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Title SYNTHESIS OF NETWORKS WITH COMPLEX TERMINATIONS

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A method for the synthesis of networks with specified complex terminations is presented. The specification function is realized within a multiplicative constant. The technique presented differs somewhat from present methods employed in the synthesis of networks with resistive terminations.

The technique is one of rearranging a transfer function and separating the specified load. The remaining portions are associated with realizable network functions which are synthesized by classical methods. The load is then added to the synthesized network.

Through the analysis of a terminated network by the use of Thevenin's and Norton's theorems, the transfer function of the network can be generated in terms of the network characteristics and the termination. Having derived the general expression for the transfer function of a network, it is possible to separate a given transfer function with a specified termination in such a manner that it can be associated with the general expression.

This separation process is the foundation of many of the operational steps of the procedure. In cases where the load is not already in the expression or not readily separable, it is inserted by multiplying and dividing or by adding and subtracting, whichever is appropriate. The separation, however, must yield network functions which are realizable. That is, driving point specifications must be positive real and transfer functions must satisfy similar requirements except that the degree of s in the numerator only has to be equal to or less than the degree of s in the denominator. The separation procedure consists of introducing a function, $X(s)$, and then dividing the numerator and denominator of the transfer function by this function. $X(s)$ is selected in such a manner that the restrictions placed on the expressions associated with the network specifications are maintained. A network can now be realized using a conventional method that is appropriate. The network is then connected to the termination and the overall transfer function is represented within a multiplicative constant.

SYNTHESIS OF NETWORKS WITH COMPLEX TERMINATIONS

by

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SYNTHESIS OF NETWORKS WITH COMPLEX TERMINATIONS

INTRODUCTION

Techniques for the realization of transfer functions are readily available in many textbooks on network synthesis. These methods, however, are only applicapable to networks with either no termination or a resistive termination. This method of synthesis presents a technique for realizing transfer functions having complex terminations.

The synthesis of transfer functions with a specified termination can be accomplished within a multiplicative constant by a separation of the function into a driving point function and a transfer function which are realizable. The procedure consists of separating the load from the overall transfer function in such a manner that the remaining portions can be associated with the driving point function and the transfer function of a network. Having obtained these network specifications, conventional methods of synthesis can then be used to realize the network within a scale factor. The network is terminated with the specified termination and the combination gives the desired transfer function. The process of separating the load requires that the transfer function or the load be of a nature such that the removal does not yield functions which are not realizable.

The separation of a transfer function refers to manipulating it into one of the following forms :

$$G_{12} = \frac{G_{12OC} Z_L}{\frac{1}{y_{22}} + Z_L} \qquad (1)$$

$$G_{12} = \frac{-y_{12}}{y_{22} + Y_L} \qquad (2)$$

$$Z_{12} = \frac{z_{12} Z_L}{z_{22} + Z_L} \qquad (3)$$

$$Z_{12} = \frac{-\alpha_{12}}{\frac{1}{z_{22}} + Y_L} \qquad (4)$$

DERIVATION OF THE TRANSFER FUNCTION EXPRESSIONS

The derivation of Equations 1, 2, 3, and 4, are made through the use of Thevenin's and Norton's theorems.

The general circuit driven by a voltage source, V_s, would appear as shown in Figure 1.

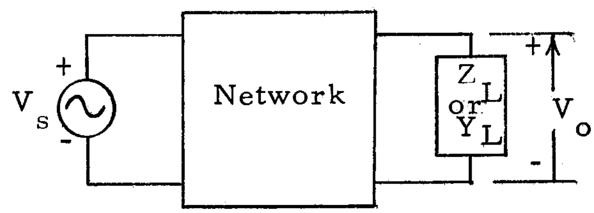

Figure 1. General circuit with a voltage source.

The portion labeled network is desired as the result of the synthesis procedure.

Thevenin's theorem states that any linear, reciprocal network with two accessible terminals may be replaced by a voltage source acting in series with an impedance, the voltage is that which appears across the output terminals when they are open-circuited; the impedance is that viewed at the open-circuited output terminals looking into the network with all independent generators in the network replaced by their internal impedances (2, p. 276).

For the circuit driven by a voltage source, the open-circuit output voltage and the output impedance are shown in Figures 2 and 3, respectively.

