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A new decentralized nonlinear controller for Robot Manipulators is presented in this thesis. Based on concepts of Lyapunov stability theory and some control ideas proposed in [3]-[7], we obtain continuous nonlinear decentralized control laws which guarantee position and velocity tracking to within an arbitrarily small error.

Assumptions based on physical constraints of manipulators are made to guarantee the existence of the controller and asymptotic stability of the closed loop system. Simulations show how well this rather simple control scheme works on two of the links of the Puma 560 Manipulator.

The main contribution of this thesis is that it extends the results of a class of complex centralized control algorithms to the decentralized robust control of interconnected nonlinear subsystems like robot manipulators.

Robust Nonlinear Decentralized Control of Robot Manipulators

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I. INTRODUCTION

The highly nonlinear dynamics and the joints interaction that characterize Robotic Manipulators, and especially the difficulties that these features embody in the control design, have received the attention of many researches during the last two decades. In the past, robot applications were rather elementary, and linear control procedures were widely used [9],[17], because of the fact that the coupling effects between joints due to Coriolis, centripetal, and gravitational forces for slow speed motion with high gear ratios do not appear to affect drastically the dynamics of the manipulator.

This decoupling-like dynamic property of manipulators when acting at slow speed has also been used in most of the work available today concerning the application of adaptive theory to solve the problem of robustness in robotic control [8],[14],[21]; which indeed, has been characterized by the use of perturbation theory to obtain suitable linear models and, therefore, apply adaptive concepts originally derived for linear systems. The common drawback, though, is that to obtain the control law and the adaptation scheme, it is assumed a time invariant inertia matrix, making it difficult to ensure robustness to unmodelled dynamics. One of the latest applications of adaptive control to manipulators that investigates time variations of the inertia matrix can be found in [18].

These previous control strategies started to fail when, later on, high performance tasks demanding characteristics such as precise trajectory tracking, high speed response for fast trajectories, global asymptotic stability, computational-time efficiency and robustness against unmodelled dynamics and unknown disturbance became part of the robot specifications. Moreover, the use of direct-drive actuators and the lack of high gear ratios demanded new control schemes that would take into account the nonlinearities of the manipulator dynamics [18].

In pursuing this high performance, researches have applied control methods such as Variable Structure Control, Adaptive Control, Computed Torque, and Lyapunov theory. In most of the cases, these technics are employed in conjunction with nonlinear cancellation methods (see for instance [15]), where the basic idea is to cancel the nonlinear part of the dynamical performance of the particular manipulator, and use a better known control theory to solve the tracking problem for the resulting linear system. The main drawback in this case is that it is commonly assumed that the mathematical model of the plant is precisely known. Although, mathematical models for robotics can be, in most of the cases, well specified by a rich robotic modelling theory [2], exact information of the model parameters, task specifications, and the environment the manipulator is to work in, are not always perfectly known. Payload Variations, for instance, in picking up or dropping actions may vary the inertia matrix substantially.

With few recent exceptions, these concepts have been applied to obtain centralized algorithms, that is, considering the robot as one system. Consequently, complex, and in some cases even impractical control laws have resulted. Recently, some authors [4], [10], [12]-[14], [19], [20], [22] have obtained rather simple control laws by considering the manipulator as a set of interconnected subsystems where each joint is controlled independently and the coupling effects of one joint on the others are treated as perturbations. This approach has been named "Decentralized Control" or "Independent Joint Control". Some of the characteristics that make this new method appealing to control designers are:

1. Due to its decentralized structure, it can be implemented by a parallel processing architecture.

2. Not being a model-based control, it is robust to a wide range of uncertainties such as time-varying parameters, unmodelled dynamics and external perturbations.

3. In contrast to stochastic approaches, no statistics of the uncertainties are required, but only the bounds of a set to which the uncertainties belong.

4. Contrary to centralized control, it is possible to design and adjust the control parameters based on specific representation of joint coupling effects, joint actuator limitations and joint motion objective.

In this thesis, we present a decentralized scheme based on the application of Lyapunov stability theory and some ideas of Variable Structure Theory. The idea appears to have been originally conceived by G. Leitmann, M. Corless and J. Martin [7]. Other related work can be found in [3] and [5]. Here we prove that this theory can be extended to design robust decentralized control of manipulators with very satisfactory results, according to the desired characteristics enumerated above.

In a standard application of Variable Structure Control, the system states are driven to a switching surface, containing the manipulator trajectories. Once the states cross this surface, the system remains within a sliding mode which makes it insensitive to parameter variations and external disturbances [1]. The main disadvantage of this approach becomes evident when we look at the control action, which virtually oscillates at infinite frequency, producing then an undesirable chattering effect [2] and the possibility of exciting high frequency resonant modes.

In this work, the chattering problem is overcome by ensuring asymptotic convergence to a neighborhood of the equilibrium state, rather than requiring the same stability of the equilibrium point itself. Therefore, we obtain a set of controllers which guarantee global ultimate asymptotic stability within some arbitrarily small neighborhood of the equilibrium state. Moreover, the synthesized control law is continuous in the states which makes possible its physical realizability.

Our approach can also be viewed as a modified Linear Multivariable Approach (LMA) according to [1], where no exact linearization of the robot is needed. That is, in LMA the globally linearized error system given by

$$z = [z_1 \; z_2]^T$$

$$z_1 = q - q_r$$

$$z_2 = \dot{z}_1$$

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{v}$$

$$\mathbf{v} = M^{-1}(\cdot)[T - U(\cdot)]$$

is utilized to find $\mathbf{v}$, usually a linear control, such that some specified closed loop performance can be achieved. However, due to the presence of the uncertainty elements in $M(\cdot)$ and $U(\cdot)$, it is impossible to cancel precisely the nonlinear terms. Some authors [1], [8], [21] employ adaptive methods and identification algorithms in either centralized or decentralized structures to obtain the estimates of these matrices, but again, giving rise to drawbacks such as lack of robustness when fast trajectory tracking are required, in which unknown parameters in M and U are no longer slow time varying, and high computational complexity.

In the approach discussed here, we do not need estimation algorithms. Moreover, since it is not a model-based control, we do not have to worry about matching a exact linearized model. The only information that is required is the knowledge of the possible size of the uncertainties.

This work is organized as follows: Chapter II contains a brief account of the manipulator dynamics. Chapter III includes the derivation of the robust nonlinear decentralized control. It also includes a Stability analysis of the composite system. In chapter IV, control laws for a two-link robot manipulator are derived and some simulations results of the closed loop system are shown. Finally, in chapter V some conclusions are drawn and open problems are stated.

II. MANIPULATOR DYNAMICS

By using the Lagrange-Euler equation [17], the dynamics of a multiple link manipulator can be represented by the second order nonlinear vector differential equation

$$M(t,q,w)\ddot{q} + U(t,q,\dot{q},w) = T, \tag{1}$$

where

$\mathbf{q}(t)$: $\Re \rightarrow \Re^n$ is the joint angular position vector.

$\mathbf{M}(\cdot)$: $\Re x \Re^n x \Re^l \rightarrow \Re^{nxn}$ is the positive definite inertia matrix.

$\mathbf{U}(\cdot)$: $\Re x \Re^n x \Re^n x \Re^l \rightarrow \Re^n$ contains Coriolis and centrifugal torques as well as gravity loading and frictional torques.

$w \in \Re^l$ is a vector of uncertainties such as unknown constant or time varying parameters and inputs. It is assumed that these perturbations belong to a closed bounded set.

$T \in \Re^n$ is a vector of control inputs.

The system modelled by (1) can also be viewed as a collection of n second-order subsystems of the form

$$m_{ii}(t,q,w)\ddot{q}_i + u_i(t,q,\dot{q},\ddot{q},e) = T_i \qquad i = 1,2,\ldots,n, \tag{2}$$

$$u_i(t,q,\dot{q},\ddot{q},w) = \sum_{\substack{i=1 \\ i \neq j}}^{n} m_{ij}(t,q,w)\ddot{q}_j(t) + U_i(t,q,\dot{q},w), \tag{3}$$

thus, $m_{ii}(\cdot)$ is the varying effective inertia seen at the i^{th} joint and $u_i(\cdot)$ is a scalar function that accounts for torques from the interaction of joint i with the others.

For our purpose, all information coming from other subsystems will be considered as perturbations at subsystem i. That is, $\{w,\ q_j,\ \dot{q}_j,\ \ddot{q}_j\} \triangleq v \in V$ are perturbation variables for subsystem $i \neq j$

Let's now obtain a state model representation for subsystem (2)

Define

$$x = [q_i \quad \dot{q}_i]^T \tag{4}$$

and

$$G(t,q,w) \triangleq m_{ii}^{-1}(t,q,w) \tag{5}$$

$$h(t,q,\dot{q},\ddot{q},w) \triangleq -m_{ii}^{-1}(t,q,w)u_i(t,q,\dot{q},\ddot{q},w), \tag{6}$$

where $m^{-1}_{ii}(\cdot)$ exists due to the positive definiteness of $M(\cdot)$. Then (2) can be written as

$$\begin{aligned} \dot{x} &= Ax + B[h(t,x,v) + G(t,x,v)T], \\ &\triangleq f(t,x,v,T), \end{aligned} \tag{7}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad T \triangleq T_i.$$

III. ROBUST NONLINEAR DECENTRALIZED CONTROL OF ROBOT MANIPULATORS

This chapter contains the description of the control design objectives in terms of some suitable stability criterion, general sufficient conditions to attain these objectives, and the synthesis of a control law satisfying the conditions for that desired stability.

