AN ABSTRACT OF THE THESIS OF

David Charles Squire for the degree of Master of Science

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Title: A Simulation Study of a Heuristic Technique for Approximating Percentiles for Cascaded Independent Systems

Abstract approved: Redacted for Privacy
Dr. John Spragins

This thesis presents work done to verify the accuracy of a heuristic method for computing availability percentiles for cascaded independent systems. Previous work has provided a model for a single line that will be used as the basis for a simulation of cascaded and parallel systems of lines. The simulation generates cumulative distribution functions that are compared with the cumulative distribution function calculated using the heuristic technique. The results of this comparison show that for independent systems with the same individual availability density functions, the heuristic technique correctly models the composite cumulative distribution function in the areas of most interest to designers.

A Simulation Study of a Heuristic Technique for Approximating Availability Percentiles for Cascaded Independent Systems

by

David Charles Squire

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Typed by Lori Ann Falkenhagen for

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I. INTRODUCTION

With the explosive growth of the data communications industry in the last decade, a great deal of effort has gone into the study of the reliability characteristics of voice grade telephone lines. Since there is a huge capital commitment in the existing voice grade telephone system, sound economics dictate that it be exploited to the hilt.

The first step in quantizing performance parameters of these lines involved the investigation of bit error rates assuming that the line under investigation was available and functioning. By the late sixties the focus of some studies shifted to the investigation and quantification of longer term failures affecting the availability of single lines and complex networks of lines. While the bit error rate studies dealt with methods of detecting and correcting a limited number of erroneous bits, the availability studies dealt with failures of the line lasting minutes or hours (1).

This paper will use Markov's definition of availability (1). That is, A, the availability, is the probability that a communication between two logical machines can be successfully carried out. If the operation cannot be carried out then the line is unavailable. Availability is a random variable representing the probability a line is operational.

Although the telephone operating companies have long recognized the need for information on the reliability of their networks the real impetus for developing a good reliability model came from IBM in the form of their "Supermarket Study" (2,3). The IBM researchers found that line reliability data could be obtained, but that no one had collected it and used it to develop a reliability model for voice grade telephone lines.

The data used by IBM came from operational systems in several countries in North America and Western Europe. These systems were monitored for periods of six months to ten years. For each line under study logs were maintained on their operation. The logs were used to record failures and their time duration. The thresholds on down time for declaring a line failure varied from one to ten minutes.

Using this data Markov et al developed the availability distribution function shown as the solid line in figure 1. Note that the function is actually a distribution function of unavailability (one minus the availability) and that the x-axis is logrithmic. This is done as a convenience and to present the distribution function in a more useful form. If a linear scale had been used the curve would have been compressed almost to a step function making the curves indistinguishable.

Once the line model was available attempts at designing highly reliable systems could be made. The first major consequence of IBM's work was their decision to go with decentralized processors rather than a single centralized processor in their supermarket system. This was a direct result of their conclusion, based on their model of phone line reliability, that the voice grade phone system would not be reliable enough for a system based on real time communication with a centralized computer.

At this stage in the evolution of the line model there are two important problems that have not been adequately solved (4). The first has to do with finding adequate techniques for computing reliabilities of systems with dependent communication line failures. Most of the work to this point assumed that failures were independent and complex systems were modeled accordingly. Assume for example that a system designer adds redundant parallel lines to his system to increase its reliability to a predetermined level. Normally these redundant lines would be in the same cable, would run through the same central office, etc., as the primary lines. If one is cut it is highly probable that the other would be too. Further, if one line is malfunctioning, there is an appreciable probability a redundant line is also malfunctioning. In the worst case the redundancy based system would be no more reliable than a non-redundant one. Spragins has addressed this problem in several papers (4,5).

The second problem has also been addressed by Spragins (4,5). This is the problem of developing techniques for handling systems with tremendously variable reliability parameters. Markov et al found that parameters such as line failure rate or percentage down time typically vary by three or four decimal orders of magnitude for different lines. A method of handling this problem has been proposed by Spragins and the validation of this technique is the subject of this thesis. The problem and its proposed solution are discussed in the following sections.

A. The Problem

It has been observed that there is tremendous variablity in the availability of different voice grade telephone lines (1,2,4). For example, Markov et al (1) found that average availability for a voice grade telephone line varies with its length, the type of data being transmitted, its country and whether it is a national or international line. The data they collected showed availabilities of individual lines ranging from greater than 99.9% to less than 90%. This corresponds to a range of about three orders of magnitude base 10 for unavailability. Spragins points out (4) that, if all availability computations are based on mean values, there is a significant probability that the availability of a system will be much worse than was computed by the system designer. As a solution to this dilemma Spragins suggests that it would be better to design systems using percentile values of availabilities so that the percentage of installations given by the percentile number can be expected to have performance at least as good as set forth in the design specifications. The percentile value refers to the value of availability which n% of the possible availabilities exceed. For example, a 90 percentile availability of 0.99 means that 90% of the time the availability will be greater than 0.99.

Finding percentile values requires integration of the density function of a function of random variables representing the way the availabilities of the individual lines combine. If availabilities of individual lines are assumed to be independent the availability of a system of cascaded or parallel lines can be written as a sum of products of the availabilities of the individual lines. Finding the density function of even the product of two random variables is a complex matter, however, as is illustrated by the general formulas below (5).

If z is a function of two random variables, x and y, then the distribution function of z is given by:

$F_z(z) = \iint_{D_z} f_{xy}(x,y)\,dxdy$ where D_z is the region where the function of x and y is $\leq z$.

The joint density function, $f_{xy}(x,y)$ is given by:

$$f_{xy}(x,y) = \frac{\partial^2 F(x,y)}{\partial x\ \partial y}$$

Since systems may have density functions based on products or sums of many random variables the evaluation of the system density function can be very difficult. Spragins (4,5) has recognized this difficulty and has proposed a solution that is described in the next section.

B. The Solution

Spragins (4,5) has proposed a heuristic approach for approximating the density function of systems of voice grade telephone lines. He has found that single line data can be fitted reasonably well with a Beta density function. The Beta density is defined as:

$$f_{A_i}(a_i) = \begin{cases} \frac{\Gamma(r+s)}{\Gamma(r)\,\Gamma(s)} \; a_i^{(r-1)} \; (1-a_i)^{(s-1)} & 0 \leq a_i \leq 1 \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

(In this paper "A" denotes a random variable and "a" denotes a specific value of the random variable used as an argument.)

Spragins assumed that if the density function for a single line can reasonably be assumed Beta then the density function of a system of lines can also be reasonably approximated by a Beta density. This is a heuristic assumption; a guess. Once having made the assumption he went on to show how to approximate the density function for a system given the densities for the individual lines. The purpose of this paper is to evaluate the validity of this heuristic assumption using the technique of simulation. Complex systems have been simulated assuming a single line is characterized by a Beta density and the resulting availability functions are compared with those computed using the heuristic technique.

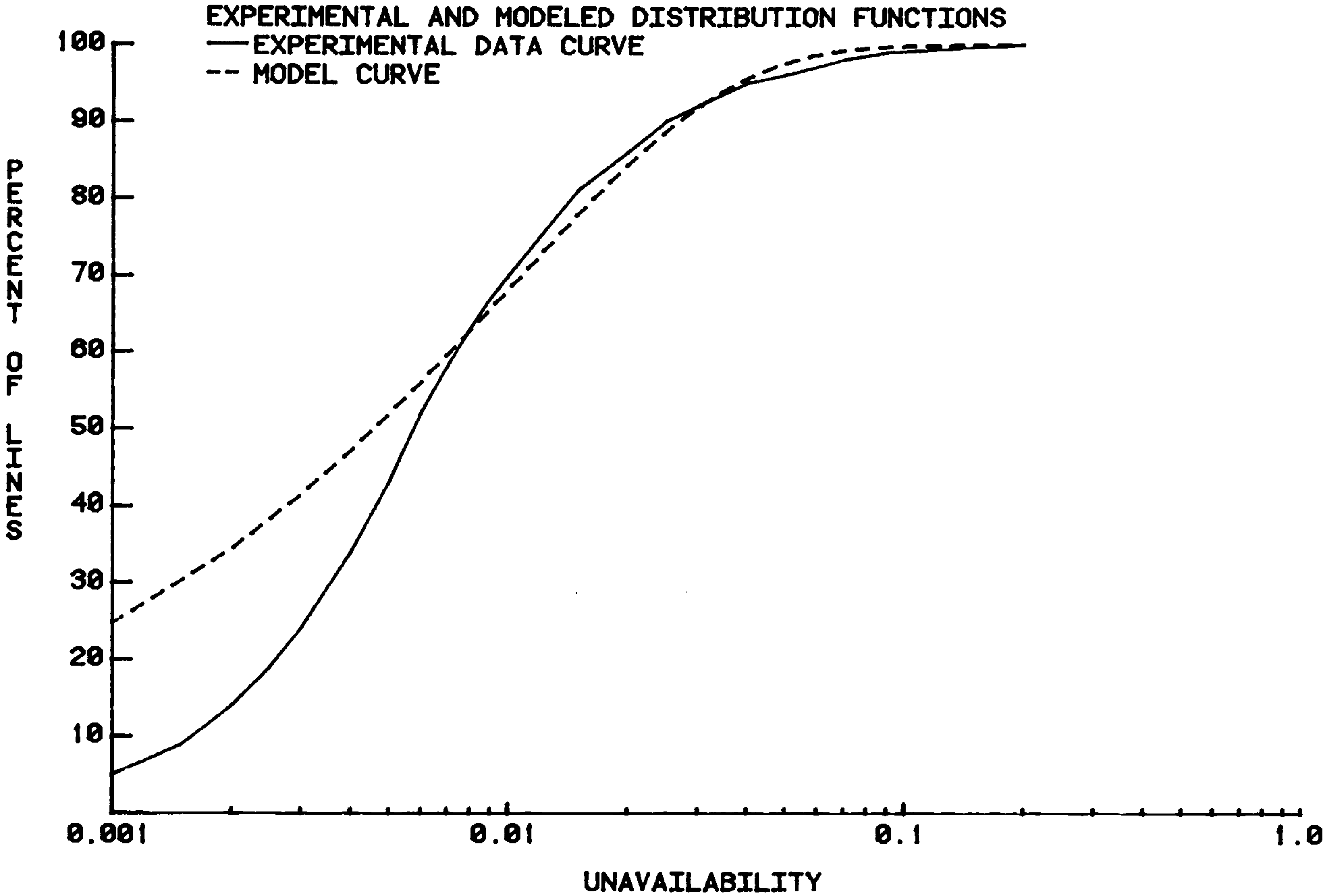
II. DESCRIPTION OF THE HEURISTIC TECHNIQUE

