AN ABSTRACT OF THE DISSERTATION OF

Matthew P. Campbell for the degree of Doctor of Philosophy in Mathematics Education presented on June 11, 2014.

Title: Responsive Pedagogies of Practice: Researching an Ambitious Secondary Mathematics Teacher Education Design.

Abstract approved: _____

Rebekah L. Elliott

Design in secondary mathematics teacher education must prepare teacher candidates to do the work of ambitious and equitable mathematics teaching with skill by situating development in the work of teaching and incorporating opportunities to investigate and enact teaching. Teacher education designs must also be responsive to the work that mathematics teachers are expected to do in school settings—which are a product of a set of goals, expectations, and communities that have formed over long histories. This dissertation pursues novel and emerging questions around what the design and implementation of a responsive and practice-focused approach to teacher education—what I call a *responsive pedagogy of practice*—entails, how those entailments are informed by the work of teaching in schools, and how those entailments inform what individuals do in teacher education programs. Three manuscripts collectively illustrate progress on these ideas, drawing upon data and analyses from design-based research in a secondary mathematics teacher education program.

The first manuscript addresses a question of what is meant by and entailed in the design and implementation of a responsive pedagogy of practice. Through an intertwined process of design, implementation, analyses, and revision, three sets of findings

informing the development of a theory of responsive pedagogies of practice emerged. First, two needs emerged in addition to the initial attention to developing teacher candidates' instructional skill—aligning with the mathematics of the secondary classroom and developing teacher candidates' mathematical knowledge for teaching (MKT; Ball, Thames, & Phelps, 2008). The negotiation of these multiple needs poses a challenge for teacher educators. This negotiation also gave rise to a second finding involving the development of instructional skill, which needs to focus on the development on multiple levels of pedagogical tools. Further, a set of pedagogical tools must be derived, in part, from the work that teacher candidates do in school settings. Ultimately, this means that responsiveness in teacher education entails preparing teacher candidates to do what is typically done in school settings while also finding the openings at which to press for more ambitious and equitable teaching practice. Finally, a third finding emerged regarding the novel roles for teacher educators and partner teachers that are constructed through a responsive and practice-focused pedagogy of teacher education.

The second manuscript highlights analyses conducted to further investigate the features of the activity of secondary mathematics teaching to which a teacher education design needs to be responsive. Data from teacher candidates' enactments across two settings—the university methods courses and their student teaching placements—were drawn upon to identify the entailments of the activity of secondary mathematics teaching. A modified analytic framework from Leont'ev (1981) and Wertsch, Minick, and Arns (1984) was used to analyze the work of teacher candidates in each setting. While the work in the methods courses emphasized providing students access to mathematics and the orchestration of goal-directed discussions, work in student teaching placements was

defined by efficient and productive work on mathematical procedures. Opportunities for more novel instruction were made available contingent on the two expectations being met. These findings have implications for what pedagogical tools should be developed through a responsive pedagogy of practice that enable efficient and procedurally focused mathematics work while also making progress on increasingly ambitious and equitable instruction.

The third manuscript highlights an example of how an emerging sense of responsive and practice-focused approaches to teacher education and the work of teacher candidates in school classrooms inform the design features of a responsive pedagogy of practice. A specific design example is put forth that situates opportunities of enactment in the work of addressing students' mathematics errors in the midst of work with students on mathematics procedures. As such, the example is derived from the work that teacher candidates do in school classrooms and also shows how a design can attend to the multiple needs related to teacher candidate and student development. The example serves as one of many activities in development—all of which are subject to further examination through a design-based research process.

Responsive Pedagogies of Practice: Researching an Ambitious Secondary Mathematics Teacher Education Design

by
Matthew P. Campbell

A DISSERTATION

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Presented June 11, 2014
Commencement June 2015

Doctor of Philosophy dissertation of Matthew P. Campbell presented on June 11, 2014.

APPROVED:

Major Professor, representing Mathematics Education

Dean of the College of Education

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Matthew P. Campbell, Author

ACKNOWLEDGEMENTS

I would first like to thank my advisor, Dr. Rebekah Elliott, who has been an incredibly supportive mentor during my five years at Oregon State University. Dr. Elliott has provided me with a wealth of professional opportunities as a researcher, teacher educator, and future university faculty member. Specific to this dissertation, Dr. Elliott gave me the opportunity to pursue an ambitious and novel study and supported me throughout the process. While I am confident that we both grew in the process, I cannot express the gratitude I have for Dr. Elliott and how she has prepared me for the next steps in my career.

I would also like to thank my other committee members—Dr. Wendy Rose Aaron, Dr. Shawn Rowe, and Dr. Mary Beisiegel. I am fortunate to have had the opportunity to draw on their expertise and experiences, both for this study and throughout my doctoral program. I would like to thank Dr. Karen Higgins and Dr. Tevian Dray for having served on my committee at earlier stages in my program and for their support as I transitioned into my dissertation work. Thank you, as well, to Dr. David Froman, who served as my Graduate Council Representative throughout my program and has made sure the process ran smoothly.

For five years in Corvallis I have also been supported by outstanding faculty, staff, and students in Science and Mathematics Education, the College of Education, and Oregon State University as a whole. I have benefitted from being here during a time of many exciting changes and have had the chance to work with many people as a result—some who have moved on to other ventures and some who are just beginning their journey at Oregon State. I am also grateful for the lasting friendships that I have

developed outside of work during my time here. Thank you, all, for the good times and for making Corvallis feel like a home.

I would like to thank all of the teacher candidates and classroom teachers with whom I have had the chance to work. That is one of the most rewarding parts of my work and what keeps me wanting to grow and improve. A special thanks goes to the participants in the work presented in this dissertation. Becoming a teacher and being a teacher is difficult work and doing those things as the focus of research is even more taxing. I truly hope that they benefitted from the experience as much as I have. It is because of their willingness, openness, and flexibility that we can continue to make positive steps in this field. I am encouraged and excited by the fact that there are eager and committed teaching professionals out there who strive to better themselves and others.

I came to Oregon State with the professional and academic support of many people, including faculty, staff, and students at North Carolina State University, where I received my bachelor's and Master's degrees. Specifically, I would like to thank Dr. Hollylynne Lee, Dr. Paola Sztajn, and Dr. Allison McCulloch for their support and guidance, even long after I left Raleigh. I would also like to thank my many former colleagues from teaching and graduate school. They continue to serve as a network of support and friendship and I have learned so much from them. I am also indebted to the Park Scholarships program and its staff, who ultimately put me on a track toward this dissertation starting my freshman year at N.C. State. I would also like to thank the teachers and supporters I have had since childhood, who have all played a role in this culminating event, and in my success moving forward.

I would like to thank my mother, Jeanine Campbell, for tirelessly working to raise my brother, my sister, and me on her own and for encouraging me during every step of my journey through school, career, and life. I would also like to thank my brother, Kevin, my sister, Katie, and the rest of my extended family and friends. They have been a strong system of support throughout my life, including through times of tragedy. I am also honored to have been welcomed into another family by Jim and Robin Hecht, Matt, Penny, and Jordie Hawkins, and Emily Moss. I am grateful for all of my family and friends' support, and driven by the hope to make them all proud.

Finally, none of this would be possible without the support and love of my wife, Heidi. She took a chance with me—in life, on this journey to Oregon, through a doctoral program, and into my career in this field. The process was not always straightforward yet she never wavered. Heidi has continued to be my biggest supporter as well as the person who pushes and challenges me to improve the most. Now we set off for the next stop, which involves another cross-country move. I am excited to continue growing our lives and our family (along with our labradoodle, Barry) wherever life takes us.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

Page

TABLE OF CONTENTS (Continued)

Page

TABLE OF CONTENTS (Continued)

Page

LIST OF FIGURES

LIST OF TABLES

For my father, Paul Clayton Campbell.

Introduction

> *Because universities are currently thought to be unsuccessful in preparing novices for practice, [teacher educators] are faced with two challenges: preparing beginning teachers to actually be able to do teaching when they get into classrooms, and preparing them to do teaching that is more socially and intellectually ambitious than the current norm.*
>
> *Lampert et al., 2013, p. 1*

In this dissertation, I highlight, define, and theorize the work of *responsive pedagogies of practice* in secondary mathematics teacher education. I propose that such pedagogies of teacher education are a direct result of and answer to recent needs, trends, and recommendations in the fields of teaching and teacher development. Broadly, such pedagogies address the two core aims of teacher education outlined in the quote above from Lampert and her colleagues. First, teacher candidates[1] must be prepared to teach mathematics ambitiously and equitably to provide *each* student with opportunities to do rigorous and authentic mathematical work and to develop mathematical proficiency (Jackson & Cobb, 2010; Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices, & Council of Chief State School, 2010). Concurrently, teacher candidates must be prepared to do the work of teaching expected of them in schools when they begin their careers after a teacher education program. Ambitious and equitable teaching is not

[1] I use the term "teacher candidate" to refer to individuals who are enrolled and progressing through a teacher education program. I will use this term consistently throughout this dissertation, recognizing that others use terms such as "preservice teacher", "novice teacher", and "student teacher". I will add a similar footnote within each manuscript that comprises this dissertation as each is being prepared for publication and would warrant a similar note.

the norm in most school settings, which makes attending to both aims a difficult task for teacher educators and for the teacher candidates and school communities with whom they work. Responsive pedagogies of practice serve as an approach to secondary mathematics teacher education that can work toward addressing this dilemma, though the design and implementation of such pedagogies lack the needed theorization and specification for success. In this introduction, I motivate a focus—in practice and in research—on responsive pedagogies of practice and provide an overview of the study.

Motivating a Study Focused on Responsive Pedagogies of Practice

University teacher education is in need of improvement—needing to better prepare skilled teachers in a time when calls to marginalize the role of teacher education programs are increasing (Darling-Hammond, 2010; Kumashiro, 2010; Wiseman, 2012). For decades, spurred by the work of NCTM and the National Research Council, goals have been established to provide each student with mathematically authentic opportunities to build mathematical proficiency, make conjectures, justify claims, make connections, and ultimately, develop a deeper understanding of and skill with mathematics (Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000). Classrooms that foster such work involve more discussion and build on students' ideas, not just the ideas from the teacher or the textbook. Furthermore, to be truly equitable teachers need to make these opportunities available to each student, not just those traditionally considered to be "good at math", such as students who complete mathematics problems or recall facts the quickest (Boaler, 2002; Horn, 2007; Jackson & Cobb, 2010). Providing equitable opportunities to engage with meaningful mathematics requires teachers to respond to students' ideas, needs, and ways of working—none of which may be the same among all

students across lines of race, ethnicity, gender, language, and socioeconomic status (NCTM, 2000). While defining the instructional work of both ambitious and equitable mathematics instruction is a central focus in the field, the concrete understandings of that work is limited.

Further limited is an understanding of how to support teachers to develop skill with such practice. This is, in part, complicated by the reality that instructional practices that are ambitious and/or equitable are not the norm in mathematics classrooms (Lampert et al., 2013; Stigler & Hiebert, 1999). In addition to developing skill with those approaches to teaching, teacher candidates must also be prepared to do the work of teaching as it is expected of them in the schools they work. Ultimately, it is the work of teacher educators and the teacher development opportunities they design and implement to prepare teacher candidates to successfully begin their careers while also finding the gradually increasing opportunities to press for and gain improvements to teaching and learning mathematics in schools.

Considering either (though, ideally, both) of these aims requires clarifying what is meant by *preparing* teacher candidates. Teachers are not born, nor are they the product of merely a passion for the work, an awareness of how the work is done well, or an expertise in the subject matter they teach (Ball & Forzani, 2009, 2011). Teaching is complex, specialized, and a culturally defined activity (Ball, Thames, & Phelps, 2008; Lampert, 2010; Stigler & Hiebert, 1999). What an individual teacher does in the classroom is not idiosyncratic or individualistic and instead is informed by the contexts in which they work. These contexts are comprised of the goals of the work, the expectations, the available tools, and the other actors in the system—all of which have

been formulated over long histories (Rogoff, 2003). Therefore developing skill as a teacher involves participation in that work and in those settings (Wenger, 1998). This is at the heart of calls in research and policy that teacher education be more closely tied to clinical practice (Ball & Cohen, 1999; Grossman & McDonald, 2008; National Council for Accreditation of Teacher Education, 2011). As the work of teaching is too complex to be learned by simply stepping into a classroom (Grossman, Hammerness, & McDonald, 2009), it is up to teacher education designs to provide teacher candidates with opportunities for participation that are more beneficial and sensible for their development.

This demand has given rise to what McDonald, Kazemi, and Kavanagh call a *pedagogy of practice* in teacher education, stemming from the recommendation from Grossman and her colleagues (2009) to provide teacher candidates with opportunities to both *investigate* and *enact* the work of teaching. Teacher education programs are typically flush with opportunities for teacher candidates to investigate the work of teaching—through observing, analyzing, and reflecting on instruction and using artifacts such as student work and classroom video. Opportunities to enact the work of teaching are less common, even though they are a core aspect of the preparation of newcomers in other professions (Grossman et al., 2009). Considering the form of such opportunities in teacher preparation requires a consideration of how to appropriately bound what teacher candidates engage with, such as what Lampert and her colleagues (Lampert & Graziani, 2009; Lampert et al., 2010) call *instructional activities* (IAs). These short lessons serve as *approximations* (Grossman et al., 2009) that maintain some of integrity to the complexity of teaching while still allowing for more productive and accessible development of what

have been called *core practices* of teaching (Ball & Forzani, 2009; Lampert et al., 2013; McDonald et al., 2013; TeachingWorks, 2014; Windschitl, Thompson, Braaten, & Stroupe, 2012). IAs can become the object of investigation and enactment in a practice-focused teacher education program. Part of this dissertation study takes on the needed work to consider the design and implementation of IAs and pedagogies of practice in secondary mathematics teacher education.

Pedagogies of practice offer an approach to teacher education that makes central the role of participation in the development of skill with a cultural practice such as mathematics teaching. However, I posit that the design and implementation of pedagogies of practice run the risk of being primarily (if not exclusively) conceptualized in the context of university teacher education. In turn, these pedagogies can end up doing little to actually support and develop teacher candidates for their work in schools because they are not informed by the work of teaching within those contexts. While teacher candidates must teach in more ambitious and equitable ways—practices that are uncommon in many schools—approaches to teacher education often assume that such novel and research-supported ideas can simply transfer from the university to schools and classrooms in a unidirectional flow of information (Borko, 2004; Peressini, Borko, Romagnano, Knuth, & Willis, 2004). From this perspective, longstanding failures to achieve such changes are attributed as a failure of the individual teacher candidates, the schools in which they teach, or the students with whom they work.

Teacher education must reframe its relationship to schools—one that is more bidirectional—as that is the setting for which teacher candidates are ultimately being prepared (Cobb, Zhao, & Dean, 2009). While a focus on core practices, the use of

artifacts of practice, and enactment situate teacher preparation in the work of teaching (Ball & Cohen, 1999), the actual designs of those experiences for teacher candidates must be *responsive* to and informed by the work that goes on in schools, while also striving to improve it (Gutiérrez & Vossoughi, 2010; Kazemi & Hubbard, 2008). Pedagogies of practice that are designed and implemented from this responsive perspective have the potential to serve the dual aims of preparing teacher candidates for the start of their careers in schools, while also preparing them to do the work in increasingly ambitious and equitable ways. However, much work remains to achieve this potential in research and in practice.

Questions to address regarding responsive pedagogies of practice. The ideas of both practice-focused and responsive teacher education are novel and in need of theorizing and specification to inform research and practice. This dissertation serves as a concerted effort to contribute to this emerging area of the field. To do so, I ask the following questions regarding responsive pedagogies of practice in secondary mathematics teacher education:

1. What does it mean for the design and implementation of a pedagogy of practice in secondary mathematics teacher education to be responsive?
2. What are the features of the activity of secondary mathematics teaching to which a teacher education design needs to be responsive?
3. In what ways does the instructional work teacher candidates do in school classrooms inform the design features of a responsive pedagogy of practice?

Addressing these questions serves as a foundation for research on and the design of responsive pedagogies of practice. For this dissertation, I present three manuscripts—

each addressing one of the questions above—that together represent the work of a *design-based research* study around the development of a responsive pedagogy of practice in secondary mathematics teacher education. The next section overviews details of the overall study and each of the three manuscripts—highlighting the distinctions that warrant the manuscripts as three separate pieces, albeit unified and representative of the entirety of this study.

Overview of the Study as Three Manuscripts

My work took place in a Master's level teacher education program for secondary mathematics teachers in which I was involved as a course instructor, design team member, and student teaching supervisor. In this particular program, efforts to develop responsive pedagogies of practice had been ongoing for a few years. Beginning with this study during the 2012-2013 academic year, these efforts were situated as a *design-based research* study, in which the context of design, implementation, analyses, and revisions serve in the development of educational innovations and the theories of development that inform them (Design-Based Research Collective [DBRC], 2003). In design-based research, design and research are intertwined, serving as an alternative to the more traditional methods of experimental design, in which a researcher would test a predetermined design against a comparison design and measure outcomes (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

The close link between design and research offers practical lessons for the improvement of education practice—providing a "relevance to practice" (Gutiérrez & Penuel, 2014) that makes it a productive path forward in educational research. It is for this reason that I see design-based research as a fruitful approach to address the following

questions:

1. What does it mean for the design and implementation of a pedagogy of practice in secondary mathematics teacher education to be responsive?
2. What are the features of the activity of secondary mathematics teaching to which a teacher education design needs to be responsive?
3. In what ways does the instructional work teacher candidates do in school classrooms inform the design features of a responsive pedagogy of practice?

Data for this study were collected during the 2012-2013 academic year from across three terms (fall, winter, and spring) and across the settings of secondary mathematics methods courses at the university and teacher candidates' student teaching placements. The three manuscripts that comprise this dissertation draw upon different subsets of these data and answer research questions tied to each of the main questions above.

The first manuscript—titled "Developing ambitious and equitable secondary mathematics teachers through a responsive pedagogy of practice: Design as a context for research"—takes on the first question above regarding what it means for the design and implementation of pedagogies of practice in secondary mathematics teacher education to be responsive. In the paper, I highlight the work of using ongoing and retrospective analyses of three design cycles during the academic year to further develop a theory of what is meant by and entailed in a responsive pedagogy of practice. In design-based research, instructional design and the hypotheses that inform those designs are refined by ongoing analyses from cycle to cycle, and retrospective analyses that involve reexamining the data from multiple iterations. What defines the strength of this process is not only the refinement of a local design, but the broader theoretical contributions that

can contribute to the field's understanding of development and design and the work of others (Barab & Squire, 2004; Edelson, 2002). I draw upon data and analyses of the evolving set of needs, design principles, and work to be done by participants in the design to build a theory of responsive pedagogies of practice while also showing an example of one design that is making progress in terms of its responsiveness to the work of teaching in schools.

Part of the findings from the work that I present in the first manuscript highlights the need for teacher educators to leverage the work that teacher candidates do in their student teaching placements and to better understand the contextual factors that inform that work. This is summed up in the second question above—What are the features of the activity of secondary mathematics teaching to which a teacher education design needs to be responsive? The second manuscript—titled, "Developing pedagogical tools for ambitious secondary mathematics instruction through responsive teacher education: An analysis of practice"—presents a set of further retrospective analyses conducted to address this question. In that work, I use a set of analytic tools modified from Leont'ev (1981) and Wertsch, Minick, and Arns (1984) from the tradition of *activity theory* in order to analyze the work of teacher candidates in the setting of a responsive pedagogy of practice and in school classrooms. Analyses from the university setting serve as a way to consider how the activity of secondary mathematics teaching was approximated across a set of IAs geared toward promoting ambitious and equitable mathematics teaching. Analyses from school classrooms (specifically the student teaching placements of a group of teacher candidates) serve in identifying what the features of the activity of secondary mathematics teaching are to which teacher education designs must be responsive.

Findings from those analyses serve as a resource for the continued evolution of the design of practice-focused and responsive teacher education pedagogies.

After further specifying the entailments of a responsive pedagogy of practice as well as the work of secondary mathematics teaching in schools in the first and second manuscript, what are the impacts on design? The third, and final, manuscript—titled, "Supporting secondary mathematics teachers in attending to errors through a responsive pedagogy of practice"—addresses the this issue by putting forth a specific example of a set of design decisions for an audience of mathematics teacher education practitioners. One instructional practice (and the way in which it was typically carried out by teacher candidates) that emerged from the analyses highlighted in the second manuscript — addressing students' mathematical errors and questions—serves as the focus of the paper. Using a structure for an IA, the ideas that have emerged from these design-based research efforts to this point are made concrete. Specifically, the IA that is discussed is proposed as a way for teacher educators and teacher candidates to work on ambitious and equitable practice, though in a form that is sensible and responsive to the work of secondary mathematics teaching. Not only, then, is there a specific activity for myself and others to use and build upon, but it serves as another way to discuss the larger, yet still domain-specific theories that are a major product of this work.

Highlighting the Intersecting Roles of Researcher and Teacher Educator

The nature of this study as a design-based research study places me in the unique position of being both a teacher educator and a researcher of my own practice. While that position is central to design-based research and is a strength to be leveraged, it also requires me to be more transparent about my role in both the research and practice

elements of this work and how that maintains a level of rigor and trustworthiness that is expected. I come to this work as a relatively new teacher educator, though one with a range of experiences in a number of settings across my graduate career. My early experiences as a teacher educator involved the central use of video of classrooms and mathematics tasks. Four years ago I became aware of the use of rehearsal in teacher development from my work on a research and development project around mathematics teacher leaders, titled *Researching Mathematics Leader Learning* [RMLL]. Dr. Elham Kazemi at the University of Washington brought this to RMLL from her ongoing work with a group of elementary mathematics teacher educators—the *Learning in, from, and for Teaching Practice* project.

This work became an object of interest for myself and my mentor and dissertation chair, Dr. Rebekah Elliott. Since the 2010-2011 academic year, each of us taught one of the two secondary mathematics methods courses that were part of the Master's program highlighted in this dissertation. During that first year, there were preliminary efforts to incorporate the investigation and enactment of IAs into the existing work of the courses. Bringing teacher candidates into classrooms to rehearse with secondary students as part of an emerging pedagogy of practice was incorporated during the next year (2011-2012)—specifically the second course during the winter term that I taught. By the 2012-2013 academic year—the year that serves as the focus of this study—efforts to transform the pedagogies across the two methods courses to be more practice-focused and responsive to school settings were now central. As a result, this study puts me in the position to research the designs and practice of my mentor and me. Furthermore, the specific work was an authentic problem of practice—one in which there was an

increasingly longstanding investment. This creates a potentially troublesome and complicated arrangement that can compromise the research aspects of the work.

The methodological affordances of design-based research offer systematic ways to frame and organize the research of one's own practice to account for these potential dilemmas. This includes the specific goals and products of the work, the use of multi-person teams (with individuals assuming a range of roles in the design and research process), and the use of multiple data sources that add transparency into the implementation and decision-making of a design and the resulting analyses. The nature and goals of design-based research, which are not to evaluate the effectiveness of a predetermined intervention or design, helps account for the close tie between the roles of researcher and practitioner in this work. Instead, in design-based research, the goal is to refine theories about development and design through an admittedly intermediate and evolving design. This offers a way to mediate some of the personal investment in a design that might compromise a research effort. A second strength of the methodology is that I was not alone in carrying out the larger scope of this teacher education work. Design-based research leverages the strength of insight of a group of individuals who work at the boundary between research and practice—each assuming different capacities in the planning, implementation, collection of data, analyses and reflection, and refinement of a design. For this study, I worked with my mentor and fellow methods course instructor, Dr. Rebekah Elliott. Joining the effort during the year of this study was another mathematics teacher educator, Dr. Wendy Rose Aaron, who added insight into the design and the research process while at the same time being new to local work and not being actively engaged in the broader planning and instruction in the methods

courses. Classroom teachers, serving as partners in the work and hosts of enactment opportunities for teacher candidates in school classrooms, also contributed to the work. The role of a partner teacher in a responsive pedagogy of practice is something I highlight in the first manuscript. Finally, a core part of this work is the maintenance of a *reflexive journal* (Altheide & Johnson, 1994). Extensive documentation is key to any design-based research process—both as a measure of transparency and trustworthiness as well as a useful source of data for considering the factors that played a role in the ongoing and in-the-moment changes to a design or a process. Such journaling was conducted by me throughout the 2012-2013 academic year and will be an aspect of my work in teacher education moving forward as I look to continue design-based research into my career. I will revisit all of the considerations of my own role as a researcher and teacher educator and the work of researching and designing responsive pedagogies of practice in the overall conclusion.

Significance of Dissertation

This dissertation study, its focus on design-based research, its responsiveness to school settings, and its theoretical (yet still domain-specific and practical) products, make for research that has "relevance to practice", which is proposed by Gutiérrez and Penuel (2014) as a key criterion for the rigor of research. The findings discussed across the three papers make meaningful progress toward answering the three questions outlined above and do so in a way that contribute to my own work as a mathematics teacher educator, as well as the work of my colleagues and others in the field. The sum of the three manuscripts also makes clear the areas for continued inquiry and effort around the design and implementation of responsive pedagogies of practice and the development of

secondary mathematics teacher candidates. This dissertation also highlights the specialized role that university teacher education can play in the practice-focused and responsive development of secondary mathematics teachers—specifically attending to the two aims of teacher development outlined in the quote at the start of this introduction.

Developing Ambitious and Equitable Secondary Mathematics Teachers Through a Responsive Pedagogy of Practice: Design as a Context for Research

Introduction

There has been greater attention from policymakers, researchers, and other stakeholders on the process of teacher development and its potential impacts on teaching quality (Darling-Hammond, 2010; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). In the United States, there is the pressing and complex need to prepare new mathematics teachers to take on the work of ensuring that *each* student—from diverse racial, linguistic, economic, and academic backgrounds—has access to opportunities for rigorous academic work to develop mathematical proficiency to meet the demands of an increasing mathematically, statistically, and technologically complex society (Darling-Hammond et al., 2009; Kilpatrick, Swafford, & Findell, 2001; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Research Council, 2010). Teachers must draw upon students' diverse cultural and linguistic resources in the mathematics classroom and position mathematics as a human practice and a tool for social change (R. Gutiérrez, 2011). These *ambitious* and *equitable* goals for student learning (Jackson & Cobb, 2010; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Newmann & Associates, 1996) have charged teacher educators with the task to prepare teacher candidates[2] with the knowledge and skills to teach in ways that are not common in school classrooms, while also preparing them to quickly step into classrooms (Lampert et al., 2013).

---

[2] We use the term "teacher candidate" to refer to individuals who are enrolled and progressing through a teacher education program. We will use this term consistently throughout the article, recognizing that others use terms such as "preservice teacher", "novice teacher", and "student teacher".

Recommendations from policy have come forth regarding the location, process, and content of teacher preparation (Ball & Cohen, 1999; National Council for Accreditation of Teacher Education, 2010). These recommendations have centered on elevating the time and attention to teacher development in the context of schools—either physically, through clinical and field-based experiences, or conceptually, through the use of artifacts of practice like classroom video and student work. To extend and refine these calls for change, teacher education researchers suggest a focus on the development of actual skill in the work of teaching—in addition to building knowledge about teaching and developing beliefs (Grossman & McDonald, 2008; Grossman, Hammerness, & McDonald, 2009). Practice-focused designs that attend to developing skilled teaching must prepare teacher candidates for the profession, making them better able to take on the work of teaching as they enter the classroom. Therefore, teacher education and its designs must be *responsive* to the context of school classrooms and to the practice of teacher candidates (Cobb, Zhao, & Dean, 2009; Kazemi & Hubbard, 2008). While the research base on teaching is robust, Grossman and McDonald (2008) contend that research on and theories of teacher development and teacher education are lacking. This can be said for the emerging movement in the field regarding responsive and practice-focused teacher education pedagogies, which is in need of further specification and theory building.

In this article, we advance and further specify a notion of responsive, practice-focused teacher education—what we call *responsive pedagogies of practice*—through a discussion of design-based research efforts (Design-Based Research Collective [DBRC], 2003; Edelson, 2002) in a secondary mathematics teacher education program. Not enough is known about what is meant by or entailed in responsive and practice-focused

teacher education. Therefore, research is needed to build basic knowledge on the entailments of such designs and the development opportunities for teacher candidates. The aim of this study is to examine the evolution of a teacher education design—drawing on data and analyses from implementation—to learn about the nature and entailments of responsive and practice-focused teacher development. While these research efforts inform our own work with secondary mathematics teacher candidates, the development of design-oriented theories of responsive and practice-focused teacher education have broader impacts in the field and motivate further innovation and research.

Focusing Teacher Development on Practice: A Review of Literature

While university teacher education has held a central position in preparing teachers for their work of supporting students, its role has been questioned due to a lack of perceived influence on preparing quality teachers (Darling-Hammond, 2010; Kumashiro, 2010; Wiseman, 2012). One means for attending to teacher quality in teacher education has been efforts to situate teacher development in the context of teaching. Linda Darling-Hammond (2010) asserts, "the central issue I believe teacher education must confront is how to foster learning about and from practice *in* practice" (p. 42). Situating teacher development in the context of teaching in schools is viewed as a way to resolve a potential disconnect between the university- and school-based components of teacher education programs (Ball & Cohen, 1999; Grossman, Hammerness, et al., 2009; Hammerness, Darling-Hammond, & Bransford, 2005; Zeichner, 2010)—what Feiman-Nemser and Buchmann (1985) called the *two-worlds pitfall*. In this review, we will highlight a number of recent recommendations that follow from this broader call for a focus on practice in teacher preparation.

Clinical practice in teacher education. The push for situating teacher preparation in the setting of schools is a core assertion from recent policy documents. The Blue Ribbon Panel on Clinical Preparation and Partnerships for Improved Student Learning commissioned by NCATE (2010) unequivocally assert:

> To prepare effective teachers for 21st century classrooms, teacher education must shift away from a norm, which emphasizes academic preparation and course work loosely linked to school-based experiences. Rather, it must move to programs that are fully grounded in clinical practice and interwoven with academic content and professional courses (p. ii).

Such calls from NCATE—and, more recently, the Council for the Accreditation of Educator Preparation (CAEP, 2013)—require dramatic change in the content and pedagogies of university teacher education. In addition to increasing the span and duration of teacher candidates' time in actual classrooms, these recommendations call for coherence and relationship building across university and school settings.

Despite these recent calls, research and reviews on clinical practice and student teaching show a lack of consistent findings to suggest teacher candidates' clinical experiences are universally beneficial, thus questioning them as productive opportunities (Anderson & Stillman, 2013a; Clift & Brady, 2005; Feiman-Nemser & Buchmann, 1987; Guyton & McIntyre, 1990; McIntyre, Byrd, & Foxx, 1996; National Research Council, 2010; Wideen, Mayer-Smith, & Moon, 1998). Valencia, Martin, Place, and Grossman (2009) concisely state that, "the power of student teaching is legend" (p. 304)—attributing this to often little connection between the goals of teacher education programs and the eventual instructional practice in schools. They point to the lack of specificity on

what is entailed in teacher development, which can then divert teacher candidates away from opportunities to develop a sense of and skill with ambitious and equitable mathematics instruction.

Anderson and Stillman (2013a) attribute the mixed contributions of clinical practice not to an absence of research or reports, but to the analytic strength of the research on the impact of clinical experiences. One concern is that work primarily focuses on constructs such as teacher candidate beliefs, knowledge, or disposition (Skott, Van Zoest, & Gellert, 2013). Anderson and Stillman suggest that studies must attend to teacher candidate practice and its connection to K-12 student learning. Further, they suggest attention be paid in studies to an ecological approach that considers the complexity of the settings in which teacher candidates learn and work and contextual factors mediating development (Clift & Brady, 2005; Valencia et al., 2009; Wideen et al., 1998). The notion that teacher candidates will develop as professionals in the midst of the complex and challenging work without direct attention paid to specifying what is important to develop or how to develop leaves too much to chance rather than purposeful design (Ball & Forzani, 2011). There are openings, though, to think purposefully about how relationships and partnerships can be forged across universities and schools and how settings and designs can be constructed in support of developing skilled teachers who hold ambitious and equitable goals for students.

Practice-based teacher education. In lieu of situating teacher preparation physically in the context of schools (with all of its complexity), longstanding recommendations for *practice-based* teacher education (Ball & Cohen, 1999; Smith, 2001; Zeichner, 2012) has given rise to the use of artifacts of practice, such as student

work, as well as representations of practice in the form of video and written cases of classroom activity. These forms of investigating teaching offer helpful supports for developing ways of noticing and communicating about practice and developing dispositions toward investigating teaching (Borko, Jacobs, Eiteljorg, & Pittman, 2008; van Es & Sherin, 2010).

Not only must teacher education make use of such *representations of practice*, they must be used to highlight, identify, and discuss the central component parts of the work of teaching—what Grossman and her colleagues (2009) call a *decomposition of practice*. Decomposing practice offers opportunities to see the complexity of the work of teaching by building language for describing component parts and elaborating aims and entailments in the work. One of the hindrances to coherence and productive teacher development across settings is the lack of specification of what good teaching looks like and entails (Ball & Forzani, 2011; Grossman & McDonald, 2008). Efforts have emerged to identify *high leverage* or *core* practices of teaching in order to specify the content of teacher education (Ball & Forzani, 2009; Lampert et al., 2013; McDonald, Kazemi, & Kavanagh, 2013; Thompson, Windschitl, & Braaten, 2013). Core practices are things that teachers do in high frequency and are shown in research to be linked to improvements in student achievement. Examples that have emerged include launching a lesson or activity (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), monitoring student work and eliciting student ideas (Lampert et al., 2013; Stein, Engle, Smith, & Hughes, 2009), and orchestrating classroom discussions (Chapin, O'Connor, & Anderson, 2009; Franke, Kazemi & Battey, 2007). A focus on core practices—and the continued and evolving work of researchers identifying and defining them—allows teacher educators to address

teaching with integrity and as a complex task, yet do so in a way that it can be taken up with teacher candidates in the limited time available in teacher education. This emerging *content* of teacher education must be mobilized with *pedagogies* of teacher education that leverage what is known about the development of skilled teachers.

Pedagogies of practice in teacher education. Foundational to understanding how teachers develop across settings is understanding teaching as a culturally defined activity (Stigler & Hiebert, 1999). This suggests that one develops skill as a teacher through participation and use of the tools that mediate the work (Rogoff, 2003; Wenger, 1998). Clinical practice and practice-based teacher education are approaches that look to shift teacher preparation into contexts of teaching. However, there must also be efforts to identify how teachers develop as practitioners and the pedagogies that support such development. Viewing teacher practice and development from this perspective offers the impetus to not just base teacher education in practice, but to focus on the development of skilled practice within contexts (Grossman & McDonald, 2008).

To develop teachers' skill with the work of teaching, Grossman and her colleagues (2009) suggest that teacher education design must emphasize two main pedagogical aspects—investigation and enactment. While teacher candidates tend to have a wealth of opportunities to observe, analyze, and reflect on the work of teaching, they less often have the opportunity to enact the work in settings of reduced complexity. Resolving a shortage of enactment opportunities is not done through increasing teacher candidates' time in student teaching placements (e.g., through traditionally-defined clinical experiences such as student teaching). Instead, Grossman, Hammerness, and colleagues (2009) suggest that teacher educators use *approximations of practice*, which

may take the form of instructional routines that simplify the work, while still being integral to the core components of teaching. These situations support teacher candidates as they develop through doing the work of teaching in authentic settings, mediated by meaningful tools, and oriented toward particular goals (Leont'ev, 1981; Vygotsky, 1978).

We utilize a frame for these approximations of practice from Lampert and Graziani (2009)—*instructional activities* (IAs). IAs are essentially short lessons or classroom activities that serve as containers for the core practices and the content knowledge for teaching that teacher candidates need to develop for and be able to use in their work with students in schools (Lampert et al., 2013). IAs structure approximated—yet still authentic—opportunities to enact the core components of the work of teaching by structuring the relationship between the teacher, students, and content by specifying mathematical goals and how individuals are expected to interact (Kazemi, Lampert, & Franke, 2009). These approximations of practice give the teacher candidate the freedom to rehearse the contingent and interactive aspects of teaching and attend to and make central use of students' ideas in the classroom, while still teaching toward a clear mathematical goal for students (Lampert et al., 2010). Given their close tie to content, the design of an IA must be specified for particular disciplines and even grade bands. To date, there is little work that exists in specifying appropriate IAs for secondary mathematics teacher education.

