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Title: A NEW METHOD FOR THE SYNTHESIS OF NETWORKS BY USING CUT-SET MATRICES

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Professor Leland C. Jensen

A method for the synthesis of bilateral networks is presented. A new algorithm for the realization of basic cut-set matrices is developed. This is done by the following procedure:

1) We form submatrices, $M_j(i)$, $j=1,2,\ldots$, of the given cut-set matrix, such that $M_j(i)$ satisfies the properties of a reduced incidence matrix.

2) These submatrices are converted to (non-reduced) incidence matrices, A_j, $j=1,2,\ldots$.

3) A_j's are combined to give an incidence matrix, A, corresponding to the given cut-set matrix.

4) From A, we can draw graph directly.

Secondly, by using the realization technique of cut-set matrices, the synthesis of RLC n-port networks with n nodes is illustrated by an example.

Thirdly, realization of (non-basic) cut-set matrices is illustrated by synthesizing a resistive network.

Finally, synthesis of single-contact networks is also illustrated by an example.

A New Method for the Synthesis of Networks by Using Cut-set Matrices

by

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A NEW METHOD FOR THE SYNTHESIS OF NETWORKS
BY USING CUT-SET MATRICES

CHAPTER I

INTRODUCTION

As an application of graph theory, realization of cut-set matrices plays a basic role in the synthesis of n-port networks as well as in switching circuits.

There exist two major approaches to the synthesis of an n-port with (n+1) nodes. The first approach is the method of determination of port structure (1,6), and the second approach is based on the decomposition of an admittance matrix, Y, into the triple product

$$Y = C_s Y_e C_s^t$$

where C_s is a seg matrix (a general definition of a cut-set matrix), Y_e is the diagonal matrix with positive elements representing edge conductances (2), and the superscript t indicates the transpose of a matrix. Thus the synthesis of Y is accomplished if C_s can be realized as a graph.

On the other hand, the synthesis of a class of unate switching networks in minimal form is directly related to the realization of a non-oriented, connected graph, that is, to the realization of the loop or cut-set matrix of a connected graph.

Thus, the bilateral network synthesis problems, whether they are RLC networks or contact networks, are actually reduced to the realization of a graph.

Several papers related to the synthesis of cut-set matrices have been published (3,4,7,10,12). However, some of them are too complex for convenient use, some of them have no algorithm, and some of them have restrictions on the size of the given matrix.

Mayeda presented necessary and sufficient conditions for the realizability of cut-set matrices (9). First, he showed the method of forming a set of v submatrices, called M-submatrices, from a given matrix consisting of v-1 rows. Then it was shown that if, and only if, all of these v submatrices are realizable as incidence matrices, the given matrix is realizable as a basic cut-set matrix. In the first step, however, we are often obliged to use trial and error method; that is, there is no rigorous method of obtaining a set of v submatrices.

In this paper, a new algorithm for the realizability of cut-set matrices is given. By studying the properties of the graph, and by applying Tutte's theorem (13), we can improve Mayeda's method and obtain the new method.

CHAPTER II

PRELIMINARY

2.1. Definitions

As a preliminary, some definitions are given.

Definition 1. The incidence matrix, denoted by $A=\left[a_{ij}\right]$, of a graph with n nodes and b edges, is the matrix with n rows and b columns. Each row corresponds to a node, and each column corresponds to an edge, such that

$a_{ij}=1$, if edge j is incident at node i and directed away from node i;

$a_{ij}=-1$, if edge j is incident at node i and directed toward node i;

$a_{ij}=0$, if edge j is not incident at node i.

If a graph under consideration is not oriented, then each element of the incident matrix of the graph will take value of 1 or 0.

As an illustration of an incidence matrix, let us consider a graph given in Fig. 1, where the nodes are denoted by numerals, and the edges are given by lower-case letters.

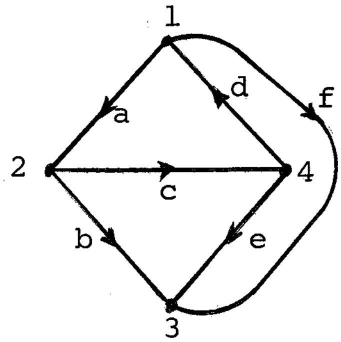

Fig. 1. An oriented graph.

The incidence matrix, A, for the graph is

$$A=\begin{array}{cc} & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & . & . & -1 & . & 1 \\ -1 & 1 & 1 & . & . & . \\ . & -1 & . & . & -1 & -1 \\ . & . & -1 & 1 & 1 & . \end{bmatrix} \end{array}$$

The incidence matrix is a basic one for topological synthesis because it completely defines the geometry of a network, that is, a matrix possessing the properties of an incidence matrix may always be realized directly. The incidence matrix with one row eliminated still defines the geometry of a graph, and is called the reduced incidence matrix. The node corresponding to the row that has been deleted is the datum node.

Definition 2. A simple cut-set of a graph is a set of edges such that the removal of the set of edges from the graph reduces the rank of the graph by one and no proper subset of the set of edges has the same property. A "cut-set" implies a simple cut-set.

Definition 3. The basic cut-sets are the (n-1) simple cut-sets in a connected graph of n nodes which are formed by each branch of a tree and some or all chords included in the basic loops (with respect to the same tree) containing the branch.

As an illustration, consider again the graph shown in Fig. 1. Some of simple cut-sets are:

$$\{a,d,f\},\ \{a,c,e,f\},\ \{a,c,b\},\ \{b,c,d,f\}$$

If we choose a tree in the graph of Fig. 1 as $\{a,d,e\}$, which is shown in bold lines in Fig. 2, we get the basic cut-sets as:

$$\{a,c,b\},\ \{b,c,d,f\},\ \{b,e,f\}$$

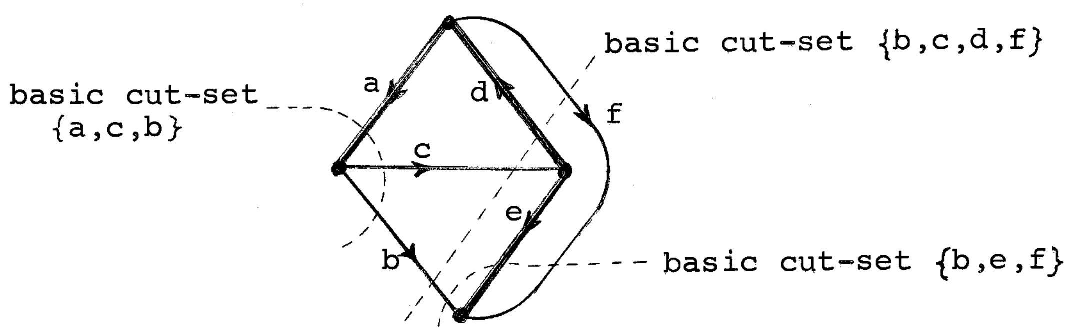

Fig. 2. An oriented graph with a chosen tree.

Definition 4. The simple cut-set matrix $Q=[q_{ij}]$, of a connected graph, has one row for each possible cut-set and one column for each edge in the graph, such that:

$q_{ij}=1$, if edge j is in simple cut-set i and their orientations are coincident;

$q_{ij}=-1$, if edge j is in simple cut-set i and their orientations are opposite;

$q_{ij}=0$, if edge j is not in simple cut-set i.

Definition 5. A basic cut-set matrix Q_f is a submatrix of a simple cut-set matrix whose rows are defined only for the basic cut-set with respect to a tree, and the orientation of each basic cut-set is defined by the branch contained in the basic cut-set.

The basic cut-set matrix for the graph shown in Fig. 2 with respect to the tree (a,d,e) is

$$Q_f = \begin{array}{c} \begin{matrix} b & c & f & a & d & e \end{matrix} \\ \begin{bmatrix} -1 & -1 & . & 1 & . & . \\ -1 & -1 & -1 & . & 1 & . \\ 1 & . & 1 & . & . & 1 \end{bmatrix} \end{array}$$

2.2. A Star-like Tree

Consider the graph shown in Fig. 3, where a chosen tree is indicated in bold lines. A tree whose branches are all incident at a common node is said to be a star-tree. The tree in Fig. 3 is a star-tree.

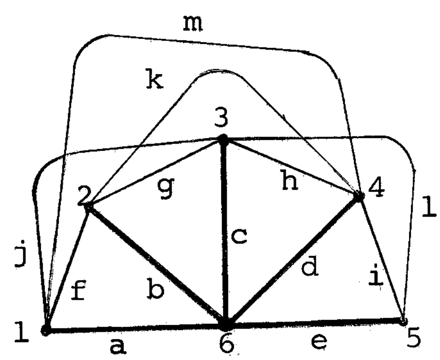

Fig. 3. A star-like tree.

From the definitions given in the preceding section, it is clear that the basic cut-set matrix with respect to a star-tree is equal to the incidence matrix which is reduced a common node. We use this fact as one of the basic ideas of our method.

CHAPTER III

MAYEDA'S METHOD

3.1. Cut of a Graph

In order to approach the star-tree, we construct two subgraphs from the given graph G in Fig. 4(a) by the following procedure (9), where the basic cut-set, S_i, consists of branches b_1, b_2, ...b_k:

1) Insert one node at the middle of each branch in S_i and coalesce the nodes together as shown in Fig. 4(b), and
2) split the node as shown in Fig. 4(c). Connected graphs G_1' and G_2' are the graphs that result from this procedure.

Now the basic cut-set, S_i, in G becomes an incidence set in both G_1' and G_2'. By repeating the procedure, G will be divided into star-tree structures.