Problem Statement

It is our goal to design controllers $T_i = p_i(t,x):\Re x \Re^2 \rightarrow \Re$ for each subsystem (7), using only that information realistically available at the subsystem, e.g., joint position and velocity, so that the local state vector $\mathbf{x}(t)$ asymptotically tracks some desired state motion $x_r(t) \in \Re^2$ within an arbitrarily small constant d, according to definition 1 below. Furthermore, to facilitate physical realizability of the controller, the function $T = p(t,x)$ is to be continuous with respect to the state.

Definition 1: $\mathbf{x}(t)$ asymptotically tracks $\mathbf{x}_r(t)$ to within d if and only if the error state model (8) defined by

$$z(t) = x(t) - x_r(t)$$

$$\dot{z}(t) = f(t,z(t)+x_r(t),v,T) - \dot{x}_r(t) \triangleq F(t,z(t),v,T) \tag{8}$$

is "globally uniformly ultimately bounded" with respect to a set $S \subset \Re^2$ whose Euclidean norm $\| S \| \leq d$. Hence, the system (8) must have the following properties:

i. For each uncertainty realization $v \in V$, for each $t_o \in \Re$, $z_o \in \Re^2$, there exists a solution to (8) defined as $z(t):[t_o,t_1) \rightarrow \Re^2$, $t_1 \geq t_o$, $z(t_o) = z_o$.

ii. For any real number $\delta > 0$, there exists a real number $d(\delta) > 0$ such that, for any solution z(t), with $z(t_o) = z_o$ and $\| z_o \| \leq \delta$, then $\| z(t) \| \leq d(\delta)$ for all $t \in [t_o,t_1)$.

iii. For each $z_o \in \Re^2$, , there exists a real number $\theta(z_o,S) > 0$ such that, for all z(t) solution to (4) and with $z(t_o) = z_o$, then $z(t) \in S$ for all $t \geq t_o + \theta(z_o,S)$. Thus, $\| z(t) \| \leq d$ for all $t \geq t_o + \theta(z_o,d)$.

Assumptions

The following properties are to be satisfied in order to sufficiently show the existence of a class of controls previously described.

i. For each $v \in V$, $G(\cdot,v)$ and $h(\cdot,v)$ are continuous functions. This is a sufficient condition for existence of solution to system (7).

ii. There exist continuous bounding functions β_o, β_1, β_2 such that

$$|h(t,q,\dot{q},\ddot{q},w)| \leq \beta_0(t,q_i,\dot{q}_i) \equiv \beta_o(t,x), \tag{9}$$

$$|G_i(t,q,w)| \leq \beta_1(t,q_i) \equiv \beta_1(t,x), \tag{10}$$

$$\min_{all v \in V} G_i(t,q,w) \geq \beta_2(t,q_i) \equiv \beta_2(t,x) > 0. \tag{11}$$

iii. The state reference $\mathbf{x}_r(t) \in \mathbf{C}^1$. Then by defining

$$T_r(t) \triangleq \ddot{q}_r(t),$$

$T_r(t)$ is a continuous function and a reference model for each subsystem can be written as

$$\dot{x}_r(t) = Ax_r(t) + BT_r(t), \tag{12}$$

with **A** and **B** as in (7).

Notice that assumptions i and iii together guarantee existence of a solution to (8).

Remark 1. Based on physical limitations of every robotic mechanism, it is possible to assume the existence of positive constants C_v and C_a for each joint, such that

$$|\dot{q}_j| \le C_{v_j} \qquad j = 1,...,n,$$

$$|\ddot{q}_j| \le C_{a_j} \qquad j = 1,...,n.$$

Remark 2. For a revolute joint, **q** appears in **M**(•) and **U**(•) only as argument of either sines or cosines; therefore, the perturbations due to q_j at joint i are always bounded for j = 1,...,n, i = 1,...,n, $i \neq j$

Remark 3. For a prismatic joint, the joint variable is a linear displacement which can also be assumed to be physically constrained. That is, if d is the displacement, there exists a positive real constant C_d so that $|d| \le C_d$ for every prismatic joint.

From these remarks and also by taking into account that **w** belongs to a closed bounded set **W** by assumption, then **V** is also a closed bounded set; hence, there exist bounding functions $\beta_o(t,x)$, $\beta_1(t,x)$ and $\beta_2(t,x)$ that satisfy (9)-(11).

Theorem 1. Sufficient Conditions for Stability

Assume there exist symmetric, positive-definite matrices $P,Q \in \Re^{2x2}$ and non-negative real numbers a, b, c, a>0, such that $V(z) = z^T P z$ is a Lyapunov function for the error model (8)

$$\dot{z} = F(t,z,v,T), \tag{8}$$

with

$$z^t PF(t,z,v,T) \leq -a\|z\|_Q^2 + b\|z\|_Q + c, \tag{13}$$

for all $t \in \Re$, $z \in \Re^2$, and $v \in V$,

where $\|z\|_Q = (z^T Q z)^{1/2}$.

Then $(q(t),\dot{q}(t))$ asymptotically tracks $(q_r(t),\dot{q}_r(t))$ within a tolerance d given by

$$d = \left[\frac{\lambda_{\max}(Q^{-1}P)}{\lambda_{\min}(P)}\right]^{1/2} d_Q, \tag{14}$$

where

$$d_Q \triangleq \frac{[-b+(b^2+4ac)^{1/2}]}{2a}. \tag{15}$$

Loosely speaking, we then want to design a continuous control law T(t,x) such that (13) is satisfied. Also, we would like that b and c be chosen arbitrarily so that d is arbitrary too.

Proof: We obtain a set for which asymptotic stability in the sense of Lyapunov is guaranteed.

Assume

$$V(z) = z^T P z,$$

$$\dot{V}(z) = 2z^TP\dot{z} = 2z^TPF(t,z,v,T).$$

Assuming also that (13) holds, then

$$\dot{V}(z) \leq 2[-a\|z\|_Q^2 + b\|z\|_Q + c]$$

and $\dot{V}(z)$ is negative definite for all $t > t_o$ if

$$\|z\|_Q > d_Q \tag{16}$$

with d_Q defined by (15). Inequality (16) implies that

$$z^TQP^{-1}Pz > d_Q^2,$$

or

$$\lambda_{min}(QP^{-1})z^TPz > d_Q^2,$$

or

$$z^TPz > [\lambda_{min}(QP^{-1})]^{-1}d_Q^2 = \lambda_{max}(Q^{-1}P)d_Q^2.$$

Hence,

$$V(z) > \lambda_{max}(Q^{-1}P)d_Q^2,$$

$$z^T\lambda_{min}(P)z > \lambda_{max}(Q^{-1}P)d_Q^2,$$

$$z^Tz > \frac{\lambda_{max}(Q^{-1}P)}{\lambda_{min}(P)}d_Q^2,$$

and

$$\| z \| > \left[\frac{\lambda_{\max}(Q^{-1}P)}{\lambda_{\min}(P)} \right]^{1/2} d_Q = d.$$

Therefore, for $\| z(t) \| > d$, the error model (8) is asymptotically stable in Lyapunov sense and, therefore, $[q(t) - q_r(t),\ \dot{q}(t) - \dot{q}_r(t)]$ asymptotically converges to a set S whose norm $\| S \| \leq d$.

Feedback Control Synthesis

A feedback control law for joint i is proposed here, which will be shown to satisfy theorem 1 and so it achieves the requirements proposed in the problem statement.

From (7), (8), and (12), we obtain

$$\dot{V}(z) = 2z^T PF(t,z,v,T), \tag{17}$$

$$\dot{V}(z) = 2z^T P[Az + B(h(\cdot) + G(\cdot)T - T_r)].$$

Let the control T(t,x) be of the form

$$T = p^o(t,x) + p^s(t,x) + p^c(t,x). \tag{18}$$

Then the closed loop error model can be written as

$$\dot{z} = Az + BG(\cdot)p^{s}(t,x) + B[e(\cdot) + G(\cdot)p^{e}(t,x)], \tag{19}$$

where $e(t,x,v) = h(\cdot) - T_r + G(\cdot)p^{o}(t,x)$ (20)

Construction of $p^{o}(t,x)$

Let $p^{o}(\cdot):\Re x\Re^{2}\rightarrow\Re$ be any continuous function. It can be chosen to reduce the effect of $e(\cdot)$ in the system (19). Thus, choose p^{o} to reduce $|e(\cdot)|$.

