Rip currents pose a serious danger to visitors of bathing beaches; they are also an important process in surf zone circulation. Haller et al. [2014] demonstrated that shore-based X-band radar can be used to compliment \textit{in situ} measurements of rip currents. However, little is known of the underlying radar imaging mechanics and the conditions under which they are observable. Herein, we analyze the expected radar backscatter characteristics of a rip current, which also contains a surf zone eddy, in order to assess their detectability by radar. The approach follows the method of Rascle et al. [2014] whereby the changes to the mean square slope (mss) of the water surface, calculated based on the surface current deformation tensor, are taken as directly related to radar backscatter intensity. The rip current and surf zone eddy were modeled using pre-existing idealized flow fields. The mss anomaly results show that the imaging mechanism of rip currents is dependent on both the surface current divergence and strain in the wind direction. The level of dependency on these two deformation tensor components varies in the rip current and surf zone eddy. The expected mss anomaly was then qualitatively compared to existing radar measurements from a field experiment and shown to have a similar brightness pattern and structure.
On the Imaging of Rip Currents in X-band Radar

by
Rebecca Kloster

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APPROVED:

Major Professor, representing Civil Engineering

Head of the School of Civil and Construction Engineering

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Rebecca Kloster, Author
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# TABLE OF CONTENTS

1. Introduction ............................................................................................................ 1

2. Wave-current interaction model............................................................................. 5
   2.1. Wave action equation ..................................................................................... 5
   2.2. Surface current deformation tensor decomposition ....................................... 7
   2.3. Mean square slope .......................................................................................... 8
   2.4. Equilibrium spectrum ..................................................................................... 9

3. Current models ..................................................................................................... 11
   3.1. Rip current .................................................................................................... 12
   3.2. Hill’s spherical vortex .................................................................................. 13
   3.3. Rip current + spherical vortex ...................................................................... 14

4. Current anomaly................................................................................................... 15
   4.1. Action anomaly ............................................................................................ 15
   4.2. Deformation tensor components .................................................................... 17
       4.2.1. Rip current ............................................................................................ 18
       4.2.2. Hill’s spherical vortex ........................................................................... 20
       4.2.3. Composite current ................................................................................. 23

5. Mean square slope................................................................................................ 25
   5.1. Projection ..................................................................................................... 26
   5.2. Current magnitude ........................................................................................ 28
   5.3. Wave number spectrum ............................................................................... 29
   5.4. Divergence versus strain .............................................................................. 32
   5.5. Polarization indexes ..................................................................................... 33

6. Comparison to radar data ..................................................................................... 36

7. Conclusion............................................................................................................ 39

References................................................................................................................... 41
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Figure 1. 60 second wave-averaged snapshot of a rip current event at Duck, NC (courtesy of D. Honegger). Green arrows represent 15-minute averaged current vectors measured in situ.</td>
<td>3</td>
</tr>
<tr>
<td>2. Figure 2. Wave action equilibrium spectrum for source function 1 by Lyzenga [2010]</td>
<td>10</td>
</tr>
<tr>
<td>3. Figure 3. Current fields for a time-averaged rip current velocity field [Haller and Dalrymple, 2001] (a), Hill’s spherical vortex velocity field, as presented by Smith [2008] (b), and composite surface current velocity field (c).</td>
<td>12</td>
</tr>
<tr>
<td>4. Figure 4. Comparison of action anomaly for the four composite current cases: case 1 (a), case 2 (b), case 3 (c), and case 4 (d).</td>
<td>16</td>
</tr>
<tr>
<td>5. Figure 5. Rip current surface deformation tensor components: divergence (a), strain (b), vorticity (c), and shear (d).</td>
<td>19</td>
</tr>
<tr>
<td>6. Figure 6. Spherical vortex surface deformation tensor components: divergence (a), strain (b), vorticity (c), and shear (d).</td>
<td>21</td>
</tr>
<tr>
<td>7. Figure 7. Composite velocity field surface current deformation tensor components: divergence (a), strain (b), vorticity (c), and shear (d).</td>
<td>24</td>
</tr>
<tr>
<td>8. Figure 8. The total mss anomaly (a) and the x (b) and y projection (c) of the mss anomaly for wind blowing offshore.</td>
<td>27</td>
</tr>
<tr>
<td>9. Figure 9. Comparison of magnitude of the mss, for the composite velocity field for case 2 (a) and case 3 (b).</td>
<td>29</td>
</tr>
<tr>
<td>10. Figure 10. The mss, anomaly due to integrating over all wave numbers (a) and over Bragg wave numbers of interest for X-band frequencies (b). See section 2.3 for details on these integration limits.</td>
<td>31</td>
</tr>
<tr>
<td>11. Figure 11. Comparison of the mss, anomaly due to current divergence (a) and strain in the wind direction (b).</td>
<td>32</td>
</tr>
<tr>
<td>12. Figure 12. Relative magnitude of divergence (a) and strain (b) contribution to the mss, anomaly when wind is blowing offshore.</td>
<td>35</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>13. Figure 13. 60 second time-averaged X-band marine radar images from Duck, NC on September 9 (a) and 10 (b), 2010 showing a rip current. Green arrows indicating in situ current measurements (courtesy of D. Honegger)</td>
<td>37</td>
</tr>
<tr>
<td>14. Figure 14. Qualitative comparison of 60 second averaged radar images from Duck, NC (courtesy of D. Honegger) (a) and the mssₜ anomaly for case 2 of the composite current when wind is blowing offshore (b)</td>
<td>38</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table                                                                                               Page
---------------------------------------------------------------------------------------------     
1. Table 1. Control variables and scaling ratio for the four rip current cases investigated            13
2. Table 2. Control variables and scaling ratio for the four spherical vortex cases investigated       14
3. Table 3. Polarization indexes (total and x projection) for the composite velocity field for various wind directions 34
1. Introduction

Rip currents are strong seaward flowing currents that return surf zone water, delivered shoreward by wave breaking, back offshore. Rip current circulation systems provide an important mechanism for the exchange of nutrients, sediment, pollutants, and biological species between the surf zone and the inner shelf [see Dalrymple et al., 2011 for a review]. Rip currents are also a natural hazard to visitors of bathing beaches. Even when beachgoers are well informed and are strong swimmers, rip currents pose a danger, as shown by the data of Drozdewski et al. [2012].

Rip currents can be generated under a variety of conditions. Transient rip currents are rips that pop up and disappear abruptly due to changes in wave conditions [Johnson and Pattiaratchi, 2004]. Morphologically driven rip currents can be more persistent and develop when there are topographical features that encourage rip development, e.g. along headlands, breaks in sand bars, and submarine canyons [e.g. Calvete et al., 2005; Long and Özkan-Haller, 2005].

