

INTERNAL REPORT 121

CASTLE LAKE MODELING REPORT

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THE HEAT BUDGET

The basic physical structure of a lake, i.e., stratification, heat content, turbulence and light regime, is related to its heat budget. The heat budget of a lake may be approximated by the balance.

$$R_n = LE + H + S + Q + C + G \quad (1)$$

where R_n is the net radiation absorbed by the lake, LE is the evaporative energy loss, H is the sensible heat loss, S is the storage of heat in the lake water, Q is the net advection of heat into and out of the lake, C is the net chemical heat production and G is the net geothermal heat flux. In Castle Lake C and G are negligible compared to the other terms and may be neglected.

Net Radiation

The net radiation received by the lake may be described by (Sellers 1965):

$$R_n = R^\downarrow (1 - \alpha) + R_{iR}^\downarrow + R_{iR}^\uparrow \quad (2)$$

where R^\downarrow is the incoming radiation, R_{iR}^\downarrow is the downward flux of longwave radiation and R_{iR}^\uparrow is the upward flux of terrestrial black-body radiation, and α is the shortwave reflected/total received or albedo. $R_{iR}^\downarrow = 9.35 \times 10^{-6} \epsilon \sigma T_a^6$ (Swinbank 1963), and $R_{iR}^\uparrow = \epsilon \sigma T_o^4$ where ϵ is surface emissivity, σ is the Stefan-Boltzmann constant, T_o is temperature at water surface ($^{\circ}\text{K}$) and T_a is temperature at screen height ($^{\circ}\text{K}$). For Castle Lake $\alpha = .05$ and $\epsilon = .97$, as constant approximations. Thus (2) is expressed as a function of shortwave radiation and air and surface temperature:

$$R_n = .95 R^\downarrow + (.97)(8.128 \times 10^{-11})(9.35 \times 10^{-6} T_a^6 - T_o^4) \quad (3)$$

Other factors, such as zenith angle (Goddard 1971) are neglected due to the short period of measurement.

Sensible Heat Loss and Evaporative Energy Loss

The vertical fluxes of heat and moisture can be described by the following relations:

$$H = -c_p \rho K_H \left(\frac{\partial T_a}{\partial z} \right) \quad (4)$$

$$E = -\rho K_E \left(\frac{\partial q}{\partial z} \right) \quad (5)$$

where H is the vertical eddy flux of sensible heat, E is the vertical eddy flux of water vapor, c_p is the specific heat of water, K_H is eddy conductivity, K_E is eddy diffusivity, q is the average specific humidity of the air, ρ is density of air, z is depth, and T_a is average potential air temperature. Dividing (4) and (5) by momentum, $\tau = \rho C_d U_a^2$ where C_d is the drag coefficient of wind on the water surface and U_a is average wind speed, and approximating the differentials yields

$$H = \rho c_p U_a C_d (T_o - T_a) \quad (6)$$

$$LE = \rho L C_d U_a (q_o - q_a) \quad (7)$$

where L is the latent heat of evaporation. This approach for finding H and LE is known as the bulk aerodynamic method (Roll 1965). To solve the equations $\rho = 0.0012 \text{ g cm}^{-3}$, $L = 585 \text{ cal g}^{-1}$, $C_d = (1.10 + .0004 U_a) \times 10^{-3}$ (Roll 1965), $c_p = 0.24 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$, $q_o = (3.74 + .0264 T_o^2) \times 10^{-3}$ (Goddard 1971) and $q_a =$ relative humidity q_o . As these calculations are based on a screen height of 10 m and the Castle Lake instrumentation was at 1 m all U_a values are multiplied by 1.33 (Powell, personal communication).

Storage of Heat in Lake Water

The energy stored in the lake is given by

$$S = \rho' c_p \int T dz = \rho' c_p \sum T_i \Delta z_i \quad (8)$$

where T_i is the temperature at each of n depths z_i . The rate of storage between time t and time $t + 1$ is

$$S_{\text{rate}} = \Delta z_i \sum (T_{i,t+1} - T_{i,t}) / (t + 1 - t) \quad (9)$$

For Castle Lake $\Delta z_i = 50 \text{ cm}$ and $t + 1 - t = 5$ days or 7200 minutes. Note that the averaging is done over 5 days, not 2 hours.

