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A combination of theoretical perspectives is used to create a rich description of student reasoning when facing a highly-geometric electricity and magnetism problem in an upper-division active-engagement physics classroom at Oregon State University. Geometric reasoning as students encounter problem situations ranging from familiar to novel is described using van Zee and Manogue's (2010) ethnography of communication. Bing's (2008) epistemic framing model is used to illuminate how students are framing what they are doing and whether or not they see the problem as geometric. Kuo, Hull, Gupta, and Elby's (2010) blending model and Krutetskii's (1976) model of harmonic reasoning are used to illuminate ways students show problem-solving expertise. Sayer and Wittmann's (2008) model is used to show how resource plasticity impacts students' geometric reasoning and the degree to which students accept incorrect results.

Geometric Reasoning in an Active-Engagement Upper-Division E&M Classroom

by
Leonard Thomas Cerny

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APPROVED:

Major Professor, representing Science Education

Dean of the College of Education

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Leonard Thomas Cerny, Author

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CHAPTER 1: INTRODUCTION

1.1 Goals and Research Questions

The main purpose of this dissertation is to develop a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. The research is divided into three distinct studies. Three different approaches to looking at one set of data are used to illuminate the thinking of upper-division physics students. The research questions for each of the three studies are:

1. What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?
2. How are students framing what they are doing? Do they see it as geometric?
3. In what ways are students using problem-solving expertise as they work through this problem?

We specifically consider junior-level physics students, in Oregon State University's Paradigms in Physics program, working in groups of three to solve for the magnetic vector potential of a spinning ring of charge. We use five different theoretical perspectives in depth when looking at the data but also tried five other theoretical models before selecting the five models used most extensively. This resulted in a secondary purpose for the research being to compare and contrast existing theoretical models and describe their usefulness in this context. The related research question is "What theoretical models are relevant and useful for considering student thinking in this case?"

1.2 Problem Statement

While there is extensive research on the thinking of introductory physics students, there is far less research addressing how upper-division students think. Upper-division students face problems that are much more complex than they faced in introductory courses and require them to use deeper and more sophisticated understanding of physical situations and the geometry involved. In the context of electromagnetism, most of our understanding of upper division students comes from a few studies involving written assessments (e.g. Kesonen, Asikainen & Hirvonen, 2011; Singh, 2006, Pepper, Chasteen, Pollock and Perkins, 2010) or problem-solving interviews (e.g. Bing & Redish, 2009; Sayre & Wittmann, 2008). This dissertation is significant because it is the first to look at the geometric reasoning that upper-division students are actually using when confronting a complex problem during class.

This dissertation is also significant because it is the most in-depth look to date at student thinking during active engagement in upper-division physics classrooms. While extensive research overwhelmingly shows active engagement produces better student understanding in introductory college physics courses (e.g. Hake, 1998), there are only a few universities studying the use of active engagement in their upper division courses. Pre-post assessments at the University of Colorado (CU) point to reformed upper-division instruction being more effective than traditional instruction (e.g. Pepper, Chasteen, Pollock and Perkins, 2010), but there has been little examination of what student thinking and engagement looks like *in situ* in these classrooms.

Another thing this dissertation accomplishes is evaluating the utility of several existing theoretic models for looking at student thinking in upper-division classrooms. The current status of upper-division physics education research (PER) is that there are several theoretical models that have had limited testing and frequently have only been used by one research group. Several theoretical models used to look at student thinking in lower-division courses have applicability to upper-division student thinking. However, many of these theoretical frameworks for looking at lower-division students are inadequate for considering student thinking when facing problems that are more complex than those in introductory courses (Bing & Redish, 2009; Manogue & Gire, 2009).

1.3 Organization of this Dissertation

1.3.1 Three Approaches

The research for this dissertation fits into three major portions involving three separate, but related studies of a single data set. The first study (Chapter 4) specifically looks at student geometric thinking as encounter problem situations ranging from very familiar to novel. The second study (Chapters 5 and 6) uses Bing and Redish's (2008) epistemic framing model to look at how students are framing the problem as they navigate through it. The third study (Chapter 7) looks at problem-solving expertise using three different theoretical perspectives.

1.3.2 Chapter 2: Literature Review

Chapter 2 reviews the literature relevant to this research. We focus on the different theoretical models and different approaches that are currently used to understand student thinking in upper-division electricity and magnetism courses. We compare and contrast these different theoretical frameworks.

1.3.3 Chapter 3: Methodology

Chapter 3 describes the Paradigms courses, the specific problem being worked by the students, the process for acquiring the video and the process of selecting the specific group problem-solving session used for this dissertation. In addition, because this dissertation uses an uncommon approach of gathering data first and then selecting a variety of theoretical frameworks to apply to the data, this chapter describes the process of selecting theoretical models and outlines the many theoretical models tested.

1.3.4 Chapter 4: Student Geometric Thinking

Chapter 4 addresses the dissertation's main purpose of providing a rich description of geometric reasoning by considering the question, "What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?" The two theoretical models used to approach the research are ethnography of communications (e.g. van Zee & Manogue, 2010) in combination with Sayre and Wittmann's (2008) model of resource plasticity.

In some cases student problem solving seems almost effortless. In other cases students are forced to use geometric concepts that they have used before, but never mastered. In yet other cases they face something entirely new. We compare student thinking in each of these cases. One of the interesting results that emerged from the data was how use of plastic resources impacts students' ability to successfully engage in sense-making.

1.3.5 Chapters 5 and 6: Epistemological Approaches

Chapters 5 & 6 address the dissertation's main purpose of providing a rich description of student reasoning when facing a highly-geometric problem by considering the questions, "How are students framing what they are doing?" and, "Do they see it as geometric?" Before students can use their geometric and physical reasoning, they first need to see that using this reasoning is part of the task at hand. If students view the task at hand to be one of calculation, then, regardless of their ability to use geometric and physical reasoning, they will not employ these resources. However, if students think that what they are supposed to be doing is connecting the physical situation to a symbolic representation, then they have the opportunity to activate the needed resources.

To understand how students are considering the problem, we use the epistemic framing model developed by Thomas Bing (Bing, 2008; Bing & Redish, 2008; Bing & Redish, 2009; Bing & Redish, 2012). We code all the transcripts using Bing's coding, which is divided into four framings. The four framings are; "calculation" which refers to symbolic manipulation; "mathematical coherence", which involves using analogous mathematical structures; "authority", which focuses on quoting or using an authoritative source; and "physical mapping", which involves taking the physical situation and turning into a symbolic representation.

We consider the overall question of how students are framing the task at hand. We also use the Paradigms data to examine the validity of several claims made by Bing. For example, Bing claims that sometimes groups of students all use a common framing during discussions and at other times will have "framing clashes," in which students are not only disagreeing about the physics and mathematics at hand, but also about what framing they use to consider it. Bing makes an additional claim about the efficiency of students using consistent framing versus being engaged in framing clashes. We find that some of Bing's claims are consistent with the Paradigms data while other claims are not entirely so.

At the end of Chapter 5, we look at the question, "Are students framing the task in ways the instructor envisioned?" We use Bing's model to analyze the alignment of student epistemological framing compared to instructor expectations.

Chapter 6 responds specifically to the sixth chapter of Bing's dissertation and a paper by Bing and Redish (2008). Bing claims that symbolic calculators can have a significant impact on student framing, leading students to focus on calculation, even when this is an inappropriate framing. Looking at students solve the ring problem yields a very different perspective. While not directly refuting any of Bing's carefully stated claims, the overall picture painted from this new data is very different than the picture painted from Bing's data.

1.3.6 Chapter 7: Models of Expertise

Chapter 7 addresses the dissertation's main purpose of providing a rich description of student reasoning by considering the question, "In what ways are students using problem-solving expertise as they work through this problem?" Once students recognize the problem as requiring physical understanding and realize the need for geometric thinking, there are then varying degrees to which students maintain the connection between the physical situation and the symbols they are using. Problem-solving expertise has been defined and described in many ways, but several of those perspectives include the idea that physical and geometric sense-making is an integral part of expertise. Making a physical or geometric connection once at the start of the problem is usually not enough. Thus, we can consider the degree to which students are demonstrating expertise by staying connected to the physical situation and the geometry of the problem as they navigate through it.

To understand student problem-solving expertise, we use Bing's epistemic framing model (Bing, 2008; Bing & Redish, 2012) along with Kuo, Hull, Gupta, and Elby's (2010) blending model and Krutetskii's (1976) model of harmonic reasoning. Each of the three models offers different insights into student expertise when solving the ring problem.

The dialogs of four different students from four different groups are used to illustrate different levels of expertise found during this problem solving session. We consider both what these models say about each student and what the application of each model indicates about the theoretical perspective.

1.3.7 Chapter 8: Comparing Theoretical Models

Chapter 8 addresses the secondary purpose for the research, which is to compare and contrast existing theoretical models and describe their usefulness in this context. It addresses the question, "What theoretical models are relevant and useful for considering student thinking in this case?" This chapter not only compares the five theoretical models used extensively in this dissertation, but also explores the utility of three additional models that show promise when considering upper-division work.

1.3.8: Chapter 9: Conclusions

This chapter summarizes the answers to the question "What can a combination of models tell us about student thinking while solving this specific problem?" In addition, there is a discussion of limitations of the research as well as implications and suggestions for future research.

CHAPTER 2: LITERATURE REVIEW

This chapter provides a review of the literature. The literature review is organized to match the overall organization of the dissertation, which contains three major studies that together create a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. The research questions for each of the three studies are:

1. What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?
2. How are students framing what they are doing? Do they see it as geometric?
3. In what ways are students using problem-solving expertise as they work through this problem?

The literature review is divided into three corresponding sections:

1. Section 2.1 – Student Geometric Understanding
2. Section 2.2 – Epistemological Models
3. Section 2.3 – Problem-Solving Expertise

2.1 Student Geometric Understanding

2.1.1 Overview of Geometric Understanding

Section 2.1 reviews the literature related to Chapter 4 and the research question, "What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?" This section specifically reviews different approaches used to look at student understanding of electromagnetism in upper-division courses. This includes student understanding of the physics content, student use of geometry, and student use of different resources. The focus is on four different approaches; testing for understanding, task analysis, resource plasticity, and ethnography of communications.

2.1.2 Testing for Understanding: Existing Upper-Division E&M Tests

One model that is common in physics education research (PER) literature is to create and administer some form of test. There are several examples of assessments that have been developed for upper-division students, including several for electricity and magnetism. In section 2.2.2 we consider some of the assumptions and limitations of this method. However, we first consider some of these tests and how they were developed.

The Colorado Upper-Division Electrostatics (CUE) 17-question diagnostic (Chasteen & Pollock, 2009) and Singh's (2006) Gauss's Law 30-question assessment were both developed using an multi-step process involving faculty input, student interviews and trying out questions on students in various ways. In addition to using the CUE test, Pollock (2009) also administered the Brief Electricity & Magnetism Assessment (BEMA) to upper-division students. BEMA was originally designed for and validated with introductory-level physics students, but was also found to be informative about the thinking of upper-division students. Another group, Kesonen, Asikainen & Hirvonen (2011) developed and administered a test on electricity and magnetism that involved seven tasks, including both closed and open-ended questions. This test was based on test questions previously used in university courses as well as questions found in physics textbooks at both the university and upper secondary level. These questions were then modified based on the input of two faculty members. In an alternate approach, Hinrichs (2010) single-handedly developed a single-question assessment of student understanding of spherical unit vectors.

Several of the written E&M assessments were accompanied by student interviews, which validated and elaborated upon the results. Hinrichs videotaped four students working on the pencil and paper test and also asked follow-up questions in which students explained their reasoning. Singh used 15 think-aloud interviews as part of her overall study. Wallace and Chasteen (2010) followed-up on CUE results with think-aloud interviews of six students specifically about Ampere's Law. Wallace and Chasteen's data were analyzed using coding developed in a multi-step process. Singh (2006) also used CUE data as a starting point and then probed student thinking about Gauss's Law using think-aloud interviews of four students. Data from the interviews were discussed and supported by outtakes from the transcripts.

In other areas of physics there have also been several assessments created. Loverude (2009) used student responses to exam questions and ungraded quizzes to evaluate student understanding of statistics in upper-division thermodynamics courses. Wittmann, Steinberg, & Redish (2002) studied student understanding of conductivity in the context of quantum mechanics using a combination of student interviews, conceptual surveys and exam questions. This is similar to the approach taken by Bao & Redish (2002) in their study of student understanding of probability as a prerequisite for understanding quantum mechanics.

These tests have been used for a variety of purposes. In addition to being a springboard for further research (e.g. Pepper, Chasteen, Pollock and Perkins, 2010), one significant application is evaluating instruction. For example, Chasteen and Pollock (2009) used data from the CUE assessments to compare reformed and traditional instruction and found that students prepared through reformed instruction outperformed students prepared through traditional instruction by a significant margin. In another example, Bao and Redish (2002) developed tutorials based on what they learned about their students. Yet another example is Loverude (2009) who, after giving his statistical assessment seven times, developed a tutorial called "counting states" that was designed to address student needs seen in the results from his assessment. These examples follow the tradition of lower-division tests such as the Force Concept Inventory (FCI), used to evaluate different instructional approaches (e.g. Hake, 1998). Data from the FCI helped reformed instruction gain respect and guided reform efforts.

2.1.3 Difficulties with "Student Difficulties": Concerns about the "Difficulties" Model

Written assessments and interviews provide a picture of the state of student knowledge. While these data tell us both about what students do know and what they do not know, it is interesting that the focus of most of the papers about these assessments is on "student difficulties". It appears that there are several underlying assumptions in the term "student difficulties" that the users of this term may not have considered. While analyzing and understanding the state of student knowledge is essential, describing student understanding in terms of "student difficulties" is problematic.

Here is a sampling of papers using the concept of student difficulties. Wallace and Chasteen's (2010) paper is titled, "Upper-division students' difficulties with Ampère's law". Pepper, Chasteen, Pollock and Perkins' (2010) paper is titled, "Our best juniors still struggle with Gauss's Law: Characterizing their difficulties". The first sentence of Singh's (2006) abstract is, "We investigate the difficulties that students in calculus-based introductory physics courses have with the concepts of symmetry, electric field, and electric flux which are important for applying Gauss's law." Kesonen, Asikainen & Hirvonen's (2011) conclusion has headings that start with phrases such as, "Insufficient presentation of…", "Lack of understanding of…" and "Difficulties in applying…" The first section after the introduction in Bao and Redish's paper on probability in quantum mechanics is called, "Student difficulties in understanding probability."

The concept of "difficulties" focuses on student deficits and has parallels with the misconception research that was a prevailing model in PER for introductory courses at the end of the last century. In a 2000 article in the *American Journal of Physics*, David Hammer noted at that time, that physics education research had a very heavy emphasis on student misconceptions and that a new perspective needed to be taken. He noted that misconception research was very important in raising awareness that student thinking was very different from what many instructors had assumed the students were thinking.

However, Hammer said it was time to move on and focus on the resources students do have and can be built upon, as opposed to solely focusing on what they do not have or what they have wrong.

There are strong parallels between the current state of upper-division PER and reformed instruction and the state 25 years ago of lower-division research and instruction. The initial focus on misconceptions eventually led to greater understanding of the nature of student thinking and also spurred reformed instruction. It could be argued that focusing on current student deficits is following the model of lower-division research and reform and thus may be a viable path for upper-division research and reform to gain greater credibility and acceptance.

However, it could be argued that simply repeating the process 25 years later in a different context is less than optimal. Let us further explore the phrase "student difficulties". The language we use carries theoretical baggage, whether or not it is recognized by the user. When authors use the phrase "student difficulties" it sometimes appears that the authors are simply looking for some words – any words – to describe that there are a substantial number of students who do not use valid physics and/or do not get the correct answer when attempting a problem. However, the phrase "student difficulties" carries certain specific assumptions. One is that the "difficulty", or problem, lies with the student. Another is that the challenges the student faces are possible but not easy.

For students who do not yet have the necessary resources to solve a problem, "difficult" may be an inappropriate word. To those students, with their current knowledge and abilities, the problem may, in fact, be impossible. If you give a calculus problem to third graders, they do not have "difficulties" in solving that problem. They simply cannot do it with their current knowledge and skills. If a person can lift 200 pounds, lifting a 195-pound rock unaided is difficult. Lifting a 2000-pound rock unaided is not. In several examples, researchers have given students unlimited time, and even hinted at various ways to reach correct solutions, but the students maintained an unsuccessful strategy. This is not evidence of a "difficulty", but is evidence that the student simply does not currently have the available resources to complete the task successfully. For those students in their current state, the task is impossible.

Consider cases in which researchers (e.g. Singh, 2006; Kesonen, Asikainen & Hirvonen, 2011) argue that distracters on multiple choice tests allow them to determine the incorrect ways in which students are thinking. The conclusion is often that this incorrect thinking prevents the student from succeeding. If a student is thinking incorrectly and this leads that student to believe that an incorrect answer is correct, this is an indication that using their current thinking, the correct solution is not possible for them to achieve. However, the term "difficulties" is often used when discussing these students.

There is also a separate concern. "Student difficulties" squarely places the "problem" with the student. Let us consider again the 2000 pound rock. When considering why a person cannot lift a 2000-pound rock, it is unproductive to conceive of the problem as being entirely a lack of strength in the person. If we wish to have the rock lifted, focusing on the weakness of the lifter is unlikely to yield the desired results. One more productive approach would be to consider the interface between the human and rock, and consider mediating it with something such as a front-end loader. Another potentially productive approach would be to break the rock into manageable pieces. These more productive approaches are analogous to providing additional cognitive or physical tools to learners or breaking the problem into pieces that they find manageable.

2.1.4 Task Difficulty: An Alternative Perspective

One alternative perspective considers the students "sufficient" and analyzes the resources they need in order to successfully tackle complex problems. Manogue, Browne, Dray & Edwards (2006) and Manogue & Gire (2009) look at two problems, one being Ampere's

Law, and the other being creating a power series for the electrostatic potential due to two point charges on an axis. It these cases the researchers do not ask, “What difficulties are the students having?” Instead they ask, “What makes this problem so hard?” Underlying their analysis is the assumption that the students have the abilities that are reasonable to expect students to have. These researchers see a mismatch between student capability and the task at hand, but they choose to focus on what makes the task difficult, not what shortcomings in the students cause them to flounder.

Let’s consider the following two paragraphs from page 348 of Manogue, Browne, Dray and Edwards’ “Why is Ampère’s law so hard? A look at middle-division physics.”

> *Line, surface, and volume densities are all different.* In our Ampère’s law problem, the current is distributed through the entire volume of a wire. In similar problems current might be considered to flow only along a surface or through an infinitely thin wire. Such problems are special cases of a volume current, where the distribution of the current in one or more dimensions is so constrained that these dimensions are idealized away. Students’ greatest level of classroom experience is with line currents, the most idealized case.
>
> Pedagogically, we can imagine two ways to handle the differences among these densities in the classroom. One is to define the different types of densities as different physical quantities, with different units, that require differing numbers of integrals to find the total value of the current. The other is to use these differences as an opportunity to exploit the sophisticated mathematics of theta and delta functions and explicitly discuss surface and line currents as limiting cases of volume currents. The first way causes the least disruption in the students’ attention to the central question of Ampère’s law. The second way seems to be the most satisfying to students who are trying to develop an understanding of current.

In this passage there is no mention of students struggling, having difficulties, or having deficiencies. Instead there is a note of the mismatch between student experience and the nature of the problem. Manogue, Browne, Dray & Edwards note that prior student experience with currents has been line currents and that they are now being asked to consider volume currents. It is not seen as a deficiency that students do not know how to do this thing. Instead, the question becomes a pedagogical one. Two choices are offered, including a discussion of the one that "seems to be the most satisfying to students who are trying to develop an understanding of current."

Compare the approach of Manogue, Browne, Dray & Edward's to the approach to Wallace and Chasteen's (2010), "Upper-division students' difficulties with Ampère's law." First consider how Wallace and Chasteen (p 1) refer to the paper of Manogue, Browne, Dray & Edwards.

> Manogue *et al.* list several difficulties they observed while teaching E&MI. They note that students may struggle to correctly determine the magnitude and direction of the magnetic field, choose an Ampèrian loop, extract $\boldsymbol{B}$ from inside $\boldsymbol{B}\cdot d$, use curvilinear coordinates, and understand current densities. Our study provides empirical support for some of these difficulties, as we discuss below.

Wallace and Chasteen continue in this same vein. On page 4, they say:

> What difficulties do students experience with Ampère's law in junior E&MI? Below, we list many of the problems we observed during the interviews. These problems can be split into two categories: difficulties connecting I_{enc} to the properties of the magnetic field, and not using information about the magnetic field.

The topics listed are the same as those covered by Manogue, Browne, Dray & Edwards, but the viewpoint is radically different. Wallace and Chasteen are focusing on the struggles and difficulties of the students. The phrase "student difficulties" does appear in Manogue, Browne, Dray & Edwards' paper, but it is not the focus. The focus is on the aspects of the problem that make it such a challenge. Manogue (personal communication, 2011) said of her students, "Of course the students don't know how to do that. Why should they?" Manogue also commented that before students learn how to do a problem it is impossible, but once they know how to do it, it is easy. She often sees education as the process of making the impossible easy.

It is interesting that neither Manogue, Browne, Dray & Edwards nor Wallace and Chasteen are explicit about their theoretical perspective in their papers. This absence of explicit viewpoints and frameworks allow the two groups to "talk past" each other without realizing the important differences in their perspectives. One is talking about "holes", the other about "barriers".

2.1.5 Implications of Where the Problem Lies

Ultimately, a thorough understanding of physics education requires there to be understanding of both student ability and the task at hand. Without knowledge of student ability, there would be no way to determine if a task were reasonable. In order to know whether a rock can be lifted, we need to know the ability of the lifter as well as the weight of the rock. Task analysis does not supplant an understanding of student ability. Furthermore, task analysis does not do many of things for which quantitative studies of student understanding are well-designed, such as comparison of different types of instruction or comparing student performance between universities.

However, it may be useful to consider the affective difference between envisioning the problem being with the task compared to being with the student. If a professor stands at a podium and sees her class as a sea of deficient students, she may take a different approach than if she looks across her class and sees highly-capable individuals who need certain new resources before they can reasonably be expected to accomplish the task at hand. The language we use in conducting and presenting our research can bias the perspectives of those consuming that research.

As mentioned before, the initial focus on deficits initially helped advance the state of both PER and reformed instruction in introductory physics. It is quite possible that a significant number of physics faculty believe that their students have extensive resources that, in reality, most of their students do not actually have. In this case, documenting the deficits may be a necessary first step to understanding student thinking and subsequently improving instruction. Furthermore, it may be initially more palatable for physics faculty to consider the unseen deficiencies in their students rather than consider the overwhelming nature of the tasks they are giving their students.

Exploring the mismatch between student capability and the task at hand, from multiple perspectives, may produce the clearest picture of student thinking and the instructional changes that are needed to create optimal learning. However, it could be beneficial to the PER community and the larger physics community to have these perspectives clarified so that the conversation can be more meaningfully advanced.

2.1.6 Resources: Hammer (2000)

A major alternative to the deficit model is resource theory. Hammer (2000) proposed the idea of “resources” using the language of computer programmers. “Resources” in computer programming refer to anything from a few lines of code to a large chunk of code that are taken and applied to a new situation. This chunk of code can be used unaltered and can be transferred as a single piece, without needing to think about any of its sub-pieces. Resource theory is based on DiSessa’s (1993) claim that knowledge comes in pieces and his description of phenomenological primitives or p-prims. While there is some indication that p-prims in their raw form are seen less commonly in upper division courses than in introductory courses, there are still “bits’ or “chunks” that students can grab from their minds and have available as resources for dealing with a problem.

Since Hammer’s 2000 paper, there has been a great deal of development in the thinking about resources, such that resource theory has become an extensively utilized theoretical framework. Epistemological resources were discussed in previous chapters, but there are a variety of other resources to consider, including mathematics resources and those involving physics content knowledge. In the next section we will consider only one particular extension of resource theory.

2.1.7 Sayre & Wittmann's Model of Resource Plasticity

One particularly interesting expansion of resource theory in the context of upper-division physics comes from Sayre and Wittmann (2008). In this paper they discuss many of the extensions and developments of resource theory and also make their own contributions to theoretical development. They then apply their expanded model to students in an upper-division classical mechanics course. Sayre and Wittmann draw from PER resource theory and also from mathematics education research's recognize/build-with/construct (RBC) model (e.g. Dreyfus & Tsamir, 2002 and Dreyfus & Tsamir, 2004), which is an extension of an overall Process/Object theory.

Sayre and Wittmann (2008) specifically consider the degree to which student resources are solid versus plastic. Solid resources tend to be older, readily available, easy to use, well consolidated and well connected to other resources. Plastic resources tend to require more effort to use, are open to re-evaluation and are often reliant on justifications from more solid resources. Sayre and Wittmann use interviews to look at the plasticity of Cartesian versus polar coordinates while students are solving for the time required for a pendulum to swing over a given arc.

In their example, one student, Derek, quickly reaches for polar coordinates, which are optimal for solving this particular problem. A second student, Wes, initially attempts to solve the problem with rectangular coordinates, even though they are ill-suited for this particular problem. Wes recognizes certain aspects of polar coordinates, but is not able to use them efficiently or effectively. Wes prefers to use rectangular coordinates because they have greater familiarity. Wes demonstrates that he has greater comfort and ease of use when it comes to rectangular coordinates. Wes needed to reconstruct concepts when he tried to use polar coordinates, but did not need to do so when using rectangular coordinates.

For Wes, polar coordinates were a more plastic resource than were Cartesian coordinates. Furthermore, for Derek polar coordinates were a more solid resource than they were for Wes. Sayre and Wittmann explore how having a resource be more solid affects the resources students choose to use. In this case choosing to use a solid resource over a plastic resource dominates the student's decision-making process.

This concept is related to ideas considered in other UDPER papers. For example, Singh, 2006 and Manogue, Browne, Dray & Edwards, 2006, although not explicitly referring to resources in this way, have considered the resources of students compared to those of a professional physicist. They provided evidence that a professional physicist may have a large resource as a single chunk whereas students must construct this larger piece from a variety of smaller resources. In several cases from the literature, as well as from looking at the ring problem (discussed in Chapter 4), it appears that upper-division students often need to draw on a large pool of resources, some solid, some plastic, just to create the resources that the professional takes for granted.

2.1.8 Ethnography of Communication

A very different approach than those mentioned previously is the one taken by Emily van Zee. She has used research rooted in ethnography of communication to analyze student understanding of physics at a variety of levels, including elementary pre-service teachers (e.g. van Zee, Hammer, Bell, Roy & Peter, 2005), high school students and their instructor (e.g. van Zee & Minstrell, 1997) and upper division students and their instructor (e.g. van Zee and Manogue, 2010).

The idea of ethnography of communication was introduced by Dell Hymes in 1972. He claims that the ways of speaking used by different cultures can be used to better understand those cultures, and reciprocally, that understanding a culture can be used to gain insights into the utterances and modes of speaking used by that culture. Gestures, written work, drawings and physical objects all contribute to this understanding. In addition, studying individuals is considered a legitimate endeavor in trying to understand the larger culture.

In this way, van Zee uses gestures, utterances and written work to gain insights into student thinking and how the students practice physics and physics learning. The ethnography of communication approach leads to very careful consideration of the words used and what those words really mean. She uses careful thought in considering each utterance. Careful analysis and thick description provide a rich understanding of specific instances of student thinking.

2.2 Epistemological Models

2.2.1 Overview of Epistemological Models

Section 2.2 reviews the literature related to Chapters 5 and 6. The related research questions are, "How are students framing what they are doing?" and, "Do they see it as geometric?"

For more than 2400 years humans have recorded their discussions about the nature of knowledge and knowing (e.g. Plato & Dyde, 1899). In 1854 James Ferrier coined the term "epistemology" to refer to the theory of knowing, which he contrasted to ontology, which he described as the study of being. In current physics education research (PER) the discussion of epistemology frequently focuses on how students understand knowledge and learning specifically in physics and how this is reflected in their approach to problem solving (e.g. Bing, 2008; Hammer & Elby, 2003; Scherr & Hammer, 2009; Tuminaro & Redish, 2007; van Zee, Hammer, Bell, Roy, & Peter, 2005).

There are numerous epistemological models present in PER, but only a few have been applied to upper-division physics. Four models that have been applied at the upper division level are "epistemic framing" (Redish & Hammer, 2009), "epistemic frames" (Bing, 2008), "modes of cognition" (Manogue & Gire, 2009) and "epistemological framing" (Scherr & Hammer, 2009). We also discuss a fifth model, "epistemic games" (Tuminaro & Redish, 2007), because it is related to several other models.

2.2.2 Epistemic Games: Tuminaro & Redish's Model

Although we have not identified examples in the literature of Tuminaro & Redish's (2007) epistemic games being applied to upper-division physics, their model for approaching epistemic thinking was used as a relevant reference point for subsequent work that Bing and others applied to upper-division students. Tuminaro & Redish describe six epistemic games; mapping meaning to mathematics, mapping mathematics to meaning, physical mechanism, pictorial analysis, recursive plug and chug, and transliteration to mathematics. These six games describe the thinking and processes used by introductory physics students as they solve a variety of problems. The different games highlight the different approaches students take and the impact of using each type of game.

Each of these games contain different "moves", which refer to specific activities undertaken during the epistemic games. Here are five moves outlined by Tuminaro & Reddish (p6) for "Mapping Meaning to Mathematics"

1. Develop story about physical situation
2. Translate quantities in physical story into mathematical entities
3. Relate mathematical entities in accordance with physical story
4. Manipulate symbols
5. Evaluate story

For comparison, here are the four moves for the game "Mapping Mathematics to Meaning"

1. Identify target concepts
2. Find an equation relating target to other concepts
3. Tell story using this relationship between concepts
4. Evaluate story

Tuminaro and Reddish produce numerous examples of students in an introductory physics course employing these different games while solving homework problems in groups. Bing (2008) considered using epistemic games for his analysis but concluded that they are inadequate for describing what students do in upper division courses. Bing uses as examples one student saying, "it's a similar process to Gauss's Law" and another student saying, "you can always take a derivative with respect to anything". Each statement is a shorthand for a much larger sequence of thinking or structure of ideas.

2.2.3 Epistemic Framing: Redish & Hammer's Model

An alternative to epistemic games is the "epistemic framing" of Redish and Hammer (2009), which includes six categories; "shopping for ideas", "restricting scope," "sense making", "choosing a foothold." "playing the implications game" and "seeking coherence/safety net." These six framings were designed to help discuss epistemic framing with non-physics majors in introductory physics courses. Here are explanations of these six ideas.

- "Shopping for ideas" occurs when students "browse" their minds for possibilities and consider whether those ideas are valid or whether other ideas should be sought.
- "Restricting the scope" occurs when students recognize and accept idealizations and simplifying assumptions which ignore certain aspects of the real world.

- "Sense making" occurs when students try to make what they are doing comprehensible to themselves, and possibly to others.
- "Choosing foothold ideas" refers to students picking ideas that they will accept and hold true, at least for the time being, and build from those ideas.
- "Playing the implications game" is described by Redish and Hammer (p. 632) as "Having chosen a foothold idea, we consider its implications; if X is true, what would that mean?"
- "Seeking coherence/safety net" involves having students realize that there should be coherence across different ways of understanding a problem. The mathematical, physical and real worlds should all align. Redish and Hammer also emphasize that students can misremember things and that cross checking against other ways of understanding is useful.

2.2.4 Modes of Cognition: Manogue & Gire's Model

Manogue and Gire (2009) attempted to use Redish and Hammer's six framings to perform a task analysis of an upper-division problem. The following problem was given to students: "For two charges $+Q$ and $-Q$ at $x = +D$ and $x = -D$ respectively, what is the fourth order approximation of the electrostatic potential, V, valid on the x-axis, for $|x| >> D$?" When coding upper-division problems, Manogue and Gire found that they needed additional categories in order to code several different aspects of the problem.

Manogue and Gire split the category of seeking coherence and employing a safety net. They also added an additional six categories. The categories are: applying learned mathematics, recognizing patterns, fleshing out formulas, applying a general principle to a specific case, translating representations/harmonic reasoning, and probing and refining intuitions.

Manogue and Gire used the term “modes of cognition” to collectively refer to their six categories combined with the six Redish and Hammer framings. One of Manogue’s goals for this task analysis was to show how many different things students have to do in order to execute a task that might seem straight forward to a professional physicist. The coding can show the variety of thinking in which students must engage in order to accomplish an overall task.

2.2.5 Behavioral Clusters: Scherr & Hammer’s Model

Scherr and Hammer (2009) took a different approach to the idea of epistemological framing and considered the behavior of students working in groups. They created four “behavioral clusters.” The blue behavioral cluster is used for students working on worksheets, hunched over their work with little conversation. Green is used for students making eye contact and actively discussing with each other. Red is used for students sitting still attentively listening to a TA. Yellow is used for students who are joking around. Scherr and Hammer compare the discussions and thinking of students in the green and blue behavioral clusters and find that in the interactive green mode students have more substantive discussions. In the green mode students use conceptual and mechanistic reasoning more than students with behaviors in the blue cluster, in which students give much shorter responses and more often reference authority.

Scherr and Hammer propose that this type of analysis could be applied to a variety of questions. Three of the questions listed were, “How do students frame classroom activities?” “In which frames do certain desirable activities (including cognitive activities) occur?” and “What precipitates shifts into (or out of) desirable frames?”

One powerful aspect of this coding is its robust nature and ease of use. Scherr and Hammer had very high inter-rater reliability and the coding can be done in real time, allowing for a large amount of data to be coded quickly.

2.2.6 Epistemic Framing: Bing's Model

Thomas Bing's 2008 dissertation and subsequent published papers (Bing & Redish, 2008; Bing & Redish, 2009; Bing & Redish, 2012) describe a new model for analyzing the mathematical thinking of upper-division physics students. Bing uses an epistemic lens as he looks at video of students working on homework problems in upper level physics courses. In looking at the video, Bing looked for different "epistemic framing" and four different frames emerged. These four frames are calculation, mathematical coherency, authority, and physical mapping.

Bing's coding, like Scherr and Hammer's behavioral cluster coding, has only four codes. This, combined with an ability to code very short chunks of dialog, allows for fairly quickly applying the coding to a transcript. Furthermore, unlike Tuminaro and Redish's (2007) epistemic games, one can consider a single student sentence and ask, "What is this student doing now?" as opposed to always needing to figure out where that student's thinking or action fits into a larger dialog. Because the coding can be done comparatively quickly (although much less quickly than the real-time coding of behavioral clusters), one can rapidly have access to a new way of looking at the data.

We now consider each of Bing's four framings. Calculation framing, also sometimes referred to as mathematical manipulation framing, occurs when a student is focused on computational correctness. Students follow algorithms to reach a reliable result. Usually the computations rely on mathematical symbols with little mention of the physical meaning of these symbols. At times students will use units, but these units function as labels, and are not used to make connections to physical understanding. One specific strong indicator of the calculation frame is equation chaining, for which a student takes the results of one equation and substitutes or "plugs it into" another equation.

The physical mapping frame is used when students connect their mathematical symbols to something in the physical world and compare the meaning of their symbolic representations to their understanding of the physical world. They seek coherence between the symbolic representations and physical world that those symbolic representations are modeling. Bing considers making gestures of physical quantities or using a drawing and diagrams to be strong indicators that a student is using the physical mapping framing.

In addition to mapping directly between algebraic symbols and the physical world, Bing also uses an example where a student compares a vector drawing to algebraic symbols and considers it “physical mapping”. The diagram is considered an intermediary between the physical world and the mathematical symbols. This dissertation will use the term similarly.

The third framing used by Bing is the mathematical coherency framing. According to Bing it is based on the idea that, “The same mathematical structure can underlie two superficially different situations.” For mathematical coherence Bing used as an example a student recognizing the similarity between the following two examples: Example one was figuring out how much $\hat{y}$ is in $\vec{r} = 3\hat{x} + 4\hat{y} - 2\hat{z}$. Example two was how much of $\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$ is in $f(x) = x(L - x)$. When students connect two different, but structurally related mathematical concepts they are using the mathematical coherency framing.

The fourth framing is authority. In the authority framing, some rule is quoted or an external source or previous result is referenced. Students using their notes, citing an authority, or stating a rule without support are indicators of invoking the authority framing. Students unequivocally use authority when they directly reference notes or an external source. However, Bing also uses an example where a student uses himself as the authority, when he says, “You can always take a derivative with respect to anything.”

Bing follows this statement with the comment, “Such a statement appeals to authority, in this case that of the student himself, for its justification.” In this quoted case, it is clear that the student isn’t analyzing the physical or mathematical situation and deriving this statement on the spot.

This dissertation embraces many of Bing’s underlying assumptions both in the chapters that specifically apply Bing’s framing model and in the subsequent chapters. One assumption is that the cognitive (e.g. Piaget, 1953) and socio-cultural models (e.g. Lave and Wenger, 1991) can be placed on a continuum as suggested by Greeno (1997). The emphasis will be primarily one of considering individual cognition, while being aware of the context in which that cognition is engaged. This dissertation will also sometimes include the group of three students as a unit of analysis, in addition to considering the students within this group. Even when the group as a whole is considered, this will not fundamentally alter the cognitive perspective, because it will be looking at the cognitive practices and epistemic viewpoints of that group more than the group’s larger participation in a physics community.

2.3 Problem-Solving Expertise

2.3.1 Overview of Problem-Solving Expertise

Section 2.3 reviews the literature related to Chapter 7. The research question for that chapter is, “In what ways are students using problem-solving expertise as they work through this problem?” We specifically consider three different models of expertise. In Chapter 7 we apply these three models to four different students solving the ring problem.

There have been many different answers to the question “What is expertise?” in physics and there have also been many different ways suggested to identify whether physics students are exhibiting expertise. The three perspectives discussed here are specifically related to students connecting their symbolic reasoning and their physical or geometric understanding. These three perspectives are Bing's (2008) model of epistemic framing; Kuo, Hull, Gupta, and Elby's (2010) concept of blending; and Krutetskii's (1976) concept of harmonic reasoning.

2.3.2 Bing's View of Expertise

In section 2.2.6 we looked at Bing's epistemic framing model. We will now consider specifically what Bing claimed about problem-solving expertise. Bing (2008) addresses problem-solving expertise in Chapter 7 of his dissertation, and in a subsequent paper with Joe Redish (Bing & Redish, 2012). Bing considers expertise to have two components; a well-organized knowledge bank and effective in-the-moment problem navigation. Bing claims that when it comes to his four framings (physical mapping, calculation, mathematical coherence, and authority) that experts will operate more fluently within each framing than novices. Experts can more deeply and accurately model a wider variety of physical situations, can calculate faster, can identify similar mathematical structures more easily, and can more readily call upon needed laws or rules.

However, Bing posits that beyond simply being good at each of these things, experts are also better at recognizing when they have reached a blockage that requires a different framing in order for additional progress to be made. He claims this frame-switching ability can be separately identified without specific consideration of the breadth and organization of the knowledge base. Bing further considers an overarching value on coherency between different approaches to show expertise, and again indicates that this can be considered separately from content knowledge.

To push his case, Bing considers a series of examples in which students all make errors and never ultimately reach a correct solution. He argues that there are important differences in their level of expertise, even though the knowledge banks of each of these students have failed them.

Bing compares the problem solving approach of different students and the degree to which students either fluidly switch framings or get “stuck”. Bing defines “stuck” not in terms of the length of time students use a particular framing, but the degree to which the problem solvers do or do not seize opportunities to switch framing when they have hit roadblocks and their current framing is proving unsuccessful. Students are often “stuck” when members of their group offer bids at reframing, but the students stick to their unsuccessful approach.

One of the examples Bing uses to illustrate expertise involves a student considering how to modify a fluid conservation equation to accommodate a chemical reaction occurring that changes the amount of chemical present. The student is wrestling with a minus sign in part of the equation. The student first makes a physical argument, which includes using a drawing to model the situation. Next, he checks for sign consistency throughout the equation. Finally, the student considers a rule about flows with which he is familiar. These fall into three different framing categories and the student fluidly switches among them, although a pair of errors result in the student not finding consistency among these results, and thus not being able to reach a conclusion. Furthermore, the student sees that all the framings should be yielding the same result and was confused and dissatisfied when they did not. Bing argues this student shows expertise, both because he attempted different framings and because he valued a consistency between framings.

The counter-examples used by Bing involved students being stuck in calculation framing. In one case a TA makes repeated bids to get students to consider physical mapping and the students continue to use calculation. In the other case, a group of upper-division students keep trying to use different calculation approaches, even though their calculations keep producing the same obviously incorrect result.

2.3.3 Blending: Kuo, Hull, Gupta & Elby

Kuo, Hull, Gupta and Elby (2010) consider, "that blending conceptual and symbolic reasoning…indicates problem-solving expertise more than adherence to 'expert' problem-solving steps." Kuo, Hull, Gupta and Elby consider two different students, each responding to two different prompts. One prompt asks students to explain the equation $v = v_0 + at$ as if they were explaining it to a fellow student from class. The other prompt asks students to consider the velocity of two balls after 5 seconds, one of which is dropped from rest, and the other which is simultaneously thrown down at 2 m/s, and decide whether the difference in velocity was more, less, or equal to 2 m/s.

Kuo, Hull, Gupta and Elby show that one student treats the equation like a "gizmo", into which data is entered and a result appears out the other end, whereas the other student sees it as a set of relationships. The first student tries to put numbers into the equation, whereas the second student recognized the "shortcut" to knowing that the two balls would have a difference of 2 m/s without having to perform the calculations. They consider this second student to be "blending" physical and conceptual understanding of the equation with an understanding of the mathematical relationships.

Kuo, Hull, Gupta and Elby consider this to be evidence for a more expert-like approach to problem solving. The authors compare the process used by the student who uses blending to prescribed problem-solving steps. The prescribed approach (e.g. Heller, Keith, & Anderson, 1992) indicates that students should solve problems in specific steps, such as: 1) visualize the problem and make a diagram create diagrams, 2) create physics descriptions and match symbols to the corresponding diagram, 3) plan a solution by considering the relevant physics principles 4) execute the plan, including performing calculations, and 5) check and evaluate. Kuo, Hull, Gupta and Elby argue that treating the "equation as gizmo" would often be consistent with what is taught in undergraduate courses as good problem solving, but is not well aligned with actual expertise in problem solving.

2.3.4 Harmonic Reasoning: Krutetskii

One alternative to some of the more recent models of expertise in physics problem solving is the perspective of V. A. Krutetskii, which is based on research from the 1950's with high achieving public school students. In his book, *The Psychology of Mathematical Abilities in Schoolchildren* (1976), Krutetskii describes three basic types of problem solvers among these high achievers. These three types are: the analytic type who uses an algebraic approach or approaches problems with symbolic manipulation; the geometric type who uses a pictorial or geometric approach; and the harmonic type, who is very capable of using a geometric, analytic, or combined approach when problem solving.

Many examples of analytic compared to geometric problem solving are offered by Krutetskii. In one example (p321), students respond to the question, "Each side of a square was increased by 3 cm and therefore its area was increased by 39 cm^2. Find the side of the resulting square." Krutetskii found that many capable sixth graders easily solved the problems in a few seconds using the equation $(x + 3)^2 – x^2 = 39$. Krutetskii called this an "analytic" approach. On the other hand, students who were primarily geometric problem solvers used a far more time-consuming approach that involved drawing a picture and doing geometric reasoning that never involved writing down an equation or performing symbolic manipulation.

Krutetskii noted that, of the 34 students he classified as "highly capable", 23 were harmonic problem solvers, who were able to easily use both geometric and analytic approaches. Harmonic problem solvers were far more common among this highly-capable group than among the general student population, indicating that being very capable is correlated with harmonic problem solving.

Krutetskii views students' problem solving as largely innate and uses the phrase "cast of mind" when referring to different types of problem solvers. However, he also asserts that mathematical flexibility can develop over time. He claims that students can learn to operate in their non-preferred problem solving mode and develop strengths. Krutetskii has many nuances to his view of what is innate versus what can be learned. However, at one point Krutetskii's briefly summarizes his viewpoint by answering the question of whether anyone can be become a mathematician or must one be born one. He answers, "Anyone can become an ordinary mathematician; one must be born an outstanding, talented mathematician."

It is interesting to consider the overlap between a student's "cast of mind" and a student's epistemic framing. This raises the issue of what is "epistemological" versus what is "cognitive". The University of Maryland's MPEX survey (Redish, 1997) asks students the degree to which they agree with the statement, "Physical laws have little relation to what I experience in the real world." This is an epistemological question. It is asking about the how students view the nature of physics knowledge. On the other hand, taking a ball of clay and rolling it into a "worm" and asking elementary students if the amount of clay has changed, is clearly a question related to cognitive structures, and not about the nature of knowledge.

In other cases it is more difficult to delineate between cognitive and epistemological questions. If we consider whether students are using symbolic manipulation or whether they are using geometric diagrams, we have entered a space where both the epistemic and cognitive perspectives both shed light on what students are doing. Using the lens of Bing's epistemic framing, we consider whether a student is framing the problem in terms of calculation or physical mapping. Using Krutetskii's lens of "mathematical cast of mind", we consider whether a student is primarily an analytic or geometric problem solver and look for evidence of this when they solve problems.

Again consider the example from Krutetskii (1976, p321), of students responding to the question, “Each side of a square was increased by 3 cm and therefore its area was increased by 39 cm^2. Find the side of the resulting square.” If we apply Bing’s (2008) epistemic framing to this situation, we see the analytic problem solvers spending almost all their time using a calculation framing, whereas the geometric problem solvers primarily use a physical mapping framing.

The epistemic approach asks the question, “What do students think they are supposed to be doing?” Kruteskii asks, “What is it that the brain naturally does?” Bings framings applied to Krutetskii’s example indicate that sometimes what a person thinks they are supposed to be doing depends on what that person’s brain is best at doing.

CHAPTER 3: METHODOLOGY

3.1 Overall Approach

The main purpose of this dissertation is to develop a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. To accomplish this I recorded numerous days of video of students solving problems *in situ* in Oregon State University's (OSU) junior-level Paradigms in Physics courses. From the available video, I chose two consecutive group problem solving sessions which I found to be especially good opportunities to look at student geometric reasoning. I then made transcripts of the dialog for each group during these sessions. Once transcripts were created, I tried a variety of existing models and theoretical frameworks to analyze the data and then chose five of these theoretical frameworks to apply in greater depth.

The approach of using multiple theoretical perspectives to examine a single data set was inspired by Rachel Scherr and Michael Wittmann's 2002 paper, *The Challenge of Listening: The Effect of Researcher Agenda on Data Collection and Interpretation*. They consider a specific student interview concerning electrical conductivity and conclude that the data are of limited value when viewing the interview from the perspective of conceptual knowledge and physical mechanism. However, when viewed from three other perspectives - source of knowledge, knowledge construction, or beliefs about knowledge – the interview is rich with information. Considering the interview from these other perspectives demonstrates how using multiple approaches can show a wealth of information about a single set of data. In the case of this dissertation, multiple theoretical perspectives are used to gain insight into how students think when approaching a highly-geometric electricity and magnetism problem.

3.2 Setting: The Oregon State University Paradigms in Physics Courses

With the aid of grants from the National Science Foundation, the OSU physics department completely reorganized and revised their junior-level physics courses and sequence (Manogue & Krane, 2003). This “Paradigms in Physics” sequence was first implemented beginning in Fall 1997. According to Manogue and Krane, the term “paradigms” refers to physical and mathematical themes that appear in several places throughout the junior year curriculum, such as the wave equation or transforming between reference frames.

Juniors consecutively take nine three-week courses for the Paradigms sequence. Prior to the start of the Paradigms sequence, students have only taken the introductory physics course sequence and modern physics. Most students enter the paradigms sequence having completed vector calculus, although some students take it concurrently. In their senior year, following the Paradigms sequence, students take “Capstone” classes, which cover more advanced topics and are more traditionally structured.

In addition to a non-traditional structure, the instructors in Paradigms courses use a variety of non-traditional instructional strategies designed to increase student involvement. These include, but are not limited to, kinesthetic activities, in which students move around the classroom or use their bodies to illustrate certain physical and mathematical concepts; small-white-board questions, for which students answer a question on small white boards and answers from around the classroom are compared and discussed; and small-group activities, in which students work in groups of approximately three students to collectively work on a specific task or problem.

During the day studied for this dissertation, students had completed the first 3-week course in the sequence, PH 320 – Symmetries, and were currently enrolled in the second course, PH422 – Vector Fields.

3.3 Method of Gathering and Selecting Data

3.3.1 Equipment and Physical Set Up

I recorded audio and video data for every class session in each of the first two Paradigms courses in Fall 2007. I also recorded several class sessions of the third class in the Paradigms sequence, which was PH421 – Oscillations.

Video and audio equipment were purchased specifically for capturing group work at tables, in addition to being able to capture the instructor's actions and overall classroom activities. Tables were arranged in a three by three array and could seat three students at each table. Three tables were along the west wall of the classroom, three tables were along the east wall, and the remaining three tables ran from the front center of the classroom to the rear of the classroom. Video and audio for the six tables along the wall were captured using a combination of webcams mounted on the walls and microphones on the tables. These were activated prior to the start of class and deactivated at the end of each class. In addition, a seventh camera, which had higher resolution, was mounted on a tripod at the back of the classroom and operated manually.

When students were not working in small groups, students were allowed to sit where they wished, which meant that students who sat in the center three tables were not being recorded except by the camera at the back of the classroom. However, when doing small group work, in order to facilitate recording, the instructor asked students to sit in groups of three and sit at tables along the walls. On the days recorded for this dissertation, there were only 17 students in class, which allowed for all the students to sit at tables along the walls. On some days students were assigned groups, but on the particular days used for this dissertation, students were allowed to choose their own groups.

Students were given a poster-board sized white board and were asked to write all their work on this board. All students had their own markers. The white board was positioned in the middle of the table, partially because white boards positioned at the end of the table away from the wall made the writing less readable when viewing the video.

While the instructor was talking, the manually operated camera was focused on the actions of the instructor, or, if a single student was speaking, then the camera was focused on that student. During small-group problem solving, on some days there were more than 18 students, at which time the manual camera was positioned to capture the seventh small group, which would work at one of the center tables. When there were 18 students or fewer, such as the day used for this dissertation, the seventh camera would be positioned at the end of one table in order to provide “double coverage” of the audio and video for one of the six student groups. At the end of each day, video was loaded onto a server and at the end of each week, DVD’s were burned with the data.

3.3.2 Choosing a Specific Problem to Examine

The data for this dissertation are video and audio taken in October 2007 of students working in small groups to solve for the magnetic vector potential of a spinning ring of charge. This particular problem is the fourth in a series of five small-group problems that students solve at various points during the first two courses of the Paradigms sequence.

The first problem students solve in PH 320 – Symmetries - is to create a power series expansion along a particular axis for the electric potential due to two point charges. The second problem is finding the electric potential in all space due to a ring of charge. The third problem is finding the electric field for the ring. In PH422 – Vector Fields – students solve two additional problems involving the ring of charge, except that now the ring is spinning. The data for this dissertation comes from this first spinning ring problem, for which students find the magnetic vector potential in all space. The fifth and final problem in this sequence is to use the Biot-Savart law to find the magnetic field.

As part of a Master's thesis (Cerny, 2007), I worked with the instructor to create instructor guides for the sequence of activities that included the four ring problems, so this particular sequence of small-group activities was of particular interest. Partial transcripts were made for several different days of small group work on the ring problems. The sequence of five activities was specifically designed to help students develop geometric thinking, so these were especially well-suited for looking at how students used geometric thinking during problem solving.

One of the reasons the magnetic vector potential problem was of particular interest was due to the specific physics and mathematics involved. This was the first time students needed to deal with current as a vector in the integrand. This provided a problem where every student would need to employ geometric thinking to a new situation. An additional factor was that students had become familiar with the customs and climate of the classroom and had an overall concept of how to work in groups to solve problems. In addition, various technical aspects had been resolved, such as not letting students use red markers, which made the writing hard to resolve in video images.

Students worked on the problem in groups during two different days. The first session, which occurred on a Friday, lasted 17 minutes and involved students trying to tackle the overall problem. The following Monday students were given some additional direct instruction and then given nine minutes to put $\hat{\phi}$ in terms of rectangular coordinates.

Once this particular problem was chosen, transcripts were made. On each of the days, full transcripts were made for five of the six groups, and, due to poor audio quality, a partial transcript was made of the sixth group. The poor audio quality happened to occur with the only group that had two students, so full transcripts exist for each of the five groups of three. Pseudonyms are used for all students and other people in the room, except for the lead instructor, Dr. Corinne Manogue, whose actual name will be used throughout.

3.4 The Ring Problem

The given problem is the fourth in a series of five activities and the third of four problems involving a ring of charge. By using the same ring for four parts of this sequence, students can focus on the differences in the physical concepts while using a familiar geometry. Each ring problem requires students to face successively harder mathematical challenges.

For this particular problem, the instructor grabs a hula hoop and holds it up and tells students the following:

> We're going to go back to the case of the ring. We have a ring with total charge *Q*, radius *R*, and now we're going to make it spin so that the charge is moving. So you have a spinning ring of charge with period capital *T*, and I want you to write an expression for the magnetic vector potential anywhere in space in a way that Maple could evaluate it.

The students are also given the general equation for the magnetic vector potenetial

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r})d\tau'}{|\vec{r} - \vec{r}'|}$$

where $\vec{r}$ denotes the position in space at which the magnetic vector potential is measured and $\vec{r}\,'$ denotes the position of the current segment.

For reference, a possible solution to this could be as follows (Cerny, 2007):

First, the general three-dimensional formula could be reduced to one dimension. Students had experience during the previous two ring problems of reducing a three dimensional formula to one dimension. In this case the result is:

$$\vec{A}(\vec{r}') = \frac{\mu_o}{4\pi} \int \frac{\vec{I}(\vec{r})d\ell'}{|\vec{r} - \vec{r}'|}$$

For the electric current, students had previously done a kinesthetic activity in which they pretended to be point charges and acted out the relationship $I = \lambda v$. They did not do it for any particular geometry. In this case:

$$\vec{I}(\vec{r}') = \lambda\vec{v} = \frac{Q}{2\pi R}\frac{2\pi R}{T}\hat{\phi}' = \frac{Q}{T}\hat{\phi}'$$

Recognizing the need to express $\hat{\phi}'$ using Cartesian basis vectors was particularly problematic for students. In this case:

$$\vec{I}(\vec{r}') = \frac{Q}{T}(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}})$$

The position vector $\vec{r} - \vec{r}'$ had been used by students in the two earlier versions of the ring problem, and was therefore not problematic for most students by the time they reached this particular problem.

$$|\vec{r} - \vec{r}'| = \sqrt{r^2 - 2rR\cos(\phi - \phi') + R^2 + z^2}$$

In cylindrical coordinates $d\ell' = Rd\phi'$

Thus:

$$\vec{A}(\vec{r}') = \frac{\mu_o}{4\pi}\int_0^{2\pi}\frac{Q}{T}\frac{(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}})\,Rd\phi'}{\sqrt{r^2 - 2rR\cos(\phi - \phi') + R^2 + z^2}}$$

$$\vec{A}(\vec{r}') = \frac{\mu_o}{4\pi}\frac{QR}{T}\int_0^{2\pi}\frac{(-\sin\phi'\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}})\,d\phi'}{\sqrt{r^2 - 2rR\cos(\phi - \phi') + R^2 + z^2}}$$

Students eventually were also asked to use power series expansions to find the magnetic vector potential along the *x*- and *z*- axes. The solutions for these are not given here because the chosen data does not include students working on this portion of the problem.

Prior to the research for this dissertation, the instructor initially created a list of some of the geometric concepts that students had to consider while finding the magnetic vector potential of a spinning ring. Among the things students had to consider were: the velocity of the rotating ring, the charge density, the magnitude of the current, the direction of the current, reducing the general formula

$$\vec{A}(\vec{r}') = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r})d\tau'}{|\vec{r} - \vec{r}'|}$$

down to one dimension, figuring out how to “chop and add” to set up the integral, expressing $\vec{r} - \vec{r}'$ in cylindrical coordinates, and expressing $\hat{\phi}$ in rectangular coordinates. The transcripts show a few additional things that some groups addressed were: *dQ*; symmetry; eliminating and changing variables in $r^2 + r'^2 - 2rr' \cos(\phi - \phi') + (z - z')^2$; angular velocity, linear velocity and related quantities; and understanding how one can integrate for “all space” while having a one-dimensional current.

3.5 Theoretical Perspectives and Methods of Analysis

Once the data were chosen and transcripts were made, I explored a variety of theoretical models that could be used for analyzing how students are thinking when facing a highly geometric problem. I chose several different models in order to give significantly different perspectives on the data. The models fit into three broad categories; epistemology, expertise in problem solving, and student use of geometric thinking.

Chapter 4 addresses the dissertation’s main purpose of providing a rich description of geometric reasoning by considering the question, “What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?” Ethnography of communications (e.g. van Zee and Manogue, 2010) was considered an effective method for providing rich description and gaining insights into student thinking. Once the data was grouped and categorized based on how familiar students were with the concepts they were using, we realized that Sayre and Wittmann’s (2008) model of resource plasticity is highly applicable to this situation.

Chapters 5 & 6 addresses the questions, “How are students framing what they are doing?” and, “ Do they see it as geometric?” To address these questions I tried five different epistemological models and tried to apply them to the available data. The five models attempted were; “epistemic games” (Tuminaro & Redish, 2007), “epistemic framing” (Redish & Hammer, 2009), “epistemic frames” (Bing, 2008; Bing and Redish, 2009), “modes of cognition” (Manogue & Gire, 2009) and “epistemological framing” (Scherr & Hammer, 2009). After considering each of these models, the epistemological model that was found to be most effective in considering students’ geometric thinking was Thomas Bing’s (2008) model for epistemic framing. All transcripts were coded using Bing’s coding prior to further analysis.

Chapter 7 addresses the question, “In what ways are students using problem-solving expertise as they work through this problem?” When examining student expertise, I use three theoretical frameworks: Bing’s epistemic framing model; Kuo, Hull, Gupta and Elby’s (2010) blending model; and Krutetskii’s (1976) model of harmonic reasoning. The three models are used in combination to consider the degree to which various students are exhibiting expertise at different times.

3.6 Choosing Specific Examples to Analyze in Greater Detail

Once theoretical perspectives were established, the entire transcripts of all the students working on this specific ring problem were considered in light of these perspectives. The process of selecting specific examples to discuss further depended on the theoretical perspective.

In Chapter 4 addressing geometric reasoning, the goal is to show a “cross section” of the class. When considering what student reasoning looks like in a highly familiar situation we are able to show the data from each of the five groups of three students. When considering student reasoning in cases where they encountered something less familiar, examples were chosen that were thought to best represent the variety of student approaches to solving the problem.

In Chapters 5 and 6, using Bing's (2008) epistemic framing perspective, the examples chosen were the ones that were thought to best illustrate a specific point. For Chapter 6, which uses Bing's perspective to consider how a calculator impacts student framing, only one student ever mentions the calculator and he does so on two occasions. In this case the class data is discussed as a whole and then I include the dialog surrounding the one student's two specific mentions of a calculator.

In Chapter 7, about student problem-solving expertise, four students are chosen to consider. One student is used as an example of an expert problem solver, based on grades, instructor opinion, and the student's progress on the ring problem. One student is identified based on a specific process involving how often a student switches framing. The two remaining students were identified as weak students both by the instructor and by their performance on the ring problem. These specific students were chosen to highlight contrasts between them. The selection of these four students is discussed in greater detail in Chapter 7.

3.7 About the Researcher

3.7.1 Personal Background

My first college degree was a Bachelor of Science (BS) in civil engineering, after which I worked for three years as a railway civil engineer. I then returned to school to get a Masters of Arts in Teaching (MAT). I worked for 18 years in a variety of public schools in the United States and Sweden teaching primarily physics, but also a variety of science and math courses. This included teaching standard high school courses as well as Advanced Placement (AP) and International Baccalaureate (IB) courses.

After taking a year as a homemaker, I then entered a science education program at Oregon State University, during which I earned a Master of Science (MS) in physics and pursued my doctoral work in physics education. During the last four years I worked on my dissertation during the summer and taught during the school year. One year I taught the introductory PH 201-202-203 sequence at a community college. The following three years I returned to the public high school setting and have taught physics along with other science courses. This remains my current employment.

3.7.2 Involvement in the Paradigms Program

I was first introduced to Oregon State University's Paradigms in Physics program as an "older student" when I took several courses in the Paradigms sequence as part of my minor for my PhD. This gave me perspectives of what it was like to be a student in the Paradigms classes. From the standpoint of a science educator, I was very interested in the instructional strategies used in the Paradigms courses, especially the strategies used by Dr. Corinne Manogue.

