AN ABSTRACT OF THE THESIS OF

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Title: AN EMPIRICAL TEST OF THE USE OF EXPECTED MARGINAL REVENUE AS AN ESTIMATE OF TOTAL COST FUNCTIONS

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The purpose of this thesis was to develop an estimation procedure for firm cost functions consistent with outcomes deduced from economic theory. The estimation procedure developed requires a minimum of data relative to previously tried estimation procedures.

The specific objectives of the thesis were the following: (1) Present those elements of economic theory which are relevant to specifying the relation between firm output and the cost function of the firm. (2) Propose a hypothesis which if not rejected would modify a portion of economic theory to more closely approximate real world conditions. (3) Develop a statistical model to quantify the relations specified in the economic model. (4) Test the hypothesis as specified in the statistical model using

cross-sectional survey data.

A firm producing at a profit maximizing level of output will produce where marginal revenue of output is equal to marginal cost of output. For most agricultural firms this is assumed to take the form $P_o = \phi'(Y)$ where $\phi'(Y)$ is the marginal cost function and P_o is the price of the output. However, the decision model for determining the profit maximizing level of output is more appropriately expressed as $E(P) = \phi'(Y)$, given that marginal cost is increasing and $E(P) > \frac{\phi(Y)}{Y}$, where $E(P)$ is defined as follows:

$$E(P) = \sum_{i=1}^{m} f(P_i)\, P_i$$

P_i is price interval i, i=1, 2, ..., m

$f(P_i)$ is the frequency with which the ith price interval occurs and m is the number of price intervals in the domain of relevant prices.

If it is possible to empirically determine $E(P)$, then the total cost function, TC, can be found by integrating.

Thus,

$$TC - \bar{b} = \int_Y \phi'(Y)dY = \int_Y E(P)\, dY$$

where $\bar{b}$ is fixed costs.

The null hypothesis tested in this thesis is that the empirical total cost function constructed (integrated) from output price expectation data taken from firms of like technology but different volume levels is identical to the cost equation of this set of firms, where the cost equation is defined to be $C' = \bar{b} + \sum_{j=}^{J} X_j r_j$ with X_j a variable input and r_j input price.

The Pacific Northwest beef feedlot industry was chosen as the economic sector from which a sample of firms was selected to provide data necessary for the hypothesis test. Two levels of technology were specified.

The Wilcoxon rank-sum statistical model was selected to provide a rule needed to decide whether or not to reject the hypothesis specified. The performance of the statistical tests failed to reject the null hypothesis for each of two technology levels of feedlot firms.

It was concluded that E(P) can be used as an estimate of $\phi'(Y)$. The estimation procedure developed will (1) allow the construction of TC not statistically different from the C' equation, (2) provide a total cost function consistent with that defined in economic theory, and (3) allow for further investigation of the nature of cost curves of agricultural firms.

An Empirical Test of the Use of Expected Marginal Revenue as an Estimate of Total Cost Functions

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AN EMPIRICAL TEST OF THE USE OF EXPECTED MARGINAL REVENUE AS AN ESTIMATE OF TOTAL COST FUNCTIONS

I. INTRODUCTION

Families of cost functions derived for groups of firms of different sizes provide a portion of the data needed by entrepreneurs contemplating a change in firm size. These families of cost functions also provide a portion of the data needed by public policy makers to assess the effects of a certain price level on the number of firms that will remain in an industry, on industry supplies of product, and to assess other intra-industry and interregional policy considerations.

The family of cost functions most commonly estimated is the family of short-run average total cost functions. Assuming that these families of curves are generated from observations taken from several different sizes of firms each observed at several levels of output, economic theory specifies the tangential relationships needed for construction of the long-run average cost function. The long-run average cost function, sometimes referred to in the economic literature as the "firm planning curve" has been estimated for several types of agricultural firms.

Doubt has been cast on the usefulness of studies which quantify long-run average cost curves for agricultural firms. Upchurch (1961) contends that despite economists' vast experience with studies of this nature, techniques used in quantifying or defining long-run average cost curves have been particularly fuzzy.

However, Upchurch (1961) makes a plea for more work in the area of defining differences in cost related to the size of the firm. He suggests a series of studies be conducted for different types of farming and the same techniques of "costing" be used throughout the series. With such a series of cost analyses, both entrepreneurs and public policy makers would have more reliable data on which to base size adjustment decisions.

Statement of Problem

A variety of estimation procedures have been used to estimate cost functions of firms. Data requirements for some of the estimation procedures are burdensome in terms of cost of data acquisition. Several of the estimation procedures employed yield results inconsistent with the expected outcomes which can be deduced from economic theory.

The purpose of the study upon which this thesis was based was to develop an estimation procedure for firm cost functions capable of describing cost functions consistent with the outcomes deduced from economic theory. The estimation procedure would be considered efficient if it requires a minimum of data relative to previously tried estimation procedures.

Objectives of Thesis

The objectives of this thesis are the following:

1. Present those elements of existing economic theory which are relevant to specifying the relation between firm output and the cost function of the firm.

2. Propose a hypothesis which if not rejected would modify a portion of the existing economic theory to more closely approximate real world conditions.

3. Develop a statistical model to quantify the relations specified in the economic model given in (2) and to provide the basis for testing the hypothesis.

4. Test the hypothesis deduced from economic theory as specified in the statistical model given in (3), using cross-sectional survey data obtained from a sample of Pacific Northwest beef feedlot firms.

Chronological Order of Research and Order of Thesis Presentation

The problem to be researched and the thesis objectives were delineated and presented in prior sections of this chapter. The economic theory underlying the hypothesis to be tested was developed as presented in Chapter II.

A statistical model was developed which provides the decision rule required to judge whether or not the hypothesis specified was rejected. This model, outlined in Chapter III, also specifies the form in which the data were to be collected and prepared to perform the statistical test.

Procedures were specified, presented in Chapter IV, for the acquisition of data from secondary sources and sample respondents. After the historical data were taken from the secondary sources, a questionnaire was designed to collect data from the sample respondents. From the sample data computations were made in preparation for the statistical test. Calculations and the tests of the statistical hypothesis are presented in Chapter V.

A summary of the conclusions that can be drawn from the tests of the hypothesis, the risky nature of the hypothesis test, the implications for use of the methodology developed, and the needs for further research are presented in Chapter VI.

II. UNDERLYING ECONOMIC THEORY OF FIRM COST FUNCTIONS

A variety of estimation procedures have been used to estimate cost functions for firms. Several of these estimation procedures yield results which are inconsistent with those outcomes which can be deduced from economic theory.

Presented in this chapter are those elements of existing economic theory which are relevant to specifying the derivation of the cost function of the firm. A hypothesis is proposed which if not rejected would modify a portion of the existing economic theory to more closely approximate real world conditions and allow the estimation of firm cost functions which are consistent with those outcomes deduced from economic theory.

Empirical estimation problems in the estimation of firm production functions are discussed--especially as related to specification error and identification problems. A review of methods commonly used in the estimation of firm cost functions is also presented.

Definition of Short-run and Long-run

A cost function expresses cost as an explicit function of the level of output achieved by a firm. Level of output per unit of time is taken as a measure of firm size. A firm can increase its level of output by intensifying production in a given plant or by increasing plant size and producing a greater volume in a larger plant.

Intensification of production in a given plant is a short-run concept. The entrepreneur can increase and vary the use of variable inputs in the production process, but the time span is too short for any modification of the fixed plant.

Increasing plant size is a long-run concept. The firm has time to increase all factors of production. If all factors of production are increased in like proportion, economies (or diseconomies) realized are economies of scale. If factors are increased by different proportions, economies (or diseconomies) realized are economies of size.

Derivation of Short-run Cost Functions

To solve for the total cost function of a firm analytically, economic theory specifies that the following information is needed:

1. The firm's production function.
2. The firm's expansion path.
3. The firm's cost equation.

Assume for simplicity the following production function, defined for one time period:

$Y_k = f(X_{1k}, X_{2k} \mid X_{3k}, \ldots, X_{nk})$, where

Y_k is the output of the k^{th} firm,

X_{1k}, X_{2k} are variable inputs one and two for the k^{th} firm,

$X_{3k}, \ldots, X_{nk}$ are fixed inputs for the k^{th} firm.

Given the production function $Y = f(X_1, X_2 \mid X_3, \ldots, X_n)$, the marginal productivities of the variable inputs may be calculated. Define $f_j = \frac{\partial Y}{\partial X_j}$, where j=1, 2. For a two variable input production function, f_1 and f_2 are the marginal productivities which can be calculated and $f_3, \ldots, f_n = 0$.

Assume that the firm buys its variable inputs in a perfectly competitive input market. That is, the variable input prices to the firm do not change with increased use of the input by the firm. If r_j is the per unit variable input price, $r_j = c_j$, where c_j are constants. The firm's cost equation may be expressed as $C = r_1 X_1 + r_2 X_2 + b$, where b is the total cost of the fixed resources for the production period.

To have variable inputs combined in optimum economic proportions, the ratios of the marginal productivities over the input prices must be equal for all variable inputs. Therefore, the condition $\frac{f_1}{r_1} = \frac{f_2}{r_2}$ must hold. Solving this condition, the firm's expansion path is then $f_1 r_2 - f_2 r_1 = 0$. Solving the following three equations simultaneously, the k^{th} firm's cost curve is determined:

$$Y = f(X_1, X_2 \mid X_3, \ldots, X_n), \quad \text{production function.}$$

$$C = r_1 X_1 + r_2 X_2 + b, \quad \text{cost equation.}$$

$$0 = f_1 r_2 - f_2 r_1, \quad \text{expansion path.}$$

Solved simultaneously, the firm's cost function is expressed as a function of output, Y:

$$TC = \phi(Y \mid X_3, \ldots, X_n).$$

Empirical Estimation Problems

One way of obtaining an empirical estimate of the cost function would be to estimate the production function and then follow the above procedure to obtain the cost function.

Attempts have been made to estimate the particular equation that represents the production function of an individual firm. Problems in estimating the production function fall into two categories: (1) specification error, and (2) identification problems. These are discussed in this order below. However, perhaps the greatest difficulty associated with the use of production functions in the analysis of a firm comes in choosing the form of the particular equation to represent the firm's production function so as to conform to reality (Toussaint, 1955). This becomes very complex as the scope of the function is enlarged.

Specification Error

The number of inputs involved in firm production function analyses often makes it necessary to group inputs into a limited number of categories before analysis can be carried out. Within any input category it is desirable that the individual inputs remain fairly constant in proportion if the category is to be meaningful. From an economic standpoint, the most meaningful proportions are the least cost combinations.

Economic theory underlying input utilization indicates that if the inputs within a category were all perfect complements, the proportions in which they would be used would not vary as output varied. If inputs within a category were good substitutes for each other, their proportions could vary widely, but there would exist a common denominator in terms of which the inputs could be measured.

Johnson has outlined three rules for the grouping of inputs for firm production function analyses (1956, p. 90-93).

1. "Group good complements together and good substitutes together, measuring the complements in terms of 'sets' and the substitutes in terms of the common denominator which makes them good substitutes.

2. "Sets of complements and sets of substitutes can be grouped into the same category very conveniently if the sets are complementary to, or substitutes, for each other.

3. "The converse of the above two rules follows: Input categories defined should neither be good substitutes nor good complements for each other."

The real input combination problems faced by a firm usually exist among categories of inputs which are neither perfect substitutes nor perfect complements for each other. The above rules leave these problem relationships among the input categories whose interrelationships are being estimated. However, such grouping of variable inputs may limit the usefulness of the statistical parameter estimates.

If inputs are grouped into categories as suggested in the three rules outlined, the problems of multicollinearity among the explanatory variables are usually minimized. The categories employed will not be highly correlated one with another and estimates of their relative effects can be obtained. However, the grouping of inputs into categories and the choice of the functional form of the equation to represent the firm production function as well as the omission of an unmeasurable input such as management all can introduce some form of specification error.

Specification error, regardless of source, introduces bias into the parameter estimates. Bias in parameter estimates of the production function would present no real problem in the derivation of empirical cost functions for a firm if the direction and/or magnitude of the bias were known. Brown (1969) points out that in general it does not appear possible to deduce very much in general about the importance of omitted variable specification bias. He states that the possible devastating effect on parameter estimation caused by specification error is too often ignored in empirical research.

Specification bias in the parameter estimates of a firm's production function would be reflected in the expansion path of the firm when derived from the first order conditions for input combinations calculated from the estimated firm production function. As the cost functions for the firm are obtained through the simultaneous solution of the production function, expansion path, and cost equation, the bias introduced into the system of equations through specification error in the firm production function would be reflected in the cost function of the firm.[1/]

1/ For example, different forms of the cost function are obtained if specification error is made in choice of production function. For a production function of the Cobb-Douglas form, representing a single output, two-input production process, cost is expressed as the positive root of output, the root being the sum of the coefficients on the input factors:

Types of specification error have induced researchers such as Paris to conclude that, ". . . the particular equation that represents the production function of an individual firm is impractical to obtain" (1960, p. 10).

Identification Problems

Paris has shown that for specific functional forms of the production function estimated for a group of firms and the n-1 first order conditions that can be derived from it, the production function is not identified (1960, p. 12).[2/] To show this for the

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$$TC = \left(Y/_{K_1}\right)^{\frac{1}{d+e}},$$

where $Y = \hat{a}X_1^{\hat{d}}\,X_2^{\hat{e}}$

and $K_1 =$ a constant.

For the production function of the polynomial form $Y = \hat{a} + \hat{d}X_1^2 + \hat{e}X_2^2$, cost is expressed as the square root of output,

$$TC = K_2\,(Y)^{1/2}.$$

The cost functions of the same production process, expressed by two production functions of different form would be identical only if d+e=2 and $\frac{1}{K_1} = K_2$.

[2/] A general description of the principle of identification of a system of equations appears in Klein's text: (1956, p. 56)

"An equation in a system of linear equations is said to be transcendental and Cobb-Douglas functional forms of a production function Paris (1960) solved simultaneously for the profit maximization output and levels of input derived from each production function and the n-1 first order conditions. Upon substituting the level of input use back into the production function, it was shown for both the Cobb-Douglas and transcendental production function that the resulting equation was indistinguishable in form from the original before substitution and hence not identified. However, when a stochastic term v_{jk} was added to each of the n-1 first order conditions, Paris (1960) showed that the production function was identified.[3/] That is, substitution of the input level from the

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> identified if it is not possible to derive another linear relation, involving exactly the same variables as the equation in question, from linear combinations of some or all the equations of the system."

