis

$$f(X_{ni}) \sim N(E(X_{ni}), \ var(X_{ni})), \quad \begin{array}{l} n = 1, 2, \ldots, N \\ i = 1, 2, \ldots, 11. \end{array}$$

was tested using the chi squared statistic. This hypothesis failed to be rejected at the 5 percent significance level. As a result, the moments (mean and variance) of the distributions $f(X_i)$ were obtained by a simple summation of the moments of the distribution $f(X_{ni})$, $n = 1, 2, \ldots, N$.

A useful initial theoretical framework for choosing between the actions in $\mathcal{A}$ was given by the mean-variance (E-V) efficiency frontier. An E-V frontier was constructed for each of the 13 in-lay periods, the producer's field of choice being conveniently reduced to those actions lying on these frontiers. The criterion of maximizing expected utility was used to select an action along an E-V frontier.

Since this study was not intended to derive utility functions for individual producers, the criterion of maximizing expected utility was illustrated with three theoretical utility functions which reflected risk preference, aversion and indifference. These utility functions, and the moments of the distribution of net income, were then combined to yield three equations for utility in terms of the mean and variance of net income. The expected utility for each action on the E-V frontier was then estimated for the 13 in-lay periods.

The results of the uncertainty analysis showed that:

1. If the producer had a risk preference he could follow action a_{28} (three force molts; at the 6, 13 and 19th month of the planning period) to maximize his expected utility.

2. If the producer had a risk aversion he could follow one of the non-force molted actions (a_4, a_5 or a_7) to maximize his expected utility. The particular action chosen was shown to be a function of the in-lay period. Thus, where the planning period commenced at one of the in-lay periods 2 through 6, action a_7 (annual replacement) was the optimum action. For all other in-lay periods, either one of actions a_4 or a_5 could be followed.

3. If the producer was indifferent to risk he could follow action a_{37} (two force molts; at the 9 and 19th month of the planning period) to maximize his expected utility.

On the basis of these results it would seem that the future importance of force molting policies in commercial egg production depends on the producer's inherent attitudes towards risk. Historical evidence has indicated that force molting policies have not found wide application. This could be due to the lack of reliable information as mentioned previously, or more significantly to an inherent risk aversion on the part of poultry producers which has resulted in the traditional choice of non-force molted actions. This would certainly be an interesting hypothesis to test in future research work.

It should also be possible to design experiments with certain testable hypotheses in mind. Two examples of such hypotheses are: (1) that the variances of the monthly egg production, egg size and egg quality distributions are constant. In this way it should be possible to detect the influence of age and force molting treatment on these factors of production, (2) that the variances of the monthly distributions of mortality are constant and not functions of age or force molting treatments. Information on (1) and (2) above should aid in a needed estimation of the stochastic nature of the production function relating egg production to such variables as feed consumption, age and force molting treatment.

Further problem areas which relate to the question of the economic feasibility of force molting programs pertain to the macro effects of the use of such programs on: (1) the supply and demand for eggs in the U. S. , i. e. , the likely changes in the distribution of market grade eggs between the large and small categories if these programs found wide application, might have a considerable influence on egg prices--especially on the price differentials between the various grades, and (2) the reaction of the large hatchery concerns to a decrease in demand for day-old chicks.

In conclusion, this study has succeeded in defining several problem areas for future research. On the basis of available information it is concluded that force molting policies can provide alternatives to traditional annual replacement policies but recommendations for their use in commercial egg production must be based on consideration of producers' attitudes towards risk.

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APPENDICES

APPENDIX A

Table I. Production data relevant to the s_{jk} states of production for the action set A.