During my third year as doctoral student I was employed as a research assistant (RA) for Corinne Manogue and worked on documenting various aspects of the Paradigms program and decided that the Paradigms courses would be the subject for both my master's thesis and my doctoral research. Dr. Manogue served as my advisor for my dissertation.

3.7.3 Influences and Perspectives

As a high school teacher I was always curious about what made things so hard to learn and so easy to forget. I also wanted to know what my students *really* learned from my course. What things would actually be important in their futures? What things would they never use again? What things would they later need and still remember? What things would they later need but find themselves insufficiently prepared?

When I first started teaching high school I was with a group of science teachers who made a habit of complaining about how poorly the middle school teachers had prepared the kids. Whether it was chemistry, physics or biology, we pretty much taught our incoming high-school students as if they knew absolutely nothing about a topic – whether it be the parts of an atom or what caused the Earth to have night and day. Since I also taught a few math classes, I found that the math teachers often did the same thing. If only those middle school teachers did their job better, they would not have to remediate so much. And so, we all went along blissfully blaming the middle school teachers for all our problems.

My viewpoint changed when I had the opportunity to work in an international school in Sweden and teach a set of students all their math and science in 7^{th}, 8^{th}, and 9^{th} grade. As I started my third year teaching these students, I realized that, as usual, the 9^{th} graders were woefully unprepared. However, the teacher I had to blame was me. I could, of course, blame their elementary school teachers for all my problems, but that had a very hollow ring to it. Ultimately my students collectively performed above international average on their IB tests, but I was still frequently amazed by some of the things students could not do, including things I greatly valued and had thought I taught well.

At the start of my PhD program I was very interested in the idea of transfer. As a high-school teacher I have limited ability to assess my students after they leave high school, and I often wonder what things transfer to students later in life. The idea of looking at upper division physics students intrigued me. Although I had never personally taught any of the students studied, it was a chance to look at how student thinking had developed since these students were in high school. Thus, I came to the research wanting to gain insights into student thinking.

My engineering background also had an influence in what I found most interesting. As a railway civil engineer, I had to make sure the track and structures could safely and reliably support trains. While theoretical models are essential for doing the job effectively, there is also an acute awareness that if the available theoretical models are idealizations that ignore important relevant factors, then additional understanding needs to be brought to bear in order to keep the trains running safely.

Thus, for me, it was always important in my engineering work as well as my work as a science and math teacher to think about the connection between mathematical models and the real world. While the "messiness" of railway engineering is not present in the idealized spinning ring problem that I consider for this dissertation, I am still very interested in the degree to which students stayed connected to the thing that their mathematical symbols are representing.

My previous participation in the Paradigms classes as a student also influenced my thinking. To some extent I was curious about whether other students "thought like me." I was not even sure what thinking like me entailed, but I was interested to see whether the thinking of other students would resonate with my own experience or seem foreign.

Although not described in this dissertation, there were a few particular instances while being a student in the Paradigms courses in which I had described my own errant thinking to Corinne Manogue and she responded that she thought I was the only student with that particular problem. I felt vindicated when I was able to show video of several other students having the same thinking error. The many aspects of my own experience in the paradigms courses were seldom universal, but they were also rarely unique.

However, my primary motivation during the research was not to find whether students thought like I did or not, but rather to explore the nature of their thinking. The combination of experience as a high school physics teacher, my experience as an engineer, and my experience as Paradigms student led me to be especially fascinated with students' thinking while solving for the magnetic vector potential of a spinning ring of charge.

3.7.4 Conflicts of Interest and Overt Biases

The subject of this research is the thinking of students in Corinne Manogue's classes. Because she is also functioning as my advisor and had substantial influence throughout the research process, it is reasonable to consider conflicts of interest. Not only is Manogue an instructor, but she also is director of the Paradigms program, which gives her a vested interest in the success and positive portrayal of the Paradigms classes. There is an active bias, of both Corinne Manogue's and mine, that the Paradigms programs are beneficial for students and specifically that small-group activities help promote students doing valuable thinking. To this extent, this research "finds what we were looking for" becausse the research finds that students have interesting thoughts during group work.

However, the goal was to discover the nature of that thinking. The goal was not to measure the effectiveness of the Paradigms programs, nor the effectiveness of the instructor, nor the effectiveness of group problem solving. Instead, there was a genuine curiosity by both Corinne Manogue and me to understand what student thinking actually was occurring.

In subsection 4.9.4 of Chapter 4, and in subsection 9.3.1 of Chapter 9, I speculate about the effect of group problem solving on student thinking. At that point, there is legitimately a potential conflict of interest. While there was no intentional (or to my knowledge, unintentional) suppression or falsification of data, these particular subsections do adopt a particular viewpoint and explicitly serve to highlight advantages of small-group problem solving. It is reasonable to consider the arguments presented in those subsections as being support for a pre-existing bias.

However, for the remainder of the dissertation, I could identify no conflict of interest or overt biases that would compromise the integrity of the research. Corinne Manogue and I both considered it in our own best interest, and the best interest of the Paradigms program, to have the most accurate possible description of what students were actually thinking as they worked on the ring problem.

CHAPTER 4: GEOMETRIC RESOURCES AND REASONING

4.1 Overview

4.1.1 Research Question and Focus

The main purpose of this dissertation is to develop a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. This chapter specifically addresses the question, "What does student geometric reasoning look like as students encounter problem situations ranging from familiar to novel?"

Students solving for the magnetic vector potential of a spinning ring of charge provided a good place to look at student geometric thinking. Students needed to use vectors extensively, including considering how to deal with current as a vector in the integrand. This chapter looks at the wide range of resources and problem-solving approaches that students use while trying to solve in this ring problem in small groups.

4.1.2 Theoretical Frameworks and Overall Approach

We use ethnography of communications (e.g. Hymes, 1972; van Zee & Minstrell, 1997; van Zee, Hammer, Bell, Roy & Peter, 2005; van Zee and Manogue, 2010) as a theoretical framework for exploring student thinking. Ethnography of communications is based on the idea that by carefully considering the communications of individuals, including their utterances, gestures and writing, we can better understand those individuals and the culture of which they are a part. In this case, we create a rich description of what students are communicating in order to gain insights into student thinking.

As we looked at what students were saying, we initially put student thinking into two categories. The first category was “things every student knew and could do without noticeable effort.” The category at the other extreme was “things that were new to all students and the instructor assumed that all students would not find easy.” However, through an iterative process of looking at the data and considering the ways in which students were using geometric concepts, we realized that there was a lot of middle ground between “got it” and “never seen it”.

One of the interesting results is that, in this problem, the number of geometric concepts that every student in the class could apply without significant effort was quite limited. In fact, the only geometric concepts related to the ring problem that all students seemed to be able to apply without any effort was that the ring’s circumference is $C = 2\pi R$ and that $\lambda = Q/\ell$.

On the other end of the spectrum two different examples emerged from the data that appeared new to all students. One was integrating while having current as a vector in the integrand, and the other was putting $\hat{\phi}'$ in terms of rectangular basis vectors. Between the extremes of unfamiliar concepts and mastered concepts lay substantial middle ground. While many potential categories could be applied to this middle territory, we identified two specific categories.

One middle-ground category is “things that students had seen multiple times, and the instructor would hope that all students could do easily, but realistically are things that many students had not yet mastered.” The term “had not mastered” includes any concepts, relationships, or processes that students misremembered, misapplied, confused, or required significant time to reconstruct. The data contained many interesting examples of geometric concepts that fit this category. One set of examples: angular velocity, linear velocity, and related concepts, are ones that many students in the class could clearly be seen addressing.

Another middle-ground category is “things students are doing for the first time in this upper-division course, but for which they had previously solved at an earlier time.” This category was not chosen for further analysis, because student reasoning when accessing recently established results was not very transparent. Frequently students simply referred to their notes or wrote down a recently memorized result. One example of this is when several students wrote

$$|\vec{r} - \vec{r}'| = \sqrt{r^2 - 2rR\cos(\phi - \phi') + R^2 + z^2}$$

Some students referred to notes, while some simply wrote it down without any obvious effort. An additional complication with this category is that retrieving a memorized result is sometimes difficult to distinguish from cases in which a student rapidly reconstructs a geometric relationship on the spot.

Examples from each of the three categories – mastered, familiar-but-not-yet-mastered, and unfamiliar –were discussed with Corinne Manogue, the course instructor. She agreed that the categorizations matched her experience with students and her expectations for those students.

Once these categories were created, we noted that there were parallels between our chosen categorizations and Sayre and Wittmann’s (2008) categorization of resources on a continuum from solid to very plastic. Sayre and Wittmann considered five criteria: ease of use, recency of construction, elaboration needed to evaluate, justification (whether a resource justifies another resource or is justified by another resource), and rejustification and rederivation needed for extended use. We consider these criteria when looking at examples of student reasoning.

This research was also informed by Manogue, Browne, Dray & Edward’s (2006) careful consideration of different aspects of the task and the demands that the task places on students. The underlying question of "what makes this problem hard?" was repeatedly considered as the research was undertaken.

4.2 The Highest Level of Familiarity: Circumference = $2\pi R$

We now consider specific examples of students using geometric reasoning at different levels of familiarity. We begin with the most familiar and consider what understanding looks like when students use a concept they have already mastered.

The following examples include what was said when each of the five groups first address the relationship for circumference, $C = 2\pi R$, to determine that the charge density $\lambda = Q/2\pi R$. Each of the groups established this relationship quickly and with little effort. In the following examples, each group had a student draw a picture of the ring on the whiteboard and this picture was in front of them during the following dialog. However, none of the students refer to the picture while making the comments below. Furthermore, no gesticulation accompanied any of these events. The following outtakes represent the entire discussion of this concept for each group. Note the rapidness of use and lack of justifications.

Group 1 – 30 seconds after the group gets together, Tom writes "Q total charge" and "$\lambda = Q/ 2\pi R$" without hesitation or comment.

Group 2 –

Tanya, "OK, so it's charge density,...which we don't have."

Bob, "Uhh,...but we could figure it out though, right?"

Tanya, "Yeah, because ρ is Q over $2\pi R$?"

Group 4- Stan says, "So lambda, lambda expands - that's charge per unit length, which is Q over $2\pi R$…"

Group 5 – Shawn, "So then our lambda equals Q over $2\pi R$ " (writes $\lambda = Q/2\pi R$) "...

Group 6 - Jack, "Um, so, we need a charge. So we have Q, over the length, which is 2..." (writes $Q/2\pi R$ at the end of his equation)

It is interesting that the word “circumference” is not used as students present the idea of $2\pi R$. One student from Group 4 and one from Group 6 mention “length”, while the other students make no mention of any linear quantity. The thinking occurs so rapidly that it is often done in the time it takes to articulate the equation in words or to write it down.

In the examples above, it is clear that students have mastered the relationship that circumference = $2\pi R$. Consider the five criteria that Sayre and Wittmann (2008) use for determining the solidity of resources. The resource is old (probably first used in middle school or earlier), it is easy for students to use (they use it in seconds without pause), no elaboration is needed when it is used, and the resource is not itself justified but it is used to create other relationships. For each of Sayre and Wittmann’s criteria, this resource appears as solid rather than plastic.

The use of $\lambda = Q/\ell$ is something that was not nearly as old for students. However, $\lambda = Q/\ell$ was used repeatedly throughout the previous few weeks of the course. Solving for the magnetic vector potential of the spinning ring is the fourth activity in a sequence of five and is the third time that students work with a charged ring. Dealing with a linear charge density was something that many students found challenging when they first encountered it in the Paradigms courses. However, by the time students encountered this third ring problem, the concept of linear charge density appeared to be understood by every student.

Purely considering the student utterances, it is not always easy to distinguish between an enduring resource, that has been used over many years in many contexts, from a resource which has been recently (and at least temporarily) mastered. This is the case when Tom and Shawn state and write that $\lambda = Q/2\pi R$ without explanation.

However, with the other students, there is some difference between use of $C = 2\pi R$ and $\lambda = Q/\ell$. For example, consider when Stan says, "So lambda, lambda expands - that's charge per unit length, which is Q over $2\pi R$…" He is explicitly explaining the relationship of charge density being expressed as charge per unit length, but he makes no similar statement about the circumference being equivalent to $2\pi R$. Similarly Jack says "…we have Q over length…" but makes no similar statement about the length being $2\pi R$. In addition, Tanya and Bob have a brief conversation about figuring out the charge density before giving their expression. Thus, while use of $C = 2\pi R$ and $\lambda = Q/\ell$ are both fairly solid resources that are easily used and dependable, $C = 2\pi R$ appears to be even more solid resource.

The substructure of student thinking is not evident in their rapid use of $2\pi R$ as the length needed for establishing the relationship $\lambda = Q/2\pi R$. While we might speculate that each student probably has some ability to specifically articulate the concept of circumference and be able to justify the relationship $C = 2\pi R$, the relationship is available to students as a single "chunk" that does not require them to probe it.

However, there was one opportunity to see that the underlying thinking did exist and it was not purely memorized by rote. This opportunity to see the underlying student thinking occurred when Dr. Alice, who was co-teaching the course, asked one group to explain their expression for current, ${}^{Q}\!/_{T}\,\hat{\phi}$. She elicits a response that gives a slightly better view of the substructure of the student's thinking. Biff says, "OK, So first off we took and we said we have a radius of 2π…or total circumference of $2\pi r$, right? And then we said that, OK, how fast is it spinning around? $2\pi r$ divided by velocity equals period." (writes $2\pi r/v = T$ and puts a box around it). When probed, Biff is explicit that $2\pi r$ represents circumference. It is reasonable to assume that all the students could state that circumference = $2\pi r$, but simply didn't find it necessary to do so when applying it rapidly in the context of an upper-division problem.

4.3 Using Geometry at Varying Levels of Familiarity: Angular Frequency, Angular Velocity, Linear Velocity, and Related Concepts

We will now move from considering the most familiar, solid, and clearly mastered concepts. We next consider things that students have seen before, but not yet fully mastered.

For this problem, many students had at least some confusion regarding both the terminology and the concepts related to the motion of a steadily spinning ring. Most of the students in the class, including at least one student from each of five groups observed, made an error involving a failure to disambiguate one or more of the following: period, frequency, angular speed, angular velocity, linear speed and linear velocity.

Rotational dynamics is often one of the last sections covered in the mechanics portion of introductory physics, and is sometimes covered in only a few days. At Oregon State University (OSU) students usually do not take classical mechanics until their junior year. Thus, for many of the students, it may be one or even two years between the time they dealt with rotational motion in introductory physics and the point at which they face it in this upper-division ring example.

Clear understanding of linear and angular speed in the context of rotating objects fits into the category of "topics that instructors really wish that all their upper-division students already knew by heart, but which many students have not yet actually mastered." The way that students deal with angular velocity and related concepts stands in stark contrast to their usage of "circumference = $2\pi R$." We will now look at how a variety of students use this concept.

4.3.1 Group 1: Using Geometry to Identify Errors

Group 1 consists of Tom, Laura, and Allen. Tom is one of the strongest students in the class. Laura and Allan are consistent performers that repeatedly make explicit geometric arguments during the solving of the ring problem. Taken as a group, these three students never "settle" for incorrect results. Any result that is not correct is either set aside or discussed until corrected.

In the following dialog, Laura and Allen will both make incorrect assertions about angular velocity and will attempt to use these incorrect relationships to create algebraic expressions. This clearly puts Allen and Laura's understanding in the category of seen-it before, but not-yet-mastered.

In the dialog, Laura and Allen correctly establish that $T = 2\pi/\omega$, but then Laura makes the claim that $\omega = Rd\theta$. This varies from the correct expression $\omega = {}^{d\theta}/_{dt}$ in two respects. One is the presence of a factor of R and the other is the absence of time in the relationship. We will now look at Laura's comments and Allen's responses.

[00:43:12.15] Laura draws a picture of a circle with a wedge (Figure 1)

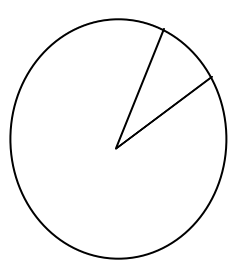

Figure 1: Laura's drawing of a circle with a wedge

[00:43:19.02] Laura writes the expression "$T = 2\pi$" while saying, "It's angular frequency, so...2π..."

[00:43:30.08] Allen says, "Over omega...isn't it?"
[00:43:35.03] Laura writes $T = 2\pi/\omega$
[00:43:44.04] Laura writes $T = 2\pi/\omega = 2\pi/R\theta$ and says, "2π over $R\theta$."
[00:43:44.04] At the same time, Allen writes $\omega = 2\pi f = 2\pi/T$ and says quietly, "Omega equals 2π over T," Then turns to Laura and says, "Yeah, it's over T. Or over omega, sorry. Um."
[00:43:51.03] Laura (looks intently at her drawing of the wedge), "Yes"
[00:43:54.04] Allen, "Yeah, that's good enough. See that's right, so yeah, OK."
[00:43:56.16] Laura says, "So now we have $Rd\theta$ so, for a little..." [looks back and forth between equation and drawing of wedge]
[00:44:03.05] Allen, "Shouldn't it be $Rd\theta/dt$? Isn't angular frequency like the change in..." Allen gestures around in a circle as Laura writes.
[00:44:08.29] Laura writes and [over Allen] says, "So dt is equal to $2\pi/Rd\theta$. Why would you have a $d\theta/dt$...[?]...?"
[00:44:24.26] Allen, "'Cause $R\theta$ would just be like your arc length,...[gestures a length]...like the circumference kind of covered...” [gestures around in a circle]
[00:44:29.10] Laura, "Yeah, so..."
[00:44:32.27] Allen, "...which it's the same thing as angular frequency...[inaudible- “over R?”]...."
[00:44:36.16] Laura, "OK $d\theta$ is,…OK say that this is $2\pi R$ we just get T equals.... $1/R$,......which is bad (laughs)."
[00:44:55.09] Allen, laughs, "...I guess we get the change in R and T."
[00:44:58.06] Tom, "Period is inverse length"

One interesting feature of the dialog is that, unlike several examples from other groups, Allen and Laura do not simply accept their initial incorrect results and move on. Instead, through use of geometric arguments and attempts at sense-making, errant assertions are brought into question and are not “settled’ upon by the group.

Allen recognizes that equating ω to $Rd\theta$ is problematic. He recognizes the need for some sort of relationship to time. In doing so he introduces his own error, proposing that ω is equivalent to $Rd\theta/dt$ which has the dimensions of linear velocity. However, Allen is clearly attempting to use geometry in his thinking, as evidenced by his motioning his hand in a circle. When faced with a situation where their understanding is not solid, Laura and Allen are both thinking about the geometry to help them reconstruct their understanding. Laura uses a drawing and Allen uses hand motions as they think about the geometric situation.

After arguing that angular frequency should be $Rd\theta/dt$, Laura is unconvinced and asks, "Why would you have a $d\theta/dt$?" Allen then points out that "…$R\theta$ would just be like your arc length,...[gestures a length]...like the circumference kind of covered..." [gestures around in a circle]. Here Allen makes the point that $R\theta$ is just an arc length, indicating it cannot be an angular velocity. However, Laura does not acknowledge that $\omega = R\theta$ is problematic and instead seizes on the idea that $R\theta$ is like the circumference. She substitutes $R\theta = 2\pi R$ into $T = 2\pi/\omega = 2\pi/R\theta$ to get the result that $T = 1/R$.

Although she did not initially take Allen's critique into consideration, Laura immediately recognizes when her calculations yield a nonsensical result and comments "…T equals.... $1/R$,......which is bad." The group finds the result literally laughable and makes humorous comments further illustrating how ridiculous it is. The combined efforts of Allen making geometric arguments and Laura trying to make sense of her algebraic manipulations leads to neither settling on an errant result.

Clearly, understanding of angular velocity in this context could not be considered a solid resource for Allen and Laura. Both students include R in their relationships, while using the symbol and language of angular frequency.

To the extent that one considers a "resource" something that can be taken as a chunk without examination of its substructure, then this would not yet be gelled into a full-blown resource. However, Allen and Laura's understanding of angular speed would fit Sayre and Wittmann's description of a plastic resource. This is something that students can call upon and for which they have some existing knowledge. They have connected angular motion concepts to the idea of a rotating ring, so this resource is easily cued, recognized, and accessed. However, it requires effort for students to use and its application is not instantaneous. Furthermore, it requires elaboration and justification from other resources and needs to be reestablished or rederived.

4.3.2 Group 2: Problems with Disambiguation Lead to Acceptance of Errant Results

We will now consider Group 2. This group consists of Nick, Bob and Tanya. Unlike Group 1 that works consistently as a collaborative group, Group 2 alternates between group discussions and students working independently without discussion.

Within the first minute of conversation, Nick, says "T is equal to 2 π r over, over v", and writes $T = 2\pi r/v$. Soon after, Bob says, "Uh, wait a sec, is it omega equals 2 π f? Omega is 2 π f, so f is one over T," (writes $\omega = 2\pi f$). These are both correct relationships, and it might seem like this group is off to a great start with respect to speed and angular speed. However, one interesting thing to note is that students refer to the quantities by the names of the variables. For example they use the word "omega" and do not give it a physical name.

The absence of a physical name becomes problematic in the next few seconds when Tanya says, ""We need veloc...that's velocity, so 2 π over T is velocity," and writes on Bob's equation, changing it to $\omega = 2\pi /T$. Further problems arise as Nick looks at $\omega = 2\pi /T$ and declares that it is incorrect. The subsequent conversation shows Nick's thinking.

For this chapter, the Inqscribe time stamp that accompanies the transcript in the appendix will be given for the first student speaking. This will be given at the end of the paragraph preceding the transcript excerpt. [00:41:05.15]

> Nick, "The units aren't right,...it's 2 π r over v....wait,..." writes $\omega = 2\pi r/v$
> Tanya, "No."
> Nick, "...*T*", erases "*v*" and writes $\omega = 2\pi r/T$.
> Bob, "Yeah."
> Nick, "Chk"
> Tanya, "Where you getting '*r*'?"
> Bob, "Yeah, where's '*r*'?"
> Nick, ERASES equations, "*R* is the radius," [points to the radius labeled *R* in the drawing] "…big *R*...What are we trying to find?"

In a span of 45 seconds the group has gone from Nick and Bob having correctly stated relationships for both speed and angular speed to having Tanya incorrectly describe "ω" as "velocity" and having Nick write incorrect relationships. Nick originally correctly claimed that $T = 2\pi r/v$, however, his lack of clarity in distinguishing between v and ω leads him to claim that $\omega = 2\pi r/T$. Tanya's use of the word "velocity" for ω may have reinforced Nick's confusion. It appears that Nick understands that linear speed needs to have a factor of R included. However, because Nick does not attempt a geometric argument, his lack of clarity about the nature of angular speed prevents him from successfully communicating with the rest of the group.

The students then spend significant time working independently. Five minutes later there are discrepancies between two different equations that Nick and Tanya have produced for magnetic vector potential. This situation is also discussed in Chapter 5, section 5.4.2 in the context of Tanya being a weak problem-solver.

Nick has

$$\vec{A} = \frac{\mu_o}{4\pi} \int \frac{\lambda \vec{v} d\phi}{\sqrt{r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2}}$$

Tanya has

$$\vec{A} = \frac{\mu_o}{4\pi} \int_0^{2\pi} \frac{\lambda RT}{|r - r'|} d\phi'$$

Nick looks at Tanya's equation and tries to reconcile it with his own. In the following dialog, Nick further shows that he is not distinguishing between linear speed v and angular speed ω. [00:46:55.19]

> Nick, "Oh wait, so you're using this for v," pointing to "$\omega = 2\pi/T$" on board,
> Tanya, "Yes."
> Bob, "OK."
> Nick says, "Where v is...2 π over T," [erases ***v*** in his own equation and writes $2\pi/T$]

It is interesting that Nick uses the word "v" while pointing at "ω". This dialog makes it even clearer that Nick is equating linear speed v and angular speed ω. For the equation for magnetic vector potential that Nick is using, linear speed was the quantity actually needed, so Nick makes an error in using $2\pi/T$ to represent his linear speed.

In the next sequence of dialog, seven minutes after Nick first claimed that the units of $\omega = 2\pi /T$ weren't correct, Nick again asserts that $\omega = 2\pi r/T$ is correct. This time he is even clearer that he is thinking that the words "angular velocity" and the variable "ω" can be used for a linear speed. He explicitly says the units for angular velocity ω should be meters per second. In addition Nick also claims that $2\pi R$ represents the radians, apparently envisioning radians as a length. [00:47:48.03]

Nick, points to $\omega = 2\pi/T$ on the whiteboard, "OK, but we just we need to add an R to this because that is not the correct units. Angular velocity is...[v?]...meters per second, so it needs to be R in here." [writes an R to get $\omega = 2\pi R/T$]

Bob, "Well this,...this is radians." [points at the equal sign in $\omega = 2\pi R/T$]

Nick, "$2\pi R$, that's, that's radians."

Bob, "Right."

Nick, "Yeah."

Tanya, "Yeah, that's, that's angular velocity."

Bob, "So, radians per time."

Nick, "Hmm."

Tanya, "Yeah."

Nick, "Oh yeah."

Tanya, "If we, if we need linear velocity we have to change that factor by...I don't know what."

Nick, erases R to return to $\omega = 2\pi/T$, and says, "OK, yeah, you're right, so...It's been awhile."

Tanya, "Ow."

For the third time Nick shows that he is thinking of ω as a linear speed. His lack of awareness that angular speed is not the same as linear speed causes a clash with the other two students who are asserting that $\omega = 2\pi/T$ is correct. Nick further adds to the communication mismatch by using the word "radians" to represent the length of the arc that corresponds to an angle measured in radians.

It should be noted that near the start of the problem solving session, Tanya referred to ω as "velocity". In the dialog above she has switched to calling ω "angular velocity" and specifically says "If we, if we need linear velocity we have to change that factor by...I don't know what." Tanya clearly recognizes that velocity and angular velocity are not identical.

Bob's thinking throughout the transcript is less evident. Bob agreed with Tanya challenging Nick's $\omega = 2\pi r/T$, when Tanya says, "Where you getting '*r*'?" and Bob adds, "Yeah, where's '*r*'?" Bob consistently shows that he thinks that $\omega = 2\pi/T$ is the correct relationship, but we get little insight as to why he thinks this. It is also unclear exactly what Bob thinks "radians" are when he says, "Well this,...this is radians," and points at the equal sign in $\omega = 2\pi R/T$. When Nick says, "$2\pi R$, that's, that's radians," Bob says, "Right." However, as seen with a variety of students in a variety of situations, students will often say, "right" or give other affirmations, even when they don't actually agree with another student's preceding statement. Bob, however, gives at least one additional insight into his thinking when Corinne Manogue stops by to talk the group.

A few minutes after the previous dialog, Corinne questions the group on their use of angular velocity. A very interesting conversation ensues that gives insights into student thinking and also shows how different instructor understanding is from student understanding. [00:51:13.15]

> Corinne, "And what is this?" points at $2\pi/T$ part of equation.
> Bob, "Uh, this is our, yeah, omega."
> Corinne, "Why do you want angular velocity?"
> Bob [gestures in a circle], "Because it's a circle."
> Corinne, "I don't care."
> Bob, "OK."
> Corinne, "This is lambda times a real velocity." [points at $\vec{I} = \lambda\vec{v}$ on whiteboard]
> Bob, "OK."
> Corinne, "...so it will be omega times *R*"
> Tanya, "That's how you go from angular to normal velocity?"

Nick, "It's..."

Corinne, "Yes"

Tanya, "We don't remember that. That was a very long time ago."

Nick, "What is, what is, no, what is the conversion from velocity to angular velocity?"

Corinne, "Eesh,...um,...."

Tanya, "v equals..."

Corinne puts her head in her hands.

(Laughter)

Nick, "I mean, no, no, no, not converting."

Corinne, "Sorry, no, sorry, I just, I don't think of it in those words, so I'm having to translate."

Nick (over Corinne), "Right, I didn't mean that, I didn't mean conversion. I didn't mean to say that."

Corinne, "OK, so...so...may I have your pen."

Corinne draws a circle and gestures around in a circle, "If you've got something going around in a circle..."

Nick, "Uh huh."

Corinne, "...it goes the whole circumference in a period."

Nick, "Uh huh, right."

Corinne, "OK, so the velocity is the circumference divided by,...or the speed...is the circumference times the period...or over the period." [writes $V = C/T$]

Nick, "OK,...so, yeah..."

Corinne, "So in your case, it's $2\pi R$..." [writes $2\pi R/T$ to get $V = C/T = 2\pi R/T$]

Nick, "Over T"

Corinne, "Over T"

Nick, "Hi Oh!" [bangs fist on table]

Bob, "Damn" [bangs fist on table]

(Laughter)

It is interesting that Bob justifies the use of angular velocity “because it’s a circle”. While using angular velocity can be a valid approach in solving this problem, Bob’s statement indicates that it is imperative. In fact, it is entirely possible to solve this problem using linear speed and not introducing angular speed. However, because the ring is circular, many students cue the concept of angular velocity and introduce it to the problem.

Nick uses the language “conversion from velocity to angular velocity,” possibly indicating he wants some formula that allows direct translation from one to the other. He does not ask how the two quantities are related nor does he request a geometric explanation. Corinne is so taken aback by the idea of “converting” between quantities of different dimensions, that she says, “Eesh…” and puts her head in her hands. Corinne then responds by using a very geometric description, including a drawing of a circle, to illustrate the relationship. It should be noted that prior to Corinne’s arrival none of the students in this groups had made any drawings or made any gestures to illustrate these geometric relationships.

Although he tried to retract the word once Corinne reacted, Nick’s use of “conversion” probably fairly accurately expresses Nick’s thinking about the relationship between the concepts of angular velocity and linear velocity. Nick and Tanya appear to think of angular velocity and linear velocity as basically the same thing, separated by some sort of conversion factor.

At the end of this dialog, Nick says a loud, exuberant, “Hi Oh!” along with banging his fist on the table, while Bob says, “Damn” and bangs his fist more softly. It appears that Nick felt vindicated after his repeated assertions with the group that the speed should be expressed as $2\pi R/T$. Nick appears to be unaware that his equating angular and linear speed was problematic.

Corinne never directly addresses the relationship between angular speed and linear speed. Furthermore, she never clarifies the difference between speed and velocity. Instead she simply clarifies the meaning of the linear speed using equations, words, a drawing, and a hand gesture in a circular motion.

From the standpoint of resources, the students in this group have a very plastic understanding of these concepts. They do not find the concept very easy to use and their understanding is open to reexamination and reinterpretation.

4.3.3 Comparison of Nick from Group 2 and Allen from Group 1: Connecting to Solid Resources Versus Plastic Resources

Nick from Group 2 shares similarities with Allen in Group 1. At one level it appears that Nick and Allen are both making similar mistakes and engaging in similar thinking. Both students use the variable ω and the word "angular" when referring to a linear speed. Allen uses $\omega = R\, {}^{d\theta}\!/_{dt}$ and Nick uses $\omega = 2\pi R/T$.

However, there are important differences between Nick and Allen's approaches to dealing with their plastic understanding. Allen attempts to construct a firm understanding by connecting to geometric ideas that are to him a solid resource. Nick connects his plastic understanding of angular velocity to yet other plastic resources.

Allen explicitly names different quantities such as circumference and arc length and uses gesticulation to show his thinking. When describing his understanding of "angular frequency", Allen uses words that are clear and unambiguous to himself and the other students in his group. Allen's use of "circumference" and "arc length" show he is connecting to things he clearly understands. Furthermore, Allen's gesticulations show he is trying to create a visual or geometric model of the relationships. Allen attempts to connect his plastic "angular frequency" resource to solid resources, and uses unambiguous physical and geometric connections to the symbols he uses.

In contrast, Nick uses “radians” to refer to the arc length and never establishes a common understanding of what “radians” are with the other students in his group. Nick never uses drawings, gesticulation or clear language that would allow him to reach a common understanding with other members of the group. Nick connects one plastic resource to yet more plastic resources, and uses language that lacks clarity and unambiguous physical or geometric interpretation. The difference between Nick and Allen is one that involves issues of depth, quality and clarity.

Redish and Hammer (2009, p632) in their section on sense-making, mention the example of a student who requests that the TA stop using analogies. The TA responds, “What do you want me to do, give you a bunch of words that you don’t know what they mean?” The student answers with a straight face, “Well, that’s what I’m used to.” The earlier example of Nick and Allen shows that upper-division students, when faced with a concept they don’t completely understand, may vary in the degree to which they are willing to have as an explanation, something that they also only partially understand.

4.3.4 Group 4: Asserting Authority

Group 4 consists of Kevin, Stan, and Robert. Stan is the strongest student and does the majority of the talking and also the majority of the writing on the whiteboard on the table.

The following dialog shows how this group responds to an incorrect assertion that $f = 2\pi\omega$. [00:44:47.08]

> Kevin [writes $f = 2\pi\omega$] "Frequency equals 2π omega"
> Stan, "Ooo, nice!"
> Kevin, "Right?"
> Stan "Yeah"

Robert, "Yeah"

Kevin, "And then, there's a, there's a formula that relates angular velocity to..."

Stan , "Wait, isn't it ω divided by 2π?...'cause it's...because in 411 we do...to get omega we get 2π times the frequency..." [writes $\omega = 2\pi f$] "...so, yeah, it's divided by 2π."

Kevin writes $f = \omega/2\pi$ and $T = 2\pi/\omega$

In the above dialog Kevin mis-remembers the formula for angular frequency and writes $f = 2\pi\omega$. Initially both of the other students affirm this equation, but then Stan eventually says that it should be $\omega = 2\pi f$, because that is the formula used in PH411. Kevin's mis-remembered statement, was asserted without justification, as if from an authority, and is eventually countered by Stan also using an authority framing, although Stan names his source. No physical, geometric, or mathematical argument is made.

Consider a second example from Group 4. In the dialog below Kevin introduces a new mistake, $\omega = vr$, and then engages in symbolic manipulation. This time Alice, the post-doc, who simply watched the student in the previous dialog, now decides to actively participate. Initially Alice simply watches and tells the group to ignore her and "keep going", however, when the group builds on their error, she decides to intervene and challenges the group as to whether the units make sense. In doing so it reveals another misunderstanding held by Stan. [00:45:27.17]

Stan, " Now we know we have the radius and....

Kevin writes $\omega = vr$

Group looks at Alice, Alice shakes her head and says, "Keep going."

Stan, "We're all trying to remember,...for omega"

Kevin, adds "$=2\pi/vr$" to his equation for T

Stan points at equation, "Then we have to just solve...That's v, right?"

Kevin, "Yeah"

Stan, "Yeah."

Kevin, "...[inaudible]..."

Stan, "No it's fine, that's great. So it's just 2π over *Tr* equals *v* ? [writes $2\pi/Tr = v$]

Alice, "Do the units of that make sense?"

Stan, "One over time....no, so it should be the inverse...That's length and that's one over a second, so right now we have seconds over length, we've got time over length, so we want to flip it."

Alice, "Hold on, what are the units of period?"

Stan, "Isn't that one over seconds?" [writes 1/s]

Alice, "No, it's just seconds."

Stan, "Oh, OK, and so we..."

Alice, "Period is how long does it take to do one cycle" (gestures in a circle), "So it's seconds. It's a time."

Stan, "So we, so we have one over *TL*"

Robert, "So you're looking at a constant here looking at L"

By asking whether the units make sense, Alice is encouraging the students to use a strategy that experienced physicists use for catching errors. In the process of checking the units, Stan reveals that he is confusing frequency (with units 1/s) with period. Alice proceeds to clarify that period has units of seconds and includes a physical description of why this is so.

There are many similarities between Alice's interaction with this group and Corinne's interaction with Group 2. In both cases students were not connecting their assertions to the geometry of the ring, and in both cases the instructor introduces an explanation that includes a physical explanation. Both instructors also gesture in a circular motion. The instructors are modeling for students that their symbols should be connected to geometric thinking.

In the sequence below, Alice continues with her questioning, by asking the students to consider their claim that $\omega = vr$. Stan responds with a correction based on authority. [00:46:40.11]

Alice, "Yeah, so what about this expression here? How confident are you in this?" [points at $\omega = vr$]

Stan, "It's divided by r, it's divided by r."

Alice, "Why is it divided by r? How does that make sense?"

Stan, "I just remember it."

Alice, "You just remember? OK, that's acceptable."

Stan, (some noise)

Alice, "What, what is this omega thing?"

Stan "The...

Robert points to an arrow on ring (which would correspond to $\hat{\phi}'$)

Alice, "Yeah, it's angular speed, it's how many angles do I go through,...[gestures, using her hand and forearm, an angle being traversed].. right? And then v is your..." [gestures holding her two fists in front of her, about shoulder width apart] "...right?"

Robert, "Divide out your radius"

Alice, "Right, so it's $r\omega = v$, right?

Robert, "Right"

Alice, “Because to get arc length...[gestures a complex series of gestures indicating radius and an angle]...it's r times θ, and this is the rate of change of θ. Right? So it's r times ω to get v, that's out on the edge. Does that make sense?

When Stan corrects $\omega = vr$ to $\omega = v/r$ and says he “just remembers it”, Alice initially tells Stan "You just remember? OK, that's acceptable." However, Alice goes on to probe whether students understand what omega really is and then launches into a geometric description regarding the relationship between angular velocity and linear velocity. Alice's description includes the use of several gestures. Again, the instructor is trying to push the students away from unsupported assertions and towards geometric reasoning.

It is interesting how Alice sometimes uses words to describe physical quantities as she talks, but also frequently uses variables to describe quantities. She initially uses the phrase "angular speed" and the word "angles", when she says, "it's angular speed, it's how many angles do I go through." However, after that she only refers to speed as "v". Similarly, in her next turn talking, she uses the words "arc length", but otherwise refers to physical quantities by the letters that represents them. She says, " …it's r times θ, and this is the rate of change of θ. Right? So it's r times ω to get v, that's out on the edge."

Throughout this dialog, Kevin and Stan are the primary participants and both are stating equations based on memory. Until Alice intervenes, there is no attempt by either student to connect the equations to any physical or geometric relationships. Even when asked to justify that $\omega = v/r$, Stan claims "I just remember it." The geometric description is done primarily by Alice, while other students watch and acknowledge.

In this case the students do not focus on correctly understanding the equations, but instead focus on correctly remembering the equations. In Kevin's case he incorrectly remembers the equations, and in Stan's case, when properly cued, he correctly remembers them. The degree to which the students truly understand these relationships geometrically is not evident in this dialog. When the incorrect equation is given, and appears to be not just a misstatement of something otherwise understood, then it is evidence of not being a solid resource. However, when students assert things using an authority framing, it is more difficult to evaluate the degree to which the resource is plastic or solid. When Stan states the correct relationships, the degree of plasticity of this resource cannot be probed with the data at hand.

In Group 4, the mis-remembering of the formula $\omega = 2\pi f$ and then also misremembering $v = \omega r$ elicits a different type of response than seen in the previous two groups. In the previous examples, students used mathematical or physical arguments to address the problem, whereas the students in Group 4 rely on authority.

4.3.5 Group 5: Angular Velocity as a Solid Resource

Group 5 consists of Shawn, Biff and Devin. Shawn, of all the students in the class, is the student who exhibits the clearest signs of having angular speed be a reasonably solid resource. As shown in the dialog below, Shawn establishes that $T = 2\pi R/V$ on his first try and without the use of diagrams or gesticulation. This example stands in contrast to the students in the previously featured groups that clearly struggled with applying the concepts of linear and angular speed.

Two minutes prior to Shawn's rapid development of the equation $T = 2\pi R/V$, Shawn and Biff are interacting. Biff makes the errant claim that $T = 2\pi f$. Shawn starts writing $T = 2\pi$, and then stops. Biff erases his own $T = 2\pi f$ equations and the group goes on to discuss the relationship between volume current density and linear charge density. Evan joins in these discussions for two minutes while "$T = 2\pi$" remains on the whiteboard in front of him.

Then, over a period of 20 seconds, Shawn goes from noting that $I = \lambda v$ to establishing that $T = 2\pi R/V$. 30 seconds later, Shawn also establishes that $\lambda = Q/2\pi R$ and uses the combination of these relationships to establish that $I = \lambda v = Q/T$. Time stamps have been used to show the speed that the dialog progresses. Shawn's name has been placed in bold letters to help emphasize Shawn's contributions.

[00:45:25.16] **Shawn**, "I equals λv"

[00:45:28.23] Biff, "...where v is the velocity of the electrons, right? [writes "$\lambda(v)$ - velocity of e-"]

[00:45:28.23] Devin writes $I = \lambda v$

[00:45:31.29] **Shawn**, "Yeah, so that'd be from the period; a period of $2\pi R$" (writes R to get $T = 2\pi R$ and then pauses, staring intently at his equation)

[00:45:38.20] Biff points at ring

[00:45:38.20] Devin, "Isn't v equal to period times frequency?"

[00:45:40.06] **Shawn** writes a division bar under the "$2\pi R$," followed by a pause, and then finishes writing $T = 2\pi R/V$.

Shawn is trying to get an expression for the period T in terms of other variables. He quickly writes "$T = 2\pi R$" and then pauses for 4 seconds while looking intently at his equation, then draws a division bar, then pauses 4 more seconds and then writes a large V in the denominator. 30 seconds later, Shawn takes a similar amount of time to write $\lambda = Q/2\pi R$. Shawn's ability to perform this quickly is an indicator that he is using fairly solid resources.

There are some differences between Shawn's creation of these expressions compared to students using $C = 2\pi R$. In the case of $C = 2\pi R$, students created the relationship in the time it took them to write or speak. In Shawn's case, the pauses suggest some degree of active consideration of the relationships. While these relationships were not instantaneously obvious, Shawn produced both of them correctly, without explicitly discussing them or making other justifications out loud, in under 10 seconds each (although peripheral thinking may have occurred over a somewhat longer period of time). He was the only student in the classroom that produced this equation error-free without extensive discussion.

When Biff sees Shawn's $T = 2\pi R/V$, he immediately checks the units.

[00:45:42.29] Biff "$2\pi R$, yeah, divided by v...That equals meters over meters per second equals seconds over meters times meters, cancel, equals seconds." [writes $2\pi R/v$ = m/(m/s) = s/m * m = s (circles the "s")]

[00:45:54.20] Devin, "That's the period."

Biff's unit-checking shows that $2\pi R/V$ has the units of seconds, which is consistent with the units of period. Once this result is achieved, all three students accept Shawn's equation and Shawn goes on to apply it to finding the current. Thus, Shawn's quickly achieved result rapidly becomes accepted and usable to the entire group.

4.3.6 A Brief Summary of Student Errors

Before considering further the difference in resource usage in the preceding groups, we will briefly consider the range of errors throughout the classroom that were related to angular frequency. Errors were widespread throughout the classroom. The majority of students in the class made errors related to angular frequency and related quantities. However, very few students made an error identical to that of another student.

Students were aware that ω could be used when considering the motion of rotating objects. However, there was widespread lack of clarity about exactly what "ω" meant and how it should be applied to this problem. Additionally, there were some students who made errors in regards to frequency f. Each error had its own nuance. Here is a list of the errors made:

Units of period T are 1/s

$\mathrm{T} = 2\pi f$

$\mathrm{T} = f/2\pi$

$f = 2\pi\omega$

$\omega = R$

$\omega = R\, {}^{d\theta}/_{dt}$

$\omega = 2\pi R/T$

$\omega = vR$

$\omega = v$

$\omega = v/2\pi$

$v = fT$

It should be noted that this list does not include any of the additional errors related to confusing scalar and vector quantities, such as referring to angular speed as "angular velocity". Six of the eleven incorrect relationships involve ω with an added, omitted, or misplaced factor of R. The remaining five errors involve misunderstandings about the frequency f or period T. It is interesting to note that although ω and f could be used to solve the problem, neither was actually required.

This list was created to clarify that the errors were pervasive, but were in no way consistent or limited to a single specific error. Collectively, these errors show that students are cuing ideas related to angular velocity, but for most students, these are not solid resources.

4.4 Discussion of Student Usage of Solid vs. Plastic Geometric Resources

While the preceding subsections document what have become commonly referred to as "student difficulties", the main purpose was not to document these difficulties. The goal was to consider the differences exhibited by students in dealing with a solid geometric resource compared to a plastic geometric resource. First, let us reconsider what a solid resource looks like when students use it.

When students used $C = 2\pi R$ and $\lambda = Q/2\pi R$ the result was easily established without significant elaboration, or justification. In the case of Shawn establishing that $T = 2\pi R/v$, he established this relationship in a few seconds without discussion or elaboration, although pauses indicate that he used at least some minor effort. It is interesting that when Shawn uses $T = 2\pi R/v$, the students in his group respond differently than the students in the groups where $C = 2\pi R$ and $\lambda = Q/2\pi R$ was established. In the case of students using $\lambda = Q/2\pi R$, this was something that every student in the class could easily access, and therefore there were no extended discussions about this result. However, in Shawn's case, it may have been clear to Shawn that $T = 2\pi R/v$, but before being accepted by the group some additional verification was needed. This verification came in the form of Biff checking the units.

Sayre and Wittmann (2008) used an interview to create a detailed analysis of how two different students used Cartesian and polar coordinates when solving for the time it takes for a pendulum to swing over a given arc. In Sayre and Wittmann's case, a TA asked questions in order to force students to consider certain things and in order to probe their thinking. With classroom video data, there is the disadvantage that we cannot probe student thinking at a deeper level, but there is the advantage that we can see how students use their plastic resources *in situ*.

Allen in Group 1 tried to take his plastic understanding of angular speed and connect it to more solid understandings by directly using geometric relationships. Nick, Bob, and Tanya in Group 2 did not use explicit geometric relationships and instead linked their tenuous understanding of ω to other concepts that they weakly understood, such as radians. Stan and Kevin in Group 4 relied almost entirely on authority to make claims and counter claims about what equation should be used. Shawn in Group 5 was able to establish the correct relationship on the first try, and Biff used his strategy of checking units to establish the validity of Shawn's equation with all the students in the group. In Group 6, which was not previously mentioned, one student introduces the incorrect relationship $\omega = 2\pi R/T$, and another student effectively uses unit analysis to convince the other two students that $2\pi R/T$ represents a "tangential velocity", and not an angular velocity.

Students in upper-division courses are asked to utilize many mathematical relationships and conceptual ideas from lower-division courses. Some of these ideas will not be fully developed in the students' thinking, and the students will be faced with repeated situations in which they are asked to draw on resources that are not solidified or not fully formed. The students in the five groups analyzed here give some reference points for the ways in which students respond when employing these plastic resources.

The preceding sections were only concerned with geometric resources. Interestingly, only Allen in Group 1 explicitly employed geometric thinking to try to resolve the uncertainties surrounding this particular geometric resource. For a large number of students geometric thinking is difficult and geometric reasoning is not always the first thing employed when encountering problems involving geometry.

4.5 The Least Familiar Level: Encountering a New Problem

We will now consider what students do when they encounter a new situation. Unlike the very familiar $C = 2\pi R$, or concepts like angular velocity that students had worked with repeatedly on previous occasions, the situations will now involve students facing what are to them new or novel aspects to the problem.

In this case two situations will be considered. The first is how students deal with the direction of the current as related to the integration. In the second situation, students needed to put $\hat{\phi}'$ in rectangular coordinates.

Unlike in introductory physics problems, the direction of the current cannot be ignored, pulled out of the integral or "tagged on" at the end of the problem. In introductory physics, students can frequently treat current like a scalar, such as when dealing with circuits. When dealing with magnetic fields, students can often rely on formulas from the text, in which the author of the text has already performed all the troublesome integrations. Students can then use the right-hand rule to find the direction of either the field or the current once they have determined the scalar magnitude of the quantity they are seeking.

Students in the Paradigms courses have faced very few problems in which it was not explicit whether they needed to include a direction. Thus, these students have limited experience in determining when the vector nature of something is or is not relevant. The students have even less experience (if any) in examining a physical situation and determining whether or not the vector quantities in the corresponding integral can be pulled out of the integral.

Student misunderstandings about vector relationships, especially in electricity and magnetism, have been documented in a variety of studies in both lower-division (e.g. Knight, 1995; Scaife & Heckler, 2011) and upper division students (e.g. Kesonen, Asikainen & Hirvonen, 2011; Manogue, Brown, Dray & Edwards, 2006; Singh, 2006; Wallace & Chasteen, 2010). The data from watching students solve the spinning ring problem lend support the conclusion that there are many "student difficulties" with vectors in upper-division E&M. However, documenting "difficulties" is not the focus of this section. Instead, the focus is on how students approach geometric problems when the situation is unfamiliar.

When students recognized that current direction was important to this problem, they struggled to figure out exactly how to address it. However, several students either didn't realize the direction was a concern at all, or thought the direction could just be "added on" at the end of the problem, similar to the way they can use right-hand rule to "tag on" a direction after solving an equation in which everything can be treated as a scalar.

This ring problem was the third in a series of four problems students had to solve that involved a ring of charge. The previous two problems were solving for the electrostatic potential and the electric field. In these cases the ring was stationary. When finding electric field due to a stationary ring, students had to deal with the direction of the field, in addition to the position vectors $\vec{r} - \vec{r}'$. However, they did not need to deal with any vector motion of the ring. Thus, there was nothing from the previous ring problems that would have specifically cued students that the vector nature of the current would be relevant for solving the problem.

Several students approached the problem by beginning with the general formula for magnetic vector potential

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r})d\tau'}{|\vec{r} - \vec{r}'|}$$

written on the blackboard at the front of the classroom. Although there are many vector symbols in the equation, many students do not cue into the importance of these. As noted in the previous sections involving angular velocity, students have not clearly disambiguated relationships such as linear speed $v = \omega r$ and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$. Vector symbols and vector language do not always sufficiently alert students to the importance of the vector nature of the quantities involved.

At one end of the spectrum, some students never discussed the direction, did not use vector signs over their current or velocity terms, and created integrals with no direction vectors in the numerator. At the other end of the spectrum, students fully recognized that the direction of the current was important and spent significant time considering how to deal with this. There was also significant middle ground. Some students considered direction briefly, but then did not put it in their equation. Some students recognized that direction was relevant but explicitly claimed that they could simply apply the right-hand rule to deal with the direction. Other students put $\hat{\phi}'$ (or "$\hat{\phi}$"), at the end of their integrand, but did not recognize that the cylindrical basis vector $\hat{\phi}'$ changes direction during integration and that this is problematic.

Students treated the direction of the current in one of five ways;

1) ignoring it and treating it as a scalar
2) initially including direction and then losing it at some point in the solving process
3) recognizing the direction and addressing it with the right-hand rule
4) putting $\hat{\phi}'$ in the equation but not recognizing it as problematic
5) genuinely recognizing that the current direction needed to be carefully considered.

We will consider examples of each of these approaches.

4.5.1 Treating Current as a Scalar

The first example to consider is Biff in Group 5 in which Biff treats current as a scalar. Biff's group includes Shawn, who in the previous examples was the only student to use angular velocity as a solid resource. However, in this case, Biff works on his own for a short time before interacting with the other students. In this example, Biff makes several conceptual and dimensional errors, in addition to his error in considering of direction. Biff initially writes $\frac{Q}{T}\hat{\phi} = Rd\phi\hat{\phi}$. How Biff got this relationship is uncertain, but we hypothesize that as Biff pictured an arc that represented *dQ*, he also pictured the physical arc in terms of *Rdφ*. When this is considered, one can see how Biff could come to think that $\frac{Q}{T} = Rd\phi$. However, the focus here is on Biff's consideration of the direction of the current.

Biff initially includes $\hat{\phi}$ in his equation $\frac{Q}{T}\hat{\phi} = Rd\phi\hat{\phi}$. In an earlier discussion with Alice, he explicitly mentions that current is in the $\hat{\phi}$ direction. However, he immediately proceeds to cross out $\hat{\phi}$ in both places and then writes

$$\frac{Q}{t}\cancel{\hat{\phi}} = Rd\phi\cancel{\hat{\phi}} = \int_0^{2\pi}\frac{QR}{T}d\phi = \frac{2\pi QR}{T}$$

Biff proceeds to check the units of $2\pi QR/T$ to see if they agree with the units of magnetic potential. This indicates he thinks he may have "solved" the problem at hand and is satisfied with a scalar answer for his result. Shawn, a student in his group, looks at Biff's equation and is concerned. Shawn immediately questions the absence of $r - r'$ in Biff's formula.

Biff never indicates why he thinks he should cross out $\hat{\phi}$, but his doing so shows that he thinks that he can ignore the direction in the final result. Biff is missing many important aspects of this problem, including ignoring the position vectors. Biff was the only student in the class to completely ignore the position vectors in his initial attempt to solve the problem. Furthermore, that his equation would imply that magnetic vector potential is exactly the same everywhere in space, does not seem to enter Biff's thinking.

Although Biff is the only one to ignore the position vectors, he is not the only one to treat current as a scalar. Another example comes from Tanya in Group 2. Group 2, with Tanya, Nick and Bob was discussed in the previous section when they struggled with linear versus angular velocity and never clear disambiguated the two concepts.

After working on her own for several minutes, Tanya creates an integral and puts a box around her result:

$$\vec{A} = \frac{\mu_o}{4\pi}\int_0^{2\pi}\frac{\lambda RT}{|r - r'|}d\phi'$$

It should be noted that she has a vector sign over $\vec{A}$, but does not have vector signs anywhere else in her equation. There is no vector direction in her integral, but she gives no indication that she considers this problematic. Tanya frequently approached this problem from the standpoint of calculation instead of employing in-depth physical or geometric reasoning. This is not atypical for Tanya, who has shown in several contexts that she enjoys algebraic manipulation and is fairly good at it, but is frequently frustrated by problems that require geometric reasoning.

In addition to omitting a direction, Tanya also made errors when equating physical quantities. For example, at one point she claimed that current could be expressed as Q/ω (instead of Q/T), which resulted in her getting the period T in the numerator of integrand, instead of in the denominator (see Chapter 5, section 5.4.2).

In the preceding examples, Biff and Tanya are not aware of the importance of the vector nature of the quantities they are considering. As evidenced by some of their other mistakes, there are other areas in which their understanding of the problem is also incomplete. Furthermore, neither attempts to understand the meaning of the equation they have obtained. Biff "checks units", but in this case he does not check to see if the units are internally consistent. Instead he only wants to know whether the units of his "answer" match the units he is supposed to have for magnetic vector potential.

Tanya does not question that her formula shows that a longer period will result in a stronger magnetic potential. Nor does she seem troubled by having a vector equal to a scalar. There is no evidence that she uses any sort of sense-making strategy. The combination of conceptual errors and failure to engage in sense-making leads Biff and Tanya both to accept nonsensical answers.

Nick, another student in Group 2 (Tanya's group), gets an equation that contains more detail and is a closer match to the correct equation. However, Nick also has equated a vector on the left-hand side of the equation to a scalar on the right. He produces the following:

$$\vec{A} = \frac{\mu_o Q}{4\pi RT}\int_0^{2\pi} \frac{d\phi'}{\sqrt{r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2}}$$

When Corinne comes to talk to the group, she initially focuses on the difference between angular speed and linear speed, which results in Nick eliminating the *R* from the constant outside the integral, to get

$$\vec{A} = \frac{\mu_o Q}{4\pi T}\int_0^{2\pi} \frac{d\phi}{\sqrt{r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2}}$$

Nick then asks, "So is this good?" Corinne responds, "Well, now, so *Q* over *T*, 4π, yes...uh...except that ***J*** is a vector." Nick responds, "Oh. This is, uh, *φ*-hat." And writes a $\hat{\phi}$ at the end of his equation to get

$$\vec{A} = \frac{\mu_o Q}{4\pi T}\int_0^{2\pi} \frac{d\phi}{\sqrt{r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2}}\hat{\phi}$$

Nick's equation now has many correct aspects. It is still off by a factor of *R* (because $d\ell' = Rd\phi'\hat{\phi}'$) and it has an omitted the prime symbol on $\hat{\phi}$. Whether Nick is aware that the direction vector $\hat{\phi}'$ must correspond to the direction of the current in the ring is unclear. However, there appears to be no awareness that $\hat{\phi}'$ changing direction during integration is problematic. While it can not be determined definitively from Nick's actions, he appears to be satisfied simply "tagging on" a direction. This is similar to how direction is added at the end of introductory physics problems by using the right-hand rule or other method.

It is interesting to note that Tanya, who was listening to Corinne during this interaction, did not add a direction to her equation. Corinne's comment, "except that ***J*** is a vector," indicated to Nick that he needed to add a $\hat{\phi}$ to his equation. It did not inspire Tanya to take a similar action. Thus, it could be argued that Nick at least has more awareness or understanding of the issue than Tanya.

4.5.2 The Right-Hand Rule: Overview

The next category of students we will consider are those who used the right-hand rule for determining direction. Because magnetic vector potential was a newly introduced concept, most students had not yet disambiguated magnetic field and magnetic vector potential. Thus, students attempted to apply the right-hand rule for magnetic fields to this magnetic vector potential problem.

Before discussing a specific example, consider the many different ways in which the term "right-hand rule" is used in physics classes. The thumb, pointer finger, and middle finger can be used to show how to establish a right-handed coordinate system. This right-hand rule is often used for showing how a right-handed coordinate system can be represented in different diagrams, even though the *x*-, *y*- and *z*-axes are sometimes drawn differently, such as the +*z* pointing up, or +*z* pointing out of the page.

The right-hand rule is also used for finding the direction when taking a cross product, such as when finding torque, $\vec{\tau} = \vec{r} \times \vec{F}$, or when finding the force on a charged particle moving through a magnetic field $\vec{F} = q\vec{v} \times \vec{B}$. Different instructors have different ways they teach the right-hand rule for a charge in a magnetic field, but in one version the pointer finger represents the motion of the charge, the middle finger represents the field, and the thumb represents the force.

This three-fingered right-hand rule system is used in multiple ways. However, for magnetic fields, there is also a right-hand rule in which the curled fingers are used. This is sometimes referred to as the "right-hand-grip rule", but is often simply called the right-hand rule. There are two versions of the curled-finger right-hand rule when it comes to magnetic fields. The most common version uses the thumb to represent the direction of the current and the curled fingers to represent the magnetic field $\vec{B}$ (Figure 2). However, there is also a version for solenoids that is sometimes used in physics and engineering classes. In this case, the curled fingers represent the current through the coils and the thumb represents the magnetic field (Figure 3).

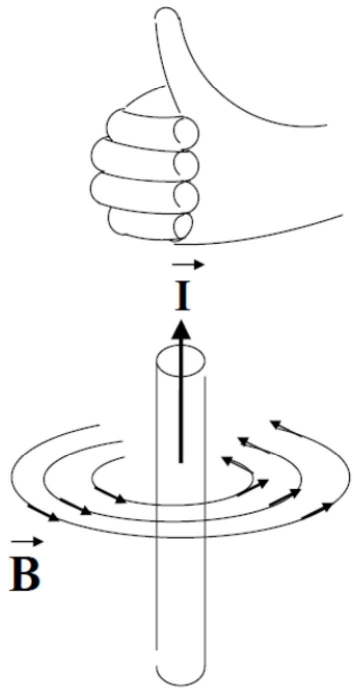

Figure 2: Right-hand rule for a straight current-carrying wire

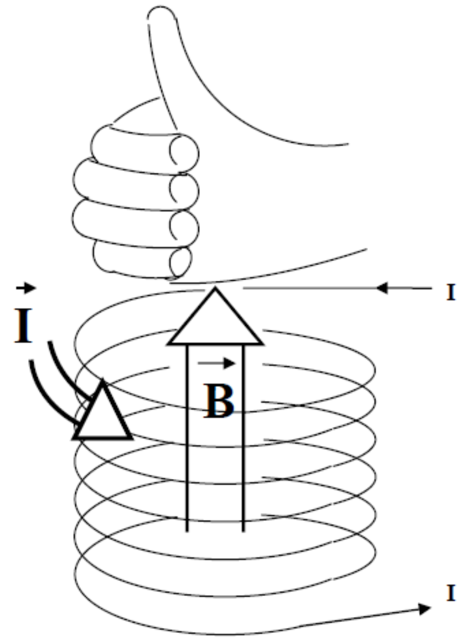

Figure 3: Right-hand rule for a solenoid

Given that there are many different uses and representations of the "right-hand rule", it is not surprising that some students are confused about the applications of the different versions of the right-hand rule. To add to the confusion, many students have not distinguished between the magnetic vector potential $\vec{A}$ and the magnetic field $\vec{B}$., and some also confuse the current $\vec{I}$ and the field $\vec{B}$. As will be shown in the next section, the combination of these muddled understandings creates a situation in which a group of students can all agree that "the right-hand rule" applies, but no one can be sure if they are communicating the same idea, and no one is entirely sure what they are asserting.

