Fatigue threshold *R*-curves predict small crack fatigue behavior of bridging toughened materials

S. Gallops [a], *T. Fett* [b], *J. W. Ager III* [c], *J. J. Kruzic* [a*]

[a] Materials Science, School of Mechanical, Industrial, and Manufacturing Engineering, Oregon State University, 204 Rogers Hall, Corvallis, OR 97331, USA

[b] Institute for Ceramics in Mechanical Engineering, Karlsruhe Institute of Technology, D-76131 Karlsruhe, Germany

[c] Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

Abstract

Small crack fatigue is a widely recognized problem in the fatigue of materials; however, there has been limited progress in developing methods for predicting small crack fatigue behavior. In this paper, small crack effects due to crack bridging are addressed. A fatigue threshold *R*-curve was measured for a 99.5% pure polycrystalline alumina using standard compact tension specimens and it was used to 1) determine the bridging stress profile for the material and 2) make fatigue endurance strength predictions for realistic semi-elliptical surface cracks. Furthermore, is has been shown that the fatigue threshold *R*-curve can equivalently be determined by measuring the bridging stress distribution, in this case using fluorescence spectroscopy, using only a long crack compact tension specimen without the need for difficult small crack experiments. It is expected that this method will be applicable to a wide range of bridging toughened materials, including composites, toughened ceramics, intermetallics, and multi-phase materials.

Keywords: fatigue; fracture; toughness; crack bridging

1. Introduction

Short and small crack fatigue, where fatigue cracks grow faster at small crack sizes than expected based on conventional ASTM standard [1] crack growth experiments (Fig. 1), has been recognized as a significant engineering problem for nearly three

* Corresponding author. Tel.: +1 541 737 7027; fax: +1 541 737 2600.

E-mail address: jamie.kruzic@oregonstate.edu (J.J. Kruzic).

decades [2-7]. Despite that fact, there has been limited progress in developing methods for accurately predicting small crack fatigue behavior [8]. This is a significant concern because most cracks that ultimately cause a fatigue failure begin at a small size. One of the challenges in predicting the behavior is that there can be many different mechanisms causing the effect, including crack closure, crack bridging, transformation toughening, excessive plasticity, and inhomogeneity/anisotropy of the microstructure at microscopic size scales [2-7]. Accordingly, a one size fits all approach to short/small fatigue crack problems may not be appropriate [8]. Furthermore, while significant attention has been given to predicting the effects of plasticity and crack closure [9-14], relatively little research has been done on how to predict effects due to the other mechanisms such as crack bridging [15].

Cracks are generally denoted as "short" when the size is restricted in only the crack length dimension, *a*, but may have a long crack front, e.g., as with a short crack emanating from a notch. The term "small crack" is generally used when the size is restricted in all dimensions, e.g., as with penny or thumbnail shaped cracks. Furthermore, the size scale over which the effect is observed depends on the specific material and mechanisms involved. Generally a small crack effect will be observed when the crack size is on the same scale as 1) the relevant length parameter associated with the salient mechanism, such as the closure, bridging, phase transformation, or plastic zone size, and/or 2) the relevant microstructural length scale, such as the grain size, composite reinforcement size, etc. As such, these crack size effects may persist over 10s of micrometers for some materials such as Si_3N_4 ceramics, and over 10s of centimeters or more for others such as concrete or fiber reinforced composites, making the terminology of short/small crack sometimes misleading.

To predict the fracture of materials, crack size effects are generally handled using a fracture resistance curve (*R*-curve) where the resistance to fracture is given as a function of crack size. It has been demonstrated that the fracture strength can be well predicted as a function of initial flaw size using carefully constructed *R*-curves [16]. Similarly, it has been proposed that fatigue threshold *R*-curves may be useful tools for explicitly incorporating small crack fatigue effects into fatigue failure predictions

[8,15,17,18]; however, to date no experimental validations of the accuracy of this approach have been conducted. A fatigue threshold *R*-curve is a simple extension of the *R*-curve concept whereby the fatigue threshold, or the stress intensity range, ΔK, below which cracks are presumed to not grow, is plotted as a function of crack extension, $\Delta K_{TH}(\Delta a)$. Accordingly, it is the goal of the present paper to examine the ability of fatigue threshold *R*-curves to accurately predict fatigue failure caused by small fatigue cracks.