A. The Model

It has been shown (4) that the Beta density function provides a good fit to the measurement data on the availabilities of single voice grade telephone lines. The experimental data was collected and analyzed by Markov et al (1) and the Beta model was developed by Spragins (4). A comparison of Markov's data and Spragins' Beta model is shown in figure 1. For convenience and ease of presentation unavailability (one minus availability) is plotted on a horizontal log scale. The vertical scale represents the probability that the unavailability is less than a specified value. This is, in fact, a cumulative distribution function on unavailability which is defined as the integral of the probability density function. Thus, if we want to find the 90th percentile availability we would go to the vertical axis at 90% and find the corresponding unavailability on the horizontal axis. This gives a percentile value for an unavailability which will not be exceeded more than 10% of the time. This is equivalent to finding an availability which will be exceeded 90% of the time.

The Beta approximation in figure 1 falls away from the experimental curve quite rapidly for availabilities greater than 99.5%, but this region is normally of little interest to the system designer who is interested in estimating the probability his design will be unsatisfactory. As availabilities go higher than 99.5%, moreover, the confidence in the measurement data goes down because of the long observation periods required to get statistically significant results. For example, an availability of 99.9% corresponds to about two hours of down time per year based on a two thousand hour year. The observation period needs to be longer than three to four years to get statistically significant results. Some of Markov's data was taken over a period of several years of continuous operation but most of it was taken over periods of less than six months of continuous operation. When Spragins computed the Beta approximation, the values of the mean and 90th percentile were used to estimate the two parameters, r and s, that describe the Beta distribution. This gave the values of r=50 and s=0.5. (The experimental data has a mean availability of 0.99 and a 90th percentile availability of 0.974. Spragins' Beta approximation has a mean of 0.990 and a 90th percentile value of 0.973.) The object was to make sure of a good fit for the high percentile availabilities (which correspond to lower line availabilities) because that is the area of most concern to system designers and the area where most confidence in experimental data exists.

FIGURE 1


EXPERIMENTAL AND MODELED DISTRIBUTION FUNCTIONS
—EXPERIMENTAL DATA CURVE
-- MODEL CURVE
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
0.001
0.01
0.1
1.0
UNAVAILABILITY

The heuristic model assumes that the density function of a complex system can also be approximated by a Beta density. A complex system is any combination of lines in series and parallel, or a more complex combination built up of series-parallel or parallel-series blocks. What follows here is a description of how Spragins computes the values of the Beta density parameters, r and s, of a system given the Beta density of a single line.

The Beta density was described in equation 1. The first and second moments of this density can be expressed in terms of the two parameters r and s as (6):

$$\overline{A_i} = \frac{r}{r + s} \tag{2}$$

$$\overline{A_i^{(2)}} = \frac{r\,(r + 1)}{(r + s)\,(r + s + 1)} \tag{3}$$

Solving these equations for r and s allows us to express r and s in terms of first and second moments as shown below:

$$r = \frac{\overline{A}_i\,(\overline{A}_i - \overline{A_i^{(2)}})}{\overline{A_i^{(2)}} - (\overline{A}_i)^2} \tag{4}$$

$$s = \frac{(1 - \overline{A_i})\,(\overline{A_i} - \overline{A_i^{(2)}})}{\overline{A_i^{(2)}} - (\overline{A}_i)^2} \tag{5}$$

The parameters (r,s) of the Beta density assumed for the complex system can then be computed if the first and second moments of the exact density for the complex system can be found. These moments for composite systems are derived using the procedures described below. Once they are known, the moments are used to compute r and s using equations 4 and 5 above. We assume here that the availabilities of different lines are independent. For functions of the availabilities of these lines this implies that:

$$\overline{f\ (A_i)\ g\ (A_i)} = \overline{f\ (A_i)}\ \ \overline{g\ (A_i)}$$

We also know that whether availabilities are independent or not:

$$\overline{f(A_1, A_2 \ldots A_n)\ +g(A_1, A_2 \ldots A_n)} = \overline{f(A_1, A_2 \ldots A_n)}\ \overline{+g(A_1, A_2 \ldots A_n)}$$

Hence, if the rth moment for cascaded lines is:

$$A^{(r)} = \int_0^1 \cdots \int_0^1 a_1^r \ldots a_n^r\ f_{A_1}\ (a_1) \ldots f_{(A_n)}\ da_1 \ldots da_n$$

then for independent cascaded lines:

$$\overline{A^{(r)}} = \prod_{i=1}^{N} \int_0^1 a_i^r\ f_{A_i}\ (a_i)\ da_i = \prod_{i=1}^{N}\ \overline{A_i^{(r)}}\ ,$$

so

$$\overline{A} = \prod_{i=1}^{N} \overline{A_i} \quad , \tag{6}$$

$$\overline{A^{(2)}} = \prod_{i=1}^{N} \overline{A_i^{(2)}} \quad . \tag{7}$$

Similarly for parallel lines:

$$\overline{A} = \overline{1 - \prod_{i=1}^{N} (1-A_i)}$$

$$= 1 - \overline{\prod_{i=1}^{N} (1-A_i)}$$

$$= 1 - \prod_{i=1}^{N} (1-\overline{A_i}) \tag{8}$$

and

$$\overline{A^{(2)}} = \overline{\{1 - \prod_{i=1}^{N} (1-A_i)\}^2}$$

$$= 1 - 2 \overline{\prod_{i=1}^{N} (1-A_i)} + \overline{\prod_{i=1}^{N} (1-A_i)^2}$$

$$= 1 - 2 \prod_{i=1}^{N} (1-\overline{A_i}) + \prod_{i=1}^{N} (1-2\overline{A_i} + \overline{A_i^{(2)}}) \tag{9}$$

Since we now have formulas for the first and second moments of availability for systems that we are heuristically assuming to be characterized by Beta densities, we can use equations 4 and 5 to compute r and s for the assumed system density function.

Systems more complex than simple series or parallel networks could be analyzed using reliability block diagram reduction techniques such as these suggested by Buzacott (8). For the purpose of this study simple series parallel systems are thought to be sufficient to demonstrate application of the techniques.

Spragins' heuristic modeling technique then consists of the following steps:

1. Fit Beta density functions to experimental data to determine the values of r and s for individual systems.
2. Compute the first and second moments of the individual systems densities using (2) and (3).
3. Find the first and second moments of the composite system densities using (6), (7), (8) and (9).
4. Compute the values of r and s for the composite system density using (4) and (5).
5. The derived density function can then be used to compute percentile availability values for the composite system since only a single density function needs to be integrated.

B. Results Prior to this Study

In two papers describing this technique (4,5), Spragins has derived the Beta density r and s parameters for a single line based on measurement data collected by Markov et al (1) and by Provetero (2). Using the density function derived for a single line the 90th percentile availabilities for one to ten cascaded lines were calculated using the heuristic technique. The results were tested by comparing them with products of the mean availabilities and products of 90th percentile availabilities. It is logical to assume that these functions represent bounds on the true 90th percentile availabilities, although this is difficult to prove rigorously. Since the heuristically derived values fell between the two product curves they thus appeared to be reasonable. Prior to the time of the study reported here no other work has been done to validate this technique.

The purpose of this study is to verify this technique by taking the density function derived by Spragins (4,5) and using it to simulate complex series and parallel systems by assuming that all of the individual lines have the same density function. The methodology and results of this simulation are discussed in the next section.

III. METHODOLOGY OF THE SIMULATION

A. System Simulation

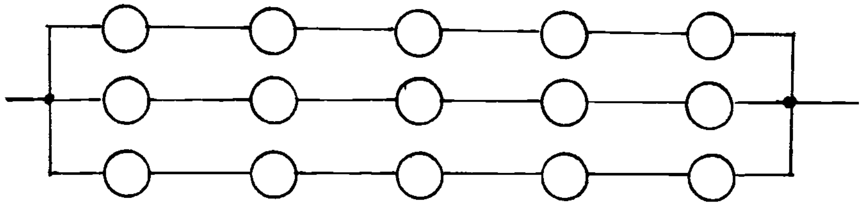
The Beta distributed lines are simulated using what Tocher calls the "Top Hat" method (7). This involves computing the inverse of the cumulative distribution function (C.D.F.) so that the availability becomes the dependent variable and the probability becomes the independent variable. If the probabilities are selected from a uniformly distributed random variable function with values between zero and one then the corresponding availabilities will follow the correct density function. Virtually any textbook on computer simulation demonstrates the validity of this technique.

