AN ABSTRACT OF THE THESIS OF

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Title: AN EVALUATION OF THE EARTH RESOURCES TECHNOLOGY SATELLITE (ERTS-1) MULTISPECTRAL SCANNER AS A TOOL FOR THE DETERMINATION OF LACUSTRINE TROPHIC STATE

Abstract approved: Redacted for Privacy
Robert E. Frenkel

This study evaluates the Earth Resources Technology Satellite-One (ERTS-1) multispectral scanner (MSS) as a means of predicting lacustrine trophic state and the magnitude of selected trophic state indicators.

Numerical classificatory methods are employed to ascertain the trophic character of 100 lakes in Minnesota, Wisconsin, Michigan, and New York using the trophic indicators: chlorophyll a, conductivity, inverse of Secchi disc transparency, total phosphorus, total organic nitrogen, and an algal assay yield. A complete linkage clustering algorithm is first used to examine the lakes for natural clusters.

The hyper-dimensional cloud of data points is then reduced in dimensionality through the ordination technique of principal components analysis. The first three principal components explain, respectively, 68 percent, 14 percent, and 8 percent of the variation in the data; the three-dimensional ordination is expressed in the form of a "ball and wire" model. The two complementary classificatory techniques show the existence of poorly defined clusters. A multivariate trophic state index (PC1) is derived from the principal component analysis.

A binary masking technique is used to extract lake-related MSS data from computer-compatible digital magnetic tapes. Data products are in the form of descriptive statistics and photographic concatenations of lake images. Linear digital contrast stretching is demonstrated.

MSS color ratio regression models are developed for the prediction of Secchi disc transparency and chlorophyll a in selected Minnesota and Wisconsin lakes. The models give good estimates of Secchi disk transparency and fair estimates of chlorophyll a levels.

Lake area estimates derived from MSS pixel counts are generally within ten percent of area values obtained from topographic maps. The synoptic view of the sensor is conducive to lake enumeration work.

The trophic state of lakes, as defined by lake position on the first principal component axis (PC1 value), is predicted using MSS color ratio regression models; each date of ERTS-1 coverage has its own model. There is a general tendency for the MSS ratios (GRNRED, GRNIR1, GRNIR2, REDIR1, and REDIR2) to decrease as the manifestations of eutrophication become more evident.

A less rigorous approach to the study of MSS data-lacustrine trophic state relationships is undertaken using three-dimensional color ratio models. The mean IR1 intensity levels of several "hyper-eutrophic" lakes exceed their mean RED levels and effectively isolate them from other lakes in the models.

An automatic image processing technique is employed to classify a group of Wisconsin lakes using MSS colors (GRN, RED, and IR1) in conjunction with the lakes' trophic state index values. The results are depicted in the form of both gray-scale and color-enhanced photographic concatenations.

The ERTS-1 MSS has utility in the assessment of the lacustrine resource. Its usefulness is most apparent when the seasonal contrasts between lakes at different points on the trophic scale are at a maximum. Excessive cloud cover, faulty or missing MSS data, and the need for some ground truth impair, but do not preclude, its use in lake monitoring and classification.

The use of computer-compatible tapes in conjunction with digital image processing techniques is essential if the maximum benefits are to be derived from the ERTS-1 MSS in lake-oriented studies.

An Evaluation of the Earth Resources Technology Satellite (ERTS-1) Multispectral Scanner as a Tool for the Determination of Lacustrine Trophic State

by

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LIST OF ACRONYMS AND SYMBOLS

AAY: algal assay yield

ADP: automatic data processing

AIP: automatic image processing

A-space: attribute space

b&w: black and white

bit: binary digit

bpi: bits per inch

CRT: cathode ray tube

CCT: computer-compatible tape

CDC: Control Data Corporation

CHLA: chlorophyll a

COND: conductivity

cm: centimeter

°C: degrees Celsius

DN: digital number

DCS: data collection system

EBR: electron beam recorder

EPA: Environmental Protection Agency (United States)

EROS: Earth Resources Observation Systems

ERTS-1: Earth Resources Technology Satellite Number One

ESB: Eutrophication Survey Branch

GRN: green (MSS Band 4)

GRNIR1: ratio of green to infrared-one

GRNIR2: ratio of green to infrared-two

GRNRED: ratio of green to red

GRAFPAC: standard graphic output subroutines

GSFC: Goddard Space Flight Center

ha: hectare

hz: hertz

IBM: International Business Machines

IFOV: instantaneous field of view

IPL: Image Processing Laboratory (Jet Propulsion Laboratory)

IR1: infrared-one

IR2: infrared-two

IR1IR2: ratio of infrared-one to infrared-two

ISEC: inverse of Secchi disc transparency

I-space: individual space

JPL: Jet Propulsion Laboratory

km: kilometer

l: liter

LARS: Laboratory for Applications of Remote Sensing (Purdue University)

LN: natural logarithm

LNAAY: natural log-transformed algal assay yield

LNCHLA: natural log-transformed chlorophyll a

LNCOND: natural log-transformed conductivity

LNISEC: natural log-transformed inverse of Secchi disc transparency

LNSECCHI: natural log-transformed Secchi disc transparency

LNTON: natural log-transformed total organic nitrogen

LNTPHOS: natural log-transformed total phosphorus

m: meter

MCR: mean composite rank

μg: microgram

μsec: microsecond

mg: milligram

MSS: multispectral scanner

N: number of observations

NASA: National Aeronautics and Space Administration

NERC: National Environmental Research Center

NES: National Eutrophication Survey

NIR: near-infrared

nm: nanometers

OD: optical density

OS: operating system

OSI: Optimum Systems Incorporated

OSU: Oregon State University

PCA: principal components analysis

PC1: principal component trophic state index

pixel: picture element

PMT: photomultiplier tube

PNERL: Pacific Northwest Environmental Research Laboratory

RBV: return beam vidicon

RED: red (MSS Band 5)

R-space: row space (i.e., attribute space)

REDIR1: ratio of red to infrared-one

REDIR2: ratio of red to infrared-two

SECCHI: Secchi disc transparency

SIPS: Statistical Interactive Programming System

S-matrix: similarity matrix

STORET: Storage and Retrieval number

TON: total organic nitrogen

TPHOS: total phosphorus

USGS: United States Geological Survey

VFC: video film converter

VICAR: Video Communication and Retrieval

XGRNIR1: mean of green to infrared-one ratios

XGRNIR2: mean of green to infrared-two ratios

XGRNRED: mean of green to red ratios

XIR1IR2: mean of infrared-one to infrared-two ratios

XREDIR1: mean of red to infrared-one ratios

XREDIR2: mean of red to infrared-two ratios

Δ: Euclidian distance

Δ^2: squared Euclidian distance

$\wedge$: predicted value

AN EVALUATION OF THE EARTH RESOURCES TECHNOLOGY SATELLITE (ERTS-1) MULTISPECTRAL SCANNER AS A TOOL FOR THE DETERMINATION OF LACUSTRINE TROPHIC STATE

I. INTRODUCTION

Homo sapiens has developed the burgeoning population and the technological level necessary to effect significant alterations in the biosphere. His impact on environment is accelerating in variety, magnitude, and geographic extent. He has failed, either through gross ignorance or a lack of concern, to give adequate consideration to the interrelatedness of the biotic and abiotic elements which compose the environment. Man-induced environmental alterations generally result in a chain of consequences, both direct and indirect, short-term and long-term, and varying in magnitude. Many of the consequences (*e.g.*, air pollution, water pollution) adversely affect his health, his economic well-being, and the pristine qualities of environment.

Man's strategies for using the earth's lacustrine resources have usually been predicated on immediate short-term economic gains with little consideration of long range environmental ramifications. It is becoming increasingly apparent that man has adversely affected many lakes, particularly those located in countries with large population densities and/or which are technologically advanced. The United States of America can serve as a prime example.

Statement of Purpose

Rational management of the lacustrine resource dictates, as the first step, an assessment of each lentic body's trophic status. Data collection for the determination of trophic status is a costly, time-consuming process, especially when thousands of lakes are to be evaluated. The need exists to find a means of rapidly assessing the trophic state of water bodies which would make it economically feasible to operate extensive systematic surveillance programs. Stewart and Rohlich (1967) have urged investigators to develop remote sensing techniques and evaluate their potential for eutrophication surveillance.

Satellite-borne sensors show promise as a means of monitoring and classifying lakes and reservoirs. The successful orbiting of the Earth Resources Technology Satellite (ERTS-1) affords the opportunity to investigate the potential of one type of satellite-borne sensor.

The purpose of this thesis is the evaluation of ERTS-1 remotely sensed multispectral scanner data as a means of determining the trophic state of a selected group of lakes located in the northern part of the conterminous United States. Specific objectives include: the development and application of a multivariate ranking system to a selected group of lakes; the formulation of empirical models for the estimation of selected trophic state indicators; and the development of empirical models for the prediction of lacustrine trophic state using ERTS-1 data.

Format of Thesis

The remainder of this chapter is devoted to a description of the Earth Resources Technology Satellite, its orbital characteristics, instrumentation, and products. In addition, the study area is described along with the criteria used in the selection of the lakes and the techniques used in the collection of the ground (*i.e.*, water) truth.

Chapter II discusses lakes as natural resources, the concept of eutrophication, trophic state indicators, and demonstrates the use of multivariate techniques in the classification of a large group of lakes on the basis of selected ground truth parameters.

Chapter III is devoted to a detailed description of the methodology used in the extraction and transformation of ERTS-1 multispectral scanner (MSS) data into forms and products which can then be used in the study of hypothesized ground truth-MSS data relationships.

Correlations between lake parameters (surface area, chlorophyll *a*, Secchi disc transparency) and MSS data are explored in Chapter IV. Regression models are developed to predict the magnitude of selected trophic state indicators.

Chapter V includes the prediction of lake trophic state using MSS color ratios in regression models. Three-dimensional models are used to illustrate qualitative differences among the lakes on the basis of MSS color ratios. An automatic data (i.e., image) processing technique is employed to generate enhanced photographic products depicting the trophic state of selected lakes.

Chapter VI is devoted to a brief discussion and summation of the potential applications and limitations of the ERTS-1 MSS in lake monitoring and classification.

The Earth Resources Technology Satellite

The Earth Resources Technology Satellite Program[1], under the sponsorship of the National Aeronautics and Space Administration, is a concerted effort to merge space and remote sensing technologies into a system which will demonstrate techniques for efficient management of the earth's natural resources. To explore the feasibility of applying earth resource data collected from satellite altitudes to resource management problems, NASA inserted an experimental satellite (ERTS-A) into a circular earth orbit. Another satellite (ERTS-B) is scheduled for launch when ERTS-A, officially designated ERTS-1 upon successful insertion, ceases to function.

ERTS-1 Orbit Parameters and Earth Coverage

ERTS-A (*i.e.*, ERTS-1) was placed into a nominal sun-synchronous near-polar orbit by a Delta launch vehicle on 23 July 1972 (Freden, 1973). Orbital parameters are listed in Table 1.

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[1] Detailed information regarding the Earth Resources Technology Program is found in the *Data Users Handbook* (NASA, 1972).

Table 1. ERTS-1 orbital parameters. Adapted from Data Users Handbook (NASA, 1972).

Orbit Parameter	Actual Orbit
Semi-major axis	7285.82 kilometers
Inclination	99.114 degrees
Period	103.267 minutes
Eccentricity	0.0006
Time at descending node (southbound equatorial crossing)	9:42 a.m.
Coverage cycle duration	18 days (251 revolutions)
Distance between adjacent ground tracks (at equator)	159.38 kilometers

The earth coverage pattern is shown in Figure 1 for two orbits on two consecutive days. Orbital parameters result in a 1.43 degree westward migration of the daily coverage swath, equivalent to a distance of 159 kilometers at the equator. The westward progression of the satellite revolutions continues, exposing all of the area between orbit N and orbit N+1 to the satellite sensors by day M. This constitutes one complete coverage cycle and consists of 251 revolutions. The cycle takes exactly 18 days and results in total global coverage between 81° N and 81° S latitude. Fourteen orbits (*i.e.*, revolutions) are completed during each of 17 days in a cycle with 13 revolutions during one day (NASA, 1972). Approximately 188 scenes are acquired on an average day (Nordberg, 1972).

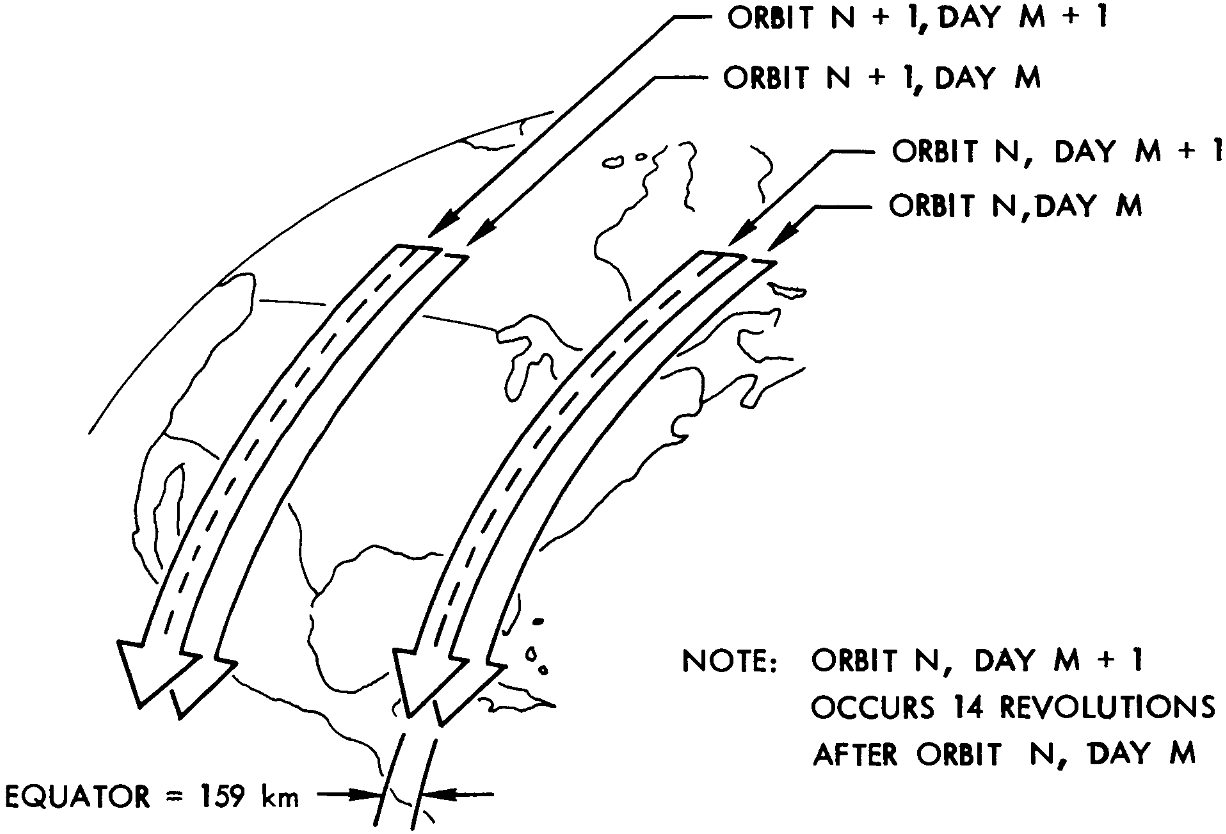

Figure 1. ERTS-1 ground coverage pattern. Adapted from <u>Data Users Handbook</u> (NASA, 1972).

ERTS-1 Instrumentation

The ERTS-1 payload consists of a Return Beam Vidicon (RBV) Camera Subsystem[2], a Multispectral Scanner Subsystem (MSS), and a Data Collection System (DCS). The RBV and MSS are designed to provide multispectral imagery of the earth beneath the observatory (i.e., satellite). The DCS serves to relay environmental information from geographically remote ground-based sensors to ERTS ground stations for processing and delivery to users. The RBV and DCS aspects of the satellite need not concern us.

The MSS is a line-scanning radiometer which collects data by creating images of the earth's surface in four spectral bands simultaneously through the same optical system. The instrument operates in the solar-reflected spectral band region from 500 to 1,100 nanometers. Scanner characteristics are listed in Table 2.

The MSS scans cross-track swaths 185 kilometers in width, simultaneously imaging six scan lines for each of the four bands. The object plane is scanned by an oscillating flat mirror positioned between the scene and a double reflector telescope-type of optical chain. An 11.5 degree (Horan, Schwartz, and Love, 1974) cross-track field of view is produced by the mirror oscillating ±2.89 degrees about its nominal position (Figure 2.).

[2]A malfunction occured on 6 August 1972 (orbit 198) in the RBV power switching circuit and the RBV cameras were turned off as only one sensor system can be used in conjunction with the one functioning video tape recorder. The second recorder aboard the observatory malfunctioned between orbits 148 and 181 (Freden, 1973).

Table 2. ERTS-1 MSS characteristics. Adapted from Horan, et al. (1974).

Item	Characteristics			
Telescope optics	22 cm (aperture diameter), f/3.6 Ritchey-Chretien			
Scanning method	Flat mirror oscillating ± 2.9 degrees at 13.62 Hz			
Scan (Swath) width	11.5 degrees (185 kilometers at 917 kilometers altitude)			
Scan duty cycle	44%			
Instantaneous field of view (IFOV)	86 microradians			
Number of bands	Four			
Number of lines (detectors) scanned per band	Six			
Limiting ground resolution from 917 kilometers altitude	40 meters			
Spectral band wavelength:				
NDPF Band Code				
Band 4 (Green)	500-600 nanometers			
Band 5 (Red)	600-700 nanometers			
Band 6 (Near-infrared One)	700-800 nanometers			
Band 7 (Near-infrared Two)	800-1,100 nanometers			
Sensor response:	Band 4	Band 5	Band 6	Band 7
Detector	PMT*	PMT	PMT	Photodiode
Nominal input for 4 V Scanner Output (10^{-4}W cm^{-2}sr^{-1})	24.8	20.0	17.6	46.0
Scanner and multiplexer weight	50 kilograms			
Signal channels	24			
Telemetry channels	97			
Command capability	72			
Scanner size	Approximately 36 x 38 x 89 cm			

*photomultiplier tube

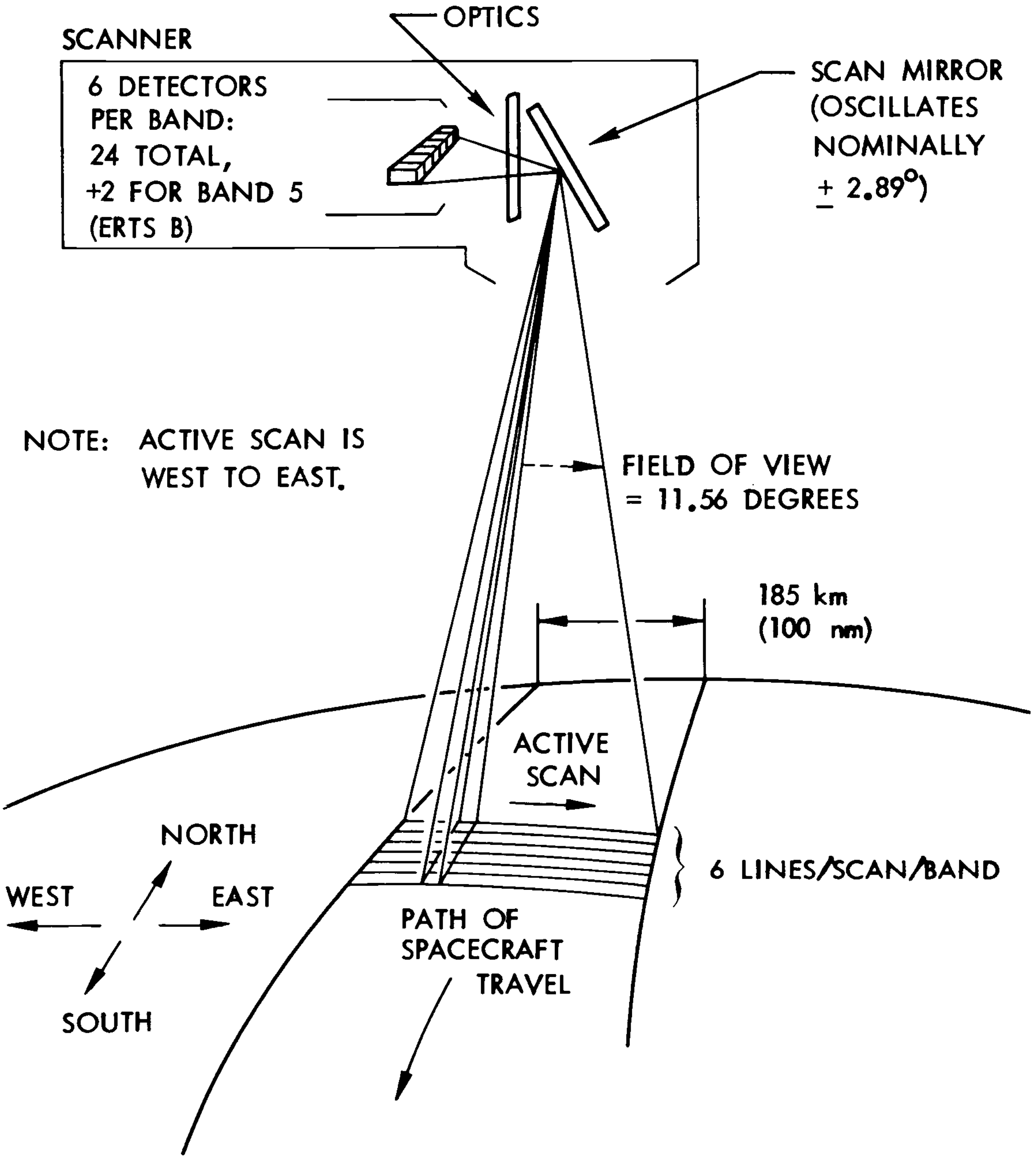

Figure 2. Schematic diagram of the ERTS-1 MSS scanning arrangement. Adapted from the Data Users Handbook (NASA, 1972).

A nominal orbital velocity of 6.47 kilometers per second, neglecting observatory perturbations and earth rotation effects, produces the requisite along-track scan. The subsatellite point moves 474 meters along the track during the 73.42 millisecond active scan-retrace cycle which is itself a consequence of the 13.62 hz mirror oscillation rate. The track distance of 474 meters synchronizes with the 474 meter along-track field of view of each set of six detectors. The line scanned by the first detector in one cycle of the active scan is in juxtaposition to the line scanned by the sixth detector of the previous scan (Figure 3).

Twenty-four glass optical fibers, arranged in a four by six (4x6) matrix in the focused area of the telescope, intercept the light from the earth scene. Light impinging on the square input end of each optical fiber is conducted to an individual detector through an optical filter unique to the respective spectral band under consideration. Photomultiplier tubes (PMT) serve as detectors for Bands Four through Six; Band Seven uses silicon photodiodes.

A video signal is produced at the scanner electronics output as the image of a line across the swath is swept across the fiber during active scan. The signal is sampled at 9.95 microsecond (μsec) intervals which correspond to a 56 meter cross-track motion of the instantaneous field of view. The sampled signal is digitized and arranged into a serial digit data stream for transmission to ground stations. Individual signals are derived from light passing through each fiber, resulting in 24 channels of output.

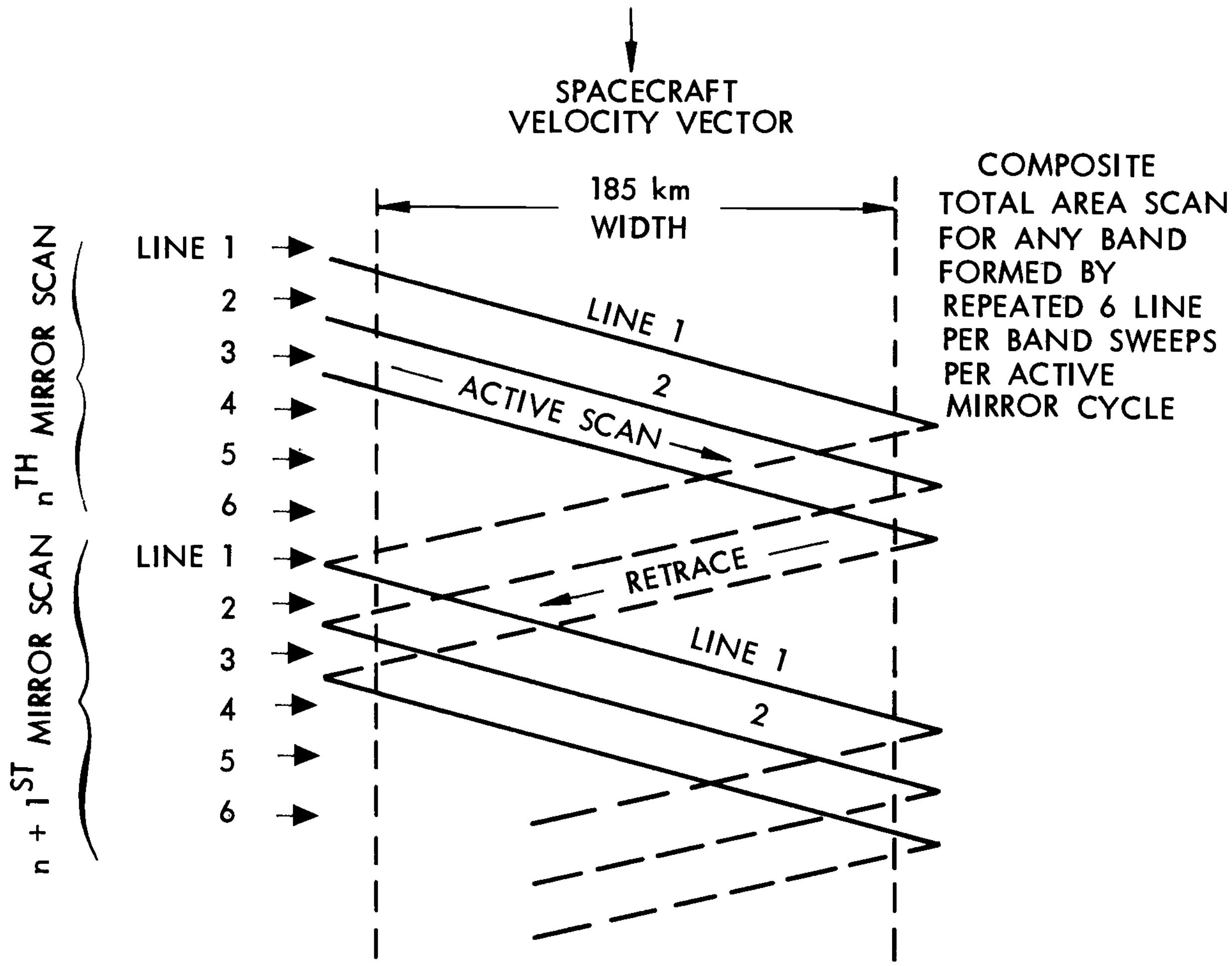

Figure 3. Ground scan pattern for a single MSS detector. Adapted from the Data Users Handbook (NASA, 1972).

ERTS-1 Products

The electronic signals from the observatory's MSS are converted into photographic and computer products at the Goddard Space Flight Center, Greenbelt, Maryland.

Third and fourth generation photographic products are available in the form of prints, positive and negative transparencies, and come in several scales including: 1:3,369,000; 1:1,000,000; 1:500,000; and 1:250,000 with the transparencies being limited to the two smaller scales. Color products are available for a relatively small number of scenes.

Computer-compatible magnetic digital tapes (CCT's) may be requested in either a 7-track or a 9-track format. Four CCT's are required for the MSS data corresponding to one scene.

Copies of the CCT's and the photographic products are placed in the public domain at the Department of the Interior's Earth Resources Observation Systems (EROS) Data Center located in Sioux Falls, South Dakota.

ERTS-1 Lake Monitoring Potential

The advantages of using remote sensing imagery systems are threefold: they afford an overall (synoptic) view, they can give a time record, and they expand the spectral limits of the human eye (Scherz, 1969; Scherz, 1971a; Scherz, 1971b). Multispectral satellite-borne imaging systems show promise as a means of monitoring and classifying the earth's lacustrine resources. This is partially due to the repetitive nature of a satellite and the tremendous synoptic view offered from orbital altitudes.

Visual examinations of select frames of ERTS-1 MSS imagery from Wisconsin, Minnesota, and Florida suggest that good correlations may exist between the trophic status of lakes and their tonal characteristics. MSS Frame 1017-16093, recorded at an altitude of approximately 917 kilometers over southeastern Wisconsin and northeastern Illinois on 9 August 1972, will serve to illustrate this point.

Figure 4 is a reproduction of an EROS Data Center photograph of the scene as recorded in the near-infrared (IR2; 800 to 1, 100 nanometers) spectral band. Water bodies, including the larger streams, stand out boldly against the lighter tones of the land features. The labelled lakes, excluding Lake Michigan, were sampled by the U. S. Environmental Protection Agency's National Eutrophication Survey (NES) during the 1972 open water season. Gray tone differences are not evident among the lakes nor are tonal patterns visible on any of the lakes. IR2 is, however, a good spectral band for the location and demarcation of water bodies. Some caution is necessary when conducting a lake enumeration on the photograph because some of the "lakes" are in reality shadows cast by cumulus clouds.

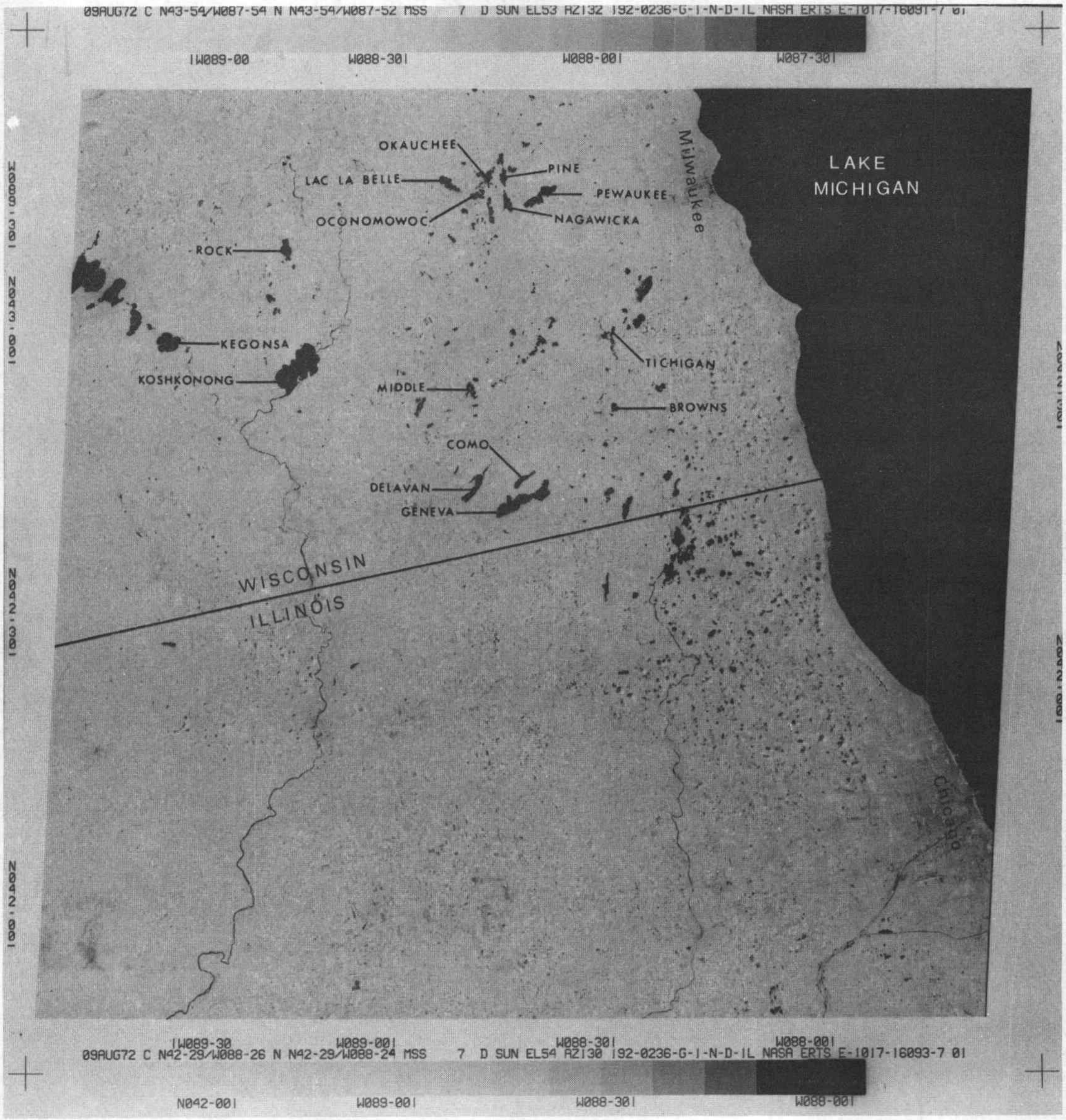

Figure 4. Reproduction of an EROS Data Center IR2 print of Frame 1017-16093 (9 August 1972). Water bodies stand out in stark contrast to the lighter-toned land features. The labelled lakes, excluding Lake Michigan, were sampled by the National Eutrophication Survey during 1972. The reproduction, originally printed at a scale of 1:1,000,000, has a scale of approximately 1:1,419,000.

Figure 5 is the same scene recorded in near-infrared one (IR1; 700 to 800 nanometers). Tonal differences are apparent, at least in the original photographic print, among the lakes, and patterns are evident on some of the lakes (e.g., Lake Koshkonong). Lakes are readily located and their boundaries delimited in this band.

Figure 6 is a red light (RED; 600 to 700 nanometers) MSS photograph of the scene. Marked gray tone differences are apparent among the lakes. Lakes commonly recognized as eutrophic (e.g., Lake Como) tend to appear light in tone and meld in with the land features. Lakes with relatively good water quality (e.g., Lake Geneva) are characterized by darker tones. Lotic bodies are not readily apparent in the photograph.

The green light (GRN; 500 to 600 nanometers) sensed by the MSS was used to construct the Figure 7 photograph. Although the lakes are difficult to discern, a result of low contrast among the scene elements, differences among the lakes can be detected with the unaided eye.

It is apparent, from the visual examination of ERTS-1 MSS Frame 1017-16093 and other frames from several additional states, that the satellite-borne multispectral scanner is collecting data which may be of value in the classification and monitoring of lentic bodies. The results of the examination suggest that GRN, RED, and IR1 contain most of the information relative to trophic status assessment.

Figure 5. Reproduction of EROS IR1 print of Frame 1017-16093 (9 August 1972). Surface patterns are evident on some of the lakes (eg., Lake Koshkonong). Each edge of the picture is equivalent to a ground distance of approximately 185 kilometers. The reproduction is printed at a scale approximating 1:1, 419, 000.

Figure 6. Reproduction of EROS RED (MSS Band 5) print of Frame 1017-16093 (9 August 1972). Variations in gray tone are readily apparent among the lakes and suggest differences in water quality. Lake Geneva, characterized by relatively high water quality, is dark in tone compared with, for example, eutrophic Lake Koshkonong. The small ball-like white objects between Milwaukee and Chicago are cumulus clouds. The reproduction is printed at a scale of approximately 1:1, 419, 000.

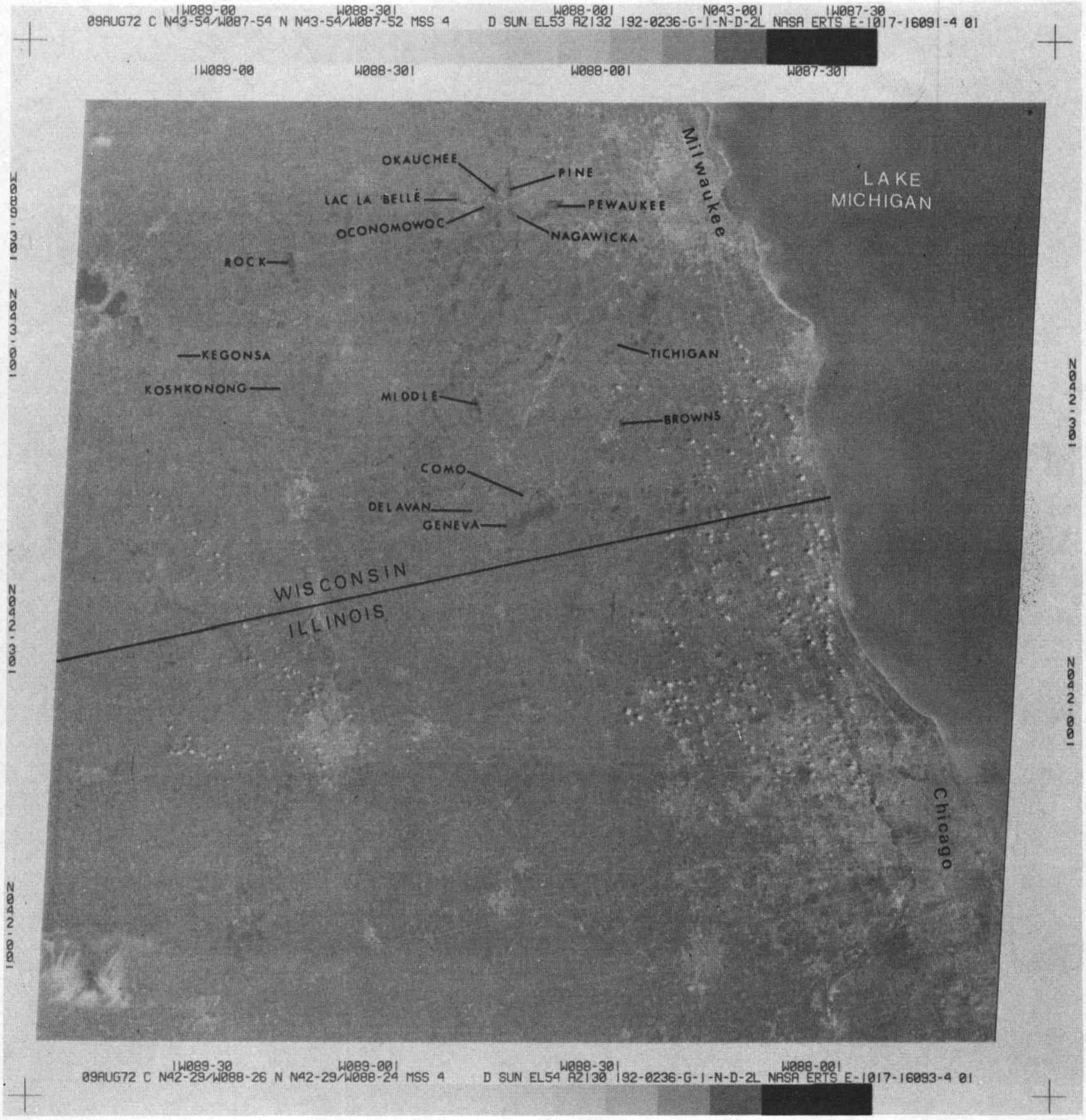

Figure 7. Reproduction of EROS GRN (MSS Band 4) print of Frame 1017-16093 (9 August 1972). Scenes recorded in the green band generally lack contrast, but contain information useful in monitoring earth resources. The reproduction is printed at a scale of 1:1, 419, 000.

Description of the Study Area

Many current efforts to determine the feasibility of using ERTS-1 MSS data in water quality monitoring tend to be intensive in nature, near-real time, and oriented toward the dynamics of pollution or eutrophication (e.g., Chase and Elliott, 1973; Lind and Henson, 1973; Lind, 1973). The investigations are very limited in geographic scope, typically involving fewer than six water bodies.

Another approach, the one used in this thesis, involves the examination of a relatively large number of lakes over a more extensive geographic area. Although it is not uncommon to find lakes of different trophic state within the same lake region (e.g., Southeastern Lake District of Wisconsin), the selection of a larger study area affords the opportunity to include lakes exhibiting a greater trophic range. In addition, assuming that lakes have characteristic multispectral signatures at particular points on a trophic scale, the use of a larger and more diverse lake population permits a more extensive evaluation of the ERTS-1 MSS's capabilities in the realm of lake monitoring and classification.

Geographic Area

The geographic area serving as a matrix for the study lakes comprises the states of Minnesota, Wisconsin, Michigan, and New York. The area was selected for its abundance of lakes, the availability of pertinent environmental (i.e. ground truth) data and concurrent or near-concurrent ERTS-1 MSS frames.

Climate

The climate of the study area is humid continental. Continental polar air masses dominate during the winter, resulting in a mean temperature of less than -10°C for the coldest month. Most of the study lakes are ice-bound for several months, two notable exceptions being the New York Finger Lakes, Seneca and Cayuga, which usually remain open and in full circulation throughout the winter. (Berg, 1963). Tropical air masses dominate during the summer and high temperatures prevail. The warmest month has a mean temperature in excess of 18°C. The temperature extremes are less severe in the eastern portion of the study area (New York).

Although precipitation is common throughout the year, a summer maximum exists. The mean annual precipitation ranges from a low of approximately 50 centimeters in northwestern Minnesota to a high of about 125 centimeters in north central New York (USGS, 1970).

Detailed descriptions of the climate are found in Trewartha (1968), USDA (1941), Visher (1954), Thornthwaite (1948), and Strahler (1969).

Geology, Soils, and Land Use

The study area extends over three physiographic divisions and includes seven subdivisions (Figure 8). The entire area, excluding a small region in western Wisconsin and southeastern Minnesota

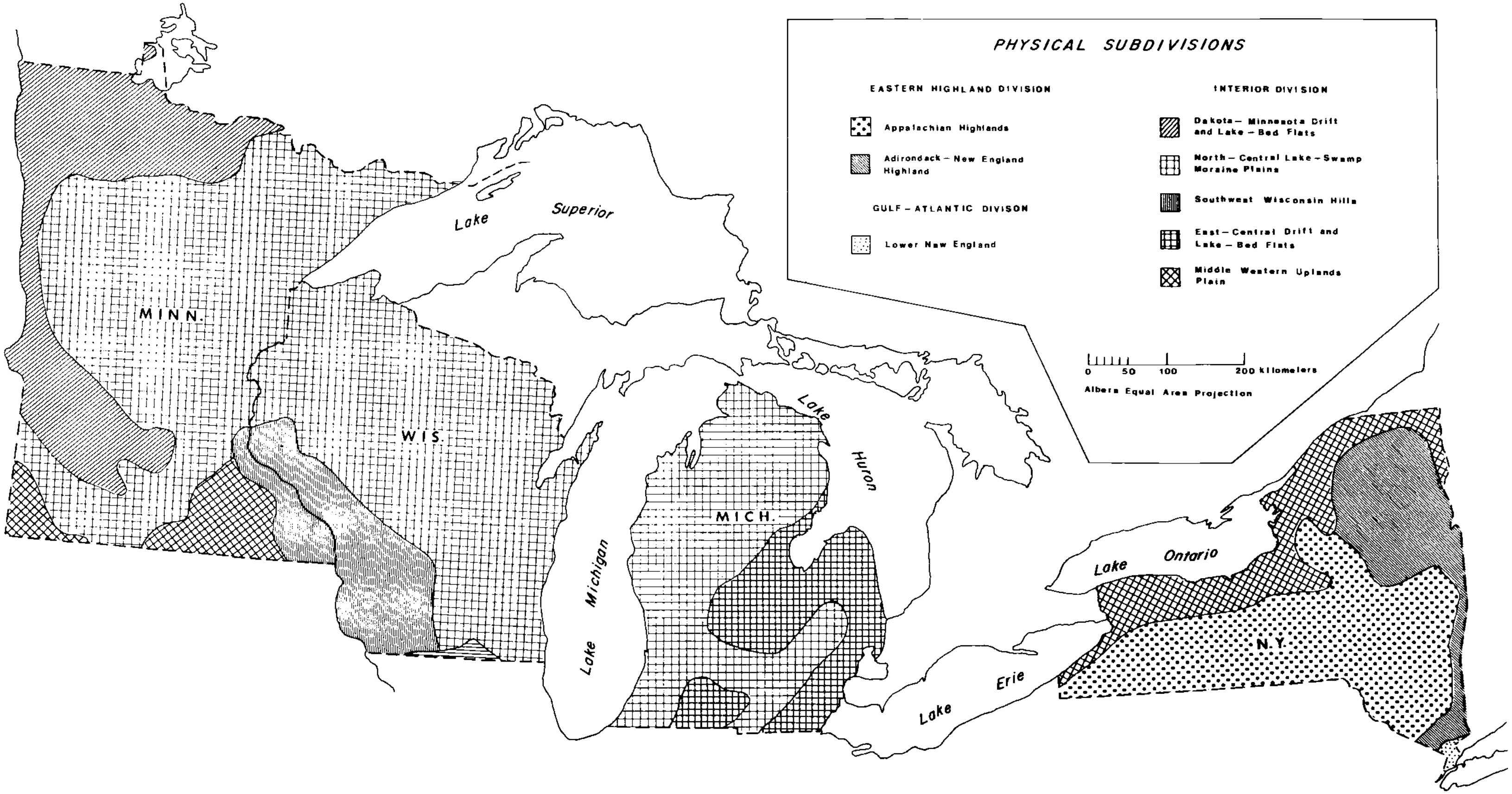

Figure 8. Physical subdivisions. Adapted from U.S. Geological Survey (1970 and Hammond (1964).

(the "Driftless Area"), was exposed to the forces of continental glaciation which were instrumental in the formation of the numerous lake basins.

The bedrock ranges in age from Precambrian to Cretaceous and includes sedimentary, metamorphic, and igneous rocks (Figure 9). Areas underlain with older Precambrian rocks and covered by a thin veil of glacial drift (*e.g.*, northeastern Minnesota) generally have lakes possessing relatively high water quality.

The landscape is dominated by the members of six soil orders (7th Approximation) including Alfisols, Entisols, Histosols, Inceptisols, Mollisols, and Spodosols (Figure 10).

Major types of land use are shown in Figure 11. Lakes seriously modified by man are generally found in areas where land use is chiefly agricultural in nature, particularly as cropland.

Lake Selection Criteria and Location

Most of the lakes incorporated into this study were selected from some 220 lakes sampled in 1972 by the U. S. Environmental Protection Agency's (EPA) National Eutrophication Survey (NES)[3]. The lakes

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[3] A detailed explanation of the National Eutrophication Survey, its objectives, lake selection criteria, and methodology are found in NES (1974), Working Paper Number 1.

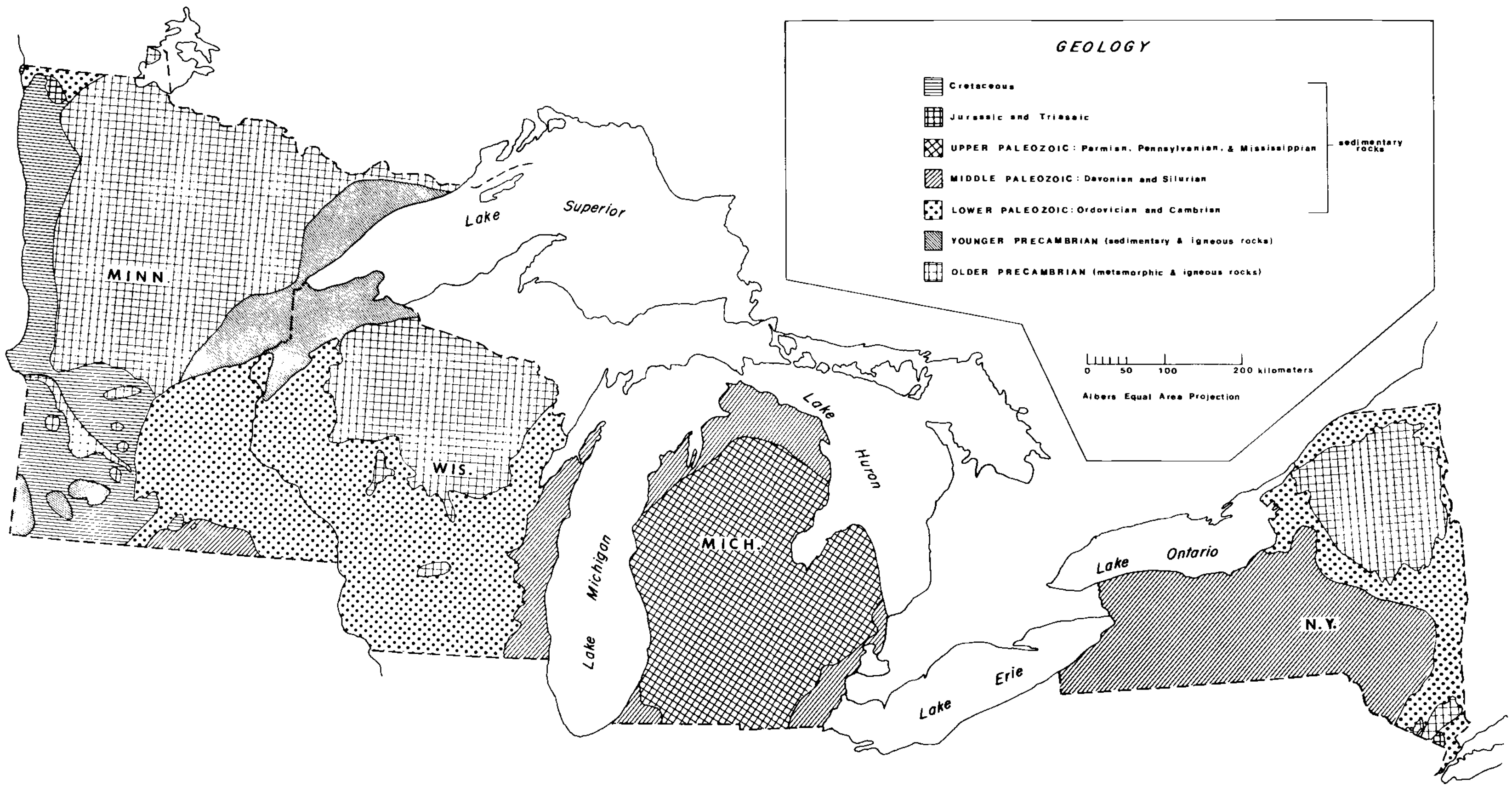

Figure 9. Geology. Adapted from U.S. Geological Survey (1970). For greater detail see U.S. Geological Survey (1965), Geologic Map of North America, 1:5,000,000.

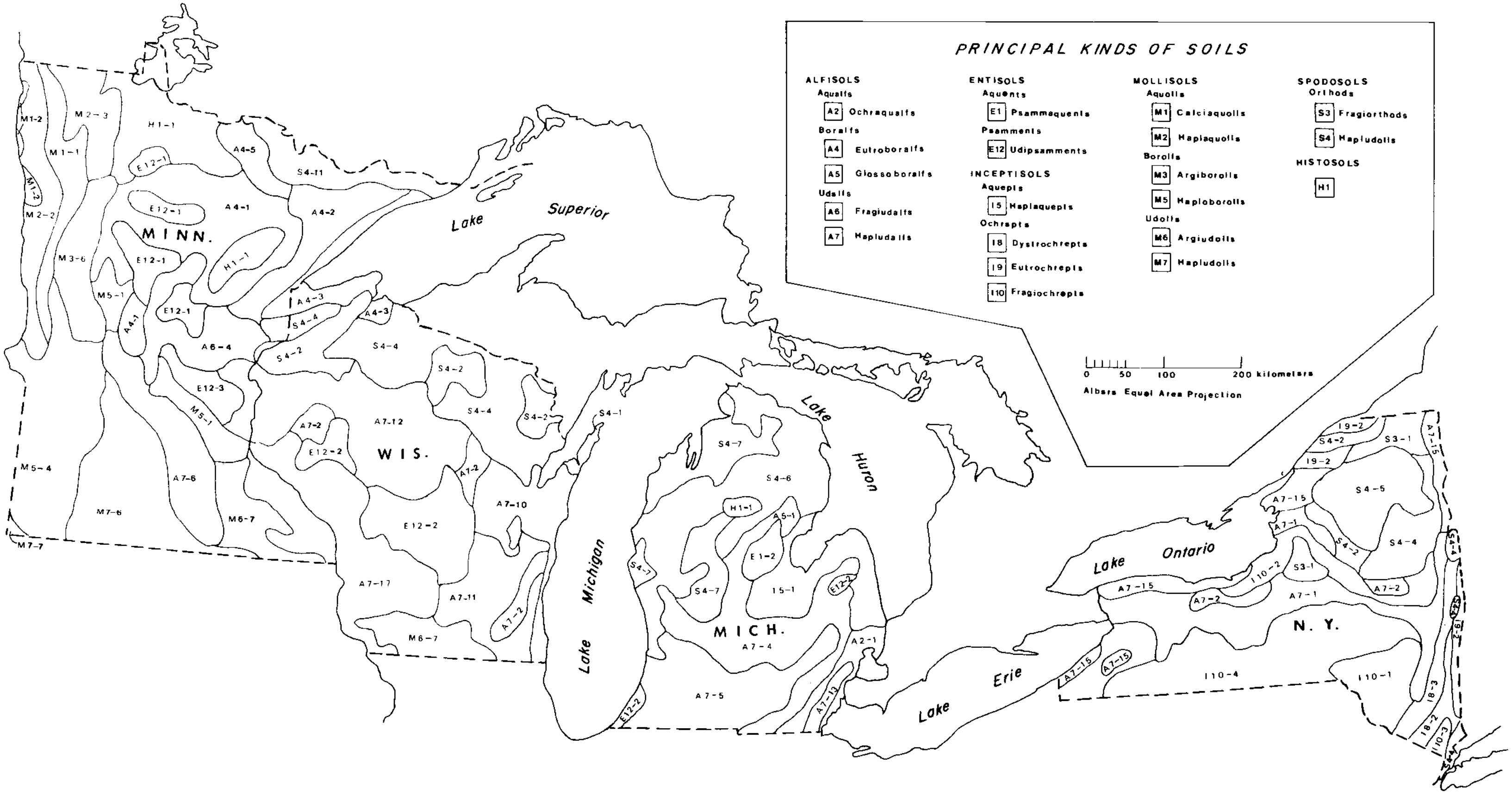

Figure 10. Distribution of principal kinds of soils: Orders, Suborders, and Great Groups. The letter-number symbols in the legend are abbreviated from those on the map. For a complete description, see U.S. Geological Survey (1970). Complete definitions of the soils are found in Soil Classification, A Comprehensive System, 7th Approximation (U.S. Department of Agriculture, 1960) and (USDA, March 1967) Supplement. The map is adapted from U.S. Geological Survey (1970).

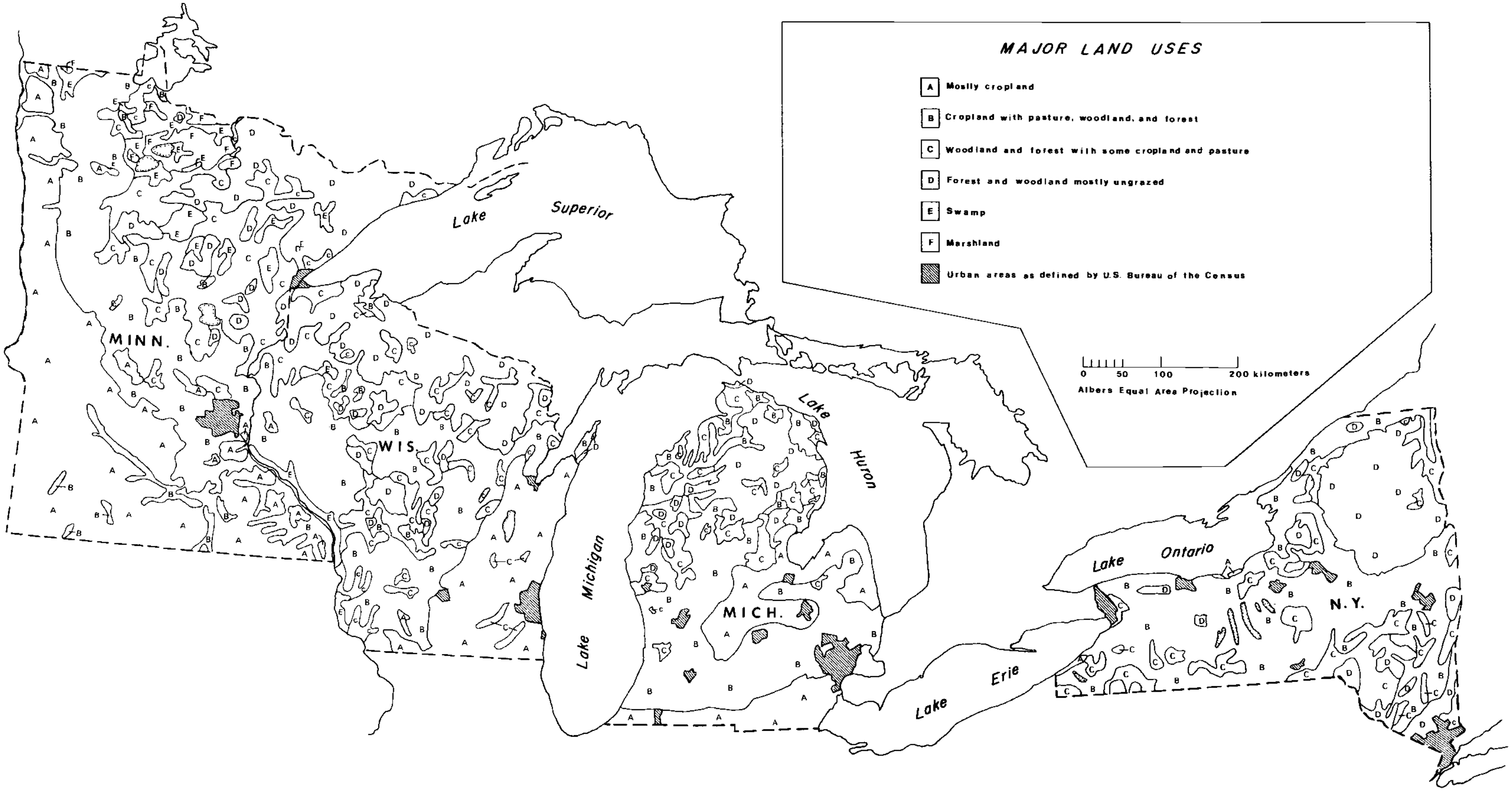

Figure 11. Major land uses. Adapted from U.S. Geological Survey (1970).

were selected, for the purposes of this thesis, to give extensive geographic coverage, trophic indicator values that covered a range representative of the 1972 NES lakes, and for completeness of sampling records. The study lakes are listed in Table 3. NES-sampled lakes have both a serial number and a STORET[4] (STOrage and RETrieval) number. Lakes with only a serial number are outside the scope of NES. Lake locations are depicted in Figure 12.

Ground Truth Collection[5]

Each NES lake (serial numbers 1-100) was sampled three times (spring, summer, fall) during the 1972 calendar year (Appendix B) by helicopter-borne sampling teams operating from pontoon-equipped Bell UL-1H aircraft.

The helicopters were equipped with a submersible pump and appropriate sensors for *in situ* measurements of conductivity, temperature, optical transmissivity, dissolved oxygen, hydrogen-ion concentration (pH), and water depth. Additional equipment included an echo sounder, 30 cm Secchi disc, and water sampling equipment.

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[4]STORET is an acronym used to identify the computer-oriented EPA management information system for the storage and retrieval of water quality, municipal and industrial waste facility inventory, water quality standards compliance, fish kill, oil spill, construction cost, and other related data.

[5]A detailed description of instrumentation, techniques, and methodology is found in NES Working Paper Number 1 (1974).

