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The Optimal Time Path for Carbon Abatement and Carbon Sequestration under Uncertainty: The Case of Stochastic Targeted Stock

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Abstract:

We explore the optimal time path of carbon sequestration and carbon abatement in stabilizing CO_2 levels under uncertainty of climate impacts. Using a two-period sequential decision making model, we analytically derive optimal rates for the two control variables, abatement and sequestration rates. Uncertainty is assumed to affect the desired future stabilization level of the CO_2 stock but is resolved prior to the decision on how much to control the stock in the second period. Contrary to recent numerical studies, we find that uncertainty can make it optimal to use carbon sequestration either earlier or later depending on the relative rates of change in both marginal cost curves and on the amount of land that can be converted to forest. Comparative statics suggest that an increase in the discount factor could either increase or decrease the optimal rate of sequestration in the first period depending on the expected rate of change of the marginal cost of sequestration in the second period and on future benefits of current sequestration.

Keywords: carbon sequestration, uncertainty, dynamic optimization, economics, climate change

1. Introduction

The Kyoto protocol initiated a broad scientific discussion concerning the role of carbon sequestration as a strategy to limit greenhouse gases (GHG) emissions. Although there is a consensus in the scientific world that carbon sequestration should be included in a portfolio of GHG mitigation strategies (Nabuurs et al,, 2007; Richards and Stokes, 2004) the optimal timing of its implementation is still debated. Some argue that carbon sequestration should be viewed as a short term reduction strategy either to buy time for other technologies to emerge (Metz et al, 2001; Feng et al, 2002) or because the attractiveness of carbon sequestration in term of its cost will decrease in the long run (Stavins, 1999). On the contrary, some argue that carbon sequestration should be delayed towards the end of the century given that carbon prices are increasing over time (Van't Veld and Plantinga, 2005; Sohngen and Mendelsohn, 2003). Other findings suggest that the rate of growth in carbon prices can influence the optimal timing of carbon sequestration (Sohngen and Sedjo, 2006).

An important feature of carbon sequestration that distinguishes it from abatement technologies is its ability to actually reduce atmospheric concentrations of CO_2, by planting trees for example. Carbon abatement, in contrast, cannot be larger than emissions at any given period. At the extreme, one could abate all emissions and hold the stock constant whereas carbon sequestration has the potential to reduce the atmospheric stock of CO_2 in absolute terms or relative to a baseline. This asymmetry may play a crucial role in determining the optimal timing of a sequestration policy. Consider the following example. Assume we would like to stabilize the atmospheric stock at B ppm at a given time in the future. But, currently, we are uncertain about the severity of impacts at that level of stabilization. For instance, choosing today a specific concentration X ppm could produce a likely global warming as low as 1.5°C, but warming could be as high as 4.5°C, increasing the severity of impacts. If sequestration is currently cheaper than abatement, should we use most sequestration capacity in the near future or should we save it as insurance in case the severity of impacts is large and we need to do more in terms of reducing the atmospheric stock?

Applying a dynamic optimization approach, this paper explores the optimal time path of carbon sequestration and carbon abatement in stabilizing the level of carbon dioxide in the atmosphere under uncertainty in climate impacts. Current international efforts to mitigate climate change are focused on stabilizing atmospheric concentrations of greenhouse gases by specific times (Den Elzen et al, 2010, National Research Council, 2010). A two-period sequential decision making model is analyzed. Expected present value costs of abatement and sequestration are minimized subject to two state variables: the level of CO_2 stock in the atmosphere and the stock of suitable land that can be converted to forest and, thereby, sequester carbon. Both controls are treated as investments where current reduction efforts yield future reduction benefits. Uncertainty regarding the desired stabilization level of the atmospheric stock is resolved prior to the decision on how much to control the stock in the second period. Our results show that uncertainty in climate impacts may lead to three different outcomes depending on the structure of both marginal cost curves and on the amount of sequestration capacity: the ***Aggressive Path*** in which uncertainty results in more deployment of abatement and sequestration in the first period, the ***Conservative Path*** in which uncertainty results in less deployment of abatement and sequestration in the first period and the ***Indeterminate Path*** in which uncertainty can lead to either more or less deployment of abatement and sequestration in the first period.

There are a handful of studies in the economics literature on the optimal time path of carbon sequestration and/or carbon abatement in controlling GHG but only a few incorporate uncertainty in the analysis. Webster (2002) shows, by using a two-period sequential decision-making model, that uncertainty in climate impacts which is resolved through time can lead to either more restrictive or less restrictive abatement reduction policies today. This author does not, however, consider tradeoffs between carbon abatement and carbon sequestration along the optimal time path. The rest of the studies explore uncertainty with respect to climate damages in a numerical analysis. Main results are consistent with the ***conservative path*** suggesting that substantial amounts of carbon could be optimally sequestered in forests especially towards the end of the century (Sohngen and Mendelsohn, 2003) and as a safety measure for future use in case of catastrophic climate events (Gitz et al, 2006). We show that these previous studies are special cases of the broader theory.

The key contribution of our paper is that we provide an analytical treatment of the optimal timing of carbon sequestration and abatement under uncertainty. We show that uncertainty can make it optimal to use carbon sequestration either earlier or later and clarify the conditions under which these outcomes are obtained. Despite the complexity of the model, we are able to express the solution in terms of a single control variable, allowing us to present intuitive graphs that clearly present our main results. The paper is organized as follows. In the next section we present the model and derive the optimal rates of sequestration and abatement in both periods. In the third section we analyze the three possible solutions. The fourth section deals with comparative analysis with regards to all parameters influencing our main results. The fifth section concludes.

2. Model Set Up and Solution

We develop a two period sequential decision making model to analyze the optimal time path of carbon sequestration and carbon abatement as a reduction strategy to control CO_2 concentrations in the atmosphere. Let X_t be the level of CO_2 stock in the atmosphere at time t where $t = \{0,1\}$ and e_t be the rate of CO_2 emissions (CO_2 tons/unit of time) that is emitted to the atmosphere at time t. e_t can be viewed as a baseline emissions path at time t and is exogenous to the model (hereinafter e) . Carbon stock can be controlled by either carbon abatement a_t and/or carbon sequestration s_t. The rate of abatement at a given time t cannot be greater than the rate of CO_2 emissions that are emitted to the atmosphere at the same period of time (i.e., $a_t \leq e$).

We denote the finite stock of suitable land which can be converted to forest (and currently not in forest use) by C_t. We assume that one unit of land sequesters one ton of carbon so that sequestration can be measured in the same units as abatement and carbon emissions. Note that the rate of carbon sequestration at any period t can be higher than the rate of CO_2 emissions (e) in the same period and thus has the capability of actually reducing the level of the CO_2 atmospheric stock.

State variables are observed at beginning of periods before the decisions on how much to control the CO_2 stock using sequestration and/or abatement are made. Therefore, X_0, corresponds to the initial level of the CO_2 atmospheric stock whereas X_1 corresponds to the stock level minus the optimal rates of abatement and sequestration reductions in time $t = 0$ (i.e., the first period) plus the rate of CO_2 emissions in that period (i.e. e).