2. Background

Fatigue-crack growth rates for long cracks often follow the classical Paris power-law relationship for a given load ratio ($R = K_{min}/K_{max}$) [19]:

$$da/dN = A(\Delta K)^m, \tag{1}$$

where A and m are scaling constants specific to the material and test conditions, da/dN is the growth rate, ΔK is the stress-intensity range ($K_{max} - K_{min}$), and K_{max} and K_{min} are, respectively, the maximum and minimum values applied during a loading cycle. Furthermore, there is often a well-defined fatigue threshold, ΔK_{TH}, below which cracks are presumed to be dormant. In the case of small fatigue cracks, design based on a fatigue life determined using Eq. 1 or based on a long crack fatigue threshold will be non-conservative (Fig. 1), leading to unexpected material failures.

Of critical concern is whether a small crack will arrest and be effectively harmless or conversely whether it will grow into a long crack and cause failure (Fig. 1). In this regard, the fatigue threshold *R*-curve should be able to predict the fatigue limit, or endurance strength, for a given initial crack size, a_i, [15,20,21]:

$$\Delta K_{app} = Y \Delta\sigma_{app}\sqrt{\pi a_i} = \Delta K_{TH}(\Delta a) \tag{2a}$$

$$\frac{d\Delta K_{app}}{d\Delta a} = \frac{d\Delta K_{TH}(\Delta a)}{d\Delta a} \tag{2b}$$

where $\Delta\sigma_{app}$ is the applied stress range and Y is the appropriate geometric factor. However, to date no experimental verification of this approach has been attempted. In conducting a validation of this approach a model material (polycrystalline Al_2O_3) has been chosen where the small crack fatigue effect occurs almost exclusively due to a crack bridging phenomenon. Specifically, fatigue cracks grow faster when the crack size is smaller than the steady state bridging zone size [22]. However, the general fatigue threshold *R*-curve concept is expected to be applicable to any situation where 1) a fatigue threshold *R*-curve can be produced for the material and 2) the crack of interest is sufficiently large compared to the microstructure such that a continuum approach is appropriate.

3. Experimental

3.1 Materials

A commercial 99.5% pure Al_2O_3 (AD995, Coors Technical Ceramics Co., Oak Ridge, TN) was chosen as a model material due to the fact that it exhibits large steady-state bridging zones (~ 2 mm) near the fatigue threshold when tested in room air [22]. This permitted direct measurements of fatigue thresholds over a range of crack sizes where short crack effects occurred, i.e., where $a < 2$ mm and the bridging zone was still being developed. Also, crack closure effects are considered negligible in this material because of its relatively low dependence of growth rates on ΔK compared to K_{max} [23]. A micrograph of the microstructure, shown in Fig. 2, was obtained from a diamond-polished (1 μm finish) and thermally etched (20 minutes at 1500°C) sample using a scanning electron microscope (SEM). Details on the grain size distribution may be found in Ref. [21], but in short the majority of the grains had areas < 63 μm^2.

3.2 Fatigue threshold experiments

Short and long fatigue-crack growth experiments were conducted using standard compact-tension, C(T), specimens (width, $W \approx 19$ mm; thickness, $B \approx 3.5$ mm) in general accordance with ASTM standard E647 [1]. Complete details of the fatigue-crack growth procedures are in Ref. [22], and a brief summary of issues pertinent to the measurement of fatigue thresholds is presented here. Fatigue cracks were initiated under cyclic loading conditions (ν = 25 Hz frequency sine wave with load ratio of R = 0.1) from straight, machined notches (length $a_0 \approx 4 – 5$ mm) that had been razor micro-notched to have root radii, $\rho \leq 10$ μm. The cracks lengths were monitored using back-face strain compliance methods [24]. Data collection did not begin until the amount of fatigue-crack extension from the notch, Δa_f, exceeded ρ, at which point the influence of the notch field on the stress intensity could be considered to be negligible [25,26].

