A Relationship of the Reynolds Stress to Local Shear and Heat Fluxes in the Planetary Boundary Layer

LARRY J. MAHRT

Dept. of Atmospheric Sciences, Oregon State University, Corvallis 97331

(Manuscript received 4 April 1973, in revised form 6 August 1973)

ABSTRACT

The steady Reynolds stress and turbulent energy equations for steady, horizontally homogeneous mean flow are used to relate the Reynolds stress $\overline{u'w'}$ to the mean wind shear and heat fluxes in the planetary boundary layer.

The resulting Reynolds stress demonstrates a $\frac{3}{2}$ power dependence on the stress Richardson number and a $\frac{1}{2}$ power dependence on the flux Richardson number. Numerical results of Deardorff are used to estimate vertical profiles of a heat flux function which results from the derivation. Such calculations and certain observations suggest that the stress depends mainly on the flux Richardson number in the upper part of the strongly heated boundary layer but more on the stress Richardson number in the lower part of the weakly heated or stable boundary layer. The simple model developed appears to be inadequate in the case of large$-z/L$ where the shear generation of stress becomes negligible and turbulent transports of stress may be significant.

1. Introduction

The parameterization of turbulent momentum transports is a fundamental problem inherent in analytical and numerical models of atmospheric flow. In the planetary boundary layer (PBL), a general approach to this problem must include the important influence of heat fluxes on turbulent momentum transports. Here we restrict the definition of the PBL to include only the subcloud layer if condensation occurs.

Theories which relate the Reynolds stress directly to the mean wind shear can plausibly be modified to include heat flux effects as long as shear generation of the stress remains significant. For example, an eddy viscosity is often parameterized in terms of the turbulent energy via a mixing length, where the turbulent energy is calculated from a parameterized turbulent energy equation which includes the heat flux term [see Zilitinkevich *et al.* (1967) for a survey]. By simplifying the turbulent energy equation further, the eddy viscosity can be conveniently related to the Richardson number. For instance, Obukhov (1946) assumed a balance among buoyancy, shear production and energy dissipation terms in a parameterized turbulent energy equation to formulate

$$\left.\begin{aligned} K/K_n &= C_\epsilon^{\frac{1}{2}}(1-\alpha\,\mathrm{Ri})^{\frac{1}{2}} \\ K_n &\equiv l^2\left|\frac{\partial \mathbf{V}}{\partial z}\right| \end{aligned}\right\}$$

where K is the actual eddy viscosity, K_n the classical "neutral" eddy viscosity, $\mathbf{V}$ the mean horizontal velocity vector, Ri the Richardson number, l a length scale characteristic of low-frequency energy containing eddies, C_ϵ a coefficient in the dissipation term, and α the inverse of the turbulent Prandtl number. When the turbulent energy terms are left in flux form, the flux Richardson number R_f becomes the appropriate stability parameter (Monin and Yaglom, 1971).

Observational studies of the surface layer (Plate, 1971; Wyngaard *et al.*, 1971) indicate that the influence of heat fluxes on the stress, as represented by the nondimensional wind shear $\phi_m \equiv kz(\partial U/\partial z)/u_*$, can be conveniently represented as a function of the similarity argument z/L where

$$L \equiv -u^3_* \Big/ \left(\frac{g}{\bar{\theta}} k\overline{w'\theta'}\,|_s\right),$$

$\overline{w'\theta'}\,|_s$ is the turbulent heat flux near the surface, $\bar{\theta}$ a mean potential temperature, u_* the surface friction velocity and k von Kármán's constant. The particular similarity function depends on the thermal stability class. Yamamoto *et al.* (1968) concluded that surface layer similarity theory could be extrapolated to reasonably describe a case study of a heated PBL. However, modification of eddy viscosity theory for the entire PBL is not always useful since shear generation of Reynolds stresses may locally become insignificant. Observations indicate that shear generation is important in stably stratified low-level flows (e.g., Metcalf and Atlas, 1972; Lettau and Davidson, 1957), but can vanish in the strongly heated PBL (Lenschow, 1970). As a result, the Reynolds stress in a convectively mixed PBL may be nearly independent of the small local mean wind shear (Deardorff, 1972a). In such cases the eddy viscosity can be extremely large and/or negative.

There is also evidence that turbulent momentum fluxes in a heated baroclinic boundary layer may be locally "countergradient" (negative eddy viscosity) above a region where the mean wind reaches a maximum (Lenschow, 1972). In fact, Lettau (1970) has shown that even in the absence of heat fluxes, the turbulence must satisfy certain coherence and similarity restrictions before the turbulent diffusion of momentum is purely gradient. Numerical calculations of Peterson (1972) indicate that horizontal inhomogeneities in the neutral boundary layer may also destroy a local shear-stress relationship.