4.5.3 Group 4: Attempting to Use the Right Hand Rule

Group 4 provides an example of how students used the right-hand rule in this problem. Within the first minute of working on the problem, the group considers the direction of the magnetic vector potential. A picture of the ring is drawn and students are adding to this drawing. [00:42:24.22]

> Stan: "Spinning..." [draws a curved arrow next to the ring],
>
> Robert, "Draw"
>
> Stan, "...current..." [draws an upward vector along *z*-axis],
>
> Robert, "Yeah."
>
> Kevin: (talking over Stan) There's got to be some moment of inertia in here.
>
> Stan, "...right hand rule..." [gestures fingers curled, thumb up] "...or, *B*..." [Stan labels vertical arrow "*B*"] "...or *A*..." [changes "*B*" to an *A* (Figure 4)]

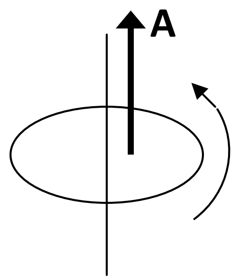

Figure 4: Stan labels vertical arrow “A”

Robert [repeatedly gestures curled fingers with thumb up] "Basically,… basically the field is going to go up...the whole right-hand-rule thing...spins that way, current up.

Stan gestures curled fingers with thumb up

Kevin draws a new, larger ring

Robert [referring to ring]: “Well yeah. So if you say it's spinning that way...” [draws arrow on ring]

Stan: "Then, then it'll be up." [labels upward on the z-axis $A(r)$] "$A(r)$"

Kevin:Yeah

Robert:Yeah

Stan changes from saying that current is up to saying that the magnetic field $\vec{B}$ is up, to saying that $\vec{A}$ is up. Similarly, Robert makes the claim that both “the field” and “current” will be “up”. The students in this group accept without question the authority of the right-hand rule, but are unsure how to apply it appropriately to this situation.

Several minutes later, Stan is concerned about the direction of the current. The students in the group briefly consider whether the direction of the current is important to the problem. However, Kevin manages to give a quick answer and the group moves on to other topics [00:48:43.01]

Stan: “It looks like we need a direction for the J. Is that true?”

Kevin: “No, it should all be in the radial direction.” [gestures by putting his two hands, palms toward each other and then opening up an angle (Figure 5)]

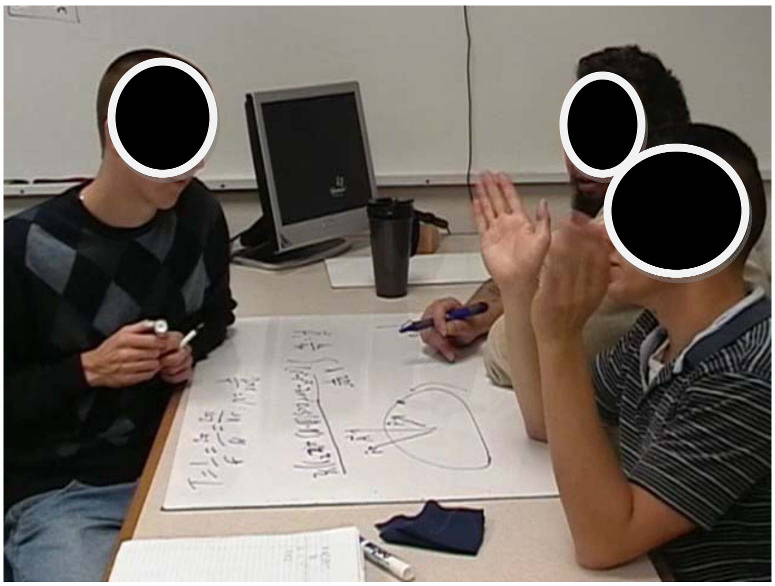

Figure 5: Kevin gestures the “radial direction”

Robert: “Yeah.” [gestures by curling fingers, thumb up (Figure 6)]

Kevin: “Yeah.”

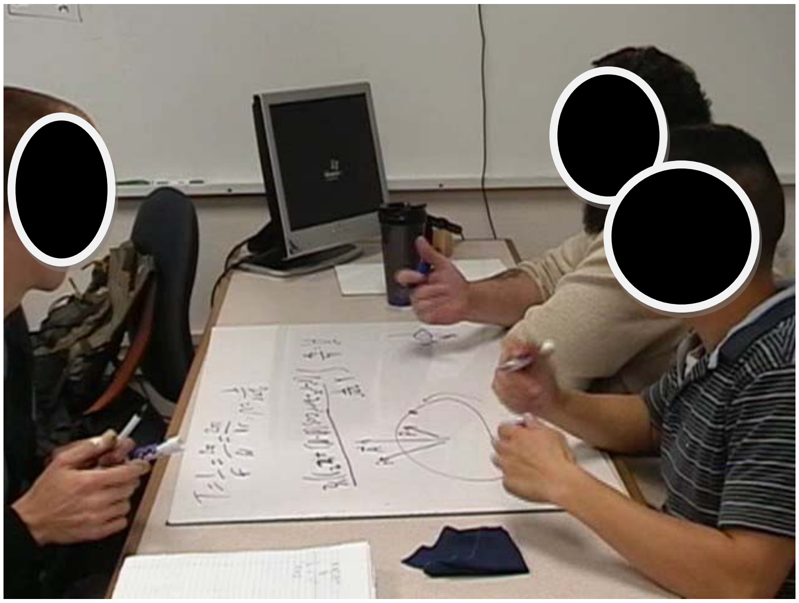

Figure 6: Robert’s gesture, responding to Kevin

Stan claims that direction is important, but the group rapidly dismisses this concern. The group somehow accepts that the current has a direction, but the nature of that direction makes it unimportant. The group has several concepts muddled. Kevin uses the term, “radial direction”, but gestures the opening of an angle. Robert affirms Kevin’s statement with a “Yeah” but gestures the curled fingers and thumb up. One could speculate that they may all have had some understanding of a current with a direction tangent to the ring. However, even if this were true, it is unclear if these arguments are implying that because the current is “constant” in some respect (in this case being “consistently” in the $\hat{\phi}'$ direction) that it does not need to be considered. The inexact language and gestures leave their understanding ambiguous. What is clear is that the students are not carefully considering the issue and resolving these ambiguities.

Three minutes later, Dr. Alice looks at their equation and validates various parts of it. However, she then raises the issue of direction. [00:52:49.03]

Alice, "OK, and what about the direction information?"

Robert, "It's supposed to be everywhere in space though..."

Alice (over Robert), "That's what you're missing"

Robert,"...right?...so"

Alice, "Right, but this thing is a vector." [points to "A" on drawing]

Robert, "Ahh, gotchya"

Alice, "How do you know that?"

Stan, "Well, it's upward, it's *z*-hat" [pulls pen to write it down]

Robert, "Ya' know the right hand rule, if it's rotating this way..." [Kevin, Robert, and Stan all gesture curled fingers with thumbs up (Figure 7)]

"...then, then it will go up."

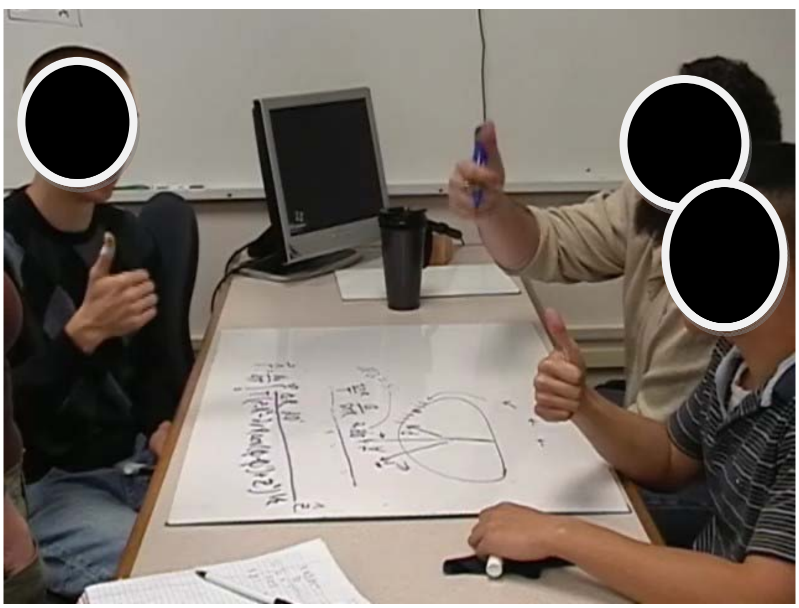

Figure 7: Kevin, Robert and Stan all gesturing the right-hand rule

Here several things can be seen. One is that students are still trying to make sense of the requirement to find the integral that represents the magnetic vector potential "everywhere in space." The "everywhere in space" language confused several students in different groups at different points in the problem. In this particular case, the student appears to make the case that because the magnetic potential is everywhere in space, then a direction can't be given (or at least isn't required).

Once students are convinced that a direction is required, they attempt to just "add it on" at the end. They use the word "it" to describe what goes up, and they confuse magnetic field with magnetic vector potential and claim the field will be in the $\hat{z}$ direction. Other groups also invoked the right-hand rule and confused magnetic field with magnetic vector potential, however, it is interesting that this group claimed that the field would only be in the $\hat{z}$ direction, which would be the direction of the magnetic field in the center of the ring and everywhere on the z-axis, but not the field direction at other points around the ring.

The instructor is able to raise these issues. The dialog continues as follows: [00:53:20.12]

> Alice, in a dramatically calm voice, "The magnetic FIELD....is that right?" [Alice nods]
>
> Robert, "And so would the current, and..."
>
> Stan, "But we're not doing magnetic field, we're doing magnetic potential."
>
> Alice (over Stan), "...doing...potential, vector potential."
>
> Robert [rubbing head], "Ahhh"
>
> Alice, "So I doubt that you'll have any intuition about the direction that will help you see this problem. In the end, we'll talk about direction."
>
> Kevin, "Does it have something to do with curl?"
>
> Alice, "I don't know how to answer that yet. It definitely has something to do with the direction of the current."
>
> Robert, "Could we write..."

Alice, "Right, because, because, so what she has up there is incomplete." [points at the blackboard with the general equation] "That $\boldsymbol{J}$ is a vector $\boldsymbol{J}$, right? The current has a direction."

Stan (over Alice), "So we need, we need a direction on this" [pointing to $2\pi R/T$ on board]

Stan, "So the, R velocity."

Alice (responding to Stan), "Yes"

Robert (over Stan and Alice),"So you could say this, goes somewhere" [points at the board]

Stan, "And that is in the phi-hat direction?" [writes $\hat{\phi}$ in the numerator of his integrand]

Kevin (over Stan), "And the velocity is tangential..."

The group suddenly becomes aware that magnetic potential and magnetic field are different and that they are solving for the magnetic potential. Alice's comment, "So I doubt that you'll have any intuition about the direction that will help you see this problem," raises the issue that not only is this ring problem hard for many reasons, but students have no intuition about magnetic potential that they can apply as they solve the problem.

Once told that he needs to consider the current direction, Stan inserts $\hat{\phi}$ into the numerator of his integrand. When it is brought to his attention, he also able to explain why it should be $\hat{\phi}'$ instead of $\hat{\phi}$. Here is the dialog in which Stan clarifies this:

Alice, "Right, and is it a $\boldsymbol{\varphi}$-hat or a $\boldsymbol{\varphi}$-prime-hat?"

Stan, "Oh, it's $\boldsymbol{\varphi}$ prime hat." [writes a prime with $\boldsymbol{\varphi}$]

Alice, "How do you know it's a prime hat?"

Stan, "'Because it's...the...with the charge. The charge is the part with...[inaudible]...not this way"

Alice, "Right, you're referring to the current, right?"

Stan, "Yes."

Alice, "Right, and so it has to be a prime." [points at $\hat{\phi}'$ on board] "Does that makes sense?...If you're going to do that, convert it to a prime...OK, can you...move forward from here? I like this. This is nice."

Unlike the students who treated the problem entirely as a scalar problem, the students who used the right-hand rule recognized that the direction mattered. They immediately cued the resource that they used most frequently in introductory physics when needing to determine a direction. Unfortunately, this convenient resource was not fruitful in this situation and student ambiguity around the physical quantities involved led to students not realizing the more sophisticated geometric issues involved in this problem. The conceptual right-hand rule prevented them from realizing that they needed to carefully consider how to “chop and add” the current in this problem.

Half of the six groups had at least one student invoke the curled-finger version of the right-hand rule. The various right-hand rules are powerful tools that allow introductory physics students to determine the direction of things like magnetic forces without actually doing sophisticated integration. It is not surprising that a significant portion of the class reached for this strategy.

None of these students raised explicit concerns about the multiple interpretations of the right hand rule. The opportunity existed for a student to say, “Whoa, what exactly are we claiming is going around and what exactly are we claiming is going up?” However, this does not occur in these groups.

4.5.4 Wrestling with Direction: Students Who Recognized the Problem

There were two groups that recognized that dealing with the current direction was not trivial. All three students in Group 1 spent significant time considering the vector nature of the current and worked collectively to reach understandings. Shawn in Group 5 also wrestled with the current direction, but in his group's case, Shawn had to spend effort convincing the other two students in his group that their attempts to oversimplify the situation were missing the mark. We will consider Shawn in Group 5 before looking at the students in Group 1.

4.5.4.1 Shawn in Group 5

When Shawn first calculated that the magnitude of the current was Q/T, he immediately expressed the current as ${}^{Q}/_{T}\,\hat{\phi}$. He was clear that he was including a direction. Shawn realizes that the direction of the current is important. However, he has difficulty figuring out how to retain the direction information during integration.

In the following dialog, Shawn struggles with how to include the direction information during integration [00:48:44.24]

> Shawn "We're going to have the integral of ***I*** dot *d**r***..." [writes$\int \vec{I} \bullet d\vec{r}$]
> Biff, "Where *d**r***..."
> Shawn (over Biff), "Where *d**r***, *d**r*** equals *Rdφ* (*φ*-hat)." [writes $d\vec{r} = Rd\phi\hat{\phi}$]
> Biff, " No, no-n-no-n-no, that's, that's magnitude *d**r***." [writes an absolute value sign around $d\vec{r}$ to get $|d\vec{r}| = Rd\phi\hat{\phi}$] "... *d**r*** is really...Oh, oh, oh, I see. You already simplified, yeah, you already simplified it then, OK, OK."

Shawn, "And then, and then when you dot those two you get a ...[points at $\hat{\phi}$ in $|d\vec{r}| = Rd\phi\hat{\phi}$]...we don't want to lose - we don't want to lose the φ-hat though."

Biff, "…and…So we have Q over t, φ-hat " [writes $Q/t\ \hat{\phi}$]

Shawn turns toward front classroom board.

Biff, "Yeah we..., why not?"

Shawn, "Because, well, yeah we do, but..."

Biff, "But we need a..."

Shawn, "We should go back,...remember, like, you have to go back to vectors for the ***A***."

Shawn using $\int \vec{I} \bullet d\vec{r}$ is a subtle and interesting error. It is completely valid to consider the current as having the direction information and the *dr′* as a scalar, or it is completely valid to consider the current as a scalar and consider the $d\vec{r}$ as carrying the direction information. However, it is not valid when Shawn writes $\int \vec{I} \bullet d\vec{r}$. The question of what quantity carries the direction information is something Shawn has never faced before in this way. It is not surprising that he is challenged by it. However, Shawn is insightful about this and quickly realizes that $\int \vec{I} \bullet d\vec{r}$ is problematic because "…we don't want to lose the φ-hat though." Thus, Shawn recognizes the need to achieve an end result that includes the direction information, but is not immediately sure how to accomplish this. Not knowing how to proceed, Shawn comments, "We should go back,...remember, like, you have to go back to vectors for the ***A***."

This comment, along with input from other students, switches the focus of the group to discussing $\vec{r} - \vec{r}'$ and other issues for several minutes. However, Shawn eventually returns to considering the direction of the magnetic vector potential. In the following dialog, Biff suggests that the right hand rule can be applied to determining the direction. Shawn does not debate whether or not the right-hand rule applies. Instead, he argues that given the right-hand rule, it does not give the direction at points not on the *z*-axis. [00:56:25.15]

Biff, "Say by the right hand rule, it's in this direction." [gestures right hand rule with thumb up]

Shawn, "But if you're, like, way up here at some weird point..." [points to a place on board away from the ring and off-axis]

Devin, "Yeah, but right-hand rule is kind of a sketch. You still have to have an exact [inaudible]."

Shawn, "Like if you're way up here," [draws an external point (Figure 8)], "like, which," [gestures from ring to external point] "I mean which way is it going to point?"

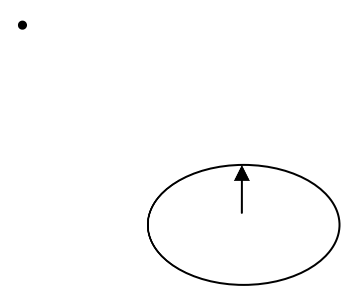

Figure 8: Shawn draws an external point

Shawn points back and forth between different locations on the ring and the external point

Biff writes *Rdθd* and says, "*R* d(phi) d-what? What, what was your thing?"

Devin, "That's a good question, where's it going to point right there?" [points at the external point on drawing]

Shawn, "I don't know. Like that way, or something like that." [draws an arrow from the external point (Figure 9)]

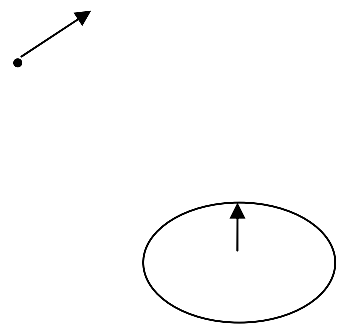

Figure 9: Shawn draws a vector from external point

Devin, "You know, right here it's going to point straight the hell up." [points to the center of the ring and gestures upward]

Shawn, "But out here it's going to point," [gestures back and forth at the external point] "I don't know."

Devin, "It's tricky"

Shawn never figures out how to address the problem until the instructor later discusses the problem with the entire class. However, Shawn clearly recognizes that determining the direction of the magnetic vector potential is not simple and he is fully aware that he does not know how to do it. At a separate point in the discussion, Shawn made a tenuous attempt to disambiguate the concepts of magnetic field and magnetic vector potential, but at this point he tentatively accepts the use of the right-hand rule and still makes the point that it doesn't solve the problem. Unlike the students in Group 4, he recognizes that even if the right-hand rule applies, the only place that finding the direction is trivial is on the *z*-axis. It does not easily resolve the problem of the direction "at some weird point."

Shawn takes two entirely different approaches for dealing with direction. One involved attempting to simply "do the math" and find $\int \vec{I} \bullet d\vec{r}$. However, when he recognizes that it will be problematic that the direction information is lost, he attempts an entirely conceptual approach based on trying to visualize what happens at some off-axis point. Neither approach is successful in achieving a solution. While Shawn never reaches an acceptable answer on his own, Shawn never settles for an incorrect answer. Shawn's symbolic and conceptual approaches provide opportunities for Shawn to demonstrate that he "knows what he doesn't know."

4.5.4.2 Group 1: Three Students Thinking about Direction

Next we will consider Group 1, with Laura, Tom and Allen. In Group 1 the students tend to pay close attention to each others' comments and frequently attempt to justify their statements.

Allen becomes the first student in class to mention using rectangular basis vectors for the numerator of the integrand in order to be able to evaluate the integral. He claims it is needed "for Maple's convenience," referring to the need to get the equation in a form that the symbolic calculator Maple can evaluate. However, Allen rapidly becomes unsure if the rectangular coordinate system is actually necessary. [00:54:00.21]

> Allen, "So then we'll write *dr* in terms of **i**'s, **j**'s, and **k**'s for Maple's convenience."
> Tom, "Yup"
> Allen, "And then we're in cylindrical...[points to denominator of integrand]...I'm gonna guess, for this portion"
> Tom, "So what is our *dr*?"
> Allen "Wait, we can write *dr* in terms of cylindrical coordinates."
> Laura (over Allen), "...because we have to...OK, so we have a...?..."
> Tom "...Wait, no,no,no, ...yuh...we are going to have a d(theta)."
> Laura, "d(phi), right?"
> Tom "d(phi)"
> Laura, [inaudible]
> Allen, "...well because this is a vector though *dr* right?..." - writes a vector sign over the dr' in Laura's formula to make

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})d\vec{r}'}{|\vec{r} - \vec{r}'|}$$

"… So then that will have components of *dr* ***r***-hat plus *rd...rdr*- ***r***′-hat."

Allen is not clear about how to properly put dr' in terms of an appropriate coordinate system, but he is clear that there needs to be direction information. Allen has the insight that many students in the class did not. He recognizes that there is a vector in the numerator of the integrand that needs to be dealt with. It should be noted that although all three group members frequently use " dr" to refer to dr', their gestures and drawings consistently show that they are referring to dr' and are thinking about the current in the ring. In addition, Laura explicitly has dr' in her equation and Allen at one point refers to $\hat{r}'$.

Laura has $\vec{I}(\vec{r})$ written as a vector and dr' as a scalar, but Allen considers the direction information to be carried by dr'. Allen suggests writing dr' in terms of **i**'s, **j**'s, and **k**'s and is explicit that dr' is a vector. Allen claims that one of the components of dr' should be $rdr\hat{r}'$, but Tom interjects a new idea before Allen has a chance to complete his thought. Regardless of the nuances or the accuracy, Allen is not simply treating the numerator like a scalar, nor trying to simply "tag on" direction at the end.

Tom now uses the ring diagram to clarify the nature of dr'. The following dialog shown above continues as follows: [00:54:42.26]

> Tom draws two marks close to each other on the ring and says, over Allen, "
>
> > Well, but there's not a ...no this is, this is *dr* around here...here to here; that's *dr*, [writes *dr* next to two marks] ..which is *Rdφ*
>
> Laura, [writes on board, $|dr| = Rd\phi'$] "Right"

Tom, [writes on board, $x = r \cos \varphi$ and $y = r\sin \varphi$ and $z = z$] "Yeah, because when we,...*x* is *r* cosine-*φ*...*r sin φ*...Right, so that's where we're going to use the phi's. Right?"

Allen, "Right."

Laura, "Alright, so magnitude of our *dr*" – writes absolute value signs around the $d\vec{r}'$ in her equation to get

$$\vec{A} = \frac{\mu_o}{4\pi} \int \frac{\vec{I}(\vec{r})|d\vec{r}'|}{|\vec{r} - \vec{r}'|}$$

"That's right, because you write it like that."

Tom draws a small distance on the ring to physically/geometrically represent *dr*. This explicit "chopping" allows for a visual representation when thinking about integration as "chopping and adding". Tom then claims that *dr* is *Rdφ*. Tom immediately starts to think about how the position can be represented in rectangular coordinates.

Unlike Allen, the other students, Laura and Tom, do not consider dr' to be carrying the direction information. Laura has a vector over her symbol for current, and in the process of changing *dr* to *Rdφ*, Tom has clearly indicated that dr' is a scalar. Laura does not directly challenge Allen's claim that $d\vec{r}'$ is a vector, but uses absolute value signs to clarify that only the magnitude is needed for this equation.

The group now switches to thinking about how to integrate the current. Early in the problem, (roughly 10 minutes before the dialog below) it is Tom who initially expresses ideas about chopping up current. When the group found that current was *Q*/*T*, Tom was concerned that this result showed the total current and not current density. He indicated that they needed the current density in order to chop it up.

As the dialog continues, the group tries to determine how to “chop and add’ the current in order to integrate. The direction of the current is not specifically discussed, but it is interesting to note that the group never loses the direction information as they wrestle with how to break up the current into bits that they can integrate. [00:55:42.04]

Laura, "Ok, and then this is just *Q*/*T*...[points to $\vec{I}(\vec{r})$ in equation]…wait,...yeah,... but if it's *Q*/*T* then that means that we've already integrated over phi."

Tom,"Oh, you're right, we needed a *dq*...which is lambda…"

Allen (over Tom), "This is for each element of charge?"

Laura, "But we don't, we don't know our interval of time?"

Allen, "From zero to *T*"

Tom (laughs)

Allen, "right, through one period"

Laura (laughs), "...and we have, we have *Q*/*T* but,....you can't, you can't...this is how..."

Tom [points to drawing], "But it's gonna to be their little individual contributions, so there's going to be [writes on drawing $dq = \lambda R d\varphi'$] uh..."

Laura, "A *dq*... yeah"

Tom (writes equation on board), "A *dq* is... $\lambda R d\varphi$ Why do we have two $d\varphi$'s?)"

Laura, "No, we don't have two $d\varphi$'s "

Tom, (over Laura), "This'd be one"

Laura writes a new expression on the far right side of the whiteboard

$$\frac{\lambda\vec{v}|d\vec{r}|}{|\vec{r}-\vec{r}'|}$$

Laura, “Cause, ok, ok, ok, ok,..so we have, we have lambda *v dr*, over **r** minus **r**-prime, right? So, our....uh....so our *dr* is $Rd\varphi$...over that,...and then our λv is just going to be *Q*/*T*....wait, now I'm confused."

Tom, "But the *Q* has to be chopped up."

Corinne starts to talk to whole class together

Allen, "Why does the Q have to be chopped up though, because we're just considering that all the current goes through?"

At the start of this portion of the dialog, Tom accepts Laura's argument that using Q/T, which is total current, indicates that the integration must have already been done. At one level they are not understanding that the magnitude of the current is constant and that the magnitude could be pulled out of the integral regardless of whether it is expressed as Q/T or whether it is expressed in terms of λ. However, at another level, they are having an important discussion about what needs to be chopped up and how it needs to be chopped up. This discussion was absent from some other groups that were willing to treat the current as a scalar and move on.

Allen questions the need to chop up current, but the group runs out of time before we can see how his thinking develops and how far this group can get on their own. The group is very explicit about the need to "chop up" things before adding during integration. Laura uses the phrase, "interval of time", Tom uses the phrase "their little individual contributions," and both Tom and Allen discuss whether Q needs "to be chopped up."

At this point the group is not clear how to solve the problem, but they are aware that there are things that are unresolved. None of the three students has settled for an incorrect result.

4.5.5 Conclusions about Students Facing a New Geometric Problem

Student responses to facing the problem of dealing with current direction fit into three general categories; students who ignored or were unaware of the problem; students who recognized that direction was important, but thought it was trivial to address with a resource they already possessed; and students who recognized this was a problem requiring careful consideration.

In order to make meaningful progress when faced with a new situation with new challenges, students first need to recognize that the problem has difficult or novel aspects. In this case, when students did not recognize that the current direction was important to this problem, they treated direction as something they could address easily, and did not seek to tackle the hard parts of the problem. Instead, they used inappropriate or oversimplified methods. When accompanied by failure to engage in sense-making, these students were satisfied with a nonsense answer. Every student who did not recognize the need to deal with current direction, also used little, if any, explicit consideration of the geometry in reaching and examining their conclusions.

For those students who did recognize that current needed to be considered, many reached for familiar resources, such as the right-hand rule, and became convinced that these familiar resources were sufficient to deal with this unfamiliar situation. Several students used the right-hand rule as if it were a gold-standard, even though it was in reality not a very solid resource.

For students who recognized the issues with current direction, this problem was very difficult. The students did not see a clear way to overcome the challenge of how to deal with current direction during integration. However, these students did not give up easily. Even when one approach failed, they were not willing to accept an over-simplified solution. They did not accept an answer they did not understand. Furthermore, these students all addressed this geometric problem as a geometric problem. They used drawings and geometric language and justifications as they wrestled with the issues at hand.

It is interesting to note that the students struggling the most were *not* necessarily the students with the poorest understanding. Instead, the students struggling the most were often the students who understood many of the complexities of the problem and were wrestling with these difficult aspects. Some of the students who did not engage in the difficult geometric reasoning reached an (incorrect) solution more rapidly. Tanya in Group 2 reached an answer she was willing to put a large box around in less than 7 minutes. On the other hand, Tom, Laura, and Allen from Group 1, along with Shawn from Group 5 were still dealing with several unresolved issues when the 17-minute work period ended.

4.6 The Problem with $\widehat{\boldsymbol{\phi}}$

We will now briefly consider students attempting to express $\hat{\phi}'$ in terms of rectangular basis vectors. This is a second example of students facing an unfamiliar geometric problem. All the previous examples in this chapter came from the work that students did during a 17-minute problem solving session on a Friday. The following Monday the class reconvened. After the instructor described many aspects of how to solve the problem, she drew students attention to the changing direction of $\hat{\phi}'$. She tells students that Maple cannot evaluate an expression with a basis vector that is changing in direction and therefore that students must create an expression for $\hat{\phi}'$ in terms of basis vectors that do not change during integration. They were given 9 minutes to complete the task. Although $\hat{\phi}'$ in the vector being considered, the distinction between $\hat{\phi}$ and $\hat{\phi}'$ is not critically important for considering the representation in rectangular basis vectors, thus the distinction will not be addressed in this section when students use either $\hat{\phi}$ or $\hat{\phi}'$.

There were many similarities between how students dealt with the issue of current direction and how students dealt with expressing $\widehat{\phi}'$ in terms of rectangular basis vectors. With current direction there were students who either treated current as a scalar or dealt with the current direction by applying the right-hand rule. Similarly, when dealing with

$\hat{\phi}'$ there were some students who did not realize that this problem required any new thinking and tried to turn the problem into something they already knew how to do easily.

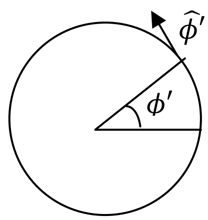

Figure 10: Diagram of ϕ', $\vec{r}'$, and $\hat{\phi}'$.

4.6.1 Quick, easy, incorrect solutions

Consider Group 5, with Shawn, Biff and Devin. As with the previous situation involving current direction, Biff tried to turn the problem into something easy that he already knew how to do. In this case he first advocates for pulling $\hat{\phi}'$ out of the integral. [00:19:29.09]

> Biff, "Can't you just pull it out, just like it's not there? And then you..."
> Devin (over Biff), "That's what I would do."
> Biff, "Yeah, that's what we did in the other problems, is we just pulled it out..." (gestures) "...[inaudible]...there, and then integrated. And then just held that constant and get direction."

Soon after, Shawn responds, "Yeah, but, well, seems like if she's asking us to figure it out it wouldn't be that easy." Soon after that, Alice stops by and convinces Biff that he needs to express $\hat{\phi}'$ using a rectangular coordinate system. Once Biff realizes this, he, along with numerous other students in the class, attempt to use what they know about the unit circle and write down $x = \cos\,\phi$ and $y = \sin\,\phi$.

Here Biff first attempts to entirely avoid the issue of using rectangular coordinates by simply pulling $\hat{\phi}'$ out of the integral, and then, when told it is not acceptable, attempts to use the already familiar unit circle relationships of $x = \cos\phi$ and $y = \sin\phi$.

In other variations that appeared around the classroom students used $x = \cos\phi\,\hat{\imath}$ and $y = \sin\phi\,\hat{\jmath}$. The correct representation is $\hat{\phi}' = -\sin\phi'\,\hat{\imath} + \cos\phi'\,\hat{\jmath}$. Students are muddling more than one concept here. First, they have set a scalar equal to a vector. Second, if by "x" and "y" students are attempting to represent the $\hat{\imath}$ and $\hat{\jmath}$ components $\hat{\phi}'$, then their answers indicate that they are confusing $\hat{\phi}'$ with either $\hat{r}'$ or the x- and y-components of the radius of a unit circle.

Another approach, used by some students, was to take an equation with which they were familiar, $\hat{r} = \vec{r}/|\vec{r}|$ and conclude that they could apply this to $\hat{\phi}$ in the form of $\hat{\phi} = \phi/|\phi|$. This approach proved unsuccessful, especially since every student who attempted to use $\hat{\phi} = \phi/|\phi|$, also confused $\hat{\phi}$ with $\hat{r}$.

Many of the students in class quickly reached incorrect solutions using familiar relationships that were not fully applicable to this situation. Students who accepted this incorrect result did not employ post-answer sense-making strategies, in order to verify their answers.

4.6.2 Carl: Establishing the correct relationship

However, there were two students who managed to successfully tackle the problem and correctly established the relationship $\hat{\phi}' = -\sin\phi'\,\hat{\imath} + \cos\phi'\hat{\jmath}$. These two students were Tom, from Group 1, who solved the problem for homework over the weekend and arrived in class Monday with a complete solution, and Carl, who had been absent the previous Friday. Because he solved it outside of class, we cannot see how Tom solved this. However, in Carl's case it was possible to watch some of his problem-solving process.

Carl drew a picture that included both $\hat{\phi}$ and a radius. He created the correct formula and then checked it for the cases in which $\hat{\phi}$ was pointing in the positive $\hat{\jmath}$ or negative $\hat{\imath}$ directions. Carl did genuine geometric reasoning, and was far more proficient than most of his classmates. Carl managed to reach a correct solution in under 3 minutes.

4.6.3 Shawn: Challenging an error

Another interesting student to consider is Shawn, from Group 5 with Biff and Devin, who earlier was one of the students trying more than one strategy when wrestling with current direction. Shawn, initially starts to write the correct $\hat{\phi} = -\sin\phi\,\hat{\imath}\,\ldots$ but then is convinced by Biff to switch to $x = \cos\phi\,\hat{\imath}$ and $y = \sin\phi\,\hat{\jmath}$.

Before examining how Shawn responded to this, consider the earlier case (presented in section 7.5.4.1) of Shawn trying to find the direction of magnetic vector potential. With the encouragement of his group members, Shawn was willing to try applying the right-hand rule to find the direction of the magnetic vector potential. However, Shawn soon created an off-axis "weird point" that provided a counter-example to the over-simplified right-hand rule model that his groupmates had advocated.

Similar to the example of direction for magnetic vector potential, Shawn again is willing to at least entertain a simple approach proposed by Biff. In this case, Shawn was willing to try Biff's equations $x = \cos\phi\,\hat{\imath}$ and $y = \sin\phi\,\hat{\jmath}$. However, when Shawn considers the case in which $\hat{\phi}$ is pointing in the negative $\hat{\imath}$ direction, he realizes that there needs to be a negative sign somewhere. Shawn has again provided a counter-example to the oversimplified models created by members of his group.

In reply to Shawn's concern, Devin suggests simply adding negative signs, but Shawn is suddenly unconvinced that the group's formula is correct and starts to re-examine it. The group's time is up before Shawn gets a correct formula, but he never settles for an incorrect one.

As was the case with students working with current direction, we can see a range of geometric problem-solving strategies when students try to use rectangular basis vectors to express $\hat{\phi}'$. At one end of the spectrum students attempt to make the problem trivial and reduce it to something they have already solved. At the other end of the spectrum, there are students who recognize that the novel problem has new and challenging aspects and use diagrams, geometric reasoning and specific examples to achieve understanding.

4.7 From Trivial to Novel: A Range of Geometric Problem Solving

We put the situations that students faced into three categories. Finding circumference and linear current density were things that all students found easy. Using angular and linear speeds was something that students had seen before, and the instructor wished would be easy, but in reality presented a challenge to most students. Dealing with current direction and re-expressing $\hat{\phi}$ were two examples of novel problems for students.

For the most familiar problem of using circumference = 2πR, students quickly produced a correct result without discussion or justification. The result was accepted by the group without question.

For dealing with angular and linear speed of the rotating ring, students had used angular speed and linear speed at some point in the past, but most had not mastered (or at least had not retained mastery) of the concept. In this case, students employed a range of strategies.

In Group 4 (section 4.3.4), Stan and Kevin simply tried to remember formulas and apply them. When they remembered correctly the formulas worked, but when their recall was flawed, they did not self-correct. In Group 2 (section 4.3.2), Tanya, Nick, and Bob do not have clear understanding of linear versus angular velocity, or their respective variable, v and ω. Furthermore there was confusion about radians versus arc length. This led to miscommunication, unresolved discrepancies, and accepting errant results. In Group 1 (section 4.3.1), Laura and Allen produce equations that are not entirely correct. However, they use drawings, gesticulation, and language that explicitly refers physical quantities and geometric relationships. The group recognizes their errors and does not accept them. In this case, the group sets aside this particular issue and instead switches to focusing on other aspects of the problem. The class period ends before all the issues related to the speed of the ring are resolved. In Group 5 (section 4.3.5), Shawn rapidly and correctly solves for the velocity and the magnitude of the current. Biff checks units to verify the result.

We also looked at students confronting two novel situations. One required students to deal with changing current direction during integration. The other required students to express $\hat{\phi}$ in terms of rectangular basis vectors. In these situations there was a wide variety of responses.

Tanya in Group 2 (section 4.5.1) ignores the vector nature of magnetic vector potential and her final equations sets the magnetic vector potential equal to a scalar quantity. She primarily tries to remember formulas that can be used to solve the problem algebraically. When she tries to find $\hat{\phi}'$ (not previously discussed), she uses previously memorized unit circle relationships to reach incorrect conclusions. This is similar to Biff's (section 4.6.1) approach. Group 4 (section 4.5.3) had relied upon authority and memory when dealing with angular speed, relied on the right-hand rule to deal with the direction of the magnetic vector potential. Shawn from Group 5 (section 4.5.4.1) uses his understanding that a direction is required when he first dismisses a scalar result. Later he uses his "weird" external point to refute the right-hand rule as sufficient to deal with the direction. When it comes to finding $\hat{\phi}'$ (section 4.6.3), he uses the specific case of $\hat{\phi}'$ pointing in the negative $\hat{\imath}$ direction to reject Biff's equations $= \cos\phi\,\hat{\imath}$ and $y = \sin\phi\,\hat{\jmath}$. Tom, Laura, and Allen from Group 1 consistently use drawings, gesticulation, and language that refer to physical quantities or geometric relationships. Although they frequently do not achieve fully correct answers, they consistently find and reject incorrect answers.

In general these results fit into three broad categories: 1) being unaware of the novel aspect of the problem; 2) recognizing that something is happening, but thinking a familiar, easily-applied strategy is sufficient; and 3) seeing the magnitude of the problem and enlisting an array of geometric problem solving approaches as they address the challenging aspects of the problem.

There were commonalities between the angular velocity cases and cases in which students faced something they had never seen before. In both cases, using explicit geometric arguments (e.g. using drawings, gesticulation and language that refer to specific geometric relationships, including whether quantities are vectors or scalars) either yielded a correct result, or at least prevented settling on an incorrect result. This was shown when Allen in Group 1 used geometry to correct an errant expression for linear velocity and when that group wrestled with the direction of current. It was also shown when Shawn prevented the group accepting the validity of the right-hand rule and when Carl and Shawn tried to find $\hat{\phi}'$ in terms of rectangular basis vectors.

However, while there were some similarities between the angular velocity case and the novel cases, there were also differences. With angular velocity, students were aware that they had seen this concept before and many students attempted recall as the first strategy. For some students, only when their memories failed them did they resort to other strategies. In the case of the direction of the current and $\hat{\phi}'$, no one thought that they had done a similar problem before and therefore no one simply tried to generate an answer purely based on recall. Each of the students who recognized that they did not have access to an easy strategy for solving the problem, reached for geometric arguments that included pictures and gesticulation, and sometimes included examining specific cases, such as Shawn's "weird point" or his counterexample of $\hat{\phi}'$ pointing in the negative $\hat{\imath}$ direction.

Sayre and Wittmann's (2008) perspective on the plasticity of student resources offers a framework for considering student geometric resources in this example. There are resources such as $C = 2\pi R$ that fit the description of a solid resource which is old, easily used, and used without elaboration or justification. There are also resources such as those relating to angular speed that are more tenuously held by students and would fit the description of a plastic resource that is more recent, used with greater effort, and requires elaboration, justification or rederivation. On the other hand, when students encounter a new aspect to a problem, students must find ways to bring in related resources in order to construct new understanding.

4.8 Sense-Making

Redish and Hammer (2009) discuss the epistemological framing of sense-making. The idea that students should make sense of their results, goes back a long way. Some sort of attempt to evaluate or make sense of a result is often the "last step" in prescriptive problem-solving steps (e.g. Heller, Keith, & Anderson, 1992).

Some students show no evidence of sense-making when they accept errant results without question. As seen with students like Tanya, from Group 2, who avoided deeper geometric thinking and used primarily symbolic manipulation, there are students who do not engage in sense-making when they reach what they perceive is a final solution to the problem at hand.

Some students like Shawn, from Group 5, showed consistent use of sense-making strategies. In two cases, Shawn temporarily "suspended disbelief" and used questionable methods advocated by his groupmates. In the case of current direction, he temporarily tried out using the right-hand rule and in the case of $\hat{\phi}'$, he temporarily tried using Biff's formulas. In both cases, after applying the suggested techniques, he engaged in strong sense-making. He applied the suggested ideas to specific cases and found that they did not work. His determination to have the applied principle be understandable in action prevented him from settling on faulty thinking.

However, most students fell between the students who employed consistent, strong sense-making strategies, and those who employed none. At the upper-division level, we see a gradation of sense-making. Many students make some attempt to interpret and understand their results, but are often willing to accept weak or partial understanding. Only a few students consistently insisted on a much firmer footing for their sense-making. "Making sense" can occasionally be a black or white proposition, but in considering students solving this ring problem, what is often seen are shades of grey.

Several students who were somewhere in the middle of the sense-making continuum attempted to justify weekly understood results with yet other weekly developed concepts. Stan, Kevin and Robert in Group 4 tied weak understandings of current, magnetic field, and magnetic potential to their weak understanding of the right-hand rule. Nick in Group 2 was willing to tie his weak understanding of angular speed to his week understanding of radians. In the examples studied, students who tried to create understanding of new concepts by building from highly plastic resources were not successful in creating a strong and valid understanding. Errant results were accepted in these cases.

In some cases, at least some minimal sense-making strategy could be identified, even when the overall approach did not show strong sense-making. For example, when dealing with the current direction, as well as when dealing with $\hat{\phi}'$, Biff was willing to accept highly over-simplified assumptions that made the problem trivial. Although he did not check for internal consistency in his work, and he did not try to make geometric sense of his answers, he at least attempted to "check units" in one case to determine if his answer had the same units as magnetic vector potential.

4.9 Instructional Implications

This section contains conclusions, recommendations and speculation. While this section would in some ways be a better fit for a conclusion chapter, the discussion presented here makes specific references to the data presented in this chapter and thus was considered to be more conveniently located at the end of this chapter instead of being in a conclusion chapter.

This dissertation does not attempt to directly analyze different instructor interactions, and therefore cannot offer recommendations concerning optimal instructional strategies based on research of the instruction itself. However, the research presented here does suggest some instructional strategies that might be helpful to students. The suggestions and ideas for instructors in the following sections are based on the research presented in this dissertation: general knowledge of reformed instruction, anecdotal observations of instructors in Paradigms courses, and the author's personal experience as a high-school teacher, community college instructor and university TA.

4.9.1 Inadvertently Discouraging Sense-Making

When students have a well-connected solid resource, students are often able to use this resource quickly and with a minimum of effort. When students are attempting to utilize a more plastic resource, they are likely to need longer times to employ that resource and may need to be reminded of relationships or be given additional time to reconstruct them.

With a plastic resource, students have used it before and have something to connect to, however tenuous or garbled that "something" may be. However, attempting to rapidly build new understandings from plastic resources can be like building a house of cards in a drafty room. This is shown when students, such as those from Group 4 and Group 5, tried to build understanding of magnetic vector potential from tenuous understanding of the right-hand rule. It is also shown when students in Group 2 are trying to figure out how to integrate current while having insufficient understand of linear and angular speed. In both cases, valid understandings were not being constructed.

When instructors push forward and rapidly try to create understanding of a new concept without students having a firm foundation, they risk having the new concept built from incorrect conceptions and thus not being correctly understood.

Perhaps a bigger systemic issue is that when professors and TA's encourage students to quickly acquire new understandings and connect them to incomplete older understandings, they may be inadvertently encouraging students to be satisfied with a poor understanding. Students may learn to see "sense-making" as connecting any new thing to any old thing, even if both are poorly understood. For example, the students in Group 4 exhibited confidence and enthusiasm while using the right-hand rule. They collectively agreed to accept a very plastic resource as fully valid. And, they collectively accepted incorrect results.

Physics instructors frequently claim they place a high value on student sense-making. However, when instructors do not insist that students connect new understanding to something they solidly understand, they are training students to accept weak (or incorrect) understanding as sufficient sense-making.

If we wish for students to employ sense-making strategies, we may need to provide active opportunities for them to genuinely connect new learning to solid resources that they already have. At a minimum, we may wish to reduce the number of times we actively encourage them to accept weak understanding as sufficient.

4.9.2 Alerting Students to Upcoming Challenges

It is important for students to recognize that new problems contain new and significant challenges compared to problems they have seen previously. This includes when students are given a new problem to be done individually, or when an instructor presents a problem and covers it with a whole class. When encountering a new situation, students need first to recognize that there is an issue with which they need to deal.

Instructionally, it may be important to specifically alert students to the difficult aspects of an upcoming problem. Failure to do this may risk having a portion of the students be unaware that the problem contains a new and challenging component. In Tanya's case she failed to realize that use of vectors and geometry was required for the problem. In addition, students in several groups failed to realize that dealing with current directly presented a significant challenge. Furthermore, several students failed to realize that finding $\hat{\phi}'$ presented a challenge beyond applying the relationships of the unit circle. Students unaware of the challenging portions of the problem may completely miss an opportunity to think about the important issues involved.

Instructors may also wish to consider ways to encourage students to engage in deeper geometric reasoning. In the examples just examined, many students reached for strategies that bypassed the hard parts of the problem. Extended wrestling with geometric issues was required to achieve deeper understanding. In the Paradigms courses, instructors employ a variety of strategies, from kinesthetic activities to group problem solving, in order to help students think geometrically. However, even with this preparation, many students still reach first for strategies that avoid the deeper geometric thinking.

4.9.3 Recognizing and Dealing with Plastic Resources

As a high school teacher, I regularly encounter students with plastic resources in a variety of areas. Prior to the research for this dissertation I was much less aware of the plasticity of students' resources and far more willing to just plow forward with my plans, regardless of how ready students were for taking in new ideas.

Based on my research I have learned to recognize in my students when their understanding is "tenuous" and not sufficient to simply "move on". If I choose to ignore this, and I plow forward at that point without addressing the underlying holes or misunderstandings, then a large percentage of students nearly always "crash and burn" later on when asked to apply their knowledge in new situations.

I have learned to recognize warning signs of insufficient or overly-plastic resources when students:

- use incorrect scientific language
- repeatedly use pronoun-laden language with words such as "it" referring to one or more concepts
- hesitantly make assertions or say things and end in a rising tone indicating a question
- directly quote what the text or I said, instead of using their own words
- incorrectly think that some idea applies to a situation

All these things are "red flags" that are put into a context of professional judgment. One red flag in one student does not necessarily indicate I need to change course. However, when a variety of these indicators are seen among several students, I now often pause to make a decision. I can either accept poor understanding of the concept and move on or I can choose to "dig in" and invest the time needed until students have a more complete understanding.

There is not enough time in a day, nor days in the course, to "dig in" and insist on deep student understanding of every concept or every topic. Sometimes I mentally imagine that I am having students "hop aboard the physics tour bus" and envision that I am showing students things, but not expecting them to have any deeper knowledge about it. I picture that this knowledge can serve several roles. The "tour bus" approach can alert students to interesting ideas they may wish to explore in the future. It can also be the first step in an eventual spiral of increasing understanding. It can also put a concept into context, so that the next time students hear about that concept, they have some idea where it "fits in" to a bigger picture. However, for "tour bus" concepts, I do not expect on a test that students will be able to apply the concept in any meaningful way.

On the other hand, if I want students to really understand something, I realize that I need to take the time to find ways for students to connect new learning to something they truly understand. Students need time and guidance to make strong connections and build solid understanding.

In the past I tended to treat topics with more the same weight, giving similar amounts of time for explaining each topic. Now I much more clearly divide my efforts into quick introductions of "tour bus" ideas and more concerted efforts for deeper understanding in other areas. The results were surprising to me. I was able to actually cover more material in a year, and even when I used the identical tests to previous years, student scores averaged higher.

4.9.4 Potential Efficiencies of Group Work when Dealing with Plastic Resources

When dealing with holes in student understanding, there may be times when a large number of students are all making nearly identical errors; however, this was not seen in this ring problem. Instead, a wide variety of different specific errors occurred, including more than a dozen different errors just when dealing with concepts related to linear and angular velocity. There were also errors related to the right-hand rule, to the idea of “all space”, to the concept of “*dq*”, to vector addition, to scalars versus vectors, to applications of delta functions, and to many other things. This wide variety of different mistakes makes it difficult for an instructor to preemptively include instruction to help students avoid specific errors.

One advantage to group work is that the students in the group will frequently be able to find and correct each other’s errors. Thus, numerous different specific errors and misconceptions can be addressed without taking inordinate amounts of class time that would be required for a single instructor to address all the different errors.

4.10 Conclusion

Differences could be seen when students faced a very familiar situation, a somewhat familiar situation, or a new situation. In very familiar situations, such as needing to find circumference form radius, students easily employed a solid resource, without discussion or delay.

When students faced a situation they had seen before, but for which they had not yet developed a solid resource, students attempted a variety of strategies. These included using recall or reference to authority; connecting to other plastic resources; or using geometric reasoning to connect to solid resources.

When students faced a new situation, their approach fell into several categories. One category was failure to recognize the difficult aspects of the problem at which point the students easily reached incorrect answers. In another category were students who recognized certain problematic aspects to the problems, but attempted to employ a plastic resource that made the problem seem comparatively trivial. These students spent significant time on the problem and often made some geometric connections, but failed to correctly address the more challenging aspects of the problem. The final category of students were those who recognized the extent of the challenges faced in the new problem and employed extensive geometric reasoning to try to solve the problem. While the students using geometric reasoning often did not reach a fully correct solution, they also did not settle on incorrect answers, unlike many of their peers.

Sense-making was not an either-or proposition. Some students employed intermediate levels of sense-making when they attempted to connect new weakly-understood concepts to older weakly understood concepts. Connecting to plastic resources did not prevent student from accepting errant results. The students who did not accept errant results were those who used geometric reasoning to connect to solid resources.

One instructional implication is that it may be problematic to allow students to connect new learning or new problems to plastic resources. Instructors may inadvertently encourage students to accept insufficient understanding when they have instruction that does not facilitate students taking the time to make connections to the solid resources they have.

CHAPTER 5: MODELS OF EPISTEMIC FRAMING

5.1 Overview of this Chapter

Chapters 5 & 6 address the dissertation's main purpose of providing a rich description of student reasoning when facing a highly-geometric problem by considering the questions, "How are students framing what they are doing?" and, "Do they see it as geometric?"

Chapter 4 took a detailed look at student geometric reasoning. However, before students can use their geometric and physical reasoning, they first need to see that using this reasoning is part of the task at hand. If a student views the task at hand to be one of calculation, then, regardless of their ability to use geometric and physical reasoning, they will not employ these resources. On the other hand, if students think that what they are supposed to be doing is connecting the physical situation to a symbolic representation, then they have the opportunity to activate the needed geometric resources.

To understand how students are considering the problem, we use the epistemic framing model developed in Thomas Bing's dissertation (2008) and later described in published papers (Bing & Redish, 2008; Bing & Redish, 2009; Bing & Redish, 2012). As discussed in the literature review (Chapter 2, section 2.2.6), Bing's four framings are calculation, mathematical coherency, authority, and physical mapping.

We consider the strengths and limitations of Bing's model and use this model to analyze the epistemic framing of students while solving for the magnetic vector potential of a spinning ring of charge. We look specifically at the thinking of students when their groups operate with consistent framing, shifting framing, or clashing framing, in which different members are simultaneously using different framings. We also analyze the extent to which different framing modes are more or less efficient. In addition, we look at the degree of alignment between how the instructor views the task and how the students are actually framing the problem.

In Chapters 5 and 6 we adopt Bing's assertion that epistemological frames can be considered "resources." Hammer (2000) proposed the idea of "resources" using the language of computer programmers, where anything from a few lines of code to a large chunk of code were taken and applied to a new situation. This chunk of code was used unaltered and transferred as a single piece, without the need to think about any of its sub-pieces. Resource theory is based on DiSessa's (1993) claim that knowledge comes in pieces and utilizes DiSessa's description of phenomenological primitives or p-prims. Authors such as Bing (2008) and Redish and Hammer (2009) have extended the resource model to epistemological thinking.

For Chapters 5 and 6 we also embrace Bing's assertion that the four epistemic framings are not stable, large-scale coherent frames that represent a student's worldview. Rather, they are resources upon which students can draw in the moment, and which change throughout the time period examined.

While Bing clearly addresses the four frames as "manifold" resources (resources that are contextual and can vary rapidly over time), it should be noted that Bing also discusses an expert "super framing"; an overarching framing that values coherency within and among frames. Thus, the four framings; calculation, physical mapping, invoking authority and math coherency; will be explicitly considered to be in-the-moment resources, while the larger value-of-coherence superframe will be used to describe students' overall approach to the entire problem-solving session. This dissertation does not examine any larger claim about whether or not superframing exhibited on this particular day in this particular activity would exhibit stability across extended periods of time or across significantly different types of activities.

5.2 Bing's Four Framing Categories

The next four subsections examine each of Bing's four framings; calculation, physical mapping, authority and math coherency; and apply them to students solving the ring problem. We look at problems encountered in using these framings and how they were resolved.

5.2.1 Applying the Physical Mapping Framing Category

The physical mapping framing, in which students directly connect their symbolic representation to the physical world, was seen repeatedly throughout the data analyzed. There are numerous examples in which students are gesturing, pointing to drawings, or explicitly discussing the physical situation while they are concurrently discussing the symbolic representations. In these cases the use of physical mapping very clearly aligns with the physical mapping situations described by Bing. However, before Bing's coding could be thoroughly applied to the data at hand, there were several issues that needed to be resolved.

While some student usage neatly aligned with Bing's examples of physical mapping, students working on the spinning ring problem posed situations that Bing did not directly address. For example, Bing considered students going directly between a physical situation and a symbolic representation or going directly from a drawing to a symbolic representation. He considers both of these cases to be "physical mapping". However, in the Paradigms data there are occasions when students are making physical to geometric arguments or are discussing the problem conceptually. Since Bing's framing is specifically designed for looking at how students use mathematics in physics, he never explicitly addresses how to consider discussions about the physical situation that are not directly linked to mathematical symbols.

The students' assignment in the ring problem is to produce a symbolic representation from a physical representation. From one perspective, any intermediaries, such as drawing could be seen as part of the overall process of going from a physical situation to mathematical symbols. An alternate approach would be to develop a separate framing for going between a physical situation and a geometric model. In addition, distinctions could be drawn for going directly between the physical situation and mathematical symbols in contrast to going between symbolic representations and geometric representations such as drawings.

While using additional framings might be helpful for analysis, the focus of this portion of the dissertation was applying the existing model instead of significant expansions of the theoretical framework. The Physical Mapping category, although large, was still highly useful for analyzing student thinking while solving the ring problem. For the purpose of this dissertation, any student discussion about the geometric or physical aspects of the problem will be considered "physical mapping".

Another issue that arose when using Bing's coding, is what to do when students refer to the units in a problem but are unclear as to how they are framing it. For example, when students make statements such as "*J* is the current density", it is sometimes not clear if they are simply thinking of "current density" as a label for the symbol "*J*" or whether they are making a physical connection between "*J*" and the physical concept of current density. Usually, comments such as these are left uncoded because of their ambiguity, but occasionally the surrounding context more clearly suggests that students are genuinely considering the physical meaning or are merely assigning labels as part of a calculation. Consider a sequence such as the following, in which a student refers to the period *T*:

> Tom, "Period is *T*"
> Laura, "So, it has to do something per *T*"
> Tom, "So it does one complete revolution..." [Gestures around in a circle]

In this sequence the comment, "Period is T," is followed by a clear connection to what "period" physically means, and thus would be considered "physical mapping". In contrast, the sequence below shows Tanya's use of "velocity" to describe "ω" appears to be using a label instead of actually making a mental connection to the physical quantity:

> Nick, "T is equal to 2 π r over, over v." [Writes $T = 2\pi r/v$.]
>
> Bob, "…or you could say frequency equals one over T." [Writes, "Period = $1/f$" and "freq = $1/T$"
>
> Nick, "Well that's..."
>
> Bob, "Uh, wait a sec, is it omega equals 2 π f? Omega is 2 π f, so f is one over T," [Writes $\omega = 2\pi f$]
>
> Tanya, "We need veloc...that's velocity, so 2 π over T is velocity." [Writes on Bob's equation, changing it to $\omega = 2\pi /T$]
>
> Bob (over Tanya), "I don't know, I don't know what that does."
>
> Bob, "Sure."

Here Tanya appears to tag "ω" with the label "velocity" and use it to chain together the equations $f = 1/T$ and $\omega = 2\pi f$ to get $\omega = 2\pi /T$. Her chaining of equations is one indication that she is thinking in terms of calculation. Another indicator is that she is using the term "velocity" instead of "angular velocity", which hints that she is at least not carefully considering the physical meaning of her term.

5.2.2 Applying the Mathematical Coherency Framing Category

With students working to solve the spinning ring problem, there were never any examples that quite fit Bing's description of students recognizing that the "same mathematical structure can underlie two superficially different situations." The student examples that Bing gives in his dissertation did not match any examples seen with students solving the ring problem. However, there were some cases that indicated students expected that two different mathematical approaches should yield equivalent results.

Here is one case in which a student checks units:

> Biff "$2\pi R$, yeah, divided by v...That equals meters over meters per second equals seconds over meters times meters, cancel, equals seconds." [Writes $2\pi R/v$ = m/(m/s) = s/m * m = s (circles the "s")]
>
> Devin, "That's the period."

Here the student is working with the units as if they are an alternate mathematical equation. It is unclear how Bing would categorize this, but for this dissertation, use of dimensional analysis in this way will be categorized as “mathematical coherence”. One reason for justifying this as part of the mathematical coherence frame instead of just a calculation frame, is that it was frequently a significant shift in student thinking compared to times when they were consistently using calculation framing. Students appeared to “switch gears” from simple calculation in order to try this approach to check for errors. Checking one’s work in this way was something the instructor often advocated when a group was unsure about their results or had an incorrect result.

Another type of student framing related to mathematical coherency occurs when students try two different approaches to get the same result. An example of this is a student using the concept of total charge and period to get the result that $I = Q/T$ and then comparing this to the result that $I = \lambda v = (Q/2\pi R)(2\pi R/T) = Q/T$. Imagining the whole charge of the ring passing a given point in period T is a different visualization than imagining a ring with a certain charge density moving at a certain velocity. Expecting that one can reconcile these two concepts goes beyond a mere manipulation of symbols.

Two groups spent more than a minute in thinking about these two ways of representing these ideas and reconciling the two concepts. In contrast, there was a group that got the two separate results and did not attempt to reconcile them. The fact that one group did not attempt this reconciliation indicates that this step is a separate step. Reconciling the two ways of looking at the problems appears to be neither pure physical mapping nor pure calculation. One approach would be to argue that this is more closely aligned with mathematical coherency than any other framing. Another approach would be to see this example is a hybrid between physical mapping and calculation framing. This dissertation will not attempt to resolve this issue. Instead, for the purposes of this dissertation, it will be indicated that there is an overall value placed on coherence, but no definitive framing will be assigned.

Another situation related to coherency is one in which one student looks over at another student's results and says, "That's what I got. What'd you get?" In the particular case from which this quote was taken, the student who asked the question made rapid attempts to reconcile the two equations, while the other student made no such attempt. While this also shows desire to establish a certain form of coherence, in this case the question itself will not be put in a particular frame. Instead, the subsequent dialog will be evaluated based on how well each statement fits a particular category.

5.2.3 Applying the Authority Framing Category

When a student makes an unsupported assertion it is often unclear whether it is a memorized result or whether it is physically established on the spot. For example, if a student declares that $\omega = 2\pi/T$, and makes no justification, we cannot tell if they stating a memorized result or are they reconstructing the relationship on the spot. Either is plausible. Occasionally there is an accompanying gesture to an assertion that would allow it to be considered physical mapping framing. However, most of the time simple assertions come with no spoken or gestured support. These statements will be simply left uncategorized.

On the other hand, sometimes an incorrect result is given, suddenly and without justification, such as "$f = 2\pi\omega$." In these cases it is reasonable, but probably not 100% reliable, to speculate that this is using an authority frame and misquoting that authority. In this dissertation these were marked with an authority code, but with a modification to the coding color to indicate that the coding was tentative.

5.2.4 Applying Bing's Coding System

At this point it is important to note that the intention of this coding is not to accurately code every line. Far from it, these codes were designed with an awareness that they would not be able to code everything, and sometimes not even the majority of a particular transcript. Bing claims about his own dissertation, "Of all the data analyzed for this dissertation, perhaps less than 50% can be cleanly coded under one of these general clusterings." He goes on to say, "Human cognition is a fuzzy process, and these four named epistemic framings were only meant to represent general clusters of similar framings." Even the transcript chosen to test inter-rater reliability yielded only 70% agreement before discussion and 80% agreement afterwards. Bing continues, "Thus, the epistemic coding scheme presented here should not be expected to yield a clean coding of most of a random transcript. This inter-rater reliability test transcript is no different. Students' thinking is simply not found to be that cleanly compartmentalized."