IAs—and the core practices and content that they contain—serve as the focus of teacher development opportunities organized in what McDonald and her colleagues (2013) call a *pedagogy of practice* in teacher education (see Figure 1). This idea is framed as a cycle that attends to both opportunities for investigation and enactment. The

work of Lampert, Kazemi, Franke, and their colleagues (Kazemi et al., 2009; Lampert et al., 2013) highlight a particular set of activities within each quadrant that we use to structure our own work in secondary mathematics teacher education. Teacher candidates first observe, decompose, and analyze an IA via video, real-time enactment, or a teaching case narrative (see Q1 in Figure 1 below). This is followed by examining a lesson plan that details the aims of the activity, elaborates teaching practices, moves and routines, and anticipates a range of ways students may respond mathematically in the activity. Teacher candidates then have multiple opportunities to enact the IA—both in the university setting with their peers playing the role of students (Q2) and with K-12 students in a school classroom (Q3). During these rehearsals, the teacher educator plays the role of an instructional coach, offering real-time feedback and support. Lampert and her colleagues (2013) offer a detailed example of how the work of coaching transpires in elementary mathematics teacher education. Enactments with K-12 students may take place in a sort of "lab classroom", where teacher candidates have the opportunity to work with small

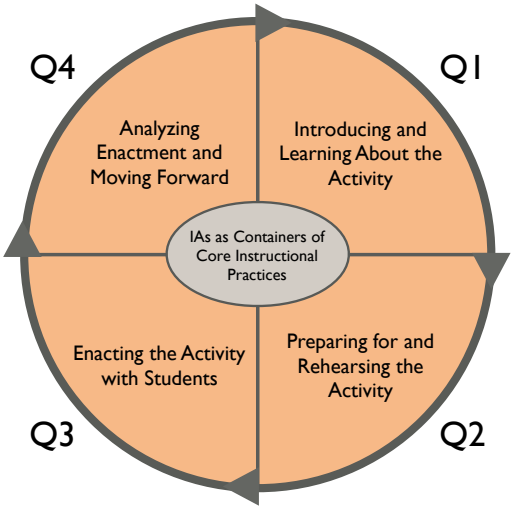


Figure 1. Cycles of investigation and enactment as a framework for a pedagogy of practice in teacher education (adapted from McDonald et al., 2013)

groups instead of a whole class and have the continued support of teacher educators and classroom teachers. Finally, video and other records of these enactments serve as a tool in analysis and reflection after enactments (Q4).

Motivating this study: Moving toward responsive pedagogies of practice. A pedagogy of practice in teacher education offers a promising approach to developing skilled ambitious teachers through opportunities to enact a repertoire of moves, practices, and routines that are attributed to more ambitious and equitable instruction and also advance mathematical goals in the classroom (Grossman, Hammerness, et al., 2009; Lampert et al., 2013; McDonald et al., 2013). Yet, truly preparing teacher candidates for their future work in schools requires accounting for the sociocultural settings of those settings (K. Gutiérrez & Vossoughi, 2010). This necessitates a reframed perspective on the relationships between professional education and school settings—from one that is seen as a unidirectional flow of information from the university, to a more responsive and bidirectional relationship (Cobb et al., 2009; Kazemi & Hubbard, 2008).

The line of teacher education approaches that situate teacher development in the practice of teaching and/or focus on the development of skilled practice through participation runs the risk of maintaining the unidirectional flow of information from the university to the school. From such a perspective, teacher candidates should be equipped with instructional theories, tools, and skills to carry with them into a new and intrinsically different activity setting (Borko, 2004; Peressini, Borko, Romagnano, Knuth, & Willis, 2004; Clift & Brady, 2005). When teacher candidates fail to take up the lessons learned from the university in their own teaching practice in schools, it is commonly viewed that such conceptions and types of teaching were overrun with what was modeled by other

teachers in the school (Gainsburg, 2012; Grossman, Smagorinsky, & Valencia, 1999; Windschitl, 2002). Despite the quality and theoretical underpinnings of a teacher education pedagogy, though, teacher candidates may not be prepared for their work in schools as they start their careers. Even Lampert and her colleagues (2013), who have been at the forefront of articulating and enacting a pedagogy of practice in elementary mathematics teacher education, concede:

> We need to know whether these principles, practices, and knowledge carry over into novices' classrooms, whether or not they are doing particular IAs. But as we conceive of the commitment to enact this kind of teaching as socially constructed, we need to understand what impact the schools and districts in which these classrooms are situated have on novices maintaining the capacity to do what they have learned (p. 15).

In this reflection from Lampert and her colleagues, they acknowledge that the work of teacher education—no matter how innovative or focused on ambitious and equitable teaching—does not exist in a vacuum. The quality and effectiveness of innovations in teacher education need to be judged on how they impact teachers' practice in schools. Further, conceptions of ambitious and equitable teaching (and the preparation of teacher candidates to do that work) must take into account the school setting, the work done in those settings, and the tools that are used.

In line with that idea, we contend that meaningful skill development among secondary mathematics teacher candidates cannot be fostered from within the prevalent, unidirectional model that views what is promoted at the university making its way into (and needing the space within) the school setting. Instead, we propose that these designs be better tied to and derived from the activity of teaching in schools—resulting in what

we will call *responsive pedagogies of practice.* Our hypothesis is that through such designs, teacher candidates would be better supported in developing skill as practitioners and do so in ways that are enabled in the school settings in which they teach. However, these concepts of responsive and practice-focused teacher education are not well defined nor commonly understood. As a result, in this paper we ask the following research question: *What is meant by and entailed in the design of a responsive pedagogy of practice in secondary mathematics teacher education?* We do so in the context of our own design efforts in secondary mathematics teacher education, through which we are interested in the evolution of the needs that arise, the resulting design principles, and the processes of implementation and refinement in responsive, practice-focused pedagogies for teacher education.

Research Methods: Design-Based Research in Teacher Education

To address the question above we see it as necessary to utilize a research methodology that matches our aim of responsiveness. To examine and further specify the evolving entailments of responsive pedagogies of practice in teacher education and how these pedagogies serve secondary mathematics teacher development, we chose to position our work as design-based research (DBRC, 2003; Edelson, 2002). We see the context of design and its implementation as a way to further our understanding—and that of the field—of responsive and practice-focused pedagogies of teacher education. Design-based research has the potential to leverage the strengths and capacities of researchers in teacher education in addressing the problems of teacher preparation, offering a novel methodological approach to addressing problems of practice.

In design-based research, instructional design and research are inextricably intertwined—research is set in the context of design, and the design is informed through ongoing and retrospective analyses (Cobb et al., 2003; DBRC, 2003). As such, design-based research is an iterative process of design, implementation, analysis, and redesign. Many researchers argue for design-based research as a form of educational research because it helps in the examination and evolution of an educational design and is a way in which researchers can be involved in the direct improvement of educational practice (Edelson, 2002; Gutiérrez & Penuel, 2014). Key to this is that the aim of design-based research is a product that does not rest in the specifics of a design or its evaluation, but the broader theories that are shaped by the work and that inform future iterations (Barab & Squire, 2004; Cobb et al., 2003; diSessa & Cobb, 2004). In our work by addressing our research question, our aim is to develop a theory of responsive pedagogies of practice in teacher education.

Context and design-based research participants. Our design-based research is set within a small Master's-level teacher licensure program for prospective secondary mathematics teachers in a very high research activity institution as classified by the Carnegie Classification of Institutions of Higher Education. Specifically, our work is set in the context of a sequence of two, ten-week secondary mathematics methods courses and co-requisite practicum placements. While our efforts to design and implement pedagogies of practice into our mathematics methods courses have been ongoing for a few years, in this paper we report on three design cycles during the 2012-2013 academic

year. With *design cycles*[3], we refer to teacher educators' construction of an IA, support of teacher candidates' preparation, implementation, and reflection of a particular IA as part of a secondary mathematics methods course, and sample observations of teacher candidates' instruction during student teaching when such experiences ran concurrently with a methods course and the use of a particular IA. In our data, teacher candidates completed a concurrent teaching practicum during the first design cycle and, later, a second, full-time practicum after the conclusion of the third design cycle.

A mathematics teacher education design team consisting of three mathematics teacher educators—including the two authors—and a partner teacher from a local school accomplished this work. For a given design cycle, one teacher educator would take the lead as the methods course instructor, with the other teacher educators assisting with the implementation (e.g., video capture, materials setup) and contributing in the planning and debrief sessions. The first author was also the teacher candidates' supervising instructor for part- and full-time student teaching. The partner teacher in a given cycle would change, working with cooperating teachers from across the teacher education program to offer both middle school and high school sites over the sum of the cycles. During the year of this study, we partnered with two teachers—a middle school mathematics teacher, Ms. Calhoun[4], and a high school mathematics teacher, Mr. Ellison—each serving as the host of the teacher candidate rehearsals with students in a given cycle.

---

[3] The term "cycle" is not to be confused with the cycles of investigation and enactment activities in a pedagogy of practice highlighted by McDonald and colleagues (2013) and depicted in Figure 1.

[4] All names used, except for those of the teacher educators, are pseudonyms.

During the 2012-2013 academic year, the teacher licensure program had a small cohort of three secondary Master's level mathematics teacher candidates—Casey, Georgia, and Susan. The teacher candidates were not involved in the design or analyses of a given IA and, thus, were not a part of the design team or all of the aspects of a design cycle. However, their enactments of an IA, feedback (both in the methods course and in an interview setting), and their practice in school classrooms during their practicum served as part of what was considered in the iterative and responsive design process—either within one design cycle or after all three cycles.

Characterizing the initial design. To address our question— *What is meant by and entailed in the design of a responsive pedagogy of practice in secondary mathematics teacher education?*—through a process of design-based research, we must share the initial design. As Edelson (2002) states,

> Design is a sequence of decisions made to balance goals and constraints. In the course of any design, the design team makes three sets of decisions that determine the results of the process. These are decisions about (a) how the design process will proceed, (b) what needs and opportunities the design will address, and (c) what form the resulting design will take (p. 108).

Edelson (2002) classifies each of these sets of design decisions as the design procedure, the problem analysis, and the design solution. The *design procedure* specifies the process of design and implementation, including the people that are involved in a design and the expertise that is necessary for the planning, development, implementation, evaluation, and refinement of a design (i.e., the design team we highlight above). The *problem analysis* characterizes the needs and opportunities for a design to address in relation to the affordances and constraints of a particular context. A design team must identify these

needs, opportunities, and contextual situations as well as track their evolution over iterations of a design. The *design solution* describes the resulting design in response to a given set of needs, affordances, and constraints, meaning the design solution also evolves. These decisions may not always be explicit or fully articulated, though they are decisions to be made in every design. In the case of the cross-setting work of teacher education as we discuss in this paper, the decisions are complex and highly contingent, thus requiring extensive investigation and iterative refinement. This is where design-based research serves our needs as teacher educators and as teacher education researchers. While we share details on our design process and design team in the previous section and share more in the data sources and analyses sections below, here we highlight the initial problem analysis and design solution from our work that became the subject of research and iterative design.

Initial problem analysis. As outlined in the introduction and review of literature, our conceptualization, use, and study of responsive pedagogies of practice are a response to the need to develop skilled practitioners for their early-career work in schools. Such skill is developed through leveraging the time and resources in university teacher education to create opportunities for teacher candidates to enact approximations of the work of teaching. In our work, we aim to develop instructional skill as a resource to be mobilized in the interactive work of teaching mathematics in secondary schools.

To focus these efforts, we specified a set of core practices of teaching, specifically the interactive work of teaching mathematics with students in the classroom. These core practices served as a focus of our efforts with teacher candidates (see Figure 2). Developing skill with these practices was addressed through pedagogies of practice rather

- Teaching toward a clear learning goal
- Representing student reasoning verbally and visually
- Constructing and organizing public records
- Eliciting and responding to student contributions
- Orienting students to one another and to the discipline
- Making sense of students' participation to inform instruction
- Positioning students as competent
- Developing and maintaining a productive learning environment
- Managing time and pacing
- Using body and voice

Figure 2. List of core practices that serve as programmatic focus (adapted from Lampert et al., 2013)

than left to candidates' time in the classroom or years of experience. These practices also best leverage the potential power of enactment opportunities instead of something like planning, which is certainly a teaching practice, but is often addressed through other opportunities in a teacher education program. One concern of ours in this work is the issue of "grain size". We see a flaw in some of the existing discourse about core practices where the phrase is used to describe a wide scope of teaching—from something that spans a lesson or multiple lessons to an instantaneous teacher move. We see the list we initially worked from as being framed at a relatively consistent grain size. While this set of practices emerges from other ongoing work in the field, we see the context of design as an opportunity to further develop and refine a set of goals that support teacher candidates develop skill with these practices.

Initial design solution. The objective of our resulting design was to provide teacher candidates with an opportunity in which to engage with core practices of ambitious and equitable teaching (as listed in Figure 2). A successful design would have teacher candidates enacting these core practices in approximated settings and then doing the work with increasing skill—in university rehearsals and in school classrooms. Our

initial design was informed by a set of principles. First, our intent was for IAs to serve as a regular opportunity for developing skilled practice across time, content, and settings. In other words, these IAs would serve as stable "containers" of teaching practice (Kazemi et al., 2009). Accordingly, we considered the way in which an IA would fit into the existing and typical classroom structures of secondary mathematics classrooms and would minimize the amount of time needed. As a result, we designed and used short activities to fill the time at the start of a lesson—what is commonly called a "warm up". Part of this initial decision was that the "warm up" was an allocation of time that teacher candidates commonly observe in classrooms and incorporate into their own plans, thus making it a recognizable act of secondary mathematics teaching.

A second principle we acted on was to develop IAs that had a quality of generalizability that was useable across mathematical domains. This was consistent with our principle for having IAs serve as containers that held stable across time, content, and settings and opportunities for teacher candidates to regularly engage with student around mathematics and enact a set of core practices. Furthermore, our initial view of responsiveness involved being as flexible as possible to the ongoing work of a teacher's classroom (e.g., partner teachers in the design-based research, cooperating teachers of the teacher candidates). This principle was partly informed by a desire to not draw on too much of a partner teacher's time in the planning of an IA. This was also informed by a goal to not interfere with or complicate the ongoing mathematical work of a classroom with our iterative and evolving designs. Thus, our initial approach looked for IAs and their planned content to remain apart from the day-to-day curriculum work of the classrooms in which all of the teacher candidates would be enacting an IA with students.

While Ms. Calhoun's middle school class might be working on measures of center and spread and Mr. Ellison's high school geometry class might be working on properties of special quadrilaterals, we did not initially aim to tie IAs to that particular content. IAs must be tied to some mathematical content, however, in order to motivate the instructional work contained within them. One idea that informed the design of the first cycle's IA was a focus on number sense and operation with Ms. Calhoun's middle school class. This allowed us to draw more on the existing work of Lampert, Kazemi, Franke, and their colleagues while also putting forth a mathematical goal that can be used across settings and can be seen as important (though not explicitly part of the standards of a given secondary grade level or course).

While the goals of our program and, in turn, the responsive pedagogy of practice looked to attend to a set of core practices, we chose to foreground the work of eliciting and responding to student reasoning and facilitating classroom discourse with mathematical ideas. We saw this instructional work as central to ambitious and equitable mathematics teaching (Lampert et al., 2013) and as something that is not explicitly worked on in the full complexity of teaching. As a result, our design did not look to formulate IAs that would center on teacher candidates leading a demonstration or giving an explanation (albeit a mathematically correct and complete one) themselves. Instead, IAs were designed to make central the work of eliciting students contributions, making sense of those contributions in the context of a mathematical point for the IA, and using those contributions as a central object in the collective mathematical work. Finally, while we saw all opportunities to rehearse and enact an IA as maintaining authenticity to the work of teaching, we had an eye on the need to consider what was reasonable and

appropriate for teacher candidates, both in general and at specific points throughout the program. As a result, we had a sense that our IAs needed to become increasingly complex over the two courses, which could take the form of longer lessons, more ambitious instructional and mathematical work, or more freedom (i.e., less support) to use one's professional skill and judgment during instruction.

Data sources. We collected and analyzed data—often in the form of video—of our design efforts from three design cycles. These included data of teacher candidates' implementation of IAs in the methods classroom and the partner teachers' classrooms Figure 3 uses the conceptual framework of pedagogies of practice to organize and display the data we collected from the events associated with the methods courses and the design. Figure 4 summarizes all of the data collected across the university and the school settings and drawn upon in our work. Data for the first design cycle included video of teacher candidates' classroom instruction (collected by the first author) during their concurrent, part-time student teaching practicum. After the three design cycles, the three teacher candidates were individually interviewed by the first author to discuss the work across

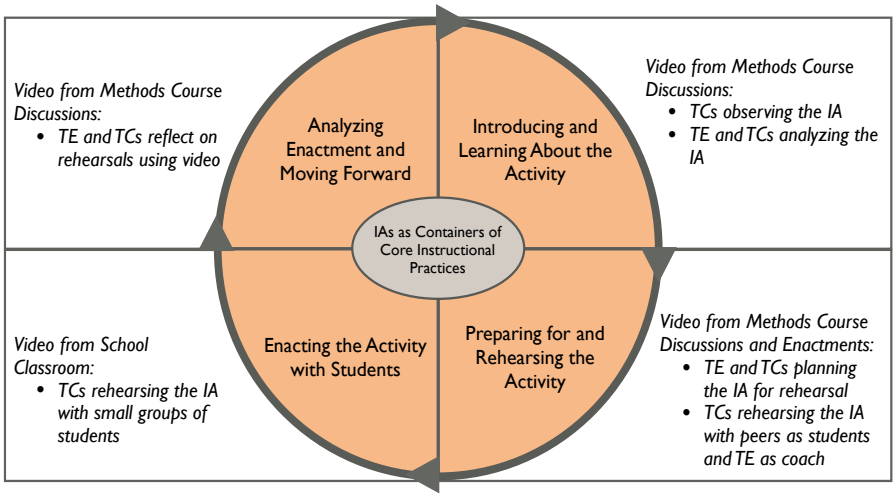


Figure 3. The data collected across the phases of the cycle of investigation and enactment

Cycle 1 – November & December 2012 - Video of investigation and enactment across settings - Artifacts from investigation and enactment - Video of one lesson during student teaching practicum (for Casey, Georgia, and Susan) - Reflexive journaling with design decisions, ongoing analyses, and reflections
Cycle 2 – January & February 2013 - Video of investigation and enactment across settings - Artifacts from investigation and enactment - Reflexive journaling with design decisions, ongoing analyses, and reflections
Cycle 3 – March 2013 - Video of investigation and enactment across settings - Artifacts from investigation and enactment - Reflexive journaling with design decisions, ongoing analyses, and reflections
After Cycles – March 2013 - Video of interviews with teacher candidates - Video of three lessons during student teaching practicum and follow-up interview

Figure 4. Summary of the data sources used across the design-based research work

the methods courses as well as their work in their school placements. The first author also collected video from three lessons that each teacher candidate taught in their full-time student teaching placement to serve in the retrospective analyses of the set of three cycles and to inform future iterations of the design and the development of a theory of responsive pedagogies of practice. Finally, a key component of the design-based research process and the maintenance of rigor and trustworthiness is the process of reflexivity (Auerbach & Silverstein, 2003). Specifically, keeping detailed records, or a reflexive journal, of the design process is essential. Throughout the year, the first author kept such a journal, with entries added after each phase of a cycle, planning and debriefing meetings with the design team, and observations of teacher candidates' instruction in school classrooms.

Data analyses. The process of design-based research involves two levels of data analyses: the ongoing analysis from cycle to cycle and the retrospective analysis of the

sequence of cycles. The ongoing analysis took place during and in between design cycles and served as the basis of subsequent design decisions, thus supporting our immediate decisions to support teacher candidate development (Cobb & Gravemeijer, 2008). Consistent with the theoretical perspective, we focused on the instructional work in which teacher candidates were participating—both within the cycles and in the school classroom when available—and made design decisions based on our design context as well as the needs of teacher candidates. Investigating teacher candidates' records of teaching—reflections on enactments in cycles and observation notes from supervision observations—were used in the ongoing analysis informing the ongoing design work. The ongoing design decisions were accounted for through reflexive journaling and represented in planning protocols, course materials, and videos of investigations and enactments, which all then serve as data for the retrospective analyses.

The retrospective analysis involved reexamining the data from the three design cycles as a whole, including data on teacher candidates' instruction in student teaching placements and from interviews conducted after the three cycles. In advancing what is meant by and entailed in the design of a responsive pedagogy of practice in secondary mathematics teacher education, we framed three analytic questions based on the three sets of design decisions as defined by Edelson (2002) to inform our analyses. Those questions were:

1. What are the needs being addressed in a given design cycle of a responsive pedagogy of practice in secondary mathematics teacher education and how did the needs evolve across cycles?
2. What are the design principles that inform the design and implementation of an

IA in respect to the needs being addressed?

3. What work was done by the various participants (e.g., teacher educators, teacher candidates, classroom teachers) as the design evolved?

For each analytic question, we drew upon different subsets of the full corpus of data—all in service of answering our research question regarding the meaning and entailments of responsive pedagogies of practice. Figure 5 aligns each analytic question with the corresponding collection of design decisions from Edelson (2002) and the data sources used in our own work.

Analytic question	**Data sources**
1. What are the needs being addressed in a given design cycle and how did the needs evolve across cycles?	• Reflexive journal • Video of classroom teaching from student teaching placements • Video of "investigation" sessions in methods • Questions around goals from teacher candidate interviews
2. What are the design principles that inform the design and implementation of an IA in respect to the needs being addressed?	• Reflexive journal • IA planning protocols and other artifacts used around the investigation and enactment of IAs
3. What work was done by the various participants as the design evolved?	• Reflexive journal • Questions around roles from teacher candidate interviews

Figure 5. Alignment of analytic questions and data sources

For the first analytic question, we drew upon data from which we could identify the needs being addressed within and across the three cycles and make sense of how and why those needs evolved. This would partially serve our larger aim to consider the entailments of a responsive pedagogy of practice. The reflexive journal served as a primary source for noting the needs that the design was intended to address and how its implementation led to revised designs for subsequent cycles. This journaling also consisted of memos crafted by the first author from the teacher candidates' student

teaching placements, which informed the direction of the design from the first to the second cycle and then upon reflecting the three cycles as a whole. There was not a concurrent student teaching practicum during the second and third design cycles, so similar observations and memos could not be conducted. However, observations were conducted and memos crafted by the first author during teacher candidates' full-time student teaching placements after all three design cycles. These were used to contribute to the retrospective analyses. Second, video of the "investigation" sessions of the methods courses (observation, planning, reflection on IA) were viewed to note any explicit mention of needs being addressed through the work or of the goals for teacher candidate or student development. To infer the focus of the work, the videos of the investigation discussions were analyzed and memos were created to describe the kind of work that the teacher educator and teacher candidates did. Finally, questions in the teacher candidate interviews geared toward the needs being addressed within the pedagogy of practice and the goals for their development contributed to answering this question.

For the second analytic question, and in response to the needs to be addressed through a responsive pedagogy of practice, we aimed to identify the design principles from within and across cycles that inform the design and implementation. Appropriately, the design principles interact with the needs for the design, thus allowing us to use much of the same data. We looked to the reflexive journaling to identify the design decisions made by the design team and the resulting design characteristics in order to infer about the design principles and their evolution. The planning protocols and other emerging artifacts served as an explicit record of the resulting design within each cycle and the changes made across cycles.

For the third analytic question, we were interested in characterizing the work done by various participants—teacher educators, partner teachers, and teacher candidates—over the course of the three cycles. Developing, implementing, and researching design—geared toward particular needs in a given setting—requires a set of expertise in order to carry out the work. In each design cycle, each group (or the individual actor representing each group) assumed a role. We characterized the responsibilities of each group for each cycle, which allowed us to note any consistencies or changes, and also begin to develop a narrative description of the work done. Identifying the evolving roles of actors in the work allows us to better understand the design process, how particular aims were achieved, and the various forms of expertise that were required. Again, the reflexive journaling served as a source of reflection and memos from the standpoint of the teacher educator and researcher. Finally, we looked to the teacher candidate interviews, specifically the questions that targeted roles, as well as other times where mention of roles and responsibilities emerged.

Overview of the IAs from Three Cycles

Table 1 below summarizes the three IAs that were designed and implemented for the three design cycles (more detail on each is provided in the form of abbreviated planning protocols in Appendix A, B, and C). These details regarding the structure of each IA, the teacher development goals, and the student development goals will serve as context in the reporting of our findings. There are additional points to highlight here as well. First, the IAs gradually increased in length—going from 18 minutes to 30 minutes, and finally 42 minutes). Each IA was also seen as more complex than the previous, both in terms of the time but also in terms of the reduction of structure in the planning

Table 1
Summary of Three IAs that Comprised the Pedagogy of Practice

Instructional Activity	Targeted grade level/course & mathematical content	Summary of Structure, Teacher Development Goals, and Student Mathematical Goals
1) String of Computational Problems	*7th Grade* Multiplication – "Halving and Doubling" Strategy	*Summary:* A string (Fosnot & Dolk, 2001; Kazemi et al., 2009) is short activity designed to highlight a particular mathematical idea, notably a computation strategy. In this string, a sequence of four multiplication problems were used to bring forth and motivate the use of a strategy for mental computation in which one factor can be halved and the other doubled to create an equivalent product.
		Teacher development goals: The IA was put forth as an opportunity for teacher candidates to do interactive instructional work toward a mathematical goal. Specifically, the structure put teacher candidates in position to elicit and respond to students' mathematical ideas, represent ideas on the board, dwell on important mathematical ideas, and orient students to one another's ideas through the use of discursive moves (e.g., Chapin, O'Connor, & Anderson, 2009)
		Student development goals: From this IA, students are expected to be able to begin to identify, use, and provide a justification for the "halving and doubling strategy" for multiplication. In terms of process, the IA is also an opportunity for students to voice their ideas about the strategy, while also being able to listen to and reason about the ideas of their peers.
2) Explaining a Concept through Connections across Representations	*High School - Algebra II* Exponential Change in Graphs, Tables, and Equations	*Summary:* Similar in some respect to the string, this IA centered around a purposefully designed sequence of prompts and representations to construct an explanation of exponential change and to visualize that change across graphs, tables, and functions. Students were shown, in sequence, the graphs of three exponential functions with the third graph providing an example that defines the boundaries of the explanation being constructed about exponential change and its relation to the closed form of the function.
		Teacher development goals: This IA positioned teacher candidates to construct a mathematical idea using a purposefully sequenced set of examples and based on contributions from students. Accordingly, the teacher candidate was in a position to elicit students' ideas and to orient students to one another and to the mathematical work at hand.
		Student development goals: This IA is designed to support the construction of the idea that an exponential functions' growth can be characterized as, for an increase of one unit in the input, the output is multiplied by a constant factor. Students further consider the role of the constant factor in the equation for the function and its graph. Students engage in mathematical practices such as constructing arguments, critiquing the reasoning of others, and attending to the precision of mathematical language.
3) Building a Definition from an Investigation	*High School - Geometry* Right Triangle Trigonometric Ratios (i.e., sine, cosine, and tangent)	*Summary:* Drawing on prior experiences with similar triangles, specifically the proportional relationship of pairs of corresponding sides, this IA focused on the development of mathematical definitions—the basic right triangle trigonometric ratios of sine, cosine, and tangent. The teacher candidate led a discussion looking across the data from groups, highlighting the constant ratios across the similar triangles in order to define the three trigonometric ratios.
		Teacher development goals: This IA required teacher candidates to manage materials and small group work. The teacher candidates worked on using precision in highlighting a problematic situation and in defining new mathematical ideas. As with the other IAs, teacher candidates had opportunities to elicit and respond to students' ideas.
		Student development goals: The work of this IA was motivated by a problem for which previous tools (i.e., using the Pythagorean Theorem) was not useful. As a result, students begin to think about the use of strategies and mathematical relationships in problem solving.

protocols. This is consistent with the initial design principle (highlighted in the Context section) that approximations of practice become more authentic (and, thus, more complex). Second, the content addressed across the three IAs varied, yet still only represents a small fraction of the scope of middle and high school mathematics. The variation—focusing one IA each on mathematical procedures, concepts, and definitions—was an effort to represent one aspect of the breadth of secondary mathematics. Finally, in addition to the planning protocol, in the second and third cycle another tool was provided to support the sense making around the mathematics of an IA. An *instructional explanation decomposition tool* (see Appendix D for an example)—modified from Patricio Herbst (n.d.) and based on the work defining instructional explanations from Leinhardt (2001)—was used in the methods course to unpack the mathematical concepts at play and link them to instructional moments in the IA.

Findings: Developing a Theory of Responsive Pedagogies of Practice

Our aim in this work is to use the context of our ongoing design of an increasingly responsive and practice-focused pedagogy in secondary mathematics teacher education to answer the question: *What is meant by and entailed in the design of a responsive pedagogy of practice in secondary mathematics teacher education?* As a design-based research study, we see this work as being in its early phases and having the potential for continued evolution. We also see the work not only impacting our own local design and populations, but also serving a larger teacher education audience in the form of domain-specific theories (Barab & Squire, 2004; Cobb & Gravemeijer, 2008; Edelson, 2002). In this section we highlight a set of findings from our analyses of the needs, design characteristics and principles, and work to be done by participants in the design—all

contributing to the development of a theory of responsive pedagogies of practice in teacher education.

First, through our design and efforts for responsiveness, we identified two needs that emerged in addition to our initial attention to developing teacher candidates' instructional skill—needing to align the mathematics of an IA to the mathematics of a partner teacher's classroom and the resulting need to attend to developing teacher candidates' mathematical knowledge for teaching (MKT; Ball, Thames, & Phelps, 2008) with that content. Attending to these concurrent goals requires a challenging process of negotiation by the teacher educator through the design—accounting for both teacher candidate development and responsiveness to the school setting (namely the partner teacher and her/his students). Second, while two additional needs emerged, our initial attention related to developing teacher candidates' instructional skill with a set of core practices was further specified and refined along two lines—the need for a focus on multiple levels of pedagogical tools (from IAs, to routines within IAs, to the practices and instructional moves that realize those routines) and the need to derive a list of core foci from the work that teacher candidates do in their student teaching placements. Ultimately, we have found that responsiveness in teacher education entails preparing teacher candidates to do what is typically done in school settings while also finding the openings at which to press for more ambitious and equitable teaching practice. This, as well as the emergence of multiple needs, had implications for our design principles, which were revised from their initial form. Finally, the set of multiple needs to which to negotiate attention is due to a novel connection across settings in a responsive pedagogy of practice. Supporting these pedagogies involves novel and collaborative roles to be taken

on by teacher educators and partner teachers. This kind of work is not typical for individuals in university teacher education but emerged as a part of the design process of responsive pedagogies of practice. This work also had implications for the design process. All three of these findings are discussed further in the ensuing sections.

Highlighting multiple needs to address through design. In discussing the needs to which a design must attend—the problem analysis—Edelson (2002) also highlights how the context provides a set of opportunities and constraints. Through our design process over three cycles, our design context—at the intersection of university teacher education and school classrooms—presented us with needs to which to attend through the design. Notably, two needs emerged in addition to our initial focus on teacher candidates' development of instructional skill. First, we confronted a need to not trivialize the mathematical development of the students in the secondary classrooms in which we situated our pedagogy of practice. In turn, we shifted toward aligning the content of an IA with the current curricular focus of a given middle or high school classroom. Second, we realized that teacher candidates needed support in developing the mathematical resources for the work of teaching in a given IA. We use the idea of MKT from Ball and her colleagues (2008) to make sense of the mathematical resources teachers need for their work and the ways in which to support its development. Negotiating these needs in addition to the development of teacher candidates' instructional skill proved to be a challenge. Across the three cycles, we found the design and implementation to be foregrounding and backgrounding attention to the three needs in different ways, resulting in a tension in the work.

Aligning to the mathematics of a partner teacher's classroom. In our initial design principles, we aimed to develop IAs that had a quality of generalizability and content-independence. We saw this as a way for IAs to serve as a regular and stable container of practice across time, content, and settings. As such, our resulting design aimed to not become intertwined with content. That was the impetus for the focus on strategies for multiplication with Ms. Calhoun's class of seventh grade students during the first cycle. This stance was informed by part of our vision of responsiveness—being as flexible as possible to the ongoing work of a partner teacher's classroom and to the time of the partner teacher outside of class. This changed after the first cycle, after which the design looked to situate an IA in the relevant mathematical work of a given class. Two factors—discussed below—led to a change in our perspective. First, the affordance of working with partner teachers and their students during their class time presented a demand to align our work in IAs with the ongoing mathematics work of the classroom. We could no longer agnostically consider the mathematical development of secondary students, instead needing to contribute to the ongoing work of a given classroom. Second, based on teacher candidates' work in school classrooms after the first cycle, our focus turned to foregrounding the work of teaching toward a mathematical goal, which was best done in the context of clear and relevant goals in the instructional settings they worked. Teacher candidates clearly articulated through their reflections during the first design cycle that they were challenged to identify how instructional work in the IA and within their own part-time student teaching supported the work of making instructional decisions based on making progress toward a mathematical goal with students. This was corroborated through the supervisory notes on their instruction taken by the first author.

We posit teaching mathematical content that was more relevant to the secondary mathematics curriculum—and to the day-to-day mathematics experiences of the students in the partner teacher's classroom—offered more authentic opportunities for teacher candidates.

Attending to the demands of working with classroom teachers. The transition from the first to the second cycle included new individuals taking on the role of methods course instructor and partner teacher. The second author was teaching the second (and final) methods course and served as the primary planner for the second and third IA. The second methods course in this particular teacher education program historically focused on high school mathematics teaching (transitioning from a focus on middle school). This led to collaboration with another partner teacher, who had worked with the program before as a cooperating teaching. Mr. Ellison served as the partner teacher for both the second and third cycles—serving a trajectory of work with middle school students in Ms. Calhoun's class to high school students in Mr. Ellison's Algebra 2 and Geometry classes. In the negotiations between the second author and Mr. Ellison regarding the timing, content, and form of the second IA, Mr. Ellison brought a different approach than Ms. Calhoun. Part of gaining access to his classroom involved designing an IA that addressed what would be the current content (projected out three weeks) of his Algebra 2 classes—exponential change. This contextual feature of leveraging relationships with local teachers to gain access to working with their students during their class time required us to make this shift, thus making it part of the design solution of a responsive pedagogy of practice.

Prior to these initial discussions with Mr. Ellison, our design team had planned to continue using the structure of short, "warm up" activities with content that could be considered beneficial across all of secondary mathematics. The resulting design was a negotiation of those original plans stemming from the first cycle with the conditions set by Mr. Ellison in the form of specifying the mathematical content. In our efforts we continued to hold onto the idea of a shorter, more generalizable activity structure (as opposed to a whole lesson for a given topic), which required further specifying the mathematical content. We referred to the *Common Core State Standards for Mathematics* for standards regarding exponential growth and also considered other potential goals (such as distinguishing between quadratic and exponential functions, which could both be characterized similarly for their nonlinear change patterns). These goals were clarified and refined to fit what could be accomplished in about 20-25 minutes, resulting in an IA that used three graphs of exponential functions to build students' ideas of how the growth factor of an exponential function is visible across graphical, tabular, and symbolic representations.

Focusing on "teaching toward a clear mathematical goal". The shift in mathematical focus posed a new challenge for the design team, though also served another need that emerged in our work with teacher candidates and their concurrent student teaching placements. Reflections of the first author found in the reflexive journaling at the end of the first cycle (which corresponded to the end of the first methods course) highlighted "leading a discussion toward a clear mathematical goal" as a prevalent need of teacher candidates. From observations of their part-time student teaching placements, teacher candidates were confronted with students' contributions and

the need to make sense of those ideas and make decisions about how to use those ideas toward a determined goal for a lesson.

Casey and Georgia both showed struggle with interpreting students' ideas, specifically incorrect ones. For Casey, in a segment of a lesson in which students were asked to match position-time graphs with different written descriptions of a bike trip, much time was spent resolving the variety of pairings proposed by students. The incorrect answers hinged on seeing the position-time graphs as if they were illustrating the topography of the ride (e.g., a line segment with a positive slope corresponds to a hill, which corresponds to slower riding), though Casey was unable to either resolve these errors or use them in productive ways through discussion among students. In turn, her move was to move on. The reflection from the first author in his memo was that the goal of the lesson was therefore not reached, instead deferring to another type of goal—getting to the predetermined set of problems for the day.

Georgia faced a similar challenge, especially in a lesson during which students were to create algebraic equations out of situations. For example, students were asked to write an equation relating minutes (m) and gallons (g) for a situation in which a pool is being filled up at a rate of six gallons per minute. While Georgia appeared ready to quickly move on after recording "g = 6m", some students raised that the equation was "backwards" and insisted that "6g = m" would be correct. After some debate, Georgia demonstrated why the correct equation would be g = 6m and stated that it is "just a different way of looking at it". While it is not clear what she meant by that, what is clear is that, upon reflecting on that segment with her, Georgia had not been able to make sense of why some students were so adamant about the equation 6g = m. It was only in

discussing this with the first author (serving as her student teaching supervisor) that students were reading the equation as, “six gallons is equal to one minute,” instead of, “six times the number of gallons equals one times the number of minutes”. As with Casey, the first author’s reflection was that the progress through a set of problems for the day, as well as Georgia’s inability to make sense of the error, trumped any further discussion of the mathematics and progress toward the goal.