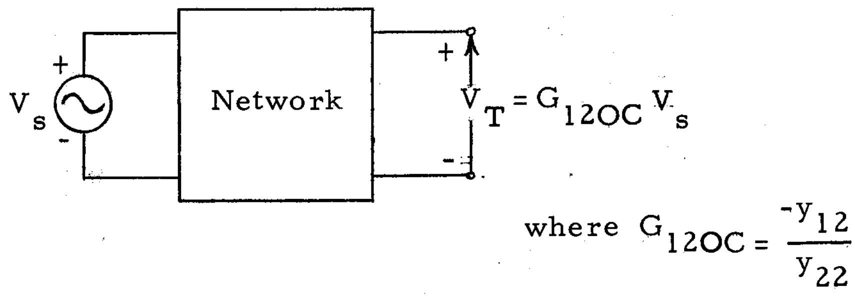

Figure 2. Circuit with output open-circuited.

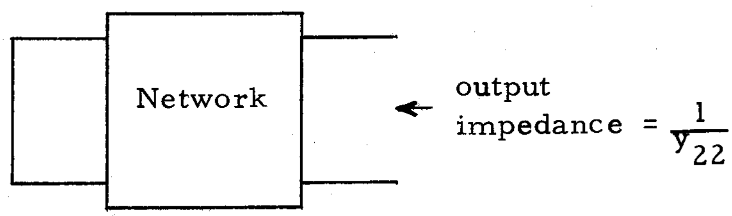

Figure 3. Circuit with voltage source replaced by a short-circuit.

Therefore, Thevenin's equivalent network will be as shown in Figure 4.

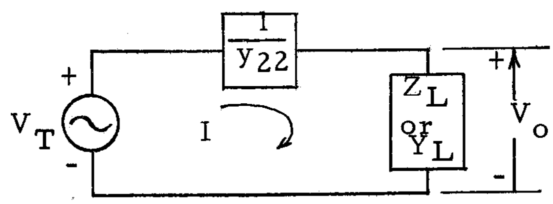

Figure 4. Thevenin's equivalent network.

Now the circuit is in a form which can be readily analyzed.

$$V_o = IZ_L$$

$$= \frac{V_T Z_L}{\frac{1}{y_{22}} + Z_L}$$

$$= \frac{G_{12OC} V_s Z_L}{\frac{1}{y_{22}} + Z_L} \qquad (5)$$

$$G_{12} = \frac{V_o}{V_s} = \frac{G_{12OC} Z_L}{\frac{1}{y_{22}} + Z_L} \qquad (6)$$

Norton's theorem states that any linear reciprocal network with two accessible terminals may be replaced by a current source acting in parallel with an admittance, where the current source is the short-circuit current delivered to the output terminals when these terminals are short-circuited, and the admittance is that of the network when all independent internal generators are replaced by their internal admittances (2, p. 278).

For the circuit with a voltage source, the short-circuit output current and the output admittance will be obtained from the networks in Figure 5 and 6, respectively.

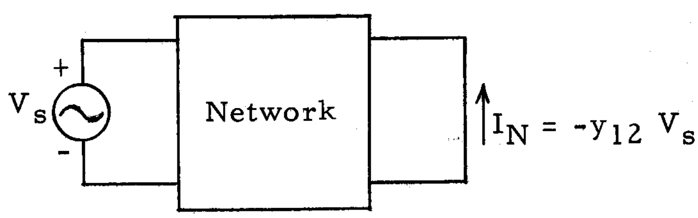

Figure 5. Circuit with output short-circuited.

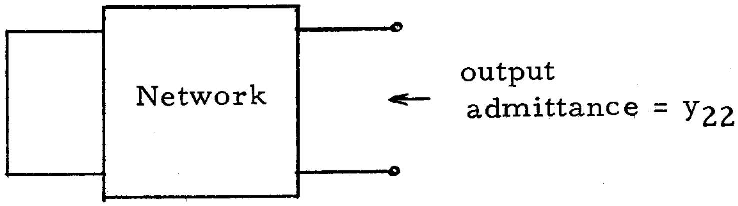

Figure 6. Circuit with voltage source replaced by a short-circuit.

Therefore, Norton's equivalent network will be as shown in Figure 7.

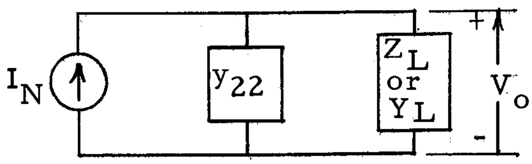

Figure 7. Norton's equivalent network.

Analyzing Norton's equivalent network, we get

$$V_o = \frac{I_N}{y_{22} + Y_L}$$

$$= \frac{-y_{12} V_s}{y_{22} + Y_L} \qquad (7)$$

$$G_{12} = \frac{V_o}{V_s} = \frac{-y_{12}}{y_{22} + Y_L} \qquad (8)$$

Now consider a general circuit with a current source, I_s, which is very much similar to the circuit with the voltage source.