Net Advection

The net advection is described by $Q = \rho' c_p \Delta V \Delta T$ where ΔV is the difference in volume between the incoming and outgoing waters and ΔT is the temperature difference of those waters. In Castle Lake this term is less than 0.1% per day after June so Q is neglected in the calculations of the heat budget from mid August until mid September.

The complete equation for calculating the Castle Lake heat budget is:

$$R_n(3) = H(6) + LE(7) + S(9)$$

Instrumentation and Procedures

To measure the above parameters a weather station was established on the sampling raft in the middle of the lake in August 1971, in conjunction with existing facilities. Humidity (for q_a and q_s) and air temperature (T_a) measurements were made with a hygrothermograph (Weather Measure Corp., Sacramento, California, Model 311) calibrated with a sling psychrometer. A contact anemometer (WM Model W164-A) with an event recorder (WM Model 5-A) was used to measure wind speed (U_a). $R+$ was measured with an Eppley pyrhelimeter (Eppley Laboratory, Inc., Newport, R.I., Model 50). A Whitney TC-5 (Monteporo Corporation, San Luis Obispo, California) thermistor was used for water column temperature measurements. Volume of inflow and outflow streams were monitored by constructing several small weirs (after Bormann and Likens, 1967). A floating thermo couple wire lying in the surface film and attached to an Esterline-Angus recorder was constructed to measure T_o .

The event recorder for the anemometer produced a tick mark for each 100 m of wind passage and the 7-day chart was marked in 2-hour intervals. The "ticks" for each of the 12 2-hour periods of the day were counted, totalled and converted to average wind speed (U_a) in cm sec^{-1} for that period. The hygrothermograph and pyrhelimeter also recorded on a 2-hour interval 7-day charts so the average relative humidity, air temperature (T_a in $^{\circ}\text{C}$), and radiation ($R+$ in ly min^{-1}) for each 2-hour interval were calculated. The thermocouple attached to the recorder did not function continuously so the diurnal variations in surface temperature (T_o in $^{\circ}\text{C}$) obtained when the recorder was working were extrapolated to produce calculated values. Temperature down the water column (T_i in $^{\circ}\text{C}$) was measured every 50 cm every 5 days (on the general sampling or "ppr" day).

Calculation of Heat Budget

The equations used, then, to calculate the heat budget are;

$$R_n = .95 R+ + (.97)(8.128 \times 10^{-11})(0.35 \times 10^{-6}[T_a + 273]^6 - [T_o + 273]^4)$$

$$H = (.0012)(0.24)(1.10 + .000532U_a)(10^{-3})(U_a)(T_o - T_a)$$

$$LE = (.0012)(585)(1.10 + .000532U_a)(10^{-3})(U_a)(q_o - q_a)$$

$$S = 0.00695 (T_{i,t+1} - T_{i,t})$$

If the heat budget is to balance the difference, Δhb of $R_n - LE - H - S$ must be small. $100 \times \Delta hb / R_n$ as the percent error in observed values was calculated. The heat budget was calculated by day, by 5 days and over the first 10 and first 20 days.

The Heat Budget, 18 August to 18 September 1972

Heat budget calculations are shown in Table 1. Results are expressed as average langlyls per minute (ly min^{-1}) over the entire day. The heat budget

here was calculated on the basis of averages for 2-hour intervals summed over 1 day and totalled over 5 days. Over the first 16 days the lake was warming from a T_0 of 17.30 to 20.25°C, then it cooled slightly over the next 5 days to 19.45°C then cooled rapidly over the last 11 days to 16.80°C. R_n was the dominant term in the equation, averaging about 0.2 ly min⁻¹. LE was generally larger than H , .03 to .17 ly min⁻¹, versus -.006 to .08 ly min⁻¹. Much of the warming energy of the first 15 days went into storage, averaging about .15 ly min⁻¹, then storage lost up to -.16 ly min⁻¹ during maximum cooling.