For Susan, the problem was a bit different. On the surface, her observed lessons ran smoothly and did not confront her with those same types of errors. However, upon further examination, this is partly because of far fewer opportunities for students to share their reasoning. Even when ideas were elicited, they were simply shared and not subject to discussion or critique. One example of this came in a lesson in which students were asked to reason about conjectures about triangles and either justify them or refute them. Susan’s aim was to not just review the ways to classify triangles, but to use those classifications to think about their properties and to engage in more authentic mathematical practices such as justification. One conjecture presented to students was, “The longest side of an obtuse triangle is always opposite the obtuse angle”. Students thought about the conjecture on their own then talked at their tables. Susan monitored students’ discussions, interjecting that “the conjecture is true” and asked the groups to record a justification why on the small whiteboards at each table. Once complete, Susan asked groups, one at a time, to share their idea, either by reading it or displaying their board so Susan could read it. While students in this class did not necessarily have all of the mathematical tools to prove the conjecture outright, Susan allowed for little discussion about the ideas while allowing for imprecise and incorrect ideas to remain

untouched. In all, while Susan's lessons seemed more efficient and streamlined than Casey and Georgia's, it was partly because there were fewer opportunities for the flow of her planned lesson to be broken.

Teacher candidates highlighted their own struggles with teaching toward a clear goal in the opening session of the second methods class—offering the desire to work on making sense of the role of the mathematical goal in a lesson and wanting to gain skill with knowing how instructional practices support advancing that goal. Here, teacher candidates were asked to identify instructional practices of focus for the cycles that would take place in the course (drawing on the list found in Figure 2). The candidates each listed "teaching toward a clear learning goal" as their top priority for the term. These ideas continued to be highlighted in the post-methods interviews. Casey expressed that, "in the moment, I haven't been very good at keeping mathematical goals in mind," referring to her own nervousness in the classroom as well as the need to keep many other considerations in mind. Susan also described the work as "hard", saying that a teacher is required to "respond to the ideas that students are putting out there," and that a teacher must "assess what is going on in the moment and make decisions," which is complex.

The second and third IA were designed to foreground the instructional work of teaching toward a clear mathematical learning goal—goals that were determined through our work with Mr. Ellison. While the first IA was built around mathematics content (i.e., a computation strategy for multiplication), the teacher candidates did not discuss the mathematical opportunities for them as teachers in their work in that first cycle, instead only focusing on the structure of a string of computation problems as something they could possibly use in their teaching (even though we do not have evidence from any of

their student teaching placements that they did). Their reflections on the second and third IAs, however, were different. When asked if she saw a difference between the first IA and the second and third IAs, Casey said, "I think [in the second and third IA] we focused on specific content in order to try to work toward a clear learning goal" and that, thought such IAs, teacher candidates were "more aware of the process of … how to plan a lesson that has a mathematical storyline and reaches a goal" (Casey, Post-Methods Interview, 3/21/2013). Susan shared that the work of responding to students' ideas is something that, "you can't learn unless you are working with actual students putting out actual ideas," which she saw the second and third IAs—with their more authentic goals for students—providing the opportunity to do. Finally, Georgia shared that the second and third IAs were still generalizable in that a focus on explaining a concept and building a definition, albeit framed in the IA around specific content, are things that teachers are always able to do. The evolving design of a responsive pedagogy of practice brought the particular mathematics of the school classroom to the university, which provided opportunities to not only work on goal-oriented instructional practice, but do so toward meaningful goals in school settings.

Developing mathematical knowledge for teaching. Focusing an IA on particular content derived from a secondary classroom required that teacher candidates be supported in understanding the mathematical ideas at play—not only as mathematical learners, themselves, but as teachers—bringing forth a third need to which to attend. Of course, this does not mean that such mathematical demands were not present in the first IA, just that our attention to them were not explicit. MKT, as described by Ball and her colleagues (2008), is the mathematical and pedagogical *knowledge-in-action* that teachers

deploy in the work of teaching mathematics. Across the three IAs, the mathematical tasks of teaching included analyzing a series of related problems for highlighting a mathematical idea, assessing the affordances of particular representations, considering mathematical structure and the underlying concepts of an idea, connecting tasks of a lesson to students' prior experiences, determining an array of ways students would reason about an idea, and posing questions that supported students making connections and extended students' reasoning to generalize ideas. While pedagogical skill is involved with these tasks, teachers develop and deploy their disciplinary knowledge in a way that is unique to the work of teaching. As such, the development of MKT must be done in specialized ways and be situated in the investigation and enactment of teaching.

In the first cycle, little time was spent in discussing the mathematics of the "halving and doubling" multiplication strategy with the teacher candidates. This decision was made, in part, because of a determination that the content of multiplication would be more familiar to teacher candidates. However, as their rehearsals with students showed, there are ways in which a teacher must know the mathematics they teach in order to do the work of teaching toward a clear learning goal with skill. Susan and Casey tandem taught a group of twelve students from Ms. Calhoun's seventh grade class. Throughout the rehearsal, there were multiple instances that ultimately complicated the enactment. First, in representing the process of halving and doubling between the two, interrelated problems at the start of the string (shown in Figure 6), Susan's representation showed a cyclical process that relied on one knowing the two expressions (the representation on the left) instead of a one-directional process of rewriting one expression in an equivalent form (the representation on the right). A teacher would need to be clear on the role that a

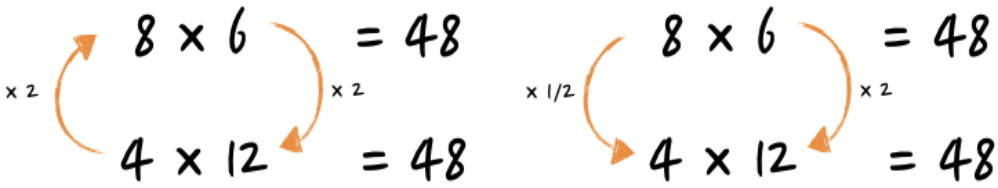


Figure 6. Representations of the "halving and doubling" multiplication strategy

particular representation plays in the construction of an idea, such as a procedure. Later in the rehearsal, Casey was confronted with a range of possible solutions to the expression 16 times 15 (which equals 240). One student offered an answer of 3000, stating that she, "did ten times ten … I took out the six and the five … which is 100. Times six is 600 and times five which is 3000." It is unclear what sense Casey was able to make of the way that student properly decomposed 16 and 15 but improperly used a series of multiplication to find the answer. Casey also dwelled in this solution, spending time having the student restate the answer, recording the strategy, and then leaving off with no clear direction for how to address this error or how to return to a focus on the halving and doubling strategy. Ultimately, teacher candidates need to be supported in specialized ways mathematically for the instructional work they do with students.

Addressing MKT through design. During the early part of both the second and third cycle, our analysis uncovered several instances where the teacher educator attempted to prepare teacher candidates for the mathematical aspects of the work, such as the use of mathematics problems given to teacher candidates in the second cycle that brought forward the relevant mathematics of exponential change. The planning protocols also included notes about the *specialized content knowledge* (SCK; Ball et al., 2008) entailed in particular segments of the IA. For example, we noted for teacher candidates

that sine, cosine, and tangent are defined using right triangles, however those ratios become tools that can use used to talk about relationships in all triangles (using the law of sines or cosines) and also have a connection to trigonometric functions. While those considerations are beyond the scope of the third IA, they are important components of what a teacher would have available in defining the ratios initially. In addition to these written artifacts, an increased amount of time and instances during the planning and analysis of an IA were devoted to addressing teacher candidates' MKT. Examples include discussion of possible student responses (and sample teacher responses to those ideas), review of what students in the class have already done, and discussion about the decisions made about the problem used, how it is displayed, and the language to be used in the IA. These discussions occurred at multiple times across the latter two IAs.

In conjunction with added content in the planning protocols around SCK and the increased discussions focused on teacher candidates' MKT, a primary artifact for foregrounding teacher candidates' mathematical development was what was called the instructional explanation decomposition tool (see Appendix D for an example). This tool served as a resource to unpack the mathematics, such as how to problematize an idea, draw upon students' prior understanding, exemplify an idea, and consider the boundaries of an idea—all of which are based on the notion of an instructional explanation from Leinhardt (2001). This tool was prepared by the teacher educators to correspond to the second and third IA—addressing the concept of exponential change and the definition of sine, cosine, and tangent. The decomposition tool was also used in a way that coincided with a planning protocol—providing connections between instructional decisions and the mathematical work the decisions accomplished. For example, in the second IA, the final

graph presented to students was motivated as a way to establish "boundaries" on an emerging idea—the way in which the growth factor of an exponential function relates to its equation. This use of the function $y = 2 \cdot 3^x$ thus supplemented the previous functions of $y = 2^x$ and $y = 3^x$ in a mathematically purposeful way. This purpose was highlighted explicitly in the section of the protocol where that final graph is introduced, and further unpacked in the decomposition tool. These connections to the instructional work that teacher candidates were doing as part of the pedagogy of practice gave the instructional explanation decomposition tool a new value, as opposed to as Georgia highlighted, "if it had just been given to us. Like, here is something that you can use" (Georgia, Post-Methods Interview, 3/21/2013).

Beyond its use in the context of work on an IA, teacher candidates expressed that the decomposition tool would be something that would support them moving forward as a way to think about the mathematics that they are teaching. Casey expressed that the tool would be useful in the future because of the way it helps,

> decompose a mathematical idea … and try to plan a lesson in a way that first problematizes an idea for students, see what their prior knowledge is and how you can build off of it. So all of these things can be used to plan a lesson around anything. That's how [the second and third IA] were more generalizable and helpful because, I mean, that's what we're going to need to do as teachers (Casey, Post-Methods Interview, 3/21/2013).

Susan also discussed how the decomposition tool would be helpful for thinking about the content of a lesson, also adding that, "most of the explanations that I see teachers give in math classes are missing a lot of these parts … I see them not having the same results with student learning as I want" (Susan, Post-Methods Interview, 4/2/2013). In sum, the decomposition tool served to support teacher

candidates in unpacking a mathematical idea, and do so in a way that had ties to their work with students.

Revising our sense of developing instructional skill. In the context of attending to two emerging needs through our design, our attention to developing instructional skill with a set of core practices was further specified and refined along two lines over the course of the three design cycles. First, we considered the way in which multiple levels of what we call *pedagogical tools* are framed and explicated in the work of investigating and enacting a set of IAs. Second, the consideration of which tools to foreground in a design must be based on what teacher candidates do in practice and the pedagogical tools drawn upon in school settings, while also looking for the instructional opportunities that serve as openings for more ambitious and equitable practice.

Developing pedagogical tools at multiple levels. From our perspective of viewing teaching as a cultural practice and skill development as a process of participation, we contend that teacher candidates do not develop actual instructional skill by having particular instructional practices modeled for them by teacher educators or practicing teachers. Even through a sequence of three IAs, though—spanning the entirety of the two methods courses and involving multiple enactments by teacher candidates—we were not content with the evidence we had (or did not have, for that matter) of teacher candidates using the activity structures or instructional practices or moves they were comprised of in their student teaching placements. As a result, we further contend that developing skilled practice is not just about enacting a lesson—no matter how well crafted.

Researchers examining the development of teacher candidates often frame analyses focused on the development and use of pedagogical tools (e.g., Grossman et al.,

2000; Newell et al., 2001; Smagorinsky et al., 2003; Windschitl et al., 2011). We define pedagogical tools as tools that mediate and enable the work of teaching in a particular setting and provide a means for a teacher to act, while also constraining other action. Pedagogical tools can be material and tangible (like a marker and also a protocol for a classroom activity) but can also be more abstract, though still practical (such as a classroom lesson structure, a question type, or a way to organize student talk). Pedagogical tools also include instructional routines, which are patterned and recurrent ways of working and interacting that shape how activity unfolds within a social group, such as a classroom (Lampert et al., 2010; Smargorinsky, Cook, & Johnson, 2003). We see there being various levels of routines, such as a full lesson or an IA, a shorter sequence of events, or even a single instructional move. As such, we have revised our consideration of developing instructional skill to consist of identifying, developing, and using of pedagogical tools and routines that enable teacher candidates to teach mathematics in ambitious ways supporting all students learning.

From this perspective, there is something to be noted about a simple observation from across teacher candidates' work in their student teaching placements—the three IAs did not become regular "containers of practice" and, thus, did not become a tool that teacher candidates continued to use within which to continue developing skill. Neither a string of computational problems (the first IA), a sequence of problems or representations to explain a concept (the second IA), or a novel way to build a mathematical definition (the third IA) were structures that teacher candidates used across their student teaching placements. This is in spite of three teacher candidates expressing that they valued the experience of investigating and enacting the IAs. However, the IAs and their

accompanying protocols did not end up being tools that teacher candidates felt that they would use in their teaching. We can only speculate that not seeing teacher candidates using the string activity in their classrooms was because of the incongruous mathematical goal of number and operation. However, the second and third IAs were not referred to concretely as structures that teacher candidates would use in their student teaching placements. While Casey and Georgia said they would look to them once they got into their placements to see if they were useful, Susan was pointed in highlighting the length of the IAs and being overwhelmed by their complexity during her post-methods interview.

What Susan also shared that the protocols for these IAs were helpful in other ways, saying, "they broke the [IA] up into different parts and we related the different parts of the [IA] to what it was going to do for students and their learning" (Susan, Post-Methods Interview, 4/2/2013). For Susan, the entirety of an IA was an overwhelming construct—"a huge document" as she said at one point during her interview. However, the component parts of an IA, such as the launch of the activity or the assigning and review of a given problem, served as milestones for Susan and the others. As we discussed in a previous section (and as Susan highlights in her quote), these segments could be paired with the specific mathematical work they accomplish toward the larger mathematical goal of the IA. Furthermore, those segments were representative of what teacher candidates did instructionally in school classrooms. They open and close their lessons (though they might not do it in a way that would be considered ambitious or equitable), give students problems to work on or prompts to consider and then elicit those ideas, and monitor students as they work in pairs or small groups. While each of these

segments must be understood as part of the larger instructional and mathematical work of an IA, a lesson, or a unit, we see the possibility of explicating these sections as a core part of teaching practice that can be carried out in particular ways to be increasingly ambitious and equitable. These core segments also became a way to provide opportunities for rehearsal in the context of longer IAs. Instead of rehearsing the IA multiple times in its entirety, the second author used the idea of "mini-rehearsals" to have teacher candidates enact an important segment of the idea, such as the launch or a discussion that connects across examples to lift the main idea of the IA.

Leveraging teacher candidates' practice in school classrooms. Our evolving notion of responsiveness and the demands of our design setting led to designs that were better aligned to the mathematical work of our partner teacher's classroom. However, we have come to realize that responsiveness does not only apply to the mathematical content. The pedagogical tools that are foregrounded in a responsive pedagogy of practice must relate to the work that teacher candidates do in school classrooms. We do not have explicit discussion from teacher candidates that tools discussed across the methods courses would not work in the school classroom. However, teacher candidates did reference the different approaches to teaching (e.g., more "traditional" or teacher-led) that their cooperating teachers enact or the goals in their placements being heavily weighted toward mathematical procedures, thus necessitating significant time spent on mathematics problems. Furthermore, when pressed during interviews on how they would carry out the work of teaching with students, teacher candidates provided vague answers or admitted that they did not yet have the answer. For instance, during her post-methods interview, Georgia responded to a number of questions about how she would use

particular artifacts and ideas with, “I don’t know. I haven’t had the chance to do that yet.” Even at the end of the academic year, when pressed to say more about how she planned to realize some of her more ambitious and equitable goals for mathematics teaching, Casey responded, “Umm … no. Well, kind of … somewhat. I think it’s pretty open as to how you do it.” Later in the interview she summed up her struggle to be more specific by saying, “I don’t know how a teacher would do it necessarily, but I know that it’s possible” (Casey, Post-Program Interview, 6/12/2013). Responses such as these suggest that, despite the tie to the work of teaching and the inclusion of enactment, teacher candidates might not have been able to see how the work would take hold in their placements.

With our view of and effort to develop responsiveness, we see the aim of the work to be developing pedagogical tools and routines for use in school settings, not for use in the university or in some ideal setting of ambitious and equitable teaching. As such, efforts around responsive pedagogies of practice must look to identify and leverage the work of secondary mathematics teaching in schools. These tools and routines need to make sense and have usefulness not only in ambitious teaching as it is defined at the university, but also in doing the work of teaching in school classrooms. In the end, teacher candidates must be supported in proficiently doing the work of teaching as it is currently defined in schools, while also having the capacity to teach mathematics more ambitiously. This is not to say that the aim of teacher education is to replicate the status quo of schools, but there needs to be an appreciation of the work that is done in schools and the tools that are used to accomplish that work—understanding schools as their own social, cultural, and historical systems.

Doing this requires teacher educators and others on a design team to understand the tools that teacher candidates do use (and the way they use them) and to better understand the activity of teaching in school settings and how various contextual factors mediate that work. This may pose a new demand on the work of teacher educators, who may traditionally remain distant from the work that goes on in schools or who engage in different types of discussions with teachers and other school stakeholders (such as managing the logistics of student teaching placements). Our continued analyses (which is highlighted in the second manuscript) include a closer examination of teacher candidates' practice in school classrooms in order to make progress on what to leverage and how to leverage it through responsive pedagogies of practice. We see such analyses as an essential aspect of teacher education that is truly responsive to the work of teaching in schools and contributing to a new terrain for teacher educators. We also see these continued research efforts contributing to the field by offering the data and analyses to better understand the practice of secondary mathematics teaching. Efforts to develop skilled teachers and approximate the work for the purposes of teacher development relies on an understanding of what the work entails, though it is an understanding that has been lacking, in part due to the common divisions between research on teacher education and research on teaching (Grossman & McDonald, 2008).

Novel terrain for teacher educators and partner teachers. The work required to plan, enact, and further refine a responsive pedagogy of practice is a product of an effort to forge a stronger and bidirectional connection between the university and school settings and to better prepare teacher candidates. In typical teacher education programs, teacher educators often have limited contact with classroom teachers, including those

who serve as cooperating teachers. As a result, there are few conversations about the needs, goals, affordances, or constraints of each setting and its stakeholders nor the actual development of a given group of teacher candidates or secondary mathematics students. One of the core elements of a push for clinical practice is the increased interaction between stakeholders at the university (especially teacher educators) and stakeholders at schools (especially classroom teachers)—interactions that have been found to be at the very least complex (Bullough & Draper, 2004; Valencia et al., 2009; Zeichner, 2010). Getting those people regularly in the same room to talk about teacher candidate development is no small feat. Furthermore, supporting the logistical aspects of these interactions does not necessarily foster a productive use of those interactions. It is naïve to assume that various stakeholders know how to and are capable of talking constructively about teaching practice and the development of mathematics students and new mathematics teachers. Through our efforts to develop more responsive and practice-focused teacher education designs, novel terrain for teacher educators and partner teachers was uncovered. For this paper, we will let the previous sections do the work of highlighting how the work of a responsive pedagogy of practice places a unique set of demands on the teacher educator—demands that move the teacher educator into a setting of clinical practice, either in a school classroom or in some newly created space at the boundary of the university and the school. For faculty at universities, this work may come in conflict with expectations for research or with lower expectations for outreach and curriculum development. In this section, we will focus on the novel role that the partner teacher can play in this work and how those entailments emerged from our design process and as a product of the evolving needs and design principles discussed in sections above.

Over the course of three cycles, we noted a novel form of participation from partner teachers. This role deviated from teachers' more typical responsibilities as classroom teachers. This was mainly due to the level of interaction between the teacher educators and a larger group of teacher candidates. Being a partner teacher involved offering up class time with one's students and welcoming in a group of teacher educators and teacher candidates into one's room. Ms. Calhoun and Mr. Ellison both corresponded with the methods course instruction to arrange a time for the rehearsal with students and to discuss—to a varying extent—the content and structure of the IA. They both secured additional space in their schools to accommodate multiple teacher candidates each working with a small group of students. During the rehearsals, Ms. Calhoun and Mr. Ellison would monitor one or more of the groups during the rehearsal time, though their lens for those observations was not predetermined as part of our design. Instead we saw that time as an opportunity for them to monitor their students, learn more about the IA, and be able to contribute broad feedback on the IA or the teacher candidates' enactments. This supervisory role is similar in some respect to the role of cooperating teacher that both Ms. Calhoun and Mr. Ellison also played for student teaching placements. However the context of that work—namely not being a traditional clinical experience—and the collaborative work with teacher educators added a new dimension to that more traditional role.

Mr. Ellison's involvement during the second and third IA was especially novel, though was mostly as a product of his own initiative as well as some advantageous scheduling. Still, as a result of that playing out through our design and in response to the evolving needs and design principles, we have now reconceptualized the role of partner

teacher as part of a responsive pedagogy of practice. Mr. Ellison made his own practice more visible by enacting the IA himself during an additional Algebra 2 or Geometry class that he had later in the day. While this was partly a pragmatic decision (e.g., to allow for all of his students to have a similar instructional experience), it offered us a new look at the design of the IA and the way in which the content coincided with that particular class of students. For instance in the second IA Mr. Ellison made a revision for his own enactment—presenting students with the graph of the function $y = x^2$ for the third and final graph instead of $y = 2 \cdot 3^x$. This slightly changed the goal of the IA from working on the connection between the constant growth factor of a function and its equation to distinguishing the differences between exponential growth versus quadratic growth. We read this change as providing insight into the mathematical goals that Mr. Ellison saw worth pursuing, thus serving as part of our effort to draw upon him as a resource even more in the conceptualization of the third IA. For that IA, Mr. Ellison became a resource for providing context for his Geometry class, such as what has been going on in the class, how students might respond to mathematical situations, and where the class is headed next in the curriculum. This resulted in more involved discussions between teacher educators and him—further distinguishing the role of partner teacher in terms of its collaboration with stakeholders at the university. His enactment also allowed us to compare the pedagogical tools he used as compared to the teacher candidates to continue thinking about the entailments of more ambitious and equitable instruction. For instance, we were struck by the way in which Mr. Ellison would narrate ideas as a way to transition between problems and highlight key ideas that emerged from students.

Finally, Mr. Ellison's involvement also had aspects that resembled a form of a teacher educator role. During the rehearsals of the third IA, Mr. Ellison engaged in a discussion with Casey during a moment in the IA when students were talking in their pairs. This interaction was not planned, nor did the teacher educators in the room prompt it. During the discussion, Mr. Ellison asked Casey about how she planned on handling the whole group discussion that would be following the small group discussions—focused on whether or not students found it surprising that the ratios of the sides of triangles (all with one 90 degree and one 55 degree angle, yet different length sides) were all about equal. Mr. Ellison provided Casey with some ideas for how to start the discussion and also how to highlight the main idea that these ratios provide a special set of values for all right triangles with a 55-degree angle. Ultimately, this type of coaching is similar to the coaching done by the teacher educator in the methods class. Mr. Ellison also took part in the reflection sessions that immediately followed a set of rehearsals. This was afforded by him having a planning period at that time, but ultimately served as an opportunity for the partner teacher to continue as part of the professional community developing around this work.

What does Mr. Ellison's involvement tell us about the role of the partner teacher in the design and implementation of a responsive pedagogy of practice? How might be prescribe this role be assumed in future iterations and for what reason? First, we acknowledge again that the involvement of both Mr. Ellison and Ms. Calhoun is an incredible asset for teacher educators and for teacher candidates. We also acknowledge that much of what Mr. Ellison brought to the role is not something we would expect from every teacher amenable to engaging in this work (though we also note that Ms. Calhoun

was not necessarily provided with the openings to showcase all that she might have brought to the role). This means that we cannot simply state that we want partner teachers to do what Mr. Ellison did. Instead we must consider how practicing teachers can be supported to develop some of the capacities that supported the design and implementation of this work, while also providing additional potential aspects of the role through continued iterations of this design-based research. Ultimately, the partner teacher—in conjunction with a teacher educator—can provide a link between the university and school settings through the design and implementation of a responsive pedagogy of practice for teacher education. She or he does this through becoming involved with the design of an IA, through making her/his own practice visible, and through providing insight on teaching and teacher development in a particular kind of teacher educator role. None of these are to be taken for granted or assumed possible, however. The partner teacher must be supported in developing an understanding of the goals for teacher candidate development that frame the work at the university and must also have more ambitious and equitable goals for students' mathematical development to offer and discuss. The partner teacher must have aspects of their teaching practice that, when made visible, offer insight into the work of teaching, though they must also be willing to grow as practitioners themselves. Finally, many teachers do not inherently have skills as teacher educators and, therefore, must be supported in order to maximize the potential and the coherence of their contributions as an instructional coach and as a member of a reflective group.

Implications and Future Directions

In this paper, we used data and analyses from three design cycles across a sequence of two secondary mathematics methods courses to make progress on the question: *What is meant by and entailed in the design of a responsive pedagogy of practice in secondary mathematics teacher education?* Design-based research provided us with the methodological and analytic tools to contribute to a theory of responsive pedagogies of practice in teacher education in the context of our own evolving efforts to create a design that is more responsive and practice-focused for secondary mathematics teacher candidates. From our initial sets of contextual needs, design principles, and design processes, we come out of this phase of the work with three findings. First, in addition to a focus on teacher candidates' development as skilled practitioners, the design of IAs must be more connected to the ongoing mathematical content and the student goals of a lesson and teacher candidates must be supported in developing the specialized mathematics resources needed for teaching that content. Second, our attention to "developing teacher candidates' instructional skill" has been further specified. Designs must explicate multiple levels of interrelated pedagogical tools and not just IAs and the individual moves that are used to ultimately carry them out. Furthermore, the set of pedagogical tools addressed in a teacher education program must be more closely tied to the work of teachers in school settings in order to prepare teacher candidates for the work they will do into their careers, while also looking for the openings in which more ambitious and equitable mathematics instruction can develop. Finally, the work of a responsive pedagogy of practice puts teacher educators and partner teachers in a novel terrain of responsibilities and collaborative work to realize the goals for teacher candidate

development and to prepare and carry out the resulting designs. In this section, we discuss some implications for this work as well as some future directions.

Considering the generalizability of design-based research. As we highlighted in our initial discussion of design-based research, its power lies in its usefulness in practice (Edelson, 2002; Gutiérrez & Penuel, 2014). As such, our intent in this paper was not to put forth the specifics of our design, but instead contribute to broader theories. Developing domain-specific theories are core to the work of design-based research, in part because of its more generalizable impact (Barab & Squire, 2004; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; diSessa & Cobb, 2004). In doing so, we see our efforts to design and implement a responsive pedagogy of practice in secondary mathematics teacher education as a phenomenon that is broader than our own context. As such, our discussion of the design problem, desired outcomes, design decisions, and processes are all specific instances of more general concerns. While our specific designs are not yet in a form that they, themselves, are repeatable across settings (in fact, we do not intend to repeat many facets of the past design ourselves), it was our attention to these broader theories that allow for impact and use across settings and contexts.

Ongoing work. What we have presented here is a snapshot in time of what we hope to be longstanding design-based research efforts, like those of others in the field (e.g., Cobb, Stephan, McClain, Gravemeijer, 2001; Cobb et al., 2009). Much of what we present here are the gains from initial efforts to systematize our design process so that it was researchable. The design and design process we discussed in this paper is not a prescription of "what works" and is instead the foundation on which further understanding of teacher development through responsive pedagogies of practice was

built. That building continues, however. Our continued work around responsive pedagogies of practice would continue to articulate and develop a theory of responsive and practice-focused pedagogies of teacher education. As Cobb & Gravemeijer (2008) discuss, this theories can become something for other researchers to take up and build upon through design processes in other settings and contexts.

Additional areas of inquiry. In addition to continued work of developing designs for and theories of teacher candidate development through responsive pedagogies of practice, we see other areas of inquiry that can support the work. First, in line with our assertion to leverage the practice of teacher candidates in schools through our designs, there must be more concerted efforts to learn about what is entailed in the work of secondary mathematics teaching. Such work must also look to understand the pedagogical tools that teachers in school settings use to accomplish the work in order to inform the tools and variations of tools that would be foregrounded in teacher education.

Second, the sense of the desired outcomes from a pedagogy of practice in teacher education is still emerging through the identification of a set of core practices and work of characterizing and measuring effective classroom practice. Our efforts at this early stage were guided by a preliminary and partial sense of how to gauge the impact of a design on teacher candidates' practice. Further efforts will need to find more concrete ways to measure teacher growth in terms of their practice. Tools such as the *mathematical quality of instruction* (MQI; Hill et al., 2008; Learning Mathematics for Teaching Project, 2011) instrument or the edTPA (developed from a partnership between the Stanford Center for Assessment, Learning, and Equity and the American Association of Colleges for Teacher Education) could be used to track progress and to further inform

the pedagogical work that is promoted in teacher education. Ongoing efforts around these designs need to use the existing instruments in the field (such as MQI or edTPA) or create new ones, though using existing instruments would leverage the power of validated tools in the field.

Third, in our work we viewed the teacher candidate and the secondary mathematics student as the "learners" in the design. However, we came to see the work of responsive pedagogies of practice as a possible development opportunity for the partner teacher and the teacher educator as well. The partner teacher's role also evolved into one that was in more constant contact with the teacher educator and became a site for the partner teacher to get involved in working on their own instruction. There was also the opportunity for the partner teacher to develop as a teacher educator through involvement in reflection sessions with the teacher educator and teacher candidates as well as their observation and support of rehearsals. For the teacher educator, the collaborative and design aspects involved in a responsive pedagogy of practice serves as novel terrain, even for the most experienced of professionals. This includes the work of instructional coaching during peer and student rehearsals, which is not a natural extension of one's skill and expertise. In turn, the teacher educator needs to develop these capacities. All of these development opportunities serve as an area for further consideration, attention, and research.

Finally, while we have discussed our work with teacher candidates, we are interested as to how similar pedagogies may find their way into the professional development of practicing teachers—beyond the role of the partner teacher as we discussed it. Work with communities of practicing teachers presents a new set of

opportunities and challenges, which would inform the needs to address, the resulting designs, and the roles and expertise needed for development and implementation. As we have started to explore those theories in our context (and, broadly, in university teacher education) it would be interesting to consider how those theories evolve when considering work with practicing teachers—who also have needs for development as professionals, yet bring a different set of resources forward.

Conclusion

By viewing our efforts in secondary mathematics teacher education as design-based research, our work is far from conclusion. Our aim in reporting on our work at this early point is to contribute to an area of the field that is gaining traction quickly, yet is in need of specification. We see the development and use of pedagogies of practice in secondary mathematics teacher education as difficult, though is work that could be supported by reports on the systematic efforts of others. Further, it is imperative that efforts in teacher education be responsive to the settings in which teacher candidates will start and continue their careers, making it important to develop a sense of what that entails, even in teacher education pedagogies that are claimed to be practice-focused. In this paper, we offer not only analysis of our own work but also a connection to larger considerations of teacher candidate development and teacher education designs.

University teacher educators and researchers need to come to the table to which teacher candidates, classroom teachers, and other clinical or practice faculty have been called to engage in more collaborative and field-based teacher preparation. Our design research efforts provide an example of how such work can be enacted and what can be learned from the work about taking teacher development out of a bubble within the

university and start preparing teachers for their work in schools, while also preparing them to do the work in more ambitious and equitable ways. This work is challenging, however it opens up a path of progress on a set of problems and desired outcomes that have long persisted. Part of what makes this work difficult is the role that the histories and communities of a context have on practice. This is why taking a theoretical stance that acknowledges, accounts for, and respects the multiple settings of teacher candidate development is a productive step forward. Paired with what is known about how teachers develop as skilled practitioners, we see responsive pedagogies of practice as a useful framework for future design and research efforts in the field.

Developing Pedagogical Tools for Ambitious Secondary Mathematics Instruction Through Responsive Teacher Education: An Analysis of Practice

Introduction

The role of university mathematics teacher education is to support the development of new teachers with the skills and resources to facilitate students meeting the demands of an increasing mathematically, statistically, and technologically complex society. The aim is for new teachers to enact *ambitious* and *equitable* goals for instruction (Jackson & Cobb, 2010; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Newmann & Associates, 1996). To do this, teacher candidates[5] need pedagogical tools that enable such instructional work in school classrooms. However, university teacher education faces a wave of criticism due to a lack of perceived influence on preparing quality teachers (Darling-Hammond, 2010; Zeichner, 2010). Instead, teachers are often found to be emulating the teaching practices prevalent in schools (e.g., Gainsburg, 2012; Lortie, 1975), leading to calls to marginalize the role of teacher preparation programs and leave certification to on-the-job training (Wiseman, 2012).

Two recent sets of recommendations offer promising paths forward. First, teacher candidates need to not only have opportunities to hear about, observe, discuss, and reflect on teaching practice, but to also have opportunities to enact teaching to develop the skill and resources required to actually do the work (Grossman & McDonald, 2008; Grossman, Hammerness, & McDonald, 2009). Calls have also emerged for more responsive professional teacher education, using the activity of teaching in schools and

[5] We use the term "teacher candidate" to refer to individuals who are enrolled and progressing through a teacher education program. We will use this term consistently throughout the article, recognizing that others use terms such as "preservice teacher", "novice teacher", and "student teacher".

the practices of teacher candidates to inform designs and evaluation (Cobb, Zhao, & Dean, 2009; Kazemi & Hubbard, 2008). Both of these recommendations are rooted in a sociocultural view of teacher development and practice—requiring a more specified understanding of teacher candidates' practice in school settings as well as the settings themselves, specifically the goals, tools, and expectations that shape what it is that teacher candidates do instructionally.

In this article, we highlight work from a design-based research study in a secondary mathematics teacher education program around the development and use of *responsive pedagogies of practice,* which provide teacher candidates opportunities to enact the work of ambitious teaching through what Grossman and her colleagues (2009) call *approximations of practice.* Through our analysis of teacher candidates' practice in these approximated enactment opportunities as well as in school classrooms, we begin to specify and defined the activity of teaching mathematics. The findings contribute to a developing sense of what is entailed in developing new teachers for that work through responsive, practice-focused teacher education designs. Such designs must attend to the dual goals of preparing teachers for the activity of teaching in schools, while also doing the work in more ambitious and equitable ways.

Review of Literature

Mathematics teacher education programs have long promoted a view of mathematics teaching and learning that is considered more rigorous, equitable, and focused on authentic disciplinary practice than is the current norm in classrooms, as outlined in numerous policy and standards documents (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000; National Governors

Association Center for Best Practices & Council of Chief State School Officers, 2010). This view of mathematics teaching and learning is realized by a range of pedagogical approaches, some of which have been identified from analyses of skilled practitioners (e.g., Franke, Kazemi, & Battey, 2007; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Lampert, 2001; Stein, Engle, Smith, & Hughes, 2008). Teaching is contingent and interactive work—work that requires mathematics teachers to be prepared with skills and resources (e.g., knowledge, dispositions, materials). However, the broader community of mathematics teachers seldom take up recommended pedagogical approaches for their work in schools, which is seen as the fault of the individual teacher, the schools in which they teach, or, ultimately, the teacher education program in which they were prepared (Clift & Brady, 2005; Gainsburg, 2012). Teacher candidates coming from teacher education programs are seen as more likely to teach in ways that are representative of their experiences as students (Kennedy, 1999; Lortie, 1975) or of the prevalent practices of their new colleagues (Clift & Brady, 2005; Grossman, Smagorinsky, & Valencia, 1999).

Often attributed to these persistent issues is the relationship between the university- and school-based components of the teacher education experience. Research and reviews continue to highlight a disconnect between university student teaching programs and schools (e.g., Clift & Brady, 2005; Guyton & McIntyre, 1990; Wideen, Mayer-Smith, & Moon, 1998)—what is often dubbed a *two-worlds pitfall* (Feiman-Nemser & Buchmann, 1985) spurred on by disparate goals across settings of teacher preparation. Recent recommendations among teacher education researchers, practitioners, and policymakers have asserted the need to situate teacher development in the context of

teaching in schools (Ball & Cohen, 1999; Grossman, Hammerness, & McDonald, 2009; Hammerness, Darling-Hammond, & Bransford, 2005; Zeichner, 2010). This assertion has given rise to calls for a greater focus on the clinical aspects of teacher preparation (e.g., Council for the Accreditation of Educator Preparation [CAEP], 2013; National Council for Accreditation of Teacher Education [NCATE], 2010). There is also increased attention to practice-based teacher education centered around the *core practices* of the work of teaching serving as the content (e.g., Ball & Forzani, 2009; Grossman, Hammerness, & McDonald, 2009). The field must consider, though, how teacher educators mobilize such content. As McDonald and her colleagues (2013) warn,

> Without a common language and a set of identified pedagogies, teacher educators are left on their own to figure out how to prepare teachers to teach the core practices, and more importantly the field itself misses and important opportunities to generate knowledge on the range of ways in which we can support teachers' learning (p. 381).