s_{jk} ^{a}		Dozens of Eggs Produced Per Hen-Month ($\bar{Q}_{jk}$)	% Hen Month Production (X_{1jk})	% Production by Grade ^{b} Grade (GD_{gjk}) 1	2	3	4	5	6	7	Feed Consump-^{n} in lbs (Y_{jk})	lbs Feed Per Dozen Eggs
1	0	1.35	58	29	67			2	1	1	6.42	4.76
2	0	2.01	86	8	70		2	17	2	1	7.08	5.24
3	0	1.95	84	2	50	1	8	34	4	1	7.03	3.61
4	0	2.00	86	1	30	1	13	46	9		7.32	3.66
5	0	1.88	81		21	2	16	47	13	1	7.12	3.79
6	0	1.82	78		16	2	18	43	20	1	6.78	3.72
7	0	1.73	74		13	1	19	40	25	2	6.71	3.88
8	0	1.74	75		9	2	19	33	32	5	7.15	4.11
9	0	1.66	71		5	2	17	27	42	7	7.12	4.29
10	0	1.58	68		2	3	15	22	48	10	6.61	4.18
11	0	1.52	65		2	4	13	19	51	11	6.95	4.57
12	0	1.58	68		2	4	12	17	52	13	6.46	4.09
13	0	1.53	66		2	5	11	16	54	12	6.44	4.21
14	0	1.43	61		2	9	9	13	54	13	6.18	4.32
15	0	1.36	58		2	10	8	12	55	13	6.29	4.63
16	0	1.25	54		1	10	8	10	55	16	6.52	5.22
7	5	1.32	57		13	2	16	43	24	2	4.68	+
8	5	0.21	*		9	1	16	39	33	2	4.64	+
9	5	1.39	60		2	1	10	26	51	10	7.79	5.60
10	5	1.88	81		2	1	11	26	50	10	7.46	3.97
11	5	1.88	81		2	1	13	28	46	10	7.65	4.07
12	5	1.82	78		2	2	15	28	48	5	7.00	3.84
13	5	1.76	75		2	2	14	28	47	7	6.17	3.51
14	5	1.54	66		2	1	14	30	44	9	3.00	+
15	5	0.53	21		3	1	11	27	49	9	6.14	+
16	5	1.77	76		4	1	11	23	51	10	8.07	4.56
17	5	1.86	80		3	1	11	20	54	11	7.25	3.89
18	5	1.79	77		2	2	12	20	55	9	7.19	4.02
19	5	1.74	75		2	2	12	20	56	8	7.10	4.08
20	5	1.27	54		2	2	11	20	57	8	4.38	+
21	5	0.11	*		2	2	11	17	58	10	5.06	+
22	5	1.24	53		2	2	10	16	59	11	7.74	6.24
23	5	1.60	69		3	2	10	16	57	12	7.47	4.66
24	5	1.68	72		3	3	9	14	58	13	6.76	4.02
25	5	1.71	73		3	3	9	14	58	13	6.84	4.00
26	5	1.68	72		4	4	9	13	56	14	6.31	3.76
10	4	1.48	62		2	2	13	25	48	10	6.06	4.09
11	4	0.19	*		2	1	11	22	52	12	4.24	+
12	4	0.84	36		1		9	20	57	13	6.73	+
13	4	1.89	81		1		10	20	57	12	7.50	3.96
14	4	1.84	79		1	1	10	19	56	13	7.02	3.81
15	4	1.78	76		1	2	11	20	55	11	7.01	3.93
16	4	1.78	76		1	2	12	20	54	11	7.30	4.10

Table I (Cont'd.)

s_{jk}[a]	Dozens of Eggs Produced Per Hen-Month ($\bar{Q}_{jk}$)	% Hen Month Production (X_{1jk})	% Production by Grade[b] Grade (GD_{gjk})							Feed Consump$^{\underline{n}}$ in lbs (Y_{jk})	lbs Feed Per Dozen Eggs
			1	2	3	4	5	6	7		
17 4	1.71	73		1	3	12	18	56	10	6.92	4.05
18 4	1.64	70		1	3	12	16	57	11	6.79	4.14
19 4	1.55	66		1	2	12	17	57	11	6.83	4.41
20 4	1.18	50			2	12	17	56	13	4.43	+
21 4	0.11	*			1	12	18	56	13	5.18	+
22 4	1.02	44			1	11	17	57	14	7.41	7.26
23 4	1.72	74			1	11	17	56	15	7.69	4.47
24 4	1.68	72			2	10	15	58	15	7.18	4.27
25 4	1.65	71			3	9	13	60	15	6.91	4.18
26 4	1.58	68			3	8	13	60	15	6.43	4.08
13 2	1.40	60		2	4	11	17	53	13	5.77	4.12
14 2	0.49	22		2	3	9	18	55	13	3.13	+
15 2	0.60	26		1	2	8	15	65	9	6.01	+
16 2	1.51	65		2	1	11	19	54	13	7.77	5.15
17 2	1.65	71		2	2	10	19	53	14	7.24	4.39
18 2	1.59	68		1	2	11	18	52	16	7.10	4.46
19 2	1.46	63		1	3	11	17	52	16	7.16	4.90
20 2	1.38	59		1	3	11	16	51	18	6.97	5.05
21 2	1.30	56		1	4	11	15	50	19	7.12	5.47
22 2	1.30	56		1	5	10	14	50	20	6.69	5.15
23 2	1.22	52		1	5	10	13	50	21	6.52	5.34
24 2	1.21	52		1	7	8	13	49	22	6.71	5.54
25 2	1.12	48		1	8	7	12	50	22	6.26	5.58
26 2	1.07	46		1	10	6	11	51	21	5.48	5.12
17 3	0.85	36		1		11	22	53	13	5.55	+
18 3	0.01	*		1	1	16	16	51	15	4.08	+
19 3	1.49	64		1	3	15	13	52	16	7.76	5.21
20 3	1.31	56		1	3	14	13	52	17	6.59	5.03
21 3	1.49	64			4	15	13	51	17	7.68	5.16
22 3	1.59	68			2	14	17	50	17	7.59	4.78
23 3	1.56	67			3	13	14	52	18	7.28	4.67
24 3	1.55	66			3	15	11	52	19	7.24	4.67
25 3	1.52	65			3	19	9	50	20	6.95	4.57
26 3	1.48	63			4	17	9	49	21	6.61	4.47
20 1	1.36	58			5	12	15	57	11	6.59	4.84
21 1	1.31	56			6	12	13	56	13	6.50	4.96
22 1	1.25	54			9	11	11	55	14	6.46	5.17
23 1	1.20	52			10	12	11	52	15	6.40	5.33
24 1	1.17	50			12	10	10	52	16	6.20	5.30
25 1	1.10	47			14	9	8	51	18	6.00	5.45
26 1	0.97	42			16	8	8	51	19	5.80	5.97

