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Larry Elwyn Shirland for the Master of Science
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Title: AN APPLICATION OF ONE-FACTOR AND TWO-FACTOR INFORMATION CHANNEL MODELS TO MARKETING RESEARCH PROBLEMS

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This thesis presents an introduction to Marketing Research concepts, relates Operations Research to Marketing Research and discusses the limitations of each. An Information Theory approach based on Shannon's fundamental theorems and extended by Kunisawa is applied to one-factor and two-factor marketing problems.

The objective of one-factor information channel models is to use a single factor such as price, tonnage, utility, or other measurable quantity to explain the behavior habits of consumers. The two-factor model may be used when two independent factors can be isolated and used to describe a particular situation.

Two FORTRAN computer programs are presented, one for solving one-factor information channel problems, and the other for solving problems involving two factors.

The Information Theory analysis as discussed in this thesis is intended for use as a mathematical model and should prove beneficial in establishing ground rules and guidelines for decision making.

An Application of One-Factor and Two-Factor Information Channel Models to Marketing Research Problems

by

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Associate Professor of Industrial Engineering
in charge of major

Redacted for Privacy

Head of Department of Industrial Engineering

Redacted for Privacy

Dean of Graduate School

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I. INTRODUCTION

The purpose of this thesis is to discuss the uses and limitations of Operations Research techniques as applied to Marketing Research problems, and to present an Information Theory approach to specific Marketing Research decision applications. Both one-factor and two-factor information channels are discussed. Also included are (1) a computer program for solving specific single factor information channel problems, and (2) a computer program for solving two-factor information channel problems. Of secondary importance, this thesis will present an introduction to Marketing Research, its functions, methods, techniques, and limitations, in order to develop a complete study of the interrelationship between Marketing Research (MR) and Operation Research (OR) and the applicability of Information Theory in solving problems in Marketing areas.

Engineering Relevance

The marketing decision maker is faced with many problems associated with the analysis of vast amounts of interrelated data. The method of attack in order to reach a decision usually begins with information gathered concerning past experience which is then projected in order to obtain an estimate of a future program.

Operations Research techniques offer a constructive framework for organizing and analyzing this myriad of data in a manner that will insure fairly consistent results. Operations Research techniques are useful in that they describe the manner in which the marketing mechanism works. Once the mechanism has been described, the interrelation between the many variables which constitute the problem may be tied together and, hopefully, a satisfactory decision will result. Through OR techniques, the decision maker may be able to project, with a certain probability of success, the inputs to the problem in terms of dollars, physical quantities, manhours, etc., in order to obtain his desired objective.

Objectives and Organization

It is the major objective of this thesis to present specific areas of applications of Information Theory techniques to Marketing Research problems which will enable the decision maker to systematically evaluate alternatives. Problems such as brand loyalty, pricing, packaging, new product development and similar areas appear to be well suited to an Information Theory analysis.

Chapter II presents a discussion of Marketing Research functions, techniques and methods. Operations Research as related to Marketing Research is discussed and the limitations of each are presented.

Chapter III explains the relevance of Information Theory in Marketing Research, and presents five basic axioms upon which Information Theory is based. Also included, are corrections to Kunisawa's (1959) theory of Two-Factor Information Channel analysis.

Chapter IV introduces a One-Factor Information Channel model based on theories developed by Kunisawa (1959) and applied to Marketing Research. Two examples are presented; one on estimating the proportion of customers purchasing airline tickets given various price classes, and the other on estimating television brand selection given the price of each brand.

Chapter V presents an analysis involving a Two-Factor Information Channel when the marginal frequencies of each level are known. An example concerning two strata of investors purchasing stocks is given.

Chapter VI summarizes the methods discussed in this thesis and draws tentative conclusions. In addition, some suggestions for future research are presented.

The appendices include the FORTRAN source programs for One-Factor and Two-Factor channel models, the definitions of relative terms, a description of the Newton Rhapson Method for determining the root of the information channel equation, and a justification for using a Chi-Square Goodness-of-Fit test to test data in the one-factor information channel models.

II. FUNDAMENTALS OF MARKETING RESEARCH AND OPERATIONS RESEARCH INTERRELATIONSHIPS

Definition of Marketing Research

Marketing Research is concerned with the marketing of products and services, selling policies, advertising, the selection and training of salesmen, and any function which deals with the disposal of a company's products or services. Specifically, Marketing Research examines the operations involved in marketing. Its aim and purpose is to get products to consumers more easily, quickly and cheaply with the greatest profit to the company. In short, Marketing Research is involved with systematic and intensive studies into some part of the process of marketing goods or services (Bradford, 1951).

Functions of Marketing Research

According to the American Marketing Association, there are sixteen basic areas in which Marketing Research may be used:

1. Analysis of consumer markets.
2. New Product Development.
3. Competitive position of companies' products.
4. Estimating potential sales.
5. Sales methods or distribution policy.
6. Market analysis by areas.

7. Customer preferences.
8. Improvement of present products.
9. Pre-testing of new products.
10. Analysis of potential old market areas.
11. Advertising policy.
12. Relative distribution costs and profits of products.
13. General business forecasting.
14. Establishment of sales territories.
15. Establishment of sales quotas.
16. Relative profitableness of markets.

<u>Applications of OR techniques to Marketing Research</u>

Operations Research is a valuable tool to the market researcher because it embraces a systems viewpoint, which is fundamental to marketing concepts. OR may be beneficial in the analysis of marketing problems in the following ways (Schwartz, 1965):

1. OR models aid in the construction of theories.
2. Simulation and experimentation can provide valuable insights.
3. OR techniques have produced some of the most successful predicting systems.
4. Complex marketing situations are presented in a simplified easier-to-analyze form.
5. Quantitative models are objective rather than subjective.

Some specific ways in which OR is valuable to the market researcher are (Montgomery, 1968):

1. Related up-to-date information for decision making purposes may be obtained.
2. Marketing resources and operations may be controlled more effectively.
3. More adequate and rational marketing objectives, policies, strategies, and programs may be developed.

In the past few years, there have been several OR techniques applied to Marketing Research problems. Some of these applications include mathematical programming, queueing theory, and Bayesian Decision Analysis. Most work, however, involves statistical models such as multiple correlation, maximum likelihood estimation theory, and factor analysis. Psychologists analyze Marketing Research problems through selective perception, cognitive dissonance, and opinion leadership.

One such OR tool that should be extremely useful in the analysis of marketing problems is Information Theory. It should prove important in that it is not simply a theory dealing with information concepts, but may be used to partition some set into subsets. In other words, its usefulness lies in its ability to divide certainty (Probability 1) into various components, none of which is certain. Using this principle of division, predictions may be made (Theil, 1967).

The objective of mathematical techniques is to inject the scientific point of view into marketing problems. The view is that problems would be attacked systematically, facts gathered by precise methods, and decisions made objectively based on these facts. Each decision should have a means to a follow-up so that it may add to, or improve the quality of information stored for future use.

The models of Marketing Research that are applicable to scientific analysis may be classed as either behavioral or analytical. Behavioral models attempt to describe how people or firms behave, while analytical models explain how people or firms ought to behave in order to achieve an explicit objective, such as maximization of profits.

Optimization models provide a link between behavioral relationships and specific analytical criterion, such as sales or profit. These models attempt to aid the decision maker to optimize his resources. However, the results from a model of this type are only as good as the behavioral information that was obtained as an input to the system. One of the weaknesses of optimization models in Marketing Research problems has been the fact that the behavioral assumptions made have not been validated in the beginning (Massy, 1964).

The major difficulty in applying OR techniques is that Marketing Research problems encompass both engineering aspects, which lend themselves to analytical interpretations, and those of Behavioral

Science; such as economics, social psychology, and sociology. Many factors are not quantifiable, and some quantifiable factors are difficult to measure.

The reason why OR has not been used more extensively in Marketing Research areas is because; traditionally, marketing management executives have been unwilling to use mathematical or statistical methods. Many of these executives are personnel oriented rather than technically oriented. They are more inclined to rely on judgment and rule of thumb techniques than on analytical tools. Secondly, many decisions made by executives must be made in a hurry, so time is an important factor. It is simply not feasible to wait several days or weeks for an analytical analysis of many management problems (Montgomery, 1968).

Operations Research is presented with immense difficulties when attempting to analytically interpret results influenced by the "human element", especially when consumer attitudes, motives, reactions, and behavior are involved. Even in a typical OR problem, there is a great deal of uncertainty as to the validity of many of the assumptions made. Couple this with marketing factors that are not concrete, and OR applicability seems formidable. However, that is not to say that OR does not have a valuable place in areas of Marketing Research. It only means that Marketing Research has its own specific hurdles to be challenged and overcome.

There are many critics of the use of OR as a tool in Marketing Research applications. With the exception of Linear Programming, present mathematical models used in solving marketing problems are quite limited. However, it is the potential applications of OR to marketing problems that make it such a valuable tool. Still, the following criticisms should be kept in mind:

1. OR studies neglect human factors, which are the very crux of marketing problems.
2. OR studies seem to tackle the least crucial problems first. Little has been done concerning the problems of selling or salesmanship, while problems dealing with mass production have had many studies. However, the sales problems are as important if not more so.
3. OR models often fail to portray a proper perspective of marketing productivity.

Another major problem confronting the OR researcher and Marketing Research personnel is lack of communication. Marketing Research people and OR people need to get more in tune to each other's problems. Many Marketing Researchers are not aware of what is happening in the OR field and are not familiar with the tools that may be useful in solving Marketing Research problems. Also, most marketing personnel do not read the many specialized journals containing OR material; therefore, a defensive attitude is generated and a general lack of communication exists. The burden of responsibility to improve communications, however, lies with both the OR and Marketing Research personnel. Those involved in OR must try to understand the Marketing Researcher's problems, and vice versa (Schwartz, 1965).

Areas of Application for OR Techniques

Basically, there are eight different types or areas of Marketing Research in which OR techniques may be applied.

Consumer Research. Consumer research deals with the influences behind consumer buying. Specifically, product users are classified as to age, sex, economic or social status. Consumer attitudes and opinions are sought. These factors are determined by nationwide studies or by isolated test community surveys.

Motivational Research. Motivational Research deals with conscious or subconscious reasons for buying certain products. Inner feelings and attitudes of consumers are analyzed.

Market Analysis. Market Analysis delves into the sales potential of market areas. It determines weaknesses and strengths of market demand, and attempts to develop more efficient methods of realizing full market potential.

Sales Analysis. Sales Analysis attempts to optimize sales techniques; thereby, increasing market potential. For example, routes for salesmen are determined and sales trends are analyzed.

Product Research. Product Research investigates the feasibility of new products and evaluates those presently being produced in order to stay ahead of competition and retain an optimum share of the market.

Distribution Cost Analysis. Distribution Cost Analysis explores methods of improving marketing efficiency. The goal of distribution cost analysis is to maintain an adequate profit margin by eliminating wasteful methods.

Advertising Research. Advertising is one of the more costly tools of marketing and is also the most difficult to appraise. Advertising Research attempts to determine which advertising method will produce maximum product demand. It also attempts to measure whether a specific advertising technique has accomplished its task.

Industrial Marketing Research. Industrial Marketing Research differs from Consumer Research, in that Industrial Marketing Research deals with the rational buying habits of industrial buyers, and Consumer Research deals with emotional buying habits of the general public (Alevizos, 1959).

Techniques of Marketing Research

Before any analytical technique can be applied to Marketing Research problems, data must be collected; however, precise measurement of market conditions is not possible, since the researcher deals with human beings instead of quantifiable parameters. If a scientific approach is used, however, and the variability of consumer behavior is kept in mind, much valuable information may be obtained. Following are some of the basic data collection methods.

The Survey Method

The Survey Method seeks facts and opinions about consumers. This is usually accomplished through a questionaire completed by a sample of the public. There are three basic methods of obtaining information through questionaires. These are: (1) Controlled Opinion, (2) Open-End Questions, and (3) Alternative Choice Questions.

Controlled Opinion. Controlled Opinion techniques involve the following:

1. Ranking
2. First Choice
3. Last Choice
4. Combination of First and Last Choice
5. Paired-Comparison
6. Objective-Scale
7. Objective-Standard

Open-End Questions. Open-End Questions are of the type: "What would you do about this problem?" These questions are asked by a trained interviewer in an attempt to obtain deep rooted opinions.

<u>Alternative Choice</u>. This method requires the consumer to choose between several alternative choices. Questions such as: "Which of these designs do you think is best, and why?" are asked (Alevizos, 1959).