The simulation was done in just this fashion. The C.D.F. of the Beta density was inverted so that the availabilities were given as a function of their probability of occurence. (Actually, a table of values stored in a computer was used to describe this function.) A random number generator computed a probability between 0 and 1 which was used to look up an availability. These availabilities then occured with probabilities predicted by the Beta density function being simulated. That is, they followed the assumed Beta density function.

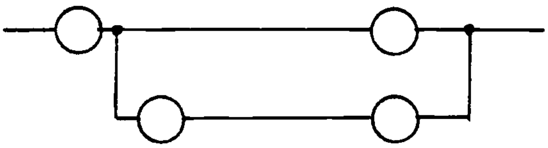
The simulation was run on a Tektronix 4051 Graphic Computing System using some modified statistical application programs. The "PLOT 50 STATISTICS" application package has a program called "Beta Tabled" (see appendix for program listings) that computes the availability given the right tail probability. This program was modified so that it would generate a table of values for probabilities from 0.001 to 0.999 in steps of 0.001. Uniformly distributed random numbers were generated using the RND function on the 4051. All individual lines were assumed to have the Beta distribution defined by r=50, s=0.5 (see section II, A).

Access to these statistical programs was invaluable to this study. There are published tables of Beta distributions (9) but their use would have been extremely cumbersome and time consuming. A random number table would have to be used and values from it used to look up values of the availability. To do this 500 times for a single system would have been painful. To do it 10,000 times for two ten-line systems in parallel would have been impossible.

The simulation program works as follows. A complex system is defined in this simulation study as any number of cascaded lines which can be paralleled any number wide. For example, a system can be defined as three five line systems in parallel.



The current program doesn't handle structures more complex than this. For example, it will not handle:



With the proper programming any system composed of arbitrary configurations of independent lines could be simulated but the program would be much more complex than was felt to be necessary for this study.

Once the system has been defined a random number is generated for each line and the corresponding availability is selected from the tabulated data via the "Top Hat" method. The system availability for this sample is then calculated using the relationship:

$$A = \prod_{i=1}^{N} A_i \qquad \text{for cascaded lines.}$$

and

$$A = 1 - \prod_{i=1}^{N} (1-A_i) \qquad \text{for parallel lines.}$$

These are formulas 6 and 8 derived in section II-A.

This process is repeated for the number of samples specified at program run time. The cumulative distribution function is then calculated by converting the availabilities to unavailabilities and accumulating their occurrences into bins which are specified at run time as "STEP SIZE". For example, the bins for Figure 2 were in steps of 0.001 from 0.001 to 0.200. The resulting distribution of availabilities is the probability density function of the system based on frequency of occurrence. To compute the cumulative distribution function the values in the bin are simply accumulated starting at 0.001 and ending at 0.200. The resulting data is then plotted as shown in Figures 2-7.

B. Heuristic Model Data Generation

The 4051 PLOT 50 Statistics package also has a program for computing right tail probabilities given availabilities for assumed Beta probability densities. This program was modified to generate the dashed curves shown in Figures 2-7. The parameters (r,s) of the Beta distribution for which right tail probabilities are evaluated are computed using the heuristic technique previously described in section II-A. The first and second moments are computed using equations 6, 7, 8, 9 and r and s are then computed using equations 4 and 5. Listings of the 4051 BASIC programs are in the Appendix.

IV. RESULTS

A. Summary of Results

The results of the simulation of six different systems are shown in Figures 2-7. Values of r and s and maximum error differences are shown in Table 1. Maximum error for any system was +6%. Error is defined as the difference between the model data and the simulation data. The match between simulation and heuristic model appears to be quite good.

Table 1. Summary of Results

System	Samples	r	s	Max. Diff.
1 cascaded	500	50	0.5	+2%
2 cascaded	500	49.75	1.0	+4%
5 cascaded	500	49.02	2.5	+6%
10 cascaded	500	47.83	5.0	-4%
2 10's in para.	500	282.19	2.55	-3%
2 20's in para.	500	171.20	5.76	-3%

FIGURE 2

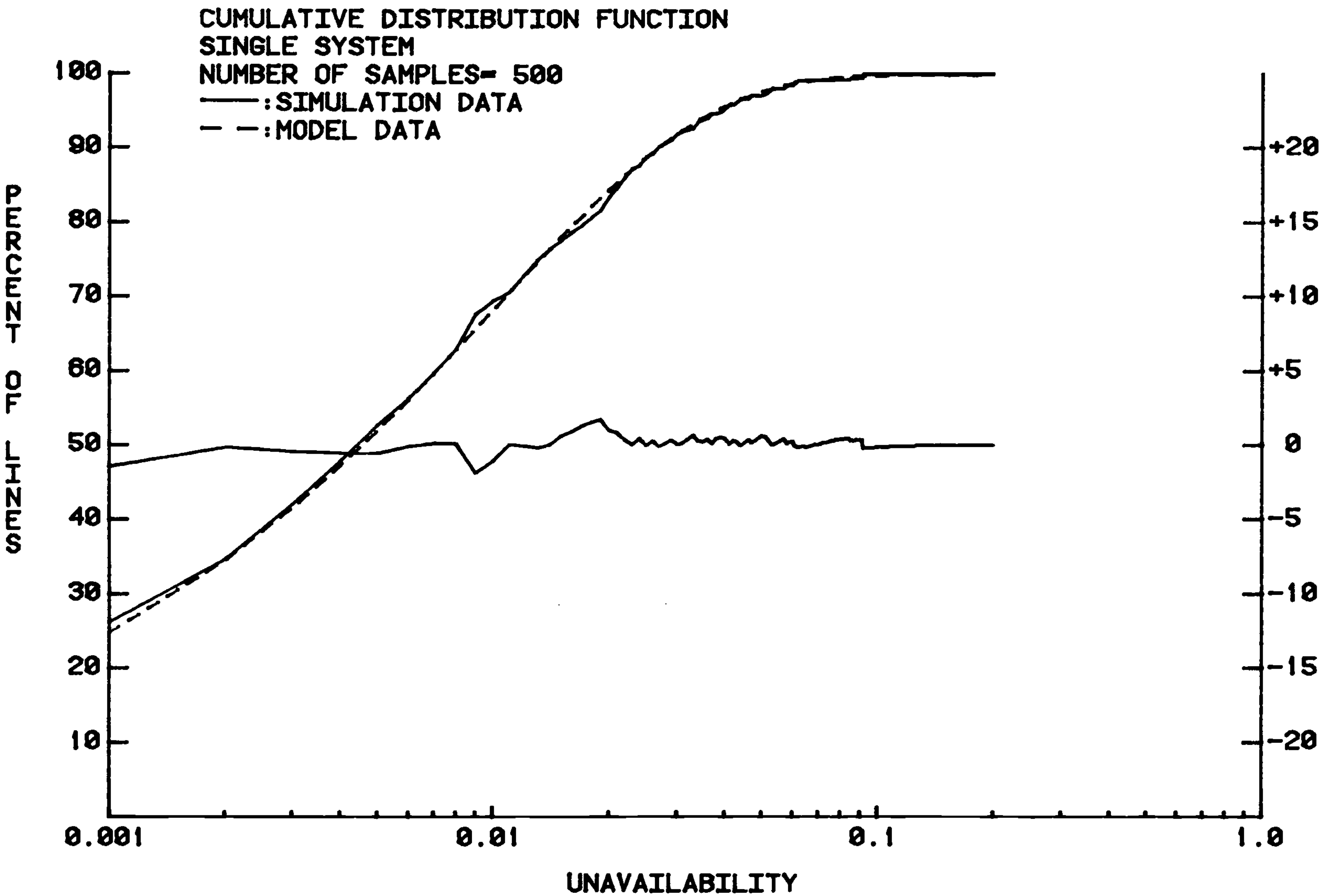
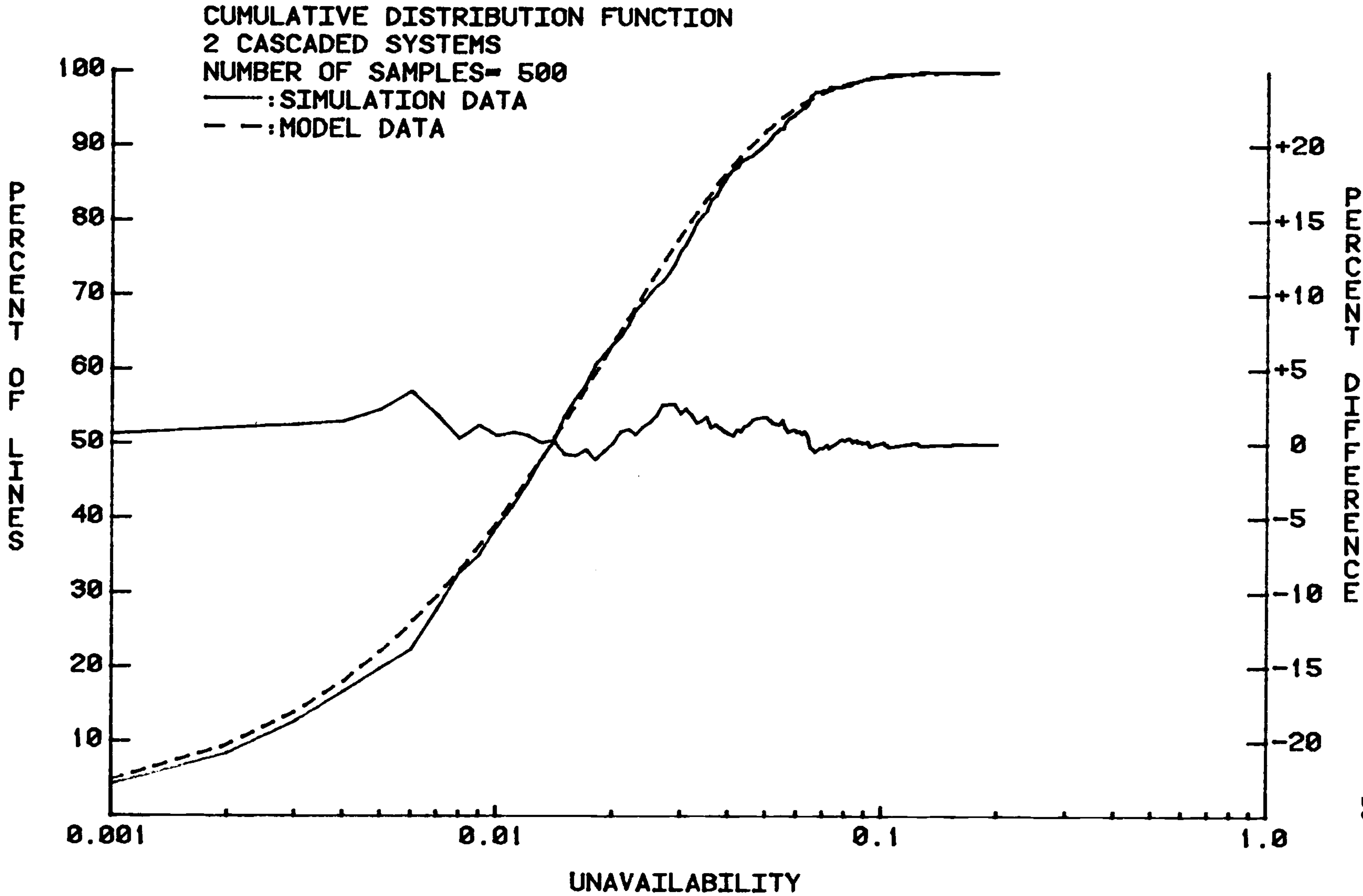