Clearly, epistemic framing analysis is not ideally suited for making a quantitative comparison between using one frame compared to another. Most transcripts will not yield the possibility of claiming "students used physical mapping x% of the time". A significant percentage of each transcript used in this dissertation is not coded. Sometimes students make statements without clear justification, sometimes there are hybrid situations, and sometimes it is unclear the approach students are taking when they make certain statements.

Consider a statement such as this one made by Allen, "Wait, we can write *dr* in terms of cylindrical coordinates." This statement could be taken as an authoritative assertion – it is simply something that Allen knows how to do and he is stating a known and memorized result. Alternately, one could see this as physical mapping, because Allen could be visualizing an actual segment of the ring, represented by *dr*, being geometrically represented in an alternate coordinate system. He might also be imagining the geometry that goes along with this. This interpretation would not be unreasonable, because this student frequently has unambiguous uses of physical mapping in other contexts. In addition, some of the discussion surrounding this quote includes physical arguments. However, yet another possibility would be to argue that this is calculation. As a student mathematically manipulates equations they substitute in different expressions. Writing a length in terms of cylindrical coordinates is a standard mathematical practice. Somewhere around half the student utterances found in the transcripts fall into this uncoded category. While this may at first appear to make it hard to draw conclusions from the data, the coding can draw attention to certain instances where the framing is unambiguous or can draw attention to certain patterns.

Later in this chapter, in section 5.6, it will be claimed that "Four of the five groups had coded instances of physical mapping more often than they had coded instances of being in any other frame." While this statement accurately reflects the data that were actually coded, there will be an unstated bridge that links it to the subsequent claim: "This indicates that the class, taken as a whole, interprets this as primarily a physical mapping problem." The unstated assumption is that the coded statements are representative of the overall framing. However, the uncoded pieces are not simply ignored when making the broader claim. Instead, there is a bit of "professional judgment" that is used in considering whether the uncoded student statements could reasonably be considered to be consistent with the surrounding coded pieces. If several uncoded bits are scattered throughout a series of clear physical mapping statements, and those uncoded bits could conceivably be in a physical mapping frame, then the general framing for the conversation can be considered to be physical mapping.

Furthermore, behind the claim that the class interprets this problem as a physical mapping problem, is the additional information that in two of the five groups, coded occurrences of physical mapping (both in terms of time and number of student turns) exceeded all other frames combined. Thus, at times there will be explicit references to coded portions of the transcript, while at other times a broader statement about the epistemic framing being used by a student or group of students will be made. In these cases, the broader statements will be based both on the coded portions of the transcript and the overall context that includes both coded and uncoded portions.

Given the frequency of uncodable student statements, and need for subjective interpretation, one might question the value of this coding. However, there is a substantial amount of the transcript that can be cleanly coded. There are places where students are gesturing, pointing at a diagram, and discussing physical quantities which clearly indicates physical mapping. There are times when students cite a specific authority, making the authority frame unambiguous. There are times when a calculation frame is apparent because students are performing calculations without referring to physical quantities or alternate approaches. By noting these clear cases, one can see certain patterns and gain insight into how students are approaching a problem. Thus, while this epistemic framing tool has its limitations, it has aspects that are robust. It is able to provide insights that are not offered by other tools for considering epistemic framing.

5.3 Extending Bing's Work

Bing's dissertation looks at students in four of the University of Maryland's physics courses: PH 401 - Quantum Mechanics I, PH 402 – Quantum Mechanics II, PH 411 – Electricity and Magnetism, and PH 374 – Intermediate Mathematical Methods. The majority of Bing's data comes from video of students working in a group on homework problems outside of class time. At the start of the semester, he asked for volunteers who would not mind being video recorded. At various points throughout the semester he recorded them with him and the camera positioned in a corner of a pre-determined room while the group of students worked on their homework. He produced 80 hours of such video and also used 25 hours of interview video. Toward the end of his dissertation, Bing suggests that it would be an interesting extension of his work to apply this technique to students working in an actual physics classroom with the instructor present. Chapters 5, 6 and 7 of this dissertation is such an extension.

The classroom setting and multiple cameras used for this dissertation allow for applications of Bing's framework to situations in which it had not been previously applied. For example, in addition to being a different group of students at a different university, the nature of this problem is different from those examined by Bing. The instructor consciously chose a problem which she realized was too difficult for the majority of students to solve alone without assistance. It was chosen as a class activity so that students could be getting support from the instructors and from fellow students. Thus, the nature of the difficulty of problem was different from what would be assigned for homework, and we can see the approaches students take when they face a very challenging problem.

Another difference in this setting is that we can consider a cross section of the class. We were able to consider the thinking of each of the 17 students present in class. While we cannot make universal conclusions from considering the 17 students in a single problem-solving session, we can nonetheless, get a clearer picture of the spectrum of what occurred in this classroom. In addition, the classroom setting allows us to consider how students interact with instructors.

5.4 Framing Clashes, Switching Framing, and Steady Common Framing

Bing uses the idea of "framing clashes" to highlight the effect that framing has on conversation. Clashes of framing occur when students are having a conversation but are framing the problem differently from each other. These are cases where students not only are in disagreement, but are in disagreement about what sort of justifications are expected in order to address the question at hand.

First, an example of a framing clash that Bing gives in his dissertation will be presented and discussed. After this an example of a framing clash from the Paradigms data will be presented, followed by two examples of groups using framing when there is no framing clash.

5.4.1 Bing's Example of a Framing Clash

The clashes of framing that Bing uses as examples all show students with a common idea being discussed, but with students approaching that idea from different frames. The students directly engage each other about the topic.

In one of Bing's examples students are working on a problem trying to find the work done on a rocket by an asteroid as the rocket moves from point A to point B. Students are given coordinate axes with the asteroid located at the origin and the rocket moving from point (1,1) to point (3,3). Students are asked to consider the diagonal path directly between the two points versus a two-stage path in the x-direction followed by the y-direction. (Figure 11)

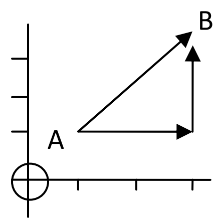

Figure 11: Diagram of a rocket moving from point A to point B via two different paths

The dialog Bing analyzes starting on page 72 of his dissertation begins as follows:

S1: what's the problem? You should get a different answer from here for this. *Points to each path on diagram.*

S2: No no no

S1: They should be equal?

S2: They should be equal

S1: Why should they be equal? This path is longer if you think about it. *Points to two-part path*

S2: Because force, err, because work is path independent.

S1: This path is longer, so it should have, this number should be bigger than… *Points to two-part path again*

S2: Work is path independent. If you go from point A to point B, doesn't matter how you get there, it should take the same amount of work.

This initiates a framing clash, in which S1 is making a physical mapping argument, pointing at the diagram and claiming that work should be more along the two-part path because the path is longer and therefore the numbers should be bigger. S2 is makes an unsupported authoritative assertion that work is path independent. After the dialog shown above, S1 switches to finding the numeric values for the integrals that the group has set up. The integrals ignored the changing angle of the force, so when they were evaluated, a larger value is found for the longer path. After substantial dialog about the calculations, the dialog continues with S1 now making a calculation argument against S2's authoritative statement. S3 also enters the discussion.

S1: See, point six one eight, which is what I said, the work done here should be larger than the work done here 'cause the path *Points to diagram*

S2: No, no no, no no no

S3: the path where the x is changing

S2: Work is path independent.

S1: How is it path independent?

S2: by definition

S3: Somebody apparently proved this before we did.

S1: OK, I don't understand the concept then, because you're saying it's path independent.

S1's earlier physical argument was never addressed by S2, and S1's new justification based on calculation is not addressed by S2 either. Instead, S2 simply reasserts that work is path independent. After the dialog shown above, S1 goes on to make a hard push to switch to a physical mapping frame and S2 eventually accepts. The discussion eventually results in a common understanding and realization that the angle of the force in relation to direction of motion is essential for solving the problem.

5.4.2 Framing Clash in Group 5

In the case of OSU students solving the ring problem, there were several cases where students were not using a common framing, however, there were no examples that had as vivid a framing clash as the one in Bing's example. When students operated in separate frames, they rarely engaged in dialog. Sometimes one student completely ignored the other group members. When dialog ensued, there was usually a rapid shift to a common frame. The closest thing to a framing clash while students worked on the ring problem occurred in Group 5.

Group 5 had established that the magnitude of the current is Q/T and is in the $\hat{\phi}$ direction. Biff decides that this information is sufficient to start evaluating the integral, while Shawn sees that before trying to evaluate the integral, there is substantial work remaining in considering the vector nature of the problem. Thus, Shawn is framing the problem as physical mapping, while Biff has moved to framing the problem as calculation. Devin joins Shawn in the physical mapping frame.

1: Shawn, "We should go back,...remember, like, you have to go back to vectors for the ***A***."

2: Biff, "Yeah, somethin'."

3: Devin? "It's gonna be..."

4: Biff, "What's that?" [Writes $\frac{Q}{t}\hat{\phi} = Rd\phi\hat{\phi}$]

5: Shawn [gestures in a circle around ring], "If you go,...here we have this thing, it's going to be like, pointing,...it's going to be pointing, like up, right?" [Draws a new line segment originating at the center of the ring to the ring]

6: Chris, head facing down towards where he is writing, continues to write to get $\frac{Q}{t}\hat{\phi} = Rd\phi\hat{\phi} = \int_0^{2\pi}\frac{QR}{T}d\phi$

7: Devin (over Shawn), "It's going to be...it's gonna...yeah...going to be pointing in the *z*-hat direction. I thought magnetic field involves a cross product...but, uh..."

8: Chris continues to write to get $\frac{Q}{t}\hat{\phi} = Rd\phi\hat{\phi} = \int_0^{2\pi}\frac{QR}{T}d\phi = \frac{2\pi QR}{T}$

9: Shawn, facing Devin, "'Cause, yeah, it's only going to be that like right there." [Traces a pre-drawn vector from the center of the ring to the ring itself, points to $Rd\phi\hat{\phi}$ in the equation] "...because like over here, isn't it going to be something like that?"

10: Biff (over Shawn), "Here's the answer that you got from our current calculation; $2\pi QR$ over T" [draws a box around $2\pi QR/T$], (Shawn and Chris turn towards Biff), "Which gives us, uh, charge, so Coulombs-distance; meter-coulombs-per second."

11: Devin, "That's pretty much, I mean that's,...yeah, that's just...[?]..."

12: Shawn (over Devin), "...[?]...what about the,…"

13: Shawn and Devin both point at Biff's equation, but it is unclear exactly what part of the equation

14: Shawn, "… what about the,...to the point though. What about this thing?" [draws an external point and a line segment from the external point to the center of the ring (Figure 12)] "Our ***r*** minus ***r′***..." [Writes $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$, next to newly drawn line segment]

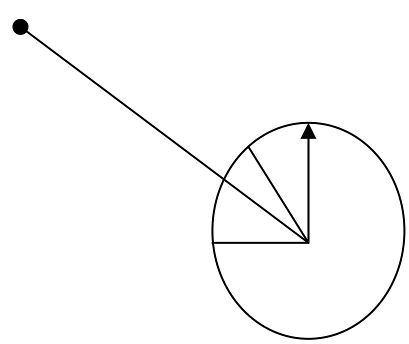

Figure 12: Shawn's line to an external point

15: Biff, (in an artificially high voice), "Please, no!"

16: Group laughs

17: Biff, "I don't wanna...I'm too young to die. (pause) Alright, let's see then. If you guys want to go through and erase all this and set it back up."

18: Everyone erases everything written on the whiteboard

Shawn's initial comment that the group should "go back to vectors for $\boldsymbol{A}$," is responded to by Biff with, "Yeah, somethin'." Biff proceeds to enter into a calculation. While doing his calculation, Biff has his head facing downward as he writes on the whiteboard in front of him, and he does not engage the members of his group in conversation. Shawn, meanwhile, is clearly in the physical mapping frame, gesturing around in a circle and adding line segments to the ring diagram in front of him.

Unlike Bing's rocket example, the students are not directly engaging each other during most of the period in question. Shawn and Devin are turned towards each other, engaging in a physical mapping discussion, while Biff temporarily focuses on his own calculation. Only when Biff gives the result of his calculation as $2\pi QR/T$, does he attempt to engage the rest of the group. Shawn and Devin both turn toward Biff. Shawn then makes a physical mapping argument, suggesting that Biff needs to consider the $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$, aspect of the problem.

Biff responds with "Please, no!" (which Biff later clarifies is referring to having to write $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$, in cylindrical coordinates), followed by claiming that the group should erase all their written work and set up the problem from the beginning. At first glance, one might conclude that Biff accepts Shawn's physical mapping bid, rejects his own $2\pi QR/T$ answer, and decides the group needs to start over in order to include the physical mapping he had earlier avoided.

While this interpretation might seem reasonable, Biff has not entirely "let go" of his former answer or his former framing. Ten seconds later when the post-doc instructor, Dr. Alice, walks past, Biff turns to her and says, "Um, what should our answer be;...the units? Like, because we came up with...$2\pi RQ$ over T which has units of meters times Coulombs per second." Alice eventually clarifies that Biff will "have to do the $\boldsymbol{r}$ - $\boldsymbol{r}'$ thing," before a valid expression can be achieved.

This subsequent retention of Biff's earlier framing illustrates how framing can impact student thinking and the conversations that students have. While he was performing calculations, Biff was framing the problem as a calculation problem while his group members were framing the current issue in a physical mapping frame. However, even after Shawn appears to have substantive communication with Biff, and Biff appears to acknowledge his viewpoint, Biff is still not entirely ready to accept the physical mapping framing.

5.4.3 Common Physical Mapping Framing in Group 1

Two different examples of dialog will be presented as a contrast to the previous example. The first will be an example where students are all using a single common frame. The second will be an example where the group is shifting framing, but they are fluidly responding to this framing shift.

In this first example, Group 1, with Tom, Laura and Allen, is considering $\vec{r} - \vec{r}$, and all three students are using a physical mapping frame.

> Tom, "So..." [draws an external point from ring]
>
> Laura, "Oh, yeah."
>
> Tom draws $\vec{r}$ and $\vec{r}\,'$ vectors on ring diagram and labels them (see Figure 13).

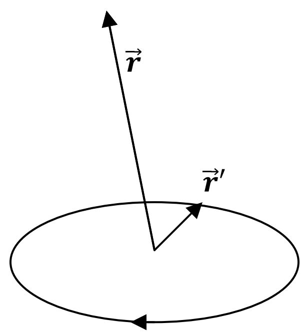

Figure 13: Tom’s drawing of position vectors

Laura writes an $\vec{A}$ on the drawing next to the external point, "I concur."

Allen draws the $\vec{r} - \vec{r}$ vector with the arrow going from the external point to the ring, then writes $\vec{r} - \vec{r}$ next to the vector (Figure 14)

Allen, "This way, right? Or is it the other way?"

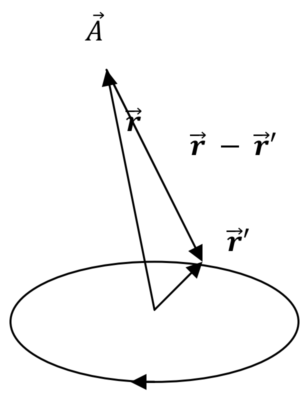

Figure 14: Allen draws and labels the $\vec{r} - \vec{r}$ vector

Laura, "It doesn't matter."

Allen, "And, I'm upside down."

Tom, "It, it is. It would be that way." [draws an arrow head on the $\vec{r} - \vec{r}$ vector at the external point on drawing, the opposite direction that Allen drew]

Allen, "Is it... I've never known which way it is...[?]..." [points at external point]

Laura, "No, because you, you add these two." [gestures, moving her finger along the *r* - *r*' vector from the ring to the external point]

Allen, "Ahhhh...O.K."

Tom, "It's *r*' plus this thing equals *r*." [points at vector from origin to ring and vector from origin to external point]

Allen, "That one, alrighty...Right, because you go tail to head, tail to head..." [points at vector from origin to ring and then to external point]

Tom, "Yeah...exactly."

Allen, "...from the original tail to the new one." [gestures in a large arc with palm facing downward]

Tom, "Yeah."

In this example, the students are establishing a common understanding of ***r***, ***r*'**, and ***r* - *r*'** vectors. Because their words are accompanied by gestures, a geometric diagram, and the corresponding symbols, all the members of the group are able to develop a clear, common understanding involving multiple representations. Students are carefully listening to and watching each other's comments, gestures, and additions to the diagram.

Allen is initially unsure about the direction of the ***r* - *r*'** vector. Group members entirely embrace the physical mapping frame and keep all their arguments as physical or geometric arguments that are demonstrated in space with a combination of the diagram and gestures.

The members of the group use different ways of articulating their arguments. For example, Tom gestures while saying, "It's ***r*'** plus this thing equals ***r***," and Allen then uses different words to describe this by saying, "That one, alrighty...Right, because you go tail to head, tail to head..." (points at vector from origin to ring and then to external point). The conversation flows smoothly and all members of the group are justifying their statements and checking for understanding. Never does any member of the group attempt to use authority, mathematical coherence, or calculation, although any of these other frames could have been used to address Allen's question. Thus, this is a "clean" example of students all maintaining a common framing.

5.4.4 Switching Framing in Group 4

The next example is one in which students are switching framing. Group 4, with Stan, Kevin and Robert, has the integral for magnetic vector potential written as [00:50:12.16]

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int \frac{\lambda \frac{2\pi R}{T}}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

Stan begins this next segment with taking λ and expressing it in terms of known variables that were given in the problem.

1: Stan, "So lambda, lambda expands - that's charge per unit length, which is Q over $2\pi R$, so that..." [erases λ and writes $Q/2\pi R$ in numerator to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int \frac{\frac{2\pi R}{T}\frac{Q}{2\pi R}}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

2: Kevin, "That's just Q over T"

3: Stan, "Oh, and then..."

4: Robert, "Yeah"

5: Stan, "And then there's the d..." (writes $d\tau'$ in numerator to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int \frac{\frac{2\pi R}{T}\frac{Q}{2\pi R}\,d\tau'}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

6: Kevin [points at $Q/2\pi R$], "This is λ...our charge density?"

7: Stan, "Yes"

8: Robert, [points at $\frac{2\pi R}{T}\frac{Q}{2\pi R}$] "Right off the bat, this and this are going to go away. This whole thing you're going to get *Q* over *T*. You're just going to get charge over period."

9: Kevin writes $I = \frac{dQ}{dt}$ on board

10: Stan, "Now, *dτ* is not a *dτ*, it's in fact a...*ds*?...*ds*, yeah...Is that true? Yeah." [changes the *τ* to and "*s*" to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int \frac{\frac{2\pi R}{T}\frac{Q}{2\pi R}\,ds'}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

11: Robert, "For everywhere...yeah, OK."

12: Stan (over Robert), "So we just want..." (draws chopping marks on ring) "...which is..."

13: Robert, "Yeah, there is no volume there."

14: Stan, "...which is...uh...*r*-prime, so big *R* times length $d\phi$?"

Stan starts in line 1 saying, "So lambda, lambda expands - that's charge per unit length, which is *Q* over 2π*R."* This could be argued to be an example a physical mapping frame seamlessly becoming a calculation frame. Stan starts with a symbol and describes it in terms of its physical dimensions and then connects those dimensions to mathematical symbols and creates an expression that is then substituted into an equation.

At this point Kevin responds using calculation framing and notes that the $\frac{2\pi R}{T}\frac{Q}{2\pi R}$ expression in the integrand can be simplified to *Q*/*T*. A few seconds later, in line 6, Kevin considers the physical meaning of an expression, when he points at $\frac{Q}{2\pi R}$ and says, "This is *λ*...our charge density?" Kevin smoothly transitions from calculation to physical mapping. Robert responds with a comment that is also a combination of calculation and physical mapping when he points at $\frac{2\pi R}{T}\frac{Q}{2\pi R}$, "Right off the bat, this and this are going to go away.

This whole thing you're going to get Q over T. You're just going to get charge over period." Kevin now writes $I = \frac{dQ}{dt}$, possibly using the calculation framing to get a new mathematical expression as an extension of the idea that $I = Q/T$.

Stan now takes the conversation more solidly into the physical mapping frame when he asks, "Now, $d\tau$ is not a $d\tau$, it's in fact a...ds?...ds, yeah...Is that true?" It may not be clear exactly what Robert is trying to communicate when he says, "For everywhere...yeah, OK," but it is clear that Robert is thinking about things spatially as opposed to using calculation. Stan and Robert then clearly stay in the physical mapping frame as Stan draws chopping marks on the ring diagram and Robert comments that there is no volume there.

In this example every member of the group is fluidly switching back and forth between calculation and physical mapping. The framing shift happens so rapidly that at all three students each manage to use both frames in a single short turn speaking.

5.4.5 Considering the Three Different Framing Modes Enacted by Groups

These three examples from three different groups illustrate three different ways that groups can be functioning. In the first case, one student was using a calculation framing and was for a time disengaged from the conversation of the other two members of the group who were using physical mapping. In the second case, students consistently used physical mapping. In the third example, the group was fluidly mixing physical mapping and calculation and then the whole group switched to more sustained physical mapping.

It should be noted that these three episodes were not necessarily representative episodes for these groups. The second example shows Group 1 sustaining a physical mapping framing. Group 1 did use physical mapping framing more than all other framing combined and was the group that used it most frequently, however, even in this group, there were multiple times when the group was switching frames. In addition there was one case in which a student from Group 1 temporarily got "stuck" in a calculation framing and was ignoring strong physical mapping bids from another group member. The three examples chosen were chosen for their clarity and duration. However, examples that were similar to all three of these could be chosen from any of the three groups. Thus while certain framing "modes" (sustained common framing, collective switching, or members with different framing) are more common in certain groups, no group consistently used one mode.

5.5 Efficiency of Different Group Framing Modes

On page 85 of his dissertation, Bing makes the claim, "When a common framing is established, the conversation tends to be richer and more efficient." Bing does not define "richer" or "efficient", and makes no explicit connection to examples. Without an operational definition of these terms it is difficult to support or refute this claim. Some possible operationalized definitions of these words are considered and then the claim is compared to the Paradigms data.

If "efficiency" means getting the problem done quickly and correctly, then in this particular case, the group that finished first with a correct answer (Group 1) did correspond to the group that had the longest sustained periods of common framing. However, throughout the class there were times when everyone in the group was using common framing, but little progress was made. There were also cases where group members were not all using common framing, but a single member of the group was temporarily "working on their own" and successfully making progress on the problem.

If "efficiency" refers to error correction, then Group 1, the group with most time with common framing, made the fewest errors and most rapidly corrected the errors that were made. Group 2, which had the highest occurrence of students "doing their own thing" had the highest rate of generating errors and had several errors that persisted and were propagated over time. When group members were checking in with each other, discussing their work, and operating in the same frame, there appeared to be a tendency to more rapidly detect and correct errors. However, there were also times when students self-corrected without input from the group, and times when the whole group collectively settled on incorrect method or answer. An example of a group all using common framing, but not catching an error, is Group 4 unanimously misapplying the “right-hand rule” to finding the direction of $\boldsymbol{A}$, the magnetic vector potential.

On the issue of richness, if “rich” conversation includes carefully justifying statements and responding to the justifications posed by other members of the group, then when students are using different framing, and especially when they are ignoring each other, there was very limited, if any, “rich” conversation.

With efficiency and richness defined in terms of the correctness of the answer and amount of justification used in discussion, then these data lend some qualified and limited support to Bing’s assertion that common framing tends to correlate with conversations that are richer and more efficient.

If, however, we think of efficiency and richness, not in terms of progress on the problem solution or quality of argumentation, but instead base it upon changes in student thinking, then there were definitely cases in which pushing hard for a student to make a frame shift resulted in important shifts in student thinking. Consider the following example from Group 6, where the instructor is pushing students to leave the calculation frame and consider the dimensions.

> Alice, "You're here" [points at $I = \lambda$] "Now you need what? What's your dimensional situation there?"
>
> Jack, "uh, we need..." (3 second pause)

Alice, "What do we measure current in?" [points at I in the $I = \lambda$ equation]

Jack, "Uh, amps"

Alice, "Which is a...? That's a...Use the units."

Jack, "Charge per time."

Alice, "It's charge per time, right? What's the dimensions of this?" [points to λ]

Jack, "Uh, charge per..."

Seth, "Charge per length."

Jack, "...per length."

Alice, "Charge per length." Right? So, how are you going to a charge per time in terms of..."

Seth (over Alice), "Per time equals charge per length times length per time." [writes $\frac{Q}{T} = \frac{Q}{L}\frac{L}{T}$ on board] "It's going to be...[?]..."

Jack, "Velocity"

Peter?, "Yeah."

Seth writes a v to make $I = \lambda v$

The instructor is pushing the students to consider their results in light of its dimensions. The students do not respond with any immediate recognition of how to use this new framing of the problem. The instructor continues to model how to use dimensions and units in order to find and correct errors. In this case, the framing used when considering dimensions is somewhat complex and hybridized. A physical mapping framing can be used when considering dimensions and units if the physical aspects of those dimensions are considered. However, as mentioned in an earlier discussion of mathematical coherence, when students put those dimensions in the form of an equation, it is used as a parallel way to mathematically address the situation. In this respect, the use of dimensions could be considered mathematical coherence. When the student is performing the dimensional calculations, he is using a calculation framing. Thus, the process of considering units and dimensions is often a hybrid of multiple frames. The preceding situation involves all these aspects of considering dimensions.

Regardless of how one chooses to define this framing, it is a different way of thinking than when the student made the previous unsupported assertion that $I = \lambda$. Jack does not immediately "see" the problem when asked, "Now you need what? What's your dimensional situation there?" No other member of the group immediately jumps in either. In the process of reframing this situation, the instructor takes the students step by step through a process that no member of the group had used on this day, prior to this instructor interaction.

Initially, this might be seen as "inefficient", the instructor's attempt to reframe the situation is initially met with a not-so-rich, not-so-efficient reply of "uh, we need..." followed by a three second pause. The instructor walking the students through the next few steps seems similarly not very rich. However, Seth, a student whose contributions to the group had been fairly limited before this point, jumps in and starts contributing.

When we look four minutes later in the conversation, we can see that Seth adopts the instructor's technique of considering dimensions and pushing for a frame shift. During this subsequent conversation, Jack claims that $\omega = 2\pi R/T$, incorrectly putting in a factor of R. Seth responds by asking him to consider units.

1: Jack, "Oh, this angular velocity. Right?" [draws a circle around the $2\pi R/T$ part of his own equation and then draws an arrow from it and writes ω]

2; Peter, "Uh, yes. That's angular velocity......Wait, so this, this has constant ω" [gestures around in a circle] "Is that why we're doing it in this class and not...[inaudible]..."

3: Jack, "Maybe, I don't know. I don't want to think about that."

4: Peter [puts head on hand], "Um, OK, so, wait, Q over $2\pi R$," [points at $Q/2\pi R$ in equation], "So that's,...That would be our λ."

5: Jack gestures in a circle, "But we need to get an integration...I think."

6: Peter, "Ah."

7: Seth [points at $2\pi R/T$ part of Jack's equation], "Wait, this is a...velocity,...because there is distance per time."

8: Peter, "Right, but it's angular velocity." [points at $2\pi R/T$ part of Jack's equation]

9: Jack (over Peter), "It's a...yeah."

10: Peter, "...it's over 2π."

11: Jack, "No."

12: Peter, "No, wait, um..."

13: Seth, "The units still don't work out though."

14: Jack, "This is tangential velocity."

15: Seth, "Yeah, there you go."

16: Jack, "It would have to be divided by 2π to be ω, right?"

17: Seth, "Divided by R, 'cause you want, like, radians per second."

18: Jack, "Yes. Yeah." (nods)

19: Seth, "The 2π/T would get us omega."

20: Peter, "Um, ...[?]...I think so, uh,..."

21: Seth, "That makes sense, 'cause it's constant..[?]..."

Seth can be seen using dimensions to respond to two different errors made by Jack. In line 7 Seth challenges Jack's $\omega = 2\pi R/T$, with "Wait, this is a...velocity,...because there is distance per time." Peter initially does not accept Seth's bid to at reframing the situation in terms of dimensions and replies with "Right, but it's angular velocity." One could speculate that Peter's statement indicates that he agrees that $2\pi R/T$ is distance per time, but having not accepted the use of dimensions as a way to approach this, still asserts that this can be the same as angular velocity. Peter continues in this line of reasoning thinking about dividing by a dimensionless factor of 2π. Seth persists and says, "The units still don't work out though." Jack finally responds that it is a tangential velocity. However, even though he has accepted $2\pi R/T$ is a linear velocity, he immediately asks, "It would have to be divided by 2π to be ω, right?" Seth responds to this new error with another argument based on dimensions, "Divided by R, 'cause you want, like, radians per second." This time, however, Jack appears to be using the same framing as Seth and immediately responds, "Yes. Yeah," and nods.

These two sequences of dialog, first between the students and instructor, and next among the students, illustrate a potential “hidden efficiency” in some cases of framing mismatch. In the first case, all the students show some lag time in responding for the instructor’s bid to start framing the problem in ways that allow for dimensional analysis. In the second case, Seth faces a similar temporary clash of framings, when Peter responds without actually considering dimensions. It cannot be established with certainty that Seth’s later use of dimensions is a direct result of the instructor’s earlier interaction, but that explanation is certainly plausible. If that were the case, then the time the instructor spent could be argued to be a learning experience for Seth. The time the instructor took to push for a shift in framing may have resulted in Seth subsequently using this same strategy when it was beneficial later on.

Similarly, the time that Seth spends pushing his framing shift when addressing the $\omega = 2\pi R/T$ problem, not only pays off in the form of getting this specific error corrected, but also pays of immediately when Jack makes the subsequent error of thinking ${}^{v}\!/_{2\pi} = \omega$. With this new error, Jack immediately switches to Seth’s framing and rapidly agrees to Seth’s proposition that ${}^{v}\!/_{R} = \omega$.

While other interpretations of these examples are possible, they at least illustrate situations where a clash of framings can have potential advantages at a later time. Essentially, the framing clash can appear to be inefficient in achieving its immediate aims while the framing clash is occurring. However, if the effort spent in establishing a common framing results in the subsequent faster switching of framing when it is advantageous, then the overall “efficiency” of the interaction is actually quite high.

5.6 Framing the Problem – Instructor and Student Alignment

One additional aspect of "common framing" is between the instructor and the students. When initially coding the transcript, one of the first impressions to emerge was the extensive usage of a physical mapping frame. Four of the five groups had coded instances of physical mapping more often than they had coded instances of being in any other frame. Here are examples of the physical mapping frame taken from three different groups:

Group 1 - Tom draws chopping marks on a drawing of the ring and says, "Well, but there's not a ...no this is, this is *dr* around here...here to here; that's dr, which is $Rd\phi$"

Group 4 - Kevin, gestures the right hand rule and says, "Well, our magnetic field points that way, right?" (gestures up), "It's constant..."(points at the *z*-axis).

Group 5 - Biff says, "So we said that the velocity of the electrons would be equal to $2\pi r$ divided by period T," and then writes "$v = \frac{2\pi R}{T}$"

Examples of making explicit references to physical things and using drawings and gestures occur repeatedly throughout all five groups. This indicates that the class, taken as a whole, interprets this as primarily a physical mapping problem. This matches the instructor's expectations. She sees this problem as an opportunity for students to think geometrically and focus on turning a physical problem into mathematical symbols.

Thus one thing that Bing's epistemic framing analysis can do is illuminate whether or not there is an alignment between what the instructor envisions as the primary focus of a particular problem and what the students primarily focus on when solving that problem. In this case, the students and instructor are in alignment. PER literature is filled with examples of students viewing a problem very differently than the instructor. The epistemic framing lens can be a tool to see if the students and instructor at least agree on the nature of the problem and what epistemic resource they expect to primarily use.

5.7 Summary

Overall, the data show that there are differences between what happens when students have a common framing compared to what happens when they don't. Similar to the situations Bing (2008) analyzed, students will sometimes interact using a steady common framing, will sometimes smoothly switch framing, and will sometimes have different members of the group framing the activity differently. Each of these framing situations result in different types of conversations.

While some of the examples hint that using a common framing can sometimes result in more quickly reaching a correct solution, there are also examples that show that this is by no means universally true. Furthermore, important changes in student thinking may occur during framing clashes that could result in learning gains. To determine if one framing situation results in a richer or more efficient conversation, would require an operational definition of those terms and would require considering not just what happens when a framing clash, or absence thereof, occurs, but also the subsequent impact later in the conversation.

In addition to student-student interaction within groups, Bing's four framings can be used to indicate whether students and the instructor are envisioning a similar overall framing for the task at hand. In the case of the ring problem, students engaged a physical mapping framing more than any other framing. This is in alignment with what the instructor had envisioned.

CHAPTER 6: DESCRIBING A CALCULATOR'S EFFECT ON STUDENT THINKING

6.1 Overview of a Calculator's Effect on Student thinking

This chapter uses Bing's (2008) epistemic framing (discussed in Chapter 5) to consider the impact of a powerful symbolic calculator on student thinking. This chapter specifically respond's to Bing's claim (Bing, 2008; Bing & Redish, 2008) that in the presence of a powerful calculator such as Mathematica, students are influenced to more frequently use calculation framing.

The data from the Paradigms course provides a different perspective. When instructing students to find the magnetic potential for the ring, the instructor explicitly asked students to create an expression that could be entered into a symbolic calculator (and the instructor did indeed do this at a later time). We found that students solving for the ring problem used physical mapping more than any other framing. This casts the use of symbolic calculators in a different light.

Bing looked at student thinking while the students were actually using a symbolic calculator, while the data for this dissertation show students working before using a calculator. Because neither Mathematica nor Maple (a similar calculator to Mathematica) was actually in use during the data for this dissertation, there is nothing in our data that either directly supports or refutes Bing's observations about what students do in the actual presence of a powerful symbolic calculator. When considering the impact on student thinking of a tool such as Mathematica, it is useful not only to consider cases during which they are actually using the tool, but also to consider the calculator's impact on their thinking at other times.

6.2 Bing's Claim that the Presence of a Calculator Influences Students to Use Calculation Framing

Bing's claim is that while using Mathematica, students retained their "math sense" and were actively engaging their mathematical thinking throughout their time using this tool. However, he asserts that the presence of Mathematica strongly influenced students to frame what they were doing as "calculation". He argues that Mathematica makes students less likely to use other frames such as physical mapping or mathematical coherence, even when switching to these frames would be more productive than focusing solely on calculation. This is especially true when the attempted calculation is based on assumptions that do not accurately correspond to the physical system at hand. He claims that Mathematica plays more than just a passive role in influencing their framing. He argues that Mathematica actively reinforces students use of the calculation frame.

The following is an example of data from which Bing draws his conclusions. Beginning on page 92 of his dissertation, Bing provides a transcript of student discussion after he describes the problem that students are considering. Bings interspersed comments throughout the transcript have been omitted. Bing describes the situation as follows:

> [Students] are working on Problem 5.6 in the [text] (Griffiths, 2004). The problem asks them to calculate $\langle(x_1 - x_2)^2\rangle$ for two particles in arbitrary stationary states of a one-dimensional infinite well, where x_1 is the coordinate of the first particle and x_2 is the coordinate of the second. Three successive parts of the problem ask them to assume the particles are distinguishable, identical bosons, and identical fermions, respectively. In the course of this calculation, the students realize they need to evaluate $\int x_1^2|\psi_n(x_1)|^2\, dx_1$. The transcript begins with a student in the group explicitly mistaking the limits of integration to be from negative infinity to positive infinity instead of just over the width, L, of the well.
>
> **S3**: The integral is from negative infinity to infinity, right?
>
> **S4**: Yeah.
>
> **S3**: So we have x squared *Types into Mathematica …one minute later…*

S3: It’s telling me it doesn’t converge. What if I tried… *Sets Mathematica aside, begins trying to integrate by parts with pencil and paper*

S5: So what’s the integral equal to?

S3: It wasn’t happy, so let me just try something else.

S5: Oh, we got undefined?

S3: It said it didn’t converge.

S5: trig substitution

S3: by parts

S5: oh, by parts

S4: Yeah.

S3: So *Starts typing again*

S6: Can you break it up into different parts and then do it on a TI-89? That’s what I usually do, a combination by hand, by calculator.

S5: Well, integrate it indefinitely and plug in.

S7: Are you not substituting a value in for n and L, or are you?

S3: Umm, no, but I just tried doing x-squared, sine of x squared, and it’s not happy.

It is reasonable for Bing to assert that these students are using a calculation frame to the exclusion of other frames. These students suggest a variety of ways in which the calculation and the tools used to perform that calculation can be altered to achieve a different result. When they become aware that the integral does not converge, they miss an opportunity to engage in physical mapping and check their assumptions. Instead, they stay immersed in a calculation framing and look to change the way in which the result is calculated.

6.3 Students Solving the Ring Problem Use Primarily Physical Mapping, While Preparing to Use a Calculator

With the ring problem, we see a very different situation. Most students spent more time in a physical mapping framing than any other framing. With the exception of a single student, calculation could not be claimed as a dominant framing while working on the ring problem. Thus, the impending use of a powerful calculator for solving the ring problem is clearly not exerting some powerful influence that consistently results in students trapped in a calculation frame.

It should be noted that in this ring example we will specifically consider Maple, which is a symbolic calculator similar to Mathematica. Maple was the calculator used during this course (in subsequent years, Mathematica replaced Maple).

6.3.1 Maple's Role in the Ring Problem

The following is how the instructor introduced the ring problem:

> "We're going to go back to the case of the ring. You have a ring with total charge *Q*, radius *R*, and now we're going to make it spin, so that the charge is moving." (holds up a hula hoop with plane of ring parallel to floor and slowly turns it). "So you have a spinning ring of charge, with period capital *T*, and I want you to write an expression for the magnetic vector potential..." (lifts right hand upwards and makes a fist above ring) "...anywhere in space, in a way that Maple could evaluate it."

A physical ring is presented and demonstrated to be spinning. Students are to take the given information about this ring and then create an expression for magnetic vector potential that a powerful symbolic calculator can evaluate. Thus, the impending use of a powerful calculator is explicit. Students know that they need to create an expression specifically to be entered into this program. However, the impact of this calculator on students' thinking is very different from what Bing saw when he observed students while actually using the calculator.

One important factor may be students' earlier experience with small group activities involving finding the electric potential and electric field of the ring. With earlier ring problems, students were not allowed to use Maple during the group problem-solving activities. Furthermore these earlier activities gave students an indication of what the instructor expected during group problem-solving.

While trying to solve for the magnetic vector potential of the spinning ring, calculation framing was used by every group, and more than one student had short periods in which they were so focused on calculation that they were missing bids by the members of their group to switch frames. However, unlike the examples Bing saw, it would be a far stretch to claim that these instances of being "stuck" in calculation were a result of the calculator. During this session, students performed 100% of their mathematical manipulations by hand, even though hand-held calculators were available. The transcripts of 15 students working on this problem revealed only a single student who explicitly mentioned a calculation tool and this was done in two separate instances. With the exception of this one student, calculators were not even mentioned during the problem-solving session.

6.3.2 Allen Mentions Maple

Because only one student explicitly mentioned the calculator, it is interesting to consider the impact the calculator had on that particular student's thinking. The student who mentions it is Allen from Group 1, that also includes students Laura and Tom. The first time that Maple is explicitly mentioned, Group 1 has an expression, Q/T, for current, and they have been having various discussions that include using dt instead of T and relating T to other variables. After Laura achieves a nonsensical result from an attempted calculation, Allen questions whether T can be validly used in the equation, without equating it to some other variables.

Allen, "Do we have to write period in terms of something else, or can we just leave it as *T*?"

Laura, "...Well, we have to..."

Tom, "Well is this...?"

Allen, "Oh, 'cause for Maple,...[?]...right?"

Tom POINTS to "Q/T" and asks, "Is this right?"

Laura, "*Q* over *T*...uh...yes...oo, for...for every...no, I mean...what, what's right..."

Tom (over Laura), "Is it charge per time? I mean is..."

Allen, "Yeah, for current. It should be."

Laura, "I mean, that is...yeah, that's alright, right? The total charge passes per *T*." [Gestures by closing hand into a fist and then moving her fist side to side]

In this case, Allen is considering whether Maple will specifically require the use of some representation other than simply "*T*." However, the other students in the group do not enter into a discussion about the calculation tool. Instead they return to whether Q/T is a correct expression for current. At this point physical connections are immediately made as Tom asks, " Is it charge per time?" and Laura responds, "…yeah, that's alright, right? The total charge passes per *T*."

It is unclear what variables Allen thinks Maple would prefer, and since the discussion does not continue about Maple, we do not get insights into how he thinks this question would impact what they need to do. However, it is clear that this reference does not get Allen, nor other members of the group, to overly focus on calculation. Instead, what is seen is that all three students rapidly switch into a physical mapping frame, and explicitly link the symbols being used to the physical situation.

The second example in which Allen mentions Maple comes as the group is trying to take the general equation given in class,

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\iiint \frac{\vec{J}(\vec{r})d\tau'}{|\vec{r}-\vec{r}'|}$$

and make it suitable for the one-dimensional charge distribution they are considering. Laura currently has written in front of her:

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})d\phi}{|\vec{r}-\vec{r}'|}$$

Allen makes a reference to Maple in line 7.

1: Allen "Um, so how do we go from $d\tau$ to $d\phi$?"

2: Laura, "Well $d\tau$ is a volume, and *dr*,...but we kind of have a..." [erases the ϕ, that was part of $d\phi$, in her equation to get

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})d}{|\vec{r}-\vec{r}'|}$$

3: Tom, "Well if you go backwards..."

4: Laura "If we already have this whole 2π*R* thing, then..."

5: Tom, "Because if you take away one of those integrals then you are d(area) and if you take away another one then it's *dr*."

6: Laura, "Yeah, I definitely think it's d..." (writes an *r* into the *dr* in her equation to get

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})dr}{|\vec{r}-\vec{r}'|}$$

7: Allen, "So then we'll write *dr* in terms of **i**'s, **j**'s, and **k**'s for Maple's convenience."

8: Tom, "Yup"

9: Allen, "And then we're in cylindrical...(points to denominator of Laura's integrand)...I'm gonna guess, for this portion"

10: Tom, "So what is our *dr*?"

11: Allen "Wait, we can write *dr* in terms of cylindrical coordinates."

12: Laura, "...because we have to...OK, so we have a...?..."

13: Tom "...Wait, no,no,no, ...yuh...we are going to have a d(theta)."

14: Laura, "d(phi), right?"

15: Tom "d(phi)"

16: Allen, "...well because this is a vector though…" [points to *dr* in Laura's formula]…*dr* right? So then that will have components of *dr* ***r***-hat plus *rd*...*rdr*-***r***-prime-hat."

17: Tom (draws two marks close to each other on the ring). Well, but there's not a ...no this is, this is *dr* around here...here to here; that's *dr*, [writes *dr* next to the two marks on the ring] ..which is $Rd\phi$."

Allen refers to the need to use rectangular basis vectors as being "for Maple's convenience". By considering the issue to be one of "making the calculator happy", Allen perhaps skirts the underlying issue of needing basis vectors that do not change during integration. These rectangular basis vectors are required in order to perform the calculation, regardless of whether the calculation was performed with a calculator or not. However, as in the previous example, mentioning Maple does not push the group into a calculation framing. By line 17 Tom is drawing two marks very close to each other on the ring and labeling it *dr*, which is a clear use of physical mapping.

6.4 Conclusion

Among the groups in the classroom, the overall extensive use of physical mapping demonstrates that Maple was not exerting an undue influence that resulted in students staying in calculation framing. In these specific examples, it can be seen that even when Maple was explicitly considered, it did not result in students framing the problem as calculation for an extended period of time.

When considering the larger issue of when, where, and how often calculators should be used in the classroom, it is important to consider the impact of these devices more than just during their actual usage. To use an analogy, consider a large-scale construction project. Imagine that the engineers are aware that during construction they will have access to a variety of heavy equipment. If we wanted to analyze the effect of using power equipment on the overall construction project, we would get a skewed view if the only data we had was from riding along with a bulldozer operator. Our ride with the bulldozer operator might lead us to conclude that using bulldozers causes operators to disproportionately view things as a “bulldozer problem” for which using a bulldozer is the preferred method for solving the problem. We would be unaware of the overall impact that using power equipment has the construction process and on what is eventually constructed.

Bing’s data provide useful insights into understanding that Mathematica can result in students disproportionately using a calculation framing while they are actively using this calculation tool. However, we should be careful not to assume that this reflects the larger picture of the effect that using Mathematica has on student thinking. Bing is careful to limit his specific claims to the effect of Mathematica while students are in the presence of Mathematica. However, because he does not examine data that show student problem solving when the use of Mathematica is imminent but not actually present, nor look at student thinking after Mathematica has been used but is no longer present, one can be left with a skewed view of Mathematica’s overall impact.

The observations of students solving the ring problem clearly show that the broader impact of powerful symbolic calculation tools is significantly different from what is seen only while those tools are in use. To obtain a more complete picture of the impact of symbolic calculators, one would need to consider a variety of situations, including looking at student reasoning before, during and after using Mathematica. While these new data do not provide a complete picture, they do provide another perspective of student thinking that was not apparent from Bing’s data. We can now see that it is possible to explicitly include calculation tools in the overall problem solving process while still having extended periods during which students integrate their physical reasoning with their symbolic manipulation.

CHAPTER 7: FRAMING AND EXPERTISE

7.1 Overview and Theoretic Frameworks

Chapter 7 addresses the dissertation's main purpose of providing a rich description of student reasoning by considering the question, "In what ways are students using problem-solving expertise as they work through this problem?" Problem-solving expertise has been defined and described in many ways, but several of those perspectives include the idea that physical and geometric sense-making is an integral part of expertise.

In the literature review (Chapter 2, section 2.3) we looked at the three different theoretical perspectives of Bing (2008), Krutetskii (1976), and Kuo, Hull, Gupta and Elby (2010). We now apply these perspectives to students solving for the magnetic vector potential of a spinning ring of charge.

In Kuo, Hull, Gupta and Elby's language, expert problem solvers blend conceptual and symbolic reasoning. In Krutetskii's language, highly capable students most often have harmonic reasoning, which is the ability to combine analytic and geometric approaches. In Bing's language, students show expertise when they value consistency among different framings and switch framing when appropriate. The commonality of the three models is that fluidity between switching approaches is important for expertise.

While there are many similarities, there are also differences. Bing considers use of authority to be a framing, whereas the other models do not directly address this. Kuo, Hull, Gupta and Elby focus on whether students are treating equations as "gizmos" instead of bringing conceptual understanding to bear on the equation while using it. Krutetskii focuses on students' underlying abilities in solving mathematics problems.

The dialogs of four different students from four different groups are used to illustrate different levels of expertise found during this problem solving session. We first consider Tom. Tom was chosen as an example because the instructor identified him as an especially strong student. This allows us to see what insights the three models offer about the expertise of a student considered to be a strong problem-solver. The second student, Biff, was selected as an example because Bing's model of frame-switching flagged Biff as exhibiting traits of expertise. However, as will be discussed later in this chapter, Biff also exhibits significant problem-solving weaknesses. The remaining students, Tanya and Kevin, are examples of two struggling students. Although Kevin and Tanya are both struggling, the models of expertise paint very different pictures of these two students.

7.2 Tom: An Example of a Strong Problem Solver

Tom was an A student and was also considered by the instructor to be a very strong student. Of the 15 students observed, Tom was the only one who, after the time spent in group work, took the problem home and reached a complete solution on his own before the next class meeting. Because students were not assigned to do this, we cannot eliminate the possibility that other students could have done this as well. However, during the problem-solving time observed, Tom repeatedly offered well-supported, clear and correct ideas. He was able to demonstrate mastery of parts of the problem that other students either misunderstood or failed to consider entirely. Many students demonstrated significant gaps in understanding that would have made successful completion of the problem on their own unlikely. Thus, Tom can be considered capable of successfully approaching this problem in a way that many of his classmates were not able to do. Based on student grades, the instructor's evaluation, and the observed data, we will consider Tom to be an expert problem solver.

7.2.1 Looking at Tom from Bing's Perspective: Sustained Physical Mapping

Let us now consider Tom from Bing's perspective. Tom stays immersed in a physical mapping framing almost the entire problem solving session. Other than stating "givens" in the problem, he never uses an authority framing. Only once does he outwardly perform a calculation, and that is only for a few seconds. Only on one occasion, in one single sentence does he consider the units of a result, which could be considered mathematical coherence. For the entire remaining time that his utterances and actions are clearly codable, they are always physical mapping.

Tom's heavy use of physical mapping is in alignment with the instructor's view of the nature of the assigned problem. The problem was designed to be primarily a physical mapping problem during which students would spend the majority of their time taking their physical and geometric understanding and creating a symbolic representation.

At the very start of the problem solving session, Tom leads his group in defining how the problem will be addressed. The following is the first minute of transcript after the group assembles, Tom comments are in bold.

> **Tom draws ring and says, "Ok, I did my part"**
> Laura, "OK"
> Allen, "Won't this more or less just be the same thing we've been doing,...except in terms of mu naught?"
> Laura writes "Period = T"
> **Tom writes "Q total charge" and "$\lambda = Q/2\pi R$"**
> Allen, "Let's see..."
> **Tom, "We have a linear charge density."**
> Allen, "We do."
> Laura, "Yeah, OK, good job."

Tom, "Period is *T*"

Laura, "So, it has to do something per *T*"

Tom, "So it does one complete revolution..." [gestures around in a circle]

Laura, "Yes"

Tom, "...every *T*"

Laura, "Yes"

Tom, "And we want to know how much charge goes through here [draws a "gate" on ring] per time,...right?"

Tom maps out the known variables, gestures the motion of the ring and draws a “gate” on the ring that represents a point through which the current will be considered. Although his statements of the givens, that Q is total charge and the period is T, could be classified as authority framing, he immediately relates the given quantities to the physics and geometry of the ring. This is physical mapping framing. In addition to using explicit language that relates the physical situation to the symbolic quantities, Tom also uses gestures and markings on a drawing, both of which Bing considers to be indicators of physical mapping.

Tom continues to use physical mapping for almost the entire 17-minute period this group works on this problem. While the entire transcript is included in the appendix, we will consider a few specific portions of that transcript here. In the next example Laura poses a question. Students are being asked to solve for the magnetic vector potential everywhere in space. Laura is trying to reconcile this with the idea that the current is contained in the ring itself. Tom responds using a physical mapping framing that includes a drawing. This leads to a discussion of how to correctly add the vectors to find $\vec{r} - \vec{r}$. A portion of this example was previously given in Chapter 3, section 3.5.3 as an example of a group that was using sustained physical mapping framing.

Laura, "But wait, a current potential can't be anywhere in space, it has to be in the conductor...Right?"

Allen, "Well the magnetic field due to the current could be anywhere."

Laura (over Allen), "No, I mean it just says right there," [points at front classroom board] "all space that has...[?]...Oh, wait, this is the magnetic..."

Tom (over Laura), "This,..So you're integrating over it, but that's not where *A* lives."

Laura, "Ohhh, right."

Tom, "So..." [draws an external point from ring]

Laura, "Oh, yeah."

Tom draws $\vec{r}$ and $\vec{r}'$ vectors on ring diagram and labels them (Figure 15)**.**

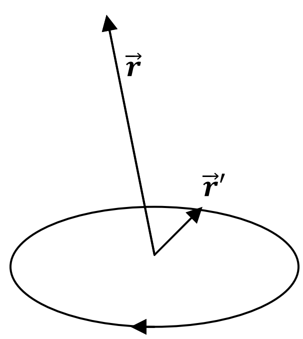

Figure 15: Tom’s drawing of $\vec{r}$ and $\vec{r}'$ vectors

Laura writes an $\vec{A}$ on the drawing next to the external point, "I concur."

Allen draws the $\vec{r} - \vec{r}$ vector with the arrow going from the external point to the ring, then writes $\vec{r} - \vec{r}$ next to the vector (Figure 16)

Allen, "This way, right? Or is it the other way?"

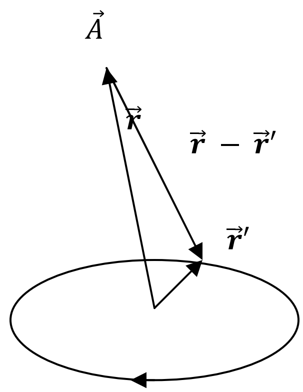

Figure 16: Allen adds the $\vec{r} - \vec{r}$ vector and labels to Tom's drawing

Laura, "It doesn't matter."

Allen, "And, I'm upside down."

Tom, "It, it is. It would be that way." [draws an arrow head on the $\vec{r} - \vec{r}$ vector at the external point on drawing, the opposite direction that Allen drew]

Allen, "Is it... I've never known which way it is...[?]..." [points at external point]

Laura, "No, because you, you add these two." [gestures, moving her finger along the $\vec{r} - \vec{r}$ vector from the ring to the external point]

Allen, "Ahhhh...O.K."

Tom, "It's *r'* plus this thing equals *r*." [points at vector from origin to ring and vector from origin to external point]

Allen, "That one, alrighty...Right, because you go tail to head, tail to head..." [points at vector from origin to ring and then to external point]

Tom, "Yeah...exactly."

Allen, "...from the original tail to the new one." [gestures in a large arc with palm facing downward]

Tom, "Yeah."

Tom uses a combination of symbols, words, and diagrams to clearly convey his ideas. His use of physical mapping is effective in communicating and clarifying ideas. In our final example of Tom using physical mapping, Tom makes a breakthrough that most students do not make during this group time. He relates cylindrical and Cartesian coordinates. This move is critical for eventually reaching a solution to the overall problem.

Allen, "...well because this is a vector though… *dr* right? " [points to dr' in Laura's formula

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})dr'}{|\vec{r} - \vec{r}'|}$$

"So then that will have components of *dr* ***r***-hat plus *rd...rdr*- ***r'***-hat."

Tom [over Allen and drawing two marks close to each other on the ring] "Well, but there's not a ...no this is, this is *dr* around here...here to here; that's *dr*, [writes *dr* next to two marks] ..which is $\boldsymbol{Rd\phi}$

Laura, [writes $Rd\phi$*'* on board] "Right"

Tom, [writes on board, and $\boldsymbol{y = \sin\phi}$ and $\boldsymbol{z = z}$] "Yeah, because when we,...*x* is *r* cosine-$\boldsymbol{\phi}$...*r sin* $\boldsymbol{\phi}$...Right, so that's where we're going to use the phi's. Right?"

While it might be claimed that Tom is taking previously known relationships such as $x = r\cos\phi$, and thus this would be an example of an authority framing, Tom later shows that he can demonstrate relationships geometrically, including putting $\hat{\phi}'$ into Cartesian coordinates. In this problem, the integral cannot be evaluated without parameterization or converting to Cartesian coordinates, because the direction of the current is tangential to the ring and changes during integration. Integrating with respect to ϕ' without considering this will yield an incorrect solution. Most students did not recognize that the changing direction of current was problematic. Seeing the need for Cartesian coordinates is indicative of geometric reasoning. Thus, it is arguable that Tom's use of these equations is more than just grabbing at some previous result.

7.2.2 Tom's Momentary Exits from Physical Mapping Framing

With numerous examples of Tom's use of physical mapping, the exceptions to using physical mapping will now be considered. In the following example, the group had previously established that $I = {}^{Q}/_{T}$. Tom is questioning whether this earlier established expression for current is the quantity that they need in order to solve the problem. Tom is concerned about the concept of "current density" compared to current, and later shows that he is also concerned about how current can be "chopped up" unless it is taken as a density instead of an overall value.

Tom POINTS to "$\boldsymbol{Q/T}$" and asks, "Is this right?"
Laura, "*Q* over *T*...uh...yes...oo, for...for every...no, I mean...what, what's right..."
Tom (over Laura), "Is it charge per time? I mean is..."
Allen, "Yeah, for current. It should be."
Laura, "I mean, that is...yeah, that's alright, right? The total charge passes per T"
[GESTURES by closing hand into a fist and then moving her fist side to side]
Tom [over Laura], "But current density though"
Allen, "So that would just be..[?]..."
Tom, "So, how do you relate, how do you relate...the...oh, so then that would be $\boldsymbol{2\pi R\lambda}$ over..." Tom finishes writing the equation $\vec{\boldsymbol{J}}(\vec{\boldsymbol{r}}) = \frac{\boldsymbol{2\pi R\lambda}}{\boldsymbol{T}}$

In the final line, Tom performs a calculation. He combines the equations $\lambda = Q/\,2\pi R$ (which is written elsewhere on the board) and $J = Q/T$. This minor calculation is preceded by physical mapping and afterwards Tom returns to physical mapping framing. It could be noted that Tom has set a vector equal to a scalar, but this is temporary, as Tom later explicitly considers the vector nature of the current.

The next example is the only other instance in which Tom could be considered to be doing something other than physical mapping. In this case Laura has performed a calculation in which she has made two different errors, one was a physical mapping error and the other was a calculation error. After performing the calculation she reaches the result $T = {}^{1}/_{R}$. As she announces the result to the group, she recognizes the result as problematic and laughs.

Laura, "OK say that this is, OK say that this is $2\pi R$ we just get T equals.... one over R,......which is bad (laughs)."
Allen, laughs, "...I guess we get the change in R and T."
Tom, "Period is inverse length"

If one considers using units as invoking a parallel mathematical structure and using mathematical coherence, then this could be considered a framing other than physical mapping. However, in this instance, Tom could plausibly be simply making a physical connection between the variables and the dimensions. Thus, even with the examples that are potential exceptions to Tom using physical mapping, Tom is clearly staying connected to the physical meaning of the symbols he uses.

7.2.3 Tom's Expertise from Bing's Perspective

What does the example of Tom say about expertise from Bing's perspective? Tom makes clear, compelling arguments using multiple representations (gestures, drawings, words, and symbols). He makes leaps in thinking that other students were unable to make. His work is almost error free. He makes important contributions to the group and the group makes substantial progress on the problem, including wrestling with issues that the instructor had hoped students would consider. However, Tom is not exhibiting repeated use of multiple framings. In fact, he uses a single framing more than any other student in the class.

Although Bing sees the use of multiple framings as a sign of expertise, he clearly states that the use of a single framing for long periods does not indicate the absence of expertise. On page 85 of his dissertation Bing comments, "A professional may set up to do a numerical simulation and spend weeks or even months working largely in Calculation." Bing repeatedly emphasizes that what is considered "too much time" using a single framing must be based on more than purely the amount of time spent. On page 118 of his dissertation, Bing states, "'Getting stuck' is defined with respect to missed bids for reframing, not with respect to a simple clock reading." The example of Tom serves to illustrate Bing's point.

Tom's consistent use of physical mapping is successful and in alignment with what the instructor viewed as productive problem solving for this problem. Tom has a productive strategy and there are no examples where Tom fails to recognize group members making strong bids to switch framings. In this case, using the same framing throughout the problem is consistent with expertise.

Among the 15 students considered, there were several cases in which students got "stuck" in a particular framing, usually calculation. These students would produce incorrect results and be unresponsive when other students attempted to use some other framing to get them to try another approach. The difference between these cases and Tom's case is that the "stuck" students were clinging to unsuccessful strategies even when multiple opportunities to switch strategies were presented.

There are also cases in which students are presented with two different results from two different framings and do not immediately seize upon this as being problematic. In the case of failing to address conflicting results, students are not exhibiting the "superframing" that Bing refers to, for which experts value not only the coherence within a frame, but also among frames.

Tom clearly values coherency. He uses drawings, words, gestures, and symbols to convey ideas and represent relationships. He insists that the physical situation is correctly modeled with symbolic notation. Although Tom is never presented with a conflict between framings, so that we may see his response in action, we certainly see no counterexample which would indicate that he does not value overall coherency.

However, it is important to note that in Tom's case, Bing's coding is not a quick way to identify Tom as an expert. If we were to use Bing's coding and assume that an increase in switching framing would correspond to an increase in expertise, then we would never have identified Tom as exhibiting expertise. Tom uses a single framing more than any other student. Bing did not provide any examples in his dissertation of students exhibiting expertise by staying in a single framing, so this example adds to the available data by providing such an example.

7.2.4 Kuo, Hull, Gupta and Elby's Perspective: Tom Clearly Shows Expertise

Tom's expertise is consistently apparent when using Kuo, Hull, Gupta and Elby's (2010) model. Tom uses a blending of symbolic reasoning with his conceptual and physical reasoning during problem solving. Tom is clearly not using an equation as a "gizmo". As he works through the problem, Tom consistently creates geometric drawings and physical descriptions of his equations and variables.

Furthermore, in the example of Tom challenging the group's previously established result that *Q/T* could be used for current density, Tom clearly isn't waiting for the end of the problem to check the physical reasonableness of his results. He is clearly not following the traditionally prescribed steps for problem solving in which students create diagrams, translate pictures into equations, solve the equations, and then check for physical meaning to see if the answer is reasonable. Instead, his actions more closely align with

Kuo, Hull, Gupta and Elby's blending, in which symbolic reasoning and physical reasoning are intermingled. The example of Tom lends support to Kuo, Hull, Gupta and Elby's hypothesis that "blending conceptual and symbolic reasoning indicates problem-solving expertise more than adherence to 'expert' problem-solving steps."