[a] s_{jk} represents the state of production. The subscript 'j' represents the age of the bird in months and 'k' represents the force molting treatment

[b] g = 1 represents small AA grade eggs; 2, medium AA grade eggs; 3, large B grade eggs; 4, large A grade eggs; 5, large AA grade eggs; 6, extra large AA grade eggs; and 7, jumbo AA gr. eggs.

* Indicates hen-month egg production of less than 10 percent.

\+ Indicates the duration of a force molting period.

APPENDIX A

Table II. Eggs produced per hen-housed over a two year period for three force molted flocks and a non-force molted flock. Percentages by market grade.[a]

Grade	All Pullet[b]	Treatment		
		6 Month (k = 5)	9 Month (k = 4)	12 Month (k = 2)
AA Small	3.12	1.94	1.83	2.03
AA Medium	22.96	15.29	14.12	14.97
AA Large	55.14	68.75	70.03	52.32
A Large	14.14	9.44	8.75	22.32
B Large	3.58	3.10	3.76	7.04
No Grade Loss	1.06	1.45	1.22	1.37

[a] After R. Hansen (1966, Table 8, p. 7)

[b] Represents two pullets housed (≅ to an annual replacement)

APPENDIX A

Table III. Average monthly salvage prices paid in cents per pound to U. S. poultry producers, 1958-1967[a].

Year	Month[b]												
	1	2	3	4	5	6	7	8	9	10	11	12	Mean
1958	14.8	15.5	16.8	17.0	16.7	16.0	15.2	14.0	12.0	11.8	12.0	12.5	14.0
1959	12.8	13.2	13.5	12.7	11.4	10.4	11.0	10.3	9.5	9.3	9.9	10.9	11.0
1960	11.4	11.8	12.5	13.3	13.4	12.9	12.2	12.1	12.0	11.9	12.1	12.6	12.2
1961	12.5	12.7	13.0	12.3	11.2	10.4	9.8	9.5	8.6	8.3	8.8	9.5	10.1
1962	10.4	11.3	11.5	11.0	10.3	9.7	9.4	9.5	9.8	9.6	9.9	10.3	10.2
1963	10.4	10.9	11.3	11.1	10.1	10.0	9.9	9.4	9.5	9.3	9.6	9.7	10.0
1964	9.8	9.8	10.0	9.6	9.3	9.0	9.1	8.8	8.8	8.7	8.7	9.0	9.2
1965	8.5	8.7	9.0	9.4	9.0	8.8	8.9	8.8	8.7	8.5	9.1	9.6	8.9
1966	9.6	10.0	10.8	10.9	10.5	10.0	9.6	9.5	9.2	8.9	9.1	8.9	9.7
1967	8.9	8.7	9.1	8.5	7.9	7.6	7.6	7.6	7.5	7.1	7.0	7.3	7.9

[a] Source: U. S. D. A. Agricultural Statistics 1958-1967

[b] Calendar month.

Table IV. Average monthly prices paid in dollars per ton for layers ration by U. S. poultry producers[a].

Year	Month[b]											
	1	2	3	4	5	6	7	8	9	10	11	12
1958	86	86	87	89	90	89	91	92	90	89	88	89
1959	90	90	90	90	90	89	89	88	87	87	86	86
1960	86	86	86	87	86	86	86	86	86	86	84	84
1961	86	86	87	87	88	88	87	87	87	86	86	86
1962	87	87	86	87	87	87	87	87	88	88	88	88
1963	85	85	85	84	84	84	85	85	85	85	84	84
1964	85	85	84	84	84	83	83	83	83	83	83	83
1965	83	83	83	83	83	83	84	84	83	83	83	83
1966	84	84	84	83	83	84	84	87	88	89	88	88
1967	88	87	88	87	86	87	86	85	85	84	83	84

[a] Source: U. S. D. A. Agricultural Statistics 1958-1967

[b] Calendar month.