Tintner defines the necessary condition for identification of a given equation in the structural model as follows: (1959, p. 157)

> "The number of variables excluded from this equation must be at least G-1, i.e., 1 less than the total number of structural equations (and also endogeneous variables) in the whole system--the system consisting of G equations and G endogeneous variables."

3/ In the Paris thesis this term was designated as v_{ij}. To conform with the subscripting used in this thesis, it is designated v_{jk}.

first order conditions including v_{jk} into the production function resulted in a functional form different from the original production function. Paris gives two sets of assumptions which justify the use of the error term v_{jk} : (1960, p. 7-8)

> "The first set of assumptions states that firms have different price expectations which deviate from observed prices. Resources are assumed to be committed at the beginning of the production period when input prices are known. Thus each firm is assumed to face the same input prices but may hold different expectations of output prices. However, when the firms are observed at the end of the production period, the output prices are known and assumed to deviate from the expected. Thus the deviation in expected points and realized points are denoted as v_{jk}.
>
> "The second set of assumptions concerns differences in expected marginal productivities. The input prices are assumed the same for all firms and inputs are committed at the beginning of the production period. However, firms are assumed to hold different expectations of resource productivity. Hence, actual input combinations may deviate from those derived from expected productivity and can be denoted by v_{jk}. The first set of assumptions, using differences in output price expectations, explains variability along the expansion path. The second set of assumptions, using differences in resource productivity expectations, explains variability about a given isoquant. Therefore, to identify the entire production function both sets of assumptions are required."

Thus in order to identify the production function by Paris's method one would need information about the output price expectation and the productivity expectations. In Paris's case output price expectations are meaningless since he was concerned with multiproduct firms and the combined output was expressed in dollars of income.

Paris worked with the following simultaneous equation system:

$Y = f(X_1, X_2 \mid b)$,	the average production function.
$0 = g(X_1, X_2) + v_{jk}$,	the expansion path of the k^{th} firm.
$r_1 = f_1(P_o - w_i)$,	first order condition for profit maximum where P_o is product price and r_1 price of input one.

where Y, X_1, X_2 are endogeneous and v_{jk} and w_i are exogeneous.

The variables v_{jk} are the deviations from the firm's observed input combination and the expansion path proportions. The variables w_i are the deviation of the market price of the output and the firm's expectation of output price. The derivative of the production function is signified by f_1.

With this system of equations the production function is identified. There are three structural equations in the system. To be identified there must be G-1 variables which are in the system that are omitted from the production function. As G denotes the number of structural equations, G-1=2. It is apparent from inspection that two variables are excluded from the production function, v_{jk} and w_i. Therefore, the production function is identified. As Paris stated, it is necessary to have both the error in the productivity estimate and the expected product price in the system to have the production function identified.

Even with an identified production function the question still remains whether or not the cost function is identified when one considers that it is the simultaneous solution of the average production function, an expansion path unique to each firm, and the general cost equation.

Now if one is attempting to estimate the cost function, one must add the cost function as a structural equation. Consider the following expanded system:

Y	$= f(X_1, X_2 \mid b)$,	the average production function.
0	$= g(X_1, X_2) + v_{jk}$,	the expansion path of the k^{th} firm.
$\phi'(Y)$	$= \frac{dTC}{dY} = P_o - w_i$,	first order condition for profit maximum.
TC	$= \phi(Y) + b$,	cost function
C	$= r_1 X_1 + r_2 X_2 + b.$,	cost equation.

There are now four structural equations and one identity, four endogeneous variables -- Y, TC, X_1, and X_2 and two exogeneous variables -- v_{jk} and w_i. The production function in this system is identified. The cost function $TC = \phi(Y) + b$ is also identified since there are more than three variables omitted -- X_1, X_2, v_{jk}, and w_i -- from the equation. However, if just the last two structural equations are considered the simultaneous system would be the following:

$$P_o - w_i = \frac{dTC}{dY} = \phi'(Y),$$

$$TC - b = \phi(Y).$$

The cost function is identified in this system, as TC and Y are the endogenous variables. The exogeneous variables are P and w_i. Therefore, G=2, and G-1=1, the minimum number of variables in the system which must be omitted from the cost function to have it identified. Thus, if $P_o - w_i = \phi'(Y)$ the cost function is identified. As will be seen in subsequent sections this is the condition that expected marginal revenue is equal to marginal cost.

Profit Maximization Conditions Expressed in Terms of Output

To develop the above condition it is assumed first that firms are not uncertain about product price, i. e., P_o is known without error or $w_i = 0$.

Secondly, if it is assumed that the firm has its inputs combined in expansion path proportions for a given size of plant, the firm's profit equation can be written in terms of output and product price as follows:

$$\pi = P_o Y - TC.$$

The total cost function, TC, is expressed in terms of output, Y. As total cost is the sum of total variable costs and total fixed costs, total cost can be expressed as $TC = \phi(Y) + b$, where $\phi(Y)$ are variable costs and b represents fixed costs.

To determine the profit maximizing level of output, the first derivative of this function with respect to Y is set equal to zero,

$$\frac{d\pi}{dY} = P_o - \phi'(Y) = 0 \text{ and solved,}$$

$$P_o = \phi'(Y).$$

The first order condition for profit maximization requires marginal revenue to be equal to marginal cost.

The second order condition for profit maximization requires that the marginal cost function be increasing at the profit maximizing output level. That is, $\phi''(Y) > 0$. The second derivative of the profit function is $\frac{d^2\pi}{dY^2}$, which for profit maximization must be negative.

Therefore, $\frac{d^2\pi}{dY^2} = -\phi''(Y) < 0.$

But $-\phi''(Y) < 0$ may be rewritten as $\phi''(Y) > 0$ by multiplication of both sides of the inequality by (-1). In summary, by expressing the profit function in terms of output the two conditions for an unconstrained profit maximization are (1) that marginal cost equals marginal revenue (output price) and (2) that marginal cost is increasing at the level of output produced.[4/]

There is one case where the firm would not operate, given the above two conditions were satisfied. That is where $P_o < \frac{\phi(Y)}{Y}$ for a particular output level. If product price will not cover short-run average variable cost, then the firm will choose not to produce.

In some instances firms are not capable of achieving the profit maximizing output level. Firms may be so restricted in operating capital that the maximum level of output they can achieve is less than that where $P_o = \phi'(Y)$, where $\phi''(Y) > 0$. In such cases the firm might be capable of producing at an output level where $P_o = \phi'(Y)$, $\phi''(Y) < 0$. However, this output level is the profit minimizing level of output. Therefore, a firm so constrained by operating capital will choose to operate at those levels of output where $P_o > \phi'(Y)$, where $\phi''(Y)$ may be less than, greater than, or equal to zero.

4/ This is assuming that the producer sells in a perfectly competitive product market.

Modification of Profit Maximization Conditions Expressed in Terms of Output

The assumption was made that the firm knows the market price of the commodity it is producing at the time the decision to produce is made. This assumption may not deviate far from reality in certain manufacturing industries where the decision to produce and the marketing of the product is separated by only a portion of an hour, a few days, or a week. However, in agricultural production, the time interval between the date of production planning and the marketing of the product is usually several weeks, a crop season, or a feeding period. As the time interval between the decision to produce is made and the sale of product is consummated lengthens, one would expect the price of the output to become less certain to the producing firm, as the forces determining price in the market have more time to adjust to conditions both internal and external to the market. Consequently, most agricultural production firms do not make production decisions based on some certain market price, but rather on expectations of the market price at the end of the production period.

Therefore, the profit function for the firm could be rewritten as follows:

$$\pi = E(P)\ Y - TC,$$

where

$$E(P) = \sum_{i=1}^{m} f(P_i)\ P_i\ , \qquad i = 1,\ 2,\ \ldots,\ m.$$

P_i is price interval i.

$f(P_i)$ is the frequency with which the i^{th} price interval occurs and m is the number of price intervals in the domain of relevant prices.

Therefore, the firm makes its production decision based on the expected value of the distribution of anticipated product prices.

Substituting $\phi(Y) + b$ for total cost into the profit equation above will give:

$$\pi = E(P)\ Y - \phi(Y) - b.$$

The first order condition for unconstrained profit maximization is:

$E(P) = \phi'(Y)$ and the second order condition remains unchanged; that is,

$\phi''(Y) > 0.$ 5/

The firm will choose to not produce if $E(P) < \frac{\phi(Y)}{Y}$.

Earlier $\phi'(Y) = P_o' - w_i$ was seen to identify the cost function.

5/ If the firm is constrained in variable capital, the first order condition is $E(P) > \phi'(Y)$, with $\phi''(Y)$ greater than, or less than, or equal to zero.

To show that $P_o - w_i = E(P)$ let $z_i = P_o - P_i$, that is, the difference between the actual market price and the i^{th} price interval from the frequency function $f(P_i)$. Now substitute for P_i into:

$$E(P) = \sum_{i=1}^{m} f(P_i)\, P_i$$

$$= \sum_{i=1}^{m} f(P_o - z_i)(P_o - z_i)$$

$$= \sum_{i=1}^{m} f(P_o - z_i)\, P_o - \sum_{i=1}^{m} f(P_o - z_i)\, z_i$$

$$E(P) = P_o - E(z_i).$$

Now only if $w_i = E(z_i) = 0$ would $E(P) = P_o$. Hence, $E(z_i)$ is what was called w_i above.

Therefore, $P_o - E(z_i) = P_o - w_i = E(P) = \phi'(Y)$ expresses the first order condition for a profit maximum.

If it is possible to empirically determine $E(P)$, then TC can be found by integrating.

Thus, $TC - b = \int_Y \phi'(Y)dY = \int_Y E(P)dY$.

The question remains whether or not a total cost function can be found by integration of the first order condition for firm profit maximization. Previous attempts to estimate firm cost functions have assumed that the firms from which data were taken had resources combined in expansion path proportions. However, most of these studies did not assume that the firms from which data were taken were operating at profit maximizing levels of output. Therefore, "cost functions" were derived from cost equation data by either synthesis or regression methods.

Statement of Hypotheses

The null hypothesis is that the empirical total cost function constructed (integrated) from output price expectation data taken from firms of like technology but different volume levels is identical to the cost equation of this set of firms.[6/]

The alternative hypothesis is that the empirical total cost function is not identical to the cost equation of this set of firms.

Review of Methods Commonly Used in the Estimation of Firm Cost Functions

Previous studies which have attempted to estimate cost functions of firms (long-run and/or short-run curves) can be categorized by the methodologies employed for estimation of the cost function. One group of studies includes those studies in which regression cost functions were fit to cost equation data

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6/ TC_k values are actually being compared with C'_k values at the same k. C'_k is considered independent of prior values of C'_k, i.e., $C'_k \neq f(C'_{k-1})$. However, throughout this analysis C' will be referred to as the cost equation of a particular set of firms.

obtained primarily from cross-sectional surveys of similar firms. The other group of studies includes those commonly referred to in economic literature as "cost synthesis" studies.

Regression Cost Functions

Numerous studies have been made of the costs of operating various plants in a given industry for a stated time period. Cost equation data are obtained for each firm through cross-sectional surveys. It has become apparent to certain researchers that such a cross-section of costs of operation for a given period must "catch" many of the firms in some sort of maladjustment which in important cases are not readily explained by the usual regression of cost against volume (Erdman, 1944).

Usually a regression line is fitted to cost-volume observations of firms grouped as similar on some _a priori_ basis. These cost-volume data are commonly presented as a scatter diagram, with an average regression line fitted to the scatter. This curve shows the average relation between plant volume and cost.

However, such a curve combines and confuses cost changes that result from the more complete utilization of a plant of a given size with the changes that accompany changes in size. Attempts to properly stratify the sample into meaningful sub-samples based on size of plant can reduce the effects on cost introduced through the confusion of size with level of realization.

Due to the nature of the regression technique, it should be clear that any average regression fitted to cross-sectional data will indicate costs above the minimum levels for a plant of given size operating at that level of output.[7/] The slope of the short-run average variable cost curve for a particular size plant will understate the change in average cost that could be realized by a change in volume of production (Bressler, 1945).

Another major disadvantage of the regression technique for deriving cost curves directly from cross-sectional farm cost survey data is one of statistical measurement often referred to as the "regression fallacy." That is, individual firms with similar fixed resources are often placed into the same subsample. However, firms with like fixed resources often operate at different levels of output because of limitations on other resources,

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7/ A similar problem exists in the use of regression techniques to estimate production functions.

As Carlson suggests ". . . if we want the production function to give only one value for the output of a given service combination, the function must be so defined that it expresses the maximum production obtainable from the combination at the existing state of technical knowledge." (1939, p. 4)

Consequently, regression estimates of the production function do not yield a function consistent with economic theory. A production function estimated by regression techniques underestimates the theoretical production function.

risk and uncertainty and related reasons; a regression equation fitted to such a scatter of cost-volume points gives a cost curve which lies above the "true" cost curve (Carter and Dean, 1961).

In summary, there seem to be three major problems in using regression analyses to estimate firm cost functions from cross-sectional firm data:

1. There is no assurance that firms observed are not in some sort of maladjustment.

2. Stratification of firms into subsamples does not eliminate the problem of "regression fallacy."

3. The statistical properties of regression analysis preclude the possibility of obtaining an estimated cost function which will coincide with the same function as defined in economic theory.

Synthesized Cost Functions

Most methods of synthesizing cost functions are designed to obtain firm (or plant) cost functions. The two most common methods are partial budgeting and complete cost synthesis.

Partial budgeting is most commonly used when plant size is given. Costs are then synthesized for various combinations of variable resources and/or for the plant operating at a given percent of total capacity. Where constraints are numerous, e.g., plant capacity defined in terms of several resources, linear programming has been used.

Complete cost synthesis involves the synthesis of both variable and fixed costs. Researchers using complete cost synthesis have allowed the combinations and levels of variable resources to change and have also changed the technical organization of the plant to assess the changes induced in the firm's cost structure.

Bressler cites the two main problems in the synthesis of cost curves: (1945, p. 536)

> "First, increasing variable costs may be overlooked, although some of the engineering data will provide a clue in this matter. Second, it is frequently held that some costs are forgotten in this process and the actual costs that will eventually characterize the plant will be higher than the estimates."

Cost synthesis techniques have been adopted from the works of engineers and architects. Their estimates of costs are made from known cost data obtained from experimental results and cross-sectional surveys of firms and tempered by their knowledge of the principles of physics and engineering. They usually assume constant marginal productivities for a variable resource used in conjunction with some fixed facility. This precludes them from recognizing the possibility of increasing variable costs, as Bressler suggests in his first point. Also these studies have dealt primarily with the synthesis of those inputs which are measurable in quantity and often can be assessed for quality. Consequently, differences in productivity and costs due to management, quality of labor, etc., are not explicitly recognized in their cost synthesis (Knutson, 1958).