Besides questionaires, there are two other major methods for obtaining information by the survey method. The first is by telephone interviews. This method is easy, quick, and if there are no long involved questions to ask, gives fairly reliable data for analysis. The second method is personal interviews. Personal contact gives the interviewer a better chance to probe for inner feelings of consumers.

<u>Limitations of the Survey Method</u>. There are several important limitations of the survey method:

1. Respondants often fail to recall needed information.
2. Respondants often fail to generalize.
3. Respondants often fail to identify their motives or reasons for acting.
4. Respondants often have personal biases.
5. Respondants often answer so as to impress the interviewer.

The Historical Method

This method is concerned with gathering statistics and facts about historical data in order to make comparisons and extrapolate this data into the future. Some typical studies concern:

1. Market Potential Measurement.
2. Sales Quota Data.
3. Price Trend Analysis.
4. Territorial Boundary Determination.
5. Saturation Demand.
6. Trading Area Analysis.
7. Consumer Indexes.
8. Facility Location.
9. Distribution Cost Analysis.
10. Sales Performance Analysis.
11. Sales Time and Duty Analysis.
12. Standard Setting for Sales Analysis.
13. Channels of Distribution.
14. Operation and Control Analysis.

Limitations of the Historical Method. There are several important limitations of the historical method which are:

1. The data used may be outdated.
2. The data may not be pertinent to the problem being studied.
3. The data may not contain complete information.

The Observational Method

The observational method involves studies to determine what people are doing, or have done. Inferences are drawn to try to determine hidden motives of consumers (Alevizos, 1959).

When using this approach, a researcher would avoid direct contact with consumers, thus eliminating much of the human factor in the analysis. The main characteristics of this method are:

1. The researcher does not talk to consumers.
2. No formal means is provided consumers to make decisions related to alternatives or to specially designed obstacles.

Limitations of the Observational Method. Some important limitations of the Observational Method are:

1. People may not do things as they normally would when they are being watched.
2. Improper conclusions may be reached from unintended actions of the subjects.
3. No means to obtain opinions from the subjects is provided.
4. The researcher may be biased.

The Experimental Method

The Experimental Method uses trial testing in certain market areas to determine various market phenomena. This method is difficult to administer since there are so many variables involved, and for a valid test, all variables except the one being considered should be held constant.

<u>Limitations of the Experimental Method.</u> Some limitations of the Experimental Method are:

1. The trial area may not be representative of the entire market.
2. The variables in the analysis are extremely difficult to hold constant.
3. The novelty effect of the product may result in false conclusions.

Limitations of Marketing Research

Just as the OR techniques themselves have limitations, the entire concepts and applicability of Marketing Research to decision making problems have their limitations.

Time

Management always wants answers "yesterday", and often, time is simply not available in which to gather data, let alone make an analysis. Data from outside sources sometimes lag by a week, month or even a year such as in the Federal Government or other political agencies.

Money

Usually the amount of data collection and the depth of analysis depends upon how much the decision maker is willing to spend. Unless funds necessary to collect data are freely available, the data will necessarily be limited in quantity and/or quality.

Skill

The level of skill of all concerned affect the quality of the Marketing Research analysis. Skill in conducting research is a combination of traning and experience. A researcher must be careful in his use of data generated by sources other than himself. There are three questions that the decision maker should ask himself: (1) "Who conducted the study? ", (2) "What was the objective of the study? ", and (3) "How was the study conducted? ".

Bias

This limitation was briefly described in the discussion of OR techniques previously. Bias is present at every stage of a research study. The previous discussed limitations of time, money and skill introduce bias. The entire manner in which the research is carried out has bias of one form or another introduced; from personal bias of the researcher himself, to bias of persons who supply the data (Alevizos, 1959).

Judgment

Judgment plays an important part in all of the data collection methods previously described. Judgment is involved when a marketing manager decides if and when a Marketing Research analysis is required, when the interviewer decides what method to use, and when a consumer responds to the stimulus provided by the researcher. All of the judgment factors create possible errors in the results obtained and should be kept in mind when making an analysis.

Validity

The data obtained by a Marketing Research method must be carefully examined for its validity. An analysis must be checked to see if it does what it was originally intended to do. A test for validity should include: (1) a determination of whether the data was obtained from a source which should be cognizant of the problem being studied, and (2) a determination of whether the problem has been solved with this data.

Reliability

Data obtained must be tested for its reliability. Questions such as: "Was the sampled population representative of the entire population in question?", or "Are the data biased?" have been used by Beckman (1962) to test data reliability.

III. INFORMATION THEORY AS APPLIED TO MARKETING RESEARCH

Relevance of Information Theory to Marketing Research

When introducing a new product on the market, there is always a great deal of uncertainty as to what the probable demand for the product will be. Once the product has been introduced, demand may be estimated using the moving average or exponential smoothing methods. However, before a new product is produced in quantity, it is desirable to have some feel for its potential demand. Most methods available today for predicting demand are subjective in nature. Methods described previously in this thesis could be used, such as, questionaires sent to segments of potential market areas, trial testing or special promotions. These methods are good, but they allow very little flexibility in analyzing alternatives such as price differentials, multiple grade products, etc.

To alleviate the weakness of these traditional approaches, an analytical model based on Information Theory is proposed to simulate market behavior. This model is an application of Shannon's (1949) fundamental theorems first proposed for electrical communication theory, but since adopted by Theil (1967) to analyze economic behavior, by Moles (1966) to construct aesthetic perception models, and by Kunisawa (1959) as an Operations Research technique. Though Theil (1967) used Information Theory to analyze problems in forecasting changes in production quantities, Kunisawa (1959) must be credited with first applying Information Theory to Marketing Research problems. Thiel (1967) used a log-linear model and logit regressions for investigating problems in economics. Unfortunately, Kunisawa's book (1959) was published only in Japanese and is essentially unknown in this country. This thesis presents Kunisawa's theory, with extentions and corrections. Also included are computer programs for solving problems that may be described using either one or two factors in the information channel.

One of the desirable features of the Information Theory approach for solving Marketing Research problems is that the probabilities computed are actually Maximum Likelihood estimates. The models that will be developed in Chapters IV and V appear beneficial because they may be used as:

1. An analytical model for consumer motivation research.
2. A planning model for new product design and development.
3. A budgeting and programming model for forecasting product acceptance and demand.
4. An evaluation model for sales performance.

The Marketing System as a Communication System

An electrical communication system is usually represented symbolically as in Figure 1.

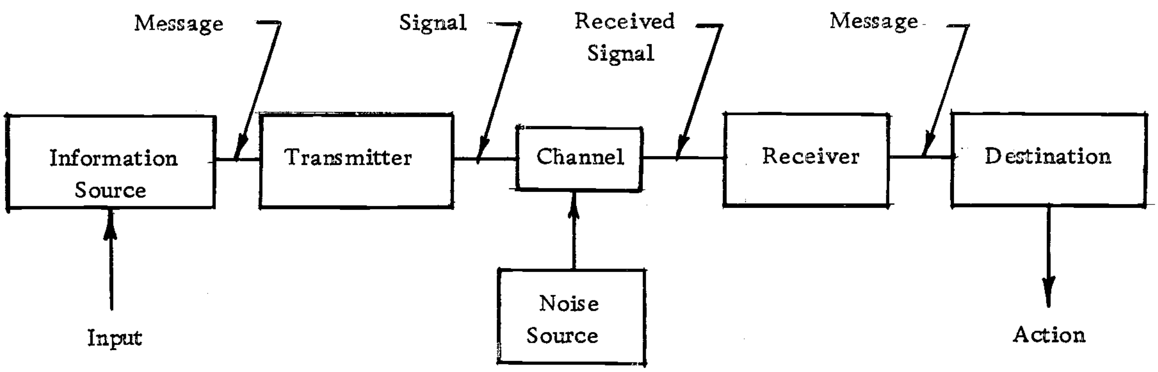

Figure 1. Electrical communication system.

The information source selects a message from a set of possible messages, the transmitter changes the message into a signal which is sent through the communication channel to the receiver. A human communication system involves the brain (information source), the voice (transmitter), and the air (channel), the ear (receiver), and the brain of the person receiving the message.

One possible analog using Marketing would appear as in Figure 2.

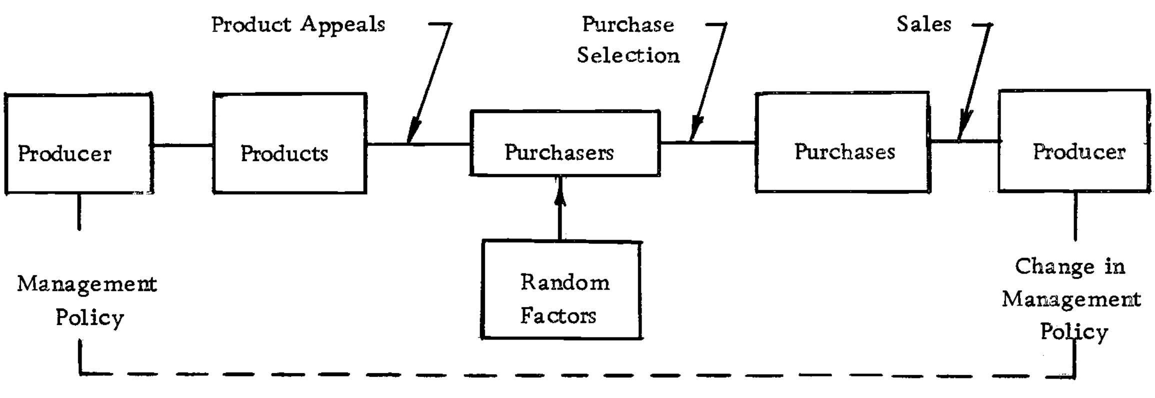

Figure 2. Marketing communication system.

The marketing analogy involves a circular information system in which the producer, who is the information source, selects a management policy which is transmitted as products. The channel is the purchaser who makes decisions. Noise, in the form of unforeseen events, such as changes in styles, economic depressions, or other unusual market conditions influences the purchaser. The message is received in the form of purchases which are passed on as sales to the producer who is the destination. The producer then makes policy decisions which are fed back into the system.

Basic Axioms of Information Theory

There are five important axioms which form a basis for the conclusions reached in the study of Information Theory (Shannon, 1949; Theil, 1967; Watanabe, 1969). The axioms are presented here without proofs.

1. Information is dependent only on the probability, $p(X)$, of a variable X.
2. The information content, $H(X)$, is a continuous function of X where:

$$0 \leq p(X) \leq 1$$

3. $H(0) = \infty$; and $H(1) = 0$.

4. $H(X)$ is a monotonically decreasing function or:

$$H(X_1) \geq H(X_2) \text{ if } 0 \leq p(X_1) \leq p(X_2) \leq 1$$

where $X_1 = x(t_1)$

$$X_2 = x(t_2) \text{ and } t_2 = t_1 + \Delta t$$

5. Additivity of independent events:

$$H(X_1 X_2) = H(X_1) + H(X_2) \qquad \text{if } p(X_1) \geq 0$$

$$p(X_2) \leq 1$$

The only expression of the information content, or entropy, satisfying the five axioms is of the form:

$$H(X) = K \sum_{i=1}^{k} p(x_i) \log p(x_i)$$

Shannon adopted the expression $H(X) = -\sum_{i=1}^{k} p_i \log p_i$, and assigned the unit of bit (binary diget; also known as binit). Since, $\log_2 A = (\log_2 e)(\log_e A)$, this thesis will use either $-\log_e p_i$ or $-\log_2 p_i$, depending on the convenience of use in numerical calculations.

Shannon's Fundamental Theorems

Associated with the concept of entropy and the technical problem of transmitting coded messages over a communication channel, Shannon proposed several theorems. One is associated with a noiseless channel and the other with a noisy channel. The two theorems are (Shannon, 1949):

Theorem I

Given an information source having entropy H bits per symbol, and a channel capacity of C bits per second, it is possible to encode messages in such a way that they may be transmitted at a rate which has a maximum value equal to C/H. Also, it is not possible to encode and transmit messages such that a rate greater than C/H may be attained without error.

Theorem II

Given a discrete noisy channel, if the entropy, H, is less than the channel capacity, C, messages may be encoded in such a way that an arbitrarily small percentage of errors are obtained at the receiver.

Shannon avoided the criticisms of other researchers of his time by specifying that his theory only applied to the technical problems of communication and not to the semantic or pragmatic problems with which social scientists were concerned. Technical problems as investigated by Shannon involve the accuracy of transmitting symbols of communication. Semantic problems, on the other hand, deal with how precisely transmitted symbols convey the desired meaning. Problems involving motivation, understanding, advertising, etc., fall into this catagory. Pragmatic problems are concerned with how successful the meaning of a message transmitted to the receiver leads to the desired result (Weaver, 1949). Kunisawa (1959) felt that the pragmatic marketing model represented in Figure 2 could be considered a communication problem and treated statistically as a noisy channel. Even though each consumer acts individually, the statistical behavior of the consumers as a body remains predictable and controllable through information content.