CUMULATIVE DISTRIBUTION FUNCTION
SINGLE SYSTEM
NUMBER OF SAMPLES= 500
——:SIMULATION DATA
— —:MODEL DATA
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
+20
+15
+10
+5
0
-5
-10
-15
-20
PERCENT DIFFERENCE
0.001
0.01
0.1
1.0
UNAVAILABILITY

FIGURE 3



CUMULATIVE DISTRIBUTION FUNCTION
2 CASCADED SYSTEMS
NUMBER OF SAMPLES= 500
——:SIMULATION DATA
– –:MODEL DATA
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
PERCENT DIFFERENCE
+20
+15
+10
+5
0
-5
-10
-15
-20
0.001
0.01
0.1
1.0
UNAVAILABILITY

FIGURE 4

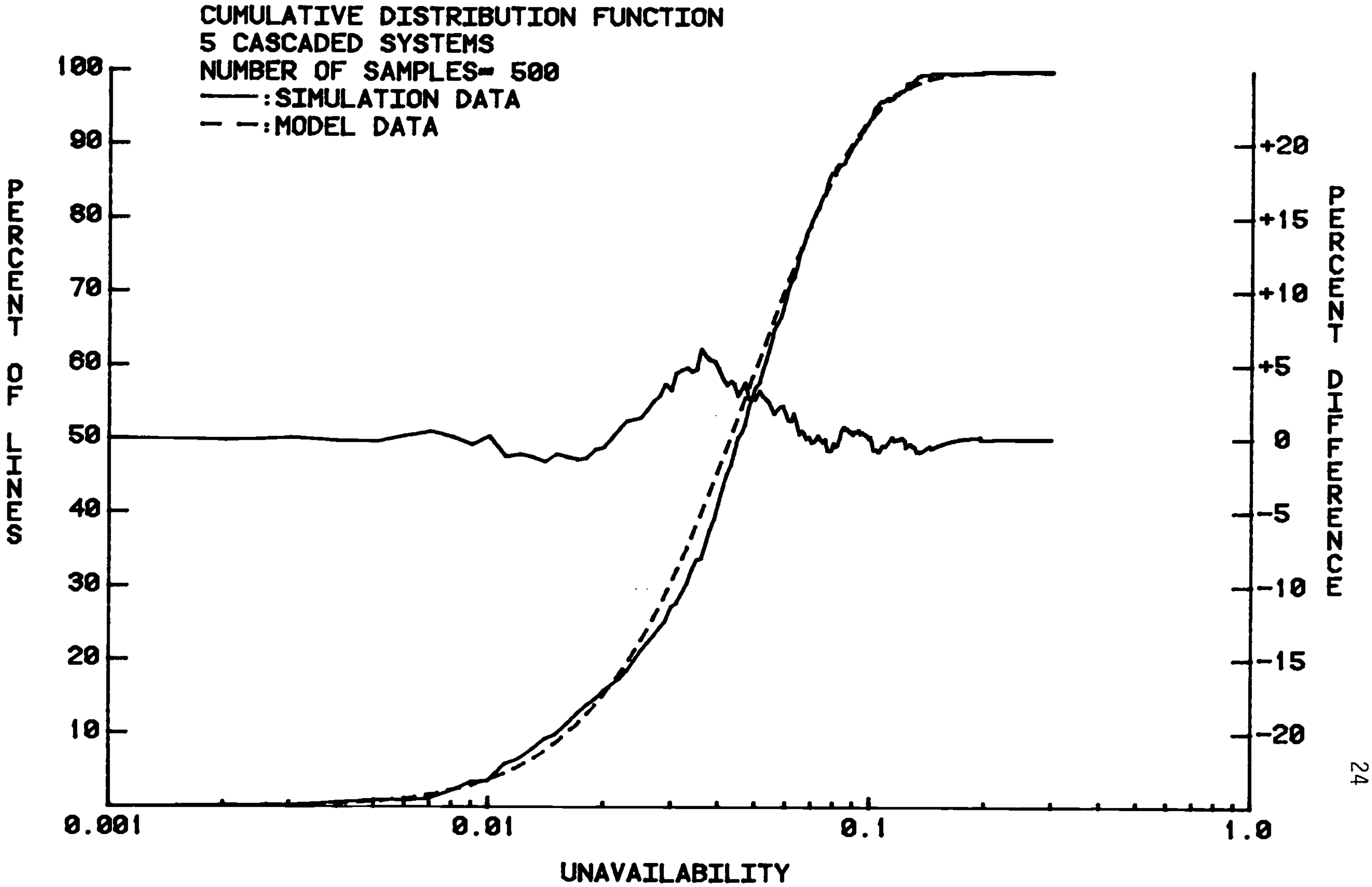

CUMULATIVE DISTRIBUTION FUNCTION
5 CASCADED SYSTEMS
NUMBER OF SAMPLES= 500
——:SIMULATION DATA
– –:MODEL DATA
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
PERCENT DIFFERENCE
+20
+15
+10
+5
0
-5
-10
-15
-20
0.001
0.01
0.1
1.0
UNAVAILABILITY