Construction of $p^{s}(t,x)$

We choose $p^{s}(t,x)$ so that the system

$$\dot{z} = Az + BG(t,q,w)p^{s}(t,x) \tag{21}$$

is globally asymptotically stable with respect to zero for all $v \in V$. To be able to apply theorem 1, we must find positive definite symmetric matrices $P, Q \in \Re^{2}$ and $p^{s}(t,x)$ such that, for $V(z) = z^{T}Pz$, the following inequality is satisfied

$$\frac{\dot{V}(z)}{2} = z^{T}P[Az + BG(t,q,w)p^{s}] \leq -\|z\|_{Q}^{2}$$

or

$$z^{T}PAz + z^{T}PBG(\cdot)p^{s} + z^{T}Qz \leq 0. \tag{22}$$

Lemma 1. For any symmetric positive definite matrix Q, any Real positive constant $\sigma > 0$, and any continuous function $\Gamma(t,x)$ which satisfies

$$\Gamma(t,x) \geq \sigma(\min_{all\,v\in V} G(t,q,w))^{-1}, \tag{23}$$

there exists a symmetric positive definite matrix **P**, solution of the algebraic Riccati equation (24)

$$\mathbf{PA} + \mathbf{A}^{\mathrm{T}}\mathbf{P} - 2\sigma\mathbf{PBB}^{\mathrm{T}}\mathbf{P} + 2\mathbf{Q} = 0 \tag{24}$$

such that with

$$p^{s}(t,x) = -\Gamma(t,x)\mathbf{B}^{\mathrm{T}}\mathbf{P}z, \tag{25}$$

equation (22) is satisfied.

Notice that assumption ii guarantees the existence of the function $\Gamma(t,x)$, and that since the pair (**A**,**B**) is controllable the existence of matrix **P** is guaranteed.

Proof: Let $p^{s}(t,x) = -\Gamma(t,x)\mathbf{B}^{\mathrm{T}}\mathbf{P}z$, with $\Gamma(t,x)$ a continuous function satisfying (23). The left hand side of equation (22) becomes

$$\begin{aligned} &z^{T}PAz - z^{T}PBG(t,q,w)\Gamma(t,x)B^{T}Pz + z^{T}Qz \\ &\leq z^{T}PAz - z^{T}PB\sigma B^{T}Pz + z^{T}Qz. \end{aligned}$$

Choose **P** for the critical case when

$$z^T PAz - z^T PB\sigma B^T Pz + z^T Qz = 0,$$

then,

$$\frac{1}{2} z^T [PA + A^T P] z - \sigma z^T PBB^T Pz + z^T Qz = 0,$$

or
$$z^T PAz + z^T A^T Pz - 2\sigma z^T PBB^T Pz + 2z^T Qz = 0,$$

which must be satisfied for all $z \epsilon \Re^2$. Consequently, **P** can be obtained by solving

$$PA + A^T P - 2\sigma PBB^T P + 2Q = 0.$$

Using this matrix **P** and the function $\Gamma(t,x)$ as defined in (23), we obtain the desired stability of (21), that is

$$z^T PAz + z^T PBG(\cdot)p^s + z^T Qz \leq 0.$$

Construction of $p^c(t,x)$

The control $p^c(t,x)$ is designed to overcome any destabilizing effect from the term e(t,x,v) in (19). Hence, $p^c(t,x)$ is chosen so that

$$z^T PB\ [e(t,x,v) + G(t,q,w)\ p^c(t,x)] \leq \epsilon \quad (26)$$

for all $t \in \Re,\ x \in \Re^2,\ v \in V.$

Lemma 2. Assume ρ and **k** are any two continuous functions satisfying

$$\rho(t,x) \geq (\min_{all\, v \in V} G(t,q,w))^{-1} \, |e(t,x,v)|, \tag{27}$$

$$k(t,x) \geq |e(t,x,v)| \qquad for\ all\ v \in V. \tag{28}$$

Let $S^{\epsilon}(\cdot):\Re \rightarrow \Re$ be any continuous function which satisfies

$$|\eta| S^{\epsilon}(\eta) = |S^{\epsilon}(\eta)| \eta \tag{29}$$

and for any $\epsilon > 0$,

$$|\eta| > 0 \rightarrow |S^{\epsilon}(\eta)| \geq 1 - |\eta|^{-1} \epsilon \qquad for\ all\ \eta \in \Re. \tag{30}$$

Then the control

$$p^{\epsilon}(t,x) = -\rho(t,x) \; S^{\epsilon}(k(t,x) B^{T} P z) \tag{31}$$

with $z = x - x_r$, guarantees the satisfaction of inequality (26).

Proof: Let

$$\alpha \triangleq B^{T} P z$$

$$L \triangleq z^{T} P B [e(\cdot) + G(\cdot) p^{\epsilon}(t,x)]$$

$$= \alpha e(\cdot) + \alpha G(\cdot) p^{\epsilon}(t,x).$$

Assume

$$p^{\epsilon}(t,x) = -\rho(t,x)\, S^{\epsilon}(k(t,x)B^{T}Pz)$$

thus,

$$p^{\epsilon}(\cdot) = -\rho S^{\epsilon}(k\alpha).$$

Assume $|k\alpha| > 0$, then from (29)

$$\begin{aligned} S^{\epsilon}(k\alpha) &= |k\alpha|^{-1}\, |S^{\epsilon}(k\alpha)|\, k\alpha \\ &= |\alpha|^{-1}\, |S^{\epsilon}(k\alpha)|\, \alpha \end{aligned}$$

and

$$\begin{aligned} \alpha G p^{\epsilon} &= -\alpha G \rho S^{\epsilon}(\cdot) \\ &= -\rho|\alpha|^{-1}|S^{\epsilon}(\cdot)|\alpha G\alpha \\ &= -\rho|\alpha|^{-1}|S^{\epsilon}(\cdot)|\alpha^{2}G \\ &\le -\rho\,|\alpha|\,|S^{\epsilon}(\cdot)|G \\ &\le -\rho\,|\alpha|\,|S^{\epsilon}(\cdot)|\min_{all\, v\in V} G(t,q,w). \end{aligned}$$

If we now use (27), we get

$$\alpha G p^{e} \le -|e|\,|\alpha|\,|S^{\epsilon}(\cdot)|.$$

Applying (30), results in

$$\alpha G p^{\epsilon} \leq -|e|\,|\alpha| + |e|\,|\alpha|\,|k\alpha|^{-1}\epsilon$$

and from (28)

$$\alpha G p^{\epsilon} \leq -|e|\,|\alpha| + \epsilon.$$

Therefore,

$$L \triangleq \alpha e + \alpha G p^{\epsilon} \leq \epsilon$$

or

$$z^{T}PB[e + Gp^{\epsilon}] \leq \epsilon.$$

In case $K\alpha = 0$, from (29), and taking into account that $S^{\epsilon}(\eta)$ is a continuous function, we obtain $S^{\epsilon}(0) = 0$. Hence, $p^{\epsilon}(t,x) = 0$, which implies $L \leq |\alpha|\,|e|$, thus, $L \leq k\,|\alpha| = 0$ and consequently, (26) holds.

Applying the control laws previously defined for subsystem (19) and choosing $V(z) = z^{T}Pz$ as a Lyapunov function, we then obtain that

$$z^{T}PF(t,z,v,T) \leq -\|z\|_{Q}^{2} + \epsilon \tag{32}$$

and by theorem 1, with $a=1$, $b=0$ and $c=\epsilon$, that the desired tracking, as stated in the problem statement, is achieved.

Composite System Stability

The stability of the composite system is based also on Lyapunov theory and consists of constructing a Lyapunov function as the sum of the ones used for each subsystem whose time derivative is proven to be negative definite to within some tolerance when evaluated on the error state trajectories of the respective composite error system.

The composite system is obtained by defining the following vectors [20]

$$X(t) \triangleq [q_1, \dot{q}_1, q_2, \dot{q}_2, \ldots, q_n, \dot{q}_n]^T,$$

$$X_r(t) \triangleq [q_{r_1}, \dot{q}_{r_1}, q_{r_2}, \dot{q}_{r_2}, \ldots, q_{r_n}, \dot{q}_{r_n}]^T,$$

$$Z(t) \triangleq X(t) - X_r(t),$$

$$A \triangleq diag(A_i), \qquad B \triangleq diag(B_i),$$

$$T \triangleq [T_1, \ldots, T_n]^T,$$

$$G(\cdot) \triangleq diag(G_i(\cdot)),$$

$$H(\cdot) \triangleq [h_1, \ldots, h_n]^T,$$

$$w \triangleq [w_1, \ldots, w_n]^T,$$

$$D \triangleq [d_1, \ldots, d_n]^T,$$

$$d_i \triangleq h_i(\cdot) + G_i(\cdot)T_i - T_i \qquad i = 1,\ldots,n,$$

$$D = H(\cdot) + G(\cdot)T - T,$$

then, the composite error system can be written as

$$\dot{Z}(t) = AZ(t) + BD. \tag{33}$$

From the assumptions made for each subsystem, each of the components of the vector **H** and matrix **G** is bounded. Therefore, there exist continuous functions $\bar{\beta}_0$, $\bar{\beta}_1$, $\bar{\beta}_2$ such that

$$\|H(t,X,w)\| \le \bar{\beta}_o(t,X),$$

$$\|G(t,X,w)\| \le \bar{\beta}_1(t,X),$$

$$\lambda_{\underset{all\ w \in W}{\min}} G(t,X,w) \ge \bar{\beta}_2(t,X) > 0,$$

for all $t \in \Re$, $X \in \Re^{2n}$ and $w \in W$.