Even when under morphologic control, rip currents are modulated by the tides and wave conditions, and the morphology on sandy beaches is, of course, mobile. Consequently, capturing rip current observations with in situ sensors is a challenge, as is forecasting them with numerical models. Real-time monitoring for the presence of rip currents is primarily done visually by beach safety personnel on public beaches [e.g. Dusek and Seim, 2013]. Such real-time monitoring could be better enabled by remote sensors that are capable of observing large nearshore areas and detecting rip currents [Haus, 2011]. Because of the benefits of using remote sensing to observe rip currents, there is a need for the development of methods to estimate rip current properties based on remote sensing observations.
Most previous remote sensing observations of rip currents have been conducted in the optical band, which can be used to identify rip current morphologies through the darker cross-shore features associated with the lack of wave breaking in the rip channel [Holman et al., 2006]. Some acoustic observations using Doppler sonar have been given by Smith and Largier [1995] and Vagle et al. [2001]. Surf zone eddies are current features that can be attributed to the pulsation of rip currents. Smith [2008] used a phased-array Doppler sonar to analyze an isolated vortex produced by a rip. Marmorino et al. [2013] also observed eddies ejected from the surf zone in airborne infrared images.

In the microwave band, da Silva et al. [2006] and da Silva [2008] gave initial observations showing features of increased backscatter, suggestive of rip currents, during low wind conditions from satellite synthetic aperture radar (SAR). Also in the microwave band, Takewaka and Yamawaka [2010] used shore-located marine radar and presented mean radar intensity images (17 minute averages) that showed cross-shore oriented bright intensity features indicative of rip currents, which were also corroborated with in situ drifter motion. Most recently, Haller et al. [2014], also using shore-based marine radar, demonstrated a regular sequence of low-tide, morphologically-driven rip current events over a 10-day period. The high backscatter, cross-shore oriented features were clearly associated with strong offshore flows measured by in situ current meters. In addition, their analysis demonstrated an imaging dependence on cross-shore wind stress, with offshore directed stress preventing rip current imaging.

The present work was motivated by two aspects of the Haller et al. [2014] rip current observations. The first is illustrated in Figure 1, showing a wave-averaged snapshot of a rip current event. The snapshot represents a 60 second time-average radar intensity image and is suggestive of a sinuous rip current with a vortex located at the rip head, i.e. the mushroom-shaped feature. The images captures by the radar are qualitatively similar to the numerical simulations of Lagrangian coherent structures produced by
rip currents in Reniers et al. [2010], which show lines of flow convergence that have similar characteristics.

Figure 1. 60 second wave-averaged snapshot of a rip current event at Duck, NC (courtesy of D. Honegger). Green arrows represent 15-minute averaged current vectors measured in situ.

The second motivational aspect is the observed dependence of rip current imaging on the wind direction [Haller et al., 2014]. In order to better understand these radar observations, the underlying radar imaging mechanism for nearshore rip currents and surf zone eddies was analyzed in this study. There is a long history demonstrating that observations of increased backscatter from X-band imaging radars (predominately satellite-based SAR systems) directly relate to the underlying current field, particularly the presence of sharp current gradients. The early work of Phillips [1984] parameterized a SAR imaging criterion based on the local one-dimensional strain rate of the currents. Alpers and Hennings [1984] analyzed the SAR imaging mechanism of underwater topography, again linking increased surface roughness to surface current strain via a perturbation to the wave action density.
Surface current gradients, such as those created from internal waves, density fronts, and rip currents, perturb the equilibrium wind-wave spectrum. The perturbation to the spectrum can alter the radar backscatter and can be modeled using the wave action equation. To model the effect on radar backscatter, Bragg scattering theory or more complicated radar backscatter models are used [e.g. Lyzenga and Bennett, 1988]. The way the current perturbs the equilibrium spectrum has been the focus of many studies. At high radar frequencies, e.g. X-band, there are discrepancies between the modeled backscatter and the radar backscatter intensity due to surface currents. To resolve these differences, extensive research has been conducted looking into various ways of including nonlinear effects in the wave action equation. Lyzenga [1998] first looked at the effect that intermediate waves have on the action spectrum at shorter wave lengths by modeling the intermediate waves as a surface current. To further address the discrepancy between simple wave-current interaction theory and observed backscatter intensities, Lyzenga [2010] introduced nonlinear wave-wave interactions in the equilibrium spectrum and the wave action equation.

Another affect that increases backscatter is the increase in surface slope, which can manifest as increased wave steepness and wave breaking. Henning and Herber [2006] demonstrated how wave steepness increases backscatter intensity and Catalán et al. [2014] examined how backscatter is also increased by wave breaking in the surf zone. Hwang et al. [2013] showed a strong correlation between the mean square slope (mss), and fraction of breaking, and radar backscatter intensity. Hence, here we treat the mss as a proxy for both Bragg scattering and breaking induced backscatter.

Rascle et al. [2014] also argued that mss is a relevant indicator of radar backscatter. Rascle et al. [2014] analyzed changes in the mss, i.e. the mss anomaly, due to the components of the surface current deformation tensor. They showed that the only components of the deformation tensor that causes an anomaly in the mss are the divergence and the current strain in the wind direction; where a negative deformation component indicates an increase in mss. These current deformations were calculated
for a rip current model to approximate what effect a rip current has on radar backscatter.

This paper is organized as follows: in section 2 we develop the wave action anomaly approach of Rascele et al. [2014] using the equilibrium wind-wave spectrum given by Lyzenga [2010]. Following Rascele et al. [2014] we utilize the mean square slope of the wind-wave spectrum as a proxy for radar backscatter. In section 3 we describe the analytic models for the rip current velocity field [Haller and Dalrymple, 2001] and a surf zone eddy [Smith, 2008]. In section 4, the action anomaly, deformation tensor components, and mss anomaly resulting from the combined rip and eddy current field are analyzed in the context of their potential effects on radar imaging. In section 5, we discuss the results with a qualitative comparison to rip current observations from the field; then in the final section conclusions from this research and suggestions for future research are presented.

2. Wave-current interaction model

To begin looking at how a rip current affects the wind-wave spectrum the wave action equation was utilized. Bretherton and Garrett [1969] developed the conservation of wave action equation in order to calculate the change in wave energy along a wave ray; the conservation of wave action is applicable in the presence of a slowly varying current.

2.1. Wave action equation

The full wave formulation of the wave action equation is time dependent and includes the changes in wave action $N$ in both spatial dimensions ($x$ and $y$ directions), and the effect of the current gradients. The wave action spectrum can then be solved forward through time until a new equilibrium is reached, using

$$
\frac{\partial N}{\partial t} + (c_{gx} + u) \frac{\partial N}{\partial x} + (c_{gy} + v) \frac{\partial N}{\partial y} - \left( k_x \frac{\partial u}{\partial x} + k_y \frac{\partial v}{\partial x} \right) \frac{\partial N}{\partial k_x} - \left( k_x \frac{\partial u}{\partial y} + k_y \frac{\partial v}{\partial y} \right) \frac{\partial N}{\partial k_y} = F_s(N),
$$

(1)
where the left hand side of the equation describes how the wave action spectrum changes in time, space, and due to gradients of the surface currents. The right hand side of the equation is the net source function $F_s$ which accounts for all sources and sinks of action in the wave field. The full wave action formulation is computationally expensive [Fusina et al., 1997] and is more complicated than is required for this first look at the effect of rip currents. The wave action equation can be simplified using the relaxation approach for steady currents and short relaxation time so that it can be solved in a form that is not dependent on time, following Rasce et al. [2014],

$$\mathcal{N} (x, y, k_x, k_y, t) = N_0(k_x, k_y) + \widetilde{\mathcal{N}} (x, y, k_x, k_y, t)$$

(2)

where $k_x$ and $k_y$ are the components of the wave number, $t$ is time, $N_0$ is the wave action equilibrium spectrum, and $\mathcal{N}$ is the disturbance to the spectrum due to the current, i.e. the action anomaly.