7.2.5 Tom in Light of Krutetskii's Model

In this particular group problem-solving session we see Tom repeatedly convert geometric ideas into symbols, and then also look at the symbols and consider the geometry they represent. This is a strong indicator of harmonic thinking. In some cases, such as adding vectors, we see that Tom can approach the problem with geometric thinking, while in other cases we see Tom going back and forth between symbolic and geometric representations.

One would have to look elsewhere to find strong evidence that Tom is also a strong symbolic problem solver. Examples of this do exist from other occasions, such as Tom's homework, tests, and class participation on other days. If these other sources were to be included in the data, then Tom's strength in both the analytic and geometric approaches is apparent. However, using only the data from this particular day, Krutetskii's model reveals Tom has strong geometric reasoning, and also suggests strong harmonic reasoning.

7.2.6 Considering the Three Models in Light of Tom's Problem Solving

Bing's model indicates that Tom has an overall value on coherency and uses appropriate framing. Krutetskii's model shows Tom is a strong geometric problem solver and also suggests he is a harmonic problem solver. However, in this particular case, it is Kuo, Hull, Gupta and Elby's model that provides the strongest evidence that Tom shows problem-solving expertise. There are several clear, unambiguous examples of Tom using blending.

7.3 Biff: A Student Who Switches Framing

We now consider Biff. After the transcripts were coded using Bing's coding, Biff stood out as someone who can rapidly switch framings. Switching framing to avoid being stuck was one of Bing's signs of expertise. I initially hypothesized that finding a student who rapidly switches framing could yield a "stand out" example of a student who does this with ease, and thus shows expertise.

However, unlike Tom, Biff is not a student that the instructor would identify as being a particularly strong student. In fact, the instructor considered Biff to be a weak problem solver whose class performance deteriorated over time. In addition, compared to Shawn, who was another student in the group, Biff's contributions to the group seemed to do comparatively little to make progress towards the problem's solution.

Thus, we have a student who at first glance from one perspective seems like he might show exceptional expertise, and at first glance from another perspective seems like a comparatively weak problem solver. Let us see what the data reveal.

7.3.1 Example 1: Biff Exhibits Expertise Finding an Expression for Current

In the case that follows, Biff's group, Group 5, is establishing the relationship between period and velocity. This portion of the transcript is abridged to highlight Biff's thinking. Biff's comments are in bold.

1: **Biff, "Alright, it's spinning at such-and-such a rate, right?" [draws two hash marks on ring]**

2: **Biff draws a vector from center of ring and labels it "*R*". On another part of the board he writes "period" and underlines it. Beneath that he writes 2π*R* and draws a line under that. It is unclear if there is anything under the line.**

40 seconds later

3: Shawn, "I equals λv"

4: **Biff, "where *v* is the velocity of the electrons, right? [writes "$\lambda(v)$ - velocity of e-"]**

5: Devin writes $I = \lambda v$

6: Shawn, "Yeah, so that'd be from the period; a period of $2\pi R$" [writes $T = 2\pi R$, still holding pen above it]

7: **Biff points at ring**

8: Devin, "Isn't v equal to period times frequency?"

9: Shawn writes "/ v" to finish $T = 2\pi R/v$

10: **Biff "2π*R*, yeah, divided by *v*...That equals meters over meters per second equals seconds over meters times meters, cancel, equals seconds." [writes $2\pi R/v$ = m/(m/s) = s/m * m = s (circles the "s")]**

11: Devin, "That's the period."

Biff starts by referring to physical aspects of the ring and adding to the ring diagram. He refers to the ring spinning and draws hash marks on the ring. He then draws a vector from the origin to the ring and labels it "R". He starts creating an expression for period by writing "period" and then writing "$2\pi R$/", but he does not immediately complete his expression. Biff is clearly using physical mapping in trying to create an expression for *T*.

After Shawn completes the expression by writing $T = 2\pi R/v$, Biff then responds by checking the units. Biff manipulates the units algebraically to show that the final units would be seconds. This shows Biff checking the result by using a different framing. Biff uses at least two, and perhaps three different framings. To the extent that checking units is using mathematical coherence to recognize a similar mathematical structure, then Biff uses this prior to actually performing a direct manipulation of the units to produce an end result (seconds) which corresponds to the physical quantity he is seeking (period).

Shortly after the transcript shown above, Shawn and Biff are both writing on separate areas of the board and simultaneously substituting in values for λv. They glance at each other's work, but also have parts where it appears they are calculating on their own.

> **Biff, under "$\lambda(v)$ - velocity of e-" on the board, writes $\lambda(2\pi RT^{-1})$**
>
> **Biff , "λ equals what?" erases λ, writes $Q/2\pi R$ in its place then crosses out each $2\pi R$ and writes "= Q/T" to get $(Q/\cancel{2\pi R})(\cancel{2\pi R}T^{-1}) = Q/T$**
>
> **Biff, "And what does $v\lambda$ equal? We're just going to call it "A" right?...we'll call it like, what, what, what,..." [writes $A = \lambda v$]**
>
> Shawn, "I think that's ***I***, right?"
>
> **Biff, "Ahh, yeah, yeah, yeah, that's *I*." [erases A from $A = \lambda v$ and writes $\vec{I} = \lambda v = Q/T$]**

Biff performs a calculation and then checks to see if he understands what his result means. In this case he is initially misunderstanding the result and is corrected by Shawn. Soon after this, when the co-instructor asks how the group about the equation written in front of Biff, $\vec{I} = \lambda v = {}^{Q}/_{T}\,\hat{\phi}$, Biff responds:

> **Biff: "OK, So first off we took and we said we have a radius of 2π… [writes $2\pi R$ above ring]…or total circumference of $2\pi R$, right? And then we said that,**

OK, how fast is it spinning around? $2\pi r$ divided by velocity equals period." [writes $2\pi r/v = T$ and puts a box around it] So we said that the velocity of the electrons would be equal to $2\pi r$ divided by period T." [writes $v = 2\pi r/T$].

Dr. Alice, "OK"

Biff, "And we plugged that into here; this little equation we had for I..." [writes an arrow on equation $I = \lambda v$] "...and now we have...[inaudible]... for that. And mind you, this is, this is in the ϕ-hat direction" [writes a $\widehat{\phi}$ after $v = 2\pi r/T$]

Biff shows that he is understanding what is happening physically as well as performing a calculation. Taken together, this set of dialogs shows that Biff is capable of fluidly switching between calculation and physical mapping framing. His also checks units as a parallel method of confirming his calculation, which could be considered mathematical coherence. Under Bing's model, Biff is exhibiting signs of expertise by fluidly switching framing to approach the problem from more than one perspective, and also by Biff assuming different approaches should yield coherent results.

When considering this example from the perspective of Kuo, Hull, Gupta and Elby (2010), we see that Biff is not simply considering the applicable physics equations to be "gizmos". He is checking the physical meaning of his symbols before, during, and after his calculation. His checking the units for $T = 2\pi R/v$ is a mid-stream check for meaning on his way to calculating $\boldsymbol{I} = \lambda v = (Q/2\pi R)\ (2\pi RT^{1}) = Q/T$. At nearly every point in his calculation he is blending his conceptual understanding with symbolic reasoning. Biff's problem solving is consistent with Kuo, Hull, Gupta and Elby's model of expert problem solving.

From Krutetskii's (1976) standpoint Biff shows signs of harmonic reasoning. Geometric thinking is combined with analytic thinking. Krutetskii actually divided harmonic reasoning into two subtypes; abstract-harmonic and pictorial harmonic. Although putting a student's thinking into a particular category requires more than a single example, this example would be most consistent with the abstract-harmonic subtype. The reason for choosing this over the more geometric pictorial-harmonic subtype is because the student never looked at the end result and clearly showed that this was geometrically consistent with the calculation performed. To the extent that harmonic problem solving is shown in this example and is associated with expertise, then Krutetskii's model also shows Biff to be consistent with an expert problem solver.

All three models point to Biff showing expertise in this example. If this were the only example we had, we might question the instructor's assessment of Biff as not being a strong problem solver. However, the data provide another example which points toward a different view of Biff than shown in this first example.

7.3.2 Example 2: Biff Does Not Show Expertise with Magnetic Vector Potential

In a Chapter 5, section 5.4.2, this example was used to highlight the idea of a framing clash. The example is repeated here for ease of reference.

Once Biff's group (Group 5) had established that the current is Q/T in the $\hat{\phi}$ direction, Biff tries to use this information to start evaluating the integral. On the other hand, Shawn sees that many additional factors involving the vector nature of the problem need to be taken into account before attempting to evaluate the integral. Thus, Shawn is framing the problem as physical mapping, while Biff has moved to framing the problem as calculation. Devin joins Shawn in the physical mapping frame.

1: Shawn, "We should go back,...remember, like, you have to go back to vectors for the ***A***."

2: **Biff, "Yeah, somethin'."**

3: Devin? "It's gonna be..."

4: **Biff, "What's that?" [writes $\frac{Q}{t}\hat{\boldsymbol{\phi}} = \boldsymbol{R d\phi \hat{\phi}}$]**

5: Shawn gestures in a circle around ring, "If you go,...here we have this thing, it's going to be like, pointing,...it's going to be pointing, like up, right?" [draws a new line segment originating at the center of the ring to the ring]

6: **Biff, head facing down towards where he is writing, continues to write to get $\frac{Q}{t}\hat{\boldsymbol{\phi}} = \boldsymbol{R d\phi \hat{\phi}} = \int_0^{2\pi} \frac{QR}{T} \boldsymbol{d\phi}$**

7: Devin (over Shawn), "It's going to be...it's gonna...yeah...going to be pointing in the *z*-hat direction. I thought magnetic field involves a cross product...but, uh..."

8: **Biff continues to write to get $\frac{Q}{t}\hat{\boldsymbol{\phi}} = \boldsymbol{R d\phi \hat{\phi}} = \int_0^{2\pi} \frac{QR}{T} \boldsymbol{d\phi} = \frac{2\pi QR}{T}$**

9: Shawn, facing Devin, "'Cause, yeah, it's only going to be that like right there." [traces a pre-drawn vector from the center of the ring to the ring itself, points to $R d\phi \hat{\phi}$ in equation] "...because like over here, isn't it going to be something like that?"

10: **Biff (over Shawn), "Here's the answer that you got from our current calculation; 2π*QR* over T" [draws a box around 2π*QR*/*T*], (Shawn and Chris turn towards Biff), "Which gives us, uh, charge, so Coulombs-distance; meter-coulombs-per second."**

11: Devin, "That's pretty much, I mean that's,...yeah, that's just...[?]..."

12: Shawn (over Devin), "...[?]...what about the,…"

13: Shawn and Devin both point at Biff's equation, but it is unclear exactly what part of the equation

14: Shawn, "… what about the,...to the point though. What about this thing?" [draws an external point and a line segment from the external point to the center of the ring (Figure 17)] "Our ***r*** minus ***r***′..." [writes $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$, next to newly drawn line segment]

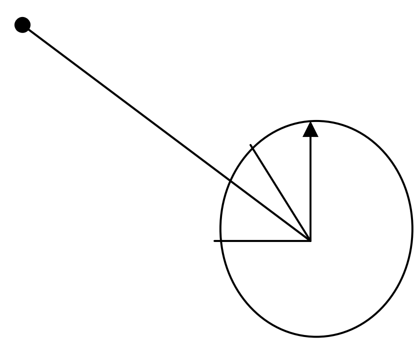

Figure 17: Shawn draws a line and external point

15: **Biff, (in an artificially high voice), "Please, no!"**

16: Group laughs

17: **Biff, "I don't wanna...I'm too young to die. (pause) Alright, let's see then. If you guys want to go through and erase all this and set it back up."**

18: Everyone erases everything written on the whiteboard

At the end of this piece of dialog, Biff appears to accept that the problem needs to be addressed as physical mapping. However, ten seconds later, he shows attachment to his previous answer when he asks the co-instructor, "Um, what should our answer be;...the units? Like, because we came up with...$2\pi RQ$ over T which has units of meters times Coulombs per second."

This example paints a very different picture of Biff than our earlier example. This example would fit Bing's description of someone who is "stuck". Biff stays in a calculation framing, even when a framing switch would be beneficial and even when fellow students were making bids to switch framing. In this case Biff is not showing expertise, even when given opportunities to do so.

From Kuo, Hull, Gupta and Elby's (2010) perspective, Biff is not exhibiting signs of blending while he is doing this calculation. Biff equates quantities with different dimensions and equates scalars to vectors. He does not make any reference to the physical situation and clearly creates an answer which ignores many relevant aspects to the problem. In this case Biff not only does not engage in blending, he also fails to engage in sense-making by comparing his final answer with the physical situation. While Biff finds the units of his answer and tries to check if these units match the units of magnetic vector potential, he does not check for internal consistency of his units and does not comment on what his answer physically means.

From Krutetskii's (1976) perspective, this example would not lead to support for the conclusion that Biff is a harmonic thinker and capable of using geometric thinking when it is optimal for solving the problem. Because Krutetskii's model requires numerous examples before one can determine the "cast of mind" of a student, neither example is conclusive evidence. However, one could at minimum say that using Krutetskii's description of geometric and algebraic reasoning, that in this example Biff did not use the type the thinking that would have best helped him advance toward a solution to this problem.

All three models indicate that Biff can show behavior indicative of expertise in one context and exhibit behavior indicating a lack of expertise in another context. This indicates that whether one considers Biff to be an expert is context dependent.

While Bing suggests that there may exist a somewhat stable overarching framing in which overall coherency is valued, he never argues that level of expertise cannot vary from one context to the next. Biff is a clear example of using varied framings in a context that is more familiar to the student and getting stuck in one frame in a less-familiar context.

Similarly, Kuo, Hull, Gupta and Elby give several examples of students who either show consistent expertise or consistent lack thereof, but also provide examples in which the exhibited level of expertise is context dependent. In one example, Kuo, Hull, Gupta and Elby point out that students who may fail to use the idea of "Base + Change" in a physics context such as applying $v = v_o + at$, may, however, successfully use "Base + Change" concepts when applied to more familiar contexts involving money. This is consistent with Biff showing blending in the more familiar context and not so in the less familiar one.

The three models now provide perspectives that are consistent with the instructor's viewpoint. In this second example, when given a complex and unfamiliar situation, Biff does not show expertise.

It should be noted that this student was selected for analysis because he showed cases of rapid frame switching when applying Bing's coding. Including the example of Tom, who was more consistently in one framing than any other student, there is no indication from these two examples that we can assume that "more is better" when it comes to considering the frequency of framing shifting on a given problem.

7.4 Applying Models of Expertise to Two Struggling Students

We now move to consider struggling students and what different models of expertise have to offer when considering the challenges these students face. Two struggling students will be considered.

To qualify as a "struggling student" in this case, students had to meet three criteria. First, their grades for this course had to be below the class average. Second, because grades can sometimes reflect lack of effort rather than lack of ability, the instructor had to describe the student as genuinely struggling with the content. Third, the number of speaking turns taken by the student had to be significantly fewer than the number of speaking turns used by the member of the group who spoke the most. Although no single criterion would necessarily indicate a student was struggling during this problem, the combination indicates that this is a student who generally struggles and is not the strongest contributor during this specific problem. While these three criteria were the initial criteria, it should also be noted that the quality and importance of the students' contributions to their groups were also considered to be below those of most of the students in the class.

Bing considers expertise to have two components that can be considered separately; one being the extent and connectedness of their knowledge base and the other the degree to which they are successfully framing a problem to be good in-the-moment problem solvers. We will consider these two aspects when thinking about our struggling students.

7.4.1 Kevin Has Content Holes, But Can Switch Framing

The first student we will consider is Kevin from Group 4. Kevin takes 59 turns speaking during the problem solving session, compared to Robert's 75 turns, and Stan's 131 turns. Kevin is an interesting student who exhibits both great difficulties and great promise. The first examples will highlight Kevin's difficulties. Times on the side are given in minutes and seconds from the time the group started discussing the problem.

Example 1: Cylindrical permeability of free space

> [00:21] **Kevin: So it's going to have cylindrical [gestures in a circle, starts to write on board] permeability of space [writes an integral with a large square root sign under it]**
>
> [00:29] Robert: (laughs) uh yeah.

Example 2: Moment of inertia

[00:46] **Kevin: There's got to be some moment of inertia in here.**

(Group never responds to this)

Example 3: Centripetal acceleration

[01:33] **Kevin, [writes $\boldsymbol{a_r = v^2/r}$], "centri,..centri,...centri-pee-tal acceleration"**

[01:37] Robert, "Centripetal?"

[01:38] **Kevin, "Centripetal acceleration....something like that"**

[01:42] Robert, "*m v*-squared over *r*...that's centripetal acceleration"

[01:47] **Kevin, "No, acceleration." [points at his equation $\boldsymbol{a_r = v^2/r}$]**

[01:48] Robert, "Oh, yeah, OK, never mind, I'm thinking for a different type object,...yeah, yes, I agree with that."

[01:56] **Kevin erases equation**

In the first two minutes of conversation Kevin has three of his ideas laughed at, ignored, or dismissed. By the time Kevin makes his centripetal force comment, Stan has been discussing the direction of the field and the idea that the current density J can be expressed as ρv. Robert has brought up the need to express this in terms of T, the period of the ring. Kevin is out of step with the rest of the group.

Using the epistemological framings of Reddish and Hammer (2009), (see Chapter 2, section 2.2.3) the other two members of the group were choosing footholds, while Kevin was still shopping for ideas. Furthermore, the ideas that Kevin was considering were not directly applicable to the problem at hand. From Bing's perspective, Kevin's knowledge base was failing him. The other two members of the group were accessing relevant knowledge and Kevin was not.

Roughly one minute later Kevin suggests a formula that the group considers relevant. Unfortunately, the relevant formula turns out to be incorrect. Kevin’s knowledge base has again failed him.

[03:05] **Kevin writes $f = 2\pi\omega$, "Frequency equals 2π omega"**

[03;08] Stan, "Ooo, nice!"

[03:10] **Kevin, "Right?"**

[03:11] Stan "Yeah"

[03:12] Robert, "Yeah"

[03:18] Stan , "Wait, isn't it ω divided by 2π?...'cause it's...because in 411 we do...to get omega we get 2π times the frequency..." [writes $\omega = 2\pi f$] "...so, yeah, it's divided by 2π."

[03:30] **Kevin writes $f = \omega/2\pi$ and $T = 2\pi/\omega$**

Now that we have a vision of Kevin’s difficulties in terms of knowledge base, let us consider Kevin in terms Bing’s epistemic framing and what that framing has to say about Kevin’s level of expertise. First we must consider that in the preceding examples there was no clash of framing. Kevin’s first three suggestions were in alignment with the group’s physical mapping frame, taking physical ideas and writing down symbolic relationships based on those ideas. Kevin’s $f = 2\pi\omega$ equation is likely misremembered and is thus using authority framing. Stan responds in kind, explicitly referencing PH 411 as the authority for the correct equation, $\omega = 2\pi f$. Thus, nothing we have seen so far indicates that Kevin is unable to use framing consistent with his group members.

Five minutes later we encounter the following situation in which the group is rapidly shifting frames between physical mapping and calculation. This example was mentioned previously in Chapter 3, section 3.5.4 in the context of frame switching. We now use this example to consider Kevin specifically.

[08:30] Stan, "So lambda, lambda expands - that's charge per unit length, which is Q over $2\pi R$, so that..." [erases λ and writes $Q/2\pi R$ in numerator to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\frac{2\pi R}{T} \frac{Q}{2\pi R}}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

[08:50] **Kevin, "That's just *Q* over *T*"**

[08:53] Stan, "Oh, and then..."

[08:55] Robert, "Yeah"

[08:56] Stan, "And then there's the d..." (writes $d\tau'$ in numerator to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\frac{2\pi R}{T} \frac{Q}{2\pi R} \, d\tau'}{(r^2 + R^2 - 2rR\cos(\phi - \phi') + z^2)^{1/2}}$$

[08:58] **Kevin [points at $\boldsymbol{\frac{Q}{2\pi R}}$], "This is λ...our charge density?"**

[09:02] Stan, "Yes"

[09:05] Robert, [points at $\frac{2\pi R}{T} \frac{Q}{2\pi R}$] "Right off the bat, this and this are going to go away. This whole thing you're going to get Q over T. You're just going to get charge over period."

[09:17] **Kevin writes $\boldsymbol{I = \frac{dQ}{dt}}$ on board**

In this case, Kevin clearly makes rapid framing shifts. At 8:50 Kevin is carefully following Stan's calculations and realizes that $\frac{2\pi R}{T} \frac{Q}{2\pi R} = \frac{Q}{T}$. Kevin uses a calculation framing. At 8:58 Kevin makes an "expert" move by switching framings and considering the physical meaning of the $\frac{Q}{2\pi R}$ part of the formula. He asks, "This is λ...our charge density?" In this example Kevin is keeping up with framing shifts and using both physical mapping and calculation.

If we were to consider nothing other than framing, we would be inclined to consider Kevin to be an expert problem solver in this case. Furthermore, this example shows that Kevin has times when his knowledge base is able to support him. Kevin is actively following the conversation and meaningfully contributing to it. He offers ideas that advance the conversation and help the group move toward their larger goal of solving this problem.

It is unclear the basis for Kevin writing down $I = \frac{dQ}{dt}$. He could be (incorrectly) building on his now confirmed realization that $\frac{2\pi R}{T} \frac{Q}{2\pi R} = \frac{Q}{T}$, or he could simply be remembering the formula $I = \frac{dQ}{dt}$, and have that memorized information be triggered by knowing that the magnitude of the current is $\frac{Q}{T}$.

Later, we run across another interesting event. The instructor is asking the group about the direction of the magnetic vector potential, and Kevin responds with, "Is this...this...this the...Well, isn't this the potential we're talking about… (draws three arrows just outside the ring that are in the $\hat{\phi}$ direction)...these arrows going around,..." (gestures around ring) "...all the way out into space?" (gestures with both arms far away from the ring).

Kevin is the only student in 15 that directly indicates he to at least some extent has a valid visualization of what the magnetic vector potential will look like in the space around the ring. This indicates he has access to some interesting resources that are not immediately accessible for most of his classmates.

Bing's epistemic framings and view on expertise give us interesting insights into Kevin that we might otherwise miss. Kevin's knowledge base has significant holes. He frequently accesses irrelevant concepts and formulas or misremembers formulas that are relevant. However, Kevin's epistemic framing resources appear to be fully adequate. He uses three different framings and adjusts his framing as the other members of the group adjust theirs. Furthermore, when his knowledge base is not failing him, he is able to offer insightful, relevant comments. Kevin's strength lies in his underlying ability as a problem navigator and occasional ability to make intuitive leaps. His weakness lies in the holes in his knowledge base and his ability to access relevant knowledge.

The limited evidence we have of Kevin using calculation and physical mapping in harmony indicate that Kevin does not view equations as "gizmos" and blends his conceptual and symbolic reasoning when he is solving a problem. Kevin also shows both geometric and algebraic thinking, suggesting he may be a harmonic thinker. Thus, all three models would lead us to seeing Kevin's potential by considering that he is exhibiting aspects of expertise in his problem solving.

7.4.2 Tanya Shows Weakness in Content and Framing

We will now consider Tanya from Group 2. Tanya has been described by the instructor as a weak student who is frustrating and confrontational. In this group exercise Tanya takes 92 turns speaking compared to 89 for Bob and 121 for Nick.

With this ring problem Tanya shows a strong preference for calculation and persists in using it even when it is not successful. Two and a half minutes into the group's work, Tanya makes the following statement:

> [02:29] Tanya, "So current equals charge over velocity,...the charge density over velocity..."

This turns out to be an error from which she never recovers. The correct relationship for current is $I = \lambda v$, however, Tanya claims that current can be expressed as Q/v or λ/v. Tanya shows with both her words and eventually a written equation that she is truly considering dividing by the velocity. At this point her comment is only spoken. 50 seconds later she puts it in writing.

> [03:18] Tanya says, "Charge over velocity," writes $J = Q/V$ and then says, "Omega," and changes it to $J = Q/\omega$

Tanya initially writes $J = Q/V$ but then changes it to $J = Q/\omega$. This is in contrast to the correct relationship $I = Q/T$. The origin of the equation $J = Q/\omega$ is entirely unclear. Tanya never makes any diagrams or makes any justification as to why she is claiming this to be true. As exhibited in later discussions, Tanya still questions whether she should be using total charge Q or charge density λ. In addition, she later considers whether angular velocity should be a linear velocity. However she never questions that she should be dividing by some sort of charge or charge density and dividing it by some sort of velocity. Once Tanya has decided that $J = Q/\omega$, she enters into calculation framing. At this point she accurately substitutes $2\pi/T$ for ω to get $J = Q/\omega = QT/2\pi$.

A little over a minute later, the following conversation between Bob and Tanya occurs:

[03:43] **Tanya, "Current is charge over velocity, or is it charge density over velocity?"**

[03:48] Nick,"Uhh,...Oh that's what J is, current."

[03:49] Bob, "...[?]...saying that J is, is...ρv."

[03:53] **Tanya, "OK, so it's charge density,...which we don't have."**

[03:58] Bob, "Uhh,...but we could figure it out though, right?"

[04:03] **Tanya, "Yeah, because ρ is Q over $2\pi R$?"**

[04:03] Bob and Tanya both separately WRITE $\rho = Q/ 2\pi R$

[04:10] Bob, "Yeah, big R."

[04:11] **Tanya, "OK."**

Tanya is unsure of whether to use charge or charge density in her equation for current. Her assertion is that current is equal to either charge over velocity or charge density over velocity (which she has already written as charge over angular velocity) and she solicits feedback from the group. Bob uses the equation he copied from his notes to assert that $\boldsymbol{J} = \rho\boldsymbol{v}$.

It is interesting that Tanya appears to accept Bob's assertion that $\boldsymbol{J} = \rho\boldsymbol{v}$. She also accepts $\rho = Q/\ 2\pi R$ and writes it down on her portion of the whiteboard. However, she does not incorporate these relationships into her equation for $\boldsymbol{J}$. Note that her original question was whether current was charge or charge density over velocity. She does not use a mathematical coherence framing or any other framing which would have allowed her to notice the discrepancies between her $J = Q/\omega$ and Bob's $\boldsymbol{J} = \rho\boldsymbol{v}$. This is clearly a missed opportunity to establish coherence between frames. This is the first clear indication that Tanya is "stuck" in calculation framing.

Half a minute later, Tanya is entirely focused on calculation.

> [04:38] **Tanya writes, starting with her earlier equation $J = QT/2\pi$, and substituting $\rho 2\pi R$ in for Q, resulting in $J = \rho 2\pi RT/2\pi$. She then cancels the 2π 's, resulting in $J = \rho RT$, saying "So, this is, ρ, 2, π, R, T over 2π, so J equals ρRT. Now we can actually put it in there."**
> [04:53] **Tanya starts to write an integral, $A = \mu_o/4\pi \int$**

Given Tanya's earlier assertions, she now performs flawless algebraic manipulation to conclude that $J = \rho RT$. Shortly after this, Nick asks, "Rho is equal to Q over $2\pi R$, when was that, when,...when did we learn that?", and Tanya replies, "Charge density equals charge over circumference." Tanya engages Nick in dialog and makes a correct assertion. However, as is typical throughout this problem-solving session, Tanya does not justify her answer. An extended conversation ensues about using *I* instead of *J* and λ instead of ρ.

Tanya's only significant comment during this discussion occurs when she refers to $J = \rho v$ and says "This is, this is the general formula for all space. We're doing it linearly, so it's all lambda instead." This appears to be a use of physical mapping framing. Tanya correctly reduces the general three dimensional formula to a linear one. She uses λ instead of ρ, $d\phi'$ instead of $d\tau$, and a single integral instead of a triple integral. Her resulting integral is:

$$\vec{A} = \frac{\mu_o}{4\pi}\int_0^{2\pi} \frac{\lambda RT}{|r - r'|}\,d\phi'$$

Half a minute later, the following dialog occurs. Tanya's comments are in bold.

[06:17] During a pause in conversation, Bob looks at Tanya's equation and gestures in a circle

[06:40] Nick, "That's what I got," and erases a former equation.

[06:44] Nick, moving over to Tanya, "What'd you get?"

[06:46] **Tanya, circling her equation, "The thing we have to solve."**

[06:51] Nick, "*RT*?" pointing at *RT* in Tanya's equation.

[06:53] Nick, "Oh"

[06:54] Bob, pointing at *T* in Tanya's equation, "*T*'s our period"

[06:55] Nick, "Oh wait, so you're using this for *v*," pointing to "$\omega = 2\pi/T$" on board,

[06:57] **Tanya, "Yes."**

[06:57] Bob, "OK."

[06:57] Nick, "Where v is...2 π over *T*," then erases **v** in his own equation and writes $2\pi/T$

[07:01] **Tanya, "Uh..."**

[07:03] Bob, "Right, and then we said lambda was *Q* over 2π*R*," POINTS at equation $\lambda = Q/2\pi R$.

[07:09] Nick, "So...", writes A = $\mu_o/4\pi * Q/RT$

[07:09] Bob, looking at Tanya's equation and at his own, $\lambda = Q/2\pi R$, "So it gives us the...no, wait......wait, shouldn't *R* be on the bottom?"

[07:23] Bob writes, λ, then erases it, then writes $(Q/2\pi R)\ 2\pi/T$.

[07:26] Nick erases A = $\mu_o/4\pi * Q/RT$ and writes $\vec{A} = \frac{\mu_o Q}{4\pi RT}\int_0^{2\pi}$

[07:29] **Tanya, gesturing at her equation $\boldsymbol{\rho = Q/2\pi R}$ using thumb and pointer finger to act as if she's grabbing that piece of equation and moving it to the top part of her $\boldsymbol{J = Q/\omega}$ equation, "'Cause you move,...you move this up here to get...[?]...and stick it in here."**

At this point there is a discrepancy between Nick and Tanya's results. Comparing the two equations would result in Nick's Q/RT being equivalent to Tanya's λRT. Both should actually be Q/T. Nick says, "That's what I got. What'd you get?" Tanya replies, "The thing we have to solve." While Nick starts comparing his equation to Tanya's, Tanya's reply contains no reference to her actual results and Tanya makes no apparent effort to reconcile her equation with Nick's. Nick comments on the *RT* in Tanya's numerator and asks, "*RT*?" Tanya neither justifies this result, nor quizzes Nick on his results. Tanya is clearly failing to seize an opportunity to consider her results from a new perspective.

Bob continues to be concerned about Tanya's equation when he compares it to his own $\lambda = Q/2\pi R$, and asks Tanya, "Shouldn't *R* be on the bottom?" Tanya responds using calculation framing, gesturing as if she is physically grabbing the $\rho = Q/2\pi R$ equation and placing it in for Q in $J = Q/\omega$.

Tanya soon faces a very strong and somewhat confrontational bid to change framing from Ken, a roving RA who sometimes runs the camera at the back of the classroom and who has previously taken this course. Ken uses a physical mapping framing and directly addresses the *T* in the numerator of Tanya's equation.

Ken, points at Tanya's equation, "In, in this formula you're telling me the longer the period, the greater the magnetic field?"

Tanya, "Don't know."

Ken, gestures rotation and pointing, "So if it takes 3 trillion years to rotate, you're going to increase...the magnetic...magnetic field?"

Nick (over Ken), pointing at $\omega = 2\pi/T$ on board, "Yeah that's not,...yeah that's not angular,...that's not angular velocity. Angular velocity would be, uh... $2\pi R/T$."

Bob writes, "$= Q/2RT$", continuing his equation for current

Nick, "But, yeah, that would be saying increase the period."

Tanya (over Nick), points at *R* in $T/2\pi R$, "*R* has never factored into it."

Nick, "No, no, no...it's, it's, it's"

Bob (over Nick), "Isn't it Q over $2RT$?"

Tanya, "No."

Bob, points at equation, "$2\pi/T$ times $Q/2\pi R$"

Ken leaves

Tanya, "Well, yeah."

Nick, "As, as, it would be as T increases..."

Tanya, "So if we're not supposed to use angular velocity, we're supposed to use linear velocity."

Nick, "Yeah, yeah, as, whoa, whoa, whoa, that's right, that's right, it should be on,...in the denominator because if T increases it it's going slower, so that the magnetic field would go down."

Bob, "OK"

Nick, pointing to "$\omega = 2\pi/T$" on board, "OK, but we just we need to add an R to this because that is not the correct units. Angular velocity is...[v?]...meters per second, so it needs to be R in here."

Bob, "Well this,...this is radians."

Nick, "2πR, that's, that's radians."

Bob, "Right."

Nick, "Yeah."

Tanya, "Yeah, that's, that's angular velocity."

Bob, "So, radians per time."

Nick, "Hmm."

Tanya, "Yeah."

Tanya never accepts Ken's bid to switch to physical mapping. Tanya does not address the concern about period and instead focuses on "R". She points to the equation and refers to it as "R", never making an explicit physical connection to the variable's meaning. When Bob asks, "Isn't it Q over $2RT$?" Tanya simply replies "No." Furthermore, when Tanya says, "So if we're not supposed to use angular velocity, we're supposed to use linear velocity," it indicates she is using authority framing. As opposed to seeking a physical or mathematical justification, she is seeking what she is "supposed to use."

It is interesting that when Tanya first wrote her equation $J = Q/V$, she was unsure of two things, one was whether it should be linear or angular velocity, and the other was whether it should be charge or charge density. In subsequent discussions these are the only two things she has entertained changing. Ken addresses T being in the numerator of her equation, yet Tanya makes no attempt to change anything that will result in correcting this problem. Tanya is not shifting out of her combination of calculation and authority framing. She has repeatedly not accepted opportunities to switch framing, even when these included direct comments from each member of her group and from an RA.

Tanya goes on to use $J = \lambda/v$ in a new calculation which again results with T in the numerator. A minute after the previous dialog, Bob directly questions why Tanya is dividing by velocity.

[09:58] Bob, pointing at Tanya's equation, "Why...why...why are you dividing that?"

[10:03] **Tanya, "Because it's lambda over......Oh, I don't know," ERASES several of her equations.**

[10:07] Nick, "Is this anywhere in space?"

[10:10] Bob, pointing at his equations, seeming to still be responding to Tanya, "This is what she told us yesterday, was that...that they were times, the charge density *times* the velocities."

[10:15] **Tanya, "And times [inaudible]..."**

[10:18] Bob, "So this is..."

[10:21] **Tanya, "Because I think of current in terms of its dimensions, so just divide it out."**

Tanya responds to Bob's questioning why she is dividing by velocity, by declaring, "Oh, I don't know." And erases the offending equation. Bob now switches to an authority framing and asserts "she told us...that they were…the charge density *times* the velocities". Tanya responds that she thinks of things in terms of dimensions, but it is unclear what she means by this statement since she has never previously indicated considering the dimensions and produces expressions which are dimensionally incorrect.

From the above discussion, one might assume that Tanya has "given up" on her expression for current with T in the numerator. However, half a minute later, more than eight minutes after her original error, Tanya makes the comment, "So $J = RT$."

To use Bing's language, Tanya is "stuck." She is missing opportunities to engage in reframing that will allow her to make progress. Her connection with a successful framing has been lost.

7.4.3 Comparing Tanya and Kevin in Light of the Three Models

Tanya is good at performing straightforward calculations. Given her initial assumptions, she flawlessly performs algebraic manipulations to get an end result. However, when her initial assumptions are flawed, the strategies she employs do not result in modifying those assumptions. Unlike Kevin, when faced with evidence that a current framing is unsuccessful, Tanya does not use flexible navigation strategies involving framing shifts. She does not fluidly switch her framing to match the people around her. For this ring problem, Tanya does not demonstrate that she values an overarching coherence. Using Bing's epistemic framing lens, while solving this ring problem, Tanya would be considered a novice in her approach to framing.

From Kuo, Hull, Gupta and Elby's (2010) perspective, Tanya's approach to equations could be considered gizmo-like. She seeks to find things to substitute for the variables that the equation requires. Once the substitution has been made, she "turns the crank" and performs a calculation with no outward signs that she is applying conceptual or physical understanding to what she is doing. Tanya does not exhibit a "blending" of conceptual and symbolic reasoning. Furthermore, Tanya makes no visible attempt to engage in sense-making for her final result. Through Kuo, Hull, Gupta and Elby's lens, Tanya appears to be a novice problem solver.

It is interesting to view Tanya through Krutetskii's (1976) lens. Tanya exhibits the characteristics of an analytic problem solver. When faced with a geometric problem, analytic problem solvers attempt to convert the problem into an algebraic problem without considering the geometric arguments beyond the minimum necessary to convert the problem into a purely symbolic one. Krutetskii does not consider analytic or geometric problem solvers to be "less than" harmonic problem solvers. Viewed through this lens, Tanya is not suffering from some sort of failure to adequately envision the nature of knowledge or suffering from lack of expertise. Instead, she is doing what her brain does best. Her "mathematical cast of mind" is analytic and this is clearly evidenced in her preference for analytic approaches.

Conceivably, had Tanya correctly remembered that $J = \rho v$, she might have, on her own, made it farther through this problem than several of the students from other groups. She was able to see the need for using the one-dimensional analogs for the three dimensional aspects of the general formula, including the triple integral, $d\tau$, J and ρ . She correctly ascertained that she could substitute $2\pi/T$ for ω. Had she also correctly accessed a relevant expression for velocity, and correctly performed all the algebraic manipulations, she would have had an integral with all the scalar components correct. With an even larger storehouse of memorized relationships (or use of notes) she could have also represented $\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'$ in cylindrical coordinates.

In introductory physics, her knowledge-based resources were sufficient to support her strong analytic problem-solving abilities. However, in this particular problem, both physical and geometric reasoning was used by the students who made the most progress toward a successful solution. Without access to harmonic reasoning, Tanya's knowledge base would have to be truly outstanding in order to be successful.

Through each of the three lenses; Bing's, Krutetskii's, and Kuo, Hull, Gupta and Elby's, Kevin and Tanya receive different diagnoses from each other. All three lenses identify Kevin as different from Tanya.

Bing's argument that we can consider someone's ability to navigate a problem, without explicitly considering their knowledge base, is consistent with what is seen here. Both Tanya and Kevin have holes in their knowledge base that lead them to make mistaken assertions. However, Kevin is able to fluidly switch framings and demonstrates an overall value on coherency, while Tanya does not exhibit these traits. From Bing's perspective

Kevin is exhibiting signs of expertise while Tanya exhibits novice-like epistemic framing.

From Kuo, Hull, Gupta and Elby's perspective, Kevin exhibits blending of symbolic and conceptual reasoning, while Tanya does not. Tanya uses an equation like a "gizmo" and once the input is in, she does not check for understanding prior to completing the calculation. Kevin is showing traits of an expert problem solver.

From Krutetskii's perspective, it is conceivable that Tanya might actually be considered the "stronger" problem solver. Her mode of problem solving is analytic, but she is consistently good at it. Kevin is a harmonic problem solver, but we have fewer opportunities to see just how strong his abilities are. In this case, Krutetskii's perspective does not contain the value-laden judgments about having or failing to have expertise. Instead, we would see these two students as having different casts of mind. From the standpoint of being successful in physics, harmonic thinking is valuable and its absence is disadvantageous for Tanya.

These three views on problem solving clarify that we cannot throw all struggling students into a common bin. The diagnoses for Tanya and Kevin are distinctly different. Tanya needs to expand her ability to think geometrically, learn to use blending when problem-solving and learn to shift framing when one particular framing is no longer fruitful. She also needs to work on building her knowledge base. For Kevin, however, we see that the primary concern is helping him build a stronger, more extensive and better connected knowledge base. His ability to blend, switch framing, and do harmonic reasoning is already in place. These existing abilities can be utilized in building a greater knowledge structure.

7.5 Reconsidering Expertise in Light of Expert and Struggling Students

We will now reconsider the three perspectives on expertise in light of the data presented. Bing's, Krutetskii's, and Kuo, Hull, Gupta and Elby each make claims on the nature of expertise. We will now re-examine these claims.

7.5.1 Reconsidering Bing's Claims on Expertise

Bing makes several claims about expertise. One is that the ability to do in-the-moment problem navigation can be done without considering the quality of knowledge base. This assertion is supported by considering Kevin and Tanya, who both have holes in their knowledge base, but exhibit markedly different problem navigation strategies. The framing differences seen between Tanya and Kevin are consistent with the types of differences Bing was observing among students he considered.

Another of Bing's claims is that switching frames when confronted with a roadblock is a sign of expertise. Here we can consider Biff and Tanya. Tanya stays "stuck" in calculation and authority framing, even when each of her group members and an RA confront her errors and make strong bids to switch to a physical mapping framing, Tanya does not make the switch. She clearly does not exhibit expertise. Biff, on the other hand, shows signs of expertise in one case but lack of expertise in another. When finding an expression for current he successfully switches framing, but when trying to find magnetic vector potential, he gets "stuck" in calculation framing. However, when Biff is confronted by group members, he (reluctantly) eventually switches framings. Thus Tanya is consistent in her novice-like approach, whereas Biff shows some ability to use expert-like framing, but is not consistent in its usage.

It is also important to also consider a student like Tom when considering an overall picture of expertise. Tom was a careful, effective and insightful problem solver who consistently employed multiple representations when solving the ring problem. Tom predominantly relied on a single framing. The challenges that Tom faced could be successfully addressed by sustaining his physical mapping approach. Unlike Biff and Tanya, Tom did not face an incorrect result that would not yield to continued usage of the same framing.

When considering whether framing shifts are beneficial or are signs of expertise, we might consider an analogy about using of a steering wheel to turn sharply when driving. When driving along a cliff-side road with sharp turns, we would rightly conclude that failure to turn the steering wheel sharply leads to undesirable consequences. However, when considering a variety of drivers on a variety of roads, we would not conclude that the driver making the most aggressive turns is necessarily the best driver. Good drivers need to know how to make sharp turns, but on a straight road, the absence of turning sharply is not a bad sign.

Thus, when considering a group of students solving a problem, we need to stay vigilant not to fall into a "more is better" trap. It is a sign of expertise to use framing shifts when facing a problem that does not yield to one's current framing. However, more shifting does not necessarily indicate greater expertise. Furthermore, absence of framing shifts may simply be a sign that the chosen framing continues to be successful in addressing challenges. When there is good alignment between the chosen framing and the nature of the problem, then consistent framing can be a sign of expertise.

7.5.2 Reconsidering the Blending Model for Expertise

Kuo, Hull, Gupta and Elby's (2010) approach had a powerful ability to identify expert problem solvers. The claim that blending physical and symbolic reasoning was a sign of expertise was consistent with what was seen across the students in this classroom. In each of the four cases examined in detail, the data lent support to the model's claims of novice or expertise. Furthermore, thinking of expertise in terms of blending was the only one of the three approaches to easily identify Tom as an expert problem solver. Tom's lack of shifting framing did not result in Bing's model highlighting Tom's expertise. Although Tom's had consistent geometric reasoning and good indicators of harmonic reasoning, the absence of watching him do analytic problem solving did not provide a full picture of Tom's abilities. However, Tom was blending symbolic and conceptual reasoning, clearly showing expertise from Kuo, Hull, Gupta and Elby's perspective.

When considering all the students in the classroom, blending of symbolic and physical reasoning tended to be the norm rather than the exception. If blending is a sign of expertise, it would be expected that we would see this more often in upper division courses than in introductory courses. Of the 15 students observed, only Tanya consistently treated equations like "gizmos" and did not use blending when problem solving. Tanya had multiple ways in which she exhibited novice like problem solving.

These data also lend support to Kuo, Hull, Gupta and Elby's assertion that blending is more expert-like than the rigid problem-solving steps sometimes taught in introductory courses. All groups did the "first step" often prescribed in problem solving, which was to visualize the problem and make a diagram. However, after that, their processes could be described as a complete mess compared to the prescribed problem solving steps.

Kuo, Hull, Gupta and Elby include an example of subsequent prescribed steps as being 2) "physics descriptions" where symbols are matched to the corresponding diagram, 3) "plan a solution" where the relevant physics principles are considered, 4) "execute the plan" including performing calculations, and 5) "check and evaluate". What was seen throughout the classroom was very different. Students often "dove into" one part of the problem without having any overall strategy. Students frequently performed mid-course checking and mapping their partial results back to physical understanding and the diagram. Students also had mid-course considerations of what additional physics principles might be relevant and what formulas they should be using. This is consistent with Kuo, Hull, Gupta and Elby's idea of blending. These data lend support to the claim that not only is blending a sign of expertise, but it is also a better sign of expertise than following prescribed steps.

Another claim by Kuo, Hull, Gupta and Elby is one that would be very relevant to students like Tanya. The claim is that teaching blending is a valid instructional target. The assertion is that students can be taught to blend. Although Kuo, Hull, Gupta and Elby claim that data supporting this viewpoint is mostly preliminary, it does hold promise for students like Tanya. We can wonder if Tanya would have been a different problem solver if she had experienced an introductory physics course that encouraged blending instead of presenting prescribed steps. These prescribed steps to some degree reinforce the idea that "executing the plan" means treating an equation as "gizmo" to which a person does not apply physical intuition until after the calculation is done.

If students can learn blending, it would also suggest that even students who have made it to the upper division courses without being good "blenders" might be able to be taught to do so. According to Kuo, Hull, Gupta and Elby, anecdotal evidence from instructors indicates that sometimes students like Tanya eventually "get it" and make the transition to being better problem solvers. Since video exists of Kevin and Tanya later in their undergraduate studies, it would be interesting to examine this for evidence of using expert-like problem solving strategies.

7.5.3 Reconsidering Krutetskii

From Krutetskii's (1976) standpoint, Tanya's analytical expertise is not well aligned with the highly geometric nature of the problem at hand. While Krutetskii often thinks about student abilities as "cast of mind", he also asserts that mathematical flexibility can develop over time. While Krutetskii might argue that Tanya is unlikely to win the Nobel

Prize in physics, he would also argue that Tanya might be able to learn to be more geometric in her thinking and gain flexibility in her ability to successfully navigate complex problems.

On the other hand, students who are already harmonic in their thinking have an advantage. In upper-division physics courses students are repeatedly presented with problems for which harmonic reasoning is better suited than either purely analytic or purely geometric reasoning.

Harmonic thinking is arguably related to blended reasoning. Students who are using a combination of geometric and analytic reasoning are doing things very similar to what Kuo, Hull, Gupta and Elby are calling “blending”. Furthermore, harmonic reasoning will frequently result in students performing calculations and then checking their answers against a geometric argument, or creating a geometric solution and verifying it algebraically. This would match Bing’s switching framing between physical mapping and calculation.

7.5.4 Synthesis of Three Models of Expertise

One aspect of being an expert problem-solver in physics is to have an extensive, well-organized knowledge base. Taken collectively, the three approaches used here provide a clearer picture of the other piece. Expertise in physics involves using harmonic thinking, blending conceptual and symbolic reasoning, switching framing when a particular framing is not resulting in making progress on the problem, and valuing overall coherency. Each of these three models provides additional insight into what constitutes expert problem solving. In combination, they can be powerful tools in considering important aspects of student expertise.

CHAPTER 8: COMPARING THEORETICAL MODELS

8.1 Overview of Theoretical Models

The main purpose of this dissertation is to develop a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. The secondary purpose of the research is to compare and contrast existing theoretical models and describe their usefulness in this context. Chapter 8 addresses this second purpose and makes conclusions about the related research question "What theoretical models are relevant and useful for considering student reasoning in this case?"

Upper Division Physics Education Research (UDPER) is a relatively new area of research and many new theoretical perspectives have been proposed in the past few years. Many of these models have only been tested on a few occasions by a single research group. The literature review (Chapter 2) looked at several of these theoretical frameworks and how have been applied in the past. At this point, we assume the reader is familiar with the various theoretical frameworks and the reader should refer to Chapter 2 if clarification is needed. This chapter (Chapter 8) discusses what we have learned about the applicability, utility, and potential utility of each of nine models when trying the employ them with our data. The focus is on what each of these models has to offer.

This dissertation primarily used five theoretical frameworks to analyze the thinking of students while they worked in small groups solving for the magnetic vector potential of a spinning ring of charge. These five models are; ethnography of communication (e.g. van Zee and Manogue, 2010), epistemic framing (Bing, 2008), blending (Kuo, Hull, Gupta and Elby, 2010), harmonic reasoning (Krutetskii, 1976) and resource plasticity (Sayre and Wittmann, 2008).

In addition, this chapter discusses three theoretical frameworks that were used far less extensively in this dissertation, but showed great promise for looking at students working in upper-division classrooms. These models are; Redish & Hammer's (2009) epistemic framing, Manogue & Gire's (2009) modes of cognition, and Scherr & Hammer's (2009) epistemological framing. One additional model is discussed (Tuminaro & Redish, 2007) primarily to illustrate the differences between upper- and lower-division student problem solving.

8.2 What the Five Models Used Extensively in this Dissertation Have to Offer

8.2.1 What van Zee's Enthnography of Communication Offers

Emily van Zee's (e.g. van Zee, Hammer, Bell, Roy & Peter, 2005; van Zee and Manogue, 2010) enthnography of communication approach is well-suited for in-depth analysis of small, rich sequences of dialog and interaction. While the method (reviewed in Chapter 2, section 2.1.8) is rooted in the study of different cultures, it can be used effectively for considering student thinking in specific cases. Detailed consideration of the utterances, gestures, writing, drawing, and other interactions can yield insights into students thinking as well as their larger participation in a culture of learning and physics. The approach can also be used to consider instructors and instructor-student interactions.

Ethnography of communication was the primary method used in this dissertation for initially gaining insights into how students used geometric reasoning. By "slowing down" and carefully considering each word, gesture and symbol chosen, combined with body language and voice tone, student thinking became more apparent.

It is possible to combine this research approach with one or more other theoretical models. In the research for this dissertation, van Zee's approach is used in combination with the resource plasticity model of Sayre and Wittmann (2008).

8.2.2 What Sayre and Wittmann's Plastic and Solid Resource Model Offers

Student resources vary in the degree to which they are fully formed. Sayre and Wittmann's (2008) model of resource plasticity is discussed in Chapter 2, section 2.1.7 and is used extensively throughout Chapter 4. Sayre and Wittmann use five criteria to determine the plasticity of a resource; ease of use, recency of construction, degree of elaboration, degree of justification, and extended use. They gave an example of two students, one who had polar coordinates as a fairly solid resource and one who had Cartesian coordinates as a solid resource but has polar coordinates as a more plastic resource. Sayre and Wittmann provide evidence that students may choose to use the coordinate system that is most solid rather than the coordinate system that is best for solving the problem.

We definitely saw similar behavior with students solving the ring problem, such as students using algebra when geometric approaches were far better suited. However, we also saw numerous students enthusiastically employing plastic resources, such as the students who tried to apply the right-hand rule to the magnetic vector potential. Further research could yield better understanding of the degree to which plasticity influences the methods that students choose for problem solving.

Sayre and Wittmann used interviews of selected students. In contrast we looked at all the students while they were participating in class. The plasticity model was helpful in both showing the degree to which individual students had solid resources and the degree to which different student resources were solid or plastic across the classroom. We found that every student appeared to have circumference = $2\pi R$ as a solid resource, but many other resources varied in plasticity from student to student.

It was intriguing that while solving the ring problem, students who used plastic resources without reconstructing them often accepted errant results, whereas every student who built their understanding from solid resources did not settle on errant results. Research into the degree to which these findings are seen elsewhere would have important implications for instruction. These implications are discussed more in section 9.3.2 which deals with extensions and suggestions.

8.2.3 What Bing's Model Offers

Bing's model (Bing, 2008; Bing & Redish, 2008; Bing & Redish, 2009; Bing & Redish, 2012), is introduced in Chapter 2, section 2.3.2 and is described in more detail and applied extensively in Chapters 5, 6, and 7. Bing divides student epistemic framing into four groups: calculation, mathematical coherency, authority and physical mapping. We found that these four groupings allow for efficient coding of data that highlights how students are framing the problem at a given time.

The Paradigms data, as well as Bing's original data, show different ways groups can operate, such as having all group members using a consistent framing and all "being on the same page," or having a group rapidly switch framings as they worked through a problem. Another possibility is having group members engage in "framing clashes" where not only was the content in dispute, but the framing used to approach it was different among different group members. Consistently being in the same framing appears to have an in-the-moment efficiency because all group members are working together in a common way toward a common goal. However, an example in the Paradigms data shows how a framing clash can potentially have a large learning value. In one case a student was stuck, but, as part of a framing clash, was given a new way to approach the problem. A few minutes later, this student used this new framing approach to catch errors and help the entire group make progress on the problem at hand.

Bing's framings are also useful when considering questions such as the impact of a symbolic calculator on student thinking. Bing's data show that when students are in the presence of a calculator like Mathematica, the presence of the calculator tends to result in students using a calculation framing, even when staying in this framing is problematic. The Paradigms data shows that it is possible to have students work on a problem, in preparation for entering equations into a calculator, and still predominantly use a physical mapping framing.

Another place Bing's framings are useful is in considering how students are approaching a problem compared to how the instructor hopes students will approach the problem. In the case of the ring problem, the instructor sees the activity as primarily one of physical mapping. Students need to take their understanding of the physical situation and use geometric thinking to translate the physical situation into a symbolic representation. In this case, the framing students used strongly matched the instructor's expectations. Collectively, students used physical mapping more than any other framing and usually more than all other framings combined.

There is additional potential for applying Bing's epistemic framings. For example, in subsection 8.2.4, we consider how Bing's model can be used to consider student expertise. One could also use Bing's model to compare different types of problems, different tasks, different instructional approaches, and even different entire physics programs to see how these impacted the framings students used.

Frequently, physicists need to use all four of Bing's framings. However, faculty concern about student thinking often lies in the physical mapping category, where students need to use geometric thinking and engage in sense-making. To the extent that students need to build their physical mapping skills, Bing's coding could be used to analyze whether students are framing the problem in a way that allows them to gain the needed experience. One could then compare the degree to which different problems or instructional approaches facilitate students using a physical mapping framing.

One could also analyze changes in student thinking over time. Anecdotal data from the Paradigms suggests that some students significantly change their framing approach over time in ways that allow for growth in the students' sophistication and ability to solve complex problems. On the other hand, some students stay entrenched in less successful ways of framing problems (such as primarily calculation) and have far less growth over time. It would be interesting to see whether further research would support this claim and also to see what insights it would yield.

An additional extension to consider would be to focus on the physical mapping category and attempt to differentiate between students going directly from the physical situation to symbolic representation compared to students using a geometric representation as an intermediary. Bing did not distinguish between these two concepts. When I initially attempted to subdivide the concept, I found it difficult to do so. Students would make statements for which it was unclear if they were referring to the actual physical situation or a representation of that situation.

However, in some cases students were spending time going from the physical situation to a geometric representation. This situation was not specifically covered by Bing and for the purposes of this dissertation it was simply lumped with "physical mapping". It could be fruitful to consider situations in which the physical mapping category could be split to facilitate a more complete analysis.

8.2.4 What Three Different Models of Expertise Offer

Three models; epistemic framing (Bing, 2008; Bing & Redish, 2012); Kuo, Hull, Gupta, and Elby's (2010) concept of blending; and Krutetskii's (1976) concept of harmonic reasoning; were used to consider student expertise while solving the ring problem. These models are introduced in Chapter 2, section 2.3, and applied extensively in Chapter 7.

In Bing's epistemic framing model, expertise is identified when students value overall coherency and also when students switch framing when they start to get stuck using an unproductive framing. With Kuo, Hull, Gupta, and Elby's blending model, students show expertise when they seamlessly go back and forth between symbolic representation and considering the physical meaning of that representation. With Krutetskii's model, students are considered to be primarily geometric thinkers, analytic thinkers, or students who easily switch between modes and are considered harmonic thinkers. Although Krutetskii did not claim that harmonic problem solvers were more expert than other types, effective solving of problems in physics is often consistent with using harmonic reasoning.

8.2.4.1 Looking at Four Students with Three Models

In one case, a student, Tom, was identified as an expert problem solver based on instructor observation, contribution to the group during problem solving, and progress on the ring problem. All three models gave some indication that Tom was an expert problem solver. However, Kuo, Hull, Gupta, and Elby's blending model most dramatically indicated he had significant expertise. Tom repeatedly went back and forth between symbolic representation and physical and geometric representations. He consistently had in mind what variables physically meant as he created equations. Using Bing's model, Tom consistently used physical mapping throughout his problem solving, which was in alignment with what the instructor hoped students would be doing. Furthermore, Tom showed that he valued coherency. However, the consistent use of a single framing did not provide opportunities for Tom to show that he could switch framings when stuck, which is one of Bing's strong indicators of expertise. From Krutetskii's perspective, Tom showed strong geometric thinking and his use of geometry and symbols indicated harmonic thinking. However, in this specific data Tom did not demonstrate the degree to which he could also solve problems analytically. Thus, each model gave insights into Tom's abilities, but only Kuo, Hull, Gupta, and Elby's blending model provided clear, repeated, unambiguous examples of Tom's expertise.

At the other end of the spectrum, all three models create a clear picture of Tanya. Kuo, Hull, Gupta, and Elby's model shows Tanya is lacking expertise by using equations as "gizmos", putting variables in equations and manipulating them without considering the physical meaning of those variables while performing the calculation. Bing's model shows Tanya repeatedly "stuck" in calculation framing even when fellow students and an RA made comments that strongly suggested that she change framings in order to correct her errors. Krutetskii's model shows Tanya as an analytic problem solver. She is a good analytic problem solver, but the highly geometric nature of the given problem does not align with Tanya's abilities. Thus, all three models shed light on different aspects of Tanya's novice-like approaches to the ring problem.

In a third case, Biff shows expertise when solving for the current of the spinning ring but does not show expertise when approaching the overall problem of solving for magnetic vector potential. Kuo, Hull, Gupta, and Elby's model shows Biff blending in the simpler case, but using an equation like a gizmo in the more complex case. Bing's model shows Biff switching framings successfully in the simpler case, but getting stuck in calculation in the more complex case. Krutetskii's model shows Biff using a combination of analytic and geometric thinking when solving for current, but only analytic thinking when solving for magnetic vector potential.

In the final case considered, Kevin has major holes in his content knowledge that lead to numerous errors, but the three models also show signs of expertise. He shows ability to switch framings, use blending, and use both geometric and analytic reasoning.

8.2.4.2 The Blending Model

When considering data from the entire class, the data provide support to Kuo, Hull, Gupta, and Elby's assertion that blending better shows expertise than does adherence to prescribed problem-solving steps such as Visualize – Describe - Plan – Execute – Check. Students throughout the classroom were seen checking as they went and "blending" their conceptual and geometric understanding with their symbolic manipulation and analytic approach. This certainly lends credibility to the idea that to create expert problem solvers, teaching students to "blend" offers more potential than teaching students the traditional problem-solving steps.

These data also suggest that Kuo, Hull, Gupta, and Elby's model is efficient at identifying student expertise, as defined by "blending" during this type of problem solving. To the extent that a quick reliable way to analyze data is desired, the blending model may also be more effective at highlighting expertise across a variety of examples than are Bing's model and Krutetskii's model.

8.2.4.3 Bing's Model

Bing's model was especially good at identifying cases where a student has an opportunity to exhibit expertise but is failing to do so. Specifically, if a student is stuck in a framing such as calculation, it can be easy to point to moments in which a student is failing to switch to a more productive framing. This insight offers some additional instructional possibilities not highlighted in the other models. For example, when students are stuck in a calculation framing, Bing's model could offer a way to both identify the problem and provide the remedy in the moment. As was demonstrated in one case in which a student was shown how to switch framing and check units, it is possible to teach this strategy in a way which leads to the strategy being repeated when the instructor is absent.

8.2.4.4 Krutetskii's Model

While some physics problems can yield to almost entirely analytic or almost entirely geometric solutions, a significant number of problems require a combination of these skills. Under Krutetskii's model, harmonic problem solvers are proficient at both analytic and geometric thinking and can use these skills in combination or can choose which skill is best aligned with a given problem. To some extent, Bing's model that students must be able to switch frames hints at the ability to solve both analytically and geometrically, as does the blending model in which students go back and forth between the physical nature of the problem and the symbolic manipulation. However, neither is nearly as explicit about the role of geometric thinking.

In the ring problem, errors in analytic thinking were far rarer than errors in geometric thinking or errors in understanding physical relationships. Considering how students use a combination of geometric and analytic thinking provides a sufficiently different lens from Bing's or Kuo, Hull, Gupta and Elby's that it is worth considering as part of creating a total picture.

There is an additional aspect to Krutetskii's model that shows a somewhat different mindset than many currently popular models. Although Krutetskii claims that students can be taught to improve the flexibility and quality of their thinking, he still views analytic, geometric and harmonic thinking primarily as "casts of mind", which individuals will use throughout their lifetime. To the degree that Krutetskii is correct that the different types of reasoning are highly stable over time, then understanding students' reasoning type could have predictive value for future student success in physics.