As an alternative to relating the stress directly to the mean shear, one can employ the Reynolds stress equations and complementary temperature (or density) variance and flux equations (Corrsin, 1956; Stewart, 1959). A number of investigators [see Reynolds (1970) for a survey] have constructed parameterized forms of the Reynolds stress and various complementary equations from which the Reynolds stresses can be numerically computed. The present study will employ a simple version of the Reynolds stress and turbulent energy equations, similar to those in Ayra (1972a), to relate the Reynolds stresses $\overline{u'w'}$ and $\overline{v'w'}$ to the local wind shear and heat fluxes, where u', v' and w' are the velocity fluctuations in the x, y and z directions. The philosophy of the present development of a Reynolds stress relationship will be somewhat analogous to Deardorff's (1972b) derivation of the countergradient heat flux formulation. Unfortunately, to apply the Reynolds stress and turbulent energy equations to PBL flows, a number of assumptions and parameterizations are necessary which cannot be rigorously justified. Since the total consequence of the various assumptions is uncertain, this development will be used only to suggest a simple format for the stress formulation. To complete the relationship, an unknown factor which has dimensions of length and absorbs certain unknown coefficients will be estimated independently from results due to Deardorff (1972a) and Lenschow (1970). It will be found that where both shear generation and turbulent heat fluxes are significant, a modified eddy viscosity can be defined which depends on the flux and stress Richardson numbers. When shear generation is not significant, a modified eddy viscosity is not appropriate.

2. The Reynolds stress equation

The behavior of various terms in the Reynolds stress equation (e.g., Monin and Yaglom, 1971) is quite uncertain for the PBL. Wyngaard *et al.* (1971) computed these terms from observations in a surface layer thought to be horizontally homogeneous. Their analysis seems to indicate an approximate balance between shear production, buoyancy production or destruction, and destruction by the pressure fluctuation term. Although the pressure fluctuation term was not measured directly, its importance was argued deductively. Observations by Lenschow (1970) in a convectively mixed PBL indicate that the shear generation term may become locally insignificant at higher elevations. Based on the above observational studies we simplify the more complete stress equation to the form

$$-\overline{w'^2}\frac{\partial U}{\partial z}+\frac{g}{\bar{\theta}_v}\overline{u'\theta_v'}-\bar{\rho}^{-1}\left(\overline{u'\frac{\partial p'}{\partial z}}+\overline{w'\frac{\partial p'}{\partial x}}\right)=0, \quad (1)$$

where $\bar{\theta}_v$ and $\bar{\rho}$ are respectively a mean virtual potential temperature and a mean density in the PBL, p' and θ_v' are the fluctuating pressure and virtual potential temperature, and U is the mean velocity in the x direction. This equation could be formally derived by making the Bousinesq assumption, assuming stationarity and horizontal homogeneity of the mean velocity field, and neglecting viscous, Coriolis and triple correlation terms. The pressure fluctuation term will be parameterized as

$$\bar{\rho}^{-1}\left(\overline{u'\frac{\partial p'}{\partial z}}+\overline{w'\frac{\partial p'}{\partial x}}\right)=C_m l^{-1} q\overline{u'w'}, \quad (2)$$

where C_m is an empirically determined coefficient and $q^2/2\equiv\frac{1}{2}(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})$ is the turbulent kinetic energy. This parameterization was originally developed by Rotta (1951) for the pressure strain rate $\bar{\rho}^{-1}\overline{p'(\partial u'/\partial z+\partial w'/\partial x)}$ part of the pressure fluctuation term. The justification of such a parameterization is based on the general belief that 1) the pressure strain rate term tends to produce isotropy, thus reducing anisotropic moments such as $\overline{u'w'}$; and 2) the rate of destruction of such anisotropic moments is proportional to the magnitude of the anisotropic moments themselves. In a more general treatment, Lumley and Khajeh-Nouri (1973) and Lumley (1970) develop a functional expansion for the pressure fluctuation term based on a "weak anisotropy" approximation. The lowest order term of this expansion is analogous to the right-hand side of (2) provided that one adopts certain scaling arguments whereby $\epsilon=q^3/(C_\epsilon l)$ (see Tennekes and Lumley 1972), where ϵ is the viscous dissipation of turbulent energy. The importance of this lowest order term is central to the present development. Higher order terms in the functional expansion of Lumley and Khajeh-Nouri as well as additional terms suggested for the pressure strain rate term (Lilly, 1967; Crow, 1968; Daley and Harlow, 1970; Reynolds, 1970) will not be considered in the present simplified analysis.