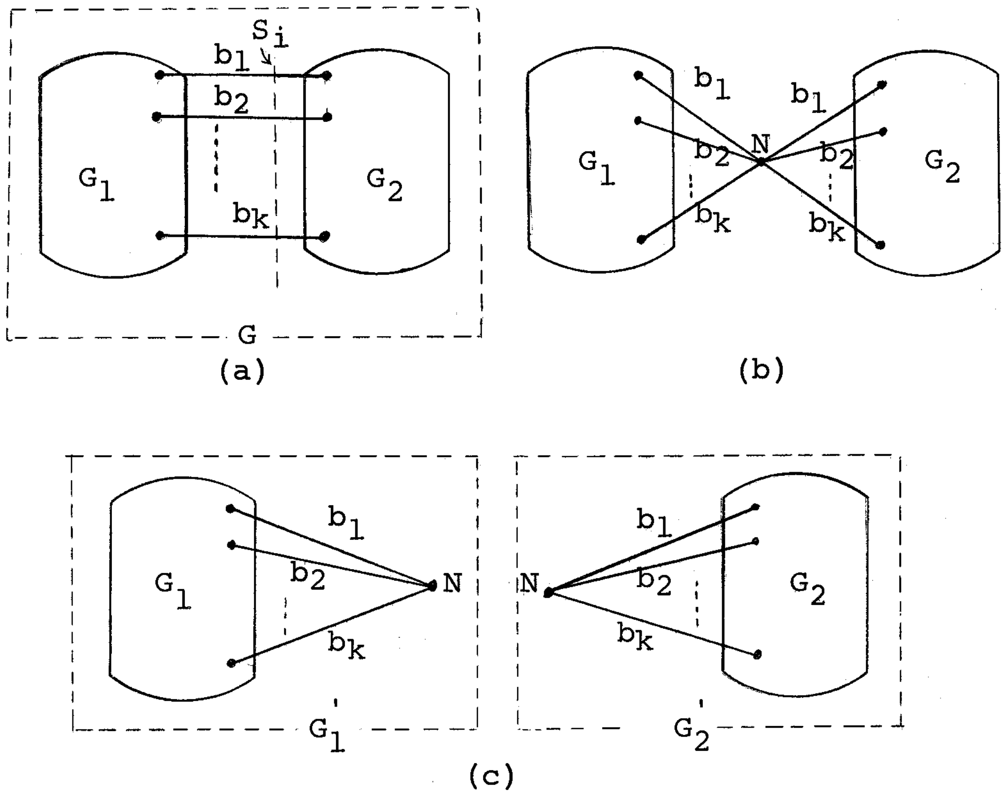

Fig. 4. Cut of a graph. (a) Given graph G; (b) producing a new node N; (c) splitting of the node.

3.2. Partitioning of a Cut-set Matrix

Supposing G_1 and G_2 in Fig. 4(a) are nonseparable when the cut-set S_i is removed from G, the above procedure on G is equivalent to the following procedure on a cut-set matrix, Q, where we suppose that Q corresponds to the graph G:

1) Remove every column which has 1 at row i, and then delete row i. The resultant matrix, H, is then partitioned, after some permutations of

rows and columns, as

$$H = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \tag{3.1}$$

2) Form new matrices, designated as $M_1(i)$ and $M_2(i)$, as follows: Construct the submatrix of Q having the rows in H_1 (or H_2) and the row S_i. This submatrix is $M_1(i)$ (or $M_2(i)$).

The row i in $M_1(i)$ and $M_2(i)$ represents an incidence set. By repeating this procedure on $M_1(i)$ and $M_2(i)$, they can be made into incidence matrices. From these incidence matrices graphs can be drawn by inspection.

Let us consider a reverse procedure. Suppose there exist two graphs G_1' and G_2' whose basic cut-set matrices are $M_1(i)$ and $M_2(i)$ which satisfy the following conditions where $M_1(i)$ and $M_2(i)$ are a pair of M-submatrices of Q with respect to row i:

1) Row i of $M_1(i)$ represents an incidence set in G_1'.

2) Row i of $M_2(i)$ represents an incidence set in G_2'.

The graph, G, whose basic cut-set matrix is Q, can be formed from G_1' and G_2' by the following procedure:

1) Join the node N in G_1' and G_2' as the graph shown in Fig. 4(b).

2) Take off the node N (Fig. 4(a)).

Following the above procedure we can combine subgraphs and get the desired graph, G. The graph, G, corresponds to the given cut-set matrix, Q. By a very simple example, Mayeda's method is illustrated.

Example 1.

Suppose the given matrix is

$$Q = \begin{array}{cc} & \begin{array}{cccccccccc} a & b & c & d & e & f & g & h & i & j \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & . & . & 1 & . & . & . & . \\ 1 & . & . & 1 & . & . & 1 & . & . & . \\ . & 1 & 1 & 1 & . & . & . & 1 & . & . \\ . & 1 & . & . & 1 & . & . & . & 1 & . \\ . & . & 1 & 1 & 1 & . & . & . & . & 1 \end{bmatrix} \end{array} \tag{3.2}$$

Removing every column which has a 1 in row 3, and then deleting row 3, we obtain

$$H = \begin{array}{cc} & \begin{array}{cccccc} a & e & f & g & i & j \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & . & 1 & . & . & . \\ 1 & . & . & 1 & . & . \\ . & 1 & . & . & 1 & . \\ . & 1 & . & . & . & 1 \end{bmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cccccc} a & f & g & e & i & j \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 5 \end{array} & \left[\begin{array}{ccc|ccc} 1 & 1 & . & . & . & . \\ 1 & . & 1 & . & . & . \\ \hline . & . & . & 1 & 1 & . \\ . & . & . & 1 & . & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \tag{3.3}$$

Forming the submatrix, $M_1(3)$, of Q which is composed of the rows of H_1 and row 3 which was earlier deleted, we obtain

$$M_1(3) = \begin{array}{c|ccccccc} & a & b & c & d & f & g & h \\ \hline 1 & 1 & 1 & 1 & . & 1 & . & . \\ 2 & 1 & . & . & 1 & . & 1 & . \\ 3 & . & 1 & 1 & 1 & . & . & 1 \end{array} \qquad (3.4)$$

Row 3 of $M_1(3)$, which represents the cut-set, must be considered as an incidence set on node 3. The cut-set matrix $M_1(3)$ is now considered as an incidence matrix, because every column of $M_1(3)$ has at most two 1's. The corresponding graph for $M_1(3)$ should have a star-tree. From $M_1(3)$, we obtain the graph as shown in Fig. 5(a), where node α represents a datum node.

Forming the submatrix, $M_2(3)$, which is made up of the rows of H_2 and row 3 which was earlier deleted, we obtain

$$M_2(3) = \begin{array}{c|ccccccc} & b & c & d & e & i & j & h \\ \hline 4 & 1 & . & . & 1 & 1 & . & . \\ 5 & . & 1 & 1 & 1 & . & 1 & . \\ 3 & 1 & 1 & 1 & . & . & . & 1 \end{array} \qquad (3.5)$$

Row 3 of $M_2(3)$, which represents the cut-set, must be considered as an incidence set on node 3. $M_2(3)$ can be considered as an incidence matrix. From $M_2(3)$, we obtain the graph as shown in Fig. 5(b), where node β represents a datum node. Combining graphs (a) and (b), by deleting node 3, we get the desired graph as shown in Fig. 5(c).

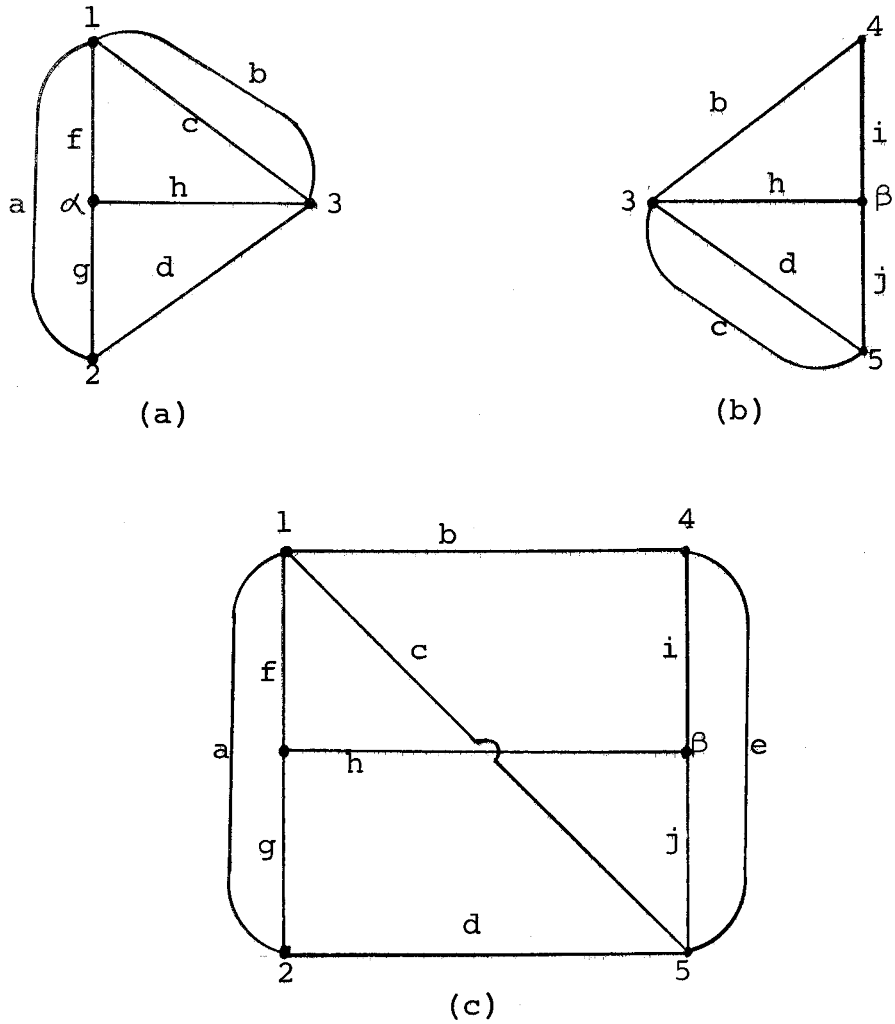

Fig. 5. Realization of the cut-set matrix of Eq. (3.2).
(a) Graph for $M_1(3)$; (b) graph for $M_2(3)$; (c) graph for Q.

CHAPTER IV

METHOD OF PARTITIONS

4.1. Separable Subgraphs

In the preceding chapter we assumed that the subgraphs G_1 and G_2 in Fig. 4 are nonseparable after removing the cut-set, S_i. By a simple graph shown in Fig. 6, it is clear this assumption loses generality. That is, if we take off the cut-set S_i, which is shown in Fig. 6(a), from the graph, one of the subgraphs, G_1, becomes a separable graph (Fig. 6(b)).