Table 3. Study lakes

Lake Name	Serial Number*	STORET Number	County	Lake Coordinates** N. Latitude	W. Longitude
Blackduck	1	2711	Beltrami	47° 45' 30"	94° 36' 00"
Bemidji	2	27C1	Beltrami	47° 29' 30"	94° 50' 00"
Andrusia	3	27C0	Beltrami	47° 26' 30"	94° 38' 30"
Wolf	4	27A2	Becker	47° 26' 30"	94° 40' 30"
Cass	5	2715	Cass	47° 27' 00"	94° 28' 30"
Leech	6	2746	Cass	47° 15' 00"	94° 13' 00"
Birch	7	2710	Cass	46° 56' 00"	94° 31' 30"
Trout	8	2793	Itasca	47° 17' 00"	93° 25' 00"
Mashkenode	9	2756	St. Louis	47° 29' 30"	94° 36' 00"
Whitewater	10	2749	St. Louis	47° 29' 30"	92° 11' 00"
Pelican	11	2756	Crow Wing/ St. Louis	48° 02' 00"	92° 50' 00"
Shagawa	12	2780	St. Louis	47° 55' 00"	91° 53' 00"
Gull	13	2737	Cass	46° 25' 00"	94° 21' 30"
Rabbit	14	2771	Crow Wing	46° 31' 30"	93° 56' 00"
Cranberry	15	2720	Crow Wing	46° 24' 00"	93° 46' 30"
Darling	16	27B4	Douglas	45° 56' 00"	95° 23' 00"
Carlos	17	27B9	Douglas	45° 56' 00"	95° 23' 00"
Le Homme Dieu	18	27B5	Douglas	45° 56' 30"	95° 21' 30"
Minnewaska	19	2761	Hennepin/ Pope	45° 36' 30"	95° 32' 00"
Nest	20	27B3	Kandiyohi	45° 16' 00"	95° 56' 00"
Green	21	27B2	Kandiyohi	45° 16' 00"	94° 52' 00"
Wagonga	22	27B1	Kandiyohi	45° 04' 00"	94° 56' 30"
Clearwater	23	2716	Wright/ Stearns	45° 20' 00"	94° 07' 00"
Mud (at Maple Lake)	24	2753	Wright	45° 13' 30"	93° 59' 00"
Cokato	25	2719	Wright	45° 07' 00"	94° 09' 30"
Buffalo	26	2713	Wright	45° 08' 30"	93° 54' 30"
Carrigan	27	2714	Wright	45° 03' 30"	93° 57' 30"
Silver	28	2782	McLeod	44° 53' 00"	94° 13' 00"
Minnetonka	29	2760	Hennepin	44° 57' 30"	93° 30' 30"
Forest	30	27A9	Washington	45° 17' 30"	92° 58' 30"
White Bear	31	27B0	Washington	45° 04' 00"	92° 58' 30"
St. Croix	32	27A7	Washington	44° 46' 00"	92° 49' 00"
Spring	33	27A6	Washington	44° 45' 30"	92° 52' 30"
Pepin	34	27A4	Goodhue	44° 23' 00"	92° 02' 00"
Madison	35	2750	Blue Earth	44° 10' 30"	93° 49' 00"
Sakatah	36	2777	Le Sueur	44° 14' 00"	93° 27' 30"
Bear	37	2706	Freeborn	43° 33' 00"	93° 30' 00"
Albert Lea	38	2702	Freeborn	43° 37' 00"	93° 17' 30"
Yellow	39	5576	Burnett	45° 55' 30"	92° 25' 00"
Wapogasset	40	5550	Polk	45° 19' 30"	92° 26' 30"

Table 3. (continued)

Lake Name	Serial Number*	STORET Number	County	Lake Coordinates** N. Latitude	W. Longitude
Long	41	5573	Price	45° 42' 00"	90° 27' 00"
Elk	42	5575	Price	45° 42' 00"	90° 24' 30"
Trout	43	5572	Vilas	46° 04' 00"	89° 40' 00"
Crystal	44	5571	Vilas	46° 00' 00"	89° 36' 39"
Tainter	45	5546	Dunn	44° 56' 00"	91° 53' 30"
Shawano	46	5539	Shawano	44° 47' 30"	88° 34' 00"
Poygan	47	5538	Winnebago	44° 06' 30"	88° 42' 30"
Butte Des Morts	48	5508	Winnebago	44° 02' 00"	88° 34' 00"
Winnebago	49	5554	Winnebago/ Fond Du Lac	44° 12' 00"	88° 27' 30"
Round	50	5566	Waupaca	44° 20' 00"	89° 10' 00"
Green	51	5519	Fond Du Lac	43° 51' 30"	88° 57' 00"
Swan	52	5545	Columbia	45° 32' 30"	89° 23' 30"
Beaver Dam	53	5503	Dodge	43° 27' 30"	88° 50' 30"
Kegonsa	54	5520	Dane	42° 58' 00"	89° 13' 30"
Rock	55	5564	Vilas	43° 04' 30"	88° 55' 00"
Koshkonong	56	5522	Dane/ Jefferson	42° 50' 00"	89° 01' 30"
Lac La Belle	57	5563	Waukesha	43° 07' 00"	88° 31' 00"
Oconomowoc	58	5532	Waukesha	43° 05' 30"	88° 28' 30"
Okauchee	59	5580	Waukesha	43° 06' 30"	88° 28' 30"
Pine	60	5536	Waukesha	43° 07' 00"	88° 23' 30"
Nagawicka	61	5531	Price	44° 04' 00"	88° 24' 30"
Pewaukee	62	5557	Waukesha	43° 05' 00"	88° 16' 00"
Tichigan	63	5559	Racine	42° 46' 00"	88° 13' 00"
Browns	64	5560	Racine	42° 41' 00"	88° 14' 30"
Middle	65	5569	Walworth	42° 46' 30"	88° 34' 00"
Delavan	66	5513	Walworth	42° 37' 00"	88° 37' 30"
Como	67	5562	Walworth	42° 37' 00"	88° 27' 30"
Geneva	68	5561	Walworth	42° 36' 00"	88° 26' 00"
Charlevoix	69	2617	Charlevoix	45° 19' 00"	85° 15' 00"
Higgins	70	2695	Roscommon/ Crawford	44° 26' 00"	84° 40' 30"
Houghton	71	2696	Roscommon	44° 24' 30"	84° 47' 30"
Pere Marquette	72	2698	Mason	43° 56' 30"	86° 27' 00"
White	73	2688	Muskegon/ Newaygo	43° 22' 30"	86° 25' 30"
Muskegon	74	2659	Newaygo/ Muskegon	43° 14' 00"	86° 20' 00"
Fremont	75	2631	Newaygo	43° 26' 30"	85° 58' 30"
Mona	76	2691	Muskegon	43° 10' 00"	86° 17' 30"
Crystal	77	2694	Montcalm	43° 16' 30"	84° 55' 30"

Lake Name	Serial Number*	STORET Number	County	Lake Coordinates** N. Latitude	W. Longitude
Jordan	78	2640	Ionia/ Barry	42° 45' 30"	85° 09' 00"
Thornapple	79	2683	Barry	42° 37' 00"	85° 21' 30"
Strawberry	80	2699	Livingston	42° 26' 30"	83° 51' 00"
Chemung	81	2618	Livingston	42° 35' 30"	83° 51' 30"
Thompson	82	2697	Livingston	42° 37' 30"	83° 55' 30"
Ford	83	2629	Washtenaw	42° 12' 30"	83° 33' 30"
Union	84	2685	Branch/ Calhoun	42° 02' 30"	85° 12' 30"
Long	85	2692	Ottawa	41° 55' 30"	84° 20' 00"
Randall	86	2671	Branch	42° 00' 30"	85° 02' 30"
Schroon	87	3624	Warren/ Essex	43° 50' 00"	73° 47' 00"
Black	88	3602	Jefferson/ St. Lawrence	44° 36' 00"	75° 28' 30"
Cassadaga	89	3607	Chautauqua	42° 20' 30"	79° 19' 30"
Chautauqua	90	3610	Chautauqua	42° 06' 00"	79° 15' 00"
Conesus	91	3639	Livingston	42° 50' 00"	77° 42' 30"
Canandaigua	92	3604	Ontario	42° 53' 30"	77° 15' 30"
Keuka	93	3617	Steuben/ Yates	42° 39' 30"	77° 09' 00"
Seneca	94	3635	Yates/ Senga/ Schuler	42° 56' 30"	76° 52' 00"
Cayuga	95	3608	Cayuga/ Tompkins/ Seneca	42° 58' 00"	76° 44' 30"
Owasco	96	3627	Cayuga	42° 47' 00"	76° 29' 00"
Cross	97	3611	Cayuga	43° 06' 00"	76° 25' 30"
Otter	98	3625	Cayuga	43° 09' 30"	76° 33' 00"
Round	99	3630	Saratoga	42° 55' 00"	73° 45' 00"
Saratoga	100	3633	Saratoga	43° 04' 30"	73° 41' 00"
Winona	101	27A1	Douglas	45° 33' 30"	95° 23' 00"
Trace	102	2792	Todd	45° 50' 30"	94° 45' 30"
Calhoun	103	27B6	Hennepin	44° 56' 00"	93° 18' 30"
Big Stone	104	2709	Big Stone	45° 18' 00"	94° 27' 00"
Zumbro	105	27A5	Olmsted	44° 14' 00"	92° 29' 00"
Oneida	106	3622	Oswego/ Oneida	43° 11' 00"	75° 45' 30"
Canadarago	107	3603	Otsego	42° 50' 00"	75° 00' 00"
Mendota	108		Dane	43° 11' 00"	89° 13' 00"
Monona	109		Dane	43° 08' 30"	89° 16' 00"
Waubesa	110		Dane	43° 01' 30"	89° 29' 30"
Cottonwood	111	27C3	Lyon	44° 36' 00"	95° 40' 00"
Maple	241		Wright	45° 13' 30"	93° 59' 30"

*The lakes with the serial numbers 1-31, 33, 35-38, 101-103, 111, and 241 are wholly contained in Minnesota. Lakes 32, 34, and 105, located on the borders of the state, are referred to as "Minnesota" lakes for convenience. Lakes 39-68 and 108-110 are Wisconsin lakes; 69-86 are in Michigan. New York lakes have been assigned the numbers 87-100 and 106-107.

**In the case of seepage lakes, the geographic coordinates represent a point on the lake surface.

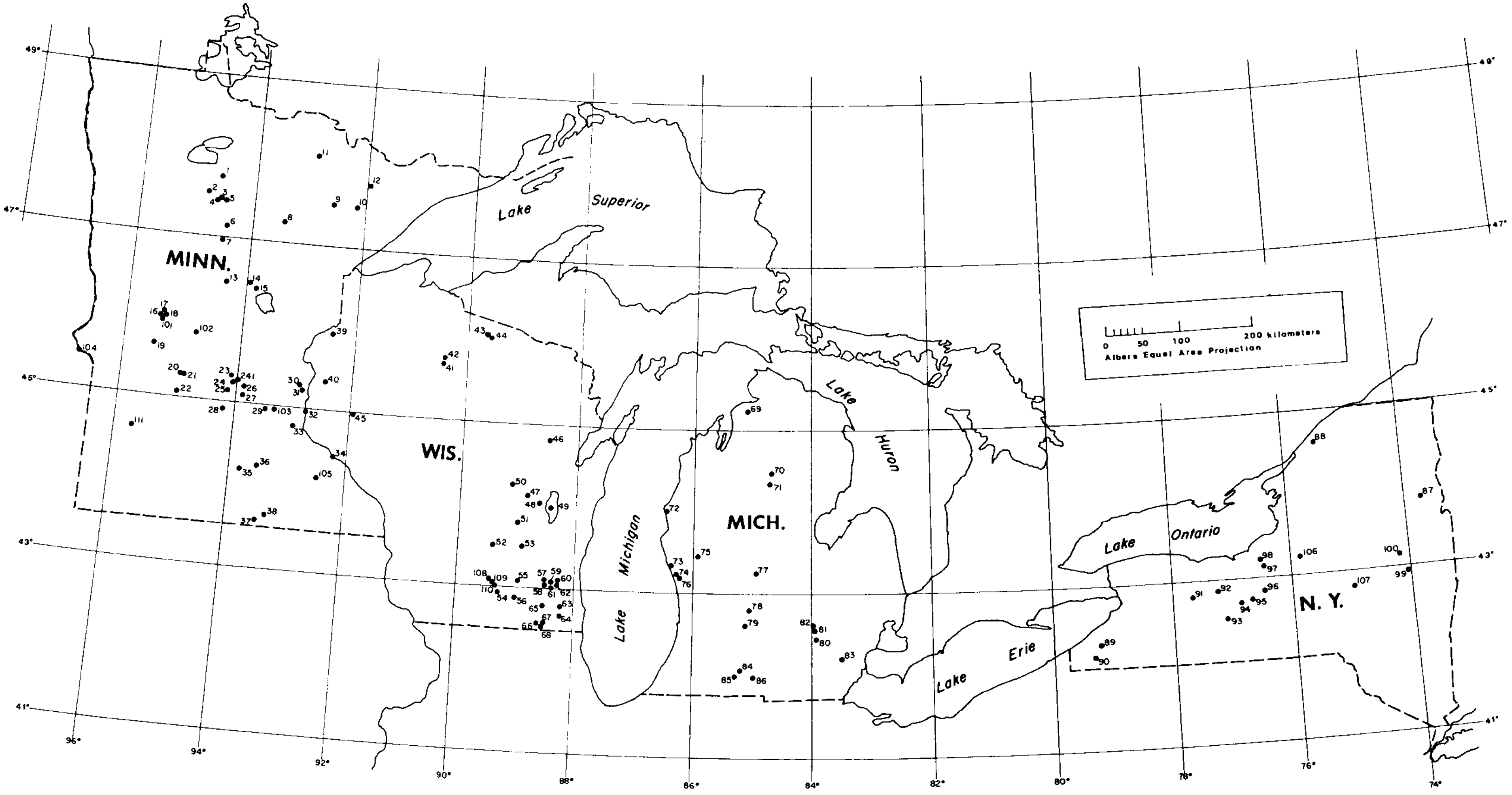

Figure 12. Location of the study lakes. The lakes with serial numbers 1-100 were used in the cluster analyses and principal component ordination analyses.

Lake Site Selection

Lake sampling sites were selected on the basis of lake morphometry, and potential major sources of nutrient input, as well as the on-site judgment of the sampling team's limnologist. The number of sampling sites varied for different lakes, ranging from one to nine (Lake Winnebago).

Lake Sampling Methods

After landing on the lake surface in the general area of a sampling site, the helicopter was taxied to locate the deepest nearby water. There a small reference buoy was deployed to aid the pilot in maintaining his station.

Observations were recorded relating to general lake appearance, phytoplankton bloom conditions, aquatic macrophytes, and shoreline developments (*e.g.*, residential units) along with magnetic compass bearings to prominent landmarks. A photograph was taken of the site, including the reference buoy and the prominent landmarks to assure return to the same site on subsequent sampling rounds.

Secchi disc transparency readings were made and water samples were bucket-dipped from the lake surface. The sensor recorders were monitored as the sensor-pump package was slowly lowered through the water column, permitting the limnologist to select depths or levels to be sampled as the package was winched to the surface.

After touching bottom, the package was raised to a point approximately 1.2 meters off the bottom. Each sensor's digital output was recorded and the submersible pump activated for the collection of water samples. The package was then raised to the next sampling level and the process was repeated. The procedure was continued until all of the selected levels were sampled.

Water samples were collected from each selected depth for nutrient, alkalinity, pH, conductivity, and dissolved oxygen determinations. The samples collected for alkalinity, nitrate-nitrogen, nitrite-nitrogen, ammonia-nitrogen, and dissolved phosphorus were preserved on-site by the addition of 40 mg/l of mercuric chloride. The samples were filtered through a 0.45 micrometer membrane filter (prerinsed with de-ionized water) and the filtrate was shipped to EPA's National Environmental Research Center (NERC), Las Vegas, Nevada. The samples for total phosphorus determination were preserved with a 40 mg/l mercuric chloride solution, but were not filtered.

Integrated samples for algae enumeration and identification purposes and chlorophyll a determination were collected by raising or lowering the package while continuing to operate the pump. The package movement was timed to provide a water sample representative of the water column from the surface to approximately 4.6 meters, or to a point just above the bottom in water less than 4.6 meters in depth. The algae samples were fixed with Lugol's solution and forwarded to NERC-Las Vegas.

A 18.9 liter water sample, composited from water collected at each sampling depth and every sampling site (*i.e.*, station) during the fall sampling period, was airmailed to the Pacific Northwest Environmental Research Laboratory (PNERL), Corvallis, Oregon, for the determination of its productivity potential under a set of standard conditions. The unpreserved sample, unrefrigerated while in transit, was stored in a freezer until the analysis could commence.

Analytical Methods

The water samples for dissolved oxygen, pH, and chlorophyll *a* determinations were analyzed in a mobile field laboratory at the end of each day of sampling.

The dissolved oxygen water samples, fixed and acidified aboard the helicopters, were titrated with phenylarsine oxide in conjunction with a starch indicator.

A Beckman Field Laboratory pH meter was used to determine the hydrogen-ion concentration of the water samples. The samples were refrigerated until they were analyzed.

The samples for chlorophyll *a* determination were refrigerated in the dark and were analyzed using the fluorometric procedure described by Yentch (1963).

Algal identification and enumeration included a total cell count (Sedgewick-Rafter) and a differential count and identification of the five most abundant genera of algae.

Ammonia-nitrogen, nitrate-nitrogen, nitrite-nitrogen, dissolved phosphorus, total phosphorus, and alkalinity were measured at NERC-Las Vegas with a Technicon Autoanalyzer II according to the general methodology described in Working Paper Number 1 (NES, 1974).

The procedures used in the algal assay test, a methodology for the determination of a water sample's productivity potential and limiting nutrients, were those outlined in EPA's National Eutrophication Research Program's publication entitled "Algal Assay Procedure Bottle Test" (Anonymous, 1971). The analyses were conducted at PNERL, NERC-Corvallis.

II. LAKE CLASSIFICATION

There is far from universal agreement as to what constitutes a lake. Veatch and Humphrys (1966) suggested that to give the word "lake" a precise, limited meaning would probably be an exercise in futility because the word has been in use for a long time and been given a diversity of applications. The word is used as a synonym for pond, reservoir, and sea. It has been applied to bodies of fresh water and saline water; to standing water and widenings in rivers; to bodies of water measuring less than a hectare and to those gauged in hundreds of thousands of hectares; to naturally occurring water bodies and man-made reservoirs; to water-filled or partially filled basins; and to basins void of water. "Lake" is generally more prestigious than other common names (e.g., pond) and is preferred by promoters of water-based tourist and recreational businesses and commercial developers of shoreline property (Veatch and Humphrys, 1966). Nevertheless, numerous attempts have been made to define and delimit the members of lentic series (i.e., lake, pond, marsh, and their intergrades).

Forel (1892) defined a lake as a body of standing water occupying a basin and lacking continuity with the sea. He defined a pond as a lake of slight depth (Welch, 1952).

Welch (1952) defined a lake as a "... body of standing water completely isolated from the sea and having an area of open, relatively deep water sufficiently large to produce somewhere on its periphery a barren, wave-swept shore." He employed the term "pond" "... for that class of very small, very shallow bodies of standing water in which quiet water and extensive occupancy by higher aquatic plants are common characteristics" and suggested that all larger bodies of standing water be referred to as lakes.

Zumberge (1952) defined a lake as an inland basin filled with water. Harding (1942) described lakes as "bodies of water filling depressions in the earth's surface."

Lakes as Natural Resources

The exact number of lakes in the United States is unknown. Welch (1952) estimated that there are at least 40,000 lakes in North America with a combined surface area of no less than 225,000 square miles (583,000 square kilometers). Hasler and Ingersoll (1968) reported a figure of 100,000 lakes in America (*i. e.*, United States).

Estimates of lake numbers from different sources or compiled at different times may exhibit wide disparity. The difference in estimates can be related to the lack of agreement as to what constitutes a lake, incompleteness of the inventories, and interpretational differences of maps and aerial photographs (Veatch and Humphrys, 1966).

For example, Minnesota, "The Land of 10,000 Lakes," completed an inventory of its surface waters in 1968. The comprehensive final report initially defines a lake as "... an enclosed basin, filled or partially filled with water..." and then extends the term to include "... all natural enclosed depressions, 10 acres or more in area, which have substantial banks capable of containing water and which are discernable on aerial photographs." Minnesota has, according to the report, 15,291 lake basins of which 3,257 are partly or completely dry (Minnesota Department of Conservation, 1968).

Fresh-water lakes are really insignificant when compared to the earth's total surface as they account for about 0.17 percent of it (Table 4). Nace (1960) estimates that only about 0.009 percent of the earth's total water supply is in the form of fresh-water lakes.

In some regions lakes give the impression of dominating the landscape. Yet, in Beltrami County (Minnesota), a place renowned for its abundance of lakes, only about 18 percent of the area is under water (Minnesota Department of Conservation, 1968). Vilas County (Wisconsin), another governmental unit well endowed with lakes, has about 15 percent of its area covered by lakes (Deevey, 1942).

However, the geographic importance of any element of the landscape is not measured merely on the basis of its areal extent. The intrinsic properties of lakes make them a natural resource whose importance is greater than is suggested by area alone.

Lakes are used as sources of municipal water, irrigation water, and cooling water for thermal-electric plants. They serve as transportation routes and as focal points for many types of recreational activity. Many lakes, particularly those in a pristine condition, are valued for their aesthetic qualities. They also provide convenient locations for dumping the organic and inorganic wastes of society.

An increasing number of lakes are viewed as merely obstacles in the "way of progress" and are being subjected to drainage or serving as land-fill sites to provide additional farmland or building sites.

Table 4. Distribution of the world's estimated water supply. Adapted from Nace (1960).

Location	Surface Area ($km^2 \times 10^3$)	Volume Of Water ($km^3 \times 10^3$)	Percentage Of Total Water
World	510,228		
Land area	148,924		
Surface water on the continents			
Polar icecaps and glaciers	17,871	30,428	2.24
Fresh-water lakes	855	125	0.009
Saline lakes and inland seas	699	104	0.008
Average in stream channels		1	0.0001
Total surface water	19,425	30,678	2.26
Subsurface water on the continents			
Root zone of the soil	129,499	25	0.0018
Ground water above depth 805m		4,168	0.306
Ground water, depth of 805m to 4,024m		4,168	0.306
Total subsurface water	129,499	8,336	0.61
World's oceans	361,303	1,321,314	97.1
Total water on land		39,014	2.87
Atmospheric moisture		12	0.001
Total, world supply of water		1,358,827	

As the world's human population increases, it is likely that attempts to rid the landscape of lakes will increase in scope.

Although drainage and land-fill schemes threaten some lakes, a problem of much greater magnitude is that of cultural (anthropro-genic) eutrophication.

Lake Succession and Eutrophication

Lake Succession

Lakes, although giving the impression of permanence when measured on the scale of the human life span, are transitory features of the earth's surface. All lakes, regardless of their origin, pass through the process of ecological succession which ultimately results in a terrestrial environment.

The ephemeral nature of lakes is a consequence of two fundamental processes, the downcutting of the outlet and, more important, the deposition of allochthonous and autochthonous materials in the basin.

Most lakes commence the successional process as bodies possessing relatively low concentrations of nutrients and, generally, low levels of productivity.[6] The importation and deposition of materials (e.g., sediment) from the shoreline and the surrounding watershed

[6] Edmondson (1974) suggested that the idea, that all lakes are born oligotrophic and gradually become eutrophic as they age, is an old misconception.

gradually decrease the lake's depth. The addition of allochthonous materials normally enriches the water and thereby stimulates the production of organic materials. The autochthonous materials increase the sedimentation rate thus accelerating succession. Marked floral and faunal changes occur. Algal blooms become more common along with submergent and eventually emergent aquatic macrophytes. Desirable game fish may be replaced by less desirable species, the so-called "rough fish". A lake will eventually become a marsh or swamp which, in turn, terminates as dry land.

Lindemann (1942) stressed the productivity aspects in relation to lake succession. Figure 13 represents the probable successional productivity relationships for a hypothetical hydrosere developing from a moderately deep lake located in a fertile humid continental region. Productivity is initially low, a consequence of low nutrient levels, but increases rapidly as nutrients become more available. The length of time required for completion of the successional process is a function of several factors including lake basin morphology, climate, and the rate of influx and nutrient value of allochthonous materials. It is readily apparent that allochthonous nutrients can drastically increase lake productivity and thereby shorten the life span of a lake.

The Concept of Eutrophication

The word eutrophication, until recently foreign to the vocabulary of the general public, has appeared in the popular and scientific

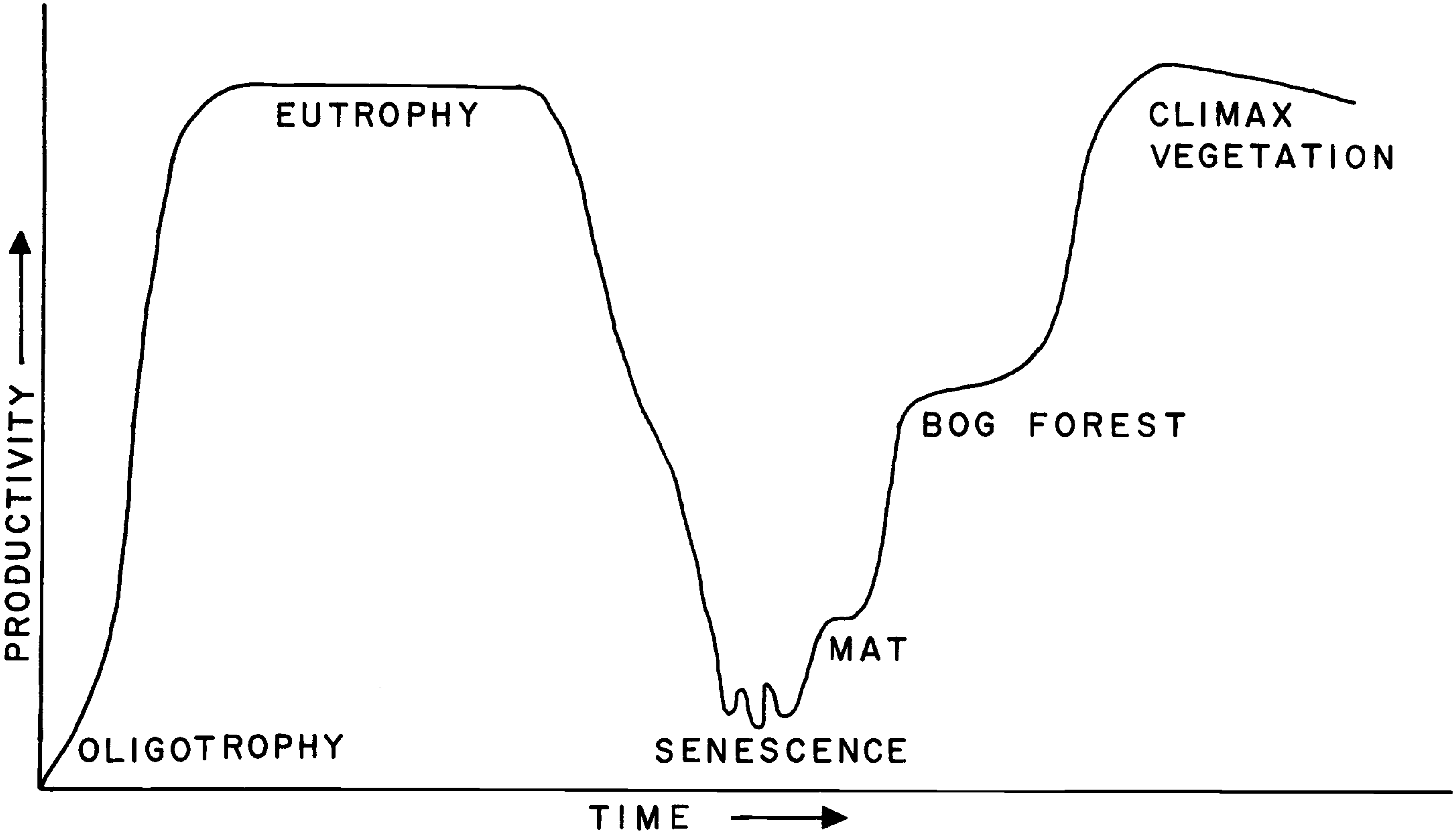

Figure 13. Hypothetical productivity growth-curve of a hydrosere.
Adapted from Lindemann (1942).

literature at a rapidly increasing rate over the past decade. The term is often used to denote the process whereby a pristine water body (*e.g.*, lake) is transformed into one characterized by dense algal scums, obnoxious odors, and thick beds of aquatic macrophytes. However, the word is applied differently, according to the respective interests of its users.

Weber (1909) used the German adjectival form of eutrophication (nährstoffreichere - eutrophe) to describe the high concentration of elements requisite for initiating the floral sequence in German peat bogs (Hutchinson, 1973). The leaching of nutrients from the developing bog resulted in a condition of "mittelreiche" (mesotrophe) and eventually "nährstoffearme" (oligotrophe).

Naumann (1919) applied the words oligotrophic (under-fed), mesotrophic, and eutrophic (well-fed) to describe the nutrient levels (calcium, phosphorus, combined nitrogen) of water contained in springs, streams, lakes and bogs (Hutchinson, 1973). Naumann (1931) defined eutrophication as the increase of nutritive substances, especially phosphorus and nitrogen, in a lake. Hasler (1947) broadly interpreted eutrophication as the "Enrichment of water, be it intentional or unintentional..." Fruh, *et al.* (1966) defined the word as the "...enhancement of nutrients in natural water..." Edmondson (1974) suggested that many limnologists seem to use the term to describe "an increase in the rate of nutrient input..."

Hasler and Ingersoll (1968) suggested that eutrophication is the "process of enrichment and aging[7] undergone by bodies of fresh water". Vollenweider (1968) summarized the eutrophication of waters as meaning "...their enrichment in nutrients and the ensuing progressive deterioration[7] of their quality, especially lakes, due to the luxuriant growth of plants with its repercussions on the overall metabolism of the waters affected..."

A search of the literature on eutrophication indicates that the meaning of the term, originally limited to the concept of changing nutrient levels, has been gradually expanded to include the consequences of nutrient enrichment.[8]

Eutrophication occurs both naturally and as a result of man's activities (cultural or anthropogenic eutrophication). Many of man's practices relating to the disposition of municipal sewage and industrial wastes and land use impose relatively large nutrient loadings on many lakes and rivers. In many cases, the enrichment results in algal blooms and other symptoms of eutrophication. The consequences of man-induced eutrophication often make the water body less attractive to potential users. More importantly, at least when a long-range viewpoint is adopted, eutrophication accelerates lake succession, thus shortening the time period before a lake loses its identity.

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[7] Emphasis added.

[8] The historical aspects and semantical problems associated with the word "eutrophication" and its companion words (oligotrophic, mesotrophic, eutrophic) are found in Weber (1909), Naumann (1919, 1931), Thienemann (1918), Rodhe (1969), Hutchinson (1969, 1973), Beeton and Edmondson (1972), and Edmondson (1974).

Trophic State and Trophic Indicators

Limnologists and other scientists concerned with lakes have used the term "trophic state" (*i.e.*, degree of eutrophy) to describe two different lake characteristics, nutrient status and productivity. Thus, trophic state is a hybrid concept as suggested by Margalef (1958).

Several different physical, biological, and chemical attributes are required to adequately describe a lake's trophic state, making the concept multi-dimensional (Brezonik and Shannon, 1971) and precluding its determination through direct measurement. However, it is possible to quantify trophic state through the use of trophic state indicators (indices) in conjunction with appropriate data reduction techniques.

There are numerous indicators of trophic state, each with its merits and shortcomings. Reviews have been written on the subject by Fruh, *et al.* (1966), Steward and Rohlich (1967), Vollenweider (1968), and Hooper (1969). A list of some common indicators or indices are found in Table 5.

A diversity of opinion exists regarding the number and kinds of indicators which should be considered in the classification of lakes. Use of a single indicator has the virtue of simplicity but may produce misleading rankings or groupings because lakes are normally too complex to be adequately gauged on such a simplified basis. On the other hand, the use of a large number of indicators may result in a problem of character redundancy.

Table 5. Trophic indicators and their response to increased eutrophication. Adapted from Brezonik (1969).

Physical	Chemical	Biological
Transparency (d) (Secchi disc reading)	Nutrient concentrations (i) (*e.g.*, at spring maximum)	Algal bloom frequency (i)
Morphometry (d) (mean depth)	Chlorophyll *a* (i)	Algal species diversity (d)
	Conductivity (i)	Littoral vegetation (i)
	Dissolved solids (i)	Zooplankton (i)
	Hypolimnetic oxygen deficit (i)	Fish (i)
	Epilimnetic oxygen supersaturation (i)	Bottom fauna (i)
	Sediment type	Bottom fauna diversity (d)
		Primary production (i)

An (i) after an indicator signifies the value increases with eutrophication; a (d) signifies the value decreases with eutrophication. The biological indicators all have associated qualitative changes (*i.e.*, species changes occur as well as quantitative (biomass) changes as eutrophication proceeds).

Multivariate Classification of Lakes[9]

A multiplicity of classificatory schemes has been devised to group and rank lakes. Examples of some approaches to lake typology are found in: Lueschow, *et al.* (1970), Rawson (1956, 1960), Margalef (1958), Hansen (1962), Jarnefelt (1958), Larken and Northcoate (1958), Moyle (1945, 1946), Pennak (1958), Round (1958), Whipple (1898), Winner (1972), Zafar (1959), Davis (1882, 1887), Beeton (1965), Donaldson (1969), and Gerd (1957). Hutchinson (1957, 1967) has reviewed many of the attempts to arrange lakes into orderly systems.

Lacustrine trophic state is a multi-dimensional concept and is, by its very nature, amenable to analysis by multivariate stastical techniques (*e.g.*, cluster analysis, principal component analysis). Multivariate techniques minimize the personal bias often present when data are examined for groups, and rankings are developed. They are of particular value in situations where large numbers of objects or parameters are to be classified.

Numerical taxonomists and quantitative ecologists have been acutely aware of the benefits which can be derived from multivariate techniques and have been very active in promoting their use. Yet, a search of the literature has yielded few publications in which the techniques have been applied to lakes (*e.g.*, Shannon, 1970; Brezonik and

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[9] The term classification is often used in the restricted sense of placing entities into distinct groups, thereby excluding arrangements showing no distinct divisions (*e.g.*, ordination). The term is used here in the broader context suggested by Sneath and Sokal (1973).

Shannon, 1971; Shannon and Brezonik, 1972a, 1972b; Sheldon, 1972). Shannon and Brezonik have devoted their efforts toward the classification and evaluation of 55 lakes in north central Florida. Sheldon reviewed the concept and functions of classification, introduced multivariate techniques of potential value in handling and synthesizing lake information, and applied the techniques to several lake populations.

Lake scientists have been slow to apply multivariate techniques to the problem of lake classification. This is probably because of a lack of familiarity with the techniques, the unavailability of large digital computers and/or the necessary software, and a shortage of comparable data from large numbers of lakes - hence, a lack of need?

The balance of this chapter is devoted to the application of two multivariate techniques (cluster analysis, principal component analysis) to a group of 100 lakes (Table 3) using selected elements of ground truth collected by NES. The resulting multivariate trophic state index will be used in Chapter IV to assist in the assessment of ERTS-1 MSS as a tool in estimating the trophic status of lakes.

Cluster Analysis

Cluster analysis is a collective term encompassing a broad spectrum of techniques for delineating natural groups ("clusters") of objects or attributes in hyperspace. A multitude of contributions have been published on the subject (*e. g.*, Ward, 1963; Lance and Williams, 1967, 1968; Gower, 1967; Padron, 1969). Three comprehensive publications, written by Sokal and Sneath (1963), Anderberg (1973), and Sneath and Sokal (1973) serve as excellent sources of information. An exposé of the various clustering techniques available to researchers is beyond the scope of this investigation, and the reader is referred to the above sources.

Clustering on objects is termed Q-technique as opposed to R-technique which leads to classifications of attributes or characters (Sneath and Sokal, 1973). Williams and Dale (1965) suggested that, when the relationships are represented in hyperspace, the kind of space that is operated on should be called A-space and I-space, not R-space or Q-space. A-space (attribute space) has p dimensions, one for each attribute (character), in which there are n points, each representing an object. I-space (individual space) has n dimensions, one for each object, in which there are p points, each representing an attribute. This study will utilize the Q-technique operating in A-space.

Objects and Attributes

One hundred lakes, sampled by NES in 1972, were selected for analysis. Each lake was assigned a serial number (1-100) which is unique to this thesis.[10] A careful examination of the physical,

[10] Each of the lakes sampled by NES carries an identification code called the STORET Number.

chemical, and biological parameters measured by NES resulted in the selection of six indicators for incorporation into both the cluster analysis and principal component analysis (PCA) ordination.[11] The indicators (i. e., lake attributes) are: conductivity (COND, μmhos cm^{-1}), chlorophyll a (CHLA, $\mu g\ l^{-1}$), total phosphorus (TPHOS, mg l^{-1}), total organic nitrogen (TON, mg l^{-1}), algal assay yield (AAY, dry-wt in mg) and Secchi disc transparency (SECCHI, m). The inverse of Secchi disc transparency (ISEC, m^{-1}) was employed so that all of the indicator values would increase as trophic status increases.

The six indicators were selected because they are quantitative, considered to be important measures of trophic state, and satisfy Hooper's (1969) criteria for trophic indices. Annual mean values for COND, CHLA, TPHOS, and ISEC were used in the analyses; AAY and TON measurements were limited to the fall overturn sample, precluding the use of an annual mean. The descriptive statistics of the six indicators are found in Table 6. A serious lack of normality in the data for each indicator necessitated a transformation prior to clustering. Natural logarithms (LN) were found to be adequate for this transformation.

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[11] A seventh indicator, an abbreviated form of the Pearsall cation ratio inverse was originally included, but was eliminated because it did not appear to contribute significantly to the classification scheme.

Table 6. Descriptive statistics of 100 lakes. The values below represent the statistics for the NES lakes which are incorporated into the cluster and principal component analyses.

Statistic	Trophic State Indicators					
	CHLA ($\mu g\ l^{-1}$)	ISEC (m^{-1})	COND ($\mu mhos\ cm^{-1}$)	TPHOS ($mg\ l^{-1}$)	TON ($mg\ l^{-1}$)	AAY (mg dry-wt)
Mean	22.3	0.85	352	0.136	0.80	7.3
Median	11.8	0.54	353	0.048	0.67	2.7
Maximum value	381.2	5.37	808	1.893	3.35	61.3
Minimum value	1.1	0.13	50	0.005	0.08	0.1
Range	380.1	5.24	758	1.888	3.27	61.2
Standard deviation	42.7	0.88	155.9	0.265	0.56	10.6
Skewness	6.4	2.86	0.3	4.264	2.20	2.4

Cluster Method

A complete linkage algorithm (McKeon, 1967) was used to examine the lakes for natural groupings. The method is also known as the furthest neighbor hierarchical strategy (Lance and Williams, 1967) and the maximum method (Johnson, 1967). The algorithm is characterized as agglomerative, nonoverlapping, and hierarchical, the latter property permitting the output to be modelled in the form of dendrograms. The program was run on an International Business Machines (IBM) Model 370-155E digital computer at Optimum Systems Incorporated (OSI), Bethesda, Maryland.

The LN-transformed trophic indicator data were entered as an N x p matrix where N is the number of observations or objects (*i.e.*, lakes) and p is the number of dimensions or attributes (*i.e.*, indicators). The data points in each column (indicator) of the data matrix were standardized by subtracting the column mean from each point and then dividing by the column's standard deviation. This was accomplished using the McKeon program option number three. Standardization was indicated because the data were measured in different units.

Euclidian distance, an extension of the Pythagorean theorem to points in hyperspace, was selected as the similarity coefficient, largely because of its intuitive appeal. The formula for the Euclidian distance, Δ_{jk}, between two objects (*e.g.*, lake "j" and lake "k") is

$$\Delta_{jk} = \left[\sum_{i=1}^{p} (X_{ij} - X_{jk})^2 \right]^{1/2}$$

The distances between all possible (N(N-1)/2) pairs of objects are computed and stored in an N x N symmetrical matrix (S-matrix). The McKeon program uses the squared Euclidian distance, Δ^2, but the same clusters could be obtained by using Euclidian distance or any monotonic function of that distance.

The clustering procedure carries out successive iterations on the S-matrix which has the maximum squared distance within cluster in the diagonal and the maximum squared distance between clusters in the off-diagonals. Initially the S-matrix is N x N with zeros in the diagonals (Figure 14). At each iteration, those two clusters are combined, which, taken together, form the most compact cluster. The measure of compactness is the maximum distance between any two points (i.e., lakes) within the cluster. The next pair to be combined are identified by finding the smallest squared distance between points in the off-diagonals.

For example, if N = 100, the program will start out with 100 clusters and successively meld the clusters, two per iteration, eventually terminating with one cluster containing 100 objects.

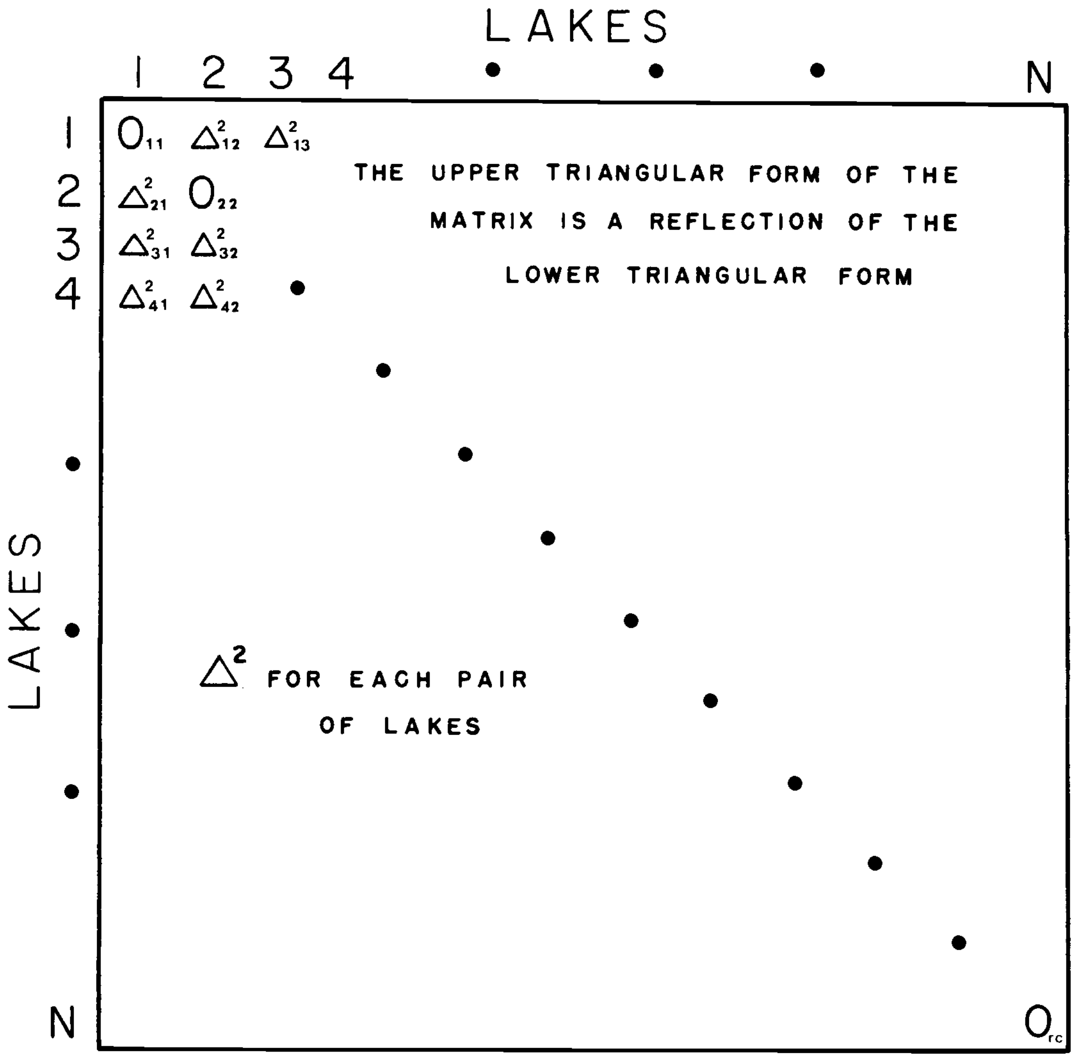

Figure 14. N x N S-matrix. Initially the matrix has only zeros on the diagonal. The off-diagonal elements represent the squared Euclidian distances between each pair of lakes. There are 4,950 distances possible for 100 lakes. Appendix F contains the matrix of Δ^2 used to examine the NES-sampled lakes for clusters.

Results and Discussion

The results of the cluster analysis are depicted as a dendrogram (Figure 15). The abscissa is scaled in Euclidian distance with the points of junction between stems implying that the maximum within-cluster distance is the value on the abscissa. Generally, the ordinate of a dendrogram has no special significance. The order of the clusters can be changed by rotating the stems, thereby producing a multitude of apparently different dendrograms. Interpretational problems arise when it is necessary to compare dendrograms developed from different data or algorithms.

Some investigators attempt to "standardize" the ordinate-induced appearance of the dendrogram by rotating the stems to keep the code or serial numbers as ordered as possible, especially if the numbers present linear arrangements of generally established taxonomic groups (Sneath and Sokal, 1973). The author has attempted to order the axis by rotating the stems to reflect the PCA ordination results found in the next section of this chapter. Consequently, the trophic status of the 100 lakes in the dendrogram generally increases along the ordinate in the "downward" direction.

It has become an accepted practice in lake studies to use the terms oligotrophic, mesotrophic, and eutrophic in reference to the trophic status of lakes. The terms, although well established in the literature, are used freely and it is difficult to quantitatively determine what is meant by them (Beeton, 1965). It may be argued that

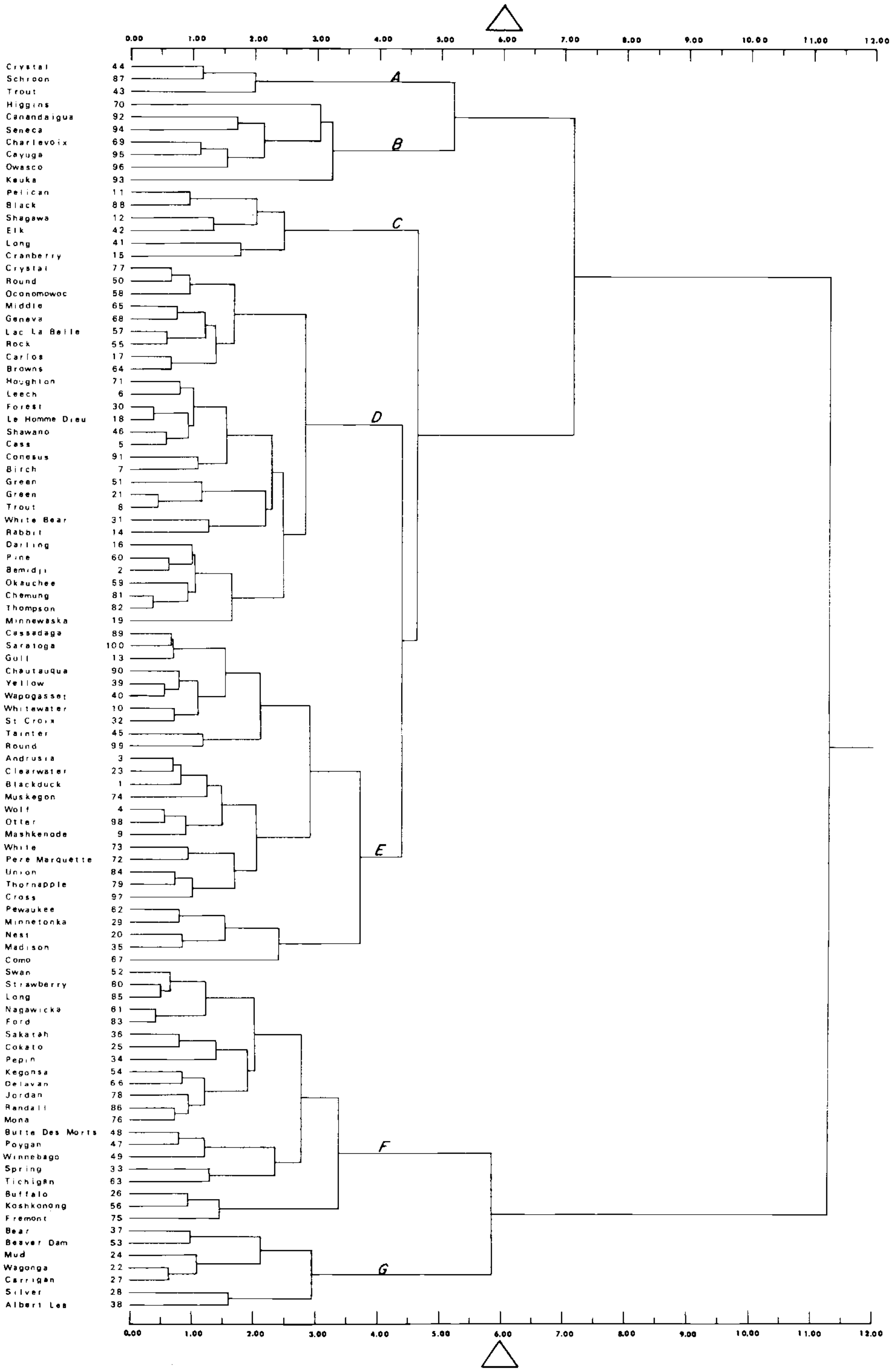

Figure 15. Dendrogram of 100 lakes sampled by the National Eutrophication Survey during 1972. The dendrogram is based on a complete linkage algorithm using generalized Euclidian distance as the measure of similarity.

the terms serve to stereotype lakes and unduly restrict, to three categories, what may be members of a trophic continuum. However, this terminology is likely to continue in vogue because, at a very minimum, it gives some indication of trophic state.

The question arises regarding the number of "good" natural clusters which have been depicted by the clustering algorithm. McKeon (1967) asserted that a sudden increase in the maximum within cluster distance suggests that the previous stage may be a good stopping point. The first "sudden" increase occurs between Δ 3.70 and Δ 4.35 indicating that there may be seven relatively coherent clusters (A, B, C, D, E, F, G). There is some difficulty involved in reconciling the seven clusters with the three classic states.

The NES, cognizant of the advantages and limitations of naming lakes according to a three-class trophic scheme, has applied the trophic names to these lakes using data collected in 1972 and information acquired from various sources, including reports and knowledgeable individuals. Using the NES assessments as a guide, Clusters A and B may be characterized as containing a mixture of oligotrophic and mesotrophic lakes. Cluster C consists of eutrophic lakes; Cluster D is comprised at both mesotrophic and eutrophic lakes. Eutrophic lakes make up Clusters E and F. Cluster G consists of lakes which are very eutrophic (hypereutrophic).

The results of the cluster analysis appear distressing in light of the three-class concept of trophic state. A more careful selection of candidate lakes could have resulted in the "discovery" of three groups matching the classic trophic states.

Nevertheless, the clustering approach is of value in showing relationships between lakes. It gives the investigator another way of perceiving his study lakes, and it is hoped will enable him to see more clearly the relationships between large numbers of lakes.

Principal Components Ordination

Hierarchical methods are a rather heavy-handed approach to the problem of reducing the dimensionality of multidimensional systems (Sheldon, 1972). They have the inherent capability to yield some form of clusters regardless of the structure of the data constellation, even if the entities to be analyzed are randomly distributed (Sneath and Sokal, 1973). Another approach that merits consideration is ordination.

Ordination is the placement of N entities in A-space varying in dimensionality from 1 to p or N-1, whichever is less. Principal components analysis, one ordination technique, will be used to examine the lakes in A-space for natural clusters and to derive a multivariate trophic state index.

Principal components analysis may be used to reduce the dimensionality of a multivariate system by representing the original attributes as functions of a smaller number of uncorrelated variates which are linear functions of the attributes. The main object is to summarize most of the variance in the system with a lesser number of "artificial" variates (i.e., principal components).

The computation of principal components can be undertaken using either a covariance matrix (S) or a p x p matrix of Pearson product-moment correlation coefficients (R). Use of the R-matrix is indicated when the variates are measured in different units (*e.g.*, grams and meters).

Computation of the R-matrix principal components involves the extraction of its eigenvalues (characteristic or latent roots) and eigenvectors (characteristic or latent vectors). The eigenvalues are a set of r nonzero, positive scalar quantities. The sum of the R-matrix eigenvalues is the matrix trace and is equal to the number of dimensions in the original system (*i.e.*, the number of variates, p). The rank of the matrix is r and is equal to p.

Normalized eigenvectors give the A-space coordinates of an orthogonal set of axes known as the principal axes. The normalized eigenvectors are commonly designated as principal components.

The first principal component of the observations of the p-variates $X_1, \ldots, X_p$ is the linear compound

$$Y_1 = a_{11} X_1 + \ldots + a_{p1} X_p$$

whose coefficients (a_{i1}) are the elements of the eigenvector associated with the largest eigenvalue of the R-matrix (Morrison, 1967). The variance of the first principal component is associated with the eigenvalue. The jth principal component of the p-variate system is the linear compound

$$Y_j = a_{1j} X_1 + \ldots + a_{pj} X_p$$

whose coefficients are the elements of the eigenvector associated with the jth largest eigenvalue extracted from the R-matrix. The jth eigenvalue is a measure of the variance of the jth principal component.

The proportion of the total sample variance in the cloud of dimensionless standard scores attributable to any component is found by dividing its eigenvalue by p. The first principal component has the innate property of explaining the greatest proportion of the sample variance with each successive component explaining progressively smaller amounts of the total sample variance. Frequently, a consequence of the decreasing order of variance, $k < r$ dimensions will adequately summarize the variability of the original variates X_1, ..., X_p. The first three components generally account for most of the variation permitting the ordination of the subjects in 1-D, 2-D, and 3-D space.

All of the dispersion in the data can be accounted for by using r dimensions, but this negates the analysis objective, the reduction of dimensionality or as Seal (1964) stated the "... parsimonious summarization of a mass of observations."

The principal components of N p-variate observations may be defined geometrically (Morrison, 1967) as "...the new variates specified by the axes of a rigid rotation of the original response coordinate system into an orientation corresponding to the directions of maximum variance in the sample scatter configuration." The normalized eigenvectors give the directions of the new orthogonal axes and the eigenvalues determine the lengths (i.e., variance) of their respective axes. The coordinate system is expressed in standard units (zero means, unit variances) when the components are extracted from the R-matrix. Figure 16 is a hypothetical bivariate example of the geometric meaning of principal components.

Detailed descriptions of the theoretical and computational aspects of principal components are found in Hotelling (1933a, 1933b, 1936), Anderson (1957), and Morrison 1967).

Methodology

The principal components analysis was accomplished using the same 100 lakes and LN-transformed trophic state indicator data used in the hierarchical cluster analysis (LNCHLA, LNISEC, LNCOND, LNTPHOS, LNTON, LNAAY). The data matrix was further standardized (zero mean, unit variance) by attributes using the relationship

$$z_{ij} = \frac{X_{ij} - \overline{X}_i}{s_i}$$

where z_{ij} is the standardized values for attribute i of observation (i.e., lake) j. X_{ij} is the LN-transformed value of observation j; and $\overline{X}_i$ and s_i are the mean and standard deviation of attribute j, respectively.

The eigenvectors and eigenvalues were extracted from a p x p correlation matrix (Table 7). All of the computational aspects were executed on a Control Data Corporation digital computer (CDC 3300)

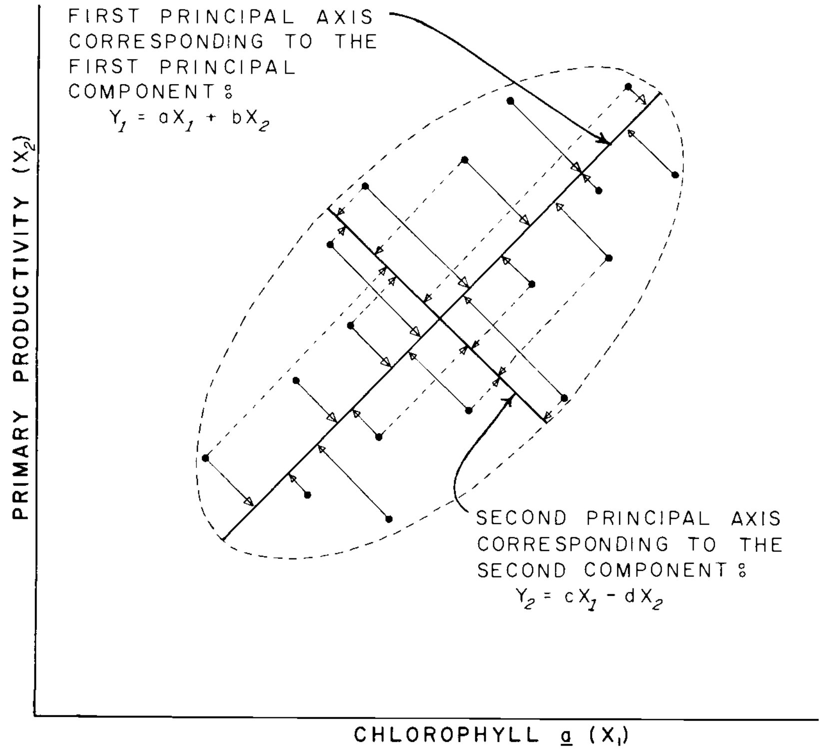

Figure 16. Geometrical interpretation of the principal components for a hypothetical bivariate system. Principal components may be interpreted geometrically as the variates corresponding to the orthogonal principal axes of observation scatter in A-space. The elements of the first normalized eigenvector (i.e., coefficients of the first principal component) define the axis which passes through the direction of maximum variance in the scatter of observations. The associated eigenvalue corresponds to the length of the first principal axis and estimates the dispersion along it. The second principal component corresponds to the second principal axis, the length of which represents the maximum variance in that direction. In our example the first component accounts for most of the dispersion in the data swarm and the original 2-dimensional system could be summarized in one dimension with little loss of information. The new variate value (PC1) for each lake is obtained by evaluating the first component

$$Y_1 = aX_1 + bX_2$$

The PC1 for each lake in 1-D A-space is its coordinate on the first component axis and is shown diagrammatically by projecting each observation to the principal axis. (Modified from Brezonik and Shannon, 1971).

Table 7. R-Mode Correlation matrix of six trophic state indicators. The coefficients were determined using LN-transformed data for the 100 NES-sampled lakes.

	LNCHLA	LNISEC	LNCOND	LNTPHOS	LNTON	LNAAY
LNCHLA	1.000	0.886	0.397	0.801	0.684	0.686
LNISEC		1.000	0.285	0.777	0.660	0.597
LNCOND			1.000	0.397	0.378	0.327
LNTPHOS				1.000	0.683	0.810
LNTON					1.000	0.576
LNAAY						1.000

at Oregon State University using the Statistical Interactive Programming System (SIPS). A detailed explanation of SIPS, and its operation is found in Guthrie, Avery, and Avery (1973).

As was asserted earlier, principal components analysis can be used to advantage because often $k \leq 3$ dimensions will explain most of the variance in the hyper-dimensional cloud of data points. The resulting ordinations can be expressed as 1-D, 2-D, and 3-D models. The ordinations are usually expressed as a sequential numerical listing (i.e., 1-D model), a scatter diagram (i.e., a 2-D model), or as a set of three 2-D scatter diagrams which, when examined carefully, may give some indication of the scatter of observations in an A-space of three dimensions. However, unless the pattern of diversity is simple, it is rather difficult for most individuals to visualize the pattern in 3-D space using a series of 2-D projections. Fraser and Kovats (1966) and Rohlf (1968) have advocated the use of stereographic projections for ordination in 3-space and have furnished the equations necessary for the development of stereoscopic models (i.e., stereograms). Some examples are found in Schnell (1970), Moss (1967), and Sneath and Sokal (1973).

The investigator has employed the techniques in Rohlf (1968) to construct 3-dimensional ordination models by plotting the lakes in the 3-D space produced by the first three principal components. The models can be examined visually for the presence of clusters. Unlike the clustering approach, no assumptions are made that the lakes must congregate into a series of clusters.

The model, as used here, consists of one member of a stereo pair; the other member is easily produced if the complexity of the data necessitates the advantage inherent in viewing objects in stereo. The lake coordinates, determined by evaluating the three components for each lake, are standardized to make the scale for the longest axis (X) run from 0.0 to 1.0. The other axes are scaled-down proportionally. The component with the smallest range is assigned to the vertical (Z) axis.

The models were plotted on an IBM Model 1724, 30 inch incremental drum plotter driven by the Oregon State University CDC 3300 computer using subroutines found in GRAFPAC, a plotting routine package developed by Rohlf (1968).

ERTS-1 MSS color ratio relationships among lakes are depicted in Chapter V, using the same modelling approach.

Results and Discussion

The normalized eigenvectors and eigenvalues are found in Table 8. Although the principal component analysis is of value in reducing the dimensionality of a multivariate system, it is sometimes difficult to interpret the new variates in terms of subject matter identities. Some indication of a principal component's meaning may be ascertained by an examination of the algebraic sign and magnitude of its coefficients.

Table 8. Normalized eigenvectors and eigenvalues. The principal components analysis was performed using an R-matrix of correlation coefficients for six trophic state indicators. The data represent 100 lakes sampled by NES during 1972.

Eigenvector Number	LNCHLA	LNISEC	LNCOND	LNTPHOS	LNTON	LNAAY	Eigenvalue	Variance (%)	Cumulative Variance (%)
1	0.457	0.435	0.251	0.458	0.403	0.408	4.081	68.02	68.02
2	-0.112	-0.249	0.952	-0.088	0.010	-0.104	0.810	13.50	81.52
3	-0.222	-0.397	-0.018	0.271	-0.383	0.757	0.477	7.95	89.47
4	-0.346	-0.395	-0.143	-0.040	0.829	0.122	0.387	6.45	96.92
5	-0.436	0.060	-0.006	0.797	-0.056	-0.410	0.152	2.54	98.46
6	-0.647	0.657	-0.099	-0.270	0.008	0.258	0.092	1.54	100.00
							5.999	100.00	

The coefficients of the first component (Table 8) are nearly equal in magnitude suggesting that it represents a general measure of trophic state, accounting for approximately 68 percent of the variation in the data. Correlations between the new variate and the LN-transformed trophic indicators are found in Table 9.

The first principal component was evaluated for each of the 100 lakes. The resultant values (PC1) are indicative of each lake's respective position on a multivariate trophic scale (Table 10). The procedure followed is essentially that of Brezonik and Shannon (1971), but the scale was not shifted into the positive domain by correcting each lake's PC1 with the PC1 obtained from a hypothetical lake. The lake lying at the negative end of the scale, Crystal Lake, is rated as having the lowest trophic status of those studied; in other words, it has relatively high water quality. Trophic state increases in the positive direction on the scale with the lake lying at the positive extreme, Albert Lea Lake, exhibiting the highest trophic state of the 100 lakes studied. Some breaks or gaps are evident on the scale, but only toward the two extremes. A very discernible gap occurs between Beaver Dam Lake (53, position 95) and Mud Lake (24, position 96); the Mud Lake - Albert Lea Lake group might be termed as hypereutrophic.

Table 9. Product-moment correlation coefficients of the trophic state indicators and the principal components.