Defining a new Lyapunov function for the composite system (33) as

$$V(Z) = \sum_{i=1}^{n} Z_i^T P_i Z_i \tag{34}$$

then from (32) we obtain

$$\dot{V}(Z) \leq \sum_{i=1}^{n} -\|Z_i\| + \epsilon_i, \tag{35}$$

where

$$Z_i = [q_i - q_{r_i} \,,\, \dot{q}_i - \dot{q}_{r_i}]^T$$

and P_i satisfies (24) for each joint i= 1, ..., n

Therefore, theorem 1 can be applied to the composite system to show that it is globally asymptotically stable within a tolerance d given by

$$d = \left[\frac{\lambda_{\max}(Q^{-1}P)}{\lambda_{\min}(P)}\right]^{1/2} d_Q, \tag{36}$$

with d_Q given by

$$d_Q = \left[\sum_{i=1}^{n} \epsilon_i\right]^{1/2}$$

and $P = \text{diag}\,(P_i)$, $Q = \text{diag}\,(Q_i)$.

IV. SIMULATION OF DECENTRALIZED CONTROL OF TWO-LINK PUMA 560 ROBOT MANIPULATOR

We apply the concept previously developed to a two-link manipulator depicted in figure 1, [20] and mathematically described by

$$\begin{aligned} T &= M(q)\ddot{q} + N(q,\dot{q}) + G(q) + H(\dot{q}) \\ &\quad + mJ^T(q)[J(q)\ddot{q} + \dot{J}(q,\dot{q})\dot{q} + g], \end{aligned} \tag{37}$$

$$M(q) = \begin{bmatrix} a_1 + a_2\cos(q_2) & a_3 + \frac{a_2}{2}\cos(q_2) \\ a_3 + \frac{a_2}{2}\cos(q_2) & a_3 \end{bmatrix},$$

$$N(q,\dot{q}) = \begin{bmatrix} -a_2\sin(q_2)(\dot{q}_1\dot{q}_2 + \frac{\dot{q}_2^2)}{2} \\ a_2\sin(q_2)\frac{\dot{q}_1^2}{2} \end{bmatrix},$$

$$G(q) = \begin{bmatrix} a_4\cos(q_1) + a_5\cos(q_1+q_2) \\ a_5\cos(q_1+q_2) \end{bmatrix},$$

$$H(\dot{q}) = \begin{bmatrix} V_1\dot{q}_1 + V_2 sgn(\dot{q}_1) \\ V_3\dot{q}_2 + V_4 sgn(\dot{q}_2) \end{bmatrix},$$

$$J(q) = \begin{bmatrix} -l_1\sin(q_1) - l_2\sin(q_1+q_2) & -l_2\sin(q_1+q_2) \\ l_1\cos(q_1) + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \end{bmatrix},$$

$$g = \begin{bmatrix} 0 \\ 9.81 \end{bmatrix},$$

$$\dot{J}(q,\dot{q}) = \begin{bmatrix} -l_1\cos(q_1)\dot{q}_1 - l_2\cos(q_{12})\dot{q}_{12} & -l_2\cos(q_{12})\dot{q}_{12} \\ -l_1\sin(q_1)\dot{q}_1 - l_2\sin(q_{12})\dot{q}_{12} & -l_2\sin(q_{12})\dot{q}_{12} \end{bmatrix},$$

$$q_{12} \triangleq q_1 + q_2,$$

$$\dot{q}_{12} \triangleq \dot{q}_1 + \dot{q}_2.$$

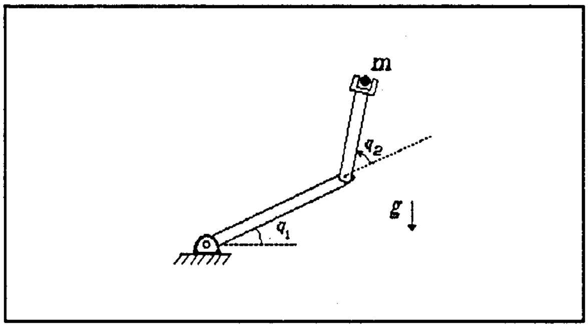

Figure 1 Two-link manipulator

In this example we consider m, V_1, V_2, V_3, V_4, as part of the uncertain elements with the following given bounds

$$m \in [0, m_{max}],$$
$$V_i \in [V_{min}, V_{max}] \quad i=1,\ldots,4.$$

From (37) we obtain the scalar equations for each joint as

$$T_1 = [M_{11} + m(J_{11}^2 + J_{21}^2)]\ddot{q}_1 \tag{38}$$
$$+ [M_{12} + m(J_{11}J_{12} + J_{21}J_{22})]\ddot{q}_2$$
$$+ N_1 + m[(J_{11}\dot{J}_{11} + J_{21}\dot{J}_{21})\dot{q}_1 + (J_{11}\dot{J}_{12} + J_{21}\dot{J}_{22})\dot{q}_2]$$
$$+ G_1 + 9.81mJ_{21} + H_1,$$
$$\triangleq \hat{m}_1\ddot{q}_1 + u_1.$$

$$T_2 = [M_{22} + m(J_{12}^2 + +J_{22}^2)]\ddot{q}_2 \tag{39}$$
$$+ [M_{12} + m(J_{11}J_{12} + J_{21}J_{22})]\ddot{q}_1$$
$$+ N_2 + m[(J_{12}\dot{J}_{11} + J_{22}\dot{J}_{21})\dot{q}_1 + (J_{12}\dot{J}_{12} + J_{22}\dot{J}_{22})\dot{q}_2]$$
$$+ G_2 + 9.81mJ_{22} + H_2,$$
$$\triangleq \hat{m}_2\ddot{q}_2 + u_2.$$

The following relatinships can be verified

$$J_{11}^2 + J_{21}^2 < (l_1 + l_2)^2,$$

$$J_{11}J_{12} + J_{21}J_{22}) < l_2^2 + 1.5l_1l_2,$$

$$J_{11}\dot{J}_{11} + J_{21}\dot{J}_{21} \leq (\frac{l_1^2}{2} + 2l_1l_2 + l_2^2)|\dot{q}_1| + (l_1l_2 + l_2^2)|\dot{q}_2|,$$

$$J_{11}\dot{J}_{12} + J_{21}\dot{J}_{21} \leq 1.5l_1l_2(|\dot{q}_1| + |\dot{q}_2|),$$
$$J_{12}\dot{J}_{12} + J_{21}\dot{J}_{21} \leq 1.5l_1l_2|\dot{q}_1|,$$

$$J_{12}^2 + J_{22}^2 = l_2,$$

$$J_{12}\dot{J}_{12} + J_{22}\dot{J}_{22} = 0,$$

$$\hat{m}_1 \le a_1 + a_2 + (l_1 + l_2)m_{\max},$$

$$\hat{m}_1 \ge a_1 - a_2 > 0,$$

$$\hat{m}_2 \le l_2^2 m_{\max},$$

$$\hat{m}_2 \ge a_3 > 0.$$

The numerical values for the manipulator parameters are

$$a_1 = 3.82,\ a_2 = 2.12,\ a_3 = 0.71,\ a_4 = 81.82,\ a_5 = 24.06,$$
$$m_1 = 15.91\text{kg},\ m_2 = 11.36\text{kg},\ m_{\max} = 10\text{kg},$$
$$l_1 = l_2 = 0.432\text{m},$$
$$V_{1\max} = V_{3\max} = 1 \text{ Nt m/rad s}^{-1},$$
$$V_{2\max} = V_{4\max} = 1 \text{ Nt m}.$$

For these parametric values, it can be proven that

$$u_1 < 7.36|\ddot{q}_2| + 6.532|\ddot{q}_1|^2 + (8.653)|\ddot{q}_2| + 1)|\ddot{q}_1| + 3.3|\ddot{q}_2|^2 + 124.2|\cos q_1| + 67.44 \triangleq u_{1m}, \tag{40}$$

$$u_2 < 2.237|\ddot{q}_1| + 1.06|\sin q_2|\,|\ddot{q}_1|^2 + 2.8|\ddot{q}_1|^2 + 67.44 + |\ddot{q}_2| \triangleq u_{2m}. \tag{41}$$

Assume for joint 1 that $|\ddot{q}_2| \leq C_{a_2}$, $|\dot{q}_2| \leq C_{v_2}$, and for joint 2 that $|\ddot{q}_1| < C_{a_1}$, $|\dot{q}_1| \leq C_{v_1}$ where C_{a_2}, C_{v_2}, C_{a_1}, C_{v_1} are real positive constant given by the manipulator physical constrains.

