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Title: ECONOMIC PRODUCTIVITY OF WATER AND RELATED INPUTS IN THE AGRICULTURE OF SOUTHERN IDAHO

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Estimates of soil and water productivity in agriculture have historically been limited both by the complexity and limited availability of the large quantity of data required. Analyses have been restricted to the use of either secondary data, or to experimental data. The experimental data are limited in the sense of restricted applicability, while secondary data do not address themselves to the productivity of the land resource.

The purpose of this study was to examine a new set of data to (1) determine if it could be used to obtain a point estimate of the marginal value of water and fertilizer, and (2) derive from the set of data, land productivity relationships which could be used over time to account for variation in land quality in production analyses using secondary data.

The new set of data is the Land Inventory and Productivity

System (LIPS) developed by the Economic Research Service and the Soil Conservation Service of U.S.D.A. The data set is based upon 1966 Conservation Needs Inventory data developed by Soil Conservation Service, U.S.D.A. From the basic land capability information, a set of observation units were developed and estimates of cropping patterns, yields, water use, and fertilizer applications made for each.

These observational units, soil resource groups (SRGs), were classified by county and land resource area. The land resource area (LRA) is defined as a geographical area with relatively homogeneous climate, precipitation, elevation, and natural cover. Eight land resource areas were used in this analysis.

A regression analysis was used to estimate a production function for each of the five crops chosen for analysis; barley, wheat, hay, sugar beets, and potatoes. The marginal value product of water was determined for each crop by LRA.

An index of productivity was determined for LRAs from the results of the regression analysis used to estimate the marginal value product of water and fertilizer. This LRA index was used to estimate a productivity index for the counties in the area. The county productivity index identifies relatively stable relationships associated with the LRAs. These index numbers will add basic productivity information to secondary data which are published on a county basis with no reference to the relative productivity of the land. The yield reported in county figures implies a level of factor input use and cannot be used directly to estimate the relative productivity of the land among counties. With the productivity index developed in this analysis, the contribution of land quality to the production of crops will be identifiable. From an econometric standpoint, the statistical equations used to estimate production functions will be more fully specified, and the specification bias will be reduced.

The productivity index was applied to a set of secondary data used in a prior analysis to estimate production functions for approximately the same area of Southern Idaho as was used in this analysis. Multicolinearity among the variables was identified as a problem in the earlier research. Ridge Regression analysis was used to circumvent the problem of multicolinearity in the data. The results of the Ridge Regression analysis were encouraging in that the estimates were obtained for all of the identified variables.

The analyses in this paper indicate that (1) the data lend themselves to production function analysis, (2) an index of productivity can be used in conjunction with published secondary data to better specify models used to estimate production functions, and (3) Ridge Regression may be used to alleviate problems of multicolinearity so that estimates for all relevant variables can be obtained.

Economic Productivity of Water and Related Inputs in the Agriculture of Southern Idaho

by

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ECONOMIC PRODUCTIVITY OF WATER AND RELATED INPUTS IN THE AGRICULTURE OF SOUTHERN IDAHO

INTRODUCTION

Irrigation and the availability of water for irrigation has historically been, and will likely continue to be, an important factor in the development of the agricultural and related sectors of the economy of the Pacific Northwest. Over six million acres were irrigated in the Pacific Northwest in 1966, accounting for over 30 percent of total cropland in the states of Idaho, Oregon, and Washington. This compares to about 8.4 percent for the United States. Population, industry, and agriculture tend to be concentrated in those areas where water is either readily available or has become available through development of water supplies for irrigation and other uses, such as recreation and electric power generation.

Water and land as natural resources are used as inputs by agricultural, industrial, residential, recreational, and other commercial users. Some of these uses directly compete for both water and land with agriculture. As population increases, the competition for these natural resources increases, and the evaluation of the economic returns from the several uses becomes increasingly important.

In the Northwest, irrigation is by far the most extensive user of water. Ruttan (1965) attests to this fact:

> In the western United States, still upwards of 90 percent of all water use is for irrigation purposes. Since irrigation is the most consumptive of water uses, an even higher percentage of total water losses is accounted for by this activity. Moreover, there are continuing large programs for further investment in irrigation by the federal government. In some areas conflicts between irrigation water use and other water uses have already developed.

Competition for available lands is also an important factor in planning resource development. Urbanization, highway and airport construction, industrial expansion, etc., all compete for the use of the land resource. As population increases, the intensity with which the land resource is used increases and competition for available agricultural land increases both by competition from outside and within the agricultural industry.

In order to plan for the needs of a growing population, planners need an accurate estimate of the value of water in alternative uses so that the most productive use of the water resource may be encouraged.

John V. Krutilla (1966) in his foreword to V. W. Ruttan's work states:

> In a country as richly endowed with water resources as the United States the task of water resource management and development should be influenced less by fears of absolute water shortage than by concern over excessive use for some purposes which serve to inhibit or exclude other more valuable uses...... Estimates of the marginal value productivity of water in alternative applications in the several supply regions, therefore, are essential for the prudent management of existing

> supplies--and as guides to the level, rate, and character of development of new supplies.

These estimates of Marginal Value Products are important on at least two levels associated with the planning process: (1) the inter firm/inter industry allocation of existing water resources, and (2) the estimation of the impact on agricultural output of new water resource developments.

Estimates of soil and water productivity in agriculture have been historically limited both by the complexity and limited availability of the large quantity of data required. Analyses have been restricted to the use of either secondary data, or to limited experimental data. The experimental data are limited in the sense of restricted applicability while the secondary data available are not able to account for the productivity of the land resource.

The land resource is an important factor in the determination of cropping patterns and output of the agricultural industry. Thomas and Grano (1972) state:

> The land resource is a major determinant of cropping patterns, management practices, and yield. The land resource is a comprehensive concept which includes the natural factors of climate, geography, geology, topography, and soils; and the cultural factors of manmade improvements and location with respect to centers of commerce and population. The use of the land resource reflects variable economic conditions as well as political and social implications, which must be taken into account in their evaluation.

It is important to note that other factors play a major role in the productivity of agriculture. These factors--capital, management, and technology--are to be held constant in this study in an effort to isolate basic productivity relationships between elements of the land resource.

The land resource is also important in the determination of the location of centers of commerce, industrial activity, and consequently population. Planners of industrial and urban expansion should consider the productivity of the land available for expansion, and make their plans according to accepted economic and environmental considerations. Too often, urban expansion consumes the most productive agricultural land because of its proximity to current population centers.

The productivity of water, fertilizer, capital, and other inputs to agricultural production is different when utilized in combination with land of varying quality. Planning for resource development in agriculture must take into account these differences in productivity if economic efficiency is to be a consideration in resource development. Farmers could possibly increase profits if the relative productivity of their inputs in combination with the land available on their farms were known.

Land and water productivity estimates are important then for land use planning, extrapolation to farm firm decisions, urban and associated expansion, and especially for resource development.

Historically the value of water has been estimated using one of four basic methods: (1) residual income, (2) linear programming analysis, (3) production function analysis, (4) residual land value. The residual income approach assumes the value of water to be the residual of the increase in income, resulting from irrigation development, after all other factor costs have been paid. Shadow prices for water, reflect the value of water and the other factors used in fixed proportions with water which are not limiting at the optimum solution of the linear programming analysis. While in production function analysis, the value of the marginal product reflects the value of the input at the margin; that is, the return in terms of the value of output obtained from the addition of the last unit of an input, while holding all other variable inputs constant. The value of the marginal product is obtained by differentiating a statistically estimated production function. The residual land value approach ascribes the residual of the increase in property value, due to irrigation development, to water after all improvements on the land associated with the irrigation development have been properly accounted for and paid.

Water values serve two distinct purposes in our economic-political system: (1) allocation of planned development expenditures and the ensuing use of the water resource among competing users using economic efficiency criterion as the basis for allocation decisions, (2) derivation of repayment rates for public irrigation development.

It is important to recognize the difference between these two purposes when appraising the various methods of estimating water values. The major difference between the two purposes of water values is that secondary and higher order benefits are often imputed to water values for allocation purposes, whereas repayment relates to the ability to pay concept which is based upon primary or direct benefits.

In this analysis, we will deal with primary benefit estimation; that is, the repayment purpose of water values. The reason for this direction is that although secondary benefits are important in the decision-making process, secondary benefits tend to increase as primary benefits increase.

The relationship between primary and secondary benefits is not the same for all alternative developments, but the secondary benefits from a particular alternative will always vary directly with the primary benefits derived from that alternative.

A workable definition is necessary to discuss primary benefits to agriculture. Stewart (1964) defines direct (primary) agricultural benefits of water development as . . . "the value of farm production estimated with project development in excess of farm production estimated without the project, less the value of additional farm inputs or associated costs" (p. 109). This definition fits all four methods of estimating value of irrigation water discussed above, including the fourth method where the residual land values assume that the value of the increase in farm production is capitalized into the value of the land.

Two sources of data for analysis of water value in agriculture have been used historically. First a value can be imputed to water by estimating the water requirements of an experimental cropping unit over time through the water year, and then determine through experiment, the impact on production of a water shortage at critical periods. The difficulty of adjusting results of controlled experimental research to apply to real-world situations, limits this type of analyses. It is a meaningful exercise to develop a value for water under experimentally ideal situations; however, the use of water is real and there are real institutional and cultural limitations which often prevent wide spread adoption to a variety of real-world situations. The value of water then, must be determined, based upon how the water is used, not how it might be used.

The second course available to estimate water values is fitting equations to secondary data which relate the level of water use to varying levels of production on an aggregate basis. This aggregate is often a county or some combination of counties. These estimated equations are interpreted as production functions from which a water value can be derived.

Lack of adequate data historically has limited the use of empirical estimation of water value. The secondary data available do not account for variations in factors affecting yields associated with physical and climatic properties of the land resource. On the other hand, results obtained from experimental data may have restricted applicability when extended to non-experimental conditions. Estimation of water values from secondary data must be done on an ex-poste[1] basis, which limits the analysis to one of what happened in the past instead of what is the potential for the future. Experimental analyses do account to a certain extent for potential, but due to the locational fixity of the results, extrapolation to the broad agricultural base is at best tenuous.

The estimation of water values should be oriented toward potential development as well as evaluation of the present situation. Consequently, a set of data is required which is based upon those relatively fixed elements of the agricultural production base which affect relative obtainable yields. This data base could be oriented toward estimation of water values for future irrigation development decisions.

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[1] Ex-poste in the sense that only the present cropland base can be considered. Using a land resource base approach, it is possible to identify the potential of yet uncultivated or undeveloped lands.

In order to make consistent evaluations of land and water productivity and their associated inputs, a consistent set of data which reflects the productive capacity of the resource base is required. Most secondary data sources do not contain the capability of determining land productive capacity except as an average for a particular political division of the land resource such as a county or township.

Several land classification systems have been developed for purposes other than productivity including, irrigability using physical or topographical criteria, determination of conservation needs from physical criteria, etc. None of these classifications have historically been constructed to reflect the relative productivity and costs of the various units within a given system. Imposition of additional criteria on one of these systems to reflect land productivity variation would require a reformulation of the intent or objectives of the classification, and becomes a colossal undertaking. New definitions must be imposed on the existing data, and without recourse to the rudimentary basic data from which the classifications were made would be an impossible task.

The Economic Research Service, Natural Resource Economics Division in conjunction with the Soil Conservation Service, has developed the necessary criteria to establish a data base which would reflect the relative productivity of groups of soils within the land resource areas defined in the 1967 Conservation Needs

Inventory. The basic building block for this inventory is a sample of the soil and land resource. The sample observations were identified by soil capability and classified by climatic areas. These sample data were expanded to the accepted acreage of land in each county to be included in the inventory. Definitions and criteria were supplied to the Soil Conservation Service to facilitate the organization of these expanded observations into a unit which would relate the characteristics of the soil to its productivity. These characteristics are explained in greater detail in the following section.

Utilizing data from this classification system, a consistent set of productivity estimates was obtained for land areas covered by the Land Inventory and Productivity System (LIPS). Water and fertilizer applications were obtained for each unit in the LIP System. The data will eventually be available on a fairly consistent basis for all of the eleven western states, with some minor modification for all states west of the Mississippi River, and eventually for many states in the east.

The development of a methodology for estimating the productivity of land, water, and related inputs from such a broad based set of data will be beneficial to planners in many types of planning capacities wherever the data are available.

Marginal value product (MVP) estimates obtained from this analysis will be useful in allocating water as well as providing information from which a price for water can be determined. The MVP information would also be useful for optimizing the allocation of inputs among crops and for maximizing value of output and benefits from a resource development.

A secondary exercise will be to obtain a productivity index for the eight land resource areas used in the analysis. Such an index may be useful to potential users of the land resource, those involved in projecting output based on projected yields, land use planners, researchers using secondary agricultural data to estimate production relationships, and others.

The productivity information and the methodology developed for this analysis would be useful in comprehensive planning in the Pacific Northwest.

Objectives

1. Estimate a production function for each of five major crops in Southern Idaho--barley, wheat, hay, potatoes, and sugar beets.

2. Analyze the parameter estimates from the production function analysis and derive the relevant economic parameters to appraise (1) the economic efficiency of current combinations of the relevant factors of production; (2) differences in productivity between homogeneous land resource areas; (3) production response of the various land areas to resource development; and (4) the usefulness

of this type of analysis in solving problems of resource development, related to the value of associated outputs.

3. Derive an index of productivity for land resource areas by county and illustrate the application of the index to economic analysis.

4. Appraise the usefulness of these results in solving problems in water resource planning, analysis concerned with making projections, and other possible uses to which it may be applied.

Procedures

1. Review and critique historical methods of estimating the values of water in irrigated agriculture.

2. Determine an appropriate model for estimating production functions for each of the five crops chosen for analysis. These functions will be estimated using the land inventory data developed for the Economic Research Service's National Inter-regional Agricultural Production Systems' Land Inventory and Productivity System (LIPS).

3. Aggregate Soil Resource Groups (SRGs) with assistance from Soil Conservation Service soil scientists to obtain a workable number of SRGs which will retain a relatively high level of homogeneity with respect to the soil factors that most affect yield.

4. Estimate production functions for each of five major crops in the area.

5. Calculate marginal value productivities for water

and fertilizer, assuming fixed output prices for all crops. The MVP curves will assume different forms depending upon the specifications of the mathematical model.

6. Examine the parameters estimated using statistical tests to draw inferences about their individual significance, and the significance of differences between the parameters estimated for the factors of production in their application to the several crops.

7. Compare results of this analysis with results from other economic studies that have estimated the marginal value productivity of water.

8. Develop subjective and objective tests to determine the accuracy, validity, and applicability of the results of data analysis based upon the Land Inventory and Productivity System.

9. Summarize and develop recommendations for water resource planners.

Source of Data

The data used in this analysis were obtained from the Land Inventory and Productivity System (LIPS) developed by the Natural Resource Economics Division of the Economic Research Service, U.S.D.A., as part of the Columbia-North Pacific Framework Study, and as an input to the Division's National Inter-regional Agricultural Production System (NIRAPS). The LIPS data were developed using the 1967 Conservation Needs Inventory (Soil Conservation Service (SCS), U.S.D.A.), as the basic source of soils information.

The SCS classified the soils into eight classes, four subclasses, and a multiplicity of units which represent the soil series from which the observations were taken. This level of classification is called the Land Capability Unit (LCU). The lands so classified are non-federal, rural lands in the following categories of ownership:

1. Private persons or corporations
2. States, counties, or municipalities
3. Indian lands (individual and tribal)
4. Corporations partly owned by the United States, such as Production Credit Associations
5. Cropland federally owned, but operated by private individuals or corporations under lease.

The Economic Research Service, Natural Resource Economics Division, developed the concept of the Soil Resource Group (SRG) to meet the land resource data needs of its projections and evaluation programs. SRGs are groups of land capability units, with a specific range of soil characteristics which influence the productivity of the land resource. The Land Capability Units (LCUs) from the Conservation Needs Inventory (CNI) were classified into homogeneous soil groups. The criteria for grouping the LCUs into SRGs, included relative homogeneity with respect to slope, texture, permeability of the sub-strata, and suitability for similar types of crops, selected input requirements, management, and yields.

Forms were developed by ERS for use by Soil Conservation Service personnel to estimate crop acreages, fertilizer application, yield, and water use for each basic SRG observation. The basic unit of observation is the SRG identified by Land Resource Area (LRA) and county. An LRA is a geographical area with relatively homogeneous climate, precipitation, elevation, and natural cover. In the Southern Idaho area for which this analysis is being made, there are portions of nine LRAs. The LRAs, delineated by the Soil Conservation Service for the Conservation Needs Inventory, are defined for the relevant area first by major land resource region, and then subdivided to the LRAs. The nine LRAs in the Southern Idaho area represent three major land resource regions defined as;(Figure 1)

(1) Region B, the "Northwestern wheat, range, and irrigated resource region."

(2) Region D, the "Western ranges and irrigated resource region," and

(3) Region E, the "Rocky Mountain range and Forest Region."

Region B is the most important irrigated region in the area and was subdivided into 11 Land Resource Areas (LRAs), seven of which occur in Southern Idaho. These seven LRAs are defined as:

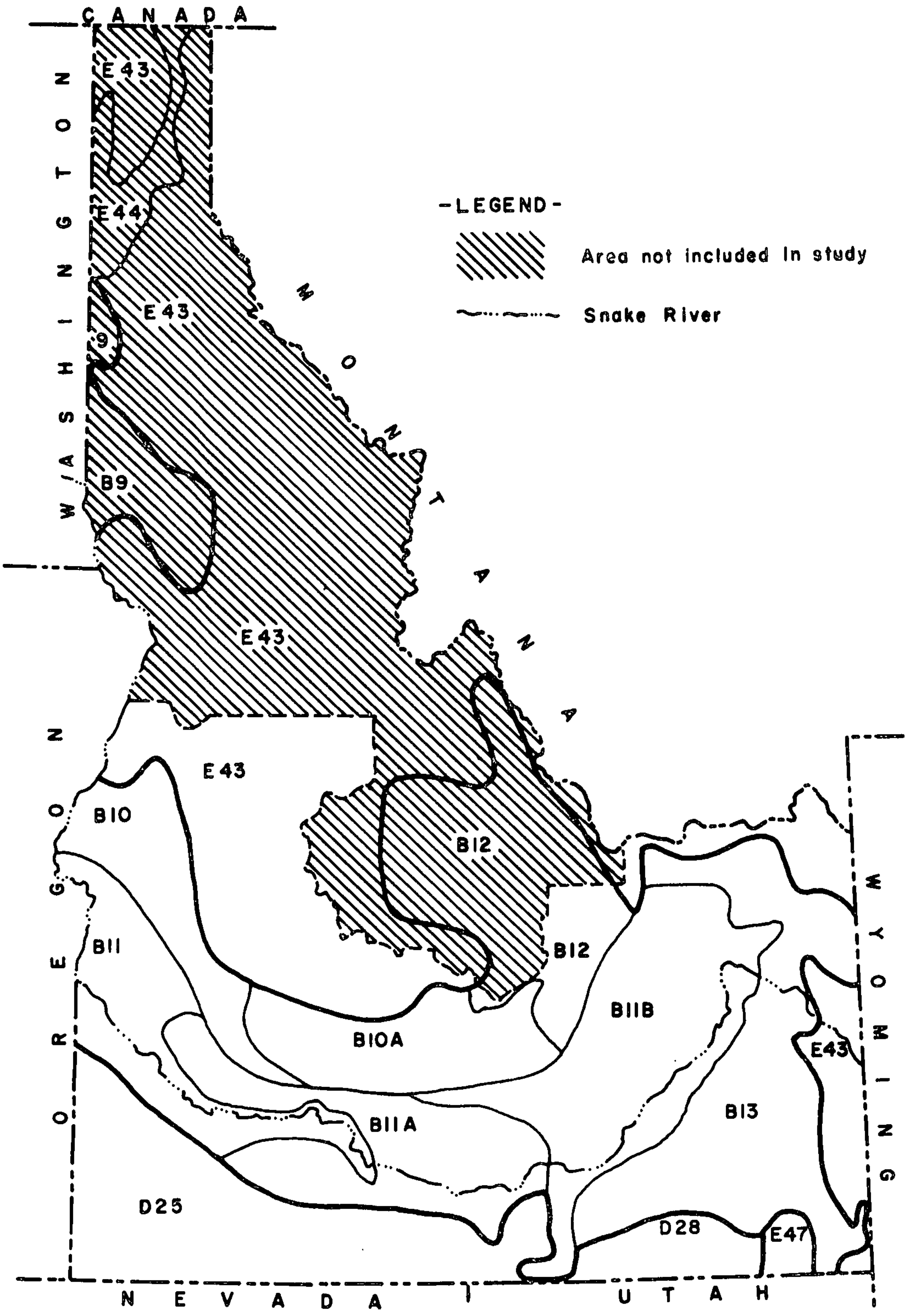

Figure I Land Resource Regions and Major Land Resource Areas, Idaho 1967

LRA 10 - Upper Snake River lava plains and hills
LRA 10A - Big and Little Wood River footslopes and plains
LRA 11 - Snake River plains
LRA 11A - Central Snake River plains
LRA 11B - Upper Snake River plains
LRA 12 - Lost River valleys and mountains
LRA 13 - Eastern Idaho plateaus

Region D is located mainly in the Great Basin drainage, but protrudes into Southern Idaho with three LRAs, two of which are important agriculturally. These LRAs are defined as: LRA 25, the Owyhee High Plateau located in the Southwestern corner of Idaho; and LRA 28, Great Salt Lake area, located in Southeastern Idaho.

The third Region, Land Resource Region (LRR) E, is divided into two LRAs within the study area which are defined as:

43 - Northern Rocky Mountains

47 - Wasatch and Uinta Mountains.

Both of these LRAs are relatively unimportant agriculturally, and for purposes of this study were combined with LRA 25 and 28, and discussed throughout as "other LRAs."

Since the macro climate associated with the LRAs has a pronounced impact on the use of the soils within each LRA, the Soil Resource Groups (SRGs) are not necessarily homogeneous across Land Resource Areas (LRAs) in terms of cropping patterns, management, and yield. Therefore, any comparison of the soils must be made in the context of a given Land Resource Area (LRA).

The relation between yield and the soil resource is influenced by a specific range of soil characteristics, consisting of soil wetness, depth of soil available for rooting, alkali, salty or acidic properties, texture and amount of coarse fragments, permeability, and organic matter. Soil Resource Groups (SRGs) were designed to relate the soils to specific cropping patterns, management, and yield. Soil Conservation Service technicians were asked to estimate the cropping patterns, water input and fertilizer input for two levels of management. The estimates of the cropping pattern were based on the "adopted" county Conservation Needs Inventory (CNI) acreage figures for each of the four major land uses. These land uses were defined by the CNI and represent generally cropland, pasture and range, forest land and other land. Cropland use was then divided into irrigated and non-irrigated land and then into several major types of cropland use such as row crops, close grown crops, hay, cropland pasture and other uses. With this set of basic acreage data the Soil Conservation Service technician apportioned the land use acreage to the specific crop by SRG using a series of tables designed to allocate the acreage consistently from the county adopted estimates through the LRAs within the county, and then to the SRGs by LRA. Once the acreage of each crop by Soil Resource Groups (SRGs) was determined, the Soil Conservation Service technicians estimated the yield and input requirements for average and high management levels as perceived by them. The average management estimates form the basic information for this analysis. The average was chosen to minimize the effect of management upon yield, which tends to reduce the error resulting from the exclusion of management as an explicit variable.

Approximately 100 SRGs were delineated. The number was large to facilitate a wide range of potential applications. As this study was formulated, it became obvious that differences in many SRGs were so small that information available from the data set would not be impaired by grouping the SRGs. The number of SRGs was reduced from approximately 100 to 11 more general groupings. These SRGs are numbered from one through eleven with no significance attached to the numerical ordering. The groupings were made in consultation with a soil scientist, and are based upon considerations of soil texture, wetness, depth of soil, and topographical location. The SRGs are identified for this study as:

SRG(1); Well drained, deep to very deep, generally moderately permeable soils with moderately fine to moderately coarse textures on terraces, foot-slopes and uplands. The most productive lands on irrigated terraces and uplands on the Snake River plains are included as well as the best grain producing lands of wind deposited silt in southern Idaho.

SRG(2); Well drained, moderately deep with moderately to slowly permeable subsoils with medium to fine textures on uplands and terraces. Similar to SRG(1) in use and management with some limitations in yield due to slope, depth of soil and restrictions in growing season due to elevation.

SRG(3); Well drained, moderately deep to very deep, generally moderately permeable soils with moderately fine to moderately coarse textures on bottomlands and low stream terraces. Units of this SRG are distributed in small bodies adjacent to stream channels throughout the area. Major problems associated with the use of this SRG are (1) in southern Idaho frost often occurs in the bottomlands adjacent to streams and (2) flood damage including deposition of silt, gravel and other debris as well as loss of soil through sheet erosion.

SRG(4); Somewhat excessively drained, shallow, medium to fine textured soils with moderately slowly to slowly permeable subsoils over hardpan on terraces. Some well drained soils with slope limitations are also included in this group.

SRG(5); Excessively drained, moderately deep to very deep, rapidly permeable soils with coarse textures on terraces and uplands. In areas receiving limited rainfall irrigation must be used for crop production. The SRG consists of mainly sandy, droughty soils with a limited cropping pattern and a high potential for water erosion.

SRG(6); This SRG consists of poorly drained lands with a permanent limitation in use and management associated with wetness. If the soils in this SRG are properly drained they may respond to a fairly wide range of crops but under natural conditions the cropping is restricted to hay and pasture.

SRG(7); These soils have highly variable properties since the amount of alkalinity or salinity overwhelms other soil characteristics. Therefore some of these soils respond well to intensive management designed to remove the salinity and/or adjust the chemical balance of the soils. The soil limitations are a permanent liability to use and productivity since the problems associated with their use cannot be permanently corrected.

SRG(8); Bottomlands, mostly excessively drained very shallow soils with medium coarse to gravelly textures. These areas are used principally for irrigated pasture and hay crops.

SRG(9); Well drained, very deep to moderately deep, fine and moderately fine textured soils with slow to moderately slow permeable subsoils on uplands and terraces. Can be characterized as clayey soils with management and use adjusted to accommodate the fine texture.

SRG(10); This SRG includes mostly lands classified as Class VI, VII and VIII. However droughty conditions on certain concentrations of the SRG can be corrected by irrigation, also drainage of certain swamp areas can be accomplished. Therefore, even though one would not expect cropping on this SRG to be extensive, some crops are grown but on a marginal basis.

SRG(11); These soils are moderately deep over igneous bedrock, hardpan or alluvium. Well and moderately well drained, medium to fine textured soils. Moderately and moderately slowly permeable in substratum. Nearly level to moderately sloping uplands and terraces with few limitations with respect to cropping and management.

Yields, estimated for each Soil Resource Group (SRG) by Land Resource Area (LRA) portion of the counties, were reported in appropriate units; e.g., wheat, barley, bushels; hay, sugar beets, tons; and potatoes, cwt. The yield reported was intended to reflect the normal yield. The method used in arriving at the normal yield was to average yields over the three years 1965, 1966, and 1967. This normalization was applied to everything estimated, except the acreage estimate which was for the year 1966. Consequently, the fertilizer input and water input are the "normal" levels associated with the "normal" yield.

Water is reported in acre feet per acre, utilizing a net disappearance concept to obtain the level of water use. The concept that Agriculture is held accountable for that amount of water made unavailable for other uses because of its diversion for irrigation, is the basis for using net disappearance rather than consumptive use or some other measure of water use.

Pounds of available nutrient applied is the unit of measurement for nitrogen, phosphorous, and potassium fertilizers used in the study. Minor nutrients were not considered.

Prices will be based on a 1965-67 average price for each of the outputs. Some commodity prices are extremely sensitive to locational factors, and these prices will be adjusted accordingly. The prices for fertilizer will be taken from data used in the Economic

Research Service, Natural Resource Economics Division's National Inter-Regional Agricultural Production System.

The Southern Idaho area was chosen as the area for analysis because of (1) the importance of irrigation in that area, (2) the large proportion of the irrigation in the Pacific Northwest that occurs in that area, and (3) the analysis may be useful to river basin investigations on the upper Snake River.