	Principal Component					
	1	2	3	4	5	6
LNCHLA	0.92	-0.10	-0.15	-0.22	-0.17	-0.20
LNISEC	0.88	-0.22	-0.27	-0.25	0.02	0.20
LNCOND	0.51	0.86	-0.01	-0.09	-0.20	0.01
LNTPHOS	0.92	-0.08	0.19	-0.03	0.31	-0.08
LNTON	0.82	0.01	-0.26	0.52	-0.02	0.00
LNAAY	0.83	-0.09	0.52	0.08	-0.16	0.08

Table 10. Principal component ordination and mean composite rank ordination of 100 lakes

Position	Lake Name	Serial Number	PC1 Value	Lake Name	Serial Number	MCR Value
1	Crystal	44	-5.04	Crystal	44	-2.17
2	Schroon	87	-4.59	Schroon	87	-1.99
3	Higgins	70	-3.92	Higgins	70	-1.56
4	Canandaigua	92	-3.63	Canandaigua	92	-1.42
5	Charlevoix	69	-3.60	Charlevoix	69	-1.41
6	Trout	43	-3.24	Trout	43	-1.39
7	Seneca	94	-2.89	Seneca	94	-1.03
8	Cayuga	95	-2.74	Cayuga	95	-1.02
9	Crystal	77	-2.52	Owasco	96	-0.98
10	Owasco	96	-2.47	Crystal	77	-0.95
11	Middle	65	-2.29	Middle	65	-0.85
12	Keuka	93	-2.14	Keuka	93	-0.85
13	Round	50	-2.09	Round	50	-0.79
14	Oconomowoc	58	-1.82	Houghton	71	-0.66
15	Geneva	68	-1.71	Pelican	11	-0.65
16	Green	51	-1.67	Oconomowoc	58	-0.64
17	Houghton	71	-1.61	Geneva	68	-0.63
18	Carlos	17	-1.55	Green	51	-0.62
19	Lac La Belle	57	-1.43	Long	41	-0.59
20	Leech	6	-1.43	Birch	7	-0.58
21	Conesus	91	-1.41	Leech	6	-0.57
22	White Bear	31	-1.41	Carlos	17	-0.57
23	Birch	7	-1.39	White Bear	31	-0.55
24	Pelican	11	-1.27	Conesus	91	-0.54
25	Forest	30	-1.22	Lac La Belle	57	-0.51
26	Rock	55	-1.21	Forest	30	-0.49

Table 10. (continued)

Position	Lake Name	Serial Number	PC1 Value	Lake Name	Serial Number	MCR Value
27	Green	21	-1.10	Cassadaga	89	-0.50
28	Long	41	-1.07	Rock	55	-0.44
29	Browns	64	-1.07	Black	88	-0.42
30	Le Homme Dieu	18	-1.06	Le Homme Dieu	18	-0.41
31	Cassadaga	89	-1.04	Green	21	-0.41
32	Trout	8	-1.01	Shagawa	12	-0.40
33	Shawano	46	-0.94	Shawano	46	-0.40
34	Cass	5	-0.90	Trout	8	-0.38
35	Black	88	-0.77	Cass	5	-0.36
36	Gull	13	-0.76	Browns	64	-0.36
37	Darling	16	-0.73	Gull	13	-0.34
38	Pine	60	-0.71	Chautauqua	90	-0.34
39	Chautauqua	90	-0.66	Cranberry	15	-0.30
40	Okauchee	59	-0.62	Pine	60	-0.26
41	Shagawa	12	-0.58	Saratoga	100	-0.25
42	Saratoga	100	-0.57	Darling	16	-0.24
43	White	73	-0.47	Whitewater	10	-0.24
44	Rabbit	14	-0.46	Elk	42	-0.22
45	Bemidji	2	-0.44	Okauchee	59	-0.18
46	Cranberry	15	-0.40	Rabbit	14	-0.17
47	Whitewater	10	-0.38	Yellow	39	-0.17
48	Minnewaska	19	-0.32	Bemidji	2	-0.16
49	Andrusia	3	-0.24	St. Croix	32	-0.14
50	Yellow	39	-0.23	White	73	-0.13
51	Pere Marquette	72	-0.21	Wapogasset	40	-0.12
52	St. Croix	32	-0.17	Andrusia	3	-0.11

Table 10. (continued)

Position	Lake Name	Serial Number	PC1 Value	Lake Name	Serial Number	MCR Value
53	Wapogasset	40	-0.16	Pere Marquette	72	-0.04
54	Elk	42	-0.08	Minnewaska	19	-0.03
55	Chemung	81	-0.06	Blackduck	1	-0.02
56	Blackduck	1	0.01	Chemung	81	0.02
57	Clearwater	23	0.01	Wolf	4	0.02
58	Thompson	82	0.03	Otter	98	0.03
59	Wolf	4	0.11	Clearwater	23	0.03
60	Otter	98	0.13	Thompson	82	0.08
61	Muskegon	74	0.18	Muskegon	74	0.09
62	Union	84	0.43	Tainter	45	0.10
63	Tainter	45	0.44	Union	84	0.22
64	Thornapple	79	0.58	Mashkenode	9	0.27
65	Pewaukee	62	0.59	Pewaukee	62	0.29
66	Swan	52	0.68	Swan	52	0.30
67	Minnetonka	29	0.73	Minnetonka	29	0.30
68	Mashkenode	9	0.74	Thornapple	79	0.30
69	Nest	20	0.77	Nest	20	0.32
70	Cross	97	0.86	Round	99	0.37
71	Strawberry	80	0.99	Cross	97	0.41
72	Round	99	1.02	Strawberry	80	0.45
73	Long	85	1.14	Como	67	0.46
74	Como	67	1.15	Butte des Morts	48	0.48
75	Butte des Morts	48	1.27	Long	85	0.50
76	Nagawicka	61	1.27	Madison	35	0.53
77	Ford	83	1.36	Nagawicka	61	0.56
78	Madison	35	1.36	Sakatah	36	0.57

Table 10. (continued)

Position	Lake Name	Serial Number	PC1 Value	Lake Name	Serial Number	MCR Value
79	Sakatah	36	1.38	Jordan	78	0.58
80	Jordan	78	1.41	Kegonsa	54	0.59
81	Kegonsa	54	1.48	Ford	83	0.60
82	Cokato	25	1.61	Poygan	47	0.65
83	Poygan	47	1.68	Cokato	25	0.71
84	Randall	86	1.92	Randall	86	0.80
85	Delavan	66	2.03	Delavan	66	0.83
86	Pepin	34	2.10	Pepin	34	0.86
87	Mona	76	2.17	Mona	76	0.87
88	Buffalo	26	2.305	Winnebago	49	0.91
89	Spring	33	2.33	Buffalo	26	0.92
90	Winnebago	49	2.36	Spring	33	0.94
91	Koshkonong	56	2.45	Koshkonong	56	1.02
92	Fremont	75	3.06	Bear	37	1.25
93	Tichigan	63	3.11	Fremont	75	1.25
94	Bear	37	3.15	Tichigan	63	1.27
95	Beaver Dam	53	3.29	Beaver Dam	53	1.29
96	Mud (at Maple Lake)	24	4.32	Mud (at Maple Lake)	24	1.70
97	Wagonga	22	4.40	Carrigan	27	1.76
98	Carrigan	27	4.40	Wagonga	22	1.79
99	Silver	28	4.79	Silver	28	1.89
100	Albert Lea	38	5.90	Albert Lea	38	2.34

The magnitude of the first component's coordinates suggests that a less elegant approach toward an ordination of the lakes might be undertaken with similar results. The approach, using the same LN-transformed standardized indicator values, involves the summation of a lake's indicator values and then dividing by p, the number of indicators. In other words,

$$MCR_j = \frac{X_{j1} + \ldots + X_{jp}}{p_j}$$

where MCR_j is the Mean Composite Rank for the jth lake, X_{j1} is the jth lake's score on the first trophic indicator, X_{jp} is the lake's score on the pth indicator and p is the number of dimensions or indicators. The results of this ranking method are found in Table 10 along with the principal components ordination. The two methods of ordinating the lakes are close agreement. However, the principal component approach has the advantage of permitting the development of 2-dimensional and 3-dimensional ordinations which explain a very high percentage of the variance.

The second component (Table 8) explains about 14 percent of the variation in the data. The LNCOND coordinate of the component is very large suggesting that the second variate is largely a measure of conductivity. The second component has a good correlation with LNCOND (Table 9).

The third component (Table 8) accounts for approximately 8 percent of the variance. LNAAY is the trophic indicator which shows the strongest correlation with the third component (Table 9), but is not readily interpretable.

Approximately 89 percent of the total sample variance can be attributed to the first three components. Figure 17 depicts the 100 lakes ordinated in 3-D space defined by the first three components. Some small clusters are apparent, but they are not well defined. The long axis is the first principal component axis (I). The axis labelled II is the second component and the vertical axis (III) is the third component.

The failure to discern well defined clusters may be partially a consequence of the trophic state of the lakes incorporated into this analysis; the NES lake population is heavily weighted toward lakes having water quality problems.

Summary

One hundred lakes were subjected to two complementary multivariate analyses, a complete linkage hierarchical cluster analysis and ordination using the technique of principal components. Well-defined clusters or natural groupings were not found through either approach. This may be a consequence, at least partially, of the "kinds" of lakes incorporated into the analyses.

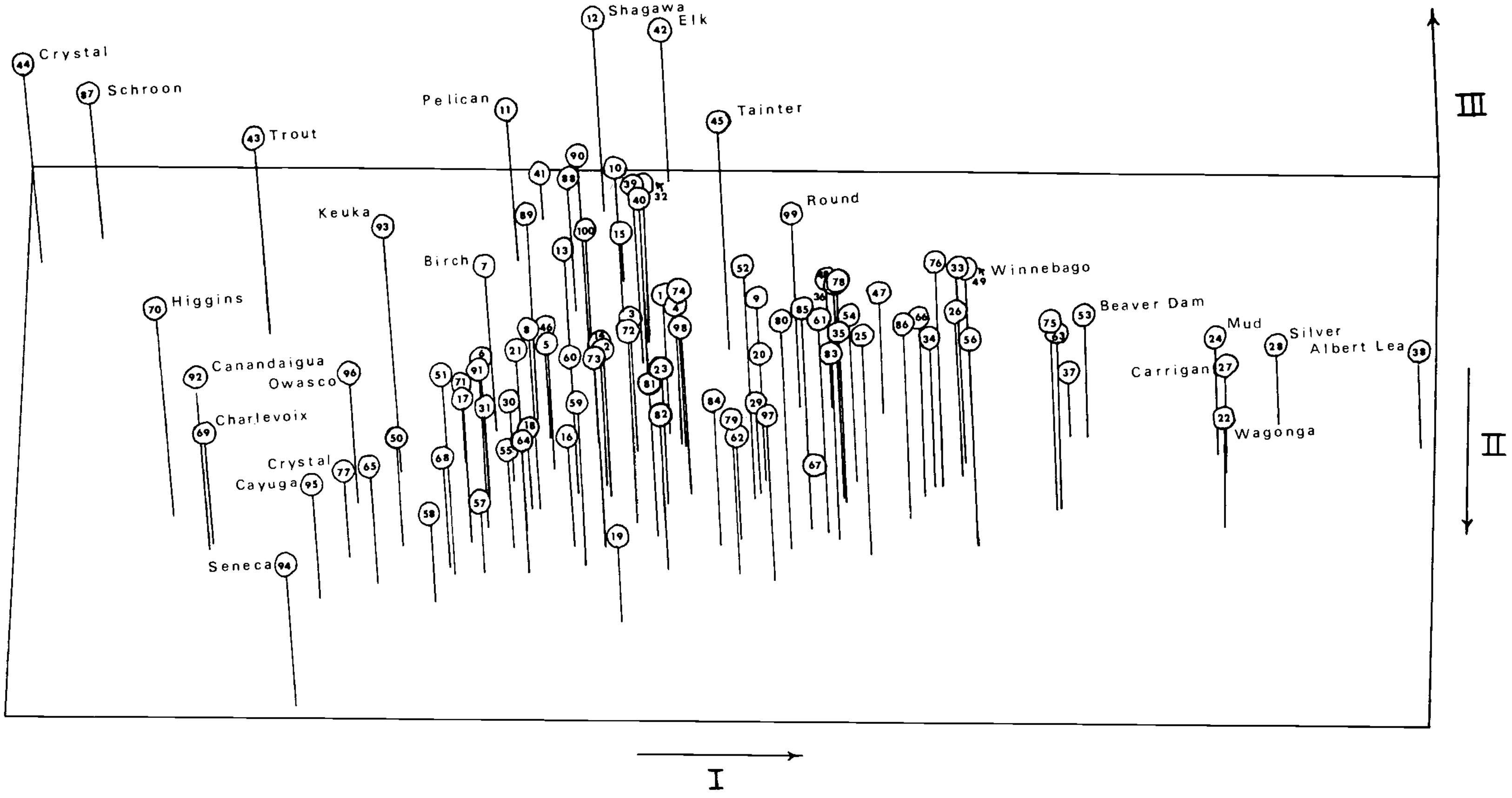

Figure 17. Three-dimensional principal component ordination of 100 lakes sampled by the National Eutrophication Survey during 1972.

The first principal component was evaluated for each lake and the resulting value (PC1), its coordinate on the axis, used as a multivariate index of the lake's trophic state. The PC1's are purported to represent a parsimonious assessment of lacustrine trophic state and will be used in Chapter V to evaluate ERTS-1 MSS color - lake relationships.

III. ERTS-1 DATA EXTRACTION TECHNIQUES AND PRODUCTS

ERTS-1 MSS data are available from EROS in the form of photographic products and computer-compatible digital magnetic tapes (CCT's). The data forms permit investigators two general approaches to the problem of data extraction and utilization, the photographic approach and the CCT approach. Each has its particular advantages and limitations.

MSS Data Extraction Approaches

Photographic Approach

The photographic products available from EROS include black and white (b&w) negative and positive transparencies, and prints. False color prints and transparencies are available for a very limited number of scenes. All are either third or fourth generation photographic products.

The photographs may be examined visually, as was done in Chapter I, for differences in gray tone, texture, shape, size, and pattern. Techniques common to the interpretation of aerial photographs may be employed to extract information pertinent to many different fields of science.

Quantification of the information contained in the undodged photographic products may be achieved through microdensitometry and photographic techniques including false color enhancement and density slicing. Manual and automatic microdensitometry can be used to digitize the information contained on the photographic positive or negative transparencies. This is easily accomplished with a microdensitometer by directing a light beam of known intensity through a small segment (e.g., lake image or portion of a lake image) of the transparency and measuring any changes in intensity on a numeric scale as percent transmission or as optical density. The quantitative data may then be used to examine correlations between MSS bands and lake indicators relating to trophic state.

Automatic scanning systems permit density slicing, the separation of the different densities on a transparency and their coding for later reproduction as b&w and color-enhanced products or symbolized listings. Reliance upon sophisticated instrumentation for the determination of areas of equal density (i.e., equidensities) can be avoided by using various photographic enhancement techniques. Nielsen (1972) summarizes a simple process reported by Ranz and Schneider (1970) which utilizes Agfacontour film for the production of equidensities.

The photographic approach is attractive because it can be accomplished with relatively simple equipment (unless a fully automated scanner is used) and is inexpensive. The four transparencies required to give all band (green, red, IR1, IR2) coverage of an ERTS-1 MSS scene cost only about eight percent of the listed (June 1974) EROS price for a comparable set of CCT's ($12.00/ $160.00). Many investigators, lacking the necessary computer and software for processing the CCT's, are able to explore possible applications of ERTS-1 data in their areas of specialization using the photographic approach.

The approach has several limitations which are particularly serious in water resource studies and which merit mention. The transparencies have a relatively small density range when compared to the sensitivity range of the MSS; this results in a scale compression when the MSS data are transformed into a film image on an electron beam recorder (EBR). The range of energy returns from water bodies is small and located at the lower end of the MSS intensity scale. Compression of the overall scale increases the difficulty of discriminating differences in water quality.

As mentioned previously, the photographic materials are third and fourth generation products (the first generation is the film image produced by the EBR), and the errors common to photographic processing the compounded. Chemical adjacency effects and the non-linearity of the density-log exposure (D-log E) curve increase the magnitude of the problem. An additional element of uncertainty is introduced by the microdensitometer.

The investigator is not trying to assert that the photographic approach is totally lacking in merit, but rather that its limitations seriously limit the investigator attempting a quantitative estimation of lake parameters relating to trophic indicators and trophic state.[12]

CCT Approach

The use of computer compatible tapes to study ERTS-1 MSS sensed phenomena is not only more expensive, but it is also more restrictive because there are relatively few research centers with the requisite computer software and output devices to use the digital data to full advantage. However, these limitations are overshadowed by the fact that the tapes contain, in digital form, the actual data point values recorded by the MSS. Investigators can avoid the numerous uncertainties introduced when the original MSS data are coded by an electron beam recorder (EBR) into photographic forms and then re-quantified through microdensitometry.

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[12] This investigation was initiated using the photographic approach. A manually operated microdensitometer was used to measure the optical density (OD) of lake images. Differences in OD were detected, but several difficulties were encountered (*e.g.*, variation in the quality of the transparencies, locating the lakes on the green band transparencies), and the approach was abandoned in favor of the CCT approach.

The CCT approach permits the rapid determination of picture element (pixel) counts, descriptive statistics (*e.g.*, means, standard deviations, and histograms), density slicing, and affords the opportunity to enhance optional photographic products through both linear and non-linear contrast stretching.

The utilization of CCT's is not without its problems, most of which are related to the quality of the information contained on the tapes. Defects commonly contained in the MSS data recorded on the tapes include missing lines and random bit-dropout for major portions of lines. The defective tapes can be repaired using existing software, but at additional cost both in time and money.

Another problem is the receipt of CCT's not meeting the specifications of the purchase order (*e.g.*, receipt of 9-track instead of 7-track tapes; 556 bpi instead of 800 bpi tapes). While this is a minor problem, it does cost the investigator valuable time.

CCT Data Extraction Techniques

The ERTS-1 MSS CCT's used in this investigation were processed at the Image Processing Laboratory (IPL), a support facility of the Jet Propulsion Laboratory (JPL), California Institute of Technology. In this section, the author will briefly discuss the image processing system and, using one lake as an example, outline the data extraction techniques employed.

Image Processing System[13]

The IPL system consists of an IBM 360/44 computer with five tape drives, four disc drives, a quick-look Polaroid pictorial output device, and a core-refreshed interactive display. A video film converter (VFC) provides offline hardcopy pictorial input and output capabilities. This device has a precision cathode ray tube (CRT), and 70-mm and 35-mm cameras.

In the pictorial output mode, the VFC reads a digital magnetic tape, containing a digital image, and displays that image on the CRT, exposing the film which is then developed and printed, producing a hardcopy output of the digital image. In the pictorial input mode, the VFC functions as a flying spot scanner producing a digital output picture on magnetic tape.

The IPL IBM 360/44 operates under the control of a special software system, VICAR (Video Communication and Retrieval). Presently, a highly stylized Operating System (OS) is operational which permits foreground-background, roll-in and roll-out capabilities. VICAR is designed to allow very flexible manipulation of

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[13]The information presented in this section is drawn from a paper by Blackwell and Boland (1974). Computer Analysis of Lakes from ERTS Imagery.

digital pictures consisting of rectangular arrays of optical measurements. The system contains a library of more than two hundred operationally executable image-processing programs. Each program operates on an input digital picture stored on magnetic tape or disc. Processing line-by-line, it generates an output picture on tape or disc, preserving the input data. Additional information regarding the system and existing programs is found in Frieden (1968), Anon (1973), and Blackwell and Billingsley (1973).

Data Extraction Technique

ERTS-1 MSS Frame 1017-16093 and one of its lakes, Lake Koshkonong (56), will be used to illustrate the methodology used to extract the digital information contained in a set of CCT's.

The extraction process commences with a change in the format of the Goddard Space Flight Center (GSFC) CCT's and an expansion to 8-bit mode giving a total of 256 digital number (DN) levels (0 to 255) of optical intensities. The system is capable of handling the data contained on two CCT's at a time, resulting in an initial output consisting of one half of an ERTS-1 frame. An example of the left half of Frame 1017-16093 is shown in Figure 18. The image has been reconstructed using the IR2 DN values and a fiducial system has been applied to the edges.

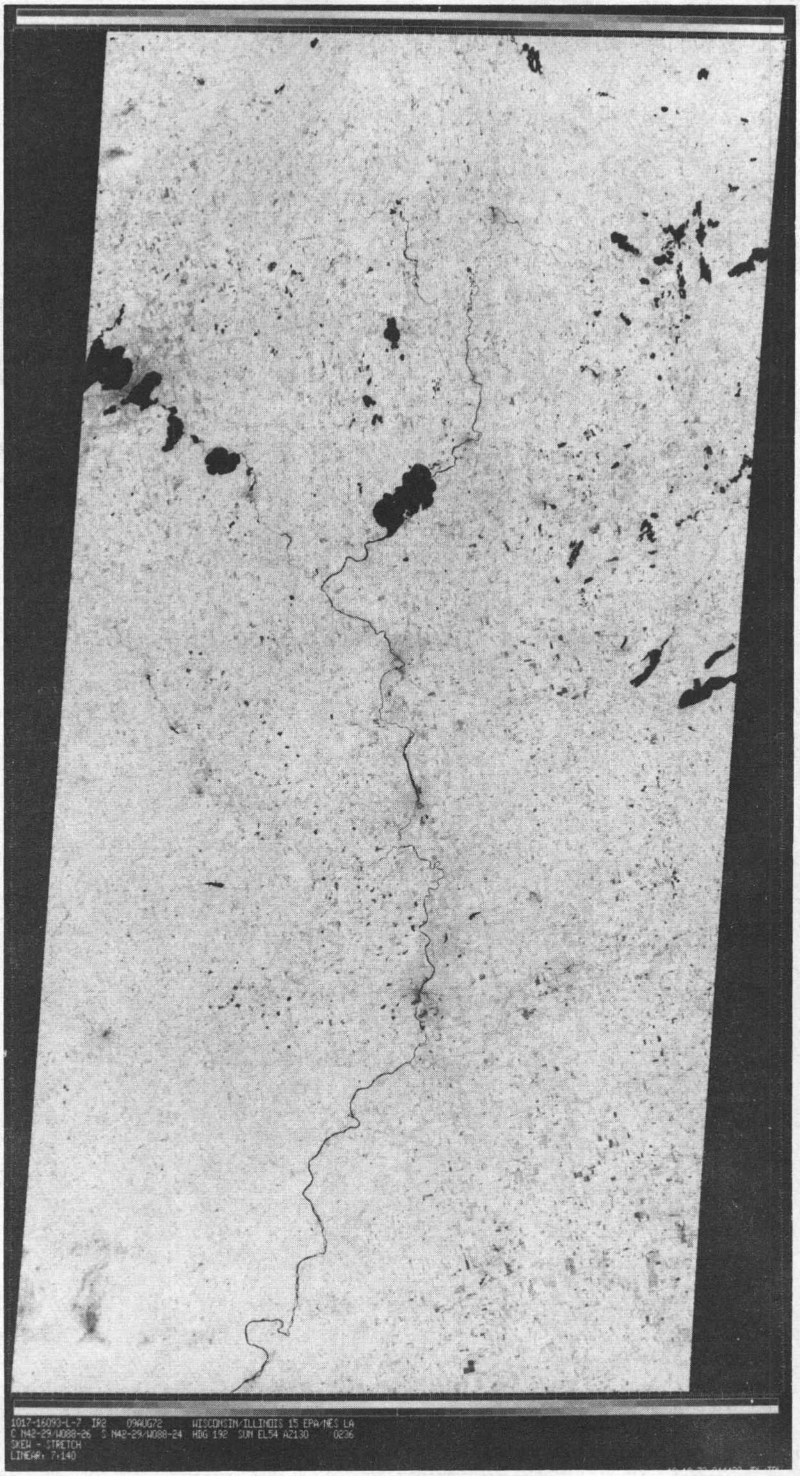

Figure 18. Left half of MSS Frame 1017-16093 reproduced using IR2 DN values. A fiducial system has been imposed along the edge of the image to aid in the determination of feature locations on the basis of line and picture element counts. Lake Koshkonong is the large dark object in the upper center of the photograph.

The section of Frame 1017-16093 containing Lake Koshkonong and a portion of the surrounding terrain is extracted from the tapes in each of the four MSS bands by supplying the computer with the appropriate coordinates. Figure 19 shows the extracted section for Band 7 (IR2). The histogram below the picture represents the full range of DN values present within the scene on a scale of 0 (black) to 255 (white). The prominent DN values, centered at approximately DN 128 and spreading symmetrically, are related to the land features within the section. The small cluster of DN values at the lower end of the DN scale is associated with water features including Lake Koshkonong and the Rock River.

In this study, the land DN values are of little interest and must be eliminated, leaving just the water body values. Previous testing has verified that the IR2 band is a good indicator of the areal extent of surface water. As was noted in Figure 19, IR2 water-related intensity values fall within the lower end of the DN scale and are essentially isolated from the land IR2 DN values. This characteristic permits the development of a binary mask which is used to eliminate the land images.

The mask is created by setting all intensity values at and below a specific numeric value equal to a value of one and setting all of the remaining DN values equal to zero. Each spatially equivalent Lake Koshkonong pixel within each band (green, red, IR1, and IR2) is multiplied by the IR2 binary mask. The net effect is to "zero out"

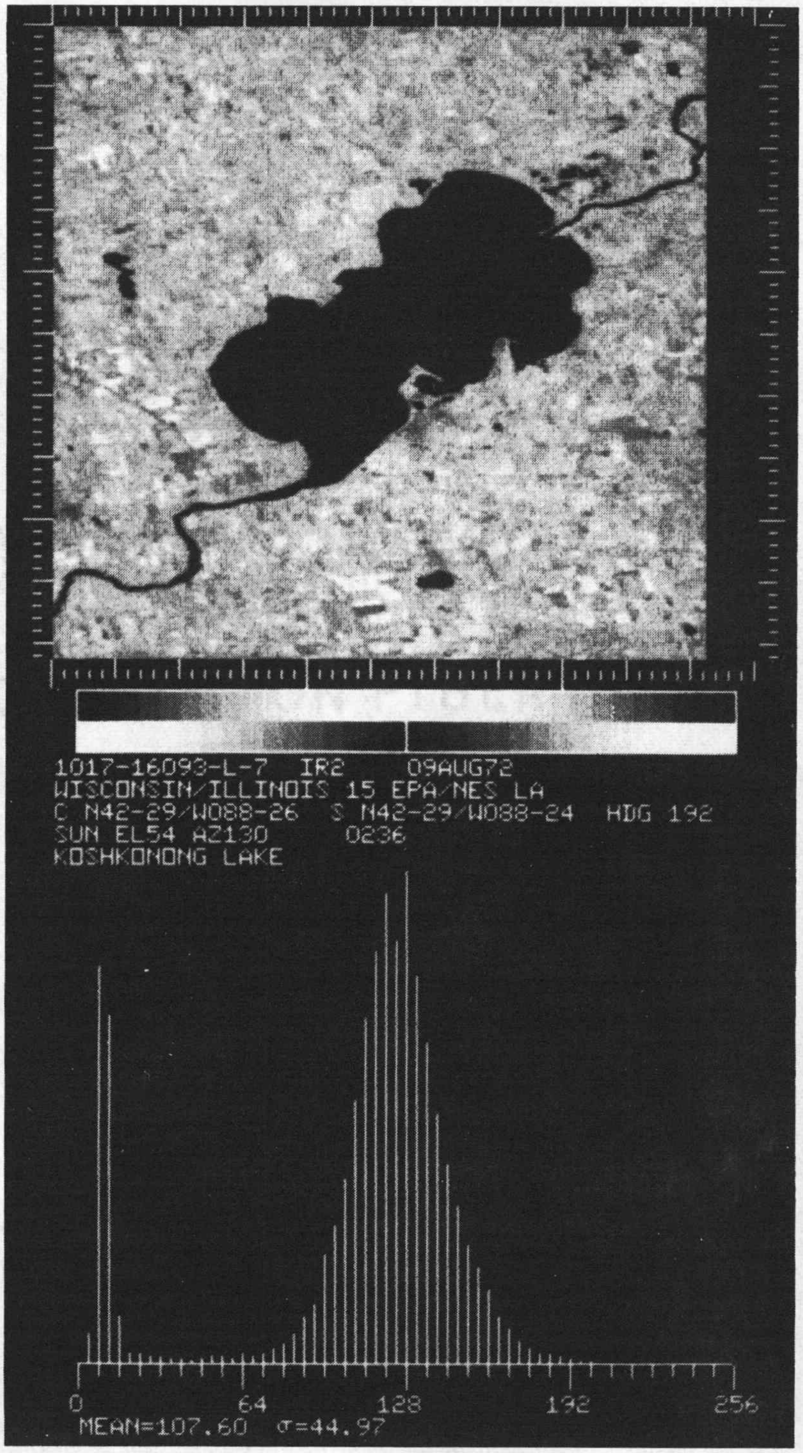

Figure 19. An extracted section of ERTS-1 MSS Frame 1017-16093. Lake Koshkonong, the Rock River, several small lakes and land features are evident. A histogram of the section, seen below the image, displays the range of DN values contained within the section. Water DN values cluster toward the lower end of the scale.

the unwanted background, leaving just the water features. Figures 20 and 21 were created by multiplication with a binary mask produced by setting all IR2 DN values of 28 or lower to a value of one. Images constructed from the IR1 and IR2 would be similar, showing only variations in the spectral signature for the water, and therefore in their pixel DN histograms.

A final cleanup is done by upgrading the binary mask prior to multiplying all of the bands. This is done by eliminating all of the smaller water bodies, streams, and swamp features which may be present. After the final cleanup and examination of a test multiplication, the mask is used on all of the bands for Lake Koshkonong. The pictorial expression of the final cleanup for the four spectral bands is depicted in Figure 22.

The concatenation technique is a very convenient method for summarizing the areal aspects of data extracted from the CCT's. Unlike line-printer copy, areal coverage afforded for several different lakes can be depicted in one simple photograph (e.g., Figure 23). Differences among the lakes in Figure 23 are not apparent because the range of IR2 DN values is very small, and no effort has been made to enhance the images.

The VICAR photographic products displayed up to this point serve mainly to aid the investigator in the extraction of digital data from the CCT's. Typical numeric output from the system includes pixel counts, DN means and standard deviations for each of the four MSS bands, and histograms of the DN distributions. Pertinent data extracted from the example, Lake Koshkonong, are listed in Table 11.

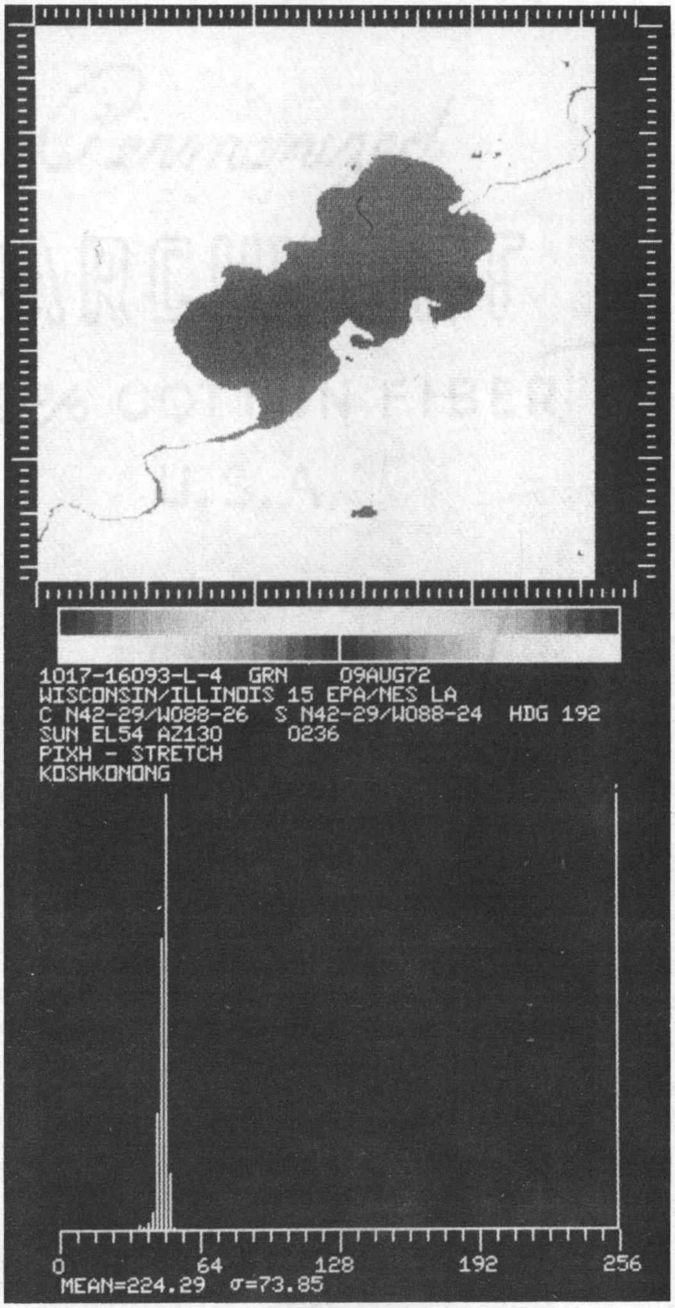

Figure 20. First-stage MSS Band 4 (GRN) cleanup picture of Lake Koshkonong. This picture was created by multiplying the Band Four scene equivalent of Figure 19 by the binary mask. All pixels corresponding spatially to IR2 pixels with DN values greater than 28 have been multiplied by zero, and thereby removed from the scene. The Rock River and miscellaneous water bodies still have to be eliminated.

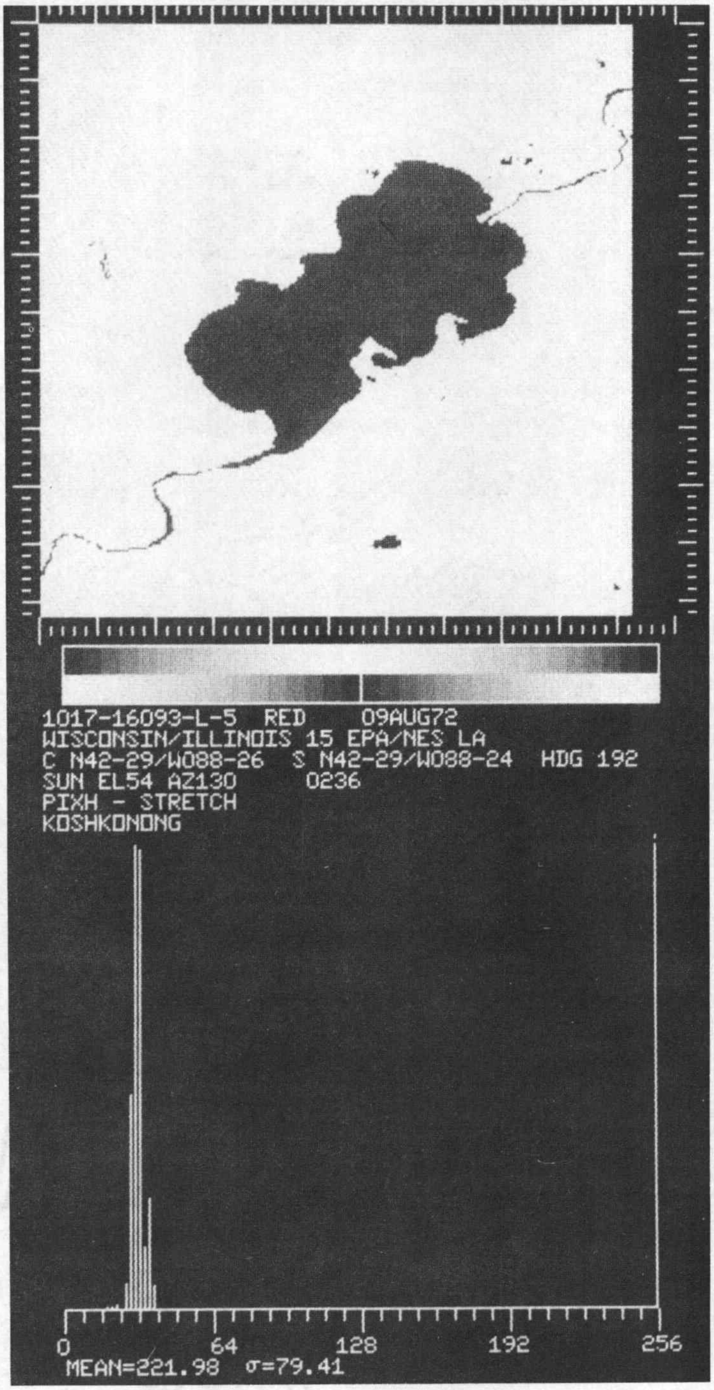

Figure 21. First-stage MSS Band 5 (RED) cleanup picture of Lake Koshkonong. This picture was created by multiplying the Band Five scene equivalent of Figure 19 by the binary mask. All pixels corresponding spatially to IR2 pixels with DN values greater than 28 have been multiplied by zero, and thereby removed from the scene.

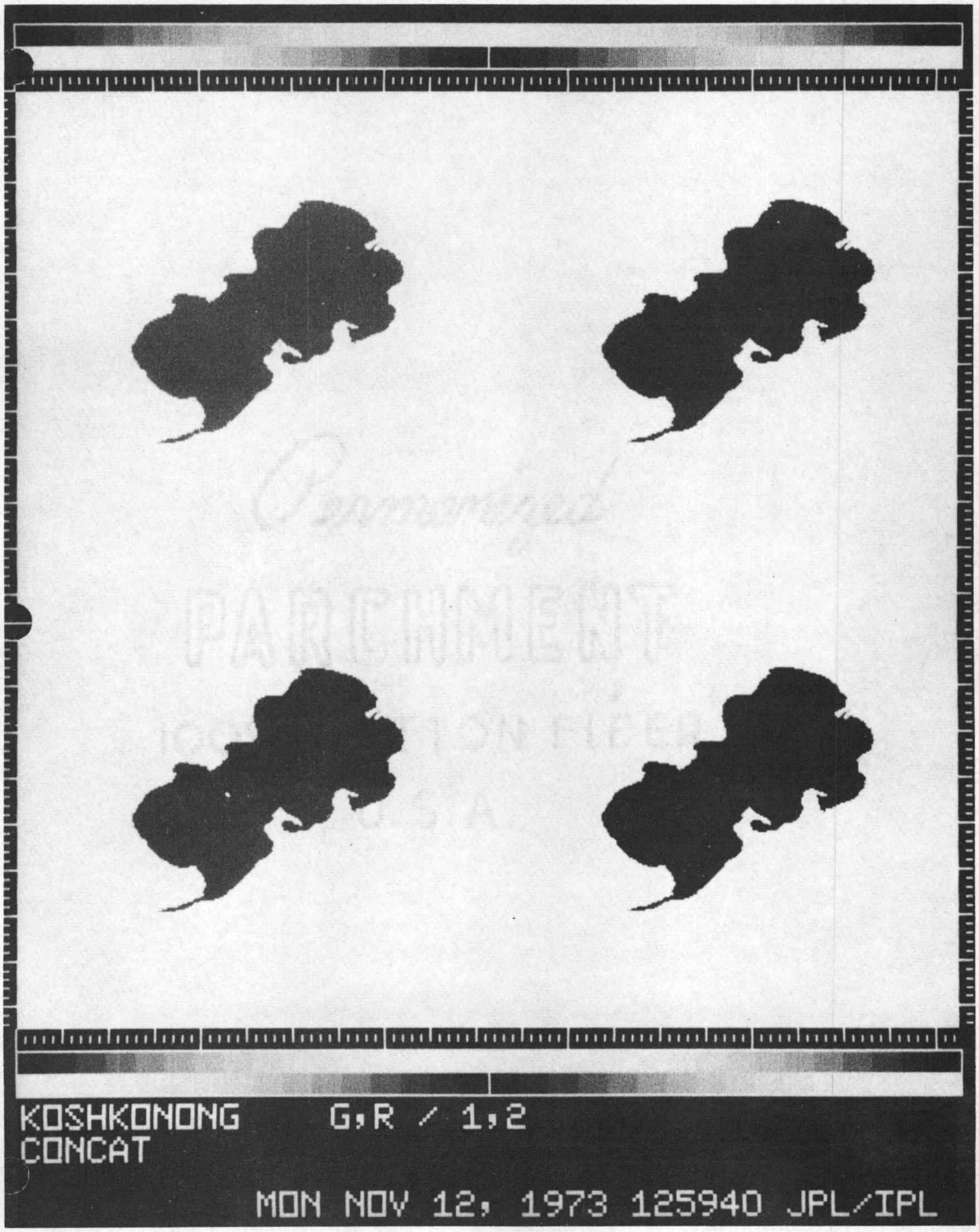

Figure 22. Four band (GRN, RED, IR1, IR2) concatenation of Lake Koshkonong after final cleanup. All of the unwanted land and water body pixels have been eliminated.

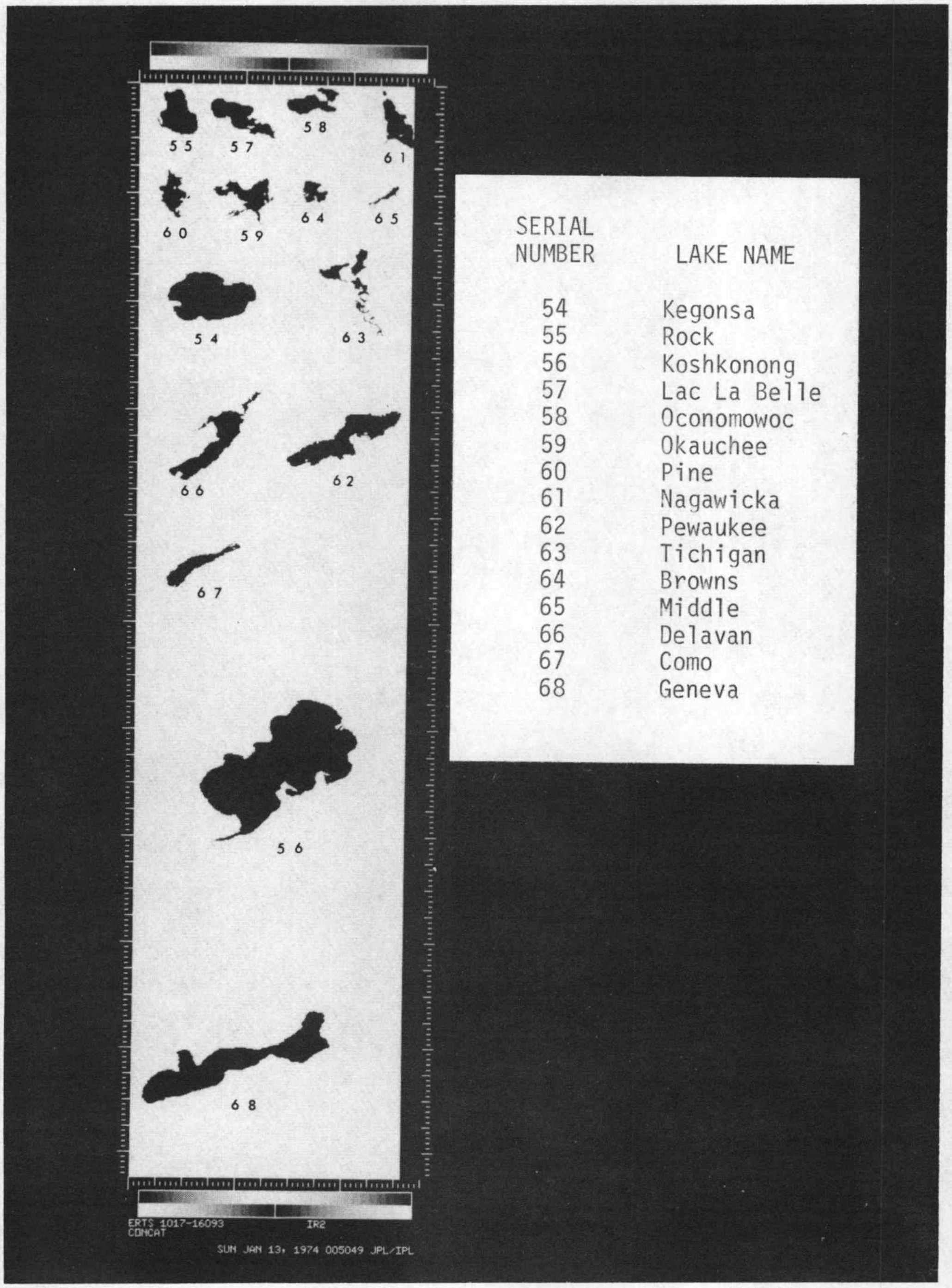

Figure 23. IR2 concatenation of 15 lakes extracted from ERTS-1 MSS Frame 1017-16093. The lake images are all reproduced at the same scale. However, they have not been skewed to correct the geometric distortion induced by the earth's rotation.

Table 11. Descriptive statistics of Lake Koshkonong MSS data extracted from Frame 1017-16093 CCT's.

	ERTS-1 BANDS			
	Green	Red	IR1	IR2
DN Mean	46.85	31.15	27.95	10.08
DN Standard Deviation	1.814	2.446	2.933	2.890
Pixel Count	9,247	9,247	9,247	9,247

The DN values are used in determining correlations with ground (i.e., water) truth, developing regression models for selected lake trophic indicators and trophic state, and for image enhancement.

Digital Image Enhancement Techniques

Up to this point, the VICAR photographic output has served as an aid in the extraction of lake MSS DN values. However, the system can also enhance the digital images to bring out latent features. Two methods commonly employed are linear contrast stretching and a technique which utilizes color ratios. The techniques are briefly explained in this section along with several examples of the photographic output.

Color Ratio Technique

The color ratio technique consists of dividing the MSS DN values of one ERTS-1 band by another ERTS-1 band, pixel by pixel (i.e., data point by data point). Water colors which are difficult to discriminate generally have spectral reflectivity curves with similar but not identical slopes (Blackwell and Boland, 1974). The difference in the curve slopes will form the basis of the discrimination rather than absolute reflectivities. Application of the digital color ratio technique between selected bands will normalize the data by removing the common brightness components and will tend to emphasize differences due to slope.

The digital image of Lake Koshkonong (Frame 1017-16093) was enhanced using the ratio technique and subsequently reproduced in the form of a concatenation (Figure 24). Compare the enhanced images with Figure 22. It is readily apparent that considerable differences in lake color exist, particularly in the GRNRED, GRNIR1, and REDIR1 images. These ratios appear to merit further consideration.

Additional information concerning ratio techniques and their applications is found in Whitaker (1965), Billingsley *et al*. (1970, and Billingsley (1972, 1973).

Linear Contrast Stretching Technique

Another technique for detecting MSS color differences involves the enhancement of digital contrast of selected MSS bands through a "stretching" process. As is evident in Figures 18-23, the images of Lake Koshkonong are of very low contrast and essentially featureless. This is a consequence of the restricted DN value-range present for the lake in all MSS bands. An expansion of the lake's DN ranges to fill the entire black to white range (0 to 255) by

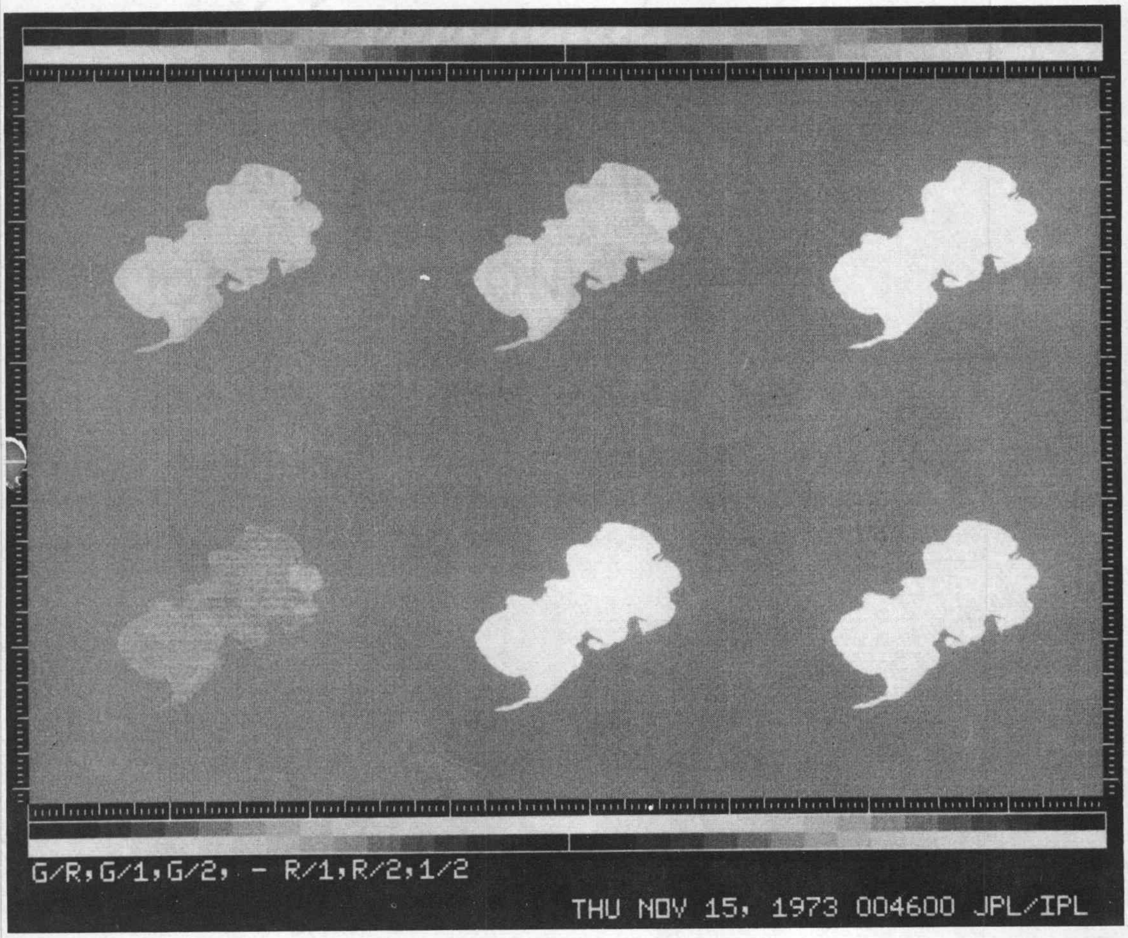

Figure 24. Lake Koshkonong image enhancement using six ERTS-1 MSS ratios. The ratios are, scanning from the top row left, GRNRED, GRNIR1, GRNIR2, REDIR1, REDIR2, IR1IR2. Differences in lake color are particularly evident among the GRNRED, GRNIR1, and REDIR1 ratios. The MSS data were extracted from Frame 1017-16093 (9 August 1972). Compare with Figure 22.

linear expansion will enhance the contrast and bring out latent features. Stretching in the digital domain has the advantage that it does not suffer from the toe and shoulder saturations encountered with photographic stretching (Billingsley and Goetz, 1973).

The contrast stretch was performed on Lake Koshkonong data extracted from two ERTS-1 MSS frames (1016-16093, 9 August 1972; 1036-16152, 28 August 1972). The results are presented as Figures 25 and 26. The very strong banding effect is an artifact created by an inherent defect in the MSS system. A comparison of the lake images suggests that the MSS is capable of monitoring spatial-temporal changes in lakes. A scarcity of ground truth precludes a full interpretation of the physical significance of the patterns. This is one area in need of additional investigation.

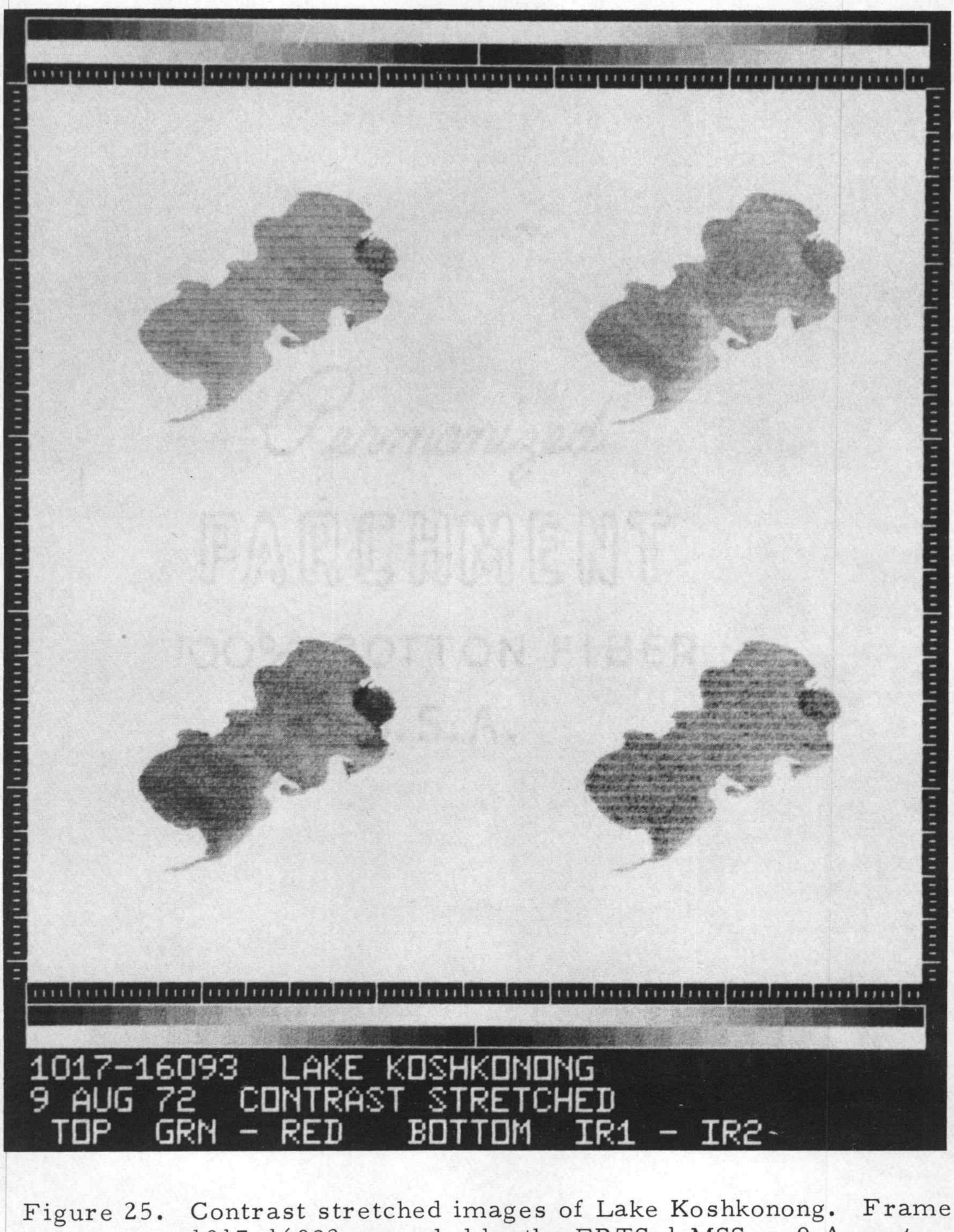

Figure 25. Contrast stretched images of Lake Koshkonong. Frame 1017-16093 recorded by the ERTS-1 MSS on 9 August 1972. Upper row, left to right: GRN, RED; lower row, left to right: IR1, IR2. The banding or striping is an artifact.

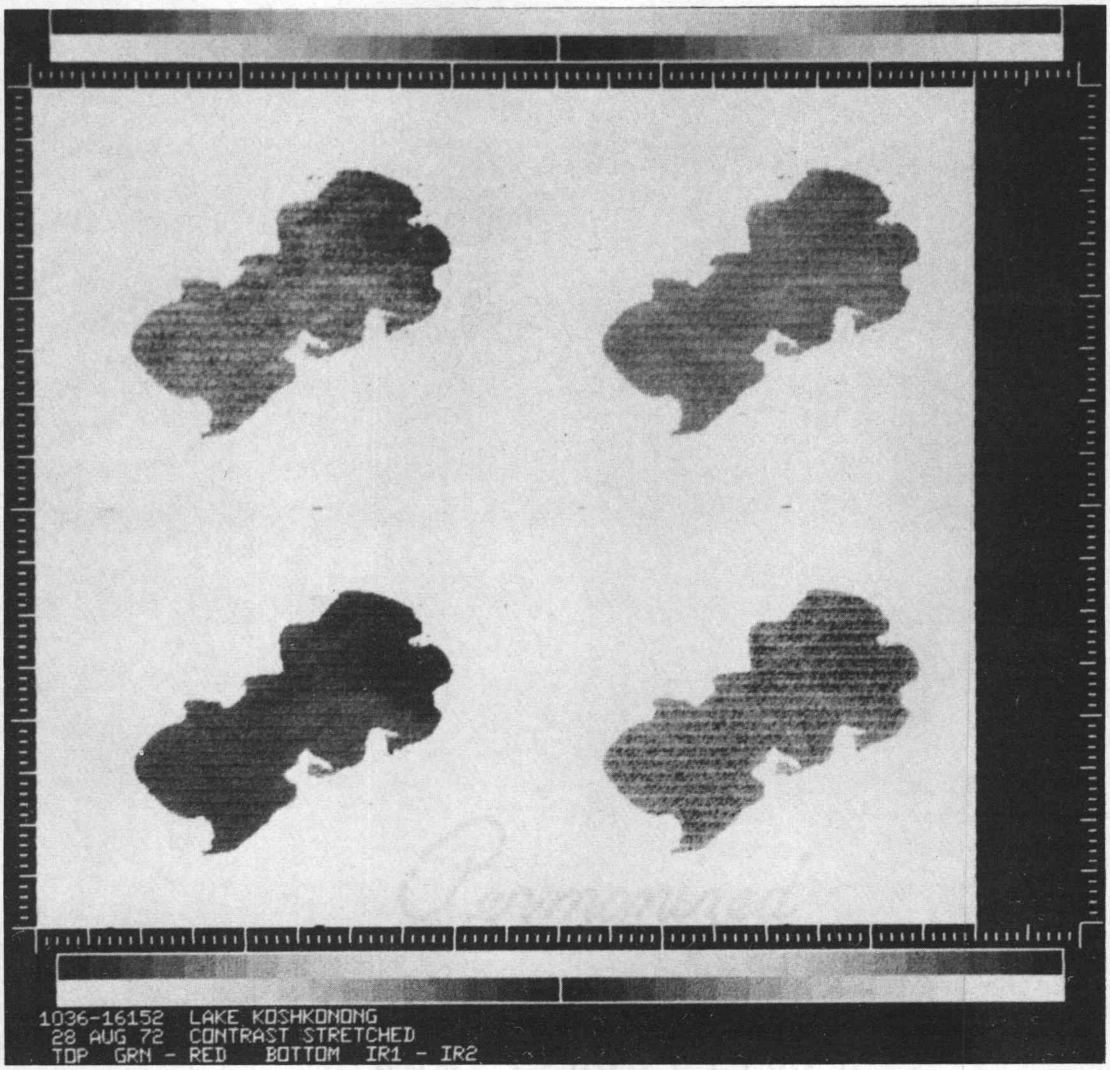

Figure 26. Contrast stretched images of Lake Koshkonong. Frame 1036-16152 recorded by the ERTS-1 MSS on 28 August 1972. Upper row, left to right: GRN, RED; lower row, left to right: IR1, IR2. Compare with Figure 25.

IV. ERTS MSS TROPHIC INDICATOR RELATIONSHIPS

This chapter is devoted to the evaluation of MSS data as a means of estimating the magnitude of trophic state indicators including Secchi disc transparency and chlorophyll a. In addition, lake surface area estimation is also examined.

Optical Properties of Pure Water and Natural Waters

It is readily apparent, even to the casual observer, particularly if he or she is looking downward from an aircraft, that lakes differ in color and brightness. Many investigations have been undertaken to develop a comprehension of the processes which result in the observed phenomena. Although a detailed discussion of the interaction of electromagnetic energy with the components of the hydrosphere and atmosphere is outside the scope of this thesis, a brief survey is essential to gain some understanding of the principles which permit remote sensing and yet constrain its use in the assessment of trophic indicator magnitudes.

The interaction between electromagnetic energy and chemically pure water has been studied by numerous investigators (e.g., Ewan, 1894; Sawyer, 1931; Collins, 1925; James and Birge, 1938; Hulbert, 1945; Raman, 1922; Dawson and Hulburt, 1937; James, 1932). The transmission of electromagnetic energy through a material medium is always accompanied by the loss of some radiant energy by absorption. Some of the energy is transformed into other forms (e. g., heat, chemical) or to some longer wavelength of radiation (James and Birge, 1938). Pure water is very transparent to violet, blue and green light. In the infrared region, the extinction coefficient is high with a complementary low degree of transmission (Table 12). The absorption spectral characteristics of pure water can be modified greatly through the addition of dissolved and particulate materials.

The absorption spectra of natural waters (e. g., lake and ocean) have been studied in detail by Jerlov (1968), Duntley (1963), Atkins and Poole (1952), Clark and James (1939), Pietenpol (1918), Smith et al. (1973), Birge and Juday (1929, 1930, 1931, 1932), and Juday and Birge (1933), to mention a few. Hutchinson (1957) has summarized the more important attempts to elucidate the interactions of light with natural waters, particularly in regards to lakes.

The attenuation of electromagnetic radiation in lake waters is a consequence of the relatively unselective effect of suspended particulate materials and the highly selective effect of dissolved coloring matter, usually of organic origin, on the electromagnetic spectrum. The dissolved matter absorbs strongly in the violet and blue wavelengths, moderately in the middle wavelengths (e. g., green), and much less strongly at longer wavelengths (Hutchinson, 1957). When the dissolved materials are present in small quantities, the water will be most transmissive in the green wavelengths. Lake waters with large amounts of

Table 12. Optical properties of pure water (room temperature). Adapted from Hutchinson (1957).

Wavelength (nanometers)	Extinction Coefficient	Percentile Absorption	Refractive Index
820 (infrared)	2.42	91.1	
800	2.24	89.4	
780	2.31	90.1	
760	2.45	91.4	1.329
740	2.16	88.5	
720	1.04	64.5	
700	0.598	45.0	
680 (red)	0.455	36.6	
660	0.370	31.0	1.331
640	0.310	26.6	
620 (orange)	0.273	23.5	
600	0.210	19.0	
580 (yellow)	0.078	7.0	1.333
560	0.040	3.9	
540	0.030	3.0	
520 (green)	0.016	1.6	
500	0.0075	0.77	
480	0.0050	0.52	1.338
460 (blue)	0.0054	0.52	
440	0.0078	0.70	
420	0.0088	0.92	
400 (violet)	0.0134	1.63	1.343
380 (ultraviolet)	0.0255	2.10	

dissolved substances will be more transmissive in the orange and red wavelengths than in the shorter wavelengths. However, the transmission of the red and orange light is still greater in pure water than in water containing particulate and/or dissolved materials. As water transparency diminishes, the detectable electromagnetic energy will be of progressively longer wavelength, at increasingly shallower depths (Hutchinson, 1957).

Scherz, et al. (1969) have investigated the total reflectance (surface reflectance plus volume reflectance) curves of pure water and natural waters under laboratory conditions using a spectrophotometer. The resulting curve for distilled water is shown in Fig. 27. They reported that the addition of dissolved oxygen, nitrogen gases, and salts (e. g., NaCl, Na_2SO_4, $Na_3PO_4 \cdot H_2O$) had no apparent effect on the reflection curve. However, water from lakes in the Madison (Wisconsin) area had reflectance curves that both differ from the distilled water curve and from each other. Scherz, et al. suggest that the differences can be attributed to the presence of different algal organisms. Filtration of the lake waters produced similar reflectance curves, though different from that of pure water (Figure 28).