The application of Information Theory to a marketing problem, therefore, must be based on a careful investigation of assumptions that are required to permit our treating a pragmatic or semantic problem as a technical problem.

Assumptions Needed to Apply Information Theory to Marketing Research

The basic assumption that must be made in order to apply Information Theory to Marketing Research problems is that consumers tend to randomize their behavior if no clear incentive (factor) is provided. This means that given a specific situation, there will be a determinable proportion of consumers which will favor each alternative course of action and that the distribution of choice over the range of possible values will follow a random process subject to the load of the controlling factor.

Another assumption is that consumers will act in a rational manner. However, in many applications, because of unforeseen events such as fluctuating market conditions, fads, changes in styles, etc., consumers may not act in a predicted manner. In these cases where the causes are not readily identifiable, the Information Theory approach will not apply.

When attempting to describe multiple factor consumer behavior using the single factor concepts, it must be assumed that the motivations of consumers by a particular factor are independent of any other motivating factor. Theil (1967) eliminated this assumption by including the interactions between various factors through the use of a regression model and weighting each level proportionately to the inverse of its variance. Multiple factor models can also be constructed (Muroga, 1953) to accomodate these interaction effects.

Corrections to Kunisawa's Theory

The chapters which follow, Chapter IV on One-Factor Information Channels and Chapter V on Two-Factor Information Channels, are bases on theories developed by Kunisawa (1959). However, there appear to be two application errors in Kunisawa's discussion of

Two-Factor Information Channels. First on page 76 of his text Kunisawa uses Pearson's Chi-Square method to determine the values for Y_1 and Y_2 which minimize the difference equations D_i given in Chapter V. The method is in error in that it uses only probabilities as values in the Chi-Square equation when it should actually use observed data also. The Chi-Square test is applicable only for counted data where n, the number of observations is relatively large. Therefore, it is not applicable as a means of determining Y_1 and Y_2 unless a sufficient amount of data is first available. The analysis in Chapter V of this thesis uses a more appropriate analysis.

Secondly, on page 79, Kunisawa describes a graphical method which he states may be used to determine the minimum of the difference equations D_i. However, his method simply solves one of the equations for Y_1 and Y_2 and does not necessarily determine the points where the total difference is minimum.

IV. ONE-FACTOR INFORMATION CHANNELS

Introduction

The simplest type of information channel involves only a single factor. In Marketing Research applications, this factor could be price, weight, income, etc.. The single factor in the analysis is used to describe the behavior of consumers and to estimate the proportion of the population interested in each level. For example, if price is used as the factor and it is desired to estimate the demand for various brands of a product, the levels will be represented by a particular brand and each will have a corresponding characteristic value equal to the price of that brand.

In the theoretical development which follows, a single factor will be used to describe a market situation. However, it should be pointed out that the model does not imply that the single factor is the only motivating force behind consumer behavior. On the contrary, the single factor is used because it best describes the behavior of consumers.

Theoretical Development

Consider an information channel containing L levels: 1, 2, 3, , , , , , K having corresponding characteristic values: t_1, t_2, t_3, , , , t_K.

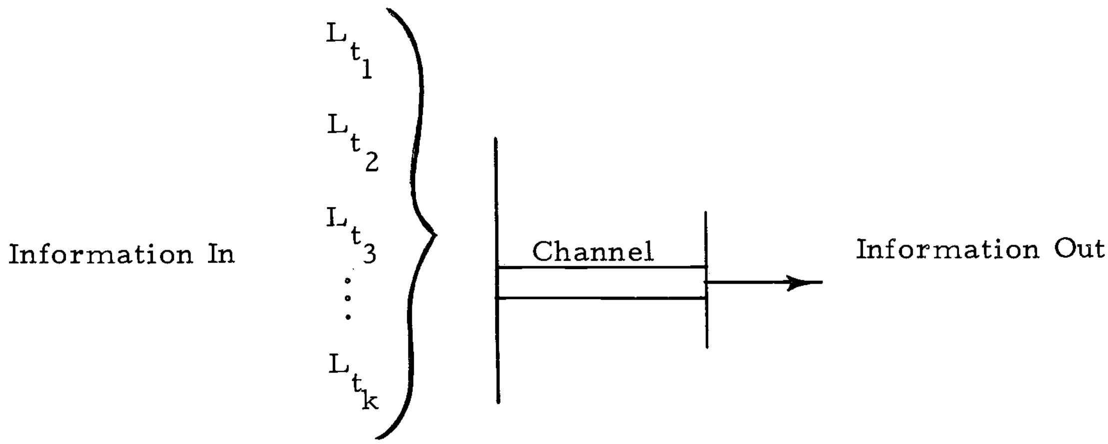

Figure 3. Input-output model.

By combining these levels, information is transmitted. The special case where ordering of any arrangement is not considered and when each level is independent from any other, will be analyzed.

The following notation and definitions will be used in the theoretical development which follows.

L_i = Level (i), i = 1, 2, , , , k.

t_i = Characteristic value of level (i), i = 1, 2, , , , k.

N = Number of positions in the message space.

n_i = Number of times value i appears in a particular arrangement.

C = Maximum channel capacity which is equal to $\mathrm{Log}_2 W_0$.

W_0 = Real root of $\sum_{i=1}^{k} W^{-t_i} = 1$.

p_i = probability of occurrence of a level or characteristic value in a message.

$$H^* = -\sum_{i=1}^{k} n_i \log p_i = -N \sum_{i=1}^{k} p_i \log p_i$$

$$H = -\sum_{i=1}^{k} p_i \log p_i$$

H_c = Information transmitted per characteristic value.

$$T^* = \sum_{i=1}^{k} n_i t_i \quad \text{and} \quad T = \sum_{i=1}^{k} p_i t_i$$

$$W^{-1} = 2^{-C}$$

If a message space contains N positions in which each level may appear, there will be k possibilities for position 1; k possibilities for position 2; etc. or:

$$\begin{array}{lcccc} & 1 & 2 & 3 & N \\ \text{Positions} & \boxed{k} & \boxed{k} & \boxed{k} & \boxed{k} \end{array}$$

In other words, in any particular arrangement, we would expect to find level 1 occuring n_1 times; level 2 n_2 times, etc. and since there exists a one to one relationship between each level and its characteristic value, each characteristic value will appear n_1, n_2, ..., n_k times respectively.

The respective probabilities of occurrence of each characteristic value, then, would be:

$$p_i = \frac{n_i}{N} \qquad i = 1, 2, , , , k \tag{1.0}$$

and

$$\sum_{i=1}^{k} p_i = 1 \quad i = 1, 2, , , , k. \tag{1.1}$$

The probability that this particular arrangement is transmitted is:

$$p = (p_1)^{n_1} (p_2)^{n_2} \ldots (p_k)^{n_k} \tag{1.2}$$

The information transmitted by this arrangement will be:

$$H^* = -\text{Log } p = -\text{Log } (p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}) \tag{1.3}$$

or

$$H^* = -n_1 \text{ Log } p_1 - n_2 \text{ Log } p_2 - \ldots - n_k \text{ Log } p_k \tag{1.4}$$

From (1.0) $n_i = p_i N$, therefore, we may write (1.4) as:

$$H^* = N(-p_1 \text{ Log } p_1 - p_2 \text{ Log } p_2 - \ldots - p_k \text{ Log } p_k) \tag{1.5}$$

or

$$H^* = N \sum_{i=1}^{k} -p_i \text{ Log } p_i \tag{1.6}$$

In order to determine the information transmitted per characteristic value, we must divide Equation (1.6) (the information transmitted by a particular arrangement) by the sum of the characteristic values appearing in that arrangement or:

$$H_c = \frac{H^*}{T^*} = \frac{N \sum_{i=1}^{k} -p_i \text{ Log } p_i}{\sum_{i=1}^{k} n_i t_i} \tag{1.7}$$

From (1.0) again we note that $p_i N = n_i$. Substituting this relationship into the denominator of Equation (1.7) gives:

$$H_c = \frac{H^*}{T^*} = \frac{N \sum_{i=1}^{k} -p_i \text{ Log } p_i}{N \sum_{i=1}^{k} p_i t_i} \tag{1.8}$$

or

$$H_c = \frac{H}{T} = \frac{-\sum_{i=1}^{k} p_i \text{ Log } p_i}{\sum_{i=1}^{k} p_i t_i} \tag{1.9}$$

In order to determine the maximum amount of information transmitted per characteristic value subject to the restriction that the sum of the individual probabilities must equal unity, we may use the Lagrange Multiplier technique. In equation form, then, we want to compute:

$$\text{Max } H_c = \frac{-\sum_{i=1}^{k} p_i \text{ Log } p_i}{\sum_{i=1}^{k} p_i t_i} \tag{1.10}$$

Taking derivatives and setting equal to zero:

$$\frac{\partial(H_c)}{\partial(p_i)} + \frac{\partial}{\partial(p_i)} [\lambda(\sum_{i=1}^{k} p_i - 1)] = 0 \tag{1.11}$$

and

$$\frac{\partial(H_c)}{\partial(\lambda)} + \frac{\partial}{\partial(\lambda)} [\lambda(\sum_{i=1}^{k} p_i - 1)] = 0 \tag{1.12}$$

Differentiating Equation (1.12) gives:

$$\sum_{i=1}^{k} p_i = 1 \tag{1.13}$$

Differentiating Equation (1.11) gives:

$$\frac{-(\log p_i + 1)\sum_{i=1}^{k} t_i p_i + t_i \sum_{i=1}^{k} p_i \log p_i}{\left(\sum_{i=1}^{k} p_i t_i\right)^2} + \lambda = 0 \tag{1.14}$$

Remembering that $T = \sum_{i=1}^{k} p_i t_i$, and $H = -\sum_{i=1}^{k} p_i \operatorname{Log} p_i$ and substituting into (1.14):

$$-T \log p_i - T - t_i H + \lambda (T)^2 = 0 \tag{1.15}$$

and

$$\log p_i = -1 - \frac{Ht_i}{T} + \lambda T \tag{1.16}$$

Multiplying both sides of Equation (1.16) by p_i and summing:

$$-H = -1 - H + \lambda T \tag{1.17}$$

Therefore:

$$\lambda = \frac{1}{T} \tag{1.18}$$

Substituting the value for λ found in Equation (1.18) into Equation (1.16) gives:

$$\operatorname{Log} p_i = -1 - \frac{Ht_i}{T} + 1 \tag{1.19}$$

or:

$$p_i = 2^{-(\frac{H}{T})t_i} \qquad (1.20)$$

if logs to base 2 are used.

Since H/T in Equation (1. 20) is the maximum amount of information transmitted, we may define max H/T to be C, the maximum channel capacity or:

$$p_i = 2^{-C\,t_i} \qquad (1.21)$$

and if we define W^{-1} to be 2^{-C}, Equation (1. 13) then becomes:

$$\sum_{i=1}^{k} W^{-t_i} = 1 \qquad (1.22)$$

There is only one real root (defined as W_0) to the polynomial of Equation (1. 22).

Therefore, from the definition of W^{-1}, the maximum channel capacity C will be:

$$C = \log W_0 \qquad (1.23)$$

Given any particular one-factor information channel application, we may now compute the respective probabilities of each level since C may be obtained from Equation (1. 23), W_0 from Equation (1. 22), and the respective probabilities from Equation (1. 21).

Information Theory Applied to Marketing Research: Example # 1

In order to illustrate the theory of single factor information channels, the following experiment was conducted using as the information source a group of 78 students at Oregon State University.

A decision was made to use the Survey Method of data collection as described in Chapter II. A questionaire was prepared and distributed to the students (See Appendix E). One of the questions asked was the following:

"Suppose that you were contemplating a plane trip from Portland, Oregon to New York City. Given the price structure as in Table 1, which fare class ticket would you purchase?"

Table 1. Plane ticket costs for example # 1.

	Costs
First Class	$350
Tourist	$200
Excursion	$150

The single factor information channel approach may be used in this example to test the following Hypothesis:

"The single factor -Price- may be used to describe the ticket purchasing habits of air travelers."

Table 2 shows the distribution of replies for the 78 students who answered the questionaire.

Table 2. Distribution of replies for example # 1.