In order to measure the fatigue threshold, the applied stress-intensity range was reduced at a roughly constant ΔK-gradient (= $[d\Delta K/da]\ /\Delta K$) of -0.08 mm^{-1}. Based on previous results [22], this ΔK-gradient was low enough in AD995 alumina to achieve steady-state bridging zones for cracks with $\Delta a_f > 2$ mm in the range of growth rates from ~10^{-8} to 10^{-10} m/cycle. In such manner, the fatigue threshold was measured as a function of crack extension for Δa_f ranging from 46 μm to 5.7 mm, with the threshold operationally defined as the lowest stress intensity at which the fatigue-crack growth rate could be measured and does not exceed ~10^{-10} m/cycle.

3.3 Bridging stress distribution determination

3.3.1 Bridging stresses from R-curves

R-curves are known to often be dependent on sample geometry [27]. Since the C(T) specimen is not a realistic crack geometry found in real applications, it is desirable to test the accuracy of the fatigue threshold *R*-curve methodology using a more realistic crack geometry. Accordingly, a better geometry insensitive parameter to evaluate bridging materials is the bridging stress distribution, $\sigma_{br}(\delta)$, where δ is the crack opening displacement. From this function, the *R*-curve for other geometries, such as small thumbnail shaped surface cracks in bending, may be determined. The detailed procedure for determining $\sigma_{br}(\delta)$ from the *R*-curve is outlined elsewhere [28], while a brief summary is given here.

From the measured *R*-curves, the bridging stress intensity factor, K_{br}, can be determined since:

$$K_R = K_0 - K_{br},\ K_{br} < 0 \quad (3)$$

(K_0 = intrinsic crack-tip toughness). Using the weight function representation, the bridging stress intensity factor can be represented by the distribution of bridging stresses, σ_{br}, acting in the wake of the crack:

$$K_{br}(\Delta a) = \int_{0}^{a_0+\Delta a} h(r,a)\,\sigma_{br}(\delta(r,a))\,dr, \tag{4}$$

with the fracture mechanics weight function *h*, the distance *r* from the tip, the initial crack length a_0 free of bridging, and the crack extension $\Delta a = a - a_0$. The bridging stresses depend on the actual crack opening displacements δ.

The total displacements in the presence of bridging stresses result from:

$$\delta = \delta_{app} - \delta_{br}, \tag{5a}$$

$$\delta_{br} = \frac{1}{E'}\int_{a-r}^{a} h(r,a')\,da' \int_{0}^{a'} h(r',a')\,\sigma_{br}(\delta(r',a))\,dr' \tag{5b}$$

with the plane strain modulus $E' = E/(1-\nu^2)$ and the "applied displacements" (the displacements under same the load in the absence of the bridging stresses):

$$\delta_{app} = \frac{1}{E'}\int_{a-r}^{a} h(r,a')\,K_{app}(a')\,da'. \tag{6}$$

The applied stress intensity factor K_{app} is given in fracture mechanics handbooks for various test specimens. The system of equations (4) and (5) can be solved by the iterative method of "successive approximation" until a converged solution is reached. More details are provided in Refs. [29,30]. Similarly, once the bridging stress distribution is known the same system of equations may be used in reverse to determine the *R*-curve for other geometries by utilizing weight function, *h*, that is appropriate for the geometry of interest. Weight functions for various crack and sample geometries are readily available [31].

3.3.2 Spectroscopy experiments

Fluorescence spectroscopy is a common method for determining stresses in Al_2O_3 by measuring the shift, from their stress-free position, in the characteristic R1 and R2 Cr^{3+} optical fluorescence lines produced by ubiquitous chromium impurities [32-35]. Bridging stresses were measured for a C(T) fatigue sample last tested near threshold and re-loaded *in situ* to ~95% of the K_{max} value at the measured fatigue threshold, K_{max}^{TH}. Specifically, a 488 nm laser was focused using an optical microscope with a 10X objective to a ~6 μm spot size on the sample surface. Laser power at the surface was kept below ~3 mW to avoid effects due to sample heating. Emitted and scattered light was directed through a holographic laser line filter and into a 640 mm single spectrometer where 1 s exposures were collected using a liquid nitrogen-cooled, back-thinned CCD camera. Bridging stresses were measured along the crack wake at 6 μm increments moving away from the crack tip. Effects due to ambient temperature variations and/or instrument drift were avoided by recalibrating after every ~10 bridging stress measurements using an internal zero stress reference (a part of the sample far away from the crack). The zero stress peak position corresponding to each bridging stress measurement was determined by linear interpolation between the appropriate calibration points. Finally, a linear extrapolation was used to estimate the bridging stresses at 100% of the $K_{\max}$ value at the measured fatigue threshold, $K_{\max}^{TH}$.