The Area

Southern Idaho for purposes of this study refers to all of the area in Idaho which is drained by the Snake River and its tributaries from where the Snake enters Idaho at the Wyoming border to a point below Oxbow dam in the Hells Canyon reach of the river. The total land area in Southern Idaho as defined exceeds 51,000 square miles, over 75 percent of which is in federal ownership.

Major tributaries of the Snake in this region include the Teton, Henry's Fork, Blackfoot, Portneuf, Big Wood, Bruneau, Owyhee, Boise, Malheur, Payette, Weiser, Burnt, and Powder Rivers. The Owyhee, Malheur, Burnt and Powder Rivers all enter the Snake from the Oregon side along the western boundary of the Region.

The major agricultural concentrations occur in the Snake River Plain in three major areas, (1) from Ashton on the Henry's Fork to American Falls, (2) from Walcott Dam on the Snake River to the

Hagerman Valley Southeast of Bliss, Idaho, and (3) in the Boise, Payette, and Weiser River valleys.

Irrigation began in Southern Idaho in the early 1860s along the Boise and Payette Rivers, and in the 1870s in the Southeastern Idaho area along tributaries of the Snake River. The irrigation in the Boise area corresponds to gold discoveries in that area while the completion of the railroad through Southern Idaho stimulated agriculture and consequently irrigation by providing transportation of farm produce to eastern and west coast markets. Southeastern Idaho was settled by people moving north from the Great Salt Lake Valley. Based upon their experience in the Salt Lake Valley, they developed irrigation using cooperatives to raise the necessary capital.

From these small beginnings, irrigation has grown in the area to over 3.2 million acres. Over 85 percent of this irrigated area is adequately supplied with water from surface and ground water sources. The irrigated acreage in Southern Idaho accounts for over 45 percent of the irrigated cropland in the three state Pacific Northwest area. Idaho accounts for over 70 percent of the irrigated land in the Pacific Northwest used in the production of field and row crops, over 60 percent of the irrigated small grains, and nearly 60 percent of the irrigated hayland.

Irrigated agriculture is the most important economic activity in the area with the two major industries, food processing and transportation, dependent upon irrigated agriculture for raw materials.

Almost 600,000 acres are irrigated from ground water sources. The Snake Plain aquifer north of the Snake River is one of the largest and most productive ground water reservoirs in the world. The aquifer discharges water into the Snake River at two major points, (1) into the American Falls Reservoir, and (2) at Thousand Springs at the entrance to the Hagerman Valley. Some return flow occurs all along the river from above American Falls Reservoir to a point below Thousand Springs. The aquifer is recharged by the Snake River above American Falls, the Big and Little Lost Rivers which sink into the ground along the northern side of the aquifer, agricultural waste water, and normal runoff which percolates into the aquifer. Some of the areas on the aquifer have experienced shortages and a lowering water table while in other areas pumping seems to have had little or no effect on the water table.

The eastern and western portions of the area have exhibited variable annual rates of increase in irrigated land since World War II. The area from King Hill to the western border has had about a 20,000 acre annual increase in irrigated land, while the eastern section from Wyoming border to King Hill, has shown an increase of about 40,000 acres of irrigated land per year.

Over 500,000 people live in the region (Southern Idaho as defined above) supported mainly by irrigated agriculture with small concentration of other industries in Southeastern Idaho related to phosphate production and mining. Most of this population is concentrated in several communities along the Snake River and its major tributaries. Those with a population of 25,000 or more include Idaho Falls and Twin Falls on the Snake River, Pocatello on the Portneuf River, and Boise located on the Boise River.

A PERSPECTIVE

The theoretical basis for estimating aggregate production is an extension of the production function concept used in economic production theory to derive the relationships between the several variable inputs and the output of a firm. The firm production function concept incorporates assumptions about institutional factors, plant size, and capital constraints such that these factors remain constant during the production period. The purpose then is to estimate the response of output to changes in the use of the several inputs given the fixed elements of the firm, and to determine from the response parameters, the economic impacts of their combination. Inferences are normally made concerning changes in the level or proportion of use of the various inputs to obtain an economically efficient combination of inputs which will maximize the firm's profits consistent with his constraint set.

Since there are no aggregate firms and no reference to the decision making unit is made in the aggregate function, the assumptions about fixed institutional, management, and capital constraints are no longer relevant.[2] These constraints play a major role in determining the production response and must be made explicit in the

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[2] Only in the case of a perfectly aggregated function would inferences drawn from the aggregate function apply to the firms.

function if it is to be truly consistent with the firm level function. However, since we are not concerned with returns to size, and profit maximization at the firm level is not of concern here, these variables may be excluded without severely damaging the results from the aggregate function as long as conclusions drawn from the aggregate function are made with full knowledge of these limitations.

Several studies in recent years have utilized aggregate secondary data to estimate production functions. Ruttan (1965) used a Cobb-Douglas type function to estimate returns to irrigation in several regions of the United States. Grunfeld and Griliches (1960) used data to estimate an aggregate production function for an investment study of several large firms and a fertilizer demand study for nine regions in the United States.

The actual selection of the algebraic functional form depends upon the objective of the analysis, the biological or technological relationships which exist between inputs and outputs, the experience and bias of the analyst, and other factors peculiar to a particular problem. In addition, availability of funds for the research must be considered. An analyst may be forced to use the algebraic form which gives the most relevant economic information for the solution of his particular problem even though it doesn't fit some of the assumptions (a priori information) about the type of relationship which exists between the inputs and outputs.

The choice of an appropriate functional form depends in part on what kind of economic information the analyst hopes to obtain. The several functions available have advantages and disadvantages computationally depending upon the economic parameter to be estimated. The Cobb-Douglas function, linear in the logarithms, requires computation of the function coefficient by observing the individual parameters for the input elasticity and then summing them for the function coefficient.

Linear functions are not only the simplest form algebraically but also provide marginal productivities in the form of coefficients to the independent variables. However if the Y intercept value is non-zero, the function coefficient is virtually impossible to obtain except for specified values of all the explanatory variables. On the other hand, linear functions lend themselves very easily to aggregation; hence, if the functions must be aggregated, use of linear equations will help satisfy the conditions for consistent aggregation.

Several alternative types of function other than linear and Cobb-Douglas exist and are often appropriate for use as algebraic forms from which productivity estimates can be made. These include the Spillman or Mitscherlich function, the square root function, quadratic functions, and other polynominal forms.

Empirical Estimation

No matter what algebraic form of the production function is chosen estimation of its parameters is an empirical problem. The characteristics of the data often preclude the use of certain algebraic forms. For instance, a power function or Spillman function doesn't allow variables to enter at the zero level because output in either case will be zero if any of the inputs are zero. Since the data with which the functions are to be estimated in this study are replete with zero inputs, power functions and Spillman functions are excluded from consideration at the outset.

Econometric theory concerns the application of mathematical and statistical concepts in estimating economic parameters from empirical data. The limitations and problems of such estimation are encountered and made explicit so that the users of a set of results are made aware of them and can make their decisions with a knowledge of the kinds of limitations inherent in the estimation procedure.

Estimation of equations linear in the parameters, is often best accomplished by using the ordinary least squares regression procedure (OLS). This procedure has been shown to give best linear unbiased estimates if certain restrictive assumptions about the independent and dependent variables and the error term are met. These assumptions include proper specification of the model, linear independence (lack of multicollinearity) in the set of independent variables, an additive random error term, and measurement of the explanatory variables without error.

If these assumptions are not acceptable in the particular data set being used, generalized least squares can be adapted to obtain parameter estimates which will aid the researcher in determining the influence of the improper assumption of the estimates from OLS.[3]

Production function estimates in the past have either been for specific soils in a limited area or more generally have ignored the impact of the micro climate, soil conditions, and climate (including rainfall, length of growing season, elevation, etc.) on the production of various crops. This is in part due to the lack of a meaningful data set which could account for these factors explicitly in a mathematical (econometric) model. An index of productivity for soils of varying productive capacity which could be tied back into a specific cropping pattern and soil distribution for a given area would increase the precision with which total product could be predicted from a given geographical area. This index of productivity would also be useful in determining the returns to the other inputs to agricultural production if it were included as an explicit variable in the estimation of a

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[3] For a review of these procedures see Johnston, Econometric Methods, Chapters 5 through 10.

production function from which MVPs would be obtained. Rather than weighting each of the observations equally, the index of productivity could be used to weight the observations to account for the inherent variability in soil or resource area (climatic area) productivity.

The procedures available for calculating an index of productivity include, (1) finding the mean yield for each soil from the LIPS data, then set the highest mean yield equal to 100 and obtain the index by dividing all of the other mean yields by the highest mean yield. (2) If an equation can be formulated for each crop within each of the climatic areas, one could obtain a yield estimate for each crop at the mean of the input levels for each area and then follow a similar procedure as that defined in (1) above to obtain the index. A third possibility (and this list is not intended to be exhaustive) would be to calculate the ratio of predicted values from a regression analysis to the actual yield values for each observation, then find the mean values of these ratios for each set of LRAs. Then one could set the highest ratio equal to 100 and find the index numbers for the other Land Resource Areas by dividing each of their respective means by the highest mean value ratio.

The model used to estimate the productivity index is assumed to be fully specified except for a Land Resource Area (LRA) variable. Management is assumed to be constant, due to the use of an average management estimate leaving the error term to reflect (1) the random error element, and (2) the variation or residual attributable to variation in Land Resource Area productivity. Since the least squares regression procedure tends to have an averaging effect upon the predicted values, a negative correlation can be hypothesized to exist between the actual values and the predicted values as the inherent productive capacity changes from relatively poor soils to the relatively good soils.

An estimate of the productivity of the land base would ideally be made based upon the variation in yield due solely to the differences in LRA quality. The index derived from procedures (1) and (2) above, reflect the compound effect of Land Resource Area quality, and the other factor inputs used to attain the yield from which the index is created. Procedure (3) uses the regression analysis to account for the effect of other structural inputs, and comes closest then to the ideal method of measuring the true variation in productivity due to differences in the land resource base.

REVIEW OF HISTORICAL ANALYSES

Many attempts have been made to determine a meaningful value of water. The purpose of this section of the analysis is to review and appraise some of the more important attempts with respect to their applicability to problems of decision making in the field of natural resource development.

The first problem to be addressed then, is what information is needed that the economist can supply, to provide decision makers with the technical data required for rational decision making. The basic problem economists face is that there is no well defined market structure for evaluating natural resources such as water, rangeland, etc. The thrust of the following arguments will be toward water related problems, but have relevance in other natural resource problems.

The economist is concerned with two major facets of the information required to make rational choices among alternatives in resource development. The major thrust of his work is to determine the economic feasibility of the alternatives from an economic efficiency posture. Secondary considerations are given to distributional consequences of each of the alternatives.

Income distribution would be interesting to study; however, Beattie et al. (1971, p. 7) state, "So long as the economist has no knowledge of the social welfare function, which alone contains the necessary information needed to combine knowledge about efficiency with knowledge about other determinants such as income distribution, he must leave the weighting of objectives to the political process . . . The economist should determine a cardinal measure of preferredness for those consequences which can be measured (economic efficiency) and is damned (to borrow Kelso's terminology) to merely describe those consequences which cannot be subjected to optimization technique (income distribution)." In this study income distribution will not be discussed. The emphasis will be on problems of economic feasibility and the information required to evaluate it.

Economic feasibility is based upon the accounting concept that benefits from a given project should exceed the cost associated with obtaining them. These benefits and costs can be classified as: primary, those benefits and costs which accrue to the direct users of the resources; secondary, those benefits and costs accruing to those who support the primary users such as the food processing industry, capital and equipment suppliers, etc.; and tertiary, those benefits accruing to other industries, especially the service industries, retail, professional, etc. Estimates of secondary and tertiary benefits are generally derived from (1) the value of the primary benefits and (2) the composition of the commodity bundle which is assumed for computation of the primary benefit.

It is apparent then that an accurate method of estimating the primary benefits for resource development alternatives with a common data base, is essential to the determination of economic feasibility, and for comparing alternatives (in terms of economic efficiency) from which priorities for development can be determined through the political process.

Choosing between alternatives involves a comparison of the marginal productivity of the inputs between said alternatives. Where a market exists the market price can be used in conjunction with estimates of physical productivity to determine the efficiency of a given situation. However, most natural resource decisions involve one or more resources for which no established market exists. Therefore, in order to evaluate the various alternatives one must determine a value for the natural resource. These values can be derived from the demand for the output of a project, assuming that the price of the resource is equal to or less than its MVP in a given use. It is then possible to construct from the physical production function a demand curve for the resource. Usefulness of the demand schedule for the resource will be limited unless the demand schedule of the output from a resource development can be determined. The demand schedule for the output is used to determine price impacts of changes in production stemming from changes in resource use. Price impacts have been largely ignored in evaluating impacts of resource development because the agricultural products are assumed to be produced in a near perfect competitive situation and that changes in output are of insufficient magnitude to have an effect on prices received. Current market prices for the various outputs will permit the derivation of the MVP of a resource from the marginal physical productivity.

Water values then are important as guides to the allocation of development funds to alternative uses of a resource. Water values developed through mathematical programming can be used to verify values of water obtained from the residual income and residual land value approaches to determining the value of water in a given development, and to verify results from other mathematical analyses.

Economic analysis is limited by the state of the art. Rather than escaping responsibility under this pretext, concern must be exercised with following all possibilities of improving not only the methods and procedures for obtaining estimates of water values, but also the data base to which those methods and procedures are applied to obtain estimates of resource values.

The remainder of this section will be devoted to a description of the methods, procedures, and data used for estimating a value of water in agriculture.

Residual Income

This procedure involves income analysis by budgeting farm situations under two assumed levels of development (1) without the project and (2) with the project. This procedure is currently used in evaluation of most federal water development projects. The residual income obtained from the difference between the estimated income from the two levels of analysis is the value attributed to the amount of the water resource required for the development. This procedure is highly sensitive to changes in the basic budgetary data and to the level of projected prices and yields. For instance, a change in the yield projection for some commodities of a unit of production could affect the benefits by several dollars per acre.

A study completed during 1966-7 at Iowa State University under the direction of E. O. Heady (1967) reviews the residual concept for valuing irrigation water used by the Bureau of Reclamation. Some of their conclusions related to budgeting and linear programming techniques for determining the value of water.

> Yield resulting from the interaction of irrigation water, fertilizer levels, other technologies, climate, microenvironments, and soil characteristics, determined by research and systematic measurement are almost completely lacking. Great need exists for obtaining yield parameters relating to these variables and characteristics through production functions fitted to experimental

> data or farm measurements designed for these purposes" (Heady, 1967, p. 38).
>
> While a market price for water could be determined only by market equation of supply and demand, valuation of water on an individual farm can be made apart from knowledge of these functions. . . . The marginal value productivity thus also expresses the effective demand for water in an individual use, by an individual farm or for an entire project" (Heady, 1967, p. 9).

The study goes on to say that the residual budgeting process used by the Bureau of Reclamation

> . . . assumes (perhaps unrealistically) that constant returns to scale prevail on the farm, and that market prices and marginal value productivities for all inputs (water, fertilizer, labor, land, seed, tractor fuel, etc.) are equal except for the one being estimated. In addition, the estimate provides the total or average return to the factors considered rather than the marginal.

Linear Programming

In the classic Linear Programming analysis, the average productivity and marginal productivity are equal since linear homogeneous production functions are implicit in the linear programming procedure. Linear programming then approximates the marginal productivity by estimating the average productivity. However, this limitation can be avoided (for diminishing marginal productivity) by approximating the declining output from successive resource increments by a separate activity for each increment.

Some linear programming estimates fail to measure marginality because several factors are allowed to shift simultaneously and hence the effect of an increase in the limiting factor cannot be separated from the total effect after adjustments in the remaining inputs. Although the shadow prices from linear programming in these instances cannot be substituted directly for value of marginal product in the economic efficiency framework, it does provide the analyst with important information as to the actual impact on total output of an increase in the use of the limiting resource. In cases where the objective is to estimate primary agricultural impacts, the output of the linear programming system may provide a more direct computational format than models designed to estimate the separable value of the marginal product.

Budgeting and linear programming are essentially similar; the procedures employed and the range of choices one can examine, are different. Linear programming is a computerized budgeting procedure that increases the range of choices and reduces the cost of the old budgeting process. The inclusion of subjective information in the linear programming model to make use of field experience and prior information is more complex and requires a higher degree of sophistication than in simple budgeting. However, such information can be included and the model sufficiently constrained to reflect real-world situations.

Paul W. Barkley (1967) participated in a ground water study of the high plains region of Colorado, then reviewed the study for the Committee on the Economics of Water Resources Development of the Western Agricultural Economics Research Council. His critique of the Colorado ground water study entitled "Research in Ground Water Problems in the High Plains Region of Colorado" provides some insight into the use of linear programming for estimating returns to water.

A representative farm approach was used to determine optimum allocations of resources and revenue producing activities from which shadow prices could be obtained for the limited resources.

Barkley was able to obtain marginal value productivities (MVPs) for irrigated land, irrigation water and sugar beet allotments. The MVPs of irrigated land range from \$30.00 to \$40.41 per acre. The MVPs of water ranged from \$0.00 to as high as \$168 per acre foot. The critique presents the results from 1 of the 5 representative farms. For the farm discussed, the MVP of irrigation water was positive for only 2 months, July (\$24.00 per acre foot) and August (\$73.51 per acre foot to \$139.94 per acre foot depending upon the assumed availability of water). Dr. Barkley concludes that

". . . conditions vary sufficiently from farm to farm to make the variations in the MVPs of water very high."[4]

The major contribution from LP is its ability to take a complex system of linear equations and find the optimum allocation of resources which will either maximize profits or minimize costs subject to the set of constraints used to provide realism in the model.

Production Functions

The empirical production function allows the economist to isolate the MVPs of the relevant factor inputs. The use of production function analysis is relatively new compared to budgeting for valuing resources.

Data availability seems to be a persistent problem associated with empirical production function analysis. Heady (1967) states:

> The literature search indicated that very little data exists for use in correlating the economic and physical factors of the soil in the agro-economic analysis of projects and in the economic principles or models of irrigated farming. While a considerable amount of literature exists in general terms, very little specific quantitative analysis has been completed.

Aggregate production functions may provide information as to the relative demand for irrigated agriculture, but provide little information for the relative productivity of alternative developments. This

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[4] See Water Resources and Economic Development of the West, page 141.

is because the data does not relate to the peculiar physical factors associated with each development alternative. Therefore, the MVPs derived from these analyses must be interpreted as some sort of average value which may bear anything but coincidental resemblance to the value associated with a new resource development.

Miller (1966) using experimental data, estimated MVPs for several crops on two soils on the Willamette catena. Irrigation in the Willamette Valley is generally characterized as supplemental irrigation. Miller estimated values of supplemental irrigation of bush beans at various levels from 2 to 15 acre inches of application. The MVPs per inch of water associated with the different levels of supplemental irrigation, range from $134.88 at 2 inches to $1.05 at 15 inches. If a 75% efficiency is assumed, these figures drop to $101.16 and $.79 respectively. For the field corn experiment, water applications ranged from 2 inches to 12 inches, the MVP of water ranged from $16.23 to $2.45 at 100% efficiency and from $12.17 to $1.84 at 75% efficiency as the water application increased from 2 to 12 acre inches applied. The MVPs per acre inch of water estimated from the survey data were much lower than those estimated from the experimental data.

Ruttan (1966) utilizing census data on irrigated agriculture, estimated MVPs of irrigated land. The MVPs estimated for the Northwest were $77.39 and $63.32 per acre foot at the geometric and arithmetic means respectively. The values were estimated at the two means because Ruttan used the Cobb-Douglas function to arrive at his estimates of water value, and the geometric mean has been considered by some to be the more meaningful point at which parameters should be estimated. It now appears that neither the geometric mean or arithmetic mean is appropriate for evaluations. The mean needs to be adjusted by a correction term as defined by Lindgren for the definition of a log normal distribution.

Lindgren (1968, p. 176) states concerning the log normal distribution:

> "The kth moment of X about zero is expressible in terms of the moment generating function of log X:
>
> $$E(X^k) = E(e^{ky}) = \psi_y(k) = \exp(K\mu + \frac{\sigma^2 K^2}{2})$$
>
> In particular
>
> $$EX = \exp\left(\frac{\mu + \sigma^2}{2}\right), \ldots"$$

This equation is identical to:

$$\ln EX = \mu + \frac{\sigma^2}{2} \ln e$$

but:

$$\ln e = 1$$

$$\therefore \ln EX = \mu + \frac{\sigma^2}{2} .$$

From this we see that μ = the geometric mean, and that it has associated with it a potentially significant term. As $\sigma^2 \rightarrow 0$ $EX \rightarrow \mu$;

but for $\sigma^2 \neq 0$, $EX \neq \mu$. Therefore, one needs to adjust the mean value by the correction term $S_x^2/2$, so that EX is approximated by $\overline{X} + S_x^2/2$.

It is difficult at this point to determine whether or not the values estimated by Miller and Ruttan are consistent since one is based upon supplemental irrigation MVPs in a comparatively high rainfall area, and the other is based upon irrigated acres in the major irrigation counties of the Region which are comparatively low rainfall areas.

To say that either of these studies were inadequate would be an injustice to both analysts. The analyses were done for particular purposes unique to each and cannot be expected to answer all the questions concerning water values. The estimates obtained from these studies are important for this analysis in that the MVPs estimated can be used as a yardstick to determine the reliability of estimates from other studies. Of course, some adjustments in the figures need to be made to make them comparable to MVPs of water based upon some other set of data.

It seems appropriate to present at this time a discussion of the implications of estimating irrigation MVPs, supplemental irrigation MVPs, and irrigated land MVPs.

It is obvious that there exists a function for each of the three estimates mentioned above. It will be the object of this discussion to identify those functions and problems associated with applying such values to other uses.

Certain areas of the U. S. are dependent upon natural rainfall for the majority of the moisture received for agricultural production. These areas and areas where the irrigation water supply is less than adequate at least part of the time, have a unique water problem. Both areas must rely on supplemental water supplies to carry them through periods when either the natural rainfall or the basic supply of water is inadequate.

In many cases, there is sufficient moisture to get the crops started and generally well along toward maturity. It is at this point that supplemental water supply becomes critical. If the supplemental water supply exists, the producer will get a full crop; if this water supply does not exist, the producer may be left with a reduced crop or no crop at all. The value associated with this supplemental supply if it's needed, is obviously very high at the margin; i.e., the producer is willing to pay a high price to save his crop.

In areas where natural rainfall is insufficient to sustain intensive cropping, the availability of water and its relative abundance enters the decision framework of the producer at a much earlier stage. The producer in many instances still has other alternatives open to him which reduce the opportunity costs associated with his decision as to whether or not the intensive water using crop should be grown. Since he makes this decision with a fair amount of prior information about the availability of irrigation water throughout the growing season, he can base his decision on all of the relevant costs. He will probably not put himself in the position of paying a high marginal cost for water during the late summer high water use months because he will have planned for that contingency in making his decision in the spring. If through some natural disaster, he were to find himself short of water when water were critical to sustain his crop, he would be in the same position as the supplemental irrigators. The fact still exists, however, that the MVP of water for the two situations could be substantially different.

The marginal value product (MVP) of irrigated land can be converted to an approximation of the MVP of the average (per acre) application of irrigation water for the area. The average MVP for the last acre foot of water is approximated by dividing through by the average application. The problem with using MVPs without knowledge of the productive capacity of the land resource is that water development has progressed in many areas to the point where marginal lands are being considered for development. If these lands can be classified as to quality and/or productive capacity, and MVPs developed which reflect those capabilities in terms of the yields and other inputs required, the decision maker would have a much better basis upon which to make his choices.

A search of literature indicated many studies relating to irrigation of certain crops in the study area. It is apparent from a review of these studies that the quantity of water is not a problem in the area. The studies focus on proper management of the water with respect to timing and maintenance of soil moisture. Consequently, no information relating quantity of water to yield has been developed in the study area.

MARGINAL VALUE PRODUCTIVITY ESTIMATES FOR WATER AND FERTILIZER

Estimates of the marginal value productivity of water and fertilizer in the production of five major crops in Southern Idaho are presented in this chapter. The MVPs were derived from equations estimated using data from the Land Inventory and Productivity System developed by the Economic Research Service and Soil Conservation Service of U.S.D.A.

Production Function Estimates

The production function relating yield to water and fertilizer inputs was estimated using two equation formulations, a linear equation and a quadratic equation (Appendix Tables I-1 through I-5). The quadratic equation resulted in a better fit for all five crops using R^2 as an indicator of goodness of fit. In order to ascertain whether or not the quadratic equation added to the reliability of the estimates, an F test was constructed to check the significance of the incremental increase in R^2 in each case. The test statistic was derived from the test used to check the significance of adding a new variable to a model. Since the specification of the quadratic function is the same as the linear except for the addition of the two squared terms, this test was expected to give an approximate test of significance. Using this test at $\alpha = .9995$, the difference $(R_Q^2 - R_L^2)$ was significant for all five pairs of equations. The calculated F exceeded the tabular F by at least 4 times[5] for each equation.

The test statistic was calculated using the following formulation:[6]

$$F = \frac{\frac{SSE_L - SSE_Q}{dfL - dfQ}}{MSE_Q}$$

where

F = the test statistic

SSE_L = residual sum of squares from the linear equation

SSE_Q = residual sum of squares from the quadratic equation

df_L = degrees of freedom associated with the error sum of squares linear equation

dfQ = degrees of freedom associated with the error sum of squares

F values calculated with the associated degrees of freedom are presented in Table 1.

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[5] See four times rule discussion on page 53 for significance of this statement.

[6] This formulation is derived as a general formula from the specific formula used to test the significance of a change in R^2 due to the inclusion of an additional variable. The numerator represents the mean square of the additional reduction in the sum of squares from the addition of the variable or conversely, the increase in regression sum of squares. The MSE_Q is used in the denominator because the more fully specified quadratic formula should result in a closer approximation of σ^2. The formula could be rewritten as:

$$F = \frac{\frac{SSR_Q - SSR_E}{\boxed{\Delta df}}}{MSE_Q}$$

Table 1. Test of the significance of the addition of quadratic terms to the linear equation.

Crop	F Value	Degrees of Freedom	Level of Significance
Barley	24.8706	(5,328)	.9995
Wheat	59.9562	(2,664)	.9995
Hay	24.7922	(3,506)	.9995
Sugar Beets	60.1214	(1,107)	.9995
Potatoes	817.0439	(1.138)	.9995

The R^2 shown for sugar beets and potatoes was calculated from the raw sums of squares;[7] consequently, the degree of fit is overestimated for those functions. Correcting estimates of R^2 for the mean involves adjusting or correcting the sums of squares used in the formula for R^2. When the equation is forced through the origin, the mean is excluded as an explanation of variation; therefore, the sum of squares obtained are raw or uncorrected for the effect of the mean. The normal interpretation of the coefficient of determination is the percent of variation in the dependent variable not explained by the mean, but explained or accounted for by the regression equation.

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[7] The equations estimated for sugar beets and potatoes were forced through the origin because it was assumed these crops could not be grown successfully in this area in the absence of irrigation. Therefore, a zero level of production must exist at zero levels of the inputs. However, the other three crops are often grown in the area at zero levels of irrigation and fertilizer.

In order then to determine the significance of the observed R^2, one must correct it for the mean. It may be that the mean actually explains more variation than the regression equation resulting in a negative value for the corrected R^2.

The significance of the coefficients and the overall equation can still be assessed by reference to the $\underline{t}$ values associated with the coefficients, the F level for regression, and a test of the significance of the correlation coefficient, r.

Draper and Smith (1966, p. 64) refer to a four times rule which they suggest can be used as a rule of thumb in determining the usefulness of a regression equation as a predictive device. The four times rule evolved from a Ph.D. thesis "Criteria for Judging Adequacy of Estimation by an Approximating Function" by J. M. Wetz under the direction of Dr. G. E. P. Box at the University of Wisconsin. The criteria "suggests that in order that an equation be regarded as a satisfactory predictor (in the sense that the range of response values predicted by the equation is substantial compared with the standard error of the response), the observed F ratio of (regression mean square)/(residual mean square) should exceed not merely the selected percentage point of the F distribution, but about four times the selected percentage point." Draper and Smith go on to say that work is not complete on the four times rule, and that the rule is still subject to confirmation. The ten equations estimated for this section of the thesis satisfied the four times rule handily at the .9995 significance level.