FIGURE 5

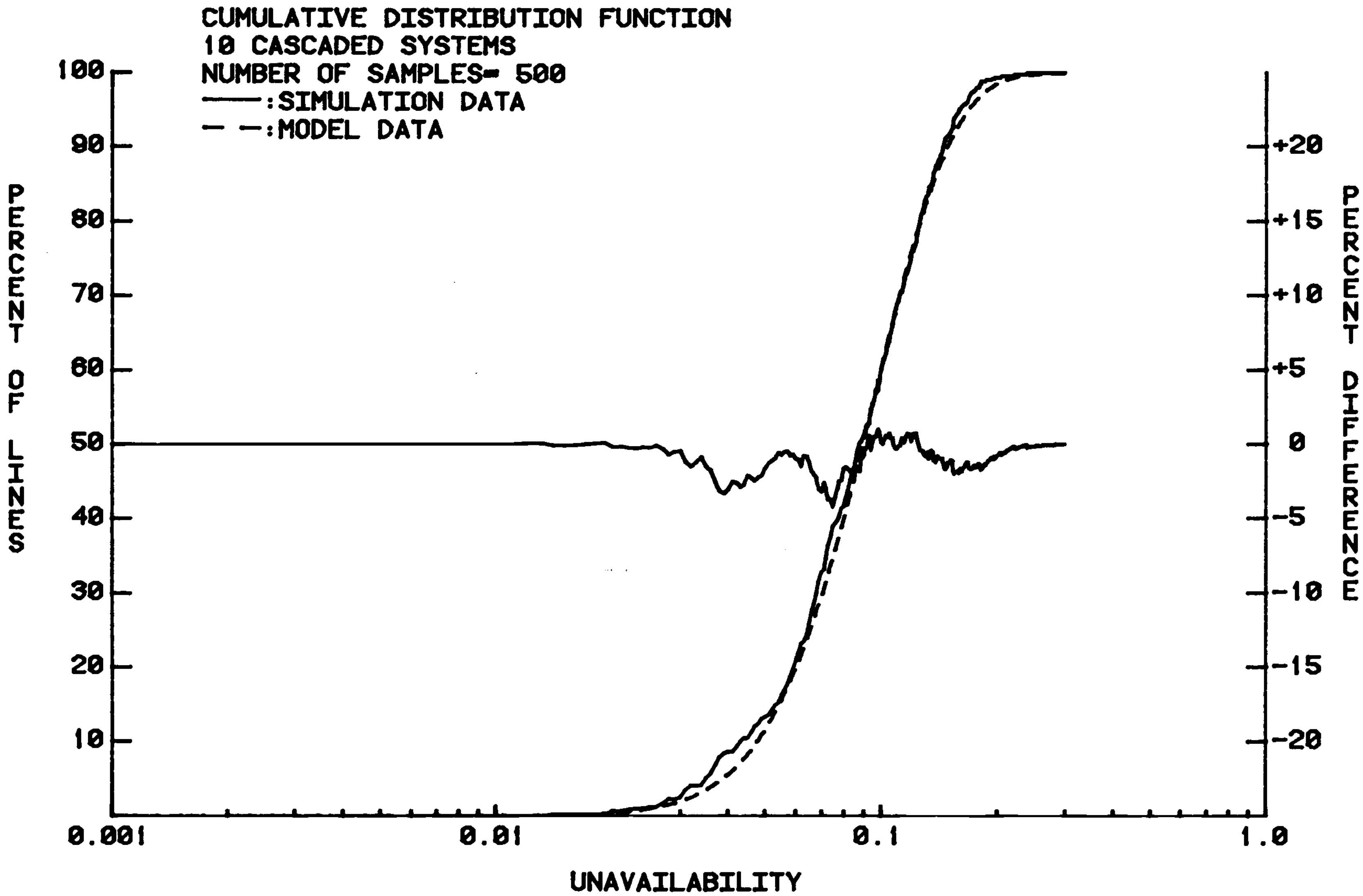

CUMULATIVE DISTRIBUTION FUNCTION
10 CASCADED SYSTEMS
NUMBER OF SAMPLES= 500
———:SIMULATION DATA
— —:MODEL DATA
100
90
80
70
60
50
40
30
20
10
PERCENT OF LINES
+20
+15
+10
+5
0
-5
-10
-15
-20
PERCENT DIFFERENCE
0.001
0.01
0.1
1.0
UNAVAILABILITY

FIGURE 6

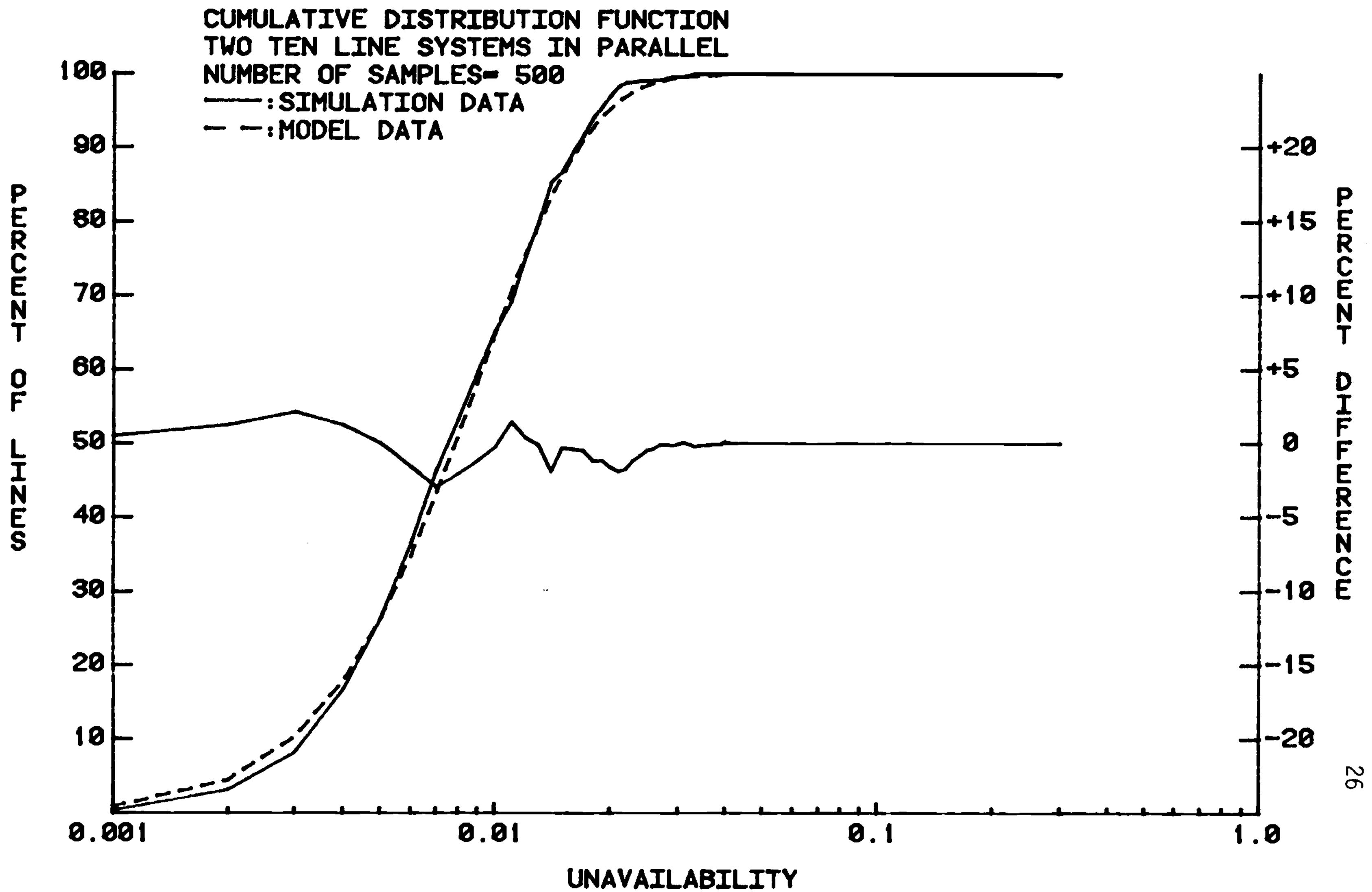
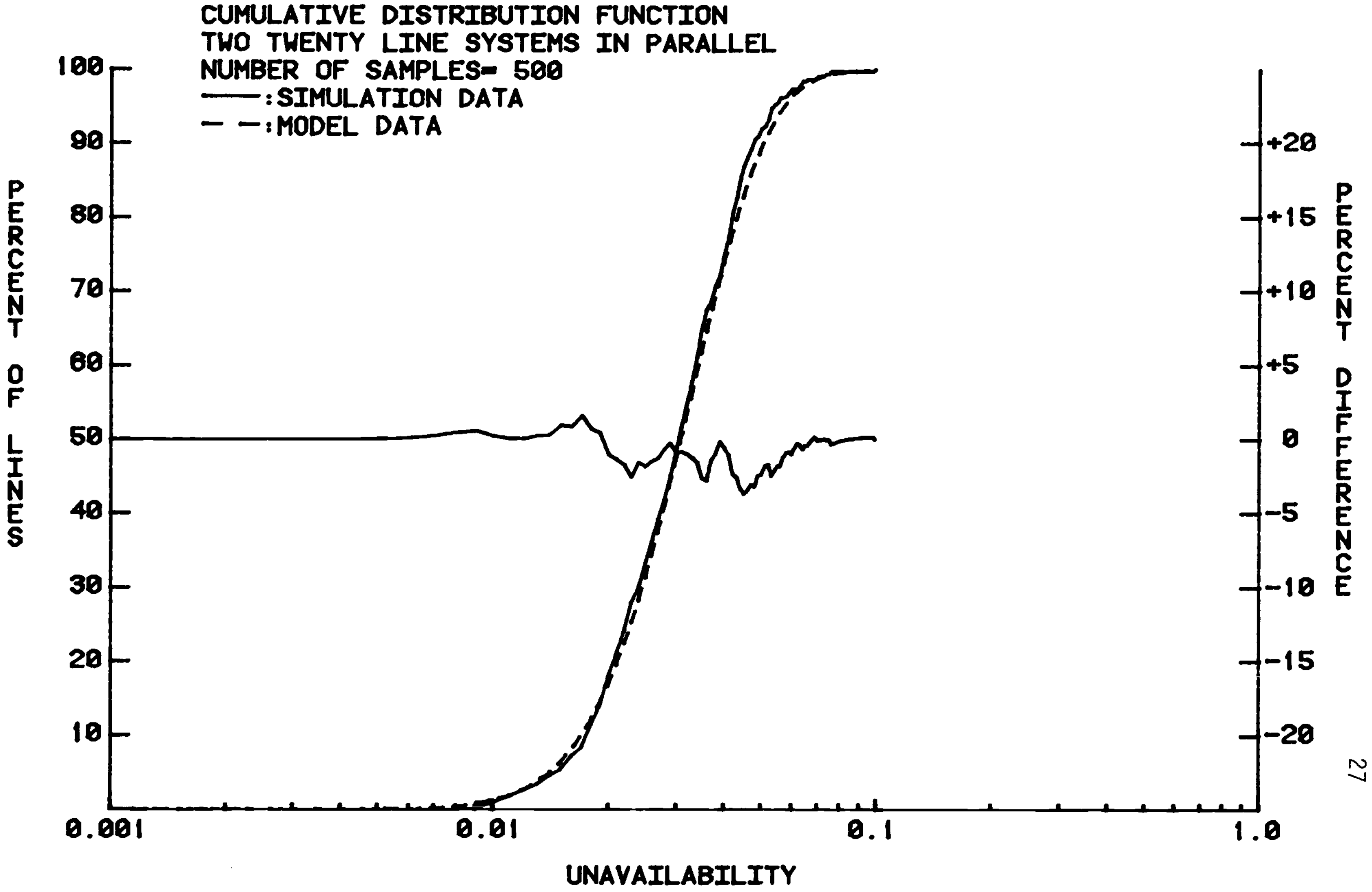

CUMULATIVE DISTRIBUTION FUNCTION
TWO TEN LINE SYSTEMS IN PARALLEL
NUMBER OF SAMPLES= 500
——:SIMULATION DATA
— —:MODEL DATA
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
PERCENT DIFFERENCE
+20
+15
+10
+5
0
-5
-10
-15
-20
0.001
0.01
0.1
1.0
UNAVAILABILITY

FIGURE 7


CUMULATIVE DISTRIBUTION FUNCTION
TWO TWENTY LINE SYSTEMS IN PARALLEL
NUMBER OF SAMPLES= 500
——:SIMULATION DATA
— —:MODEL DATA
PERCENT OF LINES
100
90
80
70
60
50
40
30
20
10
PERCENT DIFFERENCE
+20
+15
+10
+5
0
-5
-10
-15
-20
0.001
0.01
0.1
1.0
UNAVAILABILITY

B. Analysis of Results

As can be seen in Figure 2 the simulation for a single line matches the model within two percentage points for all values of unavailability. This is the verification that the simulation program does actually simulate availabilities obtained for a single line with the assumed Beta density. This case does not involve use of the heuristic model since the model is not needed for the trivial case of a single line.