Then the control functions for joint 1 can be chosen as follows

$$p^{s}(\cdot) = -13.41\sigma_1(p_{12}z_1 + p_{22}z_2), \tag{42}$$

where $P_1 \triangleq \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$ solves (24) and $\sigma_1 > 0$.

$$\begin{aligned} p^{\epsilon}(t,x) &= -\rho S^{\epsilon}[kB^{T}P_1 z], \\ &= -\rho S^{\epsilon}(k(P_{12}z_1 + P_{22}z_2)), \end{aligned} \tag{43}$$

where $S^{\epsilon}(\cdot)$ satisfies (30). From (20),

$$\begin{aligned} k(\cdot) &\geq |e(t,x,v)|, \\ |e(t,x,v)| &= |h(\cdot) - T_r + G(\cdot)p^{o}(\cdot)|, \\ &\leq |h(\cdot)| + |-T_r + G(\cdot)p^{o}(\cdot)|. \end{aligned}$$

Since

$$\begin{aligned} h(\cdot) &= -\hat{m}_1^{-1}u_1, \\ G(\cdot) &= \hat{m}_1^{-1}, \end{aligned}$$

then

$$\begin{aligned} |e(\cdot)| &\le |\hat{m}_1^{-1}u_1| + |-T_r + \hat{m}_1^{-1}p^o| \\ &\le \max(\hat{m}_1^{-1})\,|u_1| + |-T_r + \hat{m}_1^{-1}p^o| \\ &= (\min\ \hat{m}_1)^{-1}\,|u_1| + |-T_r + \hat{m}_1^{-1}p^o|. \end{aligned}$$

Let

$$p^o(\cdot) \triangleq \hat{m}_{1_0} T_r, \tag{44}$$

where

$$\hat{m}_{1_0} \triangleq [\frac{1}{2}(\max(\hat{m}_1^{-1}) + \min(\hat{m}_1^{-1}))]^{-1}$$

$$T_r(t) = \ddot{q}_{r_1}(t)$$

and let

$$\begin{aligned} k(t,x) &\triangleq (\min \hat{m}_1)^{-1} u_{1m} + |-T_r + \min(\hat{m}_1^{-1})\hat{m}_{1_0} T_r| \\ &= 0.59 u_{1m} + .79\,|T_r|. \end{aligned} \tag{45}$$

To satisfy (20) let

$$\rho(\cdot) \triangleq (\max\ \hat{m}_1)^{-1} k(\cdot) = 13.4 k(\cdot). \tag{46}$$

Then the control law for joint 1 is

$$T_1(t,x) = p^o(\cdot) + p^e(\cdot) + p^s(\cdot)$$

with p°, p^e, and p^s, given by (44), (43) and (42) respectively.

The control law for joint 2 is obtained in a similar way, that is

$$p_2^o(\cdot) = \hat{m}_{2_0} T_{r_2}, \tag{47}$$

$$\hat{m}_{2_0} \triangleq [\frac{1}{2}(\max(\hat{m}_2^{-1}) + \min(\hat{m}_2^{-1}))]^{-1},$$

$$T_{r_2} = \ddot{q}_{r_2}(t),$$

$$p_2^s(\cdot) = -1.866\sigma_2(P_{12_2}z_{1_2} + P_{22_2}z_{2_2}), \quad \sigma_2>0 \tag{48}$$

$$k_2(\cdot) = 1.43u_{2m} + .453,$$

$$\rho_2(\cdot) = 1.866k_2(\cdot),$$

$$p_2^e(\cdot) = \rho_2(\cdot)k_2(\cdot)S_2^\epsilon(k_2(\cdot)(P_{12_2}z_{1_2} + P_{22_2}z_{2_2})), \tag{49}$$

where

$$P_2 = \begin{bmatrix} P_{11_2} & P_{12_2} \\ P_{12_2} & P_{22_2} \end{bmatrix}$$

solves (24) for some symmetric positive definite matrix Q_2 and $\sigma_2 > 0$, and u_{2m} is given by (41).

In both equations (43) and (48) the function $S^{\epsilon}(\eta)$ is chosen as $S^{\epsilon}(\eta) = (|\eta| + \epsilon)^{-1}\eta$, which satisfies conditions (29) and (30).

Simulation Results

In the following simulations, the manipulator is to track these reference signals

$$q_{r_2} = \frac{\pi t}{6} - \frac{1}{4}\sin\left(\frac{2\pi t}{3}\right),$$

$$q_{r_1} = q_{r_2} - \frac{\pi}{2},$$

$$t \in [0,3](\text{sec}).$$

The design parameters are adjusted to the following values.

$$P_{1,2} = \begin{bmatrix} 5.79 & 2.01 \\ 2.01 & .532 \end{bmatrix}$$

$$\sigma_1 = 10, \quad \sigma_2 = 10, \quad \epsilon_1 = 5.3, \quad \epsilon_2 = 5.8.$$

The boundary values for velocity and acceleration of each of the joints are assumed as follows

$$C_{a_{1,2}} = 4\,\frac{rad}{\sec^2}, \quad C_{v_{1,2}} = 2\,\frac{rad}{\sec}.$$

However, it is important to point out that these variables are not restricted to these bounds during the simulations.

The control variables (torque1 and torque2), position error, and velocity error shown in the following plots are given in Newton•meters, radians, and radians/sec.

Simulation 1. A constant mass of 5 kg is considered in this case. Figures 2 and 3 show the respective joint tracking of the desired trajectories, the tracking errors and the control action required by each joint.

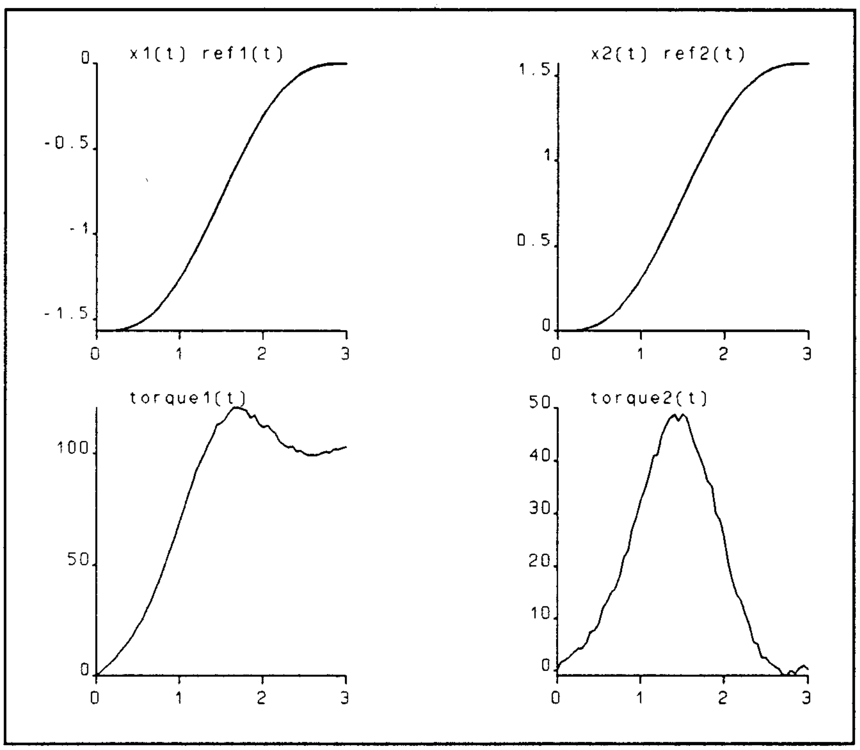

Figure 2. Simulation 1. Tracking results using a constant payload of 5 kg.

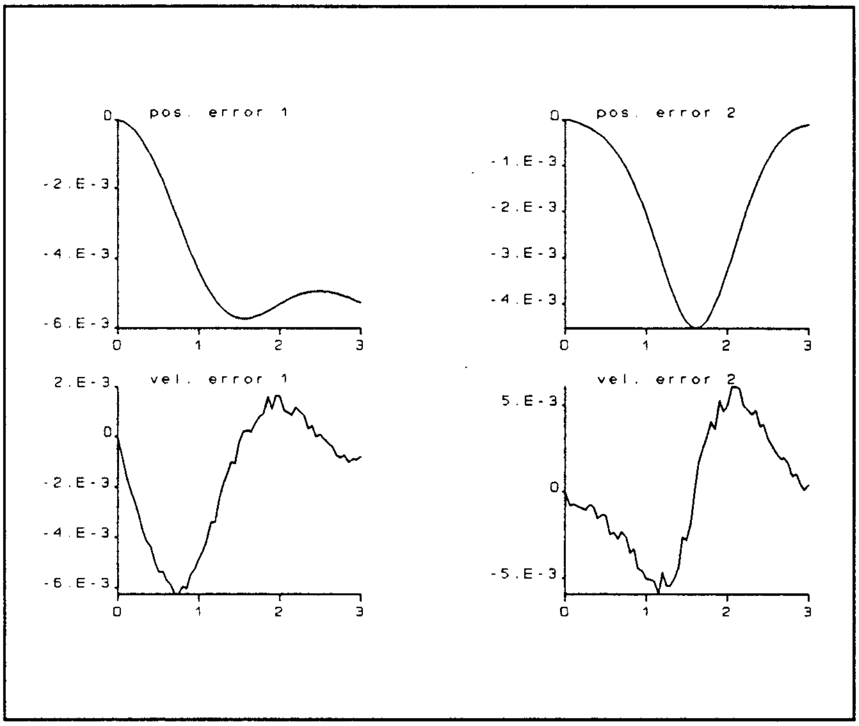

Figure 3. Tracking errors from simulation 1.