Since the equations all have large numbers of observations, it was decided to include any variable in the regression equation with a student t value greater than 1. This criteria suggests that any variable was acceptable if the coefficient of the variable exceeded the standard error of that coefficient. With this criteria in mind, the appropriate equation was selected with significance levels from the student t ranging from .800 through .9995 for variables in most equations. (See Appendix I for a detailed breakdown of the statistical tests.) Another test of the significance of the equations estimated, relates to testing the hypothesis H_o: $r = 0$. Snedecor and Cochran (1967, p. 557) present the tabular information required for performing the test displayed in Table 2. With the exception of the linear potato equation, the correlation coefficient r was significant at the 1 percent level for all equations estimated.

Since the data and analysis fail in several statistical assumptions, significance tests are not intended to cloak the estimated coefficients in a veil of validity. The validity of the coefficients depends upon the accuracy of the data as well as the estimation procedure chosen. The purpose in referring to the tests at this point, is to indicate that given the data set available, relatively good equations have been estimated. The major problem in the area of MVP estimation for water and fertilizer is that no proven set of data exists. Even such time honored sources of data as Ag Census and Ag Statistics do

Table 2. Test of the significance of correlations from the quadratic and linear equations used to estimate the production function relating yield to water and fertilizer inputs.[1]

Equation/crop	n	R^2	r	Result of Test of H_o: $\rho = 0$
Quadratic/barley	339	.716	.8462	Reject @ .01
Quadratic/wheat	676	.672	.8198	Reject @ .01
Quadratic/hay	519	.604	.7772	Reject @ .01
Quadratic/sugar beets	113	.400	.6325	Reject @ .01
Quadratic/potatoes	146	.127	.3564	Reject @ .01
Linear/barley	339	.608	.7797	Reject @ .01
Linear/wheat	676	.566	.7523	Reject @ .01
Linear/hay	519	.546	.7389	Reject @ .01
Linear/sugar beets	113	.062	.2490	Reject @ .01
Linear/potatoes	146	.000	.0000	Fail to reject

[1] Test defined in Snedecor and Cochran (1967, p. 184).

not provide us with perfectly consistent data from period to period, or even cross sectional data within a period. The major problem with Ag Census lies in constantly changing definitions of various categories reported in successive census years. This problem also hampers the consistency of Ag Statistics since it depends to a certain extent on Ag Census for periodic data control figures. The data then must be judged to some extent by the reasonableness of the results obtained from its use. This problem is lessened to some extent by the fact that the equations are used to identify relationships in the population, not in a sample or subset of the population. Therefore we are not using statistical inference in its normal use; i.e., inferring from a sample to the population. The coefficients obtained are population coefficients. Hence, their reliability can be assessed to a limited extent using the normal tests of significance, and by reference to their reasonableness.

A dummy variable was used to estimate the impact of the Land Resource Areas on the MVP of water. The dummy variables were of the form $(X_1L_1; X_1L_2 \ldots X_1L_7)$ representing a cross product term between water applied and each LRA. $L_1 - L_7$ are dummy variables indicating the location of a particular observation. For example, if $L_1 = 1$, the observation occurred in LRA 11A and all other dummy variables $L_2 - L_7 = 0$. The β value associated with L_1 is affected only by observations in LRA 11A. The partial derivative of the production function with respect to water is altered by the coefficient associated with each LRA.

The LRA classification was chosen for the analysis for basically two reasons: (1) preliminary analyses indicated differences in the soils are small and tend to lose significance in an area as large as Southern Idaho; and (2) discussions with soil scientists at Oregon

State University had indicated the climatic differences were more important than soil quality differences. A review of the data itself tended to support this conclusion, so the Soil Resource Groups (SRGs) were not isolated in the model.

It was assumed that the two relatively intensive crops could not be grown at zero input levels; therefore, the equations used to estimate the production functions for sugar beets and potatoes were forced through the origin.

Estimates of Marginal Value Product

MVP estimates for water and fertilizer were obtained for each crop by evaluating the first partial derivative of the quadratic equation with respect to each input at their respective mean application rates to obtain the marginal physical productivity, MPP (Table 3). The MPP was then multiplied times the price of the output to obtain the MVP (Table 4) using the following formula and assuming fixed product prices:

$$MVP_{ij} = P_j MPP_{ij}$$

where

Pj = Average price of product j for the three years, 1965, 66, 67 from Statistical Reporting Service data

MPP_{ij} = Marginal Physical Product of the ith input in the production of the jth crop.

Two values for R^2 are shown for sugar beets and potatoes; one

Table 3. Marginal physical product of water by crop by Land Resource Area, and fertilizer cost by crop, 1966.

Input	LRA	Marginal Physical Productivity				
		Barley	Wheat	Hay	Sugar Beets	Potatoes
Water	11	4.0474	3.4295	.3667	1.2190	10.2535
	11A	4.0474	3.4295	.2359	.7858	5.6819
	11B	4.0474	2.7792	.1817	.3511	-.4724
	10	3.8034	4.7315	.1743	--	--
	10A	4.9230	2.3299	.1469	--	--
	12	2.4431	1.9089	.1902	--	2.3506
	13	6.6326	2.3565	.1737	1.2190	4.4657
	Other	.3585	.4476	.1949	1.2190	10.2535
Fertilizer Cost		2.2296	2.2469	.1528	.1209	2.4737

calculated from the raw sums of squares, and the other corrected for the mean. Since the quadratic equation was the better estimator for all five crops, the MVPs were calculated for it. The MVPs for the linear equation may be obtained by the same procedure using the equation coefficients presented in Appendix I.

Estimates of the marginal value product of an acre foot of water applied ranged from a negative $.84 to $18.83. MVPs associated with potato production were the least stable ranging from a

Table 4. Estimates of the marginal value productivity of water and fertilizer cost (quadratic equation) in the production of five major crops by Land Resource Area for Southern Idaho, 1966.[1]

Input	Unit	LRA	Marginal Value Productivity				
			Barley	Wheat	Hay	Sugar Beets	Potatoes
Water	$/acre ft	11	3.95	4.73	7.98	18.83	14.97[2]
	$/acre ft	11A	3.95	4.73	5.14	12.14	10.17
	$/acre ft	11B	3.95	3.83	3.96	5.42	-.85
	$/acre ft	10	3.71	6.53	3.79	--	--
	$/acre ft	10A	4.80	3.22	3.20	--	--
	$/acre ft	12	2.38	2.63	4.14	--	4.21
	$/acre ft	13	6.47	3.25	3.78	18.83	7.99
	$/acre ft	Other	.35	.62	4.24	18.83	18.35
Fertilizer Cost[1]	$ investment in fertilizer		2.18	3.10	3.33	1.87	4.43[2]
Coefficient of Determination (R^2)	Percent		.716	.672	.604	.956 (.400)[3]	.934 (.127)[3]
Price	Dollar/unit		.976/bu	1.38/bu	21.77/ton	15.45/ton	1.79/cwt

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[1] MVP of fertilizer cost was not estimated by Land Resource Area.

[2] In LRA 11, the MVP of water and fertilizer cost were adjusted to reflect the lower price of potatoes ($1.46/cwt) in S. W. Idaho. The MVP of fertilizer for LRA 11 then, was 3.61.

[3] Coefficient of determination corrected for the mean.

negative $.84 in LRA 11B, to $18.35 in LRA 13. The range for hay production was smallest with a low of 3.20 and a high of 7.98 in LRAs 10A and 11 respectively.

Impact of the LRA on each crop is somewhat variable. The MVP of water in barley production was lowest in five of the eight LRAs; MVPs of water in wheat, hay, and potato production were lowest in LRAs 13, 10A and 11B respectively.

Water applied in sugar beet production had the highest return in every LRA producing sugar beets. The marginal physical product was highest in LRA 11 for three crops: hay, sugar beets, and potatoes. The MPP in the Other LRAs equaled the MPP of LRA 11 for sugar beets and potatoes as did LRA 13 in sugar beet production. The highest MVP of water in the production of barley and wheat occurred in LRAs 13 and 10 respectively.

The relatively low return to fertilizer in sugar beet production may be a result of technical assistance from the sugar companies. Fertilizer is utilized at a rate closer to the economic equilibrium condition of MVP = MC in sugar beet production than the other crops. In this case, the last unit of fertilizer, a one dollar expenditure, returns $1.87. The sugar companies provide technical guidance to the producer so that the maximum production can be obtained, whereas in the production of other crops the farmer is left to determine for himself the appropriate rate of application.

One cannot compare the optimum conditions for allocating the two factors of production since a price for water does not exist. The impossibility of this comparison is emphasized in that individual entrepreneurs in a given location will have different costs depending upon their historical water rights. Only on a firm by firm basis could the inter-factor optimum condition be tested.

Interpretation of the economic efficiency of water application rates cannot be determined in the absence of other than administered prices for water in the various areas. However, no market exists at which a price can be established through competitive bargaining between areas.

If we assume the entrepreneurs collectively have an idea about the true value of water, we can further assume that the lowest return per acre foot of water applied reflects a near optimum condition for the area; i.e., Marginal Value Product of water equals or exceeds its marginal cost.

This assumption is reasonable since no rational entrepreneur would produce a crop that did not meet the criteria of $MVP \geq MC$. This, of course, ignores the impact of joint product relationships associated with crop rotations. The marginal value product taken by itself then, cannot be used to determine a surrogate price for water.

An experimental analysis designed to identify the joint product relationship of hay and grain to sugar beets or potatoes would be useful in deriving a value for water.

A pricing policy based solely on marginal value productivities does not take into consideration the individual farmers' ability to pay. MVPs provide an estimate of the proportion of agricultural productivity associated with irrigation and associated inputs, an important piece of information needed to develop a pricing policy. If, for instance, the policy decision was made that grain should no longer be irrigated, because of the availability of grain from dry land sources, the charge to water users could be set at a level making it unprofitable to use water on grains. The MVP estimate would be used as a guide to establish the appropriate charge.

Commodity prices affect the relative MVPs when relative price changes occur. When the price level increases for all crops, the relationship between crops remains the same. When relative prices change significantly as they have in the 1972-73 crop year, the amount of water allocated to the various crops will tend to a new optimum rate of application. These changes in water allocation will eventually stabilize at a new cropping pattern. Table 5 presents the MVPs associated with crop prices in effect on October 15, 1973. Prices have generally gone up over this period as reflected by potatoes and sugar beets. Wheat, barley, and hay prices have changed to a much larger extent and have consequently affected the relationship between the marginal value products of the several

Table 5. Estimates of the marginal value productivity of water and fertilizer cost in the production of five major crops by Land Resource Area for Southern Idaho using prices in effect October 15, 1973.[1]

Input	LRA	Marginal Value Productivities				
		Barley	Wheat	Hay	Sugar Beets	Potatoes
Water	11	9.51	14.54	18.15	26.21	20.51[2]
	11A	9.51	14.54	11.68	16.89	11.36
	11B	9.51	11.78	8.99	7.55	-.94
	10	8.94	20.06	8.63	--	--
	10A	11.57	9.88	7.27	--	--
	12	5.74	8.09	9.41	--	4.70
	13	15.59	9.99	8.60	26.21	8.93
	Other	.84	1.90	9.65	26.21	20.51
Fertilizer Cost		5.24	9.53	7.56	2.60	4.95
Price (Oct 15, 1973)		2.35	4.24	49.50	21.50	2.00
% Change in price from 1966 normalized		241	307	227	139	112

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[1] Prices estimated by Statistical Reporting Service for all crops except sugar beets. The sugar beet price was estimated by the Amalgamated Sugar Company, Nampa, Idaho.

[2] Statistical Reporting Service discontinued separate estimates for Southwestern Idaho.

crops. Since it is now relatively more profitable to grow wheat, the acreage devoted to wheat production would be expected to increase at the expense of other crops. This should continue as long as the returns to wheat production remain relatively high.

ALTERNATIVE VALUE ESTIMATES

Criteria for appraising alternative means of obtaining estimates of MVPs of irrigated land can be obtained by listing data characteristics associated with an optimal or ideal set of data, and determining a priori some guidelines upon which to judge the results of each analysis.

The ideal data set would have the following characteristics:

(1) Based on a statistical sampling procedure.

(2) Contain sufficient degrees of freedom to allow hypothesis testing.

(3) Based upon a system of physical boundaries defined to account for true differences in physical factors affecting output.

(4) Broad applicability in terms of the basic units of observation being consistent in a large area.

(5) Variables measured without error so that parameters derived therefrom will be unbiased.

(6) Easily aggregable and disaggregable to increase the flexibility with which it may be used.

The results obtained from such a data set should result in estimates of economic parameters which are consistent with accepted economic theory and improve estimates of MVP for water. The results should reasonably approximate real-world information from independent sources.

The objective of this section is to establish logical arguments as to the efficacy of marginal value productivity estimates obtained from a data set such as the U.S.D.A. Land Inventory and Productivity System (LIPS).

The observations were derived from information based upon a sampling technique developed by Iowa State University for the Soil Conservation Service, U.S.D.A. (1971). The sample size in Southern Idaho was increased due to the heterogenic nature of the land resource and was expanded to fit estimates of population totals from other published sources. It was from these expanded observation values that the LIPS data were derived. The observations were expanded on a county basis so that aggregation checks on totals could be made readily.

The units of observation are defined in terms of climatic and physical boundaries which reflect relative homogeneity in rainfall, length of growing season, natural cover and elevation. These areas were identified as Land Resource Areas (LRA). The soil characteristics were used to define the basic observational unit within the broader concept of the LRA.

The soil classification proved to be a reasonable conceptual base from which to estimate productivity. The actual measurement of productivity parameters (yield, water, fertilizer) was accomplished by some 30 to 35 employees of the Soil Conservation Service, U.S.D.A. It is not possible to determine the directional bias of these estimates (if such a bias exists) since so many people were involved; however, it seems not unreasonable to assume that most of the bias associated with the estimation procedure would cancel out. Therefore, even though the variables were probably not measured without error, the error terms associated with each observation should approach randomness, and the expected value of e would be zero. This is not to say the bias is eliminated, only that its effect is diminished.

Since the productivity parameters were estimated for a relatively small unit which was well defined in terms of factors affecting productivity, the units were easily aggregable to larger still relatively homogeneous units. Using weighted averages each level of aggregation accurately reflects the composition of the individual units.

Ruttan (1965) and Stechmessar (1968) estimated marginal value productivities for selected areas including Southern Idaho. Both of these studies utilized the Cobb-Douglas formulation of the production function to estimate MVPs of irrigated and non-irrigated land. Table 6 presents MVP estimates from these studies. The Ruttan estimates (at the geometric and arithmetic means) were for the entire Pacific Northwest water resource region whereas the Stechmessar estimate

Table 6. Marginal value productivity estimates for irrigated and non-irrigated land in the Pacific Northwest Water Resource Region, 1965.and 1968.

Variable	Ruttan (1965)		Stechmessar (1968)
	Geometric Mean	Arithmetic Mean	Geometric Mean
	$/acre	$/acre	$/acre
Irrigated Land	77.39	63.32	27.32
Non-irrigated cropland	60.93	35.97	23.28

was for Southern Idaho and Eastern Oregon.

Their estimates were based upon county data from the Census of Agriculture. The results obtained from these analyses should not be perceived as accurate measures of the true value of irrigation, but as other independent estimates which are probably in the reasonable category. The MVPs are presented then, to give the reader an idea of the approximate magnitude of this type of MVP estimate.

Since neither Ruttan or Stechmessar had water use data readily available, they could not obtain a direct measure of the value of water. They used irrigated land as a proxy from which an implied relationship between values of water could be drawn between different areas.

Table 5 presented the MVPs of water in the production of five crops in eight areas of Southern Idaho. These estimates were made possible by the advent of the LIPS data. Since the values were estimated on a per acre foot of water basis, no comparable estimate to that made by Ruttan and Stechmessar was readily available.

The economic decision to irrigate or not is made normally at the beginning of a crop year with full knowledge of the reservoir storage situation. Therefore, if the acre is planted, the farmer is relatively certain he will have sufficient water to complete irrigation of it. The marginal unit under these circumstances would then be the acre and not the last acre foot of water. In order to arrive at a value of irrigation on a per acre basis comparable to those of Ruttan and Stechmessar, the following formula was applied to the water MVP estimates from Table 5.

$$V_{AJ} = \frac{\sum_i \overline{W}_{ij} MVP_{ij} A_{ij}}{\sum_i A_{ij}}$$

where

V_{AJ} = Value of an irrigated acre in the jth LRA

$\overline{W}_{ij}$ = Mean water application for the ith crop in the jth LRA

MVP_{ij} = Marginal value product of water for the ith crop in the jth LRA

A_{ij} = Irrigated acreage of the ith crop in the jth LRA.

This formulation is consistent with Theil's rules for aggregation (see Appendix II).

Table 7 and Figure 2 present the values (V_{aj}) for each LRA.

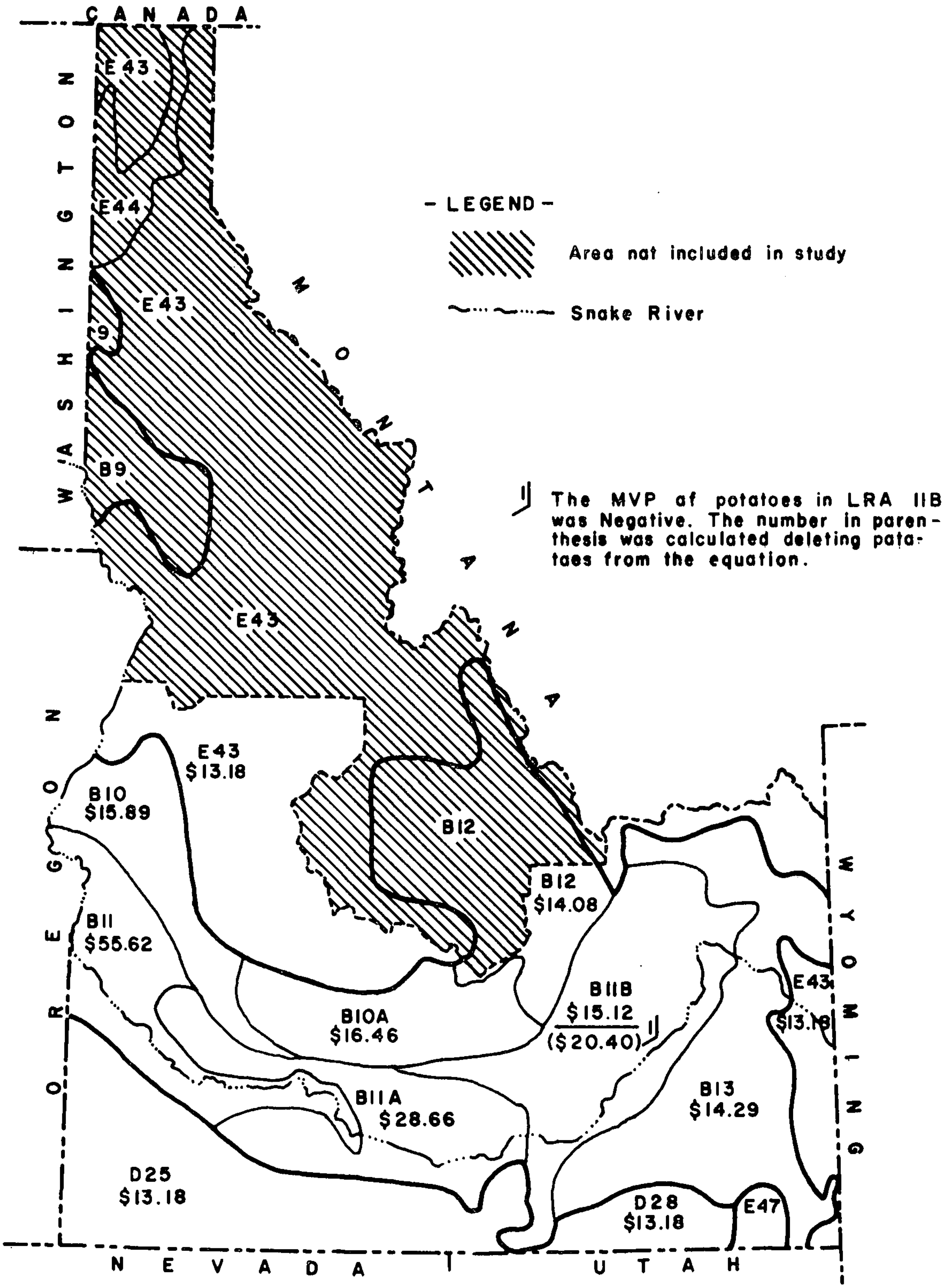
Table 7. Value of irrigation on a per acre basis for Southern Idaho, 1966.

Land Resource Area	Irrigation Value
	$/acre
11	55.62
11A	28.66
11B	15.12 (20.40)[1]
10	15.89
10A	16.46
12	14.08
13	14.29
Other	13.18
Southern Idaho Weighted Average	25.94[2] (28.63)[1]

[1] The MVP of potatoes in LRA 11B from the Quadratic equation was negative. The number in parenthesis was calculated deleting the MVP of potatoes from the equation.

[2] The value for <u>Southern Idaho</u> represents the weighted average of the MVPs for the eight LRAs using acreage of irrigated land as the weighting factor.

The major irrigated areas along the Snake River have a higher aggregate MVP reflecting the larger proportion of crop acreage devoted to the higher value specialty crops in those areas. One



1/ The MVP of potatoes in LRA 11B was Negative. The number in parenthesis was calculated deleting patataes from the equation.

Figure 2 Value of irrigatian on a per acre basis, 1966 would expect this difference in value to increase in LRAs 11 and 11A, since many high value specialty crops are grown in these LRAs. In the higher climatic zones, the five crops chosen represent a relatively larger portion of the irrigated acreage than the more climatically moderate Land Resource Areas. Table 8 presents the percentage of the irrigated land accounted for in each of the eight LRAs by the five crops chosen for this analysis.

Table 8. Percent of total irrigated land in the five crops chosen for this analysis by LRA, 1966.

LRA	Total Irrigated Acres[1]	Study Acres	Percentage
11	580,228	205,255	35.4
11A	769,800	553,105	71.8
11B	805,670	708,850	88.0
10	41,419	33,938	81.9
10A	56,923	36,836	64.7
12	130,594	112,955	86.5
13	241,902	191,557	79.2
Other	76,085	61.251	80.5

[1] Total irrigated land adjusted to account for idle land.

Cropland pasture was the largest single crop acreage not included in the analysis. Total irrigated cropland was adjusted to remove the irrigated land idled by government diversion, conservation use, and land idle more than three years. This percentage does not reflect the amount of total cropland accounted for in the study in that the five crops used accounted for a higher proportion of non-irrigated land than irrigated land.

If the soils and LRAs can be identified for new projects, its output and input requirements can be more accurately determined. Changes in technology can be incorporated into the production estimates by first estimating the impact of the technological input upon factors of production associated with the soils and/or the LRAs and then applying a factor to increase productivity where it applies. A major advantage being that the LIP System is completely computerized.

Estimates of factor costs related to the soil groupings (SRGs) would greatly increase the usefulness of land resource data such as the LIP System. The apparent failure of the soil classification system to account for differences in yield, and to significantly affect the MVP of water, stems partially from the lack of accurate measurement of input variations due to changes in the soil structure, texture, etc.

The Land Resource Areas (LRAs) have potential because they define productivity relationships not previously available and the initial task of classification, grouping and acreage estimation has been done. The job of revising and updating these acreage, yield, water, and fertilizer estimates will be minimal compared to the original effort. Obtaining good estimates of man and machine requirements for the various soils presents the biggest deterrent to successful incorporation of soil as a meaningful determinant of water value. If our assumption concerning the basic relationship of the yield to the land resource holds; i.e., that the climatic factors are the more important determinant of cropping patterns and yields, then the limitation of the soil variable declines in importance.

Data from the LIP System undoubtedly has many shortcomings, especially related to the sampling procedure, and consequently to statistical hypothesis testing. These problems could be overcome to a certain extent by a more thorough and precise method of data collection with analyses made from subsamples which meet certain statistical criteria. Hypothesis tests could then be made and inferences drawn. The question of the validity of the data set and results derived therefrom, depend upon the accuracy of the estimates made by the 30 to 35 individuals in the Soil Conservation Service. The procedure seems sound, the equations were estimated using accepted techniques, and the results are reasonable based on comparisons with results of other studies.

PRODUCTIVITY INDEX

Production functions are often estimated from secondary data with units of observation defined along political boundaries. One such method involves using the average value per acre as an indication of the response of agriculture to the set of inputs defined in the equation. These inputs are related to certain cost items, the amount of irrigated land, the total land area, etc. These functions measure the relationships between the input variables and the dependent variable, average value per acre. The procedure assumes that the land resource is a homogeneous input and hence the average value per acre does not depend upon the mix of soils and climatic factors in the land resource. This is normally done not because the researcher wants to exclude the land quality variable, but because he has had no alternative.

The data in the ERS Land Inventory and Productivity System (LIPS) provides a source of data based upon a set of homogeneous factors associated with the inherent productivity of the land resource. A major element in this set of homogeneous factors is the land resource area (LRA). The LRA is defined as a contiguous geographic area with relatively homogeneous climate, precipitation, elevation and natural cover. In the area covered by this study are portions of 9 LRAs.[8]

The productivity index was derived from an equation of the following form:

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1^2 + \beta_5 X_2^2$$

where

Y = yield in units/acre

X_1 = water input, acre feet/acre

X_2 = fertilizer input, dollars/acre

This equation form is the same as the quadratic equation presented in the section on marginal value productivity (MVP) estimates with the land resource area (LRA) dummy variables excluded.

The underlying assumptions for deriving an index of productivity from this production function are that: (1) the exclusion of the LRA variable produces a specification bias in the model. This bias becomes a major element of the error term. The error term then consists of two parts: (a) the random error and (b) the bias associated with the Land Resource Area variable. (2) The specification bias described above, causes the regression equation to underestimate the yield ($\hat{Y}$) associated with good soils and overestimate the yield ($\hat{Y}$) from the poorer soils. This assumption is based upon the

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[8] See map of LRAs on page 16.

premise that the least squares procedure tends to have an averaging effect on each observation. Then, if two observations occurred at exactly the same input levels with different Yields, the regression line would tend to go between them. Since the difference between the two levels is assumed to be the land quality (the unspecified variable) the $\hat{Y}$ associated with the better soil is biased downward, and that associated with the poorer soil is biased upward. Since we assume a random error term, the effect of the error term on an index would tend to zero for relatively large N.

Bias resulting from the exclusion of the LRA variables can be measured by calculating the ratio of the observed y values to the predicted Y for each observation. These ratios reflect both the bias and the random error terms for an individual observation. By using the mean value for each LRA the effect of the random error term diminishes as the number of observations gets large. The mean of the ratios in each LRA theoretically then reflects only the effect of the specification bias.

Variation due to factors related to the Land Resource Area (LRA variables) was measured in the MVP estimation equations by using dummy variables. Since the data from which these MVP estimates were made are for a specific normalized period 1966-67-68, their applicability in a later time period may be somewhat limited. An index for the LRAs derived in terms of their physical, climatic, and elevational impacts on production of specified crops could be applied with current or future sets of secondary data to more fully specify production function estimation equations. Differences in productivity between LRAs would then be included in an equation without being regenerated in each time period. Thus more aggregate data available from secondary sources could be better utilized in solving problems involving economics of the resource base.

A most promising aspect of this concept is that on a regional basis, physical productivity characteristics of the land resource are relatively constant over time. These characteristics involve factors that change slowly if at all, elevation, climate, precipitation, and adaptability to cover crops. Technology may be able to circumvent limitations related to these factors, but not without some cost which may preclude its adoption.

The estimation procedure used to obtain the data in the LIP system emphasizes the variation due to water and fertilizer application with the implicit assumption that the other variables--capital, labor, technology, etc.--that would normally be included in a production function are held constant.