The color of a lake is the color of the electromagnetic energy backscattered from the lake body to the sensor. Lake color ranges from the blue of pure water through greenish blue, bluish green, pure green, yellowish green, greenish yellow, yellow, yellow brown, and clear brown (Hutchinson, 1957). It is a common practice to record lake color on an empirical scale such as the so-called Forel-Ule color scale, originally devised by Forel (1889) and subsequently modified and extended by Ule (1892). Lake color need not be, and

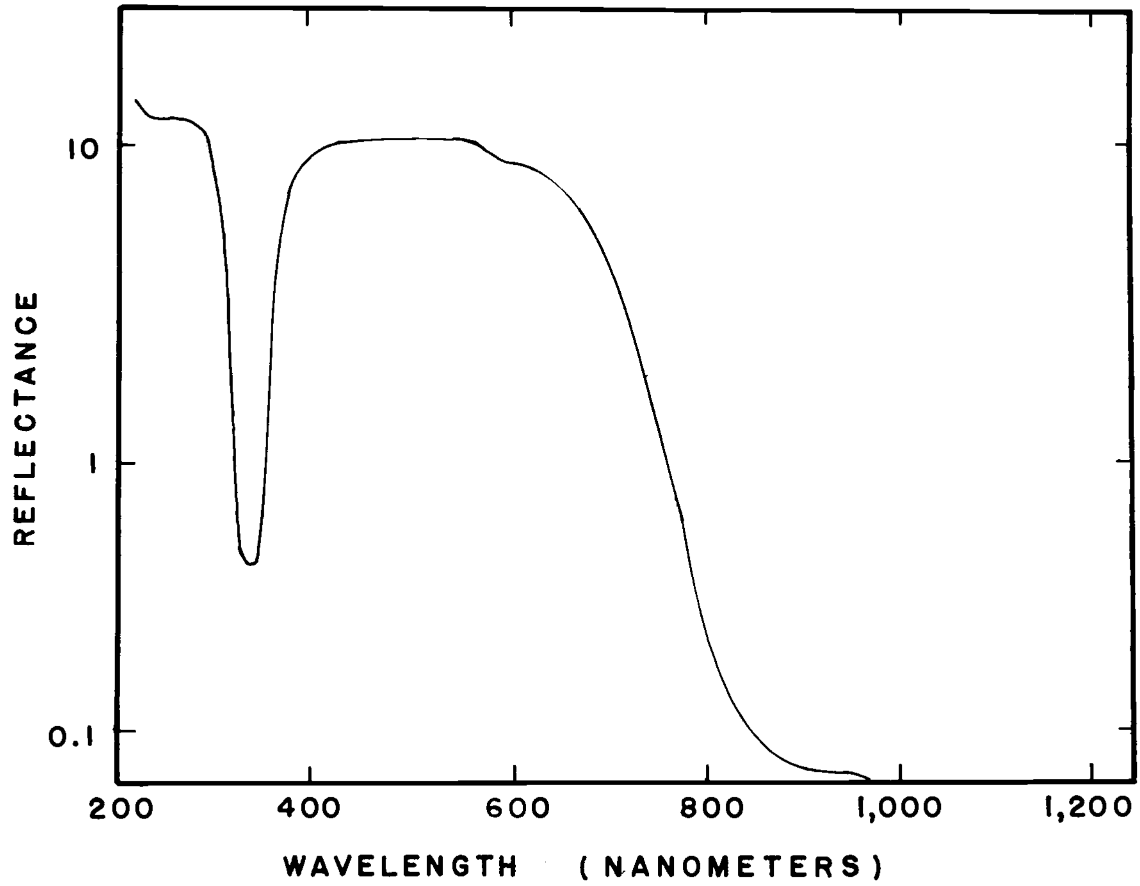

Figure 27. Reflectance curve for distilled water. Adapted from Scherz, et. al. (1969).

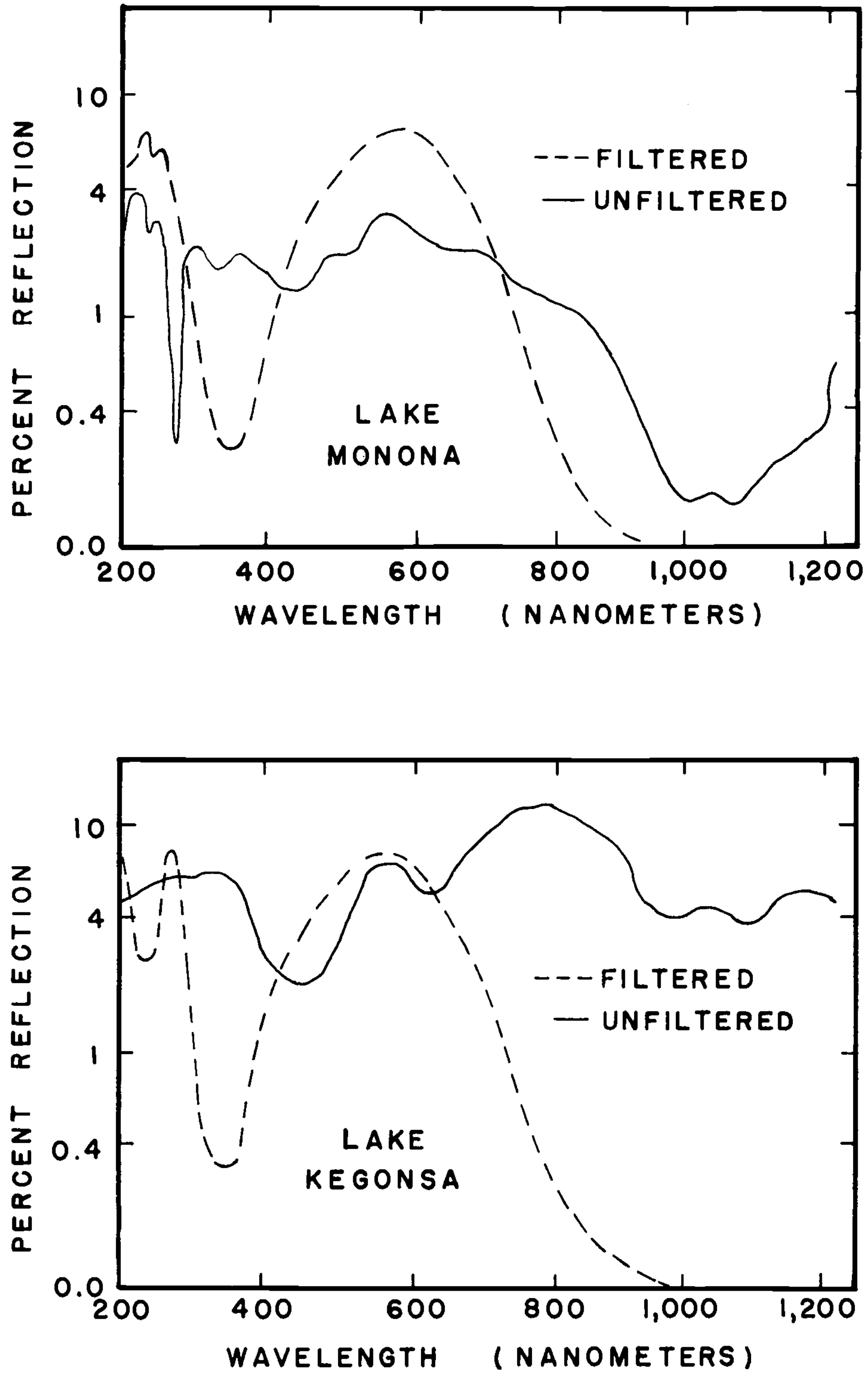

Figure 28. Reflection characteristics of filtered and unfiltered water samples from two Wisconsin lakes in the area of Madison. Adapted from Scherz, et al. (1969).

is usually not the same as, the color of the lake water.[14]

Lakes which are blue in color lack appreciable quantities of humic materials and colored materials in suspension (e.g., phyto-plankton). The bluer the lake color, the smaller the amount of free-floating organisms contained in the water (Ruttner, 1963). Waters with high plankton content possess a characteristic yellow-green to yellow color. The characteristic color may not be apparent due to masking by other materials (e.g., suspended sediments). Ruttner (1963) suggested that

> A lake with very transparent and dark blue, blue-green, or green water is always oligotrophic. On the other hand, eutrophic lakes always have a relatively low transparency and are yellow-green to yellow brown in color; but the determination of these optical properties alone will not establish the productivity type, for the turbidity can be of inorganic origin, and the color can come from humic substances.

Seston color, attributable to the reflection spectra of suspenoids of microscopic or submicroscopic size, is often observed in highly productive lakes. Lakes containing large quantities of suspended in-organic matter (e.g., silt) may acquire a characteristic seston color, but in most cases the color is related to large concentrations of phytoplanktonic organisms (Hutchinson, 1957).

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[14] Welch (1952) defines water color as "... those hues which are inherent within the water itself, resulting from colloidal substances or substances in solution" (i.e., true color). The platinum-cobalt scale (Hazen, 1892) has found favor in the United States for the determination of water color.

Peripheral Effects

The character of the electromagnetic energy impinging on the remote sensor, the ERTS-1 MSS in this case, has been shaped through interaction with numerous environmental phenomena.

The earth's atmosphere has a pronounced effect on the solar spectrum. The spectral distribution of the sun's radiation at the outer edge of the atmosphere and the normal energy distribution at the earth's surface are illustrated in Figure 29. Atmospheric conditions (e.g., degree of cloudiness; presence of fog, smoke, and dust; amount of water vapor) affect the degree of insolation attenuation. Weather conditions strongly affect the distribution of energy between sunlight and skylight (Piech and Walker, 1971), contributing a degree of uncertainty in water quality assessment through remotely sensed color measurements. Hulstrom (1973) has pointed out the adverse impact that cloud bright spots can have on remote sensing techniques which utilize reflected energy.

The degree of scattering and absorption imposed on the return signal from water bodies is related to the atmospheric transmittance and can result in changes in lake color when sensed at aircraft high flight and satellite altitudes. The attenuated return signal is also contaminated by electromagnetic radiation from the air column (path radiance). Rogers and Peacock (1973) have reported that solar and atmospheric parameters have a serious adverse impact on the radiometric fidelity of ERTS-1 data. Path radiance was found to account for 50 percent or more of the signal received by the MSS when viewing water and some land masses.

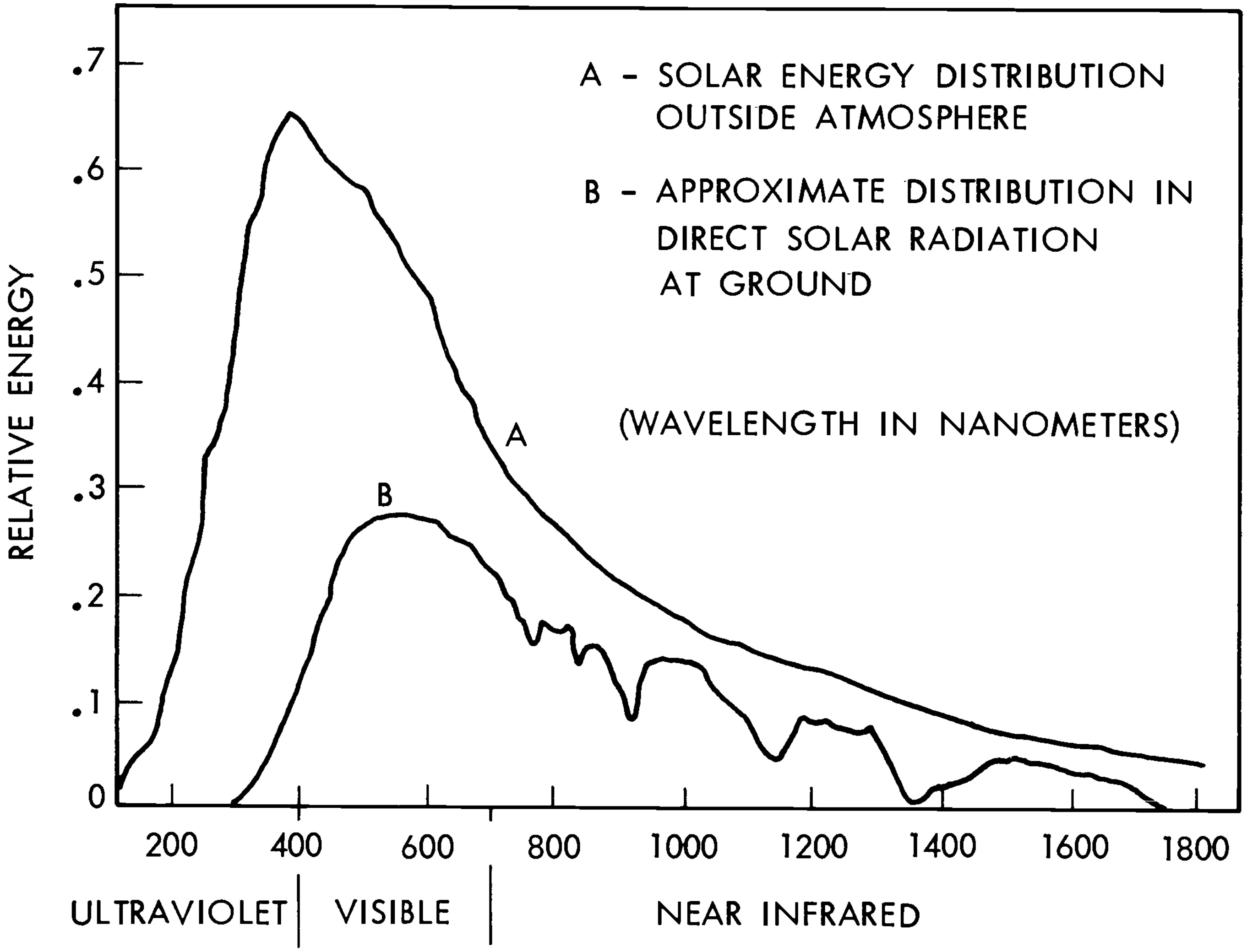

Figure 29. Spectral distribution of solar energy. Adapted from Hutchinson (1957).

The magnitude of the adverse atmospheric effects can be reduced, though not completely eliminated, by using imagery or digital data collected on clear, cloudless days. This is the approach used in this investigation.

The ERTS-1 spacecraft passes over the same point on the earth at essentially the same local time every 18 days. However, even though the time of passover will remain essentially the same throughout the year, solar elevation angle changes (Figure 30) cause variations in the lighting conditions under which the MSS data are obtained. The changes are due primarily to the north or south seasonal motion of the sun (NASA, 1972).

Changes in solar elevation angle produce changes in the average scene irradiance as seen by the sensor from space. The change in irradiance is influenced both by the change in the intrinsic reflectance of the ground scene and by the change in atmospheric backscatter (path radiance). The actual effect of changing solar elevation angle on a given scene is very dependent on the scene itself (NASA, 1972). For example, the intrinsic reflectance of sand is significantly more sensitive to changing solar elevation angle than are most types of vegetation (NASA, 1972). The effects of changing solar elevation angle are of particular importance when comparing scenes taken under significantly different angles. The use of color ratios in lieu of raw data values may be of value in reducing the magnitude of the solar angle induced effects by normalizing the brightness components. This is the approach used in this investigation.

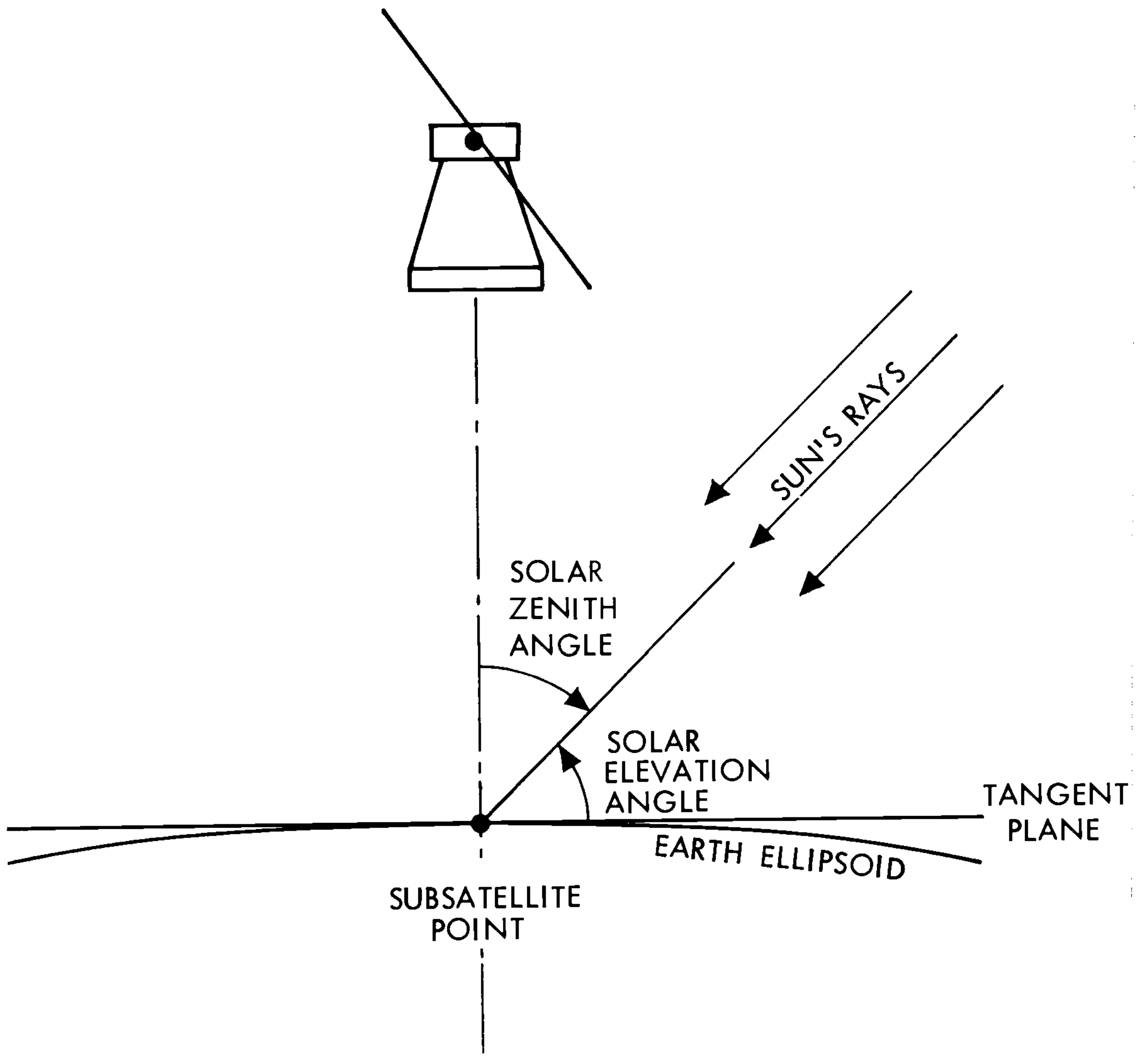

Figure 30. Solar zenith and solar elevation relationship. Source NASA (1972).

A portion of the radiation impinging on the lake surface will be reflected. The percentage of surface-reflected energy is a strong function of the angle of incidence (Figure 31). Surface roughness is known to have an effect on the percentages of light reflected and refracted at the interface (Jerlov, 1968). The effect of surface is negligible in estimating total radiation entering a water body when the solar angle is greater than 15 degrees (Hutchinson, 1957).

The light reflected from the interface is composed of diffuse light from the sky (skylight) and specularly reflected sunlight. Specular reflection areas contained in a scene are of little value in most water studies, the possible exception being the determination of surface roughness. The specularly reflected radiation exceeds, by several orders of magnitude, the reflected energy emanating from beneath the water surface (Curran, 1972). Surface reflected skylight, containing no water quality color information, can compose from 10 percent of the return signal on a clear day to 50 percent on a cloudy day (Piech and Walker, 1971). The surface reflected skylight not only increases the apparent reflectance from the water body (volume reflectance), but also affects the shape of the reflectance curve.

An electromagnetic wave impinging on the surface of a lake decomposes into two waves, one of which is refracted, proceeding into the aquatic medium and a second wave which is reflected back to the atmosphere (Jerlov, 1968). The wave entering the water is refracted as it passes through the air-water interface according to Snell's Law which may be expressed as:

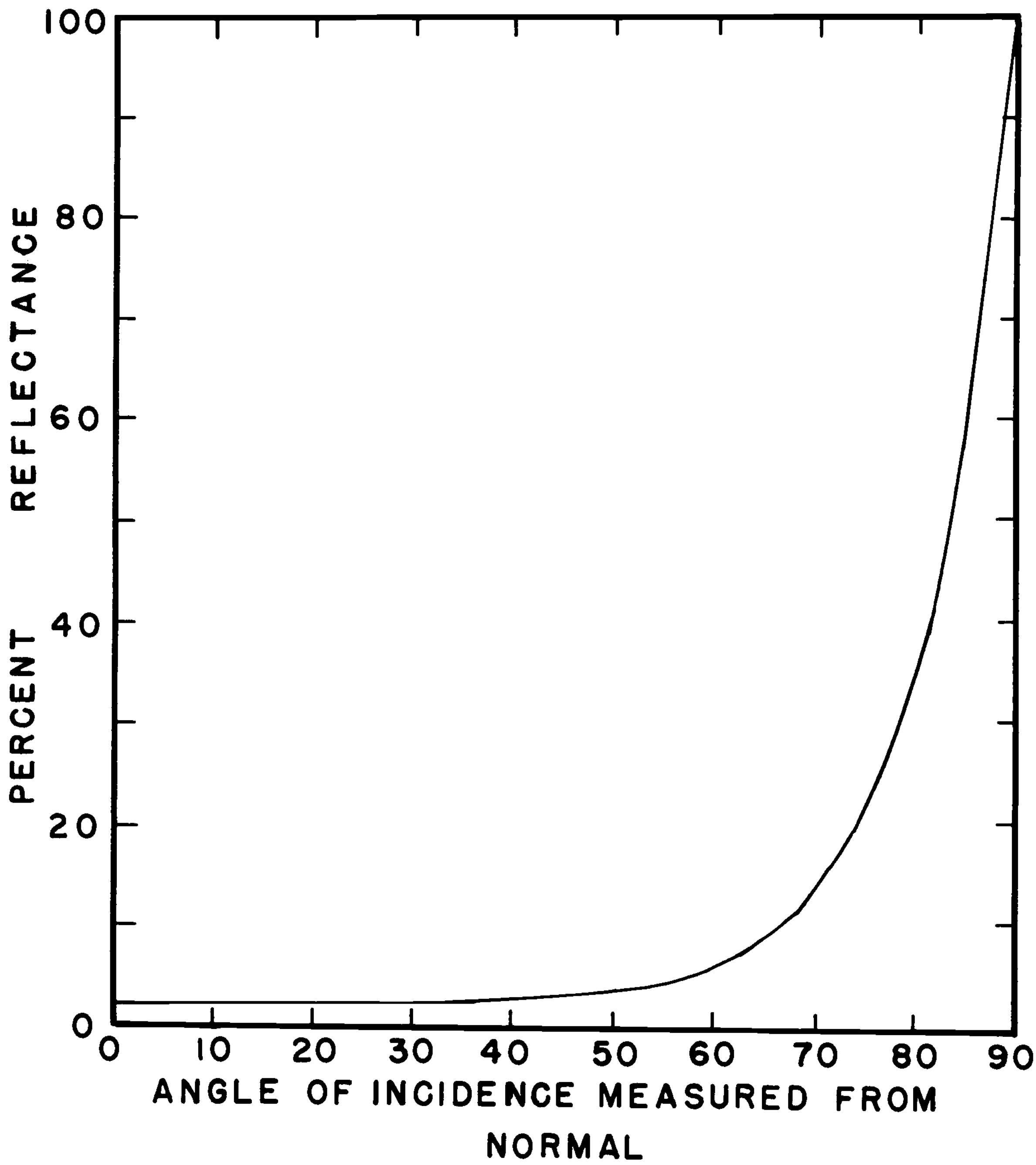

Figure 31. Percentage reflectance of the air-water interface as a function of the angle of incidence measured from normal direction. Values are for unpolarized light only. Source, Piech and Walker (1971).

$$\frac{\text{sine i}}{\text{sine r}} = n$$

where (i) is the angle of incidence, (r) is the angle of refraction, and (n) is the refractive index, which for water is approximately 1.33 (see Table 12).

Most of the electromagnetic energy entering a lake is attenuated through the process of absorption. Although only a small percentage (less than 3 percent; Davis, 1941) of the incident energy is backscattered from the lake water volume, this light (volume reflectance) is the focus of interest in the remote sensing of water quality investigations. Its spectral characteristics have been shaped by the materials found in the lake waters (dissolved and suspended materials, plankton, aquatic macrophytes, and air bubbles).

The lake bottom characteristics (color and composition) will also affect the intensity and/or the spectrum of the volume reflectance in settings where water transparency permits the reflection of a significant amount of radiation from the bottom materials. In studies involving the estimation of water depth or the mapping of bottom features, it is essential that the lake bottom be "seen" by the sensor. Bottom effects are capitalized upon and put to a beneficial use.

However, in this investigation, bottom effects are considered to be an undesirable peripheral effect. A sensor with the capabilities of the ERTS-1 MSS is not able to "see" much deeper into a lake than Secchi disc depth. The Secchi disc transparency of the selected NES lakes is, in most cases, relatively small when compared to the mean depth of each lake. The assumption is made, as a first approximation, that bottom effect is relatively insignificant when considering each of the selected lakes as an entity.

It is evident that many factors influence the intensity and spectral characteristics of the electromagnetic radiation which is collected by the sensor. A peripheral effect, desirable in one study, may have a deleterious effect in another study. Absolute quantification of remotely sensed phenomena requires that all of the effects be accounted for in the return signal.

Although the approach used here to reduce the magnitude of the undesirable peripheral effects might be criticized as simplistic or naive, it does serve as a starting point in the investigation of lake color-trophic indicator relationships using satellite-borne sensors. Although the results may be of a semi-quantitative nature, general trends in the data may be of value in the study of lake classification.

Relevant Remote Sensing Literature

A wealth of literature exists relating to the theoretical, applications, and instrumentation aspects of remotely sensed water quality. The advantages and limitations of remotely assessing water quality have been discussed by many investigators (Robinove, 1965; Hom, 1968; Kolipinski and Higer, 1968; Clarke, 1969; Fortunatov, 1969; Kiefer and Scherz, 1970, 1971; Conrod and Rottweiler, 1971; Grams and Boyle, 1971; Scherz, 1971; Clapp, 1972; Wezernak and Polcyn, 1972; Colwell, 1973).

Wezernak and Polcyn (1972) examine the question of making eutrophication assessments from the standpoint of current remote sensing technology. They suggest that of some 16 factors which often serve as measures of eutrophication (Table 13), several can be remotely sensed using operational or near-operational systems. The parameters include chlorophyll, colored water masses, suspended solids, transparency, aquatic macrophytes, and algal blooms. In addition, remote sensing technology can also provide an economic method of obtaining morphometric and land use information (e.g., shoreline development, lake surface area, and cultural impact) which has a direct bearing on the trophic state of a water body.

Most of the water-oriented investigations have focused on the oceanic environment and point source pollution of fresh waters using aerial photography (black and white, color, black and white infrared, color infrared, multispectral). Investigations utilizing airborne multispectral scanners and lasers are becoming increasingly common. Many aspects of water quality have been examined using remote sensors mounted on non-satellite platforms; pertinent aspects are discussed below.

Remotely determined estimates of chlorophyll levels in natural waters (in situ) have been made by several investigators (Clarke, et al., 1969, 1970; Mueller, 1972; Curran, 1972; White, 1969; Atwell and Thomann, 1972; Duntley, 1972; Arvensen, et al., 1971; Bressette and Lear, 1973). Curran (1972) reported a strong correlation between wavelength-dependent albedo ratios, made from measurements collected by high-flight aircraft, and phytoplankton chlorophyll concentrations, from satellite altitudes, to an uncertainty of 0.1 mg/m^3.

Table 13. Indices commonly used to assess eutrophication. Adapted from Wezernak and Polcyn (1972).

Standing crop of algae and aquatic plants*
Amount of suspended solids*
Volume of algae
Chlorophyll levels*
Number of algal blooms*
Transparency*
Plant regression*
Photosynthesis
Primary production
Aquatic plant nutrient content
Hypolimnetic oxygen concentrations
Sediment composition
Dissolved solids
Conductivity
Nutrient concentrations
Cation ratio (Na + K) / (Mg + Ca)

*Index can be remotely sensed using operational or near-operational sensors.

Blackwell and Billingsley (1973) have utilized multispectral photography in conjunction with digital computer enhancement techniques to detect algae. Crew (1973) has mapped and identified an algal species in Clear Lake (California) employing an airborne multispectral scanner.

Aerial photography, utilizing color and color-infrared films, is a relatively simple technique that has demonstrated utility in mapping aquatic macrophytes (Kolipinski and Higer, 1968; Kiefer and Scherz, 1971) and shallow water benthos (Kelly and Conrod, 1969). Photographs are used routinely to map wetlands (Anderson and Wobber, 1973). It is an accepted practice to use aerial photographs in the delineation and enumeration of lakes (Minnesota Department of Conservation, 1968).

Remote sensors are employed to estimate water turbidity (Crew, 1973; Schmer, *et al.*, 1972, Williams and Samol, 1968). Some investigators (Brown, *et al.*, 1972; Specht, *et al.*, 1973) have used remote sensors to estimate water depth and bottom topography.

Dybdahl (1973) has demonstrated an airborne remote sensing technique for the determination of dissolved oxygen levels in fresh water.

Relevant ERTS-1 Investigations

Prior to the insertion of ERTS-1 into its polar orbit, investigators interested in assessing and monitoring the earth's natural resources from satellite altitudes were restricted to using poorly suited satellite-borne sensors. The spacecraft were largely equipped with sensors designed for studies oriented toward the atmosphere or, in some cases, the ocean.

The length of the list of NASA-designated ERTS-1 principal investigators -- some 300 are found on it (Data Users Handbook, Appendix M, February, 1972) -- suggests that there is a great interest in assessing the advantages and limitations of remotely sensing the earth's resources from satellite altitudes. The major focus, as evidenced by project titles, is on the terrestrial environment; less than 10 percent of the investigations relate to water quality and the majority of these concentrate on the oceanic environment and the Laurentian Great Lakes. Some of the water-related investigations bear mentioning.

Wezernak and Polcyn (1973) have detected a municipal-industrial waste disposal area in the New York Bight. Lind and Henson (1973) and Lind, et al. (1973) have discovered and documented a pollution plume emanating from a mill located on the shore of Lake Champlain. It is apparent that the detection of large turbidity plumes and turbidity related patterns is easily accomplished using the MSS (Watanabe, 1973; Klemas, et al., 1973; Carlson, et al., 1973; Wright, et al., 1973; Pluhowski, 1973; Coker, et al., 1973; Kritikos, et al., 1973).

Enhanced and density-contoured imagery appears to permit water depth estimation to a depth of at least five meters in the clear waters around the Bahama Islands (Ross, 1973). Polcyn and Lyzenga (1973), using digital processing techniques on an ERTS-1 MSS frame taken over the Bahama Islands, have mapped shallow water features and calculated water depths to five meters. The waters in the area are known for their clarity and are conducive to remote sensor mapping activities.

Hidalgo, et al. (1973) have identified large mats of duckweeds (Lemnaceae) on Lake Pontchartrain and surrounding bayous and swamps in southeastern Louisiana. Seasonal changes have been detected. Strong (1973) has noted algal blooms in Utah Lake (Utah) and in Lake Erie.

Barr (1973) has mapped a total of 2,272 water bodies in Saline County (Kansas) where a topographic map (1955) indicates 1,056. A preliminary comparison of imagery and maps indicates that bodies larger than four hectares (10 acres) are consistently detectable. Most water bodies between about two to four hectares are usually detectable; water areas less than two hectares are occasionally resolved. Erb (1973), using nine-inch-square transparencies, has determined that ponds as small as one hectare are detectable within forested areas. The preliminary findings of Chase and Reed (1973) indicate that water bodies of about 0.5 hectares are detectable under fair conditions (haze and 70 percent cloud cover); distortion of lake size, shape, and orientation is minimal. The benefits of using ERTS-1 imagery to enumerate lakes have been demonstrated by Reeves (1973) and Work, et al. (1973).

Yarger, et al. (1973), using electronically sliced imagery, have found a good correlation between suspended load (predominantly composed of inorganic materials) and film densities of two federal reservoirs in Kansas. Bowker, et al. (1973) reported that there appears to be a positive correlation between particulate count and chlorophyll level in Lower Chesapeake Bay. A rough correlation exists between suspended sediments and MSS imagery, but no correlation is apparent for the chlorophyll bearing portion of the load. Szekielda and Curran (1973) indicate that a correlation exists between chlorophyll and ERTS-1 MSS imagery (green band) in the general area of St. John's River estuary in Florida.

Rogers and Smith (1973), reporting on their investigation of six lakes in Michigan, suggested that deep water and shallow water can be separated by a trained photo-interpreter using reflectance printout gray-scales or a cathode tube monitor. Lakes were best discriminated in band seven (IR2) due to the strong contrast between land and water (Rogers and Smith, 1973). Classifying lake eutrophication on the basis of algal scum or macrophytes in shallow water appears to be a "straight-forward" matter with ERTS-1 MSS data. Estimations of water depth may be possible in some lake areas where there are extensive shallows and the water is clear and unencumbered with vegetation (early spring).

Trophic Indicator Estimation

Although the ERTS-1 provides 18-day cyclic coverage, a point stressed in many of the reports and articles written on the spacecraft, obtaining good coverage in the study area was (and still is) a major problem. This is a problem shared with many other investigators. On numerous dates of coverage, cloud cover was excessive (greater than 10 percent) and on several occasions, when the weather conditions were conducive to monitoring the lakes, one or more of the MSS bands were either missing or rated as poor. The formation of ice cover on the study lakes, lasting for several months, further reduced the opportunity to obtain repetitive coverage.

A search of the ERTS-1 MSS imagery and CCT's for the study area (Minnesota, Wisconsin, Michigan, New York) for the period from August, 1972 through July, 1973, resulted in the selection of the frames found in Table 14. The lakes which were examined in this investigation are listed in Table 15. Several additional frames were not included because they contain less than three lakes sampled under the National Eutrophication Survey Program. Frames from the 1973 calendar year were not selected if ERTS-1 MSS coverage was not available for the 1972 sampling year.

Every effort was made to get MSS coverage which was concurrent with the collection of the ground truth by the helicopter-borne field teams. This goal was achieved in the case of only one frame (1080-15180). Unfortunately the number of NES sampled lakes contained in the frame is too small to incorporate into a regression model. The dates on which the ground truth was collected from the study lakes are found in Appendix B.

Ideally, the estimation of the magnitude of trophic state indicators should be done using concurrent data to derive the maximum benefit. However, in this investigation it is necessary to use what may be called "near-concurrent" ground truth which was collected several days before or after the day of satellite overflight. Nevertheless, models developed from such a temporal arrangement are of some

Table 14. ERTS-1 MSS frames. Lakes extracted from frames recorded on the same flightline and data are pooled to increase sample size.

Frame Number	Date	Area	Number of Lakes
1017-16091	9 August 1972	Eastern Wisconsin	5*
1017-16093	9 August 1972	Southeastern Wisconsin	15*
1036-16152	28 August 1972	South central Wisconsin	6
1323-16094	11 June 1973	Eastern Wisconsin	4*
1323-16100	11 June 1973	Southeastern Wisconsin	23*
1359-16091	17 July 1973	Eastern Wisconsin	4*
1359-16094	17 July 1973	Southeastern Wisconsin	18*
1022-16373	14 August 1972	Central Minnesota	12
1075-16321	6 October 1972	Central Minnesota	15
1077-16431	8 October 1972	West central Minnesota	10
1309-16325	28 May 1973	Central Minnesota	13
1345-16322	3 July 1973	East central Minnesota	14
1346-16381	4 July 1973	Central Minnesota	8
1027-15233	19 August 1972	Northwestern New York	7
1080-15180	11 October 1972	Northwestern New York	5

*Some of the lakes appear in both frames; e.g., Lake Winnebago is often split between two frames.

Table 15. Dates of ERTS-1 coverage.

Lake Name and Serial Number		ERTS-1 Coverage Date 1972: 9 Aug	1972: 14 Aug	1972: 19 Aug	1972: 28 Aug	1972: 6 Oct	1972: 8 Oct	1972: 11 Oct	1973: 28 May	1973: 11 June	1973: 3 July	1973: 4 July	1973: 17 July
Shawano	46									X			X
Butte des Morts	47	X								X			X
Poygan	48	X								X			X
Winnebago	49	X								X			X
Green	51	X								X			X
Beaver Dam	53	X								X			X
Kegonsa	54	X			X					X			X
Rock	55	X			X					X			X
Koshkonong	56	X			X					X			X
Lac la Belle	57	X								X			X
Oconomowoc	58	X								X			X
Okauchee	59	X								X			X
Pine	60	X								X			X
Nagawicka	61	X								X			X
Pewaukee	62	X								X			X
Tichigan	63	X								X			X
Browns	64	X								X			X
Middle	65	X								X			X
Delavan	66	X								X			X
Como	67	X								X			X
Geneva	68	X								X			X
Mendota*	108				X					X			
Monona*	109				X					X			
Waubesa*	110				X					X			
Darling	16		X				X						
Carlos	17						X						
Le Homme Dieu	18		X				X						
Minnewaska	19		X				X						
Nest	20		X				X					X	
Green	21		X				X					X	
Wagonga	22		X				X					X	
Clearwater	23		X			X			X		X		
Maple	24		X			X			X		X		
Cokato	25		X			X			X		X	X	
Buffalo	26		X			X			X		X	X	
Carrigan	27					X			X		X	X	

Table 15 (continued)

Lake Name and Serial Number		ERTS-1 Coverage Date 1972							1973				
		9 Aug	14 Aug	19 Aug	28 Aug	6 Oct	8 Oct	11 Oct	28 May	11 June	3 July	4 July	17 July
Silver	28		X			X			X		X	X	
Minnetonka	29		X			X					X		
Forest	30					X			X		X		
White Bear	31					X			X		X		
St. Croix	32					X			X		X		
Spring	33					X			X		X		
Pepin	34					X							
Madison	35					X			X		X		
Sakatah	36								X		X		
Winona	101						X						
Trace	102						X						
Calhoun	103					X			X		X		
Big Stone	104						X						
Zumbro	105					X							
Cottonwood	111											X	
Conesus	91			X									
Canandaigua	92			X									
Keuka	93			X									
Seneca	94			X									
Cayuga	95			X				X					
Owasco	96			X				X					
Cross	97			X				X					
Oneida	106							X					
Canadarago	107							X					

*This lake fell outside the scope of the National Eutrophication Survey Program, but was included here because it is a well-known lake on which considerable eutrophication research has been conducted.

value in illustrating general relationships existing between the MSS data and ground truth.

Estimations of the magnitude of two trophic state indicators, using ERTS-1 MSS data and NES collected ground truth, are demonstrated in the remainder of this chapter using two of the frames in Table 14. The frames, 1017-16091 and 1017-16093, are treated as one frame to increase the lake sample size. They are in juxtaposition on the same flightline, were collected on the same day, and, prior to Goddard Space Flight Center processing, were elements in a continuous strip. The frames were selected on the basis of temporal proximity to NES ground truth dates, quality of the MSS data (good MSS bands, little cloud cover and haze), and the presence of a relatively large number of NES lakes (N = 20). An examination of Table 15 indicates that coverage is available for the same 20 lakes on two other occasions.[15]

Data reduction was accomplished on the Oregon State University CDC 3300 computer using the Statistical Interactive Programming System (SIPS). Regression models were developed using the backward selection procedure (Guthrie, _et_ _al._, 1973).

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[15]MSS frames from 12 different dates and three states are examined in this thesis. The fragmentary ERTS-1 coverage, very evident in Table 15, makes it difficult to give a coherent demonstration of the MSS capabilities and limitations. In an attempt to reduce the magnitude of the problem, the author has taken the liberty of focusing on the Wisconsin frames and relegating the other frames to Appendix D. Appendix D is divided into a series of subappendices, one for each ERTS-1 date, which contain descriptive statistics of MSS data, ERTS-1 MSS estimates of lake area, area ratios, lake concatenations, three-dimensional color ratio models, and regression models.

ERTS-1 Lake Area Estimation

A straightforward method of estimating the area of an extracted lake involves a summation of the pixel counts in one band (e.g., IR2) and subsequent multiplication by a conversion factor of 0.48 (1 pixel = 0.48 ha = 1.18 ac)[16] to get the area in hectares. The surface areas of the 20 lakes extracted from Frames 1017-16091 and 1017-16093 are generally within 10% of values derived from U.S.G.S. topographic sheets and publications of the Wisconsin Department of Natural Resources (see Table 16).

A comparison of lake area estimations from other sources with ERTS-1-derived area values acts as a check to verify that the pixels and their respective DN's are those of the water body under scrutiny. A topographic area/ERTS-1 area ratio greater than one indicates that the extracted image is smaller than the area covered by the MSS lake image. A ratio less than one suggests that the extracted image includes wet lands and/or land features. Underestimation of lake size is preferable to overestimation (inclusion of wetland and/or land) in this study.

Discrepancies between ERTS-1 and topographic sheet-derived area estimates may be a consquence of: changes in the lake area since the topographic sheet was constructed; inaccurate delineation of the shoreline on aerial photographs or during the preparation of the sheet; inclusion of wetlands or exclusion of portions of the lake during

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[16] The conversion factor used here was obtained from R. J. Blackwell (personal communication, 1973).

Table 16. Areal aspects of 20 NES-sampled lakes extracted from ERTS-1 MSS Frames 1017-16091 and 1017-16093.

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area: ERTS Area Ratio
Poygan	47	9,177	4,382.5	4,448.5	1.015
Butte des Morts	48	7,395	3,531.5	3,584.4	1.015
Winnebago	49	114,186	54,529.1	55,730.4	1.022
Green	51	6,613	3,158.0	2,972.9	0.942
Beaver Dam	53	5,162	2,465.1	2,671.0	1.084
Kegonsa	54	2,675	1,277.4	1,099.2	0.861
Rock	55	1,009	481.8	554.8	1.152
Koshkonong	56	9,247	4,415.9	4,241.3	0.961
Lac la Belle	57	944	450.8	452.1	1.003
Oconomowoc	58	652	311.4	317.7	1.015
Okauchee	59	871	415.9	450.8	1.085
Pine	60	568	271.3	284.5	1.049
Nagawicka	61	751	358.6	415.2	1.155
Pewaukee	62	1,996	953.2	1,008.9	1.058
Tichigan	63	689	329.0	449.8	1.367
Browns	64	309	147.6	160.3	1.086
Middle	65	117	55.9	104.8	1.876*
Delavan	66	1,541	735.9	717.9	0.976
Como	67	703	335.7	383.0	1.151
Geneva	68	4,543	2,169.5	2,129.5	0.982

*Middle Lake is contiguous with Mill Lake and Green Lake; the lakes are commonly referred to as the Lauderdale Lakes. The extraction process resulted in a sample of the area mapped as Middle Lake. The Green Lake of the Lauderdale lakes is not to be confused with Green Lake (51).

the preparation of the IR2 binary mask; or failure to use a correct conversion factor.

The estimated lake areas using ERTS-1 MSS pixel counts are in good agreement for most of the lakes in Table 16. The very large error for Middle Lake is due to the contiguous nature of the Lauderdale Lakes. The computer extraction process resulted in a sample taken from the area mapped as Middle Lake. Some of the lakes, particularly those which occupy shallow basins, are known to fluctuate greatly in surface area and topographic sheets do not account for the variation.

Adding the 20 lake areas derived from the topographic sources and then comparing the resultant value with the composite ERTS-1 MSS lake area value, excluding Middle Lake, results in a ratio of 1.016: 1.000. A visual examination of the area ratios for the other frames (see the subappendices of Appendix D) indicates that the MSS can be used to give good estimates of lake surface area when a DN value of 28 is used as the "cutoff" point in extracting the lakes from their terrestrial matrix. The lake area estimation capabilities of the MSS are of value, not only in the study of lakes with established areas, but also in geographic regions for which there is no accurate topographic or aerial photographic coverage.

The photographic products (concatenations), fabricated by the VICAR system, are of immense value in extracting the lake "images" stored on the CCT's. They permit the investigator to examine the extracted lake for geometric fidelity, truncation of the water body, and for the inclusion of other water bodies. The photographic products are superior to output in the form of line printer copy in studies relating to the areal aspects of water bodies.

Secchi Disc Transparency Estimation

Although it is beyond the capabilities of the MSS to directly measure chemical indicators, its areal and spectral resolution permit the detection of phenomena related to eutrophication (such as Secchi disc transparency, chlorophyll a) together with some degree of quantification.

An R-mode Pearson product-moment correlation analysis was made using the MSS data and ground truth collected from the 20 NES lakes in Frames 1017-16091 and 1017-16093. The correlations, based on data means and transformed means, are found in Table 17.

Several multiple regression models were developed to predict Secchi disc transparency using MSS colors and color ratios as the independent variables. The use of color ratios as independent variables has appeal because the use of ratios tends to normalize the data by removing the brightness components. The best model, as measured by the magnitude of its coefficient of multiple determination (R^2) and standard error of estimate, for estimating Secchi disc transparency is:

$$\widehat{\text{LNSECCHI}} = 0.784 + 2.638\ \text{GRNIR1} - 3.731\ \text{REDIR1} - 0.754\ \text{IR1IR2}$$

The model explains approximately 87 percent of the variance about the mean (Table 18). The observed and predicted Secchi disc transparency values for the 20 lakes are given in Table 19.

The most glaring disparity occurs with Middle Lake. Several factors may individually or collectively account for the large residual,

Table 17. Correlations between ground truth and ERTS-1 MSS data (colors and color ratios) for 20 lakes in Frames 1017-16091 and 16093.

	PC1	CHLA	LNCHLA	SECCHI	LNSECCHI
GRN	0.518	0.812	0.718	-0.623	-0.662
RED	0.722	0.888	0.860	-0.788	-0.857
IR1	0.807	0.899	0.886	-0.741	-0.866
IR2	0.589	0.680	0.696	-0.492	-0.642
GRNRED	-0.823	-0.821	-0.886	0.865	0.919
GRNIR1	-0.806	-0.777	-0.838	0.685	0.803
GRNIR2	-0.470	-0.442	-0.521	0.357	0.476
REDIR1	-0.544	-0.476	-0.505	0.274	0.430
REDIR2	-0.026	0.028	-0.017	-0.156	-0.042
IR1IR2	0.516	0.522	0.474	-0.527	-0.507

Table 18. Analysis of variance table of a regression model for the prediction of Secchi disc transparency. The model was developed using MSS data from Frames 1017-16091 and 1017-16093.

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	16.340	0.860	
Regression	3	14.152	4.717	34.431
Residual	16	2.188	0.137	

R^2 = 0.8661 x 100 = 86.61% s.e. of estimate = 0.370 (1.448m)

Table 19. Secchi disc transparency residuals. The model was developed using MSS data from Frames 1017-16091 and 1017-16093.

Lake Name	Serial Number	SECCHI (m)	$\widehat{\text{SECCHI}}$ (m)	SECCHI-$\widehat{\text{SECCHI}}$ (m)
Poygan	47	0.46	0.55	-0.09
Butte des Morts	48	0.52	0.48	0.04
Winnebago	49	0.71	0.51	0.20
Green	51	5.69	4.87	0.82
Beaver Dam	53	0.38	0.58	-0.20
Kegonsa	54	0.99	0.55	0.44
Rock	55	2.39	2.68	-0.29
Koshkonong	56	0.32	0.35	-0.03
Lac la Belle	57	1.91	1.42	0.49
Oconomowoc	58	3.51	3.27	0.24
Okauchee	59	1.83	0.64	1.19
Pine	60	3.05	4.85	-1.80
Nagawicka	61	0.91	1.03	-0.12
Pewaukee	62	1.82	1.18	0.64
Tichigan	63	0.57	0.70	-0.13
Browns	64	2.13	1.97	0.26
Middle	65	5.18	2.57	2.61
Delavan	66	0.58	1.07	-0.49
Como	67	0.41	0.55	-0.14
Geneva	68	3.37	4.53	-1.16

including: bottom effects, presence of extensive beds of aquatic macrophytes, the overlap of lake pixels onto land, or some still unsuspected factor. Middle Lake has good water clarity, is relatively shallow, and does have a problem with submerged aquatic macrophytes.

Although the number of observations is limited (N = 20) and the ground truth is sparse, it is apparent that the ERTS-1 MSS can be used to estimate Secchi disc transparency in freshwater lakes.

An effort was made to estimate the Secchi disc transparency of 11 Minnesota lakes found in Frame 1022-16373. The regression model, found in subappendix D5, must be viewed with caution because the ground truth was collected approximately two and one-half weeks after the date of ERTS-1 coverage. Lakes are dynamic and significant changes may have occurred during the intervening period.

Chlorophyll *a* Estimation

Bressette and Lear (1973), using an infrared photographic technique, have detected algal blooms (primarily *Anacystis*) in the "salt wedge" area of the Potomac River near Maryland Point. The inherent characteristics of the spectral reflectance curve for chlorophyll-bearing plants (Figure 32) were used to advantage.

The reflectance of the chlorophyll-bearing plants varies greatly as illustrated by the curve with an abrupt increase at about 700 nanometers. Although the water tends to attenuate the infrared energy in a relatively short distance, the magnitude of the plant reflectance in

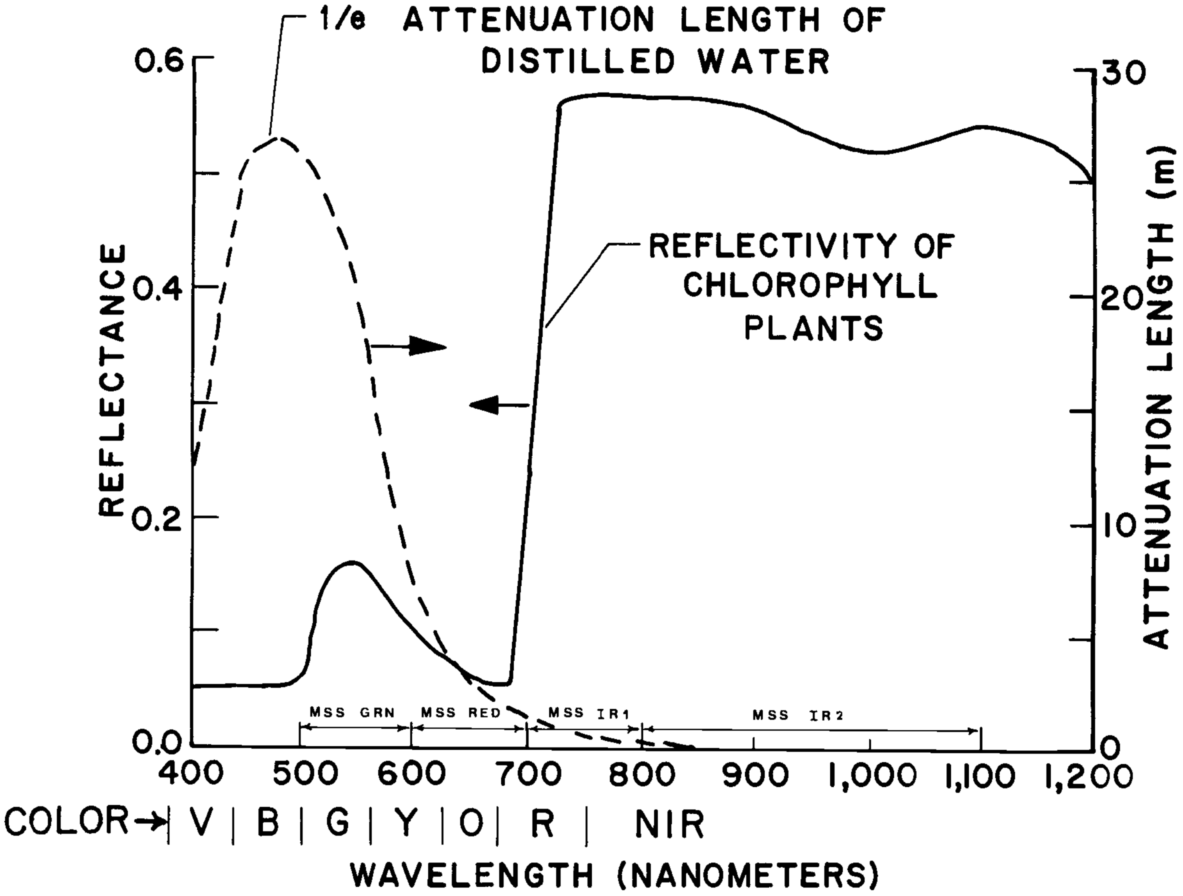

Figure 32. Comparison of the reflectance of chlorophyll-containing plants with the attenuation length of sunlight in distilled water. From Bressette and Lear (1973). The attenuation curve is based on Spiess (1970) and the plant reflectivity curve on Katzoff (1962).

the near-infrared allows the detection of chlorophyll-bearing plants on or near the water surface. Bressette and Lear (1973) proposed that "it is also probable that the magnitude of the reflected solar energy in the NIR will also depend upon the concentration of phytoplankton in the water ..." and demonstrated their infrared technique for the mapping of chlorophyll a concentrations.

An examination of the correlations between the MSS data (colors and color ratios) and the chlorophyll a data from the 20 NES-sampled lakes (Table 17) suggests that it may be possible to estimate mean chlorophyll a levels using the MSS data.

A multiple regression analysis yielded the model:

$$\widehat{\mathrm{LNCHLA}} = 6.905 - 3.662\ \mathrm{GRNIR1} + 0.636\ \mathrm{GRNIR2}$$

The model explains about 83 percent of the variance about the mean (Table 20). The observed and predicted chlorophyll a values along with the residuals are found in Table 21.[17]

The model does a fair job of estimating the mean chlorophyll a level in the 20 lakes. Large residuals are evident, primarily in the cases of lakes which have very poor water quality. Although the model is purported to estimate chlorophyll a, caution must be exercised in assuming that an absolute measurement of the chlorophyll a

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[17]An attempt to develop a regression model for the estimation of chlorophyll a levels in 11 Minnesota lakes (Frame 1022-16373) drew negative results.

Table 20. Analysis of variance table of a regression model for the prediction of chlorophyll a levels. The model was developed using MSS data from Frames 1017-16091 and 1017-16093.

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	23.998	1.263	
Regression	2	19.876	9.938	40.897
Residual	17	4.123	0.243	

$R^2 = 0.8282 \times 100 = 82.82\%$ s.e. of estimate = 0.493

Table 21. Chlorophyll a residuals. The model was developed using MSS data from Frames 1017-16091 and 1017-16093.

Lake Name	Serial Number	CHLA (µg/l)	$\widehat{\text{CHLA}}$ (µg/l)	CHLA-$\widehat{\text{CHLA}}$ (µg/l)
Poygan	47	29.7	17.0	12.7
Butte des Morts	48	26.5	29.9	- 3.4
Winnebago	49	27.2	33.7	- 6.5
Green	51	0.8	2.4	- 1.6
Beaver Dam	53	14.1	26.0	-11.9
Kegonsa	54	26.9	24.4	2.5
Rock	55	4.8	3.7	1.1
Koshkonong	56	23.0	41.3	-18.3
Lac la Belle	57	4.9	3.5	1.4
Oconomowoc	58	1.5	2.2	- 0.7
Okauchee	59	4.1	3.9	0.2
Pine	60	3.8	2.1	1.7
Nagawicka	61	13.5	9.0	4.5
Pewaukee	62	6.9	8.9	- 2.0
Tichigan	63	20.5	19.0	1.5
Browns	64	5.1	5.5	- 0.4
Middle	65	4.1	5.5	- 1.4
Delavan	66	30.3	16.9	13.4
Como	67	29.1	14.1	15.0
Geneva	68	2.1	2.2	- 0.1

level of each lake has been achieved. The spectral and spatial resolution of the scanner is low, and many factors influence the signal returned to it.

In this case, a strong inverse correlation exists between the trophic indicators LNSECCHI and LNCHLA (-0.912), and some of the return signal is undoubtedly due to the presence of chlorophyll-bearing plants. The impact of different types and concentrations of inorganic suspensoids on chlorophyll *a* estimation are not known. The estimates of chlorophyll *a* derived from the regression model are more properly treated as index numbers than as absolute values.

V. LAKE CLASSIFICATION USING ERTS-1 MSS DATA

General relationships between MSS data and the trophic state of NES-sampled lakes, as defined by lake position on the first principal component axis (PC1 value), are examined in this chapter using regression analysis, three-dimensional color ratio models, and color-enhanced photographic products generated through automatic classificatory techniques.

ERTS-1 MSS data recorded on 12 different dates (Table 14) were examined in this investigation. Although the temporal coverage of the NES-sampled lakes (Table 15) is fragmentary, a relatively coherent time series exists for 20 lakes in eastern-southeastern Wisconsin. This chapter will focus on the MSS frames obtained from the Wisconsin area on three occasions (9 August 1972, 11 June 1973, and 17 July 1973). Reference will be made to the other MSS coverage dates and frames which have been largely relegated to Appendix D.

Trophic State Index Prediction Using ERTS-1 MSS Data

As was discussed in Chapter II, trophic state is a multi-dimensional concept and can not be adequately assessed by measuring any single indicator. With this in mind, 100 NES-sampled lakes were subjected to the multivariate technique of principal component analysis using LN transformed measurements of six trophic state indicators (LNISEC, LNCHLA, LNTPHOS, LNCOND, LNAAY, LNTON). The position of each lake on the first principal component axis, the PC1 value, is purported to be a trophic state index. The larger the PC1 value, the closer the water body is to the eutrophic end of the scale and, conversely, the smaller the PC1 value, the closer the lake is to the oligotrophic end of the scale.

In Chapter IV the utility of using the ERTS-1 MSS data for the prediction of Secchi disc transparency and chlorophyll a levels was examined. While the estimation of specific trophic indicators is of value, the question arises, "Can the position of a lake on the first principal component axis be predicted using MSS data?"

Investigations relating to lake appearance as related to trophic status (Chapter IV) support the premise that the multitude of interactions occuring within a lake give the lake volume reflectance, an intensity and spectral curve which is indicative of its trophic state. While it can be argued that the MSS lacks the spectral resolution to detect some of the variables incorporated into the trophic state index (such as conductivity and total phosphorus), the elimination of the variables would make the index less stable and therefore more susceptible to a large deviation from normal for a given indicator.

PC1-MSS Regression Analyses

The Wisconsin MSS frames are treated in chronological order starting with those collected on 9 August 1972. Regression models are developed for the prediction of the trophic state index (PC1) values for 20 NES-sampled lakes for each sampling data and then for the same lakes on the basis of mean MSS values for the three dates of ERTS-1 coverage.

The product-moment correlation coefficients between the PC1 values for the 20 Wisconsin lakes and the MSS data (colors and color ratios) extracted from MSS frames recorded on the three MSS sampling dates are found in Table 22. The correlations of 9 August 1972 and 17 July 1973 are of similar magnitudes.

Numerous regression models were developed during the investigation; only the "best" models are presented in the thesis. Criteria used in the selection of the "best" models included the magnitude of the coefficient of multiple determination (R^2) and the standard error of estimate. All regression coefficients were required to be significant at the 0.05 level.

The best regression model for the prediction of the trophic state index (PC1) values of the 20 Wisconsin lakes for 9 August 1972 is:

$$\widehat{PC1} = 7.682 - 6.074\ GRNIR1 + 1.155\ GRNIR2$$

The model explains about 81 percent of the variance about the mean and has a standard error (s.e.) of estimate of 0.840 (Table 23).

Table 22. Correlations between ERTS-1 MSS data (colors and color ratios) collected on three dates and the trophic status of 20 Wisconsin lakes.

	Date of ERTS-1 Flyover		
MSS Colors & Ratios	9 August 1972	11 June 1973	17 July 1973
GRN	0.518	0.151	0.479
RED	0.722	0.533	0.721
IR1	0.807	0.512	0.836
IR2	0.589	0.174	0.628
GRNRED	-0.823	-0.712	-0.749
GRNIR1	-0.806	-0.540	-0.820
GRNIR2	-0.470	-0.091	-0.485
REDIR1	-0.544	0.084	-0.422
REDIR2	-0.026	0.298	0.109
IR1IR2	0.516	0.445	0.479

Table 23. Analysis of variance table of a regression model for the prediction of the trophic status of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1017-16091 and 1017-16093 (9 August 1972).

		Analysis of Variance		
Source	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	62.555	3.292	
Regression	2	50.572	25.286	35.867
Residual	17	11.983	0.705	

R^2 = 0.8084 x 100 = 80.84%   s.e. of estimate = 0.840

While the R^2 value is not particularly large, it must be kept in mind that the PC1 was developed using mean values of the ground truth measurements taken on three occasions during the 1972 open water season. The MSS data were collected within a few seconds on 9 August 1972.

An examination of the residuals ($PC1-\widehat{PC1}$) in Table 24 reveals relatively large absolute values for Middle, Tichigan, Pine, Beaver Dam, and Butte des Morts. Middle Lake and Butte des Morts are predicted to be in worse condition (closer to the eutrophic end of the scale) than their PC1 values indicate. The other three lakes are estimated to be in better condition (closer to the oligotrophic end of the scale) than their PC1 values suggest. The model produced negative results when it was used to predict the trophic state index values of lakes in other MSS frames.

Table 24. Trophic state index (PC1) residuals of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1017-16091 and 1017-16093 (9 August 1972).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	$PC1-\widehat{PC1}$
Poygan	47	1.68	1.41	0.27
Butte des Morts	48	1.27	2.39	-1.12
Winnebago	49	2.36	2.70	-0.34
Green	51	-1.67	-1.53	-0.14
Beaver Dam	53	3.29	2.14	1.15
Kegonsa	54	1.48	2.13	-0.65
Rock	55	-1.21	-0.85	-0.36
Koshkonong	56	2.45	2.87	-0.42
Lac la Belle	57	-1.43	-0.94	-0.49
Oconomowoc	58	-1.82	-1.82	0.00
Okauchee	59	-0.62	-0.89	0.27
Pine	60	-0.71	-1.88	1.17
Nagawicka	61	1.27	0.58	0.69
Pewaukee	62	0.59	0.45	0.14
Tichigan	63	3.11	1.51	1.60
Browns	64	-1.07	-0.43	-0.64
Middle	65	-2.29	-0.56	-1.73
Delavan	66	2.03	1.46	0.57
Como	67	1.15	1.18	-0.03
Geneva	68	-1.71	-1.78	0.06

A regression model was developed for the same 20 Wisconsin lakes using MSS data collected by the ERTS-1 on 11 June 1973. The model:

$$\widehat{PC1} = 42.761 - 18.423\ GRNRED - 18.948\ REDIR1 + 3.057\ GRNIR2$$

explains about 70 percent of the variation about the mean and has a standard error of estimate of 1.087 (Table 25). The residuals are displayed in Table 26. The model is of very limited value in the prediction of trophic state.

The MSS data collected on 17 July 1973 were also used to develop a regression model for the prediction of the PC1 values for the same lakes with the results:

$$\widehat{PC1} = 140.430 - 9.375\ GRNRED - 98.913\ REDIR1 + 45.672\ REDIR2 - 57.330\ IR1IR2$$

This model explains about 81% of the variance about the mean and has a standard error of estimate of 0.892 (Table 27). The residuals are displayed in Table 28.

No single model has been developed to predict the trophic index values for the 20 lakes on each of the three dates.