Simulation 2. In this simulation the initial state of joint 1 differs from the one given by the reference as follows

$$x_r(t_0) = [-1.5708,\ 0] \qquad x(t_0) = [-1,\ 0].$$

The convergence of the error model of each subsystems becomes more noticeable in this case as it is shown in figures 4 and 5.

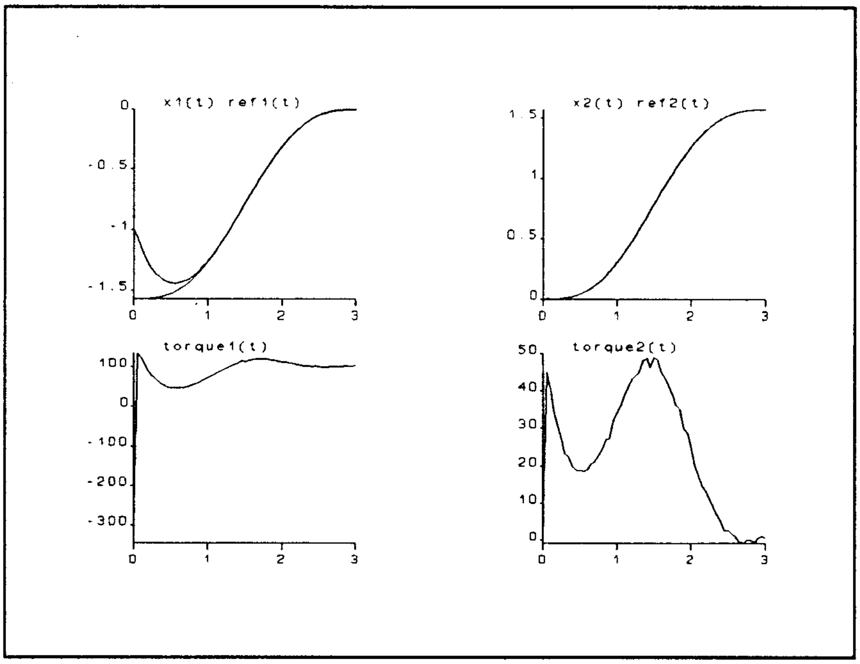

Figure 3. Simulation 2. Mismatch in initial conditions.

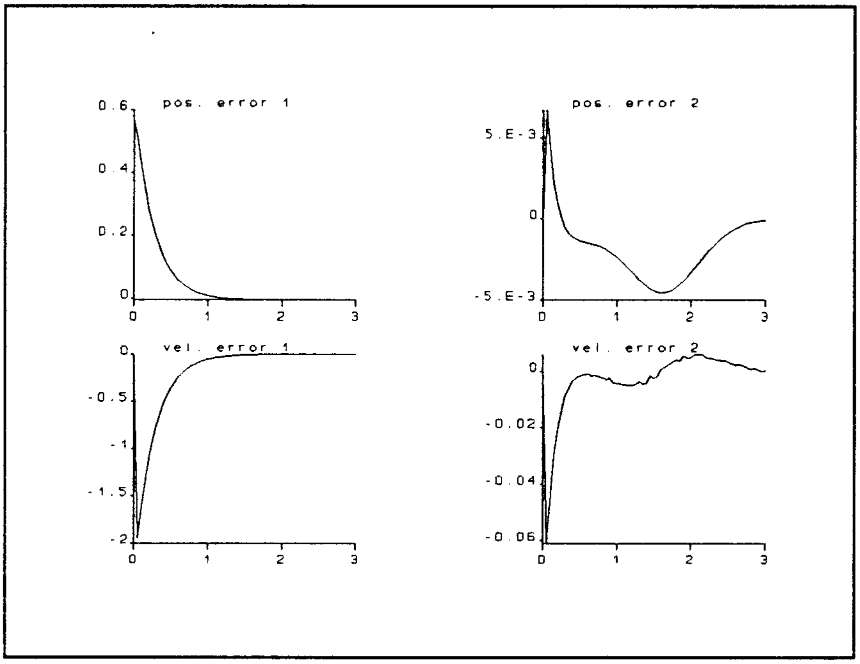

Figure 5. Tracking errors from simulation 2.

Simulation 3. In the subsequent simulation, the mass of the payload is changed from 5 kg to 0 at t=1.5sec., as in the case of an end-effector dropping the payload. Simulation results are illustrated in figures 6 and 7.

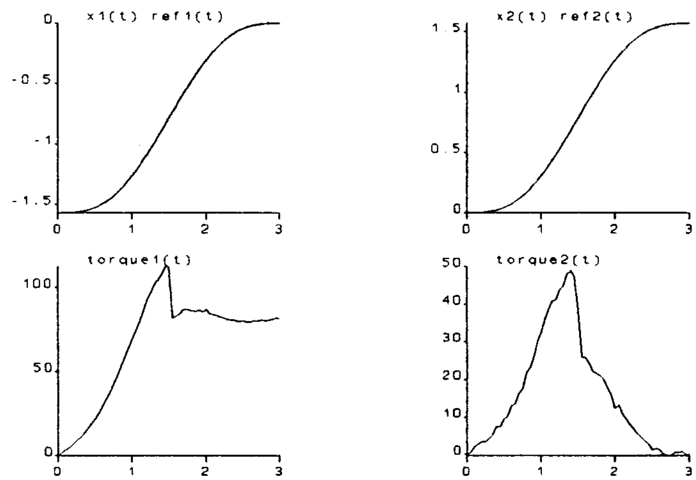

Figure 6. Simulation 3. Payload of 5 kg is dropped at t = 1.5 sec.

Simulation 4. Another interesting situation is the case in which the mass of the payload varies linearly with time according to

$$m = -\frac{5}{3}t + 5 \quad (kg.)$$

That is the case, for instance, when the end-effector pours its payload throughout the reference trajectory.

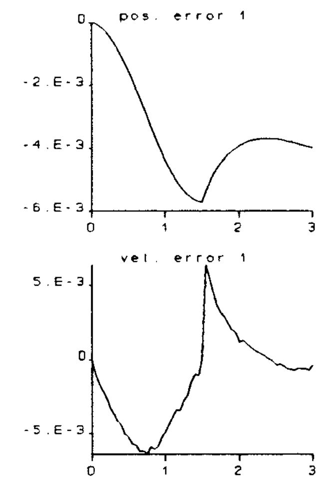

Figure 7. Tracking errors from simulation 3.

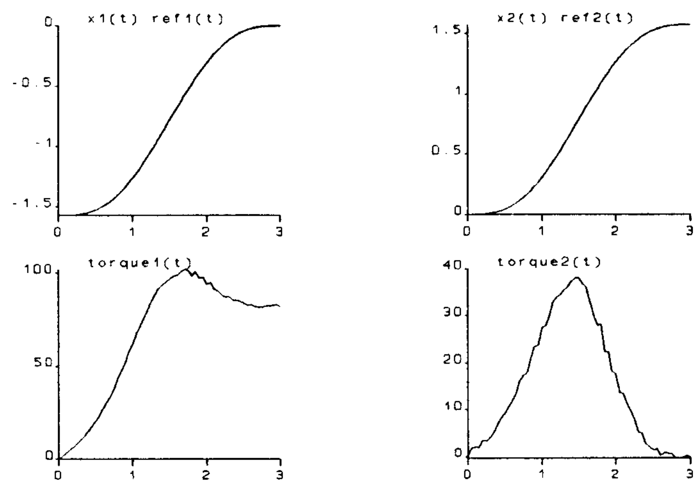

Figure 8. Simulation 4. Time varying payload.

Figures 8 and 9 show how the tracking is hardly affected by the time varying nature of the payload.

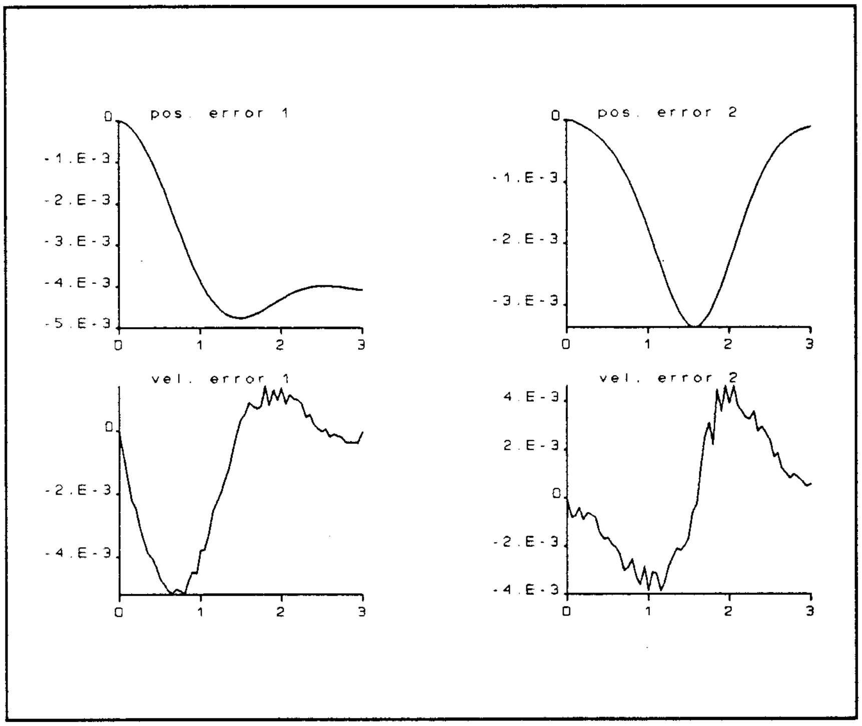

Figure 9. Tracking errors from simulation 4.