Substituting (2) into (1) and solving for $\overline{u'w'}$, we obtain

$$\overline{u'w'}=C_m{}^{-1}lq^{-1}\left(-\overline{w'^2}\frac{\partial U}{\partial z}+\frac{g}{\bar{\theta}_v}\overline{u'\theta_v'}\right).$$

This expression defines the equilibrium Reynolds stress which results from a balance between shear generation, the buoyancy term, and the parameterized pressure destruction term. Employing the stress Richardson number R_s, we obtain

$$\left.\begin{aligned}\overline{u'w'}&=-l_1q(1-R_s)\frac{\partial U}{\partial z}\\ l_1&\equiv lC_m{}^{-1}(\overline{w'^2}/q^2)\\ R_s&\equiv\frac{g}{\bar{\theta}_v}\overline{u'\theta_v'}\bigg/\left(\overline{w'^2}\frac{\partial U}{\partial z}\right)\end{aligned}\right\},\tag{3}$$

where l_1 is proportional to l except for a dependence on the anisotropy through the factor $\overline{w'^2}/q^2$. The format $K\sim lq$ has been used in a large number of boundary layer studies some of which are summarized in Zilitinkevich *et al.* (1967). The modification in (3) due to the stress Richardson number represents the direct buoyancy production or destruction of Reynolds stress.

3. The turbulent energy equation

To close the relationship between the Reynolds stress and mean wind shear, a turbulent energy relationship will now be formulated from the turbulent energy equation. Results due to Lenschow (1970, 1972) and Wyngaard and Coté (1971) indicate that the most important terms in the turbulent energy equation in the atmospheric boundary layer include shear production, buoyancy production or destruction, viscous dissipation, and the triple correlation term. Metcalf and Atlas (1972) concluded that in a stably stratified layer, unsteadiness due to Kelvin-Helmholtz type waves may also be important. Lenschow (1972) found horizontal advection of turbulent energy to be important in a case study of air mass modification. Again, we will presently consider only stationary, horizontally homogeneous flow. The pressure fluctuation term, which cannot be measured directly, is thought to be less important and is also neglected in this study.

The triple correlation term is apparently quite significant in the surface layer where it transports upward an excess of turbulent energy generated by shear production (Wyngaard and Coté, 1971), and near the top of the PBL where it deposits turbulent energy to help compensate for energy dissipation and buoyancy destruction associated with entrainment or growth of the mixed layer (Lenschow, 1970). One might expect the triple correlation term to be particularly important in a baroclinic PBL where the shear may vanish and reverse sign with height.

The triple correlation term is generally parameterized as a diffusive type term (e.g., Monin and Yaglom, 1971). However, this parameterization yields a nonlinear turbulent energy equation which prevents local solution. If such a parameterization is reasonable, then large transports of stress or turbulent energy imply that the stress is determined by shear and heat fluxes on a larger scale. In this case a local shear-stress relationship is less meaningful. In order to relate the stress to local shear and heat fluxes, the triple correlation term must presently be neglected, which may be the most serious omission of this development.

In a separate treatment of the turbulent energy equation, the diffusive type parameterization of the triple correlation term was included and the turbulent energy was numerically computed on a grid system using Newton's iteration method for a system of nonlinear equations. These results indicate that for neutral barotropic flow with zero shear at the top of the PBL, this term decreases the turbulent energy in the lowest part of the PBL and increases turbulent energy near the top of the PBL. In the rest of the PBL, percentage changes in the turbulent energy profile due to this term are generally small for a wide range of suspected values of the coefficient of the turbulent energy diffusion term. Peterson (1972) has shown that the parameterized triple correlation term may be particularly important in a horizontally inhomogeneous surface layer.

With the assumptions discussed above, the turbulent energy equation simplifies to

$$-\overline{u'w'}\frac{\partial U}{\partial z}+\frac{g}{\bar{\theta}_v}\overline{w'\theta_v'}-\epsilon=0,$$

where ϵ is parameterized as $q^3/(C_\epsilon l)$ as discussed previously. The use of a separate equation to predict ϵ or l appears to significantly improve turbulence models (Jones and Launder, 1972; Lumley and Khajeh-Nouri, 1973). For the sake of simplicity, such an equation is not included in the present study.

Employing the flux Richardson number, the turbulent energy equation reduces to the common form

$$\left.\begin{aligned}\overline{u'w'}\frac{\partial U}{\partial z}(1-R_f)+q^3/(C_\epsilon l)&=0\\ R_f&\equiv\frac{g}{\bar{\theta}_v}\overline{w'\theta_v'}\bigg/\left(\overline{u'w'}\frac{\partial U}{\partial z}\right)\end{aligned}\right\}.\tag{4}$$