Total costs of carbon sequestration in a given period t are given as $TC_t^s = \int_0^{s_t} P_s(u)du$. where $P_s(\cdot)$, the marginal cost of sequestration, is twice differentiable with both $P_s{}'(\cdot) > 0$ and

$P_s''(\cdot) > 0$, and using all available land to sequester carbon is infinitely expensive (i.e. $\lim_{s \to C_0} P_s(s) = \infty$). Similarly, total costs of carbon abatement in a given period t are given as $TC_t^a = \int_0^{a_t} P_a(v)dv$ where $P_a(\cdot)$, the marginal cost of abatement, is twice differentiable with both $P_a'(\cdot) > 0$ and $P_a''(\cdot) > 0$, and abating all emissions at a given period is infinitely expensive (i.e. $\lim_{a \to e} P_a(a) = \infty$). Furthermore, and in accordance with previous studies suggesting low costs per ton of CO_2 sequestered relative to mitigation techniques (Dudek and LeBlanc 1990; Stavins 1999; Van't Veld and Plantinga, 2005), we assume abatement is more expensive than sequestration initially (i.e., $P_a(0) > P_s(0)$).

The decision about how much to control the stock with carbon abatement is restricted in the model to the first period only. That is, the planner chooses the optimal rate of carbon abatement for both periods in the first period. The decision about how much to abate in the first period can be viewed as an investment in abatement technology which yields benefits not only at the current time but also in the future. This simplification reduces the complexity of choosing optimal reduction programs for two control variables in the second period without restricting the possible set of outcomes (derivation of the general model and how its collapses to the restricted one are available in Appendix A). Lastly, any sequestration investments done in period t, result in additional sequestration in period $t + 1$ of $\tau * s_0$, where $0 \leq \tau \leq 1$.

We are abstracting from many real world issues to simplify our model. First, the movement of land from forest to non-forest uses is not modeled. That is, we allow the conversion of non-forest land to forest but not the other way around. Second, we do not consider the permanence of sequestered carbon. In reality, carbon is stored in forests by the process of photosynthesis and CO_2 is emitted to the atmosphere when forests trees are burned and cut down.

We assume that sequestrated carbon is never released back to the atmosphere over the finite time horizon of the model. Third, we do not include the absorption of atmospheric CO_2 stock by oceans in our model.

The objective of a central planner is to minimize expected present discounted costs of sequestration and abatement while stabilizing the atmospheric stock at a level of B ppm at the end of the second period. The planner is uncertain about the desired stabilization level of the atmospheric stock, B, when making the decision about how much to control the stock in the first period. This uncertainty is due to the limited information on the severity of climate impacts that is available to the planner in the first period.[1] The planner is assumed to knows the mean of the desired stabilization level, denoted $\bar{B}$, but is uncertain about the variability around the mean, denoted σ, when making reduction decisions in the first period. We consider only two possible states of the world at the end of the second period, B_H and B_L, each with an equal probability of occurrence. If state of world H prevails, the desired level of stabilization is $B_H = \bar{B} + \sigma$ and if state of world L prevails, the desired level of stabilization is $B_L = \bar{B} - \sigma$. Information regarding the actual desired stabilization level of the stock is revealed prior to when the decision about how much to control the stock in the second period is made. Finally, we assume $C_0 \geq X_0 + 2e - B_L$ so there is enough sequestration capacity to reduce the atmospheric stock to B_L if needed.

We apply backwards induction starting with the minimization problem for the second period:

$$min_{s_1} \int_0^{s_1} P_s(u)du$$

[1] To make the story more realistic let the first period represent the current world reflecting reduction decisions in the next few decades (say 40 years) and the second period as the future world, one when uncertainty from climate change is reduced (say 80 years). The proposed length of the two periods enables the introduction of activities such as afforestation and reforestation which can be fully implemented within each one of the periods.

subject to:

$$B = X_1 + e - a^* - s_1 - \tau s_0^* \qquad X_1 \ is \ given \qquad a^* \ is \ given \qquad s_0^* \ is \ given \tag{1}$$

$$0 \leq s_1 \leq C_1 \qquad C_1 \ is \ given \tag{2}$$

The transition equation of the atmospheric CO_2 stock is given in equation (1). The stabilization level at the end of the planning horizon (B) is determined according to the CO_2 stock level at the beginning of the second period (X_1) plus the exogenous rate of emissions during the second period (e) minus the optimal rate of abatement that was set in the first period (a^*), and minus the stock reductions obtained in the first period (s_0^*) and the additional sequestration taking place in the second period due to the first period sequestration investments (τs_0^*). Constraints on the optimal rate of sequestration are given in equation (2). This is a deterministic optimization problem with a fixed end point and a single decision variable. Because the planner is forced to meet the desired stabilization level and has only one way of getting there (i.e. by sequestering more carbon) optimal rates of sequestration are derived from (1) for each one of the two possible states of the world in the second period. In particular, we solve for s_1 given a state of world B_L and a state of world B_H to get:

$$s_1^*(B_L) = \begin{cases} X_1 + e - a^* - \tau s_0^* - B_L, & X_1 + e - a^* - \tau s_0^* - B_L > 0 \\ 0 \qquad , & otherwise \end{cases} \tag{3}$$

$$s_1^*(B_H) = \begin{cases} X_1 + e - a^* - \tau s_0^* - B_H, & X_1 + e - a^* - \tau s_0^* - B_H > 0 \\ 0 \qquad , & otherwise \end{cases} \tag{4}$$

At the extreme all sequestration capacity is exploited and (2) holds with equality (i.e. $s_1 = C_1$). Substituting $s_1 = C_1$ into (3) one can form the relationship between X_1 (levels of the CO_2 stock when entering the second period) and C_1 (sequestration capacity when entering the second period).

Having found optimal rates of sequestration in the second period for both states of the world we then move backward to solve for the optimal rates of sequestration and abatement in the first period. Recall that in the first period the planner is uncertain whether state of world B_L or state of world B_H will prevail. Equation (5) is a Bellman equation[2] for the minimization problem in the first period given the expected optimized value function for the second period:

$$V(a, s_0, 0) = \int_0^a P_a(v)dv + \int_0^{s_0} P_s(u)du + \rho E\left[\int_0^{s_1^*(B)} P_s(u)du\right] \tag{5}$$

Subject to:

$$X_1 = X_0 + e - a - s_0 \qquad X_0 \; is \; given \tag{6}$$

$$0 \leq a \leq e \tag{7}$$

$$0 \leq s_0 \leq C_0 \qquad C_0 \; is \; given \tag{8}$$

The expected optimal value function for the second period is weighted according to the two states of world and ρ is the discount factor. The transition equation of the CO_2 stock in the first period is described in equation (6) and the constraints on the optimal rates of abatement and sequestration are given in equations (7) and (8), respectively.

Taking the partial derivatives of the Bellman equation (5) with respect to s_0 and a, rearranging and assuming interior solutions, we obtain the following first order conditions:

$$\frac{1}{2}P_a(a) = \left(\frac{1}{2}\right)\rho\{P_s[\beta(B_H) - 2a - (\tau + 1)s_0] + P_s[\beta(B_L) - 2a - (\tau + 1)s_0]\} \tag{9}$$

$$\frac{(1-\rho)}{\rho}\frac{P_s(s_0)}{(\tau+1)} = \left(\frac{1}{2}\right)\{P_s[\beta(B_H) - 2a - (\tau + 1)s_0] + P_s[\beta(B_L) - 2a - (\tau + 1)s_0]\} \tag{10}$$

where $\beta(B_i) = X_0 + 2e - B_i$, $i = \{L, H\}$. According to (9), at the optimum, marginal cost of reducing one ton of CO_2 by abatement over the two periods (i.e. half a unit of abatement in the first period) should be equal to the expected marginal cost of sequestering one unit of CO_2. The

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[2] A Bellman equation is a necessary condition for optimality.

first term on the RHS of (9), $\beta(B)$, represents the total CO_2 reduction needed to meet the stabilization level given the initial level of the atmospheric stock and the rate of emissions that are emitted to the atmosphere during the two periods. The second term, $2a$, is the total deployment of abatement over the two periods and the last term is the optimal rate of sequestration in the first period plus the additional sequestration taking place in the second period due to the first period sequestration investments.