Fare Class	Frequency of	Percent
First Class	8	10.2%
Tourist	24	30.8%
Excursion	46	59.0%
Total	78	100.0%

The Information Channel equation, then, is:

$$W^{-350} + W^{-200} + W^{-150} = 1.0$$

which may be reduced to:

$$W^{-2.34} + W^{-1.33} + W^{-1.0} = 1.0$$

By using an appropriate search (See Appendix C) or the computer program (Appendix B), the root of the above equation is found to be:

$$W = 2.13907$$

and the channel capacity $C = \log W = .760$ (logs to base e are used for ease of calculation).

Appropriate probabilities may be computed as:

$$P(\text{First Class}) = e^{-(.760)(2.34)} = 16.88\%$$

$$P(\text{Tourist}) = e^{-(.760)(1.33)} = 36.37\%$$

$$P(\text{Excursion}) = e^{-(.760)(1.00)} = 46.75\%$$

Therefore, if price may be used to describe the buying habits of airline customers, one would expect to sell 16.88% First Class tickets, 36.37% Tourist Class tickets, and 46.75% Excursion Class tickets.

In order to test the hypothesis stated earlier, a Chi-Square Goodness-Of-Fit test is made for a significance level of 5% and two degrees of freedom. The critical Chi-Square value is 5.99147. For a justification in favor of using the goodness-of-fit test, see Appendix F or Wine (1964).

Table 3. Chi-Square test for example # 1.

	Frequency Observed	Theoretical Estimate
First Class	8	13.2
Tourist	24	18.8
Excursion	46	36.0
Total	78	78.0

$$\chi^2 = \frac{(8 - 13.2)^2}{13.2} + \frac{(24 - 18.8)^2}{18.8} + \frac{(46 - 36.0)^2}{36.0}$$

$\chi^2 = 5.1928 < 5.99147$ so we cannot reject the hypothesis. This does not necessarily mean that the hypothesis is correct. To be more rigorous, additional data would have to be taken. However, we may conclude that the data came from a distribution not greatly different from the one formulated in the one-factor information model. It might also be pointed out that the computer program of Appendix B would perform all the necessary calculations for a single factor information channel problem including the Chi-Square Goodness-of-Fit test.

Decision Making Using Single Factor Information Channel Analysis

Suppose that the three fare classes of Example # 1 were being charged by a West Coast Airlines. Suppose also that management has set these prices based on costs, past history, competition, etc. However, it is now desired to determine if there might be a more optimum allocation of seats to the various fare classes that may result in higher profits to the company.

Given the profit curve of Figure 4, and using the single factor information channel analysis, a simple technique is available that may be used to determine total profit for various combinations of fare classes.

Suppose that management has decided that there must be at least a \$50.00 price differential between fare classes, and due to competition, no price above \$400 is desired. Also, 100 seats are available which may be allocated to the various fare classes in any way desired.

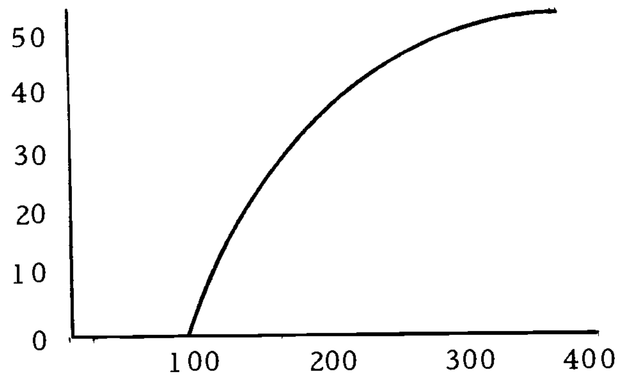

Figure 4. Profit curve for decision problem.

An analysis of this problem, then, is a matter of exploring various price combinations between $100 and $400 with at least a $50.00 price differential between classes.

For the allocation of Example # 1, the profit would be as shown in Table 4.

Table 4. Profit table for decision problem.

Class	Price	# Seats	Profit per ticket	Total Profit
First Class	$350	17	$46.40	$ 788
Tourist	$200	36	$33.00	$1189
Excursion	$150	47	$23.00	$1080
			Total Profit	$3057

For a price combination of: First Class = $400, Tourist = $250, and Excursion = $200; the probabilities would be 19.4%, 36.2%, and

44.4% for First Class, Tourist, and Excursion respectively. The total profit would appear as in Table 5.

Table 5. Profit Table # 2 for decision problem.

	Price	# Seats	Profit per ticket	Total Profit
First Class	$400	20	$50.00	$1,000
Tourist	$250	36	$39.00	$1,405
Excursion	$200	44	$33.00	$1,451
			Total	$3,856

Since in this example, the profit curve is a monotonically increasing function, it is obvious that the maximum profit will occur for prices of $400, $350, and $300 for First Class, Tourist and Excursion classes respectively. For this allocation, the profit would be determined as in Table 6.

Table 6. Profit Table #3 for decision problem.

	Price	# Seats	Profit per ticket	Total Profit
First Class	$400	28	$50.00	$1,400
Tourist	$350	33	$46.40	$1,530
Excursion	$300	39	$43.00	$1,679
			Total	$4,609

In many applications, the profit curve may not be of a nature that results in an obvious solution that maximizes profits. The above procedure, then, would prove invaluable, especially if a computer program were written which would search possible alternatives. However, this method should be used only as a guide to decision making, and not an end in itself. Special conditions may introduce restrictions into the model such as practices of competition or special cost problems, etc. In any case, a model often helps to fortify judgment in subjective decision making procedures. Neither should be used without the other.

Information Theory Applied to Marketing Research: Example #2

The questionaire of Appendix E posed the following:

"Assume that you have an evening free to do as you wish. Which of the following would you choose"? The one-factor information channel model may be used to test the following hypothesis:

"The single factor -Cost- may be used to describe students' choice of leisure time".

Table 7. Leisure time costs for example #2.

	Cost
Stay home	.65
Go out for dinner and cocktails	10.00
Go out for pizza and beer	5.00
Go to a movie	3.50
Go for a drive	1.00

The following responses were received from the 50 students answering the questionaire:

Table 8. Frequency of replies for example #2.

	Frequency of Reply	Percent
Stay home	9	18.0%
Go out for dinner and cocktails	10	20.0%
Go out for pizza and beer	20	40.0%
Go to a movie	3	6.0%
Go for a drive	8	16.0%

The one-factor channel model may be used to test the hypothesis that students will spend their leisure time according to the cost of various activities available to them.

The Information Channel equation is:

$$W^{-.65} + W^{-1.0} + W^{-3.5} + W^{-5.0} + W^{-10.0} = 1.0$$

Therefore, if cost may be used to describe the ways in which people use leisure time, one would expect the following probability distribution. Calculations were done by the computer program of Appendix B.

P(Stay home) = 55.05%

P(Dinner and cocktails) = 0.01%

P(Pizza and beer) = 1.01%

P(Go to movie) = 4.02%

P(Go for drive) = 39.91%

The data necessary to conduct a Chi-Square test is shown in Table 9.

Table 9. Chi-Square data for example #2.

Activity	Frequency Observed	Theoretical Estimate
Stay home	9	27.500
Go out for dinner and cocktails	10	.005
Go out for pizza and beer	20	.475
Go to the movies	3	2.020
Go for a drive	8	20.000
Total	50	50.000

Then:

$$\chi^2 = \frac{(9 - 27.5)^2}{27.5} + \frac{(10 - .005)^2}{.005} + \frac{(20 - .475)^2}{.475}$$

$$+ \frac{(3 - 2.02)^2}{2.02} + \frac{(8 - 20.000)^2}{20.000}$$

$$= 20,241.491$$

Comparing 20,241 with a critical value of 9.48775 with 4 degrees of freedom results in a rejection of the hypothesis. However, the necessary condition that each theoretical value must be greater than 5 is violated. Therefore, the Chi-Square test cannot be used except as a rough estimator of the closeness of fit of the distributions. A linear regression model could be used to fit the data. Since the observed data is so far from the theoretical estimates for every level except one, it may safely be concluded that cost cannot be used as the only factor in the problem.

Another factor might be the utility or personal satisfaction received from each of the various alternatives. The solution to the problem would then involve determining a numerical value for the utility derived from each alternative, and using it in conjunction with the cost factor. This would result in a two-factor model as described in Chapter V.

Information Theory Applied to Marketing Research: Example #3

A third hypothetical problem was posed in the questionaire of Appendix E. The supposition was:

"Assume that you are in the market for a new color TV, and you've narrowed your choice down to the possibilities listed below; which one would you purchase?"

	Price
Model A	$550
Model B	$500
Model C	$450
Model D	$400

In the questionaire itself, the various models were given the brand names of popular TV manufacturers such as RCA, General Electric, Sears Roebuck, etc. It was felt that this example would not be explained in terms of the one factor - Price; instead, it would seem that brand name, quality, or other factors would better explain purchasing habits of buyers. In order to investigate this possibility, the following hypothesis may be tested using results from the questionaire.

"The single factor - Price - may be used to describe the purchasing habits of TV buyers".

Table 10 shows the distribution of replies to the question posed above.

Table 10. Distribution of replies for example #3.

Model	Frequency of Replies	Percent
A	16	30.2%
B	9	17.0%
C	8	15.0%
D	20	37.8%
Total	53	100.0%

The Information Channel equation, then, is:

$$W^{-550} + W^{-500} + W^{-450} + W^{-400} = 1.0$$

or

$$W^{-1.375} + W^{-1.250} + W^{-1.125} + W^{-1.00} = 1.0$$

With the value -1.0 being associated with the low priced Model D, and -1.375 with Model A, etc.

Using an appropriate search technique or the computer program, one obtains values for $W = 3.25047$ and $C = \log_e W = 1.166$.

Respective probabilities, then, are:

$$P(A) = e^{-(1.375)(1.166)} = 19.77\%$$

$$P(B) = e^{-(1.250)(1.166)} = 22.91\%$$

$$P(C) = e^{-(1.125)(1.166)} = 26.55\%$$

$$P(D) = e^{-(1.00)(1.166)} = 30.76\%$$

The Chi-Square calculations are:

$$\chi^2 = \frac{[20 - (.3076)(53)]^2}{(.3076)(53)} + \frac{[8 - (.2655)(53)]^2}{(.2655)(53)} + \frac{[9 - (.2291)(53)]^2}{(.2291)(53)} + \frac{[16 - (.1977)(53)]^2}{(.1977)(53)}$$

$$= 7.220$$

At the .05 significance level, the critical Chi-Square value with three degrees of freedom is 7.8147. Since $7.220 < 7.8147$, we cannot reject the state hypothesis. More data would be essential if definite conclusions were to be drawn. In fact, if the hypothesis could be rejected we could then test an appropriate hypothesis involving more than one factor. The two factor analysis of Chapter V will investigate problems of this type. Also, in order to obtain a more reliable and more accurate test, questionaires should be sent to actual potential customers of the product in question rather than to a small isolated group.

This example could also be looked at as a decision making problem. Specifically, it may be desired to know what effect a price change in one brand will have on its market share, and consequently, it profits. For instance, managers of the Brand B company might want to explore the possibility of lowering their price to $450. Assuming no change in the price of competitors, Brand B's new market share and profits could be calculated. However, it is highly likely that the competitors will also react to a price change. In this case, Information Theory should be even more valuable in that it may be used in a Brand Switching Model or Brand Loyalty Model to predict various market shares to each competing company under different market conditions.

V. TWO-FACTOR INFORMATION CHANNELS

Introduction

Many problems arise in which information provided by the one-factor analysis as presented in Chapter IV is not adequate. Models with two or more factors may be required in order to describe a particular Marketing Research situation. In these problems, the same theory can be applied by assuming that the proportion of the population in each strata are motivated mainly by a particular factor and are not influenced by any other factor. For example, one group of persons who prefer to travel by ship instead of by air may be concerned mainly with safety, another with price. It would be of interest, then, to calculate the proportion of persons interested in the safety factor and those motivated mainly by the price factor. As another example, consider two types of investors purchasing stocks. One group may be motivated solely by the price to earnings ratio and another by the price of the stock itself. It would be of interest, here, to estimate the proportion of investors interested in each factor.

The theoretical development which follows assumes that a two-factor information channel with M levels is desired. It is also assumed that data is available as to the marginal frequencies of each level. The solution, then, involves determining an estimate for the marginal probability of each factor.

Theoretical Development

Suppose that in a Marketing Research situation, there are two available quantifiable factors, F_1 and F_2, containing M levels, 1, 2, ···, M. Associated with each factor and each level, there are characteristic values, t_{ij}, as in the one-factor analysis. It is assumed that data is available as to the marginal probability of each level, X_j, corresponding to the proportion of the population which is interested in a particular level.