Figures 3-7 deal with increasingly complex combinations of lines. Again the results are quite good. The conclusion must be that the heuristic model will accurately model complex systems so long as the assumptions that the single line Beta model is an accurate model of a single line and that the line availabilities are statistically independent and valid. Note also that in the regions of interest to designers, say 80th percentile or above, the differences between the model and the simulation and between single line measurement data and the assumed Beta density are both minimal.

The results of parallel combinations of systems are shown in Figures 6 and 7. The differences in availability for ten series systems (Figure 5) and for two ten line systems in parallel (Figure 6) at the same ordinate values are an order of magnitude. Remarkable improvements in system reliability might be possible if truly independent communication paths could be established with reasonable costs. Although this is not normally possible with currently available common carrier systems (which tend to use common equipment for both a primary line and its backup (1, 2, 4)) it may be more practical in the future as specialized common carrier systems, which might supply the alternate paths, become more common.

V. SUMMARY AND CONCLUSIONS

A simulation study has been done in an attempt to verify that a heuristic model for studying the availability of complex communication systems does accurately predict the cumulative distribution function of the availabilities of these networks. The results show that this is the case. With the provisos that the single lines can be modeled by Beta distributions and that they may be assumed to be independent, the heuristic model will produce accurate results.

Since designers now assume independence and that all lines are completely described by their mean availabilities this technique should result in greatly improved predictions of system availabilities. With this technique designers can use Beta density functions for each line in their system, reflecting the fact that Markov et al (1) have shown availabilities are random variables with approximately this distribution. With the model they can then derive a Beta density that approximates the statistics of the complete system. With the density function in hand they will then be able to compute percentile values for specified availabilities and thus be able to apply some level of confidence to their system specifications.

Note, however, that independence for the availabilities of different lines is assumed in this model and that in the Introduction it was stated that dependence is more likely for systems composed of telephone lines. Future work should be aimed at removing the requirement of independence. Before this can be done this dependency must be modeled. Spragins is attacking this problem with the hope that his heuristic technique can be extended to dependent systems.

Communications systems of the future will have much more stringent requirements placed on reliability because of the high cost and high visibility of down time. This will force system designers to develop redundant systems that might truly be independent. Multiple satellite links or multiple microwave links or backup links using different communications median from those used by the primary links might be examples. To prepare for this possibility future investigation should include attempts to generalize the technique to density functions that might model other than voice grade telephone lines and to study extension of the technique to systems including mixtures of different types of communications channels.

The alternative approach is to reduce the need for real-time access to remote locations by including more intelligence at the node as in (3).

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APPENDICES

APPENDIX

There are five programs listed in this appendix. All are written in Tektronix 4051 BASIC. The first program, "Beta Tabled Data File Generation", is a modified form of the Tektronix PLOT 50 "Beta Tabled" application program. This program generates the table of data that is accessed by the simulation program. The second program, "Complex System Simulation and Plot", accesses the table previously generated and does the simulation. The results of the simulation are then plotted. The third program, "Model Parameter Calculation", calculates the values of r and s for use in generating the model curves. The fourth program, "Plot of Specified Beta Unavailability", takes the calculated values of r and s, computes the appropriate Beta distribution and plots it as a dashed line. It also computes and plots the error curve. The last program, "Graph Generation Program", generates the titles and axis labels for the plotted curves.

BETA TABLED DATA FILE GENERATION

```
100 PRINT "LIBETA TABLED DATA FILE GENERATION"
110 PRINT "JA,B";
120 INPUT F1,F2
130 PRINT "JSTART,STOP,STEP SIZE";
140 INPUT C,D,E
150 G=(D-C)/E+1.5
160 G1=INT(G)
170 DIM J(2,G1)
180 C=C-E
190 FOR H=1 TO G1
200 C=C+E
210 P=C
220 P0=P
230 IF F1<0 OR F2<0 THEN 120
240 IF ABS(P-0.5)>0.5 THEN 120
250 IF F1<F2 OR F1>F2 THEN 290
260 IF P<0.5 OR P>0.5 THEN 290
270 Q=0.5
280 GO TO 410
290 A=F1
300 B=F2
310 X=A+B
320 GOSUB 1690
330 Y4=A1
340 X=A
350 GOSUB 1690
360 Y4=Y4-A1
370 X=B
380 GOSUB 1690
390 Y4=Y4-A1
400 GO TO 530
410 A1=10^(7+INT(-LGT(Q)))
420 Q=INT(A1*Q+0.5)/A1
430 J(1,H)=P0
440 J(2,H)=1-Q
450 NEXT H
460 PRINT "GJDONE.....FILE NUMBER";
470 INPUT R
480 FIND R
490 WRITE @33:J
500 PRINT "GJDONE."
510 END
520 GO TO 120
530 T7=0
540 IF P<0.5 THEN 590
```

```
550 T7=A
560 A=B
570 B=T7
580 P=1-P
590 T8=1-(P>0.5)
600 P1=P*T8+(1-P)*(T8=0)
610 T=SQR(LOG(1/(P1*P1)))
620 X=2.515517+T*(0.802853+0.010328*T)
630 Y=1+T*(1.437288+T*(0.189269+0.001308*T))
640 Y=T-X/Y
650 Y=(2*T8-1)*Y
660 X=B*ABS(1-1/(9*B)+Y*SQR(1/(9*B)))^3
670 X1=A+(B-1)*0.5
680 X2=2*X*X*(4*B-X-8)/(B+1+X)
690 X2=X2+(13*B-21)*(B+2)-10*(B-1)*(B+X+0.5)
700 X2=X2/(5760*X1^4)
710 X2=1+(B-1)*(B+1+X)*(1/(24*X1*X1)+X2)
720 X=X*X2
730 IF X/X1<50 THEN 760
740 X2=1.0E-10
750 GO TO 770
760 X2=EXP(-X/X1)
770 Q=X2
780 GO TO 1850
790 X1=X2
800 GOSUB 1300
810 Y1=P-A1
820 Y2=Y1
830 IF Y1*Y2<0 THEN 1090
840 IF Y1<0 THEN 990
850 X1=X2
860 Y1=Y2
870 X3=X2
880 IF X2=>0.1 THEN 910
890 X2=10*X2
900 GO TO 950
910 IF 1-X2<1-1/1.01 THEN 940
920 X2=X2*1.01
930 GO TO 950
940 X2=(X2+1)/2
950 Q=X2
960 GOSUB 1300
970 Y2=P-A1
980 GO TO 830
990 X2=X1
```

```
1000 Y2=Y1
1010 IF X1<0.1 THEN 1040
1020 X1=0.98*X1
1030 GO TO 1050
1040 X1=0.098*X1
1050 Q=X1
1060 GOSUB 1300
1070 Y1=P-A1
1080 GO TO 830
1090 IF ABS(Y1)>0 THEN 1120
1100 Q=X1
1110 GO TO 1270
1120 IF ABS(Y2)>0 THEN 1150
1130 Q=X2
1140 GO TO 1270
1150 Q=(Y1*X2-X1*Y2)/(Y1-Y2)
1160 IF ABS(X1-X2)/X1<1.0E-6 THEN 1270
1170 GOSUB 1300
1180 Y=P-A1
1190 IF Y*Y2>0 THEN 1230
1200 X1=X2
1210 Y1=Y2
1220 GO TO 1240
1230 Y1=Y1/2
1240 Y2=Y
1250 X2=Q
1260 GO TO 1090
1270 IF T7=0 THEN 1290
1280 Q=1-Q
1290 GO TO 410
1300 REM EVALUATE CONTINUED FRACTION
1310 V1=0
1320 A1=(A+B)*Q/(A+1)
1330 IF A1/(1-A1)>0 THEN 1390
1340 V1=B
1350 B=A
1360 A=V1
1370 Q=1-Q
1380 A1=(A+B)*Q/(A+1)
1390 A2=A1/(1-A1)
1400 I=1
1410 B1=A2
1420 B2=1+A2
1430 V=A+2*I
1440 A1=I*(B-I)*Q/(V*(V-1))
```

```
1450 A1=A1*(1+A2)
1460 A2=-A1/(1+A1)
1470 B1=B1*A2
1480 B2=B2+B1
1490 A1=-(A+I)*(A+B+I)*Q/(V*(V+1))
1500 A1=A1*(1+A2)
1510 A2=-A1/(1+A1)
1520 B1=A2*B1
1530 B2=B2+B1
1540 I=I+1
1550 IF ABS(B1/B2)>1.0E-9 THEN 1430
1560 U=B*LOG(1-Q)+A*LOG(Q)-LOG(A)
1570 A1=LOG(B2)+Y4+U
1580 IF A1<-224 THEN 1610
1590 A1=EXP(A1)
1600 GO TO 1620
1610 A1=1
1620 IF V1=0 THEN 1680
1630 A1=1-A1
1640 Q=1-Q
1650 V1=B
1660 B=A
1670 A=V1
1680 RETURN
1690 IF X<10 THEN 1720
1700 GOSUB 1820
1710 RETURN
1720 A2=10-INT(X)
1730 B1=1
1740 FOR I=0 TO A2-1
1750 B1=B1*(X+I)
1760 NEXT I
1770 X=X+A2
1780 GOSUB 1820
1790 X=X-A2
1800 A1=A1-LOG(B1)
1810 RETURN
1820 A1=(X-0.5)*LOG(X)-X+0.5*LOG(2*PI)
1830 A1=A1+1/(12*X)-1/(360*X^3)+1/(1260*X^5)
1840 RETURN
1850 N=0
1860 Q2=Q
1870 GOSUB 1300
1880 Y1=P-A1
1890 D1=Y1/EXP((A-1)*LOG(Q)+(B-1)*LOG(1-Q)+Y4)
```

```
1900 A1=((1-A)/Q+(B-1)/(1-Q))/2
1910 A2=(2*A1*A1+(A-1)/(Q*Q)+(B-1)/(1-Q)^2)/6
1920 B1=(2*(1-A)/Q^3+2*(B-1)/(1-Q)^3)/24
1930 Q1=Q+D1*(1+D1*(A1+D1*(A2+D1*B1)))
1940 N=N+1
1950 IF ABS(Q1-Q)/Q>1.0E-7 THEN 2000
1960 Q=Q1
1970 IF T7=0 THEN 410
1980 Q=1-Q
1990 GO TO 410
2000 N=N+1
2010 Q=Q1
2020 IF N>7 OR Q<0 OR Q>1 THEN 2040
2030 GO TO 1870
2040 Q=Q2
2050 X2=Q
2060 GO TO 790
```