Summary

The purpose of this chapter was to develop the economic framework underlying the estimation of the total cost curve.

It has been shown that the production function is difficult to identify. One means of identifying the production function is to use product price expectations in the first order condition. With the addition of one additional structural equation to the simultaneous system used to identify the production function the cost function is also identified. However, a more direct two equation simultaneous system was proposed. This system should yield a total cost function which is consistent with that defined in economic theory; that is, integrate the marginal cost function and thereby obtain the total cost function.

The following chapter presents the statistical model necessary for the hypothesis test specified in this chapter.

III. SPECIFICATION OF THE STATISTICAL MODEL FOR THE HYPOTHESIS TEST

The statistical model developed in this chapter provides a means by which the conjecture (hypothesis) that the empirical total cost function constructed (integrated) from output price expectation data taken from firms of like technology but different volume levels is identical to the cost equation of this set of firms can be tested. The statistical model provides the rule needed to decide whether to reject or fail to reject the hypothesis once the values of the data have been determined. The statistical model developed in this chapter provides the decision rule required to judge whether or not the hypothesis specified was rejected. This statistical model also outlines the form in which data would have to be prepared to perform the statistical test. Upon completion of the statistical model, data were taken from respondents, summarized, and the test of hypothesis performed. The latter appears in the next two chapters.

Data Series

From interviews the following data were obtained from each respondent:

1. $E(P)_k = \sum_{i=1}^{m} f(P_i)_k P_i$ where

$E(P)_k$ is the product price expectation of the k^{th} respondent,

$f(P_i)_k$ is the frequency distribution of product price associated with the k^{th} respondent, and

P_i is price interval i.

2. $C_k = \sum_{j=1} r_{jk} X_{jk} + b_k$

j=1, 2, ..., J where

C_k is the level of cost from the cost equation of the k^{th} respondent.

r_{jk} is the price of input j for the k^{th} respondent,

X_{jk} is the level of input use of the j^{th} input in the k^{th} firm, and

b_k is the level of fixed costs of the k^{th} firm.

3. Y_k is the output level of the k^{th} respondent or firm.

As previously derived, $E(P) = \phi'(Y)$, at the profit maximizing output level for the firm. Now $E(P)_k$ has been observed and $E(P)_k$ will be taken as an estimate of $\phi'(Y_k)$ and henceforth denoted as $\hat{\phi}'(Y_k)$.

The total cost function, TC, was shown to be derived from the integration of the marginal cost function $\phi'(Y)$. However, since $E(P)_k$ is obtained from each of the firms operating under like technology but different volume levels, Y_k, TC is defined as a discrete summation.

To carry out the summation let the volume levels of each respondent (Y_k) within like technologies be arranged in ascending order, i.e., from the lowest (k=1) to highest ($k=N_I$ for technology I and $k=N_{II}$ for technology II) output level. The summation is given by,

$$TC = \phi'(Y_1)(Y_1) + \phi'(Y_2)(Y_2 - Y_1) + \ldots + \phi'(Y_N)(Y_N - Y_{N-1}) + \bar{b}.$$

The above expression represents the total cost of producing at output level Y_k, where k=1, 2, ..., N.

The average level of fixed cost for firms in a particular technology is defined as:

$$\bar{b} = \frac{1}{\sum_{k=1}^{N} Y_k} \sum_{k=1}^{N} Y_k b_k .$$

Now each TC_k observation can be compared with the C_k' observation for the same volume level to determine if the empirical total cost function, TC_k, constructed from output price expectations is identical to the cost equation C_k' where C_k' is defined as follows:

$$C_k' = \bar{b} + \sum_{j=1}^{J} r_{jk} X_{jk} \text{ where } \bar{b} \text{ is defined as above.}$$

Test Statistic

Define $V_k = TC_k - C_k'$, $k=1, 2, \ldots, N$

$$W_1 = V_1$$

$$W_2 = V_2 - V_1$$

.

.

.

$$W_k = V_k - V_{k-1}$$

.

.

.

$$W_N = V_N - V_{N-1}$$

To clarify the notation problem and multiple definitions, a simple diagram may aid in interpretation of the test statistic. In Figure 1 the TC function is shown after the discrete summation, and C' is shown relative to its associated output level Y. The V_k are the deviations between TC_k and C_k'. The W_k are defined for each output level but are not shown on Figure 1. They are differences in successive V_k values except W_1 which is equal to V_1.

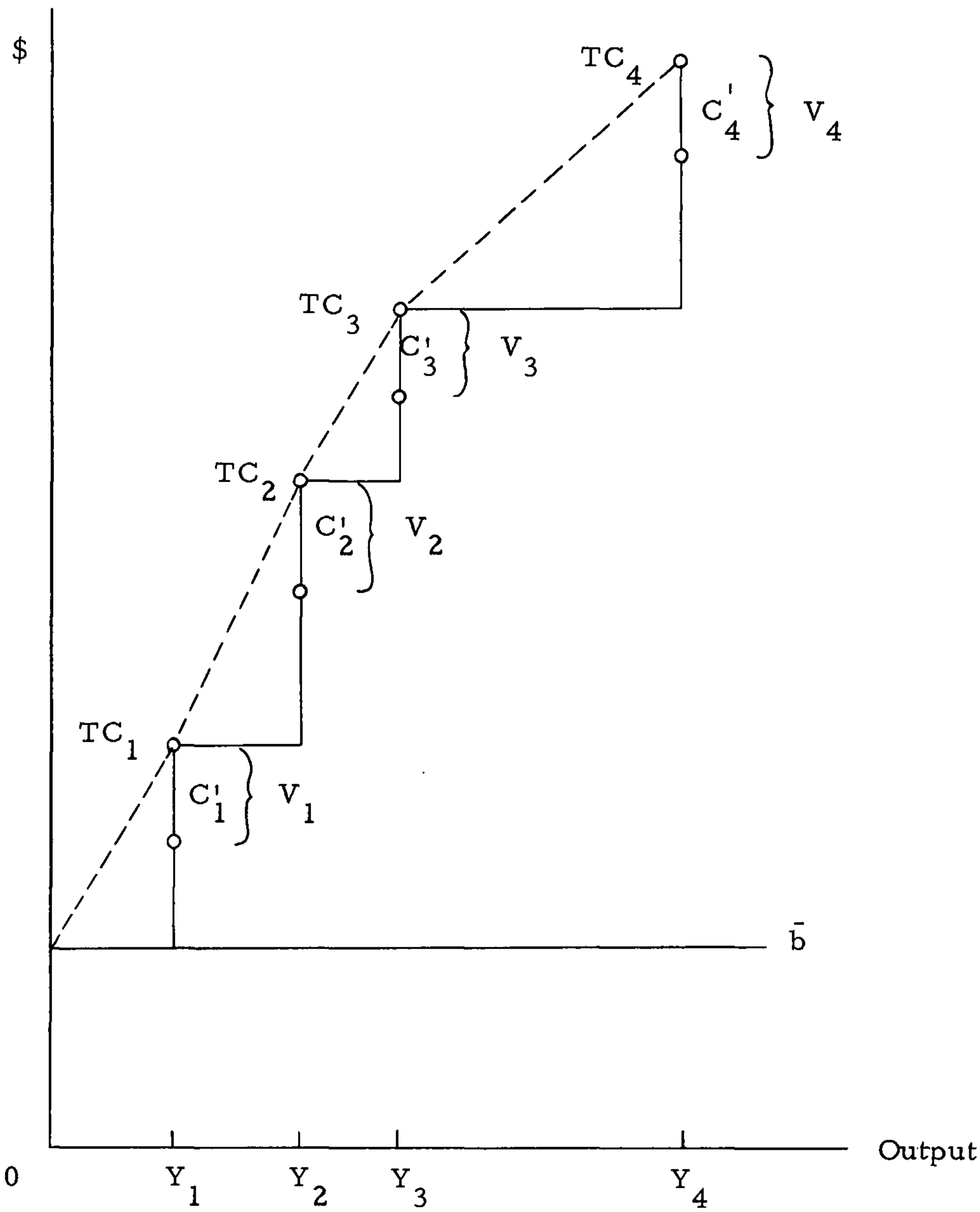

Figure 1. A hypothetical TC function, hypothetical C' equation, and V_k values for four volume levels.

By defining W_k as the difference between V_k and V_{k-1}, W_k does not include those deviations between TC and C' at levels of $Y < Y_k$. The removal of the previous deviations assures that the errors are not compounded due to the summation process.

To calculate the test statistic, the absolute values of W_k are arranged in ascending order. Each W_k value is assigned a rank number. The smallest absolute value of W_k is assigned the rank of 1, the second smallest value the rank of 2, and so forth, until the largest value of W_k is assigned the rank of N, N being the number of W_k values calculated (Wine, 1964).

Once rank numbers have been assigned to the absolute values of W_k, the W_k values are separated into two subsamples, one subsample consisting of those W_k with negative sign and the other with those of positive sign. The rank numbers of the W_k values in each subsample are then summed.

$$\text{Let } S_N = \text{minimum} \left(S^+_{n_1} , S^-_{n_2} \right)$$

where $S^+_{n_1}$ is the sum of rank numbers of all positive W_k values, $S^-_{n_2}$ is the sum of rank numbers of all negative W_k values, and $n_1 + n_2 = N$.

Providing $n_1 = n_2$, then S_N is compared with the tabled critical value of S_N.

When $n_1 \neq n_2$, a further calculation is required to obtain the test statistic.

First, find the sum of the ranks for the subsample with the smaller number of observations and call the sum S_s. Supposing n_1 were the smallest subsample, compute $S_L = n_1 (n_1 + n_2 + 1) - S_s$. The value to be compared with the tabled critical value is then $S_N = \text{minimum} \left(S_s, S_L \right)$ where S_s is the subsample with the smallest number of observations and S_L the subsample with the largest number of observations.

The test described above was developed by Wilcoxon, although it is sometimes referred to as the Mann-Whitney test. It was developed to facilitate the analysis of two-sample problems where sample observations were not paired. In this analysis, the two samples are (1) the observations at given volume levels, Y_k, from the TC function, and (2) the observations at given volume levels, Y_k, from C' equation.

In this test, it is assumed that two random and independent samples are drawn from two distributions which have the same form but possibly different values of the location parameter (e. g., mean or median). Thus under the usual null hypothesis, the random and independent samples are assumed to come from a single population. The alternative hypothesis may be expressed so that the test is either one-sided or two-sided (Wine, 1964).

The Wilcoxon test statistic is nonparametric. That is, normality is not assumed in the distribution of the deviations, i.e., of the W_k values. The Wilcoxon test statistic precludes the need for assuming the W_k values follow a normal distribution.

There are no _a priori_ reasons to assume that the W_k values defined above are normally distributed. Consequently, the Wilcoxon nonparametric rank-sum test appears a more appropriate test than the two-sample _t_ test.

Wine (1964) reports that he and other researchers have shown that if all assumptions of the two-sample _t_ test hold, the rank-sum test is valid and that the power of efficiency of the rank-sum test relative to the two-sample _t_ test is 0.95. Thus in order to provide the same power, approximately five percent more observations are required for the rank-sum test than for the _t_ test. However, for nonnormal populations the rank-sum test may be more simple than the two-sample _t_ test--especially as the two-sample _t_ test is inappropriately used when the population is nonnormal.

There are several possible advantages as well as disadvantages to using the rank-sum test. As is the case in the problem being analyzed, one advantage is that knowledge of the distribution of the population, population mean, and population variance is not necessary for the calculation of the test statistic. Secondly, the test statistic is more easily computed than alternative parametric tests. However, in this particular analysis the rank-sum test provides the decision rule needed for the test of hypothesis (Wine, 1964).

Critical Values of Wilcoxon Test Statistic

Wine (1964) has tabled critical values of S_N for both the 0.05 and 0.01 significance levels. If $n_1 \neq n_2$, n_1 as designated in the table heading is taken to be the subsample with the smallest number of observations; n_2 as designated by the table heading, is taken to be the subsample with the largest number of observations.

Given the level of significance, and n_1 and n_2, the tabled critical value is that which is common to both the n_1 column and n_2 row.

If, for a given significance level, S_N (calculated) < S_N (tabled critical value), the hypothesis is rejected.

Statement of Hypotheses

Formally the null and alternative hypotheses specified in the previous chapter may be specified as follows:

$H_o: \frac{1}{N} \sum_{k=1}^{N} W_k = 0,$ and the TC function not statistically different in location from the cost equation C'.

$H_A: \frac{1}{N} \sum_{k=1}^{N} W_k \neq 0,$ and the TC function is statistically different from the cost equation C' (two-tailed).

IV. PROCEDURES USED IN ACQUISITION OF DATA FOR HYPOTHESIS TEST

In Chapter III data needed for the test of the TC-C' hypothesis were specified. The Pacific Northwest beef feedlot industry was chosen as the economic sector from which a sample of firms was selected to provide data necessary for the hypothesis test.[8/] Presented in this chapter are sampling procedures; general characteristics of the feedlot firms; source, derivation, and use of $E(P)_k$ values; and the source of data and derivation of the cost equation.

Sampling Procedures

There were 21 beef feedlot firms selected as sample respondents for this analysis. These 21 respondents were selected because: (1) they were known to have historical records of sufficient detail from which cost of production data could be taken, (2) a preliminary estimate of their annual level of output was available from a previous survey, and (3) an indication of the types of production technology employed by these firms was available.

8/ In this analysis a beef feedlot is defined as a firm which feeds cattle to slaughter weight.

This writer interviewed the sample respondents during October, 1969. Questions asked of the respondents were framed in the context of their 1969 feeding year.[9] Several questions asked were to update information obtained by previous interviews concerning the 1967 feeding year for each of the 21 respondents.

General Characteristics of Sample Feedlot Firms

The sample of 21 feedlot firms was divided into two technology levels. One level includes those feedlot firms with "Incomplete" or "No" milling facilities. The second level includes feedlot firms with "Complete" milling facilities. Milling facility inventories were used as a proxy measure of technology to specify degree of completeness. Those firms with "Complete" milling facilities were more specialized firms; that is, either they were single enterprise firms or firms in which the feedlot was the primary enterprise.

Questions asked during the 1969 interviews required that the firms had produced or intended to feed beef to slaughter weights during the 1969 feeding year. Six firms feeding cattle during 1967 were not feeding cattle to slaughter weights in 1969.

9/ The specific questions asked are presented in the Feedlot Interview Sheet, Appendix A.