In tabular form, the above values would appear as in Table 11.

Table 11. Two-factor channel matrix.

Levels		L_1	L_2		L_m	
Factors	F_1	t_{11}	t_{12}	......	t_{1m}	Y_1
	F_2	t_{21}	t_{22}	......	t_{2m}	Y_2
	X_j	X_1	X_2	......	X_m	

Y_1 and Y_2 are the marginal probabilities associated with factors F_1 and F_2 respectively. Since there are only two factors in the analysis, we know that:

$$Y_1 + Y_2 = 1$$

or

$$Y_2 = 1 - Y_1$$

Also, since it was assumed that the two factors are independent, each may be treated as a one-factor information channel and conditional probabilities using the one-factor channel equation given in Chapter IV may be computed.

for F_1; $$W_1^{-t_{11}} + W_1^{-t_{12}} + \ldots\ldots + W_1^{-t_{1m}} = 1$$

and if logs to base e are used:

$$C_1 = \log_e W_1$$

$$p_{1j} = e^{-C_1 t_{1j}}$$

and

$$\sum_{j=1}^{m} p_{1j} = 1$$

For F_2: $$W_2^{-t_{21}} + W_2^{-t_{22}} + \ldots\ldots + W_2^{-t_{2m}} = 1$$

and

$$C_2 = \log_e W_2$$

$$p_{2j} = e^{-C_2 t_{2j}}$$

$$\sum_{j=1}^{m} p_{2j} = 1$$

The results will appear as in Tables 12 and 13.

Table 12. Probability calculations in a two-factor analysis.

Levels	L_1	L_2		L_m	
Factors F_1	$e^{-C_1 t_{11}}$	$e^{-C_1 t_{12}}$	.....	$e^{-C_1 t_{1m}}$	Y_1
Factors F_2	$e^{-C_2 t_{21}}$	$e^{-C_2 t_{22}}$	.....	$e^{-C_2 t_{2m}}$	Y_2
	X_1	X_2		X_m	

or simply:

Table 13. Results of two-factor channel analysis.

	L_1	L_2		L_m	
Factors F_1	p_{11}	p_{12}	.....	p_{1m}	Y_1
Factors F_2	p_{21}	p_{22}	.....	p_{2m}	Y_2
	X_1	X_2		X_m	

Equations for Y_1 and Y_2 may be written as:

$$
\begin{aligned}
p_{11}Y_1 + p_{21}Y_2 &= X_1 \\
p_{12}Y_1 \quad p_{22}Y_2 &= X_2 \\
\cdot \qquad\qquad & \quad \cdot \\
\cdot \qquad\qquad & \quad \cdot \\
\cdot \qquad\qquad & \quad \cdot \\
p_{1m}Y_1 + p_{2m}Y_2 &= X_m
\end{aligned}
$$

We would like to determine values for Y_1 and Y_2 which will most closely satisfy these equations. In other words, values for Y_1 and Y_2 are desired which will minimize the absolute difference of $\sum_{i=1}^{m} |D_i|$ in the following equations:

$$D_1 = X_1 - (p_{11}Y_1 + p_{21}Y_2)$$

$$D_2 = X_2 - (p_{12}Y_1 + p_{22}Y_2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$D_m = X_m - (p_{1m}Y_1 + p_{2m}Y_2)$$

The objective function, then, is:

$$\text{minimize} \quad |D_1| + |D_2| + \ldots + |D_m|$$

The computer program of Appendix D, using an iterative process, will readily carry out the necessary calculations which will minimize the total difference equation.

Two-Factor Information Channel Example #1

The data for this example was obtained from the 1969 annual reports of four common stocks listed on the New York Stock Exchange. The problem is formulated as follows: Suppose that it has been determined that investors who purchase a particular type of stock may be classified into two groups; (1) those who are motivated to buy stocks based on the Price of a stock, and (2) those who are motivated by the Price to Earnings Ratio. Four stocks are available for analysis: the numbers of shareholders, prices, and Price to Earnings Ratio are as given in Table 14. It should be noted that the prices used are the average stock prices for 1969.

Table 14. Input data for two-factor channel example.

Stock	Average Price per Share	Yearly Earnings per Share	Price of Earnings Ratio	Number of Shareholders
A	$69.00	$3.95	17.4	10,500
B	$20.00	$0.59	34.0	10,300
C	$20.50	$1.32	15.3	10,800
D	$43.00	$1.42	30.3	5,500

The two factors, then, are:

F_1 - The average yearly price of the stock.

F_2 - The Price To Earnings Ratio.

The marginal frequencies, X_j, are:

$X_1 = 28.3\%$

$X_2 = 27.7\%$

$X_3 = 29.1\%$

$X_4 = 14.9\%$

The respective characteristic values, t_{ij}, are:

$t_{11} = 69.0$

$t_{12} = 20.0$

$t_{13} = 20.5$

$t_{14} = 43.0$

$t_{21} = 17.4$

$t_{22} = 34.0$

$t_{23} = 15.3$

$t_{24} = 30.3$

The two-factor information channel data will appear as in Table 15.

Table 15. Characteristic values for two-factor stock purchase example.

Levels	A	B	C	D	
F_1 (Price)	69.0	20.0	20.5	43.0	Y_1
F_2 (P/E Ratio)	17.4	34.0	15.3	30.3	Y_2
X_j	.283	.277	.291	.149	

It is desired to determine Y_1, the proportion of investors interested solely in the price of a stock and Y_2, the proportion motivated mainly by the Price to Earnings Ratio.

For F_i, the conditional probabilities of each level, p_{ij}, (j=1, 2, 3,4)(i=1, 2) may be calculated using the computer program of Appendix B.

The results are:

$$p_{11} = 0.0450$$

$$p_{12} = 0.4088$$

$$p_{13} = 0.3998$$

$$p_{14} = 0.1461$$

$$p_{21} = 0.3413$$

$$p_{22} = 0.1216$$

$$p_{23} = 0.3888$$

$$p_{24} = 0.1483$$

Table 16 gives these results in tabular form.

Table 16. Conditional probabilities of two-factor example.

Level	A	B	C	D	
F_1 (Price)	.045	.409	.400	.146	Y_1
F_2 (P/E Ratio)	.341	.126	.389	.148	Y_2
X_j	.283	.277	.291	.149	

The equations for , D_i , i = 1, 2, 3, 4 are:

$$D_1 = 0.283 - (.045\ Y_1 + .341\ Y_2)$$

$$D_2 = 0.277 - (.409\ Y_1 + .126\ Y_2)$$

$$D_3 = 0.291 - (.400\ Y_1 + .389\ Y_2)$$

$$D_4 = 0.149 - (.146\ Y_1 + .148\ Y_2)$$

$$\text{minimize} \qquad |D_1| + |D_2| + |D_3| + |D_4|$$

The computer program of Appendix D gives results of:

$$Y_1 = .200$$

$$Y_2 = .800$$

An estimate for the strata of investors considering Price only would be 20%, and 80% for those considering the Price to Earnings Ratio.

The above analysis could easily be extended to include three or more factors. However, at present, the FORTRAN program is designed to handle only two-factor problems.

Two-Factor Information Channel: Example #2

The questionaire of Appendix E, described in Chapter IV, contained a hypothetical situation involving two factors. The following question was asked:

"Assume that you are planning to go to San Francisco for a week. Which one of the following modes of travel would you choose?"

Costs and travel times were given as in Table 17.

Table 17. Travel times and costs for two-factor example #2.

	Plane	Train	Bus
Cost	$30.00	$20.00	$10.00
Travel Time	1 hr.	4 hrs.	10 hrs.

The following responses were obtained.

Table 18. Replies for cost/time two-factor problem.

Mode of Travel	Frequency of Replies	Percent
Plane	31	62.0%
Train	16	32.0%
Bus	3	6.0%
Total	50	100.0%

The two-factor channel model may be used to determine the strata of persons influenced by cost alone, and the strata motivated mainly by the time factor.

The computer program of Appendix D gives the following probability matrix.

Table 19. Results of cost/time two-factor example.

	Plane	Train	Bus	
Cost	.160	.296	.544	Y_1
Time	.712	.255	.033	Y_2
X_j	.620	.320	.060	

The values for Y_1 and Y_2 were determined to be .166 and .834 respectively. We conclude, then, that 16.6% of the persons who answered the questionaire are motivated by cost and 83.4% by time.

VI. SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

Summary and Conclusions

It was the purpose of this thesis to present an introduction to Marketing Research concepts, to show how Operations Research may be used in marketing, and to present an Information Theory approach to a specific type of marketing problem. The One- and Two-Factor Information Channel approaches illustrated in this thesis are novel methods of solving many marketing decision problems. One idea is to use a single factor such as price, tonnage or other measureable quantity and attempt to explain purchasing habits of consumers. If it can be established that a single factor may be used to describe the behavior of consumers, decisions may be made regarding price changes, production quantities, etc.. The two-factor method may be used where two independent factors can be isolated and used to describe the system in question.

It should be noted, however, that when a single factor such as price is used in an analysis, it does not mean that consumers consider only that factor in purchasing a product. On the contrary, there are probably many factors contributing to the purchase of a product. The Information Theory analysis only attempts to explain these purchasing habits in terms of the single factor. If it should be found that the process cannot be explained by one factor, it does not necessarily mean that this factor is not being considered; it simply means that perhaps two or more factors are involved and that the problem should be analyzed using a two or multi-factor information channel approach. In any event, the Information Theory approach to solving marketing problems appears to have a tremendous amount of potential in marketing decision applications.

The Information Theory approach used here has several inherent limitations. First, Kunisawa (1959) shows that the determination of probabilities using Information Theory results in a maximum likelihood estimator. However, this procedure may not result in a "best estimator". Secondly, Information Theory looks at marketing problems on a macroscopic scale. Individual consumer behavior or potential behavior is not accounted for. Information Theory cannot account for sudden changes in market conditions or changes in styles, buying trends, competitive position, etc.. Thirdly, in the single factor analysis, we are attempting to describe a system in terms of one factor that may or may not actually be the major motivating force behind consumer behavior. In the two-factor analysis, we must assume that each factor is solely responsible for the actions of consumers and is not dependent on any other factor.

The Information Theory analysis is presented here for use as a mathematical model and it should prove beneficial in establishing groundrules and guidelines for decision making. However, no model should be used as an end in itself; sound judgment should always accompany any analysis.

Recommendations for Future Research

This thesis has only scratched the surface as far as Information Theory analysis of Marketing problems is concerned. The most useful area of study would be in an analysis of marketing problems using a multi-factor Information Theory approach where no marginal probabilities are known. For example, suppose that a company desired to analyze their product's competitive position in comparison to other firms producing a similar product. Questionaires could be sent to present or prospective customers asking some of the following types of questions.

1. Given Brand A, B, or C, which one would you buy if price were of major importance to you?
2. Considering Quality of product, which one would you purchase?
3. Considering Reliability, which would you choose?

Answers to the above questions could be then tabulated as in Table 20.

Table 20. Example of multi-factor analysis.

Brands / Characteristics		A X_1	B X_2	C X_3
Price	Y_1	p_{11}	p_{12}	p_{13}
Quality	Y_2	p_{21}	p_{22}	p_{23}
Reliability	Y_3	p_{31}	p_{32}	p_{33}

In Table 20, A, B, and C denote the various brands of the product being studied. The Characteristics, Price, Quality, and Reliability, are one word descriptions of the questions posed in the questionaire. The p_{ij}'s in the matrix are the proportions of answers favoring a particular brand for a particular question. For example, p_{11} is the proportion of consumers who favor brand A when price alone is considered as a buying criterian. p_{12} is the proportion favoring Brand B, etc. $\sum_{j=1}^{3} p_{ij} = 1$ for each i. In other words, $p_{11} + p_{12} + p_{13} = 1$, etc.

Using the above information, the market shares of each competing company may be computed using a procedure developed by Muroga, (1958). In the above example, X_1 would correspond to Brand A's market share, X_2 to B and X_3 to C. The relative weights assigned to each characteristic may also be computed as Y_1 for Price, Y_2 for Quality and Y_3 for Reliability (See Table 20).

Another area of application may be in determining Brand Loyalty. A probability matrix similar to the one illustrated in Table 20 could be generated with the values in the matrix representing the transitional probabilities of a consumer purchasing Brand B, for instance, given that Brand A was previously purchased. Using this information, the multi-factor Information Theory approach may be used to predict the proportion of customers who will remain with a product and that proportion who will switch products. This analysis could be conducted under various assumed or real market conditions in order to analyze a company's real or potential position.