The action anomaly for a steady current field is then written as

$$\mathcal{N} (x, y, k_x, k_y) = \tau_c \left[ (k_x \frac{\partial u}{\partial x} + k_y \frac{\partial v}{\partial x}) \frac{\partial N_0}{\partial k_x} + (k_x \frac{\partial u}{\partial y} + k_y \frac{\partial v}{\partial y}) \frac{\partial N_0}{\partial k_y} \right]$$

(3)

and in to calculate the action anomaly in terms of wave number magnitude and wave propagation direction, it can be written as

$$\mathcal{N} (x, y, k, \varphi) = \tau_c N_0 \left\{ \frac{\partial u}{\partial x} \left[ \cos^2 (\varphi) \frac{\partial \ln N_0}{\partial \ln k} - \sin (\varphi) \cos (\varphi) \frac{\partial \ln N_0}{\partial \varphi} \right] + \right.$$

$$\frac{\partial u}{\partial y} \left[ \cos (\varphi) \sin (\varphi) \frac{\partial \ln N_0}{\partial \ln k} + \cos^2 (\varphi) \frac{\partial \ln N_0}{\partial \varphi} \right] + \frac{\partial v}{\partial x} \left[ \cos (\varphi) \sin (\varphi) \frac{\partial \ln N_0}{\partial \ln k} - \right.$$

$$\sin^2 (\varphi) \frac{\partial \ln N_0}{\partial \varphi} \} + \frac{\partial v}{\partial y} \left[ \sin^2 (\varphi) \frac{\partial \ln N_0}{\partial \ln k} + \cos (\varphi) \sin (\varphi) \frac{\partial \ln N_0}{\partial \varphi} \right] \right\}$$

(4)
where \( k \) is the wave number magnitude, \( \omega \) is the intrinsic wave frequency, \( u \) and \( v \) are the current velocity components, \( \phi \) is the direction of wave propagation, and \( \tau_c \) is the relaxation time scale that is dependent on the wave number of interest.

According to Alpers [1985], \( \tau_c \) can range from 4.7 seconds to 47 seconds for Seasat SAR Bragg waves based on \( \tau_c \) being on the order of 10 to 100 wave periods, where a value of 30-40 seconds was used for L-band radar. For this study a value of 5 seconds was used.

2.2. Surface current deformation tensor decomposition

As demonstrated by Rascle et al. [2014], the effect of currents on the mss anomaly can be analyzed through the surface current deformation tensor. The deformation tensor can be decomposed into terms of divergence \( D \), vorticity \( V \), strain \( S_t \), and shear \( S_h \) [Rascle et al., 2014],

\[
\begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix} D + S_t & -V + S_h \\ V + S_h & D - S_t \end{bmatrix}, \text{ where}
\]

\[
D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad S_t = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y},
\]

\[
V = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad S_h = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}. \tag{5}
\]

The action anomaly can then be written as a relationship between the different forms of surface current deformation and the wave action equilibrium spectrum,

\[
N(\ , \ , k, \phi) = \tau_c N_0 \left\{ D \frac{\partial \ln N_0}{\partial \ln k} - V \frac{\partial \ln N_0}{\partial \phi} + S_t \left[ \cos(2\phi) \frac{\partial \ln N_0}{\partial \ln k} - \sin(2\phi) \frac{\partial \ln N_0}{\partial \phi} \right] + S_h \left[ \sin(2\phi) \frac{\partial \ln N_0}{\partial \ln k} + \cos(2\phi) \frac{\partial \ln N_0}{\partial \phi} \right] \right\}. \tag{6}
\]
Hence, this can be used to analyze the effects of different components of the
deformation tensor independently.

2.3. Mean square slope

By ignoring the effect of surfactants and varying wind fields, which occur on larger
spatial scales than the currents of interest [Kurdryavtsev et al., 2005, Rascle et al.
[2014] argues that the mss is a relevant indicator of the expected radar backscatter.
Because only the interaction between wind-waves and the surface current is being
considered, the mss anomaly is an appropriate measure of the change in surface
roughness due to the current,

\[ m\bar{s}_t(\gamma, x) = \int_k \int_\varphi \omega^{-1}Nk^3 \cos k \, d\varphi dk \]

\[ m\bar{s}_x(\gamma, x) = \int_k \int_\varphi \omega^{-1}Nk^3 \cos \varphi \cos k \, d\varphi dk \]

\[ m\bar{s}_y(\gamma, x) = \int_k \int_\varphi \omega^{-1}Nk^3 \sin \varphi \sin k \, d\varphi dk \] (8)

where \( m\bar{s}_x \) and \( m\bar{s}_y \) are the x and y projections of the total mss anomaly, \( m\bar{s}_t \).

Another way of studying the total mss anomaly is to write (7) in terms of a
polarization index \( \alpha_t \) [Rascle et al., 2014],

\[ m\bar{s}_t = \left[ \frac{\partial u}{\partial x} + \alpha_t \frac{\partial v}{\partial y} \right] \times \int_k \int_\varphi \omega^{-1} \tau \ k^3 N_0 \left[ \cos^2(\varphi) \frac{\partial \ln N_0}{\partial \ln k} - 
\cos(\varphi) \sin(\varphi) \frac{\partial \ln N_0}{\partial \varphi} \right] d\varphi dk. \] (9)

The polarization index is a constant that represents how sensitive the mss anomaly is
to \( \partial u/\partial \) compared to \( \partial v/\partial \).
The value of the polarization index indicates whether the mss anomaly is more dependent on the divergence or strain of the current field.

2.4. Equilibrium spectrum

The equilibrium spectrum input for the action anomaly was calculated using the net source function excluding nonlinear wave interactions, presented by Lyzenga [2010]. The net source function accounts for all sources and sinks of action in the wave field, it includes wind input and wave energy dissipation. The wind input, which includes viscous dissipation, is proportional to the “degree of saturation” [Phillips, 1985], which is an alternative, dimensionless form for studying wave action

\[ B = \frac{Nk^4}{\rho} \]  

(11)

where \( \rho \) is the density of water and \( c \) is wave celerity, and the wave energy dissipation term is proportional to the cube of the degree of saturation,

\[ F_g(B) = (\beta - 4vk^2)B - \alpha_0 \omega B^3 \]  

(12)

where \( \beta \) is the wind growth rate parameter, \( v \) is kinematic viscosity, and \( \alpha_0 \) is the scale factor for wave energy dissipation, taken to be 100.

The form of the wind growth parameter is poorly understood, not generally agreed on, and has a significant effect on the equilibrium spectrum [Donelan and Pierson, 1987]. The form used in this study was

\[ \beta = \beta_0 \max \left[ \left( \frac{U_w}{\varphi} \right) \cos(\varphi - \phi_w) - 1,0 \right] \omega, \]  

(13)
presented by Plant [1982], where $\beta_0 = 3 \times 10^{-4} \text{ s}^{-1}$, $U_w$ is the wind speed taken to be 5 m/s, $\phi_w$ is the wind direction measured clockwise from the current direction, and the wave propagation direction $\phi$ is symmetric about the wind direction.