A seventh operator was unavailable for interview during 1969. A summary of the changes in each firm's operation between 1967 and 1969 is presented in Table 1.

Of the 21 feedlot firms in operation during 1967, eleven had "Complete" milling facilities. The ten other feedlot firms either had "No" milling facilities, or a minimal investment in milling facilities. Of those firms which had ceased their cattle feeding operations in 1969, none had a "Complete" feed mill.

Two of the six firms which had ceased their feedlot operations during the 1969 year discontinued their feeding activities permanently in favor of other enterprises. Another firm was attempting to sell its feedlot facilities; the feedlot operator indicated that he expected a greater return on his capital in a non-farm enterprise. Another feedlot operator consigned his cattle to another feedlot, as he felt that due to the location of the other feedlot, it could produce gain at a lower average cost than could be achieved through his feedlot. Another feedlot operator did not anticipate that the price he received for fed cattle would allow him to break even, knowing his costs of production and the price he would pay for feeder cattle.

Table 1. Summary of 1967 and 1969 Volumes of Production by Feedlot Firm, Reason for Change in Volume Level, and Level of Technology Employed.

Firm code	1967 total cwt. of gain	1969 total cwt. of gain	Reason for change in volume of production	Level of technology
1	153	206	Increased number fed	No mill
2	325	---	Consigning cattle to other lot	No mill
3	816	---	Discontinued enterprise	No mill
4	2, 014	1, 016	Adjusted operation to finishing feeding only	Mill
5	2, 064	1, 548	Selling a portion of cattle at lighter weights	Incomplete mill
6	2, 660	2, 837. 5	Increased number fed	Incomplete mill
7	2, 922	---	Discontinued enterprise	No mill
8	3, 776	---	Feeder-fed cattle price spread too wide	No mill
9	4, 960	---	Discontinued enterprise, leased out facilities	No mill
10	6, 570	5, 085. 5	Reduced gain per animal	Incomplete mill
11	8, 870	---	Feedlot is being sold	Incomplete mill
12	11, 012	10, 410	Reduced gain per animal	Mill
13	11, 705	14, 370	Increased number fed	Mill
14	12, 587	19, 650	Increased number fed	Mill
15	20, 600	20, 875	Increased number fed	Mill
16	20, 847	18, 490	Reduced gain	Mill
17	21, 047	15, 503	Reduced gain per animal	Mill
18	26, 623	20, 898	Reduced number fed	Mill
19	48, 000	(Not available for interview)		Mill
20	84, 768	96, 238	Greater gain per animal	Mill
21	146, 250	164, 450	Increased number fed	Mill

Derivation of E(P) Values

To estimate the total cost function for firms in a given technology level, it was necessary to determine E(P) for each firm within a particular technology level. As was stated in a previous chapter $E(P)_k = \phi'(Y_k)$. That is, the expected value of the distribution of expected prices was defined to be an estimate of the marginal cost of firm k producing at output level Y within a given technology level.

The interviews conducted to obtain $E(P)_k$ were completed in two stages. First, through the use of historical frequency distributions, the respondents were asked questions concerning their sales prices on cattle marketed over the ten-year period from 1959 to 1968. Once the respondent was familiar with the concept of price frequency distributions and the interviewing techniques, he was asked to characterize his price expectations for the most recent lot of cattle placed on feed. It was assumed that the respondent had no influence on the selling price of the last lot of cattle placed on feed. See Appendix A for a discussion of questions and responses which were used to perform an independent test of this assumption.

Use of Price Data in Construction of Historical Frequency Distributions of Fed Cattle Prices

Frequency distributions of fed cattle prices, which were used in the interviews, were constructed from monthly price data reported by the Livestock Division, Consumer and Marketing Service, U. S. Department of Agriculture, Portland, Oregon (U. S. D. A., L. D., 1959-1968). Data summarized were for a ten-year period of operation at the Portland market, starting January 1, 1959, and ending December 31, 1968.

The data are reported by two weight classes for choice grade steers, good grade steers, choice grade heifers, and good grade heifers. For each of the eight weight-grade classes of cattle, price frequency distributions were established as follows:

1. The domain in the monthly average prices over the ten-year period was determined.
2. One dollar price intervals were specified within the domain.
3. The frequency of monthly prices occurring within each price interval was calculated.
4. The empirical frequency distribution was plotted on a 8-1/2 x 11" card (refer to Figure 1, Appendix A, for an example).

5. In addition to the historical frequency distribution, hypothetical frequency distributions were constructed over the same domain of prices. Each of these six hypothetical frequency distributions was plotted on an 8-1/2 x 11" card.

Use of Historical Frequency Distributions in Obtaining E(P)

The grade-weight class of fed cattle most often sold by the feedlot operator was determined at the time of the interview. The operator was shown the seven cards picturing the historical frequency distributions for that particular grade-weight class of cattle. He was asked to rank these seven distributions by visual inspection, indicating first that plot which most closely approximated the distribution of prices he received for his cattle sales of that grade-weight class over the ten-year period, 1959-1968, then indicating the one least like his, and so forth.

There was no *a priori* reason for expecting the feedlot operator to identify any particular plot, as some feeders may sell continually above the market average in all months sales are made. Contrariwise, others might sell continually below the market average.

The purpose of the question was to acquaint the feedlot operator with price frequency distributions. No further use of the historical price distributions were made in the cost of production estimates.

Source and Use of Price Data in Construction of Frequency Distributions for Feedlot Operator's Next Sale

Daily price data for the September through December period of 1968 were summarized for each of the eight grade-weight classes of cattle sold through the Portland market and country markets within the state of Oregon (U. S. D. A., L. D., daily) to obtain the possible domain of prices from the date of interview to the possible time of sale.

It was found that choice grade fed cattle prices exhibited a four-dollar price domain during the September through December period of 1968. From inspection of data available, seven frequency distributions were constructed for choice grade fed cattle prices. These seven distributions are presented in Table 2.

It was found that good grade fed cattle prices exhibited a six-dollar price domain during the September through December period of 1968. From inspection of the data available seven frequency distributions were constructed for good grade fed cattle prices. These seven distributions are presented in Table 3. Each of the frequency distributions for the choice grade and good grade cattle was plotted on a card for use during the

Table 2. Seven Frequency Distributions of Choice Grade Fed Cattle Prices, Using a Four-dollar Price Domain and One-dollar Price Intervals.

Price	Distribution						
interval	1	2	3	4	5	6	7
	Frequency						
A	.25	.16	.15	.20	.35	.10	.10
B	.25	.34	.35	.30	.15	.20	.15
C	.25	.34	.30	.35	.15	.30	.20
D	.25	.16	.20	.15	.35	.40	.55

Table 3. Seven Frequency Distributions of Good Grade Fed Cattle Prices, Using a Six-dollar Price Domain and One-dollar Price Intervals.

Price	Distribution						
interval	1	2	3	4	5	6	7
	Frequency						
A	.167	.06	.10	.05	.25	.04	.02
B	.167	.10	.30	.10	.15	.07	.06
C	.167	.34	.25	.20	.10	.14	.08
D	.167	.34	.20	.25	.10	.20	.10
E	.166	.10	.10	.30	.15	.25	.14
F	.166	.06	.05	.10	.25	.30	.60

interview. As an example, frequency distribution 2 from Table 2 is shown in Figure 2, Appendix A. The probability of a particular price occurring is shown on the vertical axis but the horizontal axis was left unspecified. The feedlot operator was asked to designate a domain of prices and this was used during the interview along the price scale.

Use of Frequency Distributions in Obtaining E(P)

Each feedlot operator interviewed was asked (1) when he placed his most recent lot of cattle on feed, (2) the grade at which he intended to sell the cattle, (3) the length of time that he intended to feed the cattle, (4) the selling weight of the cattle, and (5) a four-dollar domain of prices within which the feedlot operator expected to receive a price for his fed cattle (a six-dollar domain of expected prices was obtained for those selling good grade cattle).

Given the price domain specified by the feedlot operator for the most recent lot of cattle placed on feed, these prices were assigned along the horizontal axis (price scale) of the seven frequency distributions for the grade-class of fed beef specified. Once the prices were assigned to these plots, the feedlot operator was asked to rank the frequency distributions; indicating first, the one which most closely approximated his expectations of the prices he would receive for the most recent lot of cattle placed on feed, then indicating the second most likely, and so forth.

The frequency distribution that he selected as most likely was used to calculate the $E(P)_k$ value which provided an estimate of the marginal cost. The calculation was performed as follows:

$$E(P)_k = \sum_{i=1}^{m} f(P_i) P_i, \qquad \text{where}$$

l is minimum price expected plus \$.50,

m is maximum price expected minus \$.50,

$f(P_i)$ is the frequency with which the i^{th} price interval occurs depending on which of seven frequency distributions were selected by the feedlot operator.

P_i is the midpoint of the price interval i.

Derivation of Cost Equation

An estimate of the total hundredweight of gain, Y_k, produced by each firm in 1969 was obtained. Given the estimate of annual output, information on the total quantities of variable inputs and the prices of inputs were obtained for the 1969 production period. Prices and quantities of variable inputs were assumed observed without error. It was also assumed that each firm is so small in terms of the total market for an input that it cannot affect the price it pays for an input. See Appendix A for a discussion of the questions and responses used to make an independent test of this assumption.

As each of the sample respondents had been interviewed prior to their 1969 production period, several questions were asked to update information obtained from their 1967 records. Changes in their feeding methods and machinery inventories since the 1967 production period were obtained. This information, in conjunction with information on this period, was used to calculate each firm's 1969 level of total fixed costs. Uniform calculation procedures were used to calculate the cost equation for each firm.

Summary

Procedures for obtaining $E(P)_k$ values, volume of output levels, and cost equation data for a sample of feedlot firms were outlined. The assumptions of competitive input prices and competitive product prices were assumed for purposes of this study. (See Appendix A for a discussion of these assumptions.)

Interviews conducted to obtain $E(P)_k$ values were completed in two stages. First, through the use of historical frequency distribution cards, the respondents were asked concerning their sales prices on cattle marketed over a ten-year period from 1959 through 1968. Second, the respondent was shown another set of seven frequency distribution cards to which he assigned a domain of prices, and then ranked the distributions indicating first the one which most closely approximated his expectations of the prices he would receive for the most recent lot of cattle placed on feed, the second most likely, and so forth. From the frequency distribution he selected as most likely, an $E(P)_k$ value was calculated to provide an estimate of marginal costs, $\phi'(Y_k)$.

An estimate of the total hundredweight of gain, Y_k, produced by each firm in 1969 was obtained. Given the estimate of annual output, quantities and prices for variable inputs used were obtained. Additional questions were asked to obtain each firm's level of fixed costs. Then uniform calculation procedures were used to derive each firm's cost equation.

Chapter V presents for firms in each technology level the construction of the TC function and C' equation and the calculation of V_k and W_k values. The statistical test of hypothesis is presented for each technology level.

V. RESULTS

In the two previous chapters the form of the data series needed to calculate the test statistic and the procedures used to obtain data from sample respondents were described. In this chapter, values of the required data obtained for firms within each technology level are presented and used to test the TC-C' hypothesis.

Discussed first is the construction of the TC function and C' equation for each technology level. Then V_k and W_k values for firms in each technology level are presented. Finally, the statistical tests of hypothesis are performed for each technology level.

Construction of TC Function and C' Equation for Each Technology Level

For each of the two levels of technology, the sample observations were assembled in ascending order of annual volume for the construction of the TC function and C' equation. Data needed for construction of the TC function and C' equation for firms at the "No Mill" or "Incomplete Mill" technology level are summarized in Table 4.

Table 4. Data Needed for the Construction of TC Function and C' Equation for Firms at the "No Mill" or "Incomplete Mill" Technology Level.

Firm code	Y_k	$C_k - b_k = \sum_{j=1}^{J} r_j X_j$	$E(P)_k = \hat{\phi}'(Y_k)$	b_k
1	206	$ 6,088	$ 27.00	$ 78
5	1,548	39,995	30.50	1,336
6	2,837.5	58,906	26.73	3,950
10	5,085.5	113,375	28.50	5,418
Total	9,677.0	---	---	---

The weighted annual fixed costs for firms in this technology level are the following:

$$\bar{b} = \frac{1}{9,677} [(206)(\$78) + (1,558)(\$1,336) + (2,837.5)(\$3,950) + (5,085.5)(\$5,418)].$$

$$\bar{b} \simeq \$4,222$$

The TC_k values for each successive volume level for firms in the "No Mill" or "Incomplete Mill" technology level are the following:

<u>Firm Code</u> 1:

$$TC_1 = \bar{b} + \phi'(Y_1)(Y_1)$$
$$= \$4,222 + (\$27.00)(206)$$
$$= \$9,784$$

Firm Code 5:

$$TC_2 = TC_1 + \phi'(Y_2)(Y_2 - Y_1)$$

$$= \$9,784 + (\$30.50)(1,548 - 206)$$

$$= \$50,715$$

Firm Code 6:

$$TC_3 = TC_2 + \phi'(Y_3)(Y_3 - Y_2)$$

$$= \$50,715 + (\$26.73)(2,837.5 - 1,548)$$

$$= \$85,178$$

Firm Code 10:

$$TC_4 = TC_3 + \phi'(Y_4)(Y_4 - Y_3)$$

$$= \$85,178 + (\$28.50)(5,085.5 - 2,837.5)$$

$$= \$149,246$$

The C'_k values for each successive volume level for firms in the "No Mill" or "Incomplete Mill" technology level are the following:

Firm Code 1:

$$C'_1 = \bar{b} + (C_1 - b_1)$$

$$= \$4,222 + \$6,088$$

$$= \$10,310$$

Firm Code 5:

$$C'_2 = \bar{b} + (C_2 - b_2)$$

$$= \$4,222 + \$39,995$$

$$= \$44,217$$

Firm Code 6:

$$C_3' = \bar{b} + (C_3 - b_3)$$

$$= \$4,222 + \$58,906$$

$$= \$63,128$$

Firm Code 10:

$$C_4' = \bar{b} + (C_4 - b_4)$$

$$= \$4,222 + \$113,375$$

$$= \$117,597$$

Data needed for construction of the TC function and C' equation for firms at the "Complete Mill" technology level are presented in Table 5.

The weighted annual fixed costs for firms in this technology level are the following:

$$\bar{b} = \frac{1}{381,900} [(1,016)(\$4,256) + (10,410)(\$20,270) + (14,370)(\$23,655) + (15,503)(\$15,348) + (18,490)(\$14,488) + (19,650)(\$17,799) + (20,875)(\$34,263) + (20,898)(\$20,112) + (96,238)(\$56,445) + (164,450)(\$80,337)]$$

$$\bar{b} \simeq \$54,347.$$

The TC_k values for each successive volume level for firms in the "Complete Mill" technology level are the following:

Table 5. Data Needed for the Construction of TC Function and C' Equation for firms at the "Complete Mill" Technology Level.