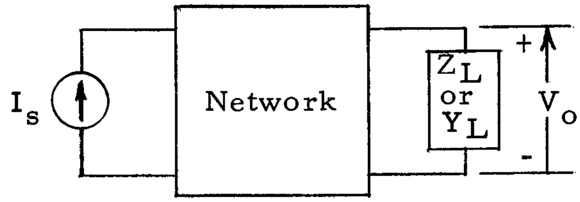

Figure 8. General circuit with a current source.

Applying Thevenin's theorem on a circuit driven by a current source, the open-circuit output voltage and the output impedance is arrived at using the networks of Figures 9 and 10, respectively.

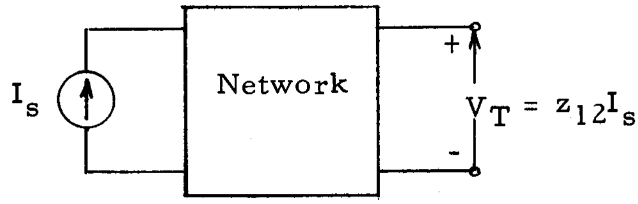

Figure 9. Circuit with output open-circuited.

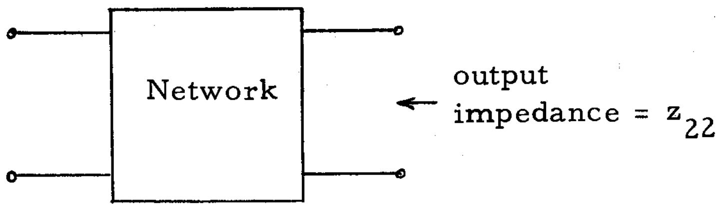

Figure 10. Circuit with current source replaced by an open-circuit.

The general circuit with a current source has a Thevenin's equivalent circuit as shown in Figure 11.

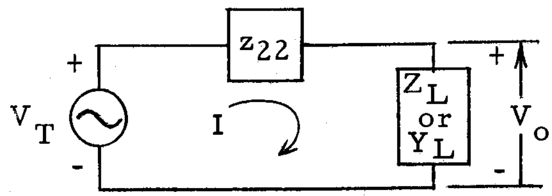

Figure 11. Thevenin's equivalent network.

Analysis of the equivalent network proceeds straight forwardly to the desired expression.

$$V_o = IZ_L$$

$$= \frac{V_T Z_L}{z_{22} + Z_L}$$

$$= \frac{z_{12} I_s Z_L}{z_{22} + Z_L} \qquad (9)$$

$$Z_{12} = \frac{V_o}{I_s} = \frac{z_{12} Z_L}{z_{22} + Z_L} \qquad (10)$$

Now let us apply Norton's theorem which is the dual of Thevenin's theorem. The short-circuit output current and the output admittance may be derived from the networks shown in Figures 12 and 13, respectively.

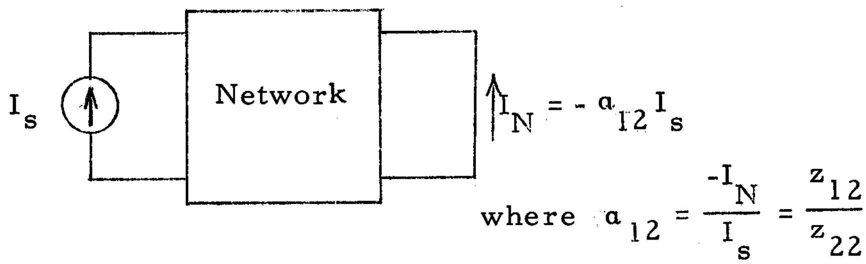

Figure 12. Circuit with output short-circuited.

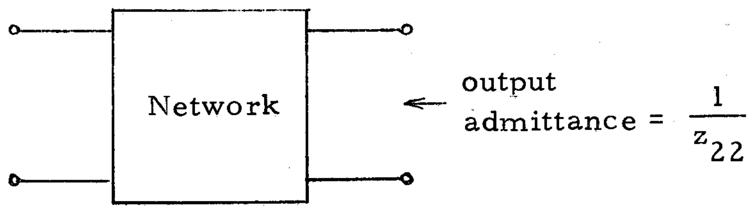

Figure 13. Circuit with current source replaced by an open-circuit.

The Norton's equivalent circuit of the general circuit with a current source appears as shown in Figure 14.

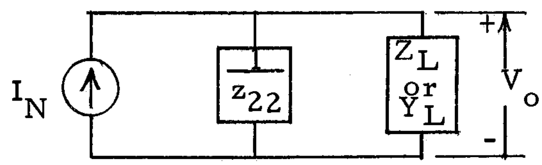

Figure 14. Norton's equivalent network.