Productivity indices should reflect the economic variation in productivity if they are to be applied in analyses involving economic indicators such as value per acre, production cost per acre, etc. from the secondary data sources. The productivity index was first determined for each observational unit within the LRA on a physical basis by the following formula:

$$PI = \frac{Y}{\hat{Y}}$$

These indexes were then aggregated to the county level where they would be more useful when applied to secondary data sources. The aggregation equation was specified as:

$$\rho_{ik} = \frac{\sum_j \frac{Y_{ijk}}{\hat{Y}_{ijk}} [A_{ijk} Y_{ijk} P_i]}{\sum_j [A_{ijk} Y_{ijk} P_i]}$$

$$i = (1\text{-}5),\ j = 1\text{-}9,\ k = 1\text{-}29$$

where

ρ_{ik} = Productivity index by crop by county

Y_{ijk} = Observed yield for the ith crop in the jth LRA and the kth county

$\hat{Y}_{ijk}$ = Predicted yield from the quadratic equation for the ith crop, jth LRA and kth county

A_{ijk} = Acres by crop, LRA, and county

P_i = Normalized price of the ith crop.

The economic productivity indexes, ρ_{ik} are presented in Table 9 for each crop by county. These figures represent the index $PI = \frac{Y}{\hat{Y}}$ weighted by the relative value of production from each observational unit. Value was chosen for the weighting factor because much of the

Table 9. Economic index of productivities for selected crops by county, 1966.

County	Barley	Wheat	Hay	Sugar Beets		Aggregate County Index
Ada	.8220	.9843	1.1390	1.1270	.9658	1.0750
Adams	.8711	1.2198	.9536		NA	.9603
Bannock	.8795	1.1677	1.1436	1.1034	1.5539	1.1788
Bingham	1.2779	1.2725	.7187	.9000	.9048	.9524
Blaine	.9449	.8611	.7137	1.0000	1.1693	.8417
Boise	1.1000	1.5879	1.2395	NA	NA	1.2796
Bonneville	.8526	.8511	.9438	.8977	.6999	.8095
Butte	.8449	.9145	.8896	NA	.9681	.9242
Camas	1.0711	.9192	1.1038	NA	NA	1.0552
Canyon	1.0901	1.2119	1.4056	1.1975	1.2489	1.2324
Caribou	1.2495	1.3476	.8585	1.000	NA	1.2007
Cassia	.9115	.9442	.9556	1.0763	1.0000	.9977
Clark	1.1629	1.0836	.9150	NA	.7680	.9306
Elmore	.9487	1.2030	1.3999	.9000	1.1903	1.1958
Fremont	1.3547	1.0405	1.2759	1.0770	1.0000	1.1651
Gem	.9962	1.3108	1.0634	.8000	1.5000	1.1335
Gooding	1.0950	.9909	1.1580	.9367	1.0000	1.1139
Jefferson	.9835	1.0769	.8661	.8000	.8000	.8740
Jerome	.9954	1.1603	.9604	1.1645	.8985	.9996
Lincoln	1.0775	1.1033	.8353	.7000	.8000	.8987
Madison	1.2164	1.0217	1.1270	.8165	.9402	.9974
Minidoka	1.0770	1.0507	1.2582	.9635	.9960	1.0464
Owyhee	1.1744	1.3857	1.1954	1.4000	1.3000	1.2885
Payette	.9025	1.1113	1.5650	1.0549	.6009	.8646
Power	.8086	.9169	1.0458	.6782	.5910	.8181
Teton	.7638	.8301	.6899	NA	NA	.7726
Twin Falls	.8601	.8902	1.1902	.9656	1.0961	1.0331
Valley	.7296	.7270	.9501	NA	NA	.8891
Washington	.8738	1.2213	.9248	1.3000	1.2000	1.0677

NA - Not Applicable

economic analysis that is done concerns the value of production. Value also gives us the common denominator by which we can aggregate to a county index. The aggregate county index is presented in the extreme right-hand column of Table 9.

Table 10 presents the county ranking according to (1) its relative value per acre by crop and (2) its ranking according to the productivity index. Rank correlation could be used to determine the degree of agreement among the ranking by crop. Regression analysis, however, will provide us with a much more meaningful measure of the relationship of the index to value per acre. It is important at this point to note that the value per acre ranking stays somewhat consistent from period to period. Asterisks indicate counties where the difference in the rankings were five or less, plus (+) marks indicate differences of 10 or less.

Appendix Table III-2 presents the average value per acre by crop by county for the two Ag Census years analyzed. Many differences in value per acre are relatively small, hence the stability of the rankings from the Ag Census data is somewhat questionable.

In order to determine if there exists a relationship between the productivity index and the average value per acre by crop, a regression model was specified to obtain an estimate of their correlation. An additional variable was included in the regression equation to account for differences in productivity related to irrigation. The

Table 10 -Relationship between the productivity index (P) and the average value per acre, by county, 1964 and 1969

County	Barley			Wheat			Hay			Sugar Beets			Potatoes		
	Pi rank	Value/acre rank 1964	Value/acre rank 1969	Pi rank	Value/acre rank 1964	Value/acre rank 1969	Pi rank	Value/acre rank 1964	Value/acre rank 1969	Pi rank	Value/acre rank 1964	Value/acre rank 1969	Pi rank	Value/acre rank 1964	Value/acre rank 1969
Ada................	26	10	5	20	24	28+	11	5	6+	5	5	5.5*	14	6	2
Adams..............	22	19	22*	7	33	26	18	25	25+	—	—	—	—	—	—
Bannock............	20	24	26*	10	22	25	10	19	18+	6	16	20	1	20	3
Bingham............	2	7	9+	5	10	11+	27	8	9	15.5	18	13*	16	15	13*
Blaine.............	17	13	23	26	12	13	28	18	14	10.5	15	NP*	7	17	17*
Boise..............	7	20	27	1	26	20	6	29	26	—	—	—	—	—	—
Bonneville.........	24	21.5	18+	27	16	16	20	11	16+	17	17	17*	21	21	18*
Butte..............	25	14	24	24	18	19*	23	14	17+	—	—	—	13	23	NP*
Camas..............	12	28	29	22	29	29+	13	27	28	—	—	—	—	—	—
Canyon.............	9	1	3+	8	1	3+	2	6	3*	3	2	1*	4	2	1*
Caribou............	3	16	19	3	15	23	25	23	21*	10.5	22	19	—	—	—
Cassia.............	18	8	11+	21	13	18+	17	9	10+	8	13	10	10	12	8*
Clark..............	6	26	17	14	28	27	22	24	24*	—	—	—	20	18	NP*
Elmore.............	16	17	12*	9	17	21	3	13	20	15.5	7	8+	6	1	8*
Fremont............	1	23	20	17	19	14*	4	21	22	7	21	21	10	22	19
Gem................	13	15	14*	4	4	7*	14	17	13*	19.5	9	5.5	2	3	NP*
Gooding............	8	12	10*	19	8	10	9	7	7*	14	8	15.5+	10	8	10*
Jefferson..........	15	18	15*	15	11	11*	24	10	11	19.5	20	12+	18.5	19	20*
Jerome.............	14	2	1	11	3	1+	16	1	1	4	10	7+	17	9	5
Lincoln............	10	9	6*	13	9	6+	26	12	8	21	14	15.5+	18.5	16	11*
Madison............	4	21.5	13	18	20	15*	12	15	15*	18	19	18*	15	13	14*
Minidoka...........	11	5	2+	16	7	5	5	2	4*	13	11	9*	12	14	16*
Owyhee.............	5	6	7*	2	6	8+	7	20	19	1	3	14	3	10	15
Payette............	19	4	8	12	4	2+	1	4	5*	9	1	3+	21	4	12
Power..............	27	27	16	23	21	24*	15	16	12	22	12	11	23	11	6
Teton..............	28	25	25*	28	25	22+	29	26	27*	—	—	—	—	—	—
Twin Falls.........	23	3	4	25	2	4	8	3	2+	12	6	4+	8	5	6*
Valley.............	29	29	28*	29	27	9	19	28	29+	—	—	—	—	—	—
Washington.........	21	11	21+	6	14	17	21	22	23*	2	4	2*	5	7	7*

NP - No production reported in Agriculture Census for 1969.

* - Indicates a difference of 5 or less among the 3 rankings.

\+ - Indicates a difference of 10 or less among the 3 rankings.

regression model then was specified as follows:

$$Y_{ik} = a + b_i \rho_{ik} + b_2 IP_{ik}$$

where

Y_{ik} = average value per acre of the ith crop in the kth county

ρ_{ik} = productivity index of the ith crop in the kth county, and

IP_{ik} = percentage of the ith crop irrigated in the kth county.

One would expect the value per acre to be highly correlated with the percent irrigated (PI) because the irrigated acres produce higher value crops. (Note: the value per acre is the weighted average value from the production of the five crops used throughout this analysis; i.e., barley, wheat, hay, sugar beets, and potatoes.)

The results of the several regression equations (one for each crop plus the county aggregate for each census year) are presented in Table 11. From the table, it would appear the productivity index has more meaning in those crops which are normally produced with a relatively large percentage of their acreage irrigated.

The index has some promise on an individual crop basis. Although the coefficients are not highly significant when used as a single independent variable, they show a larger degree of significance when used in concert with another variable.

The usefulness and significance of the index is developed in a

Table 11 -Average value per acre as a function of the percentage of crop acres irrigated (IP) and the productivity index (ρ)

Item	Barley		Wheat		Hay		Sugar Beets		Potatoes	
	1964	1969	1964	1969	1964	1969	1964	1969	1964	1969
Equation (1):										
$Y = a+b_1 IP + b_2\rho$										
a	24.3645	17.9216	19.1509	35.4040	-2.9475	-15.8138	1,395.4703		-2,533.0676	
b_1..............	27.0858	37.9285	40.9752	50.1233	54.3796	63.9279	-1,370.9527		2,760.0840	
b_2..............	3.6579	9.8029	9.6383	-4.4550	18.7110	28.5336	177.3450		110.3213	
Student's tb_1......	6.7642**	7.3875**	9.9648**	8.8255**	7.1306**	6.0593**	-.2619		3.0742*	
Student's tb_2......	.4504	1.0672+	1.2506+	-.4389	2.2434*	2.5917*	2.8411*		1.7259*	
F level regression.	23.556 **	28.884 **	52.5582**	39.2469**	29.067 **	21.226 **	4.683 *		5.2278*	
R^2..............	.6444	.6893	.8017	.7512	.6910	.6202	.3301		.3433	
Degrees of freedom.	(2,26)	(2,26)	(2,26)	(2,26)	(2,26)	(2,26)	(2,19)		(2,20)	
Correlation Coefficients:										
(Y, IP).........	.8010	.8220	.8887	.8656	.7945	.7225	-.2136	NA	.4955	
(Y, ρ)..........	.1362	.1929	.2107	.0760	.2943	.2895	.5725	.3648	.1817	
(IP, ρ).........	.1046	.0934	.1150	.1369	.0632	-.0327	-.2909	NA	-.2453	
Equation (2):										
$Y = a+b_1\rho$										
a	34.1735	37.8157	30.2215	48.1678	36.1406	37.1393	20.6622	128.1928	251.4170	350.0652
b_1..............	9.4095	16.1388	18.4674	7.8115	22.4725	26.3540	182.1002	93.5422	62.1237	35.5264
Student's tb_1......	.7146	1.0215	1.1197*	.3961	1.6003++	1.5715++	3.1226*	1.7076++	.8465	.7322
F level regression.	.511	1.0435	1.2538	.1569	2.561	2.470	9.737 *	2.9160	.7166	.5362
R^2..............	.0186	.0372	.0444	.0058	.0866	.0838	.3278	.1331	.0330	.0249
Degrees of freedom.	(1,27)	(1,27)	(1,27)	(1,27)	(1,27)	(1,27)	(1,20)	(1,20)	(1,21)	(1,21)

Levels of significance: ** = .9995; * = .9500; ++ = .9000; + = .8500.

subsequent section of the analyses. The assumption that inputs other than water and fertilizer are constant in the study area, in the LIPS data, is implicit in the derivation of the productivity index.

Example of Productivity Index Procedure

The basis for estimating or constructing the productivity index in the manner used in this analysis can be seen through the following example.

Given the following set of data:

Table 12. Productivity index test data.

Obs No.	Y	X_1	X_2	X_3
1	12	1	3	2
2	10	1	3	3
3	14	1	3	1
4	14	2	4	2
5	12	2	4	3
6	16	2	4	1
7	16	4	5	2
8	14	4	5	3
9	18	4	5	1
10	18	8	6	2
11	16	8	6	3
12	20	8	6	1
13	20	10	7	2
14	18	10	7	3
15	22	10	7	1

a simple regression analysis was run on this test data to illustrate the deletion of variable method of index construction. Repeat observations were used in variables X_1 and X_2 to exhibit all three levels of the index variable at each level of the other inputs. The index variable (X_3) has a negative correlation with the dependent variable. When the index variable (X_3) is included in the model with variable X_2 we obtain an R^2 of 1.0000 (Column 4, Table 13). Comparing the predicted values of the model $Y = b_o + b_1X_2 + b_2X_3$ with the Y column of the data set will verify that the predicted values from this model are identical with the actual observations of Y.

Variable X_3 was then dropped from the model resulting in the predicted values in Column 3 of Table 13. Column 6 presents the index obtained by dividing the observed values of Y by the predicted value of Y from the inadequately specified model. The result is an index which reflects the contribution of variable X_3 given that variable X_2 is in the model.

The same process was applied to the model $Y = b_o + b_1X_1 + b_2X_3$. The resulting index is presented in Column 5. It is interesting to note that the index associated with each level of variable X_1 is different than the one calculated using X_2 in the model, and yet the mean values for each index group are almost the same.

Productivity Index--An Application

Milton Holloway (1972, p. 48) utilized a productivity index which

Table 13. Results of regression analysis on test data.

Model 1 $Y = b_o + b_1X_1 + b_2X_2$	Model 2 $Y = b_o + b_1X_1$	Model 3 $Y = b_o + b_1X_1$	Model 4 $Y = b_o + b_1X_2 + b_2X_3$	Model 2 $Y/\hat{Y}$	Model 3 $Y/\hat{Y}$
12.8	12.8	12	12	.9375	1.6
10.8	12.8		10	.7813	.83
14.8	12.8		14	1.0938	1.17
13.6	13.6	14	14	1.02941	1.0
11.6			12	.8823	.86
15.6	13.6		16	1.1765	1.14
15.2	15.2	16	16	1.0526	1.00
13.2			14	.9211	.88
17.2	15.2		18	1.1842	1.13
18.4	18.4	18	18	.9783	1.00
16.4			16	.8696	.89
20.4	18.4		20	1.0870	1.11
20.0	20.0	20	20	1.0000	1.00
18.0			18	.9000	.9
22.0	20.0		22	1.1000	1.1
Mean Value Index Group 1				1.1283	1.130
Mean Value Index Group 2				.9995	1.000
Mean Value Index Group 3				.8708	.870

> . . . was constructed separately in each area in two steps: (1) A base county which grew crops most common to all other counties in the area was selected. Ratios of average county per acre yields for all common crops were calculated using the base county yields as the denominator. (2) The county land index was calculated by summing these ratios, weighted by the ratio of each county's acreage to the total acreage for each common crop.

This index was used to weight the acreage from each county to account for the differences between counties in quality of land.

Holloway applied the index to each county within four areas of the Northwest. He then estimated production functions from which marginal value product of certain inputs were derived. In his paper, Holloway (1972, p. 49) suggests that a shortcoming of the index which he developed was that "This index is based upon yield data which reflects, to some degree, the use of irrigation, fertilizer and the other inputs." Holloway suggests that an index might be more useful if it could be derived so that the use of irrigation and fertilization would not affect the index.

Quality of land is affected by its natural fertility, its climatic location, other location factors, response to technological improvements, etc. Therefore it would be next to impossible to determine a truly pure quality index unaffected by the inputs used. However, the index developed in this paper does attempt to measure the quality of the land after the effect of variation due to irrigation and fertilization has been removed.

The two indexes were applied to a common set of counties in Southern Idaho to determine if the index developed from the production function improved the estimates of the Marginal Value Product (MVP) of selected items. Counties chosen for the analysis were those that were common to the study area for this study and Holloway's Area A. Twenty counties were common to the two areas, including: Bannock, Butte, Camas, Canyon, Caribou, Cassia, Clark, Elmore, Fremont, Gooding, Jefferson, Jerome, Lincoln, Minidoka, Owyhee, Payette, Teton, Twin Falls, Valley, and Washington.

The correlation coefficient for the two indexes was .43576. The correlation is positive, but not high enough to indicate complete agreement between the indexes. The indexes and other input data used in the application are presented in Table 14.

Two sets of equations were estimated to compare the significance of the two indexes. (1) Using the aggregate data as presented in Table 14, and (2) adjusting the inputs to a per acre basis. Since Holloway used the aggregate approach in his analysis, it was estimated first (Table 15).

Variables in the equation using the Thomas index (ρ_k) exhibited higher t values for all but one variable (current operating expense in the second set); however, both equations resulted in a significance level of $\alpha = .9995$ for current operating expenses. The equations obtained from this analysis are presented below as equations (1)

Table 14. Selected inputs to agriculture for selected counties in Southern Idaho 1964, 1966.[1]

				Input					
County	Labor Man Years	Current Operating Expense	Service Flow of Capital	Indexed Acres of Cropland	Holloway Index I_k	Thomas Index ρ_k	AUMS	Irrigation 100 acre ft.	Value of Sales
Bannock (6)	766	2202.79	2574.67	152.25	.866	.9524	48.91	188.75	7370.20
Butte (12)	301	817.21	994.65	41.02	.817	.9242	132.91	122.99	3242.20
Camas (13)	164	498.56	757.86	104.21	1.091	1.0552	135.24	34.56	1963.90
Canyon (14)	4195	18464.42	11966.68	153.65	.775	1.2324	410.56	1195.84	57987.90
Caribou (15)	698	1924.11	2567.88	224.58	1.005	1.2007	266.49	135.08	8548.80
Cassia (16)	1816	9788.83	5491.47	242.27	.886	.9977	344.73	625.89	33106.80
Clark (17)	143	530.14	418.37	25.85	.949	.9306	199.05	45.51	1484.90
Elmore (20)	510	2047.62	1539.52	40.32	.977	1.1958	134.62	197.46	8691.70
Fremont (22)	940	3339.91	3096.53	142.95	.896	1.1651	170.04	469.05	14290.70
Gooding (24)	1061	3832.02	4036.36	57.17	.752	1.1139	164.87	413.96	12474.50
Jefferson (26)	1299	4425.78	4551.78	140.47	.892	.8470	356.84	964.73	18656.90
Jerome (27)	1462	5883.10	4695.72	108.71	.916	.9996	117.70	601.44	22067.50
Lincoln (32)	532	1693.81	1725.08	34.40	.762	.8987	112.27	220.48	5156.80
Minidoka (34)	1618	7475.47	4588.03	139.08	.897	1.0464	320.28	812.27	24743.80
Owyhee (37)	975	3606.88	3582.90	86.56	1.096	1.2885	624.50	432.68	11300.60
Payette (38)	1016	3364.49	3021.72	47.10	1.032	.8646	44.58	305.82	10486.80
Teton (41)	366	915.89	1467.45	68.14	.649	.7726	113.43	81.71	3561.50
Twin Falls (42)	2854	11694.57	9734.28	241.14	.998	1.0331	293.11	1212.27	36883.60
Valley (43)	157	343.34	639.27	15.54	.745	.8891	120.19	74.93	1379.80
Washington (44)	722	2096.90	2324.16	80.22	.948	1.0677	114.18	104.31	7878.20

[1] Data in this table taken from Holloway (1972) and used with his permission. The data was originally taken from the 1964 Census of Agriculture with the exception of the Thomas Index ρ_k and the irrigation in acre ft/acre which was derived from the USDA Land Inventory and Productivity System (1966).

Table 15. Comparison of results using alternative indexes of land quality to explain variation in value of agricultural output by county, 1964, 1967.[1]

Equation	Variable	Variable Name	Coefficient	Student's t	Significance level	
(1) Aggregate equation	X(2)	Operating Expense	3.135	12.427	.9995	R^2 = .9976,
using Thomas Index	X(3)	Capital Flow	- .928	- 1.894	.950	R^2 corrected = .9951
(ρ_k) of productivity	X(6)	ρ_k	1004.200	1.488	.900	F value for regression
	X(7)	AUMS/county	-3.104	- 1.235	.850	=981.3
	X(8)	Water applied	6.504	3.269	.995	df (6,14) - significant
	X(11)	ρ_k Acres	11.744	2.521	.975	at α = .9995
(2) Aggregate equation	X(2)	Operating Expense	3.140	11.110	.9995	R^2 = .9971,
using Holloway Index	X(3)	Capital Flow	- .725	- -1.379	.9000	R^2 corrected = .9940
(I_k) of productivity	X(4)	I_k Acres	11.135	1.987	.9500	F value for regression
	X(5)	I_k	820.800	1.029	.8000	=800.7
	X(7)	AUMS	-1.863	- .700	.7500	df (6,14) - significant
	X(8)	Water applied	5.322	2.490	.9750	at α = .9995
(3) Equation estimated		Constant	-25.099			
on a per acre basis	X(6)	ρ_k	39.154	1.482	.900	R^2 = .9692
using Thomas Index	X(13)	Operating Expense	2.619	9.057	.9995	F value for regression
(ρ_k) of productivity	X(15)	AUMS	- 2.242	- 1.331	.975	=118.03
	X(16)	Water applied	5.401	1.787	.950	df (4,15) - significant
						at α = .9995
(4) Equation estimated		Constant	7.715			
on a per acre basis using	X(5)	I_k	8.400	.280	--	R^2 = .9672
Holloway Index (I_k) of	X(13)	Operating Expense	3.055	7.677	.9995	F value for regression
productivity	X(14)	Capital Flow	- .593	- .950	.80	=82.51
	X(15)	AUMS	- 1.506	- .847	.75	df (5,14) - significant
	X(16)	Water applied	5.745	1.634	.90	at α = .9995

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[1] Data are generally from two time periods, (1) the 1964 Census of Agriculture, and (2) the 1967 Conservation Needs Inventory.

through (4). Equations (1) and (2) use the data as presented in Table 14; equations (3) and (4) were adjusted to a per acre basis by dividing each variable by the county acreage.

(1) Value = 3.135 (Operating expense) -.928 (capital flow) + 1,004.2 (ρ_k) - 3.104 (AUMS) + 6.504 (water applied) + 11.744 (acres $\cdot$ ρ_k)

(2) Value = 3.140 (Operating expense) - .725 (capital flow) + 11.135 (acres $\cdot$ I_k) + 820.8 (I_k) - 1.863 (AUMS) + 5.322 (water applied)

(3) Value per acre = -25.099 + 39.154 (ρ_k) + 2.619 (operating expense) - 2.242 (AUMS) + 5.401 (water applied)

(4) Value per acre = 7.715 + 8.4 (I_k) + 3.055 (operating expense) - .593 (capital flow) - 1.506 (AUMS) + 5.745 (water applied).

The MVP of the index in each equation in the first set is dependent upon two terms so that the MVP cannot be read directly from the equation. The MVP calculation for ρ_k and I_k for the relevant equations follow:

(1) MVP ρ_k = 1,004.2 + 11.744 (acres)

(2) MVP I_k = 820.8 + 11.135 (acres)

(3) MVP ρ_k = 39.154

(4) MVP I_k = 8.4 (not significant).

Evaluating the MVP requires that a value for acres be chosen. The mean value of the acres is normally chosen because of its favorable statistical qualities. The mean value for acres is 118.69.

Incorporating this in each of the equations, we obtain the following MVPs for the two indexes in the first set:

(1) MVP ρ_k = 2,398.10

(2) MVP I_k = 2,141.41

The MVP of the productivity index indicates the importance of the quality variable. Value of production is highly responsive to changes in the quality of the land resource. To omit this variable could seriously bias the results from a regression analysis.

The F value for the regression equation was larger for the aggregate model using ρ_k by about 23 percent. Percentage of variation explained by the regression equation was higher for the model using ρ_k than for the model using the Holloway Index. The correction for the mean (since both equations were forced through the origin) increased the difference in R^2 between the two models. The significance of the increase in R^2 cannot be assessed directly; however, a test can be made to test the hypothesis, H_o; $r_1 = r_2$, where r_1 and r_2 are the correlation coefficients from the equation using the Holloway Index (I_k) and the LIPS Index (ρ_k) respectively (Table 16).

Table 16. Test of the difference in the coefficient of correlation using two productivity indexes in production function estimates of agricultural output, 1967.

Productivity Index	No. of Observations	r	Z	$n_i - 3$
a. I_k aggregate equation	20	.9970	3.250	17
ρ_k aggregate equation	20	.9980	3.453	17
b. I_k acre equation	20	.9825	2.366	17
ρ_k acre equation	20	.9835	2.395	17

Differences in Z

a. .203

b. .029

Mean Square $\sigma_{z_1 z_2} = \frac{1}{17} + \frac{1}{17} = \frac{2}{17} = .118$

Standard Error $= \sqrt{.118} = .3435$

Test Statistic a $= \frac{.203}{.3435} = .5910$

Test Statistic b $= \frac{.029}{.3435} = .0844$

a. Critical value from Table A4 (Snedecor 1967) at ∞ degrees of freedom and 1% level of significance equals 2.576. Therefore since $.5910 < 2.576$, we fail to reject the H_o: $r_1 = r_2$.

b. Critical value is the same as in a. above. Therefore, since $.0844 < 2.576$, we fail to reject H_o: $r_1 = r_2$. The test also fails at the 5% level of significance.

The statistical conclusion from this test is that the inclusion of

ρ_k in the model in place of Holloway's Index I_k does not result in a significant change in r for either model. This should not deter us from the benefit to be gained from the use of ρ_k rather than I_k.

From a theoretical standpoint, ρ_k is superior to I_k. A look at the method of derivation will serve to point out the reasons for this superiority. ρ_k is derived from the results of a regression analysis used to measure the effects of other inputs on the production of each crop. The hypothesis was made that if the land quality variable was not specified in the model, its effects would be reflected as a measurable component of the error term. I_k was derived using the following procedure as described by Holloway (1972, p. 48).

> The cropland quality index was constructed separately in each area in two steps: (1) A base county which grew crops most common to all other crops in the area was selected. Ratios of average county per acre yields for all common crops were calculated using the base county yields as the denominator. (2) The county land quality index was calculated by summing these ratios, weighted by the ratio of each county's acreage to the total area acreage for each common crop.

The land quality index calculated by Holloway (1) uses county averages and ignores the land quality mix within each county, (2) derives the index directly from yields which are influenced by many variables not related to land quality; i.e., variation in water input, fertilizer input, management differences, etc., and (3) weights the impact on the county index of the respective crops by its share of the area acreage rather than its relative importance in the specific county.

On the other hand, ρ_k is derived from a basic unit of quality difference, the LRA, which emphasizes the quality mix in a county and its relationship to the crops adapted thereto. By deriving the index from the residuals of the purposefully misspecified equation, the effects of other inputs were largely eliminated. The county indices were weighted by the relative acreages first of crops within an LRA and then by the relative importance of each LRA to the county.

From an econometric viewpoint, the problem with the Holloway Index I_k is that I_k will not be independent of the stochastic error-term in the county value production function. This dependence occurs because I_k is derived from the same average yields involved in the dependent variable, county value of production. Since I_k will not be independent of the stochastic error term in the county value production function, some bias in the estimated coefficients would be expected (Johnston, 1963, p. 149).