Firm code	Y_k	$C_k - b_k = \sum_{j=1}^{J} r_j X_j$	$E(P)_k = \phi'(Y_k)$	b_k
4	1, 016	$ 24, 194	$30. 00	$ 4, 256
12	10, 410	239, 659	28. 26	20, 270
13	14, 370	382, 639	27. 72	23, 655
17	15, 503	441, 449	27. 73	15, 348
16	18, 490	633, 079	31. 05	14, 488
14	19, 650	472, 971	26. 02	17, 799
15	20, 875	507, 178	27. 05	34, 263
18	20, 898	496, 217	30. 36	20, 112
20	96, 238	2, 408, 772	27. 88	56, 445
21	164, 450	3, 310, 743	26. 75	80, 337
Total	381, 900	---	---	---

Firm Code 4:

$$TC_1 = \bar{b} + \phi'(Y_1)\,(Y_1)$$

$$= \$54,347 + \$30.00\,(1,016)$$

$$= \$84,827$$

Firm Code 12:

$$TC_2 = TC_1 + \hat{\phi}'(Y_2)\,(Y_2 - Y_1)$$

$$= \$84,827 + \$28.26\,(10,410 - 1,016)$$

$$= \$350,301$$

Firm Code 13:

$$TC_3 = TC_2 + \hat{\phi}'(Y_3)(Y_3 - Y_2)$$
$$= \$350,301 + \$27.72\ (14,370 - 10,410)$$
$$= \$460,072$$

Firm Code 17:

$$TC_4 = TC_3 + \hat{\phi}'(Y_4)(Y_4 - Y_3)$$
$$= \$460,072 + \$27.73\ (15,503 - 14,370)$$
$$= \$491,490$$

Firm Code 16:

$$TC_5 = TC_4 + \hat{\phi}'(Y_5)(Y_5 - Y_4)$$
$$= \$491,490 + \$31.05\ (18,490 - 15,503)$$
$$= \$584,236$$

Firm Code 14:

$$TC_6 = TC_5 + \hat{\phi}'(Y_6)(Y_6 - Y_5)$$
$$= \$584,236 + \$26.02\ (19,650 - 18,490)$$
$$= \$607,459$$

Firm Code 15:

$$TC_7 = TC_6 + \hat{\phi}'(Y_7)(Y_7 - Y_6)$$
$$= \$607,459 + \$27.05\ (20,875 - 19,650)$$
$$= \$640,595$$

Firm Code 18:

$$TC_8 = TC_7 + \hat{\phi}'(Y_8)\,(Y_8 - Y_7)$$

$$= \$640,595 + \$30.36\,(20,898 - 20,875)$$

$$= \$641,293$$

Firm Code 20:

$$TC_9 = TC_8 + \hat{\phi}'(Y_9)\,(Y_9 - Y_8)$$

$$= \$641,293 + \$27.88\,(96,238 - 20,898)$$

$$= \$2,741,995$$

Firm Code 21:

$$TC_{10} = TC_9 + \hat{\phi}'(Y_{10})\,(Y_{10} - Y_9)$$

$$= \$2,741,995 + \$26.75\,(164,450 - 96,238)$$

$$= \$4,566,666$$

The C_k' values for each successive volume level for firms in the "Complete Mill" technology level are the following:

Firm Code 4:

$$C_1' = \bar{b} + (C_1 - b_1)$$

$$= \$54,347 + \$24,194$$

$$= \$78,541$$

Firm Code 12:

$$C_2' = \bar{b} + (C_2 - b_2)$$

$$= \$54,347 + \$239,659$$

$$= \$294,006$$

Firm Code 13:

$$C'_3 = \bar{b} + (C_3 - b_3)$$
$$= \$54,347 + \$382,639$$
$$= \$436,986$$

Firm Code 17:

$$C'_4 = \bar{b} + (C_4 - b_4)$$
$$= \$54,347 + \$441,449$$
$$= \$495,796$$

Firm Code 16:

$$C'_5 = \bar{b} + (C_5 - b_5)$$
$$= \$54,347 + \$633,079$$
$$= \$687,426$$

Firm Code 14:

$$C'_6 = \bar{b} + (C_6 - b_6)$$
$$= \$54,347 + \$472,971$$
$$= \$527,318$$

Firm Code 15:

$$C'_7 = \bar{b} + (C_7 - b_7)$$
$$= \$54,347 + \$507,178$$
$$= \$561,525$$

Firm Code 18:

$$C'_8 = \bar{b} + (C_8 - b_8)$$
$$= \$54,347 + \$496,217$$
$$= \$550,564$$

Firm Code 20:

$$C'_9 = \bar{b} + (C_9 - b_9)$$
$$= \$54,347 + \$2,408,772$$
$$\doteq \$2,463,119$$

Firm Code 21:

$$C'_{10} = \bar{b} + (C_{10} - b_{10})$$
$$= \$54,347 + \$3,310,743$$
$$= \$3,365,090$$

Calculation of V_k and W_k Values for Each Technology Level

For each firm V_k was defined to be the following:

$$V_k = TC_k - C'_k, \quad k = 1, 2, \ldots, N$$

N is the total number of observations in each technology level.

V_k values for those firms at the "No Mill" or "Incomplete Mill" technology level are the following:

Firm Code 1:

$$V_1 = TC_1 - C'_1$$
$$= \$9,784 - \$10,310$$
$$= \$ -526$$

Firm Code 5:

$$V_2 = TC_2 - C'_2$$
$$= \$50,715 - \$44,217$$
$$= \$6,498$$

Firm Code 6:

$$V_3 = TC_3 - C'_3$$
$$= \$85,178 - \$63,128$$
$$= \$22,050$$

Firm Code 10:

$$V_4 = TC_4 - C'_4$$
$$= \$149,246 - \$117,597$$
$$= \$31,649$$

W_k values are defined to be the following

$$W_1 = V_1$$
$$W_2 = V_2 - V_1$$
$$\vdots$$
$$W_k = V_k - V_{k-1}$$
$$\vdots$$
$$W_N = V_N - V_{N-1}$$

The W_k and V_k values for firms at the "No Mill" or "Incomplete Mill" technology level are presented in Table 6.

Table 6. V_k and W_k Values for Firms at the "No Mill" or "Incomplete Mill" Technology Level.

Firm code	Y_k	V_k values	W_k values
1	206	$ -526	$ -526
5	1,548	6,498	7,024
6	2,837.5	22,050	15,552
10	5,085.5	31,649	9,599

V_k values for those firms at the "Complete Mill" technology level are the following:

Firm Code 4:

$$V_1 = TC_1 - C'_1$$
$$= \$84,827 - \$78,541$$
$$= \$6,286$$

Firm Code 12:

$$V_2 = TC_2 - C'_2$$
$$= \$350,301 - \$294,006$$
$$= \$56,295$$

Firm Code 13:

$$V_3 = TC_3 - C'_3$$
$$= \$460,072 - \$436,986$$
$$= \$23,086$$

Firm Code 17:

$$V_4 = TC_4 - C'_4$$
$$= \$491,490 - \$495,796$$
$$= \$ -4,306$$

Firm Code 16:

$$V_5 = TC_5 - C'_5$$
$$= \$584,236 - \$687,426$$
$$= \$ -103,190$$

Firm Code 14:

$$V_6 = TC_6 - C'_6$$
$$= \$607,459 - \$527,318$$
$$= \$80,141$$

Firm Code 15:

$$V_7 = TC_7 - C'_7$$
$$= \$640,595 - \$561,525$$
$$= \$79,070$$

Firm Code 18:

$$V_8 = TC_8 - C'_8$$
$$= \$641,293 - \$550,564$$
$$= \$90,729$$

Firm Code 20:

$$V_9 = TC_9 = C'_9$$

$$= \$2,741,995 - \$2,463,119$$

$$= \$278,876$$

Firm Code 21:

$$V_{10} = TC_{10} - C'_{10}$$

$$= \$4,566,666 - \$3,365,090$$

$$= \$1,201,576$$

The W_k and V_k values for firms at the "Complete Mill" technology level are presented in Table 7.

The W_k values for those firms at the "No Mill" or "Incomplete Mill" technology level are plotted in Figure 2. W_k values for firms at the "Complete Mill" technology level are plotted in Figure 3.

If the W_k values plotted in Figure 2 and Figure 3 oscillated from positive to negative around $W_k=0$ there would be reason to expect that the null hypothesis would not be rejected. That is, the mean of the W_k values would be expected to not be significantly different from zero, given that the magnitudes of the oscillations above and below $W_k=0$ were similar.

In Figure 2, three of the four W_k values lie above $W_k=0$, each by a greater magnitude than the only negative W_k value. In Figure 3, for the first eight Y_k values, it can be seen that the

Table 7. V_k and W_k Values for Firms at the "Complete Mill" Technology Level.

Firm code	Y_k	V_k values	W_k values
4	1,016	$ 6,286	$ 6,286
12	10,410	56,295	50,009
13	14,370	23,086	-33,209
17	15,503	- 4,306	-27,392
16	18,490	-103,190	-98,884
14	19,650	80,141	183,331
15	20,875	79,070	- 1,071
18	20,898	90,729	11,659
20	96,238	278,876	188,147
21	164,450	1,201,576	922,700

associated W_k values lie both above and below the line by similar magnitudes. The ninth and tenth W_k values lie far above W_k=0. However, these extreme observations have only a small influence on the outcome of the statistical test.

After visual inspection of W_k values for both levels of technology, the statistical test of hypothesis was performed for both technology levels to determine if the mean of the W_k values was significantly different from zero.

Performance of Statistical Tests

W_k values for firms in each of the two technology levels were ranked in ascending order of their absolute values.

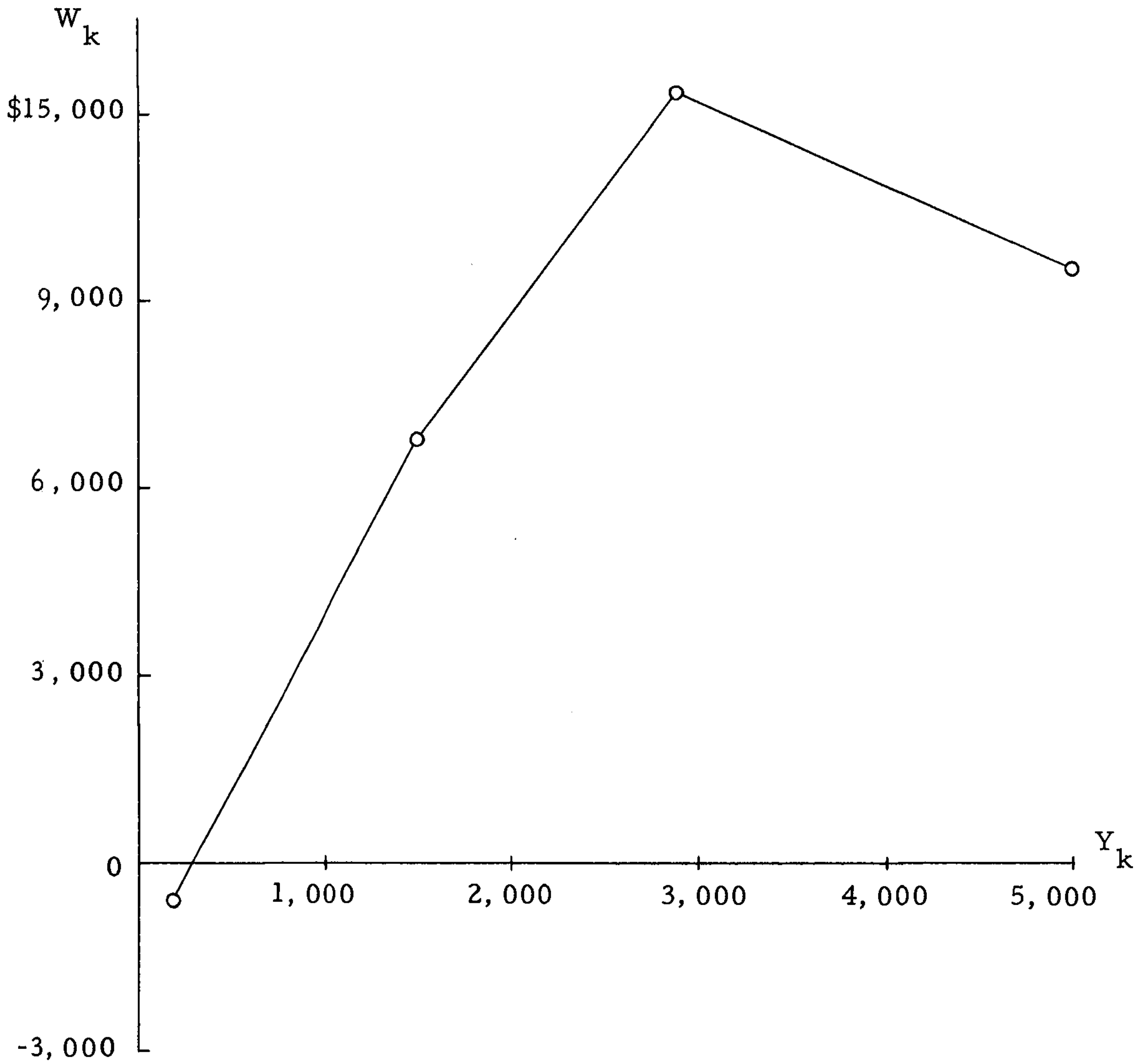

Figure 2. Plot of W_k values for firms at the "No Mill" or "Incomplete Mill" technology level.