Condition (10) is a no-arbitrage condition for sequestration deployment between the two periods. That is, no financial gains can be made by either pushing sequestration deployment to the second period or vice-verse. In addition, for an interior solution to hold $\frac{1-\rho}{\rho} > (\tau + 1)$ because for $s_0 > 0$ it is required that $P_s(s_0) < P_s(s_1^*)$.

Using conditions (9) and (10) we can find optimal rates of abatement and sequestration in the first period. Equate (9) and (10) and rearrange to get the response function of sequestration in the first period for any given rate of abatement:

$$s_0^* = P_s^{-1}\left[\frac{(\tau+1)}{2(1-\rho)}P_a(a)\right] \tag{11}$$

From (11), sequestration in the first period is increasing with abatement, with higher rates of future benefits from sequestration today and, as the discount factors rises.

Substituting (11) into (9), one obtains a single equation that fully characterizes the solution of the model in terms of the optimal rate of abatement in the first period:

$$\frac{1}{2}P_a(a) = \frac{1}{2}\rho\left\{P_s\left[\beta(B_H) - 2a - (\tau+1)P_s^{-1}\left(\frac{(\tau+1)}{2(1-\rho)}P_a(a)\right)\right]\right.$$
$$\left. + P_s\left[\beta(B_L) - 2a - (\tau+1)P_s^{-1}\left(\frac{(\tau+1)}{2(1-\rho)}P_a(a)\right)\right]\right\}$$

(12)

According to (12), one-half the marginal cost of abatement is equated to the present value expected marginal cost of sequestration in the second period. The LHS of (12) is a monotonically increasing function of a whereas the RHS of (12) is decreasing in a. Being in the interior guarantees that both sides of (12) intersect.[3]

3. The Three Paths

We now characterize the three possible solutions of the model. While the general form of the model can be analyzed, our goal is to demonstrate that the Aggressive Path, Conservation Path, and Indeterminate Path can be obtained. For this purpose, it is sufficient to evaluate a special case of the model, namely, one that employs specific, but flexible, functional forms for the marginal costs of abatement and sequestration.

We evaluate equation (12), as this equation completely characterizes the optimal solution of the model. It is instructive to first analyze two functions embedded in the RHS of (12). The first is $f_\gamma(a) = {P_s}^{-1}(\gamma P_a(a))$, a response function relating sequestration deployment in the first period to the chosen rate of abatement (see equation (11)) where $\gamma = \frac{(\tau+1)}{2(1-\rho)}$. The second is $R(a, \bar{B}, \sigma) = P_s\big(\beta(B_i) - 2a - (\tau+1)f_\gamma(a)\big)$, the marginal cost of sequestration in the second period. R is a function of abatement, the mean stabilization level and the variability of

[3] For $a = 0$, take the inverse of the marginal cost of sequestration in (12) and rearrange to get the following condition: ${P_s}^{-1}\left(\frac{1}{2\rho}P_a(0)\right) + (\tau+1){P_s}^{-1}\left(\frac{\tau+1}{2(1-\rho)}P_a(0)\right) < \beta(B_H) + \beta(B_L) = X_0 + 2e - E[B]$. The LHS is the sum of present and future sequestration. If this sum is equal or greater than the expected amount of CO_2 reduction needed (the RHS of the condition) than we are in a corner solution (i.e. we can do without abatement).

stabilization level, among other parameters. R is the key function to analyze. The optimal solution is determined at the value of a where R is equated to $\frac{1}{2\rho}P_a(a)$.

The properties of $f_\gamma(a)$, which influence the properties of $R(a, \overline{B}, \sigma)$, determine qualitatively the solution of the model when there is uncertainty in the stabilization level. In particular, the second derivative of $f_\gamma(a)$ with respect to abatement could be positive or negative and therefore the general form of $f_\gamma(a)$ could be convex, concave or a combination of both (for a detailed derivation of $f_\gamma(a)$ and $R(a, \overline{B}, \sigma)$ see appendixes B and C, respectively). $f_\gamma(a)$ is illustrated in Figure 1C where sequestration (abatement) is on the vertical (horizontal) axis. Figure 1C is constructed from figures 2A and 2B, which depict marginal costs of abatement and sequestration, respectively. As can be seen, sequestration and abatement are bounded above by the sequestration capacity C_0 (Figure 1B) and by the exogenous emissions level e (figure 1A), respectively. If sequestration is increasing (decreasing) with an increase rate in abatement then $f_\gamma(a)$ is convex (concave) as depicted in graph D (F) of Figure 1C. Finally, $f_\gamma(a)$ may exhibits an inflection point, being convex in low rates of abatement and sequestration and concave in high rates of abatement and sequestration as depicted in graph G of Figure 1C.

Before we turn to analyze the three different paths, it is instructive to examine the optimal rates of deployment of sequestration and abatement in the absence of uncertainty in climate impacts. In this case, B is known in the first period when decisions about abatement and sequestration are made. Equation (12) reduces to:

$$\frac{1}{2}P_a(a) = \frac{1}{2}\rho P_s\big(\beta - 2a - (\tau + 1)f_\gamma(a)\big) \tag{13}$$

The interpretation of (13) is similar to (12), except that the present value marginal cost of sequestration (RHS of (13)) in known with certainty in the first period. In this case, the curvature properties of $f_\gamma(a)$ do not have a role in determining the optimal solution. With no uncertainty, the optimal rate of abatement is determined by a unique point at which the marginal cost of abatement is equated to the present value marginal cost of sequestration. We denote the deployment of abatement in the absence of uncertainty by $\bar{a}$.

We proceed with specific functional forms for the marginal costs of sequestration and abatement. Let the marginal cost of sequestration be $P_s(s) = \frac{K}{(C_0-s)^q}$ where K is a positive constant, $q > 1$ and, the rate of sequestration, s, is bounded above the sequestration capacity, C_0. Similarly, let the marginal cost of abatement be $P_a(a) = \frac{J}{(e-a)^p}$ where J is a positive constant, $p > 1$ and, the rate of abatement, a, cannot exceed the rate of emissions, e. In addition, assume $\frac{J}{e^p} > \frac{K}{C_0^q}$ so that sequestration is initially cheaper than abatement. These functional forms are flexible enough to give rise to the three cases of interest. For each case, we compare the solution under uncertainty to the solution with no uncertainty in climate impacts and the desired stabilization level.

The Aggressive Path: If $\mathbf{p} > \mathbf{q}$ then uncertainty in climate impacts results in higher deployment rates of abatement and sequestration in the first period relative to no uncertainty in climate impacts.