Although the lake color ratios vary from frame to frame (date to date), it is not unreasonable to expect that over a period of time a lake would present an average color or appearance, which would be more representative of the water body's position on a trophic scale. With this in mind, a new regression model was developed using mean color ratios derived from the three dates of ERTS-1 coverage. For example, a lake's mean GRNRED color ratio was determined by establishing its GRNRED ratio for each of the three ERTS-1 dates, summing these ratios and dividing by the number of ERTS-1 dates. Stated more simply,

$$XGRNRED = (GRNRED_1 + GRNRED_2 + GRNRED_3) / 3$$

Table 25. Analysis of variance table of a regression model for the prediction of the trophic status of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1323-16194 and 1323-16100 (11 June 1973).

		Analysis of Variance		
Source	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	62.555	3.292	
Regression	3	43.649	14.550	12.313
Residual	16	18.906	1.182	

$R^2 = 0.6978 \times 100 = 69.78\%$ s.e. of estimate = 1.087

Table 26. Trophic state index (PC1) residuals of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1323-16194 and 1323-16100 (11 June 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	$PC1-\widehat{PC1}$
Poygan	47	1.68	1.19	0.49
Butte des Morts	48	1.27	1.52	-0.25
Winnebago	49	2.36	1.30	1.06
Green	51	-1.67	-0.45	-1.22
Beaver Dam	53	3.29	3.29	0.00
Kegonsa	54	1.47	0.05	1.42
Rock	55	-1.21	-0.63	-0.58
Koshkonong	56	2.45	3.64	-1.19
Lac la Belle	57	-1.43	-0.11	-1.32
Oconomowoc	58	-1.82	-0.66	-1.16
Okauchee	59	-0.62	-0.36	-0.98
Pine	60	-0.71	-2.16	1.45
Nagawicka	61	1.27	1.10	0.17
Pewaukee	62	0.59	0.55	0.04
Tichigan	63	3.11	1.36	1.75
Browns	64	-1.07	-1.30	0.23
Middle	65	-2.29	-1.22	-1.07
Delavan	66	2.03	0.71	1.32
Como	67	1.15	1.29	-0.14
Geneva	68	-1.71	-1.70	-0.01

Table 27. Analysis of variance table of a regression model for the prediction of the trophic status of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1359-16091 and 1359-16094 (17 July 1973).

		Analysis of Variance		
Source	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	62.555	3.292	
Regression	4	50.631	12.658	15.922
Residual	15	11.925	0.795	

R^2 = 0.8094 x 100 = 80.94%   s.e. of estimate = 0.892

Table 28. Trophic state index (PC1) residuals of 20 Wisconsin lakes found in ERTS-1 MSS Frames 1359-16091 and 1359-16094 (17 July 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Poygan	47	1.68	3.06	-1.38
Butte des Morts	48	1.27	1.76	-0.49
Winnebago	49	2.36	1.73	0.63
Green	51	-1.67	-1.84	0.17
Beaver Dam	53	3.29	2.15	1.14
Kegonsa	54	1.48	0.63	0.85
Rock	55	-1.21	-1.20	-0.01
Koshkonong	56	2.45	3.17	-0.72
Lac la Belle	57	-1.43	-1.48	0.05
Oconomowoc	58	-1.82	-1.15	-0.67
Okauchee	59	-0.62	-0.21	-0.41
Pine	60	-0.71	-1.59	0.88
Nagawicka	61	1.27	0.80	0.47
Pewaukee	62	0.59	0.29	0.30
Tichigan	63	3.11	2.83	0.28
Browns	64	-1.07	-0.49	-0.58
Middle	65	-2.29	-0.52	-1.77
Delavan	66	2.03	0.92	1.11
Como	67	1.15	0.70	0.45
Geneva	68	-1.71	-1.41	-0.30

where XGRNRED is the mean color ratio for a lake, $GRNRED_1$ is the ratio determined for the first ERTS-1 date, $GRNRED_2$ is the ratio for the second date of ERTS-1 coverage, and $GRNRED_3$ is the ratio for the third date of coverage. The coverage dates are 9 August 1972, 11 June 1973, and 17 August 1973.

The model which best predicts the PC1 values for the 20 lakes is:

$$\widehat{PC1} = 4.127 - 6.623\ XGRNRED - 3.511\ XREDIR2 + 8.040\ XIR1IR2$$

It explains about 80 percent of the variation about the mean and has a standard error of estimate of 0.887 (Table 29). The residuals are displayed in Table 30. Relatively large residuals (absolute values) exist for Middle, Pine, and Butte des Morts. The elimination of the 11 June 1973 data and use of average ratios developed from the two remaining dates (9 August 1972 and 17 July 1973) would result in a better model.

Regression models were also developed for NES-sampled lakes in Minnesota and New York where ERTS-1 MSS coverage was available. The best model, as measured by the criteria stated previously, for each data of ERTS-1 MSS coverage is displayed in Table 31 along with the models for the Wisconsin lakes.

Although a model is included for all of the ERTS-1 MSS coverage dates, except for the two dates with very small sample sizes, some of the models are clearly inadequate. For example, the models with an R^2 less than about 0.80 (80 percent) do not have much predictive value. This effectively eliminates a model for the

Table 29. Analysis of variance table of a regression model for the prediction of the trophic status of 20 Wisconsin lakes using mean MSS color ratios from three dates (9 August 1972, 11 June 1973, 17 July 1973).

Source	df	Analysis of Variance Sum of Squares	Mean Square	Calculated F
Total (corrected)	19	62.555	3.292	
Regression	3	50.003	16.676	22.575
Residual	16	12.529	0.783	

$R^2 = 0.7997 \times 100 = 79.97\%$ s.e. of estimate = 0.887

Table 30. Trophic state index (PC1) residuals of 20 Wisconsin lakes from a regression model incorporating mean MSS color ratios from three dates (9 August 1972, 11 June 1973, 17 July 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	$PC1-\widehat{PC1}$
Poygan	47	1.68	1.66	0.02
Butte des Morts	48	1.27	2.45	-1.18
Winnebago	49	2.36	1.77	0.59
Green	51	-1.67	-1.61	-0.06
Beaver Dam	53	3.29	3.05	0.24
Kegonsa	54	1.48	1.33	0.15
Rock	55	-1.21	-0.93	-0.28
Koshkonong	56	2.45	3.32	-0.87
Lac la Belle	57	-1.43	-1.05	-0.38
Oconomowoc	58	-1.82	-1.23	-0.59
Okauchee	59	-0.62	-0.47	-0.15
Pine	60	-0.71	-1.99	1.28
Nagawicka	61	1.27	0.42	0.85
Pewaukee	62	0.59	0.29	0.30
Tichigan	63	3.11	1.48	1.63
Browns	64	-1.09	-0.69	-0.40
Middle	65	-2.29	-0.36	-1.93
Delavan	66	2.03	1.45	0.58
Como	67	1.15	0.98	0.17
Geneva	68	-1.71	-1.74	0.03

Table 31. Regression models developed for the estimation of trophic state using ERTS-1 MSS data. Each frame or pooled frame has its characteristic model.*

Date, Frame(s) and Area	Number of Lakes	Regression Model	R^2 x 100	Standard Error of Estimate	Appendix
9 August 1972 1017-16091 1017-16093 Wisconsin	20	$\widehat{PC1}$ = 7.682 - 6.074 GRNIR1 + 1.155 GRNIR2	80.84%	0.840	D1
11 June 1973 1323-16194 1323-16100 Wisconsin	20	$\widehat{PC1}$ = 42.761 - 18.423 GRNRED - 18.948 REDIR1 + 3.057 GRNIR1	69.78%	1.087	D3
17 July 1973 1359-16091 1359-16094 Wisconsin	20	$\widehat{PC1}$ = 140.430 - 9.375 GRNRED - 98.913 REDIR1 + 45.672 REDIR2 - 57.330 IR1IR2	80.94%	0.892	D4
1017-16091 1017-16093 1323-16194 1323-16100 1359-16091 1359-16094	20	$\widehat{PC1}$ = 4.127 - 6.623 XGRNRED - 3.511 XREDIR2 + 8.040 XIR1IR2	79.97%	0.887	--
14 August 1972 1022-16373 Minnesota	11	$\widehat{PC1}$ = 13.150 - 10.626 GRNIR1 + 2.327 GRNIR2	96.06%	0.456	D5

Table 31. (continued)

Date, Frame(s) and Area	Number of Lakes	Regression Model	R^2 x 100	Standard Error of Estimate	Appendix
6 October 1972 1075-16321 Minnesota	12	$\widehat{PC1}$ = 11.533 - 7.132 REDIR1	43.91%	1.540	D6
8 October 1972 1077-16431 Minnesota	7	$\widehat{PC1}$ = 35.509 - 18.548 GRNRED	95.12%	0.247	D7
28 May 1973 1309-16325 Minnesota	11	$\widehat{PC1}$ = -16.537 + 9.844 IR1IR2	49.33%	1.525	D8
3 July 1973 1345-16322 Minnesota	12	$\widehat{PC1}$ = 10.544 - 7.240 REDIR1	70.09%	1.117	D9
4 July 1973 1346-16381 Minnesota	7	$\widehat{PC1}$ = 11.715 - 8.277 REDIR1	92.34%	0.669	D10
19 August 1972 1027-15233 New York	7	$\widehat{PC1}$ = -4.891 - 8.805 GRNIR1 + 19.301 REDIR1	82.82%	0.740	D11

*Regression models were not constructed for Frames 1036-16152 (28 August 1972, Wisconsin) and 1080-15180 (11 October 1972, New York) due to the relatively small number of observations.

Wisconsin lakes (11 June 1973) and several models for the Minnesota lakes (6 October 1972, 28 May 1972, 3 July 1973). Another model, 19 August 1972 (New York) is of dubious value because the number of observations is small (N = 7), and there are only four degrees of freedom associated with the residuals.

An examination of the remaining models indicates that they all differ from one another. Attempts to predict the PC1 values for a particular ERTS-1 coverage date, using a model from another date, drew negative results.

Although their regression coefficients are different, some models have the same variables in common. For example, the 9 August 1972 (Wisconsin) model and the 14 August 1972 (Minnesota) model incorporate the variables GRNIR1 and GRNIR2. Both models have an R^2 exceeding 0.80 and are of value in predicting the PC1 values of water bodies contained in their respective frames.

The models for 3 July 1973 (Minnesota) and 4 July 1973 (Minnesota) contain a single independent variable, IR1. Yet, the first model has an R^2 of about 70 percent and the latter model an R^2 of about 92 percent. Some of the lakes are common to both models. An attempt to use the 4 July 1973 model to predict the PC1 values of the NES-sampled lakes in the 3 July 1973 frame drew negative results.

The degree of success in the prediction of lake trophic state, as defined by a lake's position on the first principal component axis, varies considerably from date to date. Several factors could account for the variability.

Lakes are, by their very nature, dynamic and can change significantly in appearance over a matter of days or weeks. Algal blooms, the growth of aquatic macrophytes, the influx of silt carried by rain-swollen streams, and, in the case of shallow lakes with sufficient fetch, sediments churned up by wind-induced turbulence can produce changes in the volume reflectance detectable by both human observers and other sensors such as the MSS. Superimposed on the variations associated with lake dynamics are variations related to atmospheric conditions and solar angle.

The impact of antecedent precipitation on lake volume reflectance can not be ascertained with any degree of certainty in this study. The relatively poor regression model for the 3 July 1973 (Minnesota) may be a consequence of the heavy rains on 2 July 1973, but this is largely conjecture. Precipitation data for the five days prior to each date of ERTS-1 coverage is found in Appendix E.

Three-Dimensional MSS Color Ratio Models

In the preceding section, several regression models were developed to predict the trophic status of selected lakes using MSS ratios. The assumptions were made that the PC1 values adequately represent the position of the lakes on a trophic scale and that lake phenomena that correlate with the index are detectable using the MSS.

A less sophisticated but practical approach to evaluating relationships between MSS data and trophic state involves the visual examination of MSS data patterns in light of a general knowledge of the lakes as well as their trophic state index values (PC1).

Although this could be done through the use of data matrices, a graphic approach is favored because it is very conducive to pattern detection and interpretation. An examination of the various MSS ratios incorporated into the regression models in Table 31 indicates that the MSS ratios GRNIR1, GRNRED, and REDIR1 might be used to advantage.

The three-dimensional models in this section were produced using the same program and equipment as in the PCA ordination model found in Chapter II. The numerals inside the "ball" are the lake's serial number and those near the lake's name represent its PC1 value. Lakes with a serial number greater than 100 fell outside the scope of the PCA ordination and therefore lack a PC1 value. Although a MSS color ratio model was developed for each date of ERTS-1 MSS coverage, the Wisconsin lakes will serve as a focal point. The other models are found in Appendix D.

The color ratio model for 9 August 1972 (Wisconsin: 1017-16091, 1017-16093) is found in Figure 33. The model can be examined using both a general knowledge acquired about the lakes and their PC1 values as guides. There is a very definite trend for

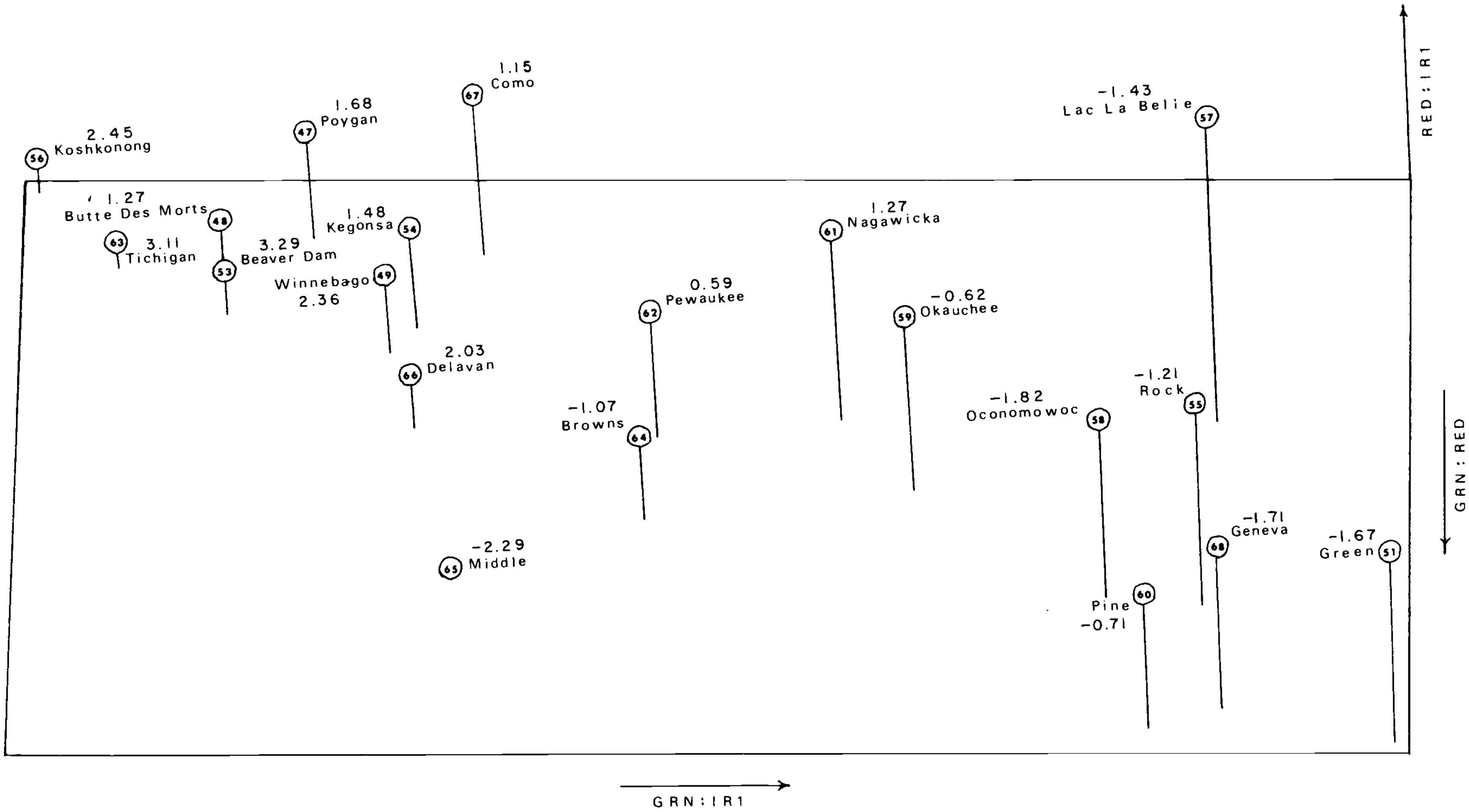

Figure 33. Three-dimensional color ratio model for 9 August 1972. The 20 Wisconsin lakes were extracted from ERTS-1 MSS Frames 1017-16093 and 1017-16091. The frames are in juxtaposition on the same flight line.

the color ratios to increase as one moves from lakes considered to be located near the eutrophic end of the scale (e.g., Beaver Dam) toward those situated more closely to the oligotrophic end (e.g., Green, 51).

It is unrealistic to expect complete agreement between the position of the lakes in the color ratio model and their respective PC1 values. In addition to the problems created by the dynamic nature of the lakes, some additional uncertainty is generated by the sampling methods. The lake MSS data for the three-dimension model were acquired by sensing the lake body, at least to Secchi depth (or to the bottom if the Secchi depth is greater than the water depth), on a pixel-by-pixel basis. The PC1 values were derived from ground truth collected at selected points (stations) ranging in number from one (e.g., Middle Lake) to nine (Lake Winnebago). As will be seen in the next section, the number of sampling sites and their location on a lake can have a significant effect on the lake's trophic index value. In addition, the sensor data represent lake phenomena at a single point in time; the PC1 values were derived using annual mean values.

Assuming that the PC1 value of Middle Lake is representative of its trophic state, it is "out of position" relative to the other lakes in the 9 August 1972 model. The lake's color ratio coordinates are indicative of a lake situated more closely to the eutrophic end of the trophic scale. Several factors may be responsible for this apparent misclassification.

As discussed previously, the trophic scale does not directly incorporate the extent of aquatic macrophytes and algal organisms in the lakes, nor does it include a direct measure of lake morphometry. The sensor may very well be "seeing" large masses of plants and/ or the lake bottom. Middle Lake is known to have weed problems and it has extensive shallows. The incorporation of some direct measure of aquatic weeds into the trophic state index would shift Middle Lake toward the eutrophic end of the scale and bring the lake index value in closer agreement with the lake's coordinates in the color ratio model.

The 11 June 1973 model of the same 20 lakes are shown in Figure 34 along with three other lakes (Mendota, Monona, and Waubesa). Many of lakes have shifted their position significantly (e.g., Winnebago, Kegonsa, Geneva, Rock, and Oconomowoc). The color ratio relationships so evident in the three-dimensional model of 9 August 1972 are not as obvious in 11 June 1973. Efforts to develop a regression model, using the PC1 values as the dependent variable in the preceding section, yielded negative results.

The Wisconsin lakes are incorporated into a three-dimensional color ratio model using MSS data from 17 July 1973 (Figure 35). Although not identical in all respects, the model bears a marked resemblance to the 9 August 1972 model. The positional change of Lake Poygan may be due in part to the large portion of cloud coverage (approximately 50 percent).

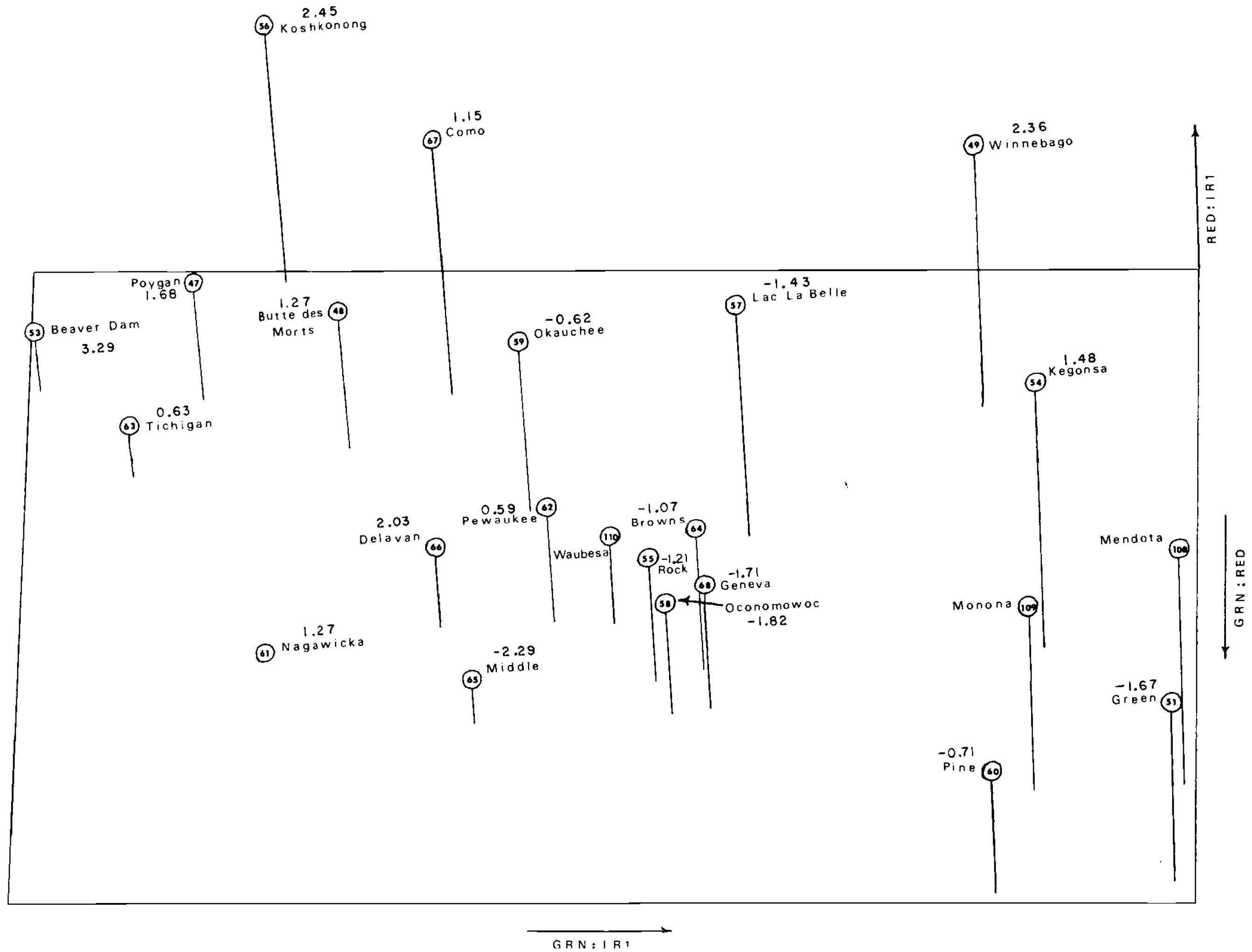

Figure 34. Three-dimensional color ratio model for 11 June 1973. The 23 Wisconsin lakes were extracted from ERTS-1 MSS Frame 1323-16100. Three of the lakes (Mendota, Monona, and Waubesa) fall outside the scope of the investigation, but are included because they are well-known by lake scientists.

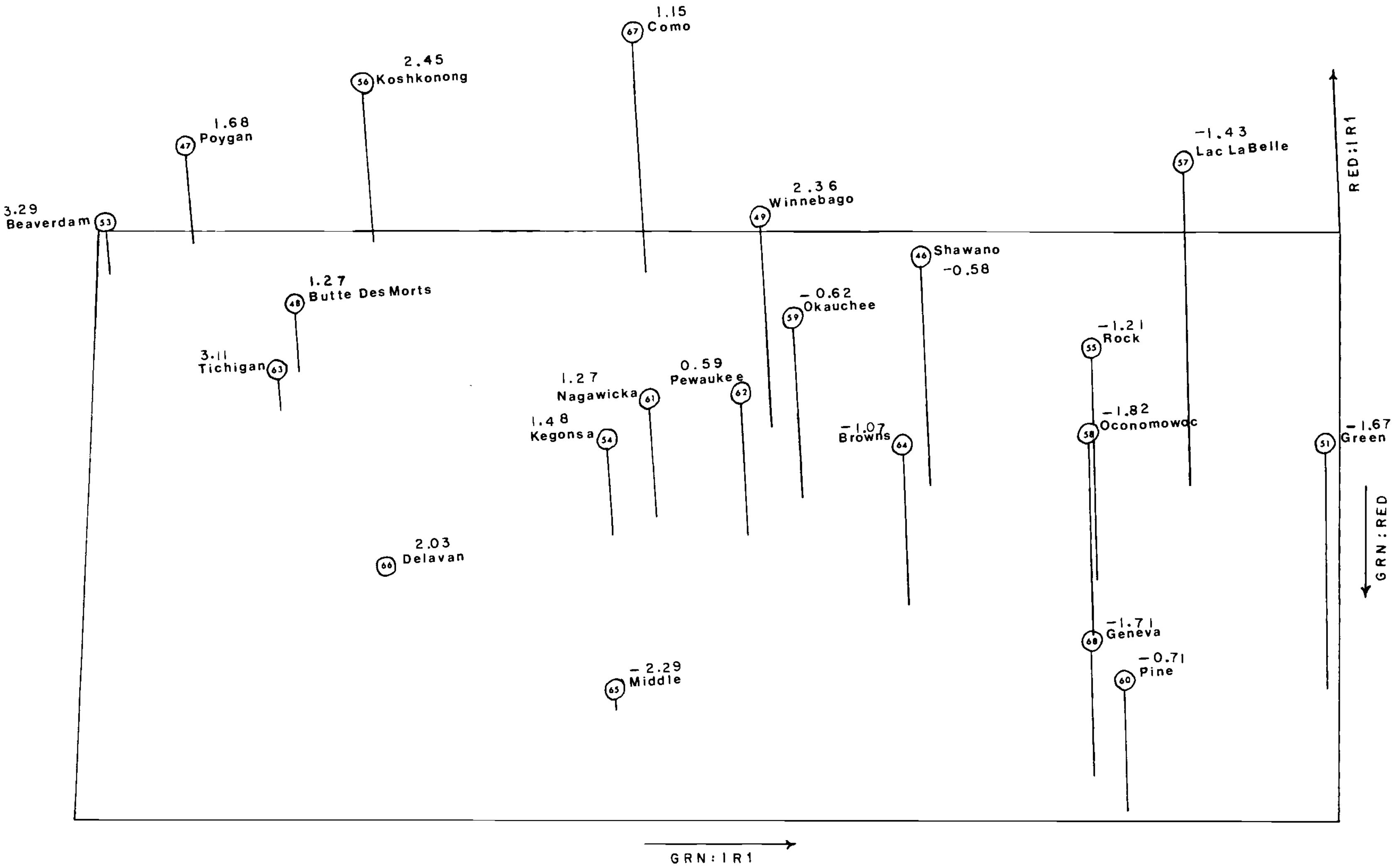

Figure 35. Three-dimensional color ratio model for 17 July 1973. The 20 Wisconsin lakes were extracted from ERTS-1 MSS Frames 1359-16091 and 1359-16094.

The model in Figure 36 represents the mean color ratio relationships between the 20 Wisconsin lakes, using MSS data for three dates (9 August 1972, 11 June 1973, 17 July 1973). The mean ratios were determined in the previous section. The model may be thought of as a representation of the general appearance of the lakes. A more extended time series is desirable, but cloud cover and poor quality MSS bands have made this impossible despite the 18-day repetitive coverage cycle.

A three-dimensional color ratio model was constructed for each of the remaining dates of ERTS-1 MSS coverage (Table 15). The models are found in Appendix D. It will be noted in some of the models of Minnesota lakes (14 August 1972, 6 October 1972, 8 October 1972) that certain lakes (Wagonga, Silver) are isolated from the other lakes. These lakes have IR1 DN values which exceed their respective RED DN values. The lakes are at the extreme end of the trophic scale and are sometimes referred to as being hypereutrophic.

An examination of both the three-dimensional color ratio models and their associated regression models suggests that the utility of the MSS for the estimation of trophic state is dependent to a substantial degree upon the time of year. The Wisconsin and Minnesota frames recorded relatively early during the open water season (28 May 1973, 11 June 1973) have spectral curves which correlate poorly with the lake PC1 values. The correlations are

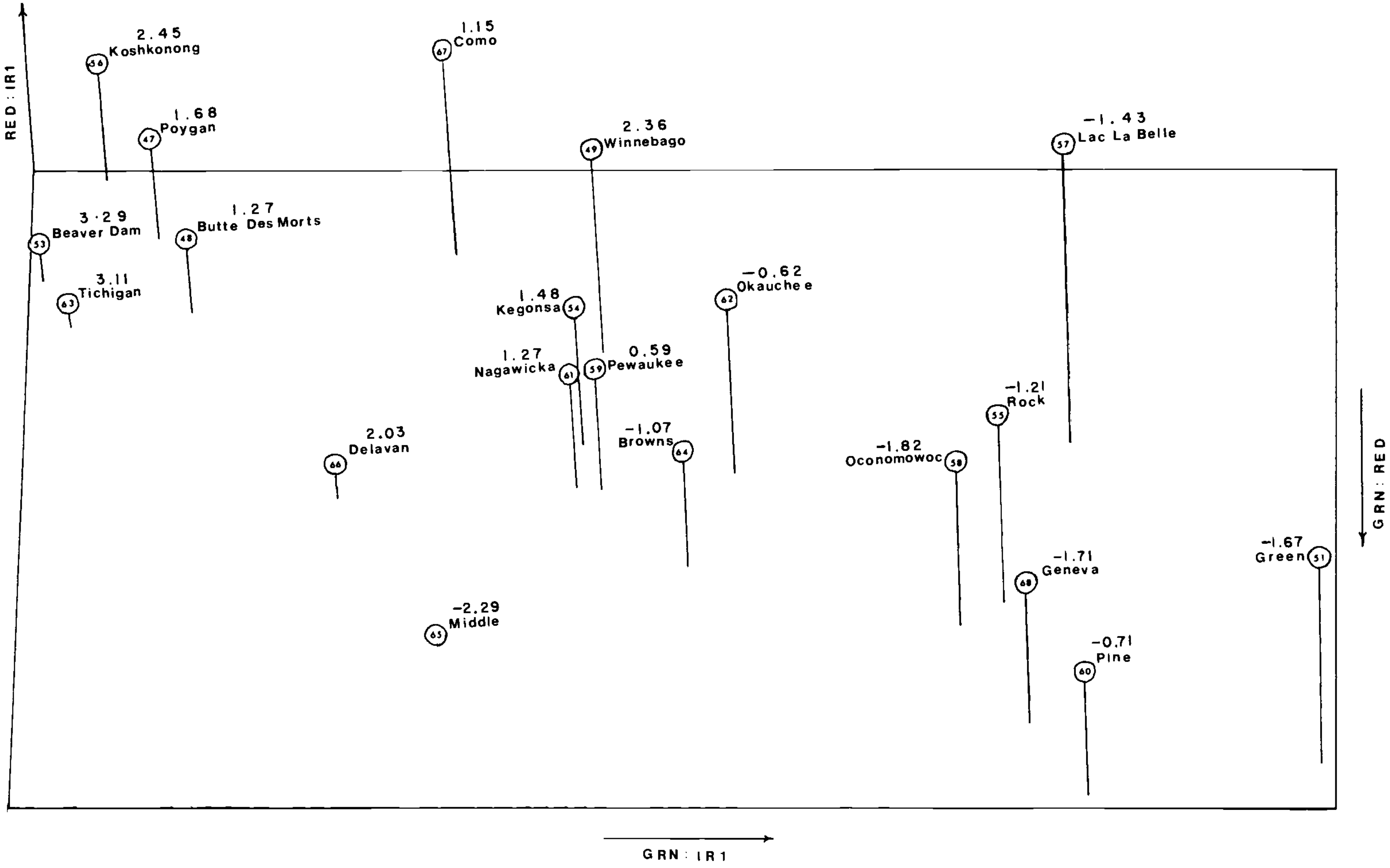

Figure 36. Three-dimensional color ratio model for three dates of ERTS-1 MSS coverage (9 August 1972, 11 June 1973, 17 July 1973). The model was developed using mean color ratios derived from MSS data collected on the three dates.

much stronger in the case of MSS frames recorded later in the season (9 August 1972, 14 August 1972, 17 July 1972, 8 October 1972) when the contrast between lakes at different points on the trophic scale tend to be at a maximum.

Lake Classification Using MSS Data in Conjunction with an Automatic Data Processing Technique

Up to this point the lake MSS data have been examined using regression techniques and three-dimensional color ratio models. The investigations were undertaken using mean color values and mean color ratios for each ERTS-1 date. The lakes were treated as entities without regard to differences which might be present within them. Automatic data processing techniques are well-suited, not only for classifying lakes, but also for classifying different types of water within individual lakes.

It is becoming increasingly common to use automatic data processing (ADP) techniques in remote sensing studies (*e.g.*, Hoffer, *et al.*, 1972). ADP techniques merit consideration because they permit the reduction of large masses of data in realistic time periods and add objectivity to the classificatory process.

With the advantages of ADP in mind, an effort was undertaken by Blackwell (1974, personal communication) to apply the techniques to the 20 lakes found in MSS Frames 1017-16091 and 1017-16093 (9 August 1972). The machinery and software at the JPL/IPL were used to process the MSS data reported here and produce hard copy in the form of black and white photographs and color-enhanced prints. The methodology employed is briefly described below.

Utilizing a LARS-developed (Laboratory for Applications of Remote Sensing, Purdue University) spectral pattern-recognition-algorithm, the IPL IBM 360/44 was trained using the GRN, RED, and IR1 MSS data in conjunction with the lake PC1 values. Preliminary processing indicated that the IR2 band data would be of little value in distinguishing one lake from another or one area of a lake from another area of the same water body. The IR2 data were not included, thereby reducing the amount of CPU time required for classification.

The number of spectral classes was set by establishing one class for each different PC1 value among the 20 Wisconsin lakes. This resulted in the formation of 19 different classes (Table 32). Butte des Morts and Nagawicka have the same PC1 value and were assigned to the same class.

The computer was statistically trained to recognize each lake as belonging to a particular class. For example, the computer was trained to perceive Beaver Dam Lake as belonging to Class 1, Tichigan as Class 2, ..., Middle Lake as Class 19. Each pixel in the 20 lakes was then classified by the computer into one of the 19 classes. The results of the classification procedure are found in Table 33.

Table 32. Lake trophic state index class assignments for the ADP technique.

Lake Name	Serial Number	PC1 Value	Computer Trained to Recognize as Class:
Beaver Dam	53	3.29	1
Tichigan	63	3.11	2
Koshkonong	56	2.45	3
Winnebago	49	2.36	4
Delavan	66	2.03	5
Poygan	47	1.68	6
Kegonsa	54	1.48	7
Butte des Morts	48	1.27	8
Nagawicka	61	1.27	8
Como	67	1.15	9
Pewaukee	62	0.59	10
Okauchee	59	-0.62	11
Pine	60	-0.71	12
Browns	64	-1.07	13
Rock	55	-1.21	14
Lac la Belle	57	-1.43	15
Green	51	-1.67	16
Geneva	68	-1.71	17
Oconomowoc	58	-1.82	18
Middle	65	-2.29	19

Table 33. ADP results for 9 August 1972 using a 19 class classification. The six trophic classes listed for each lake were assigned most of the pixels representing the lake. The percentage of pixels assigned to each class is given in parentheses.

Lake Name	Serial Number	PC1 Value	Trained as Class:	Six Major Classes Found in the Lake (Percentage)					
Beaver Dam	53	3.29	1	1 (69.4)	10 (13.0)	2 (9.8)	19 (3.2)	4 (2.5)	7 (1.3)
Tichigan	63	3.11	2	2 (41.6)	5 (17.0)	1 (16.3)	4 (8.5)	8 (8.5)	7 (3.9)
Koshkonong	56	2.45	3	7 (44.5)	17 (12.6)	16 (6.0)	12 (5.1)	2 (4.8)	3 (4.7)
Winnebago	49	2.36	4	4 (18.0)	10 (16.3)	5 (13.8)	2 (9.8)	7 (9.5)	1 (9.1)
Delavan	66	2.03	5	5 (66.0)	3 (8.0)	8 (6.1)	7 (4.2)	1 (4.0)	2 (3.5)
Poygan	47	1.68	6	7 (24.0)	6 (18.3)	9 (13.4)	8 (10.1)	5 (8.0)	4 (6.7)
Kegonsa	54	1.48	7	7 (87.4)	9 (7.4)	6 (3.0)	3 (1.2)	2 (0.1)	1 (0.1)
Butte des Morts	48	1.27	8	2 (29.3)	8 (23.3)	5 (18.4)	9 (7.8)	3 (7.0)	6 (5.8)
Nagawicka	61	1.27	8	15 (41.3)	10 (16.5)	5 (9.3)	4 (8.3)	17 (6.0)	1 (5.0)
Como	67	1.15	9	7 (39.1)	9 (31.2)	1 (7.5)	2 (5.0)	4 (3.4)	8 (3.4)
Pewaukee	62	0.59	10	10 (35.0)	14 (14.9)	1 (13.0)	11 (8.9)	13 (5.2)	19 (5.2)
Okauchee	59	-0.62	11	11 (32.7)	16 (19.5)	14 (13.0)	10 (11.8)	1 (6.6)	12 (5.8)
Pine	60	-0.71	12	12 (35.9)	16 (30.9)	11 (4.6)	17 (4.1)	19 (4.1)	13 (3.7)

Table 33. (continued)

Lake Name	Serial Number	PC1 Value	Trained as Class:	Six Major Classes Found in the Lake & (Percentage)					
Browns	64	-1.07	13	11 (22.0)	13 (17.1)	19 (15.5)	12 (10.0)	16 (6.5)	1 (6.1)
Rock	55	-1.21	14	16 (31.1)	12 (15.6)	14 (12.7)	11 (12.5)	15 (9.9)	17 (4.6)
Lac La Belle	57	-1.43	15	15 (43.0)	10 (11.9)	14 (11.8)	7 (5.3)	5 (4.9)	11 (4.8)
Green	51	-1.67	16	16 (45.0)	12 (27.8)	14 (7.7)	17 (5.7)	11 (5.2)	2 (1.9)
Geneva	68	-1.71	17	17 (38.0)	16 (18.3)	12 (15.6)	14 (12.7)	18 (5.4)	10 (2.3)
Oconomowoc	58	-1.82	18	16 (23.6)	12 (19.2)	14 (12.4)	17 (9.4)	15 (8.7)	11 (7.9)
Middle	65	-2.29	19	12 (22.2)	19 (15.4)	13 (12.8)	1 (11.1)	11 (10.2)	16 (10.2)

All of the pixels in a homogeneous lake would be classified as falling into the class for which the lake served as a training area. It is very unlikely to find a lake that has the same trophic characteristics throughout its areal extent. Some indication of the heterogeneous nature of the 20 Wisconsin lakes may be obtained by examining Table 33.

Beaver Dam Lake, for example, has 69.4 percent of its pixels classified as belonging to Class 1, 10 percent to Class 2, ..., and 1.3 percent to Class 7. Lake Kegonsa exhibits the least heterogeneity with 87.4 percent of its pixels falling in Class 7. The percentages expressed here should be treated as approximations because the "sixth line" banding affects the results of any such classification scheme.

If the classificatory results had indicated homogeneous conditions in each lake, the analysis could stop at this point. However, this is not the case, and the question arises: "What trophic-related patterns exist in each lake?" This necessitates the development of some sort of imagery.

Images of the machine-classified lakes can be produced in the form of line printer copy using different symbols to represent the various trophic classes and also as photographic prints and transparencies. Output in the form of photographs was selected because they are compact, easily handled, have much greater resolution than line printer copy of equal size, and are readily interpretable, particularly when in color.

The ADP-classified lakes (9 August 1972) are displayed in Plate I, using 19 gray-levels, one for each class. Class 1 is located toward the eutrophic end of the trophic scale (black) and Class 19 (white) is toward the oligotrophic end. It would be incorrect to call a Class 19 pixel or lake oligotrophic because none of the lakes examined are considered to possess the necessary attributes.

The pronounced linear features in Plate I are artifacts introduced by a defect in the MSS and are generally referred to as "sixth line" banding. It is important to overlook their presence when studying the patterns present in the lake images.

Plate II is a color-enhanced version of the 19-class classification of the 20 lakes (9 August 1972). Black has been assigned to represent Class 1, and each of the remaining 18 classes has been assigned its unique color. The colors were produced by using different ratios of the three primary colors: blue, green, and red. Detailed information regarding the principles involved is found in the work by Committee on Colorimetry (1966).

Differences among and within the lakes are readily apparent. Some of the lakes (Kegonsa and Beaver Dam) present a relatively homogeneous appearance. Others (Winnebago and Poygan) exhibit a diversity of trophic classes. Some of the lakes have features which bear mentioning.

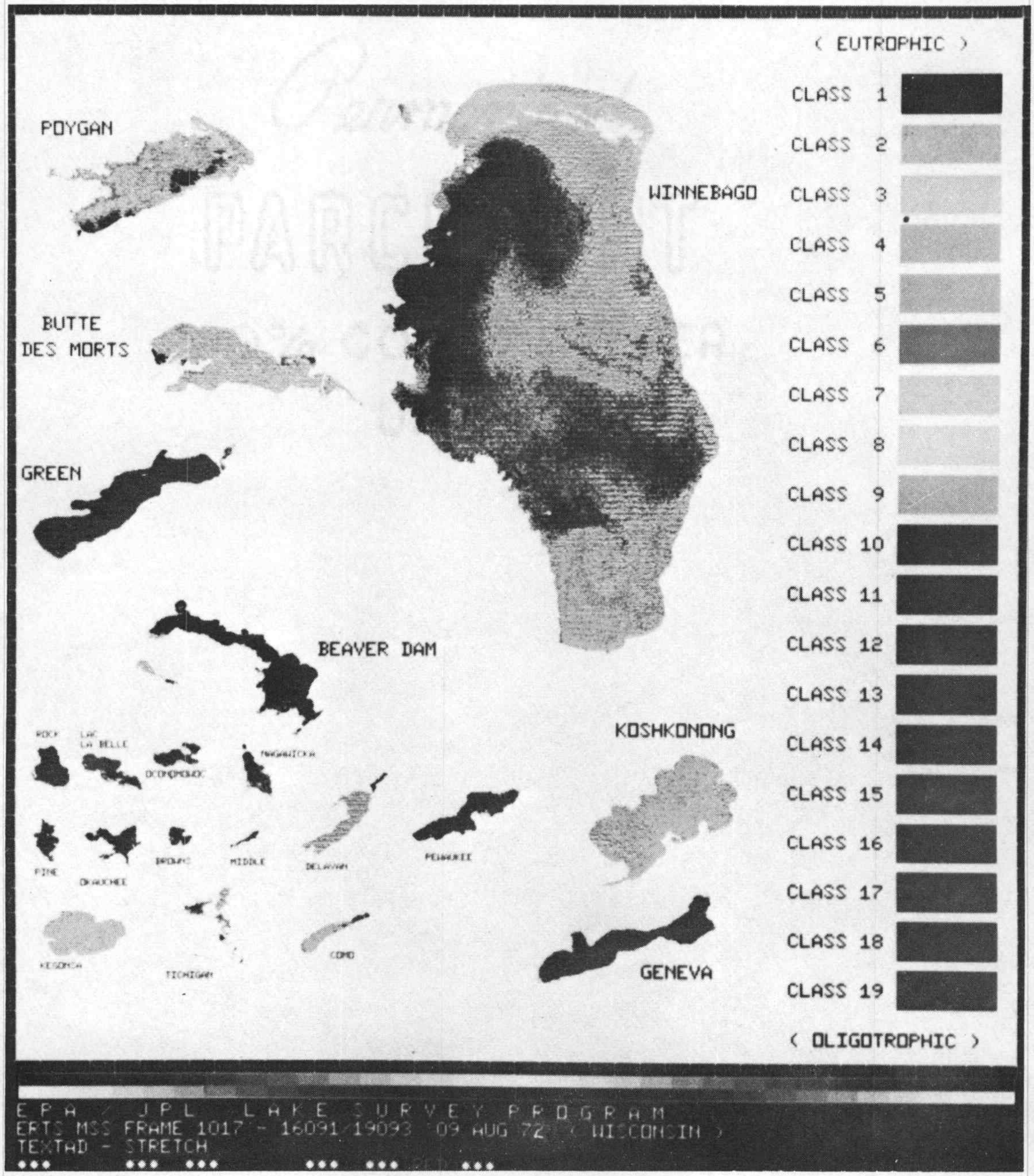

Plate I. ADP-classified lakes (9 August 1972), using 19 gray-levels, one for each class.

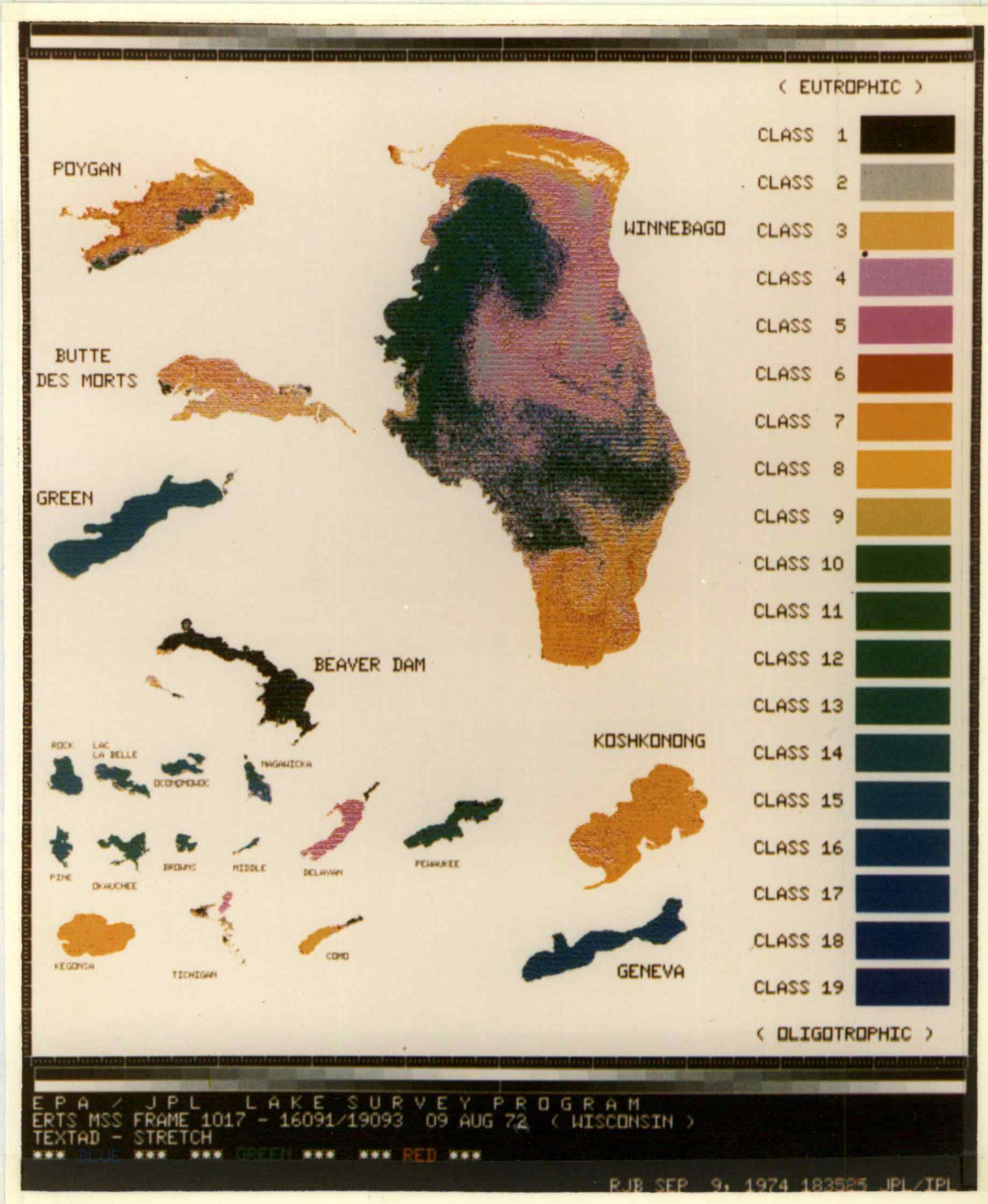

Plate II. A color-enhanced version of the 19-class classification of the 20 lakes (9 August 1972).

The appendix-like structure which appears attached to the northeast quadrant of Delavan Lake is the entry point of Jackson Creek, the major stream feeding the lake. Its waters, known to be nutrient-rich through contributions from sewage treatment plants and agricultural drainage, have been placed in Class 1.

In this study, Lake Tichigan has been defined to include the lake proper and what is commonly referred to as the "widening" in the Fox River. The lake proper has been assigned to Class 5 and the "widening" to several classes including 1, 2, and 4. Ground-truth measurements indicate that the "widening" has a lower Secchi disc transparency and a substantially high chlorophyll a level.

The Class 1 water found along the northern shore of Lake Pewaukee may be related to the presence of algae and rooted emergent plants. The helicopter-borne survey teams reported algal scum covering the surface of the northern portion of the lake on 21 June 1972 and 19 August 1972. Heavy growths of emergents covered the lake shallows.

The appendix-like portion of Green Lake located at its northeast end is the area into which Silver Creek flows. The area receives a substantial nutrient load from a sewage treatment plant and the surrounding agricultural lands. Its pixels have been classified as belonging to Class 1 and Class 2.

White areas within the lake images are indicative of either clouds or land-related phenomena. The white area in the northeastern portion of Lake Winnebago represents cloud cover. The north-south linear feature located in the eastern end of Lake Butte des Morts is a causeway.

Complete accord does not exist between the trophic index values of the 20 lakes and the results of the ADP technique. The disparity is very evident in the cases of lakes Nagawicka, Koshkonong, and Oconomowoc where few, if any, pixels were found that fell into the class for which the lake served as a training area. Middle Lake contains pixels classified as belonging to Class 19, but they constitute only 15.4 percent of the total. This is not surprising because there is an indication in the three-dimensional color ratio model and the PC1 regression model for 9 August 1972 that a disparity exists between some of the lake PC1 values and their MSS data. The use of a smaller number of classes may very well yield more consistent results. This is an area in need of additional study.

The color-enhanced imagery, produced through an ADP technique, should prove to be of value, not only in comparing lakes with each other, but also for supplying information which can be used in the selection of future lake sampling sites. Further refinement should make the imagery a valued tool in lake survey and monitoring activities.

VI. SUMMARY

This research has focused on relationships between Earth Resources Technology Satellite One (ERTS-1) multispectral scanner (MSS) data and the trophic status of selected lakes in Minnesota, Wisconsin, Michigan, and New York sampled by the U. S. Environmental Protection Agency's National Eutrophication Survey. Analyses were directed toward lake classification on the basis of ground truth, and the prediction of lacustrine trophic state indicator magnitudes and trophic state using MSS data.

Initially, 100 NES-sampled lakes were analyzed to ascertain their trophic status. Trophic indicators selected as parameters in the analyses included: chlorophyll a, conductivity, total phosphorus, total organic nitrogen, the inverse of Secchi disc transparency, and the yield of a standard algal assay procedure. Natural logarithm transformations were made on the data to produce a more-nearly normal distribution.

A complete linkage algorithm (McKeon, 1967) was used in conjunction with squared-Euclidian distance to examine the trophic status of the lake population for the presence of natural clusters or groups. The results are displayed in the form of a dendrogram (Figure 15).

The lake trophic indicator data were also subjected to a principal components analysis to reduce the dimensionality of the data from six-dimensional hyperspace to space of three or fewer dimensions. The eigenvectors and their associated eigenvalues were extracted from a p x p Pearson product-moment correlation matrix. The first principal component (normalized eigenvector) accounts for approximately 68 percent of the variation in the data. The second and third components represent about 14 percent and 8 percent of the variation, respectively. The results of the principal components analysis were used to ordinate the lakes in one-, two-, and three-dimensional space. The three-dimensional ordination is displayed as a "ball and wire" model (Figure 17).

A multivariate trophic state index was developed by evaluating the first principal component for each of the 100 NES-sampled lakes. The resultant values (PC1) are indicative of each lake's respective position on a multivariate trophic scale (the first principal axis). The larger the PC1 value, the closer the lake lies toward the eutrophic end of the scale. The coefficients of the first component are nearly equal in magnitude, suggesting that the first principal component represents a general measure of trophic state.

ERTS-1 MSS data, collected on 12 different dates from Minnesota, Wisconsin, and New York, were extracted from computer-compatible magnetic digital tapes (CCT's) using VICAR, a sophisticated software system at the Image Processing Laboratory (IPL), a support facility of the Jet Propulsion Laboratory (JPL). Each lake image was extracted from its terrain matrix through the use of a binary mask technique which employs a digital number (DN) level of 28 (8-bits of precision; 256 DN levels) as the water-land cutoff point. Descriptive statistics were computed for each lake including total number of pixels, mean DN values, standard deviations, and histograms. This was accomplished for each of the four MSS bands (GRN, RED, IR1, and IR2). A photographic concatenation was produced of the lakes extracted from each frame.

Regression models were developed to predict the magnitude of two trophic state indicators (Secchi disc transparency and chlorophyll a) using MSS data from two dates, 9 August 1972 (Wisconsin) and 14 August 1972 (Minnesota).

A regression model was developed for each date of ERTS-1 MSS coverage using the multivariate trophic state index (PC1) as the dependent variable and the MSS color ratios as independent variables. Most of the modelling effort was directed toward NES-sampled lakes in eastern-southeastern Wisconsin. The Wisconsin area was selected because good ERTS-1 MSS coverage was available for a group of 20 lakes on three different dates.

A more flexible approach toward the study of PC1-MSS relationships was undertaken using three-dimensional MSS color ratio models. The MSS ratios GRNIR1, GRNRED, and REDIR1 were used to develop the three-dimensional "ball and wire" models. A model was constructed for each date of MSS coverage. In addition, a model was created for the Wisconsin lakes using mean color ratio values for the three sampling dates (9 August 1972, 11 June 1973, and 17 July 1973) (Figure 36).

Automatic data processing (ADP) techniques were used in conjunction with the PC1 values for 20 Wisconsin lakes and MSS data

(9 August 1972) to classify the lakes. A computer was programmed to recognize 19 trophic classes using the MSS color information contained in the 20 lakes. The classification was accomplished on a pixel-by-pixel basis using the MSS GRN, RED, and IR1 data. Output is in the form of statistical data and photographic products including 19-step gray scale prints (Plate I) and 19-class color-enhanced photographs (Plate II).

Lake Classification Using Ground Truth

The examination and classification of large numbers of lakes within realistic periods of time necessitates the use of automatic data processing techniques which incorporate classificatory methods, such as cluster analysis and principal components ordination.

The complete linkage algorithm used in this research did not produce three well-isolated clusters (groups) that can be readily associated with the traditional trophic classes of lakes - oligotrophic, mesotrophic, and eutrophic. This apparent failure is due in part to the lakes used in the study. Although the 100 lakes were selected to cover the range of lakes sampled during 1972 by the National Eutrophication Survey, the NES-sampled lake population is heavily weighted toward lakes with relatively poor water quality. The three-dimensional principal component ordination does not contain large well-defined clusters. The ordination results support the findings of the cluster analysis.

It is likely that there are no sharp discontinuities which clearly separate the members of a large population of lakes into the three commonly- referred -to trophic classes. Lakes of such a population may be characterized as constituting a hyper-dimensional cloud possessing a low degree of organization.

Cluster analysis techniques are a "heavy handed" approach to the lake classification problem because they will lead to clusters even if the data are random. The principal component analysis ordination does not presume the existence of clusters. The two methods are complementary and should be used in conjunction with one another.

The concept of a multivariate trophic state index, as defined by the position of a lake on the first principal component axis, merits further consideration. However, it must be kept in mind that a less sophisticated method, the mean composite rank (MCR) index, yielded similar results as measured by lake rank. The MCR system has the advantages of conceptual simplicity and ease of computation.

MSS Estimation of Lake Area and Selected Trophic State Indicators

The ERTS-1 MSS is an effective tool for the determination of the number and the areal extent of lakes. The use of a DN level of 28 (8-bits of precision) as the water-land cutoff point for the generation of the binary mask in conjunction with a pixel conversion factor of 0.48 (1 pixel = 0.48 hectares) gives lake area estimates generally within 10 percent of values derived from topographic sheets.

While the use of near-concurrent ground truth precludes more precise estimates, it has been demonstrated that good predictions of Secchi disc tranparency can be achieved through the incorporation of MSS color ratios into regression models.

A measure of the chlorphyll a level can be obtained by using MSS color ratios as independent variables in regression models. However, caution must be used in the interpretation of the chlorophyll model predictions. The predicted values should be treated as a crude index rather than accurate estimates of chlorophyll a.

Models for the prediction of Secchi disc transparency and chlorophyll a are unique to the date of ERTS-1 MSS coverage. The models do not give good predictions when used with MSS data collected on different dates. Ground truth must be collected concurrently or near-concurrently with MSS coverage, and then the appropriate models must be constructed.

Although the need for ground truth places a restriction on the use of MSS data for the prediction of trophic state indicators, it still has utility in regions where lake concentrations result in the inclusion of hundreds and even thousands of lakes within a single MSS frame. Under such circumstances the collection of ground truth from a small number of "bench mark" lakes for the development of regression models would pay handsome dividends.

MSS Prediction of Lacustrine Trophic State

The MSS data can be used to give fair to very good estimates of lacustrine trophic state as defined by lake position on the first principal component axis. The regression models, developed using MSS color ratios as independent variables, are unique to the date of ERTS-1 coverage. The use of a model for a specific date in the prediction of trophic state, using MSS data from other dates, drew negative results. Models developed from MSS data collected early in the open-water season are inadequate; better models were constructed using MSS data collected later in the season when the contrasts between lakes at different points on the trophic scale tend to be maximized.

Extraction Techniques and Products

While it is possible to extract MSS data related to lake trophic state and trophic indicators from EROS-supplied imagery, efforts to use this imagery are seriously impeded because the photographic products lack the requisite radiometric fidelity, a consequence of the scale compression induced by condensing the MSS intensity resolution from 127 levels (7-bit precision) to 16 gray levels. Data extracted from computer-compatible tapes (CCT's) must be used if the maximum benefits are to be derived from the ERTS-1 MSS. This is particularly important in water quality-related studies because the full range of water quality differences is contained in a relatively small number of DN levels.

The computer-compatible tape approach to the problem of MSS data extraction has the added advantage of eliminating the errors introduced by differences in photographic products and the microdensitometer. CCT data extraction and processing techniques, which produce both MSS data statistics and photographic products, greatly increase the utility of the multispectral scanner.

Photographic products permit the rapid visual assessment of lake coverage by providing greater spatial resolution and a fraction of the bulk characteristic of line printer copy. It has been demonstrated that automatic digital processing techniques can produce trophic classifications which, when displayed as photographic imagery, can be incorporated into studies of lake water quality, water circulation patterns, and supply information for the selection of lake water sampling sites. The advantages of using CCT-produced photographs are particularly evident when the products are in the form of color-enhanced prints and transparencies.

ERTS-1 Coverage and Quality

Although ERTS-1 gives 18-day repetitive coverage, good frames were available for only a few days in the study area encompassing Minnesota, Wisconsin, Michigan, and New York. On many occasions, excessive cloud cover (greater than 10 percent) was present or one or more of the MSS bands was rated as poor or missing.

Good coverage was virtually nonexistent for NES-sampled lakes in Michigan during 1972.

While it is recognized that the number of days with excessive cloud cover is a function of a region's weather and climate, and therefore its geographic location, - some regions (southwestern U. S.) have viewing conditions which are consistently better than other regions (northeastern U. S.) - it is apparent that good ERTS-1 MSS coverage is available on a very erratic basis, making systematic time-series studies difficult if not impossible in the study area and, indeed, for much of the eastern United States.

Information relating to water quality is found in each of the four MSS bands (GRN, RED, IR1 and IR2). However, the DN levels sensed for water bodies are few in number and are located at the lower end of the intensity scale. Surface water resource investigations would derive substantial benefits if the instrument gain were to be increased on the MSS. It is possible to switch to high gain in the GRN and RED bands.

Research Needs

Several problems have been identified during the course of this research which merit additional consideration. One of them involves the application of automatic data processing techniques (such as cluster analysis) to the problem of lake classification. The kinds and number of trophic indicators, data transformations, and the methodology employed can produce significant differences in the resulting classifications. Very little work has been done in this area using the techniques commonly employed in numerical taxonomy. ADP techniques are well suited for the reduction of the large masses of data generated by lake survey programs.

Although atmospheric and solar angle effects were disregarded in this investigation, it is apparent that they can degrade the lake color signal received by the multispectral scanner. The development of radiometric calibration techniques to account for the peripheral effects would greatly increase the utility of the ERTS-1 MSS.

The need exists to study in greater detail the relationships between the magnitude of selected trophic state indicators and MSS data under both field and laboratory conditions. Most of the efforts in this area have involved sensors with much greater spectral resolution than that of the ERTS-1 MSS.

Bottom effects were assumed to be insignificant in this study because Secchi disc transparency was much less than water depth. However, in some lakes the bottom can be seen by the sensor. A better knowledge of bottom effects may be of value in refining the predictive capabilities of the MSS in the aquatic environment.

With the exception of some lakes in the New York frames, the remote sensing aspects of this investigation have been heavily weighted with lakes that are often referred to as "eutrophic". The need exists to examine more lakes, particularly those possessing high water quality. The sensitivity limits of the MSS have yet to be established for trophic state indicators as well as the trophic state index. The examination of lakes from other geographic regions should be undertaken to determine whether relationships found in the study area extend to lakes in other regions.

The implementation of ADP techniques, to classify lakes on the basis of PC1-MSS relationships and to display the results as gray-scale and color-enhanced photographic products, merits further consideration. The development of a more representative trophic state index may be in order.

A magnetic tape library has been established at the Image Processing Laboratory (IPL/JPL) for the lakes extracted in the course of the investigation. The lake "images" have been extracted from their terrestrial matrix and are readily available for additional statistical analyses and manipulation, using digital image-processing techniques. The enlargement of this library through continued acquisition of MSS data for NES-sampled lakes would give researchers the opportunity to study long-term changes in the lakes.

Summation Statement

The contiguous United States is estimated to have some 100,000 lakes; the exact number is not known. Maintaining a current inventory of the lacustrine resources is an immense task complicated by the dynamic nature of lakes and man's ability to both create and modify inland water bodies. Automatic classificatory techniques, such as cluster analysis and principal component ordination, merit consideration by organizations and individuals involved with lake inventory and classification work. The techniques are well suited for reducing the vast quantities of data generated by surveys while adding objectivity to the classification process.