4. The stress relationship

Solving for the turbulent energy from (4) and substituting into the Reynolds stress equation (3), we obtain

$$\left.\begin{aligned}\overline{u'w'}&=-l_2{}^2\left|\frac{\partial U}{\partial z}\right|\frac{\partial U}{\partial z}(1-R_s)^{\frac{3}{2}}(1-R_f)^{\frac{1}{2}}\\ l_2{}^2&\equiv C_\epsilon{}^{\frac{1}{2}}l^{\frac{1}{2}}l_1{}^{\frac{3}{2}}\end{aligned}\right\},\tag{5}$$

where l_2 is a parameter which has dimensions of length and will be determined later. By determining l_2 independently, the above derivation is used only to suggest a format for relating the Reynolds stress to heat fluxes and mean wind shear; thus, the employment of certain unproven parameterizations is less crucial. Formulation (5) differs from previous theories in that direct heat flux effects are represented by the stress Richardson number. This is the appropriate Richardson number representing direct heat flux effects since it is defined from the Reynolds stress equation itself, while the derivative and flux Richardson numbers can be defined only in terms of the turbulent energy equation. The flux Richardson number still enters into the formulation since the vertical heat flux influences $\overline{w'^2}$ and thus the shear generation of the Reynolds stress. Although the Reynolds stress exhibits only a square-root dependence on the flux Richardson number, changes in the flux Richardson number may dominate stress variations in certain flow situations, as will be discussed later. Relationship (5) could be simplified by assuming a relationship between R_s and R_f such as is empirically formulated in Wyngaard *et al.* (1971). Such an assumption may be necessary in modeling situations where $\overline{u'\theta_v'}$ is not generally computed.

Relationship (5) indicates that the effective eddy viscosity increases (decreases) as $-\overline{u'\theta_v'}$ and $\overline{w'\theta_v'}$ become more positive (negative). Note that if both $\overline{u'\theta_v'}$ vanishes ($R_s=0$) and $\overline{w'\theta_v'}$ vanishes ($R_f=0$), Eq. (5) reduces to the classical mixing length formulation. Neglecting the influence of heat fluxes on l_2, relationship (5) also indicates that as $R_s \to 1$ or $R_f \to 1$, turbulence cannot be maintained against buoyancy destruction. Observations indicate that R_s/R_f for stably stratified flow is greater than unity and as large as ~ 4, in which case (5) implies a critical R_f as low as 0.25. The influence of direct buoyancy destruction of stress on the critical R_f, via destruction of shear generation of turbulent energy, has been emphasized by Ayra (1972a). He predicted a critical R_f of 0.15–0.25. While shear generation directly supports only the fluctuating velocity in the mean shear direction, the other turbulent energy components depend on shear generation through the pressure fluctuation terms.

In the other extreme where buoyancy generation dominates, results due to Lenschow (1970),[1] Businger *et al.* (1971) and Deardorff (1972a) indicate that $\overline{u'\theta_v'}/\overline{w'\theta_v'}$ and R_s/R_f become small. If R_s/R_f becomes sufficiently small, (5) becomes approximately

$$\overline{u'w'} = -l_2^2 \frac{\partial U}{\partial z}\left|\frac{\partial U}{\partial z}\right|(1-R_f)^{\frac{1}{2}}.$$

This is the form predicted by Obukhov (1946) and Monin and Yaglom (1971). However, the above PBL observations and numerical simulations suggest that the R_s term can be neglected only locally in the upper part of the strongly heated PBL.

When mean wind shear nearly vanishes, such as in the interior of a convectively mixed layer, the eddy viscosity formulation is no longer appropriate. Mean shear on a larger scale and fluctuating wind shear may still generate Reynolds stresses indirectly through other terms. The role of wind shear as free convection is approached is currently a subject of debate (Ayra, 1972b; Wyngaard *et al.*, 1972). In the present simplified model, the stress is supported entirely by heat flux generation when shear generation vanishes. In this case, the resulting stress relationship may represent an intolerable over-simplification. Tennekes (1970) has suggested that for large $-z/L$, high-frequency, heat flux transporting eddies may only weakly interact with the slower momentum transporting eddies [see also Tennekes (1971) and Businger (1971) for further discussion]. Results of Wyngaard *et al.* (1971) and Deardorff (1972b) indicate that buoyancy generation of stress relative to buoyancy generation of turbulent energy decreases significantly with increasing $-z/L$. Since Lenschow (1970) found the turbulent transport of turbulent energy to be important in a case study heated PBL, one might speculate that turbulent