Using the specific functional forms in conjunction with equation (11) yields:

$$f_\gamma''(a) = Z \cdot \frac{p}{q} \cdot \left(1 - \frac{p}{q}\right)(e-a)^{\frac{p}{q}-2} \tag{14}$$

where $Z = \left(\frac{K}{\gamma J}\right)^{\frac{1}{q}}$ and $\gamma = \frac{\tau+1}{2(1-\rho)}$. Given that $p > q$, $f_\gamma''(a) < 0 \; \forall a \in (0, e)$ and according to (11) sequestration is increasing with a decreasing rate in abatement. Graphical representation of $f_\gamma(a)$ is depicted in Figure 2A. $f_\gamma(a)$ is strictly concave, bounded above by the capacity of sequestration, C_0, and restricted to rates of abatement smaller than e. Next, substituting the functional forms into $R(a, \bar{B}, \sigma)$, taking the second derivative with respect to abatement and rearranging yields:

$$\frac{R''(a,\bar{B},\sigma)}{P_S''(\cdot)} = -\frac{(\tau+1)f_\gamma''(a)}{q+1}\left(C - \left(\beta - 2a - (\tau+1)f_\gamma(a)\right)\right) + \left(2 + (\tau+1)f_\gamma'(a)\right)^2 \tag{15}$$

For $p > q$ equation (15) is always positive because $f_\gamma''(a) < 0 \; \forall a \in (0, e)$ and all other terms in (15) are positive. In this case $R(a, \bar{B}, \rho)$ is strictly convex. This is depicted in Figure 2B where abatement is on the horizontal axis and marginal cost of abatement/sequestration is on the vertical axis. As noted above, the optimal rate of abatement should be set such that at the end of the program marginal cost of abating and the expected marginal cost of sequestering one unit of CO_2 are equated. Since $R(a, \bar{B}, \sigma)$ is convex, by Jensen's inequality the expected value of $R(a, \bar{B}, \sigma)$ is not smaller than $R(\bar{a})$ as depicted in Figure 2B. Therefore, the intersection of the secant line of the expected marginal cost of sequestration, which goes through the point $(\bar{a}, E[R(a^*, \bar{B}, \sigma)])$, must intersect $\frac{1}{2}P_a(a)$ to the right of the crossing of $R(\bar{a})$ and $\frac{1}{2}P_a(a)$at a greater rate of abatement, denoted a^*, such that $\bar{a} < a^*$. Note that the distance between $a^*(B_H)$ and $a^*(B_L)$ is exactly σ, as depicted in Figure 2B,[4] as this is the only thing that differs between the two possible stabilization levels.

[4] $E[R(a, \bar{B}, \sigma)] = \frac{1}{2}R(a^*, \bar{B} - \sigma) + \frac{1}{2}R(a^*, \bar{B} + \sigma) = R\left(a^* - \frac{1}{2}\sigma, \bar{B}\right) + R(a^* + \frac{1}{2}\sigma, \bar{B})$

Therefore, under uncertainty in climate impacts more abatement is deployed, and consequently more sequestration in the first period, relative to no uncertainty in climate impacts. Note that p and q are price elasticity of excess abatement and excess sequestration capacity, respectively.[5] Thus, on the ***Aggressive Path,*** excess abatement is more elastic than excess sequestration capacity (i.e. elasticity of excess abatement with respect to excess sequestration is greater than one, $\frac{p}{q} > 1$). That is, the planner takes advantage today of the relatively small rate of increase in marginal cost of abatement, compared with the rate of increase in marginal cost of sequestration, and abates more CO_2 when facing uncertainty in climate impacts. Put differently, we would rather be on the side of protecting ourselves today from ending up at B_L than waiting for the arrival of more information on the desired level of the atmospheric CO_2 stock.

The Conservative Path: If $\boldsymbol{p < q}$ and $\boldsymbol{C_0}$ is large enough then uncertainty in climate impacts results in lower deployment rates of abatement and sequestration in the first period relative to no uncertainty in climate impacts.

Given that $p < q$, by equation (14) we have that $f_\gamma''(a) > 0 \ \ \forall a \in (0, e)$. This is depicted in Figure 3A. $f_\gamma(a)$ is strictly convex, bounded above by the capacity of sequestration, C_0, and restricted to rates of abatement smaller than e. Furthermore, for a large enough capacity of sequestration the first term of equation (15) dominants the second one and $R(a, \bar{B}, \sigma)$ is strictly concave $\forall a \in (0, e)$ as depicted in Figure 3B. Now, by Jensen's inequality $R(\bar{a})$ is not smaller than the expected value of $R(a, \bar{B}, \sigma)$ as depicted in Figure 3B. Following the same reasoning as in the previous case we can infer that $\bar{a} > a^*$. In this case less abatement is deployed, and

[5] $E_{p,(C-s)} = \frac{\frac{\Delta P_s(s)}{P}}{\frac{\Delta s}{s}} \cdot \frac{(C-s)}{P_s(s)} = q \cdot \frac{P_s(s)}{(C-s)} \cdot \frac{(C-s)}{P_s(s)} = q$ and $E_{p,(e-a)} = \frac{\frac{\Delta P_a(a)}{P}}{\frac{\Delta a}{a}} \cdot \frac{(e-a)}{P_a(a)} = p \cdot \frac{P_a(a)}{(e-a)} \cdot \frac{(e-a)}{P_a(a)} = p$

consequently sequestration in the first period is lower under uncertainty relative to no uncertainty in climate impacts. In terms of elasticities, excess sequestration capacity is more elastic than excess abatement (i.e. elasticity of excess abatement with respect to excess sequestration is smaller than one, $\frac{p}{q} < 1$). On the ***Conservative Path***, we are better off abating less and waiting until he second period to see the realization of the desired stabilization level. If it is necessary to take the stock down to B_L in the second period, we can do it with sequestration which is less responsive to price changes, compared with abatement.

The Indeterminate Path: If $\boldsymbol{p_1 < q_1}$ $\forall \boldsymbol{a} \in (\boldsymbol{0}, \boldsymbol{a_\varepsilon})$, $\boldsymbol{p_2 > q_2}$ $\forall \boldsymbol{a} \in (\boldsymbol{a_\varepsilon}, \boldsymbol{e})$ ***where*** $\boldsymbol{a_\epsilon} < e$ ***and,*** $\boldsymbol{C_0}$ ***is large enough, then for low (high) rates of abatement uncertainty in climate impacts results in lower (higher) deployment rates of abatement and sequestration in the first period relative to no uncertainty in climate impacts.***

There is no reason to assume that the relative magnitude of p and q will stay the same for each combination of abatement and sequestration deployment. More specifically, it may be the case that initially $p < q$ as in the conservative case. But, there may exist a specific rate of abatement, say a_ε, for which the relative magnitude flips so $p > q$ as in the aggressive case. That is, $f_\gamma(a)$ has an inflection point, and is convex for low rates of abatement and concave for high rates of abatement (see Figure 4A). Similarly, $R(a, \bar{B}, \sigma)$ has an inflection point, and is concave (convex) for low (high) rates of abatement, as depicted in Figure 4B.