The multi-factor Information Channel approach to solving problems of the type just described was first developed by Muroga (1958) and applied to Marketing Research problems by Kunisawa (1959). There is a basic inconsistency in Kunisawa's notation, however. In his book, Kunisawa first presents a matrix notation using Y_i to represent row marginal probabilities and X_j as column marginal probabilities, and later transposes the Y_i's and X_j's in his theoretical development.

The two-factor information channel analysis, in which the marginal probabilites X_j of each level are known, could easily be extended to include three or more factors. The computer program of Appendix D could also easily be altered to handle problems of this type.

There is a growing need today for new and improved mathematical techniques to aid the decision maker in solving many complex problems. Information Theory analysis appears to be such a technique.

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APPENDICES

APPENDIX A

DEFINITION OF RELEVANT TERMS

Information Theory: Mathematical interpretation of a measure of time or cost of transmitting messages (Raisbeck: 1963).

Marketing: As defined in the Constitution of the American Marketing Association; "The performance of business activities that direct the flow of goods and services from producer to consumer or user."

"The process whereby society, to supply its comsumption needs, evolves distributive systems composed of participants, who, interacting under constraints -- technical (economic) and ethical (social) -- creates the transactions or flows which resolve market separations and result in exchange and consumption (Bartels: 1968).

"The performance of all activities necessary for ascertaining the needs and wants of markets, planning product availability, effecting transfers in ownership of products, providing for their physical distribution, and facilitating the entire marketing process" (Beckman: 1962).

Marketing Research: The application of scientific principles to observational, experimental, historical, and survey methods in a careful search for more accurate knowledge of consumer and market behavior, so that more effective marketing may be developed

(Alevizos: 1964).

Marketing Research Analysis: Building a predictive model of some aspect of individual, group, or organizational behavior and relating that model to the problems of decision making by management (Massy: 1964).

APPENDIX B

COMPUTER PROGRAM FOR SOLVING ONE-FACTOR INFORMATION CHANNEL PROBLEMS

Computer Program

In order to apply the theory of Chapter IV, a method is needed to determine the root of Equation (1.22), page (35). A suitable method is the Newton Rhapson Method. The computer program which follows employs this method to compute the value of W for any combination of up to ten levels for any information channel.

After computing the value of W, the program then calculates the various percentages for each level using Equation (1.20), page (35). In order to determine if the information theory approach is valid, the theoretical values computed must be tested against observed data. This is accomplished by the use of a Chi-Square Goodness of Fit test. Critical values for a 95% Confidence Level are stored in the program and are compared to computed values which determine whether the theoretical values are, in truth, independent or not. It should be pointed out, however, that the mere fact that an hypothesis under a Chi-Square test cannot be rejected, does not necessarily mean that it is true. More data should be taken before final conclusions are made.

Following is the computer program, along with specific examples.

These examples, in themselves, could be solved and tested using conventional statistical tests for independence. However, the value of the Information Theory approach is in its ability to offer quick and easy solution to rather involved problems. and, more important, it allows projections to be made on problems which have no observed data. This approach should be invaluable in decision making problems.

It should be pointed out that this program was written for the CDC-3300; OS3 time sharing system with Fortran IV compiler. A unique feature of the system is the availability of the statement TTYIN which allows data to be entered directly from a remote unit without the necessity of READ statements. For example, if a program were written in which it was desired to enter an array of values for the variable X, the following statements would be sufficient.

```
      DO 1  I = 1, N
    1   X(I)  =  TTYIN   (4HX =  )
```

During execution of the program, the computer will ask for data to be entered by printing, X = . The operator will then enter his value of X_1 and the computer will agains print, X = . The process is repeated until all N values of X have been entered.

```
      PROGRAM ROOT
C     THIS PART OF THE PROGRAM CALCULATES THE ROOT OF AN
C     EQUATION OF THE FORM 1/X + 1/X**A1 + 1/X**A2 ETC = 1.0
C     FOR ANY COMBINATION OF UP TO TEN VALUES FOR X, USING THE
C     NEWTON RHAPSON METHOD.
C
C
      DIMENSION X(15),A(15),P(15),OBS(15),TOB(15)
   10 FORMAT(1H ,F10.5)
1     CONTINUE
      DO 14 I=1,10
14    X(I)=0.0
      WRITE(61,108)
108   FORMAT(' ENTER AN INITIAL VALUE TO START THE',
     1' APPROXIMATION',/,' USUALLY A NUMBER BETWEEN',
     2'  1  AND  2')
      B=TTYIN(4HINIT,4H=   )
      WRITE(61,100)
100   FORMAT(' ENTER THE POWERS TO WHICH THE VALUES ARE TO',
     1' BE RAISED'/' ENTER A ZERO FOR THE LAST VALUE IN ',
     2'ORDER TO STOP ENTERING DATA')
      DO 9 I=1,10
      A(I)=TTYIN (4H A= )
      IF(A(I) .EQ.0.0)GO TO 8
9     CONTINUE
8     DO 6 I=1,10
      IF(A(I).EQ.0.0)5,6
5     X(I)=0.0
      GO TO 12
6     X(I)=1/B**A(I)
12    FUNCT=X(1)+X(2)+X(3)+X(4)+X(5)+X(6)+X(7)+X(8)
     1+X(9)+X(10) -1.0
      IF(FUNCT.GT.  .00009)2,3
3     CONTINUE
      IF(FUNCT .LT. -.00009)2,4
2     DERIV=-A(1)/B**(A(1)+1)-A(2)/B**(A(2)+1)
     1 -A(3)/B**(A(3)+1)-A(4)/B**(A(4)+1)-A(5)/B**(A(5)+1)
     2 -A(6)/B**(A(6)+1)-A(7)/B**(A(7)+1)
     3 -A(8)/B**(A(8)+1)-A(9)/B**(A(9)+1)
     4 -A(10)/B**(A(10)+1)
      B= B- FUNCT/DERIV
      GO TO 8
4     CONTINUE
      WRITE(61,10)B
C
C
C     THIS PART OF THE PROGRAM CALCULATES THE
C     PERCENTAGES OF THE DIFFERENT LEVELS IN THE ROOT
C     EQUATION.
      WRITE(61,104)
104   FORMAT(' ENTER THE NUMBER OF VARIABLES IN THE',
     1' ROOT EQUATION.')
      NUMW=TTYIN(4HNUM=)
      C=ALOG(B)
      E=2.71828
      DO 11 I=1,NUMW
      XPON=C*A(I)
      P(I)=100./(E**XPON)
11    WRITE(61,102)I,P(I)
102   FORMAT(' P',I2,' = ',F10.2,' %')
C
```

```
C
C     THIS PART OF THE PROGRAM EXECUTES A CHI-SQUARE
C     GOODNESS OF FIT TEST.
13    WRITE(61,106)
106   FORMAT(' ENTER THE TOTAL NUMBER OF OBSERVATIONS')
      NOBS=TTYIN(4HNOBS,4H=    )
      WRITE (61,105)
105   FORMAT(' ENTER THE OBSERVED VALUES TO BE TESTED')
      CHISQRE=0.0
      DO 7 I=1,NUMW
      OBS(I)=TTYIN(4HOBS=)
      TOB(I)=NOBS*P(I)/100.
7     CHISQRE=(OBS(I)-TOB(I))**2/TOB(I) + CHISQRE
      WRITE(61,109)
109   FORMAT(' IS THE DATA JUST ENTERED CORRECT?  ENTER',
     1' 1  FOR YES OR  0  FOR NO.')
      ICHEK=TTYIN(4HCHEC,4HK=  )
      IF(ICHEK .EQ. 0)GO TO 13
      NDF=NUMW-1
      IF(NDF .EQ. 1)CRIT=3.84146
      IF(NDF .EQ. 2)CRIT=5.99147
      IF(NDF .EQ. 3)CRIT=7.81473
      IF(NDF .EQ. 4)CRIT=9.48773
      IF(NDF .EQ. 5)CRIT=11.0705
      IF(NDF .EQ. 6)CRIT=12.5916
      IF(NDF .EQ. 7)CRIT=14.0671
      IF(NDF .EQ. 8)CRIT=15.5073
      IF(NDF .EQ. 9)CRIT=16.9190
      IF(CRIT .GT. CHISQRE)GO TO 15
      WRITE(61,107)CHISQRE,NDF
107   FORMAT(1H1,' SINCE',F10.4,' IS GREATER THAN THE',
     1' CRITICAL VALUE OF',F10.4,/' WITH',I3,' DEGREES OF',
     2' FREEDOM, REJECT THE HYPOTHESIS OF INDEPENDENCE',/
     3' AT THE  .05  SIGNIFICANCE LEVEL.'//)
      GO TO 16
15    WRITE(61,110)CHISQRE,CRIT,NDF
110   FORMAT(1H1,' SINCE',F10.4,' IS LESS THAN THE CRITICAL',
     1' VALUE OF',F10.4,/' WITH',I3,' DEGREES OF FREEDOM,',
     2' ACCEPT THE HYPOTHESIS',/' OF INDEPENDENCE',
     3' AT THE  .05  SIGNIFICANCE LEVEL.'//)
16    WRITE(61,101)
101   FORMAT(' ENTER A NUMBER GREATER THAN  1  IF MORE',
     1' RUNS ARE DESIRED',/' ENTER A 1 OR 0 IF NO',
     2' EXTRA RUNS ARE DESIRED'///)
      NRUNS=TTYIN(4HRUNS,4H=      )
      IF(NRUNS.GT. 1)GO TO 1
      CONTINUE
      END
```

]

Airline Ticket Sales Example

```
ENTER AN INITIAL VALUE TO START THE APPROXIMATION
USUALLY A NUMBER BETWEEN  1  AND  2
INIT=   2.0
ENTER THE POWERS TO WHICH THE VALUES ARE TO BE RAISED
ENTER A ZERO FOR THE LAST VALUE IN ORDER TO STOP ENTERING DATA
 A= 1.0
 A= 1.33
 A= 2.34
 A= 0
   2.13900
ENTER THE NUMBER OF VARIABLES IN THE ROOT EQUATION.
NUM=3
P 1 =       46.75 %
P 2 =       36.38 %
P 3 =       16.88 %
ENTER THE TOTAL NUMBER OF OBSERVATIONS
NOBS=   78
ENTER THE OBSERVED VALUES TO BE TESTED
OBS=46.0
OBS=24.0
OBS=8.0
IS THE DATA JUST ENTERED CORRECT?  ENTER  1  FOR YES OR  0  FOR NO.
CHECK=  1




 SINCE    5.1930 IS LESS THAN THE CRITICAL VALUE OF    5.9915
WITH  2 DEGREES OF FREEDOM, ACCEPT THE HYPOTHESIS
OF INDEPENDENCE AT THE  .05  SIGNIFICANCE LEVEL.


ENTER A NUMBER GREATER THAN  1  IF MORE RUNS ARE DESIRED
ENTER A 1 OR 0 IF NO EXTRA RUNS ARE DESIRED


RUNS=   0
```

APPENDIX C

SEARCH TECHNIQUES FOR DETERMINATION OF THE ROOT OF THE INFORMATION CHANNEL EQUATION

Determination of the Root of a Single-Factor Information Channel Equation

In order to apply the theory previously discussed, a method must be developed to find the root of Equation (1. 22) page 35.

$$\sum_{i=1}^{k} W^{-t_i} = 1$$

Two methods will be described here for finding the root of the above equation; (1) The Newton-Rhapson Method, and (2) The Fibonacci Search Technique.

Newton-Rhapson Method

The Newton-Rhapson Method or simply Newton's Method is an approximation technique used to determine the point at which $f(x) = 0$. The theory is as follows. Let x_1 be an approximation of a value of x so that $f(x) = 0$. Let $y_1 = f(x_1)$ and draw a tangent to the curve $y = f(x)$ at (x_1, y_1). See Figure 5.

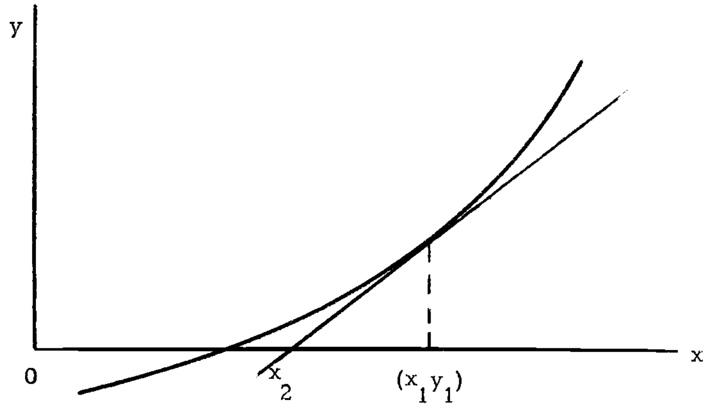

Figure 5. Curve of root equation.