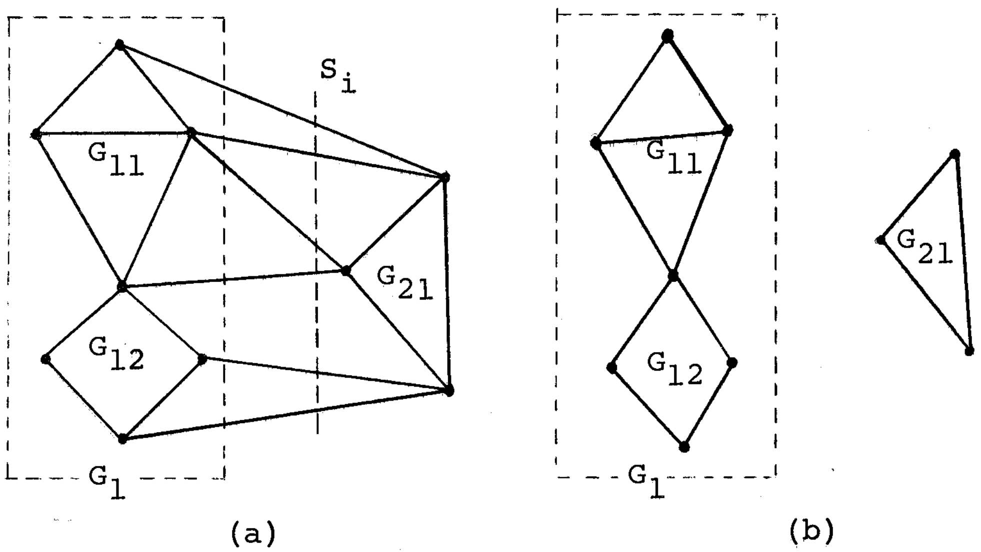

Fig. 6. Producing a separable graph.
(a) A nonseparable graph; (b) removal of a cut-set S_i.

For this case, the cut-set matrix corresponding to the graph of Fig. 6(b) will have the form

$$H = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{bmatrix} \tag{4.1}$$

Suppose H_1, H_2, and H_3 correspond to the graphs G_{11}, G_{12} and G_{21} respectively. When we partition H of (4.1) as

$$H = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{4.2}$$

where H_a consists of H_1 and H_2, and H_b equals H_3, we can synthesize the given cut-set matrix according to Mayeda's method. However, if we take other combinations, say

$$\begin{aligned} H_a &= \begin{bmatrix} H_1 & 0 \\ 0 & H_3 \end{bmatrix} \\ H_b &= \left[H_2 \right] \end{aligned} \tag{4.3}$$

we can not synthesize the given Q, because the matrices (4.3) correspond to the graphs shown in Fig. 7, and the original interconnections are changed. In the following section the method of proper partition is given.

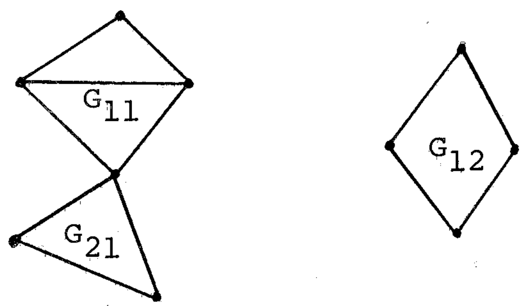

Fig. 7. Wrong interconnection of subgraphs for the graph of Fig. 6.

4.2. Sets of Parallel Branches and Tutte's Theorem

Consider the graph G shown in Fig. 8.

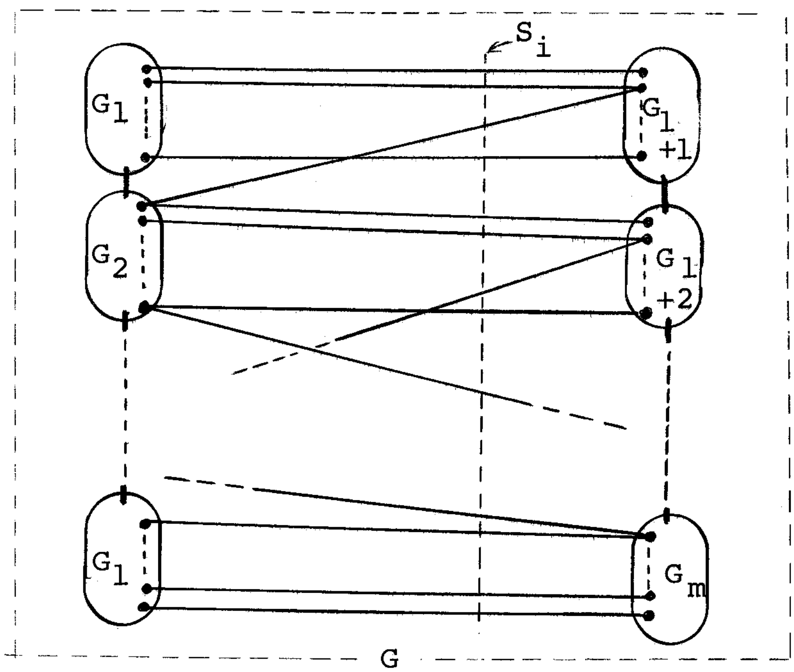

Fig. 8. A graph G which will be separable after removing a cut-set S_i from G.

By removing S_i, we get subgraphs G_1, G_2, ...G_m. The corresponding cut-set matrix, H, will have the following form after some permutations of rows and columns.

$$H = \begin{bmatrix} H_1 & 0 & 0 & \cdots & 0 \\ 0 & H_2 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & 0 & H_1 & 0 \\ 0 & \cdots & 0 & 0 & H_m \end{bmatrix} \tag{4.4}$$

By using Tutte's theorem, we can partition H such that the given matrix is realizable. In order to use the theorem, let us obtain the sets of parallel branches contained in S_i from H_j's, where j=1,2,...m, as follows:

1) Form submatrices, $M_j(i)$, from Q, where $M_j(i)$ is the same as defined in Chapter III.

2) From this submatrix $M_j(i)$, find columns which contain 1's at row S_i. Let the matrix which is made up of these columns be C. For example, let

$$M_j(i) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \!\!\begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & . & 1 & . & . \\ 1 & . & . & 1 & . & 1 & . \\ . & 1 & 1 & 1 & . & . & 1 \end{bmatrix} \end{array} \tag{4.5}$$

and let S_i = row 3.

Then the columns which contain 1's at row 3 are: b, c, d and g.

Therefore we obtain

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} b & c & d & g \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & . & . \\ . & . & 1 & . \\ 1 & 1 & 1 & 1 \end{array}\right] \end{array} \tag{4.6}$$

3) In C, find columns which are the same. For example, in C of (4.6), the columns which have the same arrangements are:

$$\{b,c\},\ \{d\},\ \{g\}$$

4) These sets of similar columns represent the sets of parallel branches contained in S_i.

Let the sets of parallel branches obtained from $M_j(i)$ and $M_h(i)$ be

$$\begin{aligned} P_j &= \{S_{j1}, S_{j2}, \ldots S_{jk}\} \\ P_h &= \{S_{h1}, S_{h2}, \ldots S_{hq}\} \end{aligned} \tag{4.7}$$

Now Tutte's theorem can be stated as follows:

Theorem 1. If we can form two sets, H_a and H_b, such that

1) $H_a \cap H_b = \Phi$ and $H_a \cup H_b = \{H_1, H_2, \ldots H_m\}$, and

2) $S_{jr} \cup S_{hp} = S_i$, for every j and h in H_a and H_b, respectively, where S_{jr} and S_{hp} are subsets of P_j and P_h respectively,

then both H_a and H_b are realizable.

If the graph G corresponding to Q contains two-terminal subgraphs, then there exist several ways to get H_a and H_b. However, these will produce trivial equivalent graphs.

CHAPTER V

REALIZATION ALGORITHM

5.1. Removal of Basic Cut-set with Respect to the Non-tip Branch

Consider the graph G in Fig. 9. If we remove the basic cut-set, T_i, from G, one of the subgraphs consists of only a node, and we can not approach a star-tree. We can avoid this by the following considerations.

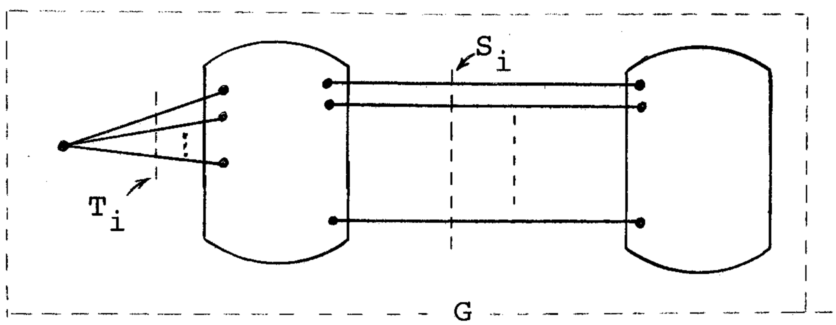

Fig. 9. A basic cut-set T_i with respect to tip-branch.

Take a column which has at least three 1's in the given basic cut-set matrix, Q. Then, from the definition of the basic cut-set matrix, there are three tree branches associated with the column. From the property of a tree, all the tree branches must be connected. Hence at least one of the three branches must be a non-tip branch. From the above discussion, we have the following property:

Property 1. A basic cut-set with respect to non-tip tree branch is found in rows which have 1's at a column having at least three 1's. By the above property, we can make our method effective. That is, when we delete a row we should select the row as follows:

1) Find a column, say the column k, which has at least three 1's. If there is no such column, then the given Matrix, Q, is an incidence matrix.
2) Delete a row which has a 1 at the column k.
3) If the resultant matrix, H, is not separable, try the other row which has 1 at the column k.
4) If H is still not separable after every row which has 1 at k is deleted, then the given matrix, Q, is not realizable, because property 1 is violated.

5.2. Manipulations on Incidence Matrices

When we get a large number of subgraphs in Mayeda's method, synthesis of the graphs is tedious. We can do it by manipulations on matrices as follows:

1) Add a row to each incidence matrix, A_j, such that the resultant matrix has exactly two 1's at every column. This row corresponds to a datum node.

2) Combine these matrices by deleting the same rows. This manipulation corresponds to deleting the same node when combining subgraphs.