Uncertainty in climate impacts in this case could lead to either less or more deployment of abatement at the optimum and, consequently, how much sequestration is done in the first period relative to the no uncertainty case depends on whether we operate on the concave or the convex part of $R(a, \bar{B}, \sigma)$, respectively and on the location of marginal cost of abatement curve (equilibrium points R and T on Figure 4B which correspond to $P_a(a)$ and $\tilde{P}_a(a)$, respectively, where $P_a(a) > \tilde{P}_a(a)\ \forall a \in [0, e]$).[6] In the case of $P_a(a)$, we operate in an area where excess sequestration capacity is more elastic than excess abatement, the marginal cost of sequestration is relatively small (low rates of abatement and sequestration in the first period), and abatement is relatively expensive, due to the location of marginal cost of abatement curve. Thus, it better to deploy less of the relatively expensive abatement today and wait for the realization of the CO_2 stock at the beginning of the second period. The relatively inexpensive sequestration is used if the realization is indeed B_L. In the case of $\tilde{P}_a(a)$, we may operate in the area where excess abatement is more elastic than excess sequestration capacity, the marginal cost of sequestration is relatively high (large rates of abatement and sequestration are deployed in the first period) and abatement is relatively inexpensive. In this case, uncertainty results in higher rates of abatement and sequestration today, as opposed to waiting for the realization of the CO_2 stock before taking an action.

4. Comparative Analysis

We next evaluate how the solution, as defined by equation (12), is affected by changes in key parameters. Applying Cramer's rule, we find that the optimal rate of abatement decreases if future benefits from today's sequestration (τ), increase. As well, optimal abatement declines with a less ambitious stabilization target (B), a decrease in the rate of emissions to the atmosphere

[6] To illustrate that the location at which the marginal cost of abatement intersects $R(a, \bar{B}, \sigma)$ could vary think of the following case. Assume the case where there is no uncertainty in climate impacts, $a = 0$ and, $\rho = 1/2$. Condition (12) is then reduces to: $f_1(a) \equiv P_s^{-1}(P_a(a)) = \frac{\beta}{2} - a$. In this case the optimal rate of abatement is found at the intersection of the straight line and $f_1(a)$. But, nothing restricts the straight line from crossing $f_1(a)$ in its concave or convex parts. The intersection could be in either place depending on the initial level of the CO_2 stock, the desired level of stabilization, as well as the rate of emissions that are emitted to the atmosphere at any given period. This should hold in the uncertain case as long as the optimal solution in the uncertain case is in the neighborhood of the certain one.

(e), and with higher levels of the initial stock of atmospheric CO_2 (X_0).[7] Lastly, and less intuitively, we find that an increase in the discount factor (ρ), all else equal, could either result in more or less deployment of abatement (and consequently, either more or less sequestration in the first period) depending on the following condition:

$$P_s(s_0^*) \lesseqgtr \phi \cdot 1/2[P_s'(s_1^*(B_H) + P_s'(s_1^*(B_L)] \tag{16}$$

where $\phi = \left(\frac{(\tau+1)}{\left(\frac{1-\rho}{\rho}\right)}\right)^2 < 1$ since $\frac{1-\rho}{\rho} > (\tau + 1)$. ϕ represents the tradeoff between generating future sequestration benefits from acting sooner $(\tau + 1)$ and capturing monetary gains from acting later $\left(\frac{1-\rho}{\rho}\right)$. According to (16), Abatement is increasing with a small increase in the discount factor if, at the optimum, the marginal cost of sequestration in the first period is greater than the fraction (ϕ) of the expected rate of change of the marginal cost of sequestration in the second period. Higher deployment rates of sequestration in the first period imply a steeper rate of change of marginal cost of sequestration in the second period (see Figure 1B). Thus, the likelihood of a wait-and-see approach increases with higher optimal rates of sequestration in the first period. In this case, abatement (and sequestration in the first period) will decrease with a small increase in the discount factor as we are better off waiting than buying protection today. In addition, as ϕ goes to 1, the likelihood increases of choosing a wait-and-see approach in favor of buying protection today. Therefore, the decision whether to act sooner or later given an increase in the discount factor involves two tradeoffs. One has to do with the optimal location of the marginal cost of sequestration in the first period and the other has to do with weighting future benefits from acting sooner and monetary gains from acting later. The RHS of (16) increases

[7] Full derivations are available from the authors upon request.

with a less discounted future reflecting the decreasing attractiveness of abatement in the first period in that case.

5. Discussion

We explore the optimal time path of carbon sequestration and carbon abatement for stabilizing CO_2 level in the atmosphere under uncertainty of climate impacts. We derive optimal rates of the decision variables in both periods and characterize the solution in terms of three possible paths: the ***Aggressive Path,*** the ***Conservative Path*** and the ***Indeterminate Path***. For the ***Aggressive Path***, uncertainty results in more deployment of abatement and sequestration in the first period whereas for the ***Conservative Path***, uncertainty results in less deployment of abatement and sequestration in the first period. For the ***Indeterminate Path***, uncertainty can lead to either more or less deployment of abatement and sequestration in the first period.

Our findings show that two important factors in determining the optimal solution are the magnitude of sequestration capacity and the ratio between the elasticities of excess abatement and excess sequestration capacity (i.e., the relative rate of change in marginal costs of sequestration and abatement). More specifically, we show that the possibility of obtaining the ***Conservative Path*** and the ***Indeterminate Path*** hinges on the availability of large volume of sequestration capacity. Recent simulation analyses find that a large amount of land is converted to forest at sufficient high carbon prices (Sohngen and Mendelsohn, 2003; Richards & Stokes, 2004; Sohngen and Sedjo, 2006). In our model, if there is a large potential for carbon sequestration, then the structure of both marginal cost curves of sequestration and abatement determines which of the three paths is optimal.

Recent numerical analysis (Sohngen and Mendelsohn, 2003; Gitz et al, 2006) have tended to find results consistent with the ***Conservative Path,*** wherein carbon sequestration is held as a safety measure for a future use in case climate impacts turn out to be severe. When the Conservation Path is followed, in our analysis, the planner limits the use of both strategies today in favor of waiting for more information on the severity of climate impacts. However, our analysis shows that this is not the only possible outcome. The structure of the marginal cost curves could be such that the planner is better off by doing more abatement and sequestration today rather than waiting for more information. On the ***Aggressive Path***, we should invest more in relatively cheap abatement, and consequently sequestration today, rather than risk having to deploy expensive sequestration in the second period in the event of severe climate impacts.

Even more interesting is the ***Indeterminate Path***, under which either of the solution the ***Aggressive Path*** or the ***Conservative Path*** applies. Here, not only the elasticity of excess sequestration with respect to excess abatement plays a role, but also the cost of abatement. The combination of both factors, together with large enough sequestration capacity, will dictate if we are optimal on the ***Conservative Path*** or the ***Aggressive Path*** or even between these cases where uncertainty in climate impacts does not affect the optimal deployment of abatement and sequestration.

In our model, the degree of uncertainty (as measured by σ) influences the marginal cost of sequestration in the second period (equation (12)) and, therefore, the tradeoff between acting sooner or later. More uncertainty, all else equal, results in a higher marginal cost of sequestration in the second period and, therefore, higher (lower) rates of abatement and sequestration in the first period when $R(a, \bar{B}, \sigma)$ is strictly convex (concave). However, more uncertainty cannot flip the decision about whether to act sooner or later in the ***Aggressive Path*** and the ***Conservative Path*** cases, because the decision is determined by the curvature of $f_\gamma(a)$. For example, in the case of the ***Aggressive Path*** (see Figure 2B), it is clear that a^* is always greater than $\bar{a}$ and that more uncertainty always increases the difference between these two terms. This is not the case, however, with the ***Indeterminate Path***. Here, due to the inflection point of $R(a, \bar{B}, \sigma)$ (see Figure 4B), it is possible that as uncertainty increases, a^* can switch from being below $\bar{a}$ to being above (or vice-versa). Thus, there is a threshold value of σ at which more uncertainty will flip the decision to act sooner or later.