Analyzing the equivalent network quickly gives an expression for its transfer function.

$$V_o = \frac{I_N}{\frac{1}{z_{22}} + Y_L}$$

$$= \frac{-\alpha_{12} I_s}{\frac{1}{z_{22}} + Y_L} \tag{11}$$

$$Z_{12} = \frac{V_o}{I_s} = \frac{-\alpha_{12}}{\frac{1}{z_{22}} + Y_L} \tag{12}$$

RESTRICTIONS FOR REALIZABILITY

The overall transfer function must have the following properties to be a function capable of being realized as a network (5, p. 258).

1) Poles must be in the left half s plane or on the imaginary axis and simple.

2) Zeros may have any s plane location, except for the positive real axis, but must always occur in conjugate pairs if complex.

3) Poles may not be located at the origin or at infinity.

4) The highest power of s in the numerator must be less than or equal to the highest power of s in the denominator.

The same properties apply to the transfer functions, $-y_{12}$ and z_{12}.

The network driving point functions, y_{22} and z_{22}, must also satisfy necessary conditions to be realizable (5, p. 72). Namely, they must be positive real. A necessary and sufficient condition for positive realness is requiring that

$$\text{Re } Z(s) \text{ or Re } Y(s) \geq 0 \text{ for Re } s \geq 0.$$

SYNTHESIS PROCEDURE

Knowing the desired terminating impedance or admittance, the problem is to separate it from the transfer function. The transfer function will be a ratio of polynomials in s.

$$T(s) = \frac{P(s)}{Q(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots}{b_0 + b_1 s + b_2 s^2 + \ldots}$$

The termination can also be expressed as a ratio of polynomials in s.

The first step is to manipulate the denominator into the desired form by dividing both the numerator and denominator of the transfer function by the denominator of the termination to form a complex fraction. At this point two cases are considered, depending on the form of the numerator desired.

Case I. If the forms of Equations 1 and 3 are of interest, dividing the transfer function by the denominator of Z_L automatically insures the correct denominator in the transfer function expression. The numerator of Z_L in the numerator of the transfer function can be introduced by multiplying both the numerator and the denominator of the transfer function by the numerator of the load impedance. Next, separate the Z_L in the denominator by adding and subtracting the term XZ_L and then dividing numerator and denominator by X. The function X is a polynomial in s which is to be defined later.

The division by the polynomial, $X(s)$, introduces the driving point and transfer ratios of the required network.

$$T(s) = \frac{P(s)}{Q(s)} \qquad Z_L = \frac{N(s)}{D(s)}$$

$$= \frac{P\,\frac{N}{D}}{Q\,\frac{N}{D}} \qquad \text{multiply by } \frac{N}{D}$$

$$= \frac{P\,\frac{N}{D}}{\frac{QN - XN}{D} + \frac{XN}{D}} \qquad \text{add and subtract } X\frac{N}{D}$$

$$= \frac{\frac{P}{X}\,\frac{N}{D}}{\frac{QN - XN}{XD} + \frac{N}{D}} \qquad \text{divide by } X \tag{13}$$

Proceed by solving for $X(s)$. Case II may be introduced at this point since the mechanics for both cases from this point on are quite similar.

Case II. Considering the forms of Equations 2 and 4, the termination factor does not appear in the numerator; therefore, the numerator can be associated with the network transfer function. However, Y_L is separated from the rest of the denominator by adding and subtracting the term XY_L and then dividing numerator and denominator by X.

$$T(s) = \frac{P(s)}{Q(s)} \qquad Y_L = \frac{N(s)}{D(s)}$$

$$= \frac{\frac{P}{D}}{\frac{Q}{D}} \qquad \text{divide by D}$$

$$= \frac{\frac{P}{D}}{\frac{Q - XN}{D} + \frac{XN}{D}} \qquad \text{add and subtract } X\frac{N}{D}$$

$$= \frac{\frac{P}{XD}}{\frac{Q - XN}{XD} + \frac{N}{D}} \qquad \text{divide by X} \tag{14}$$

Again proceed by solving for the quantity X(s).

To solve for X, begin by dividing D into the numerator directly above it and choosing X at the same time, keeping in mind the restrictions that must be satisfied. The restrictions are restated in a form that is adapted to their application here.

Case I	Case II
D) QN - XN(quotient	D) Q - XN(quotient

Find X(s) such that:

1) The division comes out even, that is, D is an even factor of the dividend.

2) The highest power of s in X does not differ from the highest power of s in the quotient by more than one.

3) There are no negative coefficients in either X or the quotient.