5.3. Algorithm

From the foregoing discussions, the algorithm for the realization of basic cut-set matrices is stated as follows:

Step 1. Find a column, k, which has at least three 1's in the given cut-set matrix, Q.

Step 2. Let the first row which has a 1 at column k be I. Delete every column which has a 1 at row I, then remove row I.

Step 3. Is the resultant matrix, H, separable? If it is not separable, find the other row which has a 1 at column k, then go to step 2.

Step 4. If H is partitioned as the matrix (3.1), go to step 6. If H is partitioned as the matrix (4.4), obtain the set of parallel branches from each submatrix H_j, $j=1,2,...m$.

Step 5. Partition H into two submatrices, H_a and H_b, by using theorem 1.

Step 6. Obtain $M_a(i)$ and $M_b(i)$ by the following procedure: Construct the submatrix $M_a(i)$ (or $M_b(i)$) of Q having the row in H_a (or H_b) and the row I which was deleted in step 2.

Step 7. Is $M_j(i)$, $j=a,b$, an incidence matrix? If it is not an incidence matrix, go to step 1, and let $Q=M_j(i)$.

Step 8. If $M_j(i)$ is an incidence matrix, then add a row such that every column has exactly two 1's.

Step 9. Combine these matrices by deleting the same rows.

Step 10. Draw the graph from the resultant incidence matrix.

The above algorithm is illustrated by the following example.

Example 2.

Suppose the given cut-set matrix is

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} a & b & c & d & e & f & g & h & i & j & k & l & m & n \end{array} \\ \left[\begin{array}{cccccccccccccc} \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ 1 & \cdot & \cdot & 1 & 1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{array}\right] \end{array} \qquad (5.1)$$

Step 1. Column a has three 1's.

Step 2. Delete every column which has a 1 at row 2, and then remove row 2. The resultant matrix, H, is found to be

$$H = \begin{array}{c|c} & \begin{array}{ccccccc} g & h & i & k & l & m & n \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} . & . & 1 & . & . & . & . \\ . & 1 & . & 1 & . & . & . \\ . & 1 & . & . & 1 & . & . \\ 1 & . & . & . & . & 1 & . \\ 1 & . & . & . & . & . & 1 \end{bmatrix} \end{array} \tag{5.2}$$

Step 3. After some permutations of columns, H is partitioned as

$$H = \begin{array}{c|c} & \begin{array}{ccccccc} i & h & k & l & g & m & n \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \left[\begin{array}{c|ccc|ccc} 1 & . & . & . & . & . & . \\ \hline . & 1 & 1 & . & . & . & . \\ . & 1 & . & 1 & . & . & . \\ \hline . & . & . & . & 1 & 1 & . \\ . & . & . & . & 1 & . & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{bmatrix} \tag{5.3}$$

Step 4. In order to partition H into two parts, we obtain the sets of parallel branches from H_1, H_2, and H_3 as follows.

First we construct the submatrix $M_1(2)$ of Q having the rows in H_1 and row 2 which was removed in step 2.

$$M_1(2) = \begin{array}{c|c} & \begin{array}{cccccccc} a & b & c & d & e & f & i & j \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} . & 1 & 1 & . & . & . & 1 & . \\ 1 & 1 & 1 & 1 & 1 & 1 & . & 1 \end{bmatrix} \end{array} \tag{5.4}$$

From $M_1(2)$, we get the parallel branches (the same columns containing a 1 at row 2) as

$$S_1 = \{\{a,d,e,f,j\}, \{b,c\}\} \tag{5.5}$$

In the same way, from the rows in H_2 and row 2, we get

$$M_2(2) = \begin{array}{c} \\ 3 \\ 4 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & h & j & k & l \end{array} \\ \begin{bmatrix} 1 & 1 & . & . & . & . & 1 & . & 1 & . \\ . & . & . & . & 1 & 1 & 1 & . & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . & 1 & . & . \end{bmatrix} \end{array} \tag{5.6}$$

From $M_2(2)$, the set of parallel branches is

$$S_2 = \{\{a,b\}, \{c,d,j\}, \{e,f\}\} \tag{5.7}$$

Again, in the same way, we get $M_3(2)$ as

$$M_3(2) = \begin{array}{c} \\ 5 \\ 6 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & g & j & m & n \end{array} \\ \begin{bmatrix} 1 & . & . & 1 & 1 & 1 & 1 & . & 1 & . \\ 1 & . & . & . & . & 1 & 1 & . & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . & 1 & . & . \end{bmatrix} \end{array} \tag{5.8}$$

From $M_3(2)$, the set of parallel branches is

$$S_3 = \{\{a,f\}, \{b,c,j\}, \{d,e\}\} \tag{5.9}$$

Step 5. From S_1, S_2 and S_3 obtained in step 4, we find

$$S_{11} \cup S_{32} = S_i \tag{5.10}$$

where

$$S_{11} = \{a,d,e,f,j\},\ S_{32} = \{b,c,j\}$$

$$S_i = \{a,b,c,d,e,f,j\} \tag{5.11}$$

Therefore H is partitioned as

$$H = \left[\begin{array}{c:c} \begin{bmatrix} H_1 & 0 \\ 0 & H_3 \end{bmatrix} & 0 \\ \hdashline 0 & H_2 \end{array}\right] = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{5.12}$$

Step 6. We construct the submatrix, $M_a(2)$, of Q having the rows in H_a and row 2 which was deleted in step 2.

$$M_a(2) = \begin{array}{c} \\ 1 \\ 5 \\ 6 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} a & b & c & d & e & f & g & i & j & m & n \end{array} \\ \begin{bmatrix} . & 1 & 1 & . & . & . & . & 1 & . & . & . \\ 1 & . & . & 1 & 1 & 1 & 1 & . & . & 1 & . \\ 1 & . & . & . & . & 1 & 1 & . & . & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . & . & 1 & . & . \end{bmatrix} \end{array} \tag{5.13}$$

Similarly, $M_b(2)$ is

$$M_b(2) = \begin{array}{c} \\ 3 \\ 4 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & h & j & k & l \end{array} \\ \begin{bmatrix} 1 & 1 & . & . & . & . & 1 & . & 1 & . \\ . & . & . & . & 1 & 1 & 1 & . & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . & 1 & . & . \end{bmatrix} \end{array} \tag{5.14}$$

Step 7. $M_b(2)$ has the property of an incidence matrix, since every column has at most two 1's. $M_a(2)$ is not an incidence matrix. Therefore we apply the step 1 - step 7 to $M_a(2)$.

Step 1. The column a in $M_a(2)$ has three 1's.

Step 2. Delete every column which has a 1 at row 5, and then remove row 5; the resultant matrix, H, is

$$H = \begin{array}{c} \\ 1 \\ 6 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccccc} b & c & i & j & n \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & . & . \\ . & . & . & . & 1 \\ 1 & 1 & . & 1 & . \end{bmatrix} \end{array} \tag{5.15}$$

Steps 3, 4. H is partitioned as

$$H = \begin{array}{c} \\ 1 \\ 2 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} b & c & i & j & n \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 1 & . & . \\ 1 & 1 & . & 1 & . \\ \hline . & . & . & . & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{5.16}$$

Since H is partitioned into two parts, we can go to step 6.

Step 6. $M_a(5)$ and $M_b(5)$ are obtained as

$$M_a(5) = \begin{array}{c} \\ 1 \\ 2 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & g & i & j & m \end{array} \\ \begin{bmatrix} . & 1 & 1 & . & . & . & . & 1 & . & . \\ 1 & 1 & 1 & 1 & 1 & 1 & . & . & 1 & . \\ 1 & . & . & 1 & 1 & 1 & 1 & . & . & 1 \end{bmatrix} \end{array} \tag{5.17}$$

$$M_b(5) = \begin{array}{c} \\ 6 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & d & e & f & g & m & n \end{array} \\ \begin{bmatrix} 1 & . & . & 1 & 1 & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . \end{bmatrix} \end{array} \tag{5.18}$$

Step 7. $M_a(5)$ and $M_b(5)$ are incidence matrices.

Step 8. We now construct the non-reduced incidence matrices. By adding a row to $M_b(2)$, such that every column has exactly two 1's, we get A_1

$$A_1 = \begin{array}{cc} & \begin{array}{cccccccccc} a & b & c & d & e & f & h & j & k & l \end{array} \\ \begin{array}{c} 3 \\ 4 \\ 2 \\ \alpha \end{array} & \begin{bmatrix} 1 & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & 1 & 1 & \cdot & \cdot & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{5.19}$$

where α corresponds to a datum node.

In the same way, from $M_a(5)$,

$$A_2 = \begin{array}{cc} & \begin{array}{cccccccccc} a & b & c & d & e & f & g & i & j & m \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 5 \\ \beta \end{array} & \begin{bmatrix} \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdot & \cdot & 1 & \cdot \\ 1 & \cdot & \cdot & 1 & 1 & 1 & 1 & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{5.20}$$

where β corresponds to a datum node.

From $M_b(5)$, we get

$$\begin{array}{cc} & \begin{array}{ccccccc} a & b & e & f & g & m & n \end{array} \\ \begin{array}{c} 6 \\ 5 \\ \gamma \end{array} & \begin{bmatrix} 1 & \cdot & \cdot & 1 & 1 & \cdot & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdot \\ \cdot & 1 & 1 & \cdot & \cdot & 1 & 1 \end{bmatrix} \end{array} \tag{5.21}$$

where γ corresponds to a datum node.

Step 9. Combining A_1, A_2 and A_3 by deleting the same rows, we get the desired incidence matrix as

$$
A = \begin{array}{c} \\ 3 \\ 4 \\ \alpha \\ 1 \\ \beta \\ 6 \\ \gamma \end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc} a & b & c & d & e & f & g & h & i & j & k & l & m & n \end{array} \\
\left[\begin{array}{cccccccccccccc}
1 & 1 & . & . & . & . & . & 1 & . & . & 1 & . & . & . \\
. & . & . & . & 1 & 1 & . & 1 & . & . & . & 1 & . & . \\
. & . & 1 & 1 & . & . & . & . & . & 1 & 1 & 1 & . & . \\
. & 1 & 1 & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & 1 & 1 & . & . & 1 & . \\
1 & . & . & . & . & 1 & 1 & . & . & . & . & . & . & 1 \\
. & . & . & 1 & 1 & . & . & . & . & . & . & . & 1 & 1
\end{array}\right]
\end{array}
\qquad (5.22)
$$

From A, the desired graph is obtained as shown in Fig. 10.