The positive dependency between the deployment of sequestration and abatement in the first period is in agreement with previous studies that have suggested that sequestration and abatement are compliments in the short run rather than substitutes (Stavins, 1999; Richards & Stokes, 2004).

Comparative statics suggest that an increase in the interest rate (less discounted future) does not necessarily result in postponing emissions reduction into the future. The decision on whether to abate and sequester more in the first period, given an increase in the interest rate, has to do with the expected rate of change of the marginal cost of sequestration in the second period and with long term benefits of current sequestration.

One clear extension to this paper would be to investigate the actual occurrence of each one of the three outcomes. For that, real world issues omitted from this stylized model should be included. Finally, this model focuses on terrestrial carbon sequestration. Other forms of sequestration, however, are available as reduction strategies to limit climate change. One example would be subsurface geological sequestration in depleted oil fields or in deep-sea formations. One possible extension to this paper may explore optimal time paths of different sequestration methods relative to abatement activities.

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7. Figures

Figure 1: Marginal cost of abatement (1A), marginal cost of sequestration (1B) and, three different shapes of equation (11), $f_\gamma(a) = P_s^{-1}\left(\frac{(\tau+1)}{2(1-\rho)} P_a(a)\right)$ (1C).

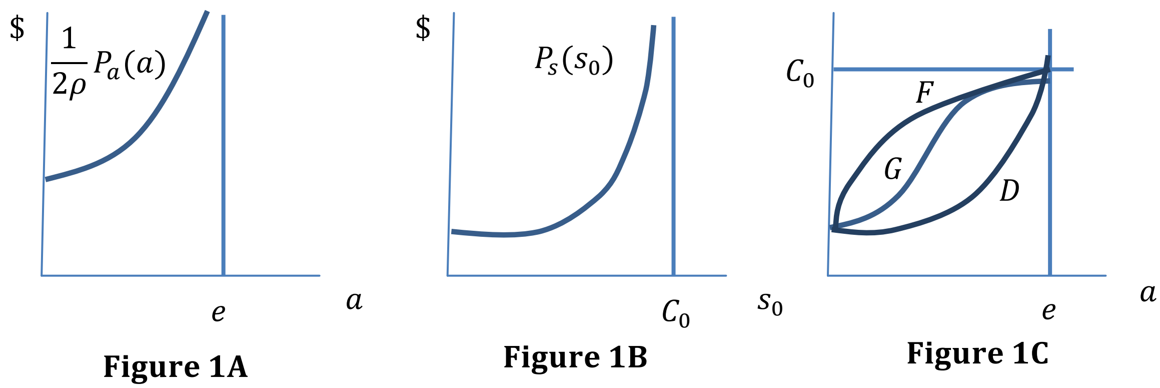

Figure 2: Graphical representation of ***the Aggressive Path***: Given that p>q then $f_\gamma(a)$ is strictly concave (2A) and $R(a, \bar{B}, \sigma)$ is strictly convex (2B). As can be seen in Figure 2B, uncertainty in climate impacts calls for higher deployment rates of abatement in the first period relative to no uncertainty in climate impacts (i.e. $a^* > \bar{a}$).

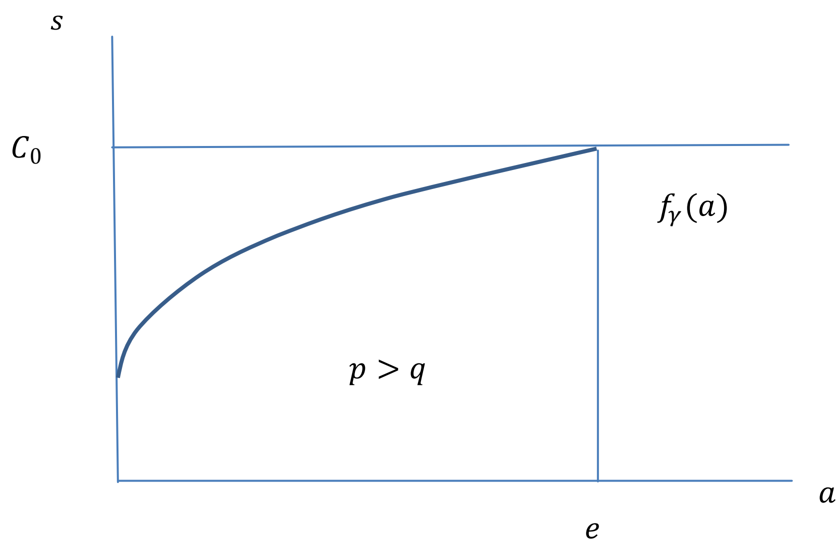

Figure 2A

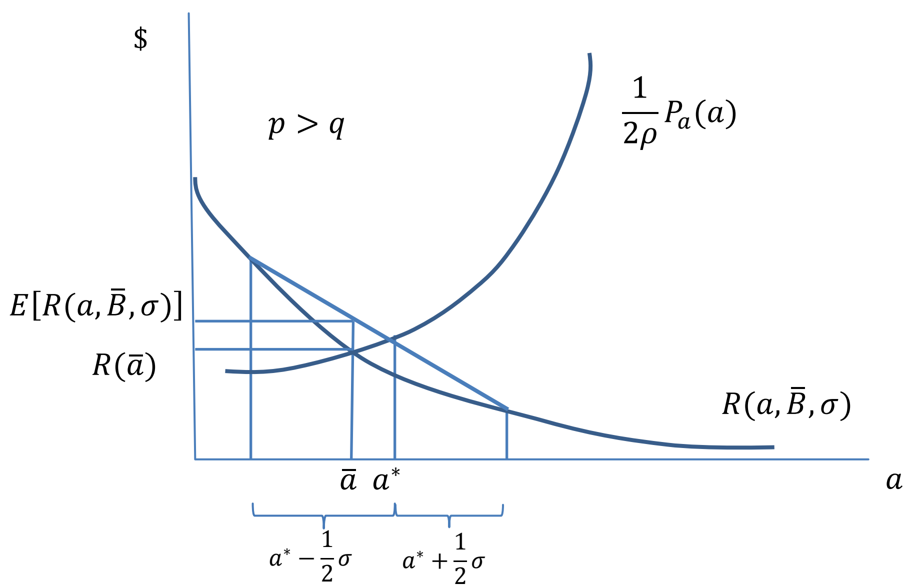

Figure 2B

Figure 3: Graphical representation of ***the Conservative Path*:** Given that p<q then $f_\gamma(a)$ is strictly convex (3A) and if C_0 is large enough then $R(a,\bar{B},\sigma)$ is strictly concave (3B). As can be seen in Figure 3B, uncertainty in climate impacts calls for lower deployment rates of abatement and sequestration in the first period relative to no uncertainty in climate impacts (i.e. $a^* < \bar{a}$).