To illustrate, let:

P = Productivity Index

Z = All other inputs

Q = County yields

$V = \lambda Q$ = County prices times county yields

u = The stochastic error term for the county value production function

$\epsilon_i = (Q_i - EQ_i)$ and $u_i = \lambda (Q_i - EQ_i) = \lambda \epsilon_i$

$$E(\epsilon_i) = 0$$

$$E(\epsilon_i \epsilon_j) = \begin{matrix} 0 \text{ if } i \neq j \\ \sigma^2 \text{ if } i = j \end{matrix}$$

Z and ϵ are independent. Then

$$P = a + \beta Q$$

and
$$V = \lambda Q = A + \gamma_1 P + Z\gamma_2 + u$$

represent the relationship between P and Q and the production function to be estimated. Assuming all elements of Z are fixed numbers, we can substitute for Q in the first equation;

$$P = a + \frac{\beta}{\lambda}(A + \gamma_1 P + Z\gamma_2 + u)$$

$$(1 - \frac{\beta\gamma_1}{\lambda})P = a + \frac{\beta}{\lambda}(A + Z\gamma_2 + u)$$

since $(1 - \frac{\beta\gamma_1}{\lambda}) = \frac{\lambda - \beta\gamma_1}{\lambda}$

$$P = \frac{\lambda a}{\lambda - \beta\gamma_1} + \frac{\beta}{\lambda - \beta\gamma_1}(A + Z\gamma_2 + u)$$

$$EP = \frac{1}{\lambda - \beta\gamma_1}(\lambda a + \beta(A + Z\gamma_2))$$

$$E[u(P - EP)] = E[u(\frac{\beta}{\lambda - \beta\gamma_1})u] = \frac{\beta}{\lambda - \beta\gamma_1} Eu^2$$

since $u = \lambda(Q - EQ) = \lambda\epsilon$

$$E[u(P - EP)] = \frac{\beta}{\lambda - \beta\gamma_1}(\lambda^2\sigma^2) \neq 0$$

Since the covariance term does not reduce to zero in the expectation, P and u are seen to be correlated. From the discussion in Johnston (1963, Chapters 6 and 9), the direct application of O.L.S. to a situation where the disturbance term and an explanatory variable are correlated, will result in a biased estimate of the regression parameters. Thus in the case of $\lambda Q = A + \gamma_1 P + Z\gamma_2 + u$, where P and u are correlated, biased estimates would result.

ρ_k was also more significant in the models using the two indexes of productivity, presented in Table 15 and Appendix Table IV-2. Thus the use of ρ_k resulted in a better empirical performance as well as being more theoretically sound, since it would be expected to be independent of the production function error term.

RIDGE REGRESSION

Holloway (1972) discussed the presence of multicolinearity in the data used in his analysis. In an effort to obtain more reasonable results from his analysis and mitigate multicolinearity effects, he used Theil's prior information model. This activity was successful in obtaining parameters more acceptable to economic theory. However, the overriding effect of the assumed prior information on the value of the new parameters, in this author's opinion, reduces the degree of objectivity and subsequently the reliability of these results. Because it injects the analyst's preconception of a value into the estimation of that value from an equation, prior values are often hypothesized and do not necessarily reflect actual parameters. If through imposition of prior information, one predetermines the results of an analysis, one may as well simply use the priors and be done with it.

There are means available that do not subject the analysis to the same degree of subjectivity as the Theil approach.

The mathematical problem encountered in the case of multicolinearity among variables relates to the behavior of the X'X matrix and the inversion of this matrix to form the equation for estimating the coefficients of the production function,

$$\hat{\beta} = (X'X)^{-1} X'Y.$$

If the (X'X) matrix is singular[9] its inverse is not defined. The usual case in economic data is that the (X'X) matrix approaches singularity when one or more of the economic variables are highly correlated. This problem can be seen more clearly if we discuss it in terms of the correlation matrix of (X'X) rather than the matrix of the sums of squares and cross products.

If the correlation matrix is nearly a unit matrix, the usual Gauss-Markov procedure for estimating a regression equation is acceptable (Hoerl and Kennard, 1970).

"However, if X'X is not nearly a unit matrix, the least squares estimates are sensitive to a number of 'errors.' The results of these errors are critical when the specification is that $X\beta$ is a true model" (Hoerl and Kennard, 1970, p. 55).

The correlation matrix will be nearly a unit matrix if the explanatory variables are independent and therefore uncorrelated. The case we are considering here is one of high correlation between one or more of the independent variables. This results in a correlation matrix which is not nearly a unit matrix, and as such is subject to the errors described by Hoerl and Kennard (1970) above. In cases of severe multicolinearity, the matrix will approximate a singular matrix resulting in a determinant small in magnitude, and an inverse

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[9] A matrix is singular if one or more of the columns (rows) can be expressed as linear combinations of other columns (rows).

matrix with elements subject to question due to the rounding errors associated with division by the small determinant.[10]

From this, it appears that a procedure whereby the X'X matrix could be adjusted to approximate more nearly a unit matrix[11] in terms of the correlations, would solve the rounding problem and result in more reasonable estimates from the given data set.

Hoerl and Kennard (1970, p. 55) state that: "Estimation based on the matrix $[X'X + KI]$, $K \geq 0$ rather than X'X has been found to be a procedure that can be used to help circumvent many of the difficulties associated with the usual least squares estimates."

This procedure has been referred to in the literature as <u>ridge regression</u>. The addition of KI to the matrix augments the main diagonal of (X'X)[12] and causes it to approach or tend toward the behavior of an orthogonal matrix as K increases in value.

The augmentation of the main diagonal in a matrix with non-zero elements off the diagonal will increase the relative size of the main diagonal elements and the augmented matrix will approximate, at some value of k, the mathematical behavior of an orthogonal matrix.

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[10] The inverse of a matrix is defined as $A^{-1} = \frac{1}{|A|}$ (Adjoint A) where $|A|$ is the determinant of the matrix A and Adjoint A is found by replacing each a_{ij} by its cofactor C_{ij} and then transposing the resulting matrix (Johnston, 1963).

[11] A unit matrix in the context of the correlation matrix is identical to an identity matrix of the same order.

[12] Using (X'X) in terms of the sum of squares and cross products matrix.

Since the use of a method such as ridge regression violates the assumptions surrounding the use of the usual least squares procedure, it will affect estimates of the true βs by a definable level of bias.

From Brown (1973, p. 15) we find that this bias is

$$E(\hat{\beta}_1^* - \beta_1) = \frac{-k}{(1 + k)^2 - r_{12}^2} [(1 + k)\beta_1 - r_{12}\beta_2]$$

where $\hat{\beta}_1^*$ is the ridge estimate of β_1, k is the augmenting constant and r_{12} is the correlation coefficient between the two variables X_1 and X_2. Brown (1973, p. 15) continues,

> . . . for the usual case of high positive correlation between economic variables, the expected bias in $\hat{\beta}_1^*$ will be lessened if the true β values have the same sign, and even more so if they are of about equal magnitude.

Three things should be considered then in making a determination about the applicability of ridge regression to a specific problem. First it must be determined whether or not precision in the estimation of equation parameters is critical. Often the purpose of an economic analysis is to predict values of the dependent variable. When this is the case, one may obtain satisfactory results using the more usual variable deletion approach to the problem of multicolinearity. The omission of one of two highly correlated variables will often not affect the prediction precision of an equation.

If, however, one is interested in drawing conclusions about the value of the parameters; i.e., differentiate to obtain marginal value product estimates, the omission of relevant variables can have serious implications (Brown, 1973).

Second, the value of the βs associated with the highly correlated variables should be determined in a theoretical context. This involves a look at economic theory and the development of a sound logical argument in support of a set of β values. The relationship between the assumed β values lead us to a conclusion about the applicability of ridge regression to a specific problem. A subjective determination must be made as to the bias that will result from Brown's (1973, p. 15) equation for estimating the expected bias, $E(\hat{\beta}_1^* - \beta_1)$.

The third consideration is an outgrowth of the second, and relates again to the relationship between β_1 and β_2. Consideration must be given to (1) the sign of β_1 and β_2, and (2) to the sign of the correlation coefficient. In the case of high positive correlation, bias will be minimized if the signs of β_1 and β_2 are the same.

The ideal situation for the use of ridge regression exists then when (1) estimates of individual parameter estimates are critical to the objective of the analysis, (2) the β values associated with the highly correlated variables are of about equal magnitude, and (3) the sign of the correlation coefficient reflects the true relationship between the respective explanatory variables.

Since the usual economic analysis involves more than two variables, the considerations listed here for the two variable case must apply to analyses involving three or more variables. For the derivation and proof of the ρ variable case, see Theorem 1, Brown, 1973, p. 31.

Holloway (1972) determined the presence of multicolinearity in the data he used to estimate the productivity of water and other factors related to agricultural production. Thiel's prior information model was applied to his data set as a means of circumventing the multicolinearity problems. It is the thrust of this section to examine the use of ridge regression, as described above, as an alternative approach to the mitigation of errors resulting from multicolinearity in the data set.

The first step in applying ridge regression is to apply the three considerations outlined above to the problem at hand.

First, multicolinearity in the data has been established by Holloway (1972, p. 77) and the necessity for individual parameter estimates is obvious since the objective of the analysis is to derive marginal value product estimates for each independent variable. Therefore, $X\beta$ is assumed to be the appropriate model and omission of any variables will result in some specification bias and loss of critical information.

The consideration given to the criteria concerning the relative magnitude and sign of the problem variables depends on an appeal to the theory of economics and must be developed.

Theoretical Marginal Value Product Values

Rational entrepreneurs will use factors of production until the marginal return from the last unit of a given factor equals the marginal return from each of the other factors. This axiom is subject in the real world to limits placed on the use of certain factors due to their relative availability to an individual entrepreneur. However, on an aggregate basis, one would expect the real world situation to approach or at least tend toward the theoretical optimum over time. Three variables in the Holloway (1972) model were highly correlated with each other; capital flow, current operating expenses, and labor input.

The rate at which these variables are applied in the production of agricultural commodities is subject to some limitations on a firm by firm basis. These limitations are reasonably flexible and over time, should tend toward the theoretical optimum levels of factor use.

From this, it is apparent that the theoretical values would (1) be of the same sign, and (2) be approximately of equal magnitude.

The model used to compare the index of productivity developed in this paper with an index developed by Holloway (1972), seems, based upon the above criteria, to present an ideal case for the application of ridge regression to current economic analysis.

In the section comparing the two indexes of productivity, it was shown that the index developed in this study was superior to that developed by Holloway (1972), from both a theoretical derivation basis, and on the basis of its performance in the production function. Therefore, the model used in this section will use the index developed in this study.

The Model

The ridge regression technique augments the diagonal elements of $(X'X)^{-1}$ such that:

$$\hat{\beta}^{*}_{*} = [X'X + KI]^{-1} X'Y; \quad K \geq 0.$$

> By adding a small positive quanity (sic) to each diagonal element, the system $X'X + K\hat{\beta}^{*} = X'Y$ acts more like an orthogonal system. When $K = KI$ and all solutions in the interval $0 \leq K \leq 1$, are obtained, it is possible to obtain a two-dimensional characterization of the system and a portrayal of the kinds of difficulties caused by intercorrelations among the predictors (Hoerl and Kennard, 1970, p. 65).

The model to predict value per county (Y) was hypothesized to be:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_6 X_6 + \mu$$

where

Y = value of production

X_1 = labor in man years

X_2 = current operating expenses

X_3 = capital flow in dollars

X_4 = AUMS

X_5 = water applied

X_6 = an index of land productivity (ρ_k)

and μ = random error term.

Using a computer program written for this purpose by David Fawcett at Oregon State University, ridge regression was applied to the data used in the previous section to compare the two indexes of productivity. The augmenting constant K was varied from $K = 0$ to $K = .9$ in increments of 0.1.

Selection of an Optimal K Value

The ridge regression technique presents us with several values of $\hat{\beta}^*$, one for each level of K selected. It has been shown that there exists a value of $K > 0$ such "that it is possible to move to $K > 0$, take a little bias, and substantially reduce the variance thereby improving the mean square error of estimation and prediction" (Hoerl and Kennard, 1970, p. 61).

The question then is which of the levels of K should be selected as the best estimator of the coefficients of the equation. Hoerl and Kennard (1970, p. 65) indicate four considerations in choosing the optimum value of K.

- At a certain value of K the system will stabilize and have the general characteristics of an orthogonal system.

> - Coefficients will not have unreasonable absolute values with respect to the factors for which they represent rates of change.
>
> - Coefficients with apparently incorrect signs at $K = 0$ will have changed to have the proper sign.
>
> - The residual sum of squares will not have been inflated to an unreasonable value. It will not be large relative to the minimum residual sum of squares or large relative to what would be a reasonable variance for the process generating the data.

To these considerations Brown (1973, p. 29) adds the following rule.

> Rule: Select a particular value of K at that point where the last ridge estimate[13] attains its maximum absolute magnitude after having attained its ultimate sign, where ultimate sign is defined as being the sign at, say, $K = 0.9$.

The four considerations discussed by Hoerl and Kennard (1970) and Brown's rule will still not lead us to a unique value of K or set of values for $\hat{\beta}^*$. Brown (1973, p. 33) stresses the importance of prior information in evaluating the results of the ridge regression analysis,

> . . . one *must* have good prior information about the true β values for $Y = X\beta + \mu$ and good information or data concerning the nature of the interrelationships among the explanatory variables. Only if one has this good information does it appear possible to evaluate one's results from ridge regression.

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[13] The last ridge estimate refers to the ridge estimate (of a variable coefficient) which attains its maximum absolute value, after attaining its ultimate sign, at the highest value of K.

Although we may not be able to evaluate the results of ridge regression, its use will generally improve estimates of β from ordinary least squares, $\hat{\beta}$. In discussing the problem of selecting a value of K and the fact no procedure existed for determining a unique value, Hoerl and Kennard (1970, p. 64) suggest "However, this is no drawback to its use because with any set of data it is not difficult to select a $\hat{\beta}^*$ that is better than $\hat{\beta}$." This statement is made in light of the fact that many restrictions exist in the normal application of linear regression to a set of data. "In fact, put in context, any set of data which is a candidate for analysis using linear regression has implicit in it restrictions on the possible values of the estimates that can be consistant with known properties of the data generator" (Hoerl and Kennard, 1970, p. 64).

Hoerl and Kennard (1970) add that these restrictions are difficult to make explicit in any form and that for linear regression it is much more difficult due to the number of parameters involved. The fact that improved estimates of β can be obtained coupled with a common situation in economic research leads to the conclusion that ridge regression can be used as long as care is taken in the selection of the problem to which it is applied.

Brown (1973) suggests that certain theoretical considerations and expectations will take the place of other good prior information in determining whether or not ridge regression can be used in a particular situation.

This leaves us with a decision to make regarding criteria to use in selecting a value of K. The ridge trace can be used to determine a relevant range of K values to consider. Then the selection rule (Brown, 1973) could be used to select from alternative K values in this predetermined range. This sequence of steps is important in most economic studies, due to the sometimes large number of parameters being estimated. Reliance upon the analyst's knowledge of the model, and the particular phenomena being analyzed is the best assurance of the selection of a proper K value.

Results of Ridge Regression Analyses

The ridge regression technique applied to the data from the comparison section of this paper resulted in the ridge trace depicted in the graph (Figure 3). The ridge trace was constructed from standardized b values resulting from 10 regression equations using $\hat{\beta}^* = X'X + KI^{-1} X'Y$ and 10 values of K ranging from 0 to .9 in steps of 0.1.

From the graph (Figure 3), the system seems to stabilize in the range K = .2 to K = .05. To apply Brown's (1973) rule, we must inspect the values of $\hat{\beta}^*$ and determine the unique value of K that will best satisfy the rule.

All of the $\hat{\beta}^*$ values increase or decrease constantly throughout

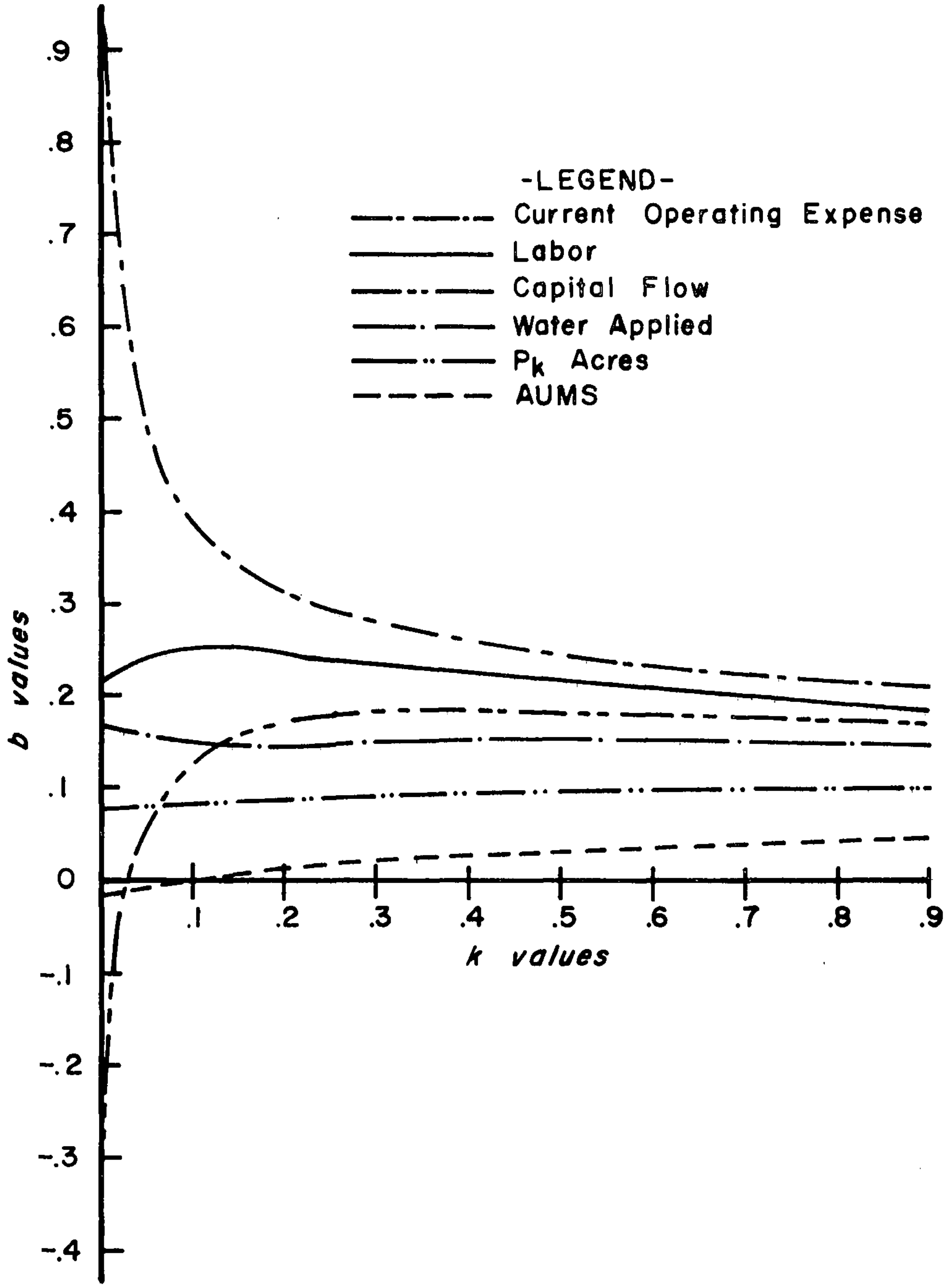

Figure 3 Ridge trace of standardized b values for county prodution model.

the range of K values selected except for $\hat{\beta}_3^*$ (capital flow). Our attention then is focussed on $\hat{\beta}_3^*$ which attains its maximum value at $K \doteq .4$.

Total value per county = a^{*}[14] + 3.30 (labor) + .84 (current expenses) + .91 (capital flow) + 2.98 (AUMS) + 5.83 (water applied) + 17.12 $\rho_k \cdot$ (acres).

Table 17. Value of standardized b values associated with increments of K.

Value of K	Standardized b Values Labor	Current Operating Expense	Capital Flow	AUMS	Water Applied	$\rho_k \cdot$ Acres
0.0	.2177	.8795	-.3001	-.0184	.1666	.0783
.1	.2584	.3935	.1400	.0060	.1260	.0817
.2	.2466	.3213	.1757	.0157	.1413	.0903
.3	.2357	.2869	.1846	.0235	.1490	.0958
.4	.2266	.2652	.1864	.0298	.1526	1.993
.5	.2188	.2497	.1856	.0349	.1540	.1015
.6	.2119	.2375	.1837	.0390	.1541	.1029
.7	.2058	.2276	.1812	.0423	.1534	.1036
.8	.2002	.2192	.1784	.0450	.1523	.1039
.9	.1951	.2118	.1755	.0472	.1509	.1038

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[14] Brown (1973) suggests that in the two variable case a^* can be estimated using $a^* = \overline{Y} - 2\hat{\beta}_i^*\overline{X}_i$, which yields $a^* = 14561.4 - [3.2999(1079.75) + .8378(4247.29) + .9123(3488.72) + 2.9825(211.22) + 5.8261(124.26)]$.

$a^* = 14,561.4 - 11,657.1$

$a^* = 2,904.3$

Table 18. Value of β^* associated with increments of K.

Value of K	$\hat{\beta}_1^*$ Labor	$\hat{\beta}_2^*$ Current Operating Expense	$\hat{\beta}_3^*$ Capital Flow	$\hat{\beta}_4^*$ AUMS	$\hat{\beta}_5^*$ Water Applied	$\hat{\beta}_6^*$ $\rho_k \cdot$ Acres
0.0	3.1704	2.7782	-1.4687	-1.8397	6.3622	13.493
.1	3.7629	1.2430	.6849	.6048	4.8106	14.077
.2	3.5901	1.0148	.8598	1.5735	5.3936	15.564
.3	3.4327	.9062	.9033	2.3526	5.6884	16.512
.4	3.2999	.8378	.9123	2.9825	5.8261	17.117
.5	3.1858	.7886	.9084	3.4911	5.8790	17.500
.6	3.0856	.7503	.8988	3.9025	5.8835	17.732
.7	2.9960	.7189	.8866	4.2360	5.8588	17.857
.8	2.9148	.6923	.8730	4.5068	5.8158	17.907
.9	2.8404	.6691	.8590	4.7269	5.7612	17.899

Since the dependent variable is measured in terms of dollars, the marginal value product for each of the six variables are directly observable from the equation.

At the level of K selected all six coefficients are positive, the return to capital flow and current operating expense are nearly equal, which one would expect if entrepreneurs are rational, and water values compare reasonably close to other water values obtained in other sections of the study. Statistical objectivity of the normal regression solution must be weighed against the gain in information concerning variables now included in the model that would otherwise, by necessity, be deleted.

The results of this exercise are very encouraging in that we have been able to observe the relative sensitivity of the correlated variables to the orthogonalizing procedure used.

Estimates of the marginal value product of each of the components of the hypothesized model have been obtained with an increase in the precision of the estimate for three of the six variables in the model. The MVPs for all six variables are reasonable from an economic theory view and from a comparison with prior information used by Holloway (1973) (Table 19).

Table 19. Prior values[1] for five variables used by Holloway in applying Theil's prior information model (Holloway, 1973, p. 92).

Variable	Ridge Estimate	Priors
Labor X_1	3.300	4.560/\$1,000 per man year
Current Operating Expense X_2	.838	1.000/return per dollar invested
Capital flow	.912	1.000/return per dollar invested
AUMS	2.983	1.650/value per AUM
Water Applied	5.826	6.000/value per acre foot applied

[1] For derivation of prior values see Holloway (1973, pp. 87-90.

SUMMARY AND CONCLUSIONS

This study was initiated as a result of the author's work in developing data for the Economic Research Service's Land Inventory and Productivity System (LIPS). A search of the literature revealed a void in published data concerning the productive relationship of water, fertilizer, and other inputs to land as a companion resource for the production of agricultural goods. The LIPS data on the other hand presented an opportunity to estimate production relationships on a broad based data system which incorporates certain physical characteristics of the land resource to aid in the identification of the relationship of water and fertilizer to yield.

Five crops were chosen for the analysis in an area covering much of Southern Idaho. These five crops accounted for slightly less than 2 million acres of irrigated cropland and a large proportion[15] of the non-irrigated cropland in the area.

The land resource was divided into Land Resource Areas (LRAs) which delineated contiguous regions of the land base with similar characteristics related to climate, elevation, natural cover, and growing season. Soils were classified into Soil Resource Groups (SRGs) to identify soils which have a relatively high degree of homogeneity with respect to cropping pattern, required cultural practices

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[15] See Appendix III for a more complete description of the data.

(i.e., drainage, strip cropping, deep tillage, etc.), water, and fertilizer requirements and yield.

The Land Resource Area was found to be the most important grouping of data for estimating production functions in this analysis. The rationale was, that management and cultural practices can compensate for deficiencies in the soil as identified by SRGs, but little can be done to change factors associated with the LRA. Thus differences in yield related to the soil classification, were overshadowed by the differences associated with the Land Resource Areas.

Equations representing production functions for each of the five crops within each LRA were estimated using least square regression techniques. The range of marginal value productivity of water estimates for each crop were from barley, \$.35 to \$6.47; wheat, \$.62 to \$6.53; hay, \$3.20 to \$7.98; sugar beets, \$5.42 to \$18.83; and potatoes, -\$.85 to \$18.35. Water attained its highest return applied in sugar beet production for every LRA that produced sugar beets.

The MVP of fertilizer was estimated over the LRAs for each crop. MVPs per dollar expenditure on fertilizer ranged from \$1.87 (sugar beets) to \$4.43 (potatoes). These values suggest that input control by sugar companies have brought the application of fertilizer closer to the point where MVP = Price than is the case for the other crops.

The low MVPs for the relatively extensive crops suggest a low cost of water. If water were more expensive, it would no longer be economically feasible to irrigate the most extensive crops and the more intensive crops would be irrigated at lower water levels. The decision the resource planner must make in determining the economic feasibility of irrigating new land then is one of whether or not the return to the least productive crop will justify the cost of providing irrigation. Any other criteria ignores the substitution of cropland between low and higher value crops. The only departure from this criteria would occur when certain extensive crops are required because of a rotation designed to maintain the integrity of the land resource. In such cases the method of calculating the benefit from irrigation development would incorporate the value imputed to the higher value crops derived from the rotation or soil building crops.

An example of this type of rotational requirement is found in LRA 11B in the Osgood area northwest of Idaho Falls. The U & I Sugar Company, owners of a large tract of land in this area, have developed a cropping rotation for their tenant farmers. This rotation consists of 1/3 of the acreage in sugar beets, 1/3 in potatoes and 1/3 in small grains. Each crop is grown for two years then rotated to the next field. In this case the value of the water applied to the small grains would exceed to some extent the MVP estimated for small grains. The amount of this difference would have to be derived from the value of water in the production of the more intensive crops. This could be done experimentally to determine the importance of such a rotation on the yield of the intensive crops. This would be an area where further research would shed light on the reasonableness of current cropping patterns and the rationale behind cropping decisions as well as on the real value or benefit of additional irrigation development in the primary or agricultural sector.

To identify the total value of water there are several site specific factors which would need to be identified and accounted for before comparisons of value between locations (sites) can be meaningful. These factors fall into three general categories: physical (technological), social and economic. Irrigation in many areas of the western U. S. is absolutely required for crop production. Owners of the land resource in these areas are physically dependent upon the water for production of any crop. Irrigation then derives a high utility because of its importance based upon physical or technological characteristics of the land resource. The degree of dependence upon water for irrigation results in differing utility from site to site. In areas with less severe physical restrictions the utility from the physical or technological standpoint would be decreased. Irrigated agriculture and farming in general approach "motherhood" status as an institution in many cases. These and other social characteristics of a given local population impinge upon the utility derived from irrigated agriculture.

Economic measures of utility are often reduced to dollars and cents through the assumption that the perfectly competitive market will reflect the aggregate utilities of the market participants. Be this as it may economic values can be quantified and displayed in terms of marginal value productivities (when market prices are not available). Secondary impacts can be estimated from Input-Output tables and values imputed to water.

Thus we can quantify the utility[16] of water from the economic factors but not the social and physical factors. Further studies of water value should incorporate the other utility factors related to the specific physical (technological), social, cultural, and economic environment associated with a given site.

An index of productivity was derived from the LIPS data. The objective for deriving an index was to make use of information available in the LIPS data in future studies where other easier to obtain data could be used. Variation in land quality due to the LRA could be measured from the LIPS data and used as a proxy for variations in land quality when using secondary data.

A comparison of an index of productivity developed by Holloway (1972) with the index developed from the LIPS data was made. The LIPS index was found to be more significant in explaining the variation in value of agricultural output among counties analyzed than the

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[16] In a dollars and cents proxy measure from the production function.

Holloway index. The LIPS index theoretically measured the variation in land quality after variation due to water and fertilizer had been accounted for by the regression equation.

The potential usefulness of the index stems from the assumption that factors related to the LRAs are relatively constant over time. Thus the index will be valid for subsequent time periods when new production parameter estimates will be needed. These parameter estimates will often be made using secondary data as in the case of Holloway's work. With the constant nature of the LIPS index, it will be possible to specify land quality as a determinant of agricultural production resulting in more fully specified models and a consequent reduction in specification bias. The relatively large MVP for land quality ($39.00/acre per percent change in the index) indicates that its omission would result in significant specification bias.