APPENDIX B

Least Squares Analysis of Flock Mortality for the Cornell Trials. Two Years of Production

First year production

Model: $Y_1 = \beta_{1,0} + \beta_{1,1} X_{1,1} + e_1$

where:

Y_1 = accumulative percent mortality

$X_{1,1}$ = age of the flock in months

e_1 = random disturbance term

Least squares regression

$$\hat{Y}_1 = \hat{\beta}_{1,0} + \hat{\beta}_{1,1} X_{1,1}$$

$$\hat{Y}_1 = -0.1140 + 0.3944 X_{1,1} \quad (R^2 = 0.97)$$

Second year production

Model: $Y_2 = \beta_{2,0} + \beta_{2,1} X_{2,1} + e_2$

Y_2, $X_{2,1}$, e_2 as for first year.

Least squares regression

$$\hat{Y}_2 = \hat{\beta}_{2,0} + \hat{\beta}_{2,1} X_{2,1}$$

$$\hat{Y}_2 = -6.498 + 0.9895 X_{2,1} \quad (R^2 = 0.96)$$

APPENDIX B

Table I. Monthly producer egg prices 1958-1967 and four hypothetical egg price structures. Prices in cents per dozen X 10.

Producer Egg Prices 1958

S	M	LB	LA	LAA	ELAA	JAA
285	318	270	350	410	430	450
210	270	210	290	355	375	395
245	325	253	340	395	415	435
248	338	258	348	398	418	438
228	318	238	338	388	408	428
180	280	213	313	363	383	403
170	270	240	345	388	408	428
180	318	275	400	450	470	490
183	345	273	428	483	503	523
200	340	245	423	478	498	518
205	285	230	355	403	423	443
253	330	325	390	425	443	463
248	313	318	368	398	418	438

Producer Egg Prices 1959

205	275	265	343	373	393	413
190	280	240	330	360	380	400
195	275	210	320	330	350	370
178	228	200	268	285	305	325
165	220	200	260	280	300	320
140	220	200	265	285	305	325
130	223	200	303	323	343	364
130	265	245	355	375	395	415
130	265	265	363	385	405	425
143	278	260	373	388	408	428
150	255	253	365	365	385	405
155	240	233	325	343	363	383
205	282	265	340	353	373	393

Producer Egg Prices 1960

240	293	263	350	360	380	400
228	290	238	335	345	365	385
193	263	200	323	333	355	375
219	296	226	354	366	386	406
225	275	245	325	338	358	378
227	275	245	325	345	365	385
210	285	245	328	348	368	388
205	330	245	375	395	415	435
210	345	295	438	450	470	490
245	393	340	455	465	485	505
280	355	365	428	438	458	478
328	423	370	468	478	498	518
333	420	415	460	485	505	525

Producer Egg Prices 1961

S	M	LB	LA	LAA	ELAA	JAA
270	340	325	390	400	420	440
228	290	258	343	353	373	393
200	265	230	330	340	360	380
198	270	243	338	348	368	388
170	260	260	335	345	365	385
180	215	215	295	305	325	345
143	243	240	335	345	365	385
148	255	255	360	370	390	410
155	275	270	400	410	430	450
145	228	255	385	398	418	438
155	298	245	360	380	400	420
205	308	270	368	388	408	428
260	340	305	385	405	425	445

Producer Egg Prices 1962

275	348	335	380	393	413	433
253	340	290	375	385	405	425
193	275	240	323	333	353	373
175	260	225	310	320	340	360
150	205	200	268	278	298	318
135	185	195	255	265	285	305
135	200	195	263	273	293	313
135	243	213	303	313	333	353
135	263	260	355	365	385	405
145	283	270	360	373	393	413
178	250	220	310	320	340	360
185	280	225	340	350	370	390
228	318	288	363	373	393	413

Producer Egg Prices 1963

238	318	288	350	368	388	408
255	330	305	370	380	400	420
240	290	255	345	355	375	395
225	263	230	325	335	355	375
178	215	195	278	288	308	328
133	193	183	273	283	303	323
120	178	165	260	270	290	310
135	220	175	305	315	335	355
130	245	235	328	338	358	378
165	295	333	373	383	403	423
175	290	310	360	370	390	410
193	283	273	380	390	410	430
203	288	255	355	365	385	405

Appendix B, Table I. (Cont'd.)