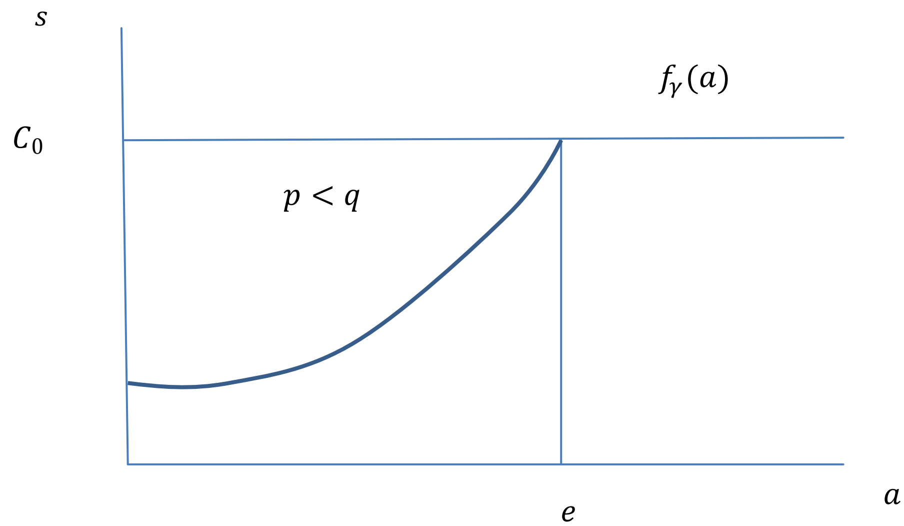

Figure 3A

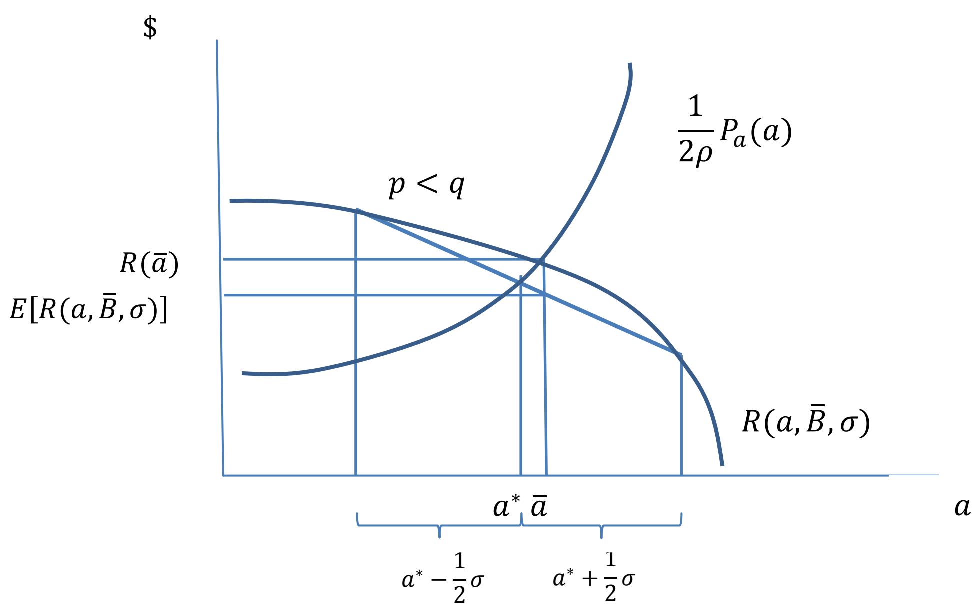

Figure 3B

Figure 4: Graphical representation of ***the Indeterminate Path***: Given $p_1 < q_1\ \forall a \in (0, a_\varepsilon)$, $p_2 > q_2\ \forall a \in (a, e)$ when $a_\epsilon < e$ then$f_\gamma(a)$ is constructed from both convex and concave regions (4A) and if C_0 is large enough then $R(a, \bar{B}, \sigma)$ is also constructed from both concave and convex regions (4B). As can be seen in Figure 4B, for low (high) rates of abatement uncertainty in climate impacts calls for lower (higher) deployment rates of abatement and, consequently, sequestration in the first period relative to no uncertainty in climate impacts

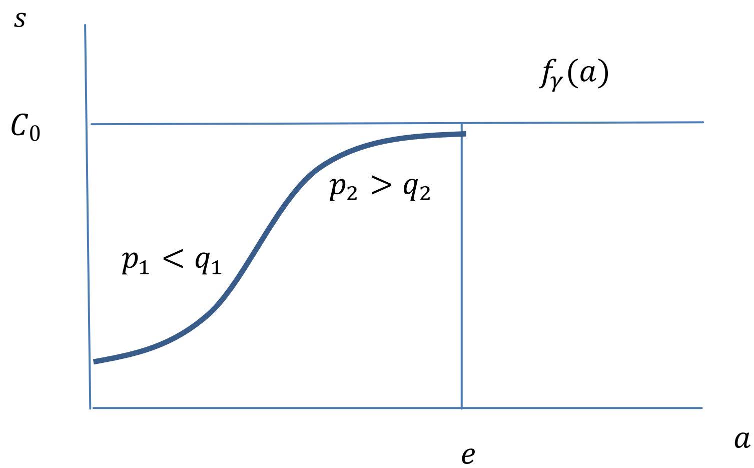

Figure 4A

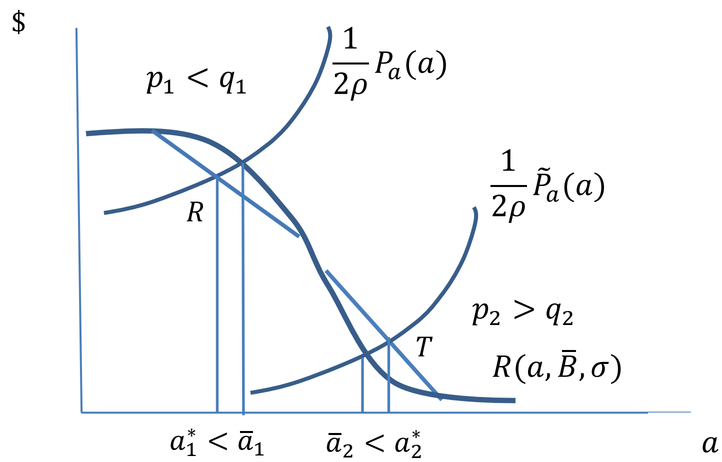

Figure 4B

Appendix A: Derivation of the general case (both abatement and sequestration are decision variables in both periods) and how it collapses to the restricted one.

Let a_1 be a decision variable in the second period:

Second period minimization problem with both abatement and sequestration as decision variables:

$min_{s_1,a_1} \int_0^{s_1} P_s(u)du + \int_0^{a_1} P_a(t)dt$

subject to:

$B = X_1 + e - a_1 - s_1 - \tau s_0^*$ $\quad X_1\ is\ given$

$0 \leq s_1 \leq C_1$ $\quad C_1\ is\ given$

$0 \leq a_1 \leq e$

The Lagrangian function can be written:

$$L(a_1, s_1, \lambda) = \int_0^{s_1} P_s(u)du + \int_0^{a_1} P_a(t)dt + \lambda(X_1 + e - a_1 - s_1 - \tau s_0^* - B_1)$$

And first order conditions of $L(a_1, s_1, \lambda)$:

$$\frac{\partial L}{\partial a_1} = P_a(a_1) - \lambda = 0 \qquad (1A)$$

$$\frac{\partial L}{\partial s_1} = P_s(s_1) - \lambda = 0 \qquad (2A)$$

$$\frac{\partial L}{\partial \lambda} = X_1 + e - a_1 - s_1 - \tau s_0^* - B = 0 \qquad (3A)$$

From (1A) and (2A) it is easy to see that the optimal rates of abatement and sequestration in the second period are determined by equating the marginal costs of the two. Equate (1A) and (2A) to get:

$$a_1^* = P_a^{-1}(P_s(s_1)) \qquad (4A)$$

Substitute (3A) to (4A) to find optimal Rate of sequestration in the second period:

$$s_1^* = X_1 + e - B - \tau s_0 - P_a^{-1}(P_s(s_1^*))^8 \tag{5A}$$

s_1 appears in both sides of (5A) and, more importantly, inside the inverse function in the RHS. Therefore there are no explicit solutions to a_1^* and s_1^* but only implicit ones in the second period.