Finally, ridge regression techniques were applied to the secondary data models used in the comparison section. Multicolinearity was found by Holloway (1972) to be a significant problem among several of the explanatory variables used in his analysis. This problem is also indicated by the high correlation among several independent variables in this analysis. Normally the occurrence of multicolinearity results in the deletion of variables from the equation so that the remaining variables form a more nearly orthogonal set. Ridge regression was applied to illustrate a method available for use in solving empirical problems related to multicolinearity.

The purpose of ridge regression is to obtain an estimate of parameters for relevant variables in a non-orthogonal set. Bias is introduced into the equation and must be weighed against the bias and lack of information resulting from the deletion of relevant variables (Brown, 1973). It must be remembered that ordinary least squares results in unbiased estimates of the equation coefficients if and only if the model is the correct model, and is fully specified. Therefore, when multicolinearity is a significant factor and variables must be deleted, the ordinary least square procedure does not result in truly unbiased parameter estimates. When information concerning deleted variables is required for decision making, it can be argued that ridge regression represents a viable alternative to ordinary least squares.

The results of the two ridge regression equations estimated above were encouraging in that parameter estimates stabilized at a relatively low level of k (the augmenting constant), and the results were meaningful in an economic sense.

The value obtained for water in the ridge regression model using secondary data is reasonably close to those obtained for the extensive crops from the quadratic equation using the LIPS data. Since the acreage of the three extensive crops (barley, wheat, and hay) account for most of the cropland, the MVPs for water associated with them would dominate the MVPs for water in the production of sugar beets and potatoes. Therefore, it is reasonable to assume that the difference between the aggregate estimate from the secondary sources, and the estimate from the LIPS data is small enough to imply that both estimates could come from the same population. This test cannot be made statistically because of the bias introduced by the ridge regression analysis.

General Observations About the Analysis

The data in the Land Inventory and Productivity System present us with new information heretofore unavailable. This information can be useful in determining the economic value of water in areas where decisions concerning the benefit of water use are going to be made. It is not enough to wait for the day when a competitive market for water will exist between areas from which prices can be observed. By making use of this data set, it would be possible to make estimates of the MVP of water for any area, consistent between areas, in the western states. These MVP estimates could then be used to compare the return to water from regions competing for the use of water.

The LIPS data is not complete for all of the states yet, but should be completed and the older data updated within the foreseeable future. A further analysis would be appropriate to determine a productivity index by county for those areas presently covered by the LIP System. The productivity index has potential as a tool in production analysis dependent upon secondary data such as Ag Census, Ag Statistics and other data sources because it disaggregates factor inputs consistant with a data pool not previously available.

Ridge regression appears, from the application herein, to have promise as a means of circumventing some data problems associated with normal economic analysis. A further application of ridge regression to a model based on a more complete set of data to be available from the LIPS data would be an appropriate extension of the application in this analysis.

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APPENDICES

APPENDIX I

Table I-1. Miscellaneous Statistical Information for Equations 1 and 2 - Barley

Equation	Var.	Variable name[1/]	Coefficient	Student t	Sig. level
01	X_0	Constant	33.723		
	X_1	Water (W)	2.750	5.1080	.9995
	X_2	Fertilizer cost (FC)	1.102	2.5940	.995
	X_3	$\sqrt{W \cdot FC}$	1.878	2.1839	.975
	X_8	$W \cdot LRA_5$	-1.697	-1.2376	.850
	X_9	$W \cdot LRA_6$	5.310	5.8555	.9995

R^2 = .60842

$\bar{X}_1$ = 2.5593; $\sqrt{\bar{X}_1}$ = 1.5998

$\bar{X}_2$ = 4.4378; $\sqrt{\bar{X}_2}$ = 2.1066

F value regression = 103,484; df (5,333) significant at (.9995)

F level enter = 1.5317; df (1,333) significant at (.750)

Eq: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 \sqrt{X_1 X_2} + \beta_8 X_1 L_4 + \beta_9 X_1 L_5$

Equation	Var.	Variable name[1/]	Coefficient	Student t	Sig. level
02	X_0	Constant	29.2857		
	X_1	Water (W)	11.1702	8.9077	.9995
	X_2	Fertilizer cost (FC)	3.7481	7.0648	.9995
	X_3	$W \cdot FC$	-.2538	-2.5324	.990
	X_6	$W \cdot L_3$	-.2440	-2.1360	.975
	X_7	$W \cdot L_4$	.8756	1.1216	.850
	X_8	$W \cdot L_5$	-1.6043	-1.3617	.900
	X_9	$W \cdot L_6$	2.5852	3.1184	.995
	X_{10}	$W \cdot L_7$	-3.6890	-3.2110	.995
	X_{11}	X_1^2	-1.1715	-5.6394	.9995
	X_{12}	X_2^2	-.0979	-5.1201	.9995

R^2 = .716

$\bar{X}_1$ = 2.5593, $\bar{X}_2$ = 4.4378

F value regression = 82.722; df = (10,328) significant at (.9995)

F level enter = 1.258; df = (1,328) significant at (.500)

Eq: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_6 X_1 L_3 + \beta_7 X_1 L_4$
$+ \cdots + \beta_{10} X_1 L_7 + \beta_{11} X_1^2 + \beta_{12} X_2^2$

[1/] $L_1 = LRA_2$; $L_2 = LRA_3$; $\cdots$ $L_6 = LRA_7$; $L_7 = LRA_8$.

Table I-2. Miscellaneous Statistical Information for Equations 1 and 2 - Wheat

Equation	Var.	Variable name[1/]	Coefficient	Student t	Sig. level
01	X_0	Constant	29.7854		
	X_1	Water (W)	1.8295	3.1051	.995
	X_2	Fertilizer cost (FC)	-.4556	-2.5624	.990
	X_3	$\sqrt{W \cdot FC}$	5.1827	9.4094	.9995
	X_4	$W \cdot L_1$	-.9082	-1.9727	.975
	X_5	$W \cdot L_2$	-1.4054	-3.1479	.995
	X_6	$W \cdot L_3$	3.4554	2.7704	.995
	X_7	$W \cdot L_4$	-1.8515	-2.4345	.990
	X_8	$W \cdot L_5$	-2.8239	-3.1034	.995
	X_9	$W \cdot L_6$	1.7427	2.1248	.975

$R^2 = .5660$

$\bar{X}_1 = 2.6688$; $\sqrt{\bar{X}_1} = 1.6336$

$\bar{X}_2 = 5.5727$; $\sqrt{\bar{X}_2} = 2.3607$

F value regression = 96.5104; df = (9,666) significant at (.9995)

F level enter = 3.8917; df = (1,666) significant at (.950)

$$Eq: Y = \beta_0 = \beta_1 X_1 = \beta_2 X_2 + \beta_3 \sqrt{X_1 X_2} + \beta_4 X_1 X_2 + \cdots + \beta_4 X_1 L_6$$

02	X_0	Constant	24.3808		
	X_1	Water (W)	8.7380	11.0820	.9995
	X_2	Fertilizer cost (FC)	3.3110	12.2246	.9995
	X_3	$W \cdot FC$	-.3394	-4.6908	.9995
	X_5	$W \cdot L_2$	-.6503	-1.6885	.950
	X_6	$W \cdot L_3$	1.3020	1.1964	.850
	X_7	$W \cdot L_4$	-1.0996	-1.7266	.950
	X_8	$W \cdot L_5$	-1.5206	-1.9413	.950
	X_9	$W \cdot L_6$	-1.0730	-1.4916	.900
	X_{10}	$W \cdot L_7$	-2.9819	-3.2276	.995
	X_{11}	W^2	-.6402	-4.9705	.9995
	X_{12}	FC^2	-.0142	-6.2750	.9995

$R^2 = .6722$

$\bar{X}_1 = 2.6688$; $\bar{X}_2 = 5.5727$

F value regression = 123.802; df = (11,664) significant at (.9995)

F level enter = 2.2247; df = (1,664) significant at (.750)

$$Eq: \quad Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_5 X_1 L_2$$

$$+ \cdots + \beta_{10} X_1 L_7 + \beta_{11} X_1^2 + \beta_{12} X_2^2$$

[1/] $L_1 = LRA_2$; $L_2 = LRA_3$; $\cdots$ $L_6 = LRA_7$; $L_7 = LRA_8$.

Table I-3. Miscellaneous Statistical Information for Equations 1 and 2 - Hay

Equation	Var.	Variable name1/	Coefficient	Student t	Sig. level
01	X_0	Constant	1.6111		
	X_1	Water (W)	.4093	17.2465	.9995
	X_2	Fertilizer cost (FC)	.0991	5.7874	.9995
	X_4	$W \cdot L_1$	-.0977	-3.4127	.9995
	X_5	$W \cdot L_2$	-.2035	-7.7521	.9995
	X_6	$W \cdot L_3$	-.1226	-2.9679	.9950
	X_7	$W \cdot L_4$	-.2731	-6.6058	.9995
	X_8	$W \cdot L_5$	-.1565	-4.0841	.9995
	X_9	$W \cdot L_6$	-.1521	-4.3110	.9995
	X_{10}	$W \cdot L_7$	-.1363	-2.6893	.9950

R^2 = .5464

$\bar{X}_1$ = 3.6160; $\sqrt{\bar{X}_1}$ = 1.9016

$\bar{X}_2$ = 2.2685; $\sqrt{\bar{X}_2}$ = 1.5062

F value regression = 68.1391; df = (9,509) significant at (.9995)

F level enter = 7.2323; df = (1,509) significant at (.990)

Eq: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_4 X_1 L_1 + \cdots + \beta_{10} X_1 L_7$

02	X_0	Constant	1.3485		
	X_1	Water (W)	.7405	12.4986	.9995
	X_2	Fertilizer cost (FC)	.2715	4.8328	.9995
	X_3	$W \cdot FC$	-.0159	-1.8313	.9500
	X_4	$W \cdot L_1$	-.1308	-4.7975	.9995
	X_5	$W \cdot L_2$	-.1850	-7.3811	.9995
	X_6	$W \cdot L_3$	-.1924	-4.7109	.9995
	X_7	$W \cdot L_4$	-.2198	-5.5525	.9995
	X_8	$W \cdot L_5$	-.1765	-4.8217	.9995
	X_9	$W \cdot L_6$	-.1930	-5.7017	.9995
	X_{10}	$W \cdot L_7$	-.1718	-3.5891	.9995
	X_{11}	W^2	-.0467	-5.6905	.9995
	X_{12}	FC^2	-.0135	-2.2752	.9750

R^2 = .6046

$\bar{X}_1$ = 3.6160; $\bar{X}_2$ = 2.2685

F value regression = 64.4672; df = (12,506) significant at (.9995)

F level enter = 3.3535; df - (1,506) significant at (.90)

$$\text{Eq: } Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1 L_1$$
$$+ \cdots + \beta_{10} X_1 L_7 + \beta_{11} X_1^2 + \beta_{12} X_2^2$$

1/ $L_1 = LRA_2$; $L_2 = LRA_3$; $\cdots$ $L_6 = LRA_7$; $L_7 = LRA_8$.

Table I-4. Miscellaneous Statistical Information for Equations 1 and 2 - Sugar Beets

Equation	Var.	Variable name 1/	Coefficient	Student t	Sig. level
01	X_1	Water (W)	1.7063	6.2254	.9995
	X_2	Fertilizer cost (FC)	.4840	4.9038	.9995
	X_3	$\sqrt{W \cdot FC}$	-.3082	-1.0848	.850
	X_5	$W \cdot L_2$	-.5970	-3.2675	.995
	X_6	$W \cdot L_6$	.7840	1.2964	.900
	X_7	$W \cdot L_7$	.8176	1.0969	.850

R^2 = .9311; adjusted for mean = .127

$\bar{X}_1$ = 5.3451; $\sqrt{\bar{X}_1}$ = 2.3120

$\bar{X}_2$ = 23.1394; $\sqrt{\bar{X}_2}$ = 4.8103

F value regression = 28.903; df = (5,107) significant at (.9995)

F level enter = 1.2031; df - (1,107) significant at .500

Eq: $Y = \beta_1X_1 + \beta_2X_2 + \beta_3\sqrt{X_1X_2} + \beta_5X_1L_2 + \beta_6X_1L_6 + \beta_7X_1L_7$

02	X_1	Water (W)	4.0563	10.1327	.9995
	X_2	Fertilizer cost (FC)	.3550	5.5247	.9995
	X_3	$W \cdot F(X_1X_2)$	-.0438	-4.0328	.9995
	X_4	$W \cdot L_1$	-.4332	-2.3459	.9750
	X_5	$W \cdot L_2$	-.8679	-4.9884	.9995
	X_8	W^2	-.1706	-4.7161	.9995

R^2 = .9559; adjusted for mean = .400

$\bar{X}_1$ = 5.3451; $\bar{X}_2$ = 23.1394

F value regression = 463.461; df = (5.107) significant at .9995

F level enter = 5.5032; df = (1,107) significant at .975)

Eq: $Y = \beta_1X_1 + \beta_2X_2 + \beta_3X_1X_2 + \beta_4X_1L_1 + \beta_5X_1L_2 + \beta_8X_1^2$

1/ $L_1 = LRA_2$; $L_2 = LRA_3$; ••• $L_6 = LRA_7$; $L_7 = LRA_8$.

Table I-5. Miscellaneous Statistical Information for Equations 1 and 2 - Potatoes

Equation	Var.	Variable name[1/]	Coefficient	Student t	Sig. level
01	X_1	Water (W)	10.1908	4.1779	.9995
	X_2	Fertilizer cost (FC)	4.7441	4.9191	.9995
	X_3	$\sqrt{W \cdot FC}$	2.7935	.9348	.800

$R^2 = .90899$; adjusted for mean = .00

$\bar{X}_1 = 5.7969$; $\sqrt{\bar{X}_1} = 2.4077$

$\bar{X}_2 = 24.7755$; $\sqrt{\bar{X}_2} = 4.9775$

F value regression = 71.4172; df = (2,143) significant at (.9995)

F level enter = 17.4552; df = (1,143) significant at (.9995)

Eq: $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 \sqrt{X_1 X_2}$

02	X_1	Water (W)	20.6339	6.3619	.9995
	X_2	Fertilizer cost (FC)	11.3408	10.0499	.9995
	X_3	$W \cdot FC$	-.4583	-3.3277	.995
	X_4	$W \cdot L_1$	-4.5716	-1.6541	.900
	X_5	$W \cdot L_2$	-10.7259	-4.4369	.9995
	X_6	$W \cdot L_5$	-7.9029	-2.3004	.975
	X_7	$W \cdot L_6$	-5.7878	-1.6303	.900
	X_{10}	FC^2	-.1424	-4.6228	.9995

$R^2 = .9337$; adjusted for mean = .127

$\bar{X}_1 = 5.2726$; $\bar{X}_2 = 22.6499$

F value regression = 277.8515; df = (7,138) significant at (.9995)

F level enter = 2.6580; df = (1,138) significant at (.750)

Eq: $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1 L_1 + \beta_5 X_1 L_2$

$+ \beta_6 X_1 L_5 + \beta_7 X_1 L_6 + \beta_{10} X_1^2$

[1/] $L_1 = LRA_2$; $L_2 = LRA_3$; ••• $L_6 = LRA_7$; $L_7 = LRA_8$.

APPENDIX II

A VISIT TO THE PROBLEM OF AGGREGATION IN PRODUCTION FUNCTION ANALYSIS

Although a general treatise of the aggregation problem is interesting and possibly highly informative, it will not be the main thrust or purpose of this paper. The intent of this paper is to glean from the body of knowledge concerning the aggregation problem those points which are relevant to the particular problems the writer is faced with in estimating production functions within the agricultural sector. Therefore, the ideas presented herein will be mainly directed at estimating and aggregating production functions given a particular set of data, and the inherent problems associated with the utilization of this set of data.

Perhaps a description of the data and the type of production functions desired would be useful at this point.

Data

The Economic Research Service developed the concept of the Soil Resource Group (SRG) to meet the land resource classification needs of its projections and evaluation program. The land resource was classified into relatively homogeneous soil groups (SRGs) with particular emphasis on slope, texture, permeability of the substrata, suitability for similar types of crops, selected input requirements, management and yields. SRGs consist of groups of land capability units (LCU), the basic land classification unit from the 1966 Conservation Needs Inventory of the Soil Conservation Service, with a specific range of soil characteristics which influence the productivity of the land resource. These characteristics consist of soil wetness, depth of soil available for rooting, etc. The grouping of LCUs into SRGs, by soil scientists of the Soil Conservation Service, was based on instructions from the Economic Research Service that the groupings be sufficiently homogeneous to permit a reasonable degree of accuracy in estimating yields, cropping patterns, fertilizer applications, and water use.

The Economic Research Service developed forms that were used by SCS personnel in estimating acreage of crops, fertilizer application, yield and water use for each basic SRG observation. The basic unit of observation is the SRG within a given geographically defined portion of a county. This geographical division was made such that the land within each division is relatively homogeneous with respect to climate, precipitation, elevation and natural cover. These divisions constitute the Land Resource Areas and will be referred to as LRAs throughout this paper.

Production Functions of Interest

The following functions are representative of the functions to be estimated from this set of data.

These functions are:

$$Y_{ijk} = b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 \qquad (1)$$

where

Y_{ijk} = Yield per acre of the ith crop, on the jth SRG and the kth LRA.

b_i = Regression coefficient

X_1 = H_2O application in acre feet per acre

X_2 = Nitrogen application in lbs. per acre

X_3 = P_2O_5 (Phosphorus) application in lbs. per acre

X_4 = K_2O (Potassium) application in lbs. per acre

X_5 = Other fertilizer inputs in lbs. per acre

X_6 = Capital input per acre for the ith crop

X_7 = Labor input per acre for the ith crop

X_8 = Productivity index of the SRGs to be estimated later

X_9 = Productivity index of the LRAs to be estimated later

Note: Variables X_6, X_7, X_8, X_9 are excluded from this initial equation. X_8 and X_9 are of course irrelevant since the equation is confined within these levels of aggregation. It is assumed that the capital (X_6) and Labor (X_7) inputs for a crop within an SRG are constant and therefore can be left out of this equation without increasing the specification bias. This can be shown by deriving the specification bias term in the general least squares equation.

Let

$$Y = X\beta + Z_{\gamma} + U \qquad \text{(i)}$$

be the equation for Y to be estimated by least squares, where

Y is the dependent variable
X the independent variables included in the model
Z the variables excluded from the equation, and
U is the disturbance term.

Let $\beta = (X'X)^{-1} X'Y$ be the ordinary least squares estimate of the regression coefficients for the perfectly specified model. Now by substituting the true expression for Y from (1) into $\beta^{*} = (X'X)^{-1} X^{1} Y$ we get

$$\beta^{*} = (X'X)^{-1} X' [X\beta + Z_{\gamma} + U)] \qquad \text{(ii)}$$

then

$$\beta^{*} = (X'X)^{-1} X'X\beta + (X'X)^{-1} X'Z_{\gamma} + (X'X)^{-1} X'U$$

$$E\beta^{*} = \beta + E\,[(X'X)^{-1} X'Z_{\gamma}] + (X'X)^{-1} X'E(U) \text{ but } E(U) = 0$$

$$\therefore E(\beta^{*}) = \beta + E\,[(X'X)^{-1} X'Z_{\gamma}]$$

hence the bias from the specification error is obviously

$$E[(X'X)^{-1} X'Z_{\gamma}] = (X'X)^{-1} X'Z\ E(\gamma)$$

since the Xs and the Zs are assumed to be constant. In order for the specification bias to be zero or at least insignificant either $(X'X)^{-1} X'Z$ must be equal to zero or $E(\gamma)$ must equal zero.

We must then either show that $(X'X)^{-1} X'Z = 0$ or that $E(\gamma) = 0$.

Solving for γ in equation (1) with the ordinary least squares procedure would give

$$\gamma^{*} = (Z'Z)^{-1}Z'Y \qquad \text{(iii)}$$

substituting for Y from i above we find

$$\gamma^{*} = (Z'Z)^{-1}Z'Z(\gamma) + (Z'Z)^{-1}Z'X(\beta) + (Z'Z)^{-1}Z'U$$

and $\quad E(\gamma)^{*} = \gamma + (Z'Z)^{-1}Z'X(E\beta) + 0$

$\therefore$ if $E(\gamma) = 0 \qquad\qquad \gamma = -(Z'Z)^{-1}Z'X(\beta)$

This result would be fortuitous indeed. Since we cannot show from this that $E(\gamma) = 0$ in general or for that matter, for any consistent set of circumstances we must show that $(X'X)^{-1}X'Z = 0$. This particular situation has been shown by Dr. William G. Brown[1] to occur when the Xs and the Zs are truly independent and perfectly uncorrelated. This is true because the matrix resulting from the multiplication of $(X'X)^{-1}$ by $(X'Z)$, where $(X'Z)$ is the matrix of corrected sums of cross products of the Zs with each of the Xs, results in a matrix such that Column 1 is the regression coefficient of each of the Z regressed on all of the X_1, Column 2 is the regression coefficient of Z regressed on all of the X_3 and so on. Thus if the Xs and Zs are perfectly uncorrelated the expression $(X'X)^{-1}X'Z$ reduces to a null matrix and the bias term in the ordinary least square estimate of β^{*} reduces to zero and β^{*} is an unbiased estimate of β.

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[1] From unpublished paper entitled "Effect of omitting relevant variables in Economic Research."

Since the economic variables in the real world are not generally perfectly uncorrelated, a further note on the characteristics of the data at hand is in order. The Z matrix in this case is the matrix formed by the Vectors X_6 and X_7 (Capital and Labor). In order for $(X'X)^{-1}X'Z$ to be a null matrix the regression coefficients of X_6 and X_7 regressed on all the X_i must be zero. These regression coefficients (b) have the following formula

$$b = \frac{\Sigma xz}{\Sigma x^2}$$

where Σxz = the mean corrected sum of cross products and Σx^2 = the mean corrected sum of squares. In order for $b = 0$ we must have $\Sigma xz = 0$. If the Zs are all constants for a given equation it can be shown that $\Sigma xz = 0$ for all X and Z. This in fact is the case for the function in equation (1) above. The cost data for Labor and Capital are constant within an LRA and consequently do not vary within a particular crop SRG, LRA unit. Hence we can say the bias term associated with variables X_6 and X_7 in equation (1) above is zero.

Function (1) represents a response function for the several crops in the model on a particular soil resource group (SRG). These functions will be estimated for these crops on several SRGs. The crops will be chosen for analysis based on their relative importance in the cropping pattern of the area. Some SRGs are used mainly for permanent pasture, range or forest and will be excluded from the analysis.

To account for the variation in soil quality within a given LRA, we would construct an index of productivity for the SRGs. This activity will involve estimating a total value product function for each SRG, then with the hypothesis that the unexplained variation in each product estimation equation is due to the omitted variable <u>Soil Quality</u> we can procede to construct the index of productivity (Variable X_8 from equation (1)).

Since the observations from which the total value product equations are to be estimated are the same as was used in equation (1) with the addition of variables X_6 and X_7 we can write:

$$Y_{jk} = f(X_{11}, X_{12}, \ldots, X_{1n_1}, X_{21}, \ldots, X_{2n_2}, \ldots, X_{71}, \ldots, X_{7n_7}) \qquad (2)$$

where

$$Y_{jk} = \sum_i P_i Y_{ij} A_{ij},$$

P_i = Price per unit of commodity i,

Y_{ij} = Observed Yield in Units per acre of commodity i, in the jth SRG,

and A_{ij} = The acreage of the ith commodity in the jth SRG.

For convenience in estimating the equation and interpretation of results we want to be able to write equation (2) in the following form

$$Y_j = F(X_1, X_2, X_3, X_4, X_5, X_6, X_7) \qquad (3a)$$

or $$Y_j = C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5 + C_6X_6 + C_7X_7 \quad (3b)$$

where equation (3b) represents the particular type of function we are interested in estimating, and merely represents the explicit functional form whereas (3a) is the more general equation.

For purposes of this paper we are only concerned with functions linear and homogeneous, however, aggregation procedures are not restricted to this type of function as long as certain requirements are met. For a discussion of these requirements and the aggregation of more complex problems the reader is referred to the books by Green (1964) and H. Theil (1954).

We must now consider the conditions under which equation (2) can be written (and estimated) in the form of equation (3b). There may exist some estimation bias due to the aggregation from (2) to (3b). However we will assume the individual disturbance terms of the elementary variables are well behaved and that $E(U_i) = E(U_j) = E(U) = 0$ where $U = \sum_i U_i$.

According to Green[2]

> . . . It is necessary and sufficient for the grouping of variables . . . , that the marginal rate of substitution between any two variables in a group shall be a function only of the variables in that group, and therefore independent of the value of any variable in any other group.

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[2] Green, H. A. J., Aggregation in Economic Analysis, An Introductory Survey, 1964, p. 12.

If this condition exists we have satisfied the weak separability condition. The weak separability condition can be expressed mathematically as:

$$\frac{\frac{\partial f}{\partial x_{ri}}}{\frac{\partial f}{\partial x_{rj}}} = f_{r \cdot ij}(X_{r1}, X_{r2}, \ldots, X_{rn_r}) \qquad (4)$$

This condition implies that X_r is a function of the X_{ri} only and independent of X_{pi} where $p \neq r$.

$$X_1 = h_1(X_{11}, X_{12}, \ldots, X_{1m_1})$$

$$X_2 = h_2(X_{21}, X_{22}, \ldots, X_{2m_2}) \qquad (5)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\text{and} \quad X_7 = h_7(X_{71}, X_{72}, \ldots, X_{7m_7}).$$

Since our assumed model is linear the weak separability condition holds, e.g.

$$\frac{\frac{\partial f}{\partial x_{11}}}{\frac{\partial f}{\partial x_{12}}} = \frac{\beta_{11}}{\beta_{12}}$$ which obviously depends only upon the coefficients from the particular inputs within the group.

With regard to economic separability Sadan[3] (1970) says

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[3] Sadan, Ezra, Partial Production Functions and the Analysis of Farm Firm Costs. AJAE, Feb. 1970.

> Two parts (of a production function) may be separated in an economic sense when, given the level of output allowed for, the two combinations of inputs employed can be determined independently.

Turning this statement around one could say that two production functions are aggregable in an economic sense if a given level of total output can be obtained either from an aggregation of the production estimates of the elementary production functions or, given the level of inputs employed, from an aggregation of the elementary functions. Thus we would have

$$Y = g(Y_i) = f(X_{1i}, X_{2i}, \ldots, X_{ri}) = F(X_1, X_2, X_3, \ldots, X_r) \qquad (6)$$

where the $(X_1, \ldots, X_r)$ in $F(X_1, X_2, \ldots, X_r)$ are a function of the X_{1i}, through X_{ri}.

$$\begin{aligned} X_1 &= h_1(X_{11}, X_{12}, X_{13}, \ldots, X_{1n_1}) \\ X_2 &= h_2(X_{21}, X_{22}, X_{23}, \ldots, X_{2n_2}) \qquad (5a) \\ X_r &= h_r(X_{r1}, X_{r2}, X_{r3}, \ldots, X_{rn_r}) \end{aligned}$$

The aggregation problem we face then is to find the proper function h_i such that the equalities in (6) hold. The Y_i are observed yield per acre in the common unit of measurement for each commodity. In order to determine a value for Y the Y_i will be weighted by their respective prices and the acreage devoted to the production of the commodities. Hence

$$Y = \sum_i P_i Y_{ijk} D_{ijk}$$

represents the total value product[4] of the jth SRG in the kth LRA.

Theil (1954) describes at least two sets of conditions for empirical aggregation: (a) the case of simple sum aggregation, and (b) the case of weighted sum aggregation. We will discuss (a) first and then proceed with (b).

To paraphrase Theil (1954, p. 140), perfect aggregation is attained when

> . . . there is no contradiction between the macroequation and the microequations corresponding to it, for whatever values and changes assumed by the microvariables. . . .

Perfect aggregation is particularly relevant in the case where the microequations are not subject to estimation or in the case where changes can be observed in the macrovariable independently of the microvariables and the corresponding microequation cannot easily be identified.