Error was calculated several different ways. The error in the observed values, as given in Table 1, was taken as $(R_n - LE - H - S)/R_n$ the percent difference between R_n and $LE + H + S$. Of the 9 days with an $R_n \geq .20$ ly min⁻¹, 6 days had an error of less than 25% and all 9 days had an error of less than 31%. Of the remaining days with an $R_n \leq .20$ ly min⁻¹, only 3 days had an error of less than 25%. The average over the first 5 days produced an error of 12% and over the second 5 days the error was 19%. Beyond that the error was much higher. For the heat budget calculations based on an average of the first 10 days the error was 2.3%, based on an average of the first 20 days the error was 1.1%, and based on an average of the entire 31 days an error of 32%. Thus it would appear that on cool days of fluctuating cloud cover the instrumentation and averaging procedures were insufficient. Under the more constant condition of sunny days and for data averaged over a longer time span than 1 day the bulk aerodynamic method of calculating the heat budget appears quite sensitive.

To further consider sources of error in the calculation of the heat budget the error contributed by the instruments was considered by using the squared sum of the terms obtained by taking the partial differentials with respect to each parameter of LE , H , R_n , S and $LE + H + S$, $(\sum [\partial x / \partial y]^2 / n - 1)^{1/2}$ (Goddard 1971). Results are shown in Table 2. Results are expressed as percent error. The pattern of error basically confirms the observed error, as shown by SRT. Over the first several days, predicted error is in the 20 to 30% range, then it increases.

Planned Use of the Heat Budget

The results of the heat budget may be used in several different ways. As it stands the heat budget describes the physical energy fluxes in Castle Lake. More work is being planned in conjunction with a group working on the heat budget of Lake Tahoe to further studies of basic physical limnology.

The R_n term will be used as the solar energy term in a photosynthesis model as part of a model of phosphorus flux in Castle Lake (Richey 1973). In the same model H will be used as the surface boundary condition for the calculation of eddy diffusivity (equation 4). These results will be compared to eddy diffusivity values calculated using the null energy flux at the bottom of the water column as the boundary condition. K_h is important in describing turbulent transport processes, affecting the distribution of nutrients.

Principle of ATP Measurements

The use of ATP measurements as an indicator of microbial biomass is based on the observation that all ATP degrades immediately upon cell death and that ATP: organic carbon ratios are remarkably constant in bacteria and algae. The constancy of ATP level apparently results from a cellular control mediated by photosynthetic phosphorylation by the chloroplasts in the light and by oxidative phosphorylation in the dark. If zooplankton are removed by filtration and phytoplankton biomass is negligible then ATP can be converted directly to bacterial biomass. In most lakes phytoplankton biomass must first be corrected for. The reliability of the bacterial community standing crop estimate has not previously been investigated, but simultaneous ATP analysis, phytoplankton counts and direct bacteria counts made it possible in this study.

The method of ATP determination used in this study is given in Strickland and Parsons (1968). Briefly, the sample is boiled in Tris buffer, frozen and then combined with luciferase in a JRB ATP photometer, which measures the light given off by the reaction of ATP + luciferase.

Results of Castle Lake ATP Bacterial Biomass Determinations

Measured ATP values, together with calculated phytoplankton ATP values, are listed in Table 3. Zooplankton were removed before ATP determinations. If we make the crude assumption that the ratio of bacterial to phytoplankton ATP is the same as the corresponding ratio of the volumes then over 90% of the ATP, on the average, will be phytoplankton ATP. Accordingly, we would expect good agreement between the total and calculated phytoplankton ATP. Table 3 shows the values usually fall within a factor of 2 of each other, but an examination of the residuals indicates that the calculated phytoplankton ATP exceeds the measured ATP by an average of about 14%. This indicates that the average ATP content of phytoplankton is being overestimated, and suggests that many of the algal cells counted were not really viable and had lowered ATP levels. A similar conclusion on viability follows from examination of ¹⁴C-uptake by individual cells in autoradiographs (Stull 1972).

If bacterial ATP were calculated from the total ATP minus the calculated phytoplankton ATP, 29 of the 48 values would be negative. This problem stems from an attempt to determine a small number by subtracting one large uncertain number from another large uncertain number. The usefulness of this method of determining bacterial biomasses depends in the proportion of the total ATP that is contained in the phytoplankton, as follows.