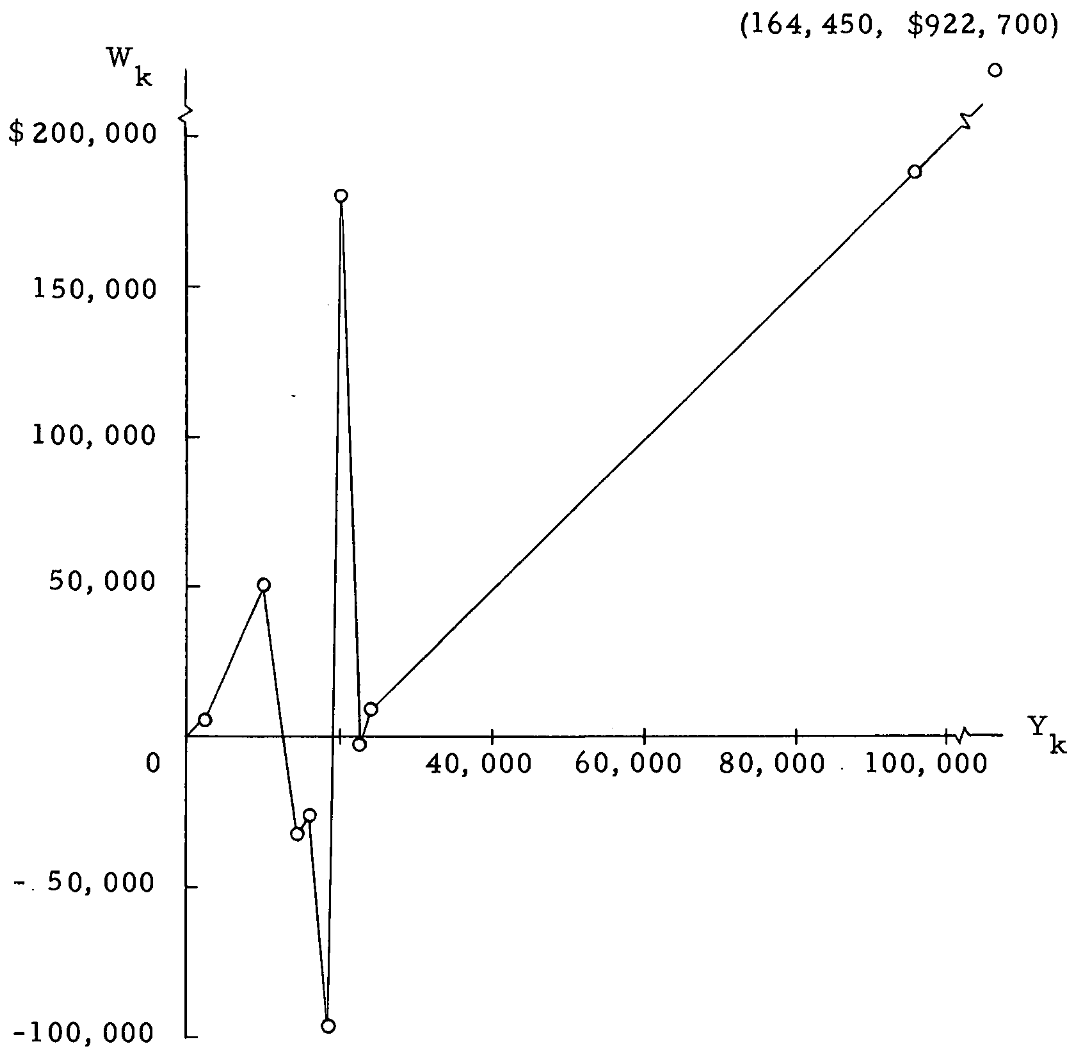

Figure 3. Plot of W_k values for firms at the "Complete Mill" technology level.

"No Mill" or "Incomplete Mill" Technology Level

The absolute values of W_k values for firms in this technology level were assigned the following ranks:

W_k values	Absolute values	Rank
$ -526	$ 526	1
7,024	7,024	2
9,599	9,599	3
15,552	15,552	4

The rank numbers were then separated into two subsamples, one subsample consisting of W_k values with a negative sign and the other with those of positive sign.

Negative sign: {1}

Positive sign: {2, 3, 4}

The rank numbers of the W_k values in each subsample were summed.

$$S^{+}_{n_1} = 9$$

$$S^{-}_{n_2} = 1$$

As $n_1 \neq n_2$, a further calculation was required to obtain the test statistic.

The subsample of ranks representing W_k values of negative sign was the smaller, $n_2 = 1$. S_s, the total of the ranks of this subsample, was 1. To compute S_L, the total of the ranks of the larger subsample, the following equation was used:

$$S_L = n_2 (n_1 + n_2 + 1) - S_s$$

$$S_L = 1 \ (1 + 3 + 1) - 1$$

$$S_L = 3$$

The S_N value to be compared with the critical value is

$$S_N = \text{minimum} \{ S_s, S_L \}$$

Therefore

$$S_N = \text{minimum} \{ 1, 3 \}$$

$$S_N = 1$$

Critical values of S_N at the 0.05 level of significance for two-sided tests where $n_1 + n_2 \geq 8$ are available in published tables, given that n_1 and n_2 are each ≥ 4. However, for $n_1 + n_2 < 8$, the critical values of S_N must be calculated (Wine, 1964). The calculated critical value of S_N for the two-tailed test at a significance level of $\alpha = 0.50$ was 1. (For details of the calculation of the S_N critical value, see Appendix B).

For rejection of the null hypothesis, it is required that the calculated value of S_N be less than the tabled critical value of S_N. For the "No Mill" or "Incomplete Mill" technology level the calculated value of S_N was equal in value to the critical value of S_N. Therefore, the test of hypothesis failed to reject the null hypothesis at the 0.50 level of significance.

"Complete Mill" Technology Level

The absolute values of W_k values for firms at this technology level were assigned the following ranks:

W_k values	Absolute values	Rank
\$- 1,071	\$ 1,071	1
6,286	6,286	2
11,659	11,659	3
-27,392	27,392	4
-33,209	33,209	5
50,009	50,009	6
-98,884	98,884	7
183,331	183,331	8
188,147	188,147	9
922,700	922,700	10

The rank numbers were separated into two subsamples, one subsample consisting of the W_k values with a negative sign and the other with those of positive sign:

Negative sign: {1, 4, 5, 7 }

Positive sign: {2, 3, 6, 8, 9, 10}

The rank numbers of the W_k values in each subsample were summed.

$$S^{+}_{n_1} = 38$$

$$S^{-}_{n_2} = 17$$

As $n_1 \neq n_2$, a further calculation was required to obtain the test statistic. The subsample of ranks representing W_k values of negative sign was smaller, $n_2 = 4$. The total of the ranks of this subsample, S_s, was 17. To compute S_L, the total of the ranks of the larger subsample, the following equation was used:

$$S_L = n_2 (n_1 + n_2 + 1) - S_s$$

$$S_L = 4 (6 + 4 + 1) - 17$$

$$S_L = 27$$

The S_N value to be compared with the critical value of S_N is

$$S_N = \text{minimum} \{ S_L, S_s \}.$$

Therefore, $S_N = \text{minimum} \{ 27, 17 \}$.

The critical value of S_N for the two-tailed test of hypothesis at $\alpha = .05$ significance level is $S_N = 12$ (Wine, 1964). As $S_N = 17$, the test fails to reject the hypothesis as S_N (calculated) > S_N (tabled critical value).

Summary

For each technology level the values of $\bar{b}$ were derived following procedures described in Chapter III. Then the TC function and C' equation values were calculated. Within each technology level the V_k and W_k values were calculated.

Once the absolute values of W_k were aligned in ascending order within each technology level they were assigned ranks, a rank of 1 designating the smallest absolute value, $N_I = 4$ designating the largest absolute value in technology I, and $N_{II} = 10$ designating the largest absolute value of W_k for technology II. Rank values for each technology levels were then sorted into two subsamples according to the sign of the associated W_k values. The test of hypothesis was then performed as outlined in Chapter III.

The test of hypothesis for each technology level failed to reject the null hypothesis that the empirical total cost function constructed (integrated) from output price expectation data taken from firms of like technology but different volume levels is identical to the cost equation of the same set of firms.

In the following chapter a discussion of the hypothesis test is presented. Also presented are its implications for economic theory, cost function estimation procedures, and for additional research.

VI. DISCUSSION OF RESULTS AND IMPLICATIONS

In the previous chapter the statistical tests of hypothesis failed to reject the TC-C' hypothesis. Through the estimating procedure employed, a total cost function was estimated which is consistent with that defined by economic theory. The estimating procedure developed uses a minimum of data relative to other estimating techniques to provide a total cost function which is not significantly different from the cost equation constructed from first principles.

Presented in this chapter are discussions of the risky nature of the hypothesis test, and implications for the use of the estimating procedure developed in estimating cost functions for other agricultural industries and facilitating additional research into other theoretical aspects of firm cost functions.

Risky Test of Hypothesis

The test of the hypothesis for firms in each of the two technology levels failed to reject the null hypothesis that the total cost function constructed from the integral of the $\phi'(Y)$ function for firms at a known technology level is identical to the empirical cost equation constructed from first principles. Therefore, it can be concluded that $E(P)_k$ is an estimate of $\phi'(Y_k)$.

To show that this is a risky test to perform, other possibilities of the relation of $E(P)_k$ to $\phi'(Y_k)$ can be considered. The expected value of the distribution of expected prices obtained from each respondent will be equal to, greater than, or less than the marginal cost of output at the volume level observed for a particular firm. Each of these possibilities is presented for a hypothetical firm in Figure 4.

At output levels Y_0 and Y_2 there exists an equality between $E(P)_k$ and $\phi'(Y)$. However, few firms would choose to operate at Y_0, as it is the profit minimizing level of output. If the firm were to produce at Y_1, $E(P)_k > \phi'(Y_1)$. If the firm were to operate at Y_3, $E(P)_k < \phi'(Y_3)$. A firm would operate at Y_1 if it were so constrained in variable capital that it could not achieve Y_2 output level. A firm would operate at Y_3 because of estimation error in its cost and/or $E(P)_k$ calculations.

At an output level of Y_1, $E(P)_k > \phi'(Y_1)$. However, as previously defined $E(P)_k$ is taken to be the estimate of $\hat{\phi}'(Y_k)$. Therefore, $\hat{\phi}'(Y_1) > \phi'(Y_1)$ by the magnitude $\overline{P_0 C_1}$. Calculation of TC_1 using $\hat{\phi}'(Y_1)$ will yield a TC_1 value which is greater than C_1'.

If a firm were observed operating at Y_3, TC_3 could be less

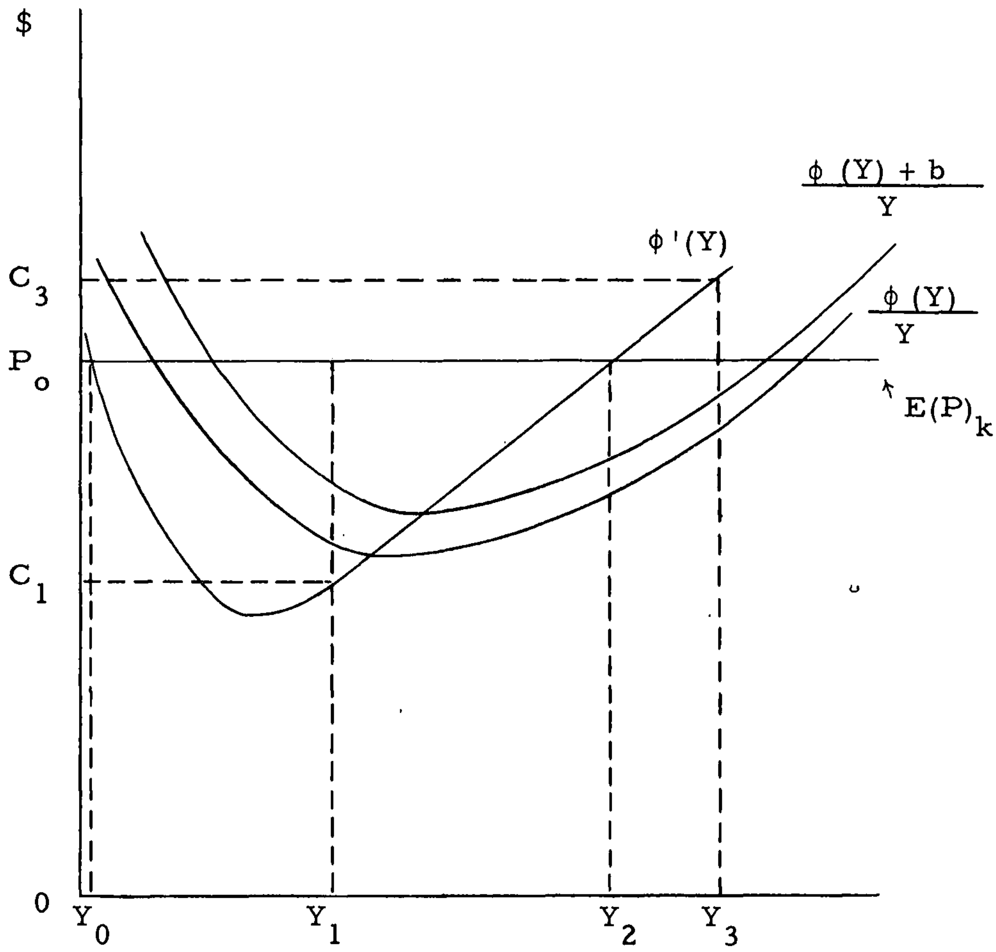

Figure 4. Short-run cost curves and $E(P)_k$ curve of a hypothetical firm.

than C_3'. At Y_3, $\hat{\phi}'(Y_3) < \phi'(Y_3)$ by a magnitude of $\overline{C_3 P_0}$. Calculation of TC_3 would yield $TC_3 < C_3'$, given that $\hat{\phi}'(Y_1) \geq \phi'(Y)$ and $\hat{\phi}'(Y_2) \geq \phi'(Y_2)$.

Therefore, the TC function expressed as an integral of the marginal cost function can only be specified under the assumption that each firm is observed where $E(P)_k = \phi'(Y)$ which for the firm in Figure 4 occurs at output Y_2. At output level Y_2, $\hat{\phi}'(Y_2) = E(P)_2 = \phi'(Y_2)$.

To demonstrate that the statistical test of hypothesis would not have had to be performed if another assumption had been made, consider the application of the statistical test under the assumption that $E(P)_k > \phi'(Y_k)$ for k=1, 2, . . ., N. (That is, the inequality exists for all observations in a particular technology level.)

Under the assumption of $E(P)_k > \phi'(Y_k)$ for all k, it can be shown that the statistical test used in this thesis will reject the hypothesis that the TC function integrated from the $\phi'(Y)$ is equal to the cost equation for a given set of firms. To show this, five hypothetical firms where $E(P)_k > \phi'(Y_k)$ will be used. Suppose the firms were observed at increasing levels of output and let Y_1, Y_2, Y_3, Y_4, and Y_5 represent equal increments of output.