Although the Earth Resources Technology Satellite-One (ERTS-1) multispectral scanner (MSS) has relatively low spectral and spatial resolution, it has the capability to provide information relevant to survey-type programs. Its synoptic view of the earth's surface makes it an ideal tool for undertaking rapid lake enumerations and estimating the extent of lake surface area.

MSS color ratios can be used to predict the magnitude of selected trophic state indicators and a multivariate trophic state index. However, the accuracy of the prediction values varies considerably from date to date. Its utility is reduced by the need for some ground truth and, even more important, the lack of good coverage on a cyclic basis. Maximum benefits will be derived from the ERTS-1 MSS only if the computer-compatible magnetic digital tapes (CCT) are used in conjunction with digital image processing techniques.

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APPENDICES

APPENDIX A

APPENDIX A

Trophic Indicator Data for 100 NES-Sampled Lakes

Lake Name	Serial Number	SECCHI	ISEC	COND	TPHOS	TON	CHLA	AAY
Blackduck	1	1.72	0.58	244	0.051	0.80	14.6	2.2
Bemidji	2	2.15	0.47	315	0.049	0.69	9.5	1.3
Andrusia	3	1.37	0.73	276	0.035	0.58	13.0	1.8
Wolf	4	1.19	0.84	384	0.068	0.48	17.2	1.6
Cass	5	1.88	0.53	272	0.027	0.53	9.8	0.8
Leech	6	2.17	0.46	262	0.020	0.53	6.2	0.5
Birch	7	2.44	0.41	205	0.024	0.42	7.1	1.4
Trout	8	2.81	0.36	325	0.045	0.43	7.0	1.6
Mashkenode	9	1.05	0.95	301	0.100	0.56	25.3	2.6
Whitewater	10	1.71	0.59	139	0.066	0.67	9.8	2.9
Pelican	11	1.71	0.59	88	0.033	0.39	11.4	1.3
Shagawa	12	1.77	0.57	71	0.084	0.53	11.3	4.5
Gull	13	2.32	0.43	204	0.031	0.51	12.5	2.4
Rabbit	14	2.71	0.37	276	0.036	1.25	6.7	2.0
Cranberry	15	1.18	0.85	107	0.038	0.78	30.2	0.2
Darling	16	2.56	0.39	391	0.019	0.80	11.8	0.8
Carlos	17	2.74	0.37	353	0.014	0.56	4.6	0.9
Le Homme Dieu	18	2.31	0.43	315	0.022	0.67	12.4	0.2
Minnewaska	19	1.71	0.59	638	0.035	0.90	7.6	0.4
Nest	20	1.36	0.74	353	0.050	1.03	21.4	3.8
Green	21	2.79	0.36	353	0.051	0.46	4.9	1.3
Wagonga	22	0.29	3.41	808	0.940	1.54	94.5	13.9
Clearwater	23	1.57	0.64	362	0.033	0.77	12.7	2.4
Mud (at Maple Lake)	24	0.35	2.88	530	1.893	1.31	132.3	8.5
Cokato	25	1.34	0.54	540	0.208	1.39	10.7	29.0
Buffalo	26	1.27	0.79	597	0.224	1.65	38.1	16.2
Carrigan	27	0.28	3.58	595	1.164	1.81	84.3	13.2
Silver	28	0.25	3.94	470	0.540	3.35	126.1	25.9
Minnetonka	29	1.38	0.73	361	0.070	1.45	16.6	1.2
Forest	30	2.16	0.46	273	0.024	0.61	10.5	0.2
White Bear	31	3.45	0.29	253	0.019	1.32	5.2	0.3
St. Croix	32	1.11	0.90	159	0.054	0.57	10.2	3.4
Spring	33	0.47	2.15	450	0.237	0.64	21.8	34.5
Pepin	34	0.96	1.05	417	0.187	2.13	14.9	14.0
Madison	35	0.86	1.16	305	0.064	1.24	30.4	3.8
Sakatah	36	1.89	0.53	389	0.324	1.21	10.8	14.8
Bear	37	0.28	3.58	431	0.191	1.57	61.2	5.5
Albert Lea	38	0.19	5.37	650	1.002	2.50	381.2	61.3
Yellow	39	1.49	0.67	174	0.071	0.43	13.7	3.3

APPENDIX A (Contd)

Lake Name	Serial Number	SECCHI	ISEC	COND	TPHOS	TON	CHLA	AAY
Wapogasset	40	1.80	0.56	198	0.053	0.45	16.6	5.5
Long	41	0.88	1.13	72	0.046	0.87	7.1	0.2
Elk	42	0.93	1.07	66	0.140	0.83	7.1	2.8
Trout	43	4.12	0.24	96	0.042	0.24	2.7	0.7
Crystal	44	8.03	0.13	50	0.007	0.15	1.5	0.2
Tainter	45	1.39	0.72	173	0.115	0.50	13.7	16.8
Shawano	46	2.00	0.50	220	0.021	0.66	11.9	0.7
Poygan	47	0.48	2.07	306	0.077	1.09	19.4	8.9
Butte Des Morts	48	0.58	1.73	289	0.063	0.70	25.4	6.7
Winnebago	49	0.55	1.82	316	0.156	1.02	48.4	13.4
Round	50	3.40	0.29	323	0.018	0.60	3.5	0.2
Green	51	5.78	0.17	386	0.043	0.43	4.8	1.0
Swan	52	1.91	0.53	378	0.158	0.71	8.2	17.2
Beaver Dam	53	0.43	2.34	422	0.388	0.31	69.5	12.7
Kegonsa	54	1.10	0.91	403	0.140	0.73	30.9	8.0
Rock	55	2.32	0.43	378	0.019	0.63	8.1	0.3
Koshkonong	56	1.58	0.63	580	0.333	1.29	36.1	29.0
Lac La Belle	57	2.04	0.49	434	0.013	0.57	7.9	0.2
Oconomowoc	58	3.34	0.30	437	0.015	0.93	3.1	0.2
Okauchee	59	2.49	0.40	446	0.016	0.70	8.4	3.5
Pine	60	1.84	0.54	308	0.026	0.67	7.5	1.4
Nagawicka	61	1.30	0.77	503	0.135	0.77	12.0	21.8
Pewaukee	62	1.72	0.58	435	0.036	1.63	15.5	2.1
Tichigan	63	0.58	1.71	599	0.479	0.97	44.7	26.0
Browns	64	2.78	0.36	435	0.021	0.68	6.4	0.8
Middle	65	3.59	0.28	399	0.012	0.39	4.7	0.2
Delavan	66	1.26	0.79	451	0.144	1.02	43.9	19.9
Como	67	0.52	1.94	390	0.064	1.05	36.4	0.3
Geneva	68	3.22	0.31	386	0.015	0.59	5.8	0.3
Charlevoix	69	3.78	0.27	289	0.007	0.19	3.0	0.1
Higgins	70	6.21	0.16	231	0.040	0.13	1.1	0.1
Houghton	71	2.01	0.50	236	0.015	0.52	9.2	0.2
Pere Marquette	72	1.31	0.77	438	0.035	0.29	11.8	5.5
White	73	2.09	0.48	442	0.031	0.42	9.2	4.5
Muskegon	74	1.61	0.52	347	0.081	0.61	9.5	7.3
Fremont	75	1.48	0.68	486	0.482	3.35	28.5	44.0
Mona	76	1.23	0.82	446	0.428	0.79	27.3	31.6
Crystal	77	3.40	0.29	327	0.009	0.62	2.9	0.1
Jordan	78	1.84	0.54	416	0.187	0.84	20.5	22.2
Thornapple	79	1.45	0.69	576	0.050	0.71	14.7	4.5

APPENDIX A (Contd)

Lake Name	Serial Number	SECCHI	ISEC	COND	TPHOS	TON	CHLA	AAY
Strawberry	80	1.97	0.51	405	0.168	0.75	11.2	16.9
Chemung	81	2.43	0.41	402	0.037	0.83	13.5	2.7
Thompson	82	2.34	0.43	490	0.040	0.83	12.0	2.9
Ford	83	1.11	0.90	539	0.111	0.81	14.7	14.6
Union	84	1.13	0.88	510	0.055	0.51	15.7	2.7
Long	85	1.93	0.52	452	0.197	1.00	10.1	17.4
Randall	86	1.08	0.92	518	0.203	0.78	27.2	18.8
Schroon	87	3.73	0.27	51	0.005	0.16	2.1	0.1
Black	88	1.86	0.54	116	0.036	0.67	13.1	1.3
Cassadaga	89	2.57	0.39	204	0.029	0.36	9.7	4.4
Chautauqua	90	2.00	0.50	152	0.036	0.41	13.3	5.4
Conesus	91	3.19	0.31	341	0.022	0.36	9.9	0.9
Canandaigua	92	4.33	0.23	318	0.014	0.08	4.3	0.1
Keuka	93	3.57	0.28	246	0.010	0.18	5.9	6.0
Seneca	94	4.14	0.24	778	0.012	0.11	6.1	0.1
Cayuga	95	2.80	0.36	442	0.015	0.23	3.0	0.1
Owasco	96	2.71	0.37	280	0.010	0.19	8.5	0.2
Cross	97	1.36	0.74	641	0.127	0.50	19.5	3.2
Otter	98	1.14	0.88	283	0.078	0.63	13.3	1.1
Round	99	1.17	0.86	262	0.073	0.58	28.3	18.4
Saratoga	100	2.52	0.40	227	0.028	0.51	11.8	7.5

Indicator acronyms: SECCHI = Secchi disc transparency (m)*
ISEC = inverse of Secchi disc transparency (m^{-1})*
COND = conductivity (micromhos cm^{-1})*
TPHOS = total phosphorus (mg l^{-1})*
TON = total organic nitrogen (mg l^{-1})**
CHLA = chlorophyll a (μg l^{-1})*
AAY = algal assay control yield (mg dry wt)**

Lakes 1-38 are located in Minnesota; 39-68 are in Wisconsin; 69-86 are found in Michigan; 87-100 are in New York. The serial numbers are unique to this thesis.

*Mean values based on three sampling periods.

**Values based on composite fall sample.

APPENDIX B

APPENDIX B

Sampling Dates for 100 NES-Sampled Lakes (1972)

Lake Name	Serial Number	1st Sample	2nd Sample	3rd Sample	Lake Name	Serial Number	1st Sample	2nd Sample	3rd Sample
Blackduck	1	07/12	09/08	10/21	Cokato	25	06/30	08/29	10/26
Bemidji	2	07/11	09/08	10/21	Buffalo	26	06/30	08/29	10/26
Andrusia	3	07/11	09/08	10/21	Carrigan	27	06/30	08/29	10/26
Wolf	4	07/11	09/08	10/21	Silver	28	07/03	08/29	10/26
Cass	5	07/11	09/07	10/21	Minnetonka	29	06/29	09/05	10/29
Leech	6	07/11	09/08	10/21	Forest	30	06/29	08/27	11/05
Birch	7	07/12	09/03	10/24	White Bear	31	06/29	08/27	11/05
Trout	8	07/11	09/08	10/22	St. Croix	32	06/28	08/26	11/04
Mashkenode	9	07/10	09/09	10/19	Spring	33	06/28	08/26	11/04
Whitewàter	10	07/06	09/02	10/25	Pepin	34	06/28	09/03	11/04
Pelican	11	07/10	09/07	10/22	Madison	35	07/01	08/30	10/29
Shagawa	12	07/08	09/07	10/22	Sakatah	36	07/01	08/30	10/29
Gull	13	07/02	09/05	10/24	Bear	37	07/01	08/30	10/29
Rabbit	14	07/02	09/04	10/24	Albert Lea	38	07/01	08/30	10/29
Cranberry	15	07/02	09/04	10/24	Yellow	39	06/26	08/27	11/03
Darling	16	07/06	09/01	10/25	Wapogasset	40	06/26	08/26	11/03
Carlos	17	07/10	09/02	10/28	Long	41	06/25	08/25	11/04
Le Homme Dieu	18	07/06	09/02	10/28	Elk	42	06/25	08/25	11/04
Minnewaska	19	07/06	09/01	10/25	Trout	43	06/25	08/23	11/04
Nest	20	07/02	08/31	10/25	Crystal	44	06/25	08/23	11/04
Green	21	07/02	08/31	10/25	Tainter	45	06/26	08/26	11/03
Wagonga	22	07/02	08/31	10/25	Shawano	46	06/22	08/24	11/08
Clearwater	23	07/03	08/29	10/27	Poygan	47	06/22	08/21	11/08
Mud (at Maple)	24	06/30	08/29	10/26	Butte Des Morts	48	06/22	08/20	11/09

APPENDIX B (Contd)

Lake Name	Serial Number	1st Sample	2nd Sample	3rd Sample	Lake Name	Serial Number	1st Sample	2nd Sample	3rd Sample
Winnebago	49	06/24	08/20	11/09	Fremont	75	06/13	09/15	11/13
Round	50	06/23	08/22	11/08	Mona	76	06/13	09/19	11/14
Green	51	06/22	08/21	11/08	Crystal	77	06/15	09/17	11/14
Swan	52	06/22	08/20	11/10	Jordan	78	06/15	09/18	11/15
Beaver Dam	53	06/20	08/21	11/09	Thornapple	79	06/13	09/18	11/14
Kegonsa	54	06/22	08/20	11/10	Strawberry	80	06/17	09/19	11/13
Rock	55	06/23	08/20	11/10	Chemung	81	06/16	09/19	11/15
Koshkonong	56	06/22	08/17	11/10	Thompson	82	06/16	09/19	11/15
Lac La Belle	57	06/23	08/19	11/09	Ford	83	06/16	09/19	11/13
Oconomowoc	58	06/21	08/19	11/11	Union	84	06/14	09/16	11/12
Okauchee	59	06/21	08/19	11/10	Long	85	06/13	09/17	11/12
Pine	60	06/21	08/19	11/09	Randall	86	06/14	09/16	11/12
Nagawicka	61	06/21	08/19	11/10	Schroon	87	06/01	07/25	10/10
Pewaukee	62	06/21	08/19	11/10	Black	88	05/20	07/25	10/10
Tichigan	63	06/21	08/17	11/10	Cassadaga	89	05/26	07/27	10/13
Browns	64	06/21	08/16	11/10	Chatauqua	90	05/26	07/27	10/12
Middle	65	06/22	08/19	11/10	Conesus	91	05/27	07/27	10/13
Delavan	66	06/23	08/17	11/10	Canandaigua	92	05/27	07/21	10/14
Como	67	06/21	08/16	11/10	Keuka	93	05/27	07/21	10/14
Geneva	68	06/21	08/16	11/10	Seneca	94	05/16	07/23	10/14
Charlevoix	69	06/16	09/14	11/12	Cayuga	95	05/16	07/23	10/13
Higgins	70	06/15	09/16	11/12	Owasco	96	05/28	07/24	10/12
Houghton	71	06/15	09/20	11/14	Cross	97	05/28	07/24	10/13
Pere Marquette	72	06/17	09/18	11/13	Otter	98	05/17	07/24	10/13
					Round	99	05/28	07/25	10/10
White	73	06/13	09/18	11/14	Saratoga	100	05/15	07/25	10/11
Muskegon	74	06/13	09/19	11/14					

APPENDIX C

APPENDIX C

Morphometry and Hydrology of Study Lakes*

Lake Name	Serial Number	Area (ha)	Mean Depth (m)	Maximum Depth (m)	Volume $(m^3) \times 10^6$	Shore Line (km)	Retention Time**
Blackduck	01	1,110	4.6	8.5	50.733	16.0	4.3 y
Bemidji	02	2,598	9.6	23.2	253.407	23.8	268 d
Andrusia	03	614	7.9	18.3	48.426	14.7	47 d
Wolf	04	425	8.5	17.7	36.299	12.1	37 d
Cass	05	6,312	7.6	36.6	480.934	63.1	313 d
Leech	06	45,326	4.7	45.7	214.132		5.2 y
Birch	07	519	3.1	13.7	15.826	22.7	
Trout	08	765	15.2	41.2	116.564	21.7	17.4 y
Mashkenode	09	41	2.1	4.3	0.872	3.06	9 d
Whitewater	10	490	6.1	22.3	29.701	21.1	
Pelican	11	4,429	3.7	11.6	162.005	56.1	5.0 y
Shagawa	12	955	5.7	14.7	54.436	29.0	0.9 y
Gull	13	3,812	9.1	26.2	348.508	64.2	2.9 y
Rabbit	14	340	3.7	12.8	43.520	9.81	
Cranberry	15	8.1	3.4	10.4	0.275		
Darling	16	386	6.2	18.9	23.888	10.8	
Carlos	17	1,020	13.1	49.7	133.660	19.2	3.7 y
Le Homme Dieu	18	706	6.4	25.9	45.175	15.3	7.9 y
Minnewaska	19	2,877	6.0	9.8	171.893	29.6	12.7 y
Nest	20	382	4.6	12.2	17.485		190 d
Green	21	2,187	6.4	33.5	140.032	19.8	3.7 y
Wagonga	22	654	1.2	4.6	7.978	20.9	1.3 y
Clearwater	23	1,288	5.2	22.9	66.724	27.9	1.4 y
Mud (at Maple Lake)	24						

APPENDIX C (Contd)

Lake Name	Serial Number	Area (ha)	Mean Depth (m)	Maximum Depth (m)	Volume $(m^3) \times 10^6$	Shore Line (km)	Retention Time
Cokato	25	220	7.6	15.2	16.775	7.15	413 d
Buffalo	26	611	4.2	9.1	27.007	9.25	1.4 y
Carrigan	27	66	0.9	2.4	0.594		
Silver	28	170	1.2	2.1	2.040	6.87	3.5 y
Minnetonka	29	5,855	4.3	27.8	401.727	175.0	15 y
Forest	30	893	3.4	11.3	30.204	23.1	6.5 y
White Bear	31	1,077	6.9	24.4	74.152	21.6	
St. Croix	32	3,322	8.7	23.8	291.619		23 d
Spring	33	2,392		3.1		33.8	
Pepin	34	10,118	5.1	17.1	514.979		9 d
Madison	35	541	4.0	18.0	17.847	15.8	3.3 y
Sakatah	36	497	2.1	3.7	7.607		44 d
Bear	37	556		1.1			
Albert Lea	38	985	1.1	1.8	10.591	36.0	73 d
Yellow	39	926	5.8	9.8	53.330	11.3	
Wapogasset	40	480	5.2	9.8	25.248		194 d
Long	41	169	3.4	16.5	5.632		15 d
Elk	42	36	1.8	6.4	0.691		2 d
Trout	43	1,544	11.7	35.1	180.276	30.6	
Crystal	44	36	9.8	21.0	3.552	2.25	---***
Tainter	45	709	4.0	11.3	27.806		45 d
Shawano	46	2,491	3.2	10.7	80.231	26.6	1.5 y
Poygan	47	4,449	2.1	3.4	94.305		13 d
Butte des Morts	48	3,584	1.8	2.7	65.550		6 d
Winnebago	49	55,730	4.0	6.4	2,208.184	116.9	210 d
Round	50	5.7	5.7	17.1	0.321		
Green	51	2,972	31.7	71.9	939.021	34.1	20.7 y

APPENDIX C (Contd)

Lake Name	Serial Number	Area (ha)	Mean Depth (m)	Maximum Depth (m)	Volume (m^3) x 10^6	Shore Line (km)	Retention Time
Swan	52	164	9.8	25.0	15.908		179 d
Beaver Dam	53	2,671	1.4	3.4	37.840	63.7	155 d
Kegonsa	54	1,099	5.2	8.8	58.234	15.4	139 d
Rock	55	555	4.9	17.1	26.325	14.2	3.6 y
Koshkonong	56	4,241	1.5	2.1	68.829	48.9	24 d
Lac la Belle	57	452	4.7	14.0	21.080	14.0	8 m
Oconomowoc	58	318	6.3	18.9	20.043	11.3	
Okauchee	59	451	9.5	28.7	40.320	24.1	10 m
Pine	60	285	11.8	25.9	33.448	11.8	
Nagawicka	61	415	10.1	27.4	41.147	13.8	1.5 y
Pewaukee	62	1,009	3.1	13.7	45.698	22.1	4.2 y
Tichigan	63	450	1.8	19.2	8.096	20.0	19 d
Browns	64	160	2.4	13.4	3.867	8.04	
Middle	65	72	4.6	13.7	3.293	8.53	
Delavan	66	718	7.6	17.1	54.810	28.5	2.8 y
Como	67	383	1.2	2.7	4.795	13.5	1.6 y
Geneva	68	2,130	18.6	41.2	395.928	32.5	29.9 y
Charlevoix	69	6,985	16.8	37.2	1,170.944	54.1	3.2 y
Higgins	70	3,885	14.9	41.2	580.230		15.6 y
Houghton	71	8,112	2.3	6.4	187.901	49.6	1.3 y
Pere Marquette	72	224		11.6			
White	73	1,040	6.7	21.3	71.468	20.8	56 d
Muskegon	74	1,680	7.0	21.0	120.295	18.8	23 d
Fremont	75	320	10.1	26.8	32.157	8.66	1.9 y
Mona	76	281	4.0	12.8	11.487	18.6	76 d
Crystal	77	293		21.3			

APPENDIX C (Contd)

Lake Name	Serial Number	Area (ha)	Mean Depth (m)	Maximum Depth (m)	Volume (m^3) x 10^6	Shore Line (km)	Retention Time
Jordan	78	174	7.3	17.7	12.995	7.19	304 d
Thornapple	79	166	4.3	9.5	7.063		11 d
Strawberry	80	104	6.7	15.2	7.006	6.34	13.2 d
Chemung	81	126	8.5	21.3	10.821	7.93	4.2 y
Thompson	82	106	2.7	15.8	2.909	7.35	152 d
Ford	83	425	4.4	11.9	18.521		15.2 d
Union	84	213	0.9	4.3	1.943	23.2	2 d
Long	85	85	5.2	12.5	4.425		31 d
Randall	86	208	5.5	14.9	11.263	5.89	41 d
Schroon	87	1,671	14.3	46.3	239.315	39.9	153 d
Black	88	3,380	2.7	5.2	92.718	92.7	0.1 y
Cassadaga	89	85	3.1	8.8	2.603	8.21	4 m
Chautauqua	90	5,720	6.7	23.5	392.266	68.3	1.4 y
Conesus	91	1,347	11.5	18.0	154.760	29.7	2.3 y
Canandaigua	92	4,300	39.0	83.5	1,677.377	57.8	15 y
Keuka	93	4,740	22.4	56.7	1,063.356	94.0	7.8 y
Seneca	94	17,252	88.4	188.4	15,249.165	121.3	33.1 y
Cayuga	95	17,198	52.4	132.6	9,375.141	136.5	10.9 y
Owasco	96	2,746	29.3	54.0	802.997	39.8	
Cross	97	881	5.5	16.8	49.339	20.0	7 d
Otter	98	114				5.86	
Round	99	127				4.35	
Saratoga	100	1,632	7.9	29.3	129.308	37.1	150 d
Winona	101	73	1.2	2.4	0.876	2.74	1.3 y
Trace	102						
Calhoun	103	170	8.2	25.0	13.940		

APPENDIX C (Contd)

Lake Name	Serial Number	Area (ha)	Mean Depth (m)	Maximum Depth (m)	Volume (m^3) x 10^6	Shore Line (km)	Retention Time
Big Stone	104	5,103	3.4	4.9	17.35	96.4	1.6 y
Zumbro	105	345					
Oneida	106	20,721		16.7			
Canadarago	107						
Mendota	108	3,983					
Monona	109	1,350					
Waubesa	110	855					
Cottonwood	111	150	1.6	2.4	0.240	5.63	347 d
Maple	241	240	5.8	23.2	13.920		

* This table has been compiled from information contained in the files of the Eutrophication Survey Branch, Pacific Northwest Environmental Research Laboratory (NERC-Corvallis).

** y = year, m = month, d = day

*** Seepage lake.

APPENDIX D

APPENDIX D

ERTS-1 MSS Models, Concatenations, Areal Relationships and Descriptive Statistics

This appendix contains information relevant to each ERTS-1 MSS frame examined during the investigation. It is divided into a series of subappendicies, one for each frame or pair of frames. Frames in juxtaposition on the same flightline are treated as one. Each subappendix is in turn divided into five sections:

Section D_. 1. Regression Models and Correlation Coefficients

D_. 2. Three-dimensional Color Ratio Model

D_. 3. Concatenation of Extracted Lakes

D_. 4. MSS-Lake Surface Area Relationships

D_. 5. Lake MSS Descriptive Statistics

APPENDIX D1. 9 August 1972 (1017-16091, 1017-16093)

Section D1. 1 Regression Models and Correlation Coefficients

The regression models for the prediction of Secchi disc transparency and chlorophyll a are in Chapter IV along with the coefficients of correlation between the MSS data (colors and color ratios), the trophic indicators and the multivariate trophic state index. The model for the prediction of trophic state is in Chapter V.

Section D1. 2. Three-dimensional Color Ratio Model

The color ratio model is in Chapter V (Figure 33).

Section D1. 3. Concatenation of Extracted Lakes

The concatenation of lakes extracted from Frame 1017-16093 is in Chapter III; the Frame 1017-16091 concatenation is in this appendix (Figure 37).

Section D1. 4. MSS-Lake Surface Area Relationships

The estimates of lake surface area using MSS data are in Chapter IV (Table 16).

Section D1. 5. Lake MSS Descriptive Statistics

The MSS statistics for Frame 1017-16093 are in Table 34 and those for Frame 1017-16091 are in Table 35.

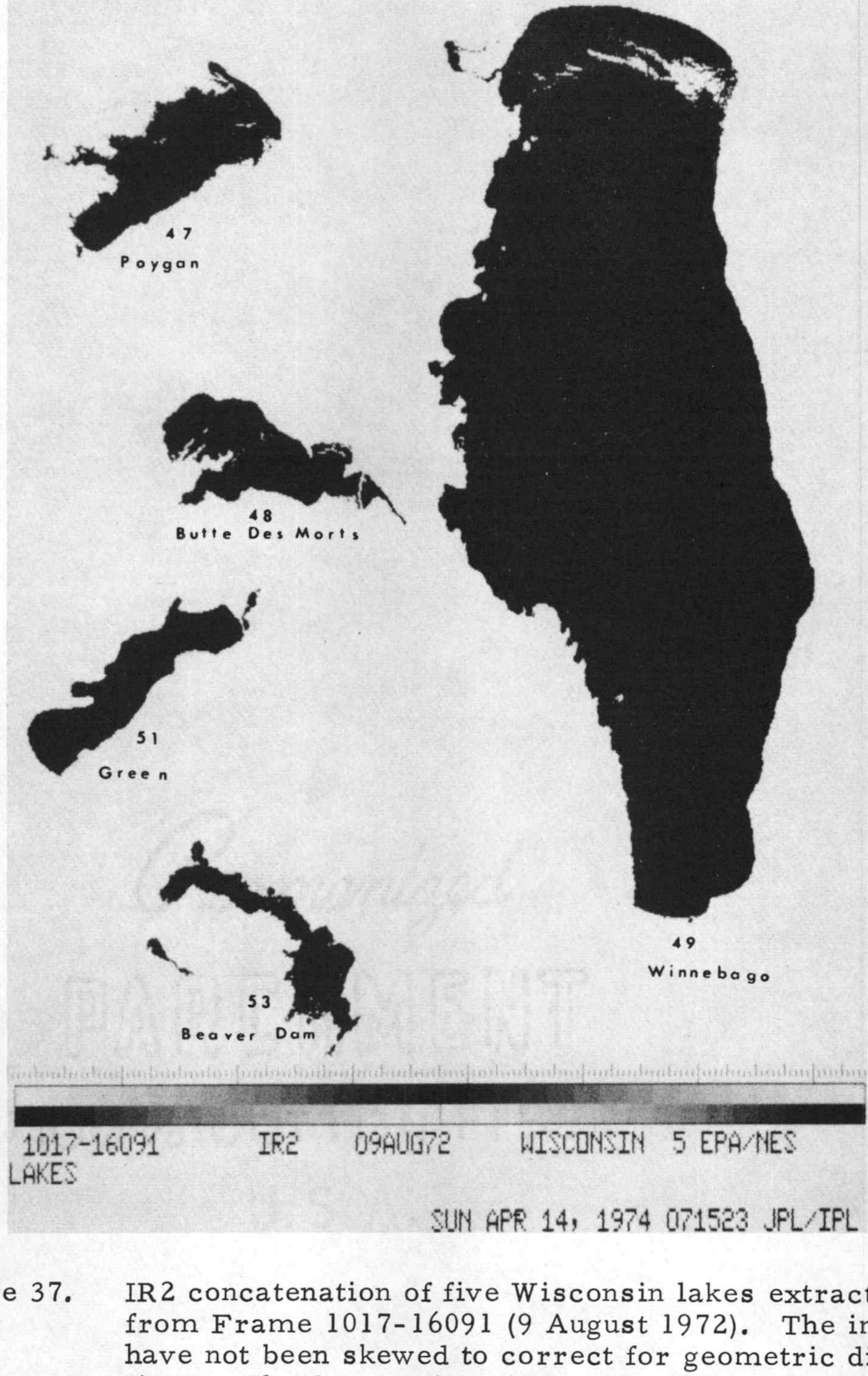

Figure 37. IR2 concatenation of five Wisconsin lakes extracted from Frame 1017-16091 (9 August 1972). The images have not been skewed to correct for geometric distortions. Cloud cover is evident over the northern (upper) end of Lake Winnebago.

Table 34. MSS descriptive statistics for 15 Wisconsin lakes extracted from Frame 1017-16093 (9 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Kegonsa	54	2,675	50.18*	30.08	24.38	8.34
			1.63**	1.50	3.30	5.24
Rock	55	1,009	38.86	19.39	13.62	5.10
			3.27	3.07	3.38	3.74
Koshkonong	56	9,247	46.85	31.15	27.95	10.08
			1.81	2.45	2.93	2.89
Lac La Belle	57	944	43.30	24.27	15.11	5.69
			4.81	4.07	3.35	3.73
Oconomowoc	58	652	39.87	19.99	14.46	6.35
			4.90	4.27	4.48	4.72
Okauchee	59	871	36.78	19.70	14.35	6.07
			1.73	2.17	4.27	4.68
Pine	60	568	36.54	16.99	13.04	5.66
			2.89	2.70	3.95	4.58
Nagawicka	61	751	42.46	23.85	17.06	6.12
			2.78	1.83	3.71	3.94
Pewaukee	62	1,996	37.35	20.75	16.19	6.36
			2.54	2.34	4.44	4.01
Tichigan	63	689	40.78	25.56	23.21	10.47
			3.06	1.67	4.28	5.26
Browns	64	309	36.23	19.06	15.78	7.17
			2.59	2.87	3.99	4.63
Middle	65	117	36.49	18.54	17.38	9.35
			2.77	3.27	6.53	6.91
Delavan	66	1,541	45.42	25.39	22.07	8.35
			2.62	1.66	3.00	3.98
Como	67	703	47.86	30.27	22.51	8.62
			5.20	4.24	3.25	4.39
Geneva	68	4,543	39.42	18.53	13.71	5.69
			2.12	2.16	2.81	3.25

*Mean DN value for the lake.

**Standard deviation for the lake pixels.

Table 35. MSS descriptive statistics for 5 Wisconsin lakes extracted from Frame 1017-16091 (9 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Poygan	47	9,177	46.05*	29.50	23.62	9.55
			4.82**	5.07	5.25	5.24
Butte Des Morts	48	7,395	42.99	26.74	23.08	8.25
			2.55	1.83	2.78	3.59
Winnebago	49	114,186	42.38	24.95	20.86	6.65
			3.22	2.67	5.50	3.92
Green	51	6,613	36.90	17.03	12.12	4.59
			2.00	2.32	3.21	3.18
Beaver Dam	53	5,162	37.57	22.76	20.12	7.48
			2.09	2.00	4.24	4.57

*Mean DN value for the lake.
**Standard deviation of the lake DN values.

APPENDIX D2. 28 August 1972 (1036-16152)

Section D2. 1. Regression Models and Correlation Coefficients

No models were developed because the sample size is insufficient; only three NES-sampled lakes were extracted from the frame. The three lakes (Rock, Kegonsa, and Koshkonong) have higher mean DN levels than on 9 August 1972, but have maintained their relative positions.

Section D2. 2. Three-dimensional Color Ratio Model

No model was constructed.

Section D2. 3. Concatenation of Extracted Lakes

The concatenation of extracted lakes is displayed as Figure 38.

Section D2. 4. MSS-Lake Surface Area Relationships

The estimates of lake surface using the MSS pixel counts and a conversion factor of 0. 48 (1 pixel = 0. 48 hectares) are in Table 36.

Section D2. 5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 37.

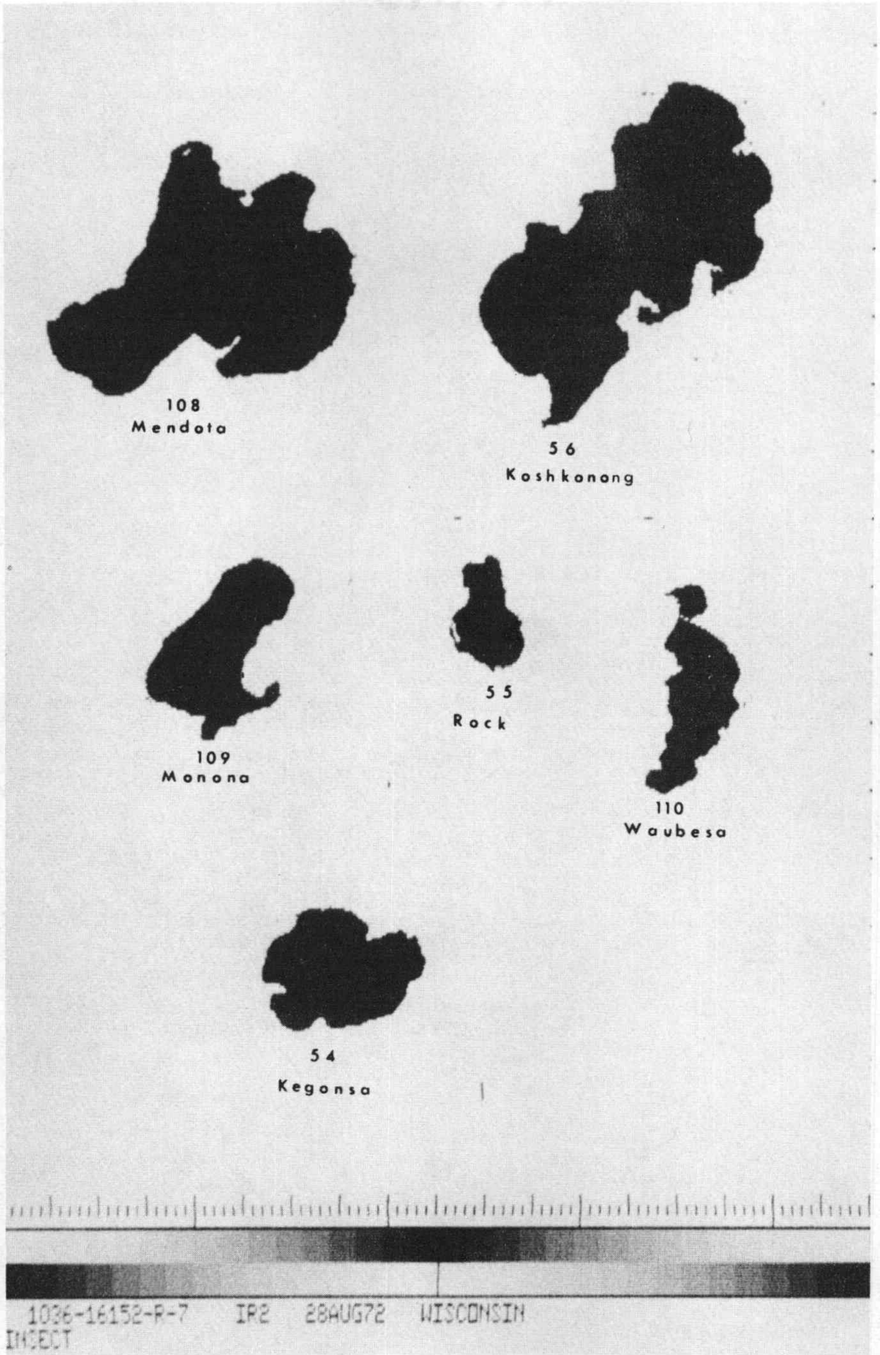

Figure 38. IR2 concatenation of six Wisconsin lakes extracted from Frame 1036-16152 (28 August 1972).

Table 36. Areal aspects of 6 Wisconsin lakes extracted from Frame 1036-16152 (28 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Kegonsa	54	2,805	1,340.8*	1,099.2	0.820
Rock	55	1,039	496.6*	554.8	1.117
Koshkonong	56	9,364	4,476.0*	4,241.5	0.948
Mendota	108	8,460	4,043.9	3,937.7	0.974
Monona	109	2,729	1,304.5	1,349.7	1.035
Waubesa	110	1,938	926.4	855.1	0.923

*The estimated areas of these lakes using the MSS pixel counts from Frame 1017-16093 are, respectively: 1,277.4 hectares, 481.8 hectares, and 4,415.9 hectares.

Table 37. MSS descriptive statistics for 6 Wisconsin lakes extracted from Frame 1036-16152 (28 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Kegonsa	54	2,805	46.97*	26.63	17.88	6.57
			2.01**	1.40	2.64	3.41
Rock	55	1,039	42.90	22.93	15.22	6.78
			2.38	1.74	3.92	4.49
Koshkonong	56	9,364	43.47	27.82	22.23	8.19
			2.72	1.64	2.55	3.15
Mendota	108	8,460	40.07	21.71	13.88	5.24
			2.01	1.83	2.97	3.31
Monona	109	2,729	39.24	21.84	14.29	6.05
			2.84	2.17	3.38	3.70
Waubesa	110	1,938	46.78	26.50	20.28	7.92
			4.01	2.07	3.08	4.03

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D3. 11 June 1973 (1323-16100, 1323-16094)

Section D3.1. Regression Models and Correlation Coefficients

The regression model for the prediction of trophic state is found in Chapter V; it is of little practical value.

Section D3.2. Three-dimensional Color Ratio Model

The color ratio model is in Chapter V (Figure 34).

Section D3.3. Concatenations of Extracted Lakes

The 23 lakes extracted from the frames are displayed in the form of two concatenation, Figures 39 and 40. The fragmented appearance of Lake Geneva is a result of cloud cover. Lake Winnebago is partially truncated, a consequence of separating the continuous MSS strip into discrete frames. Cloud cover is responsible for the mottled appearance of Lake Poygan. Middle Lake has not been excised from the Lauderdale Lakes as was the case for 9 August 1972.

Section D3.4. MSS-Lake Surface Area Relationships

The estimates of lake surface area using the MSS pixel count are found in Table 38. Area estimates have not been determined for truncated water bodies or those partially covered by clouds.

Section D3.5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 39.

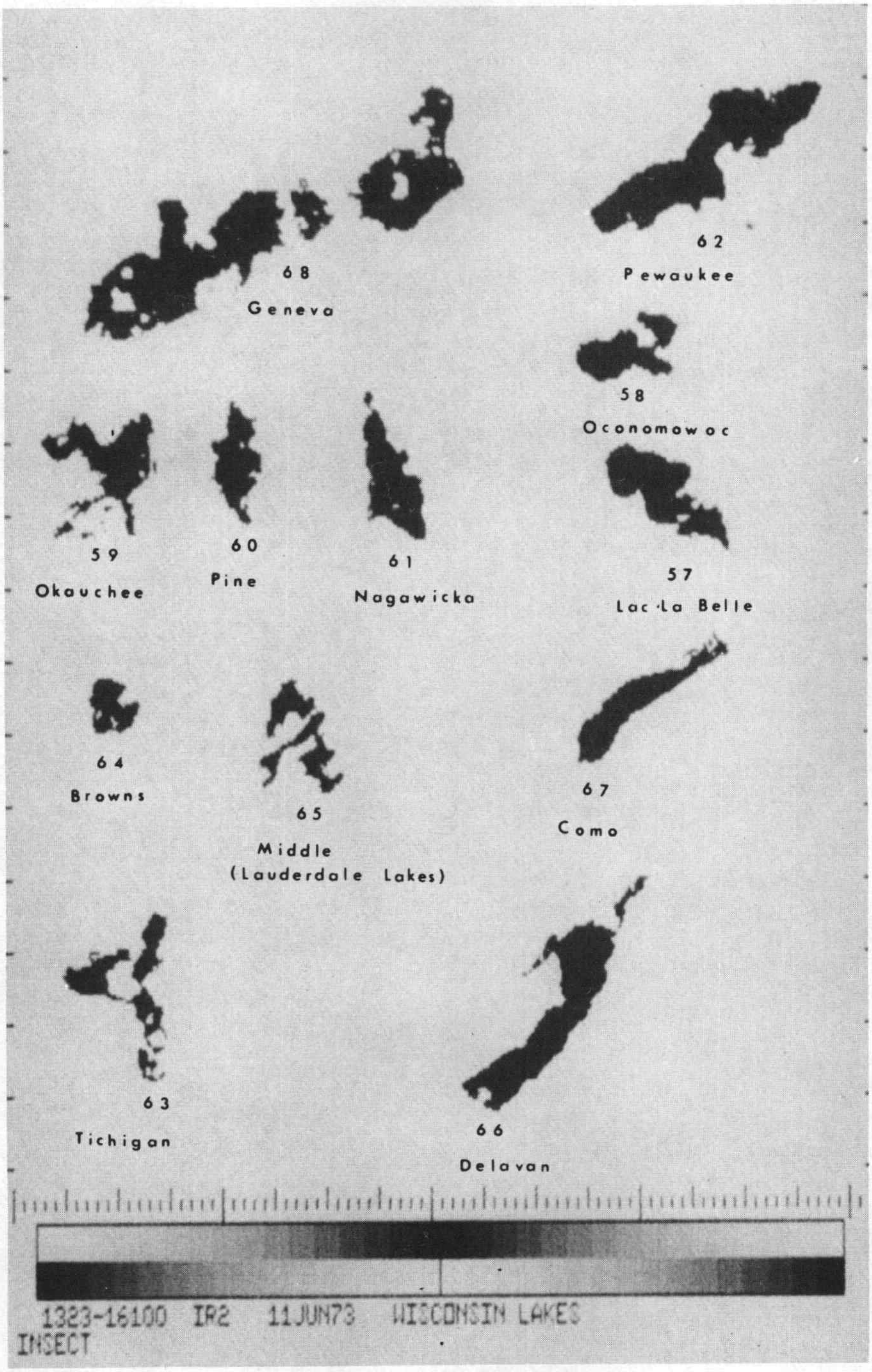

Figure 39. IR2 concatenation of 12 Wisconsin lakes extracted from Frame 1323-16100 (11 June 1973). The 11 other lakes extracted from the frame are in Figure 40.

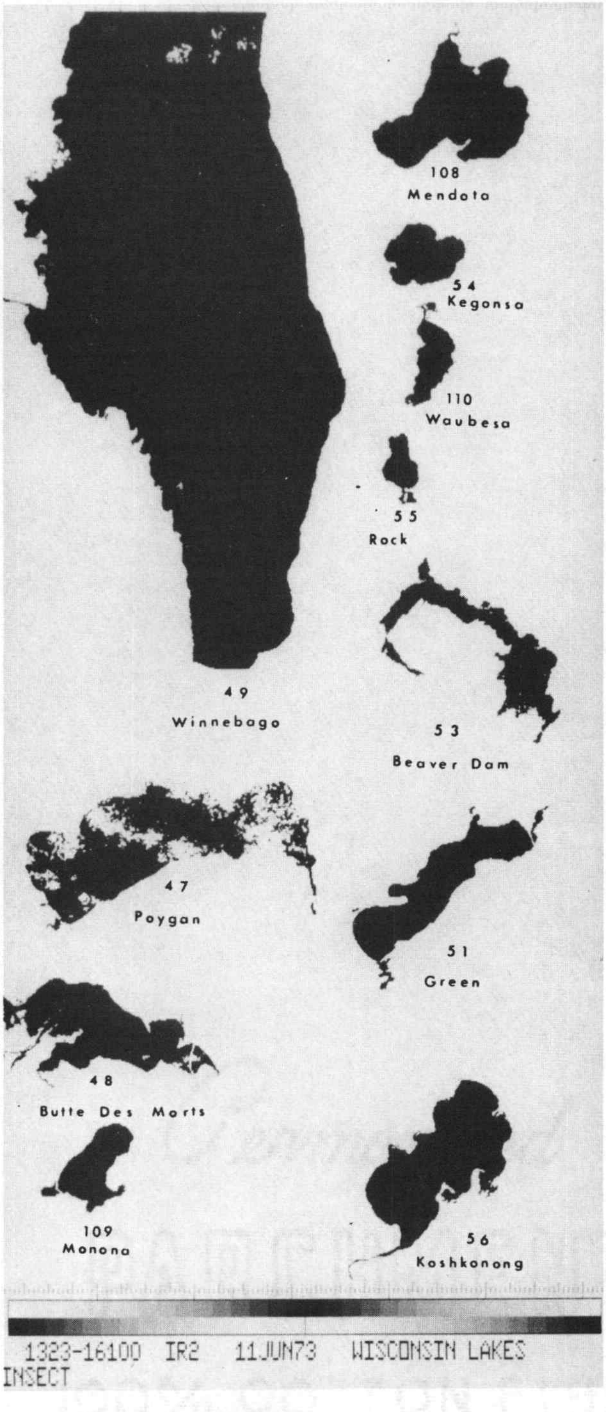

Figure 40. IR2 concatenation of 11 Wisconsin lakes extracted from Frame 1323-16100 (11 June 1973). The other 12 lakes extracted from the frame are in Figure 39.

Table 38. Areal aspects of 23 Wisconsin lakes extracted from Frames 1323-16094 and 1323-16100 (11 June 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Poygan	47	11,829	5,648.9	4,448.5	---*
Butte Des Morts	48	8,053	3,845.3	3,584.4	0.932
Winnebago	49	98,821	47,206.8	55,730.4	1.181**
Green	51	6,742	3,220.7	2,972.9	0.923
Beaver Dam	53	5,485	2,618.8	2,671.0	1.010
Kegonsa	54	2,706	1,292.7	1,099.2	0.850
Rock	55	1,127	538.4	554.8	1.031
Koshkonong	56	9,516	4,545.8	4,241.3	0.933
Lac La Belle	57	939	448.6	452.1	1.008
Oconomowoc	58	627	299.5	317.7	1.061
Okauchee	59	928	443.3	450.8	1.017
Pine	60	550	262.7	284.5	1.083
Nagawicka	61	775	370.2	415.2	1.122
Pewaukee	62	1,982	946.8	1,008.9	1.066
Tichigan	63	707	337.7	449.8	1.332
Browns	64	304	145.2	160.3	1.104
Middle	65	563	268.9	337.6	1.254***
Delavan	66	1,517	724.7	717.9	0.991
Como	67	740	353.5	383.0	1.084
Geneva	68	3,555	1,698.2	2,129.5	--- ****
Mendota	108	8,488	4,054.7	3,937.7	0.971
Monona	109	2,791	1,333.3	1,349.7	1.012
Waubesa	110	1,882	899.0	855.1	0.951

*The extracted image includes Lake Winneconne and substantial cloud cover.
**The northern end of the lake is outside the sensor field of view.
***The Lauderdale lakes complex was extracted and treated as a single lake.
****The presence of cloud cover has resulted in the low ERTS area estimate.

Table 39. MSS descriptive statistics for 23 Wisconsin lakes extracted from Frame 1323-16100 (11 June 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Poygan	47	11,829	50.53	33.74	25.52	13.95
			4.97	5.24	5.89	5.69
Butte Des Morts	48	8,053	47.28	30.78	22.83	11.47
			2.83	2.73	4.06	4.14
Winnebago	49	98,821	49.43	32.87	20.14	8.80
			2.98	3.01	2.65	2.80
Green	51	6,742	43.53	23.59	16.93	7.75
			2.59	2.67	3.67	3.75
Beaver Dam	53	5,485	51.55	34.55	27.40	13.25
			1.99	2.57	3.34	3.75
Kegonsa	54	2,706	49.01	29.20	19.67	9.18
			2.48	2.03	3.50	3.49
Rock	55	1,127	50.02	29.41	22.14	11.66
			3.07	2.21	3.35	3.73
Koshkonong	56	9,516	51.94	36.81	25.61	11.26
			2.40	3.31	2.97	3.43
Lac La Belle	57	939	52.02	32.55	22.49	11.37
			4.28	4.34	3.49	3.92
Oconomowoc	58	627	46.24	26.80	20.38	10.65
			3.68	3.15	3.72	4.09
Okauchee	59	928	49.49	31.32	22.68	11.75
			2.49	2.28	4.13	4.69
Pine	60	550	51.83	27.97	21.03	10.99
			2.45	2.20	3.47	4.20
Nagawicka	61	775	46.71	27.70	23.09	11.77
			2.65	2.18	5.17	4.74
Pewaukee	62	1,982	53.19	32.02	24.20	12.08
			5.65	3.70	4.24	4.15
Tichigan	63	707	48.19	31.03	24.85	13.54
			2.82	2.37	5.11	5.04
Browns	64	304	50.40	29.75	22.03	12.11
			2.49	2.37	3.84	4.31
Middle	65	563	48.94	28.25	22.74	13.05
			3.04	2.46	5.31	5.22
Delavan	66	1,517	51.76	31.16	24.30	12.29
			3.05	2.72	5.01	4.10
Como	67	740	60.33	40.42	28.28	14.07
			5.56	5.37	4.02	4.22
Geneva	68	3,555	54.15	31.44	23.62	13.26
			5.71	4.87	5.23	4.90
Mendota	108	8,488	51.01	28.74	19.80	8.59
			2.58	2.36	3.19	3.16
Monona	109	2,791	49.27	27.68	19.81	9.36
			3.08	2.39	4.39	3.90
Waubesa	110	1,882	47.56	27.53	21.31	10.38
			2.76	2.16	6.56	5.28

APPENDIX D4. 17 July 1973 (1359-16091, 1359-16094)

Section D4. 1. Regression Models and Correlation Coefficients

The regression model for the prediction of the multivariate trophic state index is in Chapter V.

Section D4. 2. Three-dimensional Color Ratio Model

The MSS color ratio model is in Chapter V (Figure 35). It is very similar in appearance to the model constructed from 9 August 1972 data.

Section D4. 3. Concatenations of Extracted Lakes

Eighteen of the 21 lakes extracted from the frames are found in Figures 41 and 42. Lake Winnebago is common to both frames. Cloud interference is noted over some of the lakes.

Section D4. 4. MSS-Lake Surface Area Relationships

The estimates of lake surface area using the MSS pixel counts are found in Table 40.

Section D4. 5. Lake MSS Descriptive Statistics

The MSS statistics for Frame 1359-16091 are in Table 41 and those for Frame 1359-16094 are in Table 42.

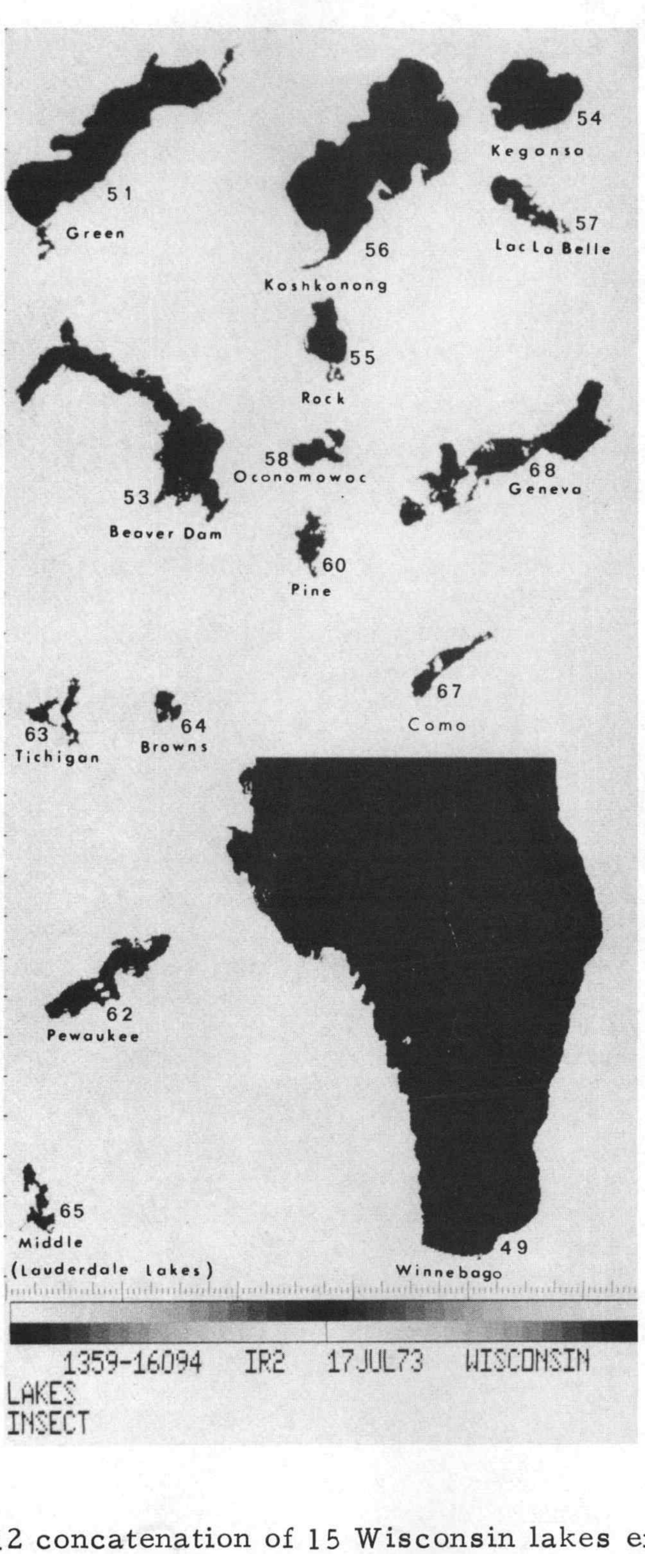

Figure 41. IR2 concatenation of 15 Wisconsin lakes extracted from Frame 1359-16094 (17 July 1973).

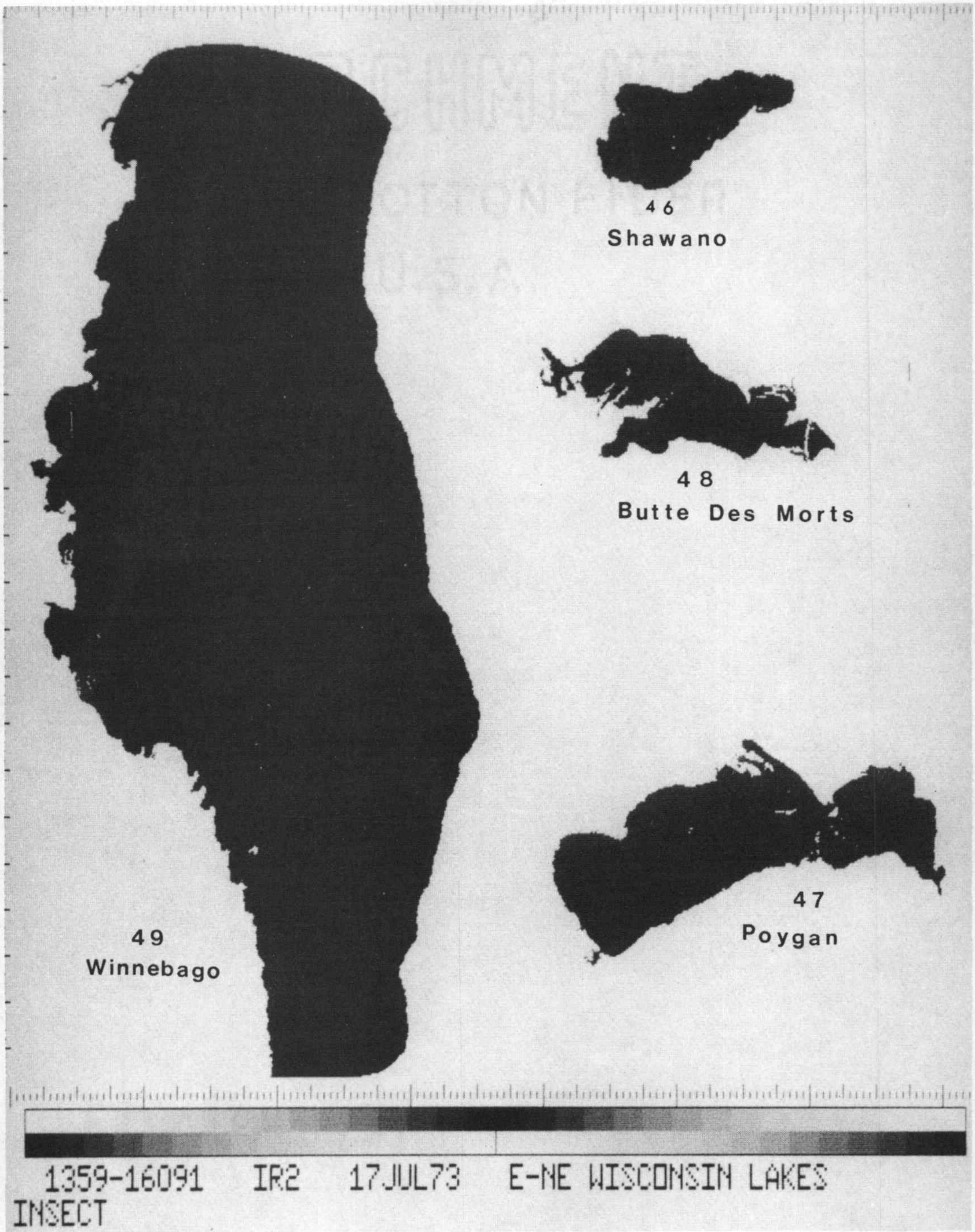

Figure 42. IR2 concatenation of 4 Wisconsin lakes extracted from Frame 1359-16091 (17 July 1973).

Table 40. Areal aspects of 21 Wisconsin lakes extracted from Frames 1359-16091 and 1359-16094 (17 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Shawano	46	5,263	2,514.1	2,491.3	0.991
Poygan	47	15,836	7,564.9	4,448.5	--- *
Butte Des Morts	48	7,600	3,630.5	3,584.4	0.987
Winnebago	49	112,887	53,926.1	55,730.4	1.034
Green	51	6,594	3,150.0	2,972.9	0.944
Beaver Dam	53	5,057	2,415.7	2,671.0	1.106
Kegonsa	54	2,716	1,297.4	1,099.2	0.847
Rock	55	1,102	526.4	554.8	1.054
Koshkonong	56	9,029	4,313.2	4,241.3	0.983
Lac La Belle	57	927	442.8	452.1	1.021
Oconomowoc	58	613	292.8	317.7	1.085
Okauchee	59	791	377.9	450.8	1.193
Pine	60	563	269.0	284.5	1.058
Nagawicka	61	505	241.2	415.2	--- **
Pewaukee	62	1,851	884.2	1,008.9	1.141**
Tichigan	63	561	268.0	449.8	--- ***
Browns	64	317	151.4	160.3	1.059
Middle	65	630	301.0	337.6	1.122****
Delavan	66	1,487	710.3	717.9	1.011
Como	67	615	293.8	383.0	1.304**
Geneva	68	3,684	1,759.9	2,129.5	1.210**

*Pixel count includes Lake Winneconne.
**Cloud interference.
***A portion of the lake body was truncated during the extraction process.
****Includes the entire Lauderdale lake complex.

Table 41. MSS descriptive statistics for 4 Wisconsin lakes extracted from Frame 1359-16901 (17 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Poygan	47	15,836	56.74	39.27	32.09	16.74
			2.96	3.45	4.37	3.32
Butte Des Morts	48	7,600	53.31	34.49	28.99	13.44
			3.04	3.45	4.51	3.67
Winnebago	49	112,887*	51.41	32.40	23.94	10.74
			3.29	3.31	3.62	3.35
Shawano	46	5,263	46.38	28.38	20.61	10.16
			2.69	2.89	3.57	3.54

*Southern end of Lake Winnebago is outside the sensor field of view.

Table 42. MSS descriptive statistics for 17 Wisconsin lakes extracted from Frame 1359-16094 (17 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Green	51	6,594	52.65	29.40	21.00	10.69
			2.27	2.10	3.37	3.35
Beaver Dam	53	5,057	56.88	38.74	33.17	14.70
			2.40	2.72	3.80	3.40
Kegonsa	54	2,716	60.79	36.33	29.71	13.97
			2.59	2.25	4.06	3.83
Rock	55	1,102	51.85	30.38	21.99	11.42
			3.04	3.13	4.44	4.37
Koshkonong	56	9,029	54.72	37.93	29.03	12.26
			2.79	3.52	3.89	3.21
Lac La Belle	57	927	56.55	34.60	23.29	12.00
			5.43	5.24	4.48	4.64
Oconomowoc	58	613	49.58	28.35	21.04	10.97
			3.45	3.31	3.86	4.17
Okauchee	59	791	50.77	30.91	23.42	12.99
			3.57	3.46	5.13	5.27
Pine	60	563	53.20	28.25	22.35	12.95
			4.07	3.37	4.87	5.51
Nagawicka	61	505	51.92	31.30	25.04	13.91
			6.11	5.26	5.73	5.46
Pewaukee	62	1,851	52.27	31.26	24.50	13.04
			3.75	2.88	5.26	4.41
Tichigan	63	561	52.04	33.00	28.48	15.30
			4.75	3.60	5.43	5.08
Browns	64	317	53.91	31.25	24.09	13.07
			2.12	2.28	3.39	3.72
Middle	65	630	46.91	25.95	22.87	10.73
			2.95	2.71	5.07	6.94
Delavan	66	1,487	57.85	33.98	30.45	14.74
			3.49	2.87	4.78	4.03
Como	67	615	59.61	40.60	28.89	15.20
			4.50	4.35	4.43	4.35
Geneva	68	3,684	52.22	28.12	22.14	12.67
			4.11	3.68	4.57	4.71

APPENDIX D5. 14 August 1972 (1022-16373)

Section D5.1. Regression Models, and Correlation Coefficients (Table 43)

Twelve Minnesota lakes were extracted from the frame; Maple Lake (241) was not included in the final regression model efforts.