4) All the terms in X and the quotient are present unless all the odd or all the even terms in X and the quotient are missing.

5) For Case I each of the coefficients of X must be equal to or greater than each of the coefficients of P.
For Case II each of the coefficients of the quotient must be equal to or greater than each of the coefficients of P.

6) The highest power of s in X is equal to or greater than the highest power of s in P.

An important set of relationships between the coefficients of a polynomial and its factors can be obtained by expanding the factored form and collecting various powers of s. Since two polynomials in s are identical if and only if the coefficients of corresponding powers of s are the same, general relationships can be obtained by comparing the result of expanding the factored form with the polynomial form. The polynomial form is

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0 \tag{15}$$

while the factored form is

$$P(s) = a_n (s + z_1)(s + z_2) \ldots (s + z_n). \tag{16}$$

A fundamental theorem of polynomials is stated: The ratio $\frac{a_{n-i}}{a_n}$ is the sum of the products of the z's taken i at a time (3, p. 18).

Since the dividend of our problem is a product of the factors D and the quotient which are functions of s, the general relationships just mentioned are applicable. The two factors involved may be considered as complex factors comprised of one or more simple factors. Therefore, applying the rule leads to associating the product of D and the quotient term by term with the dividend. By equating the coefficients of the like powers of s, the coefficients of X and the quotient can be solved simultaneously. Before associating the product with the dividend, rewrite the X in the dividend as its equivalent polynomial in s. The highest power of s present being sufficient to accommodate the highest power of s in the dividend. Likewise, the highest power of s needed in the quotient is one sufficient to accommodate the highest power of s in the dividend.

As an illustration consider

$$X = x_2 s^2 + x_1 s + x_0$$

$$d_1 s + d_0)\ n_3 s^3 + n_2 s^2 + (n_1 - X) s + n_0 (\ q_2 s^2 + q_1 s + q_0$$

The dividend can then be written as

$$(n_3 - x_2) s^3 + (n_2 - x_1) s^2 + (n_1 - x_0) s + n_0$$

which will be associated with

$$d_1 q_2 s^3 + (d_1 q_1 + d_0 q_2) s^2 + (d_1 q_0 + d_0 q_1) s + d_0 q_0 \ .$$

Now equating like powers of s

$$d_1 q_2 = n_3 - x_2$$

$$d_1 q_1 + d_0 q_2 = n_2 - x_1$$

$$d_1 q_0 + d_0 q_1 = n_1 - x_0$$

$$d_0 q_0 = n_0 \ .$$

The equations are now solved simultaneously. In cases where there are more unknowns than equations, the restrictions that have to be satisfied may eliminate some of them. However, if non-unique solutions do occur, it may be possible to realize several networks having the required transfer function and termination. When no solution exists, a network meeting the specifications can not be realized.

Some suggestions which may simplify the solution.

1) The cofficients must all be positive as per Restriction 3, p. 13.

2) If the denominator of the terminations is of greater order than the numerator, the first term in the dividend should be reduced by the first term in the quotient in order to satisfy Restrictions 2, p. 13. Therefore, the first term in X can be deleted.

3) If the numerator of the termination is of greater order than the denominator, the first term in the dividend should be reduced by the first term in X in order to satisfy Restriction 2, p. 13. Therefore, the first term in the quotient can be deleted.

4) If an internal coefficient vanishes, this suggests that either all odd or all even coefficients are missing in the quotient with all even or all odd coefficients missing in X as per Restriction 4, p. 14.

After X has been properly determined and the dividend replaced by its factors D and the quotient, the ratio of D's in the denominator will cancel which leaves a ratio of the quotient to X as shown in Equations 17 and 18.

Case I

$$T(s) = \frac{\frac{P}{X}\ \frac{N}{D}}{\frac{(D)(\text{quotient})}{XD} + \frac{N}{D}}$$

$$= \frac{\frac{P}{X}\ \frac{N}{D}}{\frac{\text{quotient}}{X} + \frac{N}{D}} \tag{17}$$

Case II

$$T(s) = \frac{\frac{P}{XD}}{\frac{(D)(\text{quotient})}{XD} + \frac{N}{D}}$$

$$= \frac{\frac{P}{XD}}{\frac{\text{quotient}}{X} + \frac{N}{D}} \tag{18}$$

If the specification function is such that it can be manipulated into the proper form, the use of surplus factors becomes necessary. A given specification function may be realized by either of the two methods presented. For a given termination, one method may lead to a more convenient solution than the other.

With the transfer function rearranged in the form of Equation 17 or 18, the terms of the equation can be associated with the terms of Equation 1, 2, 3, or 4. The specification function can now be realized by conventional synthesis methods. The transfer and driving point functions are realized, and the network is terminated with the specified complex termination.