Simulation 5. We now consider a combination of perturbations. The initial state of the manipulator are set as for simulation 2, and we let also the payload be time varying as in the previous case. The simulation results are illustrated in figures 10 and 11.

Simulation 6. To illustrate the robustness of the closed loop system to the unknown viscous and Coulomb friction coefficients V1, V2, V3, and V4, we would let these parameters have any constant values within the bounds

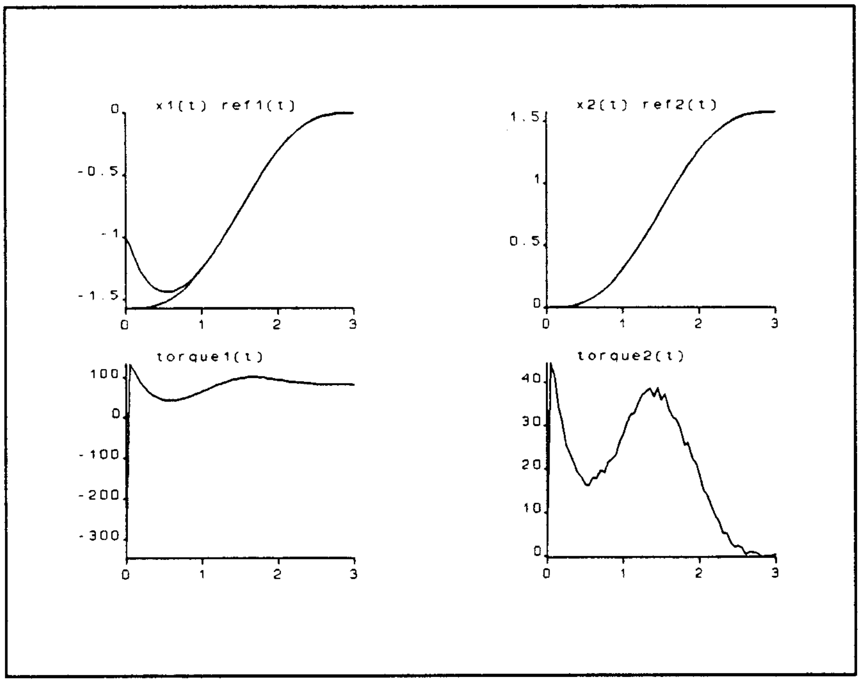

Figure 10. Simulation 5. Mismatch in IC. and time varying payload

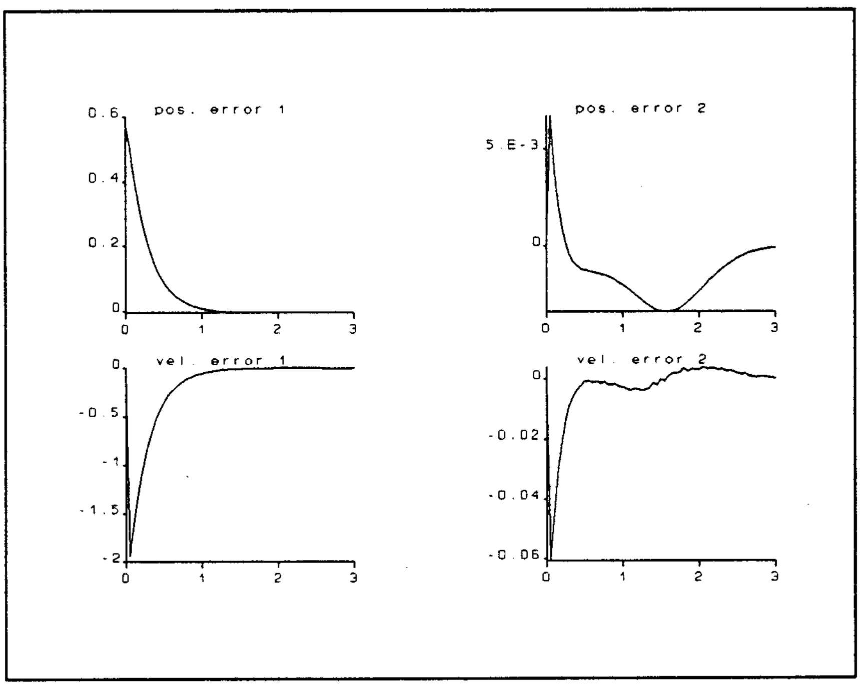

Figure 11. Tracking errors from simulation 5

assumed to derive the controller. However, in this simulation, these parameters are set as random variables with normal distribution, mean .5, and standard deviation equal to .1666. Although, random friction coefficients do not occur in most real situations, by assuming so, we account for most real cases in this regard. The smoothness of the control action is degraded by doing so though, as can be observed in figure 11. Figures 12 and 13 show the respective errors and the values of the friction coefficients during the simulation. The payload m is set to a constant value of 5 kg.

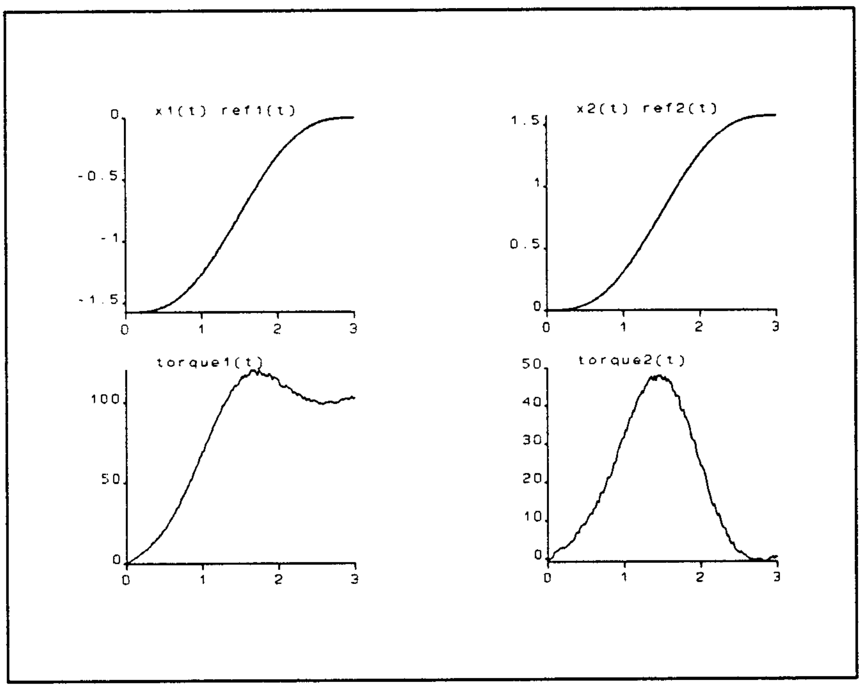

Figure 12 Simulation 6. Random friction coefficients.

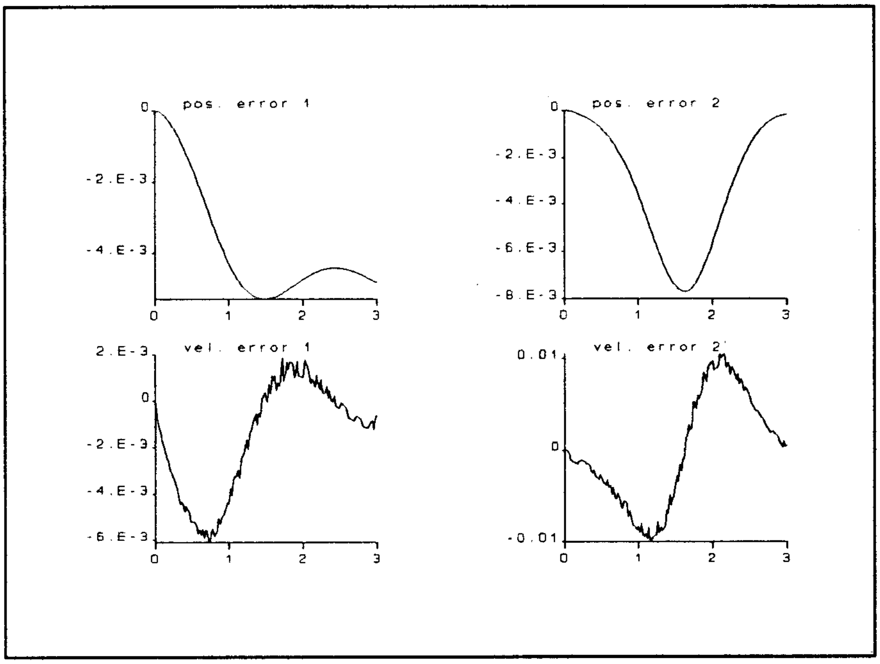

Figure 13. Tracking errors from simulation 6.