Producer Egg Prices 1964

233	320	290	368	378	398	418
225	308	285	355	365	385	405
220	273	275	345	355	375	395
185	263	245	323	333	353	373
170	245	235	305	315	335	375
145	203	213	283	293	313	333
130	188	195	265	275	295	315
130	210	215	285	295	315	335
130	245	255	325	335	355	375
135	260	268	340	350	370	390
130	240	240	320	340	360	380
140	225	225	305	315	335	355
150	245	240	310	320	340	360

Producer Egg Prices 1967

243	283	200	333	343	363	383
170	260	200	310	320	340	360
145	260	195	310	320	340	360
125	210	180	260	270	290	310
103	203	180	253	263	283	303
095	180	180	230	240	260	280
095	180	180	230	240	260	280
095	190	180	268	278	298	318
095	180	170	270	280	300	320
090	170	165	273	283	303	323
085	160	150	230	240	260	280
095	173	150	243	253	273	293
113	203	150	270	280	300	320

Producer Egg Prices 1965

148	243	243	293	303	323	343
125	208	208	265	275	295	315
120	195	198	258	268	288	308
145	210	220	280	290	310	330
128	190	205	263	273	293	313
120	185	195	255	265	285	305
120	188	208	268	278	298	318
120	218	218	298	308	328	348
120	250	233	330	340	360	380
130	275	255	350	360	380	400
130	270	250	330	340	360	380
165	280	263	343	378	398	418
223	338	303	383	393	413	433

Hypothetical Price Year Pext1

19	20	19	31	33	35	37
19	22	20	33	34	36	38
20	22	21	32	34	36	38
19	21	20	31	33	35	37
15	20	19	30	32	34	36
14	19	18	29	30	32	34
12	18	16	28	29	31	33
11	19	18	29	30	32	34
11	19	15	26	28	30	32
12	20	16	25	26	28	30
14	21	18	26	27	29	31
16	23	19	27	28	30	32
18	24	21	29	30	32	34

Producer Egg Prices 1966

243	353	330	388	398	418	438
250	355	315	395	405	425	445
240	350	325	405	415	435	455
215	330	310	390	400	420	440
135	243	228	315	325	345	365
125	218	208	298	308	328	348
125	213	203	298	303	323	343
135	263	230	353	363	383	403
148	295	235	378	388	408	428
168	330	268	418	428	448	468
165	300	220	370	380	400	420
188	313	233	383	393	413	433
203	323	243	380	390	410	430

Hypothetical Price Year Pext2

19	25	24	30	31	33	35
19	26	25	31	32	34	36
18	27	26	32	33	35	37
17	28	25	32	33	35	37
14	26	24	32	33	35	37
11	20	20	30	32	34	36
09	17	15	29	30	32	34
10	17	15	24	26	28	30
11	18	16	24	26	28	30
12	19	17	25	26	28	30
13	20	18	26	28	30	32
15	21	19	28	30	32	34
16	24	22	30	31	33	35

Appendix B, Table I (Cont'd.)

Hypothetical Price Year Pext3

S	M	LB	LA	LAA	ELAA	JAA
16	25	24	31	33	35	37
17	26	24	31	34	36	38
17	26	23	31	33	35	37
16	24	20	30	31	33	35
14	21	17	29	30	32	34
10	19	14	26	27	29	31
09	17	11	22	23	25	27
08	17	11	21	22	24	26
08	17	11	22	23	25	27
09	18	12	23	24	26	28
10	19	13	24	25	27	29
12	20	16	25	26	28	30
14	21	18	26	27	29	31

Hypothetical Price Year Pext4

15	23	19	27	28	30	32
15	22	18	26	27	29	31
14	23	20	28	29	31	33
11	23	20	28	29	31	33
10	23	18	28	29	31	33
09	22	14	27	28	30	32
08	20	12	26	27	29	31
07	18	10	24	25	27	29
07	16	10	20	22	24	26
08	15	10	19	20	22	24
09	14	11	18	19	21	23
10	14	12	19	20	22	24
11	16	13	20	21	23	25

APPENDIX B

Table II. Figures used to derive the distribution of mortality for 80 California flocks.