Let τ , β , and ϕ be total, bacterial, and phytoplankton ATP, respectively, and let $\gamma = \phi/\tau$. Then $\beta = (1 - \gamma)\tau$ and $(\sigma_\beta/\beta)^2 = \sigma_\gamma^2/(1 - \gamma)^2 + (\sigma_\tau/\tau)^2$. But $\sigma_\gamma^2/\gamma^2 = (\sigma_\phi/\phi)^2 + (\sigma_\tau/\tau)^2$, so $(\sigma_\beta/\beta)^2 = (\gamma/[1 - \gamma])^2 \{ (\sigma_\phi/\phi)^2 + [\sigma_\tau/\tau]^2 \} - (\sigma_\tau/\tau)^2$. Since 95% confidence limits are given approximately by $\pm 2J$, then these limits for β as a percentage of the mean will be given by $\pm 100 \times (2\sigma_\beta/\beta) = \pm 200 \times \{ (\gamma/[1 - \gamma])^2 \{ (\sigma_\phi/\phi)^2 + [\sigma_\tau/\tau]^2 \} + (\sigma_\tau/\tau)^2 \}^{1/2}$. Twelve replicate determinations of total APR at 15 m yielded a value of $0.381 \pm 0.064 \mu\text{g}^{-1}$, implying that $(\sigma_\tau/\tau)^2 = 0.028$. If we make the optimistic assumption that the only error in ϕ is the counting error, and that the community is made up of one species

for which 200 cells are counted in each sample, then $(\sigma_\phi/\phi)^2 = (\sqrt{200}/200)^2 = 0.005$. Thus, $\pm 100 \times (2\sigma_\beta/\beta) = \pm 200 \times \{0.028 + 0.033 (\gamma/1-\gamma)^2\}^{1/2}$ and this relationship between 95% confidence limits and γ is plotted in Figure 1.

Estimating β from $\tau-\phi$ is obviously a futile enterprise in Castle Lake, where 95% confidence limits will be over $\pm 200\%$ of the mean, and the same conclusion is likely to hold in all lakes but the deepest, where bacterial biomass in the depths might overshadow phytoplankton biomass. According to Figure 1, the best obtainable 95% confidence limits are $\pm 33\%$ of the observed value when phytoplankton are absent. Most of the uncertainty probably arises in sampling, since Strickland and Parsons (1968) state 95% confidence limits of 6% at the 0.050 g l^{-1} level. Precision would undoubtedly improve in regions where the ATP gradient with depth is not as large as in Castle Lake.

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OVERVIEW OF CASTLE LAKE MODELING

Modeling efforts at Castle Lake were initiated to provide a framework to work out ideas concerning lake processes. The ultimate goal is to quantitatively predict lake behavior, given changing inputs of nutrients and energy.

It would be quantitatively impossible and theoretically unnecessary to do a complete step-by-step deterministic model involving every process and nutrient. An alternate, more manageable approach taken for Castle Lake is to select some of the most important processes and limiting nutrients bounding the lake and studying them in depth in a series of discrete submodels. Eventually when a sufficient number of submodels have been gathered their inputs and outputs may be integrated into a more general model describing the structure and function of the lake.

The limnology of Castle Lake has been under continuous study by Dr. Charles R. Goldman and his students at the University of California, Davis, since 1959, with particular emphasis on primary production and nutrient limitation. The regular, detailed sampling program in effect plus individual research allows formulation of models of finer resolution than possible with models based on a general survey of several lakes. Combined, the two approaches should produce very powerful models.

Using the past data base at Castle Lake to delineate the important processes five submodels are currently being developed, including studies of the plankton community, bacterial production, the phosphorus cycle, benthic carbon flux and the benthic macrofaunal community. All models are being developed from extensive fieldwork designed in advance to measure the necessary pools, fluxes and external influences over depth and time. We feel that in this way meaningful, surprising models may be developed. Our initial rather optimistic goals have been somewhat modified as the complexities of the lake present themselves. The following is a synopsis of where we are. Please contact the individual researchers for further detail and results.