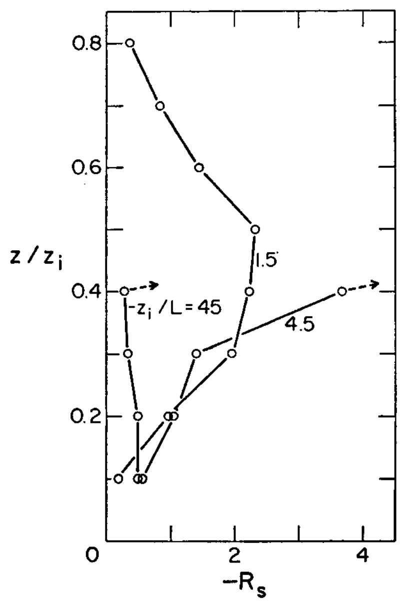

FIG. 1. The stress Richardson number R_s for various stability classes (as indicated by the ratio of z_i, the inversion height, to L, the Monin-Obukhov length for moist unsaturated air), as calculated from Deardorff's (1972a) nondimensional profiles of $\overline{u'\theta'}$ and $\overline{w'^2}\,\partial U/\partial z$, where U is the flow parallel to the surface stress. Values are shown only for $z/z_i<0.8$, since values in the upper part of Deardorff's model may be strongly influenced by the zero shear upper boundary condition at $z/z_i=1$. Terminated dashed arrows indicate that R_s approaches a large number at next level of computation due to very small wind shear. Values for flow perpendicular to the surface stress are not shown as they were considerably smaller and were strongly influenced by the zero shear upper boundary condition.

[1] Lenschow's horizontal heat fluxes and $\overline{w'^2}$ data are unpublished.

transport of stress also becomes important for large $-z/L$. The present simplified development indicates that the vertical shear of the mean wind and Reynolds stress vector $\bar{\rho}\overline{u'w'}\mathbf{i}+\bar{\rho}\overline{v'w'}\mathbf{j}$ are aligned when the horizontal heat flux perpendicular to the shear vanishes. Analyses due to Johnston (1970) indicate that the stress and shear vectors may not necessarily be closely aligned. There is no general agreement on the relationship between these shear and stress directions in the PBL, as is indicated by conflicting analyses of the "Leipzig Wind Profile" (Lettau, 1950; Swinbank, 1970; Carson and Smith, 1973).

While the stress and flux Richardson numbers represent direct effects of heat fluxes on the stress and turbulent energy, heat fluxes may also influence the stress indirectly by modification of coefficients in the pressure fluctuation and turbulent energy dissipation terms. Due to this type of coupling, neglect of turbulent transports, and other approximations in the above development, we cannot expect l_2 to be completely independent of the heat fluxes. In the next sections l_2 and the heat flux function will be estimated independently of the above development.

5. The heat flux function

The heat flux function, $\Phi\equiv(1-\mathrm{R}_s)^{\frac{3}{2}}(1-\mathrm{R}_f)^{\frac{1}{2}}$, represents the direct modification of the stress due to turbulent heat fluxes. In the unstable case (upward heat fluxes), the increase of $-\mathrm{R}_f$ with height and heating may be largely responsible for the increase of Φ with height and heating, even though Φ exhibits only a square-root dependence on R_f. This is due to the fact that $-\mathrm{R}_f$ increases much more rapidly with height than $-\mathrm{R}_s$. This behavior is evident in Figs. 1–3 which show R_f, R_s and Φ calculated from PBL numerical simulations of Deardorff (1972a). In fact, in the middle and upper portions of the strongly heated PBL, $\mathrm{R}_s/\mathrm{R}_f$ decreases to $O(10^{-3})$ which is also consistent with observations of Lenschow (1970). Similarly, surface layer measurements analyzed by Wyngaard *et al.* (1971) indicate that $\mathrm{R}_s/\mathrm{R}_f$ becomes much less than unity as $-z/L\rightarrow\infty$ $z\ll z_i$.

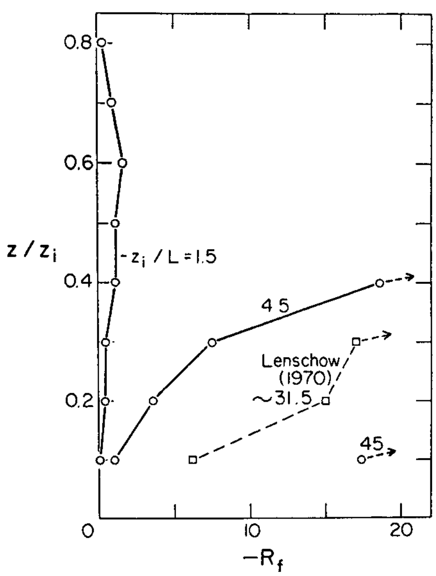

FIG. 2. The flux Richardson number R_f as calculated from Deardorff's (1972a) nondimensional profiles of $\overline{u'w'}\;\partial U/\partial z$ and Deardorff's specified nondimensional heat flux $\overline{w'\theta'}=[1-(z_i/z)]^2$. See Fig. 1 for further explanation. Lenchow's (1970) case study corresponds approximately to $-z/L=31.5$.