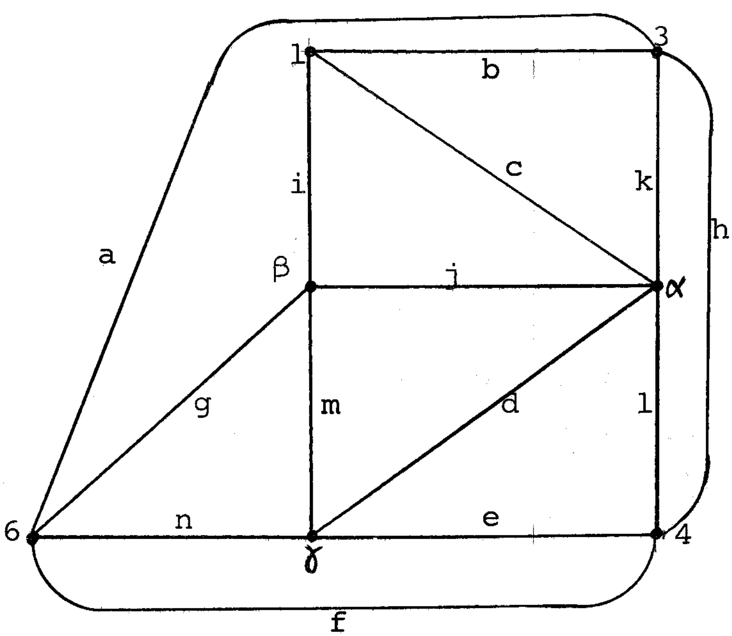

Fig. 10. The graph corresponding to the cut-set matrix of Eq. (5.1).

CHAPTER VI

SYNTHESIS OF BILATERAL NETWORKS

6.1. Realization of Admittance Matrices

By the following example, realization of an admittance matrix of order n as an RLC n-port with n+1 nodes is illustrated.

Example 3.

Let us consider the realization of the following Y-matrix with an RLC network with five nodes.

$$Y = \begin{bmatrix} 2+3s+\frac{1}{s} & 3s+\frac{1}{s} & 3s & 0 \\ 3s+\frac{1}{s} & 5+3s+\frac{2}{s} & 3s & \frac{1}{s} \\ 3s & 3s & 1+7s & -4s \\ 0 & \frac{1}{s} & -4s & 3+4s+\frac{1}{s} \end{bmatrix} \tag{6.1}$$

For real and positive values of the complex variable, s, an RLC n-port behaves as a resistive n-port. We therefore arbitrarily assign a real, positive value to the variable s. If we let s=1, the matrix becomes

$$\overline{Y} = Y\Big|_{s=1} = \begin{bmatrix} 6 & 4 & 3 & 0 \\ 4 & 10 & 3 & 1 \\ 3 & 3 & 8 & -4 \\ 0 & 1 & -4 & 8 \end{bmatrix} \tag{6.2}$$

where the matrix with s set equal to 1 is denoted by $\overline{Y}$.

By using Cederbaum's algorithm, we decompose $\overline{Y}$ into the triple product:

$$\overline{Y} = C_s D\, C_s^t \tag{6.3}$$

where C_s is a seg matrix and D is a diagonal matrix with positive elements representing edge conductances. Let $C_i]$ be the i-th column matrix of C_s, d_i be the i-th diagonal element in D.

Now, we choose in $\overline{Y}$ the element with the smallest absolute value, say the element of (2,4)-position which has the value of 1. Thus the first diagonal element, d_1, is determined as $d_1=1$. Next we determine the first column $C_1]$ of C_s by the following procedure:

1) If the smallest absolute value element, say k_{mn}, is at (m,n) position, then the m-th row and n-th row of $C_i]$ should have nonzero elements, as

$$C_i] = \begin{array}{c} 1 \\ \vdots \\ m \\ \vdots \\ n \\ \vdots \end{array} \left[\begin{array}{c} \vdots \\ \vdots \\ 1 \\ \vdots \\ \pm 1 \\ \vdots \end{array} \right] \tag{6.4}$$

where the element at n-th row should have the same sign as the sign of k_{mn}.

2) Other elements of $C_i]$ can be obtained by the following rule:

If

$$k_{mn}k_{mj}k_{nj} > 0 \qquad \text{for } j \neq m,n$$

then the element of the j-th row of the column is nonzero and the sign of this element is the same as the sign of k_{mj}, and if

$$k_{mn}k_{mj}k_{nj} \leq 0 \qquad \text{for } j \neq m,n$$

then the element on the j-th row of $C_i]$ is zero.

By the above procedure we can obtain $C_1]$ as

$$C_1] = \begin{bmatrix} \cdot \\ 1 \\ \cdot \\ 1 \end{bmatrix} \tag{6.5}$$

We define

$$G_i = C_i]\,[d_i]\,C_i]^t \tag{6.6}$$

$$\overline{Y}_i = \overline{Y}_{i-1} - G_i \tag{6.7}$$

where $\overline{Y}_{i-1} = \overline{Y}$ if $i=1$ (6.8)

By the definitions, we obtain

$$G_1 = \begin{bmatrix} \cdot \\ 1 \\ \cdot \\ 1 \end{bmatrix} [1] [\cdot \quad 1 \quad \cdot \quad 1] = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 \end{bmatrix} \tag{6.9}$$

$$\overline{Y}_1 = \begin{bmatrix} 6 & 4 & -3 & 0 \\ 4 & 10 & -3 & 1 \\ -3 & -3 & 8 & -4 \\ 0 & 1 & -4 & 8 \end{bmatrix} - \begin{bmatrix} . & . & . & . \\ . & 1 & . & 1 \\ . & . & . & . \\ . & 1 & . & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 & -3 & 0 \\ 4 & 9 & -3 & 0 \\ -3 & -3 & 8 & -4 \\ 0 & 0 & -4 & 7 \end{bmatrix} \qquad (6.10)$$

From $\overline{Y}_1$, we choose the second diagonal element as $d_2=3$, and the second column, $c_2]$, of C_s is found as

$$c_2] = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \end{bmatrix} \qquad (6.11)$$

G_2 and $\overline{Y}_2$ are found as

$$G_2 = c_2] \left[d_2 \right] c_2]^t = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \end{bmatrix} \left[3 \right] \left[1 \quad 1 \quad 1 \quad . \right]$$

$$= \begin{bmatrix} 3 & 3 & 3 & 0 \\ 3 & 3 & 3 & 0 \\ 3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (6.12)$$

$$Y_2 = Y_1 - G_2 = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 1 & 6 & 0 & 0 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & -4 & 7 \end{bmatrix} \qquad (6.13)$$

In the same way, we can determine d_i and $C_i]$ successively. These are tabulated together with Y_i and G_i in Table I.

TABLE I. Decomposition of $\bar{Y}$

$\bar{Y}$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$
6 4 3 0	6 4 3 0	3 1 0 0	2 0 0 0
4 10 3 1	4 9 3 0	1 6 0 0	0 5 0 0
3 3 8 −4	4 3 8 −4	0 0 5 −4	0 0 5 −4
0 1 −4 8	0 0 −4 7	0 0 −4 7	0 0 −4 7
G_1	G_2	G_3	G_4
0 0 0 0	3 3 3 0	1 1 0 0	0 0 0 0
0 1 0 1	3 3 3 0	1 1 0 0	0 0 0 0
0 0 0 0	3 3 3 0	0 0 0 0	0 0 4 −4
0 1 0 1	0 0 0 0	0 0 0 0	0 0 −4 4
$C_1] = \begin{bmatrix} \cdot \\ 1 \\ \cdot \\ 1 \end{bmatrix}$	$C_2] = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \end{bmatrix}$	$C_3] = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \end{bmatrix}$	$C_4] = \begin{bmatrix} \cdot \\ \cdot \\ 1 \\ -1 \end{bmatrix}$
$d_1 = 1$	$d_2 = 3$	$d_3 = 1$	$d_4 = 4$

(continued)

TABLE I (continued)

$\overline{Y}_4$	$\overline{Y}_5$	$\overline{Y}_6$	$\overline{Y}_7$
$\begin{matrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{matrix}$
G_5	G_6	G_7	G_8
$\begin{matrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{matrix}$
$c_5] = \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$	$c_6] = \begin{bmatrix} \cdot \\ 1 \\ \cdot \\ \cdot \end{bmatrix}$	$c_7] = \begin{bmatrix} \cdot \\ \cdot \\ 1 \\ \cdot \end{bmatrix}$	$c_8] = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$
$d_5 = 2$	$d_6 = 5$	$d_7 = 1$	$d_8 = 3$

Now Y is decomposed as

$$\overline{Y} = \begin{bmatrix} . & 1 & 1 & . & 1 & . & . & . \\ 1 & 1 & 1 & . & . & 1 & . & . \\ . & 1 & . & 1 & . & . & 1 & . \\ 1 & . & . & -1 & . & . & . & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & \cdots & & & & 0 \\ 0 & 3 & 0 & \cdots & & & & 0 \\ 0 & 0 & 1 & & & & & \vdots \\ \vdots & \vdots & & 4 & & & & \\ & & & & 2 & & & \\ & & & & & 5 & & \\ & & & & & & 1 & 0 \\ 0 & 0 & \cdots & & & & 0 & 3 \end{bmatrix}$$

$$\times \begin{bmatrix} . & 1 & . & 1 \\ 1 & 1 & 1 & . \\ 1 & 1 & . & . \\ . & . & 1 & -1 \\ 1 & . & . & . \\ . & 1 & . & . \\ . & . & 1 & . \\ . & . & . & 1 \end{bmatrix} \qquad (6.14)$$