In response, McDonald and colleagues suggest that teacher educators must develop pedagogies of teacher education that focus on ways of identifying, specifying, and developing instructional practice linked to particular content—what they call *pedagogies of practice*. This is not a call for greater regulation of teacher education, rather an invitation to further the field of teacher education research and development.

Teacher development through enactments. A core aspect of a pedagogy of practice in teacher education is opportunity for teacher candidates to actually do the work, albeit in supported ways (Grossman, Hammerness, et al., 2009). Opportunities for enactment have been found to be lacking in university teacher education, especially when compared to the preparation of individuals in other professions (Grossman et al., 2009). Grossman, Hammerness, and colleagues (2009) suggest that teacher educators design

approximations of practice that simplify the work, while still being integral to the core components of teaching. In our work constructing pedagogies of practice in secondary mathematics teacher education, we have designed and used *instructional activities* (IAs), tasks enacted in classrooms that structure the work between the teacher and students around content, (Lampert & Graziani, 2009; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010) and that serve as the focus of teacher candidates' opportunities for development.

Understanding Teaching and Teacher Development Using Activity Theory

Becoming a teacher is not a solitary and idiosyncratic process and instead is a socially mediated process that occurs in activity settings. Teacher candidates develop instructional skill by teaching students within school contexts that are imbued with normative practices, working with teachers of record who host and apprentice candidates, and engaging with teacher educators whose role is to translate licensing policy into rigorous curriculum and sound professional education pedagogy. The actors within this enterprise move across settings negotiating participation and myriad roles demanded of them by the work.

To understand teacher development and practice, one must start with an examination of social phenomena, steeped in historical and cultural activity, oriented toward a particular goal (Vygotsky, 1978). Specifically, we use activity theory (Leont'ev, 1981; Wertsch, Minick, & Arns, 1984) as a way to conceptualize the situated development of teachers. In the case of teaching, an interacting set of tools, roles, and expectations ultimately shape—while also being shaped by—what teachers do in their work and, in turn, how they develop. An increasing number of researchers of teaching

and teacher education have taken up a lens of activity theory to consider teacher development and practice, in part because of how it allows researchers to consider the complexity of collective activity and how the theory explicitly attends to development mediated across settings (Anderson & Stillman, 2013b; Grossman, Smagorinsky, & Valencia, 1999; Newell, Gingrich, & Johnson, 2001; Roth & Lee, 2007; Roth & Tobin, 2002; Smagorinsky et al., 2003; Valencia et al., 2009).

In line with a focus on the culturally defined settings in which the activity of teaching occurs, teacher education design also needs to be *responsive* to the work of teaching in schools (e.g., Cobb, Zhao, & Dean, 2009; Kazemi & Hubbard, 2008). Teacher education designs have historically focused on and been based in assumptions that teacher candidates can acquire tools, practices, and resources at the university and use them in schools (Cochran-Smith & Lytle, 1999; Nolen, Horn, Ward, & Childers, 2010). This unidirectional approach means teacher education designs are often not informed by practice in schools. In our work of constructing pedagogies of practice and designing IAs to support the development of teacher candidates, we are committed to designs informed by an evolving understanding of secondary mathematics teaching. As such, we have specified our design-based research efforts as developing *responsive pedagogies of practice* for teacher education.

Developing pedagogical tools in teacher education. Key to understanding a system of activity is to understand the tools that mediate the work (Wertsch, 1991). Researchers examining the development of teacher candidates often frame analyses focused on the development and use of tools (e.g., Grossman et al., 2000; Newell et al., 2001; Smagorinsky et al., 2003; Windschitl et al., 2011). Tools enable particular forms of

practice in that they provide a means to act; simultaneously they can constrain other actions because of their defined use. We define *pedagogical tools* as tools that mediate and enable the work of teaching in a particular setting. Pedagogical tools can be material and tangible, and can be more abstract, though still practical (such as a classroom lesson structure, a question type, or a routine for interaction). Pedagogical tools enable the work teacher candidates take up across settings and shape how that work gets done as tools are put into action.

The design and implementation of responsive pedagogies of practice in secondary mathematics teacher education must look to develop the skilled use of pedagogical tools, specifically tools that have purchase in secondary school classrooms. We contend that teacher educators and researchers need to understand the tools that teacher candidates do use (and the way they use them) to better understand the activity of teaching in school settings and how various contextual factors mediate that work. This view of tool use is more consistent with a activity theory perspective, which instead of viewing the movement of "packages" of knowledge and skills (as is the case from more prevalent, cognitive perspectives) teacher educators would interpret, modify, and reconstruct around mutually relevant practices and tools (Tuomi-Gröhn & Engeström, 2003).

Routines. One type of pedagogical tool in which we are interested is that of the principled sequences, or routines, that structure culturally defined practice (Smagorinsky, Cook, & Johnson, 2003). Routines are patterned and recurrent ways of working and interacting that shape how activity unfolds within a social group. Teaching routines have been highlighted by a number of research groups uncovering the challenges and benefits of teacher-to-teacher and teacher-student interactions mediated by routines (Horn &

Little, 2010; Kazemi & Hubbard, 2008; Lampert et al., 2010). In teaching, we see that there are various levels of routines, such as a full lesson or an IA, a shorter sequence of events, or even a single move. We do not see a routine as belonging to an individual (and, specifically, a skilled individual) teacher, as the routine is part of the larger activity of the setting. As a pedagogical tool, certain routines are made more feasible to carry out based on the activity of a setting and the resources available to the teacher.

Motivating This Study

Through our design-based research we pursued two tenants. The first was identifying pedagogical tools that were both accessible and supportive of teacher candidates developing instructional skill. The second was that the pedagogical tools employed in teacher education must leverage the understanding of the activity of secondary mathematics teaching in school classrooms. Contributing to these aims, the work presented in this paper presents part of the retrospective analysis of the design-based research process focused on understanding the activity and tools of teaching in our constructed instructional setting across methods courses and the instructional setting across student teaching placement classrooms. Specifically, we address the following questions:

1. What is the activity of secondary mathematics teaching as defined throughout a responsive pedagogy of practice in a university methods course?
2. What is the activity of secondary mathematics teaching as defined across teacher candidates' student teaching placements?

By working to define the activity of teaching in each setting of teacher development, we are enabled to compare what is done instructionally in each setting and to help us make

sense of what teacher candidates were doing in their teacher education program in service (or not) to what they were called to do in their student teaching placements. Moreover, the implications of the second question have bearing on the future designs of responsive pedagogies of practice that look to leverage the work and tools of teaching in schools in the construction of approximations of practice.

Methods

Our work is set within ongoing design-based research in a sequence of two, ten-week secondary mathematics methods courses and subsequent student teaching experiences. The courses are part of a small Master's level teacher licensure program at a very high research activity institution as classified by the Carnegie Classification of Institutions of Higher Education. In all, the program spans ten months—from late August through the end of the spring quarter in mid-June. Teacher candidates in the program have two, ten-week student teaching experiences—a part time experience in the fall (concurrent with the first of the two methods courses) and a full time experience during the spring quarter (after the second course).

The data analyzed for this project were collected in the 2012-2013 academic year, during which we had a very small cohort of three teacher candidates—Casey, Georgia, and Susan[6]. Both authors served as the members of the mathematics teacher education design team[7] at the university. In this particular year, the first author taught the first

[6] All names used, except for those of the teacher educators, are pseudonyms.
[7] A third mathematics teacher educator is also part of the design team but was only occasionally involved during the academic year we discuss in this article. She did not serve as an instructor for one of the two mathematics methods courses but did observe class sessions and enactments and also took place in some of the planning and debrief sessions.

methods course and the second author taught the second course, though for the previous two years that arrangement was reversed. The teacher educator who was not teaching in a given term would regularly observe the course and would be actively involved in the planning and debriefing. The curriculum across the two courses focused on teacher candidates developing skilled practice through investigation and enactment. This meant candidates not only read and discussed articles and case studies about teaching, they participated in supported rehearsals of IAs which were vehicles for developing ambitious teaching practice designed to promote core content and practices for secondary mathematics students. This design served to create an instructional setting, which we look to define further through pursuing our first research question. To support teacher candidates during student teaching phases of the program, the first author also served as a supervisor during both the fall and spring practicum experiences. This arrangement provided an additional link between the university- and school-based components of the teacher education program, while also facilitating the collection of data from school classrooms.

Data sources. For our first research question, we focused on the work of teaching in the instructional setting that was created through a series of enactment opportunities in the methods courses. We looked at three "cycles of investigation and enactment" (Lampert et al., 2013; McDonald et al., 2013), each centered on a new IA designed for secondary mathematics that was enacted in a university-based rehearsal setting as well as in a local middle school or high school classroom. We summarize the three IAs from that academic year in Table 2 and provide a modified planning protocol for each in Appendix A. For each IA, we specified the structure of the activity, as well as the teacher and

Table 2
Summary of Three IAs that Comprised the Pedagogy of Practice

Instructional Activity	Targeted grade level/course & mathematical content	Summary of Structure, Teacher Development Goals, and Student Mathematical Goals
1) String of Computational Problems	*7th Grade* Multiplication – "Halving and Doubling" Strategy	*Summary:* A string (Fosnot & Dolk, 2001; Kazemi et al., 2009) is short activity designed to highlight a particular mathematical idea, notably a computation strategy. In this string, a sequence of four multiplication problems were used to bring forth and motivate the use of a strategy for mental computation in which one factor can be halved and the other doubled to create an equivalent product.
		Teacher development goals: The IA was put forth as an opportunity for teacher candidates to do interactive instructional work toward a mathematical goal. Specifically, the structure put teacher candidates in position to elicit and respond to students' mathematical ideas, represent ideas on the board, dwell on important mathematical ideas, and orient students to one another's ideas through the use of discursive moves (e.g., Chapin, O'Connor, & Anderson, 2009)
		Student development goals: From this IA, students are expected to be able to begin to identify, use, and provide a justification for the "halving and doubling strategy" for multiplication. In terms of process, the IA is also an opportunity for students to voice their ideas about the strategy, while also being able to listen to and reason about the ideas of their peers.
2) Explaining a Concept through Connections across Representations	*High School - Algebra II* Exponential Change in Graphs, Tables, and Equations	*Summary:* Similar in some respect to the string, this IA centered around a purposefully designed sequence of prompts and representations to construct an explanation of exponential change and to visualize that change across graphs, tables, and functions. Students were shown, in sequence, the graphs of three exponential functions with the third graph providing an example that defines the boundaries of the explanation being constructed about exponential change and its relation to the closed form of the function.
		Teacher development goals: This IA positioned teacher candidates to construct a mathematical idea using a purposefully sequenced set of examples and based on contributions from students. Accordingly, the teacher candidate was in a position to elicit students' ideas and to orient students to one another and to the mathematical work at hand.
		Student development goals: This IA is designed to support the construction of the idea that an exponential functions' growth can be characterized as, for an increase of one unit in the input, the output is multiplied by a constant factor. Students further consider the role of the constant factor in the equation for the function and its graph. Students engage in mathematical practices such as constructing arguments, critiquing the reasoning of others, and attending to the precision of mathematical language.
3) Building a Definition from an Investigation	*High School - Geometry* Right Triangle Trigonometric Ratios (i.e., sine, cosine, and tangent)	*Summary:* Drawing on prior experiences with similar triangles, specifically the proportional relationship of pairs of corresponding sides, this IA focused on the development of mathematical definitions—the basic right triangle trigonometric ratios of sine, cosine, and tangent. The teacher candidate led a discussion looking across the data from groups, highlighting the constant ratios across the similar triangles in order to define the three trigonometric ratios.
		Teacher development goals: This IA required teacher candidates to manage materials and small group work. The teacher candidates worked on using precision in highlighting a problematic situation and in defining new mathematical ideas. As with the other IAs, teacher candidates had opportunities to elicit and respond to students' ideas.
		Student development goals: The work of this IA was motivated by a problem for which previous tools (i.e., using the Pythagorean Theorem) was not useful. As a result, students begin to think about the use of strategies and mathematical relationships in problem solving.

student development goals. The IAs became more complex over time, in part due to increased length. By design, the sum of the three IAs also captured a range of instructional work focused on mathematical procedures, concepts, and definitions.

The data (summarized in Table 3) on the three cycles consisted primarily of video and field notes inventorying events during each design cycle. From the methods course, our data included teacher candidates observing and analyzing an IA, planning the IA, and reflecting on their enactments using video. We also collected video of the enactments—in the methods course and in the approximated secondary mathematics classroom. Across the three cycles, a reflexive journal (Altheide & Johnson, 1994; Auerbach & Silverstein, 2003) was maintained by the first author to contribute to the transparency into the process of data collection and design work. Given both authors' tightly knit roles as both

Table 3
Summary of Data Sources for Both Research Questions

Research Question	**Fall**	**Winter**	**Spring**
1. What is the activity of secondary mathematics teaching as defined throughout a responsive pedagogy of practice in a university methods course?	Video and artifacts from one cycle of investigation and enactment (11/2012) Reflexive journaling	Video and artifacts from one cycle of investigation and enactment (2/2013 & 3/2013) Post-coursework interview with each teacher candidate (3/2013) Reflexive journaling	
2. What is the activity of secondary mathematics teaching as defined across teacher candidates' student teaching placements?	Video from two lessons from each teacher candidates in their part-time student teaching placement (see Table 4 for more detail)		Classroom data sets consisting of two to three lessons in sequence and a post-lesson interview (see Table 4 for more detail) Post-program interview (6/2013)

researchers and teacher educators, which is a strength of the design-based research process, measures taken such as the maintenance of a reflexive journal is key to ensuring trustworthiness and rigor in the research process (Auerbach & Silverstein, 2003; Barab & Squire, 2004). The journal served as a data source in its own right, which we discuss further in the section on our analyses. Finally, an interview was conducted with each of the three teacher candidates after the second methods course (and before their full-time student teaching practicum in the spring term). The focus of this interview was to recap the work of across the three cycles, including what they perceived to be the goal of the work and what they felt enabled to bring into their teaching.

For the second research question, we focused on each teacher candidate's teaching experiences (summary of these data also found in Table 3). During their part-time practicum in the fall, two lessons were video recorded by the first author. Casey and Georgia were in the same middle school classroom during this experience and Susan was in a high school Geometry classroom. In the spring, during teacher candidates' full-time practicum, data sets consisting of two sequential lessons and a post-lesson interview were collected. The decision to collect two lessons in sequence was to allow for a focal mathematical idea to unfold in more detail and with the possibility for a wider range of instructional work. Furthermore, the lessons were recorded no sooner than three weeks after the start of their placement and no later than two weeks before the end of the school year to give teacher candidates time to become acquainted with their new instructional setting without the potential irregularities that might occur at the end of a year. The first author video recorded these lessons and arranged with each teacher candidate regarding the selection of a two-day arc of lessons on a common topic. More detail on the lessons

from the student teaching placements is provided in Table 4. Teacher candidates were also interviewed by the first author toward the end of the program in order to gain their insight into the instructional work they saw themselves doing in both the university and classroom setting. The first author conducted these interviews, as well as the interviews within the data sets.

Table 4
Summary of Lessons Collected from Student Teaching in Middle School (MS) & High School (HS) Placements

	Casey	**Georgia**	**Susan**
Lessons Collected from Part-time Practicum	MS Lesson 1: Order of Operations (11/6/2012) MS Lesson 2: Interpreting Qualitative Position-Time Graphs (12/3/2012)	MS Lesson 1: Writing Algebraic Expressions to Represent a Situation (11/6/2012) MS Lesson 2: Solutions to Linear Equations Across Representations (12/3/2012)	HS Lesson 1: Classifying triangles (11/27/2012) HS Lesson 2: Triangle congruence postulates (11/29/2012)
Lessons Collected from Full-time Practicum	HS Lesson 1: Measures of Center and Spread (5/10/2013) HS Lesson 2: Solving Problems Involving the Mean (5/13/2013) Interview (5/15/2013)	HS Lesson 1: Defining Pi as the Ratio of Circumference and Diameter (5/21/2013) HS Lesson 2: Area of Circle, Area of Sectors, and Arc Length (5/23/2013) Interview (5/24/2013)	MS Lesson 1: Stem-and-Leaf Plots (4/22/2013) MS Lesson 2: Evaluating the Use of Stem-and-Leaf Plots (4/24/2013) MS Lesson 3: Properties of Exponents (5/24/2013)[8] Interviews (4/24/2013 & 5/28/2013)

Analytic framework for parsing instruction. To begin to define the activity of mathematics teaching in each of the two instructional settings constructed in teacher preparation, we look to our data—specifically the video of instruction—to find out what it is that teacher candidates do instructionally in each setting. An affordance of activity

[8] Due to extenuating circumstances in the field placement, video from a third lesson was collected one month after the first two lessons. An interview was conducted after the second lesson, though the interview conducted after the third lesson brought in video from all three lessons.

theory for investigating teacher candidate practice is the nested levels at which social phenomena can be analyzed as outlined by Leont'ev (1981) and elaborated by Wertsch and his colleagues (1984): *activity*—the level of a social system; *action*—the level of an individual agent acting within that activity; and *operation*—the level of concrete procedure and behavior. Activity is organized toward a motive—what is to be maximized in a setting—and is specified and defined by "the socioculturally defined milieu in which it occurs" (Wertsch et al., 1984, p. 155). In addressing our two research questions, we look to begin to define the activity of teaching in particular settings. Part of doing that involves identifying the ways in which the activity of teaching is carried out. This is where the rest of the framework—consisting of actions, subactions, and operations—proves to be useful.

Within an IA or classroom lesson, we characterize the action-level with what we called *instructional episodes*, defined by changes in the work that a teacher and students are engaged in, such as launching mathematical work, eliciting a solution to a procedural problem, or eliciting connections across problems or mathematical ideas. Thus, an IA or classroom lesson[9] is a sequence—potentially purposefully organized—of these episodes. Episodes are not unique to a given IA or lesson. For instance, teachers quite often elicit solutions to problems in a mathematics class, but they do that for any number of purposes. However, the range of goals for eliciting solutions to problems in a secondary mathematics class is tied to the larger motive of the activity of teaching.

[9] We do not contend that a lesson or an IA is the "activity" of teaching, as what is meant by activity captures something much larger. Based on how we define "action", a lesson or an IA—as a collection or sequence of instructional episodes—could be defined as a "super-action".

A given episode is carried out, at the sub-action level, with what we called *instructional practices*. The term *practice* has become pervasive in research and development on teaching, as well as problematic because of its varying meanings (Lampert, 2010) and grain sizes (Ball & Forzani, 2011; Boerst, Sleep, Ball, & Bass, 2011). In our work, an example of a "practice" would be managing engagement, such as explicitly defining participation expectations. Another example would be dwelling on a mathematical idea, when a teacher lingers on an idea that may need further unpacking or is key to the big ideas of the lesson. Teachers also may represent mathematical ideas shared by students, or need to address an error that emerges. In our framework, practices are of a larger grain-size of analysis than individual *instructional moves*, which is how we classified the operation level. While instructional practices are how an episode is carried out, a move is how a practice is carried out. So a teacher might dwell on an idea by prompting a student to revoice another student in the class. A teacher might manage engagement by providing students with time to think to themselves after assigning a problem or a prompt. When eliciting ideas, a teacher might call on a particular student for mathematical or participatory reasons. Like instructional episodes and practices, instructional moves can be used to realize multiple practices and as part of multiple kinds of episodes. Their use, though, is contingent on the conditions at play. For example, a teacher may do more telling or ask fewer follow-up questions if time is short.

We outline these three levels of instructional episodes, practices, and moves within the activity of teaching and provide examples of each in Table 5. The process for fleshing out the entire set of codes and their descriptions at each level (i.e., developing a code book) began by looking across the existing literature on decomposing the work of

Table 5
Analytic Framework for Coding Instructional Practice

Analytic Level	Unit of Analysis of Teaching	Codes (Examples)
Action	Instructional episodes	• Launching a Lesson • Assigning a Prompt • Elicit Solution to Problem • Elicit Connections • Facilitating Individual or Small Group Work • Non-Mathematical Work
Subaction	Instructional practices	• Managing Engagement • Managing Mathematical Goal • Eliciting Student Contribution • Dwelling on Mathematical Idea • Representing / Recording Idea • Addressing an Error
Operation	Instructional moves	• Prompting Student to Revoice Other Student • Providing Private Reasoning Time • Calling on a Particular Student • Pressing a Student for More Information

mathematics teaching (Ball & Forzani, 2009; Ball, Thames, & Phelps, 2008; Lampert et al., 2013), instruments measuring the mathematical quality of instruction (Hill & Ball, 2008; Learning Mathematics for Teaching Project, 2011), as well as more public displays of the core practices of teaching (TeachingWorks, 2014). Looking across these sources, we identified codes at each level and also began a process of merging similar ideas. This initial framework was used in an early set of coding of the enactment videos from the first cycle (the "Strings" IA) and one lesson from each teacher candidate from the fall student teaching practicum. We used the qualitative video analysis software Studiocode© for our analyses, which allowed us to manage the large set of data, tie codes directly to the video, and look across subsets of data to further stages of analyses. We discuss our use of this software further and how it contributed to our analyses in the next section. The early set of coding serve as an opportunity to (i) further develop the codebook and code window in Studiocode, and (ii) construct inter-rater reliability. Through the analysis of additional enactments and lessons, new codes emerged at each analytic level and others

were collapsed and refined. After these changes were discussed and agreed upon, the new scheme was applied to the rest of the enactment and lesson data, as well as the data that was previously coded.

Data analysis. Using the analytic framework outlined above with the video of IA enactments and classroom lessons allowed us to ask the question of our data: What do teacher candidates do in each instructional setting? This served as part of our analyses contributing to our research questions aiming to detail the activity of secondary mathematics teaching as it was defined within our responsive pedagogy of practice and across student teaching placements in school classrooms. Accordingly, we looked across the three IAs as a collective set to represent the work done in the methods courses and we combined all of the lessons analyzed across the three teacher candidates to represent the work done in student teaching placements.

The process using the framework in Studiocode first involved "chunking" an enactment or classroom lesson into instructional episodes. We used the term *episode* from our analytic framework to mean an *idea unit* (Sherin & van Es, 2009) that entailed a unique and particular work of teaching. With the software, this entails segmenting an interval of the video as an instance. We then labeled each instance with the type of episode it was, such as "assigning a problem" or "eliciting ideas". Each idea unit was associated with one episode type. When the type of episode changed a new idea unit was demarcated. In a second round of coding, we labeled each episode with instructional practices that emerged, such as "posing a problem or prompt", "managing engagement", and "representing mathematical ideas". For this second round of coding, episodes could have many practices. The number of practices within episodes ranged from one (often in

"non-mathematical work" episodes involving a practice such as "managing materials") to 12.

After completing the coding process we employed the software analytics of Studiocode with which we were able to examine an inventory of counts of episodes and practices in a matrix. Further the matrix allowed for sorting data and examining video across clips with the same code. From the matrix we first looked at the range of episodes within a setting and then identified patterns of high frequency for the instruction in that given setting. We reasoned that having an inventory of the range and, more importantly, high frequency episodes would provide us insights on what work was being accomplished within a setting across teacher candidates. Since we were most interested in what is the activity of secondary mathematics teaching within each setting, examining the range of work, or episodes, would give us systematic data across the three teacher candidates. Our analysis continued with a focus on teaching practices employed within episodes. We similarly looked for the most frequently occurring practice codes, with the most frequent occurring in 50% to 75% of all episodes. We used this as a way to identify the most common ways in which the work of teaching was carried out across teacher candidates' placements at the subaction-level—both within and across episode types.

Looking across the two instructional settings, we attended to episodes and practices that were surprising, meaning they occurred with moderate to high frequency in one setting with few to no instances in the other. This allowed us to capture episodes and practices that were not necessarily the most frequent, but serve as a distinguishing characteristic of the instruction in one setting versus another in service of the emerging definitions of the activity in each setting. In lieu of coding every move across our data,

we instead looked across sets of similar episodes and practices of interest to identify instructional moves that were prevalent. For example, across a set of instances of the practice of "eliciting student contribution", it was noted and recorded that teacher candidates in their student teaching placement would most often use a move of broadcasting questions so as to welcome volunteers or calling out. These were accounted for in analytic memos that served in capturing these analyses and in the selection of representative examples and vignettes that contribute to the definitions of the activity in each setting.

We also drew upon reflexive journaling and two interviews with each teacher candidate to contribute to an emerging definition of the activity of secondary mathematics teaching in each instructional setting and to situate our emerging findings from our analysis of instruction. The reflexive journal served as a way to identify the goals and foci of the design team across the three cycles of the responsive pedagogy of practice, which when considered in conjunction with what teacher candidates actually did across the IAs, supported identifying the entailments of the activity and contributed to an emerging definition of the activity in that setting. Similarly, in regard to the activity of secondary mathematics teaching in student teaching placements, questions from the interviews with teacher candidates that focused on how they perceived (or were informed) the goals and expectations of their student teaching placement. These served in the construction of definitions of the activity of teaching in secondary school classrooms and as a way to make sense of the trends we saw emerge from our analysis of the actual instructional work. These analyses and the work to define the activity of teaching in each

setting provide implications for continuing evolution of the design and implementation of responsive pedagogies of practice in secondary mathematics teacher education.

Identifying the Entailments of Teaching in a Pedagogy of Practice

Through a sequence of three IAs, we created an instructional setting in which the work of teaching is defined in a particular way, is oriented toward particular motives, and is mediated by a particular set of pedagogical tools. The analyses we discuss here works to define the activity of teaching created in this approximated setting. Two definitions of the activity of teaching emerged that capture the work that was constructed in the pedagogy of practice—a focus on providing students access to the mathematics and a focus on the orchestration of whole class discussions that build on students' ideas and move toward a clear mathematical goal. Our analyses identified the instructional episodes, practices, and moves that comprised what it was that teacher candidates did instructionally across the three IAs within this particular definition of secondary mathematics teaching.

Providing students access and orienting students to the mathematics. An overarching theme in the instructional work across the three IAs—both in investigation and enactment—was the importance of providing students access to the mathematics and the expectations at hand through instruction. The structure of the three IAs provided multiple opportunities for teacher candidates to do instructional work that was consistent with this theme. Furthermore they were provided with pedagogical tools to do the work across the IAs, with the goal of those tools being usable across settings. These tools were specified in planning protocols given to teacher candidates to support their enactments. From our analyses, we saw two broad types of episodes in the enactments—launching an

IA and assigning a problem or prompt—as well as their constituent parts as attending to this aim.

Launching an IA. Jackson and her colleagues (2013) have identified the features of an effective *launch*, including how a teacher supports students' reasoning about the contextual features of a task as well as the mathematical relationships at play while maintaining the cognitive demand. While none of the IAs across our three enactment cycles featured notably complex mathematics tasks, especially in terms of contextual features, there was a constant instructional focus on the introduction of an IA. At the start of each IA, teacher candidates had the launch specified for them in the planning protocol, including scripted segments to support teacher candidates in addressing two main features—the mathematical goal for the IA and the expectations for participation. Further, teacher candidates were advised to check in with students, such as through asking a student to restate those expectations, especially when the activity structure or the mathematics was unfamiliar.

The work of launching is not just the stating of expectations and objectives by the teacher candidate, however, as it could also include how a mathematical idea was problematized and how students' prior understanding of an idea was elicited. The third IA around building a definition for the three basic right triangle trigonometric ratios positioned teacher candidates to do this work. Both of these additional launching strategies were highlighted by teacher candidates in their interviews as tools they felt they could use moving forward into their teaching practice.

Problematizing a mathematical idea. In the Building a Definition IA (the third IA), students were presented a "zip line problem" (see Figure 7) to motivate the content of the IA. To that point, the students in the rehearsal classroom had solved similar problems in terms of the context and the figure, though problems that provided two of the lengths (instead of one length and one angle, in addition to the right angle). Students could solve the previous problems using the Pythagorean Theorem. Teacher candidates were prompted to ask students to consider what was similar and, then, what was different about this problem as compared to the previous zip line problems they had done. After some discussion, as well as a realization that a strategy was not available for solving the problem, the teacher candidate would orient students to the focus of the IA, such as in this excerpt from Casey's enactment:

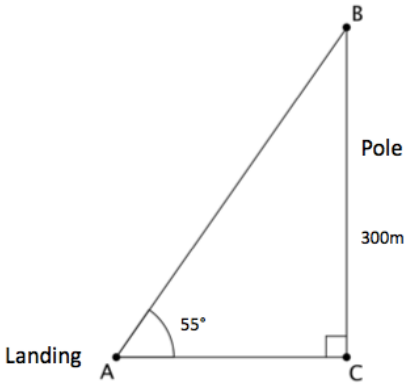


Figure 7. The zip line problem

Casey: Today we're going to try to build a tool that will help us solve this problem, this new zip line problem—a tool that will help us relate the angle measure to the side length. So … [gathering materials for next section of IA]

Student: (quietly) If only we had the length of the zip line … [some laughter in the room]

Mr. Ellison[10]: What was that?

Student: I said if only we had the length of the zip line then we could solve the problem.

The second part of the excerpt illustrates a common occurrence in the enactments—both with peers and with secondary students—where the teacher educator or classroom teacher would interject. In this instance, Mr. Ellison simply asks a student to restate a comment that could have been leveraged to further highlight the distinction between the new problem and previous ones as well as the fact that previous strategies are not useful for this new problem.

Surfacing students' prior understandings. In the case of the Building a Definition IA, the main activity and the resulting definitions relied on what students knew about similar triangles, specifically about the proportional relationship between corresponding ratios of side lengths. For instance, Figure 8 depicts two similar triangles. Based on that, the ratio of a to b is equal to the ratio of d to e. In the IA, teacher candidates posted the images in Figure 8 and asked

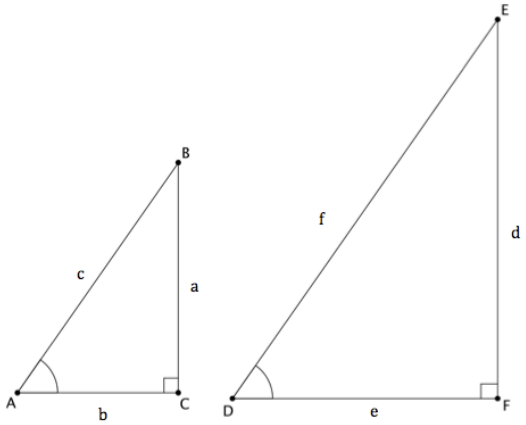


Figure 8. Two triangles presented to students to elicit prior understanding

[10] Mr. Ellison (a pseudonym) was one of our partner teachers in our responsive pedagogy of practice. We talk more about Mr. Ellison's role as a partner teacher in the design process of our pedagogy of practice in the first manuscript.

students to write down everything they knew about the two triangles. Students were then instructed by the teacher candidate to share their ideas with a partner. The teacher candidate then discussed some observations in the whole group setting. The excerpt below is from Casey's enactment:

> Casey: I heard, many of you actually, classify these triangles by their angle measure. Would you guys like to share?
>
> Student 1: They are both right triangles.
>
> Casey: Right triangles [writes on board]. OK … I also heard some groups specify the relationship between the two triangles. How about this group, would you like to specify the relationship you saw?
>
> Student 2: They're similar.
>
> Casey: [writes on board] OK, how do you know they're similar?
>
> Student 3: A-A Similarity.
>
> Casey: Can somebody restate what I just heard?
>
> Student 4: They are similar because of the Angle-Angle Similarity.
>
> Casey: OK, so we know the Angle-Angle Similarity postulate and we have these two congruent angles. So by the A-A Similarity postulate we know that these must be similar triangles, right? … I also saw some of you set up proportions. Would you like to share one of the proportions you set up?

It was at this point that Casey elicited and recorded a proportion such as the one referenced above. An episode such as this, centered on eliciting what students know already about particular mathematics, plays a key role in orienting students to the mathematics of an upcoming IA or lesson. We will discuss more about the way in which Casey elicited those ideas in the section on orchestrating whole class discussions.

Assigning a problem or a prompt. Throughout an IA, teacher candidates were supported in continuing to provide students access and orient them to the mathematics of

an IA, as well as the expectations for participation. Across the IAs, a common episode type that we coded was around the assigning of a problem or a prompt for students to reason about individually or in small groups. In addition to the way in which the problem was assigned, students' access to the mathematics benefited from the time given for students to reason about a problem or prompt before discussing it in whole group. Across the IAs, teacher candidates were provided with prompts to specify expectations like the use of a silent thumb[11] as a signal as well as structuring the participation format, such as specifying a structure and process for partner talk.

The way in which a problem or prompt is framed relative to both the work that has already been done and the goal of the work moving forward proved to be important as well. While the specifics of the narration by the teacher candidate are quite contingent on the moment, the notion of setting up a problem in this way serves as a pedagogical tool. For example, in the second IA around exponential change the protocol called for a third graph to be put forth to press on the boundaries of an emerging explanation. The excerpt below shows how this problem could have been assigned quite mechanically and how a teacher educator steps in to provide more productive narration:

> Susan: (stepping in after students had been talking in pairs about the relationship between the constant growth factor and the closed form of the function) So, keep in mind that general pattern you've been talking about with your partners. We are going to go ahead and look at Graph C [$h(x) = 2 \cdot 3^x$]. We don't have the time to look at it individually *and* talk in groups, so go ahead and look at it. I have a copy of it over here [moves to display Graph C].

[11] The "silent thumb" (i.e., a thumb turned up in front of one's chest) is a move intended to be less distracting that a raised or waving hand. This is seen as more accessible and equitable, especially for students who are slower to arrive at an answer or less forthcoming with their reasoning.

> Rebekah: (to the whole class) So one of the things you're going to want to think about with Graph C is you've now seen a general pattern for [Graph A and B]—that exponential functions seem to have this constant factor that is multiplied to the output and the input increases by one. And I heard somebody talk about the equation and said, 'Oh, it's the base.' What we want you to do now in Graph C is think about, is that always the case? Is that constant factor always the base? And does it help you make sense out of what the equation is? So use this third example to think about the pattern you noticed and how it relates to the equation.

In this excerpt, we see the way in which a teacher can continue to orient students to the mathematics of the IA, or the lesson, when introducing new problems or prompts. Both Casey and Georgia leveraged the third graph prompting students to refine ideas generated from the previous two graphs similar to what we see the teacher educator prompt for in Susan's enactment. These key transition points became objects for discussion on how a lesson is designed to move a math goal forward.

Using student contributions in mathematical goal-oriented discussions. A second way in which the activity of teaching was defined across the three IAs was how student contributions can be used in whole class discussions aimed toward a clear mathematical goal. Of course, this was not by chance, as the IAs were designed, in part, to highlight and emphasize this complex aspect of the work of teaching. From our analyses, the central focus on this aspect of the work came in the form of multiple instances of certain episodes, such as eliciting solutions to problems and ideas in response to prompts. Further, there were practices and moves that emerged that managed student engagement and participation (namely moves that oriented student to one another as

opposed to the teacher) and made progress toward a mathematical goal (through dwelling on and highlighting important mathematics ideas).

Monitoring student work. While monitoring is a way to manage a classroom, it was framed across the IAs as an opportunity for teacher candidates to see what students are doing or saying in order to structure the way in which ideas get elicited. This is much like the work of monitoring, selecting, and sequencing outlined by Stein and her colleagues (2009) in their extensive work around orchestrating the use of complex tasks. In planning for the third IA, teacher candidates anticipated (or had anticipated for them in the planning protocol) the types of ideas they might hear from students as they examined the zip line tasks and further into the IA when examining similar triangles. The IA listed the type of ideas that would be important to monitor for and elicit from students to be built on in the whole group discussion. We see evidence of those determinations in Casey's excerpt from the third IA, which we described above and will highlight again next.

Eliciting solutions and ideas. In most cases the work of eliciting solutions or ideas was preceded by an "assigning" episode (along with the practice of "monitoring"), meaning across the IAs teacher candidates were seldom eliciting contributions immediately after a problem or prompt was initially posed, rather they had opportunities to monitor students' written work or partner discussion. As a result, within the eliciting episodes themselves we found emerging pedagogical tools to support the work. The way in which students' contributions were elicited broke from what might be considered to be more traditional approaches, such as broadcasting a question and allowing for calling out from students or calling on students more randomly and spontaneously (typically

characterized as "cold calling"). Once strategies or ideas were offered by students, teacher candidates were encouraged to use discursive moves, such as revoicing an idea, prompting a student to restate, reason about, or add on to a peer's idea, or simply wait time (Chapin et al., 2009). This kind of move is seen in the excerpt from Casey's work in eliciting ideas about the two similar triangles (specifically, asking someone to restate how the two triangles were known to be similar). We saw this move and others being used as a way to orient students to one another and to make progress on a mathematical goal.