The following expresses the TC function values for each firm, assuming $\bar{b} = 0$.

$$TC_1 = E(P)_1 Y_1$$

$$TC_2 = E(P)_1 Y_1 + E(P)_2 Y_2$$

$$TC_3 = E(P)_1 Y_1 + E(P)_2 Y_2 + E(P)_3 Y_3$$

$$TC_4 = E(P)_1 Y_1 + E(P)_2 Y_2 + E(P)_3 Y_3 + E(P)_4 Y_4$$

$$TC_5 = E(P)_1 Y_1 + E(P)_2 Y_2 + E(P)_3 Y_3 + E(P)_4 Y_4 + E(P)_5 Y_5$$

Under the same assumption that $\bar{b} = 0$, the C' equation values are expressed by:

$$C'_1 = \phi'(Y_1) Y_1$$

$$C'_2 = \phi'(Y_1) Y_1 + \phi'(Y_2) Y_2$$

$$C'_3 = \phi'(Y_1) Y_1 + \phi'(Y_2) Y_2 + \phi'(Y_3) Y_3$$

$$C'_4 = \phi'(Y_1) Y_1 + \phi'(Y_2) Y_2 + \phi'(Y_3) Y_3 + \phi'(Y_4) Y_4$$

$$C'_5 = \phi'(Y_1) Y_1 + \phi'(Y_2) Y_2 + \phi'(Y_3) Y_3 + \phi'(Y_4) Y_4 + \phi'(Y_5) Y_5$$

The successive V_k values are defined in general to be $TC_k - C'_k$. For these five firms they can be expressed as follows:

$$V_1 = E(P)_1 Y_1 - \phi'(Y_1) Y_1$$

$$V_2 = \sum_{k=1}^{2} E(P)_k Y_k - \sum_{k=1}^{2} \phi'(Y_k) Y_k$$

$$\vdots$$

$$V_5 = \sum_{k=1}^{5} E(P)_k Y_k - \sum_{k=1}^{5} \phi'(Y_k) Y_k$$

All value of W_k, except $W_1 = V_1$, are defined as the difference in successive V_k values, that is $W_k = V_k - V_{k-1}$, k=2, 3, 4, 5.

The text statistic is calculated from the rank values attached to the absolute values of the W_k values. If S_N calculated is less than the critical value, then the hypothesis is rejected. To prevent the calculated value of S_N from being equal to zero and hence assuring rejection requires that at least one value of W_k be negative. Therefore since

$$W_1 = V_1 > 0 \quad \text{assume that}$$

$$W_2 = V_2 - V_1 < 0.$$

Under the assumption of $E(P)_k > \phi'(Y_k)$ for all k,

$$W_1 = V_1 = E(P)_1 Y_1 - \phi'(Y_1) Y_1$$

$$W_2 = V_2 - V_1 = E(P)_1 Y_1 + E(P)_2 Y_2 - \phi'(Y_1) Y_1$$

$$- \phi'(Y_2) Y_2 - E(P_1) Y_1 + \phi'(Y_1) Y_1$$

The value of W_2 expressed in terms of the V_k components reduces to

$W_2 = E(P)_2 Y_2 - \phi'(Y_2) Y_2$. Factoring out Y_2, the expression becomes

$$W_2 = Y_2 [E(P)_2 - \phi'(Y_2)].$$

For W_2 to be negative would require $\phi'(Y_2) > E(P)_2$ which contradicts the previous assumption that $E(P)_k > \phi'(Y_k)$ for all k. Hence $W_2 > 0$.

Now suppose that $W_2 > 0$, $W_3 > 0$, $W_4 > 0$ and $W_5 < 0$.

For W_5 to be less than zero, the following conditions would have to exist:

$$W_5 = V_5 - V_4 < 0$$

$$\begin{aligned} W_5 = {} & E(P)_1 Y_1 + E(P)_2 Y_2 + E(P)_3 Y_3 + E(P)_4 Y_4 \\ & + E(P)_5 Y_5 - \phi'(Y_1) Y_1 - \phi'(Y_2) Y_2 - \phi'(Y_3) Y_3 \\ & - \phi'(Y_4) Y_4 - \phi'(Y_5) Y_5 - E(P)_1 Y_1 - E(P)_2 Y_2 \\ & - E(P)_3 Y_3 - E(P)_4 Y_4 + \phi'(Y_1) Y_1 + \phi'(Y_2) Y_2 \\ & + \phi'(Y_3) Y_3 + \phi'(Y_4) Y_4 \end{aligned}$$

The value of W_5 expressed in terms of V_k components reduced to

$W_5 = E(P)_5 Y_5 - \phi'(Y_5) Y_5$. Factoring out Y_5, the expression becomes

$$W_5 = Y_5 [E(P)_5 - \phi'(Y_5)].$$

For W_5 to be negative would require $\phi'(Y_5) > E(P)_5$, which contradicts the assumption that $E(P)_k > \phi'(Y)$ for all k. Hence W_5 0.

Thus, it has been shown that to get any reversal in the sign of W_k as k increases would require that $E(P)_k < \phi'(Y_k)$ for some k, which is contrary to the assumption that $E(P)_k > \phi'(Y_k)$. A similar situation would arise if the assumption is made that $E(P)_k < \phi'(Y_k)$ for all k. Therefore, any other assumption than $E(P)_k = \phi'(Y_k)$ for all k causes the statistical test to reject the TC-C' hypothesis.

Under the assumption that $E(P)_k = \phi'(Y_k)$ the error in estimating TC from the integral of $E(P)_k$ provides the alternations in the sign of W_k. The test statistic provides the means for testing the significance of the errors. Thus, given that the test does not reject the hypothesis it says that TC estimated by integrating $E(P)_k = \phi'(Y_k)$ is a "good" fit in the statistical sense to C'.

Implications

With the particular sample used, the conclusions presented in this chapter are not contradicted by the data obtained. Several implications about the nature and future use of the estimating procedure are presented.

Use of Methodology

The integration of the marginal cost function to obtain the total cost function for a group of firms provides an estimate of the total cost function which is consistent with that defined in economic theory. Previous studies which used regression procedures gave a biased estimate of the cost function. Regression procedures gave a best fit to a scatter of points but denies the definition of the cost function given by economic theory.

The discrete summation procedure used in this study to obtain the total cost function is also more efficient than previously used procedures in that empirical observations need be made only of the Y_k and $E(P)_k$ values rather than the complicated task of collecting data on input levels and input prices. Thus, cross-sectional data, easily obtained can be used to make rapid calculations of the total cost functions for a group of firms. It would also be possible to use a time series of E(P) values for one firm, assuming no change in technology, and obtain an estimate of one firm's cost function by the same procedure.

It has been demonstrated how the methodology applies to firms operating under two different technologies but within the same industry. The beef feeding industry was used as the testing ground. Previous attempts at estimating cost functions for this industry have been wrought with difficulty. Therefore, it should not be exceeding the bounds of reality to conclude that the procedure developed here should be applicable to several other agricultural industries.

Further Research

The estimating procedure developed is readily adaptable to several other agricultural industries comprised of single enterprise firms. With some modification of the procedure a means for estimating firm cost functions for an agricultural industry comprised of firms which produce outputs through joint production processes or for multiple enterprise firms could be developed. Traditional methods of enterprise accounting violate economic theory in attempting to estimate cost functions for a single product (or enterprise) produced in a multiple product firm. These methods have not provided a means for obtaining a meaningful or useful joint product cost function. It is not meaningful from the standpoint of economic theory and is useless for decision making. Research should be initiated using the basic methodology of this study to develop a procedure for combining the marginal cost estimates, once they have been obtained for each product produced, into a total cost function. The marginal cost functions developed in this study by themselves without further modification provide the information to decide on the level of output for a single product firm, and given the level of one product provide the decision information for the level of output of the other product for a multiple enterprise firm. The total cost function for a group of joint product firms would provide the marginal cost information for the profit maximization product mix decision.

In addition to the possible use in firm management decision making, the methodology should have broader application to such empirical problems as economies of size. With an estimating procedure yielding an estimate of the cost function that is consistent with the fundamentals of economic theory, the issue of economies of size in agricultural production can be readdressed. Procedures for tests of hypotheses like those developed here should be researched and tested.

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APPENDICES

APPENDIX A

FEEDLOT INTERVIEW SHEET

NAME OF OPERATOR ______________________________

A. Would you please provide the following information on the most recent lot of cattle you placed on feed?

1. When did you place your most recent lot of cattle on feed?

   Date ______________________

2. How many head were placed on feed?

   No. of head ________________

3. Were the feeders purchased all steers or heifers -- or was it a mixed lot?

   No. of steers ______________

   No. of heifers _____________

4. What was the average purchase weight of feeders?

   Average purchase weight (steers) ________________

   Average purchase weight (heifers) ________________

5. What was the average price per hundredweight paid for these feeders?

   Average purchase price (steers) ________________

   Average purchase price (heifers) ________________

6. How many days do you plan to have these cattle on feed?

   Days on feed (steers) __________

   Days on feed (heifers) __________

7. At what average weight do you plan to sell these cattle?

Average sale weight (steers) ______________

Average sale weight (heifers) ______________

8. What grade do you expect your fed cattle to reach?

Grade (steers) ______________

Grade (heifers) ______________

B. (Use the cards of expected prices to determine $E(P)_k$ for the particular lot of cattle.)

Historical Price Frequency Distributions

These are graphs based on the prices received by feedlot operators selling through Portland, Oregon, during the ten-year period, 1958-1968.

Take for example this graph (use #1, 700-900 lb. Choice heifers). It shows that on the average about 8.4% of the prices received were in the \$20.00-\$21.00 interval, 7.4% of the prices were in the \$21.00-22.00 interval, 11.2% of the prices were in the \$22.00-\$23.00 interval, 28% were in the \$23.00-\$24.00 interval, 15% in the \$24.00-\$25.00 interval, 19.8% in the \$25.00-\$26.00 interval, 9.3% in the \$26.00-\$27.00 interval, and 0.9% in the \$27.00-\$28.00 interval.

From your knowledge of the market and your cattle sales during this period, would you rank these seven graphs, starting with the one which most closely approximates what you recall about cattle prices over this ten-year period?

Heifers ______________

Steers ______________

Quality grade ______________

Weight class ______________

Ranking ____ ____ ____ ____ ____ ____ ____
1 2 3 4 5 6 7

Would you please explain your ranking?

____

____

Now I would like to ask you the following on the most recent lot of cattle you have placed on feed?

Good Cattle Price Expectation Frequency Distributions

Good cattle prices during the September-December period of each year tend to exhibit about a $6.00 "range".

For example, good steer prices in the Portland market during 1968 varied from $21.01-$27.00 during the September-December period. During the same period good heifer prices varied from $20.00-$26.00.

Would you give a $6.00 "range" of the prices you might receive for the most recent lot of cattle you placed on feed?

I have placed the set of prices you gave me on seven different graphs similar to those we worked with for prices over the last ten years.

From your knowledge of the market conditions, would you rank these graphs, starting with the one which most closely approximates your expectations of the prices you might receive for the most recent lot of cattle placed on feed?

"Range" of prices ________________

Heifers ____________

Steers ____________

Weight class ____________

Ranking	____	____	____	____	____	____	____
	1	2	3	4	5	6	7

Would you please explain your ranking?

____

____

Now I would like to ask you the following on the most recent lot of cattle you placed on feed:

Choice Cattle Price Expectation Frequency Distributions

Choice cattle prices during the September-December period of each year tend to exhibit about a $4.00 "range".

For example, choice steer prices in the Portland market during 1968 varied from $25.00 to $29.00 during the September-December period. During the same period choice heifer prices varied from $24.00-$28.00.

Would you give a $4.00 "range" of the prices you might receive fro the most recent lot of cattle you placed on feed?

I have placed the set of prices you gave me on seven different graphs similar to those we worked with for prices received over the last ten years.

From your knowledge of the market conditions, would you rank these graphs, starting with the one which most closely approximates your expectations of the prices you might receive for the most recent lot of cattle placed on feed?

"Range" of prices ______________

Heifers ____________

Steers ____________

Weight class ______________

Ranking ____ ____ ____ ____ ____ ____ ____
1 2 3 4 5 6 7

Would you please explain your ranking?

____

____

The price "range" you selected as most likely was $__________.

Suppose you had the following choices:

(1) You can pick a ball from a box with 50% red and 50% black balls.

If you pick a black ball you will win the value of the lot of finished cattle today.

(2) You can wait for the time of sale and receive a price from the $__________ interval for the same lot of cattle.

Which is your choice? Choose from box ____________

Wait for sale of cattle ________

[If the operator chooses from the box, this implies a probability of price interval < .5. Repeat the question, lowering the number of black balls.]

Suppose you had the following choice:

(1) You can pick a ball from a box with ____% red and ____% black balls.

If you pick a black ball you will win the value of the lot of finished cattle today.

(2) You can wait for the time of sale and receive a price from the $_________ interval for the same lot of cattle.

Which is your choice? Choose from box ______________

Wait for sale of cattle __________

If the question is continuing -

(1) How many black balls would there have to be in the box before you would choose to wait for the sale of your cattle at a price in the $_________ interval?

Number of black balls __________

or

(2) What do you think is the probability of receiving a price in the $__________ interval?

Probability __________________

C. Would you please give me the following information on your ration on a per animal basis?

Days fed	Ration ingredient	Amount fed/day	Price/unit
________	________________	________________	__________
________	________________	________________	__________

(If you do not feed the same ration the entire feeding period, denote how many days each ration ingredient is fed.)

D. Could you please provide the following information on your 1969 cattle feeding program?

1. How many cattle will you feed during the 1969 feeding year?

   Number of steers ______________

   Number of heifers ______________

2. What will be the average purchase weight of the feeder cattle?

   Average purchase weight of steers

   ____

   Average purchase weight of heifers

   ____

3. What will be the average price per hundredweight that you will pay for feeders?

   Average price of steers __________

   Average price of heifers __________

4. What will be the average sale weight of your fed cattle:

   Average sale weight of steers

   ____

   Average sale weight of heifers

   ____

5. Will the total hundredweight of gain produced be approximately the following?

(a) (Number of steers) [(average sale weight) ______ (average purchase weight)] = ______ lbs. of gain

(b) (Number of heifers) [(average sale weight) ______ (average purchase weight)] = ______ lbs. of gain

Total feedlot gain (a+b) ____________

6. What is the annual interest rate charged on your operating capital?

Operating capital interest rate ______

7. If your ration ingredients and/or length of feeding period differ considerably during other seasons from those of your most recent lot placed on feed, would you please outline how they differ?

____

____

E. Would you please provide the following information on the changes in your feeding operation since our discussion of your October 1966-October 1967 feeding period?

1) Have you changed your method of feeding since the 1966-1967 period?

____

____

2) Have you added any additional feedlot facilities, milling facilities, or equipment since the 1966-1967 feeding period?