To identify the element values, let

$$Y = \begin{bmatrix} 2 + 3s + \frac{1}{s} & 3s + \frac{1}{s} & 3s & 0 \\ 3s + \frac{1}{s} & 5 + 3s + \frac{2}{s} & 3s & \frac{1}{s} \\ 3s & 3s & 1 + 7s & -4s \\ 0 & \frac{1}{s} & -4s & 3 + 4s + \frac{1}{s} \end{bmatrix}$$

$$= \begin{array}{c} \begin{matrix} d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 \end{matrix} \\ \begin{bmatrix} \cdot & 1 & 1 & \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & \cdot & \cdot & 1 & \cdot \\ 1 & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} d_1 & 0 & 0 & \cdots & & & & 0 \\ 0 & d_2 & 0 & \cdots & & & & 0 \\ 0 & 0 & d_3 & & & & & \vdots \\ \vdots & \vdots & & d_4 & & & & \vdots \\ \vdots & \vdots & & & d_5 & & & \vdots \\ \vdots & \vdots & & & & d_6 & & \vdots \\ \vdots & \vdots & & & & & d_7 & 0 \\ 0 & 0 & \cdots & & & & & d_8 \end{bmatrix}$$

$$\times \begin{bmatrix} \cdot & 1 & \cdot & 1 \\ 1 & 1 & 1 & \cdot \\ 1 & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & -1 \\ 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} = \begin{bmatrix} d_2+d_3+d_5 & d_2+d_3 & d_2 & 0 \\ d_2+d_3 & d_1+d_2+d_3+d_6 & d_2 & d_1 \\ d_2 & d_2 & d_2+d_4+d_7 & -d_4 \\ 0 & d_1 & -d_4 & d_1+d_4+d_8 \end{bmatrix} \qquad (6.15)$$

From the above equation we get the element values as

$$d_1 = \frac{1}{s},\ d_2 = 3s,\ d_3 = \frac{1}{s},\ d_4 = 4s,\ d_5 = 2,\ d_6 = 5$$
$$d_7 = 1,\ d_8 = 3 \tag{6.16}$$

The next step is to realize

$$P = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccccc} d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 \end{array} \\ \begin{bmatrix} . & 1 & 1 & . & 1 & . & . & . \\ 1 & 1 & 1 & . & . & 1 & . & . \\ . & 1 & . & 1 & . & . & 1 & . \\ 1 & . & . & -1 & . & . & . & 1 \end{bmatrix} \end{array} \tag{6.17}$$

This is a basic cut-set matrix of an oriented graph, where the letters a, b, c and d are assigned for cut-sets (in this case these also represent ports).

Although we have developed the method of realization of non-oriented cut-set matrix, the method is also applicable to the oriented cut-set matrix as follows: Ignoring the negative signs in P, we can use our algorithm and get the non-oriented graph as shown in Fig. 11(a). Recalling definition 5, we can orient the graph as shown in Fig. 11(b).

Next we must describe the ports. From P, we know that branches d_5, d_6, d_7 and d_8 correspond to tree branches. In the cut-set schedule, the tree branches are considered as independent voltages (5, p. 13-17), i.e., port voltages correspond to tree branch voltages.

By these considerations we get the final network as shown in Fig. 11(c).

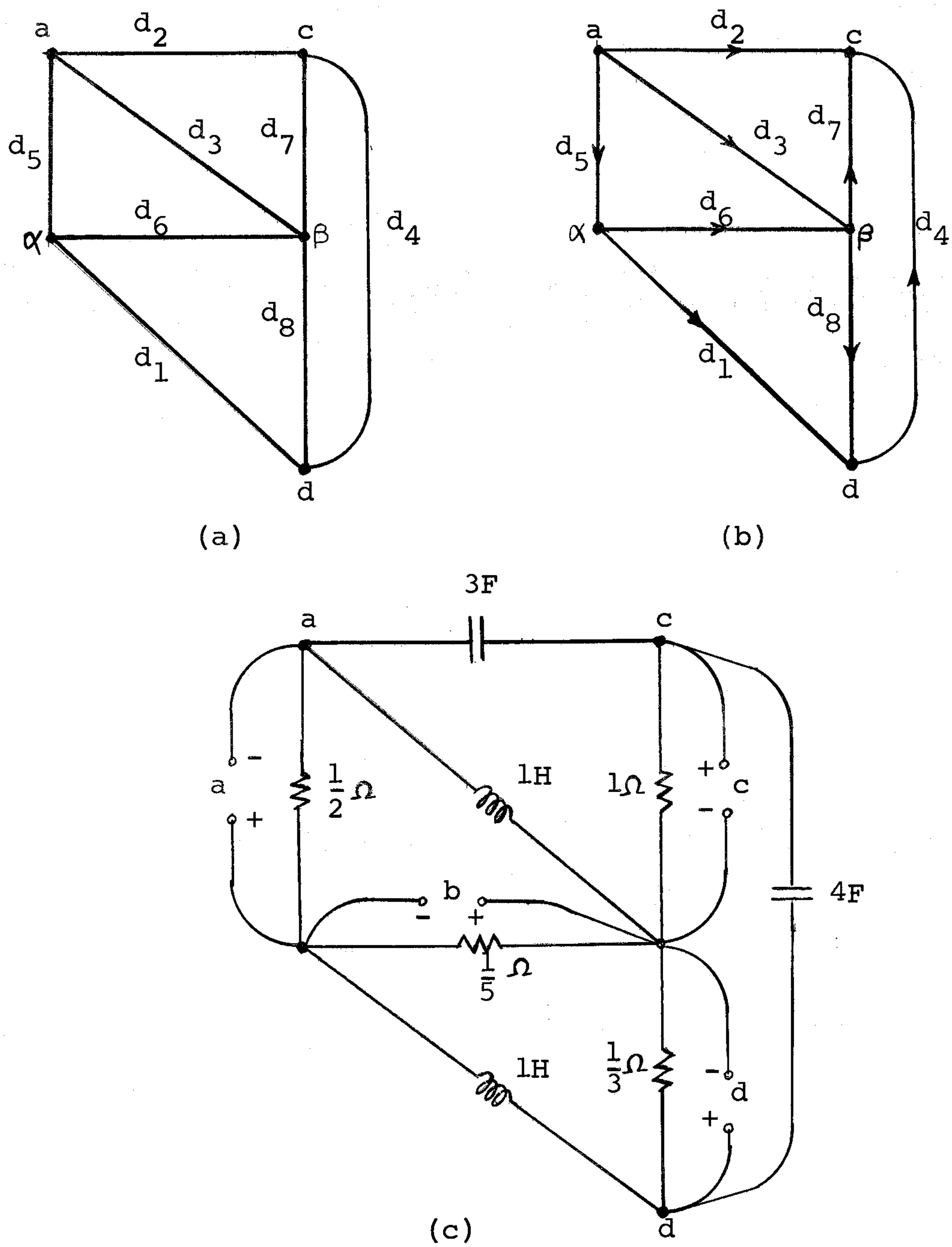

Fig. 11. Realization of the Y-matrix of Eq. (6.1).
(a) Non-oriented graph; (b) oriented graph;
(c) final graph.

6.2. Realization of Non-basic Cut-set Matrices

So far, we have considered the realization of basic cut-set matrices. In practical applications, however, we must often realize non-basic cut-set matrices. The method is illustrated by the following example.

Example 4.

Consider the realization of the following 5-port Y-matrix with six nodes.

$$Y = \begin{bmatrix} 21 & -21 & -6 & -10 & -10 \\ -21 & 35 & 7 & 21 & 10 \\ -6 & 7 & 25 & -15 & -7 \\ -10 & 21 & -15 & 49 & 26 \\ -10 & 10 & -7 & 26 & 26 \end{bmatrix} \tag{6.18}$$

Again by using Cederbaum's algorithm, Y is decomposed as

$$Y = C_s \; D \; C_s^t \tag{6.19}$$

where

$$C_s = \begin{array}{c} \begin{matrix} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1 & . & . & . & 1 & 1 & . & . & . & . & . \\ -1 & 1 & . & . & -1 & -1 & 1 & . & 1 & . & . \\ -1 & 1 & 1 & 1 & . & . & . & . & . & 1 & . \\ . & . & -1 & -1 & -1 & . & 1 & 1 & . & . & 1 \\ . & . & -1 & . & -1 & . & . & 1 & . & . & . \end{bmatrix} \end{array} \tag{6.20}$$

$$D = \begin{bmatrix}
6 & 0 & 0 & \cdots & & & & & & & 0 \\
0 & 1 & 0 & \cdots & & & & & & & 0 \\
0 & 0 & 7 & & & & & & & & \vdots \\
\vdots & \vdots & & 8 & & & & & & & \\
 & & & & 10 & & & & & & \\
 & & & & & 5 & & & & & \\
 & & & & & & 11 & & & & \\
 & & & & & & & 9 & & & \\
 & & & & & & & & 2 & & \vdots \\
\vdots & \vdots & & & & & & & & 3 & 0 \\
0 & 0 & \cdots & & & & & & & 0 & 4
\end{bmatrix} \tag{6.21}$$

where the numbers on C_s represent the corresponding element values.

Since C_s does not have the form:

$$Q = \left[\; Q_{11} \;\; U \;\right] \tag{6.22}$$

where U is a unit matrix, C_s is not a basic cut-set matrix (11, p. 75).

Let us augment C_s such that C_s has unit matrix of order 5, and call this Q_1.

$$Q_1 = \begin{array}{c} \begin{array}{ccccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & d_1 & 2 & 3 & 4 & d_2 \end{array} \\ \left[\begin{array}{ccccccccccccc} 1 & . & . & . & 1 & 1 & . & . & 1 & . & . & . & . \\ -1 & 1 & . & . & -1 & -1 & 1 & . & . & 1 & . & . & . \\ -1 & 1 & 1 & 1 & . & . & . & . & . & . & 1 & . & . \\ . & . & -1 & -1 & -1 & . & 1 & 1 & . & . & . & 1 & . \\ . & . & -1 & . & -1 & . & . & 1 & . & . & . & . & 1 \end{array}\right] \end{array} \tag{6.23}$$

where d_1 and d_2 represent the element values of augmented edges.

Discarding the negative signs, we get the following non-oriented cut-set matrix, Q.

$$Q = \begin{array}{c|c} & \begin{array}{ccccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & d & 2 & 3 & 4 & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \left[\begin{array}{ccccccccccccc} 1 & . & . & . & 1 & 1 & . & . & 1 & . & . & . & . \\ 1 & 1 & . & . & 1 & 1 & 1 & . & . & 1 & . & . & . \\ 1 & 1 & 1 & 1 & . & . & . & . & . & . & 1 & . & . \\ . & . & 1 & 1 & 1 & . & 1 & 1 & . & . & . & 1 & . \\ . & . & 1 & . & 1 & . & . & 1 & . & . & . & . & 1 \end{array}\right] \end{array} \tag{6.24}$$

We realize Q by using our method.