3.4 Small crack fatigue experiments

Controlled small thumbnail shaped cracks were induced in beam specimens with dimensions 3 mm × 3.5 mm × 50 mm in general accordance with ASTM Standard C1421-01b sample preparations for a surface crack in flexure [36]. A Knoop indenter was used to indent the middle of the polished (1 μm diamond finish) surface of the beams at a load of 100 N and a full-force dwell time of 30 seconds. The specimens were then lapped using SiC and re-polished with diamond, removing ~70 μm of material so that both the indent and the residual stress field were eliminated leaving only the cracked surface. To accurately measure the cracks a fluorescent dye penetrant (Zyglo ZL-27A, Magnaflux, Glenview, IL) was applied to the induced cracks both before and after the indent was polished away. The cracks were then viewed and measured using a fluorescence microscope. Induced cracks had surface crack lengths ranging from c = 102 μm to 224 μm (Fig. 3a). Based of examination of fracture surfaces in a scanning electron microscope (Fig. 3b) the exact determination of the ratio of crack depth to surface length was somewhat subjective for each crack; however, based on multiple observations a value of a/c ~ 0.5 was considered to be a reasonable estimate.

The cracked specimens were fatigue tested in four-point bending using a computer controlled electro-magnetic test system (ElectroForce 3200, Bose Corporation, Eden Prairie, MN) using a support span of 60 mm and an inner loading span of 28 mm. A sine wave form was used with a test frequency v = 20 Hz and load ratio R = 0.1. Cyclic stress levels were applied above and below the endurance strengths predicted using Eq. 2. Testing was suspended for samples not failing within 10^7 cycles and those experiments were considered run-outs.

3. Results

Fig. 4 shows the fatigue threshold R-curve for AD995 alumina, measured from C(T) specimens. In bridging ceramics it is well known that K_{max} is more dominant in controlling the fatigue crack growth behavior than ΔK [37]; thus, the R-curve is plotted in terms of K_{max}. Furthermore, by plotting in terms of $K_{max}(\Delta a)$, the R-curve can be assumed to begin at zero crack extension, Δa = 0, at the intrinsic toughness of the material, K_0 = 1.4 MPa√m, taken from a previous study [38].

The calculated bridging stress distribution is shown in Fig. 5, along with the measured bridging stresses from the spectroscopy experiments. There is good agreement between the data from both methods suggesting either can be used equivalently to predict the fatigue threshold R-curve for sample geometries of interest. Using Eq. 4, with the weight function for a semi-elliptical surface crack and the bridging stress distribution from Fig. 5, the R-curve for a semi-elliptical surface crack was calculated to see the effect on the predicted fatigue endurance strengths. The R-curves are compared in Fig. 6.

Using both the measured R-curve for C(T) specimens in Fig. 4 and the calculated R-curve for a semi-elliptical surface crack, Eq. 2 was applied along with the correct geometrical function [39] to calculate the expected fatigue endurance strength as a function of initial flaw size for a semi-elliptical surface crack and a load ratio of R = 0.1. Stress intensities were calculated at the sample surface using [39] and a crack shape of a/c = 0.5 was assumed based on measurements of the crack shape on the fracture surfaces using a scanning electron microscope (Fig. 3). The predictions are displayed in Fig. 7 comparing both cases: C(T) and surface crack. The maximum difference between the predictions is less than 3% at all crack sizes; thus, for this specific material those extra calculations to convert from C(T) to surface crack were deemed unnecessary.

The results of the four-point bending small crack fatigue experiments are plotted in Fig. 8 with the predictions for crack lengths between 100 μm and 400 μm. Induced cracks were targeted for this region of interest because at these initial crack sizes the *R*-curve was observed to have a significant effect on the endurance strength. Samples tested at stress/crack length combinations below the line of predicted endurance strength exhibited no failures within 10^7 cycles, while those tested above the predicted endurance strength had a high number of failures (56%) within 10^7 cycles. Thus, the fatigue threshold *R*-curve appears to be quite capable of distinguishing the safe operating stresses for a given crack size.