Let

$$dX_r = \sum_i dX_{ri}$$

be the observed change in the macrovariable. Assuming $dX_{ri} = 0$ for all $i \geq 3$ we can write $dX_r = dX_{r1} + dX_{r2}$. Since the relation of

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[4] Represents total value product of the m crops in the analysis.

the endogeneous variable, in the macroequation, to X_r can be written as $\beta_r X_r$ we know that the change in the macro endogenous variable is equal to $\beta_r dX_r$ given the ceterus perabus assumption with respect to $X_{r'}$, $r' \neq r$. Due to the relationship between changes in the macrovariable with respect to changes in the microvariables we can write

$$\beta_r dX_r = \beta_{r1} dX_{r1} + \beta_{r2} dX_{r2}. \tag{7}$$

Now the rule of perfection requires no contradiction between the macro change and the sum of the micro changes corresponding to it for whatever the change in the microvariables. This requires the macro change to be invariant with respect to the size of dX^1_{ri} as long as

$$\sum_i dX_{ri} = dX_r.$$

Assuming $\beta_r \neq \beta_{r2}$ and that $dX_{r2} = 0$ then (7) above reduces to

$$\beta_r dX_r = \beta_{r1} dX_{r1}. \tag{8}$$

Now since

$$dX_r = \sum_i dX_{ri}$$

and $dX_{ri} = 0$ for all i except $i = 1$ we know that

$$dX_r = dX_{ri} \tag{9}$$

which => $\beta_r = \beta_{r1}$ but the rule of perfection requires no contradiction for "whatever values and changes assumed by the microvariables."

Then assuming $dX_{r1} = 0$ and

$$\sum_i dX_{ri} = 0$$

for all $i \geq 3$ we have from (8)

$$\beta_r dX_r = \beta_r dX_{r2}. \tag{10}$$

We also know from (9) that

$$dX_r = dX_{r2} \quad \text{which} \Rightarrow \beta_r = \beta_{r2}$$

but our original assumption was that

$$\beta_r \neq \beta_{r2} = \beta_r \neq \beta_r$$

which is an obvious contradiction.

Therefore we have shown that for the rule of perfection to hold

$$\beta_{r1} = \beta_{r2} = \beta_{r3} = \dots \beta_{rn_m} = \beta_r \tag{11}$$

which says simply that the slopes of all the microequations for input r must be equal to each other and to the slope of the macroequation for input r. "This condition is both necessary and sufficient for satisfying the rule of perfection."[5]

Weighted Sum Aggregation

We continue to require that the aggregate dependent variable be

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[5] Theil (1954), p. 10.

a simple sum of the micro dependent variables

$$Y = \sum_i Y_i \tag{12}$$

However we now allow the aggregate independent variables to be weighted sums of the micro independent variables.

$$X_r = \sum_i w_{ri} X_{ri} \tag{13}$$

where the w_{ri} are the suitably selected weights which permit consistent aggregation as per "Theil's theorem that a suitable selection of weights W_{rs} makes consistent aggregation possible for any set of linear functions f_s."[6]

To determine what in fact this weight is, we refer to and paraphrase Greens' Theorem 7.[7]

Since Y can be written either as

$$Y = F(X_1, X_2, \ldots, X_m) \tag{14a-1}$$

or

$$Y = \sum Y_i = \sum f_i (X_{1i}, X_{2i}, \ldots, X_{mi}) \tag{14b-1}$$

the theorem states that

$$\frac{\partial F}{\partial X_r} \frac{\partial X_r}{\partial X_{ri}} = \frac{\partial Y}{\partial Y_i} \frac{\partial f_i}{\partial X_{ri}} .$$

In order for this equality to hold we must have from (14a-1) and

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[6] Green (1964), p. 41.

[7] Green (1964), p. 36.

(14b-1) respectively

$$dy = \sum_{r=1}^{m} \frac{\partial F}{\partial X_r} dX_r = \sum_{r=1}^{m} \sum_{r=1}^{n} \frac{\partial F}{\partial X_r} \frac{\partial X_r}{\partial X_{ri}} dX_{ri} \qquad (14a-2)$$

$$dy = \sum_{i=1}^{n} \frac{\partial Y}{\partial Y_s} dys = \sum_{r=1}^{n} \sum_{i=1}^{n} \frac{\partial Y}{\partial Y_i} \frac{\partial f_i}{\partial X_{ri}} dx_{ri} \qquad (14b-2)$$

since $dy \equiv dy$ and $dX_{ri} \equiv dX_{ri}$ from the two equations it must be that

$$\sum_{r=1}^{m} \sum_{i=1}^{n} \frac{\partial F}{\partial X_r} \frac{\partial X_r}{\partial X_{ri}} = \sum_{r=1}^{m} \sum_{i=1}^{n} \frac{\partial Y}{\partial Y_i} \frac{\partial f_i}{\partial X_{ri}} \qquad (15)$$

Then since

$$X_r = \sum_{i} w_{ri} X_{ri},$$

$$Y_i = \sum_{r} br_i X_{ri} = f_i(X_{1i}, X_{2i}, \ldots, X_{ri}, \ldots, X_{mi})$$

$$Y = \sum_{i} Y_i = F(X_1, X_2, \ldots, X_r, \ldots, X_m)$$

we have from the aggregate function

$$\frac{dY}{dX_{ri}} = \frac{\partial F}{\partial X_r} \frac{dX_r}{dX_{ri}} = \frac{\partial F}{\partial X_r} w_{ri} \qquad (16)$$

and from the micro function

$$\frac{dY}{dX_{ri}} = \frac{\partial Y}{\partial f_i} \frac{\partial f_i}{\partial X_{ri}} = (1) \cdot b_{ri} \qquad (17)$$

We know from equation (15) that we can equate the results of equations (16) and (17) thus we have shown that in order to obtain consistent aggregation we must have

$$\frac{\partial F}{\partial X_r} w_{ri} = b_{ri}. \tag{18}$$

Equation (18) implies that

$$\frac{\partial F}{\partial X_r} = \frac{b_{ri}}{w_{ri}} = \frac{b_{rs}}{w_{rs}} = \frac{b_{rt}}{w_{rt}} \tag{19}$$

We can now write the aggregate equation the following ways

$$Y = \sum_i Y_i = \sum_{r=1}^{m} \sum_{i=1}^{n} b_{ri} X_{ri} \tag{20a}$$

$$= \sum_{r=1}^{m} \frac{b_{rs}}{w_{rs}} \quad \sum_{i=1}^{n} w_{ri} X_{ri} \tag{20b}$$

from equation (18)

$$= \sum_{r=1}^{m} C_r X_r \quad \text{where } C_r = \frac{b_{rs}}{w_{rs}} \tag{20c}$$

In the case of exact functions

$$C = \frac{b_{rs}}{w_{rs}} = 1$$

and the suitable weight for aggregation would be $w_{rs} = b_{rs}$ = marginal product of the r input in the production of the sth crop.

A test of this aggregation procedure would be to re-estimate equation (3a) using the aggregate X_r from equation (20c), then testing the coefficients C_r to determine whether or not $C_r = 1$.

APPENDIX III

Table III-1. Acreage, Production, and Value of Five Crops in Southern Idaho, by County, 1959, 1964, and 1969

			1959			1964				1969			
County	Crop	Prod. unit	Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
ADA:													
	Barley........	bu.	6,104	270.7	43.33	3,943	2,320	184.2	45.58	4,940	4,613	355.7	70.27
	Wheat.........	bu.	6,604	328.1	68.59	8,366	2,816	208.2	34.36	10,451	2,227	227.1	29.95
	Hay...........	tons	34,943	113.5	70.68	36,498	34,299	130.8	78.02	30,301	25,469	121.5	87.27
	Sugar beets...	tons	1,333	32.5	321.35	1,898	1,898	37.1	256.82	2,807	2,807	51.5	241.01
	Potatoes......	cwt.	416	87.4	306.60	137	137	36.0	383.10	1,413	1,413	477.6	493.48
	TOTAL......		49,400	832.2		50,842	41,470	596.3		49,912	36,529	1,233.4	
ADAMS:													
	Barley........	bu.	1,657	56.3	33.18	928	184	38.9	40.89	592	149	26.1	43.04
	Wheat.........	bu.	740	20.0	37.26	499	32	12.7	35.05	746	284	19.2	35.47
	Hay...........	tons	16,237	25.9	34.76	18,874	9,719	34.4	39.70	13,296	8,869	26.3	43.03
	Sugar beets...	tons	—	—	—	—	—	—	—	—	—	—	—
	Potatoes......	cwt.	2	.1	102.20	7	6	1.2	260.46	—	—	—	
	TOTAL......		18,636	102.3		20,308	9,941	87.2		14,634	9,302	71.6	
BANNOCK:													
	Barley........	bu.	23,100	716.7	30.26	16,144	3,764	559.1	33.77	23,811	7,232	955.5	39.14
	Wheat.........	bu.	49,296	1,179.7	32.98	54,544	4,553	1,481.2	37.54	53,573	4,259	1,426.1	36.71
	Hay...........	tons	21,717	53.2	53.30	24,390	18,116	63.0	56.22	24,547	19,633	68.0	60.33
	Sugar beets...	tons	3,365	57.9	226.52	5,197	5,197	67.1	169.89	3,890	3,890	49.2	165.94
	Potatoes......	cwt.	3,066	540.3	315.40	1,847	1,751	252.4	244.69	4,374	4,374	1,131.6	463.07
	TOTAL......		100,544	2,547.8		102,122	23,381	2,422.8		110,195	39,388	3,630.4	
BINGHAM:													
	Barley........	bu.	14,681	554.9	36.89	18,020	15,465	939.3	50.85	42,845	40,171	2,708.3	62.17
	Wheat.........	bu.	50,412	2,369.8	64.86	50,258	39,674	2,204.0	60.44	41,166	31,738	2,071.8	69.41
	Hay...........	tons	63,278	199.3	68.57	74,567	73,253	240.8	70.32	60,085	57,754	202.2	73.24
	Sugar beets...	tons	6,083	93.0	201.50	16,920	16,842	703.0	158.04	12,744	12,744	195.3	201.50
	Potatoes......	cwt.	44,591	8,515.3	340.28	42,714	42,624	6,806.6	285.33	49,714	49,714	10,962.5	394.70
	TOTAL......		179,045	11,732.3		202,479	187,858	10,393.7		206,194	192,121	16,140.1	
BLAINE:													
	Barley........	bu.	2,460	96.8	38.45	2,104	1,788	94.6	43.92	4,173	3,038	180.2	42.16
	Wheat.........	bu.	6,300	240.6	52.72	4,329	3,892	160.1	51.06	2,831	2,544	127.9	62.38
	Hay...........	tons	20,476	52.4	55.70	20,301	18,750	55.4	59.39	21,067	19,393	65.1	67.30
	Sugar beets...	tons	158	3.4	280.52	138	138	1.8	171.21	—	—	—	—
	Potatoes......	cwt.	795	195.9	441.06	78	78	11.2	258.12	564	564	112.2	356.03
	TOTAL......		30,189	589.1		26,950	24,646	323.1		28,635	25,539	485.4	

(continued)

Table III-1. (continued)

County	Crop	Prod. unit	1959			1964				1969			
			Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
BOISE:													
	Barley........	bu.	654	20.3	30.26	247	98	10.2	40.21	317	141	10.4	32.01
	Wheat.........	bu.	370	9.5	35.60	182	43	4.1	31.33	348	72	11.2	44.44
	Hay...........	tons	6,748	8.0	25.91	8,501	3,580	12.8	21.77	4,881	2,008	8.5	37.73
	Sugar beets...	tons	38	1.1	368.75	--	--	--	--	--	--	--	--
	Potatoes......	cwt.	110	18.8	250.10	3	3	.4	204.84	--	--	--	--
	TOTAL......		7,290	57.7		8,933	3,724	27.5		5,546	2,221	30.1	
BONNEVILLE:													
	Barley........	bu.	25,062	784.2	30.45	26,671	7,887	1,050.4	38.45	42,964	18,530	2,178.8	49.48
	Wheat.........	bu.	84,069	2,326.6	38.23	72,272	21,829	2,367.0	45.26	63,366	21,102	2,285.4	49.82
	Hay...........	tons	41,330	125.2	65.95	48,100	38,504	141.2	63.90	44,649	35,271	131.7	64.21
	Sugar beets...	tons	2,704	37.9	184.38	4,804	4,804	60.0	164.62	3,491	3,491	52.4	175.16
	Potatoes......	cwt.	25,099	4,371.3	311.82	35,487	35,487	4,766.9	240.40	35,199	35,199	6,915.8	351.74
	TOTAL......		178,264	7,645.2		187,334	108,511	8,385.5		189,669	113,593	11,564.1	
BUTTE:													
	Barley........	bu.	4,339	184.8	41.58	5,540	5,102	245.0	43.14	7,063	5,868	302.1	41.77
	Wheat.........	bu.	7,173	223.5	43.06	7,407	4,792	236.2	44.02	5,031	3,245	165.1	45.26
	Hay...........	tons	14,650	33.0	48.73	16,383	16,298	47.0	63.13	25,784	24,833	72.2	60.95
	Sugar beets...	tons	--	--	--	--	--	--	--	--	--	--	--
	Potatoes......	cwt.	2,705	426.2	281.92	3,536	3,536	390.4	197.62	3,832	3,832	599.8	280.14
	TOTAL......		28,867	867.5		32,866	29,728	918.6		41,710	37,778	1,139.2	
CAMAS:													
	Barley........	bu.	8,200	105.1	12.49	5,410	289	141.1	25.38	6,921	652	200.4	28.21
	Wheat.........	bu.	28,732	569.1	27.32	19,568	691	341.1	24.01	12,722	756	195.5	21.25
	Hay...........	tons	16,744	18.0	23.39	33,225	3,717	52.1	34.17	40,728	4,887	656.5	35.09
	Sugar beets...	tons	--	--	--	--	--	--	--	--	--	--	--
	Potatoes......	cwt.	--	--	--	--	--	--	--	25	25	8.8	626.15
	TOTAL......		53,676	692.2		58,203	4,697	534.3		60,396	6,320	1,061.2	
CANYON:													
	Barley........	bu.	3,369	196.2	56.80	8,370	8,300	580.0	67.64	24,155	23,401	1,085.8	73.00
	Wheat.........	bu.	20,590	1,231.4	82.52	16,348	16,318	992.1	83.77	10,335	9,606	647.0	86.39
	Hay...........	tons	55,376	196.0	77.04	56,361	56,059	198.9	76.82	31,613	30,551	132.5	91.25
	Sugar beets...	tons	22,124	580.6	345.05	31,224	31,224	706.9	297.64	34,837	34,837	848.9	321.35
	Potatoes......	cwt.	6,321	1,834.4	423.69	8,487	8,487	2,478.7	426.32	12,594	12,594	4,073.1	578.89
	TOTAL......		107,780	4,038.6		120,790	120,388	4,956.6		113,534	110,989	7,507.3	

(continued)

Table III-1. (continued)

			1959			1964				1969			
County	Crop	Prod. unit	Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
CARIBOU:													
	Barley........	bu.	63,174	1,742.6	26.74	50,179	9,660	2,184.1	42.46	58,721	17,614	2,961.0	49.14
	Wheat.........	bu.	46,651	1,177.9	34.78	43,891	5,317	1,447.0	45.54	38,793	6,139	1,096.6	39.05
	Hay...........	tons	35,742	71.4	43.49	38,395	27,647	85.7	48.58	36,111	27,647	83.7	50.84
	Sugar beets...	tons	474	6.5	181.75	2,674	2,674	23.4	114.58	427	427	5.5	168.58
	Potatoes......	cwt.	1,126	258.3	410.63	2,459	2,459	340.1	247.56	3,599	3,599	769.2	382.52
	TOTAL......		147,707	3,256.7		137,598	47,757	4,080.3		137,651	55,426	4,916.0	
CASSIA:													
	Barley........	bu.	8,277	251.0	29.51	15,105	10,622	780.9	50.46	25,744	22,436	1,580.7	59.93
	Wheat.........	bu.	53,643	1,861.5	47.89	47,910	19,290	1,670.9	48.16	47,142	13,946	1,565.0	45.82
	Hay...........	tons	53,416	150.7	61.40	61,193	60,045	194.1	69.07	56,903	55,211	189.3	72.41
	Sugar beets...	tons	7,169	136.0	250.23	20,215	20,215	276.7	180.43	19,021	19,021	310.9	214.67
	Potatoes......	cwt.	15,975	3,561.6	399.17	23,460	23,460	3,861.8	294.63	18,835	18,835	4,351.2	413.49
	TOTAL......		138,480	5,960.8		167,883	133,632	6,784.4		167,645	129,449	7,997.1	
CLARK:													
	Barley........	bu.	2,297	62.8	26.64	2,221	980	64.5	28.30	2,195	1,882	113.2	50.36
	Wheat.........	bu.	3,159	78.8	34.36	4,088	391	79.3	26.77	6,749	1,674	148.6	30.36
	Hay...........	tons	8,598	14.2	35.86	9,654	6,227	18.5	41.73	13,651	6,884	30.1	47.96
	Sugar beets...	tons	—	—	—	—	—	—	—	—	—	—	—
	Potatoes......	cwt.	550	65.8	613.25	285	285	40.4	253.82	140	140	—	—
	TOTAL......		14,604	221.6		16,248	7,883	202.7		22,735	10,580	291.9	
ELMORE:													
	Barley........	bu.	4,334	121.0	27.23	4,429	3,043	192.2	42.36	11,247	10,441	688.3	59.73
	Wheat.........	bu.	5,699	143.8	34.78	4,324	1,805	138.3	44.16	2,838	978	87.7	42.64
	Hay...........	tons	17,213	40.6	51.36	19,294	15,609	56.2	63.37	19,976	12,583	51.1	55.74
	Sugar beets...	tons	297	5.7	251.55	245	245	4.3	233.11	10,170	10,170	175.2	226.52
	Potatoes......	cwt.	155	29.4	277.40	3,263	3,263	1,219.2	545.46	7,841	7,841	2,250.8	419.02
	TOTAL......		27,698	340.5		31,555	23,965	1,610.2		52,072	42,013	3,253.1	
FREMONT:													
	Barley........	bu.	22,414	703.6	30.65	18,681	5,238	662.7	34.65	27,998	8,992	1,387.5	48.41
	Wheat.........	bu.	43,726	1,403.8	44.30	40,118	13,990	1,240.9	42.64	34,996	10,785	1,404.5	55.34
	Hay...........	tons	21,136	45.0	46.34	26,483	19,403	62.2	51.10	22,894	16,431	52.9	50.32
	Sugar beets...	tons	137	2.1	200.18	1,238	1,238	13.3	135.65	1,080	1,080	12.7	155.41
	Potatoes......	cwt.	12,980	2,356.3	324.88	16,063	15,065	2,160.7	240.76	17,635	17,635	3,320.8	337.06
	TOTAL......		100,393	4,510.8		102,583	54,934	4,139.8		104,603	54,923	6,178.4	

(continued)

Table III-1. (continued)

			1959			1964				1969			
County	Crop	Prod. unit	Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
GEM:													
	Barley........	bu.	1,562	51.8	32.40	1,672	1,231	73.1	42.65	1,966	1,475	109.9	54.56
	Wheat.........	bu.	2,331	93.8	55.48	1,306	984	64.3	67.90	1,113	949	60.2	74.66
	Hay...........	tons	18,333	48.5	57.56	20,087	14,831	54.9	59.45	14,938	12,032	47.7	69.50
	Sugar beets...	tons	328	7.3	293.69	752	752	12.2	213.35	169	169	3.1	241.01
	Potatoes......	cwt.	9	1.6	263.68	36	35	8.0	324.85	41	41	—	—
	TOTAL......		22,563	203.0		23,853	17,833	212.5		18,227	14,666	220.9	
GOODING:													
	Barley........	bu.	1,561	68.5	42.75	1,399	897	64.1	44.70	2,649	2,625	165.6	61.00
	Wheat.........	bu.	10,234	450.6	60.72	7,505	6,706	338.8	62.24	4,832	4,036	245.9	70.24
	Hay...........	tons	29,757	96.4	70.55	36,829	31,531	124.2	73.39	27,338	24,568	105.0	83.63
	Sugar beets...	tons	1,257	28.2	295.01	1,602	1,602	26.5	217.30	2,394	2,394	34.5	189.65
	Potatoes......	cwt.	651	136.7	375.72	807	807	164.0	363.73	2,520	2,520	572.0	406.33
	TOTAL......		43,460	780.4		48,142	41,543	717.6		39,738	36,143	1,123.0	
JEFFERSON:													
	Barley........	bu.	13,117	449.5	33.48	14,370	12,977	603.3	40.99	22,223	20,330	1,240.0	54.46
	Wheat.........	bu.	29,091	1,201.3	56.99	28,236	24,387	1,177.3	57.55	17,816	13,780	810.0	62.79
	Hay...........	tons	51,821	156.8	65.86	63,372	60,924	191.1	65.65	58,143	56,823	190.1	71.16
	Sugar beets...	tons	1,123	12.3	143.55	2,132	2,132	23.4	144.87	1,350	1,350	20.7	202.82
	Potatoes......	cwt.	18,409	3,509.0	341.17	23,150	22,471	3,178.4	245.77	20,544	20,544	3,840.9	334.73
	TOTAL......		113,561	5,328.9		131,260	122,891	5,173.5		120,076	112,627	6,101.7	
JEROME:													
	Barley........	bu.	927	53.8	56.71	3,386	3,386	231.2	66.66	7,265	7,191	567.3	76.23
	Wheat.........	bu.	15,763	1,012.3	88.60	14,016	14,016	796.4	78.38	10,450	10,250	666.3	88.04
	Hay...........	tons	29,165	118.2	88.22	34,172	34,172	139.7	89.03	32,383	32,163	147.7	99.32
	Sugar beets...	tons	2,866	68.6	314.76	8,715	8,715	140.4	212.04	7,577	7,577	135.5	235.74
	Potatoes......	cwt.	9,843	2,425.0	441.41	7,633	7,633	1,524.0	357.28	10,509	10,509	2,672.0	455.02
	TOTAL......		58,555	3,677.9		67,922	67,922	2,831.7		68,184	67,690	4,188.8	
LINCOLN:													
	Barley........	bu.	392	23.1	57.49	1,182	1,182	60.4	49.87	3,418	3,412	233.5	66.66
	Wheat.........	bu.	8,238	441.4	73.87	7,101	7,101	312.6	60.72	3,513	3,354	192.1	75.49
	Hay...........	tons	15,681	51.8	71.94	16,080	16,080	47.1	63.70	16,099	16,033	58.7	79.37
	Sugar beets...	tons	681	10.1	194.92	1,852	1,852	24.3	172.53	836	836	12.0	189.65
	Potatoes......	cwt.	1,666	371.4	398.99	1,716	1,716	253.5	264.38	2,941	2,941	659.8	401.50
	TOTAL......		26,658	897.8		27,931	27,931	697.9		26,807	26,576	1,156.1	

(continued)

Table III-1. (continued)

County	Crop	Prod. unit	1959			1964				1969			
			Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
MADISON:													
	Barley........	bu.	19,748	571.2	28.21	20,563	5,322	810.4	38.45	32,760	18,533	1,844.0	54.95
	Wheat.........	bu.	50,202	1,584.2	43.61	47,458	11,164	1,452.1	42.23	35,101	11,305	1,330.2	52.30
	Hay...........	tons	19,593	60.4	67.14	22,156	19,061	63.8	62.72	16,321	14,620	49.3	65.75
	Sugar beets...	tons	950	11.8	163.13	2,436	2,436	28.2	152.77	828	828	10.7	169.89
	Potatoes......	cwt.	11,261	1,886.7	299.82	15,667	15,620	2,548.7	291.23	24,910	24,910	5,311.0	381.63
	TOTAL......		101,754	4,114.3		108,280	53,603	4,903.2		109,920	70,196	8,545.2	
MINIDOKA:													
	Barley........	bu.	2,506	131.5	51.24	10,728	8,765	579.9	52.70	26,838	26,454	2,089.8	76.03
	Wheat.........	bu.	25,889	1,339.1	71.35	21,160	18,433	995.2	64.86	11,464	11,294	674.9	81.28
	Hay...........	tons	29,409	108.5	80.32	36,465	35,383	140.1	83.64	31,402	30,020	129.9	90.02
	Sugar beets...	tons	12,040	233.9	255.50	26,955	26,955	394.9	192.28	21,555	21,555	365.1	222.57
	Potatoes......	cwt.	22,107	4,902.2	396.84	16,677	16,677	3,017.0	288.01	18,274	18,274	3,727.1	365.16
	TOTAL......		91,951	6,715.2		111,985	106,213	5,127.4		109,533	107,597	6,986.8	
OWYHEE:													
	Barley........	bu.	2,673	147.2	53.68	3,784	3,554	202.9	52.31	9,671	9,528	658.2	66.47
	Wheat.........	bu.	5,077	303.8	82.52	4,129	3,829	198.7	66.38	4,156	4,059	221.3	73.42
	Hay...........	tons	36,081	85.8	51.78	41,024	40,927	99.9	53.02	51,903	50,293	141.1	59.19
	Sugar beets...	tons	4,475	107.8	317.40	4,690	4,690	101.9	285.79	7,349	7,349	111.7	200.18
	Potatoes......	cwt.	1,224	295.0	351.86	2,291	2,291	507.0	323.10	4,371	4,371	1,121.1	374.50
	TOTAL......		49,530	939.6		55,918	55,291	1,110.4		77,450	75,600	2,253.4	
PAYETTE:													
	Barley........	bu.	1,874	67.4	35.04	1,512	1,462	90.5	58.36	2,703	2,388	176.4	63.64
	Wheat.........	bu.	4,449	213.4	66.24	3,650	3,017	199.9	75.62	2,677	2,557	169.2	87.22
	Hay...........	tons	16,451	60.6	80.19	17,085	16,531	61.9	78.85	11,586	11,084	48.4	88.94
	Sugar beets...	tons	1,851	47.4	337.15	2,727	2,727	61.9	298.96	1,644	1,644	38.1	305.54
	Potatoes......	cwt.	1,174	257.7	320.47	773	773	208.9	394.49	767	767	207.8	395.66
	TOTAL......		25,799	646.5		25,747	24,510	623.1		19,377	18,440	639.9	
POWER:													
	Barley........	bu.	35,989	639.3	17.37	28,515	2,524	823.7	28.21	14,625	8,503	773.3	51.63
	Wheat.........	bu.	82,798	1,895.2	31.60	81,469	7,162	2,326.5	39.47	102,991	13,253	2,757.8	36.98
	Hay...........	tons	10,148	28.4	60.85	11,515	9,394	31.8	60.14	10,633	8,649	34.4	70.38
	Sugar beets...	tons	700	11.6	218.62	6,550	6,550	91.1	183.06	11,099	11,099	176.6	209.40
	Potatoes......	cwt.	4,661	995.2	382.16	8,686	8,466	1,457.8	300.36	15,734	15,734	3,862.5	438.19
	TOTAL......		134,296	3,569.7		136,735	34,096	4,730.9		155,082	57,238	7,604.6	

(continued)

Table III-1. (continued)