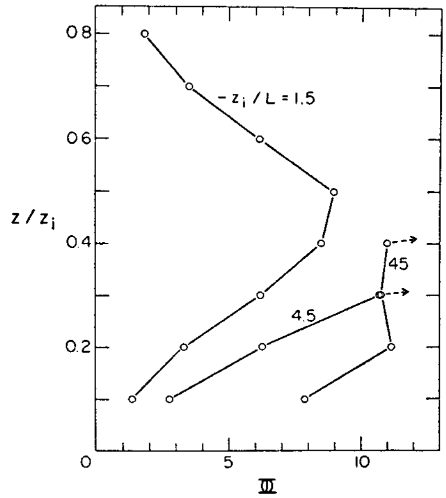

FIG. 3. The heat flux function Φ calculated from R_s and R_f profiles (Figs. 1 and 2). See Fig. 2 for further explanation.

It is noteworthy that in the strongly heated PBL's, the importance of shear generation decreases dramatically with height in a transition region. The height of this transition relative to $-L$ is greater than 1 and increases with increasing $-z_i/L$. This transition divides the heated PBL into two layers. In the upper layer 1) shear generation is unimportant, 2) $\mathrm{R}_s/\mathrm{R}_f$ is small, 3) free convection scaling is useful (Deardorff, 1972a), and 4) an eddy viscosity formulation is inappropriate. In the lower layer, shear generation is generally significant although its significance decreases considerably with increasing heat flux.

If the stress is large at the top of the PBL one might expect a third layer adjacent to the PBL top where shear generation is again significant. Deardorff's data do not reflect this feature because of the zero shear upper boundary condition. The importance of allowing nonzero stress at the top of the PBL has been recently emphasized by Deardorff (1973). Significant stress at the top of the PBL may be induced by entrainment of different velocity fluid from above an inversion capping the PBL, or may result from moist convection-induced turbulent transports across a lifted condensation level capping the PBL.

In contrast to unstable cases, $\mathrm{R}_s/\mathrm{R}_f$ in stable flow remains approximately constant or increases with

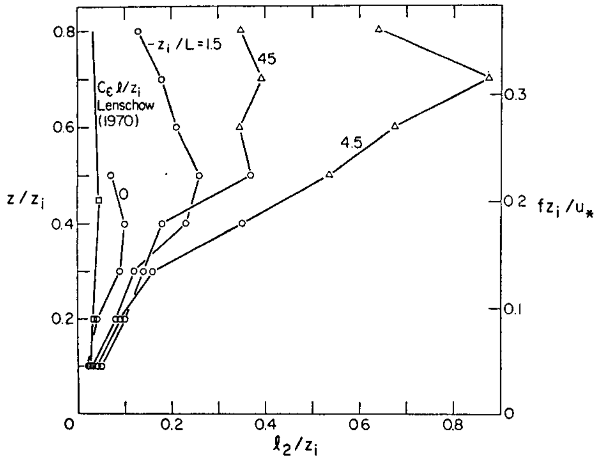

FIG. 4. The function l_2 calculated from Φ (Fig. 3) and Deardorff's (1972a) numerical values of $\partial U/\partial z$ and $\overline{u'w'}$. The triangles indicate that shear generation of the stress is less than 5% of the buoyancy generation. The right ordinate zf/u_* is for the neutral case only; the neutral case l_2 is not shown for $zf/u_*>0.275$ because the stress was insignificant. See Fig. 2 for further explanation.

increasing height and increasing stability, as is indicated by measurements of Ayra and Plate (1969), Webster (1964) and Wyngaard *et al.* (1971). As a result, Φ depends more on R_s than R_f. These measurements also indicate that R_s increases (and therefore Φ decreases) with increasing height and increasing negative heat flux. In summary, the heat function Φ is more dependent on R_s at lower levels and near neutral or stable conditions while Φ is more dependent on R_f at higher elevations with stronger heat fluxes.

6. The function l_2

Values of l_2 calculated from data due to Deardorff (1972a) are shown in Fig. 4. In the weakly heated PBL, both the magnitude of l_2 and the height of maximum l_2 increase with increased heat fluxes. In the strongly heated PBL, further increases in heat fluxes result in a decrease in l_2, as the heat flux function increases faster than the stress. The dependence of l_2 on heat fluxes is related to the various assumptions and parameterizations leading up to (5). For example, one might expect the relative importance of turbulent transports of both turbulent energy and Reynolds stress to increase with increasing turbulent energy. In (5) such effects would have to be absorbed by variations in l_2.

In the lowest part of the PBL, where shear generation is important, l_2 increases approximately linearly with height with a slope of about 0.35 in neutral conditions and about 0.5 in the heated PBL. Above this layer in the transition where the importance of shear generation decreases rapidly with height, $\partial^2 l_2/\partial z^2>0$. This feature of l_2 is generally not present in mixing length profiles developed for the neutral boundary layer (e.g., Lettau, 1962; Blackadar, 1962).