Darling (16)	Clearwater (23)
Le Homme Dieu (18)	Maple (241)
Minnewaska (19)	Cokato (25)
Nest (20)	Buffalo (26)
Green (21)	Silver (28)
Wagonga (22)	Minnetonka (29)

Table 43. Correlations between ground truth and MSS data (colors and color ratios) for 11 Minnesota lakes in Frame 1022-16373 (14 August 1972).

	PC1	CHLA	LNCHLA	SECCHI	LNSECCHI
GRN	0.422	0.487	0.461	-0.749	-0.734
RED	0.547	0.613	0.610	-0.766	-0.801
IR1	0.890	0.763	0.620	-0.791	-0.920
IR2	0.764	0.758	0.698	-0.626	-0.801
GRNRED	-0.585	-0.646	-0.725	0.415	0.492
GRNIR1	-0.920	-0.704	-0.539	0.557	0.728
GRNIR2	-0.644	-0.602	-0.558	0.268	0.475
REDIR1	-0.940	-0.670	-0.454	0.562	0.728
REDIR2	-0.636	-0.568	-0.495	0.220	0.439
IR1IR2	0.536	0.195	-0.052	-0.597	-0.518

The best regression model for the prediction of trophic state is:

$$\widehat{PC1} = 13.150 - 10.626 \text{ GRNIR1} + 2.327 \text{ GRNIR2}$$

It explains about 96 percent of the variation about the mean (Table 44). The observed and predicted PC1 values are in Table 45.

Table 44. Analysis of variance table of PC1 regression model for 11 Minnesota lakes in Frame 1022-16373 (14 August 1972).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	10	42.423	4.242	
Regression	2	40.750	20.375	97.488
Residual	8	1.673	0.209	

R^2 = 0.9606 x 100 = 96.06%   s.e. of estimate = 0.456

Table 45. PC1 residuals of 11 Minnesota lakes in Frame 1022-16373 (14 August 1972).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Darling	16	-0.73	-0.34	-0.39
Le Homme Dieu	18	-1.06	-0.84	-0.22
Minnewaska	19	-0.32	-0.68	0.36
Nest	20	0.77	1.04	-0.27
Green	21	-1.10	-1.00	-0.10
Wagonga	22	4.40	4.65	-0.25
Clearwater	23	0.01	0.08	-0.07
Cokato	25	1.61	1.05	0.56
Buffalo	26	2.31	2.92	-0.61
Silver	28	4.79	4.23	0.46
Minnetonka	29	0.73	0.29	0.44

The best model for the prediction of Secchi disc transparency is:

$$\widehat{\text{LNSECCHI}} = -4.105 - 154.13\ \text{GRNIR1} + 76.641\ \text{GRNIR2} + 265.290\ \text{REDIR1} - 130.200\ \text{REDIR2}$$

It explains about 87 percent of the variation about the mean (Table 46). The observed and predicted Secchi disc transparency values are in Table 47. Caution is advised in using this model because a relatively large number of variables are incorporated into it.

Table 46. Analysis of variance table of the Secchi disc transparency regression model for 11 Minnesota lakes in Frame 1022-16373 (14 August 1972).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	10	9.529	0.953	
Regression	4	8.324	2.081	10.353
Residual	6	1.205	0.201	

$R^2 = 0.8735 \times 100 = 87.35\%$ s.e. of estimate = 0.448

Table 47. Secchi disc transparency residuals of 11 Minnesota lakes in Frame 1022-16373 (14 August 1972).

Lake Name	Serial Number	SECCHI	$\widehat{\text{SECCHI}}$	SECCHI-$\widehat{\text{SECCHI}}$
Darling	16	2.79	1.87	0.92
Le Homme Dieu	18	1.73	2.45	-0.72
Minnewaska	19	1.55	1.73	-0.18
Nest	20	0.97	0.97	0.00
Green	21	2.29	2.68	-0.39
Wagonga	22	0.23	0.16	0.07
Clearwater	23	1.80	1.25	0.55
Cokato	25	2.74	2.82	-0.08
Buffalo	26	1.98	1.44	0.54
Silver	28	0.15	0.31	-0.16
Minnetonka	29	1.47	1.60	-0.13

Efforts to construct a regression model for the prediction of chlorophyll _a_ drew negative results.

Section D5. 2. Three-dimensional Color Ratio Model.

The color ratio model is displayed in Figure 43. Wagonga Lake and Silver Lake are isolated from the other lakes because their IR1 values exceed their RED DN values. The two lakes are often referred to as being hypereutrophic.

Section D5. 3. Concatenation of Extracted Lakes

The Frame 1022-16373 concatenation is in Figure 44. Portions of Lake Minnetonka and Lake Le Homme Dieu were outside the sensor field of view, resulting in the linear shore line effects. Minnetonka is a complex lake consisting of 15 large "bays".

Section D5. 4. MSS-Lake Surface Area Relationships

The areal aspects of Frame 1022-16373 are in Table 48.

Section D5. 5. Lake MSS Descriptive Statistics

The MSS statistics for Frame 1022-16373 are in Table 49.

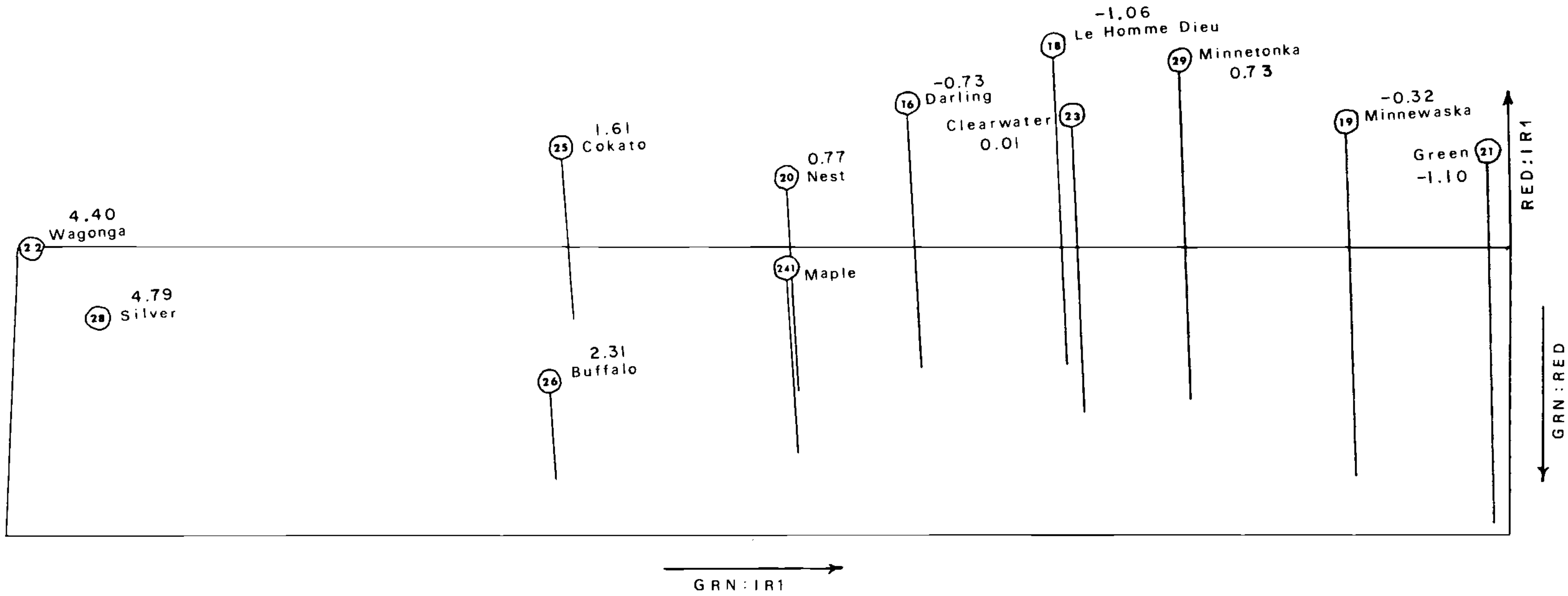

Figure 43. Three-dimensional MSS color ratio model of 12 Minnesota lakes extracted from Frame 1022-16373 (14 August 1972).

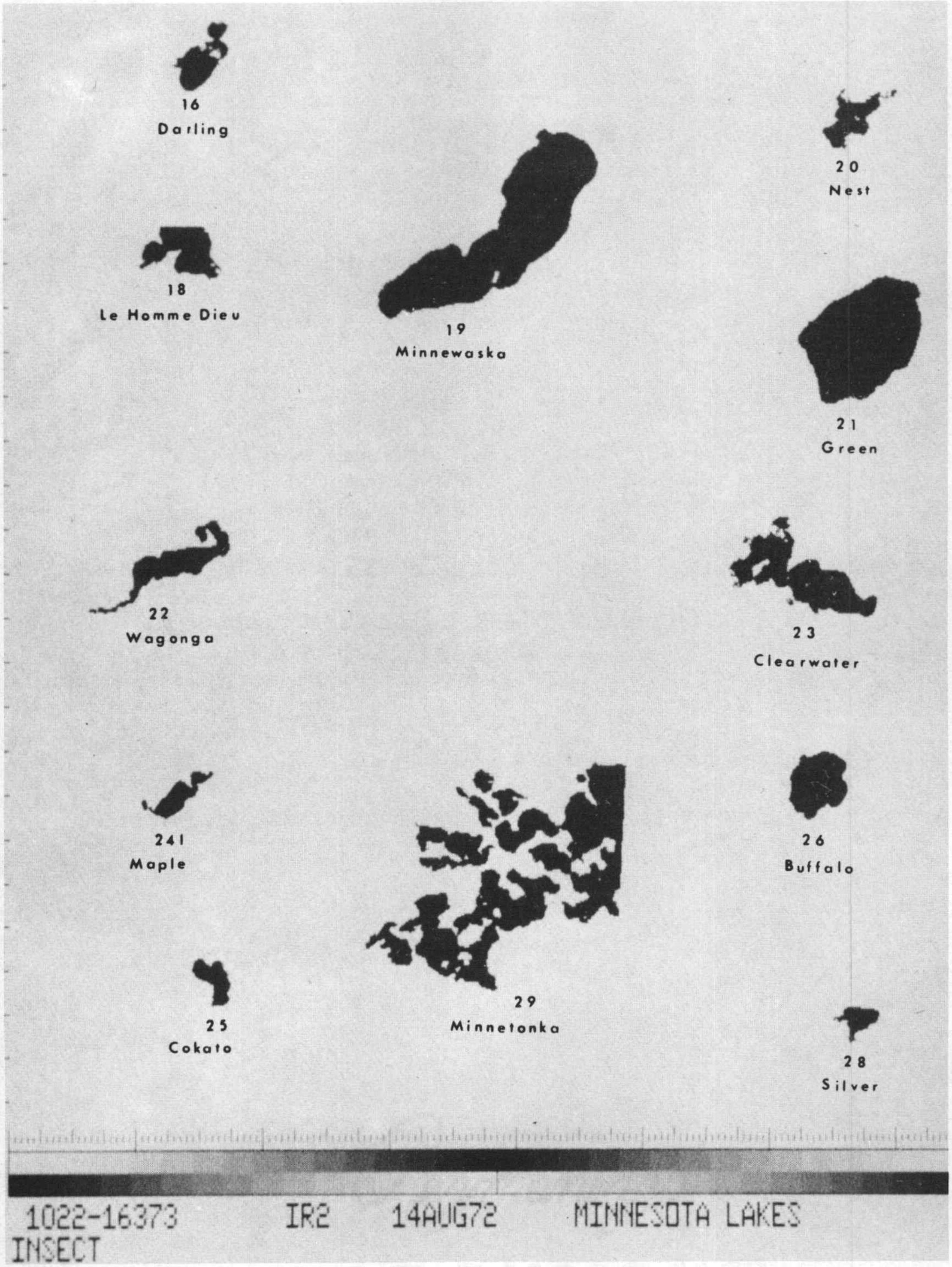

Figure 44. IR2 concatenation of 12 Minnesota lakes extracted from Frame 1022-16373 (14 August 1972).

Table 48. Areal aspects of 12 Minnesota lakes extracted from Frame 1022-16373 (14 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Darling	16	824	393.5	386.1	0.981
Le Homme Dieu	18	1,127	538.2	765.7	--- *
Minnewaska	19	6,996	3,340.9	3,144.5	0.941
Nest	20	778	376.3	382.4	1.016
Green	21	4,846	2,314.2	2,355.8	1.018
Wagonga	22	1,339	639.4	725.2	1.134
Clearwater	23	2,561	1,223.0	1,287.8	1.053
Maple	241	518	247.4	287.3	1.161
Cokato	25	470	224.5	220.2	0.981
Buffalo	26	1,334	637.1	611.1	0.959
Silver	28	367	175.3	170.8	0.974
Minnetonka	29	9,787	4,673.7	5,855.6	--- *

*A portion of the lake was outside the sensor field of view.

Table 49. MSS descriptive statistics for 12 Minnesota lakes extracted from Frame 1022-16373 (14 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Darling	16	825	49.30*	29.37	23.89	13.59
			4.00**	4.27	5.28	5.57
Le Homme Dieu	18	1,163	51.76	30.90	24.02	13.51
			3.01	3.33	4.45	5.03
Minnewaska	19	6,996	50.56	28.74	21.66	10.72
			2.65	2.11	3.32	3.97
Nest	20	788	46.81	27.60	23.54	12.08
			2.67	1.98	4.62	5.48
Green	21	4,846	46.25	25.78	19.12	9.31
			2.06	1.59	2.74	3.34
Wagonga	22	1,339	55.77	34.92	36.92	17.18
			3.48	2.96	3.33	3.90
Clearwater	23	2,561	47.64	27.82	22.00	11.15
			2.47	2.09	4.56	5.14
Maple	241	518	44.22	25.38	22.23	12.43
			2.16	1.80	4.81	5.26
Cokato	25	470	41.37	25.19	22.37	12.74
			2.87	1.80	4.19	5.39
Buffalo	26	1,334	46.68	26.49	25.36	11.64
			2.28	1.46	4.37	3.88
Silver	28	376	55.75	33.86	35.89	17.10
			2.46	2.30	2.72	4.10
Minnetonka	29	9,787	53.81	31.60	24.11	11.53
			5.30	4.31	4.87	4.85

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D6. 6 October 1973 (1075-16321)

Section D6. 1. Regression Models, and Correlation Coefficients (Table 50)

Sixteen Minnesota lakes were extracted from the frame. Lakes Sakatah*, Calhoun, Zumbro, and Maple were not used in the construction of the regression model.

Clearwater (23)	Minnetonka (29)	Madison (35)
Maple (241)	Forest (30)	Sakatah (36)
Cokato (25)	White Bear (31)	Calhoun (103)
Buffalo (26)	St. Croix (32)	Zumbro (105)
Carrigan (27)	Spring (33)	
Silver (28)	Pepin (34)	

Table 50. Correlations between ground truth and MSS data (Color ratios) for 12 Minnesota lakes in Frame 1075-16321 (6 October 1972).

	PC1	CHLA	LNCHLA	SECCHI	LNSECCHI
GRNRED	-0.375	0.067	-0.197	0.582	0.517
GRNIR1	-0.793	-0.391	-0.494	0.706	0.827
GRNIR2	-0.620	-0.425	-0.305	0.424	0.545
REDIR1	-0.663	-0.471	-0.425	0.425	0.606
REDIR2	-0.393	-0.410	-0.173	0.105	0.250
IR1IR2	0.220	-0.146	0.205	-0.349	-0.353

*MSS data are not reliable; Lake Sakatah not included in analyses.

The best model for the prediction of trophic state is:

$$\widehat{PC1} = 11.553 - 7.132\ \mathrm{REDIR1}$$

This model explains about 44 percent of the variation about the mean (Table 51). It is of little practical value. The observed and predicted PC1 values are in Table 52.

Table 51. Analysis of variance table for the PC1 regression model for 12 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	11	42.274	3.843	
Regression	1	18.561	18.561	7.827
Residual	10	23.713	23.713	

$R^2 = 0.4391 \times 100 = 43.91\%$ s.e. of estimate = 1.540

Table 52. PC1 residuals of 12 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Clearwater	23	0.01	0.95	-0.94
Cokato	25	1.61	0.84	0.77
Buffalo	26	2.31	0.14	2.17
Carrigan	27	4.40	3.11	1.29
Silver	28	4.79	4.81	-0.02
Minnetonka	29	0.73	1.06	-0.33
Forest	30	-1.22	0.45	-1.67
White Bear	31	-1.41	1.31	-2.72
St. Croix	32	-0.17	1.40	-1.57
Spring	33	2.33	1.22	1.10
Pepin	34	2.10	0.87	1.23
Madison	35	1.36	0.67	0.69

Section D6. 2. Three-dimensional Color Ratio Model

The color ratio model is displayed in Figure 45.

Section D6. 3. Concatenation of Extracted Lakes

The 15 lakes are displayed in two concatenations, Figure 46 and Figure 47. A portion of Lake Pepin is outside the sensor field of view, accounting for the linear shore line. The lake images are in scale and this results in the very small image of Lake Zumbro. Lake St. Croix was truncated during processing.

Section D6. 4. MSS-Lake Surface Area Relationships

The areal aspects of the lakes extracted from Frame 1075-16321 are in Table 53.

Section D6. 5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 54.

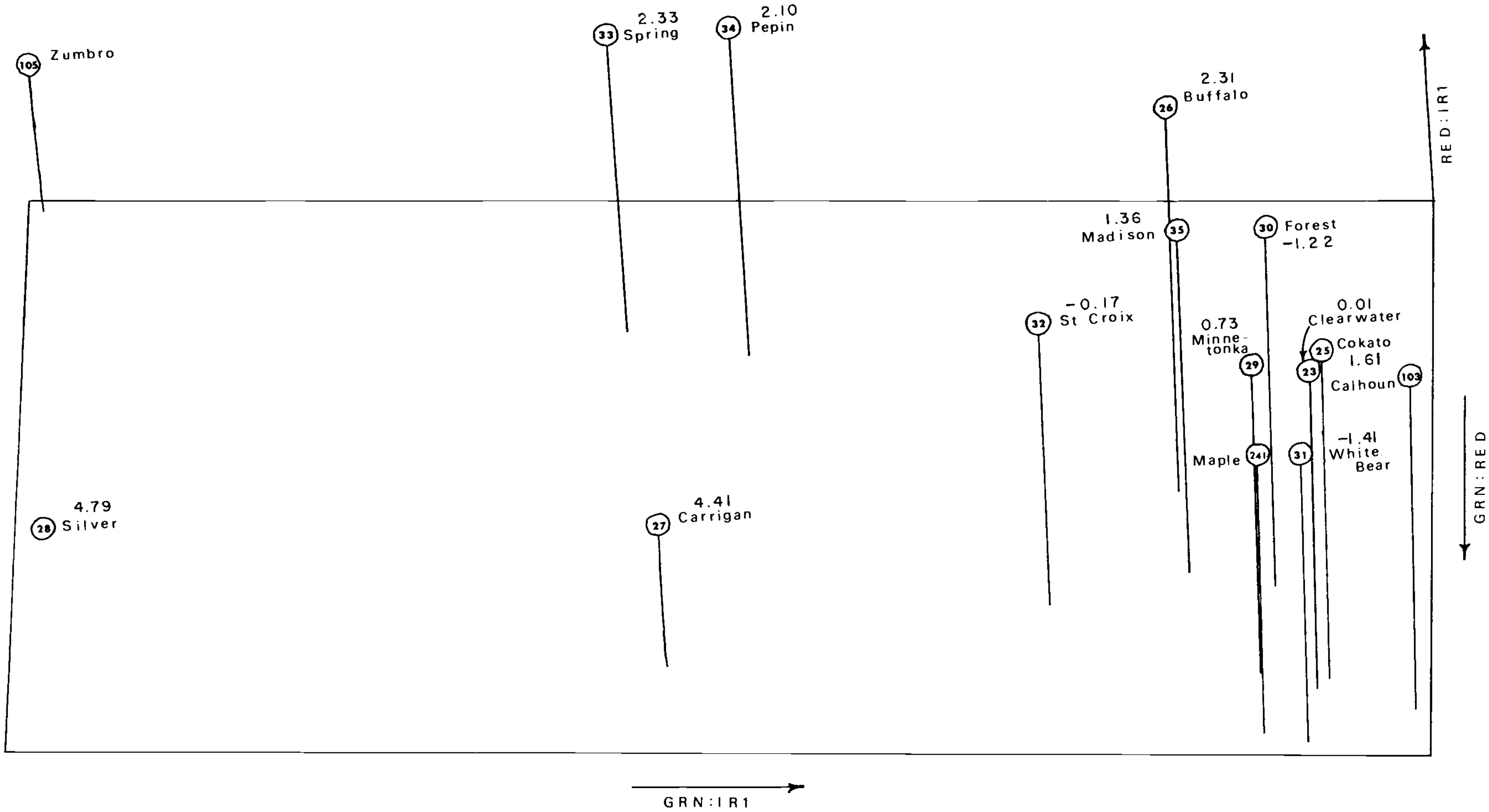

Figure 45. Three-dimensional MSS color ratio model of 15 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

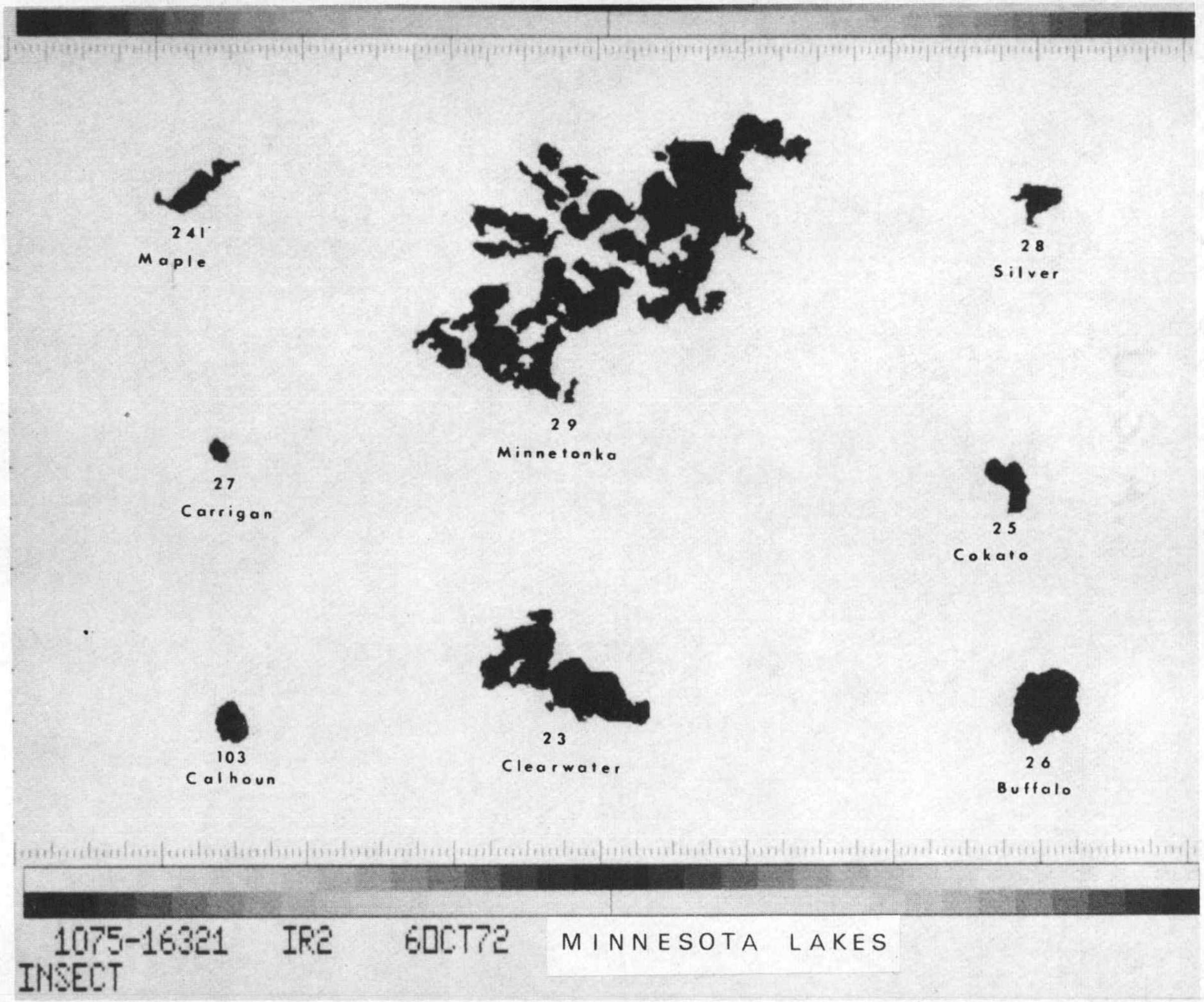

Figure 46. IR2 concatenation of 8 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972). Figure 47 contains 8 additional lakes extracted from the same frame.

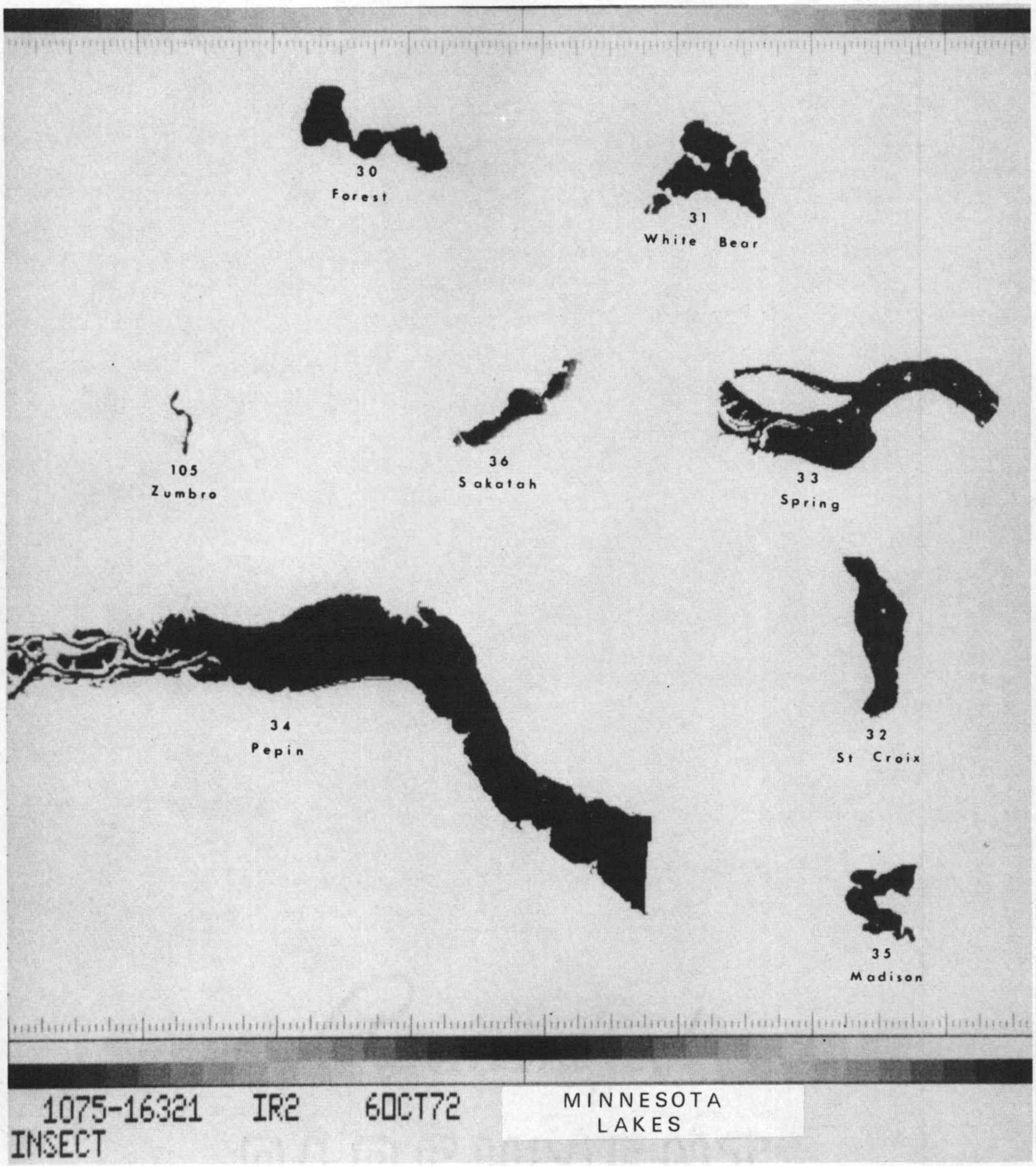

Figure 47. IR2 concatenation of 8 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

Table 53. Areal aspects of 15 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Clearwater	23	2,797	1,333.7	1,287.8	0.966
Maple	241	561	268.0	287.3	1.072
Cokato	25	481	229.8	220.2	0.958
Buffalo	26	1,350	644.9	611.1	0.948
Carrigan	27	125	59.7	65.6	1.099
Silver	28	386	184.4	170.8	0.926
Minnetonka	29	12,131	5,795.0	5,855.6	1.011
Forest	30	1,945	929.1	892.8	0.961
White Bear	31	2,233	1,066.7	1.076.5	1.009
St. Croix	32	2,202	1,051.9	3,322.2	--- *
Spring	33	5,338	2,550.0	2,391.8	0.938
Pepin	34	16,910	8,077.9	10,117.5	--- **
Madison	35	1,224	584.7	541.1	0.925
Calhoun	103	363	173.4	169.6	0.978
Zumbro	105	129	61.6	344.8	--- ***

*The entire image was not extracted from the CCT's.

**A portion of the lake was outside the sensor field of view.

***The disparity may be related to the long sinuous shape of the water body.

Table 54. MSS descriptive statistics for 15 Minnesota lakes extracted from Frame 1075-16321 (6 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Clearwater	23	2,797	33.13* 2.42**	17.03 2.93	11.46 5.70	5.14
Maple	241	561	32.95 2.42	16.49 2.81	11.59 6.11	5.97
Cokato	25	481	31.53 2.90	16.31 2.72	10.86 5.66	5.16
Buffalo	26	1,350	35.22 2.16	20.43 1.95	12.77 4.25	4.46
Carrigan	27	125	32.12 2.34	16.71 2.75	14.12 6.40	7.11
Silver	28	386	41.15 1.61	23.20 1.64	24.56 2.95	8.83
Minnetonka	29	12,131	35.90 2.49	18.61 2.54	12.65 5.23	5.09
Forest	30	1,945	34.35 2.09	18.76 2.39	12.05 4.84	4.99
White Bear	31	2,233	33.36 2.98	16.63 3.60	11.57 6.25	5.20
St. Croix	32	2,202	31.19 2.64	16.84 3.46	11.83 4.98	5.17
Spring	33	5,338	34.64 2.28	22.52 2.47	15.55 4.76	6.60
Pepin	34	16,910	33.72 2.09	21.56 2.27	14.39 4.40	5.88
Madison	35	1,224	35.37 2.23	19.50 2.35	12.78 5.24	5.67
Zumbro	105	129	38.29 5.44	27.39 2.83	23.05 5.20	14.17
Calhoun	103	363	33.79 2.18	17.15 3.12	11.32 5.28	4.93

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D7. 8 October 1972 (1077-16431)

Section D7.1. Regression Models, and Correlation Coefficients (Table 55)

Ten lakes were extracted from the frame; Lake Winona, Trace Lake, and Big Stone Lake were not used to develop the regression model.

Darling (16)	Minnewaska (19)	Winona (101)
Carlos (17)	Nest (20)	Trace (102)
Le Homme Dieu (18)	Green (21)	Big Stone (104)
	Wagonga (22)	

Table 55. Correlations between ground truth and MSS data (color ratios) for 7 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

	PC1	CHLA	LNCHLA	SECCHI	LNSECCHI
GRNRED	-0.975	-0.936	-0.945	0.876	0.935
GRNIR1	-0.941	-0.962	-0.968	0.738	0.858
GRNIR2	-0.770	-0.786	-0.918	0.474	0.579
REDIR1	-0.918	-0.963	-0.953	0.679	0.826
REDIR2	-0.675	-0.716	-0.864	0.370	0.473
IR1IR2	0.453	0.466	0.168	-0.581	-0.660

The best model for the prediction of trophic state is:

$$\widehat{PC1} = 34.509 - 18.548\ GRNRED$$

This model explains about 95 percent of the variation about the mean (Table 56). The observed and predicted PC1 values are in Table 57.

Table 56. Analysis of variance table of the PC1 regression for 7 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	6	25.269	4.212	
Regression	1	24.035	24.035	97.433
Residual	5	1.233	0.247	

$R^2 = 0.9512 \times 100 = 95.12\%$ s.e. of estimate = 0.497

Table 57. PC1 residuals of 7 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Darling	16	-0.73	-0.92	0.19
Carlos	17	-1.55	-1.18	-0.37
Le Homme Dieu	18	-1.06	-0.68	-0.38
Minnewaska	19	-0.32	-0.06	-0.26
Nest	20	0.77	1.09	-0.32
Green	21	-1.10	-1.90	0.80
Wagonga	22	4.40	4.07	0.33

Section D7.2. Three-dimensional Color Ratio Model

The MSS color ratio model is displayed in Figure 48. The PC1 values are in very good agreement with the model.

Section D7.3. Concatenation of Extracted Lakes

The 10 lakes are shown in Figure 49.

Section D7.4. MSS-Lake Surface Area Relationships

The areal aspects of the lakes extracted from Frame 1077-16431 are in Table 58.

Section D7.5. Lake MSS Descriptive Statistics

The MSS statistics are presented in Table 59.

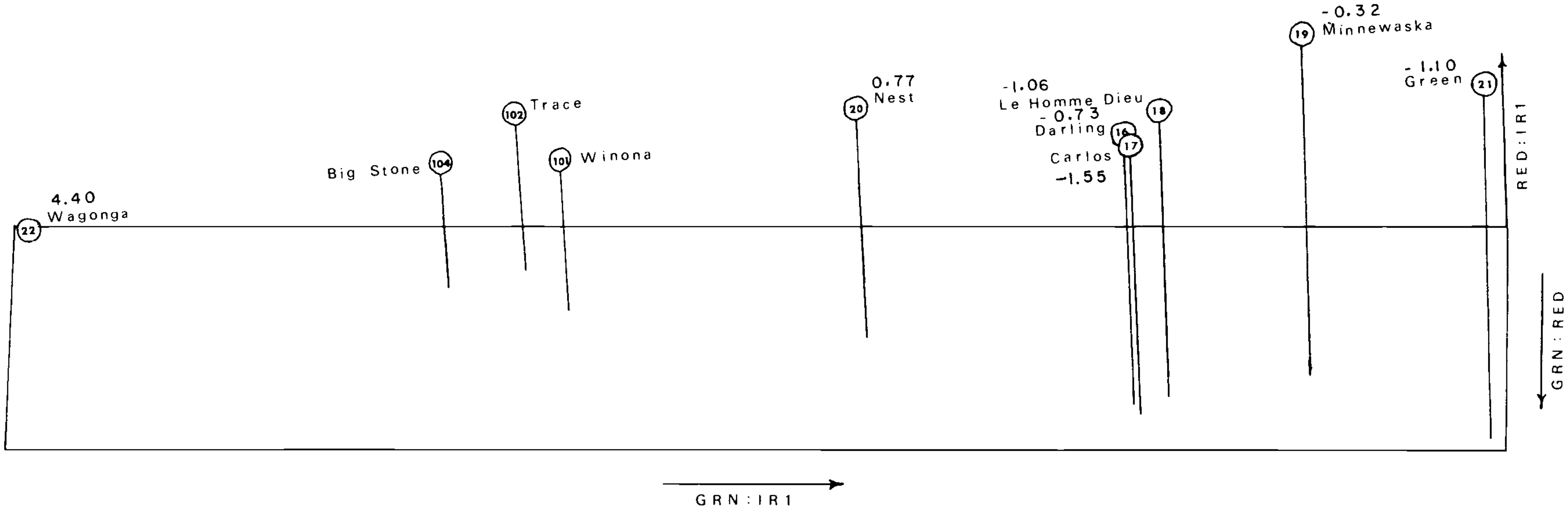

Figure 48. Three-dimensional MSS color ratio model of 10 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

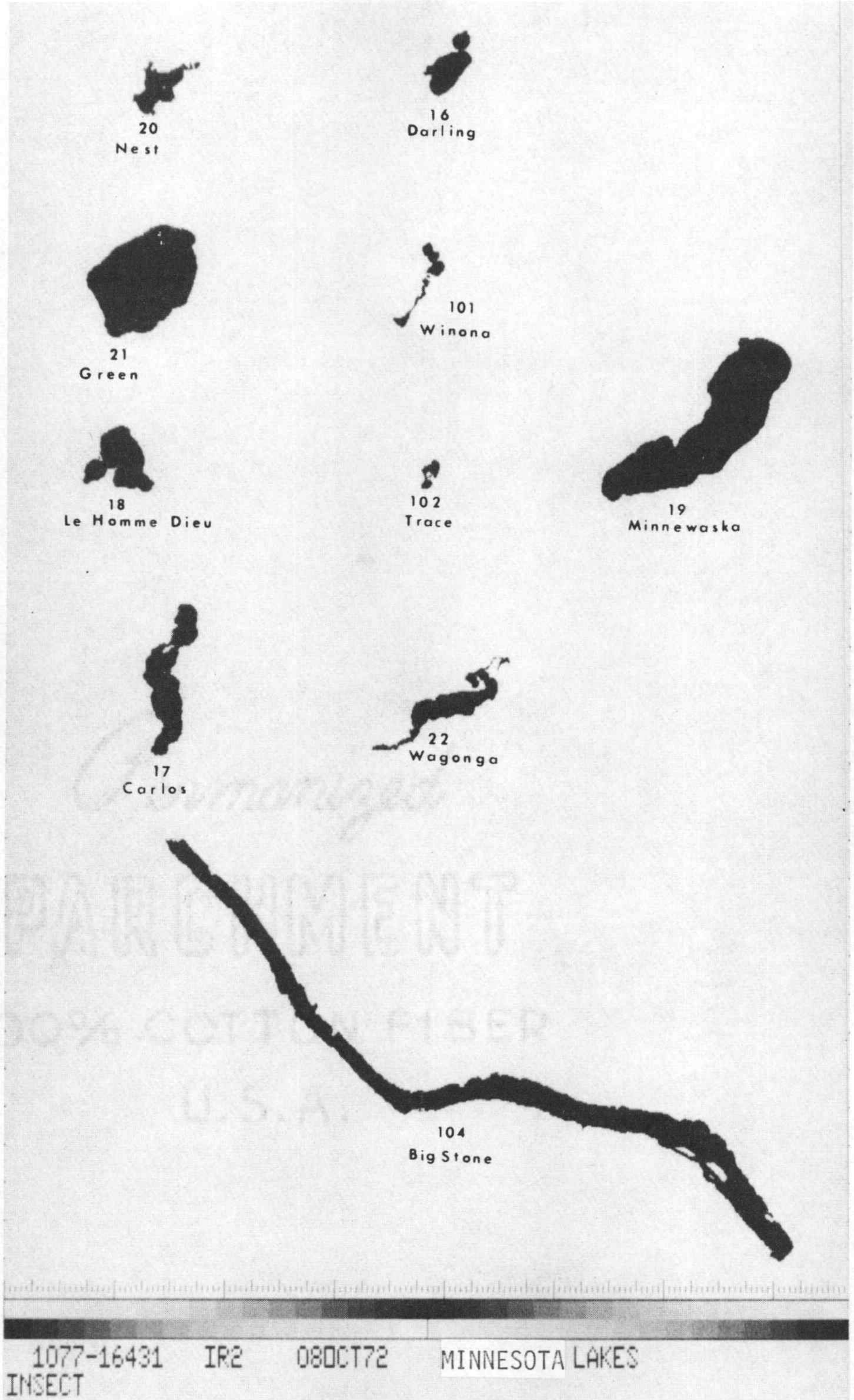

Figure 49. IR2 concatenation of 10 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

Table 58. Areal aspects of 10 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Carlos	17	2,200	1,050.9	1,019.8	0.970
Le Homme Dieu	18	1,561	745.7	705.8	0.947
Darling	16	923	440.9	386.1	0.876
Minnewaska	19	7,028	3,357.3	2,877.4	0.857
Nest	20	872	416.6	382.4	0.918
Green	21	4,986	2,381.8	2,187.8	0.919
Wagonga	22	1,501	717.0	654.4	0.913
Winona	101	425	203.0	73.3	--- *
Trace	102	102	89.8		
Big Stone	104	10,407	4,971.4	5,103.3	1.027

*The ERTS image includes pixels from Lake Agnes and Lake Henry.

Table 59. MSS descriptive statistics for 10 Minnesota lakes extracted from Frame 1077-16431 (8 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Carlos	17	2,200	33.19*	17.25	10.58	4.74
			2.86**	3.36	4.56	
Le Homme Dieu	18	1,561	33.27	17.54	10.48	4.50
			2.56	2.95	4.16	
Darling	16	923	31.72	16.61	10.14	4.64
			1.91	2.76	4.29	
Minnewaska	19	7,028	37.48	20.11	11.14	4.173
			1.68	2.07	3.05	3.46
Nest	20	872	31.94	17.73	11.53	5.79
			1.95	2.26	4.87	
Green	21	4,986	33.69	17.16	9.35	3.87
			2.27	2.12	2.81	3.20
Wagonga	22	1,501	40.97	24.96	24.91	9.28
			2.47	2.11	3.20	4.96
Trace	102	188	36.20	21.38	15.68	9.49
			4.62	5.21	7.02	8.05
Winona	101	425	35.02	19.92	14.76	7.57
			2.40	3.09	6.02	6.91
Big Stone	104	10,407	38.76	22.56	17.58	6.11
			1.93	3.05	3.78	4.50

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D8. 28 May 1973 (1309-16325)

Section D8.1. Regression Models, and Correlation Coefficients (Table 60)

Thirteen Minnesota lakes were extracted from the frame. Lakes Calhoun and Maple were not used in the development of the regression model.

Clearwater (23)	Carrigan (27)	Spring (33)
Maple (241)	Silver (28)	Madison (35)
Cokato (25)	Forest (30)	Sakatah (36)
Buffalo (26)	White Bear (31)	Calhoun (103)
	St. Croix (32)	

Table 60. Correlations between MSS data (colors and color ratios) and PC1 values for 11 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

	PC1
GRN	-0.068
RED	-0.040
IR1	-0.014
IR2	-0.259
GRNRED	0.061
GRNIR1	0.017
GRNIR2	0.316
REDIR1	-0.036
REDIR2	0.434
IR1IR2	0.702

The best model for the prediction of trophic state is:

$$\widehat{PC1} = -16.537 + 9.844\ IR1IR2$$

This model explains about 49 percent of the variation about the mean (Table 61) and is not adequate. The observed and predicted PC1 values are in Table 62.

Table 61. Analysis of variance table of the PC1 regression model for 11 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	10	41.326	4.133	
Regression	1	20.385	20.385	8.761
Residual	9	20.941	2.327	

R^2 = 0.4933 x 100 = 49.33% s.e. of estimate = 1.525

Table 62. PC1 residuals of 11 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Clearwater	23	0.01	2.88	-2.87
Cokato	25	1.61	2.07	-0.46
Buffalo	26	2.31	2.29	0.02
Carrigan	27	4.40	1.78	2.62
Silver	28	4.78	3.89	0.89
Forest	30	-1.22	-0.19	-1.03
White Bear	31	-1.41	-0.04	-1.37
St. Croix	32	-0.17	-0.82	0.65
Spring	33	2.33	1.65	0.68
Madison	35	1.36	1.51	-0.15
Sakatah	36	1.38	0.38	1.00

Section D8. 2. Three-dimensional Color Ratio Model

The MSS color ratio model is displayed in Figure 50. There appears to be little agreement between lake position in the model and trophic state as defined by the PC1 value.

Section D8. 3. Concatenation of Extracted Lakes

The concatenation of 13 lakes is displayed as Figure 51. Sakatah Lake includes both Upper Sakatah Lake and Lower Sakatah Lake.

Section D8. 4. MSS-Lake Surface Area Relationships

The areal aspects of the lakes are in Table 63.

Section D8. 5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 64.

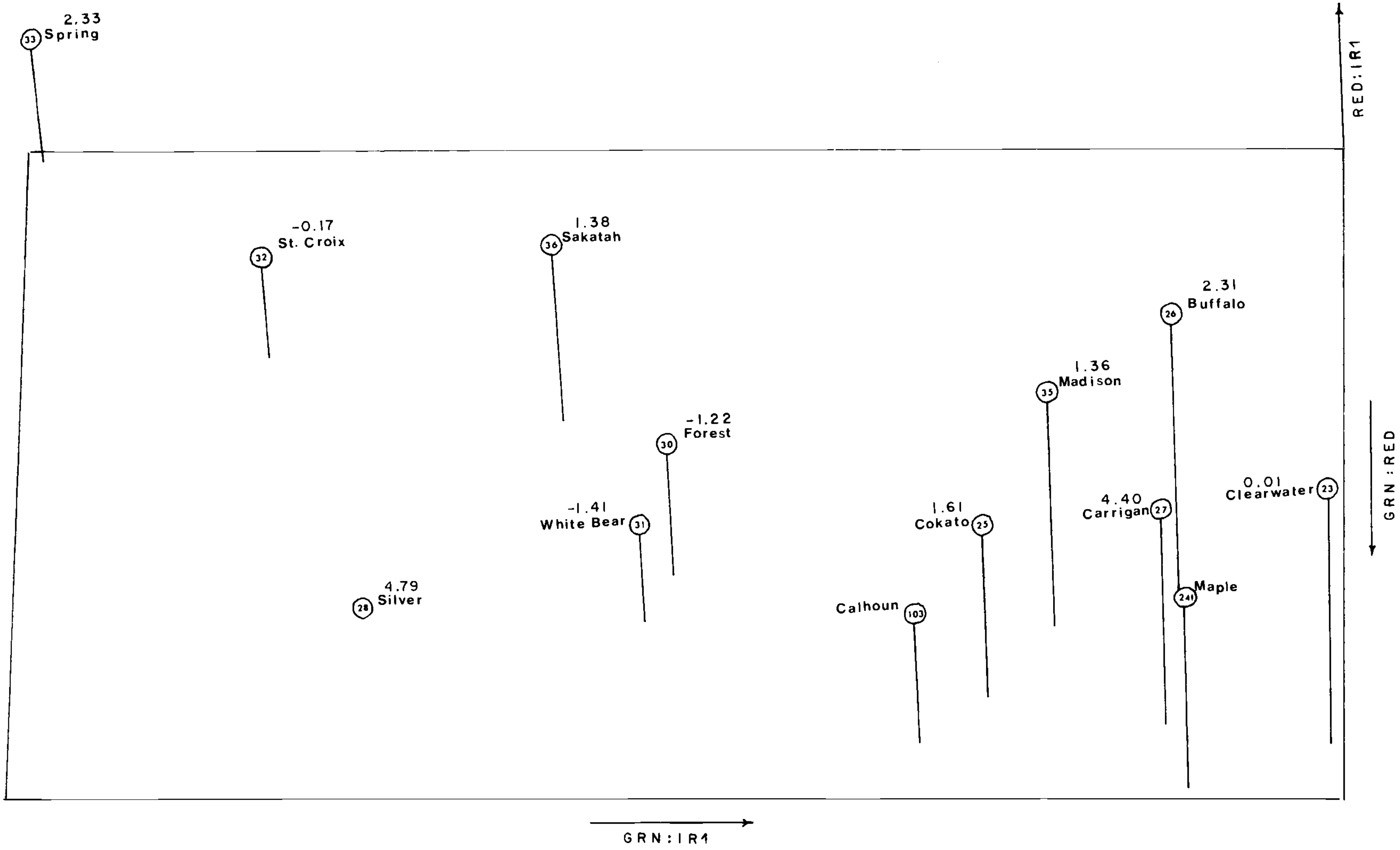

Figure 50. Three-dimensional MSS color ratio model of 13 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

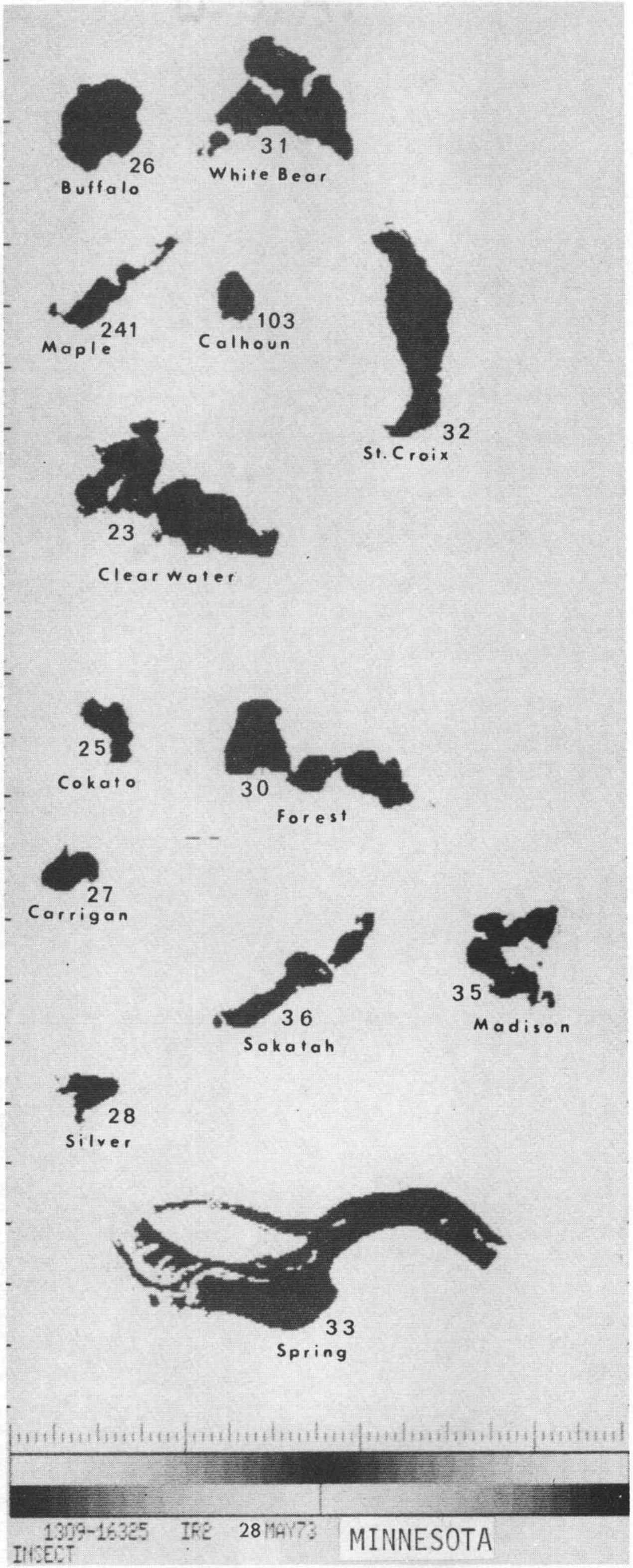

Figure 51. IR2 concatenation of 13 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

Table 63. Areal aspects of 13 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Clearwater	23	2,689	1,284.5	1,287.8	1.003
Maple	241	584	279.0	287.3	1.030
Cokato	25	442	211.1	220.2	1.043
Buffalo	26	1,327	633.0	611.1	0.965
Carrigan	27	401	191.6	65.0	--- *
Silver	28	368	175.8	170.8	0.972
Forest	30	1,871	893.8	892.8	0.999
White Bear	31	2,093	999.8	1,076.5	1.077
St. Croix	32	2,133	1,018.9	3,322.2	--- **
Spring	33	4,856	2,319.7	2,391.8	1.031
Madison	35	1,177	562.3	541.1	0.962
Sakatah	36	977	466.7	497.0	1.065
Calhoun	103	339	161.9	169.6	1.048

*A portion of the lake was outside the sensor field of view.

**The entire lake image was not extracted from the CCT's.

Table 64. Descriptive statistics for 13 Minnesota lakes extracted from Frame 1309-16325 (28 May 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Clearwater	23	2,689	45.89* 3.24**	26.74 2.94	19.09 3.92	9.68 4.18
Maple	24	584	49.09 3.04	28.11 2.48	21.15 4.40	11.19 4.91
Cokato	25	442	44.73 2.47	26.60 2.08	20.32 3.51	10.75 4.05
Buffalo	26	1,327	51.01 2.06	31.60 2.17	22.04 2.82	11.52 3.24
Carrigan	27	401	48.12 2.37	28.29 2.12	20.86 3.49	11.21 4.26
Silver	28	368	51.79 1.71	31.92 1.58	28.36 3.04	13.67 3.80
Forest	30	1,871	52.71 6.32	33.06 2.59	26.18 3.04	15.76 3.31
White Bear	31	2,093	53.58 2.31	32.92 2.80	26.85 3.47	16.02 3.15
St. Croix	32	2,133	53.50 2.11	37.24 2.62	30.31 2.76	18.98 2.77
Spring	33	4,856	61.44 2.89	47.49 3.61	37.79 2.91	20.46 3.01
Madison	35	1,177	50.78 2.10	31.15 2.19	22.67 3.33	12.36 3.89
Sakatah	36	977	53.09 2.16	35.79 2.98	27.31 3.27	15.90 3.78
Calhoun	103	339	51.72 1.95	30.17 2.21	23.93 2.89	13.81 3.32

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D9. 3 July 1973 (1345-16322)

Section D9.1. Regression Models, and Correlation Coefficients (Table 65)

Fourteen Minnesota Lakes were extracted from the frame. Lakes Calhoun and Maple were not used in developing the regression model.

Clearwater (23)	Silver (28)	St. Croix (32)
Maple (241)	Minnetonka (29)	Spring (33)
Cokato (25)	Forest (30)	Madison (35)
Buffalo (26)	White Bear (31)	Sakatah (36)
Carrigan (27)		Calhoun (103)

Table 65. Correlations between MSS data (color ratios) and PC1 values for 12 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

	PC1
GRNRED	0.239
GRNIR1	-0.788
GRNIR2	-0.641
REDIR1	-0.837
REDIR2	-0.642
IR1IR2	0.172

The best model for the prediction of trophic state is:

$$\widehat{PC1} = 10.544 - 7.240 \text{ REDIR1}$$

This model explains about 70 percent of the variation about the mean (Table 66), but is of little practical value. The observed and predicted PC1 values are in Table 67.

Table 66. Analysis of variance table of the PC1 regression model for 12 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	11	41.744	3.795	
Regression	1	29.260	29.260	23.438
Residual	10	12.484	1.248	

$R^2 = 0.7009 \times 100 = 70.09\%$ s.e. of estimate = 1.117

Table 67. PC1 residuals of 12 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Clearwater	23	0.01	-0.08	0.09
Cokato	25	1.61	0.33	1.28
Buffalo	26	2.31	3.05	-0.74
Carrigan	27	4.40	3.13	1.27
Silver	28	4.79	4.90	-0.11
Minnetonka	29	0.73	-0.31	1.04
Forest	30	-1.22	-0.54	-0.68
White Bear	31	-1.40	0.75	-2.15
St. Croix	32	-0.17	0.76	-0.93
Spring	33	2.33	1.25	1.08
Madison	35	1.36	0.98	0.38
Sakatah	36	1.38	1.92	-0.54

Section D9. 2. Three-dimensional Color Ratio Model

The MSS color ratio model is displayed in Figure 52. Silver Lake's IR1 DN level exceeds its RED DN level; this isolates it from the other lakes.

Section D9. 3. Concatenation of Extracted Lakes

The concatenation of the 14 lakes is in Figure 53.

Section D9. 4. MSS-Lake Surface Area Relationships

The areal aspects of the 14 lakes are in Table 68.

Section D9. 5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 69.

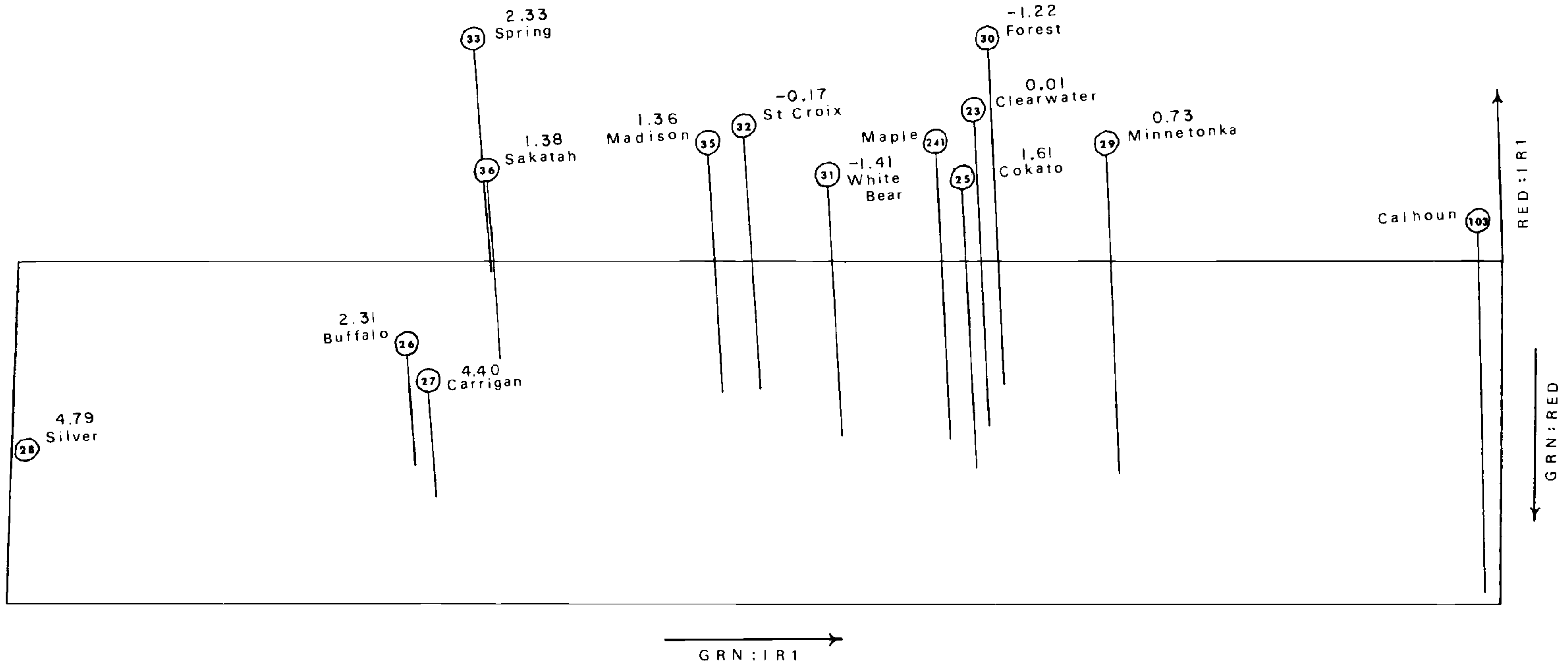

Figure 52. Three-dimensional MSS color ratio model of 14 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

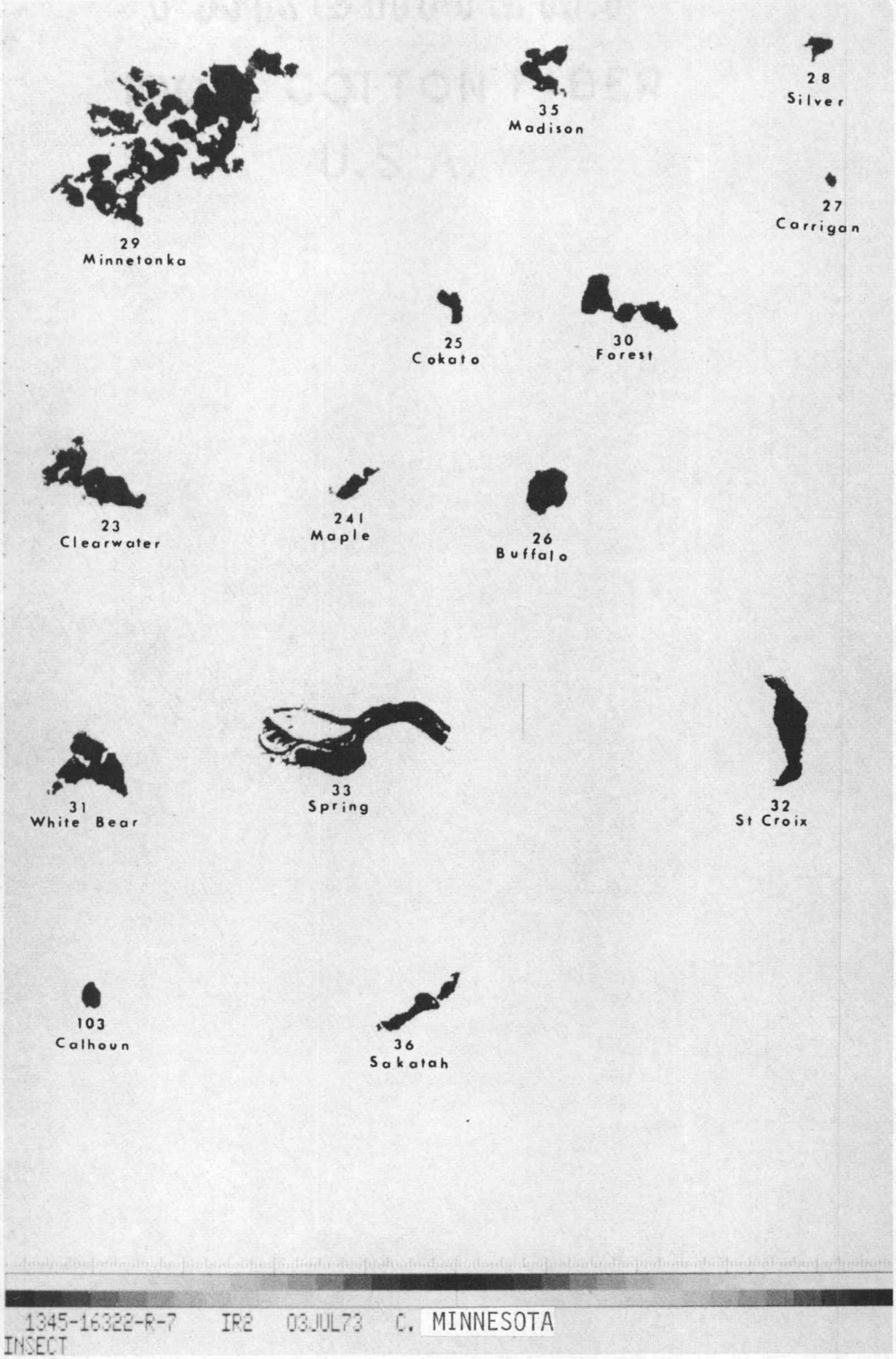

Figure 53. IR2 concatenation of 14 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

Table 68. Areal aspects of 14 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Clearwater	23	2,485	1,187.1	1,287.8	1.085
Maple	241	505	241.2	287.3	1.191
Cokato	25	437	208.8	220.2	1.055
Buffalo	26	1,285	613.8	611.1	0.996
Carrigan	27	106	50.6	65.6	1.296
Silver	28	346	165.3	170.8	1.033
Minnetonka	29	11,239	5,368.9	5,855.6	1.091
Forest	30	1,818	868.5	892.8	1.028
White Bear	31	2,094	1,000.3	1,076.5	1.076
St. Croix	32	2,145	1,024.7	3,322.2	--- *
Spring	33	4,945	2,362.2	2,391.8	1.013
Madison	35	1,151	549.8	541.1	0.984
Sakatah	36	986	471.0	497.0	1.055
Calhoun	103	342	163.4	169.6	1.038

*The entire lake image was not extracted from the CCT's.