SCALE FACTOR CONSIDERATION

Upon realization, a multiplying constant will be associated with the network transfer function. Even though the driving point specification is realized exactly, a multiplying constant is introduced since the transfer function is realized only within a scale factor, K, over which there is no control. Network realizations do, however, always realize the poles and zeros of the transfer function in developing the driving point specification. The frequency-independent scale function is defined by the equations

$$K = \frac{z_{12} \text{ realized}}{z_{12} \text{ specified}} \tag{19}$$

$$K = \frac{-y_{12} \text{ realized}}{-y_{12} \text{ specified}} \tag{20}$$

This problem of a scale factor can be remedied by the use of a constant phase amplifier whose gain is sufficient to compensate for the scale factor (5, p. 273-274).

EXAMPLES OF THE SYNTHESIS METHODS

Perhaps the mechanics of the procedure can be best shown by means of some examples.

Example 1. Synthesize a network with a third order pole at -1, terminated in a parallel RC configuration. The voltage gain would be

$$G_{12} = \frac{V_o}{V_s} = \frac{1}{s^3 + 3s^2 + 3s + 1} \tag{21}$$

with a termination of

$$Z_L = \frac{1}{s + 1} \tag{22}$$

Using the steps of Case I,

$$G_{12} = \frac{1}{s^3 + 3s^2 + 3s + 1}$$

$$= \frac{\frac{1}{s+1}}{\frac{s^3 + 3s^2 + 3s + 1}{s+1}}$$

$$= \frac{\frac{1}{s+1}}{\frac{s^3 + 3s^2 + 3s + 1 - X}{s+1} + \frac{X}{s+1}}$$

$$= \frac{\frac{1}{X} \quad \frac{1}{s+1}}{\frac{s^3 + 3s^2 + 3s + 1 - X}{X(s+1)} + \frac{1}{s+1}}$$

$$X = x_2 s^2 + x_1 s + x_0$$

$$s + 1) s^3 + 3s^2 + 3s + 1 - X(q_2 s_2 + q_1 s + q_0$$

$$s^3 + (3 - x_2) s^2 + (3 - x_1) s + 1 - x_0 =$$

$$q_2 s^3 + (q_1 + q_2) s^2 + (q_0 + q_1) s + q_0$$

$q_2 = 1$	$q_2 = 1$	$x_2 = 0$
$q_1 + q_2 = 3 - x_2$	$q_1 = 2$	$x_1 = 1$
$q_0 + q_1 = 3 - x_1$	$q_0 = 0$	$x_0 = 1$
$q_0 = 1 - x_0$		

Selecting $X = s + 1$ satisfies all the conditions mentioned. The separation now continues to

$$G_{12} = \frac{\left(\frac{1}{s+1}\right)\left(\frac{1}{s+1}\right)}{\frac{s^2 + 2s}{s+1} + \frac{1}{s+1}} = \frac{G_{12OC} Z_L}{\frac{1}{y_{22}} + Z_L} \tag{23}$$

Identifying the terms,

$$y_{22} = \frac{s+1}{s^2 + 2s} \tag{24}$$

$$G_{12OC} = \frac{1}{s+1}$$

$$-y_{12} = \frac{1}{s^2 + 2s} \tag{25}$$

$$Z_L = \frac{1}{s+1}$$

The zero of transmission is at infinity so we can use Cauer's continued fraction expansion at infinity in developing y_{22}.

$$
\begin{array}{l}
s+1)\; s^2+2s\;(\; s \\
\qquad\;\; \underline{s^2+\;s} \\
\qquad\qquad\quad s)\; s+1\;(\; 1 \\
\qquad\qquad\qquad\;\; \underline{s} \\
\qquad\qquad\qquad\quad 1)\; s\;(\; s \\
\qquad\qquad\qquad\qquad\;\; \underline{s}
\end{array}
$$

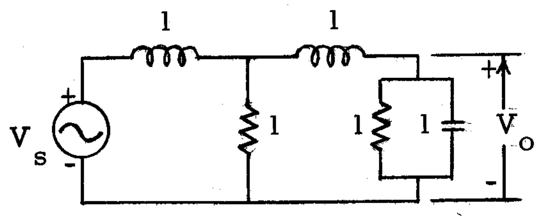

Figure 15. Realized network of Example 1.

Now K may be computed

$$K = \frac{-y_{12} \text{ realized}}{-y_{12} \text{ specified}}$$

Since K is frequency-independent, we choose a frequency that simplifies the computation as much as possible such as $s = 1$.

$$-y_{12} \text{ realized} = \frac{1}{1 + \frac{1}{1+1}} \quad \frac{1}{1+1} = \frac{1}{3}$$

$$-y_{12} \text{ specified} = \frac{1}{3}$$

$$K = 1$$

The scale factor for this problem turned out to be unity which means that the specifications are realized exactly.