By setting the net source function equal to zero, the sources and sinks of action are balanced or there are no sources or sinks of action in the wave field. This is called the equilibrium spectrum [Lyzenga, 1998]. Because the degree of saturation is then the only unknown in (12) the wave action equilibrium spectrum can be solved analytically. Figure 2 shows the analytic solution to the equilibrium spectrum, consistent with the source function excluding nonlinear effects presented by Lyzenga [2010].

![Figure 2. Wave action equilibrium spectrum for source function 1 by Lyzenga [2010]](image)

For a local fully developed sea the equilibrium spectrum is symmetrical about the wind direction, in the same fashion as the wave propagation direction is. Though this
spectrum only takes into account local wind conditions, because we are focused on the Bragg waves, which are all locally generated, this is an acceptable spectrum.

3. Current models

To model the wave action anomaly induced by a composite rip current and surf zone eddy flow field we utilized an existing, idealized rip current model and model for a spherical vortex. It is generally known that rip currents often eject vortices and vortex pairs from the surf zone that can then travel alongshore or offshore [e.g. Chen et al., 1999; MacMahan et al., 2004; Marmorino et al., 2013]. In addition, we used the model for a spherical vortex as given by Smith [2008], which he chose as representative of a rip current generated eddy observed at the same field site (Duck, NC) as the observations presented by Haller et al. [2014].

First, the time-averaged rip current was modeled, then the spherical vortex was looked at, and finally the composite model was examined for a more comprehensive view on what is expected to be imaged in radar data. Figure 3 presents the characteristic velocity fields of the models used for this study. These three current fields are discussed in further detail below.
3.1. Rip current

The model for a time-averaged flow of a rip current, developed by Haller and Dalrymple [2001], is a two dimensional model. It takes into account a sloping bottom, non-parallel flow, turbulent mixing, and bottom friction. The flow is treated as non-divergent in a depth-integrated sense. The variables that control the rip current scales
are the alongshore width $b_0$ and centerline velocity $U_0$, values of which are shown in Table 1. The beach slope was specified to be 1/100 to be representative of the beach slope near the U.S. Army Corps of Engineers Field Research Facility pier in Duck, NC.

Table 1. Control variables and scaling ratio for the four rip current cases investigated

<table>
<thead>
<tr>
<th>Case</th>
<th>$b_0$ (m)</th>
<th>$U_0$ (m/s)</th>
<th>$L_{10%}$ (m)</th>
<th>$U_0/L_{10%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.25</td>
<td>86</td>
<td>0.0029</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0.50</td>
<td>215</td>
<td>0.0023</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>1.00</td>
<td>215</td>
<td>0.0047</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>1.00</td>
<td>430</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

The scaling ratio used to examine how changes in the control variables affect the rip current model was the ratio of the centerline velocity to the distance to where the centerline velocity had decayed to 10% of the original value $L_{10\%}$. For cases 1, 2 (Figure 3a), and 4 the velocity fields look very similar but with different spatial extents; while for case 3 the velocity vectors are larger but the shape is consistent with the other three cases.

3.2. Hill’s spherical vortex

Smith [2008] applied Hill’s spherical vortex model [Batchelor, 1967], to describe a horizontally divergent flow field measurement by phased-array Doppler sonars. Hill’s spherical vortex is a two-dimensional surface current model for a three-dimensional spherical vortex, or vortex ring. This type of flow field is plausible for a vortex pair ejected by a pulsed rip current that has detached from the seafloor [Smith, 2008]. The control variable for the model are the radius $a$ and the velocity scale $U$.

The ratio of velocity scale to radius is the scaling ratio that was used to study how changes in the control variables affects the resulting mss anomaly.
Table 2. Control variables and scaling ratio for the four spherical vortex cases investigated

<table>
<thead>
<tr>
<th>Case</th>
<th>a (m)</th>
<th>U (m/s)</th>
<th>U/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>0.05</td>
<td>0.0031</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>0.10</td>
<td>0.0025</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.20</td>
<td>0.0050</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>0.20</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

The control variables used in case 2 are the same values that Smith [2008] used to compare Hill’s spherical vortex to the divergent velocity field that was extracted by the potential-solenoidal flow decomposition of an isolate feature. Case 2 is also the characteristic spherical vortex velocity field presented in Figure 3b. The velocity field for the other three cases are similar but with different spatial extend and different magnitude of velocity vectors for case 3.

3.3. Rip current + spherical vortex

To couple the two flow field models the spherical vortex velocity fields was linearly added to the rip current velocity fields. To ensure that the spherical vortex was an isolated feature, as observed by Smith [2008], the vortex velocities were added to the rip current centered at the cross-shore location where the rip current centerline velocity had decayed to 10% of the original value, L10% in Table 1. This location of the spherical vortex also ensures some interaction between the rip current and vortex velocity fields, observed in the action anomaly in section 4.1.

As with the rip current and spherical vortex models, four composite velocity field cases were studied. These four cases correspond to the four case conditions for the rip current and spherical vortex model. Figure 3c shows a characteristic composite velocity field, which corresponds to case 2. As discussed previously, the other three cases had similar structures. However, they had different spatial extents and case 3, specifically, had larger velocity vectors relative to the other cases.
4. Current anomaly

The action anomaly for the composite current field was examined for the four current cases to determine the effect of the current model scaling on radar backscatter. Next, the deformation tensor components of the three current fields were examined to determine which components have the greatest effect on the action anomaly.

4.1. Action anomaly

Only the action anomaly for the composite current was examined in order to gain a better understanding of how changing the scaling ratios in both the rip current and spherical vortex models, $U_0/L_{10\%}$ and $U/a$, affects the current field. The action anomaly being examined here is the anomaly at a characteristic wave number for X-band radar, 62 m$^{-1}$, caused by wind blowing offshore, in the same direction as the current. The wave number used is associated with the frequency of the X-band radar used by Haller et al. [2014], 9.45 GHz. All derivatives were taken numerically, using central differencing, to ensure consistency between the three current models. The derivatives of the rip current and spherical vortex were compared with the analytical solutions and were determined to be the equivalent.

Figure 4 shows how the magnitude of the action anomaly changes with the scaling of the spatial and velocity constants in the current model.
Figure 4. Comparison of action anomaly for the four composite current cases: case 1 (a), case 2 (b), case 3 (c), and case 4 (d)

It is apparent that for cases 2 and 4 the magnitude of the action anomalies are the same (Figure 4 a and b). This indicates that the magnitude of the anomaly does not change when the scaling ratios are kept constant. This is not surprising since the action anomaly is dependent on the gradients of the current velocity. When the velocity and spatial constants are scaled by the same amount, the gradients of the velocities do not change.
Case 1 has smaller spatial constants, $b_0$ and $a$, than would be necessary to maintain the same scaling ratios as cases 2 and 4. The spatial constants are scaled to be 2.5 times smaller than case 2 while the characteristic velocity is only 2 times smaller. The resulting scaling ratio is 1.25 times larger than those for cases 2 and 4 (Table 1 and Table 2). The increase in the scaling ratios is relatively small but has a significant impact on the magnitude of the action anomaly. By looking at the maximum positive anomaly in the rip neck it is apparent that the larger scaling ratio for case 1 (Figure 4a) translates into the same magnitude of difference in the action anomaly.