4. Discussion

4.1. Experimental scatter and failure probability

Although the fatigue threshold *R*-curve appears to be quite good at predicting the safe operating stresses for a given crack size, it is noted from Fig. 8 that not all samples tested above the predicted endurance strength failed within 10^7 cycles. Several factors may contribute to this discrepancy. First, to keep the time of the fatigue experiments reasonable testing was arrested after 10^7 cycles. Thus, it is impossible to say if those samples would have failed if cycling continued. Next, it must be noted that the endurance strength predictions are based off *R*-curve data that contains some degree of experimental scatter. Such scatter will exist whether the predictions are made from an *R*-curve that was measured directly using short crack experiments or from an *R*-curve that was calculated using a measured bridging stress profile (e.g., by spectroscopy). Because of inherent experimental scatter in the data, specimens should not be expected to consistently fail just at the predicted endurance strength; rather, there will be bands of failure probability around the prediction line. This further explains why some specimens did not fail when loaded above their predicted endurance strength (Fig. 8).

From a practical standpoint the failure probability for applied stresses above the predicted endurance strength is very high, on the order of 56% in this study. Thus, engineering designers will need to use loads below the prediction line, with some safety factor applied, to ensure the probability of failure is acceptably small. In this manner it is expected plots like Figs. 7-8 can be effective engineering design tools.

4.2. Determining the R-curves

It is noted that for the present Al_2O_3 material the difference between the endurance strength predictions from the *R*-curve for a compact tension specimen and the *R*-curve for a surface crack was negligible (< 3%). However, in general it is important to use the appropriate *R*-curve in order to make accurate predictions since *R*-curves are dependent on the sample/crack geometry [27]. Such differences may lead to erroneous endurance strength predictions. In this regard, the utilization of weight functions provides a straightforward mathematical way to transform data from a geometry that is convenient to test in a laboratory, e.g., compact tension, to one that is relevant to engineers, e.g., a surface crack.

Similarly, in many materials the collection of short or small crack data can be difficult or inconvenient for all sample geometries. Thus, when possible it would be advantageous to deduce the small crack behavior based solely on long crack experiments. In this paper it is shown how a clear understanding of the micromechanics causing the small crack effects can allow the *R*-curve to be predicted from other methods, such as bridging stress measurements via spectroscopy combined with weight function calculations. The good agreement between the measured bridging stresses and the calculated bridging stress profile in Fig. 5 demonstrates the possibility of using these methods equivalently to calculate an appropriate *R*-curve in materials where bridging is the dominant mechanism. Indeed, the same small crack endurance strength prediction in this study could have been arrived at from the spectroscopic data in Fig. 5 *using only long crack experiments*. Furthermore, a similar methodology can be extended beyond the case of bridging to other mechanisms, such as transformation toughening, that also may be modeled as a stress distribution in the crack wake.

4.3. Combined small crack effects

Overall it is expected this methodology may be extended to incorporate multiple small crack mechanisms. For example, while crack closure is considered negligible in this model material [23], for other bridging materials, e.g., titanium aluminides [40], both crack bridging and closure simultaneously cause small crack effects. In a material where closure is also a concern and the *R*-curve can be measured directly, the effect of closure will be explicitly included in the above predictions. This holds for all continuum level mechanisms: crack closure, crack bridging, transformation toughening or excessive plasticity. In such cases, though, deducing the endurance strengths from independent measurements, such as spectroscopy to measure bridging stresses, will require independent measurements of each effect. However, a continuum-based approach like this would not be appropriate for small cracks on the size scale of the microstructural features. In such cases a probabilistic method will likely be required to determine the probability of fatigue failure based on the likelihood of finding a crack in various locations and orientations within the microstructure.

4.4. Practical applications

For bridging toughened ceramic materials the application of this methodology in an industrial setting would be straightforward. Details of the Weibull statistics for the strength and the fracture toughness *R*-curve would likely be known for a commercial bridging toughened ceramic. From that information the natural flaw size distribution can be determined [41]. One could then choose an acceptable failure probability and determine the flaw size with the corresponding probability [41]. Using that flaw size a design tool such as Fig. 8 could be used to define the nominal safe operating stress.