Using the steps of Case II,

$$G_{12} = \frac{1}{s^3 + 3s^2 + 3s + 1}$$

$$= \frac{1}{s^3 + 3s^2 + (3 - X)s + 1 - X + X(s + 1)}$$

$$= \frac{\frac{1}{X}}{\frac{s^3 + 3s^2 + (3 - X)s + 1 - X}{X} + s + 1}$$

$$X = x_2 s^2 + x_1 s + x_0$$

1) $s^3 + 3s^2 + (3 - X)s + 1 - X(q_2 s^2 + q_1 s + q_0$

$$(1 - x_2)s^3 + (3 - x_1 - x_2)s^2 + (3 - x_1 - x_0)s + 1 - x_0 =$$

$$q_2 s^2 + q_1 s + q_0$$

$0 = 1 - x_2$	$x_2 = 1$	$q_2 = 0$
$q_2 = 3 - x_1 - x_2$	$x_1 = 2$	$q_1 = 1$
$q_1 = 3 - x_1 - x_0$	$x_0 = 0$	$q_0 = 1$
$q_0 = 1 - x_0$		

Therefore,

$$G_{12} = \frac{\frac{1}{s^2 + 2s}}{\frac{s + 1}{s^2 + 2s} + s + 1} = \frac{-y_{12}}{y_{22} + Y_L}$$

where

$$-y_{12} = \frac{1}{s^2 + 2s}$$

$$y_{22} = \frac{s+1}{s^2 + 2s}$$

$$Y_L = s + 1$$

which are the results obtained from Case I.

Example 2. As a second example, consider the transfer function

$$G_{12} = \frac{V_o}{V_s} = \frac{18(s+1)}{3s^3 + 55s^2 + 187s + 174} \tag{26}$$

having the termination

$$Z_L = \frac{s+1}{s^2 + s + 1} \tag{27}$$

Utilizing Case I,

$$G_{12} = \frac{\dfrac{18}{X} \; \dfrac{s+1}{s^2 + s + 1}}{\dfrac{3s^3+55s^2+(187-X)s+174-X}{X(s^2+s+1)} + \dfrac{s+1}{s^2+s+1}} \tag{28}$$

$$X = x_1 s + x_0$$

$$s^2 + s + 1\overline{)\,3s^3 + 55s^2 + (187 - X)s + 174 - X(q_1 s + q_0}$$

$$3s^3 + (55 - x_1)s^2 + (187 - x_0 - x_1)s + 174 - x_0 =$$

$$q_1 s^3 + (q_0 + q_1)s^2 + (q_0 + q_1)s + q_0$$

$$\begin{aligned} q_1 &= 3 & q_1 &= 3 & x_1 &= 10 \\ q_0 + q_1 &= 55 - x_1 & q_0 &= 42 & x_0 &= 132 \\ q_0 + q_1 &= 187 - x_0 - x_1 \\ q_0 &= 174 - x_0 \end{aligned}$$

An acceptable X turns out to be $X = 10s + 132$; therefore,

$$G_{12} = \frac{\dfrac{18}{10s + 132} \quad \dfrac{s + 1}{s^2 + s + 1}}{\dfrac{3s + 42}{10s + 132} + \dfrac{s + 1}{s^2 + s + 1}} = \frac{G_{12OC} Z_L}{\dfrac{1}{y_{22}} + Z_L} \tag{29}$$

where

$$y_{22} = \frac{10s + 132}{3s + 42} \tag{30}$$

$$G_{12OC} = \frac{18}{10s + 132} \tag{31}$$

$$-y_{12} = \frac{18}{3s + 42} \tag{32}$$

Cauer's continued fraction expansion at infinity may be used to develop y_{22}.

$$10s + 132 \,\big)\, 3s + 42 \,\Big(\frac{3}{10}$$
$$3s + \frac{396}{10}$$
$$\frac{24}{10}\Big)\, 10s + 132 \,\Big(\frac{100}{24}\, s$$
$$10s$$
$$132 \Big)\, \frac{24}{10} \,\Big(\frac{24}{1320}$$
$$\frac{24}{10}$$

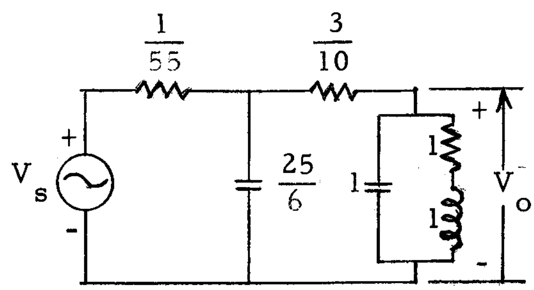

Figure 16. Realized network of Example 2.