Research into valid selection of qualified physics candidates is not the most common focus of physics education research. However, if we ignore the selection process and solely focus on teaching the students we have, then we abdicate our responsibility in the selection process and allow producers of tests like the SAT, ACT and GRE to fill the void. It may be that traits such as use of harmonic reasoning could be one way to identify candidates with high potential to be successful physicists.

8.3 Looking at Models Used Less Extensively in this Dissertation

8.3.1 What Redish & Hammer's "Epistemic Framing" Offers

Prior to adopting Bing's (2008) model the primary epistemic framing model for this dissertation, Redish and Hammer's 2009 epistemic framing model (see Chapter 2, section 2.2.3) was seriously considered. Examples of each of the six epistemic framings could clearly be identified in the Paradigms data. The following six paragraphs contain examples from the Paradigm course of each of these six framings.

"Shopping for ideas" occurs when students "browse" their minds for possibilities and consider whether those ideas are valid or whether other ideas should be sought. For the ring problem, students would suggest useful ideas such as charge density, cylindrical coordinates, vector addition, and using a single integral, along with less-applicable ideas such as angular momentum and the right-hand rule. Students are frequently tentative when introducing an idea, such as, "Shouldn't it be $Rd\theta/dt$?", "Do we have any symmetry?" and "Is omega v over r? Is that relevant?" At other times students simply insert a new idea into conversation, such as "$\boldsymbol{J}$ is equal to, like, ρ". The person suggesting the idea along with other members of the group then respond to these ideas in a variety of ways, including ignoring the idea, rejecting and discarding the idea, immediately incorporating the idea, and discussing and considering the idea.

"Restricting the scope" occurs when students recognize and accept idealizations and simplifying assumptions that ignore certain aspects of the real world. With students solving the ring problem, this occurred frequently. One example is that students were easily able to consider a spinning hula-hoop of charge as having a one-dimensional linear charge density. However, in other cases, students struggled with certain idealizations. Examples that are taxing for some students include solving for the magnetic vector potential in “all space”, considering the effect of a ring of charge that is “far away” from the measurement point, and determining how many terms of a series expansion should be included to be “sufficient”. Issues connected to restricting the scope are discussed in the upper-division PER (UPDER) literature, including describing the challenges faced by upper-division students when using symmetry arguments as related to Gauss’s Law and Ampere’s Law (Manogue, Browne, Dray & Edwards, 2006; Sing, 2006; Wallace and Chasteen, 2010).

“Sense making” occurs when students try to make what they are doing comprehensible to themselves, and possibly to others. For upper-division students solving the ring problem there was a wide variety in the degree of sense-making. At one end of the spectrum one particular student searched for quantities to plug into a formula, performed calculations, got a nonsensical answer, and made no attempt to consider the meaning of the answer produced. At the other end of the spectrum several students engaged in blending, as described by Kuo, Hull, Gupta and Elby’s (2010), in which students were consistently intermixing their conceptual understanding and their calculations, having sense-making be an integral aspect of their problem solving. There were also many students that had intermediate levels of sense-making. The nature and degree of student sense-making is discussed throughout Chapter 4, and specifically section 4.8, which focuses on the concept of sense-making in light of the data.

“Choosing foothold ideas” refers to students picking ideas that they accept and hold true, at least for the time being, and build from those ideas. With the ring problem, one group spent a significant amount of time reconciling the concepts of charge density, current density, and total current before finally settling on the idea that the current I could be expressed as Q/T. Once they had accepted this idea, they moved on to consider other aspects of the problem and then used the Q/T result in later parts of the problem.

Redish and Hammer (p. 632) describe "playing the implications game" as "Having chosen a foothold idea, we consider its implications; if X is true, what would that mean?" In several cases students considered the implications of certain assumptions, but in the data considered for this dissertation, there were no cases that would neatly fit into the category of establishing a foothold and then considering the implications.

With "seeking coherence/safety net" students realize that there should be coherence across different ways of understanding a problem. The mathematics, physics, and real world should all align. Redish and Hammer emphasize that students can misremember things and that cross checking against other ways of understanding is useful. Of the 17 students working to solve the ring problem, only one repeatedly ignored strong evidence of disagreement between the claimed result and other information. However, there is a wide range in the degree to which students passively or actively seek coherence or try alternative approaches. We used the concept of seeking coherence in Chapter 7 when considering student expertise.

We found that these six epistemic framings are relevant at the upper-division level and are specifically relevant for analyzing the video used for this dissertation. Several of these six frames are discussed at various points in this dissertation, but especially in Chapters 4 and 7. However, these six framings were not sufficient to encompass the variety of epistemic framing that the upper-division students were using in the ring problem. This led to consideration of an expanded version, proposed by Manogue and Gire (2010).

8.3.2 What Manogue & Gire's "Modes of Cognition" Offer

The combination of Redish and Hammer's framings and Manogue and Gire's additional modes of cognition (see Chapter 2, section 2.2.4) would allow for coding or categorizing a large percentage of the video used for this dissertation. The codes are designed for task analysis and have been used by Manogue and Gire (2009) and Manogue, Browne, Dray & Edwards (2006) allow for asking the question, "What makes this task so hard?"

In the case of the students solving for the magnetic vector potential of the spinning ring of charge, these modes of cognition work well when considering the question, "What types of student thinking are required for students to successfully complete this problem?" Students were seen engaging in every category of thinking proposed by Manogue and Gire. For example, "applying a general principle to a specific case" clearly fits students taking a general three-dimensional formula $\vec{A} = \frac{\mu_o}{4\pi}\iiint \frac{\vec{J}(\vec{r}')d\tau'}{|\vec{r}-\vec{r}'|}$ and applying it to the specific one-dimensional case of the rotating ring. From the standpoint of task analysis, these codes highlighted various types of thinking that students need to do in order to complete the task at hand.

Manogue and Gire designed their codes for task analysis. Using these to code transcripts requires considering several additional factors. When looking at a given piece of transcript, a student can be doing something that could potentially fall into several categories at once. For example, consider students going back and forth between thinking of the geometry of the ring and how to "chop and add" the current. In some ways this might most neatly fit into the "translating representations/harmonic reasoning" category. However, as students go back and forth between their formula and the physical situation, they are often engaging in a form of "sense-making," and as they catch errors and think about what it means, they might be considered to be "probing and refining intuitions". In addition, there is some additional overlap with aspects of "recognizing patterns", "applying learned mathematics", "seeking coherence", "employing a safety net", "playing the implications game", and even "shopping for ideas".

There could be benefits to having multiple codes overlapping. It could highlight the multiple things that students are sometimes doing simultaneously, which could give further insight into the range of things students are doing and how those different things interact. Furthermore, it could be an opportunity to probe how upper-division student thinking is different from student thinking in introductory courses.

For this dissertation we wanted to consider the thinking of all 17 students in class for the entire group problem-solving time. From a pragmatic standpoint, considering each student statement in light of 13 codes makes the task of looking at multiple hours of transcripts quite formidable. Because Bing had developed a framework specifically designed to be used for coding transcripts, his coding was chosen for this dissertation.

8.3.3 What Scherr and Hammer's Behavioral Cluster Model Offers

Scherr and Hammer's (2009) four behavioral clusters; blue for being hunched over worksheets, green for active discussion, red for listening to a TA and yellow for joking around; provide a way for very quickly getting an overview of what students are doing. Scherr and Hammer's coding is discussed in Chapter 2, section 2.2.5.

Prior to settling on Bing's (2008) coding for Chapters 5 and 6, the transcripts of three different groups of three students were coded using Scherr and Hammer's (2009) behavioral cluster coding. The ability to code the transcripts in "real time" (i.e. the time it takes to watch the video at normal speed) allowed for very rapid accumulation of coded data. The coding system is sufficiently clear and easy to use that it is not surprising that Scherr and Hammer had very high inter-rater reliability.

After comparing this coding for the same transcript coded for Bing's epistemic framings, the first impression was that there were notable differences between types of thinking and the progress made based on which behavior cluster groups were most frequently exhibited. Because no detailed analysis was done, the following comments are largely speculative based on first impressions, but they provide an example of potential applications for this type of analysis. Based on these first impressions, groups that were almost entirely in the animated green behavioral cluster tended to have discussions that involved more explicit connections between the geometry of the problem compared to those that were frequently in the blue cluster, in which students are typically hunched over their work with little discussion. Furthermore, these green-coded groups tended to have all three students "on the same page" instead of having individual students doing their own work that diverged from other group members.

The behavioral clusters also highlighted a particular interaction between students and the co-instructor. While the instructor was interacting with one particular group, all the students continued to have animated whole-group interactions and were thus in the "green" mode instead of the more typical "red" mode with student eyes focused on the instructor. In this case, the instructor managed to interact with the group more as a group member than as the sole figure of focus. She often made use of what van Zee and Minstrell (1997) refer to as a "reflective toss", in which questions are deflected back to the students. This results in the students talking amongst each other instead of solely focusing on the instructor. This coding offers great potential for locating instructor interactions that do not fit the standard instructor-focused pattern.

While we saw Bing's model of epistemic framing to be a better match for specifically considering the types of thinking that students were doing during this particular problem, Scherr and Hammer's approach is potentially powerful for a variety of purposes, and appeared to be a yielding fruitful analysis.

This approach could be used to answer questions such as how students frame classroom activities, and what precipitates shifts to desirable frames. Because the analysis technique is so efficient, it could be used on large data sets, such as an auditorium full of students, or numerous students doing homework problems during an entire course. In addition the possibility exists for instructors to use this tool during class time, while they are teaching, in order to assess whether students are currently doing the types of activities they want students to do.

8.3.4 Applications of Tuminaro & Redish's Epistemic Games

The data from the Paradigms courses shows that epistemic games, as defined by Tuminaro & Redishi (2007), do sometimes occur at the upper-division level, but also provided evidence to support Bing's (2008) concern that upper division students' work is often hard to fit into these "games" (see Chapter 2, section 2.2.2). Epistemic games could sometimes be identified among upper-division students, such as when some students played "recursive plug and chug" in which students "identify a target quantity," "find an equation," "plug in known quantities," and then perform calculations. The recursive plug and chug game was seen on several occasions.

On the other hand, consider when one student, Stan, says, "This is, OK,...Oh, I forgot my $d\varphi$-prime, I'm sorry" and writes $d\varphi'$ in numerator of integrand to get

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int_0^{2\pi}\frac{QR}{T}\frac{d\phi'}{\sqrt{r^2+R^2-2rR\cos(\phi-\phi')+z^2}}$$

One would have to engage in speculation as to which resource or array of resources Stan engages when performing a step like this. That he "forgot" indicates he thinks it is obvious. Is he instantly seeing that the $d\tau'$ from the general formula at once reduces to $d\varphi'$? Is he constructing this based on physical arguments? Is he remembering that a previous problem had a $d\varphi'$? Even if his shorthand could be unequivocally deciphered, it is but one step in a complex series of actions that would not fit neatly into the “epistemic games” that are at most six moves.

While the overall problem solving could be approached as a series of nested games, it would then become very difficult to determine if the math was coming before a physical argument or vice versa, which is the type of determination required to differentiate between different epistemic games. While students were solving the ring problem, they frequently invoked larger concepts that contain aspects of several games, are nesting games within each other, or are using “blending” (Kuo, Hull, Gupta and Elby, 2010), in which students are intermixing their conceptual reasoning with their calculations.

8.4 Possible Additional Models and Perspectives

While this dissertation used numerous theoretical models to consider a single set of data, the models used are but a small subset of the many possible models. This dissertation focused on epistemology, expertise, and geometric thinking. However, even given this specific data set, there are numerous other lenses that we could use to examine the data.

For example, there are a host of theoretical perspectives for considering instruction and student-instructor interaction. In the Paradigm courses instructors are using a variety of innovative instructional techniques. It would be interesting to know what insights we could gain by considering the Paradigms instruction from one or more of these different theoretical perspectives.

We could also tap into the extensive body of research on group interactions. In what ways are students exchanging ideas and encouraging or discouraging each other’s participation and thinking?

We could look at the broader picture and apply models of apprenticeship and students' growing participation in the physics community. Or we could go the other direction and pick a specific concept, such as students' use of vectors and apply theoretical models used in mathematics education.

With each additional lens, new insights would be gained. Fortunately, the data used for this research have no expiration date and are available to other researchers. In addition, video of this same course taught over many years is available, as are data from other Paradigms courses. This provides many future research opportunities.

CHAPTER 9: CONCLUSIONS ABOUT STUDENT THINKING ABOUT THE RING PROBLEM

The main purpose of this dissertation is to develop a rich description of student reasoning when facing a highly geometric problem in an upper-division active-engagement physics classroom. The research is divided into three distinct studies designed to collectively address this purpose. Chapter 4 looked at student geometric reasoning as students encounter problem situations ranging from familiar to novel. Chapters 5 and 6 looked at how students were framing what they are doing and the degree to which they saw it as geometric. Chapter 7 looked at how students using problem-solving expertise as they work through the problem of trying to find the magnetic vector potential of a spinning ring of charge. In addition to the primary goal of examining student reasoning, we had a secondary goal of examining the utility of nine different theoretical perspectives for considering students working on problems such as this ring problem. This is done in Chapter 8.

9.1 Overview of Student Reasoning While Solving the Ring Problem

Applying several different theoretical frameworks to the same data setallowed understanding from several perspectives. The following is a synthesis of what was learned from these models, combined with some general observations.

Students were given a problem that was essentially too hard for them to solve by themselves and then asked to solve the problem. The instructor expected meaningful progress but was not anticipating that groups would reach a correct solution in the allotted time. The task required students not only to recall substantial amounts of specific mathematical and physics content, but also to apply them in a new context. By itself, the task of dealing with current as a vector in the integrand provided a formidable challenge. In addition, the problem required students to reduce a general formula from three dimensions to one, solve for the magnitude of the current, use the concept of magnetic vector potential, describe the current and position vectors in terms of curvilinear coordinates and then translate those to Cartesian coordinates.

Overall, students were extremely engaged in the activity. There was very little "off-task" behavior in which students were not focused on the ring problem. Two students were somewhat marginalized in their groups, while the remaining students all actively participated and made important contributions to the group.

Not only were students engaged, but they were engaged in the type of thinking the instructor desired. For the majority of the class time, students used their understanding of physics and geometry to create symbolic representations - a process Bing (2008) calls "physical mapping".

There appeared to be an in-the-moment efficiency when all the students worked together using a common framing. However, in at least one case, framing clashes, in which students were mismatched in their framing, appeared to have an important value for student learning and resulted in one student eventually using an effective strategy that he had not been using previously.

When considering the class as a whole, using ethnography of communications (e.g. van Zee and Manogue, 2010) combined with Sayre and Wittmann's (2008) resource plasticity model, we realized that there were only two concepts that it appeared that every student had solidly, which was that circumference = $2\pi R$ and $\lambda = Q/\ell$. On the other hand, no students showed that they were already familiar with how to effectively deal with current as a vector in the integrand. Nor did any student have memorized how to represent $\hat{\phi}$ in rectangular coordinates.

When students faced a highly familiar problem, such as finding circumference, students quickly produced correct expressions with little or no discussion. On the other hand, when facing an aspect of the problem such as angular speed or related concepts, different students showed significantly different abilities in how to deal with it. In one case a student quickly and correctly used the concept of linear speed to achieve an accurate expression for current. In some cases students had incorrectly memorized relationships that yielded incorrect results. Viewed through the lens of plastic resources (Sayre & Wittmann, 2008), most students had concepts related to angular and linear velocity as a plastic resource. Students who relied on plastic resources without sufficient sense-making strategies often settled on incorrect answers, whereas students who connected their understanding to a solid resource did not accept errant results.

When students faced a situation that was new to them, such as dealing with current as a vector in the integrand, most students made errors and did not employ the strategies needed to recognize that they had made an error. In order to not accept an incorrect result students had to first recognize that there was a difficult aspect to the problem. Students who did not recognize the challenging aspect to the problem, or who thought there was an easy way to solve it, did not engage in the thinking needed to solve the problem. On the other hand, students who recognized the difficult aspects of the problem did not reach successful solutions in the time given, but they also did not settle on incorrect results.

In addition to some of the concepts on which nearly every group of students spent time, there were also concepts that different groups and different students within groups addressed to varying degrees. These included delta functions, the concept of "all space", the nature of magnetic vector potential, the applicability of the right-hand rule, vector addition, choosing coordinate systems, creating geometric models of the physical situation, and "chopping and adding" in order to integrate.

Students were rarely explicit about why they chose a specific way to approach the problem. However, as indicated by the variety of concepts addressed, each group found different ways to break the problem into pieces.

From the standpoint of expertise, the majority of the students exhibited a significant degree of expertise, as shown by using three models related to expertise. Most students used “blending” (Kuo, Hull, Gupta, and Elby, 2010) by going back and forth between the physical meaning of their symbols and the symbols themselves. Students were able to use a framing (Bing, 2008) that was appropriate to the problem at hand and were able to switch framings before being “stuck” for an extended period of time. They also demonstrated that they valued coherence between different ways of approaching the problem. While Krutetskii’s (1976) lens provided very limited direct evidence for harmonic reasoning, we could see most students demonstrating use of both geometric and analytic thinking. Only a single student of the 17 students consistently showed the traits of a novice problem solver based on all three models. However, for some students, expertise was inconsistent and context dependent.

Overall, students broke a complex problem into smaller, more manageable pieces and then used a combination of geometric and analytic thinking to make progress on the problem. The group problem-solving process gave students an opportunity for students to be very engaged in tackling relevant pieces of a very challenging problem.

9.2 Limitations

This study is designed to give a rich description of a particular group of student reasoning in the context of a specific problem. The limitations to broader applicability are numerous.

This study is done in the context of Oregon State University’s Paradigms in Physics program. This study provides the richest description to date of student reasoning while actually participating in an upper-division class. This offers insights into this particular junior-level active-engagement class in this particular setting, and to the reasoning students do during this class. However, the results may or may not be applicable to students in traditional courses or in other active-engagement classes. It would be an interesting subject of future research to consider in-class student reasoning in other programs.

Furthermore, we considered student thinking in the second course in the Paradigms sequence, and solving the third in a sequence of four ring problems. Looking at students solving this specific problem was chosen in part to avoid watching students as they adjust to active engagement and small group problem solving. Thus this study should not be considered an accurate reflection of what student problem-solving looks like when they first attempt group problem solving or when they first encounter junior-level material, or when they first encounter a problem with a ring geometry. It would be interesting for future research to consider longitudinal studies of students across several courses. The video data currently exist to look at many of these students over time.

The study only looked at a single group of 17 students over the course of 26 minutes. The advantage of this study over several other studies is that we were able to look at the reasoning of every student in class. This resulted in fewer issues of sample bias related to which students were considered. However, 17 students is a small sample size and this particular 26 minutes gives us only a short look at how students are reasoning. It would be interesting to see if the same types of reasoning would exist when different students solve the same problem during different years. The video data currently exist that would allow future research to pursue this question.

This study only involved a single researcher, which prevents establishing inter-rater reliability and limits the number of perspectives while considering the data. While the major professor, Corinne Manogue, also looked at the data, the degree to which she did this was not consistent throughout the research process.

We used nine different theoretical frameworks to consider this particular set of data, and applied five of those extensively. While this is far more than most studies, which rely on a single theoretical framework, it is still only a partial set of the many theoretical models available. Examining these data from other theoretical perspectives, such as those addressing student-instructor interactions, would be an exciting area for future research. This could give insights into how different types of instructor input affect student reasoning and problem-solving.

9.3 Discussion, Extensions and Suggestions for Future Work

9.3.1 Using Small-Group Work in Upper-Division Classes

Does small-group problem solving 'work' for teaching upper division students? This question is ill-defined, but at some level, it is what instructors want to know. The research methods employed for this dissertation do not directly address this question, but the data do provide some intriguing insights. As discussed in Chapter 3, section 3.7.4, both Corinne Manogue and I have advocated for the use of small-group problem solving and value its use in the Paradigms program. The following discussion serves primarily to highlight some of the potential advantages to small-group problem solving.

Students were very actively engaged and doing the type of problem-solving the instructor hoped to see. Students primarily used physical mapping in this problem-solving session. In Bing's (2008) data, he had several examples of students stuck in calculation at times when switching to physical mapping would have been beneficial. To the extent that students need to practice going back and forth between a physical problem and the symbolic representation, then this problem solving session clearly provided a good opportunity to do so.

Students each had different “holes” in their understanding, and the group problem-solving process allowed individual students to get their individual questions answered either by the other group members or by the roving instructors. Comparatively, attempts to use direct instruction to fill this wide variety of individualized gaps in understanding could be highly inefficient.

On the other hand, students sometimes spent time applying invalid processes, such as students who enthusiastically embraced the right-hand rule as a way for finding the direction of the magnetic vector potential. If the desire during an instructional session is to never have students making mistakes, then it would probably be optimal for the instructor to lead the session and have students not do any thinking other than what is dictated by the instructor.

However, if we wish for students to develop strengths in tackling difficult novel problems, then allowing students to do this in an environment where they can receive some guidance and support is potentially beneficial. Overall, students did not spend most of their time floundering, nor did they spend most of their time re-enforcing errant ideas. Instead, students were often focusing on important physics concepts. Sometimes these physics concepts were precisely the ones that the instructor had hoped students would address. At other times students spent significant time exploring ideas that were perhaps not on the instructor’s agenda, but were important for their understanding the task at hand. Frequently instructors were able to intervene when students were on the wrong track and were able to quickly address student misunderstandings.

The group problem solving needs to be in context. The problem-solving session on the first day was 17 minutes of a 50-minute class period and on the second day was 9 minutes of a 50-minute class. The majority of the class time both days involved direct instruction. The Paradigms instructors do not attempt to supplant direct instruction with small-group problem solving, but instead use small-group problem solving as a valuable part of an effort to create an optimal mix.

One way to further study the question of whether small-group problem solving is effective would be to use some of the available analysis tools to determine differences over time and differences between instructional programs. For example, if using small groups is considered one way to help students "think like a physicist", we may wish to consider part of "thinking like a physicist" as using physical mapping, employing harmonic reasoning, switching framing when stuck, using "blending" when problem-solving, and building sense by connecting to solid resources. We have the tools to check for students doing these things. These tools could be used to see how these aspects of student "thinking like a physicist" develop over time, or vary from course to course, or vary from program to program. As part of the process we could examine the impact of small-group problem solving.

9.3.2 Connecting Students to Solid Resources

The piece of this research that has had the most significant impact on my own teaching was the realization that during solving for the ring problem, students who insisted on having a solid understanding of what they were doing never settled on errant conclusions. On the other hand, students were willing to accept and settle on incorrect results when they relied on tenuous knowledge or plastic resources without going through the process of verifying or reconstructing that knowledge from something with which they were sufficiently familiar.

If these results are supported by additional research, there are numerous implications for the concept of "sense-making". If students are willing to base their sense-making on insufficient knowledge, then they are susceptible to accepting nonsense. Deeper sense-making may require students asking not only "Does an answer seem to make sense?" but also asking whether they can establish the validity of the answer based on things they solidly understand.

To the extent that we teach students that a poorly understood answer is sufficient, then we are teaching students to accept shallow sense-making. Furthermore, when we teach new concepts, and use as justification other concepts that students only tenuously understand, then we are teaching students to accept their inadequate understanding as sufficient.

As part of the learning process students often need to work with new, tenuous plastic ideas or processes. However, it may be important to differentiate between when students are to be exploring and testing new ideas and when students are expected to be truly able to ground their understanding on a solid foundation.

The idea that students need to build from solid resources in order to not accept invalid results has potential broader implications. The idea that this can be applied to general human thinking in areas as diverse as politics, economics, and the environment is intriguing, but highly speculative. However, the links to other aspects of physics education are less tenuous. Anecdotally, when holding this idea in my head while making pedagogical decisions in my high school class, I ended up getting student work and student understanding that more closely matched what I had hoped my own students would do, compared to the results I had gotten previously, before this realization.

It appears that I am now more able to recognize signs of plastic or tenuous understanding in students. Furthermore, I now operate under the assumption that if I want students to be able to successfully tackle novel problems, I need to make sure their underlying knowledge is sufficiently solid and that they have been taught how to infuse sense-making into their problem-solving process. I have seen higher average test scores than in past years and have also seen what I perceive to be greater depth of understanding and greater problem-solving confidence in my students.

Further research could determine the degree to which various adjustments in teaching based on these assumptions result in improved student understanding and problem solving ability. Understanding how students use plastic and solid resources offers fruitful possibilities for both research and development of curriculum and instruction.

9.4 Why the Findings in this Research Matter

9.4.1 Why Resource Plasticity Matters

Sayre and Wittmann (2008) claimed that the degree to which a resource is plastic or solid affects which resources a student will employ when solving a problem. In chapter 4 we took the resource plasticity model and used it as way of considering how students reasoned when encountering problems of different levels of familiarity. This allowed us to accomplish several things.

First it allowed us to look at the degree to which different concepts were solid across the classroom. We found that all students had circumference, $C = 2\pi R$, as a solid resource but none had dealing with current as a vector in the integrand as a solid resource. We also found that for concepts related to angular speed, there was a wide variation in the plasticity of this resource from student to student. This matters because it provides a language and perspective for dealing with the current state of knowledge in students. It also matters because we found that there are very few concepts that at least some of the students will not have as solid resources, which identifies a significant challenge for instruction.

Second, in the Paradigms data we saw that every student who connected their understanding to solid resources did not accept errant results, whereas the majority of the students used plastic resources or incorrect resources and were willing to accept errant results. This matters because these data show students having limited sense-making capabilities and accept incorrect results when they don't connect the resources they are using to solid resources. The implication is that if we want students to engage in deep sense-making then we need to provide sufficient time and opportunities for students to build and connect to the necessary solid resources.

9.4.2 Why Epistemic Framing Matters

Bing's (2008) four epistemic framings allowed us to see when students were using a framing that resulted in meaningful understanding and progress in solving the problem at hand. In Chapter 5 we saw that how students were framing affected how they solved the problem. When students were viewing this highly geometric problem as physical mapping they were often more successful in terms of making progress and not accepting errant results. We also saw a case in which an instructor showed a student how to switch framings and that student went on to use this strategy successfully later in the problem. The implication is that it is important to help students frame the problem productively.

In Chapter 6 the data showed us that under the right conditions, the impending use of a calculator such as Maple does not cause students to inappropriately frame the problem as calculation. This matters because it provides a very different perspective than Bing and Redish's (2008) data that showed students inappropriately framing a problem as calculation when in the presence of a calculator. When instructors are considering using a powerful calculator like Mathematica, this can be very important. The implication is that although instructors may need to be vigilant for students overusing a calculation framing while actually using a calculator, it is possible to create an environment in which students are not overly influenced by the impending use of a calculator.

9.4.3 Why Problem-Solving Expertise Matters

Our data support Kuo, Hull, Gupta, and Elby's (2010) assertion that traditionally taught expert problem-solving steps do not reflect optimal problem-solving at the upper-division level. Our data show that students who stay connected to the physical situation using geometry as an intermediary are more successful in solving complex physics problems. The implication is that we should switch from promoting traditional problem-solving steps in favor of developing and using instruction that encourages students to use geometric reasoning and maintain connection between the physical situation and the symbols they are using.

9.4.4 Why Understanding Upper-Division Active Engagement Matters

While lower-division research overwhelming shows that active engagement results in stronger student understanding than traditional instruction, the current reality is that active engagement is infrequently used in upper-division classrooms. The research that is starting to appear (e.g. Pepper, Chasteen, Pollock and Perkins, 2010) suggests that active engagement is more effective at the upper-division as well. For instructors to embrace reformed instruction, it would be helpful to have research that shows what student thinking looks like during active engagement. This research provides a rich description of what student reasoning looks like in an upper-division active-engagement classroom.

REFERENCES

Bao, L. & Redish, E.F. (2002). Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics. *American Journal of Physics, 70*(3), 210-217.

Bing, T.J. (2008). *An epistemic framing analysis of upper level physics students' use of mathematics.* (Doctoral dissertation, University of Maryland, 2008) Retrieved from http://www.physics.umd.edu/perg/dissertations/Bing/BingDissertation.pdf

Cerny, L.T. (2007), *Using inter-related activities to develop student understanding of electric and magnetic fields.* (Master's thesis, Oregon State University, 2007). Retrieved from http://physics.oregonstate.edu/portfolioswiki/whitepapers:sequences:emsequence:start

Chasteen, S.V. & Pollock, S.J. (2009). Tapping into juniors' understanding of E&M: The Colorado Upper-Division Electrostatics (CUE) diagnostic. In C. Henderson, M. Sebella, & C. Singh (Eds.) *2009 Physics Education Research Conference Proceedings* (109-112). Melville, NY: AIP Press.

diSessa, A. (1993). Toward an epistemology of physics. *Cognition and Instruction, 10*(3), 105-225.

Dreyfus, T. & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *Journal of Mathematics Behavior*. *23*, 271-300.

Ferrier, J.F. (1854). *Institutes of metaphysic: The theory of knowing and being*. Edinburgh, William Blackwood and Sons.

Greeno, J.G. (1997). On claims that answer the wrong questions. *Educational Researcher, 26,* 5-17.

Griffiths, *D. J*. (2004) *Introduction to Quantum Mechanics*. *2nd ed*. Upper Saddle River, NJ: Pearson Prentice Hall.

Hake, R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66,* 64-74.

Hammer, D. (2000). Student resources for learning introductory physics, *American Journal of Physics*. *68*, S52.

Hammer, D., & *Elby*, A. (2003). *Tapping epistemological resources for learning physics. The Journal of the Learning Sciences, 12*(1), 53-90.

Heller, P., Keith, R. & Anderson, S. (1992). Teaching problem solving through cooperative grouping: Part 2. *American Journal of Physics, 60*(7), 627-636.

Hinrichs, B.E. (2010). Writing position vectors in 3-D space: A student difficulty with spherical unit vectors in intermediate E&M. In M. Sabella, C. Singh, N.S. Rebello (Eds.) *2010 Physics Education Research Conference Proceedings* (173-176). Melville, NY: AIP Press.

Hymes, D. (1972). Models of the interaction of language and social life. In J. Gumperz & D. Hymes (Eds.), *Directions in sociolinguistics: The ethnography of communication*. 35-71 New York: Holt, Rhinehart & Winston.

Kesonen, M.H.P., Asikainen, M.A. & Hirvonen, P.E. (2011). University students' conceptions of the electric and magnetic fields and their interrelationships. *European Journal of Physics 32***,** 521-534**.**

Knight, R.D. (1995), The vector knowledge of beginning physics students. *The Physics Teacher, 33*(2), 74-77.

Krutetskii, V. A. (1976), *The Psychology of Mathematical Abilities in Schoolchildren*, Chicago, IL: University of Chicago Press.

Kuo, E., Hull, M., Gupta, A., & Elby, A. (2010). *A Problem with Current Conceptions of Expert Problem Solving.* Manuscript submitted for publication.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge.

Loverude, M.E. (2009). Student understanding of basic probability concepts in an upper-division thermal physics course. In C. Henderson, M. Sebella, & C. Singh (Eds.) *2009 Physics Education Research Conference Proceedings*, (189-192). Melville, NY: AIP Press.

Manogue, C., Browne, K., Dray, T., Edwards, B. (2006). Why is Ampère's law so hard? A look at middle-division physics. *American Journal of Physics, 74*(4), 344-350.

Manogue, C.A. & Gire, E. (2009). Cognitive Development at the Middle-Division Level. In C. Henderson, M. Sebella, & C. Singh (Eds.) *2009 Physics Education Research Conference Proceedings* (109-112). Melville, NY: AIP Press.

Manogue, C.A. & Krane, K.S. (2003). Paradigms in physics: Restructuring the upper level. *Physics Today, 56*(9), 53-58.

Pepper, R.E., Chasteen, S.V., Pollock, S.J.& Perkins, K.K. (2010). Our best juniors still struggle with Gauss's Law: Characterizing their difficulties. In M. Sabella, C. Singh, N.S. Rebello (Eds.) *2010 Physics Education Research Conference Proceedings* (245-248). Melville, NY: AIP Press.

Piaget, J. (1953). How children form mathematical concepts. *Scientific American, 189,* 74-79.

Plato & Dyde, S. W. (1899). *The Theaetetus of Plato*. Glasgow: J. Maclehose.

Pollock, S.J. (2009). Longitudinal study of student conceptual understanding in electricity and magnetism. *Physics Review Special Topics - Physics Education Research, 5* (020110), 1 -8

Redish, E.F. (1997, January). *Measuring Student Expectations in University Physics: The MPEX Survey*. Invited talk, presented at the AAPT Winter Meeting, Phoenix, AZ.

Redish, E.F., & Hammer, D. (2009). Reinventing college physics for biologists: Explicating an epistemological curriculum. *American Journal of Physics, 77*(7), 629-642

Sayre, E.C. & Wittmann, M.C. (2008). Plasticity of intermediate mechanics students' coordinate system choice. *Physics Review Special Topics - Physics Education Research, 4*(020105), 1-14

Scaife, T. M. & Heckler, A .F. (2011). Interference between electric and magnetic concepts in introductory physics, *Physical Review Special Topics-Physics Education Research*, *7*(010104), 1-11.

Scherr, R. E., & Hammer, D. (2009). Student Behavior and Epistemological Framing: Examples from Collaborative Active-Learning Activities in Physics. *Cognition and Instruction, 27*(2), 147-174.

Scherr, R.E. & Wittmann, M.C. (2002). The challenge of listening: The effect of researcher agenda on data collection and interpretation. In S. Franklin, K. Cummings, & J. Marx (Eds.) *2002 Physics Education Research Conference Proceedings.* Melville, NY: AIP Press.

Singh, C. (2006). Student Understanding of Symmetry and Gauss's Law of Electricity. *American Journal of Physics, 74* (10), 923-936.

Tsamir, P. & Dreyfus, T. (2002). Comparing infinite sets—a process of abstraction: The case of Ben. *The Journal of Mathematical Behavior*, *21*(1), 1-23.

Tuminaro, J. & Redish, E.F. (2007). Elements of a cognitive model of physics problem solving: Epistemic games**.** *Physical Review Special Topics-Physics Education Research, 3*(020101)**,** 1-22.

van Zee**,** E., Hammer, D., Bell, M., Roy, P. & Peter, J. (2005). Learning and teaching Science as inquiry: A case study of elementary school teachers' investigations of light. *Science Teacher Education, 89* (6), 1007-1042.

Van Zee, E.H. & Manogue, C.A. (2010). Documenting and interpreting ways to engage students in 'thinking like a physicist'. In M. Sabella, C. Singh, N.S. Rebello (Eds.) *2010 Physics Education Research Conference Proceedings* (61-64). Melville, NY: AIP Press.

van Zee, E. & Minstrell**,** J. (1997). Using questioning to guide student thinking. *Journal of the Learning Sciences, 6* (2), 227-269.

Wallace, C.S. & Chasteen, S.V. (2010). Upper-division students' difficulties with Ampere's law. *Physical Review Special Topics-Physics Education Research, 6* (020115), 1-8.

Wittmann, M.C., Steinberg, R.N., & Redish, E.F. (2002). Investigating student understanding of quantum physics: Spontaneous models of conductivity. *American Journal of Physics,70*(3), 218-226.

APPENDICES

Appendix 1: Transcripts of Group 1 with Bing Coding

Group 1 - Tom, Allen, Laura

Coding Scheme

Physical Mapping

Mathematical manipulation

Mathematical coherence

Authority

Student makes a notable error

NOTE: Solid color indicates that comments fit the coded category, color just over part of the time stamp indicates that the particular coding was suspected, but it was less clear than the solidly coded pieces.

Transcript

[00:40:59.12] Group 1 starts

[00:41:04.01] Tom DRAWS ring and says, "Ok, I did my part"

[00:41:07.29] Laura, "OK"

[00:41:07.16] Allen, "Won't this more or less just be the same thing we've been doing,...except in terms of mu naught?"

[00:41:15.17] Laura WRITES "Period = T"

[00:41:31.22] Tom WRITES "Q total charge" and "$\lambda = Q/2\pi R$"

[00:41:43.15] Allen, "Let's see,...[point equals T?]..."

[00:41:46.16] Tom, "We have a linear charge density."

[00:41:47.27] Allen, "We do."

[00:41:49.13] Laura, "Yeah, OK, good job."

[00:41:50.09] Tom, "Period is T"

[00:41:52.15] Laura, "So, it has to do something per T"

[00:41:57.25] Tom, "So it does one complete revolution..." [GESTURES around in a circle]

[00:41:59.22] Laura, "Yes"

[00:42:00.22] Tom, "...every T"

[00:42:01.27] Laura, "Yes"

[00:42:03.10] Tom, "And we want to know how much charge goes through here [DRAWS a "gate" on ring] per time,...right?"

[00:42:16.00] Allen, "And solving for our J of [r? our?]equation."

[00:42:18.23] Tom, "Yeah, right."

[00:42:20.02] Allen, "For current density."

[00:42:21.00] Tom, "Yeah, it'd be a current density."

[00:42:26.03] Allen points at Tom's $\lambda = Q/2\pi R$ equation and says, "So,...will it just be lambda over T?"

[00:42:31.26] Tom ,"...Lambda over T? No, I don't think so."

[00:42:33.09] Allen, "Ya' know, 'cause there's our length, there's our time....Yeah and there's....[POINTS at $\lambda = Q/2\pi R$] …"2π here"…where circumference would come in, so that's got to be right."

[00:42:40.05] Laura, "Wait, wouldn't Q pass through in T? In time T?"

[00:42:49.10] Tom "It should be all of Q"

[00:42:52.17] Allen, "So'd be all the Q's coming past."

[00:42:54.18] Laura, "Yeah, Q/T" [WRITES "Q/T"]

[00:42:58.19] Tom, "I think..."

[00:43:00.28] Laura, "But you have to..."

[00:43:02.04] Tom, "But we end up, but we can write T out as more..."

[00:43:03.26] Laura, [over Tom] "You wanna, you wanna, yeah, right."

[00:43:05.29] Laura, "Um, T...[?]..."

[00:43:10.18] Allen "What is it, Q = lambda times $2\pi R$?"

[00:43:12.15] Laura DRAWS picture of a circle with a wedge

[00:43:19.02] Laura WRITES expression "$T = 2\pi$" while saying, "It's angular frequency, so...2π..."

[00:43:30.08] Allen says, "Over omega...isn't it?"

[00:43:35.03] Laura WRITES $T = 2\pi/\omega$

[00:43:44.04] Laura WRITES $T = 2\pi/\omega = 2\pi/R\theta$ and says, "2π over *Rθ*."

[00:43:44.04] At the same time, Allen WRITES $\omega = 2\pi f = 2\pi/T$ and says quietly, "Omega equals 2π over *T*," Then turns to Laura and says, "Yeah, it's over *T*. Or over omega, sorry. Um."

[00:43:51.03] Laura, "Yes"

[00:43:54.04] Allen, "Yeah, that's good enough. See that's right, so yeah, OK."

[00:43:56.16] Laura says, "So now we have *Rdθ* so, for a little..."

[00:44:03.05] Allen, "Shouldn't it be *Rdθ/dt*? Isn't angular frequency like the change in..." Allen GESTURES around in a circle as Laura WRITES.

[00:44:08.29] Laura WRITES and [over Allen] says, "So *dt* is equal to 2π*/Rdθ*. Why would you have a *dθ dt*...[?]...?"

[00:44:24.26] Allen, "'Cause *Rθ* would just be like your arc length,...[GESTURES a length]...like the circumference kind of covered...(GESTURES around in a circle)

[00:44:29.10] Laura, "Yeah, so..."

[00:44:32.27] Allen, "...which it's the same thing as angular frequency...[inaudible]...."

[00:44:36.16] Laura, "OK say that this is, OK say that this is 2π*R* we just get *T* equals.... 1/*R*,......which is bad (laughs)."

[00:44:55.09] Allen, laughs, "...I guess we get the change in R and T."

[00:44:58.06] Tom, "Period is inverse length"

[00:45:04.03] Allen, "Do we have to write period in terms of something else, or can we just leave it as *T*?"

[00:45:08.06] Laura, "...Well, we have to..."

[00:45:08.06] Tom, "Well is this...?"

[00:45:10.03] Allen, "Oh, 'cause for Maple,...[?]...right?"

[00:45:11.22] Tom POINTS to "Q/T" and asks, "Is this right?"

[00:45:13.14] Laura, "Q over T...uh...yes...oo, for...for every...no, I mean...what, what's right..."

[00:45:22.01] Tom (over Laura), "Is it charge per time? I mean is..."

[00:45:23.19] Allen, "Yeah, for current. It should be."

[00:45:25.08] Laura, "I mean, that is...yeah, that's alright, right? The total charge passes per T" [GESTURES by closing hand into a fist and then moving her fist side to side]

[00:45:28.03] Tom [over Laura], "But current density though"

[00:45:30.28] Allen, "So that would just be..[?]..."

[00:45:33.17] Tom, "So, how do you relate, how do you relate...the...oh, so then that would be $2\pi R\lambda$ over..." Tom finishes writing the equation $J(r) = 2\pi R\lambda/T$

[00:45:36.00] Laura [over Tom], WRITES as she says, "I mean this, lambda over $2\pi R$ over 2π over $Rd\theta$ so these cancel, then you write $4\pi^2$ [and that's that?]...$\lambda d\theta$ over $4\pi^2$. That's kind of exciting."

[00:46:04.03] Allen "Wait, so we are keeping the d(theta) in there?"

[00:46:08.16] Laura, "I don't know."

[00:46:12.00] Tom "Well we're,...Lets just get a, a general thing first,..."

[00:46:15.21] Laura, "OK"

[00:46:17.09] Allen, "And then expand."

[00:46:17.24] Tom "...and then we can try to make it a small thing."

[00:46:19.17] Laura, ERASES all her previous work

[00:46:21.01] Allen, "So just stick with T for now...for the period?"

[00:46:24.01] Tom, "So..."

[00:46:26.08] Laura, "But we do know we have a rate."

[00:46:30.01] Tom, "I think that looks right for the general J."

[00:46:35.03] Allen, "I agree"

[00:46:37.02] Laura, "$2\pi R\lambda$ over T"

CORINNE ARRIVES

[00:46:40.07] Tom, (to Corinne) "Are we on the right track?"

[00:46:43.01] Corinne, "I don't know, say more."

[00:46:45.18] Tom, "We're just trying to express current density."

[00:46:48.26] Allen "In terms of..."

[00:46:49.14] Tom, "J"

[00:46:50.00] Allen, "Right, in terms of how long it takes to cover the circumference." (GESTURES in a circle)

[00:46:51.29] Tom, "So we have a... this is Q...per period, and we have it expressed in terms of lambda. So does that make it...."

[00:47:00.04] Corinne, "Lambda is Q over $2\pi R$, so why is it Q over T?"

[00:47:08.17] Allen, "Should it just be lambda over T then? Since we have charge density per time. Since that'...it's the density of charge"

[00:47:15.18] Allen GESTURES circles on the board for 18 seconds

[00:47:17.17] Corinne, "Do you want charge density? How do you...that's why you were marched around and around and around and around yesterday...so how do you measure current?"

[00:47:26.26] Tom, "Charge per time."

[00:47:31.27] Corinne, "Charge per time is its dimensions, how do you measure it?"

[00:47:34.18] Laura,"So we're measuring how much passes each point on the ring at..."

[00:47:40.08] Corinne, "Right, so what you want to do is tell me, here's a point on your ring, you want to tell me how much passes there in a second. In your unit of time. So how are you going to figure out how much passes this point in a second?"

[00:47:56.12] Laura, "Well we know that the total charge passes that point in a time equal to the period."

[00:48:03.02] Corinne, "OK"

[00:48:05.01] Laura, "And that's why we have Q/T."

[00:48:08.27] Corinne, "Ok, so there's the total. That's how much charge passes in that amount of time." (pointing to Tom's equation $J(r) = 2\pi r\lambda/T$).

[00:48:13.27] Laura, "Right."

[00:48:14.23] Corinne, "So why are you even bothering to ask me?"

[00:48:17.07] Tom, "We didn't realize we were wrong."

[00:48:22.02] Corinne, "OK. The point is that, that if you're trying to say it's charge per time then you have...if,if you just remember the dimensions that someone...something has then in any problem that has more than one charge and more than one time you have to think really hard about which ones you mean. If have an operational definition which says," (POINTS at ring) "how much charge is passing a particular point in a second, in a unit of time, and you use an operational definition, then you can check yourself. Alright?

[00:48:56.01] Tom (nods), "Yes"

[00:48:57.08] Corinne (GESTURES in circle), "So all of the charge passes in time T. So if you divide all of the charge by T that tells you how much passes in one unit of time."

[00:49:04.15] Tom, "Right."

[00:49:05.09] Corinne, "OK. Now I gave you," (POINTS at front classroom whiteboard) "Q, R, and T not lambda, R, and T."

[00:49:13.01] Tom(POINTS at equation), "Well..."

[00:49:14.24] Laura, "We can just...we'll just leave it as Q."

[00:49:16.28] Tom, "I thought we were going to need a lambda to...to do J."

[00:49:19.06] Laura, "Well, but lambda is constant, so we can just..."

[00:49:22.17] Tom, "Sure." (ERASES part of equation)

[00:49:24.20] Corinne, "Well, don't keep erasing everything, or you don't know where you've been."

[00:49:28.09] Laura, "Well we know where we've been."

[00:49:30.00] Corinne, "OK"

[00:49:30.26] Group laughs

[00:49:33.24] Tom WRITES equation for I again

[00:49:34.28] Corinne, "OK. One way I like to think about it...Can I have a pen that works better?...is that the current density is the, um, charge density, which is in this case is a linear charge density, times the velocity." (WRITES λv)

[00:49:50.20] Allen, "Oh, that's right."

[00:49:52.03] Corinne, "OK, and then λ is Q over $2\pi R$," (WRITES $Q/2\pi R$) "which you already found out, and the velocity is..."

[00:49:59.00] Allen, "2 pi R over T"

[00:50:00.23] Corinne, "$2\pi R$ over T" (WRITES $2\pi R/T$ so that it now reads $Q/2\pi R \;\; 2\pi R/T$, making the cancelation of the $2\pi R$ to yield Q/T obvious)

[00:50:03.20] Allen, "Huhn. Ohh, yeah."

[00:50:04.27] Laura, "That makes me so much happier... with the λ and the v."

[00:50:07.25] Corinne, "It makes me happier too. It's easier for me to think about it that way. I actually think about λ's times v's."

[00:50:13.22] Allen (POINTS at Corinne's equation), "And this is J? Current."

[00:50:17.13] Corinne, "Yes...Well, no. See J..."

[00:50:19.19] Tom (over Corinne), "Well, Q over T is current."

[00:50:21.03] Corinne (over Tom), "Uh, well this is a linear current density, which is I." (WRITES an I) "J is a volume current density so J would be..."

[00:50:31.27] Allen, "Oh, right."

[00:50:32.09] Corinne, "...rho v" (WRITES) "and K, this on a surface, just gets sigma v."

[00:50:40.00] Allen (POINTS at front classroom board), "So that up there is like the most general...We're not solving for $J(r)$, just $I(r)$...."

[00:50:47.00] Laura, "Yeah, we were, we just wanted dr-prime."

[00:50:48.27] Corinne, "Yes. So, I, I, I always put" (GESTURES an expansive gesture with both hands going above her head and moving outwards) "the volume one down, figuring that you can..." (GESTURES, brings her outstretched hands inward and brings her hands together cupped towards each other)

[00:50:53.23] Allen (over Corinne), "Remember which is which."

[00:50:53.23] Corinne, "...figure out from that, that if you have,..." (POINTS at expressions for I, J, K) "...if it's just a surface," (GESTURES a plane with her hand horizontal moving in a horizontal cutting motion) or just a line (GESTURES a vertical motion with her pointer finger), you restrict it."

[00:50:58.21] Laura (over Corinne and to Corinne), "You have a...[?]..."

[00:51:01.16] Corinne (to Laura), "I do."

[00:51:01.01] Allen, "So, yeah, if it's general, it's going to be J of r, but specifically it's going to be I of r" (POINTS at Corinne's expressions) "for us, because it's linear charge density."

[00:51:07.18] Laura starts to WRITE integral for ***A***

[00:51:07.18] Corinne (nods), "Yes!"

[00:51:08.11] Allen, "Alright...[?]...Wow. OK. Thank you."

CORINNE LEAVES

[00:51:18.03] Tom ERASES equation for I, "...[Made that point?]..."

[00:51:22.15] Laura "Huh?" continues to WRITE integral $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{I}(\boldsymbol{r})\, dr' / |\boldsymbol{r} - \boldsymbol{r}'|$ (with vector signs over the ***A***, both the ***I*** and ***r*** in ***I***(***r***) and the ***r*** and ***r'*** in the denominator)

[00:51:31.26] Allen, "That's for anywhere in space? Is that right?"

[00:51:40.00] Laura, "Well, it might be."

[00:51:41.03] Tom? Allen?, "Well we...."

[00:51:41.29] Laura, "Because, like, last time we did everywhere in space we kind of had *dφ*. (POINTS at *dr* in equation) "Because we've kind of already done our *dr*."

[00:51:50.06] Allen, "Hmm?"

[00:51:51.19] Laura, "I kind of suspect that this will be *dφ*" (WRITES a *φ* in equation in place of the *r'* in *dr* to get $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{I}(\boldsymbol{r})\, dr' / |\boldsymbol{r} - \boldsymbol{r}'|$

[00:51:57.24] Laura, "Because...we...wait."

[00:52:03.15] Tom, "Well, we're going to want to end up with the..."

[00:52:04.12] Allen (over Tom), "*dφ*...[?]..."

[00:52:07.06] Laura, "But wait, a current potential can't be anywhere in space, it has to be in the conductor...Right?"

[00:52:16.16] Allen, "Well the magnetic field due to the current could be anywhere."

[00:52:19.16] Laura (over Allen), "No, I mean it just says right there," (POINTS at front classroom board) "all space that has...[?]...Oh, wait, this is the magnetic..."

[00:52:25.10] Tom (over Laura), "This,..So you're integrating over it, but that's not where A lives."

[00:52:28.04] Laura, "Ohhh, right."

[00:52:30.14] Tom, "So..." (DRAWS an external point from ring)

[00:52:33.26] Laura, "Oh, yeah."

[00:52:36.16] Tom DRAWS $\vec{r}$ and $\vec{r}\,'$ vectors on ring diagram and labels them (see Figure 18).

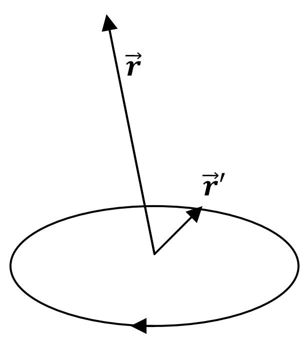

Figure 18: Tom’s drawing of position vectors

[00:52:39.25] Laura WRITES an $\vec{A}$ on the drawing next to the external point, "I concur."

[00:52:43.19] Allen DRAWS the $\vec{r} - \vec{r}$ vector with the arrow going from the external point to the ring, then WRITES $\vec{r} - \vec{r}$ next to the vector (Figure 19), "This way, right? Or is it the other way?"

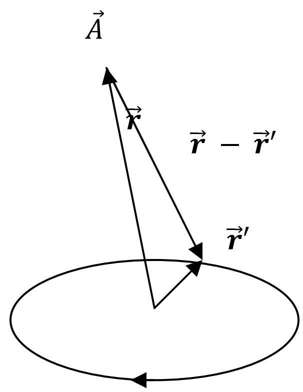

Figure 19: Allen draws and labels the $\vec{r} - \vec{r}$ vector

[00:52:50.08] Laura, "It doesn't matter."

[00:52:53.27] Allen, "And, I'm upside down."

[00:52:56.07] Tom, "It, it is. It would be that way." (DRAWS an arrow head on the ***r* - *r'*** vector at the external point on drawing, the opposite direction that Allen drew)

[00:52:59.11] Allen, "Is it... I've never known which way it is...[?]..." (POINTS at external point)

[00:53:01.04] Laura, "No, because you, you add these two." GESTURES, moving her finger along the ***r* - *r'*** vector from the ring to the external point)

[00:53:03.05] Allen, "Ahhhh...O.K."

[00:53:09.27] Tom, "It's ***r'*** plus this thing equals ***r***." (POINTS at vector from origin to ring and vector from origin to external point)

[00:53:12.28] Allen, "That one, alrighty...Right, because you go tail to head, tail to head..." (POINTS at vector from origin to ring and then to external point)

[00:53:17.22] Tom, "Yeah...exactly."

[00:53:18.21] Allen, "...from the original tail to the new one." (GESTURES in a large arc with palm facing downward)

[00:53:21.03] Tom, "Yeah."

[00:53:24.15] Allen "Um, so how do we go from $d\tau$ to $d\varphi$?"

[00:53:28.27] Laura, "Well $d\tau$ is a volume, and *dr*,...but we kind of have a..." ERASES φ with the $d\varphi$ in her equation to get $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{I}(\boldsymbol{r})\, d\_ / |\boldsymbol{r} - \boldsymbol{r}'|$

[00:53:37.25] Tom, "Well if you go backwards..."

[00:53:40.20] Laura "If we already have this whole $2\pi R$ thing, then..."

[00:53:46.10] Tom, "Because if you take away one of those integrals then you are d(area) and if you take away another one then it's *dr*."

[00:53:55.01] Laura, "Yeah, I definitely think it's d...?..." WRITES an *r* into the *dr* in her equation, returning to $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{I}(\boldsymbol{r})\, dr / |\boldsymbol{r} - \boldsymbol{r}'|$ (what she had at [51:22] except there is no prime with the *r* in the *dr*)

[00:54:00.21] Allen, "So then we'll write *dr* in terms of **i**'s, **j**'s, and **k**'s for Maple's convenience."

[00:54:07.10] Tom, "Yup"

[00:54:12.02] Allen, "And then we're in cylindrical...[points to denominator of integrand]...I'm gonna guess, for this portion"

[00:54:19.17] Tom, "So what is our *dr*?"

[00:54:24.23] Allen "Wait, we can write *dr* in terms of cylindrical coordinates."

[00:54:27.18] Laura (over Allen), "...because we have to...OK, so we have a...?..."

[00:54:28.21] Tom "...Wait, no,no,no, ...yuh...we are going to have a d(theta)."

[00:54:32.00] Laura, "d(phi), right?"

[00:54:32.23] Tom "d(phi)"

[00:54:34.07] Laura, [?]

[00:54:33.28] Allen, "...well because this is a vector though…" (WRITES a vector sign over *dr* in Laura's $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{I}(\boldsymbol{r})\, dr / |\boldsymbol{r} - \boldsymbol{r}'|$ formula)…*dr* right? So then that will have components of *dr* **r**-hat plus *rd...rdr*- ***r'***-hat."

[00:54:42.26] Tom (over Allen and DRAWS two marks close to each other on the ring). Well, but there's not a ...no this is, this is *dr* around here...here to here; that's dr, (WRITES dr next to two marks) ..which is $Rd\varphi$

[00:54:56.22] Laura, (WRITES on board, R$d\varphi$' ?) "Right"

[00:55:01.15] Tom, (WRITES on board, $x = r\ cos\ (\varphi)$ and $y = rsin(\varphi)$ and $z = z$) "Yeah, because when we,...x is r cosine-φ...$r\ sin\ \varphi$...Right, so that's where we're going to use the phi's. Right?"

[00:55:24.12] Allen, "Right."

[00:55:24.12] "Alright, so magnitude of our dr" – WRITES absolute value signs around the $d\vec{r}'$ in her equation to get

$$\vec{A} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}(\vec{r})|d\vec{r}'|}{|\vec{r} - \vec{r}'|}$$

[00:55:31.17] Laura, "That's right, because you write it like that."

[00:55:42.04] Laura, "Ok, and then this is just Q/T...[points to $\vec{I}(\vec{r})$ in equation]…wait,...yeah,... but if it's Q/T then that means that we've already integrated over phi."

[00:56:00.12] Tom,"Oh, you're right, we needed a dQ...which is lambda"

[00:56:04.28] Allen (over Tom), "This is for each element of charge?"

[00:56:07.21]Laura, "But we don't, we don't know our interval of time?"

[00:56:16.26] Allen, "From zero to T"

[00:56:16.26] Tom (laughs)

[00:56:20.14] Allen, "right, [though one?] period"

[00:56:22.12] Laura (laughs), "[?]...and we have, we have Q/T but,....you can't, you can't...this is how..."

[00:56:28.14] Tom (POINTS to drawing), "But it's gonna to be their little individual contributions, so there's going to be (WRITES on drawing $dq = \lambda Rd\varphi$?) uh..."

[00:56:36.10] Laura, "A dq... yeah"

[00:56:36.27] Tom (WRITES equation on board), "A dq is... $\lambda Rd\varphi$ Why do we have two $d\varphi$'s?)"

[00:56:48.03] Laura, "No, we don't have two $d\varphi$'s "

[00:56:48.29] Tom, (over Laura), "This'd be one"

[00:56:53.07] Laura (WRITES expression on board, (λv $|dr|$ / $|\mathbf{r} - \mathbf{r}'|$), "'Cause, ok, ok, ok, ok,..so we have, we have lambda *v dr*, over **r** minus **r**-prime, right? So, our....uh....so our *dr* is $Rd\varphi$...over that,...and then our λv is just going to be Q/T....wait, now I'm confused."

[00:57:39.13] Tom, "But the *Q* has to be chopped up."

[00:57:50.22] Corinne starts to talk to whole class together

[00:57:44.03] Allen, "Why does the *Q* have to be chopped up though, because we're just considering that all the current goes through?"

[00:59:44.06] Laura DRAWS a vector on ring drawing

[01:00:16.08] End of class

Appendix 2: Transcripts of Group 2 with Bing Coding

Group 2 - Tanya, Bob, Nick

Coding Scheme

Physical Mapping

Mathematical manipulation

Mathematical coherence

Authority

Student makes a notable error

NOTE: Solid color indicates that comments fit the coded category, color just over part of the time stamp indicates that the particular coding was suspected, but it was less clear than the solidly coded pieces.

Transcript

[00:39:50.04] Group gets their whiteboard

[00:40:08.10] Nick draws three sides of a rectangle

[00:40:22.25] Corinne, "That is the squarest ring I ever saw."

[00:40:27.26] Bob, "What the hell is that?"

Group members proceed to collectively draw a ring with axes

[00:40:54.14] Bob, DRAWS on diagram "And this is our radius R"

[00:40:58.12] Tanya, "Mm hmm."

[00:41:00.07] Bob, "...with charge Q, and it's spinning." and DRAWS arrow for direction of spin.

[00:41:03.00] Tanya (over Bob), "Period T"

[00:41:05.15] Nick, (over Bob) says "T is equal to 2 π r over, over v", and WRITES $T = 2\pi r/v$.

[00:41:06.16] Bob, "[inaudible, over Nick]..., or you could say frequency equals one over T." WRITES, "Period = $1/f$" and "freq = $1/T$"

[00:41:16.27] Nick, "Well that's..."

[00:41:17.22] Bob, "Uh, wait a sec, is it omega equals 2 π f? Omega is 2 π f, so f is one over T," WRITES $\omega = 2\pi f$

[00:41:27.00] Tanya, "We need veloc...that's velocity, so 2 π over T is velocity." and WRITES on Bob's equation, changing it to $\omega = 2\pi /T$.

[00:41:29.21] Bob (over Tanya), "I don't know, I don't know what that does."

[00:41:36.16] Bob, "Sure."

[00:41:42.26] Nick, "Both you guys aren't right,...it's 2 π r over v....wait,..." writes $\omega = 2\pi r/v$

[00:41:48.03] Tanya, "No."

[00:41:48.25] Nick, "...T", ERASES "v" and WRITES $\omega = 2\pi r /T$.

[00:41:49.19] Bob, "Yeah."

[00:41:50.07] Nick, "Chk"

[00:41:50.20] Tanya, "Where you getting 'r'?"

[00:41:51.13] Bob, "Yeah, where's 'r'?"

[00:41:51.29] Nick, ERASES equations, "*R* is the radius," [POINTS to the radius labeled *R* in the drawing] "…big *R*...What are we trying to find?"

CORINNE PASSES BY

[00:41:57.00] Tanya, "Maybe.."

[00:41:58.00] Corinne, "**A**"

[00:41:59.08] Nick, "We're trying to find **A**."

[00:42:00.15] Tanya, "OK."

[00:42:01.10] Nick, "...[after this?]..."

[00:42:02.23] Corinne, "Yes."

[00:42:02.23] Bob starts to look at notes

[00:42:04.03] Tanya, "OK, so....*J* equals current...", WRITES "*J* ="

[00:42:07.26] Nick (over Tanya), "So, uh,...[?]...", POINTS at origin on whiteboard.

[00:42:10.15] Bob, looking through notes, "How come I don't have what ***A*** equals?"

[00:42:14.14] Nick, "Cylindrical, or?"

[00:42:15.27] Tanya, "Probably."