Bacteria Cell Concentration Model - Alan Jassby

A literature search and experience suggest that the rate of change of bacteria cell numbers in the water column may be given by

$$\frac{dB}{dt} = T_h + T_v + S + G + P + D$$

where T_h is horizontal transport, T_v is vertical transport, S is sinking, G is grazing, P is cell division and D is death. The model is for the hypolimnion, where bacterial gradient experiments showed that T_h is negligible. T_v is given by

$$T_v = -\frac{\partial}{\partial z} k_z \frac{\partial B}{\partial z}$$

where k_z is the coefficient of eddy diffusivity and z is depth. k_z was

solved for by assuming a null energy flux at the bottom of the water column as the boundary condition. S is given by Stoke's law, as verified by experimentation. G is given by literature results for zooplankton grazing, with the Castle Lake zooplankton data divided into three species and three size classes used. P was measured by using a combined ^{35}S - ATP biomass technique and dissolved organic carbon and temperature data. D was solved for by running the simulation initially without death and assuming the discrepancy was due to death.

Multispecific Model of a Plankton Community - Noel Williams

This model discusses the changes by species of the phytoplankton and zooplankton communities of Castle Lake. Data is derived from phytoplankton and zooplankton counts, measurements of particulate, dissolved inorganic and organic carbon and other chemical and physical parameters. Equations are based on conservation of mass and the form derived from physiological and behavioral evidence. Parameters are best fit using the Marquardt algorithm and are non-linear functions of environmental conditions such as light temperature and nutrients. The general form of the equations is shown in Figure 1.2, where P is phytoplankton, Z is zooplankton, N is nutrients and lower case letters are parameters.

Phosphorus Dynamics - Jeff Richey

The phosphorus cycle approach for Castle Lake is to take the model proposed in Figure 2, consider it repeated for a series of cubes down the water column and measure regularly the changes in the different components over time and depth. The different phosphorus pools will be considered as state variables and their size determined chemically, with the flows between measure by isotope flux. Exogenous variables such as temperature, light and excess nutrients will be handled through a set of independent experiments. A heat budget will be calculated to establish the boundary conditions for (physical) energy flux across the surface. Equations are shown in Figure 1.3. Terms represent conditions expressed in Figure 2, and will be derived from the blocks of data described above. Possible directions for generalizing the model to other systems and to models of eutrophication will be explored.

Benthic Carbon Flux - Frank Sanders

As yet the benthic carbon flux model is still being developed, with the emphasis on measuring the different compartments. Figure 1.4 is a summary with Figure 3 showing crudely the basic model and how the different measurements are being done.

Benthic Macrofaunal Community - Peter Neame

This model was originally to be a model of the benthic insect community, including temperature, growth and predator-prey interactions. However this proved to far too difficult for one person, so efforts turned to species distribution, which showed clumping to be rather dubious and perhaps even an artifact. Work is progressing on nutrient regeneration from the sediments and population distribution.

SUMMARY

So far the separate submodels are not finished, the pelagic models perhaps being further along. The ties between the bacteria and plankton model would be through grazing. Both of these models contribute to the phosphorus model through statements of nutrient regeneration and uptake and of metabolic activity. The fate of the nutrients in turn affects the plankton community. Heat budget calculations may be used to set up a general physical matrix for the entire biotic system. The benthic models depend on each other through grazing and nutrient behavior, particularly carbon. Once they can predict the affects of the sediments on the overlying waters, the benthic models may be linked with the pelagic models. Some of these links have already been made, most are waiting to be done.

Figure 1. Summary of Castle Lake submodels.

1. Bacteria cell concentration model (Alan Jassby)

$$\frac{\partial B}{\partial t} = T_h + T_v + S + G + P + D$$

2. Multispecific model of a plankton community (Noel Williams)

$$\frac{\partial P}{\partial t} = KP + \frac{\partial k_z}{\partial z} \frac{\partial P}{\partial t} - S \frac{\partial P}{\partial t} - GPZ$$

$$\frac{\partial Z}{\partial t} = (g-r)PZ - dZ$$

$$\frac{\partial N}{\partial t} = -a(kP) + bZ$$

3. Phosphorus dynamics (Jeff Richey)

$$\frac{\partial P_i}{\partial t} = f(G)Z_s + f(Au, Lk) + f(AP) - f_1(B) - f_1(PS)$$