Managing engagement by orienting students to one another. The promotion and use of discursive moves— especially ones that prompt students to grapple with their peers' ideas—was seen as one way to break the pattern in a classroom in which the teacher serves as the primary authority and conduit for discussion. In the coding, this emerged as a practice of "managing engagement", which was further realized by discursive moves and structures that gave students the opportunity to share ideas with peers. The participation structure advanced (Lampert, 2001) was intended to engage multiple students rather than typical structures, which position the teacher as the "hub" of discussion. Efforts to break that pattern were emphasized and regularly discussed and enacted for those purposes. Teacher candidates were regularly prompted to allow for students to talk in pairs or in small groups. In addition to doing this when assigning new problems or prompts, the use of the small group format was used as a tool when a novel idea was shared or when there was some confusion on a point. In these instances, the teacher candidate would prompt students to talk about a given idea with their partner for a short amount of time. This allows students to reason about an idea and do so with a

peer, thus breaking the cycle in which the teacher might be doing much of the mathematical work, especially in times of struggle.

Teaching toward a clear learning goal. Discursive moves and the use of small group talk formats were not positioned as a tool to use at any time (or all the time) during instruction. Instead, in addition to the work of orienting students to one another, these moves were promoted for use in important moments, mathematically. For instance, in the String IA, when a student made a comment that brings forth the focal strategy of "halving and doubling", teacher candidates were prompted to use discursive moves as a way to *dwell on* and *highlight* those ideas in order to make progress on the mathematical goal for the IA. While the teacher candidate doing the revoicing was also seen as a way to highlight important mathematical ideas, the use of discursive moves and "turn-and-talk" was more notable as a change to the pattern of talk in the classroom.

The excerpt from Casey's enactment around the discussion of the two similar triangles highlights another way in which teacher candidates were supported in orchestrating discussions based on students' ideas toward a mathematical goal. In the excerpt, Casey does not simply ask for a student to share. Instead she frames particular categories of observations and claims and chooses specific students or pairs to share. She does this in a way that does not have her sharing the ideas herself, but structuring and sequencing the contributions in order to elicit particular ideas toward a particular end—that similar triangles have a proportional relationship among their corresponding sides. However, we see few examples across the three teacher candidates' enactments of discursive moves—especially more complex moves such as prompting students to reason about and build on peers' ideas. We attribute this to the lack of specification for such

moves in the planning protocol for the third IA. Our systematic look at our design has highlighted the intended and productive use of these moves. Future iterations of the design would not only specify these discursive moves but also link them to the instructional and mathematical work that they accomplish in order to provide teacher candidates with rationale for using the move (beyond it being an expectation of their course instructors).

Summary. Two main definitions of the activity of teaching in our responsive pedagogy of practice emerged from our analysis—providing students access to the mathematics at hand and orchestrating whole class discussions using students' ideas toward a clear mathematical goal. Both of these are key to a vision of ambitious and equitable mathematics teaching and are bolstered by the specification of the episodes, practices, and moves that carry the work out. This work created an image of mathematics instruction and allowed for teacher candidates to realize that image through action. In our broader work, we are left with two core questions. First, how do teacher educators and teacher candidates develop through these practice-focused opportunities? In the case of teacher candidates, how do those experiences result in developing skilled practice for the work in secondary school classrooms? That gives rise to a second question, which we discuss in the next section: What is the activity of secondary mathematics teacher in school classrooms and how does the opportunities to investigate and define that activity impact the design of a responsive pedagogy of practice?

Identifying the Entailments of Teaching in Teacher Candidates' Secondary Mathematics Classrooms

Here we focus on three main definitions that emerged from the sum of our analyses of the three teacher candidates' work in secondary mathematics classrooms in addition to their interviews conducted by the first author. We base these claims on a total of 13 lessons across four middle or high school classrooms, in addition to multiple interviews with each teacher candidate. As a result, we make progress—albeit a start—on identifying and defining the activity of secondary mathematics teaching in school classrooms. It is this aspect of our research that allows us to reflect on how the activity of teaching was defined in our responsive pedagogy of practice, and how future designs can better account for what it is that teachers are called to do in school settings. First, a central aspect of the work of secondary mathematics teaching revolves around preparing students to be able to carry out a range of procedures. Second, teacher candidates commonly felt the pressure of maintaining a certain pace in terms of the content that they led students through, which led to many efforts toward more time-efficient pedagogical decisions, such as moments in which the teacher candidate would do more of the telling or leading or the way that mathematical errors and questions were typically handled. Finally, assuming those two conditions were met, a third way that teaching was defined was as a space to try out other things instructionally. For the three teacher candidates, this led to interesting instructional moments that serve as exceptions to the norm. Our data show, though, that teacher candidates often could not capitalize on this newfound space because of a lack of clarity on alternative pedagogical tools—either complete IAs,

individual episodes, practices, or moves— and, instead, a maintenance of the instructional work that persisted the in the classroom otherwise.

A focus on mathematical procedures. Without question, the most prevalent aspect of teacher candidates' work, and of their comments about what they were expected to do, related to a focus on mathematical procedures. These expectations came from cooperating teachers, other teachers, administrators, and even students and were reified in items such as common assessments given by departments. These expectations were also made apparent through the instructional work that was prevalent across the lessons we analyzed. Across all thirteen lessons from the three teacher candidates from the fall and spring, 125 of 208 episodes fell into a grouping we will refer to as "going over mathematics problems"—demonstrating how to solve a problem, assigning a problem to students, or eliciting solutions to problems. While our analyses are not designed to look at those counts rigorously[12], it is apparent that a considerable portion of teacher candidates' work consists of going over mathematics problems. We discuss the work of assigning problems as part of our second definition. Here we discuss the trends across far and away the most frequently occurring (82 of 208 episodes total) single episode type—eliciting solutions to problems.

[12] One consideration is that episodes could have been "chunked" in a biased way. For instance, in the event of a problem with multiple parts or a sequence of problems, individual episode instances were created, which could lead to an inflated total. In addition to work on mathematics problems (specifically ones that emphasize procedures) was a stated central goals, the sum of lengths of instances related to "going over a problem" accounts for more than half of the total instructional time, providing an additional rationale for considering this work as central to secondary mathematics teaching.

Our analyses allowed us to take a more detailed look at how these episodes were carried out by teacher candidates. From the perspective of looking at instructional practices, teacher candidates were commonly eliciting student contributions (and, about half of the time, eliciting contributions from multiple students), representing mathematical ideas publicly, and highlighting mathematical ideas through their own revoicing or comments. In about half of the "eliciting solutions" episodes, teacher candidates asked follow-up questions of students regarding their ideas. From that standpoint, with multiple students participating and ideas being highlighted and recorded, it would be reasonable to believe that these episodes were quite dynamic, like those that were approximated in the strings IA. However, with an even closer look, our analyses tell a different story of the prevalent patterns of instruction around going over problems. Figure 9 puts forth a representative example of an "eliciting solutions" episode from each teacher candidate. In all of these instances, students had time to complete the problem prior to it being discussed among the whole class. Across these examples, we see problems that are not complex and, furthermore, these are all problems that were some sort of a review— either content from previous lessons (or previous years in the case of calculating a median) or problems that are similar to something that was just demonstrated by the teacher candidate. This was the case for nearly all of the problems done across the lessons.

The transcript excerpts in Figure 9 illustrate the kinds of participation structures—routines of teacher and student interaction—that were regularly used by teacher candidates across these episodes. In characterizing these, we found that none of them make a student's contribution an object for discussion. Instead, the teacher candidate

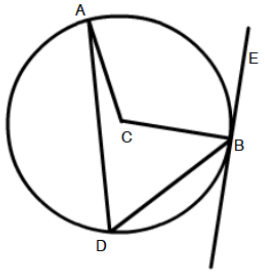
Teacher Candidate and Mathematics Problem	Transcript Excerpt
Casey Students are asked to find the median of a set of bowling scores—104, 117, 104, 136, 189, 109, 113, and 104	*After instructing a student to list out the scores in order from least to greatest* Casey: OK, yes, we have them all from least to greatest. Student 1: Can I give you the median? Casey: Just a second. So the median, if we go back to that definition, is going to be the number in the middle once it's ordered from least to greatest (pauses). So [Student 1], why don't you help me out, how do I find the median here? Student 1: Well there is no middle number so you have to divide – Casey: What do you mean there's no middle? Student 1: It's not an odd number, so – Casey: OK, so let me just get to the middle first. We have eight total so we have (counting from left to right) one, two, three, four … we have four [on the left] and four [on the right]. So there's nothing right in the middle, that's what you mean? Student 1: Yeah. Casey: So we've found the middle and how do we find the median exactly? Yeah, [Student 2]. Student 2: If I remember right, don't you add them up and divide by two? Casey: Yeah, we're going to average these two numbers (continues on to talk through computing that average, while recording on the board)
Georgia Given that C is the center and BE is tangent, what is m<CBE?  	*Transitioning from another question in reference to the same figure* Georgia: What about angle CBE? (waits) [Student 1], what'd you get? Student 1: I think it might be 90 degrees. Georgia: Why do you think it might be 90 degrees? Student 1: Because it says that BE is tangent (Georgia records "tangent" next to segment BE). And since CB is … kind of like a radius … so it would be a right angle. Georgia: You guys agree? I thought that was going to be tricky – hoping you would look at that arc length. You guys are just like, "we have a radius and a tangent, that's 90." Cool!
Susan Simplify: $(3xy)^2$	Susan: [Student 1], can you help me with this first one? Student 1: You would write out $3^2x^2y^2$. Susan: (recording student solution) Uh-huh. Yep. Student 1: And that would be $9x^2y^2$. Susan: (recording) Yep, $9x^2y^2$. (referring back to recording) So [Student 1] squared every one of the factors that was in here. She squared the three, she squared the x, and she squared the y (pauses). [Student 2], can you help me with the next one?

Figure 9. Representative transcript excerpts from eliciting solution episodes

simply elicits the components of a solution in order to arrive at a conclusion. Further, these components often come from one student in a back-and-forth with the teacher candidate, while the rest of the students in the class are presumably expected to follow along. Additional student contributions are not always elicited within the discussion around a single problem and when they are it is often to pick up where the previous student left off. This is in contrast to moves, such as discursive moves, that orient students to one another and foster discussion that does not necessarily go through the teacher candidate. It is worth noting, however, that if students' contributions are largely short and computational or procedural, they are not necessarily worth dwelling on in discussion or having other students reason about.

Time-efficient instruction. While it is common to disparage a nearly exclusive focus on mathematical procedures in a mathematics class, focusing on procedures is not inherently bad teaching. The strings activity provides a structure of how a procedure can be talked about in a more rewarding and powerful way. Even outside of that structure, questions that press students to justify, explain, and evaluate the utility of procedures would bolster such conversations. However, that is rarely what we found to be happening in these episodes. Instead, a second definition of the activity of teaching seems to influence just how these episodes and practices were carried out—a focus on coverage and the efficient use of time. The teacher candidates entered into settings where groups of teachers based their actions on a scope-and-sequence document that was created in-house or provided by another entity (e.g., district office). These documents tended to align with the textbook used by the teachers of a given course. These expectations came down to teacher candidates from their cooperating teachers and we continually referenced as

determining what could and could not be done in the classroom. As might be expected based on the first definition we describe above, the bulk of what was outlined in these documents focused on mathematical procedures. Casey expressed this the best, saying in her post-program interview, "I felt like I had to present content [what she later clarified as a set of procedures] to students so they had that exposed to them … So a successful student can memorize the procedures and use the correctly" (Casey, Post-Program Interview, 6/12/2013). When asked what determined those goals (and, specifically, that list of procedures), Casey referenced, "the outline from my [cooperating teacher] … [the teachers in the department] did a scope-and-sequence document that they connected to standards from the Common Core. So basically it was the outline and we followed the textbook."

In reference to patterns like those highlighted in Figure 9, Susan shared that, "I felt that doing it [that way] would be a way to get the information across to [students] in a way that was clear and give them opportunities to practice with immediate feedback on if they were able to do it right" (Susan, Post-Lesson Interview, 5/28/2013). In this quote, Susan expresses a commitment to her students having opportunities to hear and experience information about mathematics clearly, to have practice with relevant procedures, and to get immediate feedback. At one level, these are laudable goals for a teacher. However, the way in which they are realized instructionally—while typical of instruction in the U.S. (Stigler & Hiebert, 1999)—do not provide students with the opportunity to make sense of the mathematics and develop understanding. Instead, there was a sense in teacher candidates' instruction (though mostly with Casey and Susan, a difference we describe more with our third definition) that the goal was not only to

emphasize procedures, but also to do so in a way that maximized efficiency and accuracy. In fact, Susan shared that, "when you've got kids for 45 minutes every day, you want to use their time well, it might be most productive to just tell them [an answer or a strategy] and go on with it" (Susan, Post-Program Interview, 6/10/2013). From our analyses, we see this expectation having had an impact on much of what teacher candidates did, though we highlight two central ones here—the work of assigning and transitioning between problems and the way in which teacher candidates' addressed errors and questions from students.

Assigning problems and transitioning between problems. One trend across the data from student teaching placements (as compared to the work across the three IAs) is that there were far fewer episodes during which a problem was assigned and students worked on it individually or in small groups than episodes where solutions to problems were elicited and discussed. Instead, teacher candidates would often introduce a problem then immediately discuss its solution (often by leading students through the solution process) or would elicit solutions to a sequence of problems after one episode during which students could work on them individually. We see this as an implication of efforts to be more efficient with time and to place more value in the number of problems that are reviewed in a given class. One concern with this trend is that students could have less access to or opportunity to grapple with problems (and the ensuing discussion) without that time to work on the problems themselves.

We also found that the work of assigning that was done was largely focused on participatory aspects of working individually or with a partner. We found that problems were not often assigned with the kind of narration (or attention to launching) we saw

teacher candidates being supported in enacting across the IAs. Teacher candidates seldom unpacked the mathematics of a problem before sending students off to work on it. In sum, we did not see ways in which students were being provided access or being oriented to the mathematics at hand, even though that was a point of emphasis across the set of IAs. Even we, as researchers, had a difficult time determining what the goal of a problem or set of problems was, aside from them being more examples of a particular problem type or using a given procedure. Given the common emphasis on covering a particular amount of content, simply moving through a set of problems seemed to be the primary objective.

Addressing error and questions during instruction. A second area of teaching on which we see this definition of teaching having an impact is how errors and questions were addressed in the classroom. In general, this label (an instructional practice) struck us due to its moderate frequency across the classroom lessons and its lack of frequency from across the three IAs. We take this to mean that our approximations of practice did not account for this aspect of the work, leaving teacher candidates without the pedagogical tools to addressing mathematical errors and student questions as they emerged in instruction. While these practices are not exclusively tied to particular instructional episodes, nor are they always predictable, they are certainly part of the dynamic and interactive work of mathematics teaching, especially teaching that looks to make student reasoning and sense making central.

We identified patterns by looking across all instances in which "addressing error" or "taking up student question" was coded (as an instructional practice) and looking at the ways in which the error or question was responded to once it emerged. In our data we saw a recurring set of routines in which the teacher candidate would quickly resolve the

error or question through correcting or answering it themselves, prompting its resolution, or moving on to a different student. For example, in a lesson from Casey during her fall student teaching placement, she asked for a student to come to the board to work through a warm up problem that drew on the rules for the order of operations. Specifically, students were to simplify the expression, $8(4 – 5)^3$. As a student walked to the front of the room, Casey reminds the student that "we want to know each step" and reminds the class to be listening to the student. At the board, the student starts to explain her process, with Casey standing nearby:

> Student 1: You have to do the parentheses first so five minus four is one (*student moves to record on the board*) --
>
> Casey: Let me stop you there. So I heard you say five minus four is one. So what is four minus five?
>
> Student 1: Oops.
>
> Casey: Negative one. Yeah. Alright.
>
> Student 1: OK (*records* $8(-1)^3$ *on board)*. So eight times negative one is negative eight (*records on the board*). And --
>
> Casey: So did you do the multiplication next or … what did you do after the parentheses?
>
> Student 1: (*slowly erases her latest recording*) After parentheses … exponents. So neg –
>
> Casey: OK, so after the parentheses is exponents. And what is negative one to the third power?
>
> Student 1: One?
>
> Casey: Is it positive or negative one? (*turns to class; students in class provide mixed responses of "positive" and "negative"*). OK (*grabs pen from student*), let me just point something out –

At this point, Casey began to show students how to expand $(-1)^3$ as (-1)(-1)(-1) to make sense of how it is negative one. She also contrasts this with -1^3, which is negative one for all values of the exponent while the other expression is only negative one for exponents that are odd numbers. In this example, we see how quickly Casey corrects errors (or even

possible errors) through quickly correcting them herself or through asking a leading question. In this instance she stood only a few feet from the student presenting, adding to the immediacy of the interjections and even enabling Casey to quickly take the pen. We saw these kinds of behaviors in most instances when errors and questions emerged across lessons. As with the discrepancies with "assigning problems or prompts", we see the prevalent actions taken by teacher candidates in response to student errors and questions as a sign of an effort to continue making efficient progress through a lesson, even though the common ways of dealing with errors and questions often led to more confusion—what Georgia called in her post-program interview, "chaos". Furthermore, these actions are taken in the midst of rhetorical statements made by teachers both in and out of the classroom that mistakes and questions are valuable and opportunities to develop.

Available space for novel instructional work. While demands of coverage and a focus on procedures impacted much of teacher candidates' work in the classroom and was evident in their own reflections, there was also a sense that being a teacher candidate in a placement allows for some openness to try new things instructionally. Across our data, we saw the root of this space taking three different forms. First, Casey shared that her cooperating teacher was cognizant that what is often done in teacher education is different from the kind of teaching work she sees herself doing:

> Casey: [My cooperating teacher] is aware of broad things that I am expected to do [from the program]. I'm not sure how much she knows about what we've done here. But she has said that she is a very traditional teacher – I mean that's how she learned math and that's how she teaches it.
>
> MPC: So, could describe what [traditional teaching] means in two sentences?
>
> Casey: Lecture, examples, and having the students work through problems … She does ask questions to the class sometimes,

> but they're usually very low-level, like a simple procedural question, like "What do I get when I distribute the four?" or something like that. It's usually just open to a volunteer and sometimes she goes through and asks everyone a part of a question to get people involved. So it's very focused on procedural stuff.

While we do not subscribe to the view that a single teacher has that much autonomy over the "preferred" methods of teaching, what Casey describes is consistent with and indicative of the activity of secondary mathematics teaching that we began to define through our analyses. However, what we see from her comments is that there is some agreement (sometimes unspoken) that teacher candidates have the opportunity to incorporate novel instructional approaches in the classroom. For Susan, space was created to make some changes to her cooperating teachers' classroom (specifically incorporating more small group work) during the early part of her full-time, middle school practicum. This partly stemmed from the "agreement" highlighted above that teacher candidates be able to bring novel ideas from their teacher education program. However another notable factor was that by the time Susan began teaching, students in this middle school had already taken their state tests. Susan and her cooperating teacher viewed this as the opening for Susan to "try new things"—an opening that might not have been available even a few weeks earlier.

Finally, in the midst of expectations to prepare students for procedurally focused department-level exams given at the school, Georgia felt as though her cooperating teacher had been making changes to his own instruction in the context of those assessments to focus on having more discussion in class and building on students' ideas. Allowing students some opportunity to reason about

mathematical relationships (not just procedures) was seen as a way to still prepare students for the assessments while also being more responsive to students' needs and to expectations coming from documents such as the *Common Core State Standards for Mathematics.* As a result, Georgia claimed she felt supported to teach in ways that were consistent with what was emphasized in the teacher education program. While Georgia would give students problems to do in class and then discuss solution strategies, she also had time for students to reason and conjecture about geometric relationships, which she called "time well spent". It is interesting to think about the explicit mention of "time" in this way in the midst of apparent concerns about efficiency and the inherent lack of time. Across our data we saw examples of instruction that served as exceptions to the prevalent work we describe in sections above and serving as the way in which teacher candidates found to capitalize on these available moments of time for more novel instructional work in the classroom. Some of these moments gave rise to novel episode types (what we called "eliciting ideas") while others provided examples of similar episodes ("eliciting solutions") and practices ("addressing errors") that played out in potentially more ambitious ways.

A novel type of episode: Eliciting ideas. In the midst of a majority of time dedicated to work assigning and eliciting solutions to procedural problems, teacher candidates did have some opportunities to elicit ideas to prompts. While the frequency of these episodes was relatively low, these episodes were long and, in sum, comprised about one-fifth of the time across the lessons. We found these episodes of interest because they are potential spaces for more ambitious

mathematics discussion. However, they also draw on a complex constellation of practices and moves and something for which teacher candidates would need support in developing skill. One example of these instances comes from Susan's lesson during which she presented students with a stem-and-leaf plot (see Figure 10) and prompted them to list what they can say about the data set based on the display. The goal was for students to consider the utility of a stem-and-leaf plot—namely the relevant information about a data set, such as measures of center and spread. Students were provided time to record their ideas in writing and share them with a partner. The transcript below highlights part of the discussion that followed after students shared with a partner as Susan facilitated a whole class discussion:

Susan: I saw a lot of things written down on people's papers. I saw that there were more numbers in the 80s than there were in anything else. I saw somebody say that the … lowest number was 58 and the highest number was 98. Is the highest and lowest value something that you're always going to be able to tell from a stem-and-leaf plot? Or are there sometimes where you won't know what that is? (pauses) [Student 1]?

Student 1: If it's an ordered, completed stem-and-leaf plot, you'll always be able to tell. But if it's not ordered it won't be that quick.

Susan: Tell me more about it not being quick if it's not ordered.

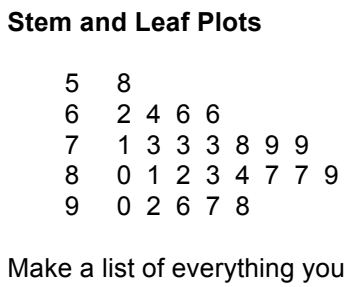


Figure 10. The data display and prompt given to students during Susan's second lesson

Student 1: You'd be able to tell from a raw set of data that's not ordered but it will take a lot longer. This [display] has it ordered lowest to highest on each row. So the highest is on the right of the bottom row and the lowest is the left at the top. If it were unordered you wouldn't know where the highest would be.

Susan: Can somebody restate what [Student 1] said? [Student 2]?

Student 2: In an ordered stem-and-leaf plot, the highest value will always be on the bottom row farthest to the right and the lowest value will always be on the top row and the farthest to the left. But in an unordered one you'd have to figure out where the highest one actually was.

Susan: (pauses) So [Student 2], it sounds like you're saying you *will* always be able to tell the lowest number and the highest number.

Student 2: Yeah.

Susan: I think she's right! So the highest and lowest value is something that you can always tell from a stem-and-leaf plot. If you have the highest value and the lowest value and you do a little bit of math, what's something else that you can tell?

After this discussion the class moved on to talk about the range of the data and, eventually, the median. Susan later recorded these features on a poster that summarized what a stem-and-leaf plot can tell the reader, and what kinds of data sets stem-and-leaf plots are useful for displaying.

In this example we see a discussion of a data display with a focus on evaluating the information that can be determined from it and, eventually, its optimal use. In the excerpt we see Susan build from her monitoring of students' work, posing a question to the class about the ability to always find the highest and lowest value of a data set in a stem-and-leaf plot. A student offers a conditional statement, as well as its inverse ("If it's an ordered, completed stem-and-leaf plot, you'll always be able to tell. But if it's not ordered it won't be that quick."). While the student raises an interesting point, Susan's practices of questioning and dwelling upon the latter idea of the "unordered" stem-and-

leaf plot turns the focus to the construction of a stem-and-leaf plot and not about the information that can be gleaned from one. This idea is what Susan moved to have another student restate. Susan also appears to need to make a jump back to the original goal through her affirmation that the highest and lowest value can be found in a stem-and-leaf plot and her directed question to move on to a discussion about the range. These distinctions are subtle, and we point them out not to criticize Susan's performance, but more to acknowledge the complexity of facilitating mathematical discussions and the affordances and drawbacks of Susan's choice of lifting a particular phrase in Student 1's contribution.

Not all student contributions have the same role in making progress toward a mathematical goal. In turn, the instructional moves that get utilized by teacher candidates (especially ones coming from their teacher education program that are deemed to be part of "ambitious and equitable" instruction) are not always useful. Teacher candidates make judgments in the moment as to whether to use particular moves. In turn, they must be supported with not only the tools (such as a discursive move or the episode of structuring a discussion around "eliciting ideas") but also the role they play in making progress on the important mathematics of a classroom. Teacher candidates must be supported in noticing these moments and provided with opportunities to practice interpreting students' contributions and lifting different pieces of a contribution relative to a particular mathematical goal.

Alternatives for going over mathematics problems. In addition to how mathematics problems were typically assigned and discussed in student teaching placements, most of the problems given were an application of previously provided

procedures. Georgia provided one of the few exceptions to that trend by providing a problem for which there was not a prescribed (or even implied) procedure. In the second lesson of Georgia's spring data set, she presented students with a problem that asked them to find the area of shaded region bounded by a central angle within a circle (see Figure 11). This came after a discussion earlier in the lesson during which Georgia led students through an activity of conceptualizing a proof of the formula for the area of a circle. This was followed by the defining of two terms—"sector" (such as the shaded region in Figure 11) and "arc length". Students were then given time to work on the problem below. The following transcript highlights the discussion that ensued after Georgia asked for solutions:

Student 1: Our group got it because 120 is one-third of 360 and when you find the area of a circle you are finding 360 degrees. So when you are finding 120 you are just finding one-third of the circle. So we just found the area of the circle –

Student 2: (jumping in) Yeah, so it would just be pi times 81 and then divide that by three.

Georgia: (recording the idea on the board) Where did the 81 come from?

Student 2: That's just nine squared

Georgia: And what is this finding?

Find the area of the shaded region if the radius of the circle is 9

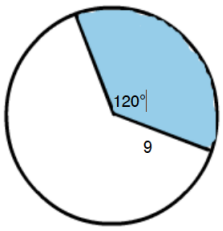


Figure 11. Mathematics problem given to students in Georgia's second lesson

Student 2: The area of the sector (Georgia writes "area of the sector" next to the solution)

Georgia: (after a pause) [Student 3], I saw your hand come up. What were you thinking about?

Student 3: I just had this little formula for it. πr^2 in parentheses, times whatever the central angle is, over 360 (Georgia records this formula on board)

Georgia: Nice! Way to take us to the general, [Student 3]. What do you guys think? Is that going to work every time? [Student 4], what do you think?

Student 4: Yeah.

In this instance, we see a more dynamic discussion that involves multiple students and welcomes the emergence of a new idea. Students had not been introduced to a formula for finding the area of a sector (as might be done upfront in many textbooks) so giving this problem not as an application of a known process allowed for actual reasoning about the solution and the emergence of an accessible generalization. However, the problem itself is essentially the same problem that would otherwise be given to students after being introduced to such a formula. Future work would need to consider how a pedagogical tool that highlights giving secondary mathematics students accessible problems *prior to* a formal discussion of an efficient strategy is a usable structure in the classroom. We understand why the prevalent pattern of giving problems that apply known procedures occurs—a perceived benefit in the name of efficiency.

Such a tool at the level of an episode would need to be further articulated in order to highlight the kinds of instructional practices and moves that would carry out the episode in the more ambitious and equitable way possible. While Georgia does ask questions on the initial students' ideas, those questions focus on finding the area of the circle with a radius of nine, and on repeating the formal name of what is being calculated

(i.e., "the area of the sector"). What Georgia does not unpack nor highlight through her questioning is the idea of taking one-third of the total area. As in the instance with Susan and the stem-and-leaf plots, the moves Georgia uses in this instance come off as ambitious (e.g., questioning students' contributions) but miss the mark when it comes to the core mathematics of the problem. We also do not know what would have happened in this instance had Student 3 not contributed the general formula. Was the goal to elicit or co-construct a general formula? Would Georgia have just moved to the next problem once the solution was stated? Would she have involved any other students after the contribution from Student 2? Even once the idea was offered, Georgia does little to engage other students with the idea, other than broadcasting a question as to whether it is "going to work every time". If teacher candidates are going to have the opportunity to assign and elicit solutions and strategies to problems that do not have prescribed procedures, they need to have ways to carrying that work out with students that targets the mathematical goal and makes the mathematics accessible to all students through discourse. While similar expectations of questioning and goal-directedness come with problems that employ a known procedure, the example above highlights an opportunity to leverage through a responsive pedagogy of practice.

Alternatives to addressing mathematical errors. A final way in which we saw the instructional space in the mathematics classroom leveraged was with the way in which mathematical errors were used as opportunities for further discussion. This was in contrast to the prevailing work of quickly remediating or moving on from errors that allowed for more efficient instruction. Like with the example above, a primary example of this comes from Georgia's instruction. Given her sentiment that her cooperating

teacher was trying to make similar shifts to his own practice, this is not surprising. However, it is important to note that Georgia and her cooperating teacher were still held to larger expectations of preparing students for tests that were largely procedural, meaning that these exceptions are not wholly unrealistic in other settings.

In this example, Georgia was in the midst of eliciting solutions to a multi-step problem related to the diagram[13] seen on the left in Figure 12. One of the problems had students making sense of the length DF. One student began offering a possible theorem to utilize that would rely on both BE and AF being chords (as opposed to AF extending outside of the circle), which Georgia illustrated with an image like the one at the right side of Figure 12. The student then recanted, saying:

Student 1: Oh! That's a different one – never mind.

Georgia: Well, that's nice – Thinking about the theorems we do have available to us in this unit. [Student 1], why do you think that might not be as helpful as you originally thought?

Student 1: Because the bottom triangle goes out of the circle. It's not inscribed in the circle.

Georgia: Yeah, it's outside the circle. Do we have any theorems that help us see relationships when there are parts that go outside the circle?

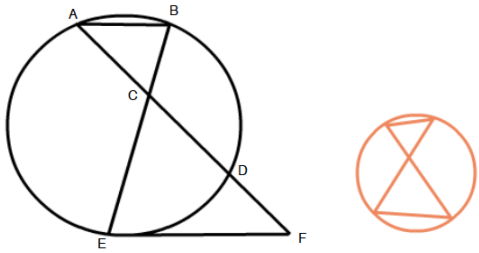


Figure 12. Problem discussed in Georgia's first lesson and an idea from a student

[13] The diagram given to students had more information, such as arc and segment lengths and angle measures. We omit those here for simplicity and provide the modified diagram for context.

In this instance we see Georgia interpreting a students' conjecture in the form of a visual representation on the board. The distinction between the two figures becomes an object for comparing the student's idea to the problem at hand and likely caused the identification of the error. Across our experiences working with teacher candidates (including across our data), we regularly see instances that would end with Georgia's first statement (i.e., "Well that's nice …")—commending a student for being willing to share an incomplete idea. Her additional move made the incomplete idea more transparent, and opened up an opportunity to compare and contrast relevant approaches given the situation. Georgia attributed this move to open up a space for discussion to comfort in "stopping instruction". What is interesting about that comment is that it implies that "instruction" is something to be stopped, though that is consistent with the view of teaching as something that is efficient and has some sort of momentum. Even in the case of Georgia and the classroom in which she was working, these extended opportunities are seen as some form of a deviation, possibly because of the extra time they might take. In this example we see Georgia use a move that prompts an additional turn of talk from the student. This move plays a role in an opportunity for error analysis. Again, this work comes in the context of eliciting solutions to a problem—specifically one that that was meant to draw on a known procedure (though not one that was specifically named for the problem). Students in the class not quickly offering that procedure and a student offering, with uncertainty, an incorrect strategy cannot be predicted. However a response such as the one used by Georgia serves as a tool to break the normal patterns of addressing errors and can serve as a tool to promote through responsive pedagogies of practice—

highlighting the practice of addressing errors and the moves through which a teacher candidate can leverage those events in more ambitious and equitable ways.

Implications in the Design of Responsive Pedagogies of Practice

While we saw instructional episodes, practices, and moves that served as a deviation from the prevailing work and an indication that teacher candidates (and, possibly, classroom teachers) have openings for novel instructional work in the classroom, those opportunities were contingent on particular expectations being met. We capture this idea with this excerpt from Casey's post-program interview, picking up at a point when she was asked about her goals as a teacher moving forward:

> Casey: I want to be the kind of math teacher that is more of a facilitator, that picks awesome tasks for students to work through … [Students] are working in groups and working on mathematical ideas that are targeted to the task. And it all comes together in the end. [laughs]
>
> MPC: Just like that? [laughs] … So do you have an idea of how you go about doing any of that?
>
> Casey: Umm … no. Well, kind of … somewhat. I mean I'm assuming the requirements are to address the state standards, right? So as long as you're going with that and you can justify that's what you're doing in your classroom, then I think it's pretty open as to how you do it.

Continued press for specifics on the kind of work she aimed to do instructionally and how she would negotiate those expectations lead to similar uncertainly, summed up with the comment, "I don't know how a teacher would do it necessarily, but I know that it's possible" (Casey, Post-Program Interview, 6/12/2013). There are two things we look to raise from comments such as these.

First, in spite of aspirations to teach in more ambitious and equitable ways (such as using "awesome" tasks, structuring the classroom around group work on those tasks,

and having it "come together" toward a mathematical goal), the teacher candidates we worked with were beholden to expectations to cover a wide base of discrete, mostly procedural, mathematical ideas in an efficient way. These expectations stem from an interpretation of new policies (e.g., the Common Core State Standards for Mathematics) merging with previous iterations of state standards and prevalent instructional practices. A broader interpretation of recent calls from research and policy around mathematics instruction—one that calls for more authentic mathematics work and equitable opportunities for each student—must be negotiated with the normative expectations and resulting instructional work. Teacher candidates (who then become new teachers in their own classrooms) must fulfill their role of emphasizing mathematical procedures and providing instruction and feedback efficiently. Many of the instructional episodes, practices, and moves we saw teacher candidates doing are the things that get that work done—thus explaining their resilience and stability. For teacher educators to construct responsive pedagogies of practice these normative episodes, practices, and moves—deployed in response to particular expectations—need to be better understood. The research that we outline in this paper begins to uncover this and, more importantly, begins to coordinate these norms with other episodes, practices, and moves that fulfill those expectations while offering tools for more ambitious and equitable instruction. A focus on solving mathematics problems—especially ones that are not individually considered to be cognitively demanding—is often attributed to mathematics teaching that is focused on mathematics procedures and ultimately quite "traditional". In response, focusing on doing such instructional work may not seem compelling by teacher educators and may be seen as reinforcing the status quo in schools. However, a responsive

pedagogy of practice would look to foreground this work and to provide teacher candidates with pedagogical tools that enable them to do the work, and do it in increasingly ambitious and equitable ways.

The second point we raise is that, once that instructional space was made available for teacher candidates through satisfying other expectations, they did not necessarily have the pedagogical tools to capitalize on it. This manifests itself through uncertainty in talking about teaching (such as with Casey) or instructional episodes that have many of the attributes of what would be considered more ambitious and equitable, thought that are still carried out with many of the prevalent practices and moves from the rest of their instruction in their student teaching placements. The design of responsive pedagogies of practice in secondary mathematics teacher education must look at these "openings" as work to foreground pedagogical tools that enable teacher candidates to leverage those opportunities. However, those pedagogical tools must also be commensurate with their daily work in school classrooms. For example, efforts to foster more thorough assigning of problems, more productive handling of errors, and more rich discussions of prompts must still honor the need for coverage and efficiency, so long as those expectations persist. This also means that the pedagogical tools that teacher candidates begin to develop cannot be things that, when used initially, are clumsy and arduous. We have wondered in our own work as teacher educators if stakeholders' aversion to more ambitious and equitable teaching practices is not because of the nature of the strategy, but because of the time it takes a newcomer to implement it.

Limitations and Future Directions

In this work, we have set out to begin to define the activity of secondary mathematics teaching in school classrooms. However, we did so relying on data from a small cohort of teacher candidates. Furthermore, we did not leverage the perspectives of others in the settings of teaching and teacher education—classroom teachers, administrators, parents, and students. As a result, we are not making general claims about the activity of secondary mathematics teaching. However, the range of examples we had even in this small set of participants make for interesting opportunities to consider what might be entailed in the work. The examples also show that the perceived constraints on teacher candidates and their ability to do more ambitious and equitable teaching as often promoted in teacher education are not always a product of a reluctant cooperating teacher. Even in the case of Georgia and her cooperating teacher, expectations from the department, school, and in response to policy documents shaped what occurred "on the ground". Ultimately, they were beholden to many of the same expectations that shaped Casey and Susan's experiences. While it would be easy to say that Georgia had a more accommodating student teaching placement in the spring, we see this example as not illustrating what *would* happen in a classroom that is more aligned with the teacher education program, but what *could* happen in a mathematics classroom that operates within a context of particular goals, expectations, and tools. Furthermore, we use this example to then shape what should be leveraged through a responsive pedagogy of practice in secondary mathematics teacher education.