The most apparent difference in magnitude of the action anomalies is in case 3 (Figure 4c), where the magnitude is significantly larger than those for the other three cases. This is due to the fact that the scaling ratios are 2 times larger for case 3 than cases 2 and 4, see Table 1 and Table 2. This supports that the magnitude of the action anomaly, and therefore the mss anomaly, scales with the ratios of $U_0/L_{10\%}$ and $U/a$. The largest positive anomaly, in the neck of the rip, is about 2 times larger in case 3 than in cases 2 and 4. Since the differences in the maximum action anomaly for cases 1 and 3 versus 2 and 4 are linearly scaled with the differences in the scaling ratios it is apparent that the radar backscatter intensity will scale with the scaling of the velocity and spatial constants of the rip current, consistent with the conclusions of Phillips [1984] about scaling current features.

4.2. Deformation tensor components
In order to explore how each deformation tensor component affects the action anomaly, the components are presented below for case 2 of the three current models. To examine how the components of the deformation tensor show up in the action anomaly, the wind was set to be blowing offshore to match the results examined in section 4.1. Because there is no wave number dependency for the deformation tensor the results presented in this section are independent of the radar properties. It is apparent that the action anomaly is inversely related to the sign of the current deformations. This is as expect, since divergence has no signature in radar backscatter
while convergence, negative divergence, has a positive relationship with increased backscatter [Chang et al., 2008].

The smallest surface current deformation component for each of the three current models is the divergence. This suggests that the action anomaly is enhanced by the divergence, but is more dependent on the strain in the wind direction; therefore the imaging of rip currents in radar is dependent on wind direction. This dependency on wind direction will be explored further in section 5.

4.2.1. Rip current

Figure 5a shows that the magnitude of the divergence of the rip current is significantly smaller than the other three components of the deformation, it is not large enough to appear on the same colorbar scale as the other three components; while the strain, shear, and vorticity of the model have similar magnitudes (Figure 5 b, c, and d). The magnitude of the divergence was expected to be small due to the rip current models dependence on water depth and its assurance of continuity through the water column. The farther from the base of the rip neck the deformation tensor is calculated, the smaller the components get. This is due to the slowing of the current as the water depth increases and the current spreads.
Since the rip current model is symmetric about the rip current centerline the
divergence and strain, which are dependent on $\partial u / \partial x$ and $\partial v / \partial y$, are also symmetric
about the rip current centerline. The rip current model has both convergent flow (not
apparent on the colorbar scale chosen) and negative strain, i.e. compression of the
flow, in the rip current neck. Convergent flows are known to produce an increase in
radar backscatter [Chang et al., 2008] while there has been no study looking at how
the compression of a flow affects radar backscatter. It is hypothesized that the
horizontal compression on a flow would increase surface roughness, as shown in Figure 4b by the positive action anomaly in the rip neck. Because divergence and strain are the only current deformations that cause a signature in the radar backscatter [Rascle et al., 2014], the backscatter should also appear symmetric about the rip current centerline when wind is traveling parallel or normal to the current.

Both the vorticity and shear of the surface current are symmetric, but opposite, about the rip current centerline. Though vorticity does not create a backscatter anomaly [Rascle et al., 2014], it does provide a check to ensure the resulting deformation tensor components are as expected. By looking at the rip current velocity field (Figure 3a) it is apparent that the flow has slight rotation counterclockwise upshore and clockwise downshore of the rip current centerline. This can also be seen in the vorticity component of the deformation tensor where positive vorticity means the flow is cyclonic and negative vorticity means the flow is anticyclonic. The vortical flow of the model is supported by field experiments investigating the use of floating as an escape strategy from rip currents. McCarroll et al. [2014] showed that water in the rip current recirculates into the surf zone where waves then carry the water back to shore.

When wind is not blowing parallel or normal to the current, the current shear is also taken into account with respect to strain in the wind direction. Because the current shear is of similar magnitude to the strain, it can have a large effect on the strain in the wind direction. See Rascle et al. [2014] for the method of axis rotation when the wind is not blowing along the x-axis, the direction of the current.

4.2.2. Hill’s spherical vortex
The magnitudes of the four deformation tensor components for Hill’s spherical vortex, shown in Figure 6, are much more similar than those of the rip current model. The largest deformation component for the spherical vortex is the vorticity (Figure
6c). This is not surprising since by definition a spherical vortex is a vortical flow field.

The method of stitching together two flow fields to create the spherical vortex velocity field, see Smith [2008], has a significant effect on the continuity of the deformation tensor across the radius of the vortex. The effect is most apparent in the vorticity component of the deformation tensor where there is no vorticity outside the radius of the vortex while the vorticity is rather larger near the center of the individual eddies inside the radius of the vortex.

To ensure there were no errors in the calculations of the spherical vortex velocity field, the calculated divergence and vorticity for case 2 were compared to the results of Smith [2008] and were determined to be consistent.

**Figure 6.** Spherical vortex surface deformation tensor components: divergence (a), strain (b), vorticity (c), and shear (d)
Along the axis of origin in the along-current direction there is zero divergence and strain. This is due to divergence and strain only being dependent on $\partial u / \partial \varphi$ and $\partial v / \partial \varphi$. This indicated that if wind is blowing parallel or normal to the along-current flow direction then there should be no radar backscatter along this axis of origin. Though this is an idealized flow fields, this feature has been imaged in radar data of divergent flows, see Ivanoc and Ginzburg [2002].

The large divergence in the spherical vortex model is not surprising based on the assumptions that were made about the flow field that was extracted by the potential-solenoidal flow decomposition of an isolate feature measured by Smith [2008]. Based on the work of Rascle et al. [2014], since the spherical vortex is divergent it should show up in the radar backscatter regardless of the wind direction. When the wind direction is aligned with the current then the strain should enhance the signal, while when the wind direction is normal to the along-current direction it may destructively interact with the divergent field.

There is zero vorticity and shear along the cross-current axis of origin. This is due to vorticity and strain being dependent on $\partial u / \partial \varphi$ and $\partial v / \partial \varphi$ and the magnitudes of $u$ and $v$ are not changing in these dimensions along the axis of origin. This can be seen in models of surface vorticity for isolated vortices that are shed off of rip current. Though vorticity has the largest magnitude of deformation, it does not exhibit a signature in the radar imagery [Rascle et al., 2014]. The shear on the other hand affects the relative strain in the wind direction when the wind is not blowing parallel or normal to the cross-current direction. The strain and shear are of similar magnitudes so the shear will have a large impact on the mss anomaly. Since the axis of zero shear is normal to the axis of zero strain, when wind blow in a direction that is not normal or parallel to the reference axes, e.g. $\varphi_w$ of 45°, then the center of the vortex will appear to be rotated from where it actually is. This could significantly
alter any assumptions about the rotation direction and propagation of a set of vortices that are observed in the nearshore via radar.