Choose x_2 as the point where the tangent line crosses the x axis. Now, x_2 may be used as a second approximation to $y = f(x)$. The equation for the tangent line is:

$$y - y_1 = f'(x_1)(x - x_1)$$

and $y = 0$ at the point of intersection with the x axis and $x = x_2$, so

$$-y_1 = f'(x_1)(x_2 - x_1)$$

Therefore,

$$x_2 = x_1 - \frac{y_1}{f'(x_1)}$$

or

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

The above equation may be written in a more generalized form as:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{2.0}$$

To find the root of $\sum_{i=1}^{k} W^{-t_i} = 1$, the following procedure may be used.

1. Set $y_1 = \sum_{i=1}^{k-1} W^{-t_i}$

   Set $y_2 = 1 - W^{-t_k}$

2. Graph y_1 and y_2 and approximate the point of intersection. Let this value be x_1.

3. Tabulate

Table 21. Sample table for determining the root of single-factor equation.

i	x_i	$f(x_i)$	$f'(x_i)$

4. Compute $f(x_1)$ and $f'(x_1)$ where $x_i = W$, and enter the values obtained in the table.

5. If $f(x) = 0$, the solution is complete and x_1 is the root being sought.

6. If $f(x)$ does not equal zero, find $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

7. Compute and check $f(x_2)$. If $f(x_2) \neq 0$, continue to find x_{i+1} until a point is reached where $f(x) = 0$. The value of x_i at that point is the root of the equation in question.

An Example

Suppose we wish to find the root of:

$$W^{-1} + W^{-2} + W^{-3} = 1$$

then $f(x) = W^{-1} + W^{-2} + W^{-3} - 1$ and $f'(x) = -W^{-2} - 2W^{-3} - 3W^{-4}$.

Step (1). Set $y_1 = W^{-1} + W^{-2}$

Set $y_2 = 1 - W^{-3}$

Step (2). Graph y_1 and y_2 .

Table 22. Root determination example.

W	y_1	y_2
1	2	0
2	3/4	7/8
3	4/9	26/27

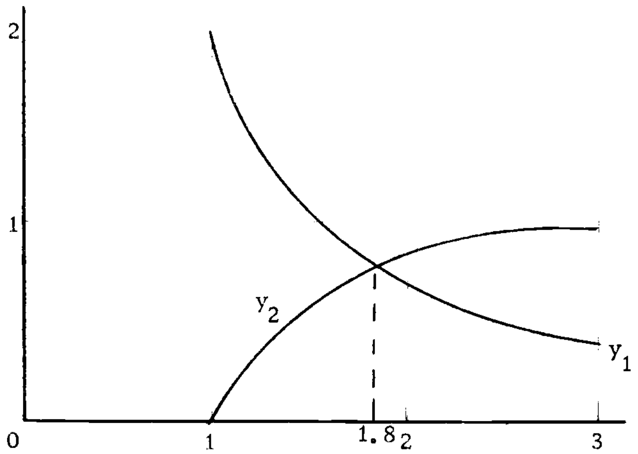

Figure 6. Graph of root equation example.

It may be noted that these two curves intersect at approximately W = 1.8.

Choose $x_1 = 1.8$

Step (3).

Table 23. Step 3 table for root determination.

i	x_i	$f(x_i)$	$f'(x_i)$
1	1.8		

Step (4). Compute $f(x_i)$ and $f'(x_i)$ where $x_i = W$.

$$f(x_1) = 1/1.8 + 1/1.8^2 + 1/1.8^3 - 1$$

$$f(x_1) = +0.03565$$

$$f'(x_1) = -1/1.8^2 - 2/1.8^3 - 3/1.8^4$$

$$f'(x_1) = -0.93731$$

Step (5). f(x) does not equal zero, so go to step 6.

Step (6). $x_2 = 1.8 - \frac{(0.03565)}{(-0.93731)}$

$$x_2 = 1.83803$$

Step (7). Repeat previous steps.

$$f(1.83803) = +0.00110$$

$$f'(1.83803) = -0.88091$$

$$x_3 = 1.83803 - \frac{(0.00110)}{(-0.88092)}$$

$$x_3 = 1.83927$$

$$f(1.83927) = 0.000000$$

Table of values using the Newton-Rhapson Method

Table 24. Results of Newton-Rhapson search technique.

i	x_i	$f(x_i)$	$f'(x_i)$
1	1.8	0.03565	-0.93731
2	1.83803	0.00110	-0.88091
3	1.83927	0.00000	

Finally, then, the root of the equation:

$$W^{-1} + W^{-2} + W^{-3} = 1$$

is equal to 1.83927. Any other desired root of any combination of levels may be computed in this manner. However, when larger values are sought or when more levels are desired, this computational procedure becomes quite lengthy. Therefore, a computer program could be implemented to perform these calculations.

Single Variable Search - Fibonacci Search Technique

An alternative method for finding the root of an equation of the type: $W^{-1} + W^{-2} + \cdots + W^{-n} = 1$, is by the Fibonacci Search Technique (Gue and Thomas, 1968). Usually, this technique is used for finding maxima or minima of unimodal equations, but without loss of generality, may be applied to find the root of the equation defined above.

The procedure is as follows:

1. Graph the function as in Step (1) of the Newton-Rhapson Method.
2. Determine the interval Δx within which the intersection of the two curves occur.
3. Choose x_1 according to the Fibonacci search techique, and evaluate the function.
4. Determine the value of x for the next evaluation of the function. Evaluate the function.

5. Compare the value of the function just obtained with the value for the previous point.

6. Eliminate one of the intervals that do not contain $f(x) = 0$, and return to Step 4.

7. Repeat the above procedure until $f(x) = 0$ or until the last value of x allowed by the Fibonacci process has been applied. Take the value of x at this point as the root of the equation.

An illustration:

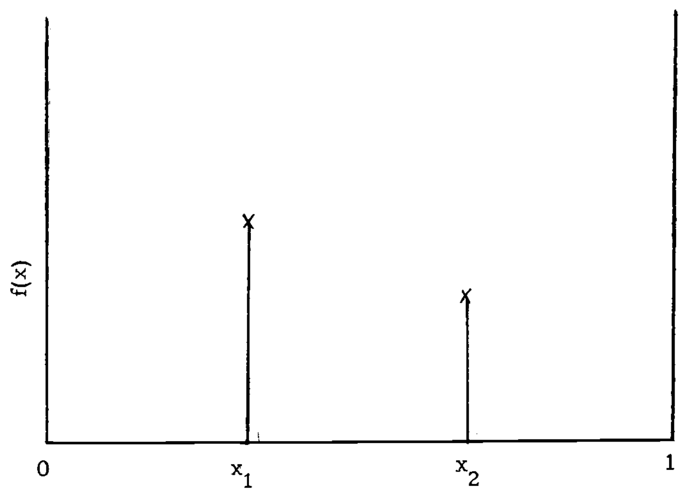

Figure 7. Fibonacci search technique.

In the illustration, the first two values for $f(x)$, as determined by the search technique are determined. Let's assume that the problem is to maximize a unimodal function. Then it is obvious that the interval from x_2 to 1 cannot contain this maximum, since the function is decreasing from x_1 to x_2. Therefore, we may eliminate this interval from x_2 to 1 and proceed according to the Fibonacci technique to eliminate other areas until the optimum value is found.

This procedure may be used quite effectively to find the root of $W^{-1} + W^{-2} + \ldots + W^{-n} = 1$. It may be helpful, at this point, to look at the graph of this equation.

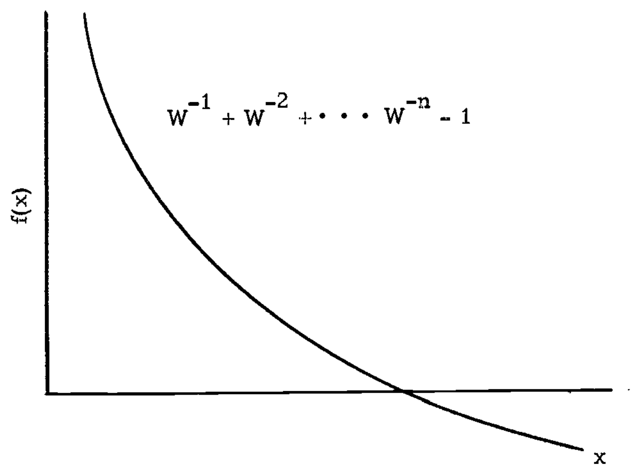

Figure 8. Graph of root equation.

In order to use the Fibonacci process, we must assume that one and only one root exists. In other words, that the function crosses the x axis at only one point. This is the case, since any combination of $W^{-a_1} + W^{-a_2} + \cdots + W^{-a_n} = 1$ has the form of the graph above.

We may, then, use successive approximation according to the Fibonacci method to find a value of x for which $f(x) \sim 0$. Our problem is first to locate points x_1 and x_2. This is done by placing x_1 according to the Fibonacci search technique and then placing x_2 symmetrically within the interval Δx. The next point x_3 is then placed symmetrically within the current search interval. This process is repeated for as many values of x as initially specified in the Fibonacci process.

We have no loss of generality if we normalize the interval Δx so that its length is 1. We then must select a Fibonacci number depending on the number of steps through which we wish to search. The first eight Fibonacci numbers F_n are:

Table 25. Fibonacci numbers.

n	0	1	2	3	4	5	6	7	8
F_n	1	1	2	3	5	8	13	21	34
x_1	0	0	1/2	1/3	2/5	3/8	5/13	8/21	13/34

It may be noted from the table above that for each successive n_i $i = 1, 2, 3, \ldots, 8$. The value of x_1 has for its numerator, the sum of the numerators of the values of x_1 for n_{i-1} and n_{i-2}, and for its denominator, the sum of the denominators of the values of x_1 for n_{i-1} and n_{i-2}. This sequence is called a Fibonacci sequence (Gue, 1968). Therefore, if we choose to evaluate through n steps, the first value x_i should be located at:

$$x_1 = \frac{F_{n-2}}{F_n} \qquad (2.1)$$

An Example

Suppose we wish to find the root of:

$$W^{-1} + W^{-2} + W^{-3} = 1$$

therefore:

$$f(x) = W^{-1} + W^{-2} + W^{-3} - 1$$

Step (1). Since this is the same equation as used for the Newton-Rhapson Method, we may use the curves for that example.

Step (2). It may be seen that the intersection of y_1 and y_2 lies between $x = 1\ 1/2$ and 2. Choose this interval for Δx.

Step (3). Assume that we wish to conduct our Fibonacci search through 5 steps. Then, from Equation (2.1) and the table of Fibonacci numbers: For $n = 5$,

$$X_1 = \frac{F_3}{F_5} = \frac{3}{8}$$

We place x_1 then 3/8ths of the way from $x = 1\ 1/2$ to $x = 2$. We evaluate the function at $x_1 = 1.625$.

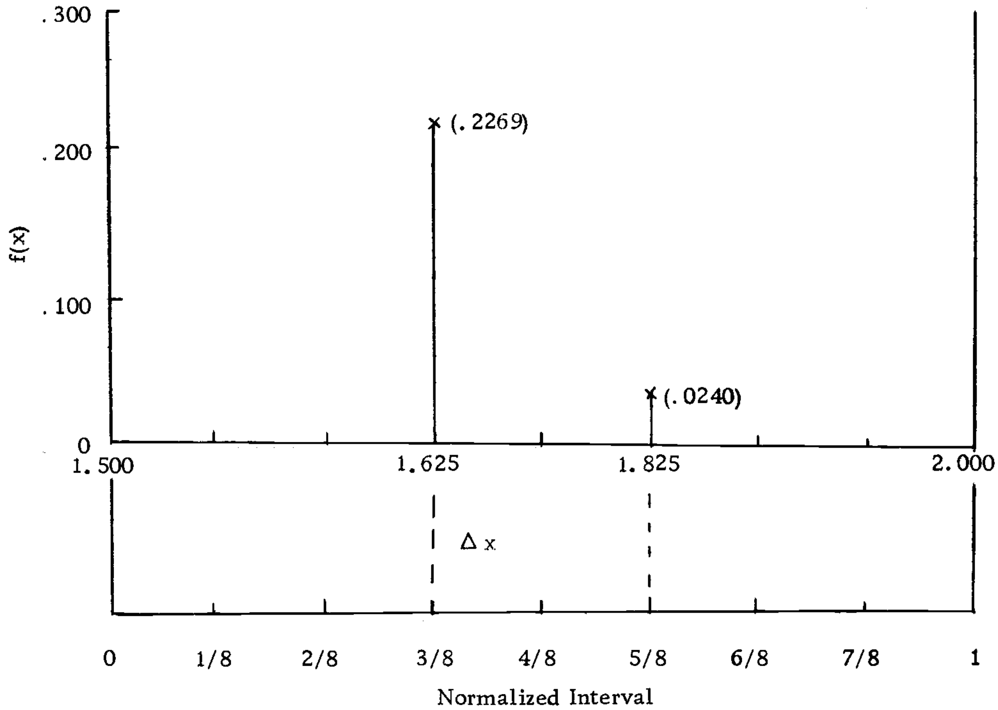

Figure 9. Fibonacci search example - step 1.

$$f(1.625) = \frac{1}{1.625} + \frac{1}{(1.625)^2} + \frac{1}{(1.625)^3} - 1$$

$$f(1.625) = +0.2269$$

Enter this value on the figure above.