[00:42:16.20] Bob, "How come I don't know what ***A*** is? Do we know what ***A*** is?"

[00:42:18.29] Tanya, "We do not, that's what we're going for."

[00:42:20.24] Bob, "Well, no, I mean, like, the formula I don't remember...[?]..."

[00:42:23.21] Nick, "It's right up there," POINTS at class blackboard.

[00:42:25.22] Bob, "Oh, I did, I just wrote it down. Sorry."

[00:42:20.10] Nick points athe equation on the board and then WRITES $\mu_o/4\pi \int$ [note that the overall equation for magnetic vector potential. On the classroom blackboard is written $\mu_o/4\pi \iiint J(r)\, d\tau \,/\, |\mathbf{r}\text{-}\mathbf{r}'|$.]

Bob continues to look at his notes

[00:42:29.10] Tanya, "So current equals charge over velocity,...the charge density over velocity..."

[00:42:36.00] Nick, "The volume of what? [looking at the front classroom whiteboard] The volume of what you're integrating...volume, all space that has..."

[00:42:38.00] Bob, copying from notes, WRITES $\boldsymbol{J} = \rho\boldsymbol{v}$, (with vector signs over J and v)

[00:42:40.21] Tanya, DRAWS chopping marks on ring, "The volume of all these little bits."

[00:42:46.00] Nick, "OK, So it's actually just a double integral."

[00:42:49.23] Tanya, "Um, yes."

[00:42:53.07] Nick, "Just, over, actually just a single integral..."

[00:42:58.20] Bob, "...[?]...the integral"

[00:43:02.03] Nick WRITES in numerator of integral $\boldsymbol{A} = \boldsymbol{J}(\boldsymbol{r}')\, d\varphi'$ (including vector signs over **A**, ***J*** and ***r***)

[00:43:04.25] Ken asks group to move whiteboard, group makes comments

[00:43:18.25] Tanya says, "Charge over velocity," WRITES $J = Q/V$ and then says, "Omega," and changes it to $J = Q/\omega$

[00:43:26.22] Tanya WRITES $QT/2\pi$ and says "QT over 2π"

[00:43:26.22] Nick WRITES denominator for integral, completing $\boldsymbol{A} = \mu_o/4\pi \int \boldsymbol{J}(\boldsymbol{r'})\, d\varphi' / \sqrt{r + r'^2 - 2rr'\cos(\varphi-\varphi') + (z - z')^2}$

[00:43:28.29] Bob comments about cameras

[00:43:43.10] Tanya, "Current is charge over velocity, or is it charge density over velocity?"

[00:43:48.10] Nick,"Uhh,...Oh that's what J is, current."

[00:43:49.15] Bob, "...[?]...saying that J is, is...ρv."

[00:43:53.10] Tanya, "OK, so it's charge density,...which we don't have."

[00:43:58.07] Bob, "Uhh,...but we could figure it out though, right?"

[00:44:00.23] Nick ERASES picture

[00:44:03.00] Tanya, "Yeah, because ρ is Q over $2\pi R$?"

[00:44:03.07] Bob and Tanya both separately WRITE $\rho = Q/ 2\pi R$

[00:44:10.21] Bob, "Yeah, big R."

[00:44:11.13] Tanya, "OK."

[00:44:12.22] Nick, "Rho is 2 Q over uh $2\pi R$."

[00:44:15.09] Bob, "$2\pi R$."

[00:44:16.14] Tanya, "Yeah"

[00:44:18.11] Nick, "Q."

[00:44:20.25] Bob, "No." [POINTS to ρ in $\rho = Q/ 2\pi R$]

[00:44:21.08] Nick, "Q...[?]..."

[00:44:21.25] Tanya, "No."

[00:44:21.27] Bob, "Rho."

[00:44:22.07] Nick, "Wait, what rho?"

[00:44:23.01] Bob, "Rho is, 'cause Q is our total charge." [POINTS at ρ and then Q in $\rho = Q/ 2\pi R$]

[00:44:25.05] Tanya, "Yeah."

[00:44:26.00] Bob, "Rho is our density."

[00:44:27.03] Tanya, "So,..."

CORINNE ARRIVES

[00:44:27.23] Corinne, "I'm just amused because every group is starting somewhere different."

[00:44:31.00] Nick, "Alright."

[00:44:31.26] Corinne, "Yeah, you're only getting a laugh because as I compare the different groups."

CORINNE LEAVES

NOTE: Nick and Tanya are working simultaneously and talking independently of each other, so their transcripts are divided for these 21 seconds, with Nick first.

Nick

[00:44:33.12] Nick DRAWS a new ring with a radius

[00:44:35.14] Nick, WRITES, changing *r′* to *R* in the denominator of his integrand, saying, "OK, so this is big *R*"

[00:44:37.25] Bob WRITES, labels radius with *R* as Nick crosses out the **z'** at end of the denominator of the integrand $\sqrt{\ } r + r' - 2rr'\cos(\varphi - \varphi') + (z - z')^2$.

[00:44:39.26] Nick, "Over that, where,...anywhere in space,...OK", POINTS first to the φ' and then the r' part of the denominator $\sqrt{\ } r + r' - 2rr'\cos(\varphi - \varphi') + (z - z')^2$ and then toward origin in the drawing, then back at crossed out z'.

[00:44:48.04] Nick starts to WRITE a new equation for magnetic vector potential, $\mathbf{A} = \mu_o/4\pi\ {}_0\!\int^{2\pi}$

Tanya

[00:44:38.00] Tanya WRITES, starting with her earlier equation $J = QT/2\pi$, and substituting $\rho 2\pi R$ in for Q, resulting in $J = \rho 2\pi RT/2\pi$. She then cancels the 2π 's, resulting in $J = \rho RT$, saying "So, this is, ρ, 2, π, R, T over 2π, so J equals ρRT. Now we can actually put it in there."

[00:44:53.06] Tanya starts to WRITE an integral, $A = \mu_o/4\pi \int$

GROUP RETURNS TO 3-WAY INTERACTION

[00:44:59.12] Nick, "Rho is equal to Q over $2\pi R$, when was that, when,...when did we learn that?"

[00:45:04.26] Tanya, "Charge density equals charge over circumference"

[00:45:05.10] Bob, "It's charge over area, right?" uses his pen to POINT at $\rho = Q/2\pi R$ and taps his pen at Q when he says “charge” and at $2\pi R$ when he says “area” and then moves from pointing at $2\pi R$ to the drawing and uses his covered pen to GESTURE in a circle around the ring drawing, he says something that sounds like “which is this”

[00:45:08.01] Nick (over Bob), POINTS at $\rho = Q/2\pi R$, "Charge,...charge density,...this isn't,...we want lambda."

[00:45:12.00] Tanya, "Oh, right, lambda."

[00:45:13.13] Nick, "Yeah."

[00:45:15.11] Nick POINTS to ρ in $J = \rho v$ equation and says, "..and if and J is equal to something different if you want lambda," and briefly opens and closes notes.

[00:45:17.17] Bob, looks at his notes, starts talking over Nick and WRITES $I = \lambda v$, and says, "...well, I is equal to,….I,…well I wrote this down, I is lambda v."

[00:45:22.22] Nick, POINTS back and forth between his integral equation and Bob's $I = \lambda v$ equation, "Right, so we want I, we want I instead."

[00:45:25.00] Tanya, "Right."

[00:45:25.17] Nick, "So, oh, it's lambda here, OK, so it's *I*," and WRITES, changing $J(r)d\varphi$ to $I(r)d\varphi$ in his integrand.

[00:45:27.10] Tanya, "This is, this is the general formula for all space. We're doing it linearly, so it's all lambda instead."

[00:45:33.03] Nick, "OK, OK, that's, that's good. I can work with that."

[00:45:38.02] Bob, POINTS at equation $J = \rho v$, "Someone said *J*, so I looked at my notes and wrote that down."

[00:45:39.15] Nick ERASES integral sign and limits of integration,

[00:45:41.07] Nick, "So it's, uh, lambda...[?]..."

[00:45:44.01] Tanya WRITES her integrand, "lambda,"...writes $\lambda 2$, then erases the 2, and writes $\lambda RT/|\mathbf{r}-\mathbf{r}'|\ d\varphi'$ "λ, *R*, *T* over **r** minus **r**-prime...uh...$d\varphi$,...zero to 2π,...yes." writing in limits of integration, to make $\boldsymbol{A} = \mu_o/4\pi\ {}_0\int^{2\pi} \lambda RT/|\mathbf{r}-\mathbf{r}'|\ d\varphi'$

[00:45:52.09] Nick ERASES $I(r)$ portion of integrand and WRITES $\lambda\mathbf{v}$ (with a vector sign over **v**)

[00:46:01.24] Nick, for his second equation, WRITES $Q/2\pi R\ \boldsymbol{v}$ prior to writing the integral.

[00:46:12.01] Nick, "Now I'm confused,"

[00:46:13.01] Nick WRITES out rest of integral, $Q/2\pi R\ \boldsymbol{v}\ {}_0\int^{2\pi} 1/\sqrt{(r^2+R^2-2rRcos(\varphi-\varphi')+z^2)}$

[00:46:13.01 Tanya WRITES in her notebook.

[00:46:17.27] Bob looks at Tanya's equation and GESTURES in a circle

{pause in conversation}

[00:46:40.02] Nick, "That's what I got," and ERASES former equation.

[00:46:44.18] Nick, moving over to Tanya, "What'd you get?"

[00:46:46.14] Tanya, circling her equation, "[The thing we have to solve?]"

[00:46:51.00] Nick, "*RT*?" pointing at *RT* in Tanya's equation.

[00:46:53.11] Nick, "Oh"

[00:46:53.11] Tanya, points at class blackboard, says "[A ring?]".

[00:46:54.14] Bob, pointing at T in Tanya's equation, "T's our period"

[00:46:55.19] Nick, "Oh wait, so you're using this for v," pointing to "$\omega = 2\pi/T$" on board,

[00:46:57.02] Tanya, "Yes."

[00:46:57.16] Bob, "OK."

[00:46:57.26] Nick, "Where v is...2 π over T," then ERASES **v** in his own equation and WRITES $2\pi/T$

[00:47:01.28] Tanya, "Uh..."

[00:47:03.06] Bob, "Right, and then we said lambda was Q over $2\pi R$," POINTS at equation $\lambda = 2\pi R$.

[00:47:09.06] Nick, "So...", WRITES $A = \mu_o/4\pi * Q/RT$

[00:47:09.16] Bob, looking at Tanya's equation and at his own, $\lambda = Q/2\pi R$, "So it gives us the...no, wait......wait, shouldn't R be on the bottom?"

[00:47:23.04] Bob WRITES, λ, then ERASES it, then WRITES $(Q/2\pi R)\ 2\pi/T$.

[00:47:26:00] Nick ERASES $A = \mu_o/4\pi * Q/RT$ and writes $\mathbf{A} = \mu_o Q/4\pi RT\ {}_0\int^{2\pi}$

KEN ARRIVES

[00:47:29.28] Jessie, GESTURING at equation $\rho = Q/2\pi R$ using thumb and pointer finger to act as if she's grabbing that piece of equation and moving it to the top part of her $J = Q/\omega$ equation, "'Cause you move,...you move this up here to get...[?]...and stick it in here."

[00:47:37.26] Ken, POINTS at Tanya's equation, "In, in this formula you're telling me the longer the period, the greater the magnetic field?"

[00:47:45.16] Tanya, "Don't know."

[00:47:46.03] Ken, GESTURES rotation and pointing, "So if it takes 3 trillion years to rotate, you're going to increase...the magnetic..."

[00:47:48.03] Nick (over Ken), pointing at $\omega = 2\pi/T$ on board, "Yeah that's not,...yeah that's not angular,...that's not angular velocity."

[00:47:52.03] Ken,"...magnetic field."

[00:47:53.03] Nick, "Angular velocity would be, uh... $2\pi R/T$."

[00:47:57.26] Bob writes, "$= Q/2RT$", continuing his equation for current

[00:48:00.10] Nick, "But, yeah, that would be saying increase the period."

[00:48:00.10] Tanya (over Nick), "[?] has never factored into...[?]..."

[00:48:04.03] Nick, "No, no, no...it's,it's,it's"

[00:48:04.03] Bob (over Nick), "Isn't it Q over $2RT$?"

[00:48:07.29] Tanya, "No."

[00:48:08.28] Bob, POINTS at equation, "$2\pi/T$ times $Q/2\pi R$"

[00:48:13.06] Ken, GESTURING a spinning motion, "But now it's spinning so you're changing, you're changing what's going on when it's spinning. You can't, you can't ignore the fact that it's spinning in terms of what changes."

KEN LEAVES

[00:48:21.13] Tanya, "Well, yeah."

[00:48:22.20] Nick, "As, as, it would be as T increases..."

[00:48:24.10] Tanya, "So if we're not supposed to use angular velocity, we're supposed to use linear velocity."

[00:48:27.20] Nick, "Yeah, yeah, as, whoa, whoa, whoa, that's right, that's right, it should be on,...in the denominator because if T increases it it's going slower, so that the magnetic field would go down."

[00:48:37.00] Bob, "OK"

[00:48:37.24] Nick, pointing to "$\omega = 2\pi/T$" on board, "OK, but we just we need add an R to this because that is not the correct units. Angular velocity is...[v?]...meters per second, so it needs to be R in here."

[00:48:47.00] Bob, "Well this,...this is radians."

[00:48:48.22] Nick, "2πR, that's, that's radians."

[00:48:50.14] Bob, "Right."

[00:48:51.10] Nick, "Yeah."

[00:48:51.10] Tanya, "Yeah, that's, that's angular velocity."

[00:48:52.09] Bob, "So, radians per time."

[00:48:53.28] Nick, "Hmm."

[00:48:54.18] Tanya, "Yeah."

[00:48:55.20] Nick, "Oh yeah."

[00:48:55.20] Tanya, "If we, if we need linear velocity we have to change that ...[factor by?]...I don't know what."

[00:49:00.10] Nick, "OK, yeah, you're right, so...It's been awhile."

[00:49:03.26] Tanya, "Ow."

[00:49:06.04] Nick works on new version of magnetic vector potential

[00:49:17.12] Nick, "I thought I'd just do this for practice."

[00:49:21.05] Bob and Nick laugh

[00:49:23.10] Nick, "It was like, you know, you, that's wrong,..[points at omega = 2(pi)/T]...that there, so as it increases, that's wrong?"

[00:49:30.20] Nick, "Whah? Whahsa? Whah? Vuh-vuh-vuh-vuh?"

[00:49:31.18] Tanya (over Bob), "So, so, is, is, is current charge over velocity or charge density. It's charge density, so this is...I messed this part up."

[00:49:41.13] Tanya WRITES $(Q/2\pi R)/(2\pi/T)$ and says, "This is Q over $2\pi R$ over 2π over T..."

[00:49:50.18] Tanya WRITES (algebraically manipulates this), "...which is a 4 pi-squared R," writing $QT/4\pi^2R$.

[00:49:58.25] Bob, pointing at Tanya's equation, "Why...why...why are you dividing that?"

[00:50:03.29] Tanya, "Because it's lambda over......Oh, I don't know," ERASES several of her equations.

[00:50:07.17] Nick, "Is this anywhere in space?"

[00:50:10.00] Bob, pointing at his equations, seeming to still be responding to Tanya, "This is what she told us yesterday, was that...that they were times, the charge density *times* the velocities."

[00:50:15.21] Tanya, "...[?]..[and 'times' means x's?]..."

[00:50:18.24] Bob, "So this is..."

[00:50:21.08] Tanya, "Because I think of current in terms of its dimensions, [so just divide it out?]"

[00:50:21.27] Bob (over Tanya), "So this is what...yeah, well, so, so if we go with that we want the lambda part because we're thinking...uh...because we're not thinking of an area because it's a ring. But this is what I is, it's $Q/2RT$," WRITES box around $Q/2RT = I$ on board.

[00:50:38.10] Bob, "So then, so then we just do..."

[00:50:41.12] Nick, "I erased all that work up to this."

[00:50:43.29] Bob, "Did ya?"

[00:50:44.18] Nick, "Yeah."

[00:50:45.00] Tanya, "So $J = RT$."

[00:50:47.11] CORINNE ARRIVES

[00:50:51.00] Bob, POINTS at Nick's equation, and looking at his notes and own equation, "Q...4 pi..."

[00:50:55.02] Tanya, "So," and ERASES her entire integrand

[00:50:56.22] Bob, "Isn't there...but there's another 2. Did you, did you use this?...Shouldn't this be 8?" POINTS at a 4 in the denominator of Nick's equation.

[00:51:02.00] Nick, "No, it'd be, the 2's cancel along with π."

[00:51:05.06] Corinne, "OK, so $Q/2\pi R$ is what?"

[00:51:08.04] Bob, "Uhhh..."

[00:51:09.10] Corinne, "You have $Q/2\pi R$."

[00:51:11.20] Tanya, "Lambda."

[00:51:12.05] Corinne, "That's λ."

[00:51:12.14] Bob, "This is our λ."

[00:51:13.15] Corinne, "And what is this?" points at $2\pi/T$ part of equation.

[00:51:14.22] Bob, "Uh, this is our, yeah, omega."

[00:51:17.03] Corinne, "Why do you want angular velocity?"

[00:51:18.09] Bob (GESTURES in a circle), "Because it's a circle."

[00:51:19.23] Corinne, "I don't care."

[00:51:21.08] Bob, "OK."

[00:51:22.25] Corinne, "This is lambda times a real velocity."

[00:51:26.22] Bob, "OK."

[00:51:27.00] Corinne, "...so it will be omega times R"

[00:51:31.02] Tanya, "That's how you go from angular to normal velocity?"

[00:51:33.25] Nick, "It's..."

[00:51:34.00] Corinne, "Yes"

[00:51:35.00] Tanya, "We don't remember that. That was a very long time ago."

[00:51:36.22] Nick, "What is, what is, no, what is the conversion from velocity to angular velocity?"

[00:51:40.20] Corinne, "Eesh,...um,...."

[00:51:44.15] Tanya, "V equals..."

[00:51:44.15] Corinne puts her head in her hands.

[00:51:45.12] Laughter

[00:51:46.06] Nick, "I mean, no, no, no, not converting."

[00:51:46.15] Corinne, "Sorry, no, sorry, I just, I don't think of it in those words, so I'm having to translate."

[00:51:50.13] Nick (over Corinne), "Right, I didn't mean that, I didn't mean conversion. I didn't mean to say that."

[00:51:51.22] Corinne, "OK, so...so...may I have your pen."

[00:51:55.12] Corinne DRAWS a circle and GESTURES around in a circle, "If you've got something going around in a circle..."

[00:51:56.24] Nick, "Uh huh."

[00:51:57.11] Corinne, "...it goes the whole circumference in a period."

[00:51:59.18] Nick, "Uh huh, right."

[00:52:00.21] Corinne, "OK, so the velocity is the circumference divided by,...or the speed...is the circumference times the period...or over the period." (WRITES V = C/T)

[00:52:07.29] Nick, "OK,...[?]..."

[00:52:08.21] Corinne, "So in your case, it's $2\pi R$..."

[00:52:11.02] Nick, "Over T"

[00:52:12.07] Corinne, "Over T"

[00:52:13.01] Nick, "Hi Oh!" (bangs fist on table)

[00:52:14.04] Bob, "Damn" (bangs fist on table)

[00:52:14.24] Laughter

[00:52:16.13] Nick WRITES, changes constant on integral to Q/T

[00:52:16.13] Corinne, "Alright, but if you want angular velocity, you want to know what angle it goes through [DRAWS an angle on ring], instead of what distance it goes through."

[00:52:23.14] Bob, "Right."

[00:52:23.14] Corinne, "Remember back to some early things that we did, we were talking about an arc length?...An arc on a piece of a circle that, that the arc length was just the radius times the angle that you went through?"

[00:52:35.00] Tanya, "uh huh"

[00:52:35.26] Corinne, "So the relationship between lengths and angles is the angle times the, the radius that you're on is the arc length. So if you want an, an angle per time instead of distance per time you just cross that out."

[00:52:50.00] Tanya, "But we want a distance?"

[00:52:51.01] Corinne, "But, you want a distance. This, in this formula [POINTS at $I = \lambda v$] the dimensions are real velocities, ...with R's in it."

[00:52:56.25] Bob, "OK."

[00:52:57.12] Corinne, "Genuine velocities, not angular velocities."

[00:53:00.05] Bob, "OK."
[00:53:00.23] Nick, "So is this good?"
[00:53:03.00] Corinne, "Well, now, so Q over T, 4π, yes...uh...except that $\boldsymbol{J}$ is a vector."
[00:53:10.29] Nick, "Oh."
[00:53:15.18] Nick, "This is, uh, **v**-hat."
[00:53:17.09] Bob, "But is it $\boldsymbol{J}$ or is it I?"
[00:53:20.02] Nick, (POINTS at equation) "...[that's v?]..."
[00:53:20.08] Corinne, "It's I in this case. So I wrote down the general. If you, if your current is distributed through the entire volume you use J."
[00:53:28.29] Bob, "Right."
[00:53:29.13] Corinne, "You use a similar equation, but with just either $\boldsymbol{K}$ or $\boldsymbol{I}$, if your current is a surface or a line current.
[00:53:36.00] Bob, "Right."
[00:53:37.00] Corinne, "So you did, you adjusted for that just fine."
[00:53:39.27] Bob, "OK, OK."

CORINNE LEAVES

[00:53:40.20] Tanya WRITES a new integral with (Q/T) |r-r'| dφ for integrand,
[00:53:46.16] Bob WRITES, working on his equation for $I = \lambda v$
[00:53:50.26] Bob, "Whoops"
[00:53:59.18] Tanya ERASES |r-r'| in numerator and correctly WRITES it in the denominator
[00:54:08.24] Bob WRITES, completing his equation and gets result $I = Q/T$, "Well then, it's just...did I screw this up?"
[00:54:11.25] Tanya, "Q over T."
[00:54:12.16] Bob, "OK. Sorry, I'm a little slow."
[00:54:14.20] Bob ERASES Corinne's ring picture.
[00:54:16.06] Tanya, "And then I...this is $d\varphi$."

[00:54:23.00] Bob, "OK, now you've got it.......OK...now we just need to...do...this..."

[00:54:33.13] Bob, POINTS at z in Nick's equation, "But in this case, *z* is zero, so we don't need that... "

[00:54:36.20] Bob POINTS to equation, Tanya crosses out the last term, z^2, from equation) "Right? And then, uh,......uh...well it's moving,...and I'm kind of confused about what to do with the phi's,...It seems like, it seems like we should cancel one of the phi's, but, it's aparen..."

[00:54:55.20] Nick, "No." (shakes head)

[00:54:56.21] Bob, "No?"

[00:54:59.00] Tanya, "Well, one of them is a constant when you integrate."

[00:54:59.00] Nick (while his right hand POINTS at the φ' in the denominator, he GESTURES in a large sweeping motion high above the table with his left hand), "'Cause you could be anywhere. It could be anywhere in space...and we're also integrating."

[00:55:02.02] Tanya (over Nick), "Yeah, your other one."

[00:55:04.03] Nick (while his right hand still POINTS at φ' in the equation, with his left hand he GESTURES in a large motion), "But this is everywhere in space, so you want to be there (POINTS up and outward to the left), you might want to be there (POINTS outward to a different location up and to the right), you might want to be there." (POINTS to a third location up and behind himself),

[00:55:08.06] Tanya (over Nick), "And this is what we're integrating." (POINTS to φ' Michael's pointing)

[00:55:08.06] Nick, "...and then we're integrating this (his right hand still POINTS at φ', while his left hand first points at phi-prime then at $d\varphi'$in the numerator). That's the ring." (use left hand and GESTURES in a circle on the whiteboard where the ring had previously been drawn, but where there is currently just white space)

[00:55:10.25] Bob, "OK."

[00:55:11.27] Nick, "So we want both of them."

[00:55:11.27] Tanya (Using her pen above the surface of the board, POINTS at $r^2 + R^2$ part of denominator) and says (over Nick), "All of these are..."

[00:55:12.27] Nick, "So the only thing we can cancel is that" (POINTS to crossed out z^2) "the one z".

[00:55:15.27] Tanya, (POINTS and makes a circling motion around $r^2 + R^2$ part of denominator)"These are constants"

[00:55:16.18] Nick, "Wait, no,no,no,no,..."

[00:55:18.16] Tanya, "Yeah"

[00:55:18.29] Nick, "Yeah"

[00:55:19.08] Bob, "Yeah"

[00:55:19.14] Nick, "No, I already cancelled the *z*-prime." (ERASES cross-out marks and re-writes z^2)

[00:55:21.11] Tanya, "Oh, right, right, you cancelled the *z*-prime"

[00:55:22.20] Bob, "Oh, sorry, I got cancel happy. Thank you. I like that."

[00:55:35.28] Bob WRITES and darkens and lengthens the division bar

[00:55:43.03] Nick, "This is like every other problem we were doing."

[00:55:45.03] Bob, "Every other problem?"

[00:55:47.00] Nick, "I like, I like this. I like magnetism like this." (POINTS at whiteboard)

[00:55:50.28] Bob, "Like what?"

[00:55:51.07] Nick, "I hated magnetism in lower division. 'Cause it was like I had to remember, I had to remember the freakin' unit...[?]..."

[00:55:56.26] Bob, "4 π?"

[00:55:58.28] Tanya, "I like magnetism."

[00:56:02.16] Nick, "Oh, no, I like it now too."

[00:56:03.03] Tanya (over Nick), "But I had a pretty good teacher."

[00:56:07.08] Nick, "Was it your teacher here?"

[00:56:09.02] Tanya (shakes head), "I had a good high school physics teacher..."

[00:56:11.19] Nick, "Oh, OK."

[00:56:12.24] Tanya, "...so I didn't really learn much in college. I learned all of it in high school. And then I just got more complicated math in college."

{pause in conversation}

[00:56:51.04] Bob, "I'm trying to write this down, because I always tend to like not write it down in my notes and then I get screwed for when I get ready to do homework."

{pause in conversation}

[00:57:08.01] Nick DRAWS new ring picture

[00:57:14.10] CORINNE STARTS TALKING TO WHOLE CLASS, Nick still drawing

[00:57:53.04] Nick DRAWS "chopping" lines on ring

[00:58:21.03] Nick WRITES a new integral - (presumably from board) - $\mathbf{A} = \mu_o/4\pi \int_0^{2\pi} \lambda v\, d\varphi' \bullet \boldsymbol{\varphi}\text{-hat} / |\mathbf{r} - \mathbf{r}'|$

[00:59:32.09] End of class

Appendix 3: Transcripts of Group 4 with Bing Coding

Group 4: Robert, Kevin, Stan

Coding Scheme

Physical Mapping

Mathematical manipulation

Mathematical coherence

Authority

Student makes a notable error

NOTE: Solid color indicates that comments fit the coded category, color just over part of the time stamp indicates that the particular coding was suspected, but it was less clear than the solidly coded pieces.

Transcript

NOTE: The portable main camera was also focused on Group 4. Time stamps given are for the small wall-mounted camera.

[00:41:42.09] Group assembles

[00:41:48.20] Stan WRITES $\boldsymbol{A} = \mu_o/4\pi \iiint$ (with vector sign over the A) on white board with marker. Kevin picks up marker to write as well.

[00:41:54.00] Kevin: Obviously *r* and *r*' are going to be whatever our object... Did she say what our object was?

[00:42:00.25] Stan: Oh...

[00:42:00.25] Kevin (GESTURES in a circle), "Oh, the ring."

[00:42:01.21] Stan, "The ring"

[00:42:02.07] Robert, "Yeah"

[00:42:01.21] Stan: ......uh..yeah (wheels chair over, looks in his notebook and then WRITES $\rho\boldsymbol{v}$ into integrand to make $\boldsymbol{A} = \mu_o/4\pi \iiint \rho\boldsymbol{v}$, and then looks back at notes)

[00:42:03.01] Kevin: So it's going to have cylindrical (GESTURES in a circle, starts to WRITE on board.) permeability of space (WRITES an integral with a large square root sign under it)

[00:42:11.24] Robert: [laughs] uh yeah.

[00:42:14.09] Kevin: so this...

KEN ARRIVES

[00:42:17.29] Ken: Draw pictures! Do geometry before you write down anything!

[00:42:21.21] Stan quickly DRAWS a ring with an axis through it, quips back: Done! [group laughs]

[00:42:22.25] Ken: Draw. Think, think, think, think.

KEN LEAVES

[00:42:24.22] Stan: "Spinning..." (DRAWS a tangential vector),

[00:42:25.13] Robert, "Draw"

[00:42:26.04] Stan, "...current..." (DRAWS an upward vector along z-axis),

[00:42:26.25] Robert, "Yeah."

[00:42:28.11] Kevin: [talking over Stan] There's got to be some moment of inertia in here.

[00:42:27.10] Stan, "...right hand rule..." (GESTURES right hand rule for magnetic field around a current carrying wire - or is it B field through a solenoid) "...or, B..." (now WRITES (labels) vertical arrow "B") "...or A..." (changes "B" to an A)

[00:42:30.19] Kevin starts to ERASE integral

[00:42:30.25] Robert (repeatedly GESTURES right hand rule for magnetic field through a solenoid), "Basically,...basically the field's going to go up...the whole right hand rule thing...spins that way, [something - current? curving? curl?]...up.

[00:42:37.00] Stan GESTURES right-hand-rule

[00:42:37.00] Kevin DRAWS a new, larger ring

[00:42:40.08] Robert (referring to ring): Well yeah. So if you say it's spinning that way... (DRAWS arrow on ring)

[00:42:43.00] Stan: "Then, then it'll be up." (WRITES -labels upward on the *z*-axis $\boldsymbol{A}(r)$)

Kevin:Yeah

Robert:Yeah

[00:42:45.14] Stan: [A?]...Which'll make it easier.

[00:42:47.06] Robert & Kevin: Nods and gestures

[00:42:48.09] Robert:Yeah

[00:42:48.09] Kevin:Yeah

[00:42:48.16] Stan: Let's do that.

[00:42:51.00] Stan: OK, so we have the charge density of this [POINTS with left hand to $\rho\boldsymbol{v}$ in integrand, and POINTS with right hand to a specific point on ring and draws with marker]. So this is normally *J*. (GESTURES with right hand in a circle around $\rho\boldsymbol{v}$ in numerator of integrand while left hand continues to point at it) The charge density times the velocity (GESTURES - right hand moves back to ring and moves back and forth along it), which is . . . right...I remember this is in our notes from yesterday (POINTS at notes).

[00:42:58.19][Corinne comes and stands watching the group, but not interacting,]

[00:43:03.03] Kevin, "Yeah."

[00:43:03.20] Robert, "Right."

[00:43:04.02] Stan, "But, so, we need to relate this to the period, I think."

[00:43:07.01] Robert, "...It should be...[?]...OK, period...We weren't given a period, were we?"

[00:43:11.28] Stan, "*T*, capital *T*"

[00:43:12.09] Robert, "Yeah, but, OK, it's a hollow variable then"

[00:43:15.06] Stan, "Yeah."

[00:43:15.13] Robert, "...said it could be anything then"

[00:43:15.13] Kevin, (WRITES $a = v^2/r$), "centri,..centri,...centri-pee-tal acceleration"

[00:43:19.11] Robert, "Centripetal?"

[00:43:20.24] Kevin, "Centripetal acceleration....something like that"

[00:43:24.07] Robert, "*m v*-squared over *r*...that's centripetal acceleration"

[00:43:29.26] Kevin, "No." (points)

[00:43:30.07] Robert, "Oh, yeah, OK, nevermind, I'm thinking for a different type object,...yeah, yes, I agree with that."

[00:43:38.09] Kevin ERASES equation

[00:43:38.09] Robert, "But it's omega"

[00:43:42.03] Kevin, "Yeah, we wanted omega, we want, we wanted omega, omega"

[00:43:43.23] Robert, (WRITES "$\omega_v = v^2/r$)

[00:43:53.08] Robert, "Well, I had an eraser."

[00:43:56.01] Stan, "We can definitely do our *r* minus *r*-prime, right? "

[00:43:59.22] Kevin, "Yeah, that part's easy"

ALICE ARRIVES

[00:44:01.23] Stan "This is *r*'...(DRAWS an *r*' vector on ring) "...and here" (DRAWS a vector to outside the ring)"

[00:44:13.10] Stan (to Alice): "How do we find *v*?"

[00:44:14.10] Alice, "How do you find what?"

[00:44:15.27] Stan, "Velocity."

[00:44:16.23] Alice, "Uh, you're given period, right?"

[00:44:18.14] Stan, "yes."

[00:44:19.05] Alice, "You should be able to figure it out from information about the period...and the radius."

[00:44:23.25] Kevin, "Period..." (ERASES previous equation and WRITES $T = 1/f$)

[00:44:23.25] Stan, "Oh, we have radius..."

[00:44:25.11] Alice (over Stan), "But I have a question,...about this..." (POINTS to rho in numerator of integrand) "...what's rho?"

[00:44:28.25] Robert, (ignores Alice's question about rho and WRITES $T = 1/f$ on board) "T equals 1 over f, right?"

[00:44:31.28] Alice, "So to me ρ means volume charge density. Do you have a volume charge density?"

[00:44:35.13] Stan, (WRITES - replacing ρ with λ to become $\boldsymbol{A} = \mu_o/4\pi \iiint \lambda \boldsymbol{v}$), "Lambda"

[00:44:37.13] Alice, "Yeah,...lambda makes me happy."

[00:44:40.04] Group laughs

[00:44:42.00] Stan, "Fair enough...uhhh...."

[00:44:47.08] Kevin (WRITES $f = 2\pi\omega$) "Frequency equals 2π omega"

[00:44:49.02] Stan, (Some comment to Alice)

[00:44:49.24] Alice, "I know."

[00:44:50.28] Stan, "Ooo, nice!"

[00:44:52.27] Kevin, "Right?"

[00:44:53.16] Stan "Yeah"

[00:44:54.02] Robert, "Yeah"

[00:44:56.06] Kevin, "And then, there's a, there's a formula that relates angular velocity to..."

[00:45:00.17] Stan , "Wait, isn't it ω divided by 2π?...'cause it's...because in 411 we do...to get omega we get 2π times the frequency..." (WRITES $\omega = 2\pi f$) "...so, yeah, it's divided by 2π."

[00:45:12.14] Kevin WRITES $f = \omega/2\pi$ and $T = 2\pi/\omega$

[00:45:23.18] Stan, " Now we know we have the radius and....

[00:45:27.17] Kevin WRITES $\omega = vr$

[00:45:33.13] Group looks at Alice, Alice shakes her head and says, "Keep going."

[00:45:37.04] Stan, "We're all trying to remember,...for omega"

[00:45:43.18] Kevin, adds "= $2\pi/vr$ to his equation for T

[00:45:47.20] Stan POINTS at equation, "Then we have to just solve..[?]..That's v, right?"

[00:45:52.14] Kevin, "Yeah"

[00:45:52.26] Stan, "Yeah."

[00:45:53.08] Kevin, "...[?]..."

[00:45:54.12] Stan, "No it's fine, that's great. So it's just 2π over Tr equals v ? (WRITES $2\pi/Tr = v$)

[00:46:05.09] Alice, "Do the units of that make sense?"

[00:46:07.14] Stan, "one over time....no, so it should be the inverse...That's length and that's one over a second, so right now we have seconds over length, we've got time over length, so we want to flip it."

[00:46:23.26] Alice, "Hold on, what are the units of period?"

[00:46:26.16] Stan, "Isn't that one over seconds?" (WRITES 1/s)

[00:46:28.02] Alice, "No, it's just seconds."

[00:46:29.06] Stan, "Oh, OK, and so we..."

[00:46:30.02] Alice, "Period is how long does it take to do one cycle" (GESTURES in a circle), "So it's seconds. It's a time."

[00:46:36.16] Stan, "So we, so we have one over TL"

[00:46:37.22] Robert, "So you're looking at a constant here looking at L"

[00:46:40.11] Alice, "Yeah, so what about this expression here? How confident are you in this?" (POINTS at $\omega = vr$)

[00:46:44.28] Stan, "It's divided by r, it's divided by r."

[00:46:47.02] Alice, "Why is it divided by r? How does that make sense?"

[00:46:50.14] Stan, "I just remember it."

[00:46:52.28] Alice, "You just remember? OK, that's acceptable."

[00:46:55.05] Stan, (some noise)

[00:46:55.29] Alice, "What, what is this omega thing?"

[00:46:58.23] Stan "The...

[00:46:58.23] Kevin "...[?]..."

[00:47:00.02] Robert POINTS to arrow on ring (which would correspond to linear velocity)

[00:47:00.21] Alice, "Yeah, it's angular speed, it's how many angles do I go through,...(GESTURES, using her hand and forearm, an angle being traversed).. right? And then v is your..." (GESTURES holding her two fists in front of her, about shoulder width apart) "...right?"

[00:47:10.05] Robert, "Divide out your radius"

[00:47:12.16] Alice, "Right, so it's $r\omega = v$, right?

[00:47:15.20] ?, "Right"

[00:47:16.11] Alice, 'Cause to get arc length...(GESTURES a complex series of gestures indicating radius and an angle)...it's r times θ, and this is the rate of change of θ. Right? So it's r times ω to get v, that's out on the edge. Does that make sense?

[00:47:19.14] Stan ERASES λv and writes $\lambda(2\pi r/T)$ into integrand to make $\boldsymbol{A} = \mu_o/4\pi \iiint\lambda(2\pi r/T)$

[00:47:27.21] Robert, "This is what happens when it's been a little while since I've taken dynamics, where they constantly deal with it." (GESTURES with hand moving forward and fingers spreading) "...[?]...where $v = \omega r$."

[00:47:31.14] Stan WRITES $|r\text{-}r'|$ into denominator to get $\boldsymbol{A} = \mu_o/4\pi \iiint\lambda(2\pi r/T)/|r\text{-}r'|$

[00:47:33.05] Alice (over Robert), "Right, because then…[?]...look at the dimensions. Dimensions help...ummmm...OK" (walks away)

ALICE LEAVES

[00:47:42.23] Stan, "OK, so then,...uh...we know this..." (POINTS at $|r - r'|$ in denominator) "...It's cylindrical, yeah?"

[00:47:52.29] Kevin, "Yeah, it's cylindrical"

[00:47:54.17] Stan, "So that is r squared minus r-prime squared plus $2rr'cos(\varphi\text{-}\varphi') + z - z'$...... (WRITES denominator for integrand of $\boldsymbol{A} = \mu_o/4\pi \iiint\lambda(2\pi r/T) \;/\, (r^2 - r'^2 + 2rr'cos(\varphi\text{-}\varphi') + z^2 - z'^2)$

[00:48:11.00] Stan: Oh yeah, we only have one *z*."

[00:48:12.07] Robert, "Yes."

[00:48:13.19] Stan, "And its normal *z*"

[00:48:13.19] Kevin ERASES *z′*

[00:48:14.16] Robert, "Yes, because we picked...constant rotation"

[00:48:19.17] Stan, "...all to the 1/2" Finishes WRITING $\boldsymbol{A} = \mu_o/4\pi \iiint \lambda(2\pi r/T)/(r^2 - r'^2 + 2rr'cos(\varphi\text{-}\varphi') + z^2)^{1/2}$.

[00:48:22.15] Stan, "And then we need *dt*...d(tau)."

[00:48:30.24] Robert, "Uhhh."

[00:48:33.16] Kevin, "Is there anything else we have to have for...[?]...for our *J*?"

[00:48:37.11] Stan, "Uh, it looks like we need a direction"

[00:48:40.05] Robert, "*dτ* is gonna go up on top" (POINTS to numerator of integrand)

[00:48:43.01] Stan, "It looks like we need a direction for the ***J***. Is that true, is that...[?]..."

[00:48:48.29] Kevin, "No, it should all be in the radial direction" (GESTURES an angle with his hand)

[00:48:51.13] Robert, "Yeah." (GESTURES right-hand rule)

[00:48:51.24] Kevin, "Yeah."

[00:48:56.04] Robert, "So....." (WRITES *dτ* on his side of the whiteboard) "... *dτ* is the complete thing. You have to shove that up on top up here, right?" (POINTS to numerator of integrand)

[00:48:57.11] Stan WRITES in his notes

[00:49:06.25] Kevin, "Should we replace all those *r*-primes with just *r*?" (POINTS at an *r′* in formula)

[00:49:09.29] Stan, "Yeah, and this is a capital *R*" (POINTS to numerator of integrand)

[00:49:14.10] Stan, "Equals the radius,….so this is capital *R*"

[00:49:16.23] Robert, "So to get this, we need to fully expand that."

[00:49:16.23] Stan WRITES in *R*'s on formula to get $\boldsymbol{A} = \mu_o/4\pi \iiint \lambda(2\pi R/T)/(r^2 - R^2 + 2rRcos(\varphi\text{-}\varphi') + z^2)^{1/2}$.

[00:49:23.01] Stan, "Now, we can,...Or, do we have any symmetry?"

[00:49:28.24] Robert, "Should."

[00:49:29.15] Stan, "Well, 'cause it's anywhere in space,...it's anywhere in space; the point we're measuring from..." (POINTS to point in space on drawing)

[00:49:36.04] Stan, "No symmetry on this..." (POINTS to numerator) "...but there will be symmetry on this because there's no *z*, which we took out, and does *φ*-prime matter?"

[00:49:46.00] Robert, "I think...one of those go away that we were...cancelled this all out."(POINTS to $cos(\varphi\text{-}\varphi')$ in the denominator)

[00:49:54.20] Stan, "I'm not sure."

[00:49:57.03] Robert, "There was something that left us with this part..." (POINTS to $r^2 - R^2$) "...and this part..." (POINTS to z^2)

[00:50:03.00] Stan, "Yeah, but...that was when we were only on the *z*-axis. Right now we're anywhere in space."

[00:50:12.16] Stan, "So lambda, lambda expands - that's charge per unit length, which is *Q* over 2π*R*, so that..."(ERASES λ and WRITES $Q/2\pi R$ in numerator to get $\boldsymbol{A} = \mu_o/4\pi \iiint(2\pi R/T)(Q/2\pi R) / (r^2 - R^2 + 2rRcos(\varphi\text{-}\varphi') + z^2)^{1/2}$.)

[00:50:32.10] Kevin, "That's just *Q* over *T*"

[00:50:35.04] Stan, "Oh, and then..."

[00:50:37.01] Robert, "Yeah"

[00:50:37.16] Stan, "And then there's the d..." (WRITES $d\tau'$ in numerator to get $\boldsymbol{A} = \mu_o/4\pi \iiint(2\pi R/T)(Q/2\pi R)\, d\tau' / (r^2 - R^2 + 2rRcos(\varphi\text{-}\varphi') + z^2)^{1/2}$.))

[00:50:40.12] Kevin (POINTS at $Q/2\pi R$), "This is *λ*...our charge density?"

[00:50:44.26] Stan, "Yes"

[00:50:47.09] Robert, (POINTS at $(2\pi R/T)(Q/2\pi R)$) "Right off the bat, this and this are going to go away. This whole thing you're going to get *Q* over *T*. You're just going to get charge over period."

[00:50:59.18] Kevin WRITES $I = dQ/dt$ on board

[00:50:59.18] Stan, "Now, *dτ* is not a *dτ*, it's in fact a...*ds*?...*ds*, yeah...Is that true? Yeah." (WRITES *ds* on board to get $\boldsymbol{A} = \mu_o/4\pi \iiint(2\pi R/T)(Q/2\pi R)\, ds / (r^2 - R^2 + 2rRcos(\varphi\text{-}\varphi') + z^2)^{1/2}$.))

[00:51:11.00] Robert, "For everywhere...yeah, OK."

[00:51:12.17] Stan (over Robert), "So we just want..." (DRAWS chopping marks on ring) "...which is..."

[00:51:15.26] Robert, "Yeah, there is no volume there."

[00:51:17.13] Stan, "...which is...uh...*r*-prime, so big *R* times length $d\varphi$?"

ALICE ARRIVES/LEAVES

[00:51:25.02] Robert, "OK, and.... $d\varphi$ is going where, where?"

[00:51:32.23] Stan, "$d\varphi$" (Stan DRAWS a very thin angle on ring)

[00:51:36.10] Kevin, "Zero to 2π"

[00:51:37.06] Stan, "Zero..."

[00:51:37.13] Robert, "Thank you...I mean over..."

[00:51:40.10] Stan, "Oh yeah."

[00:51:40.20] Robert, "...over here" (POINTS at integral sign)

[00:51:41.18] Stan, "Oh yeah, ...[?]..." (WRITES $\boldsymbol{A} = \mu_o/4\pi \ {}_0\int^{2\pi}$)

[00:51:45.03] Stan mumbles about what he's writing

[00:51:48.28] Robert, "*Q* over *T*"

[00:52:18.05] Kevin ERASES old integral

[00:52:18.05] Stan finishes WRITING a new integral on board $\boldsymbol{A} = \mu_o/4\pi \ {}_0\int^{2\pi} QR\,/\,T(r^2 - R^2 + 2rR\cos(\varphi\text{-}\varphi') + z^2)^{1/2}$.

[00:52:08.24] ALICE RETURNS

[00:52:19.15] Stan,(turns to Alice), "Is it right?"

[00:52:21.06] Alice, "Is it right? I don't know, I gotta look..."

[00:52:23.27] Stan laughs

[00:52:25.01] Alice, "Alright, so this is your lambda..."

[00:52:26.13] Stan, "Yes."

[00:52:26.13] Alice, "And what is this?"

[00:52:28.06] Stan, "This is, OK,...Oh, I forgot my $d\varphi$-prime, I'm sorry" (writes $d\varphi'$ in numerator of integrand to get $\boldsymbol{A} = \mu_o/4\pi \ {}_0\int^{2\pi} QR\ d\varphi' / T(r^2 - R^2 + 2rRcos(\varphi\text{-}\varphi') + z^2)^{1/2})$

[00:52:32.01] Stan, "So this is lambda." (points to $Q/2\pi R$ on remaining part of earlier equation)

[00:52:35.19] Alice "Mm-hm"

[00:52:36.08] Stan, "This is our *d*...our little length" (POINTS at $Rd\varphi'$ in equation, then at chopping marks on ring)

[00:52:38.06] Alice "Mm-hm"

[00:52:38.25] Stan, "And this is...our...velocity"(POINTS at $2\pi R/T$)

[00:52:40.24] Robert (over Stan), "Yeah because we set, we set this equal to *ds* and set *ds* to $Rd\varphi$." (WRITES something on board)

[00:52:46.13] Alice, "Good,...alright."

[00:52:47.11] Stan, "This and those cancel..." (referring to the $2\pi R$ parts of $2\pi R/T{*}Q/2\pi R$)

[00:52:49.03] Alice, "Ok, and then this is the r-r' in absolute value" (points at denominator on board and gestures an absolute value sign) "OK, and what about the direction information?"

[00:52:59.01] Kevin, "...[?]..."

[00:53:00.27] Robert, "It's supposed to be everywhere in space though..."

[00:53:02.13] Alice (over Robert), "That's what you're missing"

[00:53:03.14] Robert,"...right?...so"

[00:53:04.16] Alice, "Right, but this thing is a vector." (POINTS to $\boldsymbol{A}$)

[00:53:07.01] Robert, "Ahh, gotchya"

[00:53:10.09] Alice, "How do you know that?"

[00:53:11.29] Stan, "Well, it's upward, it's z-hat" (pulls pen to WRITE it down)

[00:53:14.00] Robert, "Ya' know the right hand rule, if it's rotating this way..."(Kevin, Robert, and Stan ALL GESTURE right-hand rule)"...then, then it will go up."

[00:53:20.12] Alice, in a dramatically calm voice, "The magnetic FIELD....is that right." (Alice nods)

[00:53:24.23] Robert, "And so would the current, and..."

[00:53:27.05] Stan, "But we're not doing magnetic field, we doing magnetic potential."

[00:53:29.00] Alice (over Stan), "...doing...potential, vector potential."

[00:53:31.14] Robert (rubbing head), "Ahhh"

[00:53:32.03] Alice, "So I doubt that you'll have any intuition about the direction that will help you see this problem. In the end, we'll talk about direction."

[00:53:40.22] Kevin, "Does it have something to do with curl?"

[00:53:42.20] Alice, "I don't know how to answer that yet. It definitely has something to do with the direction of the current."

[00:53:47.22] Robert, "Could we write..."

[00:53:48.10] Alice, "Right, because, because, so she has up there is incomplete." (POINTS at main front classroom whiteboard) "That ***J*** is a vector ***J***, right? The current has a direction."

[00:53:59.00]Stan (over Alice), "So we need, we need a direction on this" (pointing to $2\pi R/T$ on board)

[00:54:01.19] Stan, "So the, our velocity."

[00:54:03.10] Alice (responding to Stan), "Yes"

[00:54:01.19] Robert (over Stan and Alice),"So you could say this, goes somewhere" (POINTS at the blank space in the end of the numerator)

[00:54:05.04] Stan, "And that is in the φ-hat direction?" (WRITES φ-hat on numerator of old integral)

[00:54:05.15] Kevin (over Stan), "And the velocity is tangential...[inaudible]..."

[00:54:08.29] Ken, "You can't just throw a direction out at the end."

[00:54:13.02] Robert (sarcastically), "We can't?"

[00:54:15.14] Stan, "Looks,...it looks good there."

[00:54:16.29] Robert, "Can't we, can't we,...We've got three to choose from."

[00:54:17.27] Alice (over Robert), "Wa-wa-wait, OK, so, I think, wait, don't erase this, I think this is right, but you've got to say some words around it, so it convinces me that it's right."

[00:54:25.02] Stan, "The velocity is going...up..." (POINTS up)

[00:54:28.11] Kevin, "Tangential."

[00:54:29.28] Stan, "...yeah, to the *r* direction, so it's just the φ-hat." (Stan points and GESTURES in the φ-hat direction)

[00:54:32.07] Alice, "OK, so you're going to put a φ-hat here?" (POINTS to the end of the numerator of the integrand)

[00:54:36.03] Stan, "Yeah, sure, I'll throw it in right there." (WRITES a φ-hat in integrand)

[00:54:38.13] Alice, "Right, and is it a φ-hat or a φ-prime-hat?"

[00:54:41.25] Stan, "Oh, it's φ prime hat." (WRITES a prime)

[00:54:43.00] Alice, "How do you know it's a prime hat?"

[00:54:44.24] Stan, "'Because it's...the...with the charge. The charge is the part with...[inaudible]...not this way"

[00:54:52.20] Alice, "Right, you're referring to the current, right?"

[00:54:53.28] Stan, "Yes."

[00:54:55.08] Alice, "Right, and so it has to be a prime." (POINTS at φ-hat-prime on board) "Does that makes sense?...If you're going to do that, convert it to a prime...OK, can you...move forward from here? I like this. This is nice."

[00:55:06.10] Stan (over Alice), "So does that...does that mean the magnetic potential is in...[the field? d(phi)?]...after we do it? Is that what you're saying?

[00:55:19.13] Alice, "I'm not saying that...yet...'cause, ..."

[00:55:23.04] Stan (over Alice), "Will we eventually find out by the end of class?"

[00:55:23.04] Alice, "This...this part is tricky." (POINTS at $d\varphi' * \varphi'$-hat in numerator) "Right, because you're integrating over φ'..."

[00:55:29.22] Stan, "Uh huh"

[00:55:30.17] Alice, "...and you have a φ'-hat here" (still pointing at numerator), right? Does φ'-hat change when you change φ' ?"

[00:55:39.14] Robert, (over Kevin), "Yes"

[00:55:40.12] Alice, "What do you mean?"

[00:55:40.12] Kevin (over Alice), "Is this...this...this the...Well, isn't this the potential we're talking about...[say?]...these arrows going around,..." (GESTURES around ring) "...all the way out into space?" (GESTURES large arm motion)

[00:55:49.23] Alice, "That's what it will look like...Yes."

[00:55:52.00] Kevin, "So..." (POINTS at equation)

[00:55:52.27] Alice, "But I want to talk about this. Does φ'-hat change when you change φ'?

[00:55:57.27] Stan, "Yeah. yeah. when you..." (POINTS at $d\varphi$ wedge on drawing)

[00:55:58.14] Kevin (over Stan), "yeah, because you..."

[00:55:58.14] Alice (over Kevin), "How do you see that?"

[00:55:59.25] Kevin,"...because you change your, I mean,..this...this direction is different from that direction..." (GESTURES in two tangential directions from ring that are perpendicular) "...they're different angles."

[00:56:05.19] Alice, "Yes, yeah."

[00:56:06.11] Stan, "Always perpendicular to your *r*"

[00:56:08.08] Alice, "Yeah, so when you do this integration you have to be careful....Do you know...how..."

[00:56:15.08] Stan, "How would you be careful?"

[00:56:17.02] Alice, "So do you remember that we had the same problem when we were doing electric field."

[00:56:19.17] Stan, "...[?]..."

[00:56:21.21] Alice, "You wrote it up!" [laughs]

[00:56:23.19] Stan, "i's j's and k's"

[00:56:26.19] Alice, "i's j's and k's"

[00:56:28.01] Robert, "Oh, then we switched back to Cartesian, and then switched back."

[00:56:30.22] Stan, "OK"

[00:56:31.05] Alice, "Right. The reason is because when you change φ, **i**-hat is still in the same direction."

[00:56:36.00] Robert, "Yeah."

[00:56:36.16] Stan, "So we need something that is constant, has a constant direction, doesn't change. A direction that doesn't change."

[00:56:41.21] Robert WRITES expression on board (for $\boldsymbol{r} - \boldsymbol{r}'$ in cylindrical coordinates?)

[00:56:42.00] Alice, "A direction that doesn't change,....

[00:56:44.15] Stan, "When you change....

[00:56:45.11] Alice, "...when you change your integration variable."

[00:56:47.09] ?, "OK"

[00:56:47.09] Robert mumbles as he continues to WRITE his expression

[00:56:47.09] Kevin "That only means we need...[one thing?]...magnetic field, right?"

[00:56:51.00] Alice, "What?"

[00:56:52.27] Kevin, (GESTURES right hand rule), "Well, our magnetic field points that way, right?" (GESTURES up), "It's constant...[?]..."(POINTS at the *z*-axis)

[00:56:58.29] Stan, "Yeah, well we just..."

[00:57:00.16] Kevin, "I guess I missed the point."

[00:57:02.23] Alice, "Yes. Yes, but I'm not sure how that's related to this. I'm talking when you actually evaluate this integral, right? There are two options." (POINTS at integral) "One is that you can take this φ'-hat and pull it on the outside of the integral and then do the integral and then your answer just has φ'-hat in it."

[00:57:19.13] Stan, "Is that bad?"

[00:57:20.20] Alice, "That is bad,..

[00:57:22.05] Group laughs

[00:57:22.04] Alice, "... because when you integrate over...the other option is you keep the φ'-hat in here and you have to be careful when your evaluating this integral because φ'-hat changes as you're going from zero to 2π (POINTS at limits of integration)… φ' (POINTS at φ' in numerator)...Does that make sense, what I mean?"

[00:57:37.15] Robert, "Yeah."

[00:57:37.15] Alice, "That. So I would do what Stan was saying, which is write φ'-hat in a basis, using basis vectors...so that they don't change direction..."

[00:57:45.21] Kevin (over Alice), "So like, [that will have?], that over the magnitude?

[00:57:50.27] Stan, "We want to change it to i, j, and k direction."

[00:57:54.09] Robert, "Yeah, remember, switch back over, to Cartesian,...and I really wish I had my write-up...then I could find it quickly"

[00:58:00.08] Stan (over Robert), "Yeah"

[00:58:01.01] Alice (over Robert), "Find a way..."

[00:58:01.24] Stan (over Alice), "Oh, there's, it's like sine-cosine, sine-sine,..."

[00:58:04.04] Robert (over Stan), "Yeah...we'd switch to Cartesian..."

[00:58:04.22] Alice (over Robert), "Find a way to convince Kevin of what I was saying..."

[00:58:07.21] Stan, "I will."

[00:58:08.10] Alice, "...because I can't think of another way,...I'll think of another way to say it next time."

ALICE LEAVES

[00:58:11.23] Robert, "If I could bring my write-up, up, because I have it in that."

[00:58:14.00] Stan, "So,..."

[00:58:15.11] Robert, "But it, it's..."

[00:58:16.11] Kevin, "...[?]..."

[00:58:17.10] Stan, "Remember when we did the electric field stuff and we ended up with an electric field in the direction of cylindrical coordinates, right? And it had like φ-hat and also z-hat and r-hat, but the problem is that as you go around our circle..." (GESTURES in a circle around ring drawing) "...all those change, except for ***r***-hat....[inaudible]...so we had to change it into **i**, **j**, and **k**. **i**, **j**, and **k** are always in the same direction, no matter where you are..." (GESTURES back and forth with hands) "...so we're going to do the same thing with this."

[00:58:47.00] Kevin, "I don't, I don't, I don't remember that, but...[?]..."

[00:58:49.01] Robert, "Well, I'm looking to see if I can find it in the notes somewhere and show you...what we did and how we did it...blah, blah, blah blah."

[00:59:00.18] Ken, "You're just trying to turn φ-hat into **i**, **j**, and **k**..." (GESTURES angles with hand) "...I mean, that shouldn't be too hard."

[00:59:06.29] Stan, "Well, we're trying to explain why. He doesn't remember doing for the electric field...remember why we...[did that?]..."

[00:59:15.03] Ken, "Why you did it into...

[00:59:16.09] Stan, "Yeah."

[00:59:17.03] Ken, " Because φ-hat is different..."

[00:59:21.00] Corinne starts addressing whole class

[00:59:23.10] Ken, "...Does that make sense?"

[00:59:28.13] Kevin, "Yeah, yeah, I think so."

Appendix 4: Transcripts of Group 5 with Bing Coding

Group 5: Biff, Devin, Shawn

Coding Scheme

Physical Mapping

Mathematical manipulation

Mathematical coherence

Authority

Student makes a notable error

NOTE: Solid color indicates that comments fit the coded category, color just over part of the time stamp indicates that the particular coding was suspected, but it was less clear than the solidly coded pieces.

Transcript

[00:41:23.20] Group 5 assembles

[00:42:00.28] Biff, "What's the question again?"

[00:42:00.28] Discussion about engineering physics majors

[00:42:06.08] Shawn DRAWS ring in plane of board

[00:42:33.05] Shawn WRITES a "Q"

[00:42:38.06] Biff, "What's the, what's question?"

[00:42:40.12] Devin,"...[do?]...involves *dr*

[00:42:42.23] Biff, "So, a constant *I*, right?"

[00:42:45.25] Shawn, "Supposed to like, weren't we supposed to like,....[?]..."

[00:42:47.25] Devin, "...[?]...gravitational...[?]..."

[00:42:50.11] Biff WRITES "I = constant" on board

[00:42:51.07] Shawn, (WRITES "T =" on board), "So T equals period. J?"

[00:42:58.17] Shawn?, "T is period, I think, right?"

[00:43:00.13] Biff, "Yeah."

[00:43:01.13] Shawn, "equals v, or?"

[00:43:04.23] Biff, (WRITES on board $T = 2\pi f$"), "2π times frequency,...right?"

[00:43:09.09] Biff, (WRITES on board "$f/2\pi = T$"), "No, wait, frequency over 2π = T, Yeah"

[00:43:13.13] Shawn?, "So, I think we want it in terms of v, right?"

[00:43:17.00] Biff ERASES equations from board

[00:43:25.09] Biff, "We're trying to solve for $\boldsymbol{J}(\boldsymbol{r})$, is that what you're saying?"

[00:43:27.04] Shawn?, "Yeah."

[00:43:28.07] Biff, "Alright, no problem, so...."

[00:43:29.14] Shawn?, "Is J (lambda)v,...or..."

[00:43:33.01] Biff (WRITES $J(\boldsymbol{r})$ with a vector over r and a box around the expression on board), "This is our current charge density, right?" (DRAWS an arrow and writes "current Q density")

[00:43:39.22] Devin, "J is the density"

[00:43:43.00] Biff (POINTS to board), "Right. Isn't this J? Current charge density?"

[00:43:46.00] Devin, "Yeah." (WRITES $J = \rho\boldsymbol{v}$, (with a vector arrow over v))

[00:43:46.03] Biff, "So that's current,...uh..."

[00:43:47.20] Devin, "So like..."

[00:43:49.08] Biff, "... per unit volume, right? (WRITES "I/V" on board)...Yeah."

[00:43:51.08] Devin, finishes writing $J = \rho\boldsymbol{v}$, says (over Biff), "Is that correct?"

[00:43:53.16] Biff ERASES equation

[00:43:54.16] Shawn (POINTS at Devin's equation), "It's like ρ times velocity"

[00:43:56.18] Devin, "I think it's..."

[00:43:56.10] Biff, "yeah, yeah, yeah, yeah, it's λ, right?"

[00:44:00.06] Devin, "It's just ρ."

[00:44:00.28] Biff, "I think, I think this one's the three-dimensional one. The three-dimensional one is ρ." (WRITES "$\rho\mathbf{v}$" and circles "ρ")

[00:44:05.03] Devin, "Yeah."