$$\frac{\partial P_o}{\partial t} = T_o + f_2(PS) - f_3(PS) - f_2(B)$$

$$\frac{\partial P_p}{\partial t} = T_p + gP_p - f(P_o, PS) - bP_p(G) - sP_p$$

$$\frac{\partial P_b}{\partial t} = T_b + gP_b - \alpha P_b G$$

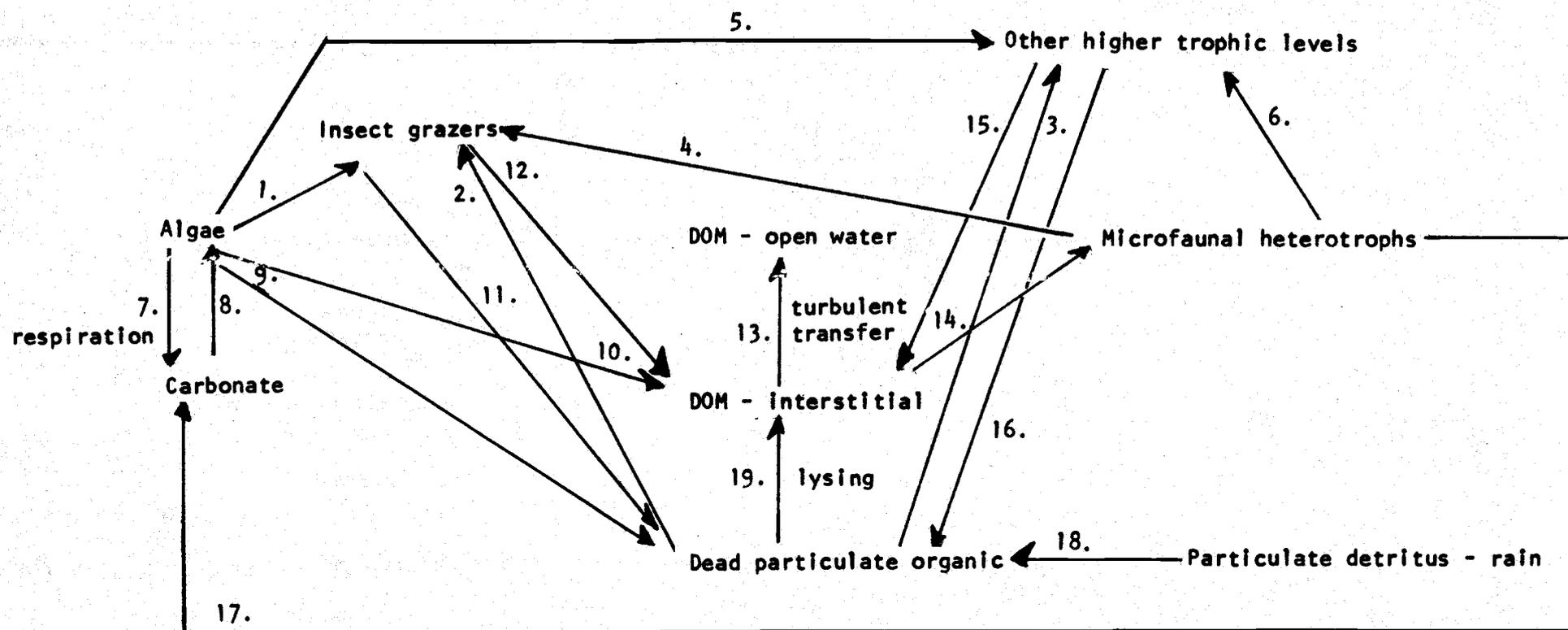
$$\frac{\partial P_z}{\partial t} = FEG - (dT \frac{eP_i}{C}) Z_s + f(VM)$$

4. Benthic carbon flux (Frank Sanders)

The following pools and fluxes between are currently being measured: dead particulate organic, particulate detritus, interstitial DOM, open water DOM, carbonate, microfaunal heterotrophs, algae and insect grazers.

5. Benthic macrofaunal community (Peter Neame)

Started to model species-specific insect communities but found them too complex; currently considering species distribution and nutrient regeneration.



1. Measured
2. Measured
3. Measured or assumed relationship
4. Measured
5. Assumed relationship
6. Assumed relationship
7. Measured
8. Measured
9. Extracted from data and assumed relationship
10. Measured
11. Measured (?)

12. Measured (?)
13. Measured (?)
14. Measured (?)
15. Assumed relationship
16. Assumed relationship
17. Measured
18. Measured
19. Measured

Figure 3. Benthic carbon flux.