Quantile	Number of Observations	Probability	Accumulative Probability
0- 6.9	0	0.0000	0.0000
7- 7.9	0	0.0000	0.0000
8- 8.9	3	0.0375	0.0375
9- 9.9	0	0.0000	0.0375
10-10.9	1	0.0125	0.0500
11-11.9	4	0.0500	0.1000
12-12.9	7	0.0875	0.1875
13-13.9	17	0.2125	0.4000
14-14.9	13	0.1625	0.5625
15-15.9	10	0.1250	0.6875
16-16.9	9	0.1125	0.8000
17-17.9	3	0.0375	0.8375
18-18.9	7	0.0875	0.9250
19-19.9	1	0.0125	0.9375
20-20.9	1	0.0125	0.9500
21-21.9	1	0.0125	0.9625
22-22.9	0	0.0000	0.9625
23-23.9	0	0.0000	0.9625
24-24.9	1	0.0125	0.9750
25-100	2	0.0250	1.0000

APPENDIX C

The Simulation Procedure

Definitions of Model Components

Before embarking upon a description of the simulation procedure[1] the components of the model will be defined. These were, in the main, identical to those defined in Chapter III and in Chapter VI in equations 6.2 through 6.11. These components are redefined below under the headings of exogenous, and endogenous variables, parameters, operating characteristics, and identities used in the simulation model.[2] These components are defined for the a_ith action in $\mathcal{A}$.

Exogenous variables

$\ln M_i^1$ = the log of percentage flock mortality over the first year of the planning period; a stochastic variate with a known probability distribution, expected value equal to $\ln \overline{M}_i^1$ and variance equal to $\ln \hat{\sigma}^2_M$, i. e.

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[1] For a discussion of computer simulation and simulation models see Naylor, et al. (1967, p. 1-12), Orcutt (1960) and Suttor and Crom (1964).

[2] For the definitions and examples of these components see Naylor, et al. (1967, p. 10-13) and Forrester (1964, p. 67-72).

$$\ln M_i^1 \sim N(\ln \overline{M}_i^1, \ln \hat{\sigma}_M^2), \; i = 1, 2, \ldots, 11$$

$$\text{and} \quad M_i^1 \sim \Lambda(\overline{M}_i^1, \hat{\sigma}_M^2)$$

$\ln M_i^2$ = the log of percentage flock mortality over the second year of the planning period; a stochastic variate with a known probability distribution, expected value equal to $\ln \overline{M}_i^1$ and variance equal to $\ln \hat{\sigma}_M^2$, i.e.

$$\ln \overline{M}_i^2 \sim N(\ln \overline{M}_i^2, l_n \hat{\sigma}_M^2)$$

$$\text{and} \quad M_i^2 \sim \Lambda(\overline{M}_i^2, \hat{\sigma}_M^2)$$

Q_{ni} = total hen-month egg production in the nth stage of production; a stochastic variate with a known probability distribution, expected value equal to $\overline{Q}_{ni}$ and variance equal to $\hat{\sigma}_q^2$, i.e.

$$Q_{ni} \sim N(\overline{Q}_{ni}, \hat{\sigma}_q^2), \quad n = 1, 2, \ldots, 26$$

S_Y = a stochastic variate describing the hen-month variation in feed consumption where

$$S_Y \sim N(0, \hat{\sigma}_Y^2)$$

$F(P_{1t})$ = the cumulative probability distribution of small AA grade eggs in month t of the price year, t = 1, 2, . . . , 13.

S_2 = a stochastic variate with a known probability distribution; expected value equal to zero and variance equal to $\hat{\sigma}^2_{P_2}$, i.e.

$$S_2 \sim N(0, \hat{\sigma}^2_{P_2})$$

where P_2 signifies the price of medium AA grade eggs.

S_3 = a stochastic variate with a known probability distribution; expected value equal to zero and variance equal to $\hat{\sigma}^2_{P_3}$, i.e.

$$S_3 \sim N(0, \hat{\sigma}^2_{P_3})$$

where P_3 signifies the price of large B grade eggs.

S_4 = a stochastic variate with a known probability distribution; expected value equal to zero and variance equal to $\hat{\sigma}^2_{P_4}$, i.e.

$$S_4 \sim N(0, \hat{\sigma}^2_{P_4})$$

where P_4 signifies the price of large A grade eggs.

X_{1ni} = percentage hen-month egg production in the nth stage of production.

X_{2ni} = the age of the hen in months in the nth stage of production.

X_{3ni}, X_{5ni}, X_{7ni} = dummy (0, 1) variables to account for the influence of force molting on egg production.

l_i = the number of replacement flocks entering the laying unit over the planning period.

Endogenous variables

M^1_{ni} = the percentage flock mortality in the nth stage of production over the first year of the planning period.

M^2_{ni} = the percentage flock mortality in the nth stage of production over the second year of the planning period.

m_{ni} = accumulative percentage flock mortality up to and including the nth stage of production.

p_{ni} = total flock mortality up to and including the nth stage of production.

Y_{ni} = hen-month feed consumption in the nth stage of production.

Q_{gni} = hen-month production of eggs laid in gth grade in the nth stage of production.

P_{2t} = the price of medium AA grade eggs in month t, $t = 1, 2, \ldots, 13$.