Take derivatives of (5) and (5B) with respect to a_0 and s_0 to get:

$$0 = -\frac{\partial a_1^*}{\partial a_0} - 2 - \frac{\partial P_s^{-1}(P_a(a_1^*))}{\partial u}_{|u(a_1^*)} \cdot \frac{\partial a_1^*}{\partial a_0} \tag{6A}$$

$$0 = -\frac{\partial a_1^*}{\partial s_0} - (\tau + 1) - \frac{\partial P_s^{-1}(P_a(a_1^*))}{\partial u}_{|u(a_1^*)} \cdot \frac{\partial a_1^*}{\partial s_0} \tag{7A}$$

$$0 = -\frac{\partial s_1^*}{\partial s_0} - (\tau + 1) - \frac{\partial P_a^{-1}(P_s(s_1^*))}{\partial u}_{|u(s_1^*)} \cdot \frac{\partial s_1^*}{\partial s_0} \tag{8A}$$

$$0 = -\frac{\partial s_1^*}{\partial a_0} - 2 - \frac{\partial P_a^{-1}(P_s(s_1^*))}{\partial u}_{|u(s_1^*)} \cdot \frac{\partial s_1^*}{\partial a_0} \tag{9A}$$

Moving backward to solve for the first period:

$$V(s_0, a_0, 0) = \min_{a_0, s_0} \int_0^{a_0} P_a(t)dt + \int_0^{s_0} P_s(u)du + \rho \int_{a_0}^{a_1^*} P_a(t)dt + \rho \int_{s}^{s_1^*} P_s(u)du \tag{10A}$$

Take first order derivatives with respect to a_0 and s_0 to get:

$$\frac{\partial V}{\partial a_0} = P_a(a_0) + \rho\left(\frac{\partial a_1^*}{\partial a_0} P_a(a_1^*) - P_a(a_0)\right) + \rho\left(\frac{\partial s_1^*}{\partial a_0} P_s(s_1^*) - 0\right) = 0 \tag{11A}$$

$$\frac{\partial V}{\partial s_0} = P_s(s_0) + \rho\left(\frac{\partial a_1^*}{\partial s_0} P_a(a_1^*) - 0\right) + \rho\left(\frac{\partial s_1^*}{\partial s_0} P_s(s_1^*) - P_s(s_0)\right) = 0 \tag{12A}$$

Restricting the general model to our current model (abatement is a decision variable only in the first period) i.e. let $a_1^* = a_0$ and $s_1^* = s_1^*(s_0, a_0)$ reduces (11A) and (12A) to:

$$\frac{\partial V}{\partial a_0} = P_a(a_0) + \rho \frac{\partial s_1^*}{\partial a_0} P_s(s_1^*) = 0 \tag{13A}$$

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8 The same exercise can be done to find A_1^* as a function of only the parameters and the first period decision variables. Then, $a_1^* = X_1 + e - B - \tau s_0 - P_s^{-1}(P_a(a_1^*))$ (5B)

$$\frac{\partial V}{\partial s_0} = (1-\rho)P_s(s_0) + \rho \frac{\partial s_1^*}{\partial s_0} P_s(s_1^*) = 0 \qquad (14A)$$

Because $\frac{\partial a_1^*}{\partial a_0} = 1$ and $\frac{\partial a_1^*}{\partial s_0} = 0$.

Equations (13A) and (14A) are identical to equations (9) and (10) from the restricted model. This is because the optimal rate of sequestration in the second period is reduced to $s_1^* = X_0 + 2(e - a_0) - B - (\tau + 1)s_0^*$. Plugging the derivatives of s_1^* with respect to s_0 and a_0 and divide equation (13A) by 2 yields:

$$\frac{\partial V}{\partial a_0} = \frac{1}{2} P_a(a_0) - \rho P_s(\beta - 2a_0 - (\tau + 1)s_0) = 0 \qquad (15A)$$

$$\frac{\partial V}{\partial s_0} = (1-\rho)P_s(s_0) - \rho(\tau + 1)P_s(\beta - 2a_0 - (\tau + 1)s_0) = 0 \qquad (16A)$$

Where $\beta = X_0 + 2e - B$

Appendix B: Detailed derivation of equation (11) with respect to abatement ($\boldsymbol{a}$)

Let's $f(a) = P_s^{-1}\big(f_\gamma(a)\big)$ then:

$$f_\gamma'(a) = \gamma P_a'(a) \cdot \frac{dP_s^{-1}}{dy}_{|\gamma P_a(a)} > 0 \qquad (2A)$$

Because $P_a'(a) > 0 \ \forall a \in [0, e]$ by assumption and $P_s^{-1}(\cdot) > 0 \ \forall a \in [0, e]$.

$$f_\gamma''(a) = \gamma P_a''(a) \cdot \frac{dP_s^{-1}}{dy}_{|\gamma P_a(a)} + \big(\gamma P_a'(a)\big)^2 \cdot \frac{d^2 P_s^{-1}}{dy^2}_{|\gamma P_a(a)} \lesseqgtr 0 \qquad (2B)$$

Equation (2B) could go either way. This is because $\frac{d^2 P_s^{-1}}{dy^2} < 0 \ \forall a \in [0, e]$ and all other terms of (2B) are positive.

Appendix C: Detailed derivation of the RHS of equation (12) with respect to abatement ($\boldsymbol{a}$)

Let's $R(a, \bar{B}, \sigma) = \rho P_s\big(\beta - 2a - (\tau + 1)f_\gamma(a)\big)$ then:

$$R'(a, \bar{B}, \sigma) = \left(-2 - (\tau + 1)f_\gamma'(a)\right) \cdot P_s'\left(\beta - 2a - (\tau + 1)f_\gamma(a)\right) < 0 \tag{3A}$$

Because $P_s'(\cdot) > 0 \forall a \in [0, e]$ and the first term of (3A) is always negative (recall that $f_\gamma'(a) > 0 \ \forall a \in [0, e]$ from (2A)).

$$R''(a, \bar{B}, \sigma) = -(\tau + 1)f_\gamma''(a) \cdot P_s'(\beta - 2a - (\tau + 1)f_\gamma(a)) + \left(-2 - (\tau + 1)f_\gamma'(a)\right)^2 \cdot P_s''(\beta - 2a - (\tau + 1)f_\gamma(a)) \tag{3B}$$

Equation (3B) could go either way. This is because the first term of (3B) could go either way depending on $f_\gamma''(a)$ and the second term of (3B) is always positive.