COMPLEX SYSTEM SIMULATION AND PLOT

```
100 REM THIS PROGRAM SIMULATES COMPLEX SYSTEMS AND PLOTS THEIR
110 REM CUMULATIVE DISTRIBUTION FUNCTION.
120 INIT
130 Z=0
140 PRINT "LISERIES PARALLEL SYSTEM SIMULATION"
150 PRINT "JNUMBER OF SERIES SYSTEMS ";
160 INPUT N
170 PRINT "JNUMBER OF PARALLEL SYSTEMS ";
180 INPUT N1
190 PRINT "JSTARTING UNAVAILIBILITY ";
200 INPUT S
210 PRINT "JENDING UNAVAILIBILITY ";
220 INPUT E
230 PRINT "JSTEP SIZE ";
240 INPUT S1
250 PRINT "JNUMBER OF SAMPLES ";
260 INPUT M
270 PRINT "JFILE ";
280 INPUT F8
290 DIM D(2,999)
300 FIND 11
310 READ @33:D
320 C=(E-S)/S1
330 DIM T(C)
340 DIM T1(C)
350 FOR H=1 TO C
360 T(H)=0
370 T1(H)=0
380 NEXT H
390 REM GENERATE THE M SAMPLES OF THE CASCADED SYSTEMS
400 FOR I=1 TO M
410 A1=1
420 A2=1
430 REM CALCULATE THE CASCADED AVAILIBILITY
440 FOR L=1 TO N1
450 FOR J=1 TO N
460 REM SELECT A SAMPLE VIA A UNIFORM RANDOM VARIABLE.
470 I1=INT(1000*RND(-1))
480 IF I1=0 OR I1=1 THEN 470
490 S2=1-D(2,I1)
500 REM COMPUTE THE   AVAILABILITY
510 A1=A1*S2
520 NEXT J
530 A2=A2*(1-A1)
540 A1=1
```

```
550 NEXT L
560 A2=1-A2
570 REM ROUND THE UNAVAILABILITY.
580 U1=INT(1000*(1-A2)*1.0E-3/S1)
590 U2=U1/1000
600 IF U2<S THEN 650
610 IF U2>E THEN 410
620 T(U1)=T(U1)+1
630 GO TO 660
640 REM COUNT THE SAMPLES THAT ARE OUT OF THE PLOT WINDOW.
650 Z=Z+1
660 NEXT I
670 REM INTEGRATE THE DENSITY FUNCTION
680 T1(1)=Z
690 FOR L=2 TO C
700 T1(L)=T(L-1)+T1(L-1)
710 NEXT L
720 REM GENERATE THE SEMILOG PLOT OF % OF LINES WITH EQUAL OR BETTER
730 REM AVAILIBILITY(UN) AS A FUNCTION OF LOG(UNAVAILIBILITY).
740 VIEWPORT 15,140,10,90
750 WINDOW LGT(1.0E-3),0,0,100.001
760 MOVE @1:LGT(S),T1(1)*100/M
770 FOR P=2 TO C
780 S=S+S1
790 DRAW @1:LGT(S),T1(P)*100/M
800 NEXT P
810 FIND F8
820 WRITE @33:T1
830 END
```

MODEL PARAMETER CALCULATION

```
100 PRINT "LJMODEL PARAMETER CALCULATION"
110 PRINT "↓NUMBER OF CASCADED SYSTEMS ";
120 INPUT N
130 PRINT "↓NUMBER OF PARALLEL SYSTEMS ";
140 INPUT N1
150 PRINT "↓PARAMETERS OF SINGLE SYSTEM MODEL (A,B) ";
160 INPUT A,B
170 A1=(A/(A+B))^N
180 A2=(A*(A+1)/((A+B)*(A+B+1)))^N
190 A3=1-(1-A1)^N1
200 A4=1-2*(1-A1)^N1+(1-2*A1+A2)^N1
210 A0=A3*(A3-A4)/(A4-A3^2)
220 B0=(1-A3)*(A3-A4)/(A4-A3^2)
250 PRINT "↓MODEL A=";A0
260 PRINT "↓MODEL B=";B0
270 END
```

PLOT OF SPECIFIED BETA UNAVAILABILITY

```
100 PRINT 'LIPLOT OF SPECIFIED BETA UNAVAILIBILITY'
110 INIT
120 PRINT 'JA,B ';
130 INPUT A,B
140 PRINT 'JSTARTING UNAVAILIBILITY ';
150 INPUT S
160 PRINT 'JENDING UNAVAILIBILITY ';
170 INPUT E
180 PRINT 'JSTEP SIZE ';
190 INPUT S1
200 PRINT 'JFILE ';
210 INPUT F8
220 G=INT((E-S)/S1)
230 DIM T(G)
240 DIM T1(G)
250 FOR H=1 TO G
260 T(H)=0
270 T1(H)=0
280 NEXT H
290 Z=S-S1
300 FOR R=1 TO G
310 Z=Z+S1
320 Q=1-Z
330 IF A<0 OR B<0 OR Q<0 OR Q>1 THEN 120
340 IF A>500 AND B>500 THEN 1250
350 X=A+B
360 GOSUB 1000
370 Y4=A1
380 X=A
390 GOSUB 1000
400 Y4=Y4-A1
410 X=B
420 GOSUB 1000
430 Y4=Y4-A1
440 GOSUB 700
450 IF A1=0 THEN 480
460 A2=10^(5+INT(-LGT(A1)))
470 A1=INT(A1*A2+0.5)/A2
480 T(R)=A1
490 NEXT R
500 N1=-1
510 N2=0.5
520 N9=1
530 V0=80/125
540 W0=100.1/-LGT(1.0E-3)
```

```
550 VIEWPORT 15,140,10,90
560 WINDOW LGT(1.0E-3),0,0,100.1
570 X4=LGT(S)
580 Y4=T(1)*100
590 GOSUB 1600
600 FOR P=2 TO G
610 S=S+S1
620 X4=LGT(S)
630 Y4=T(P)*100
640 GOSUB 1730
650 NEXT P
660 FIND F8
670 WRITE @33:T
680 END
690 REM EVALUATE CONTINUED FRACTION
700 V1=0
710 A1=(A+B)*Q/(A+1)
720 IF A1/(1-A1)>0 THEN 780
730 V1=B
740 B=A
750 A=V1
760 Q=1-Q
770 A1=(A+B)*Q/(A+1)
780 A2=A1/(1-A1)
790 I=1
800 B1=A2
810 B2=1+A2
820 V=A+2*I
830 A1=I*(B-I)*Q/(V*(V-1))
840 A1=A1*(1+A2)
850 A2=-A1/(1+A1)
860 B1=B1*A2
870 B2=B2+B1
880 A1=-(A+I)*(A+B+I)*Q/(V*(V+1))
890 A1=A1*(1+A2)
900 A2=-A1/(1+A1)
910 B1=A2*B1
920 B2=B2+B1
930 I=I+1
940 IF ABS(B1/B2)>10^-9 THEN 820
950 U=B*LOG(1-Q)+A*LOG(Q)-LOG(A)
960 A1=LOG(B2)+Y4+U
970 IF A1<-224 THEN 1000
980 A1=1-EXP(A1)
990 GO TO 1010
```

```
1000 A1=1
1010 IF V1=0 THEN 1070
1020 A1=1-A1
1030 Q=1-Q
1040 V1=B
1050 B=A
1060 A=V1
1070 RETURN
1080 IF X<10 THEN 1110
1090 GOSUB 1210
1100 RETURN
1110 A2=10-INT(X)
1120 B1=1
1130 FOR I=0 TO A2-1
1140 B1=B1*(X+I)
1150 NEXT I
1160 X=X+A2
1170 GOSUB 1210
1180 X=X-A2
1190 A1=A1-LOG(B1)
1200 RETURN
1210 A1=(X-0.5)*LOG(X)-X+0.5*LOG(2*PI)
1220 A1=A1+1/(12*X)-1/(360*X^3)+1/(1260*X^5)
1230 RETURN
1240 REM NORMAL APPROXIMATION
1250 B1=(B-1/3+0.02/B)*Q-(A-1/3+0.02/A)*(1-Q)+(0.02*Q+0.01)/(A+B)
1260 X=(B-0.5)/((A+B-1)*(1-Q))
1270 GOSUB 1430
1280 A2=Q*B2
1290 X=(A-0.5)/((A+B-1)*Q)
1300 GOSUB 1430
1310 A2=A2+(1-Q)*B2
1320 X=B1*SQR((1+A2)/((A+B-5/6)*(1-Q)*Q))
1330 IF ABS(X)>20 THEN 1410
1340 T=1/(1+0.2316419*ABS(X))
1350 A1=T*(0.31938153+T*(-0.356563782+1.781477937*T))
1360 A1=A1+T^4*(-1.82155978+1.33027429*T)
1370 A1=SQR(1/(2*PI))*EXP(-X*X/2)*A1
1380 IF X=>0 THEN 480
1390 A1=1-A1
1400 GO TO 450
1410 A1=0
1420 GO TO 1380
1430 IF X=0 OR X=1 THEN 1460
1440 B2=(1-X*X+2*X*LOG(X))/(1-X)^2
```

```
1450 RETURN
1460 B2=0
1470 RETURN
1480 REM ... DASHED LINE FOR X AND Y ENTRIES ...
1490 REM
1500 REM     X4 ... X
1510 REM     Y4 ... Y
1520 REM
1530 REM     N1 ... Dash length
1540 REM     N2 ... Dash/(dash+space)
1550 REM
1560 REM     N9 ... Display address
1570 REM     V0 ... Y to X viewport ratio
1580 REM     W0 ... Y to X window ratio
1590 REM
1600 REM ... Initialization for starting with dash ...
1610 N4=0
1620 GO TO 1650
1630 REM ... Initialization for starting with space ...
1640 N4=N2
1650 MOVE @N9:X4,Y4
1660 X5=X4
1670 Y5=Y4
1680 X8=X4
1690 Y8=Y4
1700 N8=ABS(N1)/((N1>0)+(N1<0)*W0)/N2
1710 RETURN
1720 REM ... Branch point for drawing dashes and spaces ...
1730 X6=0
1740 Y6=SQR(((Y4-Y8)*V0/W0)^2+(X4-X8)^2)
1750 IF Y6=0 THEN 1780
1760 X6=(X4-X8)/Y6
1770 Y6=(Y4-Y8)/Y6
1780 N5=SQR(((Y4-Y5)*V0/W0)^2+(X4-X5)^2)
1790 N6=(N2*(N4<N2)+(N4=>N2)-N4)*N8
1800 X5=X5+X6*(N5 MIN N6)
1810 Y5=Y5+Y6*(N5 MIN N6)
1820 IF N4=>N2 THEN 1870
1830 REM ... Dash ...
1840 DRAW @N9:X5,Y5
1850 GO TO 1880
1860 REM ... Space ...
1870 MOVE @N9:X5,Y5
1880 N4=N2*(N6<=N5)*(N4<N2)+(N4+N5/N8)*(N6>N5)
1890 IF N6<=N5 THEN 1780
```

```
1900 N4=N4*(N4<1)
1910 X8=X4
1920 Y8=Y4
1930 RETURN
```