P_{3t} = the price of large B grade eggs in month t.

P_{4t} = the price of large A grade eggs in month t.

P_{5t} = the price of large AA grade eggs in month t.

P_{6t} = the price of extra large AA grade eggs in month t.

P_{7t} = the price of jumbo AA grade eggs in month t.

TR_{ni} = total revenue from egg sales in the nth stage of production.

TCF_{ni} = total feed cost in the nth stage of production.

LD_{ni} = total cost of livestock depreciation up to and including the nth stage of production.

$E(X_{ni})$ = the expected net income accruing to the flock in the nth stage of production.

$Var(X_{ni})$ = the variance of net income in the nth stage of production.

$E(X_i)$ = the expected net income accruing to the flock over the planning period N.

$Var(X_i)$ = the variance of net income accruing to the flock over the planning period N.

Parameters

W = the weight of a culled bird in pounds.

P^c = salvage price per pound liveweight for culled birds.

C^r = cost of the point-of-lay pullet.

C^f = cost of a pound of layers ration.

C^c = per bird cost of cleaning out the laying unit at the end of the planning period.

C^t = per bird cost of transferring a point-of-lay pullet from the rearing to the laying quarters.

V = the flock size.

GD_{gni} = the percent of total eggs (Q_{ni}) laid in the gth grade in the nth stage of production where

$$\sum_{g=1}^{8} GD_{gni} = 100\%$$

Operating characteristics

$$Y_{ni} = 6.27280 + 0.01555X_{1ni} - 0.01902X_{2ni} - 0.52251X_{3ni} + 0.19453X_{5ni} + 0.73363X_{7ni} + S_Y$$

$$P_{2t} = 10.34027 + 0.92837P_{1t} + S_2$$

$$P_{3t} = 5.16122 + 0.10409P_{1t} + 0.64742P_{2t} + S_3$$

$$P_{4t} = 8.16419 - 0.31381P_{1t} + 0.78122P_{2t} + 0.40368P_{3t} + S_4$$

Identities

$$Q_{gni} = Q_{ni} \cdot GD_{gni}$$

$$X_{1ni} = (Q_{ni}/2.33) \cdot 100\%$$

X_{2ni} = j--the age of a bird in the nth stage of production being defined by the s_{jk}th state of production for a bird in that stage.

$$M_i^1 = e^{\ln M_i^1}$$

$$M_i^2 = e^{\ln M_i^2}$$

$$M_{ni}^1 = M_i^1/13$$

$$M_{ni}^2 = M_i^2/13$$

$$m_{ni} = n \cdot M_{ni}^1, \qquad n = 1, 2, \ldots, 13$$

$$m_{ni} = M_i^2 + (n - 13) \cdot M_{ni}^2, \qquad n = 14, 15, \ldots, 26$$

$$p_{ni} = V \cdot m_{ni}$$

$$TR_{ni} = (V - p_{ni}) \left(\sum_{g=1}^{7} P_{gt} \cdot Q_{gni} \right)$$

$$TCF_{ni} = (V - p_{ni}) (Y_{ni} \cdot P^f)$$

$$LD_{ni} = [l_i \cdot V \cdot (C^r - (1 - (M_i^1 + M_i^2)/100) \cdot P^c \cdot W] /26$$

$$E(X_{ni}) = (V - p_{ni}) (TR_{ni} - TCF_{ni}) - (LD_{ni} + V \cdot C^c + l_i \cdot V \cdot C^t)$$

$$E(X_i) = \sum_{n=1}^{N} X_{ni}$$

$$E(X_{ni}) = \sum_{m=1}^{M} E(X_{mni})/M$$

$$\mathrm{var}(X_{ni}) = \sum_{m=1}^{M} (E(X_{mni}) - E(X_{ni}))^2, \qquad m = 1, 2, \ldots, 100$$

where M represents the number of generated 'observations' or simulation runs per stage of production (n).

The use of the equations

$$E(X_i) = \sum_{n=1}^{N} E(X_{ni})$$

and

$$\mathrm{var}(X_i) = \sum_{n=1}^{N} \mathrm{var}(X_{ni})$$

to estimate the expected value and variance of net income for each of the actions in $\mathcal{A}$ was contingent upon a failure to reject the hypothesis

$$f(X_{ni}) \sim N(E(X_{ni}), \quad \mathrm{var}(X_{ni})) \text{ for all } n = 1, 2, \ldots, N.$$

The simulation routine included a statistical test of this hypothesis based upon the chi squared statistic. The derivation of the test statistic under the above hypothesis is described by Hogg and Craig (1967, p. 303). In the actual simulation runs this hypothesis was tested at the 5 percent significance level, and at this level the hypothesis was not rejected for any of the $f(X_{ni})$ distributions tested.