Step 1. The first column (numbered 6) has three 1's.

Step 2. Delete every column which has 1 at the row b, and then remove the row b. The resultant matrix H is

$$H = \begin{array}{c} \\ a \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccccc} 7 & 8 & 9 & d_1 & 3 & 4 & d_2 \end{array} \\ \left[\begin{array}{ccccccc} . & . & . & 1 & . & . & . \\ 1 & 1 & . & . & 1 & . & . \\ 1 & 1 & 1 & . & . & 1 & . \\ 1 & . & . & . & . & . & 1 \end{array}\right] \end{array} \qquad (6.25)$$

Steps 3,4. H is partitioned as

$$H = \begin{array}{c} \\ a \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{c|cccccc} d_1 & 7 & 8 & 9 & 3 & 4 & d_2 \end{array} \\ \left[\begin{array}{c|cccccc} 1 & . & . & . & . & . & . \\ \hline . & 1 & 1 & . & 1 & . & . \\ . & 1 & 1 & 1 & . & 1 & . \\ . & 1 & . & . & . & . & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \qquad (6.26)$$

We can go to step 6.

Step 6.

$$M_a(b) = \begin{array}{c} \\ a \\ b \end{array}\begin{array}{c} \begin{array}{ccccccc} 6 & 1 & 10 & 5 & 11 & d_1 & 2 \end{array} \\ \left[\begin{array}{ccccccc} 1 & . & 1 & 1 & . & 1 & . \\ 1 & 1 & 1 & 1 & 1 & . & 1 \end{array}\right] \end{array} \qquad (6.27)$$

$$M_b(b) = \begin{array}{c} \\ c \\ d \\ e \\ b \end{array}\begin{array}{c} \begin{array}{cccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & d_1 & 2 & 3 & 4 & d_2 \end{array} \\ \left[\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & . & . & . & . & . & 1 & . & . \\ . & . & 1 & 1 & 1 & . & 1 & 1 & . & . & 1 & . \\ . & . & 1 & . & 1 & . & . & 1 & . & . & . & 1 \\ 1 & 1 & . & . & 1 & 1 & 1 & . & 1 & . & . & . \end{array}\right] \end{array} \qquad (6.28)$$

Step 7. $M_a(b)$ is an incidence matrix, but $M_b(b)$ is not an incidence matrix. Therefore we apply step 1-step 7 to $M_b(b)$.

Step 1. In $M_b(b)$, the column numbered 7 has three 1's.

Step 2. Delete every column which has 1 at the row d, then remove row d.

$$H = \begin{array}{c} \\ c \\ e \\ b \end{array} \begin{array}{c} \begin{array}{cccccc} 6 & 1 & 5 & 2 & 3 & d_2 \end{array} \\ \begin{bmatrix} 1 & 1 & . & . & 1 & . \\ . & . & . & . & . & 1 \\ 1 & 1 & 1 & 1 & . & . \end{bmatrix} \end{array} \tag{6.29}$$

Steps 3,4. H is partitioned as

$$H = \begin{array}{c} \\ e \\ c \\ b \end{array} \begin{array}{c} \begin{array}{cccccc} d_2 & 6 & 1 & 5 & 2 & 3 \end{array} \\ \left[\begin{array}{c|ccccc} 1 & . & . & . & . & . \\ \hline . & 1 & 1 & . & . & 1 \\ . & 1 & 1 & 1 & 1 & . \end{array}\right] \end{array} = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{6.30}$$

We can skip step 5.

Step 6.

$$M_a(d) = \begin{array}{c} \\ e \\ d \end{array} \begin{array}{c} \begin{array}{ccccccc} 7 & 8 & 10 & 11 & 9 & 4 & d_2 \end{array} \\ \begin{bmatrix} 1 & . & 1 & . & 1 & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . \end{bmatrix} \end{array} \tag{6.31}$$

$$M_b(d) = \begin{array}{c} \\ c \\ b \\ d \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & . & . & . & . & . & 1 & . \\ 1 & 1 & . & . & 1 & 1 & 1 & . & 1 & . & . \\ . & . & 1 & 1 & 1 & . & 1 & 1 & . & . & 1 \end{bmatrix} \end{array} \tag{6.32}$$

Step 7. $M_a(d)$ and $M_b(d)$ are incidence matrices.

Step 8. From $M_a(b)$, $M_a(d)$ and $M_b(d)$, we get A_1, A_2, and A_3, respectively.

$$A_1 = \begin{array}{c} \\ a \\ b \\ \alpha \end{array} \begin{array}{c} \begin{array}{ccccccc} 6 & 1 & 10 & 5 & 11 & d_1 & 2 \end{array} \\ \left[\begin{array}{ccccccc} 1 & . & 1 & 1 & . & 1 & . \\ 1 & 1 & 1 & 1 & 1 & . & 1 \\ . & 1 & . & . & 1 & 1 & 1 \end{array}\right] \end{array} \tag{6.33}$$

$$A_2 = \begin{array}{c} \\ e \\ d \\ \beta \end{array} \begin{array}{c} \begin{array}{ccccccc} 7 & 8 & 10 & 11 & 9 & 4 & d_2 \end{array} \\ \left[\begin{array}{ccccccc} 1 & . & 1 & . & 1 & . & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & . \\ . & 1 & . & 1 & . & 1 & 1 \end{array}\right] \end{array} \tag{6.34}$$

$$A_3 = \begin{array}{c} \\ c \\ b \\ d \\ \gamma \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{ccccccccccc} 1 & 1 & 1 & 1 & . & . & . & . & . & 1 & . \\ 1 & 1 & . & . & 1 & 1 & 1 & . & 1 & . & . \\ . & . & 1 & 1 & 1 & . & 1 & 1 & . & . & 1 \\ . & . & . & . & . & 1 & . & 1 & 1 & 1 & 1 \end{array}\right] \end{array} \tag{6.35}$$

Step 9. Combining A_1, A_2 and A_3 by deleting the same rows, we get the desired incidence matrix as

$$A = \begin{array}{c} \\ a \\ \alpha \\ e \\ \beta \\ c \\ \gamma \end{array} \begin{array}{c} \begin{array}{ccccccccccccc} 6 & 1 & 7 & 8 & 10 & 5 & 11 & 9 & d_1 & 2 & 3 & 4 & d_2 \end{array} \\ \left[\begin{array}{ccccccccccccc} 1 & . & . & . & 1 & 1 & . & . & 1 & . & . & . & . \\ . & 1 & . & . & . & . & 1 & . & 1 & 1 & . & . & . \\ . & . & 1 & . & 1 & . & . & 1 & . & . & . & . & 1 \\ . & . & . & 1 & . & . & 1 & . & . & . & . & 1 & 1 \\ 1 & 1 & 1 & 1 & . & . & . & . & . & . & 1 & . & . \\ . & . & . & . & . & 1 & . & 1 & . & 1 & 1 & 1 & . \end{array}\right] \end{array} \tag{6.36}$$

Step 10. From A, we get the non-oriented graph as shown in Fig. 12(a). Considering the oriented cut-set matrix, Q_1, of Eg. (6.23), we get the oriented graph as shown in Fig. 12(b). Describing the ports, letting $d_1 = 0$ and $d_2 = 0$, and finally discarding the orientations of edges (since the orientations of only the ports are important) we get the network corresponding to Y as shown in Fig. 12(c).

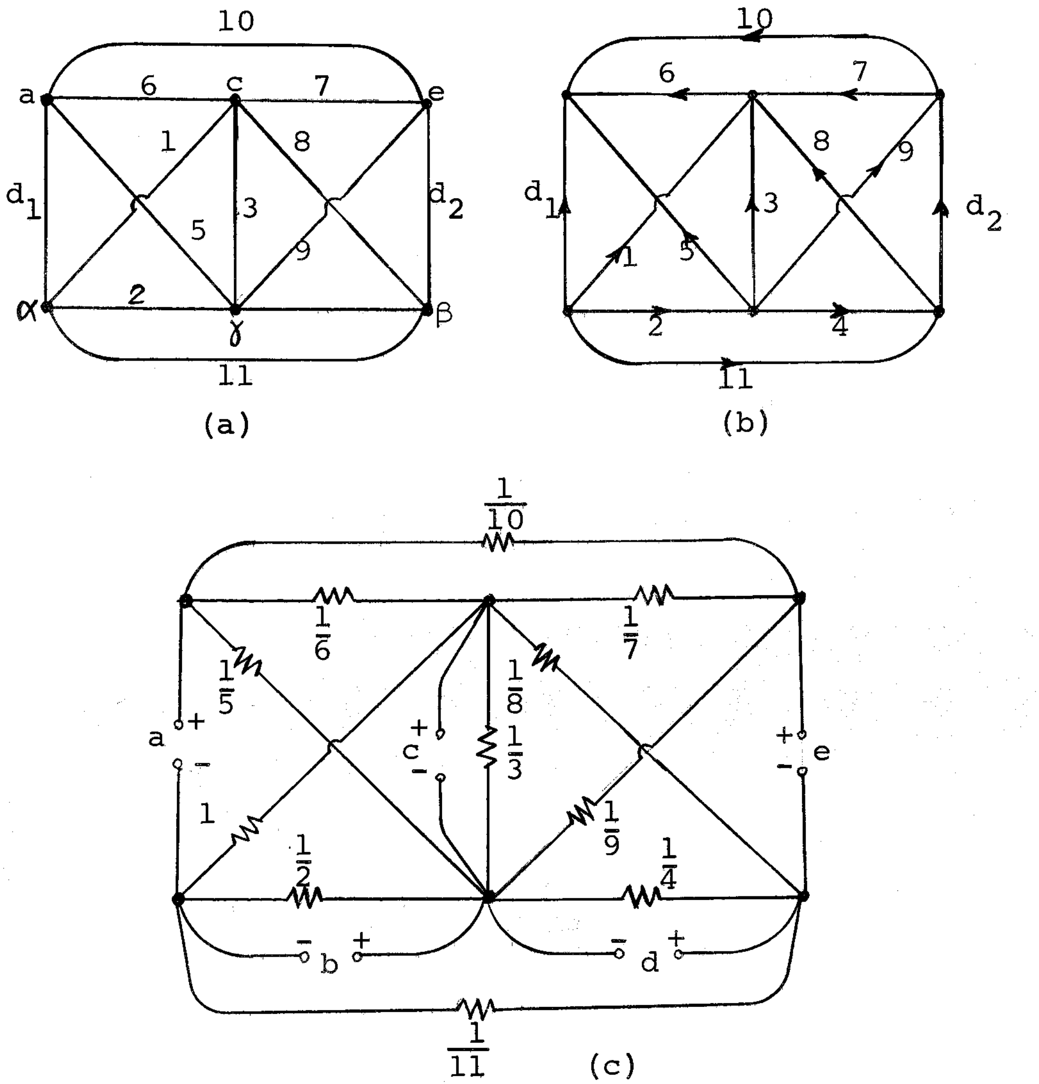

Fig. 12. (a) Non-oriented graph, (b) oriented graph; (c) network for Y. (Values in ohms)