GRAPH GENERATION PROGRAM

```
100 REM THIS PROGRAM GENERATES THE BASIC GRAPH FOR THE CUMULATIVE
110 REM DISTRIBUTION FUNCTIONS
120 WINDOW 0,150,0,100
130 VIEWPORT 0,150,0,100
140 MOVE @1:25,95
150 PRINT "LIPLOT ROUTINE"
160 PRINT "JNUMBER OF SAMPLES ";
170 INPUT S4
180 PRINT "JSTARTING UNAVAILIBILITY ";
190 INPUT S
200 PRINT "JENDING UNAVAILIBILITY ";
210 INPUT E
220 PRINT "JSTEP SIZE ";
230 INPUT S1
240 PRINT @1,7:
250 PRINT @1:"CUMULATIVE DISTRIBUTION FUNCTION"
260 MOVE @1:25,92
270 PRINT "JTITLE ";
280 INPUT T$
290 PRINT @1:T$
300 PRINT "JFILES ";
310 INPUT F8,F9
320 MOVE @1:25,89
330 PRINT @1:"NUMBER OF SAMPLES=",S4
340 MOVE @1:15,10
350 DRAW @1:15,90
360 MOVE @1:15,10
370 DRAW @1:140,10
380 DRAW @1:140,90
390 MOVE @1:15,10
400 FOR I=1 TO 9
410 A=8*I+10
420 MOVE @1:15,A
430 DRAW @1:17,A
440 MOVE @1:9,A-0.6
450 PRINT @1:I*10
460 NEXT I
470 MOVE @1:15,90
480 DRAW @1:17,90
490 MOVE @1:7.218,89.4
500 PRINT @1:100
510 MOVE @1:4,76
520 PRINT @1:"P"
530 MOVE @1:4,73.5
540 PRINT @1:"E"
```

```
550 MOVE @1:4,71
560 PRINT @1:"R"
570 MOVE @1:4,68.5
580 PRINT @1:"C"
590 MOVE @1:4,66
600 PRINT @1:"E"
610 MOVE @1:4,63.5
620 PRINT @1:"N"
630 MOVE @1:4,61
640 PRINT @1:"T"
650 MOVE @1:4,56
660 PRINT @1:"O"
670 MOVE @1:4,53.5
680 PRINT @1:"F"
690 MOVE @1:4,48.5
700 PRINT @1:"L"
710 MOVE @1:4,46
720 PRINT @1:"I"
730 MOVE @1:4,43.5
740 PRINT @1:"N"
750 MOVE @1:4,41
760 PRINT @1:"E"
770 MOVE @1:4,38.5
780 PRINT @1:"S"
790 FOR I=1 TO 9
800 MOVE @1:140,8*I+10
810 DRAW @1:138,8*I+10
820 MOVE @1:140.9,8*I+10-0.6
830 Q=(I-5)*5
840 IF Q<0 THEN 900
850 IF Q=0 THEN 880
860 PRINT @1:"+";Q
870 GO TO 910
880 PRINT @1:" ";Q
890 GO TO 910
900 PRINT @1:"-";ABS(Q)
910 NEXT I
920 MOVE @1:10.5,7
930 PRINT @1:"0.001"
940 MOVE @1:15+125/3,10
950 DRAW @1:15+125/3,11
960 MOVE @1:11.4+125/3,7
970 PRINT @1:"0.01"
980 MOVE @1:15+83.33,10
990 DRAW @1:15+83.33,11
```

```
1000 MOVE @1:12.3+83.33,7
1010 PRINT @1:"0.1"
1020 MOVE @1:12.3+125,7
1030 PRINT @1:"1.0"
1040 MOVE @1:62.5+2.5,2
1050 PRINT @1:"UNAVAILABILITY"
1060 MOVE @1:25,86
1070 MOVE @1:25,87
1080 DRAW @1:31,87
1090 MOVE @1:31.5,86
1100 PRINT @1:":SIMULATION DATA"
1110 MOVE @1:25,84
1120 DRAW @1:27,84
1130 MOVE @1:29,84
1140 DRAW @1:31,84
1150 MOVE @1:31.5,83
1160 PRINT @1:":MODEL DATA"
1170 MOVE @1:15,10
1180 FOR I=1 TO 3
1190 FOR K=1 TO 10
1200 MOVE @1:15+125/3*LGT(K)+125/3*(I-1),10
1210 RDRAW @1:0,0.5
1220 NEXT K
1230 NEXT I
1240 MOVE @1:148.5,76
1250 PRINT @1:"P"
1260 MOVE @1:148.5,73.5
1270 PRINT @1:"E"
1280 MOVE @1:148.5,71
1290 PRINT @1:"R"
1300 MOVE @1:148.5,68.5
1310 PRINT @1:"C"
1320 MOVE @1:148.5,66
1330 PRINT @1:"E"
1340 MOVE @1:148.5,63.5
1350 PRINT @1:"N"
1360 MOVE @1:148.5,61
1370 PRINT @1:"T"
1380 MOVE @1:148.5,58
1390 PRINT @1:"D"
1400 MOVE @1:148.5,53.5
1410 PRINT @1:"I"
1420 MOVE @1:148.5,51
1430 PRINT @1:"F"
1440 MOVE @1:148.5,48.5
```

```
1450 PRINT @1:"F"
1460 MOVE @1:148.5,46
1470 PRINT @1:"E"
1480 MOVE @1:148.5,43.5
1490 PRINT @1:"R"
1500 MOVE @1:148.5,41
1510 PRINT @1:"E"
1520 MOVE @1:148.5,38.5
1530 PRINT @1:"N"
1540 MOVE @1:148.5,36
1550 PRINT @1:"C"
1560 MOVE @1:148.5,33.5
1570 PRINT @1:"E"
1580 FIND F8
1590 C=(E-S)/S1
1600 DIM T(C)
1610 DIM T1(C)
1620 READ @33:T
1630 FIND F9
1640 READ @33:T1
1650 VIEWPORT 15,140,10,90
1660 WINDOW LGT(1.0E-3),0,0,50.001
1670 MOVE @1:LGT(S),100*(T1(1)-T(1)/S4)+25
1680 FOR P=2 TO C
1690 S=S+S1
1700 DRAW @1:LGT(S),100*(T1(P)-T(P)/S4)+25
1710 NEXT P
1720 END
```