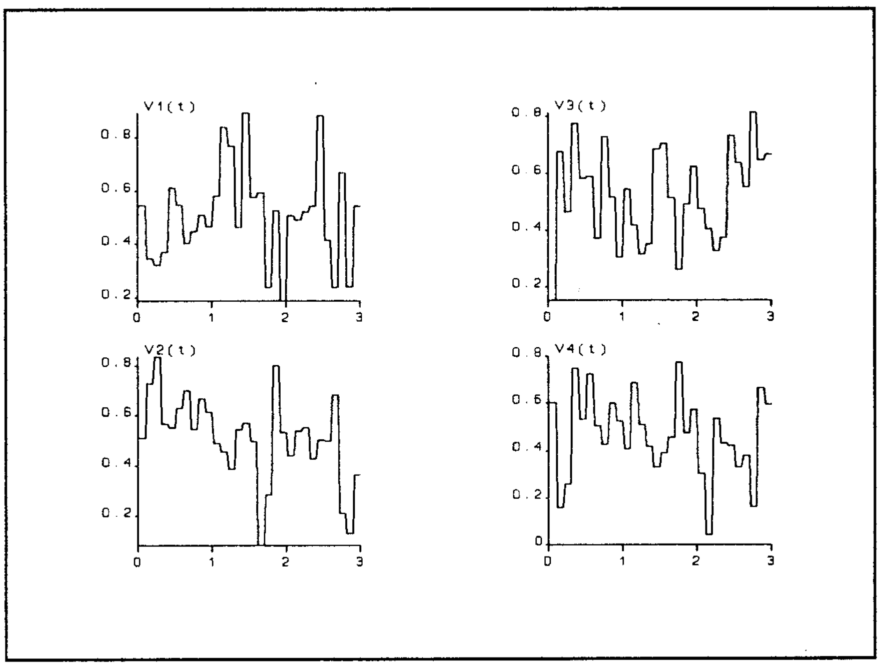

Figure 14. Viscous and Coulomb friction coefficients used in simulation 6.

The controller design parameters for all the previous simulations were the same, thus reassuring in this way the robustness of this control scheme. It is important to note that the control effort, the smoothness of the control action and the magnitude of the tracking errors could be improved if these controller parameters were properly adjusted for each one of the simulations. Consider as an example the following simulation.

Simulation 7. In this instance, we simulate the same situation as described in simulation 6, however, we now adjust the parameters of the controller to the following values so that the tracking errors are smaller in magnitude (see figures 15 and 16).

$$P_{1,2} = \begin{bmatrix} 15.0179 & 1.1147 \\ 1.1147 & .2126 \end{bmatrix}$$

$$\sigma_1 = 10, \quad \sigma_2 = 10, \quad \epsilon_1 = 5.3, \quad \epsilon_2 = 5.0. \tag{50}$$

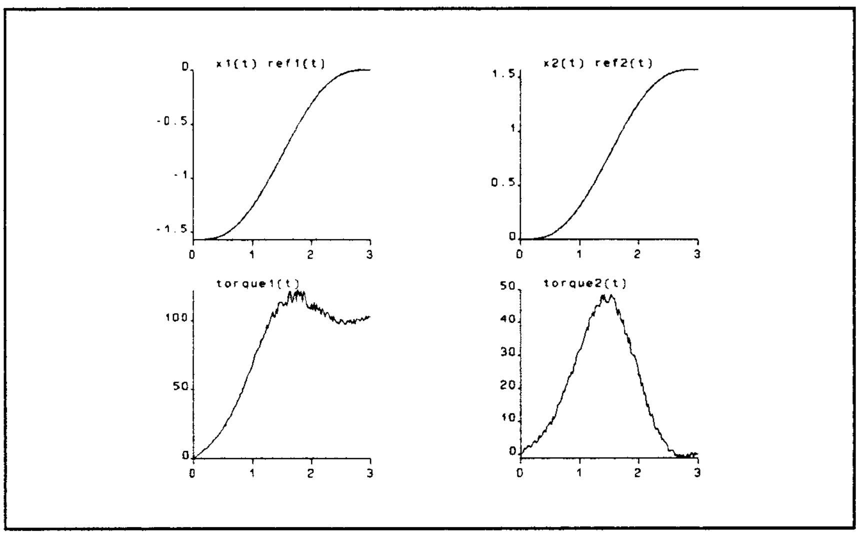

Figure 15. Simulation 7. Simulation 6 with controller parameters given by (50).

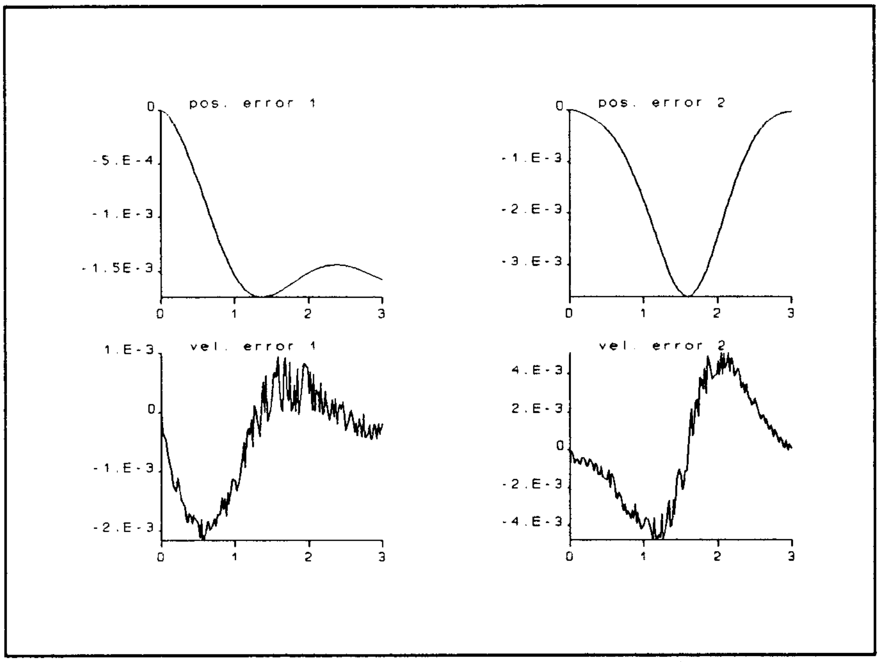

Figure 16. Tracking errors from simulation 7.

V. CONCLUSIONS

By means of Lyapunov theory, and by taking advantage of physical constraints, such as limited velocity and acceleration, of every real manipulator, we obtained a continuous nonlinear decentralized controller for which perturbation bounds and joint position and velocity is the only knowledge required to drive the joint position and velocity to desired reference trajectories within an arbitrary small error.

The control law obtained is composed of three parts, one defined as an arbitrary continuous function, another defined to overcome any possible instability effect due to the perturbation variables present in the inertia matrix. The third part ensures stability against perturbations from both the inertia matrix and the coupling terms among joints.

The scheme considers the coupling effects among joints as perturbations and accounts for any model uncertainty or external disturbance, thus ensuring its robustness. Moreover, the designed controller is also memoryless and computationally very simple.

The control scheme obtained in this work, guarantees physical realizability through continuous feedback control laws and offers design flexibility through parameter adjustment.

In contrast to a centralized structure, the controller can be designed and adjusted according to the specific joint dynamics, joint actuator limitations and joint task specifications optimizing the control effort at each joint.

Simulations employing a two-link manipulator shows the simplicity and efficiency of the controller when a fast reference trajectory is to be tracked by the manipulator under time varying uncertainties such as payload mass changes and unknown joint friction constants.

The dependence of the controller on physical constrains can be viewed as a limitation, for example, in cases where the boundary values are very large, a relatively large control effort would be required to mathematically ensure stability. However, it is important to keep in mind that the condition used in Lyapunov theory are only sufficient and as it has been shown through simulations, in many cases, these conditions can be violated and still obtain the desired stability. Nevertheless, in regard to this issue, further research needs to be done to make the controller of a particular joint independent of the velocity and acceleration of the other joints. We think that by establishing a closed bounded domain for the initial state within the physical constrains of the joint variables, and employing upper and lower bounding functions of the Lyapunov function defined for that joint, it may be possible to prove the existence of a control which does not depend explicitly on physical constrains of other subsystems state variables. In this case our approach could be used to control other complex nonlinear systems where either the bounds for velocity and acceleration are too large or in the case that they are not constrainable.

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