Step (4). We locate x_2 symmetrically within the interval Δx, or x_2 is located at 5/8 on the normalized interval and the function is evaluated at $x_2 = 1.8125$.

$f(1.8125) = +0.024$ (Enter this value on the figure above)

Step (5). Comparing the value of $f(x)$ at x_1 and x_2, we see that the point $f(x) = 0$ cannot occur in the interval from $x = 1.5$ to $x = 1.625$. Therefore, we eliminate the area from x = 1.5 to X = 1.625.

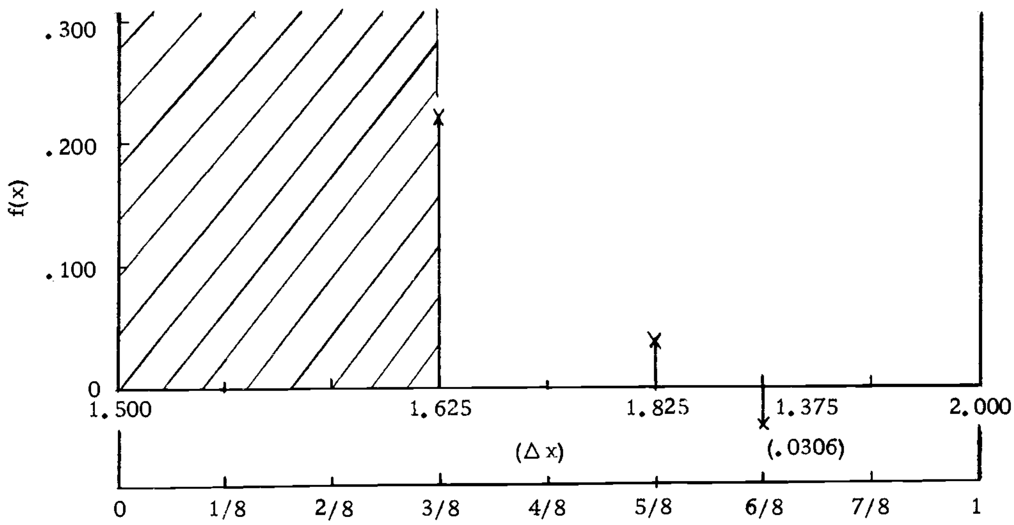

Figure 10. Fibonacci search technique - 2nd iteration.

Step (6). Eliminate area as shown above.

Step (7). We now locate x_3 symmetrically in the interval that remains or at $x_3 = 6/8$ on the normalized interval or at $x_3 = 1.875$ in the Δx interval. $f(1.875) = -.0306$ (Enter this value in the figure above).

Step (8). We now know that $f(x) = 0$ occurs between $x = 1.8125$ and $x = 1.8750$, and we have two approximations left to make. We therefore choose x_4 midway between these two values of x or $x_4 = 1.8437$.

$f(1.8437) = -.0041$

Step (9). Now locate the final value x_5 midway between x = 1.8125 and $x = 1.8437$ or $x_5 = 1.8281$.

$f(1.8281) = 0.0000$

Therefore the root of the equation in question is $x = 1.8281$ which closely agrees with the value found by the Newton-Rhapson Method.

It may be noted that the Fibonacci Search Technique is nothing more that a systematic trial and error method that does not guarantee an accurate value for the root of the equation in question. However, if slide rule calculations are being made, this approximation technique will be good enough.

Both the Newton-Rhapson Method and the Fibonacci Search Technique involve many calculations; especially when more complicated equations are evaluated. In fact, the calculations are quite formidable in many instances. However, a fairly simple computer program has been written (Appendix B) that may be used to find the root of any combination of exponents for W for as many as ten levels.

APPENDIX D

COMPUTER PROGRAM FOR SOLVING TWO-FACTOR INFORMATION CHANNEL PROBLEMS

This program is written in Conversational form and is designed to be used with the OS-3 system at Oregon State University. Through an iterative process, it systematically searches for the minimum absolute total difference of a set of equations of the form:

$$\begin{array}{ccc} a_{11}Y_1 + a_{21}Y_2 & = & C_1 \\ a_{12}Y_2 + a_{22}Y_2 & = & C_2 \\ \vdots & & \vdots \\ a_{1m}Y_1 + a_{2m}Y_2 & = & C_m \end{array}$$

where the a_{ij} and the C_i are given, and it is desired to determine the values of Y_1 and Y_2 which will most closely fit the above equations.

```
      PROGRAM MINVALUE
      DIMENSION P(2,50),X(50)
16    WRITE(61,200)
200   FORMAT(' THIS PROGRAM CALCULATES THE VALUES OF'
     1' THE VARIABLES',/,' Y1 AND Y2 IN A TWO FACTOR',
     2' INFORMATION CHANNEL PROBLEM.'///)
      WRITE(61,201)
201   FORMAT(' ENTER THE NUMBER OF TERMS IN EACH ',
     1' ROW OF THE CHANNEL MATRIX.'///)
      M=TTYIN (4HNOEQ,4HS=   )
      DO 15 K=1,2
      DO 15 L=1,25
15    P(K,L)=0.0
      DO 18 K=1,25
18    X(K)=0.0
      WRITE(61,202)
202   FORMAT(' ENTER THE VALUES FOR THE P(I,J)',
     1' IN THE ORDER',/' P(11),P(12) . . . P(1M)',
     2' P(21),P(22), . . ., P(2M).'///)
12    DO 10 I=1,2
      DO 10 J=1,M
10    P(I,J)=TTYIN (4HP=   )
      WRITE(61,203)
203   FORMAT(' ENTER THE OBSERVED MARGINAL PROBABILITIES',
     1' AS',/,' X(1),X(2) . . . X(M).'///)
      DO 11 J=1,M
11    X(J)=TTYIN (4HX=   )
      WRITE(61,300)
300   FORMAT(' IS THE DATA ENTERED CORRECT?',
     1' ENTER A  1 FOR YES  OR  0 FOR NO.')
      NDATA =TTYIN(4H? = )
      IF (NDATA .EQ. 0)GO TO 12
      Y1=0.0
      Y2=1.0-Y1
2     CALL GUESS (M,P,Y1,Y2,X,DIFF1)
      CALL PROB (0.1,Y1,Y2)
      CALL GUESS(M,P,Y1,Y2,X,DIFF2)
      IF(DIFF2 .LT. DIFF1)2,1
1     Y1=Y1-0.1
      Y2=1. - Y1
4     CALL GUESS(M,P,Y1,Y2,X,DIFF1)
      CALL PROB(0.01,Y1,Y2)
      CALL GUESS(M,P,Y1,Y2,X,DIFF2)
      IF(DIFF2 .LT. DIFF1)4,3
3     Y1=Y1 - 0.01
      Y2=1.0-Y1
5     CALL GUESS(M,P,Y1,Y2,X,DIFF1)
      CALL PROB(0.001,Y1,Y2)
      CALL GUESS(M,P,Y1,Y2,X,DIFF2)
      IF(DIFF2 .LT. DIFF1)5,6
6     Y1=Y1-0.001
      Y2=1-Y1
      WRITE(61,100)Y1,Y2
100   FORMAT(' Y1 = ',F6.4,' Y2 = ',F6.4)
      WRITE(61,400)
400   FORMAT(' IF MORE RUNS ARE DESIRED TYPE',
     1' A  1  IF NOT TYPE A  0')
      NRUNS=TTYIN(4HNRUN,4HS=   )
      IF(NRUNS .EQ. 1)GO TO 16
      END
C
```

```
C
C
      SUBROUTINE GUESS(M,P,Y1,Y2,X,DIFF)
      DIMENSION P(2,50),X(50)
      DIFF=0.0
      DO 1 K=1,M
      DIF=P(1,K)*Y1+P(2,K)*Y2-X(K)
      D=ABS(DIF)
1     DIFF= D  + DIFF
      WRITE(61,500)DIFF,Y1,Y2
500   FORMAT(1H ,F8.5)
      RETURN
      END
C
C
C
      SUBROUTINE PROB(Z,A,B)
      A=A+Z
      B=1.0-A
      RETURN
      END
```

Stock Investment Example

```
THIS PROGRAM CALCULATES THE VALUES OF THE VARIABLES
Y1 AND Y2 IN A TWO FACTOR INFORMATION CHANNEL PROBLEM.


ENTER THE NUMBER OF TERMS IN EACH  ROW OF THE CHANNEL MATRIX.


NOEQS=  4
ENTER THE VALUES FOR THE P(I,J) IN THE ORDER
P(11),P(12) . . . P(1M) P(21),P(22), . . ., P(2M).


P=  .045
P=  .409
P=  .400
P=  .146
P=  .341
P=  .126
P=  .389
P=  .148
ENTER THE OBSERVED MARGINAL PROBABILITIES AS
X(1),X(2) . . . X(M).


X=  .283
X=  .277
X=  .291
X=  .149
IS THE DATA ENTERED CORRECT? ENTER A  1 FOR YES  OR  0 FOR NO.
? = 1


Y1 =  .2000 Y2 =  .8000
IF MORE RUNS ARE DESIRED TYPE A  1  IF NOT TYPE A  0
NRUNS=  0
```

APPENDIX E

RESEARCH QUESTIONAIRE

Please answer the following questions as truthfully as you can.

1. Assume that you are planning to take a plane trip to New York City. Your choice of Class of Air travel is as below. If you were paying the fare yourself, which class would you choose?

PLANE TICKET COST	
First Class	$350.00
Tourist	$200.00
Excursion	$150.00

Check One

First Class ________

Tourist ________

Excursion ________

2. Assume that you have an evening free to do as you please. Which of the following would you choose?

	Cost
a. Stay Home	$ .65
b. Go out to dinner and cocktails	$10.00
c. Go out for pizza and beer	$ 5.00
d. Go to a movie	$ 3.50
e. Go for a drive	$ 1.00

Check One

a. ________

b. ________

c. ________

d. ________

e. ________

3. Assume that you are planning to go to San Francisco for a week. Which mode of travel would you choose? The costs and travel times are given below.

	Plane	Train	Bus
Cost $	$30.00	$20.00	$10.00
Travel Time	1 hr.	4 hrs.	10 hrs.

Check One

Plane ________

Train ________

Bus ________

RESEARCH QUESTIONAIRE

Please answer the following question as truthfully as you can.

1. Assume that you are in the market for a new color TV, and you've narrowed your choice down to the possibilities listed below. Which one would you purchase?

		Cost
a.	RCA	$550
b.	General Electric	$500
c.	Emerson	$450
d.	Sears Roebuck	$400

Check One

a. ________
b. ________
c. ________
d. ________

APPENDIX F

CHI-SQUARE GOODNESS OF FIT TESTS

A Chi-Square Goodness-of-Fit test may be used to analyze counted data. The χ^2 statistic is used to determine if observed frequencies match corresponding theoretical frequencies. The only assumption necessary is that the individual categories be mutually exclusive. In the Information Theory models included in this thesis, the categories involved are represented by the respective levels of a factor. The appropriate statistic is:

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - np_i)^2}{np_i}$$

where o_i is the observed frequency of a level i, n is the total number of observations, and, p_i, is the theoretical probability of level, i. The above statistic is approximately distributed as χ^2 with $k-1$ degrees of freedom if $np_i > 5$ for every i.

Some common situations in which a Chi-Square goodness-of-fit test may be applicable are (Wine, 1964):

1. Most opinion polls.
2. Checking characteristics of insects against theoretical values.
3. Comparing defective parts produced by various machines.

.

4. Studies of most human characteristics such as: income, hair color, occupation, etc..
5. Research in which a relatively small number of categories are meaningful or when results are needed in a hurry.
6. Studies involving traffic, weather, amount of sleep, working conditions, etc..