Fig. 4 also shows $C_\epsilon l=q^3/\epsilon$ calculated from ϵ values for the heated PBL due to Lenschow (1970). Lenschow used the Kolmogoroff hypothesis for the inertial subrange to determine the turbulent energy dissipation. $C_\epsilon l$ is considerably smaller than the corresponding l_2 for the strongly heated PBL, and is less height-dependent. Physical interpretation of the differences between l_2 and $C_\epsilon l$ is not appropriate, since the existence of a physical meaning of l_2 is not obvious.

7. Further discussion

The intent of the above analysis is to relate the Reynolds stress to local wind shear and heat fluxes in a manner which is simple yet consistent as possible with the physics of the Reynolds stress equation. It is clear that formulation (5) has more general application than classical eddy viscosity and mixing length theories, especially when the heat flux terms are important. However, when turbulent transports of the Reynolds stress and turbulent energy (triple correlation terms) and certain pressure fluctuation terms are important, one cannot categorically claim usefulness for relating the stress to the local shear and heat fluxes. Here the use of bulk boundary layer properties (Csanady, 1972) or layer integration (e.g., Geisler and Kraus, 1969; Lavoie, 1972) may be more profitable.

A second disadvantage of (5) is that the behavior of horizontal heat fluxes and $\overline{w'^2}$ cannot be easily related to typical model variables since their behavior in the PBL is not well known.

Greater availability of measurements of PBL fluctuating quantities in the future may increase our knowledge of the behavior of these variables as well as allow refinement of the more complete parameterized Reynolds stress and turbulent energy equations. The resulting improvement of the more complete models may suggest improvements for simple stress relationships.

Acknowledgments. I express my appreciation to Dr. James Deardorff who offered many helpful suggestions during this research and to Drs. Clayton Paulson and James Riley who critically read the manuscript. This study was completed with the support of the Atmospheric Sciences Section, National Science Foundation, under Grant GA-37571. The Oregon State University Computing Center provided the computing time.

REFERENCES

Ayra, S. P. S., 1972a: The critical condition for the maintenance of turbulence in stratified flows. *Quart. J. Roy. Meteor. Soc.*, **98**, 264–273.

——, 1972b: Comments on "Local free convection, similarity, and the budgets of shear stress and heat flux." *J. Atmos. Sci.*, **29**, 1230–1231.

——, and E. J. Plate, 1969: Modeling of the stably stratified boundary layer. *J. Atmos. Sci.*, **26**, 656–665.