			1959			1964				1969			
County	Crop	Prod. unit	Crop acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)	Crop acres	Irrigated acres	Production (thousands)	Value/acre ($)
TETON:													
	Barley........	bu.	14,504	345.7	23.23	12,043	2,521	361.0	29.28	22,485	6,046	909.6	39.43
	Wheat.........	bu.	23,657	585.7	34.22	23,556	2,238	553.6	32.43	21,013	2,439	600.8	39.47
	Hay...........	tons	23,355	28.7	26.75	25,860	18,937	40.7	34.30	21,425	18,092	34.7	35.25
	Sugar beets...	tons	--	--	--	--	--	--	--	--	--	--	--
	Potatoes......	cwt.	1,572	163.8	186.52	2,777	2,663	251.7	162.17	2,913	2,913	474.7	291.77
	TOTAL......		63,088	1,123.9		64,236	26,359	1,207.0		67,836	29,790	2,019.8	
TWIN FALLS:													
	Barley........	bu.	3,957	212.6	52.41	6,216	5,886	388.8	61.00	12,594	12,296	937.7	72.61
	Wheat.........	bu.	33,956	2,209.1	89.70	32,051	29,970	1,838.9	79.21	25,340	22,520	1,564.2	85.15
	Hay...........	tons	60,784	231.3	82.82	63,553	63,173	241.6	82.76	58,167	56,534	247.2	92.53
	Sugar beets...	tons	13,887	310.5	295.01	19,736	19,736	363.7	242.33	23,086	23,086	465.0	264.72
	Potatoes......	cwt.	9,034	2,266.0	448.93	5,864	5,864	1,271.1	388.07	7,415	7,415	1,893.5	457.17
	TOTAL......		121,618	5,229.5		127,420	124,629	4,104.1		126,602	121,851	5,107.6	
VALLEY:													
	Barley........	bu.	1,640	46.9	27.91	843	172	21.4	24.79	96	41	3.1	31.43
	Wheat.........	bu.	500	10.3	28.15	287	31	5.8	27.74	734	145	38.2	71.76
	Hay...........	tons	7,481	7.0	20.43	9,896	3,995	12.5	27.41	6,557	3,778	8.2	27.21
	Sugar beets...	tons	--	--	--	--	--	--	--	--	--	--	--
	Potatoes......	cwt.	309	43.5	205.71	375	374	39.0	151.99	387	387	74.9	282.66
	TOTAL......		9,930	107.7		11,401	4,572	78.7		7,774	4,351	124.4	
WASHINGTON:													
	Barley........	bu.	8,368	306.4	35.72	6,659	1,629	310.1	45.48	5,876	2,986	284.5	47.24
	Wheat.........	bu.	14,040	427.6	41.95	11,526	2,632	397.4	47.61	10,015	2,181	355.7	48.99
	Hay...........	tons	32,045	67.7	46.00	35,966	11,724	83.8	50.73	31,023	9,320	69.5	48.78
	Sugar beets...	tons	2,309	62.7	358.22	2,997	2,969	64.6	284.47	3,334	3,334	80.6	318.71
	Potatoes......	cwt.	613	120.4	286.74	295	295	76.9	380.62	903	903	271.0	438.15
	TOTAL......		57,375	984.8		57,443	19,249	932.8		51,151	18,724	1,061.3	

Table III-2. Acreage, Irrigation, and Value of Production for Five Crops in Southern Idaho, by County, 1964, 1969

	1964				1969				
County	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	Percent change in value/acre
				BARLEY					
Ada.............	3,943	2,320	58.83	45.58	4,940	4,613	93.38	70.27	+54.16
Adams............	928	184	19.82	40.89	592	149	25.16	43.04	5.25
Bannock..........	16,144	3,764	23.31	33.77	23,811	7,232	30.37	39.14	15.90
Bingham..........	18,020	15,465	85.82	50.85	42,485	40,171	94.55	62.17	22.26
Blaine...........	2,104	1,788	84.98	43.92	4,173	3,038	72.80	42.16	- 4.01
Boise............	247	98	39.67	40.21	317	141	44.47	32.01	-20.40
Bonneville.......	26,671	7,887	29.57	38.45	42,964	18,530	43.12	49.48	28.68
Butte............	5,540	5,102	92.09	43.14	7,063	5,868	83.08	41.77	- 3.18
Camas............	5,410	289	5.34	25.38	6,921	652	9.42	28.21	11.15
Canyon...........	8,370	8,300	99.16	67.64	24,155	23,401	96.87	73.00	7.92
Caribou..........	50,179	9,660	19.25	42.46	58,721	17,614	30.00	49.19	15.85
Cassia...........	15,105	10,622	70.32	50.46	25,744	22,436	87.15	59.93	18.77
Clark............	2,221	980	44.12	28.30	2,195	1,882	85.74	50.36	77.95
Elmore...........	4,429	3,043	68.71	42.36	11,247	10,441	92.83	59.73	41.00
Fremont..........	18,681	5,238	28.04	34.65	27,998	8,992	32.12	48.41	39.71
Cem..............	1,672	1,231	73.62	42.65	1,966	1,475	75.03	54.56	27.92
Gooding..........	1,399	897	64.12	44.70	2,649	2,625	99.09	61.00	36.47
Jefferson........	14,370	12,977	90.31	40.99	22,223	20,330	91.48	54.46	32.86
Jerome...........	3,386	3,386	100.00	66.66	7,265	7,191	98.98	76.23	14.36
Lincoln..........	1,182	1,182	100.00	49.87	3,418	3,412	99.82	66.66	33.67
Madison..........	20,563	5,322	25.88	38.45	32,760	18,533	56.57	54.95	42.91
Minidoka.........	10,728	8,765	81.70	52.70	26,838	26,454	98.57	76.03	44.27
Owyhee...........	3,784	3,554	93.92	52.31	9,671	9,528	95.73	66.47	27.07
Payette..........	1,512	1,462	96.69	58.36	2,703	2,388	88.35	63.64	9.05
Power............	28,515	2,524	8.85	28.21	14,625	8,503	58.14	51.63	83.02
Teton............	12,043	2,521	20.93	29.28	22,485	6,046	26.89	39.43	28.48
Twin Falls.......	6,216	5,886	94.69	61.00	12,594	12,296	97.63	72.61	3.10
Valley...........	843	172	20.40	24.79	96	41	42.71	31.43	109.36
Washington.......	6,659	1,629	24.46	45.48	5,876	2,986	50.82	47.24	107.77
TOTAL........	290,864	126,248			448,495	286,968			

(continued)

Table III-2 (continued)

County	1964				1969				Percent change in value/acre
	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	
	WHEAT								
Ada...............	8,366	2,816	33.66	34.36	10,451	2,227	21.31	29.95	-12.83
Adams.............	499	32	6.41	35.05	746	284	38.07	35.47	1.20
Bannock...........	54,544	4,553	8.35	37.54	53,573	4,259	7.95	36.71	- 2.21
Bingham...........	50,258	39,674	78.94	60.44	41,166	31,738	77.10	69.41	14.84
Blaine............	4,329	3,892	89.91	51.06	2,831	2,544	89.86	62.38	22.17
Boise.............	182	43	23.63	31.33	348	72	20.69	44.44	41.84
Bonneville........	72,272	21,829	30.20	45.26	63,366	21,102	33.30	49.82	10.07
Butte.............	7,407	4,792	64.70	44.02	5,031	3,245	64.50	45.26	2.82
Camas.............	19,568	691	3.53	24.01	12,722	756	5.94	21.25	-11.50
Canyon............	16,348	16,318	99.81	83.77	10,335	9,606	92.95	86.39	3.13
Caribou...........	43,891	5,317	12.11	45.54	38,793	6,139	15.83	39.05	-14.26
Cassia............	47,910	19,290	40.26	48.16	47,142	13,946	29.58	45.82	- 4.86
Clark.............	4,088	391	9.56	26.77	6,749	1,674	24.80	30.36	13.41
Elmore............	4,324	1,805	41.74	44.16	2,838	978	34.46	42.64	- 3.44
Fremont...........	40,118	13,990	34.87	42.64	34,996	10,785	30.82	55.34	29.78
Gem...............	1,306	984	75.34	67.90	1,113	949	85.27	74.66	9.96
Gooding...........	7,505	6,706	89.35	62.24	4,832	4,036	83.53	70.24	12.85
Jefferson.........	28,236	24,387	86.37	57.55	17,816	13,780	77.35	62.79	9.11
Jerome............	14,016	14,016	100.00	78.38	10,450	10,250	98.09	88.04	12.32
Lincoln...........	7,101	7,101	100.00	60.72	3,513	3,354	95.47	75.49	24.32
Madison...........	47,458	11,164	23.52	42.23	35,101	11,305	32.21	52.30	23.85
Minidoka..........	21,160	18,433	87.11	64.86	11,464	11,294	98.52	81.28	25.32
Owyhee............	4,129	3,829	92.73	66.38	4,156	4,059	97.67	73.42	10.61
Payette...........	3,650	3,017	82.66	75.62	2,677	2,557	95.52	87.22	15.34
Power.............	81,469	7,162	8.79	39.47	102,991	13,253	12.87	36.98	- 6.31
Teton.............	23,554	2,238	9.50	32.43	21,013	2,439	11.61	39.47	21.71
Twin Falls........	32,051	29,970	93.51	79.21	25,340	22,520	88.87	85.15	7.50
Valley............	287	31	10.80	27.74	734	145	19.75	71.76	158.69
Washington........	11,526	2,632	22.84	47.61	10,015	2,181	21.78	48.99	2.90
TOTAL.........	657,554	267,103			582,302	211,477			

(continued)

Table III-2. (continued)

County	1964				1969				Percent change in value/acre
	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	
	HAY								
Ada.............	36,498	34,299	93.96	78.02	30,301	25,469	84.05	87.27	11.86
Adams............	18,874	9,719	51.50	39.70	13,296	8,869	66.70	43.03	8.39
Bannock..........	24,390	18,116	74.28	56.22	24,547	19,633	79.98	60.33	7.31
Bingham..........	74,567	73,253	98.24	70.32	60,085	57,754	96.12	73.24	4.15
Blaine...........	20,301	18,750	92.36	59.39	21,067	19,393	92.05	67.30	13.32
Boise............	8,501	3,580	42.11	21.77	4,881	2,008	43.39	37.73	73.31
Bonneville.......	48,100	38,504	80.05	63.90	44,649	35,271	79.00	64.21	0.49
Butte............	16,383	16,298	99.48	63.13	25,784	24,833	96.31	60.95	- 3.45
Camas............	33,225	3,717	11.19	34.17	40,728	4,887	12.00	35.09	2.69
Canyon...........	56,361	56,059	99.46	76.82	31,613	30,551	96.64	91.25	18.78
Caribou..........	38,395	27,647	72.01	48.58	36,111	27,647	76.56	50.84	4.65
Cassia...........	61,193	60,045	98.12	69.07	56,903	55,211	97.03	72.41	- 4.84
Clark............	9,654	6,227	64.50	41.73	13,651	6,884	50.43	47.96	14.93
Elmore...........	19,294	15,609	80.90	63.37	19,976	12,583	62.99	55.74	-12.04
Fremont..........	26,483	19,403	73.27	51.10	22,894	16,431	71.77	50.32	- 1.53
Gem..............	20,087	14,831	73.83	59.45	14,938	12,032	80.55	69.50	16.90
Gooding..........	36,829	31,531	85.61	73.39	27,338	24,568	89.87	83.63	13.95
Jefferson........	63,372	60,924	96.14	65.65	58,143	56,823	97.73	71.16	8.39
Jerome...........	34,172	34,172	100.00	89.03	32,383	32,163	99.32	99.32	11.56
Lincoln..........	16,080	16,080	100.00	63.70	16,099	16,033	99.59	79.37	24.60
Madison..........	22,156	19,061	86.03	62.72	16,321	14,620	89.58	65.75	4.83
Minidoka.........	36,465	35,383	97.03	83.64	31,402	30,020	95.60	90.02	7.63
Owyhee...........	41,024	40,927	99.76	53.02	51,903	50,293	96.90	59.19	11.64
Payette..........	17,085	16,531	96.76	78.85	11,586	11,084	95.67	88.94	12.80
Power............	11,515	9,394	81.58	60.14	10,633	8,649	81.34	70.38	17.03
Teton............	25,860	18,937	73.23	34.30	21,425	18,092	84.44	35.25	2.77
Twin Falls.......	63,553	63,173	99.40	82.76	58,167	56,534	97.19	92.53	11.81
Valley...........	9,896	3,995	40.37	27.41	6,557	3,778	57.62	27.21	- 0.73
Washington.......	35,966	11,724	32.60	50.73	31,023	9,320	30.06	48.78	- 3.84
TOTAL........	926,279	777,974			834,404	691,433			

(continued)

Table III-2. (continued)

County	1964				1969				Percent change in value/acre
	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	Crop acres	Irrigated acres	Percent irrigated	Value/acre ($)	
	SUGAR BEETS								
Ada	1,898	1,898	100.00	256.82	2,807	2,807	100.00	241.01	- 6.16
Adams	—	—	—	—	—	—	—	—	—
Bannock	5,197	5,197	100.00	169.89	3,890	3,890	100.00	165.94	- 2.33
Bingham	16,920	16,842	99.54	158.04	12,744	12,744	100.00	201.50	27.50
Blaine	138	138	100.00	171.21	—	—	—	—	—
Boise	—	—	—	—	—	—	—	—	—
Bonneville	4,804	4,804	100.00	164.62	3,491	3,491	100.00	175.16	6.40
Butte	—	—	—	—	—	—	—	—	—
Camas	—	—	—	—	—	—	—	—	—
Canyon	31,224	31,224	100.00	297.64	34,837	34,837	100.00	321.35	7.97
Caribou	2,674	2,674	100.00	114.58	427	427	100.00	168.58	47.13
Cassia	20,215	20,215	100.00	180.43	19,021	19,021	100.00	214.67	18.98
Clark	—	—	—	—	—	—	—	—	—
Elmore	245	245	100.00	233.11	10,170	10,170	100.00	226.52	- 2.83
Fremont	1,238	1,238	100.00	135.65	1,080	1,080	100.00	155.41	14.57
Gem	752	752	100.00	213.35	169	169	100.00	241.01	12.96
Gooding	1,602	1,602	100.00	217.30	2,394	2,394	100.00	189.65	-12.72
Jefferson	2,132	2,132	100.00	144.87	1,350	1,350	100.00	202.82	40.00
Jerome	8,715	8,715	100.00	212.04	7,577	7,577	100.00	235.74	11.18
Lincoln	1,852	1,852	100.00	172.53	836	836	100.00	189.65	9.92
Madison	2,436	2,436	100.00	152.77	828	828	100.00	169.89	11.21
Minidoka	26,955	26,955	100.00	192.28	21,555	21,555	100.00	222.57	15.75
Owyhee	4,690	4,690	100.00	285.79	7,349	7,349	100.00	200.18	-29.96
Payette	2,727	2,727	100.00	298.96	1,644	1,644	100.00	305.54	2.20
Power	6,550	6,550	100.00	183.06	11,099	11,099	100.00	209.40	14.39
Teton	—	—	—	—	—	—	—	—	—
Twin Falls	19,736	19,736	100.00	242.33	23,086	23,086	100.00	264.72	9.24
Valley	—	—	—	—	—	—	—	—	—
Washington	2,997	2,969	99.07	284.47	3,334	3,334	100.00	318.71	12.04
TOTAL	165,697	165,591			169,688	169,688			

(continued)

Table III-2. (continued)

County	1964 Crop acres	 Irrigated acres	 Percent irrigated	 Value/acre ($)	1969 Crop acres	 Irrigated acres	 Percent irrigated	 Value/acre ($)	Percent change in value/acre
					POTATOES				
Ada..............	137	137	100.00	383.10	1,413	1,413	100.00	493.48	28.81
Adams............	7	6	85.71	260.46	--	--	--	--	--
Bannock..........	1,847	1,751	94.80	244.69	4,374	4,374	100.00	463.07	89.25
Bingham..........	42,714	42,624	99.79	285.33	49,714	49,714	100.00	394.70	38.33
Blaine...........	78	78	100.00	258.12	564	564	100.00	356.03	37.93
Boise............	3	3	100.00	204.84	--	--	--	--	--
Bonneville.......	35,487	35,487	100.00	240.40	35,199	35,199	100.00	351.74	46.31
Butte............	3,536	3,536	100.00	197.62	3,832	3,832	100.00	280.14	41.76
Camas............	—	—	—	—	25	25	100.00	626.50	--
Canyon...........	8,487	8,487	100.00	426.32	12,594	12,594	100.00	578.89	35.79
Caribou..........	2,459	2,459	100.00	247.56	3,599	3,599	100.00	382.52	54.52
Cassia...........	23,460	23,460	100.00	294.63	18,835	18,835	100.00	413.49	40.34
Clark............	285	285	100.00	253.82	140	140	100.00	--	--
Elmore...........	3,263	3,263	100.00	545.46	7,841	7,841	100.00	419.02	-23.19
Fremont..........	16,063	15,065	93.79	240.76	17,635	17,635	100.00	337.06	40.00
Gem..............	36	35	97.22	324.85	41	41	100.00	--	--
Gooding..........	807	807	100.00	363.73	2,520	2,520	100.00	406.33	11.71
Jefferson........	23,150	22,471	97.07	245.77	20,544	20,544	100.00	334.73	36.20
Jerome...........	7,633	7,633	100.00	357.28	10,509	10,509	100.00	455.02	27.36
Lincoln..........	1,716	1,716	100.00	264.38	2,941	2,941	100.00	401.50	51.86
Madison..........	15,667	15,620	99.70	291.23	24,910	24,910	100.00	381.63	31.04
Minidoka.........	16,677	16,677	100.00	288.01	18,274	18,274	100.00	365.16	26.79
Owyhee...........	2,291	2,291	100.00	323.10	4,371	4,371	100.00	374.50	15.91
Payette..........	773	773	100.00	394.49	767	767	100.00	395.66	0.10
Power............	8,686	8,466	97.47	300.36	15,734	15,734	100.00	438.19	46.89
Teton............	2,777	2,663	95.89	162.17	2,913	2,913	100.00	291.77	79.92
Twin Falls.......	5,864	5,864	100.00	388.07	7,415	7,415	100.00	457.17	17.81
Valley...........	375	374	99.73	151.99	387	387	100.00	282.66	85.97
Washington.......	295	295	100.00	380.62	903	903	100.00	438.15	15.11
TOTAL..........	224,573	222,236			267,994	267,994			
OVERALL TOTAL....	2,264,957	1,559,157			2,302,888	1,627,660			

SOURCE: Census of Agriculture

Table III-3. Distribution of the Value of Production, by Crop, for Counties in the Study Area, 1964, 1969

County	Barley		Wheat		Hay		Sugar beets		Potatoes	
	1964	1969	1964	1969	1964	1969	1964	1969	1964	1969
	percent									
Ada...........	4.64	7.42	7.45	6.69	73.88	56.49	12.67	14.49	1.36	14.90
Adams..........	4.59	3.93	2.17	4.25	92.91	91.81	—	—	.22	—
Bannock........	10.30	13.21	38.60	27.90	25.89	20.99	16.68	9.18	8.53	28.71
Bingham........	3.81	8.23	12.64	8.91	21.79	13.71	11.11	8.01	50.64	61.13
Blaine.........	5.91	8.92	14.14	8.95	77.16	71.93	1.51	—	1.29	10.19
Boise..........	3.35	4.86	1.93	7.38	94.52	87.76	—	—	.20	—
Bonneville.....	6.15	10.02	19.53	14.87	18.43	13.51	4.74	3.25	51.15	58.35
Butte..........	10.45	9.31	14.25	7.19	44.76	49.61	—	—	30.54	33.89
Camas..........	7.90	10.24	27.00	14.12	65.10	74.81	—	—	—	.82
Canyon.........	2.95	7.78	7.13	3.94	22.56	12.73	48.50	49.32	18.86	26.24
Caribou........	30.85	37.65	28.90	19.72	26.99	23.75	4.45	.94	8.81	17.94
Cassia.........	4.27	7.83	12.92	10.96	23.68	20.91	20.41	20.78	38.72	39.52
Clark..........	9.71	10.91	16.91	20.24	62.20	64.64	—	—	11.18	4.21
Elmore.........	5.46	8.96	5.55	16.14	35.56	15.36	1.66	2.23	51.78	79.26
Fremont........	10.66	12.83	28.21	18.36	22.29	10.91	2.89	1.58	35.95	56.31
Gem............	4.67	8.36	5.81	6.48	78.24	80.95	10.51	3.17	.77	1.04
Gooding........	.84	3.54	6.28	7.43	36.28	50.07	4.68	9.95	51.92	22.42
Jefferson......	4.76	9.60	13.13	4.43	33.63	32.80	2.49	2.16	45.99	54.50
Jerome.........	2.52	4.92	12.29	8.17	34.02	28.57	20.67	15.85	30.50	42.49
Lincoln........	2.57	7.33	18.85	8.52	44.76	41.09	14.00	5.09	19.82	37.97
Madison........	8.80	12.54	22.30	12.79	15.46	7.48	2.66	.98	50.77	66.22
Minidoka.......	3.63	11.81	8.81	5.39	19.56	16.36	33.36	27.83	34.64	38.61
Owyhee.........	4.19	9.01	5.80	4.28	46.00	43.10	28.37	20.64	15.65	22.96
Payette........	3.12	7.60	9.74	10.31	47.57	46.55	28.81	22.15	10.77	13.39
Power..........	9.44	5.47	37.70	27.58	8.13	5.42	14.08	16.85	30.64	50.10
Teton..........	14.36	26.72	31.13	24.96	36.15	22.74	—	—	18.36	25.58
Twin Falls.....	2.49	5.09	16.65	12.01	34.51	29.95	31.43	34.08	14.93	18.86
Valley.........	5.78	.87	2.21	15.34	75.04	51.94	—	—	15.77	31.85
Washington.....	8.32	7.43	15.07	13.13	50.15	40.48	23.38	28.38	3.09	10.58

SOURCE: Census of Agriculture.

Table III-4. Summary of Data Used for Marginal Value Product Analysis, by Crop and Land Resource Area, 1966

LRA	Non-irrigated acreage	Irrigated acreage	Total water	Average water	Number of observations
BARLEY					
11	0	37,190	143,040	4.0	18
11-A	8,753	49,140	166,932	3.6	12
11-B	8,680	75,090	327,923	4.8	14
10	5,500	1,720	5,685	2.9	7
10-A	14,100	4,790	18,210	4.6	7
12	0	560	3,120	5.5	2
13	165,770	23,680	43,542	2.1	12
43	2,820	4,800	9,934	3.1	8
TOTAL.........	203,623	196,970	589,650	3.0	80
WHEAT					
11	1,480	82,420	319,500	4.0	17
11-A	2,800	130,430	543,040	3.9	13
11-B	17,120	213,840	1,054,296	5.0	15
10	8,860	2,160	6,285	3.0	9
10-A	43,470	8,490	32,180	3.9	11
12	7,440	3,650	21,434	5.9	5
13	380,350	45,680	87,656	2.2	12
43	20,150	5,880	20,683	3.7	8
TOTAL.........	481,670	492,550	2,085,074	4.2	90
HAY					
11	5,510	164,850	774,778	5.1	20
11-A	0	233,660	1,273,717	4.8	15
11-B	5,660	265,810	1,513,209	5.2	14
10	46,970	25,650	108,360	4.2	15
10-A	106,270	35,130	160,090	5.2	11
12	0	23,830	142,594	5.8	10
13	38,260	81,270	199,985	3.2	15
43	11,960	17,480	68,725	4.4	10
TOTAL.........	214,630	847,680	4,241,468	5.0	110
SUGAR BEETS					
11	0	47,260	322,920	6.3	13
11-A	0	57,590	275,130	4.7	7
11-B	0	36,500	194,900	5.4	13
10	0	0	0	0.0	0
10-A	0	0	0	0.0	0
12	0	0	0	0.0	0
13	0	3,200	9,252	3.0	5
43	0	100	450	4.5	1
TOTAL.........	0	144,650	802,652	5.5	39
POTATOES					
11	0	45,170	251,615	5.9	14
11-A	0	78,160	307,746	4.5	8
11-B	0	156,340	970,388	5.7	13
10	0	180	630	3.5	1
10-A	0	0	0	0.0	0
12	0	5,200	39,000	7.5	7
13	500	26,770	82,556	3.4	8
43	0	300	1,260	4.2	2
TOTAL.........	500	312,120	1,653,195	5.3	53

APPENDIX IV

Table IV-1. Economic Index of Productivities for Selected Crops, by County, 1966

County	Barley	Wheat	Hay	Sugar beets	Potatoes	Aggregate county index
Ada..............	.8220	.9843	1.1390	1.1270	.9658	1.0750
Adams............	.8711	1.2198	.9536	--	n.a.	.9603
Bannock..........	.8795	1.1677	1.1436	1.1034	1.5539	1.1788
Bingham..........	1.2779	1.2725	.7187	.9000	.9048	.9524
Blaine...........	.9449	.8611	.7137	1.0000	1.1693	.8417
Boise............	1.1000	1.5879	1.2395	--	n.a.	1.2796
Bonneville.......	.8526	.8511	.9438	.8977	.6999	.8095
Butte............	.8449	.9145	.8896	n.a.	.9681	.9242
Camas............	1.0711	.9192	1.1038	n.a.	n.a.	1.0552
Canyon...........	1.0901	1.2119	1.4056	1.1975	1.2489	1.2324
Caribou..........	1.2495	1.3476	.8585	1.0000	n.a.	1.2007
Cassia...........	.9115	.9442	.9556	1.0763	1.0000	.9977
Clark............	1.1629	1.0836	.9150	n.a.	.7680	.9306
Elmore...........	.9487	1.2030	1.3999	.9000	1.1903	1.1958
Fremont..........	1.3547	1.0405	1.2759	1.0770	1.0000	1.1651
Gem..............	.9962	1.3108	1.0634	.8000	1.5000	1.1335
Gooding..........	1.0950	.9909	1.1580	.9367	1.0000	1.1139
Jefferson........	.9835	1.0769	.8661	.8000	.8000	.8740
Jerome...........	.9954	1.1603	.9604	1.1645	.8985	.9996
Lincoln..........	1.0775	1.1033	.8353	.7000	.8000	.8987
Madison..........	1.2164	1.0217	1.1270	.8165	.9402	.9974
Minidoka.........	1.0770	1.0507	1.2582	.9635	.9960	1.0464
Owyhee...........	1.1744	1.3857	1.1954	1.4000	1.3000	1.2885
Payette..........	.9025	1.1113	1.5650	1.0549	.6009	.8646
Power............	.8086	.9169	1.0458	.6782	.5910	.8181
Teton............	.7638	.8301	.6899	n.a.	n.a.	.7726
Twin Falls.......	.8601	.8902	1.1902	.9656	1.0961	1.0331
Valley...........	.7296	.7270	.9501	n.a.	n.a.	.8891
Washington.......	.8738	1.2213	.9248	1.3000	1.2000	1.0677

Table IV-2 Comparison of results using alternative indexes of land quality to explain variation in average value per acre for selected counties in Southern Idaho, 1964-1966

Equation	Variable	Variable name	Coefficient	Student's t	Significance level	
Aggregate equation using Thomas index of productivity (ρ_k)	X_2	Operating expense	3.135	12.427	.9995	R^2 = .9976, corrected R^2 = .9951.
	X_3	Capital flow	-.928	-1.894	.950	F value for regression - 981.3.
	X_6	Thomas index (ρ_k)	1004.200	1.488	.900	df (6,14), significant at α = .9995.
	X_7	AUMS/county	-3.104	-1.235	.850	
	X_8	Water applied	6.504	3.269	.995	
	X_{11}	ρ_k acres	11.744	2.521	.975	
Aggregate equation using Holloway index of productivity (l_k)	X_2	Operating expense	3.140	11.110	.9995	R^2 = .9971, corrected R^2 = .9940.
	X_3	Capital flow	-.725	-1.379	.9000	F value for regression - 800.7.
	X_4	I_k acres	11.135	1.987	.9500	df (6,14), significant at α = .9995.
	X_5	Holloway index (I_k)	820.800	1.029	.8000	
	X_7	AUMS	-4.329	-2.275	.975	
	X_8	Water applied	5.322	2.490	.9750	
Equation estimated on a per-acre basis, using Thomas index (ρ_k) of productivity	X_6	Thomas index (ρ_k)	41.420	2.951	.975	R^2 = .9937, corrected R^2 = .9759.
	X_{11}	ρ_k acres	-.130	-2.079	.950	F value for regression = 368.6.
	X_{12}	Labor	-5.242	-1.975	.950	df (6,14) significant at α = .0995.
	X_{13}	Operating expense	3.829	6.403	.9995	
	X_{15}	AUMS	-4.329	-2.275	.975	
	X_{16}	Water applied	5.858	2.078	.950	
Equation estimated on a per-acre basis, using Holloway index, I_k, of productivity	X_4	(l_k)(acres) x (ρ_k)	-.123	-1.612	.900	R^2 = .9926, corrected R^2 = .9718.
	X_5	l_k	39.762	2.327	.975	F value for regression = 313.7.
	X_{12}	Labor	-5.228	-1.717	.900	df (6,14), significant at α = .9995.
	X_{13}	Operating expense	3.860	5.772	.9995	
	X_{15}	AUMS	-3.655	-1.792	.950	
	X_{16}	Water applied	6.054	1.954	.950	