Compute K at s = 0.

$$K = \frac{\dfrac{1}{\dfrac{1}{55} + \dfrac{3}{10}}}{\dfrac{18}{42}} = \frac{22}{3} \qquad (33)$$

The network realized must be amplifed by a constant factor of $\frac{22}{3}$ to meet the specification desired.

It is not necessary to use Case II to work the problems; however, it is of demonstrative value to go through and illustrate different aspects of the procedure in more detail.

$$G_{12} = \frac{\frac{18}{X}}{\frac{3s^3 + (55-X)s^2 + (187-X)s + 174 - X}{X(s+1)} + \frac{s^2+s+1}{s+1}} \quad (34)$$

$$X = x_1 s + x_0$$

$$s + 1)\ 3s^3 + (55 - X)s^2 + (187 - X)s + 174 - X(q_1 s + q_0$$

$$(3 - x_1)s^3 + (55 - x_0 - x_1)s^2 + (187 - x_0 - x_1)s + 174 - x_0 =$$

$$q_1 s^2 + (q_0 + q_1)s + q_0$$

$$0 = 3 - x_1 \qquad x_1 = 3 \qquad q_1 = 10$$

$$q_1 = 55 - x_0 - x_1 \qquad x_0 = 42 \qquad q_0 = 132$$

$$q_0 + q_1 = 187 - x_0 - x_1$$

$$q_0 = 174 - x_0$$

Therefore,

$$G_{12} = \frac{\frac{18}{3s + 42}}{\frac{10s + 132}{3s + 42} + \frac{s^2 + s + 1}{s + 1}} = \frac{-y_{12}}{y_{22} + Y_L} \quad (35)$$

giving

$$-y_{12} = \frac{18}{3s + 42}$$

$$y_{22} = \frac{10s + 132}{3s + 42}$$

As the termination becomes more complex, the complexity of determining X is magnified tremendously. However, given an appropriate transfer function; a termination of any complexity can, with patience, be synthesized. Also, the X determined may not be a unique one. Different networks may be realized that have the same transfer function. In this case the simplest network may be of interest; that is, the one having an X and quotient of lowest degree and therefore fewer elements.

PRACTICAL APPLICATIONS

It is frequently necessary to terminate a network with a system having a complex input impedance. Therefore, there is a great variety of uses for networks capable of being terminated in a complex load. An example would be an electric motor which is definitely not pure resistive in nature. It may also be an integral part of a control system, feeding into a complex network within the system. There are an unlimited number of other situations.

Scaling may be a useful tool in working practical problems. By either magnitude scaling or frequency scaling, a network problem can be somewhat simplified to prevent working with exceedingly large or exceedingly small numbers. A network is frequency scaled by a factor a, if each inductance and each capacitance is multiplied by $\frac{1}{a}$ and a network is magnitude scaled by a factor b, if each resistance and inductance is multiplied by b and each capacitance is divided by b. (5, p. 48-53).

$$R' = bR$$

$$L' = \frac{b}{a} L$$

$$C' = \frac{1}{ab} C$$

To allow for the non-idealness of a real situation, predistortion may be of interest in realizing a network (3, p. 709-712; 5, p. 92-95).

FUTURE CONSIDERATIONS

Further investigation of the problem could lead to a computer solution. The task would be to write a program of the problem which may call for a slight revision into a programable form depending upon the particular machine. The nature of the problem makes it a difficult one to program because of the generalities involved. However, once programed, a computer will provide a quick and easy solution.

SUMMARY AND CONCLUSIONS

This method of realization of transfer functions synthesizes networks with either real, imaginary, or complex terminations. The transfer function is realized within a constant. A unity scale factor indicates an exact realization of the function.

The procedure consists of removing a specified termination from a transfer function in such a manner that the remaining functions are network specifications. In order to implement a solution, definite requirements are placed on the types of functions that the separation must yield. These restrictions are stringent enough to specify a network that satisfies the required transfer function; however the network may not be unique. The transfer function and the termination must both individually and mutually satisfy certain restrictions in order to have a solution. The restrictions are namely those which insure realizability of the resulting functions.

The complexity of the termination, which can be synthesized, depends on the transfer function given. Also, the complexity of the algebra increases tremendously in proprotion to that of the termination.

Due to the combination of elements present in any practical situation, a technique accounting for complex termination will provide a better approximation to what is called for than one which does not.

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