The fact that we have only just started a process of identifying and understanding motivates continued work in this area—including work that begins to address the

limitations of our work described here. For example, beyond increasing the sample size, this work would be strengthened by accounting for the perspectives of other stakeholders—teachers, principals, students, parents—who are central in the teacher education and teaching process. This would lead to more responsive work and more detailed and accurate definitions of the activity of secondary mathematics teaching. Activity theory calls for analyses that take into account the history and multiple voices of a system, to better represent and explicate the goals, roles, rules, and communities from a variety of impactful perspectives.

Another resource from activity theory was the analytic tools that provided us a way to specify the work that teacher candidates did in school classrooms and allowed us to identify the component parts of the work of secondary mathematics teaching, even across different sites. We propose that such an analytic framework can serve to clarify the problematic issues of "grain size" in recent talk about the work of teaching (Boerst et al., 2011). We struggle with how terms such as "practice" get used in recent work in the field—meaning anything from leading a whole class discussion to asking a specific question to a student. We hope that advancing these theoretical and analytical tools emerging from activity theory serve the field in this time of focus on specifying the work of mathematics teaching and considering the development of newcomers for the work.

Finally, these kinds of analyses also provide a novel way to evaluate the development of teacher candidates and the effectiveness of the teacher education programs that developed them. To assert that a teacher candidate has developed skilled practice as an ambitious mathematics teacher based on their use of a particular activity or episode structure, the kinds of questions they ask, or the materials that they use isolates

all of those pedagogical tools from the activity itself. Conversely, saying that a teacher candidate has not developed skill because they do not do particular things focused on in a teacher education program ignores considering whether or not those pedagogical tools have a place in the work of teaching as it is defined in those school settings. If anything, teacher education programs should be evaluated on their responsiveness and their evolving efforts to prepare teacher candidates to do the work of teaching in schools early in their careers while also mobilizing a bounded set of tools that allow for increasing ambitious and equitable mathematics instruction.

Conclusion

While the field is making progress on how to characterize and prepare ambitious teachers through pedagogies of practice, we will not have a full vision of how to support teacher candidates without a closer look at the tools and practices that one draws upon in the activity of teaching secondary mathematics in schools. This should still be done with an eye on increasingly ambitious and equitable mathematics teaching, however those strides must be made from the starting point provided in schools. The design of practice-focused pedagogies is a novel idea for most teacher educators, though being attentive to the work that teacher candidates do in school settings is an even more unfamiliar terrain. We see our design-based research efforts—as mathematics teacher educators and researchers—as making progress on developing a sense of the design and use of responsive pedagogies of practice in secondary mathematics teacher education. We hope for the findings from this study to serve as a way to focus emerging efforts around designing and using pedagogies of practice and to develop a common language and scope among secondary mathematics teacher educators.

Supporting Secondary Mathematics Teachers in Attending to Errors Through a Responsive Pedagogy of Practice

Introduction

To support the development of teacher candidates[14] as *ambitious* and *equitable* instructors (Jackson & Cobb, 2010; Newmann & Associates, 1996), there has been increasing discourse in the field of mathematics teacher education around the design and implementation of *pedagogies of practice* (Kazemi, Lampert, & Franke, 2009; McDonald, Kazemi, & Kavanagh, 2013). Such teacher education pedagogies are situated in the work of teaching and look to have teacher candidates actually enact teaching to develop skill with the practices and tools of the complex and demanding work of the classroom (Grossman, Hammerness, & McDonald, 2009). Pedagogies of practice are seen as a way to mobilize the emerging content of mathematics teacher education in the form of *core practices*, which have been building from the large body of work of research on mathematics teaching (Ball & Forzani, 2009).

The focus on core practices and enactments of teaching in teacher education has gained popularity in the field, but is also an area that is in need of research and development, especially around how such pedagogies are designed and used in secondary mathematics teacher education. There is also an additional need to consider how teacher education designs are *responsive* to the settings in which teacher candidates work and the work they do in those settings. In line with broader recommendations for teacher professional development (Cobb, Zhao, & Dean, 2009; Kazemi & Hubbard, 2008),

---

[14] We use the term "teacher candidate" to refer to individuals who are enrolled and progressing through a teacher education program. We will use this term consistently throughout the article, recognizing that others use terms such as "preservice teacher", "novice teacher", and "student teacher".

responsive pedagogies of practice must support teacher candidates developing pedagogical tools (and the opportunity to enact teaching with them). Such tools must connect to the instructional work that takes place in classrooms, as well as teacher candidates' efforts to teach in increasingly ambitious and equitable ways.

In this article, we highlight how we have taken up and conceptualized a notion of responsive pedagogies of practice in a secondary mathematics teacher education program. Based on design-based research (Design-Based Research Collective [DBRC], 2003; Edelson, 2002) on the teacher education design, we highlight emerging findings from our work using an example focused on the instructional practice of addressing mathematical errors and student questions. We discuss how the work with teacher candidates foregrounds not only beneficial mathematical content and practice for students, but also the mathematical skills and pedagogical tools for teacher candidate development. In sharing the example, we aim to provide readers with a specific tool for their own practice as teacher educators, while also providing broader design considerations to inform ongoing development within a range of contexts.

Pedagogies of Practice in Teacher Education

Research on teaching and teacher education is predominantly situated in paradigms focused on teachers' knowledge, beliefs, and identity (Grossman & McDonald, 2008; Skott, Van Zoest, Gellert, 2013). However, teaching is a culturally defined activity and one develops skill in such work through participation and through using meaningful tools (Rogoff, 2003; Stigler & Hiebert, 1999; Wenger, 1998). Viewing teacher practice and development from a sociocultural perspective puts forth the notion that teacher candidates must be supported in their preparation to actually do and develop

skill with the work of teaching. The work of Grossman and her colleagues (Grossman & McDonald, 2008; Grossman, Compton, et al., 2009; Grossman, Hammerness, et al., 2009) has highlighted that teacher education design must consist of two main pedagogical features—investigation and enactment. While teacher candidates tend to have a wealth of opportunities to observe, analyze, and reflect on the work of teaching in the context of university courses, they less often have the opportunity to enact the work in settings of reduced complexity.

Resolving a shortage of enactment opportunities is not done through increasing teacher candidates' time in student teaching placements. Instead, Grossman and her colleagues suggest that teacher educators use *approximations of practice*, which may take the form of instructional routines that simplify the work, while still being integral to the core components of teaching. A popular conception of how to frame these approximations of practice comes from Lampert and Graziani (2009) who offer the idea of designing and using *instructional activities* (IAs) in teacher education. IAs serve as containers for the core practices and principles of ambitious teaching and the content knowledge for teaching that teacher candidates need to develop for and be able to use in their work with students in schools (Lampert et al., 2013). IAs structure the relationship between the teacher, students and content by specifying learning goals and how individuals are expected to interact, while still giving teacher candidates the freedom to enact the more contingent and interactive aspects of teaching, mainly around eliciting and responding to students' ideas in the classroom (Kazemi et al., 2009; Lampert et al., 2010). Given their close tie to content the design of an IA must be specified for particular disciplines and even grade bands. To date, there is little work that exists in specifying

appropriate containers of practice, content, and teacher and student development for secondary mathematics. That need serves as part of the motivation for our work as teacher educators and as researchers.

IAs—and the core practices and content that they contain—serve as the focus of teacher development opportunities organized in what McDonald and her colleagues (2013) call a *pedagogy of practice* in teacher education (see Figure 13). This idea is framed as a cycle, though all quadrants of the cycle are focused on practice. We see the complete cycle as important because, through investigating *and* enacting IAs, teacher candidates are able to develop skilled practice through authentic and supported approximations of the work of teaching. In our work, we incorporate activities that fill in each quadrant. Teacher candidates first observe, decompose, and analyze an IA via video, real-time enactment, or a teaching case narrative. This is followed by examining a lesson plan (what we call a protocol) that details the aims of the activity, elaborates teaching routines and practices, and anticipates a range of ways students may respond mathematically in the activity. Teacher candidates then have multiple opportunities to

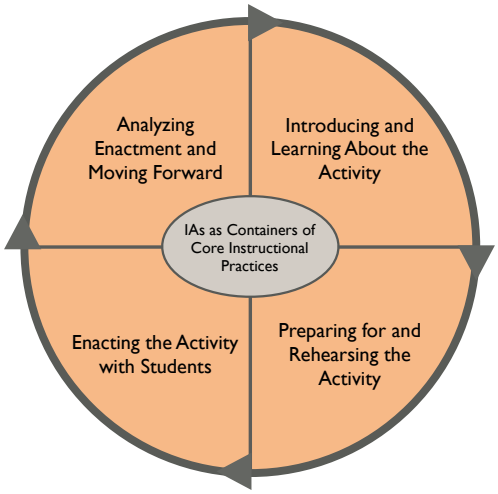


Figure 13. Cycles of investigation and enactment as a framework for a pedagogy of practice in teacher education (adapted from McDonald et al., 2013)

enact the IA—both in the university setting with their peers playing the role of students and with secondary students in a school classroom. During these rehearsals, the teacher educator plays the role of an instructional coach, offering in-the-moment feedback and support. Enactments with secondary students take place in a sort of "lab classroom" arranged through a partner teacher, where they have the opportunity to work with small groups instead of a whole class and have the continued support of teacher educators and the classroom teacher. Finally, video and other records of these enactments serve as supports for analysis and reflection after enactments. A pedagogy of practice offers a promising approach to developing skilled mathematics teachers through opportunities to enact a repertoire of routines and practices that are attributed to more ambitious and equitable instruction and also advance mathematical goals in the classroom.

Meaningful skill development of teacher candidates cannot be fostered without accounting for the sociocultural settings of schools in which teachers do their work (Cobb, Zhao, & Dean, 2009; Gutiérrez & Vossoughi, 2010; Kazemi & Hubbard, 2008). We propose that practice-focused design be better tied to and derived from the activity of teaching in schools—resulting in what we will call *responsive pedagogies of practice.* From this, we can expect that not only will teacher candidates be better supported in developing skill as practitioners, but that they will do so in ways that are enabled in the school settings in which they teach. A notion of responsive teacher education challenges the prevalent view that the disparity between what is promoted in teacher education and what teachers do in schools is the result of barriers that exist in schools and classroom or the lack of development or will among individual teachers. Instead of viewing preferred instructional practices as a complete package that should be moved across settings,

teacher education design needs to be more adaptive to school settings and the notion that the pedagogical tools promoted in teacher education may not be deemed useful or usable in the work of teaching. That work is defined at the intersection of particular expectations, rules, and goals that have been agreed upon and established in schools.

The idea of designing and implementing responsive pedagogies of practice in teacher education is new and emerging, leaving much ground to cover to specify what the work entails. As teacher educators and researchers interested in the notion of responsive pedagogies of practice, we see the context of design as a way to further our understanding—and that of the field—of the goals and design products and processes of such pedagogies. We have used our own efforts as teacher educators as a setting of design-based research (DBRC, 2003; Edelson, 2002). The strength of design-based research as a form of education research lies in the practical lessons offered by the process (Edelson, 2002; Gutiérrez & Penuel, 2014). In the next section, we highlight details of our design-based research around responsive pedagogies of practice in secondary mathematics teacher education.

Details of Our Design-Based Research

Context. Our work is set within a sequence of two, ten-week secondary mathematics methods courses and subsequent student teaching experiences. The courses are part of a small Master's level teacher licensure program at a very high research activity institution as classified by the Carnegie Classification of Institutions of Higher Education. Teacher candidates enter the program with an undergraduate degree in mathematics, 60 hours of practicum experiences, and prerequisite courses in adolescent psychology, mathematical practice and current standards, and educational technology for

science and mathematics. The program spans ten months—from mid-August through mid-June. The program offers teacher candidates two methods courses in which there were field-embedded assignments enacting IAs through a cycle of investigation and enactment. Teacher candidates had opportunities to decompose a lesson, construct planning notes, rehearse in front of peers with coaching, rehearse the lesson with secondary mathematics students, and investigate learning opportunities to reflect on the experiences. Teacher candidates in the program also have two, ten-week student teaching experiences—a part time experience in the fall (concurrent with the first of the two methods courses) and a full time experience during the spring quarter (after the second course).

Our design-based research efforts around responsive pedagogies of practice involve a design team that consists, in part, of mathematics teacher educators, including both authors, at the university. At a given time, one of these individuals serves that the instructor for one of the mathematics methods courses, with the other teacher educators regularly observing the course and participating in the planning and debriefing of the work. The efforts to implement opportunities for enactment in the methods courses have been in place for four years, though our work to systematize and research the efforts are more recent.

Because design-based research involves at least two layers of analysis—ongoing and retrospective—we saw it as a useful series of approaches to develop and refine a design for responsive pedagogies of practice. Further through these analyses we are continually building a set of theories, tools, and practices that account for how teacher candidates participated in pedagogies of practice. Edelson (2002) offers us helpful

structures for building a set of theories related to: (i) the context in which pedagogies take place (including the affordances and constraints), (ii) the desired outcomes from the design, (iii) the inventory of design decisions, and documenting the design process (including roles to be assumed). A second line of inquiry that supplements our design process is an investigation of teacher candidates' practice in their student teaching placements. In an effort to be responsive to the development of teacher candidates and to the work of mathematics teaching as it is defined in schools, it is imperative to analyze and develop understanding of the work in secondary mathematics classrooms. In this paper, we put forth a subset of those considerations. Specifically, we highlight our investigation and developing understanding regarding the following questions:

1. What is the instructional work that teacher candidates do in school classrooms?
2. How can the instructional work teacher candidates do in school classrooms inform the goals, content, and design features of a responsive pedagogy of practice?

Data sources. For this study we collected video data during the 2012-2013 academic year—from methods courses, teacher candidates' teaching, and interviews. In 2012 the program had a small cohort of three teacher candidates in the secondary mathematics program. We collected data from each of the candidate's two teaching practicum experiences. Specifically, we collected video of two lessons from the fall teaching practicum and larger data set of lessons from the spring practicum. Each data set consisted of two or three lessons in sequence as well as a summative interview (conducted by the first author) that elicited ideas from the candidate based on viewing video clips from the lessons. These data served in our investigation of the instructional

work teacher candidates enacted in their school classrooms (Research question 1).

In order to develop a sense of the goals of a responsive pedagogy of practice and the resulting design features, we analyzed data from the methods courses (including teacher candidates' enactments of IAs with students). During that year, we implemented three design cycles focused on the field-embedded methods work using IAs. We collected video from the investigation and enactment phases as well as artifacts from these cycles. The first author also maintained a reflexive journal (Altheide & Johnson, 1994; Auerbach & Silverstein, 2003) to document the ongoing design decisions. The journal also served as a data source in its own right. Finally, the first author conducted two interviews with each teacher candidate—one after the third and final cycle (after the second methods course but prior to their full-time student teaching practicum) and one at the end of the program.

Data analyses.

Framework for analyzing mathematics instruction. In order to analyze teacher candidates' instruction in school classrooms, we modified an analytic framework emerging from the tradition of *activity theory*—specifically the work of Leont'ev (1981) and the elaboration by Wertsch, Minick, and Arns (1984). One dilemma of the recent wave of literature on the core practices of teaching is the widely varied way in which the work "practice" gets used, mainly around the notion of what it is that teachers do for their work in the classroom with students and content. Of specific concern are the varying grain sizes that are referenced with the one word (Boerst, Sleep, Ball, & Bass, 2011; Lampert 2010). As such, we sought out a framework that would account for the various

grain sizes of the work of teaching, which led us to the work of Leont'ev and of Wertsch and his colleagues.

In the framework, we define three levels of instruction that realize the broader *activity* of teaching in a particular setting—instructional episodes, instructional practices, and instructional moves. Instructional episodes were seen as a way to characterize the active work in a classroom at a given moment, such as launching mathematical work, eliciting a solution to a procedural problem, and facilitating individual or small group work. Therefore, a sequence of instructional episodes is what makes up a longer instructional event, such as an IA or a lesson. While there may be consistency across instructional settings regarding episodes, how those episodes are carried out may be quite different. At a finer grain size, we characterized instructional practices, such as managing materials, dwelling on mathematical ideas, representing mathematical ideas, and addressing errors. Each of these practices are specified through instructional moves, which is the smallest level at which we parsed the work of teaching. For example, a teacher candidate might dwell on a mathematical idea by using discourse moves of revoicing an idea or prompting another student to reason about peer's idea. The benefit of examining instruction at these three levels allows for parsing the work, showing the range of work within and across levels that is getting done, and detailing nuances in the work at each level.

This initial framework was discussed between the researchers, drawing on existing work in the literature on decomposing the work of mathematics teaching (Ball & Forzani, 2009; Ball, Thames, & Phelps, 2008; Lampert et al., 2013), instruments measuring the mathematical quality of instruction (Hill & Ball, 2008; Learning

Mathematics for Teaching Project, 2011), as well as more public displays of the core practices of teaching (TeachingWorks, 2014). Using qualitative video analysis software, Studiocode ©, we coded a subset of IA enactments and classroom lessons in order to: (i) continue developing a codebook and refine the code window in Studiocode, and (ii) construct inter-rater reliability. From this coding, new codes emerged at each analytic level and others were collapsed and refined. This new scheme was applied to the rest of the enactment and lesson data, as well as the data that was previously coded. As potential new or revised codes emerged, these were discussed and applied.

To answer the question regarding teacher candidates' work in school classrooms, we first "chunked" classroom lessons into instructional episodes—segmenting an interval of the video as a single instance and tagging it as a particular type of episode. We also tagged each instance with the instructional practices carried out. Looking across the lessons of the three teacher candidates, we looked for frequent episode types to bring forth the common ways in which time is spent in the secondary mathematics classroom. We looked across common episodes to characterize the routine ways in which particular episodes were carried out. We also looked at prevalent or unexpected trends in practices to identify the common instructional moves used in teaching (in lieu of tagging each instance of every move across the data). The findings from these analyses contribute to answering our research questions: (1) what was the instructional work teacher candidates do in school classrooms, and (2) how can the instructional work teacher candidates do in school classrooms inform the goals, content, and structure (design features) of a responsive pedagogy of practice.

Illustrating Our Developing Sense of Responsive Pedagogies of Practice

From our research of teacher candidates' work in secondary classrooms, we were able to identify the common instructional episodes, practices, and moves that serve as the realization of what the work of teaching entails in school settings. Through our analyses, including interviews with teacher candidates, we moved to understand these patterns in the context of how the activity of teaching was defined in school settings—what are the goals for the work, what are the rules that guide the work, and what are the available tools for the work. In the following discussion we highlight one finding from our analysis—teacher candidates' attention and handling of student errors and questions—because it represents a high leverage idea important for teacher candidate development as well as student development. Further, the finding presents a fruitful space for furthering discussion of teacher education design for supporting teacher candidate development. It is our intent to use this finding to consider our second research question, how does teacher candidates' instructional work inform the goals, content, and structure of responsive pedagogies of practice. We do so by offering an example of how to account for the work of addressing errors and questions through a responsive pedagogy of practice. We start by discussing why addressing mathematical errors and student questions should be considered a core practice and how it emerged in our analyses.

Addressing mathematical errors and student questions. Mathematical errors in the classroom and the way those errors are responded to play an important role in the development experience of students. The emergence of errors and questions in the classroom is inevitable, even in classroom where students are primarily giving short answers to closed questions. Many point to the emergence and handling of errors as a key

opportunity in a mathematics classroom for more authentic inquiry and discussion about mathematics (Borasi, 1994; National Council of Teachers of Mathematics, 1991). Making errors and asking questions are part of authentic mathematical practice, which contributes to the development of mathematical proficiency and supports students to meet the demands of an increasing mathematically, statistically, and technologically complex society (Kilpatrick, Swafford, & Findell, 2001; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). This is a shift from historic views of mathematics errors as something to be eradicated for fear of misleading students (Schleppenbach, Flevares, Sims, & Perry, 2007).

The likely emergence of errors and questions as well as the benefit of their proper handling and use makes this a core practice of the mathematics teaching. In the classroom, teachers play a key role in the management and use of mathematical errors and questions. This impact goes beyond the cultivation of a classroom environment that is deemed as safe for mathematical risk-taking (such as sharing incomplete thoughts). While that is a necessary condition, it is far from sufficient. Teachers' responses to student errors and questions can vary greatly—with different types of responses being more productive and equitable than others. The *mathematical quality of instruction* (MQI) instrument has as one of set of codes an attention to the way in which a teacher interprets students' productions and uses student errors (Hill & Ball, 2008; Learning Mathematics for Teaching Project, 2011). A teacher who is able to understand students' ideas in the context of instruction and, in the case of an error, substantively respond by using the error in instruction is providing a more mathematically rich and equitable environment for students. While teachers might rhetorically place value in mathematical

errors and questions in the development process, their responses to errors and questions typically overshadow that intent. Teachers (especially in the U.S.) are often found responding to errors and questions by doing the correcting or answering themselves or disregarding them completely, thus shutting down extended opportunities for discussion and the mathematical development of students (Kazemi, 1998; Santagata, 2005; Schleppenbach et al., 2007; Stigler & Hiebert, 1999; Tulis, 2013).

Our interest in teachers' attention and response to errors emerged through our analyses, in part because of multiple examples we saw of instances in which a teacher candidate was responding to an error or a question in ways that we would not characterize as ambitious or equitable. For example, in a lesson taught by one teacher candidate, Casey (a pseudonym), students were presented with a prompt to pair a short written narrative of a bike trip with a position-time graph (see Figure 14). The class had been working on graphing distance traveled in relation to time based on data in a table and considering linear versus nonlinear graphs. This extension of considering nonlinear graphs and their meaning caused much confusion in the class. After giving students time to think about the prompt themselves and discuss with a partner, Casey inventoried which

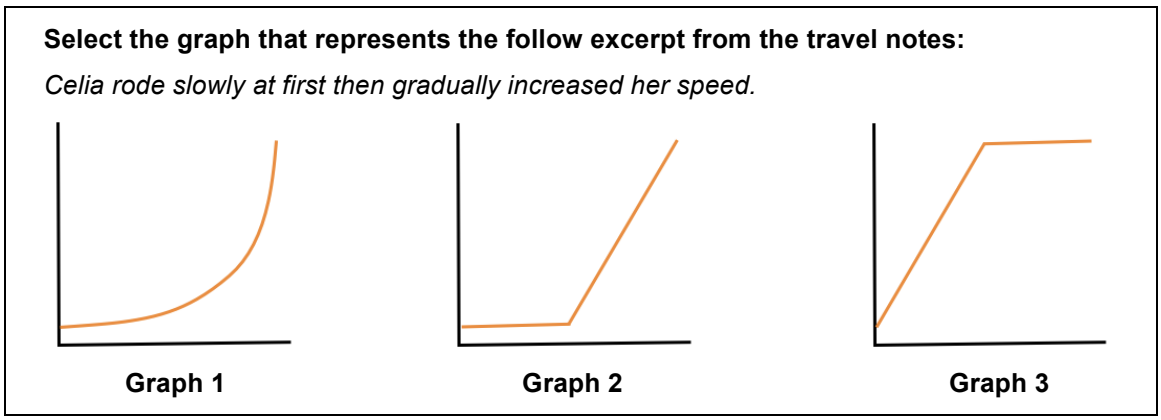


Figure 14. Prompt given to students in Casey's lesson

graph students decided on by asking for a show of hands, with Graph 1 being the correct pairing and a majority of students indicating as such. Casey elicited an explanation from a student who indicated she selected Graph 1. She then asked the rest of the class if they agreed with that answer, which led to differing opinions emerging as students offered other ideas publically without being elicited by the teacher candidate. Bringing the class back together, Casey called on a student who did not agree:

Casey: [Student 2], why do you say it's [Graph 3]?

Student 2: Well, because they never said that it has to go up (*other students in the class comment aloud*). So it would go slow, then they will gradually increase speed (*students comment aloud)*

Casey: So, [Student 2], let me say one thing … So the distance is going to increase over time. So if she starts out slow and then gradually increases her speed over time, how is her total distance going to change over that time period?

Student 2: But you never said you had to go straight up. You could go down.

Casey: OK. But it says she rides slowly at first. How could it – [Student 1] could you explain again why you said [Graph 1]?

Student 1: Because it's not just a straight line – since (*gesturing with hands to model Graph 1*) the bottom part of the line is kind of lower, you can tell that she would cover less ground in a larger amount of time.

Casey: (*pointing to graph*) So [Student 2], here she's covering less ground in the same amount of time. Is that what you said, [Student 1]?

Student 1: She's covering less ground in a larger amount of time, then she increases speed. She goes farther, faster.

Student 2: Yeah, but you never said you had to go up.

Casey: So the distance is increasing, so it has to go up. (*students in room start commenting aloud*). Alright, [Student 3] what do you have to say?

Student 3: I think it's [Graph 2], because she starts slowly –

Casey: OK, let me ask you a question. How far does she go during this part (*points to left part of graph*)? OK, it's flat – how far is she going? (*long pause*) She hasn't gone anywhere, right? Her

distance is not changing until this point. So does that match the description – that she starts out slowly at first? If she's going slowly, she's still increasing her distance, just not fast. Does that make sense?

Student 3: Not really. Because it's going slow at first –

Casey: It's not going anywhere at first, right? So, actually, let's look at the other ones and then come back through and see what we think

There are many things to highlight with this episode of instruction. Our analyses attempted to note the conditions under which teacher candidates used particular moves and the response within the classroom. Casey then elicits reasoning from a student who chose the correct answer—the first graph in the set—which was the choice for the overwhelming majority of students in the classroom. Her subsequent move to ask the class, "Do you all agree?" after Student 1 initially shared her reasoning might have been rhetorical, with the intent being to move on to the next prompt with the correct answer and reasoning having been stated. However, a student with a differing idea interrupted this pattern. Casey's response to this was to provide some explanation, then to have the original student (with the correct answer) restate her reasoning. When the confusion was not resolved (and, in fact, further enhanced by the contribution of another student), Casey suggested moving on to the next problem. Perhaps not surprisingly, this confusion persists throughout the other prompts, which had students pairing the graphs with other narratives of the bike trip.

We were interested in the instructional moves Casey employed in her work and what Casey knew mathematically—not in terms of what the correct answer was and why, but in terms of what she would need to know for the work she was doing as a teacher. As a result, we considered the *mathematical knowledge for teaching* (MKT; Ball, Thames, &

Phelps, 2008) entailed in this instance. The dissenting students in this excerpt were relying on some conception that these position-time graphs instead were an indication of the topography of the bike ride, as illustrated by comments such as, “You never said you had to go up”. Using this conception to consider Graph 3, the student seemed to be communicating that a bike rider would in fact go slow up a hill (imagining the first part of the graph as a hill), and then faster once the route levels out (the flat portion of the graph). From this same point of view, Graph 1 would not seem to indicate a rider going faster given the increasing steepness of the “hill”. What is not clear is whether or not Casey knew this common conception of position-time graphs and whether or not she anticipated any possible errors for these prompts. There are also less overt issues, such as she used the word “distance”, which would rest on the assumption that the bike riders were moving in one direction. Left implicit is that this needed to be defined as the distance from a given spot. At no point was the confusion opened up for further discussion—involving more students and more precise language. In this case, the conception that some students had to look at the graph as the topography of a route was never addressed nor resolved.

We also observed instances in which errors were quickly disregarded. In a lesson by a third teacher candidate, Susan, students were sharing ideas regarding the information that could be culled from a given stem-and-leaf plot (see Figure 15). After a discussion about being able to identify the highest and lowest data value in the set, one student offered that one could determine the range, to which Susan responded:

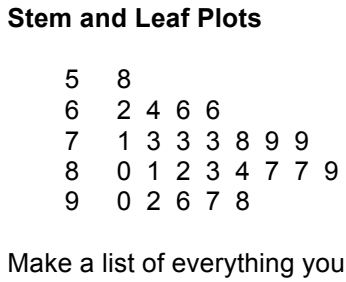


Figure 15. The data display and prompt given to students during Susan's second lesson

Susan: What's the range, [Student 1]?

Student 1: That is the highest number subtracted by the lowest number.

Susan: Uh-huh. So, [Student 2], we've got a high number of 98 and a low number of 58. Can you tell me what the range is?

Student 2: 20? … 30? … 40?

Susan: 20, 30, 40 – I like the way you just keep revising your answer and keep getting closer and closer until you get it. Nice. (pauses) So, [Student 2] took 98 minus 58 and got 40. That's the range.

In this example, we see Susan elicit a definition from one student, then transfer her line of questioning to a second student using a leading recap of the relevant information for finding the range. While we do not know the reasoning behind the second student's set of answers, we do know that Susan's response does not surface any underlying confusion. Instead, she makes a comment that praises (somewhat jokingly) the idea of revising one's answer then states how one would find the range, framed as if Student 2 were the one to share that reasoning initially. We saw many instances in which teacher candidates would pick the correct answer out of a range of offerings and continue to progress through the lesson. Similarly efficient was when teacher candidates would note an error, though immediately correct it. At that point, the teacher candidate would either prompt the student to continue, call on another student to resume the solution process, or pick up the work themselves as a demonstration to the class.

In the cases we highlight above and many across our data, the errors that emerged were not truly resolved for students, nor were they used to prompt the discussion of new ideas or the strengthening of correct strategies. Our findings are in line with findings from other studies that have focused on teachers' handling of mathematical errors. While pressures to cover a large amount of content or make progress on large sets of problems may be informing these instructional ways of working, teacher education designs must take these realities into account and design pedagogies that support teacher candidates in developing skill with new pedagogical tools to deploy during instruction. We see the context of a responsive pedagogy of practice as a space in which to focus on developing teacher candidates' skill for the work of teaching, including addressing errors and student questions. In the sections that follow, we will use an example of an IA targeted at the instructional practice of addressing an error to illustrate the findings from our larger design-based research work. We see this example as an immediate resource for mathematics teacher educators entering into work focused on the investigation and enactment of practice. We see the larger findings as a framework for teacher educators to develop responsive pedagogies of practice in their own settings—thus opening the door for broader contributions to this emerging area of the field.

The activity: "My Favorite No". There is no shortage of instructional ideas for mathematics teachers and teacher educators from blogs and websites with extensive video libraries and related resources. One classroom idea that has gained popularity comes from a clip from the Teaching Channel (http://www.teachingchannel.org), specifically the warm-up activity titled, "My Favorite No" (clip found at http://www.teachingchannel.org/videos/class-warm-up-routine). In the clip we see the

teacher, Ms. Alcala[15], introducing an activity to her middle school class. We also benefit from the narration and reflection provided by the classroom teacher, in addition to seeing segments of instruction and student activity.

In the activity, the teacher presents students with a problem, specifically one that would be familiar to students in terms of the strategies used. The problem is often procedural, such as factoring, expanding, and simplifying in an Algebra 1 class (see Figure 16 for an example from one of our teacher candidates). The teacher passes out note cards for students to complete the problem individually. During this time, the teacher monitors students as they work, answers questions that arise, and collects cards as students complete their work. Once all of the cards are collected, the teacher quickly sorts through the pile, making a pile of correct answers and a pile of incorrect answers. From the latter pile, the teacher selects his or her "favorite no"—the incorrect answer that is chosen for further analysis as a class. The selection of this card is based on some set of criteria, such as a common mistake or an error that highlights some larger mathematical concept. The chosen solution also typically has a number of correct decisions, which are also to be discussed in the class.

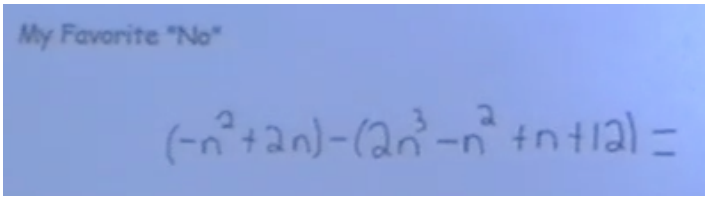


Figure 16. Example problem given to students during My Favorite No activity

[15] We are indebted to Ms. Alcala and other teachers who make their own teaching practice public for the development of others. We use her name here to express our gratitude and to give credit for the idea. Our continued discussion of the activity will be more general, as we highlight some modifications that are not necessarily part of the video.

The teacher records the entire solution strategy, which is announced as incorrect, on the board so as to conceal the identity of the student. He or she then starts by prompting students to state some of what is correct about the solution. In addition to eliciting statements of what was done correctly, the teacher may ask follow-up questions such as how one knows the student did that correctly, why it is correct, or what about that particular move might be a common pitfall for others. The teacher may also revoice the ideas shared by students and also represent ideas on the existing recording of the solution. The teacher may also prompt other students to restate or reason about the ideas from their peers, instead of always doing the revoicing, summarizing, and highlighting. After some time spent discussing what was done correctly, the teacher then prompts students to identify the error. Students can be given time to analyze the problem themselves, then discuss their analysis with a partner, before sharing ideas in the whole group. As with the correct aspects, the teacher may ask students to offer what the common pitfall is, what the correct answer would be instead, and why that alternative solution is correct. As this revision is shared, the teacher re-records the solution.

Since we learned of the My Favorite No activity, we have informally recommended the activity structure to our teacher candidates for work in classrooms—to bring in novel types of conversations, and to address errors in more productive ways (as opposed to casting them aside or quickly resolving them). The comments tied to this video on the Teaching Channel (at http://www.teachingchannel.org/videos/class-warm-up-routine) and on YouTube (at https://www.youtube.com/watch?v=Rulmok_9HVs), as well as the references to this activity on numerous blogs and professional development forums, show others praising the activity—as a way to engage in formative assessment,

to give students feedback, and to demonstrate that errors are valuable in the classroom. Our intent in using this example here is to highlight how a popular and appealing mathematics classroom activity can be framed for purposefully and explicitly not only as a benefit for student development, but also for teacher candidate development. Our design-based research around responsive pedagogies of practice has highlighted three concurrent goals to attend to through the work—developing students mathematically, developing teacher candidates' MKT, and developing teacher candidates' instructional skill. We organize the follow sections to highlight how the My Favorite No activity can be conceptualized to serve as an IA based on a developing notion of how to attend to each of those three goals.

Developing students mathematically. Our initial design of IAs focused on mathematical goals for students that were not as closely tied to the day-to-day work of the partner teacher's classroom. We did this to be as unobtrusive to a classroom as possible, while also being more focused on putting forth generalizable aspects of the work, not the specific content. However, seeing teacher candidates struggle in teaching toward the mathematical goals of their student teaching settings while also being responsible to the ongoing needs of partner teachers led us to design IAs that addressed or could be modified for particular mathematical goals. My Favorite No is an example of the latter—a routine that could be used for a range of mathematical problems, making it malleable to a range of purposes. But no matter the content, the activity follows a similar structure and is used to bring forth a particular error.

The problem in Figure 16 would be an example of a problem to give to Algebra 1 students, especially those in the midst of work on simplifying expressions. A problem

such as that has a few potential pitfalls. First, the problem involves dealing with positive and negative coefficients, including coefficients of 1 and -1 and, once simplified, a coefficient of zero. The problem also involves the subtraction of a polynomial, which can also be handled through multiplying each term by -1. These kinds of errors would be highly contingent on the type of problem presents as well as the students and classroom for which the activity is used. The assumption would be that students had already done a number of problems of this type, though particular errors such as the ones highlighted above might still be prevalent. In turn, the goal for students is to identify and justify errors and correct solutions, especially ones that are more pervasive than a single problem. The activity also engages students in authentic mathematics practice, such as making sense of problems, constructing mathematical arguments, critiquing the reasoning of others, and attending to precision (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In a responsive pedagogy of practice, teacher candidates would be supported in modifying the IA for a range of content and settings, while enacting specific versions of the IA that are relevant to the students and partner teaching in a given lab classroom.

Developing teachers mathematically. A goal that emerged across our three design cycles related to the specified mathematical goals for students. Teacher candidates, themselves, must be supported mathematically. However, that development is not for them as learners of mathematics but as prospective teachers of mathematics, which required a specialized set of mathematical resources (Ball et al., 2008). To enact this activity, a teacher must not only know how to get the correct answer to the problem, but he or she must also complete tasks such as quickly diagnosing the solutions of

students (both for accuracy and, in the case of errors, what the root of the mistake is), drawing upon an understanding of the core mathematical ideas at play in a problem as well as a set of possible errors that could be made. Teachers must make a decision on the error to highlight during the rest of the activity, and ask questions of students that press for more explanation and justification of both the correct and incorrect components of the solution. Even before the activity, the teacher must make appropriate decisions regarding the problem to present to students, asking questions such as what common pitfalls or core ideas the problem might elicit. While such *knowledge-in-use* is developed and deployed in the context of teaching, we have found that responsive pedagogies of practice must also look for other practice-focused ways to address the development of teacher candidates' MKT. This is, in part, necessary so as to not interfere with other novel aspects of such pedagogies, like the repeated opportunity for rehearsal and enactment.