6.3. Synthesis of Single-contact Networks

Let us consider the realization of single-contact networks. We assume that the state of each contact is independent of the state of all the other contacts in the network. Thus, the switching function of a single contact network is a proper function in which none of the variables are vacuous, and in which no variable appears both negated and unnegated. Therefore, we can evaluate the switching function of a two terminal single-contact network as the Boolean sum of all the path products between the terminal nodes. For example, the switching function, F, of a single contact network shown in Fig. 13 is obtained as

$$F = ad + ace + be + bcd \tag{6.37}$$

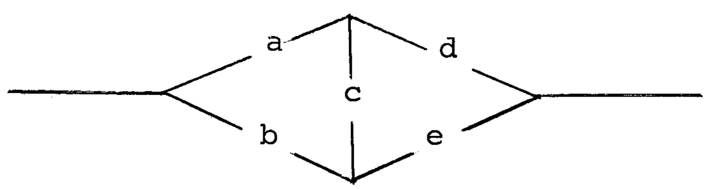

Fig. 13. A single-contact network.

By the following example, the realization method is illustrated.

Example 5.

Let us consider the following single-contact switching function given in a normal form,

$$F = adcg + adf + ae + bcf + bcde + bg \tag{6.38}$$

The path matrix is

$$P = \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & 1 & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & 1 & \cdot & 1 & \cdot \\ 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & 1 & \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & 1 & 1 & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix} \end{array} \tag{6.39}$$

The converted loop matrix corresponding to F is

$$B = \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & x \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & 1 & \cdot & \cdot & 1 & 1 \\ 1 & \cdot & \cdot & 1 & \cdot & 1 & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & 1 \\ \cdot & 1 & 1 & \cdot & \cdot & 1 & \cdot & 1 \\ \cdot & 1 & 1 & 1 & 1 & \cdot & \cdot & 1 \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & 1 \end{bmatrix} \end{array} \tag{6.40}$$

We now eliminate the dependent rows in B as follows: Adding (mod 2) the first row to the second and third rows, and again adding the fourth row to the fifth and sixth rows, we obtain the resultant matrix, B_1, as

$$B_1 = \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & x \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & 1 & \cdot \\ \cdot & \cdot & 1 & 1 & 1 & \cdot & 1 & 1 \\ \cdot & 1 & 1 & \cdot & \cdot & 1 & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & 1 & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & 1 & \cdot \end{bmatrix} \end{array} \tag{6.41}$$

In B_1 we add the third row to the fifth row, and then we add the second row to the fourth and fifth rows; the resultant matrix, B_2, is obtained as

$$B_2 = \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & x \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & 1 & \cdot \\ \cdot & \cdot & 1 & 1 & 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & 1 & \cdot \end{bmatrix} \end{array} \tag{6.42}$$

The fifth row in B_2 can be eliminated, because it contains only 0's. The second row and the sixth row are identical; the sixth row can therefore be deleted. The resultant matrix, B_3 is

$$B_3 = \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & x \end{array} \\ \begin{bmatrix} 1 & . & 1 & 1 & . & . & 1 & 1 \\ . & . & 1 & . & . & 1 & 1 & . \\ . & . & 1 & 1 & 1 & . & 1 & . \\ . & 1 & . & . & . & . & 1 & 1 \end{bmatrix} \end{array} \qquad (6.43)$$

After some permutations of columns, we obtain the loop matrix, B_4, in basic form, as

$$B_4 = \begin{array}{c} \begin{array}{cccccccc} a & f & e & b & c & d & g & x \end{array} \\ \begin{bmatrix} 1 & . & . & . & 1 & 1 & 1 & 1 \\ . & 1 & . & . & 1 & . & 1 & . \\ . & . & 1 & . & 1 & 1 & 1 & . \\ . & . & . & 1 & . & . & 1 & 1 \end{bmatrix} \end{array} \qquad (6.44)$$

The above elimination procedure is called Jordan's elimination procedure (8, p. 225-227). The corresponding cut-set matrix to be realized is then found to be

$$Q = \begin{array}{cc} & \begin{array}{cccccccc} a & f & e & b & c & d & g & x \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & . & 1 & . & . & . \\ 1 & . & 1 & . & . & 1 & . & . \\ 1 & 1 & 1 & 1 & . & . & 1 & . \\ 1 & . & . & 1 & . & . & . & 1 \end{bmatrix} \end{array} \qquad (6.45)$$

After deleting every column which has a 1 in row 1, and then removing row 1, Eq. (6.45) becomes

$$H = \begin{array}{c} \\ 3 \\ 4 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} b & g & x & d \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & \cdot & \cdot \\ 1 & \cdot & 1 & \cdot \\ \hline \cdot & \cdot & \cdot & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{6.46}$$

The pair of submatrices are

$$M_a(1) = \begin{array}{c} \\ 3 \\ 4 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & f & e & b & g & x & c \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & \cdot & \cdot \\ 1 & \cdot & \cdot & 1 & \cdot & 1 & \cdot \\ 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \end{bmatrix} \end{array} \tag{6.47}$$

$$M_b(1) = \begin{array}{c} \\ 2 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccccc} a & f & e & d & c \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & 1 & \cdot \\ 1 & 1 & 1 & \cdot & 1 \end{bmatrix} \end{array} \tag{6.48}$$

$M_b(1)$ became an incidence matrix.

From $M_1(1)$, deleting every column which has a 1 at row 3, we get

$$H = \begin{array}{c} \\ 4 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} x & c \end{array} \\ \left[\begin{array}{c|c} 1 & \cdot \\ \hline \cdot & 1 \end{array}\right] \end{array} = \begin{bmatrix} H_a & 0 \\ 0 & H_b \end{bmatrix} \tag{6.49}$$

The pair of submatrices are

$$M_a(13) = \begin{array}{c} \\ 4 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & f & e & b & x & g \end{array} \\ \begin{bmatrix} 1 & \cdot & \cdot & 1 & 1 & \cdot \\ 1 & 1 & 1 & 1 & \cdot & 1 \end{bmatrix} \end{array} \tag{6.50}$$

$$M_b(13) = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & f & e & b & c & g \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & \cdot & 1 & \cdot \\ 1 & 1 & 1 & 1 & \cdot & 1 \end{bmatrix} \end{array} \tag{6.51}$$

$M_a(13)$ and $M_b(13)$ are incidence matrices. Adding a row corresponding to a datum node to $M_b(1)$, $M_a(13)$ and $M_b(13)$ we get non-reduced incidence matrices as follows.

From $M_2(1)$

$$A_1 = \begin{array}{c} \\ 2 \\ 1 \\ \alpha \end{array} \begin{array}{c} \begin{array}{ccccc} a & f & e & d & c \end{array} \\ \begin{bmatrix} 1 & . & 1 & 1 & . \\ 1 & 1 & 1 & . & 1 \\ . & 1 & . & 1 & 1 \end{bmatrix} \end{array} \quad (6.52)$$

From $M_1(13)$

$$A_2 = \begin{array}{c} \\ 4 \\ 3 \\ \beta \end{array} \begin{array}{c} \begin{array}{cccccc} a & f & e & b & x & g \end{array} \\ \begin{bmatrix} 1 & . & . & 1 & 1 & . \\ 1 & 1 & 1 & 1 & . & 1 \\ . & 1 & 1 & . & 1 & 1 \end{bmatrix} \end{array} \quad (6.53)$$

From $M_2(13)$

$$A_3 = \begin{array}{c} \\ 1 \\ 3 \\ \gamma \end{array} \begin{array}{c} \begin{array}{cccccc} a & f & e & b & c & g \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & . & 1 & . \\ 1 & 1 & 1 & 1 & . & 1 \\ . & . & . & 1 & 1 & 1 \end{bmatrix} \end{array} \quad (6.54)$$

Combining A_1, A_2 and A_3 by deleting the same rows we get the resulting incidence matrix

$$A = \begin{array}{c} \\ 2 \\ \alpha \\ 4 \\ \beta \\ \gamma \end{array} \begin{array}{c} \begin{array}{cccccccc} a & f & e & b & g & d & c & x \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & 1 & 1 & \cdot \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1 \\ \cdot & 1 & 1 & \cdot & 1 & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & 1 & \cdot & 1 & \cdot \end{bmatrix} \end{array} \qquad (6.55)$$

From A, we get the contact network corresponding to F as shown in Fig. 14.

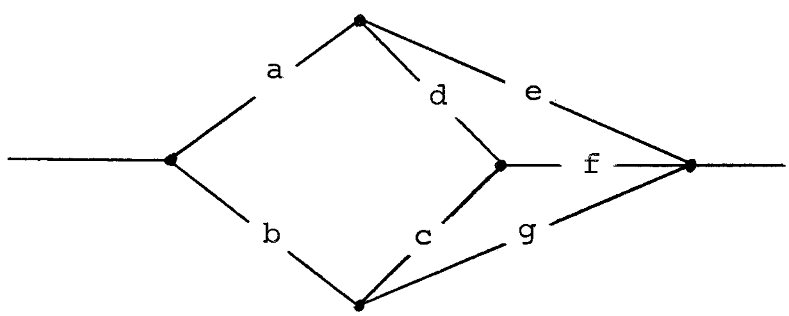

Fig. 14. Realization of the function of Eq. (6.38).

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