Blackadar, A. K., 1962: The vertical distribution of wind and

turbulence exchange in a neutral atmosphere. *J. Geophys. Res.*, **67**, 3095–3102.
Businger, J. A., 1971: Comments on "Free convection in the turbulent Ekman layer of the atmosphere. *J. Atmos. Sci.*, **28**, 298–299.
——, J. C. Wyngaard, Y. Izumi and E. F. Bradley, 1971: Flux-profile relationships in the atmospheric surface layer. *J. Atmos. Sci.*, **28**, 181–189.
Carson, D. J., and F. B. Smith, 1973: The wind-stress relationship. *Quart. J. Roy. Meteor. Soc.*, **99**, 171–177.
Corrsin, S. 1956: Some current problems in turbulent shear flow. *Naval Hydrodynamics Symposium*, Publication 515, Nat. Acad. Sci., Nat. Res. Coun., 373–400.
Crow, S. C., 1968: Viscoelastic properties of fine-grained incompressible turbulence. *J. Fluid Mech.*, **33**, 1–20.
Csanady, G. T., 1972: Geostrophic drag, heat and mass transfer coefficients for the diabatic Ekman layer. *J. Atmos. Sci.*, **29**, 488–496.
Daly, B. J., and F. H. Harlow, 1970: Transport equations in turbulence. *Phys. Fluids*, **13**, 2634–2649.
Deardorff, J. W., 1972a: Numerical investigation of neutral and unstable planetary boundary layers. *J. Atmos. Sci.*, **11**, 91–115.
——, 1972b: Theoretical expression for the countergradient vertical heat flux. *J. Geophys. Res.*, **77**, 5900–5904.
——, 1973: An explanation of anomalously large Reynolds stresses within the convective planetary boundary layer. *J. Atmos. Sci.*, **30**, 1070–1076.
Geisler, J. E., and E. B. Kraus, 1969: The well-mixed Ekman boundary layer. *Deep-Sea Res.*, **16** suppl., 73–84.
Johnston, J. P., 1970: Measurements in a three-dimensional turbulent boundary layer induced by a swept, forward facing step. *J. Fluid. Mech.*, **42**, 823.
Jones, W. P., and B. E. Launder, 1972: The prediction of laminarization with a two-equation model of turbulence. *Intern. J. Heat Mass Transfer*, **15**, 301–310.
Lavoie, R., 1972: A mesoscale numerical model of lake-effect storms. *J. Atmos. Sci.*, **29**, 1025–1040.
Lenschow, D. H., 1970: Airplane measurements of planetary boundary layer structure. *J. Appl. Meteor.*, **9**, 874–884.
——, 1972: Airborne measurements of air mass modification over the Great Lakes in late fall. *Proc. Symp. Air Pollution, Turbulence and Diffusion*, Atmos. Sci. Lab., White Sands Missile Range, N. M., 84–92.
Lettau, H. H., 1950: A re-examination of the "Leipzig Wind Profile" considering some relations between wind and turbulence in the frictional layer. *Tellus*, **2**, 125–129.
——, 1962: Theoretical wind spirals in the boundary layer of a barotropic atmosphere. *Beit. Phys. Atmos.*, **35**, 195–212.
——, 1970: Note on eddy diffusivities. Final Report, Dept. of Meteorology, University of Wisconsin, 1–11.
——, and B. Davidson, Eds., 1957: *Exploring the Atmosphere's First Mile, Vol. 2.* New York, Pergamon Press, 578 pp.
Lilly, D. K., 1967: The representation of small-scale turbulence in numerical simulation experiments. *Proc. IBM Scientific Computing Symp. Environmental Science*, IBM Form No. 320–1951, 195–210.
Lumley, J. L., 1970: Toward a turbulent constitutive relation. *J. Fluid Mech.*, **41**, 413–434.
——, and B. Khajeh-Nouri, 1973: Computational modeling of turbulent transport. *Advances in Geophysics* (in Press).
Metcalf, J., and D. Atlas, 1972: Microscale ordered motions and atmospheric structure associated with thin echo layers in stably stratified zones. Presented at IUCRM Colloquium on Waves and Turbulence in Stratified Layers and the Effects on Electromagnetic Propagation, La Jolla, Calif.
Monin, A. S., and A. M. Yaglom, 1971: *Statistical Fluid Mechanics*, J. L. Lumley, Ed. The MIT Press, 769 pp.
Obukhov, A. M., 1946: Turbulence in an atmosphere with inhomogeneous temperature. *Tr. Akad. Nauk SSSR Inst. Teoret. Geofiz.*, No. 1, 95–115 (reprinted in *Boundary-Layer Meteor.*, **2**, 7–29).
Peterson, E. W., 1972: Relative importance of terms in the turbulent-energy and momentum equations as applied to the problem of a surface roughness change. *J. Atmos. Sci.*, **29**, 1470–1476.
Plate, E. J., 1971: *Aerodynamic Characteristics of Atmospheric Boundary Layers.* AEC Critical Review Series, 190 pp.
Reynolds, W. C., 1970: Computation of turbulent flows state-of-the-art. Rept. MD-27, Dept. Mech. Eng., Stanford University, 90 pp.
Rotta, J. C., 1951: Statistische Theorie nichthomogener Turbulenz. *Z. Phys.*, **129**, 547–572.
Stewart, R. W., 1959: The problem of diffusion in a stratified fluid. *Advances in Geophysics*, Vol. 6, 303–310.
Swinbank, W. C., 1970: Structure of wind and the shearing stress in the planetary boundary layer. *Arch. Meteor. Geophys. Bioklim.*, **A19**, 1–12.
Tennekes, H., 1970: Free convection in the turbulent Ekman layer of the atmosphere. *J. Atmos. Sci.*, **27**, 1027–1034.
——, 1971: Reply (to J. A. Businger, 1971). *J. Atmos. Sci.*, **28**, 300–301.
——, and J. L. Lumley, 1972: *A First Course in Turbulence.* The MIT Press, 300 pp.
Webster, C. A. G., 1964: An experimental study of turbulence in a density-stratified shear flow. *J. Fluid Mech.*, **19**, 221–245.
Wyngaard, J. C., and O. R. Coté, 1971: The budgets of turbulent kinetic energy and temperature variance in the atmospheric surface layer. *J. Atmos. Sci.*, **28**, 190–201.
——, —— and Y. Izumi, 1971: Local free convection similarity, and budgets of shear stress and heat flux. *J. Atmos. Sci.*, **28**, 1171–1182.
——, —— and ——, 1972: Reply (to S. P. S. Ayra, 1972b). *J. Atmos. Sci.*, **29**, 1231–1233.
Yamamoto, Giichi, N. Yasuda and A. Shimanuki, 1968: Effect of thermal stratification on the Ekman layer. *J. Meteor. Soc. Japan*, **47**, 442–454.
Zilitinkevich, S. S., D. L. Laikhtman (Leichtmann) and A. S. Monin, 1967: Dynamics of the atmospheric boundary layer. *Izv. Atmos. Oceanic Phys.*, **3**, 297–333.