We have considered how specialized mathematics tasks—such as those conceptualized by Suzuka and her colleagues (2009)—could be designed and used with teacher candidates to develop MKT in the context of a responsive pedagogy of practice. These specialized tasks, like IAs, look to put teacher candidates in the position to use and develop their mathematical and instructional skill. With mathematics tasks, this work can be done more readily, though not fully replacing the affordances of enactment and the interactive components of teaching. The specialized nature of an "MKT task" leads to tasks that move beyond having teacher candidates solve a given problem, though that is certainly a baseline. For a task to access and develop teacher candidates' MKT, it must also have features that model the work of teaching. This could include presenting teacher candidates with a solution (or a set of solutions, like they will receive during the My

Favorite No activity), which would involve not just determining whether the solution was correct or incorrect, but also determining what is the error and what is the fundamental idea at play in that error. In the case of Casey's response to the confusion regarding the interpretation of qualitative graphs, she could have been supported with a hypothetical student's response that built on the conception of graphs as showing the topography of a route in order to make her aware of that common mistake and to be able to think about possible responses. Teacher candidates could be presented with a range of problems for them to evaluate in terms of their solutions but, more importantly, the common errors they may elicit. The IA planning protocol can also play a role by specifying a set of anticipated strategies and errors, as well as rationale for focusing on a given one. While this kind of work can only be done for a small set of mathematical content over the course of a teacher education program, the hope would be that teacher candidates become more clear of the ways in which they need to know mathematics for their work as teacher and the ways in which they can plan and prepare to develop and use that knowledge.

Developing teachers' instructional skill. The design of an IA puts the teacher candidate in the position to do particular instructional work with students and mathematics. In turn, teacher candidates are thought to develop certain ambitious and equitable instructional skill through those experiences. In the case of My Favorite No, a teacher candidate would launch the activity, assign a problem, monitor students' work, present an incorrect strategy, elicit and respond to ideas on how the strategy is correct and how it is incorrect, orient students to the ideas peers and to the key mathematical ideas at play when considering what is correct and incorrect in the error, and close the activity. During that time, the teacher candidate targets a particular goal, records ideas, distributes

and manages materials, and manages discussion in the classroom. However, teachers could carry out that same set of work in vastly different ways, mostly through the specific instructional moves that are used. For instance, a teacher may or may not provide enough time for students to work on the problem, or may or may not provide students with time to think individually or with a partner about what is correct or incorrect about the posted solution. This becomes inequitable—advantaging the students who regularly participate first and work "the fastest". A teacher may or may not use follow up questions that press for more explanation or justification, instead doing much of the explanation him or herself, or letting some important ideas go unstated so as to not slow the pace of the activity. A teacher may or may not use moves that prompt students to reason and talk about ideas from their peers, such as the talk moves discussed by Chapin, O'Connor, and Anderson (2009). This results in classroom discourse that is always routed through the teacher. And finally, a teacher may or may not lift the key mathematical ideas at play via students' ideas or pressing questions to highlight the differing import and contours of the mathematical ideas within an error. So in thinking about the potential for My Favorite No as an IA that attends to teacher development, the instructional skills and pedagogical tools that maximize the effectiveness of this activity must be identified and explicated.

Teacher candidates should be supported in their planning and enactment of My Favorite No (and others IAs) with a protocol that specifies the sequence of episodes and the practices and moves that should be used to carry out those events. This is in addition to the work the protocol does in providing teacher candidates with a problem to use, a mathematical focus, and anticipated student responses. In the case of My Favorite No, the protocol would be divided into episodes specifying the launch of the activity, monitoring

student work, sorting the cards, eliciting ideas about what was done correctly, eliciting ideas about the error, and some form of closure. Within each episode, teacher candidates would be supported with scripts and example prompts. For example, when eliciting ideas about what was done correctly in the solution, teacher candidates would be provided with specific support to break the routine of the teacher asking for an idea, a student offering the ideas, and the teacher revoicing or simply accepting the idea and moving on. This version of the "Initiation-Response-Feedback" questioning pattern (Herbel-Eisenmann & Breyfogle, 2005; Mehan, 1979) would be common to expect from teacher candidates without further structuring. Instead, the planning protocol should offer teacher candidates with a follow-up move that presses for more explanation or justification (e.g., "What do you see in the solution that suggests the student did that correctly?" or "Why is that the correct strategy?") or promotes discourse among students before giving one's approval.

These specified pedagogical tools need to be supported with a rationale that lifts their importance as part of a given activity, while not being inextricably linked to that one activity. In addition to offering a move, we have found that planning protocols need to explicitly address the rationale for that move in order to highlight its importance and use and not have it be considered as just one piece of the whole that is the IA. For example, providing students with time to think individually and then discuss with a partner is not some exercise that is specific to an IA, nor is it something a teacher should do at every turn. But such a move is a way to provide students with equitable access to important mathematics and the participation in the classroom. Time to work individually and/or with a partner might be planned with the assigning of a new problem or prompt, and must be accompanied by a clear transition, link to the mathematical work at hand, and

expectations for participation. Providing the time to "turn-and-talk" might be given more spontaneously, following a difficult to follow, yet important, idea from a student. This type of dwelling on a particular idea can also be done through whole-class discourse moves, such as asking a student to agree or disagree with a peer's claim, or to simply restate the idea.

Conclusion

Important in the work of responsive pedagogies of practice and the design and use of IAs is considering how what teacher candidates develop in the way of skill can be applicable to their work as classroom teacher and can be generalized outside of a given IA. If the instructional and mathematical lessons for teacher candidates are too closely tied to a specific IA or specific content, then its broader impact is compromised. In the case of My Favorite No, we aim to consider how that IA can be used not only to bring a novel activity into the classroom and into a teacher candidates' repertoire, but also to highlight broader work around how teacher candidates elicit and respond to mathematical errors made by students. For the latter need, tools provided in an IA should serve as an alternative to the more common routines that have teachers dismissing or quickly resolving errors that emerge naturally in the classroom. For example, the questions a teacher can ask to further unpack an idea or promote broader discussion about the idea (e.g., asking a student to agree, disagree, or expand on an idea from a peer) as part of the My Favorite No activity can also be used in response to emerging errors in the classroom. As such, mathematical errors serve as a true opportunity for student inquiry and discussion about mathematics, just as is recommended by researchers and policy documents.

The My Favorite No activity is one of several IAs we are conceptualizing and developing in response to the theories that have been refined throughout our design-based research efforts. Of course, through continued implementation and systematic analysis these theories will continue to be reshaped, and so too will the resulting designs. Engaging teacher candidates in opportunities to enact teaching has become an increasingly popular aspect of teacher education pedagogy, though one for which there has yet to be much in the way of specification and theory building. In this paper we have used an example of an IA from our own developing practice to illustrate our evolving sense of designing and implementing responsive pedagogies of practice in secondary mathematics teacher education. Specifically, the way in which practice-focused teacher education pedagogies are responsive to student development goals as well as teacher candidates' development—mathematically and pedagogically—must be considered. We see this as the beginning of a discussion in the field. Between specific examples and broader theories, we hope that other teacher educators can be supported by this work—thus developing a larger community from which to develop and grow.

Conclusion

In this dissertation, I set out to highlight, define, and theorize the work of *responsive pedagogies of practice* as an approach to addressing the needs and trends in the fields of secondary mathematics teaching and teacher development. In the introduction, I used the following quote from Lampert and her colleagues (2013) to capture the dual aims that I propose such responsive and practice-focused pedagogies support:

> Because universities are currently thought to be unsuccessful in preparing novices for practice, [teacher educators] are faced with two challenges: preparing beginning teachers to actually be able to do teaching when they get into classrooms, and preparing them to do teaching that is more socially and intellectually ambitious than the current norm (p. 1).

While teacher education programs set out to develop skilled teachers—specifically with increasingly ambitious and equitable approaches to mathematics teaching—the central role of these university programs has been questioned due to a perceived ineffectiveness in preparing teacher candidates for their immediate work in schools (Darling-Hammond, 2010; Kumashiro, 2010; Wiseman, 2012). Teacher education designs must take seriously the work that mathematics teachers are expected to do in school settings—which are a product of a set of goals, expectations, and communities that have formed over long histories (Rogoff, 2003). Responsive pedagogies of practice as an approach to teacher education offer a solution to preparing skilled practitioners through foregrounding the importance of participation in an activity while also seeking out what are the core constellations of episodes, practices, and moves for the preparation of new teachers. Through this work, responsiveness is not a matter of succumbing to and maintaining the status quo in mathematics teaching. Instead, it is a recognition of the need to prepare

teachers for the work of teaching is it is currently defined in schools while also leveraging the opportunities to extend the boundaries into more ambitious and equitable forms of mathematics teaching.

The practical work of responsive pedagogies of practice is also a needed area of further research in order to theorize and specify these pedagogies, their design features and implementation processes, and their impact on teacher candidate development. That pressing need is what motivated this study. With this dissertation, I aimed to make progress on three questions that have arisen from my own take up, consideration, design, and implementation of responsive pedagogies of practice in secondary mathematics teacher education:

1. What does it mean for the design and implementation of a pedagogy of practice in secondary mathematics teacher education to be responsive?
2. What are the features of the activity of secondary mathematics teaching to which a teacher education design needs to be responsive?
3. In what ways does the instructional work teacher candidates do in school classrooms inform the design features of a responsive pedagogy of practice?

The three manuscripts that comprise this dissertation contributed to each of these questions and, in general, help developed a more robust and informed notion of teacher candidate development and responsive pedagogies of practice in secondary mathematics teacher education. They, collectively, represent the entirety of the beginnings of a design-based research effort. Separately, they offer insight into different aspects of the larger work as well as unique products for researchers and practitioners in teacher education.

Review of the Three Manuscripts

The first manuscript took on the first of the questions above—addressing what is meant for a design and implementation of a pedagogy of practice to be responsive. I drew upon data and analyses of the evolving set of needs, design principles, and work to be done by participants in the design to build a theory of responsive pedagogies of practice while also showing an example of one design that is making progress in terms of its responsiveness to the work of teaching in schools. From these analyses came three sets of findings. First, two needs emerged in addition to the initial attention paid to developing teacher candidates' instructional skill—considering the mathematics with which secondary students need to engage as part of their curriculum and developing teacher candidates' MKT (Ball, Thames, & Phelps, 2008) with that content. The negotiation of these multiple needs poses a challenge for teacher educators. This negotiation also gave rise to a second finding involving the development of instructional skill, which needs to focus on the development on multiple levels of pedagogical tools. Further, a set of pedagogical tools must be derived, in part, from the work that teacher candidates do in school settings. Ultimately, this means that responsiveness in teacher education entails preparing teacher candidates to do what is typically done in school settings while also finding the openings at which to press for more ambitious and equitable teaching practice. Finally, a third finding emerged regarding the novel roles that are constructed through a responsive and practice-focused pedagogy of teacher education, namely for teacher educators and partner teachers.

Further pursuing one of these findings, the second manuscript highlights retrospective analyses of the work that teacher candidates do in their student teaching

placements. Using a modified analytic framework based on activity theory (Leont'ev, 1981; Wertsch, Minick, & Arns, 1984), these analyses helped identify the pedagogical tools used in the work and the features of the activity of teaching as characterized in those schools settings. The analysis of both the activity of secondary mathematics teaching as constructed within the methods courses and within teacher candidates' work in their student teaching placements highlighted discrepancies between what was worked on through investigation and enactment at the university and what was actually called for in schools. The takeaway for future design work comes from the second part of the analyses from the student teaching placements. It was found that an attention to mathematical procedures was prevalent as was a push for efficient instructional work that puts forth a glut of these procedures—often as specified in scope-and-sequence documents and pacing guides in schools and districts. These expectations had an impact on what teacher candidates actually did in classrooms, even after their experience in the methods courses. A responsive pedagogy of practice would look to leverage these expectations and provide teacher candidates with tools that support this efficient, procedurally focused work while making progress on increasingly ambitious and equitable instruction. Once these expectations are realized, there was a space for more novel instructional work, though teacher candidates similarly need the pedagogical tools to fully take advantage of those opportunities.

The third manuscript brought many of the research findings put forth in the first two manuscripts to life through a practical example geared toward an audience of mathematics teacher education practitioners. The focus was on one of the instructional practices that emerged from the analysis of the activity of mathematics teaching—

addressing students' mathematical errors and questions—motivated by examples from the data that show teacher candidates' prevalent ways of dealing with such moments. Using a popular classroom activity—"My Favorite No"—as an example, the third manuscript specified how an IA is to be developed to attend to the multiple needs of a responsive pedagogy of practice in secondary mathematics teacher education and can be derived from the work that teacher candidates for in school settings. Readers are provided with both a concrete example to incorporate into their practice and build upon as well as an example that speaks to the larger developments around responsive pedagogies of practice as a result of this dissertation study.

Collectively, these three manuscripts represent products from one year of design-based research around a responsive pedagogy of practice in secondary mathematics teacher education. This work is ongoing and evolving, thus sets me up to continue this work as a core aspect of my research agenda. The findings and recommendations from this dissertation are an intermediate step in a process that will continue onward. They are not final and instead are just what would next be implemented and considered in future cycles of research. The emerging theory of responsive pedagogies of practice, approach to analyzing teaching practice and defining the activity of teaching, and design of IAs must all be considered and potentially further refined relative to making progress on the dual aims of preparing teacher candidates to teach more ambitiously and equitably while also being able to do so in school contexts.

Revisiting My Role as Researcher and Teacher Educator

In the introduction to this dissertation, I discussed the intersection of the roles of researcher and practitioner that design-based research created for me. I shared how I

came to this problem of practice from my own work and in the field of teacher education and how I saw design-based research as a methodology providing the provisions for mediating the potential complications of taking on these roles simultaneously. In this conclusion, I revisit those reflections to talk about my own evolution as a practitioner and researcher and to forecast ahead to my continued efforts with responsive pedagogies of practice in my work as a teacher educator and as a focus of my research.

In short, balancing the roles of practitioner and researcher in the context of design-based research is difficult. The design, implementation, and theorizing of responsive pedagogies of practice in secondary mathematics teacher education is a central area of focus in my future work as a researcher and teacher educator and the experience of this dissertation study has provided a set of considerations that will inform my efforts moving forward. The experience of this study has also highlighted some possible constraints to be addressed in my own work in the future as well as the work of others interested in this area of teacher education practice and/or research. Over the course of this particular study, this role of researcher and practitioner took on multiple forms—changing with each academic term—that offered a different set of affordances and challenges.

During the fall term, I served as the instructor of the secondary mathematics methods course as well as a supervisor for the part-time student teaching placements. This put me in the closest position possible to both the design and the work of the teacher candidates, though it poses a number of dilemmas. While I was in position to have a grasp of the design changes and processes and their rationale, fully articulating those changes and reasons is difficult work. This makes the reflexive journaling all the more

important as a way to synthesize what are otherwise very immediate and potentially implicit design decisions. As a result, it is important for individuals in this type of position to get in a habit of journaling and documentation, despite the time it requires. For me, I feel it is necessary to have this practice be immediate and be the way in which an instructor reflects on a lesson or on some other phase of a design. Another point to acknowledge—similarly related to the negotiations of time, resources, and foci that a course instructor must manage—a clear focus for the design-based research is needed. Without a clear focus, an instructor can get lost in the myriad concerns, dilemmas, and needs that arise in teaching. The focus can emerge and evolve, however. In my work, the emerging focus on pedagogical tools at a range of analytic levels now serves as clear framework for being "practice-focused" in the design, implementation, and reflection of a responsive pedagogy of practice.

While in this position in the fall, I benefited from the input of a larger design-based research team comprised, mainly, of two other secondary mathematics teacher educators. These individuals observed aspects of the implemented design and brought insights from those observations forward in debriefing meetings. These discussions were valuable in balancing the day-to-day and immediate demands of being a course instructor with the larger goals of the design-based research. However, such resources may not be readily available for all in this position, which would make the work a challenge. In such a position, a course instructor looking to engage in design-based research would need to find other ways to get feedback and insight from others who may not be as deep into the implementation of the design and other demands of teaching that particular course or set of courses. Furthermore, even with the resource of colleagues and collaboration available,

what I have learned is the importance to have recorded these conversations to be able to return to those data in the retrospective analyses. During the year of work discussed throughout this dissertation, though, such recordings were not captured, resulting in claims that could not ultimately be made because of the lack of concrete evidence. Recordings should have also been collected from interactions between the teacher educator(s) and the partner teacher. Recollections, no matter how seemingly accurate, are not sufficient in backing claims made in design-based research, which is held to the same standards of rigor and transparency as other research.

In the winter term, I was still involved in the active, ongoing design and implementation in the second methods course, but I did the work as a member of the design-based research team and not as the course instructor. While this position allows a researcher to take a slightly more holistic view of the ongoing design decisions, it also requires one to collect data in addition to one's own journaling in order to fully capture the design process. In the case of this study, I served as the primary researcher—it was my dissertation. However, to maintain integrity to a design-based research process and given the arrangement of trading the role of methods course instructor with another teacher educator, the instructor of a given course should have been keeping a journal. That did not happen during the year discussed in this dissertation. This would not preclude others from also journaling. This would also not be resolved had the debriefing meetings had been recorded. Ultimately, there is a specificity of note taking from the fall that is missing from the winter. If anything, given the nature of this as a doctoral dissertation, there should have been four concurrent pieces of data being collected during the winter term—my own journaling, the instructor's journaling, video or audio recording

of the debriefing meetings, and even interviews of the instructor conducted by me. Without those additional data, I was left without the adequate warrants for claims made about the design or the developing theories. For design-based research to be conducted with integrity and in a trustworthy manner, these data must be collected and managed in a sustainable way. As stated above, though, some teacher educators may not be in the position in which teaching duties can be divided, so this is a potentially unique role. Furthermore, even in the event of another teacher educator serving as the course instructor of a particular course, the ability to plan, observe, and debrief with a design-research team is contingent on the other responsibilities one has. During this particular winter term I was the instructor for another course, which gave me the experience of trying to maintain this balance.

The spring term did not have a methods course as teacher candidates took part in their full-time student teaching practicum. This, in itself, is a potentially unique situation as other contexts may have a methods course running concurrently with even a full-time student teaching practicum. This would be more similar to the organization of the fall term I described above and would, in turn, come with those considerations. During this particular spring term, however, my responsibilities were limited to student teaching supervision, which allowed for collecting video of lessons and conducting interviews. I was also able to begin the process of coding lessons—the results of which were discussed in the second manuscript. My ability to spend time in school settings (as was also somewhat the case in the fall term) was key to this research and would continue to support future design and research efforts in this area.

The work of designing and researching responsive and practice-focused teacher education pedagogies relies on collecting data from teacher candidates' concurrent teaching experiences (such as a student teaching practicum) and their future teaching into their careers. However, that is not a role I may maintain as a teacher educator and researcher. What this study has shown, however, is that the work of university teacher educator must include continued contact with the activity of teaching in schools—whether observations of the work of teacher candidates and practicing teachers, or discussions around the goals and expectations of the work with the variety of stakeholders in those settings. Teacher education programs and the people who work in them must recognize the importance of school contexts and must work to better understand the work that is called for there. This involves actually spending time in schools and with its stakeholders, which may be novel responsibility for university faculty working in teacher education programs as well as a challenging one to balance with the other demands of their work.

While continuing this work may have its challenges, having had the opportunity to engage in design-based research and interactions across university and school settings has changed me as a researcher and a practitioner. Early forms of the design of a pedagogy of practice looked to foreground pedagogical tools that seemed to fit into the work of secondary mathematics teaching in schools. As this work progressed, however, that still-distant stance was no longer acceptable. Researchers and practitioners alike must dig deep into school settings in order to better understand them and be responsive to them. This involves talking and working with classroom teachers and other stakeholders (such as administrators and students) in order to continue building a picture of the activity

of secondary mathematics teaching. By immersing myself in thinking about responsiveness in teacher education my perspective has changed greatly—not only in thinking about the research aspect of this work but also my continuing work as a teacher educator. When I observe a teacher candidate in the classroom, I think about what in the system is informing those actions, not about what the teacher candidate is or is not doing based on what is promoted at the university or in policy or research. When I make recommendation to a teacher candidate, I think not only of what would provide more authentic and accessible mathematical opportunities for each student, but also what is sensible for a teacher candidate to do in the classroom in order to meet the larger expectations of their work in school contexts.

Limitations of This Study and Future Considerations

As with any study, the work described throughout this dissertation has a number of limitations. As was discussed across the three manuscripts, this work around responsive pedagogies of practice is new and developing. I came to this work through problems from and questions about my own practice as a mathematics teacher educator. So, as a result, these limitations are not so much flaws as they are bounds set on the claims that can be made from these particular data and analyses and motivation for future efforts. In this closing section, I will highlight these limitations—summarizing ideas from across the three manuscripts as well as the previous section—and discuss future directions that the limitations motivate.

Paper 1. As a design-based research study, this work around responsive pedagogies of practice is in its infancy. This specific study highlighted the start of these systematic efforts—analyzing only a short span of time (i.e., one academic year) and a

small number of cycles within that span. This limits the overall progress made in this study on the development of a theory of responsive pedagogy of practice, though provides a start for future iterations. My hope is that design-based research in the area of secondary mathematics teacher education can serve as a longstanding and systematic effort. **C**ontinued work would continue to articulate and develop a theory of responsive and practice-focused pedagogies of teacher education—a theory that can become something for other researchers to take up and build upon through design processes in other settings and contexts. While this work is far from conclusion, the goal in reporting on this work at this point is to contribute to an area of the field that is gaining traction quickly, yet is in need of specification.

As was discussed in the previous section, this early phase of the work has taught me lessons about the kind of data that must be collected to maintain the rigor and trustworthiness of design-based research and to help support the claims made. Reflexive journaling is a nontrivial act to maintain, though is of central importance as a documentation of a design process and as a source of data and warrants for claims. A limitation of the work highlighted in the first manuscript is that others in the design team did not conduct such journaling, namely the other methods course instructor. This was an oversight at the time, though manifested as a limitation because of claims that could not be supported because of a lack of concrete data. The same can be said for the lack of recording of debriefing meetings with the design team and interactions between teacher educators and the partner teacher. Another type of data that would need to be collected in future iterations of this work is a way to measure teacher practice and their resulting growth through a teacher education program and/or over time. Examples of existing

instruments are the *mathematical quality of instruction* (MQI; Hill et al., 2008; Learning Mathematics for Teaching Project, 2011) instrument or the edTPA (developed from a partnership between the Stanford Center for Assessment, Learning, and Equity and the American Association of Colleges for Teacher Education).

Finally, the products of a design-based research effort are limited, though that is the nature of the methodology. Theories regarding development and the designs that support development that are built out of such work are domain-specific, yet not entirely tied to only the context discussed throughout this dissertation. The extensive documentation that accompanies design-based research (and that could have been further supplemented) serves a role in providing enough information on the context in which a design was implemented and revised. From this, others can make appropriate sense of and modification to the products from this work.

Paper 2. The second manuscript focused on identifying the entailments of secondary mathematics teaching in two settings—enactments that were part of the pedagogy of practice in two methods courses and the work that teacher candidates did in their student teaching placements. Regarding the former setting, the same issues of only having three design cycles to analyze could limit the claims made about how mathematics instruction was promoted through enactment. Regarding the work of teaching in schools, there was a limited view of the activity for similar reasons but, more notably, because of the subset of perspectives that were unpacked through these particular analyses. In thinking about the use of activity theory to understand the motives of a system and the corresponding actions of actors, further work would need to involve the voices of more stakeholders—such as classroom teachers, school and district

administrators, students, and parents—to better represent and explicate the goals, roles, rules, and communities from a variety of impactful perspectives. Data of teaching could also be collected from a larger number of mathematics teachers, of varying experience and across a number of years. Such continued research efforts—such as following teachers for multiple years—would be a way to continuing thinking about both the work of teaching and, in the case of individuals who were first observed as teacher candidates, about their progress and development. The theoretical and analytic approaches put forth in the second manuscript provides a foundation for such additional work.

Paper 3. My colleagues and I have only just started to mobilize the early findings of this design-based research effort into a set of design decisions (e.g., IAs, frames for discussion and reflection)—resulting in a small set to use in future iterations of the work. To think about the design of a full experience for teacher candidates over a teacher education program, work needs to be done to think about the collection of experiences and how those experiences represent an adequate amount of the work of teaching to support a new teacher. Furthermore, those designs are subject to the same limitations outlined in the previous sections and will, thus, be subject to the ongoing work described across the manuscripts and in this section. Through continued implementation and systematic analyses, a theory of responsive pedagogies of practice will continue to be reshaped, and so too will the resulting designs.

Closing comment. All of these limitations and need for future research and development are the product of novel work that is just beginning. Even with those limits, the work put forth in this dissertation highlights important progress made in the conceptualization of designing practice-focused and responsive pedagogies of teacher

education. The emphasis on using the context of design as a context for systematic inquiry is important in the field of mathematics teacher education (and educational research fields, in general) because of its "relevance to practice" (Gutiérrez & Penuel, 2014). The limitations of the work thus far are, instead, areas that motivate continued design-based research in this field. By virtue of my own professional trajectory, future work will benefit from an increased number of sites in which efforts to design, implement, and research responsive pedagogies of practice take place. Continued progress made in my own work, that of my colleagues, and that of others in the field will address the limitations of this study. However, it is this study that provides firm footing from which to make progress.

References

Altheide, D. L., & Johnson, J. M. (1994). Criteria for assessing interpretive validity in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 485-499). Thousand Oaks, CA: SAGE Publications.

Anderson, L. M., & Stillman, J. A. (2013a). Student teacher's contribution to preservice teacher development: A review of research focused on the preparation of teachers for urban and high-needs contexts. *Review of Educational Research, 83*(1), 3-69.

Anderson, L., & Stillman, J. (2013b). Making learning the object: Using cultural historical activity theory to analyze and organize student teaching in urban high-needs schools. *Teachers College Record, 115*(3).

Auerbach, C. F., & Silverstein, L. B. (2003). *Qualitative data: An introduction to coding and analysis*. New York, NY: New York University Press.

Ball, D. L. & Cohen, D. K. (1999). Developing practice, developing practioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the Learning Profession: Handbook of Policy and Practice* (3-32). San Francisco: Jossey-Bass Publishers.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497-511.

Ball, D., & Forzani, F. (2011). Building a common core for learning to teach and connecting professional learning to practice. *American Educator*, 35(2), 17–21, 38–39.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Barab, S., & Squire, K. (2004). Design-based research: Putting a stake in the ground. *The Journal of the Learning Sciences, 13*(1), 1-14.

Boerst, T., Sleep, L., Ball, D., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record, 113*(12), 2844-2877.

Borasi, R. (1994). Capitalizing on errors as "springboards for inquiry": A teaching experiment. *Journal for Research in Mathematics Education, 25*, 166–208.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24,* 417-436.

Bullough, R. V., & Draper, R. J. (2004). Making sense of the failed triad: Mentors, university supervisors and positioning theory. *Journal of Teacher Education*, 55(5), 407–420.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussion: Using math talk to help students learn.* Sausalito, CA: Math Solutions.

Clift, R. T., & Brady, P. (2005). Research on methods courses and field experiences. In M. Cochran-Smith & K. M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 309–424). Mahwah, NJ: Lawrence Erlbaum.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 68-95). New York: Routledge.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences, 10*, 113-163.

Cobb, P., Zhao, Q., & Dean, C. (2009). Conducting design experiments to support teachers' learning: A reflection from the field. *The Journal of the Learning Sciences, 18*, 165-199.

Cochran-Smith, M., & Lytle, S. L. (1999). The teacher research movement: A decade later. *Educational Researcher, 28*(7), 15-25.

Council for the Accreditation of Educator Preparation. (2013). *CAEP accreditation standards and evidence: Aspirations for educator preparation.* Retrieved from http://caepnet.files.wordpress.com/2013/09/final_board_approved1.pdf.

Darling-Hammond, L. (2010). Teacher education and the American future. *Journal of Teacher Education, 61*(1-2), 35-47.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the United States and abroad.* Retrieved from http://www.nsdc.org/news/NSDCstudy2009.pdf.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5-8.

diSessa, A. A., & Cobb, P. (2004). Ontological innocation and the role of theory in design experiments. *The Journal of the Learning Sciences, 13*(1), 77-103.

Edelson, D. C. (2002). Design research: What we learn when we engage in design. *The Journal of the Learning Sciences, 11*(1), 105-121.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), 133-155.

Feiman-Nemser, S., & Buchmann, M. (1985). Pitfalls of experience in teaher preparation. *Teachers College Record, 87*, 53-65.

Feiman-Nemser, S., & Buchmann, M. (1987). When is student teaching teacher education? *Teaching and Teacher Education, 3*(4), 255–273.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work*. Portsmouth, NH: Heinemann.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 225-256). Charlotte, NC: Information Age Publishing.

Gainsburg, J. (2012). Why new mathematics teachers do or don't use practices emphasized in their credential program. *Journal of Mathematics Teacher Education, 15*(5), 359-379.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. W. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055-2100.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: theory and practice, 15*(2), 273-289.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*(1), 184-205.

Grossman, P., McDonald, M., Hammerness, K., & Ronfeldt, M. (2008). Constructing coherence: Structural predictors of perceptions of coherence in NYC teacher education programs. *Journal of Teacher Education*, 59(4), 273–287.

Grossman, P. L., Smagorinsky, P., & Valencia, S. W. (1999). Appropriating tools for teaching English: A theoretical framework for research on learning to teach. American Journal of Education, 108, 1–29.

Grossman, P. L., Valencia, S. W., Evans, K., Thompson, C., Martin, S., & Place, N. (2000). Transitions into teaching: Learning to teach writing in teacher education and beyond. *Journal of Literacy Research, 32*(4), 631-662.

Gutiérrez, K. D., & Penuel, W. R. (2014). Relevance to practice as a criterion for rigor. *Educational Researcher, 43*(1), 19-23.

Gutiérrez, K., & Vossoughi, S. (2010). Lifting off the ground to return anew: Mediated praxis, transformative learning, and social design experiments. *Journal of Teacher Education, 61*(1–2), 100–117.

Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17-33). New York: Springer.

Guyton, E., & McIntyre, D. J. (1990). Student teaching and school experience. In W. R. Houston, M. Haberman, & J. Sikula (Eds.), *Handbook of research on teacher education* (pp. 514–534). New York, NY: Macmillan.

Hamnerness, K., Darling-Hammond, L., & Bransford, J. (2005). How teachers learn and develop. In L. Darling-Hammond & J. Bransford (Eds). *Preparing teachers for a changing world* (pp. 358-389). San Francisco: Jossey Bass.

Herbel-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26,* 430-511.

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics' teachers conversations. *The Journal of the Learning Sciences*, *16*(1), 37-79.

Horn, I. S., & Little, J. W. (2010). Attending to problems of practice: Routines and resrouces for professional learning I nteachers' workplace interactions. *American Educational Research Journal, 47*(1), 181-217.

Jackson, K., & Cobb, P. (2010, May). Refining a vision of ambitious mathematics instruction to address issues of equity. Paper presented at the annual meeting of the American Educational Research Association, Denver, CO.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between settings up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646-682.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4*, 410–414.

Kazemi, E., & Hubbard, A. (2008). New directions for the design and study of professional development: Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education, 59*, 428- 441.

Kazemi, E., Lampert, M., & Franke, M. (2009). Developing pedagogies in teacher education to support teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Math Education Research Group of Australasia*. Palmerston North, NZ: MERGA.

Kennedy, M. M. (1999). The role of preservice teacher education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 54-85). San Francisco: Jossey-Bass.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up.* Washington, DC: National Academy Press.

Kumashiro, K. K. (2010). Seeing the bigger picture: Troubling movements to end teacher education. *Journal of Teacher Education, 61*(1-2), 56-65.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1-2), 21-34.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional explanation in the disciplines*. New York: Springer.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226-243.

Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning. *The Elementary School Journal, 109*(5), 491-509.

Learning Mathematics for Teaching Project. (2011). Measuring the mathematical quality of instruction. *Journal of Mathematics Teacher Education, 14,* 25-47.

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and locating contrast. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 333-337). Washington, DC: American Educational Research Association.

Leont'ev, A. N. (1981). The problem of activity in psychology. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology*. Armonk, NY: Sharpe.

Lortie, D. C. (1975). Schoolteacher: A sociological study. Chicago: University of Chicago Press.

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for common language and collective activity. *Journal of Teacher Education, 64*(5), 378-386.

McIntyre, D. J., Byrd, D. M., & Foxx, S. M. (1996). Field and laboratory experiences. In J. Sikula, T. Buttery, & E. Guyton (Eds.), *Handbook of research on teacher education: A project of the Association of Teacher Educators* (pp. 171–193). New York, NY: Macmillan Library Reference.

Mehan, H. (1979). *Learning Lessons.* Cambridge, MA: Harvard University Press.

National Council for Accreditation of Teacher Education. (2010). *Transforming teacher education through clinical practice: A national strategy to prepare effective teachers (Report of the Blue Ribbon Panel on Clinical Preparation and Partnerships for Improved Student Learning).* Retrieved from http://www.ncate.org/LinkClick.aspx?fileticket=zzeiB1OoqPk%3d&tabid=715

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards*. Washington, DC: Authors.

National Research Council. (2010). *Preparing teachers: Building evidence for sound policy*. Retrieved from http://www.nap.edu/catalog.php?record_id=12882

Newell., G. E., Gingrich, R. S., & Johnson, A. B. (2001). Considering the contexts for appropriating theoretical and practical tools for teaching middle and secondary English. *Research in the Teaching of English, 35*(3), 302-343.

Newmann, F., & Associates. (1996). *Authentic achievement: Restructuring schools for intellectual quality.* San Francisco: Jossey Bass.

Nolen, S. B., Horn, I. S., Ward, C. J., & Childers, S. A. (2010). Novice teacher learning and motivation across contexts: Assessment tools as boundary objects. *Cognition and Instruction, 29*(1), 88-122.

Peressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics, 56,* 67-96.

Rogoff, B. (2003). *The cultural nature of human development*. New York: Oxford University Press.

Roth, W.-M., & Lee, Y.-J. (2007). Vygotsky's neglected legacy": Cultural-historical activity theory. *Review of Educational Research, 77*(2), 186-232.

Roth, W.-M., & Tobin, K. (2002). Redesigning an "urban" teacher education program: An activity theory perspective. *Mind, Culture, and Activity, 9*(2), 108-131.

Santagata, R. (2005). Practices and beliefs in mistake-handling activities: a video study of Italian and US mathematics lessons. *Teaching and Teacher Education, 21,* 491-508.

Schleppenbach, M., Flevares, L. M., Sims, L., & Perry, M. (2007). Teacher responses to student mistakes in Chinese and U.S. mathematics classrooms. *Elementary School Journal, 108,* 131-147.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education*, *60*(1), 20–37.

Skott, J., Van Zoest, L., Gellert, U. (2013). Theoretical frameworks in research on and with mathematics teachers. *ZDM Mathematics Education, 45*, 501-505.

Smagorinsky, P., Cook, L. S., & Johnson, T. S. (2003). The twisting path of concept development in learning to teach. *Teachers College Record, 105*(8), 1399-1436.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practice for helping teachers move beyond show and tell. *Mathematics Thinking and Learning, 10,* 313-340.

Stigler, J.W. & Hiebert, J. (1999). The teaching gap: Best ideas from the world's teachers for improving education in the classroom. New York, NY: The Free Press.

Suzuka, K., Sleep, L., Ball, D. L., Bass, H., Lewis, J. M., & Thames, M. H. (2009). Designing and using tasks to teach mathematical knowledge for teaching. In D. Mewborn & H. S. Lee (Eds.), Association of Mathematics Teacher Educators Monograph VI: Scholarly practices and inquiry in the preparation of mathematics teachers. San Diego, CA: AMTE.

TeachingWorks. (2014). The work of teaching: High-leverage practices. Retrieved from http://www.teachingworks.org/work-of-teaching/high-leverage-practices.

Thompson, J., Windschitl, M., & Braaten, M. (2013). Developing a theory of ambitious early-career teacher practice. *American Educational Research Journal, 50*(3), 574-615.

Tulis, M. (2013). Error management behavior in classrooms: Teachers' responses to student mistakes. *Teaching and Teacher Education, 33,* 56-68.