4.2.3. Composite current

In order to study how the rip current and spherical vortex model interact when linearly added together, the components of the deformation tensor for the composite flow field were examined (Figure 7). The most noticeable trait in the composite model is how different the magnitudes of divergence are in the rip current and the spherical vortex.
Figure 7. Composite velocity field surface current deformation tensor components: divergence (a), strain (b), vorticity (c), and shear (d)

Because the divergence is so much smaller than the other deformation components, it suggests that the imaging of rip currents is dependent on wind direction. Though the convergence of the rip current is very small, it is still apparent that it is destructively interacting with the divergent zone and constructively interacting with the convergent zone of the spherical vortex (Figure 7a). The convergent zone of the spherical vortex may enhance the radar backscatter while the slight convergence of the rip neck may
not have a magnitude large enough to affect the radar backscatter above the noise level.

There is also constructive and destructive interaction in the strain field. The dominate feature in the strain field is the neck of the rip current. It is expected that there will be a change in backscatter intensity between the rip current and spherical vortex even before the vortex becomes an isolated feature. This could indicate that a similar pattern of dark and bright backscatter intensities can be been between the rip neck and the rip head as those shown between the convergent and divergent surface signatures of internal waves and ocean front [Lyzenga, 1998; 2010]. Because most increased backscatter in surface current imaging is assumed to be due to convergent flow fields [Rascle et al., 2014], the wrong interpretation of the flow field could be done.

Since the shear of both features in the current field are large, the rip current should be visible regardless of the wind direction, but the current may appear to have a different shape depending on the wind direction.

5. Mean square slope
There are many things that contribute to the magnitude of the mss anomaly. In this section, the projection of the mss anomaly will be investigated along with the effect of the magnitude of the spatial and velocity constant ratios, the wave number integration limits, and the contributions from divergence and strain in the wind direction. To further examine the contributions of divergence and strain in the wind direction to the mss anomaly, the polarization indexes were calculated for various wind directions. All integrations were taken using the trapezoidal numerical integration method; the wave direction integration was taken over all directions.

In order to further investigate the effect that wind direction has on radar backscatter, the mss anomaly is calculated taking into account the effect that wind direction has on the equilibrium spectrum and the signature of the deformation tensor. In order to
calculate the equilibrium spectrum for wind directions that are not directed offshore the range of wave propagation directions are shifted to be centered on the wind direction, where there are 180 degrees of propagation directions in either direction from the wind direction. Then the equilibrium spectrum is calculated using (12), where the only parameter that changes is the wind growth parameter (12). To incorporate the effect that the change in wind direction has on the signature of the current, the deformation tensor components are rotated so that the calculate strain and shear are in the wind direction. Because there is no directional dependency on the signatures of divergence and vorticity, these two components are not rotated. To calculate the strain and shear in the wind direction, both the strain and shear deformation tensor components are rotated to a new coordinate system [Rascle et al., 2014],

\[
\begin{align*}
S_t' &= \cos 2\varphi_w - \sin 2\varphi_w S_t \\
S_h' &= \sin 2\varphi_w \cos 2\varphi_w S_h
\end{align*}
\]

(14)

where \(S_t'\) and \(S_h'\) are the new strain and shear in the wind direction and \(\varphi_w\) is the wind direction measured clockwise from offshore. This method of calculating the strain in the wind direction maps the strain in the new coordinate system back to the original coordinate system. Based on these new deformation tensors, the action anomaly and mss anomaly can be calculated using (7) and (8), respectively. Because the new strain field is mapped to the original coordinate system, the x-axis, and the x projection, is still in the offshore direction and the y projection of the mss anomaly is the alongshore component of the total mss anomaly.

5.1. Projection

The first factor that was investigated was differences in the magnitude of the mss anomaly depending on the projection being studied. Because of the assumption that the mss anomaly is directly related to the surface roughness, and therefore the radar backscatter, it is important that the correct projection of the mss anomaly is
examined. The projection that is modeled is dependent on the radar and the radar look direction. The radar that was set-up at Duck, NC was facing offshore. Since rip currents are offshore direction flows, and to maintain consistency with Rascle et al. [2014], the direction of interest is along the x-axis. Though the x projection is the projection of importance for this study, the total mss anomaly and the y projection were also investigated (Figure 8). To maintain consistency with the action anomaly results, the wind was directed offshore for the study.

Figure 8. The total mss anomaly (a) and the x (b) and y projection (c) of the mss anomaly for wind blowing offshore
By looking at the total mss anomaly and how it compares to the x and y projections, it is apparent that the x and y projections are constructively interacting so that the magnitude of the total mss anomaly is similar to, but smaller than, the x projection. The y projection of the mss anomaly is much smaller than the total and x projection. It is also negative, whereas the total mss anomaly and the x projection are positive. This is due to the effect that the projection term ($\cos^2\phi$ or $\sin^2\phi$) has on the action anomaly. Before taking the integrations in (7), the action anomaly multiplied by the projection is large and positive for the x projection, while small and negative for the y projection. When the wind is blowing parallel to the shore, 90° or 270°, this effect is switched so that anomaly for the x projection is small and negative while the y projection is large and positive (not shown).

When the radar look direction, or the projection of the mss anomaly, is normal to the wind direction, as seen in Figure 8c where the wind is blowing offshore but the y projection is being calculated, the majority of the surface roughness is not in the direction that enhances backscatter. This results in the characteristic features that enhance backscatter, such as wave breaking and increased steepness, being oriented in a direction that does not scatter the electromagnetic waves back towards the sensor.

Figure 8 shows that the vortex has a larger mss anomaly than the rip current; this is unexpected based on the magnitude of the divergence versus strain presented in section 4 and is investigated further in section 5.5.

5.2. Current magnitude
By scaling the ratio of the spatial and velocity constants in the composite velocity field it was shown that the action anomaly scaled by the same factor (section 4.1). Figure 9 shows that the same trend appears in the mssx anomaly, where the magnitude of the maximum mssx anomaly in case 3 is twice as larger as in case 2. This indicates that two rip currents of the same spatial extend with different velocities will have different backscatter intensities. It also means that depending on the current and wind
conditions a radar may not be able to sense a rip current that is present in the nearshore.

![Image](image.png)

**Figure 9.** Comparison of magnitude of the mss\textsubscript{x} for the composite velocity field for case 2 (a) and case 3 (b)

Comparing Figure 9 and Figure 4 (b and c) it is apparent that the mss\textsubscript{x} anomaly is not a scaled feature of the action anomaly. In order to analyze the action anomaly, it was examined for one wavelength while the mss anomaly takes into account how the spectrum is changing at a set of wave numbers. By comparing the mss\textsubscript{x} anomaly for cases 2 and 3, the significance of the magnitude of the anomaly on how the current looks in the radar data becomes apparent. For the range of magnitudes of the mss\textsubscript{x} anomaly examined in Figure 9, case 2 makes the spherical vortex look like an isolated feature, whereas case 3 makes the vortex appear to be the head of the rip, similar to what is seen in Figure 1.

### 5.3. Wave number spectrum

Because each radar is sensitive to surface roughness at different wavelengths, it is important to understand the effect that different wavelength limits, i.e. wave number limits, have on the resulting mss\textsubscript{x} anomaly. Here the resulting mss\textsubscript{x} anomaly for two
sets of integration limits are compared to investigate how changing the integration limits effect the radar backscatter intensity.