More broadly, industries that inspect parts for flaws, such as aerospace, could easily adopt this methodology for a wide range of bridging materials. In that case the minimum detectable flaw size would be used with a design tool such as Fig. 8 to determine a conservative safe operating stress.

5. Conclusions

Based on a study of the fatigue threshold behavior, bridging stress measurements, and fatigue life tests of 99.5% pure polycrystalline alumina, the following conclusions can be made:

1. The fatigue threshold *R*-curve accurately predicts safe operating stresses for semi-elliptical small cracks. Indeed, no failures occurred within 10^7 cycles below the line of predicted fatigue endurance strength, and a high

number of failures (56%) occurred within 10^7 cycles above the predicted endurance strength.

2. Not all of the samples tested above the predicted endurance strength failed, though it is impossible to say whether those samples would have failed had cycling continued beyond 10^7 cycles. In general bands of failure probability are expected around the prediction line since the predicted endurance strengths are based off data that contains some degree of experimental scatter.
3. Accurate fatigue threshold *R*-curves can be deduced from methods other than direct measurement *using only long crack experiments*. In this case it was shown that bridging stress measurements by fluorescence spectroscopy combined with weight function calculations can be used. Thus, small crack endurance strength predictions can be made from the measured bridging stresses with out the need for small crack fatigue experiments.
4. It is expected that this methodology may be extended to incorporate multiple, continuum level, small crack mechanisms where the *R*-curve can be measured directly or where the effect of each mechanism may be measured independently.

Acknowledgements

S. Gallops would like to acknowledge support from the Karlsruhe House of Young Scientists, while both J. J. Kruzic and S. Gallops would like to acknowledge support from the National Science Foundation CAREER Award No. 0547394. The authors would also like to acknowledge the assistance of Rawley Greene in helping with the spectroscopy experiments and data analysis and Joseph Ferron in helping make the indentation cracks.

References

[1] ASTM Standard E647, 2008e1, Standard Test Method for Measurement of Fatigue Crack Growth Rates. ASTM International, West Conshohocken, Pennsylvania, USA, 2008. DOI: 10.1520/E0647.
[2] Ritchie RO, Lankford J. Mater. Sci. Eng. 1986;A84:11.
[3] Ritchie RO, Suresh S. Mater. Sci. Eng. 1983;57:L27.