[00:44:05.24] Biff, "Yeah, that was another thing."

[00:44:07.15] Shawn, "Wait, isn't it, isn't it this?" (looking at front classroom whiteboard) "Right? It's just a ring, right?...So wouldn't that have a line...linear charge density?"

[00:44:14.24] Devin, (over Shawn), "Well isn't that what T is though?"

[00:44:17.12] Biff, "T was the density, huh?"

[00:44:19.09] Devin, "...or it was K...J, K, and, uh, I."

[00:44:24.22] Biff ERASES equations

[00:44:33.00] Biff, "Alright, it's spinning at such-and-such a rate, right?" (DRAWS two hash marks on ring)

[00:44:35.24] Devin, "v equals period times frequency, right?" (WRITES $v = Tf$)

[00:44:41.12] Biff DRAWS vectors from center of ring, and WRITES something on board

[00:44:49.17] Shawn, "If you have a ring you can't do, like a.....How do you do a surface integral on a ring?"

[00:44:57.03] Biff ERASES part of board

CORINNE ARRIVES

[00:44:58.04] Corinne (talking to a different group?), "V minus one is A."

[00:45:00.24] Biff, "I was curious."

[00:45:01.26] Shawn, "So...[hard to understand]...period."

[00:45:01.26] Biff (over Shawn), "Oh wait, that just says all space, and it's current, so maybe we're just not $d\tau$."

[00:45:08.00] Biff (turns to Corinne and POINTS at the board), "It's not necessarily that formula, right?"

[00:45:10.19] Corinne, "That's right. That's a formula if your current density is spread everywhere in space."

[00:45:11.12] ? (over Corinne), "Oh,…so we're not looking at J"

[00:45:14.24] Biff, "OK that's what we're, we're trying to get the [free? three?] space here, and I'm like, ya' know,...that doesn't work; nope, that doesn't work."

[00:45:14.24] Devin ERASES equation

[00:45:21.09] Shawn, "So that's actually,...that's λ..."

CORINNE LEAVES

[00:45:24.09] Biff, "Yeah."

[00:45:25.16] Shawn, "I equals λv"

[00:45:28.23] Biff, "...where v is the velocity of the electrons, right? (WRITES "$\lambda(v)$ - velocity of e-")

[00:45:28.23] Devin WRITES $I = \lambda v$

[00:45:31.29] Shawn, "Yeah, so that'd be from the period; a period of $2\pi R$"

[00:45:38.20] Biff POINTS at ring

[00:45:38.20] Devin, "Isn't v equal to period times frequency?"

[00:45:42.29] Biff "$2\pi R$, yeah, divided by v...That equals meters over meters per second equals seconds over meters times meters, cancel, equals seconds." (WRITES $2\pi R/v$ = m/(m/s) = s/m * m = s (circles the "s"))

[00:45:54.20] Devin, "That's the period."

[00:45:55.25] Biff, "I'm not a nerd...not at all"

[00:45:57.13] Biff ERASES equation

[00:45:57.13] Shawn, "So, v equals $2\pi R$ over T, right?" (WRITES $v = 2\pi R/T$)

[00:46:07.21] Biff, "Mm-hm"

[00:46:09.10] Shawn, "So then our lambda equals Q over $2\pi R$ " (WRITES $\lambda = Q/2\pi R$) "...so then $v\lambda$ equals $2\pi R$ over T times Q over $2\pi R$." (WRITES $v\lambda = (2\pi R\ /T)(Q/2\pi R)$, then cancels $2\pi R$) "...so we just have $v\lambda$ equals Q over T." (WRITES.... $v\lambda = Q/T$)

[00:46:12.26] Biff WRITES $\lambda(2\pi RT^{-1})$

[00:46:19.22] Biff (over Shawn), "λ equals what?" ERASES λ, WRITES $Q/2\pi R$ in its place then crosses out each $2\pi R$ and WRITES "$= Q/T$" to get $(Q/\cancel{2\pi R})\,(\cancel{2\pi R}T^{-1}) = Q/T$

[00:46:42.02] Biff, "And what does $v\lambda$ equal? We're just going to call it "A" right?...we'll call it like, what, what, what,..." (WRITES $A = \lambda v$)

[00:46:47.04] Shawn, "I think that's ***I***, right?"

[00:46:50.17] Biff, "Ahh, yeah, yeah, yeah, that's ***I***." (ERASES A from $A = \lambda v$ and WRITES $\boldsymbol{I} = \lambda v = Q/T$, with a vector sign over the ***I***)

[00:46:51.21] Shawn, "...we've got to have a vector across too, right?" (WRITES a vector symbol over I) "So wouldn't it be...φ-hat?"

[00:46:56.23] Shawn WRITES a φ-hat at end of equation

[00:46:56.23] Biff ERASES "T" and WRITES "t" and says, "I'm going to make it little t, is that cool?"

[00:47:00.15] Devin WRITES a φ at the end of his equation

[00:47:00.15] Biff WRITES a φ-hat at end of his equation, "Alright, so, um, is it φ -hat?"

[00:47:04.10] Devin, "Little t's cool.]

[00:47:08.12] Devin?, "Q over T, φ-hat" (Devin WRITES a hat on his φ)

[00:47:10.07] Biff, "Alright, so then dI, right?" (WRITES $dI = d$)

ALICE ARRIVES

[00:47:16.12] Shawn ERASES earlier equations

[00:47:17.00] Alice, "How you guys doin'?"

[00:47:18.15] Shawn (POINTS at $\boldsymbol{I} = \lambda v = Q/T\ \varphi$-hat), "Does that look good for and ***I***?"

[00:47:21.02] Shawn WRITES a vector symbol over his I

[00:47:23.24] Alice, "Tell me how you figured that out."

[00:47:26.17] Biff, "OK, So first off we took and we said we have a radius of 2π… (WRITES $2\pi R$ above ring)…or total circumference of $2\pi R$, right? And then we said that, OK, how fast [is it spinning around?]. $2\pi r$ divided by velocity equals period." (WRITES $2\pi r/v = T$ and puts a box around it)

[00:47:39.18] Biff,"So we said that the velocity of the electrons would be equal to $2\pi r$ divided by period T." (WRITES $v = 2\pi r/T$)

[00:47:47.05] Alice, "OK"

[00:47:48.08] Biff, "And we plugged that into here; this little equation we had for $\boldsymbol{I}$..." (WRITES an arrow on equation $\boldsymbol{I} = \lambda v$) "...and now we have...[?]... for that. And mind you, this is, this is in the φ-hat direction" (WRITES a φ-hat after $v = 2\pi r/T$)

[00:47:58.00] Alice, "Right on" (gives a thumbs up)

[00:47:59.23] Biff, "That's..."

[00:48:00.13] Alice, "That's [good?]"

[00:48:01.04] Biff, "Hey, I saw a couple of eye..."

[00:48:01.17] Alice, "That's exactly what I did."

[00:48:03.03] Biff, "Oh, as I said, I saw a couple of eye rolls and then all of a sudden you're just like, 'Yeah, you're doin' great!' and I was like, [Ay, ta-buz-uh?]"

[00:48:07.10] Alice, "No, they weren't eye rolls."

ALICE LEAVES

[00:48:08.06] Laughs

[00:48:08.27] Biff, "Oh, OK"

[00:48:08.27] Devin, "I'm so confused!"

[00:48:10.00] Shawn, "That's the way I read it too."

[00:48:11.19] Biff, "Was it? 'Cause I was reading it like I'm doing the wrong thing."

[00:48:14.11] Devin, "That's bullshit."

[00:48:15.23] Laughs

[00:48:17.10] Biff, "OK"

[00:48:17.10] Shawn, "So now...now we have to do..." (turns to look at front classroom whiteboard)

[00:48:20.06] Biff, "So, dl, so dl, uh,...(DRAWS a second radius on the ring to form a wedge, Figure 20)...well our only variable is phi, right? Hey look, check this out, Buddy, *dI* equals, um, *Q*, *t*, *dφ* is that right?" (WRITES *d**I*** = *Q*/*t d(φ-hat)*, with a vector sign over the ***I***)

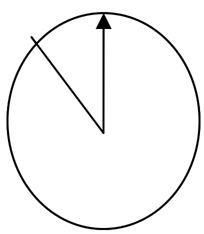

Figure 20: Biff draws a second radius on the ring

[00:48:37.11] Biff, "This seems like *dQ*/*dt*" (ERASES "= *Q*/*t d*(*φ-hat*)")…"how do we set that up as a differential equation?"

[00:48:44.24] Shawn "We're going to have the integral of ***I*** dot *d**r***..." (WRITES ∫ ***I*** • *d**r***, with vector signs over ***I*** and ***r***)

[00:48:53.16] Biff, "Where *d**r***..."

[00:48:53.16] Shawn (over Biff), "Where *d**r***, *d**r*** equals *Rdφ* (*φ*-hat)." (WRITES *d**r*** = *Rdφ* (*φ*-hat), with a vector sign over the ***r*** in *d**r***)."

[00:49:03.05] Biff, " No, no-n-no-n-no, that's, that's magnitude *d**r***." (WRITES absolute value sign around *d**r*** to get |*d**r***| = *Rdφ φ*-hat) "... *d**r*** is really...Oh, oh, oh, I see. You already simplified, yeah, you already simplified it then, OK, OK."

[00:49:12.19] Shawn, "And then, and then when you dot those two you get a ...(POINTS at *φ*-hat in |*d**r***| = *Rdφ φ*-hat)...we don't want to lose - we don't want to lose the *φ*-hat though."

[00:49:13.11] Biff (over Shawn), "...[…and…?]..."

[00:49:18.00] Biff, "So we have *Q* over *t*, *φ*-hat " (WRITES *Q*/*t φ*-hat)

[00:49:22.03] Shawn turns toward front classroom board.

[00:49:22.03] Biff, "Yeah we..., why not?"

[00:49:24.21] Shawn, "Because, well, yeah we do, but..."

[00:49:27.13] Biff, "But we need a..."

[00:49:27.13] Shawn, "We should go back,...remember, like, you have to go back to vectors for the ***A***."

[00:49:32.28] Biff, "Yeah, somethin'."

[00:49:34.28] Devin? "It's gonna be..."

[00:49:35.18] Biff, "What's that?" (WRITES Q/t φ-hat = $Rd\varphi$ φ-hat)

[00:49:36.21] Shawn (GESTURES in a circle around ring), "If you go,...here we have this thing, it's going to be like, pointing,...it's going to be pointing, like up, right?" (DRAWS a new line segment originating at the center of the ring to the ring)

[00:49:41.11] Biff continues to WRITE to get Q/t φ-hat = $Rd\varphi$ φ-hat = $\int_0^{2\pi} QR/T\, d\varphi$

[00:49:41.11] Devin (over Shawn), "It's going to be...it's gonna...yeah...going to be pointing in the z-hat direction. I thought magnetic field involves a cross product...but, uh..."

[00:49:56.01] Biff continues to WRITE to get Q/t φ-hat = $Rd\varphi$ φ-hat = $\int_0^{2\pi} QR/T\, d\varphi = 2\pi QR/T$

[00:49:56.01] Shawn, facing Devin, "'Cause, yeah, it's only going to be that like right there." (Traces a pre-drawn vector from the center of the ring to the ring itself, then POINTS to $Rd\varphi$ φ-hat in equation) "...because like over here, isn't it going to be something like that?"

[00:50:01.25] Biff (over Shawn), "Here's the answer that you got from our current calculation; $2\pi QR$ over T" (WRITES a box around $2\pi QR/T$), (Shawn and Biff turn towards Biff), "Which gives us, uh, charge, so Coulombs-distance; meter-coulombs-per second."

KEN ARRIVES, LEAVES

[00:50:15.24] Devin, “That's pretty much, I mean that's,...yeah, that's just...[?]..."

[00:50:18.02] Shawn (over Devin), "...[?]...what about the,…”

[00:50:19.25] Shawn and Devin both point at an Biff's equation. It is unclear where they are pointing.

[00:50:19.25] Shawn, "…what about the,...to the point though. What about this thing?" (DRAWS an external point and a line segment from the external point to the center of the ring – Figure 21) "Our ***r*** minus ***r***′..." (WRITES $|\vec{r} - \vec{r}'|$, next to newly drawn line segment)

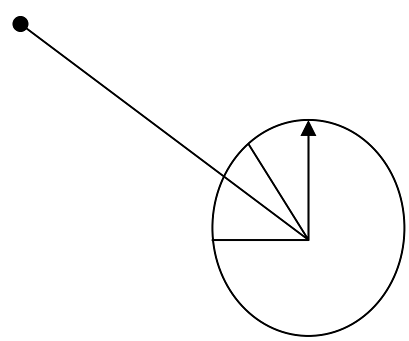

Figure 21: Shawn's line to an external point

[00:50:29.02] Biff, (in an artificially high voice), "Please, no!"

[00:50:30.18] Group laughs

[00:50:33.00] Biff, "I don't wanna...I'm too young to die."

[00:50:36.05] Biff, "Alright, let's see then. If you guys want to go through and erase all this and set it back up."

[00:50:44.06] Everyone ERASES EVERYTHING

[00:50:44.06] Devin? "Let's get it started."

[00:50:45.01] Biff, "Yeah."

ALICE ARRIVES

[00:50:49.06] Biff, "We got mad at it."

[00:50:50.03] Alice, "You got mad?

[00:50:50.24] Biff, "We're not doing it anymore. That's what we decided...Just kidding."
[00:50:54.05] Biff (turns to Alice), "Um, what should our answer be;...the units?"
[00:50:57.08] Alice laughs
[00:50:58.00] Biff, "Like, because we came up with...[?]...$2\pi RQ$ over T which has units of meters times (Coulombs?) per second."
[00:51:07.22] Alice, "OK, well look at your equation up there for vector potential."
[00:51:11.06] Biff, "Uh, 'K, that's for 3-D though"
[00:51:15.18] Alice, "Is that bad?"
[00:51:17.00] Biff,"...[?]...and not for 3-D."
[00:51:18.18] Alice, "Are you asking about the units of vector potential?"
[00:51:20.29] Biff, "We're going to need that r-r' thing, right?"
[00:51:22.18] Devin (over Shawn), "...[?] ...was vector potential, right?"
[00:51:26.24] Alice, "You're going to need that, yes...But what was your question? I don't think I understood."
[00:51:30.20] Biff, "I just wanted to make sure we had to do the r-r' thing. That sucks."
[00:51:33.24] Alice, "You do have to do the r-r' thing."
[00:51:35.27] Biff, "Indeed."

ALICE LEAVES, KEN ARRIVES

[00:51:37.22] Devin, "Well it's not that hard really, I mean..."
[00:51:38.24] Biff, "No, it's just..." (WRITES $R^2 + r^2 + 2Rr\,cos(\varphi\text{-}\varphi') +$)
[00:51:39.28] Ken, "Your charge distribution is a curved line, but in terms of thinking about where, what's happening in all space you can't ignore the fact there's three dimensions."
[00:51:50.27] Biff, "Oh."
[00:51:50.27] Ken, "You can't keep yourself in less than three dimensions."

KEN LEAVES

[00:51:53.08] Shawn, "So..."

[00:51:53.08] Biff, "...[?]...dimension right."

[00:51:56.23] Shawn, "So are we going to need the, uh,..."

[00:51:58.08] Biff, "I'm noticing there's a μ_o over 4π. Add that right here?" (WRITES an integral sign with $\mu_o/4\pi$ in front)

[00:52:07.01] Biff, "Is it like some standard constant?"

[00:52:07.17] Devin (WRITES the rest of the expression for r-r'(it's hard to read, but looks like $(z\text{-}z')^2$ and then puts a square root sign over the whole thing)

[00:52:12.01] Shawn, "So like, the direction that we want, that *A*-hat thing, is going to be,...so like if you're in the middle... (DRAWS a new ring picture)

[00:52:17.07] Devin ERASES his expression for *r*-*r*'

[00:52:23.12] Biff? Shawn?, "So we're at..."

[00:52:24.04] Biff WRITES $I(\boldsymbol{r})\ d\boldsymbol{r}$ / $|\boldsymbol{r}\text{-}\boldsymbol{r}'|$ (with vector signs over all the *r*'s)for integrand to get $\mu_o/4\pi \int I(\boldsymbol{r})\ d\boldsymbol{r}$ / $|\boldsymbol{r}\text{-}\boldsymbol{r}'|$

[00:52:24.04] Shawn WRITES- labels *r*-*r*' on vector on the ring drawing, "This is your *r* minus *r*-prime"

[00:52:29.06] Biff, "This what we need, right? Right here."

[00:52:30.26] Devin, "Yeah, that is, that's just our *r*, isn't it?

[00:52:34.19] Shawn, "But, like, if, if you were looking at it right."

[00:52:37.19] Devin, "Oh."

[00:52:38.23] Shawn, "I mean stuff would cancel out, but, like, what you'd want your *A* to be..." (POINTS and then DRAWS a vector on ring) "...in the *j*-hat direction."

[00:52:44.09] Devin "...[?]...[going in that direction?]

[00:52:46.27] Shawn, "So, what do you do to this..." (POINTS at diagram) "...to get it like that? You do, you need a cross product somewhere."

[00:52:53.15] Devin (over Shawn), "Yeah, well, how'd'ya, yeah you need a cross product to do it."

[00:52:55.21] Shawn (over Devin), "Where do you put that cross product?"

[00:52:58.13] Devin, "I do not know."

[00:53:03.24] Biff WRITES "= D ∫ (Q/T φ-hat)" to get $\mu_o/4\pi \int I(\boldsymbol{r})d\boldsymbol{r} \,/\, |\boldsymbol{r}\text{-}\boldsymbol{r}'|$ = D ∫ (Q/T φ-hat), "Anyone got their Griffiths, ya' know, Mama..." (referring to their textbook)

[00:53:06.04] Devin, "Yeah." gets textbook and gives it Biff

[00:53:16.26] Biff starts looking at text

[00:53:33.20] Devin, "So..."

[00:53:43.28] Devin, "So, wait, the current..."

[00:53:54.19] Devin, "So..." (POINTS at ***I*** in Biff's equation)

[00:53:56.07] Biff turns text so group can see it, "Here. It's this."

[00:53:57.12] Devin "So what do you..."

[00:53:58.21] Biff (over Devin), "Magnetic field due to a circular loop."

[00:54:02.04] Shawn, looks at text, "Ooooooo"

[00:54:05.22] Biff, "we are looking for magnetic field, right?"

[00:54:07.14] Devin, "Yes, sir."

[00:54:09.10] Shawn POINTS at text

[00:54:14.20] Biff (quietly), "Fuck."

[00:54:16.11] Shawn, "I...so you can,...they pull the ***I*** out. Yeah, that makes sense."

[00:54:21.28] Shawn? Devin?, "It's just constant."

[00:54:22.24] Biff, "It's just constant." ERASES part of his equation

[00:54:24.09] Shawn, "So *dl*..."

[00:54:25.17] Devin, "Problem is it's a vector isn't it?"

[00:54:27.29] Shawn, "...*dl* prime," (POINTS at text) "that our, that's our *d*." (POINTS at equation on whiteboard) "this is this...over scripty r squared. So that's going to be..."

[00:54:39.00] Devin, "[Well?] scripty *l* and scripty r are the same thing as the, uh, *r* minus *r*-prime"

[00:54:33.10] Biff WRITES $\mu_o/4\pi \int d\boldsymbol{\theta}' \,/\, |\boldsymbol{r}\text{-}\boldsymbol{r}'|$

[00:54:51.03] Biff, "Go find θ, then"

[00:54:52.24] Shawn, "So, is this the...?"

[00:54:53.23] Devin, "Yeah, what's that shit?"

[00:54:54.19] Shawn POINTS at text, "Is this their magnitude? And then is this the..." (POINTS to new spot in text)

[00:54:57.14] Devin, "It's that whole cosine θ business."

[00:55:04.07] Biff looks in back of text

[00:55:06.03] Shawn, "θ is the...θ is this angle. They did some funky thing to figure out, like, which direction it would go in."

[00:55:16.02] Devin, "Yeah, that's uncool."

[00:55:17.24] Shawn, "we need to do something with this." (POINTS at text)

[00:55:19.24] Devin, "...[?]..."

[00:55:21.06] Shawn, "No, that's, that's for surface. Never mind."

[00:55:30.12] Shawn, "We need a constant...[?]..."

[00:55:34.08] Biff, "Where'd you get your ...[?]...in the cross product from?"

[00:55:37.23] Shawn, "'Cause, well, like, if you look at..."

[00:55:38.05] Devin (over Shawn), "...[?]...would be in the k-hat direction?"

[00:55:40.19] Shawn, "Yeah, if you look at, like, the, a point in the center of the loop."

[00:55:44.08] Biff, "And why does it need a k up here?"

[00:55:46.11] Shawn, "'Cause that's the way the magnetic field..."

[00:55:46.26] Devin (over Shawn), "It's a magnetic field" (GESTURES with hand open, thumb up and then curls fingers in slightly (perhaps a right-hand-rule gesture))

[00:55:48.11] Biff (GESTURES right-hand rule), "Opposite of current."

[00:55:50.01] Shawn, "Yeah, like..." (GESTURES a right-hand rule)

[00:55:51.03] Devin "Perpendicular"

[00:55:51.26] Biff, "I'm kinda half asleep."

[00:55:53.12] Devin, "So am I."

[00:55:54.26] Shawn, "So, I mean, like, we just did..." (POINTS at drawing) "Like if we just do this" (POINTS at equation) "then we're just going to have some value right here, but we're not going to have in the **k**-hat direction."

[00:56:07.20] Shawn, "So what we need is, like, something that says, right here..." (POINTS at ring and the WRITES) "**i**-hat cross **j**-hat...we need something like that"

[00:56:16.08] Biff, "Maybe we should just add that."

[00:56:17.25] Devin, "For..."

[00:56:18.11] Shawn, "But we don't know what to add to that..."

[00:56:19.12] Biff (over Shawn), "Maybe we could..[?]... we can add that by the right hand rule...or, yeah...or not"

[00:56:19.12] Devin over Shawn and Biff? "...[?]...**r**-hat cross θ-hat."

[00:56:25.15] Biff, "Say by the right hand rule, it's in this direction." (GESTURES right hand rule with thumb up)

[00:56:27.19] Shawn, "But if you're, like, way up here at some weird point..." (POINT to place on board)

[00:56:29.24] Devin, "Yeah, but right-hand rule is kind of a sketch. You still have to have...[an exact reason?]..."

[00:56:36.01] Shawn, "Like if you're way up here," (DRAWS an external point), "like, which," (GESTURES from ring to external point) "I mean which way is going to point?"

[00:56:44.14] Shawn GESTURES (or POINTS) back and forth between different spots on the ring and the external point

[00:56:46.07] Biff WRITES $Rd\theta d$,"R d(phi) d-what? What, what was your thing?"

[00:56:50.17] Devin "That's a good question, where's it going to point right there?" (POINTS at external point on drawing)

[00:56:53.11] Shawn, "I don't know. Like that way, or something like that." (GESTURES perpendicular to line drawn from the center of the ring to a far external point)

[00:56:55.24] Devin,"You know, right here it's going to point straight the hell up." (POINTS to center of ring and GESTURES upward)

[00:56:59.06] Shawn, "But out here it's going to point," (GESTURES back and forth at the external point) "I don't know."

[00:57:03.13] Devin, "It's tricky"

[00:56:57.17] Biff starts to WRITE a denominator for his integrand "$|r^2$ - R^2 - $2Rr$"

[00:57:04.18] Shawn, So we've got have some kind of like a..."

[00:57:06.18] Devin (looks at front classroom board), "Well, we know one thing, don't we? She didn't finish that thing up there." (POINTS at front board) "It's, it's this." (WRITES I(r)) "...but I wouldn't know how the hell to write that out."

[00:57:18.14]Devin, "...['cause?]...somebody showed that to me written out in component form [perfect?]..."

[00:57:25.01] Biff ERASE his entire integral equation

[00:57:26.19] Devin, "How can you, how can you have a vector that is...[?]... with respect to another vector?"

[00:57:33.28] Biff, "Mm-hm."

[00:57:34.17] Devin, "That doesn't make any sense to ...[?]..."

[00:57:36.24] Biff, "Let's see here." (WRITES $V(r)$)

ALICE APPROACHES GROUP 6, BUT SHAWN RAISES HIS HAND

[00:57:46.05] Alice (to group 6), "How's it going over here?"

[00:57:47.11] Shawn raises hand.

[00:57:47.11] Nick (in Group 6) replies to Alice, "Not good."

[00:57:48.12] Alice (to group 6), "Not good. OK."

[00:57:49.14] Shawn, "We have a question."

[00:57:50.27] Alice walks away to go to other side of group 6's table, "I'll be right there."

[00:57:51.15] Shawn, "Oh, OK, I thought you were leaving them."

[00:57:53.04] Alice, "Nope."

[00:57:55.12] Biff WRITES $V(r)$ = (something hard to read)

[00:57:58.19] Biff ERASES this

[00:57:59.01] Shawn looks at front classroom board, "So, A equals zero, oh wait." (WRITES "A =" and starts to write a zero for limit of integration)

[00:58:10.23] Shawn ERASES limits of integration

[00:58:13.13] Shawn WRITES $\mu_o/4\pi$ while looking at board.

[00:58:13.13] Devin, "Oh, we're solving for potential."

[00:58:16.28] Shawn, "Yeah."

[00:58:19.03] Shawn, "Zero, two pi...all of this is going to be...Q over T" WRITES integral sign and limits of integration to get $\mu_o/4\pi \, {}_0\int^{2\pi}$ Q/T

[00:58:22.22] Devin (over Shawn), "Potential's also going to have...So we don't know if it's in the **k**-hat because it's potential, not magnetic field."

[00:58:28.28] Biff, "...[?]...again..."

[00:58:31.08] Biff?, "So I had it right,... originally"

[00:58:33.06] Biff, "That is a bitchin' thing. You didn't really want to do that."

[00:58:36.02] Shawn?, "What?"

[00:58:36.26] Biff, "That" (POINTS to denominator) "that's, uh,"

[00:58:39.10] Devin, "It's just ugly."

[00:58:39.10] Biff (over Devin), "...[?]...could find it a little bit."

[00:58:42.08] Shawn WRITES more on equation, "I'm just going to kind of write out what we have, just so she can help us....and then we have an $Rd\varphi$ φ-hat"

[00:58:48.15] Biff WRITES "θ =", then says, "If θ equals,...why don't,...hey, why, yeah."

[00:58:52.03] Corinne addresses whole class

[00:59:09.24] Biff WRITES on board - "cylindrical; $\boldsymbol{r} = <r, \theta, z>$; $\boldsymbol{r}' = <R, \theta, 0>$", and then circles it

[00:59:34.03] Shawn WRITES on small whiteboard, appears to initially copy equation from large whiteboard but then seems to be adding new things

[01:00:38.11] Biff ERASES entire large whiteboard and Shawn's small whiteboard

[01:01:08.24] Devin, "Psychotic."

CORINNE PULLS WHOLE CLASS TOGETHER

[01:01:22.00] Class ends

Appendix 5: Transcripts of Group 6 with Bing Coding

Group 6: Bryan, Nick, Paul

Coding Scheme

Physical Mapping

Geometric mapping (where it is contrasting with physical mapping) – See Note 2

Mathematical manipulation

Mathematical coherence

Authority

Student makes a notable error

NOTE 1: Solid color indicates that comments fit the coded category, color just over part of the time stamp indicates that the particular coding was suspected, but it was less clear than the solidly coded pieces.

NOTE 2: The “Geometric mapping” category is not an official Bing category. It should be considered “Physical mapping” using Bing’s coding. The additional color was an initial attempt to create a separate fifth framing, but was not further pursued after attempting it with Group 6.

Transcript

[00:40:39.03] Group starts to assemble

[00:41:07.27] Paul, "The written expression for the magnetic field anywhere in space...[?]...current distribution...that really bothers me, by the way; current distribution; isn't current supposed to be in wires?"

[00:41:25.04] Nick, "How did it escape?" (laughs)

[00:41:28.01] Paul (laughs)l, "How do we get out into free space?"

[00:41:31.07] Bryan arrives, "Alright. Let's have some fun. Well, I don't even remember what the question was and it sounded really good."

[00:41:37.12] Paul, "Sounded really hard."

[00:41:39.00] Bryan, "We're going to have a lot of fun doing this."

[00:41:40.11] Paul, "Yes."

[00:41:40.19] Nick, "By 'fun' you mean you suffer more but you learn to get an answer ...[?]..."

[00:41:46.23] Bryan, "Which the teacher could give to ya' "

[00:41:49.26] Paul, "Yeah."

[00:41:50.07] Bryan, "Alright, so..."

[00:41:50.25] Nick, "We have A" (WRITES A (vector))

KEN ARRIVES

[00:41:52.03] Ken, "...[?]...and draw pictures before you write down formulas."

[00:41:55.14] Nick, "Well, I don't even know what the picture is supposed to be."

[00:41:58.08] Bryan, "I don't remember..."

[00:41:59.07] Ken, "It's the same ring that you've been doing for the last..."

[00:42:01.08] Nick?, "Oh yeah, yeah, yeah, right, right, it's a ring, it's a ring."

[00:42:03.00] Bryan, "I have an idea."

[00:42:03.22] Ken, "Think about what it's saying at least once"

KEN LEAVES, ALICE ARRIVES

[00:42:05.25] Alice (laughs), "OK."

[00:42:06.18] Nick?, "I don't even..."

[00:42:07.04] Alice, "Ring of charge."

[00:42:08.13] Nick, "OK, ring of charge."

[00:42:08.04] Bryan DRAWS ring with axes

[00:42:09.18] Alice, "Spinning around."

[00:42:10.09] Paul, "Oh, it's spinning."

ALICE LEAVES

[00:42:11.20] Both Bryan and Nick DRAW on ring diagram

[00:42:14.17] Bryan, "This is really hard to draw from this angle. What should I do? OK, hold on. Let's do this"

[00:42:17.24] Paul appears to WRITE "$\omega = \infty$"

[00:42:19.29] Group laughs

[00:42:21.13] Nick (laughs), "The velocity equals infinity"

[00:42:24.28] Paul WRITES on drawing

[00:42:27.00] Bryan, "Alright."

[00:42:28.05] Paul?, "Therefore v equals infinity" (WRITES $v = \infty$)

[00:42:31.10] Nick (laughs), "Well at least we got that case done."

[00:42:35.07] Paul, "Alright, alright. We got one case. This is one case."(POINTS)

[00:42:39.11] Nick, "Wait, wait, what about omega equals zero?"

[00:42:44.00] Paul WRITES more

[00:42:45.07] Nick, "Clearly,...[?]... OK"

[00:42:48.13] Nick, "...infinity minus 2, and as you can see..." (laughs) "...[?]...as you go down...(laughs)...it's always equal to infinity."

[00:42:57.07] Nick, "I like this argument and I think we should go with it."

[00:43:03.02] Bryan, "So what's the...[?]..." (turns to look at front classroom board)

[00:43:09.04] Nick, "Say what?"

[00:43:09.15] Bryan, "What's the actual question being asked?"

[00:43:11.08] Nick, "Well, we're trying to find..."

[00:43:12.23] Paul, "...the expression for..."

[00:43:14.09] Nick, "...magnetic potential, uh..."

[00:43:16.14] Bryan, "That?" (POINTS to ***A*** (with a vector over it))

[00:43:17.15] Paul, "Potential, wait..."

[00:43:18.08] Nick, "No." ERASES ***A***

[00:43:18.24] Bryan, "No, it's not, it's..."

[00:43:19.11] Paul, "No. No. An expression for the magnetic field."

[00:43:22.18] Nick (looks at notes), "No, magnetic potential. She said magnetic potential."

[00:43:25.09] Paul (over Nick), "Wait, wait, what's magnetic potential?"

[00:43:27.27] Nick (bangs pen on table), "I don't even know. I don't even know what these letters mean, so..."

[00:43:36.25] Nick laughs, "Neither do you."

[00:43:38.19] Bryan (looking around, trying to talk to instructor or other student), "What's...Do you guys?.....wooo, wooo...Hey, Evan, what was the question being asked?"

[00:43:45.02] (no reponse)

CORINNE ARRIVES

[00:43:48.04] Bryan (turns to Corinne), "Can you restate the question, please?"

[00:43:49.10] Corinne, "Find the magnetic vector potential due to the spinning ring."

[00:43:53.04] Bryan, "OK, I don't why, just we missed, we all missed it."

[00:43:57.07] Corinne, "OK"

[00:43:58.07] Bryan, "OK...um"

[00:43:59.24] Corinne, "You know, it's one of these find it at every point in space so that you have an expression that MAPLE can evaluate. Find ***A*** for that ring."

[00:44:08.08] Bryan, "Oh, ***A***."

[00:44:10.02] Corinne, "***A***"

[00:44:10.02] Nick, "Ohhhh."

[00:44:11.14] Corinne, "***A***, that ***A***"

[00:44:12.09] Nick, "Ohhh, OK"

[00:44:13.03] Paul, "Ohhh"

[00:44:13.03] Group laughs

[00:44:13.03] Corinne, "That, evaluate that."

[00:44:15.23] Paul, "Oh, OK, OK, I think we can do that."

[00:44:17.21] Bryan, "Find ***A***," (laughs) "That's all you've got to say."

CORINNE LEAVES

[00:44:25.21] Paul starts to WRITE equation from board, "Wait, what's μ_o, do you know?" (WRITES $\boldsymbol{A} = \mu_o/4\pi\iiint \boldsymbol{J}(\boldsymbol{r})/|\boldsymbol{r}-\boldsymbol{r}'|$)

[00:44:30.16] Bryan, "Uhhh,..."

[00:44:31.10] Nick, "It's definitely a constant."

[00:44:33.28] Bryan, "Yeah."

[00:44:34.14] Paul, "Oh, OK"

[00:44:35.04] Bryan, "Well, yeah, it's got to be,...[?]...all the integrals."

[00:44:35.04] Paul (over Bryan), "What's J?"

[00:44:38.22] Nick, "Yeah."

[00:44:39.20] Bryan, "It's like the permeability of...something or other"

[00:44:45.00] Nick?, "...[wait a minute?]..."

[00:44:48.04] Paul, "Um, so we want to do this in cylindrical coordinates, right?"

[00:44:50.08] Nick, "Yes."

[00:44:51.10] Paul, "OK, so that's um" (WRITES $r\,dr\,d\varphi\,dz$ onto equation to get $\boldsymbol{A} = \mu_o/4\pi\iiint \boldsymbol{J}(\boldsymbol{r})/|\boldsymbol{r}-\boldsymbol{r}'|\ \ r dr d\varphi dz$) note that this is a direct substitution for $d\tau$, which is written on the front classroom whiteboard)

[00:44:56.09] Bryan, "Doot, da doot, doooo. r, dr, $d\varphi$, dz"

[00:44:58.09] Nick POINTS at equation and then POINTS at drawing, "However, the r is constant and z is zero."

[00:45:05.06] Bryan DRAWS a line from the center to the edge of the ring, DRAWS another radius, then ERASES original line, then DRAWS another radius

[00:45:05.06] Paul, "Right...um, constant...when *z*,...*z* is zero?" (POINTS at *dr* in equation and looks at ring picture) "Oh, that is"

[00:45:13.03] Nick, "It's a delta function of, uh...."

[00:45:16.00] Paul, "What's with your coordinate system?"

[00:45:18.13] Bryan, "What?"

[00:45:19.07] Paul, "I'm just..."

[00:45:21.07] Bryan, "What's with the coordinate system?" (DRAWS a line from point on ring outward)

[00:45:22.26] Paul, "It just looks weird because this is z," (POINTS along the drawn vertical axis) "and you drew it so that," (GESTURES a ring with both hands (as if holding a bowl) and then uses right hand to gesture back and forth along the horizontal axis) "this is the...[?]...plane instead of..."

[00:45:26.04] Bryan (over Paul), "You know I, I couldn't figure it out from being,...from, you know I couldn't figure it out from being from this angle so..."

[00:45:30.14] Paul, "Let's do this." (ERASES a *y* on vertical axis and WRITES in a *z*) "There we go"

[00:45:34.15] Nick, turning head to view it from another angle, "Yeah, OK....so...No, it's still not right." (GESTURES first on vertical axis then on diagonal axis) "This has to be *x*, that has to be *y*."

[00:45:41.19] Bryan, "How does that matter?"

[00:45:43.06] Paul, "It doesn't matter."

[00:45:43.14] Nick (over Paul), "Right hand rule." (GESTURES right-hand rule with thumb up and fingers curling, then bangs hand on table) "It does matter!"

[00:45:46.08] Paul (POINTS at drawing, but does not appear to be pointing at a specific thing in the drawing) "No! We're not in Cartesian coordinates."

[00:45:48.16] Nick grabs his head and makes a noise.

[00:45:49.27] Paul, "Here." (ERASES labels on *x* and *y* axes)

[00:45:52.26] Bryan, "There's..."

[00:45:53.15] Paul, "No, just, just leave that."

[00:45:54.08] Nick, "Just, just," (laughs) "...screw it dudes."

[00:45:55.02] Paul, "That's z," (POINTS at horizontal axis), "make up the other ones. OK, what's J?"

[00:45:57.05] Bryan WRITES question marks for axes labels

[00:45:58.08] Nick, "you've got question marks every time." (laughs)

[00:46:00.14] Paul, "What's $\boldsymbol{J}$?" (POINTS at J in equation)

[00:46:01.04] Nick, "$\boldsymbol{J}$ is the..., uh, density, uh, charge density."

[00:46:05.14] Paul, "Why is it $\boldsymbol{J}$?"

[00:46:06.23] Nick, "Not charge density, current density."

[00:46:08.16] Paul, "Oh, OK."

[00:46:10.02] Bryan, "Why is it $\boldsymbol{J}$?" (laughs)

[00:46:12.01] Paul, "Why is it $\boldsymbol{J}$?" (laughs)

[00:46:13.12] Nick, "OK, so, we can rewrite $\boldsymbol{J}$ as, uh, the current," (WRITES an $\boldsymbol{I}$ on board) "The linear density, right? The linear current density."

[00:46:24.12] Paul, "Yeah."

[00:46:24.12] Nick, "Times the delta z, right?" (WRITES "$\delta(z)$" with a space between it and I, to get $I\ \ \delta(z)$)

[00:46:28.03] Paul, "Um..."

[00:46:33.24] Nick, "Yes."

[00:46:34.19] Paul, "But, is it also...."

[00:46:36.27] Nick, "Um, because its z component."

[00:46:36.27] Paul, "Wait do we need...

[00:46:38.10] Nick, "Wait...z...um"

[00:46:42.10] Paul, "Wait, the z-component, yes. But then, don't, don't we also need to restrict it to R, our constant" (POINTS first at the "$\delta(z)$" expression, then with finger and thumb about two inches apart GESTURES at ring drawing from the center to ring itself)

[00:46:50.09] Nick, "Big R?" (WRITES a $\delta(R)$ to get $I\ \ \delta(z)\delta(R)$)

[00:46:50.27] Paul, "Yeah. This is...[?]..." (ERASES small r's and WRITES capital R's to get $\boldsymbol{A}=\mu_o/4\pi\iiint \boldsymbol{J}(\boldsymbol{r})/|\boldsymbol{r}-\boldsymbol{r}'|\ RdRd\varphi dz$)

[00:47:01.28] Paul, "R"

[00:47:04.00] Nick, POINTS at ring, "It's infinitely thin."

[00:47:08.04] Nick, "Yeah."

[00:47:09.05] Paul, "Yes."

[00:47:09.23] Nick, "Density is infinite. There the ring is infinitely thin along the z and along the r." (GESTURES z and r directions) "...directions."

[00:47:14.25] Paul, "...um..."

[00:47:16.03] Nick, "Right?"

[00:47:20.05] Paul, "Uh, yes, yes...uh,...wait, so...so..." (POINTS first at R, then at triple integral in his own equation for $\boldsymbol{A}$)

[00:47:23.18] Nick (over Paul), "And this, this is 2π...or is this $d\varphi$ over 2π?" (POINTS at $I\ \delta(z)\delta(R)$ and then uses pen to mark a φ in the air above the end of his expression)

ALICE ARRIVES

[00:47:30.05] Bryan, "Where does the, where does the I come from?"

[00:47:32.15] Nick, "I is, uh, uh, lambda." (WRITES $I = \lambda$)

[00:47:38.03] Bryan, "Is this, is this (POINTS at $J(r)$ in Paul's equation) going to be going like this (POINTS at I in Nick's expression), or what does this equal?"

[00:47:44.00] Nick, "Uh, we're trying to say J is equal to..." (WRITES "J =" in from his expression to get $J = I\ \delta(z)\delta(R)$)

[00:47:47.06] Bryan, "Oh, OK"

[00:47:48.11] Nick, "...this" (GESTURES in a circle above the $I\ \delta(z)\delta(R)$ part of his expression)

[00:47:48.23] Bryan, "I just had no idea what I was doing."

[00:47:51.25] Nick looks at Alice, "We're not quite done yet so don't...[?]..."

[00:47:53.27] Alice, "Don't worry about it. I'm just trying to figure out where you are." (Gestures and laughs)

[00:47:57.20] Nick, "OK, ummm, and then the linear density is...uhhh..."

[00:48:14.17] Nick, "You need some sort of φ component here." (WRITES a φ with a rotated "{" under it above an empty spot at the end of his $J = I \ \delta(z)\delta(R)$ expression) "That's going to be dependent on the rotation."

[00:48:27.15] Bryan, "Does that come out in the omega? (POINTS at Paul's $\omega = \infty$ equation)

[00:48:30.10] Paul ERASES his $\omega = \infty$ and $v = \infty$ equations.

[00:48:31.15] Alice laughs

[00:48:32.21] Group laughs

[00:48:34.04] Alice, "It's like, 'Don't let her see it!'" (Gestures)

[00:48:36.16] Bryan, "Those were our idealizations for this problem."

[00:48:40.07] Paul, "Yeah."

[00:48:39.07] Alice (over Bryan and Paul), "OK, so, you're here," (POINTS at $I = \lambda$ equation) "Right? I like all this stuff" (POINTS at $J = I \ \delta(z)\delta(R)$ and I like this." (POINTS at φ) "You're here" (POINTS at $I = \lambda$) "Now you need what? What's your dimensional situation there?"

[00:48:50.02] Nick, "uh, we need..."

[00:48:53.12] Alice, "What do we measure current in?" (POINTS at I)

[00:48:56.15] Nick, "Uh, amps"

[00:48:57.19] Alice, "Which is a...? That's a...Use the units."

[00:48:59.02] Nick, "Charge per time."

[00:49:01.04] Alice, "It's charge per time, right? What's the dimensions of this?" (POINTS to λ)

[00:49:04.03] Nick, "Uh, charge per..."

[00:49:05.05] Bryan, "Charge per length."

[00:49:05.15] Nick, "...per length."

[00:49:06.00] Alice, "Charge per length." Right? So, how are you going to a charge per time in terms of...[?]..."

[00:49:08.29] Bryan (over Alice), "Per time equals charge per length times length per time." (WRITES $Q/T = Q/L \ \ L/T$ on board) "It's going to be...[?]..."

[00:49:16.17] Nick, "Velocity"

[00:49:17.10] Paul?,"Yeah."

[00:49:18.25] Bryan WRITES a v to make $I = \lambda v$

[00:49:18.04] Alice, "Ok, and velocity happens to be a vector quantity, so you put a direction there too."

[00:49:24.04] Paul, "Yes."

[00:49:25.21] Nick taps table, puts hand to head, makes noise, "Ahhh"

[00:49:27.09] Alice, "Does that make sense now?

[00:49:28.23] Nick, "Yeahh...Maybe."

[00:49:30.11] Alice, "Maybe?"

[00:49:31.20] Nick, "Um, so, we need like total charge here?" (POINTS in a circle to the space at the end of his $J = I \ \ \delta(z)\delta(R)$ expression, under where φ is written)

[[00:49:36.00] Bryan, "That's going to make this problem a lot more interesting."

[00:49:39.10] Nick (looks at front classroom board and POINTS), "Are trying to get it in terms of Q, R, and T?"

[00:49:43.17] Alice, "Yes."

[00:49:44.02] Nick, "OK. What's R?"

[00:49:45.13] Alice, "The radius."

[00:49:46.02] Nick, "Oh, it's the radius. Right."

[00:49:47.10] Bryan (laughs), "Oh yeah. That's what that means."

[00:49:49.25] Paul, "This, this one right here" (POINTS to a radius on drawing) "we actually have it labeled."

[00:49:51.29] Nick (laughs), "OK"

[00:49:53.18] Alice, "Yes. Your picture's very nice. Use your picture."

[00:49:55.25] Nick, "Um, so, we need a charge. So we have Q, over the length, which is 2..." (WRITES $Q/2\pi R$ at the end of his equation, to get $J = I\ \delta(z)\delta(R)\ Q/2\pi R$)

[00:50:01.28] Paul DRAWS a point on the ring at the end of the drawn radius

[00:50:07.22] Paul, "Wait." ERASES the point he just drew.

[00:50:10.10] Paul POINTS to $Q/T = Q/L\ \ L/T$ at the bottom of the vertical axis and says (over Nick), "Why...This is, this is bad."

[00:50:11.16] Bryan, WRITES $\omega = v/r$, and says, "v over... Is omega v over r?"

ALICE LEAVES

[00:50:14.06] Paul, "Yeah."

[00:50:17.00] Bryan, "Is that relevant?"

[00:50:17.26] Paul, "Umm...."

[00:50:20.08] Bryan, "Don't we have to...We have to incorporate this somehow, don't we?" (POINTS at $\omega = v/r$ equation)

[00:50:23.00] Paul, "I would think so, because, we have a circle," (GESTURES a circle with both hands as if holding a bowl) "that's moving, so" (POINTS at $\omega = v/r$ equation) "that sounds like omega to me."

[00:50:31.14] Bryan, "OK"

[00:50:32.12] Paul, "Um..."

[00:50:33.00] Bryan, "Should J..."

[00:50:33.29] Paul, "This is bothering me." (WRITES something small on board near ring)

[00:50:37.18] Nick, "Yeah this is...[?]..." (WRITES $2\pi R/T$ at end of his equation to get $J = I\ \ \delta(z)\delta(R)\ Q/2\pi R\ 2\pi R/T$)

[00:50:39.09] Paul, "Dude, yesterday I hit my head on my driveshaft so hard."

[00:50:43.25] Nick, "What?"

[00:50:45.16] Paul, "I was taking my transmission out..."

[00:50:46.20] Nick, "Yeah."

[00:50:47.16] Paul, "Oh yeah. I got the clutch off."

[00:50:48.25] Nick, "Yeah. Did you get another clutch on?"

[00:50:51.06] Paul, "No. It's so fucked. The pressure plate has cracks that go all the way through the pressure plate."

[00:50:56.29] Nick. "Ahhh."

[00:50:58.08] Paul, "The disk is like (gestures) so, it's like just started to scrape the buttons, you know, that hold it down. The flywheel is probably OK, but..."

[00:51:08.06] Nick nods

[00:51:10.20] Paul, "Anyways, um..."

[00:51:13.09] Bryan, "Why is it that we're using,..." (POINTS to delta functions in Nick's equation) "...I mean like I understand that it's infinitely thin, but why do we have to use those?"

[00:51:18.21] Nick, "Uh, Because we're going to be integrating over this." (POINTS to Paul's equation then his own equation), "this is the only way define that it's a..." (GESTURES in a circle above the ring drawing) "...a...disk...I mean a circle." (GESTURES in a circle above the ring again) "The problem we had," (POINTS at front classroom board) "is that she only gave us the, uh, ***A*** equation." (POINTS at Paul's equation) "like the big density equation, so we don't have any, we don't have the..." (GESTURES a complex gesture with hands initially about six inches apart, palms towards each other, and then moves them together) "...lower level, like, points or linear stuff that we'd used before."

[00:51:46.14] Bryan, "Sure."

[00:51:47.14] Nick, "So instead of building up," (GESTURES with both hands moving away from himself) "...we have to build it down." (GESTURES with both hands moving towards himself) "So to do that we need the, uh, the, uh, yeah."

[00:51:53.13] Bryan, "So..."

[00:51:53.27] Paul, "Wait, so then our limits of integration...wait, wait...let's get our limits of integration straight. So for R is it..."

[00:51:59.10] Nick, "Zero to infinity"

[00:52:01.08] Paul, "Right...Oh, well, it just, yeah, needs to potentially cross where our actual R value is" (POINTS at both equations, then WRITES in limits of integration on his own equation) "And then φ is zero to 2π. And z is..."

[00:52:16.12] Nick, "Zero to infinity."

[00:52:17.26] Paul, "Zero to infinity." (WRITES limits on integral)

[00:52:23.00] Nick, "Right." (nods)

[00:52:23.26] Bryan, "Yep."

[00:52:25.06] Paul, "Great."

[00:52:25.06] Nick, "Ummm..."

[00:52:29.00] Paul, "Um, oops." (WRITES primes on his r and R in denominator)

[00:52:31.20] Bryan, "So what is J fundamentally? Like I times...er, yeah..."

[00:52:37.01] Paul, "Yeah, yeah, well,..."

[00:52:37.04] Bryan, "...J equals I times..."

[00:52:38.14] Paul, "Wait, J, J is the current density; the current per..."

[00:52:40.08] Nick, "J is equal to, like, ρ" (WRITES $J = \rho$)

[00:52:43.18] Bryan, "It's equal to ρ times velocity, isn't it?"

[00:52:46.04] Paul (over Bryan), "No, well, no, it's, it's..." (POINTS at $J = \rho$ equation)

[00:52:46.04] Nick (WRITES to modify the equal sign (to some sort of proportional sign maybe?)), "They're related"

[00:52:49.00] Bryan, "Well, yeah, but I want to know, like, what's the exact...I think it's ρ times velocity, isn't it?"

[00:52:54.11] Nick, "I don't know. This is where this thing comes in here." (POINTS at some part of Bryan's $Q/T = Q/L \;\; L/T$ equation)

[00:52:57.20] Bryan, "Yeah, 'cause,...ρ, or,...yeah ρ is, like,...L,Q..." (POINTS at Q/L in his $Q/T = Q/L \;\; L/T$ equation)

[00:53:05.16] Paul, "Yeah, yeah, that's, that's ρ, but..." (POINTS at Q/L in Bryan's $Q/T = Q/L \;\; L/T$ equation)

[00:53:08.26] Bryan, "Doesn't, didn't J have, like, the same units as...."

[00:53:14.17] Nick, "Well, it's length over time, so that the, uh,..." (POINTS at the L/T part of Bryan's equation)... "Oh, this angular velocity. Right?" (WRITES a circle around the $2\pi R/T$ part of his own equation and then draws an arrow from it and writes ω)

[00:53:22.26] Paul, "Uh, yes. That's angular velocity......Wait, so this, this has constant ω" (GESTURES around in a circle) "Is that why we're doing it in this class and not...[inaudible]..."

[00:53:42.03] Nick, "Maybe, I don't know. I don't want to think about that."

[00:53:46.10] Paul (puts head on hand), "Um, OK, so, wait, Q over $2\pi R$," (POINTS at $Q/2\pi R$ in equation), "So that's,...That would be our λ."

[00:53:54.00] Nick GESTURES in a circle, "But we need to get an integration...I think."

[00:53:56.22] Paul, "Ah."

[00:53:57.23] Bryan (POINTS at $2\pi R/T$ part of Nick's equation), "Wait, this is a...velocity,...because there is distance per time."

[00:54:04.09] Paul, "Right, but it's angular velocity." (POINTS at $2\pi R/T$ part of Nick's equation)

[00:54:05.16] Nick (over Paul), "It's a...yeah."

[00:54:06.25] Paul, "...[it's over?]...2π."

[00:54:10.12] Nick, "No."

[00:54:11.00] Paul, "No, wait, um..."

[00:54:11.13] Bryan, "The units still don't work out though."

[00:54:14.02] Nick, "This is tangential velocity."

[00:54:16.07] Bryan, "Yeah, there you go."

[00:54:18.29] Nick, "It would have to be divided by 2π to be ω, right?"

[00:54:24.13] Bryan, "Divided by R, 'cause you want, like, radians per second."

[00:54:28.00] Nick, "Yes. Yeah." (nods)

[00:54:29.08] Bryan, "The 2π/T would get us omega."

[00:54:33.14] Paul, "Um, ...[?]...I think so, uh,..."

[00:54:38.25] Bryan, "That makes sense, 'cause it's constant..[?]..."

[00:54:42.01] Nick, POINTS to ($2\pi R/T$)($Q/2\pi R$) part of expression, "But this has to be with respect to $d\varphi$ right? Charge over length is,…(GESTURES a small distance with thumb ring finger)...this has to go, (WRITES a circle around $Q/2\pi R$)...we need this related to $d\varphi$, so for,...we've got small sections." (GESTURES a small distance with thumb and pointer finger, DRAWS a small wedge and WRITES labels on the end of the wedge as dQ and the angle as $d\varphi$ – Figure 22)

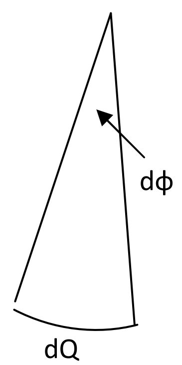

Figure 22: Nick's drawing of *dQ*

[00:55:08.16] Bryan, "So $Rd\varphi$ equals Q?"
[00:55:15.07] Nick (mumbles), " $Rd\varphi$ equals Q."

{pause}

[00:55:29.03] Paul, POINTS to $J = I\ \delta(z)\delta(R)\ Q/2\pi R\ 2\pi R/T$ and says, "This equals J, huh?"
[00:55:31.11] Bryan, "This equals J, huh?"
[00:55:32.09] Paul, "Is this true?"
[00:55:33.11] Nick, "Not quite"
[00:55:36.01] Paul, "What are we missing?"
[00:55:38.04] Nick, "We need a $d\varphi$"

[00:55:45.16] Bryan, "We need somehow to, like, incorporate, like, an $Rd\varphi$ right?", (WRITES "$Rd\varphi$" on board next to ring drawing)

[00:55:50.15] Nick, "Yeah."

[00:55:51.27] Bryan, POINTS at Nick's wedge drawing and the $Rd\varphi$ expression, "So, there's your...so we got our dz, our dr, we need $Rd\varphi$. $Rd\varphi$ is dq, right? (Pauses, looks at Nick, Nick nods) For example. So, how can we make this look like a dq?"

[00:56:16.11] Bryan, POINTS to Q in $Q/2\pi R$ part of expression, "So this is the charge,...total charge divided by..."

[00:56:22.16] Nick, "The length."

[00:56:23.29] Bryan GESTURES around ring and says, "The total length, that makes sense."

[00:56:26.03] Nick, "So..."

[00:56:26.03] Bryan, "That looks,...this to me looks like a dQ, right?"

[00:56:28.10] Nick, "Oh, OK, so, yeah, dQ over (POINTS at drawing of wedge),...then our partial length is going to be $rd\varphi$ right?", (WRITES $dQ/ rd\varphi$)

[00:56:37.00] Bryan, "So dQ over dQ?"

[00:56:38.24] Nick, POINTS at $d\varphi$ in $dQ/ rd\varphi$ expression, "Uh, $d\varphi$"

[00:56:40.28] Bryan, "But then it...but, like, $rd\varphi$ is dQ, so, like, that'd be dQ over dQ." (POINTS at $dQ/ rd\varphi$ expression)

[00:56:44.27] Nick, "Wait."

[00:56:47.21] Paul, "Uh, that'd be a big R, by the way, just ...[?]..."

[00:56:52.13] Bryan (laughs)

[00:56:54.15] Nick, "Um, well, no, really it has to be a little r, because it's changing. No, wait, no, it's not, it's got to be a big R,..." (WRITES a capital R into expression to get $dQ/ Rd\varphi$)

[00:57:04.03] Paul, "Yes."

[00:57:04.14] Nick, "...because it's not changing."

[00:57:06.01] Group laughs

[00:57:07.03] Bryan, "That was really good intuitive..."

[00:57:09.20] Nick, "Um..."

ALICE ARRIVES

[00:57:11.17] Alice, "How's it going over here?"

[00:57:13.01] Nick, "Not good."

[00:57:13.25] Alice, "Not good? OK."

[00:57:15.09] Shawn from Group 5 says, "We have a question."

[00:57:16.12] Alice (to Shawn), "I'll be right there."

[00:57:17.11] Shawn from Group 5 says, "Oh, OK, I thought you were leaving them."

[00:57:18.04] Alice (to Shawn), "Nope."

[00:57:18.21] Alice, "I've come in so that I can read this side."

[00:57:21.06] Bryan (POINTS at Nick's $J = I \ \delta(z)\delta(R) \ Q/2\pi R \ 2\pi R/T$ equation), "So we've got, for, $J = I$ times, here's our z component." (POINTS at $\delta(z)$)

[00:57:27.08] Alice, "Mm-hm."

[00:57:28.19] Bryan POINTS at $\delta(R)$, "Here's our R component."

[00:57:30.05] Alice, "OK"

[00:57:30.20] Bryan POINTS at $Rd\varphi$ written near ring, "And we still need our $Rd\varphi$ so we decided that $Rd\varphi$ equals dQ, so we..."

[00:57:37.23] Alice, "Wa, wa, wa, wait. I am confused. I think you're convolving some things." (Gesticulates) "So first is" (Gestures by using both hands to make brackets around $J = I \ \delta(z)\delta(R) \ Q/2\pi R \ 2\pi R/T$ equation) "J you're saying is this."

[00:57:46.12] Nick, "Sort of. We're not sure."

[00:57:49.06] Bryan, "we're trying to make this" (POINTS in general at J equation) "a J, but we know that J equals this (POINTS at I) times this (POINTS at $\delta(z)$) times this (POINTS at $\delta(R)$) times an $Rd\varphi$ component. Is that right?"

[00:58:00.01] Alice, "Why do you need an $Rd\varphi$ component?"

[00:58:02.05] Bryan, "Oh, man, I thought that you wanted one."

[00:58:04.23] Alice, "You will when you're doing the integral." (POINTS at integral) "I mean, that's where your $d\varphi\ dr\ d\theta$'s are going to come in."
[00:58:09.15] Bryan (over Alice), "So you need a...so you just need a φ component?"
[00:58:13.00] Nick, "So we don't need a..."
[00:58:14.09] Alice (over Nick), "Why do need a..."
[00:58:14.09] Bryan, "...[We don't need a tau?]..." (shakes head)
[00:58:15.22] Nick (POINTS at $Q/2\pi R\ 2\pi R/T$ part of equation), "You just,...Is this good? Are we good?"

CORINNE TRIES TO PULL WHOLE CLASS TOGETHER, BUT ALICE CONTINUES TALKING TO GROUP 6, IGNORING CORINNE

[00:58:17.03] Corinne, "OK, I'm hearing a lot of different groups wondering about - you're getting - there's a lot of getting the units confused..."
[00:58:18.06] Alice, "If you have..." (WRITES vector signs over J) , "...doing that, (WRITES vector signs over I) and doing that...(uses hand to cover up the $Q/2\pi R\ 2\pi R/T$ part of equation) and getting rid of that, you're good."
[00:58:29.10] Nick, "Why are we..."
[00:58:30.10] Bryan, "What are the dimensions of I?"
[00:58:33.26] Alice looks at Bryan, "Dimensions of I? Ummm..." (WRITES Q/T)
[00:58:43.11] Bryan, "Oh, that's this" (POINTS at $Q/T = Q/L\ \ L/T$ equation)

ALICE STEPS AWAY

[00:58:50.12] Nick ERASES $dQ/\ Rd\varphi$, his wedge drawing, and the $Q/2\pi R\ 2\pi R/T$ part of the $J = I\ \ \delta(z)\delta(R)\ Q/2\pi R\ 2\pi R/T$ equation, so that only $J = I\ \delta(z)\delta(R)$ and Q/T are left.

{pause while Corinne talks}

[00:59:10.11] Nick to Bryan, "...[?]..."
[00:59:14.29] Bryan, "...[?]...Time to go."

ALICE COMES BACK

[00:59:28.26] Alice WRITES $I = \lambda v$, POINTS at what she's written and says (quietly), "...[?]...lambda v."
[00:59:35.25] Nick (quietly), "I equals λv? Yeah."
[00:59:40.21] Alice WRITES and POINTS, showing $\lambda = Q/2\pi R$ and says, "This is just..."
[00:59:49.03] Alice POINTS at v in equation and WRITES something, then ERASES it.
[00:59:58.21] Nick, "$2\pi R$ over T"
[01:00:02.17] Alice WRITES $2\pi R/T$ and POINTS at v in equation.
[01:00:06.28] Alice, "...[?]...the direction..."
[01:00:11.08] Nick, "Uh,...[?]..." (GESTURES)
[01:00:14.01] Alice WRITES φ-hat
[01:00:17.01] Nick, "Sure."

[01:00:18.23] ALICE AND GROUP MEMBERS LEAVE