The first wave number integration was taken over all wave numbers, from 0.31 m\(^{-1}\) to 81 m\(^{-1}\), and second was taken around the Bragg wave numbers for X-band radar, taken to be from 52 m\(^{-1}\) to 81 m\(^{-1}\). The larger integration limits were studied in order to include the effect that waves, on the order of a meters long, have on the action density spectrum; introducing a simple method of acknowledging that nonlinear interactions between wave numbers effect surface roughness. The limits for the Bragg wave numbers were based on the frequency range specified by IEEE Standard 521 [2002] for X-band radar and the relationship between the radar incident electromagnetic wave number \(k_i\) and the wave number of the surface wave sensed \(k\), 
\[
k = 2k_i \sin \theta,
\]
where \(\theta\) is the angle of incidence [Phillips, 1984] taken to be between 87° and 89° based on the set-up at Duck, NC [Haller et al., 2014].

It is important to note that the larger integration limit studied, which is being defined as the integration over all wave numbers of interest, is a limited integration range. Due to the fetch limit associated with wind blowing offshore, the maximum wavelength was set to be on the order of meters rather than 10s or 100s of meters. When studying the mss anomaly associated with wind blowing in other directions the wave number integration limits were kept the same for consistency.
It is apparent that when longer waves are included the \( mss_x \) anomaly is larger (Figure 10). This is due to the equilibrium action being larger for long waves (Figure 2, lower wave numbers) along with a larger perturbation at smaller wave numbers, see Lyzenga [1998] Figure 1. There is an order of magnitude difference between the equilibrium actions at the upper wave number limit compared to the lower wave number limit. This has a significant effect on the \( mss_x \) anomaly and indicates that remote sensors that are sensitive to a broader band of wave numbers, e.g. in sun glitter images, would have a stronger signature than those sensitive to a narrow band, e.g. marine radar and SAR. Lyzenga [1998; 2010] argues that the longer waves have a significant effect on the backscatter at Bragg wavelengths due to nonlinear interactions between wave numbers; these effects are not being modeled in this study.

The integration limits used in this study are not based on physical or numerical research; they are limits set based on the expectations of the researchers and is not intended to be the final say on the wave numbers that are associated with imaging currents in the nearshore. An investigation to determine what appropriate limits for the wave number cut-offs should be done in order to better understand the difference.

Figure 10. The \( mss_x \) anomaly due to integrating over all wave numbers (a) and over Bragg wave numbers of interest for X-band frequencies (b), see section 2.3 for details on these integration limits.
in the wave action spectrum based on the wind direction. The purpose of this comparison is to look at the effect that the wave number range has on the predicted backscatter results.

5.4. Divergence versus strain

Though it is apparent that the deformation tensor is dominated by the strain in the wind direction, it is not as apparent whether the divergence or strain in the wind direction would have a larger effect on the mss anomaly. In order to investigate which deformation component is more important to the imaging of a rip current, the x projection of the mss anomaly was calculated considering only the divergence, then calculated considering only the strain in the wind direction. Figure 11 shows the resulting mssx anomalies.

![Figure 11. Comparison of the mssx anomaly due to current divergence (a) and strain in the wind direction (b)](image)

The mssx anomaly in the rip neck due to divergence (Figure 11a) is significantly smaller than that due to strain in the wind direction (Figure 11b), but the anomalies in the spherical vortex are of similar magnitudes. This indicates that there is important constructive interactions in the spherical vortex that makes it the dominate feature in
the mss, anomaly (Figure 8b). Because the signature of the spherical vortex appears to be equally dependent on the divergence and the strain in the wind direction, it should be imaged regardless of the wind direction; it would also be difficult to make any conclusions about the current deformation if attempting to deconstruct the current field of the spherical vortex from radar images. The effect of the divergence of the surface compared to the strain in the wind direction is investigated further by calculating the polarization indexes.

5.5. Polarization indexes

By calculating the polarization index $\alpha$, (9), for wind directions at every 45° increments for 360° starting with wind blowing offshore ($\varphi_w = 0°$) it is possible to analyze how the mss anomaly for various wind directions changes. The polarization index values presented in Table 3 are for case 2 of the composite current model.

An alternative way of examining the polarization indexes is by calculating $\beta$, where $\beta = (1 - \alpha)/(1 + \alpha)$. $\beta$ is a constant that defines the relationship between the effect of divergence and strain in the wind direction on the mss anomaly, $D + \beta S_t$ [Rascle et al., 2014]. When wind is blowing at 45°, 135°, 225°, and 315° the total mss anomaly has no dependence on strain in the wind direction though the magnitude of the strain in the wind direction does not go to zero, indicated by $\beta$ equal to zero (Table 3).
Table 3. Polarization indexes (total and x projection) for the composite velocity field for various wind directions

<table>
<thead>
<tr>
<th>φw (deg)</th>
<th>αt</th>
<th>β</th>
<th>αx</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.86</td>
<td>0.07</td>
<td>0.38</td>
<td>0.45</td>
</tr>
<tr>
<td>45</td>
<td>1.00</td>
<td>0.00</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>90</td>
<td>1.16</td>
<td>-0.07</td>
<td>0.15</td>
<td>0.74</td>
</tr>
<tr>
<td>135</td>
<td>1.00</td>
<td>0.00</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>180</td>
<td>0.86</td>
<td>0.07</td>
<td>0.38</td>
<td>0.45</td>
</tr>
<tr>
<td>225</td>
<td>1.00</td>
<td>0.00</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>270</td>
<td>1.16</td>
<td>-0.07</td>
<td>0.15</td>
<td>0.74</td>
</tr>
<tr>
<td>315</td>
<td>1.00</td>
<td>0.00</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>360</td>
<td>0.86</td>
<td>0.07</td>
<td>0.38</td>
<td>0.45</td>
</tr>
</tbody>
</table>

According to Rascle et al. [2014], when α is close to 1, i.e. β is close to 0, the mss anomaly is equally dependent on ∂u/∂x and ∂v/∂y; indicating that the roughness pattern is due to the divergence of the surface current. When α is much less than 1, i.e. β is about equal to 1, the mss anomaly is more dependent on ∂u/∂x than ∂v/∂y; the roughness pattern is polarized in the wind direction. Whereas, when α is much larger than 1, i.e. β is about equal to -1, the mss anomaly is polarized in the crosswind direction. Based on the value of α, or β, it can be determined whether the mss anomaly is more dependent on the divergence or strain in the wind direction. These values, combined with the relative magnitudes of the divergence and strain in the wind direction, determine what current deformation component the mss anomaly is most dependent on. To examine this more closely, Figure 12 shows the relative divergence and strain in the wind direction when wind is blowing offshore.
When wind is blowing offshore, the contribution of strain to the mss$_t$ anomaly is 0.18 (Table 3). Therefore, although the magnitude of strain in the wind direction is larger than the magnitude of divergence, the relative contributions to the radar backscatter are the opposite in the spherical vortex. The dominant imaging mechanism in the spherical vortex is due to the current divergence, whereas in the rip neck the dominant imaging mechanism is the strain in the wind direction (Figure 12).