Description of item	New cost	When purchased
____________	____________	____________
____________	____________	____________

3. What is the current interest rate that is charged on your capital improvement loans?

Capital improvement interest rate ______________

F. Would you please provide the following information on feedlot utilization?

1) Could you be feeding more cattle at this time than you have in your lot?

Yes ________

No ________

2) If yes, would you give those reasons why you choose not to feed more?

____

____

3) If no, what factors in your current operation restrict the feeding of additional cattle?

____

____

G. Could you provide the following information on your buying and selling activities?

1) When you buy concentrates, does the volume purchased at any one time or yearly volume affect the price you pay?

____

____

2) Is the same true for your roughage purchases?

____

____

3) What factors are most important in selling your fed cattle (lot size, even flow, annual volume)?

____

____

Assumption of Competitive Output Price

Data were taken from the feedlot operators interviewed to determine if there were any selling economies associated with feedlot size. The question asked of each feedlot operator was "What factors are important in selling your fed cattle (lot size, even flow, annual volume)?" Response to the three factors suggested in the question were ranked, assigning "1" to the most important, "2" to second most important, and "3" to the third most important of these factors affecting selling price. Where two or more factors were felt of identical importance, the same rank was assigned to each. For those feedlot operators suggesting that none of the factors suggested had a measurable effect on selling price, "0" was assigned for the rank of each factor. The responses of all operators are summarized in Appendix Table 1.

Lot size was viewed as the least important factor in determining the selling price for fed cattle. Most operators suggested that as long as truck load lots of cattle were available for sale, no greater price would be received by having more than one truck load ready for shipment to slaughter at any one time. Several feeders who have less than a truck load available for sale at one time (less than 40 head) suggested that this was an

Appendix Table 1. Ranking of Factors Viewed by Feedlot Operators as Important in Determining the Selling Price of Fed Cattle.

Firm code	Lot size	Even flow	Annual volume
1	1	1	1
4	2	1	2
5	3	1	2
6	0	0	0
10	0	0	0
12	3	2	1
13	3	2	1
14	3	2	1
15	0	0	0
16	3	2	1
17	3	1	2
18	0	0	0
20	2	1	2
21	2	1	2

accommodation to some buyers--especially small local packing plants.

Annual volume and an even flow of cattle from a feedlot were about equally important factors in determining selling price and both more important than lot size in determining selling price. Several smaller volume operators stated that their production schedule is well known by the primary buyers, and that these buyers do not offer them less than the market price for their cattle. Their small annual volume is marketed unevenly throughout the year--but in a pattern that their primary buyers know.

There are few if any apparent internal selling economies related to size of feedlot, given that a firm is capable of selling truck load lots of cattle, and its annual volume of output and production schedule is known to primary buyers. Large volume producers may attract a larger group of effective buyers, but there is no indication that this increases the price paid to them for their fed cattle.

Assumption of Variable Input Prices

Questions were asked to determine if the feedlot operators could affect the price of two purchased inputs--concentrates and roughages.

If purchase price was decreased by the quantity purchased at one time or the total quantity purchased annually this was shown as (-) entry in Appendix Table 2. If the input referred to was produced by the feedlot firm, the entry was designated by (H) in Appendix Table 2. If there was no affect, this was designated by "0" entry in Appendix Table 2.

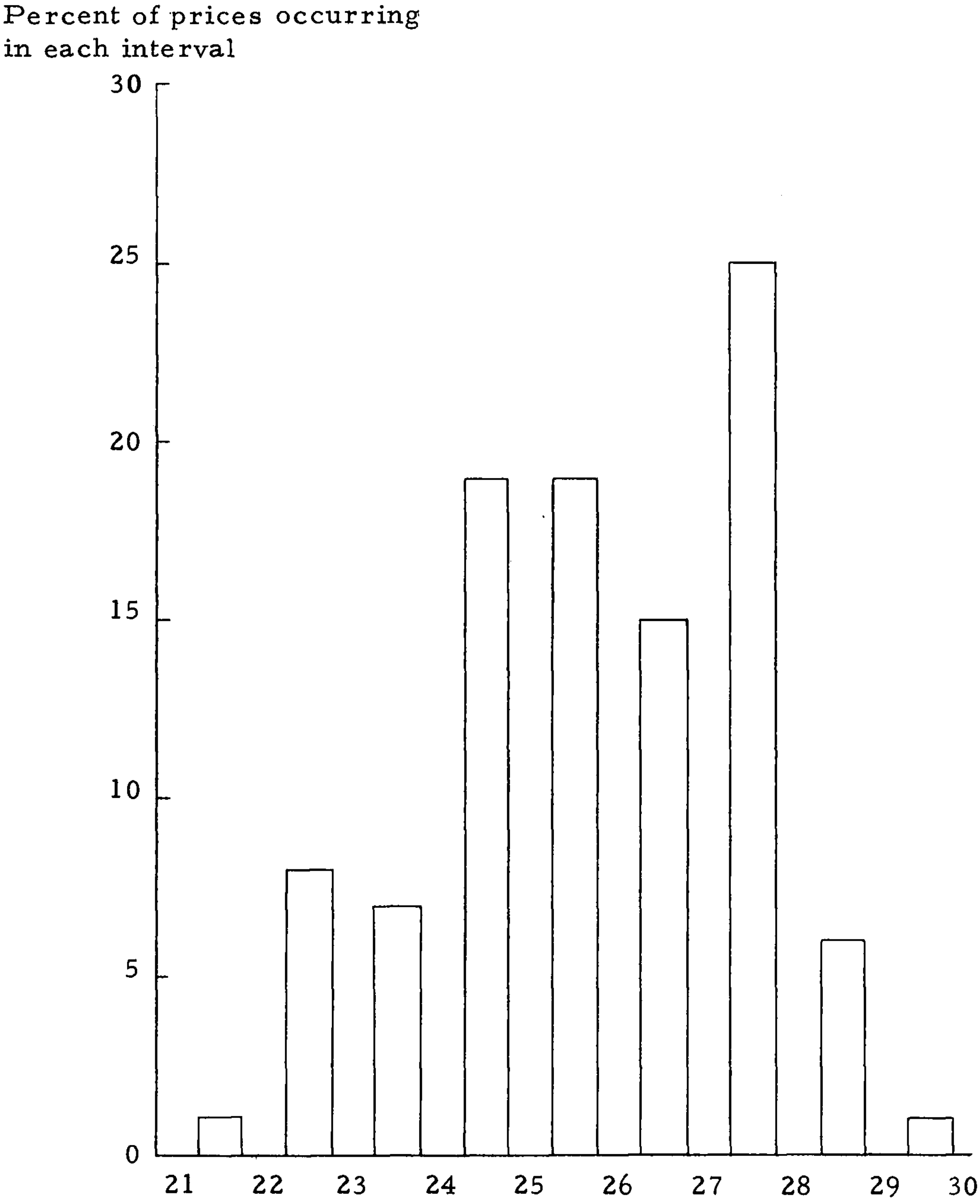
Few if any internal pecuniary buying economies were evident in the purchase of concentrates. On certain supplements, up to five percent price discounts were received by those purchasing in truck load lots. These were only reported by the smaller volume operators. Evidently, price discounting is discontinued on

Appendix Table 2. Affect of the Size of a Single Purchase and Volume Purchased Yearly on Input Prices of Concentrates and Roughages.

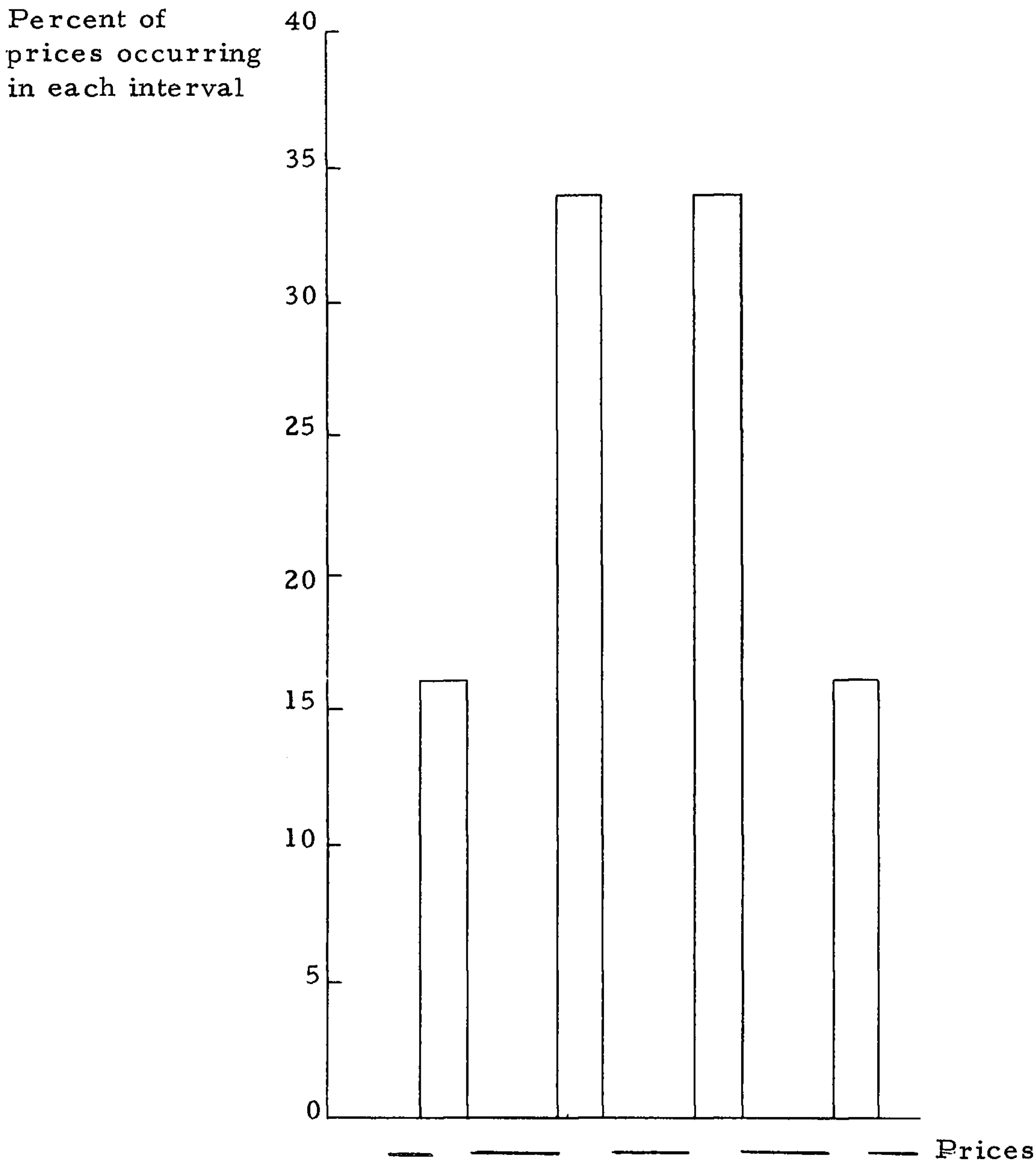
Firm code	Concentrates		Roughages	
	Single purchase	Yearly volume	Single purchase	Yearly volume
1	0	0	H	H
4	0	0	H	H
5	-	0	H	H
6	-	0	H	H
10	0	0	0	0
12	-	0	0	0
13	0	0	0	-
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0
20	0	0	0	0
21	0	0	0	0

supplements at greater than local delivery truck load quantities, i.e., 4-8 ton loads. Larger volume operators are evidently receiving the "delivery truck" discount, but no additional discounts for larger single delivery purchases.

One operator reported receiving a 15% price discount on the purchase of his annual requirements of low quality hay at harvest. No other internal pecuniary buying economies were achieved in roughage purchases.



Appendix Figure 1. 900-1100 Pound choice steer prices.



Appendix Figure 2. Choice cattle prices.

APPENDIX B

Calculation of Critical Value for "No Mill" or "Incomplete Mill" Technology Level.

If T denotes the total of the ranks in the smaller subsample (n_2 in this case), the smallest value of T is

$$1 + \ldots + n_2 = \frac{n_2 (n_2+1)}{2} .$$

In this case $n_2 = 1$. Therefore, the smallest value of T is as follows:

$$\text{Minimum } T = \frac{1\ (1+1)}{2} = 1.$$

The largest value that T can take is defined to be

$$\frac{n_2 (n_2 + 2n_1 + 1)}{2} .$$

Therefore, the largest value of T is as follows:

$$\text{Maximum } T = \frac{1\ (1+2.\ 3+1)}{2} = 4,$$

where $n_1 = 3$. Therefore, the possible values of T are 1, 2, 3, and 4.

It is then necessary to determine the number of ways in which a specified T can be obtained. There are $N_I = 4$ distinct ranks assigned to the W_k values, where $N_I = n_1 + n_2$. To determine the total number of combinations possible in selecting n_2

objects from N_I objects, the following expression was used:

$$\binom{N_I}{n_2} = \frac{N_I!}{n_2!(N_I - n_2)!}$$

From above $n_2 = 1$, $N_I = 4$. Therefore,

$$\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4.3.2.1}{1.3.2.1} = 4.$$

The frequencies of specific rank sums presented in Appendix Table 3 were determined by exhaustive listings of the sums of samples of size one.

Appendix Table 3. Sampling Distribution of Rank Totals T of Samples Size One in Combination with Samples of Size Four.

Rank total T	Frequency	Relative frequency	Cumulative relative frequency
1	1	.25	.25
2	1	.25	.50
3	1	.25	.75
4	1	.25	1.00

The relative frequencies may be considered to be probabilities, since each of the $\binom{N_I}{n_2}$ ways in which T is computed is considered to be equally likely.

Due to the symmetry property of the T distribution, the lower part of the distribution may be used either for a one-sided or two-sided test. In order to apply a two-sided test, a critical point T_0 for which $\alpha/2$ of the rank sums in the appropriate sampling distribution lie below is required.

The minimum level of significance for a two-tailed hypothesis test that can be constructed from Appendix Table 2 is $\alpha = .5$, as the minimum $\frac{\alpha}{2} = .25$.

From the previous calculations, $S_N = \text{minimum}\ \{S_L, S_s\}$ is analogous to the calculated value of T.

Therefore $T = S_N = \text{minimum}\ \{3, 1\}$

$T = 1$.

The critical value of T for the two-tailed test at a significancy level of $\alpha = 0.50$ is $T_0 = 1$. Referring to Appendix Table 3 the tabled T value at $\alpha/2 = .25$ is $T_0 = 1$.