Tuomi-Gröhn, T., & Engeström, Y. (2003). Conceptualizing transfer: From standard notions to developmental perspectives. In T. Tuomi-Gröhn, & Y. Engeström (Eds.), *Between school and work: New perspectives on transfer and boundary-crossing* (pp. 19-38). Oxford, England: Elsevier Science.

Valencia, S. W., Martin, S. D., Place, N. A., & Grossman, P. (2009). Complex interactions in student teaching. *Journal of Teacher Education, 60*(3), 304–322.

van Es, E. A., & Sherin, M. G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13,* 155-176.

Vygotsky, L. S. (1978). *Mind and society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, England: Cambridge University Press.

Wertsch, J. V. (1991). *Voices of the mind. A sociocultural approach to mediated action.* Cambridge, MA: Harvard University Press.

Wertsch, J. V., Minick, N., & Arns, F. J. (1984). The creation of context in joint problem-solving, ln B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context.* Cambridge, MA: Harvard University Press.

Wideen, M., Mayer-Smith, J., & Moon, B. (1998). A critical analysis of the research on learning to teach: Making the case for an ecological perspective on inquiry. *Review of Educational Research, 68*(2), 130–178.

Windschitl, M. (2002). Framing constructivism in practice as the negotiation of dilemmas: An analysis of the conceptual, pedagogical, cultural, and political challenges facing teachers. Review of Educational Research, 72(2), 131–175.

Windschitl, M., Thompson, J., & Braaten, M. (2011). Ambitious pedagogy by novice teachers: Who benefits from tool-supported collaborative inquiry into practice and why? *Teachers College Record*, *113* (7), 1311-1360.

Windschitl, M., Thompson, J., Braaten, M., & Stroupe, D. (2012). Proposing a core set of instructional practices and tools for teachers of science. *Science Education, 96*(5), 878-903.

Wiseman, D. L. (2012). The intersection of policy, reform, and teacher education. *Journal of Teacher Education, 63*(2), 87-91.

Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college- and university-based teacher education. *Journal of Teacher Education, 61*(1-2), 89-99.

Zeichner, K. (2012). The turn once again toward practice-based teacher education. *Journal of Teacher Education, 63*(5), 376-382.

APPENDICES

Appendix A
IA Planning Protocol: String of Computational Problems

General Description of Activity:
This instructional activity, also known as a "minilesson" (Fosnot & Dolk. 2001), asks teachers to engage a group of students in mentally solving a sequence of related computation problems, designed to highlight certain strategies and develop efficiency. In this string, a sequence of four multiplication problems were used to bring forth and motivate the use of a strategy for mental computation in which one factor can be halved and the other doubled to create an equivalent product.

Student Development Opportunities:
Students are expected engage in mathematical discussions toward beginning to identify, use, and provide a justification for the "halving and doubling strategy" for multiplication. Students share and defend their solutions and strategies using precise language and are also provided with opportunities to reason about other students' ideas, the efficiency of strategies, and the relationships between problems.

Teacher Development Opportunities:
Teachers choose a sequence of problems that would be productive and accessible for their students, while also foregrounding a particular strategy or characteristic of interest. The teacher purposefully and clearly represents students' strategies, pressing and probing through questioning in order to unpack the numerical relationships involved and make progress toward a specified mathematical goal. The teacher also facilitates students' sense making of their peers' ideas. It is important for the teacher to position students as competent mathematical thinkers in a shared enterprise as students may share tentative, incomplete, or incorrect reasoning.

Credit/Resources: Fosnot & Dolk, 2001; Kazemi et al., 2009; Parrish, 2010

Step 0: Planning and Preparation

What the Teacher Will Do	Notes to the Teacher
• Consider what mathematical ideas you would like highlighted and choose a sequence of problems that best attends to those goals. • Think ahead to how you will represent students' ideas in an organized way. • Anticipate ways in which students might solve the problem mentally, including incorrect answers.	String: 8 x 6 = 4 x 12 = 16 x 15 = 50 x 42 = Instructional Objective(s): • Students understand that multiplying one of the factors of a multiplication problem by one-half and multiplying the other factor by two will yield the same product and that this strategy is advantageous with certain multiplication problems, specifically ones with one or two even factors • Students will be able to use the "halving and doubling" strategy to efficiently solve certain multiplication problems mentally

Step 1: Introduce Activity to Students

What the Teacher Will Do	Notes to the Teacher
• **SAY:** *"We are going to do a String, which will help us think about efficient and accurate ways to do math mentally. We are going to work on a series of multiplication problems. I would like for you to mentally solve each problem and when you have an answer and a strategy, show me a silent thumb. Once we have all had enough time to think, we will discuss your solutions and strategies and listen to one another's ideas."*	The level of detail you give in these launches will depend on students' familiarity with this work. This will also impact whether or not you ask for students to restate the expectations and goals for the activity. Through a launch, paired with students' previous experiences with similar activities, the expectations for participation should be clear.

Step 2: Post the First Problem and Private Reasoning Time

What the Teacher Will Do	Notes to the Teacher
• Once students are clear on the purpose and expectations of the activity, write the first problem horizontally at the top left corner of your writing space. • Give students time to think about the problem, waiting for them to raise their thumbs until at least majority of students show they have a solution and a strategy.	The first problem in your string should be one that you are pretty certain students can easily solve. Given the nature of this activity, you want to start with an increased likelihood that students get the correct answer, as you do not want to spend your time in the way you spend your time with a Number Talk (for instance). Recording the problem horizontally takes away some of the assumption that students should use the traditional (U.S.) algorithms. It is also a way to make connections across multiple problems. **As a result, you will be listing all of the problems on the left side of your writing space—problems written horizontally, listed vertically.**

Step 3: Elicit Solutions

What the Teacher Will Do	Notes to the Teacher
• Once at least a majority of students have a thumb raised, elicit just the solutions from students, asking for different solutions, and recording the totals vertically alongside the original problem • If you elicit more than one total, acknowledge that your ensuing discussion about people's strategies will help to make sense of the solutions and may cause some to revise their solutions.	As needed, assert that students should not call out. For the first solution, you may choose to call on a student based on the fact that they had their thumb raised. You might make this decision based on what you know of a group of students and if you find you have the same subset of students volunteering. It is critical that you do not evaluate the solutions given as they are shared. You should also use wait time after asking for more solutions to allow for more students to potentially volunteer.

Step 4: Discussing and Recording Strategies for the First Problem

What the Teacher Will Do	Notes to the Teacher
• Ask for a strategy for one of the totals	As with the other activities, you will make a decision about whether to start with the incorrect solution or the correct solution. You may also choose to first call on the student that originally offered the solution to share their strategy.
• As student explains their strategy, record their thinking in a way that represents their method, using appropriate representations. Do this starting to the right of the growing list of problems. Record the student's name next to the strategy. Respond to the provided explanation to further unpack an idea or to highlight an idea.	This first problem is meant to be quick and easy so it is not meant to take all of your time. The strategies shared may be more "in students' heads" (e.g., basic facts, etc.) so make sure students are clear but do not spend all of your time on this first problem. You will also not likely spend too much time with having students repeat or reason about the strategies of others at this point, unless you are still working on those expectations and norms.
• If there are multiple solutions offered, after eliciting one strategy on the first solution discussed, go back to the list to acknowledge the other ideas, seeing if people would like to revise their thinking or share their idea. Continue eliciting strategies accordingly.	Strategies should be recorded top to bottom, left to right, while being mindful of how much space you have and wanting to keep all of the work visible throughout the activity. For this first problem, limit the number of correct strategies you elicit to two.

Step 5: Work on Second Problem

What the Teacher Will Do	Notes to the Teacher
• Write your second problem <u>horizontally</u>, <u>below</u> the first problem on the left side of your writing space and give students time to think about the problem, like above. • Elicit solutions, like above	The second problem may also be easy but is crafted to begin to highlight the strategy you are focused on in the string. As a result, if a strategy is shared that is central to your goal, you may want to name it (e.g., "Emily's strategy", "a doubling strategy"), have other students name it, or otherwise focus on it. You will be prompting students to use this strategy in subsequent problems.

• Ask for strategies, like above • After a couple of strategies, if your central goal is not addressed, you need a way to focus the discussion toward that goal. • It is important that the discussion emphasize that: (a) the relationship observed offers a potential strategy for solving more complicated problems easily and (b) that the manipulations involved in the strategy result in a problem that is equivalent to the original (and why) • Once the focal strategy is shared and discussed, move on to the next problem, having students utilize this new strategy.	One method for this is to introduce a strategy as if it were used by a student "in another class". You may also ask students, *"Did anyone use the first problem to help them solve this one?"* For this string, an array can be used to explain and justify this strategy as can the use of he idea of multiplicative inverses and identities. The discussion around this second problem is an optimal place to use student-to-student talk moves to emphasize and name a particular strategy. The name of the strategy should become more refined over the string (for instance, it might start as being "Emily's strategy" and then eventually become "the Halving and Doubling strategy", connected to the work that has been represented)

Step 6: Work on Third Problem

What the Teacher Will Do	Notes to the Teacher
• Emphasize that students are to solve this next problem using this new, named strategy. (**SAY:** *"Let's see if we can use [Emily's strategy; the doubling strategy] to solve this next problem"*) • Write the third problem below the first two and give students time to think, like above. • Elicit solutions, like above. • Ask for someone who was able to solve the problem using the new, named strategy.	In the event of multiple solutions, use your judgment on which solution to start with. Whatever the case may be, be sure you are pressing students to connect their strategy to the focal strategy. You are not trying to elicit a range of strategies here like you do in a Number Talk. Instead, use this time to get students discussing the focal strategy using talk moves and to explain why it works. You are also trying to get the students to create a description or explanation of what the strategy is as you move through the problems.

Step 7: Work on Fourth Problem

What the Teacher Will Do	Notes to the Teacher
• Again emphasizing the new strategy, give students the final problem, give them time to think, elicit solutions, and ask for someone to share.	Again, you are continuing to focus on your goal, getting students to discuss the focal strategy using talk moves and to take opportunities to explain and justify why the strategy works. By the end of your last problem, before your closure, you will want to arrive at some concise way to describe the strategy using students ideas from the activity.

Step 8: Closing the Activity

What the Teacher Will Do	Notes to the Teacher
• **SAY:** *"So in this activity we saw how the [Halving and Doubling] strategy can help us make multiplication problems easier to solve mentally. So, think to yourself for a moment about whether the Halving and Doubling would always be a helpful strategy with multiplication problems."* • Then, **SAY:** *"Now turn to your neighbor to discuss your thinking"* and give students a structure for who talks first (i.e., Partner A) using some characteristic (e.g., the person sitting closest to the door). Give students a few moments to share. • Elicit ideas from groups to come to some shared understanding of the utility of the strategy • End activity	Instructional Objective(s): • Students understand that multiplying one of the factors of a multiplication problem by one-half and multiplying the other factor by two will yield the same product and that this strategy is advantageous with certain multiplication problems, specifically ones with one or two even factors • Students will be able to use the "halving and doubling" strategy to efficiently solve certain multiplication problems mentally As you are able, quickly move about the room to listen in on what students are saying. A conversation like this is an opportunity to not only prompt students to repeat one another's ideas but also reason about (i.e., agree/disagree) and add on to ideas.

Appendix B
IA Planning Protocol: Explaining a Concept Through Connections Across Representations

General Description of Activity:
This IA has students investigate a sequence of graphs to examine the nature and structure of change and co-variation in exponential functions. In the full IA, students explore the uniqueness of this change, as compared to other functions, and the sufficiency of the change for predicting the function's equation. Students were shown, in sequence, the graphs of three exponential functions with the third graph providing an example that defines the boundaries of the explanation being constructed about exponential change and its relation to the closed form of the function.

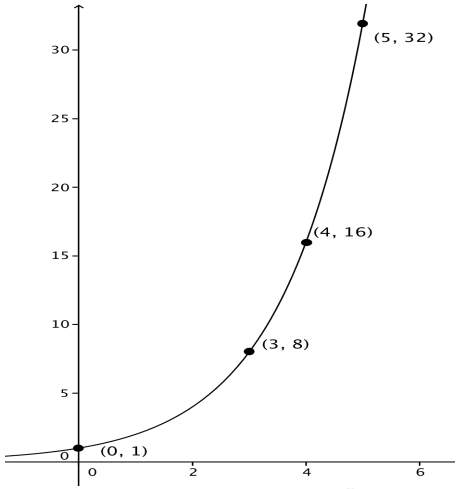
Student Development Opportunities:
Explaining a concept using a representation is meant to support developing students' reasoning on a big idea in mathematics—co-variation. Students are supported in the construction of the idea that an exponential functions' growth can be characterized as, for an increase of one unit in the input, the output is multiplied by a constant factor. Students further consider the role of the constant factor in the equation for the function and its graph. Students engage in mathematical discourse and other authentic mathematical practices such as constructing viable arguments and critiquing the reasoning of others, looking for and making use of structure, looking for and expressing regularity in repeated reasoning, as well as attending to precision of terminology and reasoning. Students share and defend their solutions linked to using key mathematical ideas of co-variation. They examine structure and pattern of co-variation in graphs. They are also provided with opportunities to reason about other students' ideas, examine the nature of evidence used to make a convincing argument, and move toward developing precise language.

Teacher Development Opportunities:
During the enactment of this instructional activity, the teacher strategically selects examples to investigate what would be mathematically productive and accessible for their students, while also opening up opportunities for students to reason about structures and patterns of change of functions represented in graphical form. The teacher purposefully poses questions focused on key ideas, elicits students' reasoning, pressing and probing through questioning in order to unpack the key relationships involved. The teacher also facilitates students' sense making of peers' ideas and uses discourse moves to assure access to ideas for all students. It is important for the teacher to position students as competent mathematical thinkers in a shared enterprise as students may share tentative, incomplete, or incorrect reasoning. This activity requires the teacher to make decisions based on the particular goal, making decisions on ideas to pursue and ideas to navigate around.

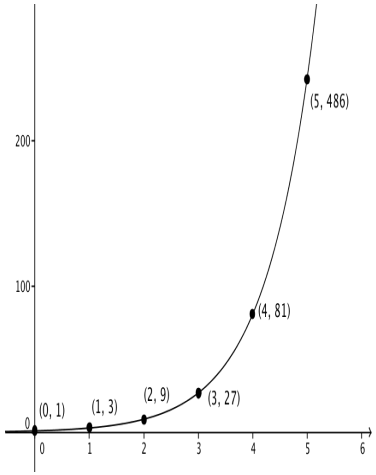
Credit/Resources: P. Herbst, 2011; Leinhardt, 2001, 2010; Leinhardt & Steele, 2005

Step 0: Planning and Preparation

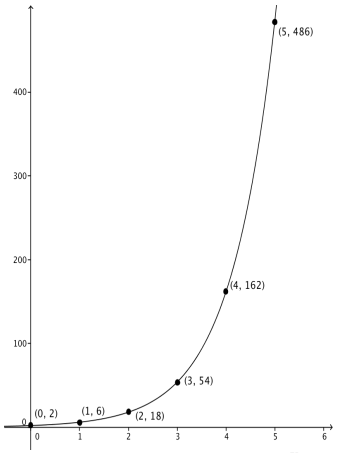
What the Teacher Will Do	Notes to the Teacher
Before enacting this instructional activity, you should: • Consider what mathematical ideas that can be accessed from a graphical representation. Consider what affordances and drawbacks a graph may present to students related to the key mathematical point of the lesson. • Think ahead to how you will represent students' ideas in an organized way. • Anticipate ways in which students might solve the problem mentally, including incorrect answers. • Create large format graphs and tables for lesson.	Instructional Objective(s): • Students learn how exponential functions change examining the structure and pattern of co-variation. Students learn that an exponential functions' growth can be characterized as, for an increase of one unit in the input, the output is multiplied by a constant factor. Students explore the nature of co-variation for an exponential noting how this constant factor is multiplied by the outputs, but not necessarily the difference between the y values, and not necessarily the equation for the graph (could be a translation or dilation of an exponential). • Students explore the uniqueness of exponential function's growth/decay by seeing how it differs from quadratic and linear change.



Graph A: $f(x) = 2^x$



Graph B: $f(x) = 3^x$



Graph C: $f(x) = 2 \cdot 3^x$

Step 1: Launch the Activity

What the Teacher Will Do	Notes to the Teacher
Introduce activity to the student Initial question: *"How do you see the function changing? Describe this change using mathematical ideas, graphs, tables, and equations. What patterns do you see?"*	The launch provides access for all students to reason. The level of background and support you give in a launch will depend on students' familiarity with this work. Your launch should make the goal and expectations for students' participation clear. You may ask students to restate the question under consideration or the expectations for participation. You may also ask students to hold off looking at the graphs and focus on the goal to establish that the focus is on using mathematical tools to reason.

Step 2: Reasoning on Graphs A and B

What the Teacher Will Do	Notes to the Teacher
Prompt students to examine Graph A (individually then in pairs). Briefly share patterns that students see in growth Prompt students to examine Graph B (individually then in pairs).	The first graph in this activity was chosen because it is one that students can be expected to know. The point of this step is to provide access to exponential functions, focus on students' reasoning on the function, and highlight students' use of specific language and tool use.

Step 3: Discussing and Recording Reasoning on Graphs A and B

What the Teacher Will Do	Notes to the Teacher
Lead whole group discussion around patterns of growth in and the equations for Graphs A, then Graph B, using ideas from students. Elicit hypothesis for characterization of exponential growth	Goal for this section is to collect ideas on how Graphs A & B's change. Moving students to see the general property about exponential functions by exploring the property for $y=2^x$ & $y=3^x$. They may say for exponential functions the output values increase by a factor of 2 or 3 as input increases by 1. Support students seeing this pattern and ask them to

Transition to Graph C	consider how the pattern relates to the equation for each function. This supports reasoning on exponential functions to prepare them to see the uniqueness of the function as compared to quadratics.

Step 4: Reasoning on Graph C

What the Teacher Will Do	Notes to the Teacher
Prompt students to examine Graph C (individually then in pairs) Lead whole group discussion around patterns of growth in and the equation of Graph C, using ideas from students. Elicit comparison between change in Graph C and change in Graphs A & B. As needed elicit refined hypothesis for characterization of exponential growth. Prompt students to examine Graph D, first individually then in pairs Lead whole group discussion around pattern of growth in and the equation of Graph D, using ideas from students. Compare the growth of Graph D to that of Graphs A, B, and C. As needed elicit refined hypothesis for characterization of exponential growth. Elicit the equation for each as it emerges from observations from the graph or table. Compare relationships between change factor and equation across graphs. Elicit hypothesis for connection between change factor and equation of exponential functions.	Goal: Students explore non-example of exponential change. Graph C (quadratic) presses on the uniqueness of exponential change. Like Graphs A & B, Graph C has changes that increase for every increase of 1 in the input. It also has a predictable change, but not a constant multiplicative change (exponential), nor a constant change (linear). Students may use first and second differences to look for a pattern of change. They may also look for a pattern in output to describe growth. Goal: To have students reason if this graph is exponential and how the change factor is a unique property of exponential functions. In general, students may test first differences and see non-constant. Students are also encouraged to connect how the property of change relates to the equation of the function. Students might think that the factor multiplied to the previous to get the next is the closed form equation for the function. This doesn't hold for all exponentials.

Step 5: Generalizing and Closing the Task

What the Teacher Will Do	Notes to the Teacher
Prompt students to individually write a conjecture of how exponential graphs change and their equations. Lead whole group closing discussion, using ideas from students.	Goals: • To have students reason and write a conjecture about the general property for exponential change. • To have students reason about the property for exponential functions, its uniqueness, and insufficiency to fully identify the equation for the function.

Appendix C
IA Planning Protocol: Explaining a Definition from an Investigation

General Description of Activity:
This IA focused on the development of mathematical definition—the basic right triangle trigonometric ratios of sine, cosine, and tangent—by investigating a set of within figure ratios that are proportional across a set of similar right triangles. Drawing on prior experiences with similar triangles, specifically the proportional relationship of pairs of corresponding sides, students work in pairs to construct a right triangle of an assigned size with a 55-degree angle. Students measure the sides and found the ratios of the three pairs of sides within their figure. The teacher candidate leads a discussion looking across the data from groups, highlighting the constant ratios across the similar triangles in order to define the three trigonometric ratios.

Student Development Goals:
The work of this IA was motivated by a problem for which previous tools (i.e., using the Pythagorean Theorem) was not useful. As a result, students begin to think about the use of strategies and mathematical relationships in problem solving. Building a definition is meant to support students' accessing prior reasoning of similar triangles and the idea that the ratios of corresponding sides within a triangle are proportional to the ratios of corresponding sides in a dilated or reduced triangle. When this set of ratios is generated in a right triangle and coordinated with respect to an angle the ratios have particular names, sin= opposite side/ hypotenuse, cos=adj/hyp., tan=opp/adj. The definition's use will be explored in subsequent lessons as students conjecture, through discussion and exploration, how trig. ratios are useful for calculating particular side lengths and angle measures on right triangles.

Teacher Development Goals:
The teacher surfaces students' prior reasoning by eliciting ideas about similar triangles. The teacher then leverages ideas about proportional sides and connects this to definitions of trigonometric ratios. The teacher purposefully poses questions focused on key ideas of what it means to be similar (congruent angles, proportional side lengths), elicits students' reasoning about the within ratios of similar rt. triangles and presses and probes to unpack the key relationships that determine trigonometric ratios. The teacher also facilitates students' sense making of peers' ideas and uses discourse moves to assure access to ideas for all students. Important in this IA is the attention to precision, with measurement, to develop insight on how similar right triangles have proportional within figure ratios. Teachers will need to position students as competent mathematical thinkers as students generate different ideas about similar triangles to launch the task at the same time they guide the discussion toward the mathematical point of the lesson. As in typical in all IAs the teacher will need to make decisions based on the lesson goal, about ideas to pursue that evolve in the moment, and which ideas to navigate around to support all students making meaning of the mathematics and pacing the lesson to completion.

Credit/Resources: P. Herbst, 2011; Leinhardt, 2001, 2010; Leinhardt & Steele, 2005; Mod4 materials, University of Michigan

Step 0: Planning and Preparation

What the Teacher Will Do	Notes to the Teacher
Consider what mathematical ideas are entailed in defining the trigonometric ratios of sine, cosine, and tangent. Consider how work with similar triangles can be leveraged to build the trig definitions and to make explicit the class of triangles for which trig. ratios exist. In addition, in this chapter, the trig ratios are tools for solving for missing dimensions (angle measures/side lengths) of right triangles. Think ahead to how you will represent students' ideas in an organized way. Anticipate ways in which students might solve the problem, issues of precision that may arise, how to support students keeping pace with the ideas as they unfold, consider places that you will need to listen carefully to students, and how you will move students to identifying the appropriate trig. ratios.	For your rehearsals, you will be provided with a series of tasks and the instructional objectives. Instructional Objective(s): • Students will see the context of a problem that they can not solve. This will motivate the need for a new tool hat relates angle measures and side lengths. Students will activate prior knowledge of similar triangles and potential everyday understandings of similarity, to explore within figure ratios of set of similar right triangles. Students will learn or confirm that similar triangles have two proportional sets of ratios (across figure and within figure) that are useful mathematical tools. Students will learn that in a right triangle, particular within ratios are equivalent for an infinite set of similar rt. triangles and have particular names, sine, cosine, tangent.

Step 1: Problematizing the Content

What the Teacher Will Do	Notes to the Teacher
SAY: *Today we are going to work on another zipline problem. This time we want to know the length of the line so that we know where to put a landing. In this zip line you see what have only one line. Can someone say what is the mathematically the same or different than the last time we worked on a zipline problem?* **SAY:** *How is the information provided the same and what might you have done with it? How is the information different?*	Goal: Intro the problematize context – elicit students' ideas on "givens" clearly articulating what are previous ideas • Anticipate Ss response: One zipline, distance along the ground instead of height. We have an angle measure .

What do you think we might do with the information and how does it relate to our problem? We don't currently have the tools to solve this problem, because we don't know how to relate side lengths and angle measures. Today we are going to develop some tools that relate angle measures and side lengths.	

Step 2: Eliciting Prior Ideas

What the Teacher Will Do	Notes to the Teacher
SAY: *In the previous unit you worked on similar triangles I want us to take a couple minutes to remember what we know about similar triangles. Take about a minute and half and write down what you know about these two triangles up here on the poster. Be ready to talk about how you know what you know. After you feel like you have enough on your own turn to your partner and share the different things you know about these two triangles.* [show figure and question written on board]	Goal for this step: Students will activate prior knowledge of similar triangles and potential everyday understandings of similarity, to explore numerous ideas about figures. The level of detail you give in these launches will depend on students' familiarity with this work. Your launch should make the goal and expectations for students' participation clear. On Board: Goal: Today we are going to develop a tool that is helpful for working with triangles.

Step 3: Sharing Ideas

What the Teacher Will Do	Notes to the Teacher
Pair Share (keep track of time – two minutes) SAY: In pairs, please share your thinking about the two triangles. Talk about how you know what you know about the triangles. Also take a minute to think about a question you	Goal for this step: The launching activity allows the teacher to pre-assess where students are with a mathematical defensible rationale for similar triangles. In addition, the task lets the teacher see if anyone in the class considers within figure ratios. The goal is to generate a list of some ideas (not exhaustive) and transition to main activity in IA.

might have about the triangles. **Listen** to pairs share ideas. (keep track of time – two minutes) **Monitor and select**: SAY: I heard this (point to grp) group classify the triangles by angles? . (right triangles) Can you share that? I heard this group (point to group) identify the relationship between the two triangles. (similar, AApostulate) Can you share that and tell us how you know the relationship? I heard this group (point to group) identify proportions that you set up using the side lengths of similar triangles. Can you share one? (record one) In the last unit you classified proportions as related to scale factor on Wed./Thurs. you talked about another set of proportions that can be set up with similar triangles. Can someone share a ratio they thought was equivalent to another ratio? Is this one related to scale factor? What is one that is not? Can someone share the other kind? **Record** on Board: Right Triangle Similar Triangle (no elaboration on what this definition includes) Record the first proportion: label scale or not scale (keep track of time – two minutes) **Transition** *Say: What we working on today is developing a tool for relating side lengths and angle measures. We are going to spend more time on considering the ratios of sides of triangles to develop this tool.*	*Consider what you will do if you hear someone resist examining the figures or if someone is not providing a rationale for what he/she knows. *Plan how you will support: <u>Public Access</u> – use of board, recording ideas quickly No need to translate ratios to side names. <u>Pacing</u> -- 6minutes for this step <u>Monitoring Considerations</u> 1) Listen as students share ideas or questions. If they are not specific, publically highlight examples of specific ideas after both partners have shared. For example: you could say *I hear people listing facts but not talking about how they know the idea is mathematically valid. I would like you to take another minute with your partner to share why one of the ideas is mathematically valid or a question you have about why something might be valid. 2) Listen to understand students' reasoning connected to lesson goal <u>Anticipate Students Reasoning</u> (i) similar rt. triangles because of AA, (ii) from AA know across figure ratios proportional, (iii) exists a scale factor related to ii, (iii) from AAA know within figure ratios proportional, (iv) can find the third angle measure, (v) acute triangles, (vi) can use pythagorean thm. because it is rt. triangle if we knew two sides could find third, (vii) identify side names. Students will learn or confirm that similar triangles have two proportional set of ratios (across figure and within figure) that are useful mathematical tools. <u>SCK</u> Classify triangles, similar triangles – relationship between the triangles, and there are within triangle ratio

Step 4: Pairs Exploring Similar Right Triangles

What the Teacher Will Do	Notes to the Teacher
SAY In a minute I am going to distribute a set of directions and graph paper. In the class we are going to be drawing right triangles with a 55º angle. Each pair will be assigned a length adjacent to the 55º angle to make the triangle. Step two is to find ratios of side lengths that we are going to record on a chart. You will need to use a ruler and protractor for this and carefully measure You have about 5min. to complete this so please work accurately and efficiently. Please put your pair's names on the graph paper and the directions. [poster or smart board slide] **Listen** to pairs work on ideas. (keep track of time – 5-7min.) **Monitor/Time out as needed**: As pairs determine ratios have them convert to decimal values. Prompt students to record on chart as they complete task. Have one partner record both ratio and decimal equivalent for each ratio on chart.	Goal for this section Students move into heart of lesson. Pairs of students construct an acute right triangle with one angle at 55º and both sides at least 10cm long. Public Access – use of board, recording ideas quickly, Need to think about how to lay out the chart to get info and make it accessible. Pacing – 10min Monitoring Considerations Need to keep tabs on students' measuring precision and decimal equivalents. Anticipate Students Reasoning sinA=BC/AB 0.819 cos A = AC/AB 0.574 tan A = BC/AC 1.428

Step 5: Noticing Patterns in Table

What the Teacher Will Do	Notes to the Teacher
Say: I would like you to take a couple minutes to write about what you notice about the values on the chart Is this surprising,	Goal for this step: Collect measures and discuss question: will all pairs have the same results for ratios? Elaborate why or why not.

why or why not? . **Writing Prompt on Board** **Monitor** writing time **Whole group discussion** Say: I heard people talk about how the values are very close. Was that surprising? Why mathematically might the measures be close to one another? What contributed to them not being close? [listen] [USE discourse moves here to provide access and elaboration that presses on mathematical sense making.] Say: In mathematics these ratios of opposite /hypotenuse, adj/hyp, opp/ have specific names. And they help us related the angle measure to the side lengths which is what we were trying to do in with the new zipline problem. For our triangle we found, [Write and say] Sin of 55^a = BC/AB, Cosine of 55^a = AC/AB, Tangent of 55^a= BC/AC in the top row. Also write the sin, cos, tan equations for each column.	*Plan how you will support: <u>Public Access</u> – <u>Pacing</u> -- 15min <u>Monitoring Considerations</u> <u>Anticipate Students Reasoning</u>: 1) Notice that have a series of similar rt triangles all examine the same set of within fig. ratio. According to the AAA similarity and definition of similarity the ratios should be equivalent. Measurement error may impact the precision of the values. <u>SCK</u> We use the trig functions in triangle problems that are not right triangles, e.g., law of cosines, sine. However, the definition of trig ratios

Step 6: Closing the Activity

What the Teacher Will Do	Notes to the Teacher
Say: Returning to the zip line task, you saw this problem at the beginning of class and you weren't able to solve it. Now that	<u>Goal for this step:</u> Individual and group accountability <u>Anticipate Students Reasoning</u>:

we have these tools of sin, cos, tan. what do you notice about this problem. Take a minute to think for your self about what you now might be able to do with this or you have a question when thinking about it. Give me a silent thumb when you have had enough time to think about it. Share a few ideas. For homework I would like you to write about how you could make progress on this problem using the tools we developed today. End activity	*Plan how you will support: <u>Public Access</u> – use of board, recording ideas quickly, <u>Pacing</u> -- 5minutes for this step <u>Monitoring Considerations</u> You will need to then use your judgment on how to handle conflicting ideas and ideas that are not wrapped up. You may want to hear what is generally accepted.

Appendix D
Sample Instructional Explanation Decomposition Tool

To explain that an exponential function's rate of change is the value of the function multiplied by a constant factor for an increase of 1 unit in the input.		
A teacher	**Which means**	**For example**
Problematizes the concept	Problem is posed so that students have access to the questions drawing on various mathematical ideas to develop more connected, faster, more elegantly, more comprehensively, etc. understanding of a concept.	Students examine co-variation on an exponential function from a graph connected to other representations. Students can use the graph to reason about co-variation of a function that does not have a constant rate of change. Like previous graphs, rates of change are change in y with a consistent change in x, which can be seen as the change in the output values. For an exponential function students will need to use previous understanding of change and reason about the patterns of how the changes in y are changing, both specifically and generally.
Connects to **prior knowledge**	*Draws on students' prior understanding and experiences relevant to the new concept. * Uses that prior understanding and experience with the class as a collective to construct and connect to new concepts	Before getting started on the exponential function remind students that they have reasoning about linear graph's growth. For an exponential function students will need to use previous understanding of change, reasoning with tables, connections to structure of exponential equations..
Represents the concept	Uses multiple representation of the same concept while explaining a new idea, examining correspondences, building reasoning based on examining various representations	The problem starts with a unique entry point, a graph, to thinking about co-variation. Students examine the graph to consider the change in y values over a consistent x interval. The changes in y over this consistent interval of x change by a common factor. This common factor is seen by looking at the change in outputs over a consistent change in inputs on the graph. This can also be identified on a table. Some students may find the first differences and see that the values are not constant, therefore not linear. They also may see for $y = 2^x$ the first difference has the same values as the outputs. With the next graph they will see that this second pattern in first differences does not hold for all exponential functions. They also may think that the constant factor is the equation for an exponential. This constant factor is the rate of change, but not necessarily the equations, e.g., $y= a \bullet b^x$. Graph C brings this into relief for students.

Exemplifies the concept	Provides examples and non examples chosen to emphasize different features of the concept	The two exponential functions allow students to make sense of the pattern of the changing changes in y over the consistent interval of x. In general students explore the property: exponential functions' growth can be characterized as, for an increase of one unit in the input, the output is multiplied by a constant factor. Graph C presses on the generalization to think about this property in relation to the equation of the exponential. An extension to this activity (exploring quadratic growth), is a non-example of this kind of change. The quadratic graph provides the students to see a non-linear graph that does not change by a constant factor over a consistent interval.
Identifies **core principles** of the concept	Defines the concept in general explains what each components of the concept mean and why the various components of the concept are relevant (or true).	Co-variation of exponential functions: for an increase of one unit in the input, the output is multiplied by a constant factor. This idea is explored across representations and pressed on to relate to the function's equation. Extended examples and non examples are a part of extensions to this activity e.g., $y = a \bullet bx$ quadratic . The parent quadratic function changes by consecutive odd integers over a consistent change in x.
Identifies **key errors**	Creates contexts for conceptual errors to appear and be discussed. Shows connections between the new concept and reasonable but error full ways of dealing with situations that the concept clarifies.	Uses graphs to support students reasoning about changes in y that may not be coordinated with changes in x (consistent x interval). Any examination across graph and table may uncover only looking at a pattern now the graph without thinking about it as a rate.
Establishes the **range/ boundaries** of the new concept	*Presses students to consider the concept in general for a class and to other classes. *When the concept does not apply *Investigates with students some of those questions making connections to the concept	Questions posed to students to consider the class of functions for which the pattern in co-variation holds and in the full or extended activity to make sense of non examples.
Assesses and holds students **accountable**	Asks questions of students to gauge what they understand of the new concept. Expects students to think about those questions and participate in the explanation	Students are asked to provide written conjecture on co-variation for exponential functions in general. They are also asked to participate in a summary conversation to generalize ideas about co-variation for exponential functions and connections to the equation for these functions.

Credit/Resources: Herbst (2011); Leinhardt (2001, 2010)

Appendix E
Sample Rubric for Final Written Reflection

	0 Points	1 Point	2 Points
Elaborating on eliciting Ss ideas	Missing explicit attention to key Ss contributions in rehearsal. Missing or vague attention to components of the question (a-e). Misplaced connections to how examples support the mathematical storyline of the lesson.	Section is complete. Some elaboration with some vagueness on how clip illustrates components of question (a-e). Claims supported with evidence and some misplaced rationale for moves. General discussion of connections/lack of connections to mathematical storyline of the lesson.	Meets criteria for 1 point, plus: all claims supported with evidence and logical rationale. Narrative provides thorough and specific connections/or lack of connections to how examples support the mathematical storyline of the lesson.
Discussion of Growth	Vague focus on growth without specific evidence (timestamp) cited. Vague or missing discussion of growth in specific way related to practices or areas of teaching and learning. Area for growth not connected to central features of high quality instruction	Section is complete. Explains how clip illustrates success. Claims supported with appropriate evidence and some misplaced logical rationale. General statements about practices. Area for growth connected but may be somewhat vague to central features of high quality instruction.	Meets criteria for 1 point, plus: all claims supported with evidence and logical rationale and provides thorough and specific connections between example and evidence. Area for growth thoughtfully connected to central features of high quality instruction.
Discussion of Features of Inst. Explain.	Section is missing or without major components. Missing or vague characterizing of challenge using features of instructional explanation. Missing or vague connections to important Practices.	Section is complete. Correctly ties rehearsal to conception of instructional explanation using appropriate evidence. Highlights important Practices to the construction of explanation. Some general statements and connections allowed.	Meets criteria for 1 point , plus provides specific and accurate connections between features of instructional explanations, teaching practices, and evidence from student rehearsal.
Quality of Writing	Writing is not acceptable with sentence fragments and incomplete thoughts or numerous errors in spelling or grammar.	Response is written in prose and contains complete thoughts but has errors in style, spelling or grammar.	Writing is acceptable with complete sentences and paragraphs and few or no errors in style, spelling or grammar.