The following sections of this Appendix will detail the steps used in the simulation routine and present a flow diagram of the simulation procedure used.

Outline of the Simulation Procedure

STEP 1. Read $\hat{\sigma}^2_q$, $\hat{\sigma}^2_{P_2}$, $\hat{\sigma}^2_{P_3}$, $\hat{\sigma}^2_{P_4}$, $\hat{\sigma}^2_Y$, $\hat{\sigma}^2_M$, and the cost and price parameters W, P^c, C^r, C^f, C^c, C^t, and flock size V^3.

STEP 2. Read the probability intervals for the cumulative distributions $F(P_{1t})$.

STEP 3. Set $i = 1$ and $r = 0$, where i designates the action, and r is used as an index to change the form of the egg price cycle according to the in-lay period.

STEP 4. Read $\bar{Q}_{ni}$, X_{2ni}, X_{3ni}, X_{5ni}, X_{7ni}, GD_{gni} for each of the N stages of production for the ith action.

STEP 5. Read $\ln\overline{M}^1_i$ and $\ln\overline{M}^2_i$.

STEP 6. Set index $t = 1$.

STEP 7. Set the DO LOOP index $n = 1, 2, \ldots, 25$ where n designates the stage of production.

STEP 8. Set $t = t + r$ and test $t > 13$. If $t > 13$ set $t = 1$. If $t \leq 13$ procede direct to step 9.

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[3] The parameter levels used in the simulation runs were set at: $W = 5.0$ pounds, $P^c = 6.0$ cents, $C^r = \$1.50$, $C^f = 2.7$ pounds, $C^c = 2.0$ cents, and $C^t = 1.0$ cents. Flock size V was set at 100 birds housed at point-of-lay.

STEP 9. Set the DO LOOP index $m = 1, 2, \ldots 100$, where m designates the mth simulated 'observation' on X_{ni}.

STEP 10. Generate values for the normal variates $\ln M_i^1$ and $\ln M_i^2$.

STEP 11. Calculate M_{ni}^1 and M_{ni}^2.

STEP 12. Calculate m_{ni}.

STEP 13. Generate a pseudorandom number $0 \leq N \leq 1$ and use this number to select a probability interval $x \leq P_{1t} < y$ from the distribution $F(P_{1t})$. Having selected the interval a second pseudorandom number is then used to select the price of small AA grade eggs from this interval.

STEP 14. Generate values for the normal variates S_2, S_3, and S_4.

STEP 15. Calculate P_{2t}, P_{3t}, P_{4t}, . . . , P_{7t}.

STEP 16. Generate a value for the normal variate Q_{ni}.

STEP 17. Calculate X_{1ni}.

STEP 18. Calculate the egg grade distribution for the nth stage of production.

STEP 19. Calculate TR_{ni}.

STEP 20. Generate a value for the normal variate S_Y.

STEP 21. Calculate Y_{ni}.

STEP 22. Calculate $TCF_{ni\cdot}$.

STEP 23. Calculate the value for $E(X_{mni\cdot})$; the mth 'observation' on expected net revenue for the nth stage of production.

STEP 24. Test $m > M$. If $m \leq M$ procede to step 9 and repeat to step 23, otherwise procede to step 25.

STEP 25. Calculate $E(X_{ni\cdot})$ and $var(X_{ni\cdot})$.

STEP 26. Test the hypothesis $f(X_{ni\cdot}) \sim N(E(X_{ni\cdot}),\ var(X_{ni\cdot}))$ using the chi squared test statistic, at the 5 percent significance level. If Ho is rejected plot $E(X_{mni\cdot})$, $m = 1, 2, \ldots, 100$ and print $E(X_{ni\cdot})$ and $var(X_{ni\cdot})$ and STOP. If fail to reject Ho procede to step 27.

STEP 27. Test $n > 25$. If $n \leq 25$ procede to step 7 and repeat to step 26, otherwise, procede to step 28.

STEP 28. Calculate $E(X_{i\cdot})$ and $var(X_{i\cdot})$ and print $E(X_{ni\cdot})$, $var(X_{ni\cdot})$, $E(X_{i\cdot})$ and $var(X_{i\cdot})$.

STEP 29. Set $r = r + 1$. Test $r > 12$. If $r \leq 12$ procede to step 6 and repeat to step 28. If $r > 12$ procede to step 30.

STEP 30. Set $i = i + 1$. Test $i > 11$ to see if all actions have been considered. If $i \leq 11$ procede to step 4 and read in values for the next action. If $i > 11$ then STOP.

APPENDIX C

SIMULATION FLOW DIAGRAM