[4] Ritchie RO, Yu W. Short crack effects in fatigue: a consequence of crack tip shielding. In: Ritchie RO, Lankford J, editors. Small Fatigue Cracks: TMS-AIME, Warrendale, PA, 1986. p.167.
[5] Miller KJ. Fatigue Eng. Mater. Struct. 1982;5:223.
[6] Suresh S, Ritchie RO. Int. Metals Rev. 1984;29:445.
[7] Lankford J. Fatigue Eng. Mater. Struct. 1985;8:161.
[8] Kruzic JJ. Science 2009;325:156.
[9] Endo M, McEvily AJ. Mater. Sci. Eng. A 2007;468:51.
[10] Tanaka K, Akiniwa Y. Eng. Fract. Mech. 1988;30:863.
[11] McEvily AJ, Minakawa K. Scripta Metall. 1984;18:71.
[12] Zaiken E, Ritchie RO. Metall. Trans. A 1985;16:1467.
[13] Santus C, Taylor D. Int. J. Fatigue 2009;31:1356.
[14] Tanaka K. JSME Inter. Journal 1987;30:1.
[15] Kruzic JJ, Cannon RM, Ager JW, Ritchie RO. Acta Mater. 2005;53:2595.
[16] Kruzic JJ, Satet R, Hoffmann MJ, Cannon RM, Ritchie RO. J. Am. Ceram. Soc. 2008;91:1986.
[17] Pippan R, Berger M, Stuwe HP. Metall. Trans. A 1987;18:429.
[18] Tanaka K, Akiniwa Y. Eng. Fract. Mech. 1988;30:863.
[19] Paris PC, Erdogan F. J. Basic Eng. 1963;85:528.
[20] Pippan R, Hageneder P, Knabl W, Clemens H, Hebesberger T, Tabernig B. Intermetallics 2001;9:89.
[21] Gallops S, Fett T, Kruzic JJ. J. Am. Ceram. Soc. 2011:in press DOI: 10.1111/j.1551.
[22] Kruzic JJ, Cannon RM, Ritchie RO. J. Am. Ceram. Soc. 2004;87:93.
[23] Gilbert CJ, Ritchie RO. Fatigue Fract. Eng. Mater. Struct. 1997;20:1453.
[24] Deans WF, Richards CE. J. Test. Eval.1979;7:147.
[25] Dowling NE. Fatigue Eng. Mater. Struct. 1979;2:129.
[26] Palazotto AN, Mercer JG. Eng. Fract. Mech. 1990;37:473.
[27] Steinbrech RW, Reichl A, Schaarwächter W. J. Am. Ceram. Soc. 1990;73:2009.
[28] Fett T, Munz D, Thun G, Bahr HA. J. Am. Ceram. Soc. 1995;78:949.
[29] Fünfschilling S, Fett T, Gallops SE, Kruzic JJ, Oberacker R, Hoffmann MJ. J. Eur. Ceram. Soc. 2010;30:1229.
[30] Fett T, Munz D, Thun G, Bahr H-A. J. Am. Ceram. Soc. 1995;78:949.
[31] Fett T, Munz D. Stress Intensity Factors and Weight Functions: Computational Mechanics Publications, Southampton, UK, 1997.
[32] He J, Clarke DR. J. Am. Ceram. Soc. 1995;78:1347.
[33] Dauskardt RH, Ager JW. Acta Mater. 1996;44:625.
[34] Pezzotti G. J. Raman Spectroscopy 1999;30:867.
[35] Pezzotti G, Sbaizero O, Sergo V, Muraki N, Maruyama K, Nishida T. J. Am. Ceram. Soc. 1998;81:187.
[36] ASTM Standard C1421, 2010, Standard Test Methods for Determination of Fracture Toughness of Advanced Ceramics at Ambient Temperature. ASTM International, West Conshohocken, Pennsylvania, USA, 2010. p.DOI: 10.1520/C1421.
[37] Ritchie RO. Int. J. Fract. 1999;100:55.
[38] Kruzic JJ, Cannon RM, Ritchie RO. J. Am. Ceram. Soc. 2005;88:2236.
[39] Newman Jr. JC, Raju IS. Eng. Fract. Mech. 1981;15:185.

[40] Kruzic JJ, Campbell JP, Ritchie RO. Acta Mater. 1999;47:801.
[41] Munz D, Fett T. Ceramics: Mechanical Properties, Failure Behaviour, Materials Selection: Springer-Verlag Berlin Heidelberg New York, 2001.

Figures Captions

Fig. 1. Schematic of short and long fatigue crack growth curves. Short or small cracks may grow faster than long cracks and also below the long crack fatigue threshold, causing unexpected fatigue failures.

Fig. 2. Scanning electron micrograph of the 99.5% pure alumina, Coors AD995, used in this study.

Fig. 3. a) Schematic showing the crack shape for a semi-elliptical surface crack and b) a back scattered electron micrograph of the crack shape after indenting and grinding the beam specimens.

Fig. 4. Measured fatigue threshold *R*-curve from compact tension specimens, plotted in terms of the maximum stress intensity, at a load ratio $R = 0.1$ and a frequency $\nu = 25$ Hz.

Fig. 5. Bridging stress distribution for AD995 (solid line) calculated from weight functions and the fatigue threshold *R*-curve. Also plotted are bridging stresses measured using fluorescence spectroscopy (solid circles) with error bars indicating ±1 standard deviation based on the standard deviation of the linear calibration fit.

Fig. 6. Measured fatigue threshold *R*-curve from compact tension specimens (solid line) compared with the calculated fatigue threshold *R*-curve for a surface crack (dashed line).

Fig. 7. Fatigue endurance strength predictions based off the *R*-curves for both C(T) specimens and surface cracks assuming a crack shape of $a/c = 0.5$. The difference between the predictions was < 3% at all points.

Fig. 8. Results from the four-point bending fatigue experiments plotted with the fatigue endurance strength predictions for crack sizes from 100 μm to 400 μm. No failures occurred below the prediction line, and a high number of failures (56%) occurred above the prediction line.