It is apparent from the polarization indexes that the mss anomaly is the same when wind is blowing from direction that are 180° apart. This trend was seen when modeling the mss anomaly for wind blowing offshore and onshore. This does not follow the observations made by Haller et al. [2014] for rip currents at Duck, NC; their results compared to the modeled mss$_x$ anomaly are discussed in the following section.

Though the total mss anomaly is dominated in the spherical vortex by the divergence while in the rip neck by the strain, there is also the dependence on the projection. Because we are interested in the x projection, the polarization indexes were calculated

**Figure 12.** Relative magnitude of divergence (a) and strain (b) contribution to the mss$_t$ anomaly when wind is blowing offshore.
for the x projection. In order to calculation the x projection of \( \alpha \), a similar method was used as determining the x projection of the mss anomaly:

\[
\alpha_x = \frac{\int_k \int_{\varphi} \omega^{-1} k^2 N_0 \left[ \sin^2(\varphi) \frac{\partial \ln N_0}{\partial \varphi} + \cos(\varphi) \sin(\varphi) \frac{\partial \ln N_0}{\partial k} \right] \cos^2(\varphi) d\varphi dk}{\int_k \int_{\varphi} \omega^{-1} k^2 N_0 \left[ \cos^2(\varphi) \frac{\partial \ln N_0}{\partial k} - \cos(\varphi) \sin(\varphi) \frac{\partial \ln N_0}{\partial \varphi} \right] \cos^2(\varphi) d\varphi dk}. \tag{15}
\]

To check the method of determining \( \alpha_x \), the mss\(_x\) anomaly, using the polarization index method, was evaluated and determined to be equivalent to the x projection of the mss anomaly when the deformation tensor approach was used (8).

Based on the values of \( \alpha_x \) (Table 3) it is apparent that the mss\(_x\) anomaly is not polarized in the wind direction, but it has a larger effect than when concerned with the total anomaly. The range of values of \( \alpha_x \), associated with \( \beta \) ranging from 0.37 to 0.60, and the difference in magnitudes of the divergence and strain in the wind direction (Figure 7 a and b) indicated that the strain will still be the dominant current deformation that enables rip currents to be images by radars.

6. Comparison to radar data

To conduct a qualitative comparison of the mss\(_x\) anomaly of the modeled current field and radar data from Duck, NC, two 60 second time-averaged radar images were used, shown in Figure 13.
Figure 13. 60 second time-averaged X-band marine radar images from Duck, NC on September 9 (a) and 10 (b), 2010 showing a rip current. Green arrows indicating *in situ* current measurements (courtesy of D. Honegger)

The comparison looks at how the shape of the idealized rip current compares to the imaged rip current, along with how the pattern and magnitude of mss$_x$ anomaly compares to the backscatter intensity. Observations about the benefits of using radar to compliment *in situ* measurements of rip currents is also discussed.

To better compare the modeled roughness and the observed radar backscatter, Figure 14 shows both an observed rip current and calculated mss$_x$ anomaly for case 2 of the composite current field. It is apparent that the brightness pattern of the rip current imaged at Duck, NC is similar to the pattern of positive and negative mss$_x$ anomaly. This is a good indication that though an idealized flow field was used to determine the mss$_x$ anomaly, this simple method of looking into the conditions necessary to image rip currents with radar is an appropriate starting point.
Figure 14. Qualitative comparison of 60 second averaged radar images from Duck, NC (courtesy of D. Honegger) (a) and the mss\(_x\) anomaly for case 2 of the composite current when wind is blowing offshore (b)

Figure 14a shows a high backscatter intensity along the rip neck, since the modeled mss\(_x\) anomaly is not dependent on the surface current divergence, then the high backscatter along the rip neck is due to strain in the wind direction. The rip head also shows a high level of backscatter, indicating that there is a surface current deformation pattern that increases backscatter on the offshore side of the head, while there is a decrease in backscatter on the onshore side due to a positive deformation, divergence or positive strain. Unlike in the modeled mss\(_x\) anomaly, it appears that the strain in the wind direction along the neck is stronger than the divergence and strain in the vortex head. Since the backscatter intensity along the rip neck is brighter than that of the rip head, either the strain in the neck is larger than is modeled in the mss\(_x\) anomaly or the convergence and strain in the wind direction in the vortex is modeled to be higher than is seen in radar data. The mushroom shape of the rip head is similar to the large mss\(_x\) anomaly on the offshore side of the spherical vortex; though it
appears to have a brighter pattern along the edge of the vortex than is modeled in the mss_{x} anomaly.

Though the flow field is idealized, the shape in mss_{x} anomaly and radar image are similar but it is apparent in Figure 13b that there is current instability and wind affects that cause the rip to not always be directed normal to shore. In order to better model the mss_{x} anomaly, the unstable components of the model developed by Haller and Dalrymple [2001] could be included. The mss_{x} anomaly also predicts that the radar images of the rip current will change depending on the wind direction. When the wind is blowing parallel to the current, the mss_{x} anomaly along the rip neck goes negative while when the wind is blowing in directions that are not normal or parallel to the current, the mss_{x} anomaly models the rip current being images so it looks tilted.

The observations by Haller et al. [2014] highlight how X-band radar can be used to extend in situ measurements, and how difficult it is to continuously measure rip currents with in situ devices since the rip current can move along the coast. Figure 13 shows how the large spatial extend of remote sensing with X-band radar can capture rip currents regardless of where they travel along the shore, whereas the in situ devices used in that study were stationary, green arrows indicated in situ current measurements in Figure 13 and Figure 14a. This again emphasizes that more research needs to be conducted in order to estimate rip current properties from radar backscatter.

7. Conclusion
Based on this preliminary study on how rip currents are imaged by radar, it was shown that the increased backscatter observed near rip currents can be attributed to the surface current divergence and strain in the wind direction. Though both components are important for the imaging of rip currents, it is apparent that sensing the rip neck is more dependent on the strain in the wind direction while a vortex at the rip head is imaged through both the divergence and strain in the wind direction. By
comparing the model results to data from Duck, NC, it was shown that while the $mss_\chi$ anomaly modeled the roughness of the vortex, or the rip head, to be greater than the rip neck, this was not seen in the data from Duck, NC. A more sophisticated numerical model the current field could be used to improve the comparison between the modeled $mss$ anomaly pattern and radar images of rip currents. An alternative, potentially more accurate, method of modeling the $mss$ anomaly of the rip currents imaged in the radar data would be to use the velocity field measured by infrared particle image velocimetry from the field experiment in Duck, NC as the current input. Though the anomaly and backscatter compare qualitatively well, the limited knowledge on the effect of fetch for offshore wind compared to onshore wind could significantly affect the modeled anomaly. To better understand the effect that wind direction has on radar backscatter intensity a more detailed comparison of the radar imagery to wind direction and mean wind velocity data should be completed. To further the understanding of how radars image rip currents, nonlinear effects and wave breaking should be taken in to account in the equilibrium spectrum model, a comparison between the $mss_\chi$ anomaly modeled for unstable rip currents and radar data should be conducted, and a study on how the equilibrium spectrum affects the model results could shed light on the importance of including long waves in the model while looking at nearshore features.
References