Table 69. MSS descriptive statistics for 14 Minnesota lakes extracted from Frame 1345-16322 (3 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Clearwater	23	2,485	47.29* 3.25**	26.59 2.49	18.11 9.85	7.21 5.19
Maple	241	505	43.43 2.49	24.15 2.21	16.94 7.55	7.86 5.43
Cokato	25	437	41.50 2.45	22.54 2.27	15.98 3.67	6.88 4.42
Buffalo	26	1,285	47.77 2.01	26.01 1.63	25.12 4.68	9.04 3.72
Carrigan	27	106	47.41 2.04	25.16 1.37	24.56 4.44	13.35 3.92
Silver	28	346	50.44 1.68	27.61 1.25	35.42 3.30	13.43 3.94
Minnetonka	29	11,239	47.05 3.79	25.44 3.26	16.96 9.07	6.89 4.66
Forest	30	1,818	42.24 2.27	24.61 2.24	16.07 3.99	7.82 4.22
White Bear	31	2,094	40.94 3.05	22.78 3.27	16.84 8.33	7.94 4.41
St. Croix	32	2,145	38.78 2.26	22.50 2.77	16.66 7.24	6.59 4.26
Spring	33	4,945	43.93 2.51	28.36 2.27	22.09 1.68	8.03 4.55
Madison	35	1,151	46.84 2.23	27.10 1.70	20.51 8.51	7.79 4.63
Sakatah	36	986	44.46 2.25	26.43 1.87	22.18 4.13	9.22 4.40
Calhoun	103	342	50.45 2.22	24.91 2.57	15.65 3.75	6.29 4.13

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D10. 4 July 1973 (1346-16381)

Section D10. 1. Regression Models, and Correlation Coefficients (Table 70)

Eight Minnesota lakes were extracted from the frame; Cottonwood Lake was not used in developing the regression model.

Nest (20)	Buffalo (26)
Green (21)	Carrigan (27)
Wagonga (22)	Silver (28)
Cokato (25)	Cottonwood (111)

Table 70. Correlations between MSS data (colors and color ratios) and PC1 values for 7 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

	PC1
GRN	0.874
RED	0.829
IR1	0.903
IR2	0.829
GRNRED	-0.595
GRNIR1	-0.956
GRNIR2	-0.800
REDIR1	-0.961
REDIR2	-0.839
IR1IR2	0.573

The best model for the prediction of lake trophic state is:

$$\widehat{PC1} = 11.715 - 8.277\, REDIR1$$

This model accounts for about 92 percent of the variation about the mean (Table 71). The observed and predicted PC1 values are in Table 72.

Table 71. Analysis of variance table of the PC1 regression model for 7 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	6	29.233	4.872	
Regression	1	26.994	26.994	60.255
Residual	5	2.238	0.448	

$R^2 = 0.9234 \times 100 = 92.34\%$ s.e. of estimate = 0.669

Table 72. PC1 residuals of 7 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Nest	20	0.77	0.33	0.44
Green	21	-1.10	-0.34	-0.76
Wagonga	22	4.40	4.04	0.36
Cokato	25	1.61	0.86	0.75
Buffalo	26	2.31	3.07	-0.76
Carrigan	27	4.40	4.71	-0.31
Silver	28	4.79	4.49	0.30

Section D10. 2. Three-dimensional Color Ratio Model

The MSS color ratio model is displayed in Figure 54. Lakes Wagonga, Carrigan, and Silver have IR1 values which exceed their RED DN values.

Section D10. 3. Concatenation of Extracted Lakes

The concatenation of eight lakes is in Figure 55.

Section D10. 4. MSS-Lake Surface Area Relationships

The areal aspects of the eight lakes are in Table 73.

Section D10. 5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 74.

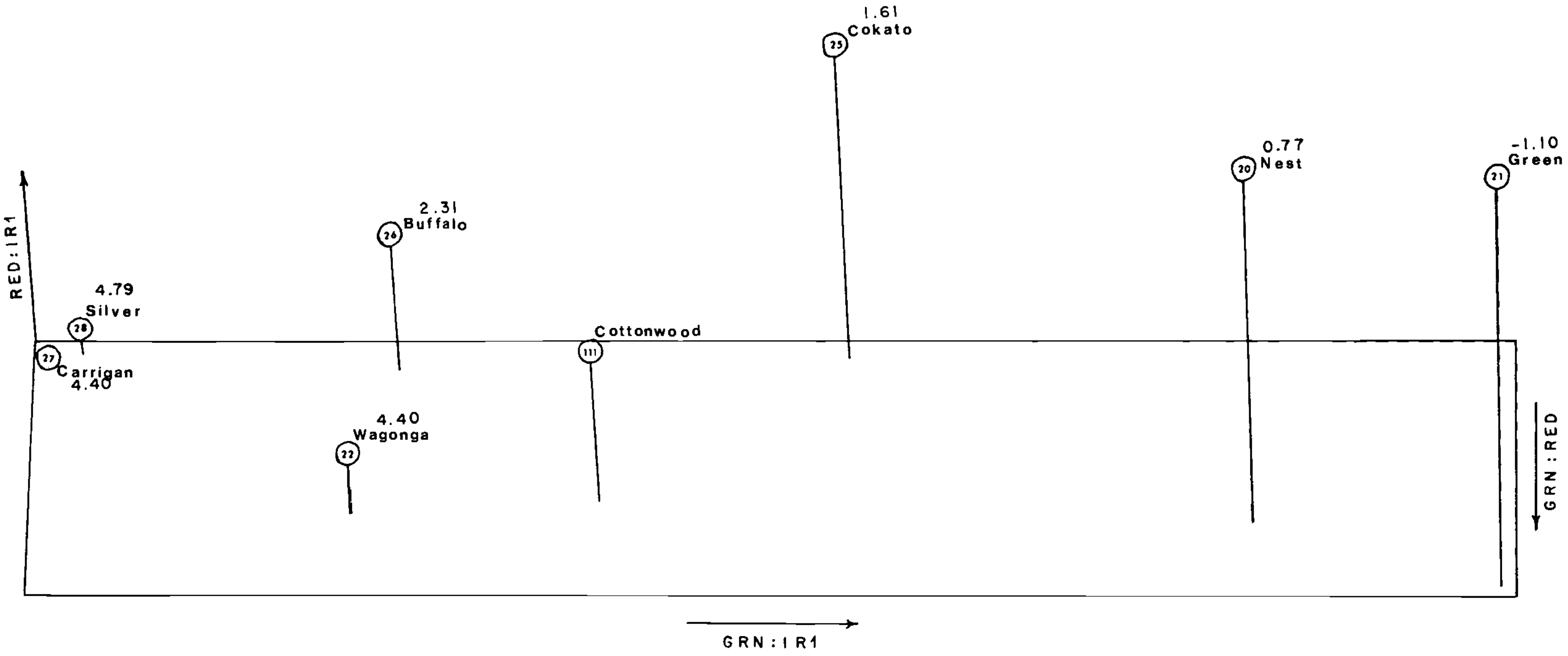

Figure 54. Three-dimensional MSS color ratio model of 8 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

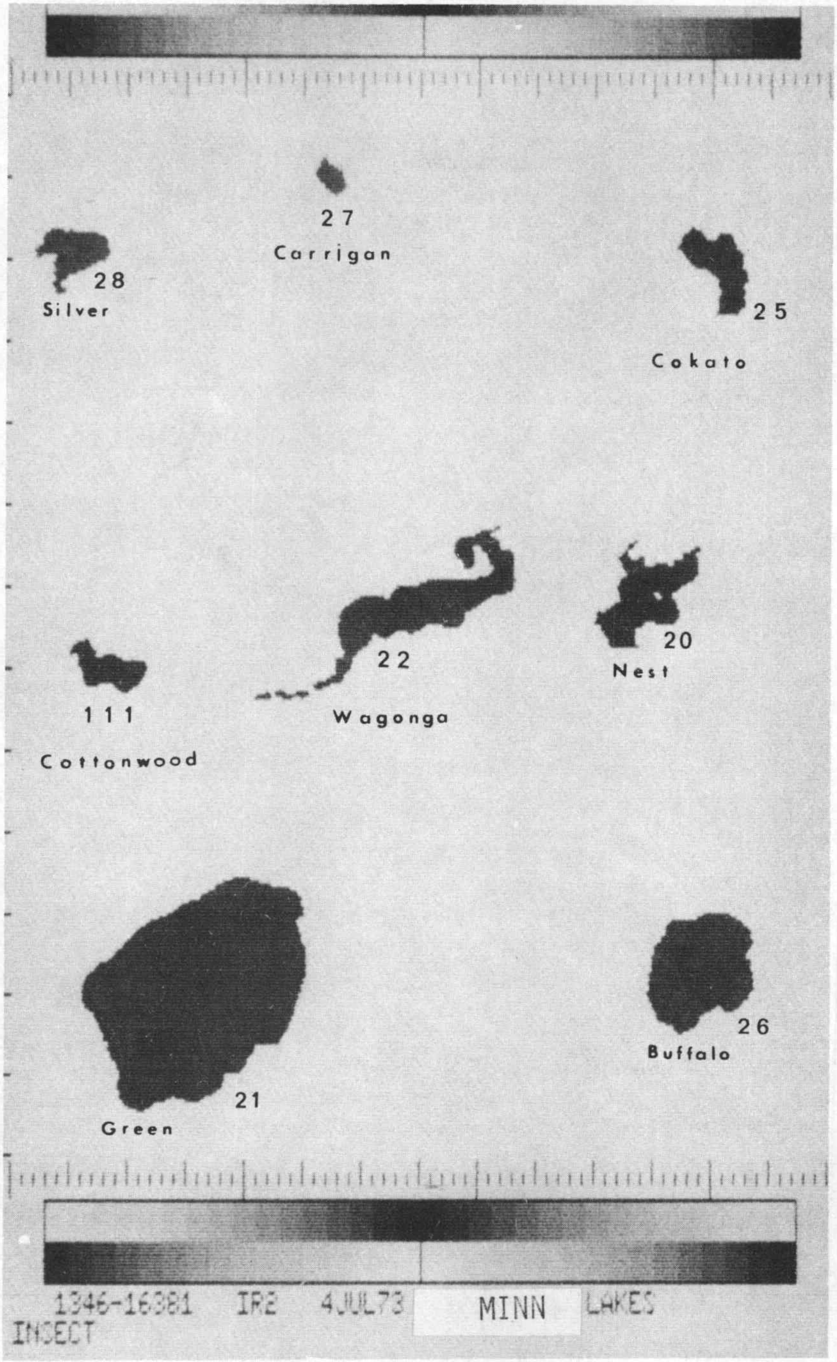

Figure 55. IR2 concatenation of 8 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

Table 73. Areal aspects of 8 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Nest	20	724	345.9	382.4	1.106
Green	21	4,701	2,245.7	2,187.8	0.974
Wagonga	22	1,275	609.1	654.4	1.074
Cokato	25	434	207.3	220.2	1.062
Buffalo	26	1,300	621.0	611.1	0.984
Carrigan	27	98	46.8	65.6	1.402
Silver	28	346	165.3	170.8	1.033
Cottonwood	111	293	140.0	149.7	1.069

Table 74. MSS descriptive statistics for 8 Minnesota lakes extracted from Frame 1346-16381 (4 July 1973).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Nest	20	724	45.37*	27.24	19.79	10.34
			2.86**	2.38	3.61	4.12
Green	21	4,701	47.34	27.54	18.91	10.06
			3.48	2.93	2.71	3.08
Wagonga	22	1,275	58.42	35.47	38.25	15.45
			2.79	3.09	3.04	3.39
Cokato	25	434	52.32	35.20	26.85	16.04
			3.83	4.22	3.72	4.02
Buffalo	26	1,300	59.55	39.73	38.07	18.97
			3.90	4.23	4.13	3.52
Carrigan	27	98	59.77	39.97	47.26	24.08
			3.37	3.42	3.73	3.28
Silver	28	346	60.43	40.78	46.73	22.61
			3.67	4.02	3.33	3.39
Cottonwood	111	293	52.35	32.04	30.10	12.13
			1.46	1.38	3.50	3.91

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D11. 19 August 1972 (1027-15233)

Section D11.1. Regression Models, and Correlation Coefficients (Table 75)

Seven New York lakes were extracted from the frame; all were incorporated into the regression model.

Conesus (91)

Canandaigua (92)

Keuka (93)

Seneca (94)

Cayuga (95)

Owasco (96)

Cross (97)

Table 75. Correlations between ground truth and MSS data (colors and color ratios) for 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

	PC1	CHLA	LNCHLA	SECCHI	LNSECCHI
GRN	0.245	0.601	0.463	-0.792	-0.804
RED	0.772	0.833	0.664	-0.507	-0.572
IR1	0.752	0.632	0.550	0.063	-0.013
IR2	0.890	0.599	0.510	0.116	0.023
GRNRED	-0.906	-0.717	-0.580	0.072	0.158
GRNIR1	-0.596	-0.331	-0.307	-0.419	-0.356
GRNIR2	-0.740	-0.337	-0.284	-0.389	-0.315
REDIR1	-0.286	-0.032	-0.084	-0.621	-0.583
REDIR2	-0.645	-0.207	-0.186	-0.490	-0.423
IR1IR2	-0.860	-0.349	-0.268	-0.239	-0.152

The best model for the prediction of the lake trophic state index is:

$$\widehat{PC1} = -4.981 - 8.805\ GRNIR1 + 19.301\ REDIR1$$

This model explains about 83 percent of the variation about the mean (Table 76). The observed and predicted PC1 values are in Table 77.

Table 76. Analysis of variance table of the PC1 regression model for 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

Source	Analysis of Variance			
	df	Sum of Squares	Mean Square	Calculated F
Total (corrected)	6	12.741	2.124	
Regression	2	10.552	5.276	9.645
Residual	4	2.189	0.547	

$R^2 = 0.8282 \times 100 = 82.82\%$ s.e. of estimate = 0.740

Table 77. PC1 residuals of 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

Lake Name	Serial Number	PC1	$\widehat{PC1}$	PC1-$\widehat{PC1}$
Conesus	91	-1.41	-1.82	0.41
Canandaigua	92	-3.63	-2.67	-0.96
Keuka	93	-2.14	-2.82	0.68
Seneca	94	-2.89	-2.58	-0.31
Cayuga	95	-2.74	-3.28	0.52
Owasco	96	-2.47	-1.98	-0.49
Cross	97	0.86	0.73	0.13

Section D11.2. Three-dimensional Color Ratio Model

The MSS color ratio is displayed in Figure 56. Lake Canandaigua appears to be misplaced if its PC1 value is an accurate assessment of its trophic state.

Section D11.3. Concatenation of Extracted Lakes

The concatenation of seven lakes is found in Figure 57.

Section D11.4. MSS-Lake Surface Area Relationships

The areal aspects of the seven lakes are in Table 78.

Section D11.5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 79.

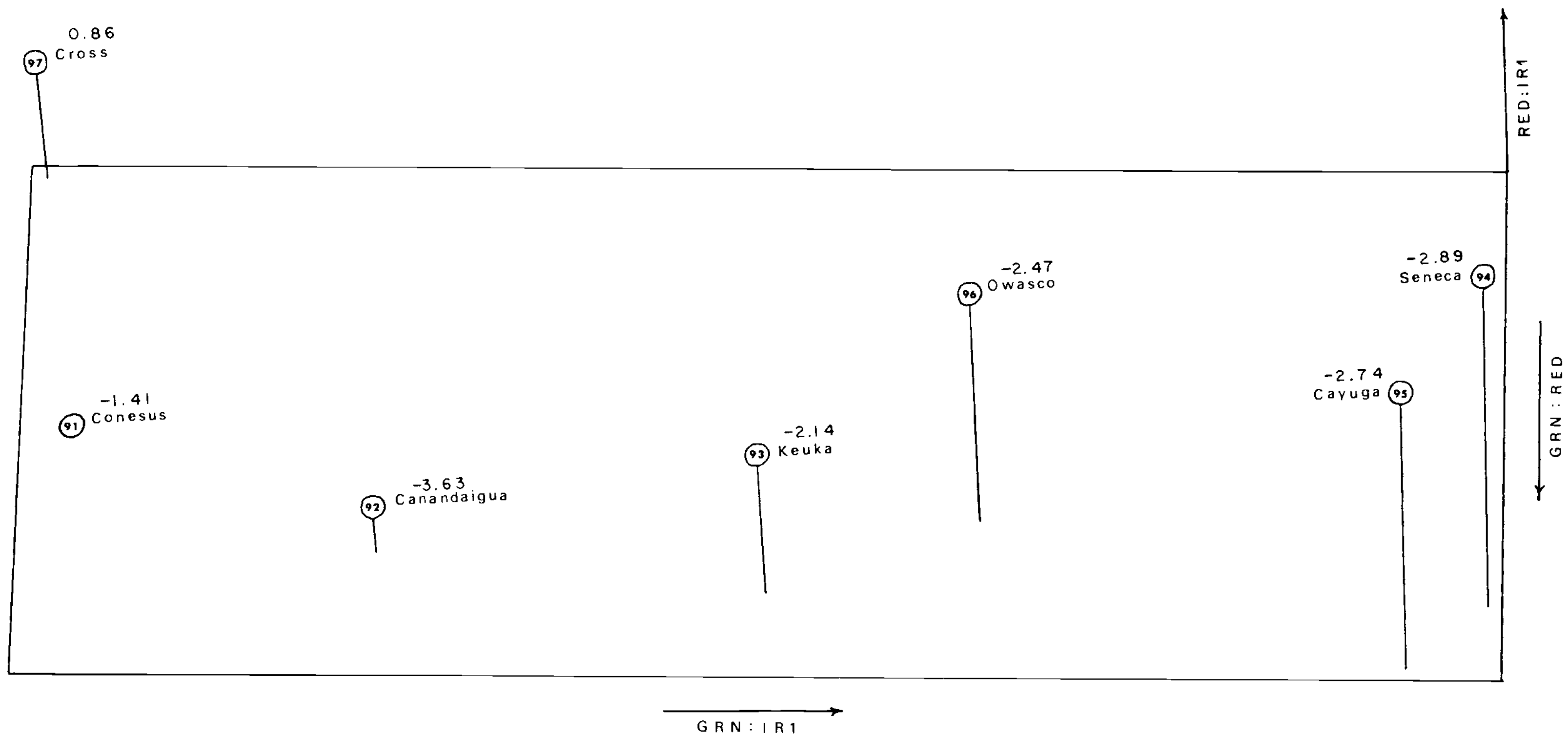

Figure 56. Three-dimensional MSS color ratio model of 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

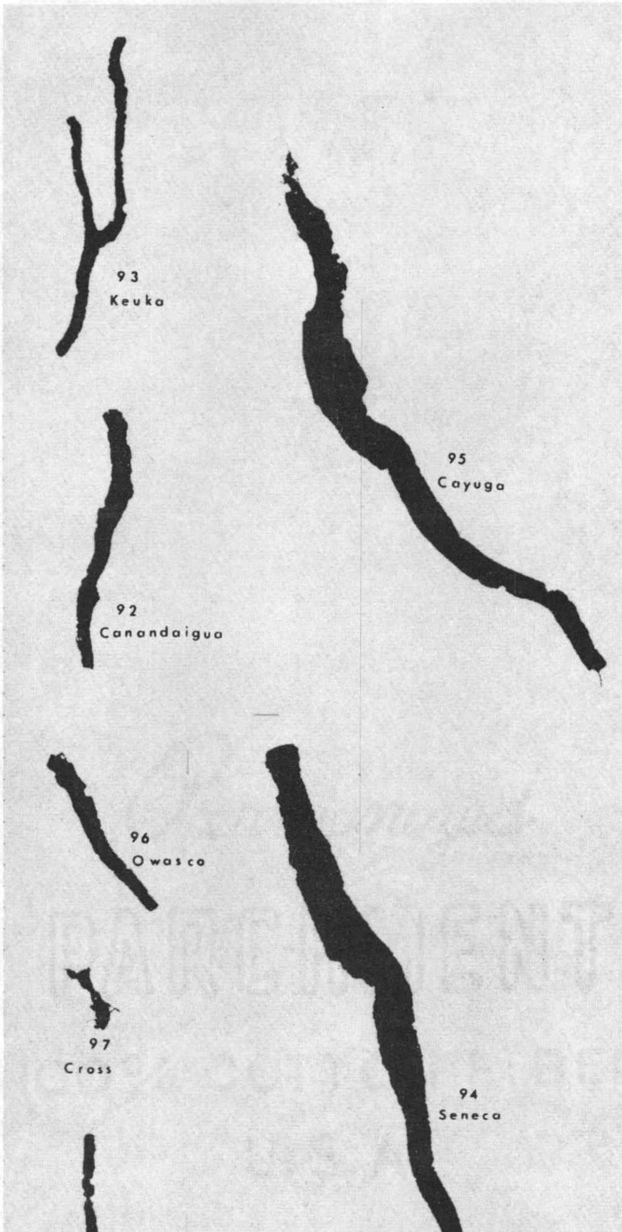

Figure 57. IR2 concatenation of 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

Table 78. Areal aspects of 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Conesus	91	2,764	1,319.9	1,347.3	1.021
Canandaigua	92	9,009	4,302.2	4,291.8	0.998
Keuka	93	9,896	4,725.8	4,739.9	1.003
Seneca	94	37,782	17,583.9	17,252.5	0.981
Cayuga	95	37,339	17,831.1	17,319.9	0.971
Owasco	96	5,835	2,786.5	2,745.5	0.985
Cross	97	1,712	817.6	844.4	1.033

Table 79. MSS descriptive statistics for 7 New York lakes extracted from Frame 1027-15233 (19 August 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands			
			Green	Red	IR1	IR2
Conesus	91	2,764	37.38*	19.67	16.47	7.83
			2.12**	2.21	3.49	4.14
Canandaigua	92	9,009	40.17	20.20	16.32	6.05
			3.14	2.68	2.04	3.83
Keuka	93	9,896	38.49	19.09	14.23	5.99
			2.32	2.19	3.37	4.09
Seneca	94	37,782	42.53	21.02	13.49	5.04
			2.68	1.99	3.61	3.45
Cayuga	95	37,339	41.11	19.86	13.25	5.11
			1.92	3.01	3.41	3.42
Owasco	96	5,835	40.79	20.78	14.38	5.94
			4.00	4.05	3.72	3.81
Cross	97	1,712	43.25	25.34	19.26	9.11
			3.62	3.16	4.10	4.68

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX D12. 11 October 1972 (1080-15180)

Section D12.1. Regression Models and Correlation Coefficients

Five lakes were extracted from the frame. No regression models were constructed due to the small number of observations. Correlation coefficients between ground truth and MSS data were not determined for the same reason.

Section D12.2. Three-dimensional Color Ratio Model

The MSS color ratio model is displayed in Figure 58. The large shift in position of Lake Cayuga (95) - compare with Figure 56 - may be a consequence of the thin cloud deck over it.

Section D12.3. Concatenation of Extracted Lakes

The extracted lakes are in Figure 59. Lakes Cross and Cayuga are only partially in the sensor field of view; this accounts for the linear "shore lines".

Section D12.4. MSS-Lake Surface Area Relationships

The areal aspects of the lakes are in Table 80.

Section D12.5. Lake MSS Descriptive Statistics

The MSS statistics are in Table 81.

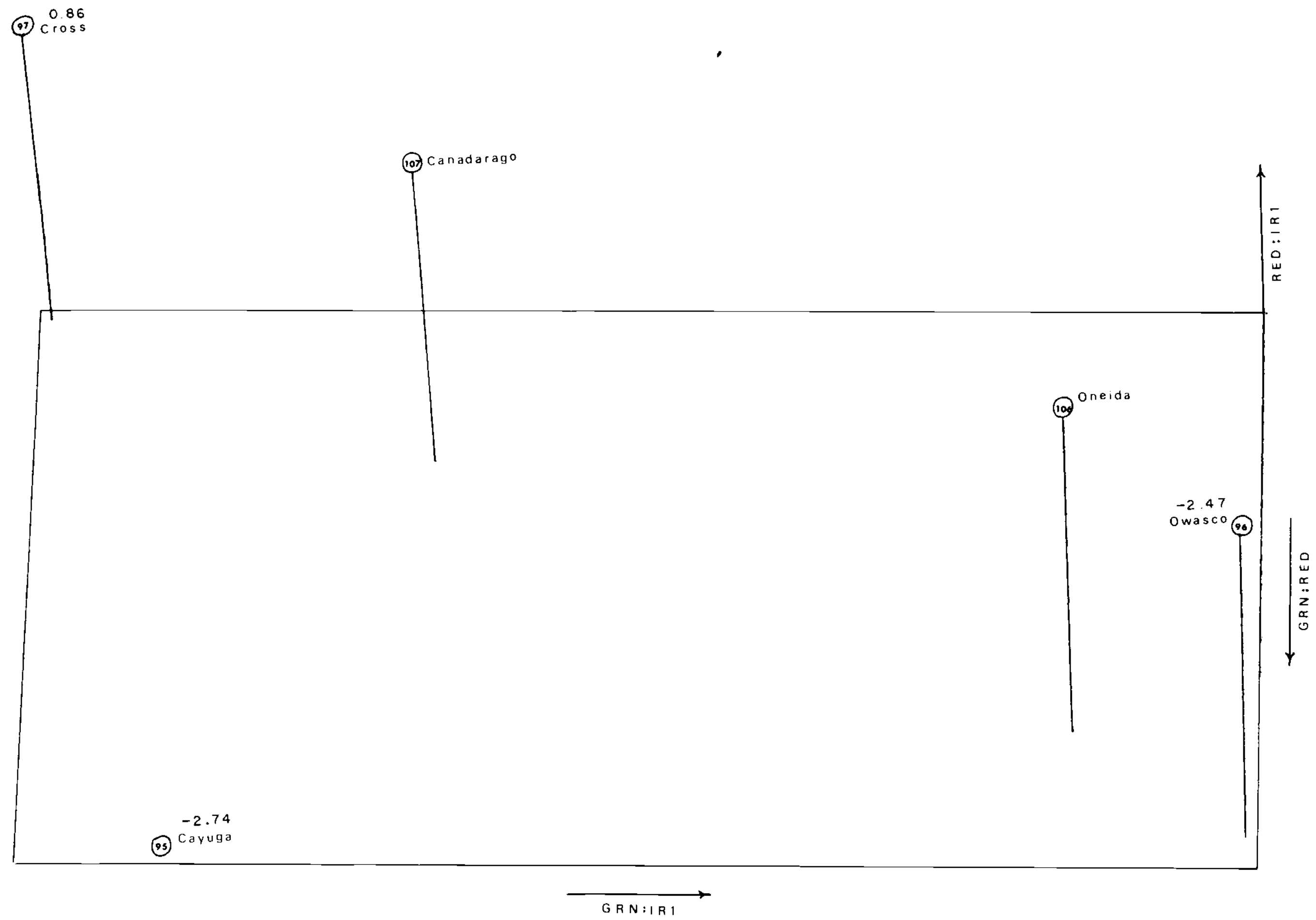

Figure 58. Three-dimensional MSS color ratio model of five New York lakes extracted from Frame 1080-15180 (11 October 1972).

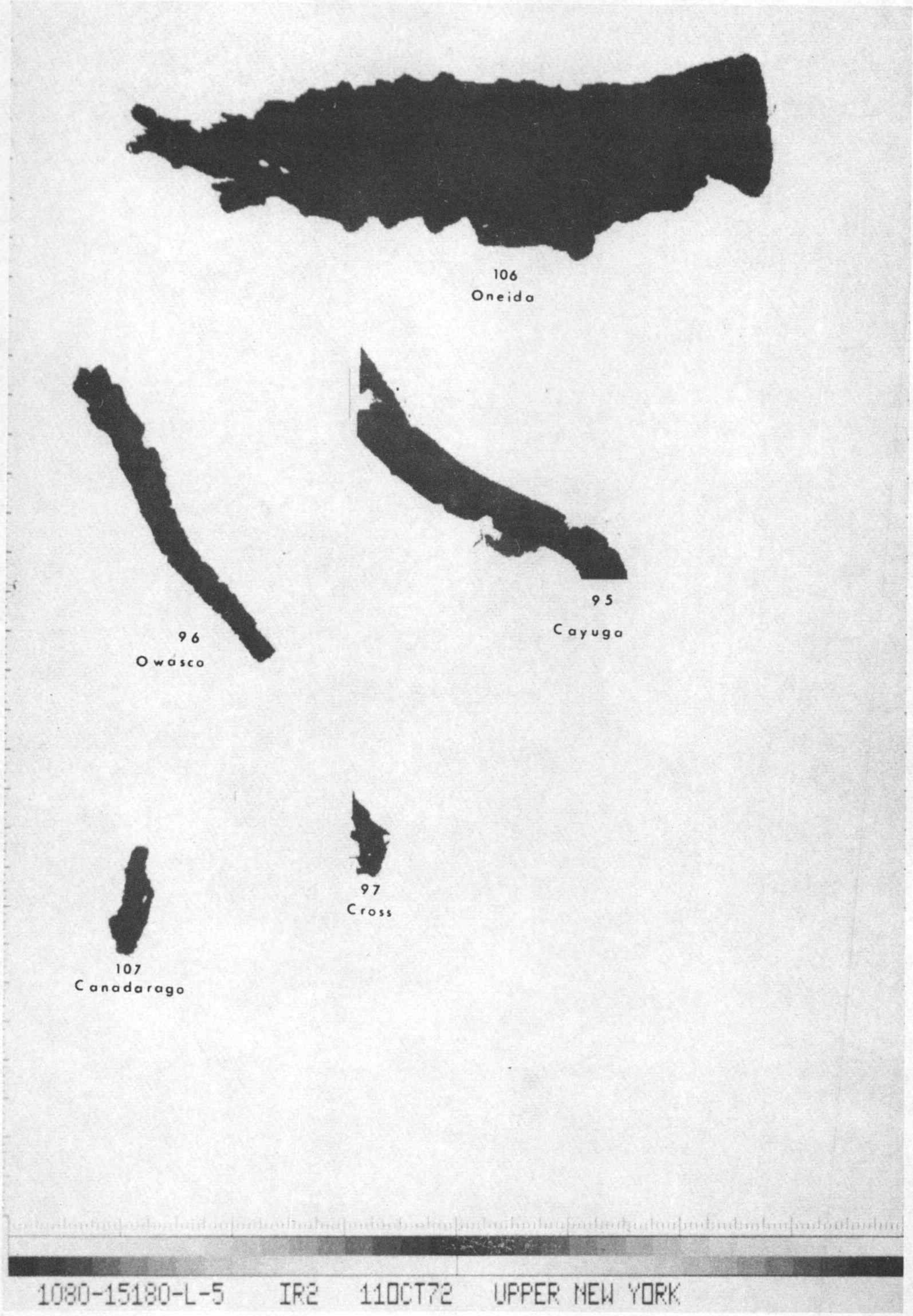

Figure 59. IR2 concatenation of five New York lakes extracted from Frame 1080-15180 (11 October 1972).

Table 80. Areal aspects of 5 New York lakes extracted from Frame 1080-15180 (11 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 Lake Area (ha)	Map Lake Area (ha)	Map Area:ERTS Area Ratio
Cayuga	95	8,799	4,206.0	17,319.9	--- *
Owasco	96	6,018	2,876.6	2,745.5	0.955
Cross	97	1,072	512.4	844.4	--- *
Oneida	106	44,934	21,478.5	20,720.6	0.963
Canadarago	107	1,644	785.8		

*Only a fraction of the lake surface is in the sensor field of view.

Table 81. MSS descriptive statistics for 5 New York lakes extracted from Frame 1080-15180 (11 October 1972).

Lake Name	Serial Number	Pixel Count	ERTS-1 MSS Bands Green	Red	IR1	IR2
Cayuga	95	8,799	36.31*	18.18	12.49	6.23
			2.72**	2.89	4.88	5.63
Owasco	96	6,018	38.66	19.46	11.40	4.53
			2.94	3.63	3.75	4.00
Cross	97	1,072	38.68	22.63	13.58	4.93
			1.28	2.01	3.87	4.68
Oneida	106	44,934	34.89	18.07	10.53	4.17
			2.11	2.15	2.77	2.93
Canadarago	107	1,664	35.53	19.89	11.75	4.63
			2.18	2.62	4.20	

*Mean DN value for the lake.

**Standard deviation of the lake DN values.

APPENDIX E

APPENDIX E

Study Area Precipitation Data*

This appendix contains precipitation data for the five days antecedant to the date of ERTS-1 coverage. The U.S. National Weather Service stations were selected to give good areal coverage of the study lakes in the MSS frame(s). The dimensional unit of precipitation is the inch, (T) represents a trace of precipitation, and a blank ( ) indicates that no precipitation was recorded.

ERTS-1 MSS Date and Frames: 9 August 1972 (1017-16093, 1017-16093)

Station	August 1972					
(Wisconsin)	4	5	6	7	8	9
Appleton			0.34	0.39	0.17	
Oshkosh			0.33	0.27	0.44	
Fond du Lac		T	0.54	0.13	0.28	
Ripon 5 NE			0.23	2.35	0.43	
Beaver Dam			0.12	0.21	0.09	
Madison		0.07	0.65	0.03	0.02	
Oconomowoc			0.17	0.40	0.02	T
Burlington			0.30	0.13	0.02	
Fort Atkinson	0.01		0.07	0.09	0.03	

*Source: Climatological Data (By Sections). U. S. Department of Commerce, National Oceanic and Atmospheric Administration, Environmental Data Service.

New York: 84(8):152.
Minnesota: 78(8):138-139; 78(10):168-169; 79(5); 79(6); 79(7).
Wisconsin: 77(8):125-126; 78(6); 78(7).

APPENDIX E (Contd)

ERTS-1 MSS Date and Frames: 11 June 1973 (1323-16094, 1323-16100)

Station (Wisconsin)	June 1973 6	7	8	9	10	11
Appleton	0.05	T	T			0.35
Oshkosh		0.02	0.10			T
Fond du Lac	T	T	0.03			0.21
Ripon		0.03				
Beaver Dam						0.30
Madison		0.01	T			0.10
Oconomowoc		0.05				
Burlington	0.04			0.04		
Fort Atkinson	0.02		0.02			

ERTS-1 MSS Date and Frames: 17 July 1973 (1359-16091, 1359-16094)

Station (Wisconsin)	July 1973 12	13	14	15	16	17
Appleton	0.07					
Oshkosh		T				
Fond du Lac	T	T				
Ripon	T	T				
Beaver Dam						
Madison						
Oconomowoc						
Burlington						
Fort Atkinson						

APPENDIX E (Contd)

ERTS-1 MSS Date and Frames: 14 August 1972 (1022-16373)

Station	August 1972					
(Minnesota)	9	10	11	12	13	14
Alexandria		T			T	
Glenwood			T			
Willmar						
Hutchinson			T			
Cokato						
Buffalo			T			T
Maple Plain						

ERTS-1 MSS Date and Frame: 6 October 1972 (1075-16321)

Station	October 1972					
(Minnesota)	1	2	3	4	5	6
Buffalo			0.05			0.30
Forest Lake			0.77			0.23
Cokato			0.08			0.23
Hutchinson			0.01			0.29
Hastings Dam			0.25	0.50		0.15
Lake City			0.15	1.23		0.30
Rochester			0.29	T	0.32	0.04
Minn.-St. Paul		T	0.13		0.15	

APPENDIX E (Contd)

<u>ERTS-1 MSS Date and Frame:</u> 8 October 1972 (1077-16431)

Station	October 1972					
(Minnesota)	3	4	5	6	7	8
Alexandria	0.02		0.49			
Glenwood	0.29			0.26		
Ortonville				0.33		
Willmar	0.06			0.40		
Melrose	0.26			0.41		

<u>ERTS-1 MSS Date and Frame:</u> 28 May 1973 (1309-16325)

Station	May 1973					
(Minnesota)	23	24	25	26	27	28
Buffalo	0.04	2.16	0.35		0.02	T
Cokato		1.20	0.47			
Hutchinson		0.45	0.36		0.22	
Forest Lake	0.02	1.02	0.75	T	T	
Hastings Dam	0.07	0.20	0.76	0.10	0.19	0.04
Minn.-St. Paul	0.02	0.60	0.02	0.05	0.05	
North Mankato	T	T	0.47	0.02	0.40	0.39

APPENDIX E (Contd)

ERTS-1 MSS Date and Frame: 3 July 1973 (1345-16322)

Station	June-July 1973					
(Minnesota)	29	30	1	2	3	4
Buffalo				1.00		
Cokato				0.90		
Hutchinson				1.04		
Forest Lake				0.25		
Hastings Dam				0.63		
Minn.-St. Paul				0.14		
North Mankato				0.79		

ERTS-1 MSS Date and Frame: 4 July 1973 (1346-16381)

Station	June-July 1973					
(Minnesota)	29	30	1	2	3	4
Buffalo				1.00		
Cokato				0.90		
Hutchinson				1.04		
Willmar				1.01		
Marshall				0.61		

APPENDIX E (Contd)

ERTS-1 MSS Date and Frame: 19 August 1972 (1027-15233)

Station	August 1972					
(New York)	14	15	16	17	18	19
Canandaigua		0.60		0.12		
Penn Yan	0.31	0.47	T	0.11		
Mount Morris		0.21		0.17	0.07	
Cayuga Lock 1		0.95		0.05	T	
Skaneateles	0.01	0.85		0.05	0.02	T
Aurora Rsch Fm		0.03		0.06		0.07
Newark		1.02		0.10	T	
Wolcott			T	T	0.42	
Ithaca Cornell U.		0.56		0.05	T	

APPENDIX F

APPENDIX F

N x N Squared Euclidian Distance Matrix

The dendrogram of 100 NES-sampled lakes (Figure 15) was created using the output of the McKeon hierarchical cluster analysis program. The clustering procedure was carried out using the matrix in this appendix. Only the lower triangular form of the matrix is reproduced here.

MCKEON CLUSTER ANALYSIS VERSION I.1

CONTROL INPUT: 100 3 6 2 2 1 2
:(3X,6F10.5)

FIRST ROW OF DATA FOR VERIFICATION-
2.678E 00 -5.413E-01 5.497E 00 -2.976E 00 -2.206E-01 7.655E-01
SQUARED DISTANCES BETWEEN POINTS BEFORE CLUSTERING

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0														
2	.633	0													
3	.511	.728	0												
4	1.052	1.490	.505	0											
5	1.191	.571	.542	1.511	0										
6	2.536	1.271	1.730	3.307	.395	0									
7	2.258	1.551	1.658	2.961	.695	.679	0								
8	2.244	.760	1.756	2.522	.951	1.244	.962	0							
9	1.564	2.666	1.439	.431	3.280	5.833	5.290	4.401	0						
10	1.319	2.450	2.050	2.568	2.589	3.605	2.155	3.515	3.528	0					
11	4.698	6.044	4.580	5.190	4.318	4.884	2.840	6.188	7.145	1.847	0				
12	5.546	7.828	6.684	6.858	7.412	8.634	5.540	8.561	7.854	1.642	1.421	0			
13	.943	1.131	.896	1.774	.785	1.617	.566	1.314	3.177	1.278	2.697	4.419	0		
14	1.646	1.209	2.736	4.672	2.396	2.571	3.070	2.707	5.927	3.232	7.943	8.772	2.622	0	
15	4.553	6.822	5.087	5.238	5.425	6.758	6.605	9.630	6.036	4.071	3.375	5.346	5.156	8.601	0
16	2.032	1.002	1.856	3.473	1.110	1.441	2.634	1.968	5.008	5.363	8.718	12.168	2.370	1.741	8.134
17	3.929	1.900	2.991	5.214	1.409	.696	1.552	1.361	7.975	5.845	8.189	12.181	2.790	2.568	11.240
18	2.501	1.469	2.072	3.294	.851	.952	2.373	2.379	5.216	5.128	6.931	11.120	2.526	3.021	5.661
19	4.325	2.385	3.705	4.818	3.191	3.481	6.090	3.777	6.143	8.991	14.415	18.137	6.215	3.638	12.471
20	.936	1.898	1.477	1.840	3.097	5.192	5.376	4.397	1.444	4.005	9.164	9.789	3.156	2.907	7.214
21	2.998	1.033	2.487	3.402	1.436	1.375	1.497	.204	5.512	4.293	7.498	9.894	2.281	2.689	11.161
22	22.133	25.375	23.284	19.575	29.635	35.780	36.401	31.353	14.676	28.494	39.817	36.582	30.258	29.357	30.971
23	.657	.597	.489	1.350	1.129	2.342	2.727	1.980	2.011	3.337	7.454	9.325	1.713	1.802	7.334
24	22.071	25.930	23.830	19.299	30.077	36.507	36.287	31.833	14.242	26.961	36.974	32.902	29.806	30.524	28.023
25	6.173	6.436	7.887	7.886	10.346	12.829	12.407	8.642	6.556	9.523	19.213	16.559	9.711	6.218	19.590
26	5.819	8.049	7.981	7.092	11.475	15.212	14.650	11.985	4.604	9.423	17.974	15.137	10.320	8.627	14.305
27	21.351	25.121	23.003	19.342	29.424	35.476	35.836	31.508	14.417	26.603	37.647	33.579	29.636	28.481	28.754
28	23.864	29.064	26.324	23.642	33.403	39.725	40.050	37.009	18.172	29.290	40.641	35.882	32.866	30.494	29.962
29	1.530	2.119	2.532	3.000	3.684	5.317	6.562	5.317	2.742	4.821	10.684	11.343	4.669	2.445	7.009
30	2.564	1.555	2.020	3.166	.691	.630	1.885	2.275	5.310	4.457	5.688	9.832	2.323	3.261	4.990
31	4.547	3.223	5.379	8.133	3.440	2.394	4.110	4.569	10.609	6.205	9.321	12.213	4.862	1.609	8.987
32	1.422	2.689	1.360	1.635	2.430	3.739	2.467	3.829	2.453	.522	2.256	2.626	1.578	4.316	4.293
33	8.760	10.724	8.534	6.602	12.953	16.994	15.817	13.246	4.447	11.355	18.630	16.682	12.429	14.228	18.229
34	5.909	7.541	7.815	7.722	11.043	13.889	14.202	11.679	5.839	9.154	18.537	15.790	10.899	6.965	15.510
35	2.281	4.371	3.025	2.886	5.592	8.361	8.564	8.050	1.738	5.215	10.509	10.454	5.554	5.548	6.664
36	4.586	5.083	6.512	6.034	8.679	11.067	10.305	7.167	4.890	6.644	15.121	12.203	7.782	5.311	15.560
37	12.202	15.716	12.798	10.813	17.877	22.567	23.367	21.542	7.558	16.692	24.474	23.099	18.586	18.445	16.543
38	36.761	42.864	38.964	34.615	47.677	55.855	55.514	51.164	27.557	43.687	55.890	50.561	46.610	46.502	43.655
39	1.373	2.334	1.313	1.078	2.183	3.748	2.074	2.811	1.818	.746	2.241	2.732	1.016	4.827	4.710
40	1.169	2.111	1.211	1.258	2.145	3.857	2.025	2.491	1.853	1.208	3.100	3.631	.657	4.315	5.728
41	8.347	10.062	8.767	9.644	8.384	8.271	8.426	12.584	11.652	5.286	4.148	5.601	8.733	10.557	3.121
42	7.268	9.988	8.649	8.696	10.007	11.161	9.036	12.184	9.345	3.002	4.073	1.778	7.980	10.196	6.141
43	12.322	10.826	11.275	13.706	8.212	6.323	4.519	8.003	18.326	8.577	5.147	8.843	7.279	11.877	13.910
44	29.181	26.758	27.907	31.161	22.466	18.838	16.490	22.066	37.900	22.704	15.198	20.440	21.185	27.697	27.000
45	2.714	4.360	3.246	2.756	5.223	7.487	4.854	5.083	2.640	1.726	4.825	3.247	2.945	6.309	8.351
46	1.149	1.078	.938	2.255	.340	.692	.972	2.012	3.986	2.280	3.551	6.517	.798	2.191	3.794
47	4.370	6.701	4.492	4.095	7.917	10.875	10.778	10.326	2.796	6.572	12.328	11.622	7.966	8.248	9.885
48	3.220	5.337	2.837	2.169	5.815	8.777	8.212	8.045	1.242	5.216	9.341	9.592	5.610	8.041	7.748
49	6.350	9.497	7.102	5.545	11.356	15.559	14.697	13.484	3.082	9.105	15.342	13.567	10.444	12.016	11.571
50	6.285	3.503	5.557	7.835	2.812	1.290	3.058	2.949	11.291	7.932	9.643	14.419	5.125	4.043	11.675
51	5.998	3.099	5.836	7.215	3.665	3.065	3.122	1.344	9.885	7.664	10.679	13.600	4.285	4.345	15.287
52	3.211	3.118	3.972	3.857	5.566	7.443	6.252	3.784	3.659	4.703	11.341	9.857	4.686	4.197	14.366
53	11.689	15.151	12.904	10.344	18.115	23.223	22.882	20.228	6.717	15.684	24.023	21.256	17.644	18.125	17.746
54	3.046	4.377	3.487	2.318	6.202	9.451	8.760	6.617	.947	6.121	12.033	11.459	5.702	7.119	10.479

55	3.091	1.405	2.293	3.842	.893	.643	2.250	1.818	6.095	5.932	8.278	12.722	2.932	2.795	8.224
56	8.208	9.624	10.169	8.836	13.723	17.709	16.927	12.687	6.167	12.659	22.389	19.388	12.575	10.932	19.969
57	5.089	2.986	3.651	5.336	1.915	1.403	3.691	3.316	7.954	8.560	10.384	16.150	4.813	4.859	9.717
58	7.649	4.562	7.192	10.031	4.522	2.853	5.616	4.780	13.281	10.666	14.168	18.984	7.580	4.006	14.787
59	2.756	1.568	2.264	4.167	1.984	2.399	2.850	1.816	5.676	5.911	10.057	12.864	2.657	2.081	12.070
60	1.100	.378	.666	2.029	.345	.654	1.145	1.022	3.665	2.817	5.803	8.364	1.204	1.330	7.120
61	4.125	4.405	4.494	4.097	6.933	9.477	8.761	5.804	3.227	7.031	14.515	13.370	6.545	6.019	15.821
62	2.252	2.495	3.207	4.523	4.140	5.514	6.898	5.491	4.467	6.372	12.752	13.851	5.087	1.978	9.640
63	11.530	13.762	12.389	9.802	17.274	22.154	21.519	17.452	6.481	15.922	25.254	22.465	16.825	17.103	21.589
64	3.111	1.170	2.587	4.385	1.368	1.134	2.385	1.271	6.495	6.092	9.695	13.297	2.979	1.910	10.997
65	7.676	4.485	6.079	8.156	3.188	1.845	3.448	3.062	11.803	10.096	10.935	17.014	5.620	6.344	13.624
66	5.095	6.972	6.282	5.260	9.647	13.461	12.542	9.894	3.101	8.981	16.387	14.746	8.491	8.831	14.029
67	6.039	7.416	5.608	4.759	7.555	10.051	12.318	11.640	4.030	10.327	14.479	17.172	10.054	10.522	6.979
68	4.870	2.448	4.027	6.109	1.850	.961	2.597	2.128	9.011	7.532	9.627	14.506	3.946	3.404	10.909
69	15.745	11.669	12.830	15.059	8.457	5.997	7.412	8.502	20.389	16.450	13.643	21.929	11.344	14.926	18.792
70	21.170	16.064	19.025	20.417	14.144	11.215	11.141	11.073	26.140	19.146	17.160	22.834	15.942	19.369	27.082
71	3.713	2.687	2.697	4.181	1.051	.633	1.787	3.045	6.854	5.025	4.980	9.862	2.713	4.515	5.065
72	4.033	3.334	2.202	2.104	3.078	4.668	3.914	2.639	3.082	6.323	9.098	11.846	3.425	7.283	12.529
73	2.656	1.495	1.666	2.415	1.813	2.772	2.442	.983	3.692	5.173	8.792	11.266	2.171	3.778	12.024
74	1.381	1.159	1.586	1.809	2.547	4.038	3.216	1.637	2.214	2.962	7.884	8.000	2.103	2.637	10.401
75	12.380	14.671	15.909	15.185	20.061	24.045	23.653	19.561	11.817	16.540	28.537	23.338	18.612	13.367	24.353
76	7.037	8.526	8.319	6.432	11.917	15.846	14.254	10.698	4.155	9.801	17.975	14.936	10.460	11.029	17.671
77	8.931	5.796	7.785	10.787	4.478	2.368	4.672	5.134	14.864	10.796	11.808	17.767	7.338	5.963	13.835
78	3.841	4.706	5.095	4.381	7.571	10.541	9.217	6.353	3.062	6.512	13.793	11.820	6.219	6.155	14.376
79	2.562	2.126	2.165	2.403	3.455	5.312	5.827	3.401	2.401	6.681	12.569	14.172	4.337	4.121	12.266
80	4.035	3.785	4.792	4.443	6.602	8.855	8.046	4.691	3.828	6.847	14.380	13.141	6.048	5.188	16.232
81	1.107	.650	1.439	2.450	1.712	2.847	3.147	1.758	3.096	4.220	8.962	10.576	2.015	1.409	9.074
82	1.813	.985	1.978	2.943	2.300	3.430	4.021	2.013	3.546	5.478	10.925	12.654	2.987	1.860	11.052
83	3.869	4.269	4.031	3.607	6.549	9.200	8.947	6.079	2.644	7.375	14.721	14.181	6.611	6.119	14.756
84	2.520	2.177	1.508	1.196	2.781	4.709	5.110	3.156	1.389	6.052	10.592	12.813	3.889	5.411	10.384
85	4.043	4.103	5.346	5.178	7.290	9.488	8.789	5.608	4.391	6.557	14.663	12.679	6.623	4.645	15.953
86	5.388	6.541	6.000	4.615	9.258	12.849	12.029	8.761	2.756	9.006	16.645	15.145	8.650	9.150	15.617
87	25.252	23.701	23.250	26.207	18.687	15.566	13.874	20.152	32.518	19.321	11.366	17.378	18.195	25.397	20.835
88	1.983	3.329	2.604	3.515	2.608	3.352	2.150	4.561	4.871	.642	.929	1.850	1.400	4.061	2.161
89	2.441	2.220	1.939	2.815	1.677	2.368	.631	1.301	4.583	2.209	3.152	4.934	.489	4.189	8.028
90	2.103	3.063	2.039	2.583	2.495	3.710	1.501	2.985	3.787	1.071	1.688	2.481	.645	4.903	5.631
91	3.453	1.759	2.430	3.486	1.068	1.184	1.194	.698	5.789	5.406	6.696	10.732	1.743	4.241	9.569
92	20.187	15.770	16.489	17.186	12.349	10.675	10.606	10.767	22.346	20.451	16.548	24.834	14.391	21.830	24.288
93	9.146	7.447	7.158	8.843	5.710	5.460	3.325	4.187	12.075	8.825	7.960	11.902	4.671	10.224	16.852
94	19.975	14.725	16.114	16.875	12.376	11.162	12.874	10.636	21.252	24.083	23.336	32.578	16.040	20.739	27.895
95	12.260	8.077	9.468	10.978	6.069	4.296	6.208	5.562	15.361	14.256	14.044	21.369	9.533	11.626	18.028
96	9.001	6.702	6.417	7.569	3.928	3.243	3.470	4.432	11.254	10.109	7.877	14.630	5.424	10.853	11.799
97	4.214	3.641	3.651	2.515	5.226	7.711	7.971	4.558	2.066	8.410	14.340	15.686	6.192	7.415	13.748
98	.810	1.123	.595	.326	1.442	2.927	3.192	2.696	.815	2.354	5.631	7.093	2.193	3.555	4.819
99	2.549	4.488	2.806	2.228	5.441	8.461	6.535	6.057	1.355	3.925	7.944	7.238	3.694	6.873	8.848
100	1.541	1.785	1.575	2.666	1.869	2.922	1.319	1.696	3.829	2.031	4.282	5.334	.471	2.889	8.039

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
16	0														
17	1.386	0													
18	.702	1.996	0												
19	1.685	3.091	2.458	0											
20	2.764	6.084	4.223	4.032	0										
21	2.348	1.226	2.785	3.320	5.268	0									
22	29.282	38.305	31.824	24.940	15.775	32.522	0								
23	1.116	2.739	2.244	2.349	.709	2.518	21.452	0							
24	30.948	40.444	32.318	27.824	16.918	33.366	1.193	23.002	0						
25	9.144	12.010	12.858	8.385	4.123	8.674	14.082	5.507	16.184	0					
26	10.573	16.371	13.277	11.031	3.065	13.054	8.076	6.291	8.540	2.530	0				
27	29.668	38.631	31.841	25.792	15.598	32.649	.401	21.529	.808	14.044	7.539	0			
28	32.638	42.921	35.564	30.272	17.285	38.617	2.722	24.329	3.634	16.850	8.475	1.750	0		
29	2.876	6.504	3.783	2.964	.799	5.661	16.358	1.519	17.338	4.767	3.797	15.779	17.597	0	
30	1.238	2.009	.136	3.078	4.849	2.615	33.256	2.623	33.356	13.830	14.362	32.988	37.041	4.350	0
31	2.347	2.586	2.387	4.408	7.046	4.234	40.173	4.718	41.193	13.446	16.095	39.186	41.382	5.323	2.452
32	5.607	6.024	5.415	8.483	3.466	4.669	25.427	2.807	24.626	9.588	9.196	23.905	26.759	4.764	4.729
33	15.017	18.404	17.342	13.694	6.044	14.160	7.091	8.204	8.460	5.716	4.527	7.147	10.720	8.428	17.698

34	10.198	14.511	13.098	9.087	3.352	11.886	9.856	5.829	11.646	1.747	1.324	8.980	9.888	3.137	13.917
35	5.749	10.262	6.900	6.790	.746	9.181	12.022	2.544	12.720	5.633	2.440	11.361	12.064	1.397	7.458
36	8.631	11.347	11.208	8.387	3.710	7.260	14.563	5.027	15.057	.654	2.374	13.857	17.293	3.950	11.731
37	18.389	25.628	19.743	16.386	7.886	22.820	3.395	12.076	4.497	11.406	5.157	2.884	3.374	8.345	20.555
38	46.942	59.482	50.343	44.161	28.235	53.596	3.924	36.895	4.545	27.159	16.303	3.855	2.551	29.920	52.307
39	5.304	5.983	5.024	8.562	3.539	3.879	25.285	2.835	23.859	9.406	8.959	24.181	28.047	5.243	4.458
40	4.463	5.460	4.882	8.238	2.817	3.756	25.067	2.242	24.163	8.260	7.972	24.362	27.685	5.025	4.652
41	12.821	13.044	9.567	16.107	12.961	13.011	41.286	11.732	38.324	24.562	21.935	37.736	39.634	11.413	7.909
42	15.241	15.438	13.835	19.100	11.140	12.874	33.497	11.261	30.127	17.135	15.393	29.632	31.451	11.212	12.237
43	12.363	7.529	10.606	18.641	19.324	8.360	64.816	14.094	63.137	29.094	33.650	63.232	68.483	20.762	9.013
44	28.144	20.703	24.415	37.013	39.846	22.345	98.801	32.330	95.095	53.247	59.026	96.407	102.886	40.935	21.938
45	8.450	9.364	9.369	11.769	4.112	6.241	22.010	4.338	20.771	6.908	6.788	20.701	23.954	6.516	8.926
46	1.240	2.068	.806	4.274	3.238	2.717	31.298	1.589	31.343	11.655	11.726	30.659	33.400	3.630	.662
47	9.308	12.775	10.786	9.215	2.538	11.191	10.671	4.295	12.100	6.124	3.798	9.859	10.950	3.661	11.011
48	7.719	10.904	8.629	8.644	2.036	9.293	12.489	3.261	13.228	7.308	4.623	12.034	13.988	3.873	8.762
49	12.593	18.073	14.175	13.124	3.729	15.030	6.530	6.885	6.697	6.730	2.205	5.898	7.001	5.431	14.730
50	2.781	1.148	2.129	3.942	9.602	2.359	44.735	5.602	46.000	16.678	21.245	44.680	49.925	8.467	1.875
51	3.547	2.111	4.169	5.463	8.907	1.115	41.114	5.455	41.752	11.948	17.562	41.640	48.155	9.243	4.243
52	6.354	7.297	8.746	6.861	3.161	3.973	18.759	3.114	19.708	1.621	4.616	18.526	22.850	4.489	9.008
53	18.800	26.067	20.618	17.507	7.625	21.677	2.275	12.112	2.417	8.735	3.074	1.795	3.161	8.678	21.492
54	6.820	10.801	8.411	7.345	1.438	7.748	10.065	3.002	10.450	3.601	1.888	10.175	13.334	3.032	9.027
55	.559	.890	.354	1.611	4.716	1.869	33.225	2.137	34.699	12.186	14.325	33.577	37.836	4.371	.511
56	12.382	17.940	15.845	12.013	4.901	13.609	7.876	7.946	8.865	1.869	.884	8.173	10.622	6.142	17.204
57	1.417	1.590	.853	1.925	6.628	3.279	35.925	3.559	37.943	15.534	17.691	36.663	41.255	6.115	1.030
58	2.980	1.889	3.139	2.966	9.885	3.814	43.727	6.146	46.390	15.712	20.916	43.946	48.362	8.040	3.308
59	.986	1.248	2.953	2.684	3.069	2.119	29.560	1.152	32.379	7.162	10.466	30.308	33.344	4.321	3.561
60	.868	1.005	1.399	2.381	2.471	1.231	28.368	.620	29.723	8.075	10.173	28.180	31.650	2.916	1.400
61	7.156	9.224	10.011	6.729	2.554	6.181	13.366	3.216	15.307	1.202	3.134	13.694	17.465	4.191	10.634
62	2.150	5.549	4.038	2.643	1.015	5.833	19.115	1.434	21.359	4.514	4.595	19.039	20.040	.632	4.984
63	17.662	23.651	20.371	15.633	7.410	18.523	2.730	11.073	3.724	5.641	2.816	3.050	6.185	9.015	21.384
64	.537	.451	1.384	1.466	4.361	1.146	32.700	1.796	34.862	9.351	12.979	33.244	37.473	4.366	1.733
65	3.106	1.329	2.496	4.576	10.750	2.811	46.475	6.265	48.122	18.931	23.542	47.336	53.402	10.631	2.418
66	9.113	14.302	11.811	10.005	2.367	11.264	8.630	4.938	9.447	2.960	.736	8.849	10.635	4.243	12.916
67	7.920	13.140	7.055	6.621	4.449	12.469	13.326	5.751	13.943	13.176	8.398	13.107	15.412	3.581	7.443
68	1.236	.586	1.140	2.598	7.146	1.944	39.762	3.733	41.471	14.311	17.967	40.242	44.947	6.725	1.267
69	10.296	6.131	7.884	12.721	21.229	8.200	64.857	14.662	65.548	33.124	38.607	65.634	73.635	21.355	7.003
70	17.845	11.246	15.327	19.771	28.733	9.960	73.305	21.374	72.202	35.322	44.422	73.274	84.543	28.520	13.698
71	2.148	2.139	.673	4.603	6.610	3.460	38.048	3.784	38.173	17.170	17.801	37.695	41.657	6.402	.309
72	4.801	4.758	5.905	5.886	4.341	3.429	24.115	2.609	25.816	8.952	11.063	25.231	30.757	7.161	6.005
73	2.256	2.189	3.776	3.627	3.227	1.445	26.533	1.316	28.469	6.826	9.981	27.436	32.250	5.186	4.090
74	3.404	4.197	5.030	4.620	1.857	2.037	21.005	1.198	21.859	3.330	5.611	20.917	25.083	3.213	5.209
75	17.814	24.332	22.008	17.032	8.504	20.096	9.386	12.799	10.561	3.212	1.907	8.540	8.662	8.394	23.488
76	12.631	16.804	15.286	12.481	4.861	11.647	8.442	7.297	8.554	2.474	1.788	8.404	12.193	6.689	16.003
77	4.109	1.911	3.328	5.434	12.488	4.437	51.186	7.798	53.150	21.062	26.077	51.273	56.115	11.214	3.071
78	7.569	10.805	10.266	8.500	2.479	7.189	13.181	3.889	13.758	1.285	1.761	13.193	16.422	4.163	11.020
79	2.792	4.987	4.808	2.566	1.230	3.854	16.658	.955	19.116	3.669	5.059	17.605	21.318	2.434	5.598
80	6.487	8.367	9.268	6.386	3.030	4.954	15.999	3.381	17.429	.999	3.663	16.345	20.746	4.372	9.925
81	.832	2.687	2.387	2.408	1.145	2.274	23.084	.433	24.559	4.788	6.199	23.377	26.299	1.867	3.032
82	1.104	2.868	2.956	1.867	1.445	2.327	22.190	.645	24.243	4.198	6.285	22.802	26.160	2.085	3.717
83	6.514	9.140	9.164	5.812	1.905	6.545	11.776	2.727	13.943	1.667	2.777	12.263	15.726	3.389	9.891
84	3.310	5.205	4.375	3.020	1.733	3.738	16.726	1.183	18.489	5.835	6.487	17.601	22.209	3.098	4.811
85	7.029	9.158	9.892	6.856	3.037	5.730	15.617	3.761	16.922	.432	2.926	15.473	19.154	3.843	10.518
86	9.454	13.503	12.003	9.035	2.932	9.684	8.247	4.857	9.337	2.333	1.651	8.667	12.152	4.723	12.849
87	24.838	18.297	20.625	32.871	34.964	20.661	89.703	28.053	86.525	50.775	54.403	87.306	92.880	36.076	18.098
88	5.252	6.059	4.311	10.028	5.167	5.662	33.120	4.272	31.170	13.644	12.280	31.119	33.022	5.887	3.616
89	3.869	3.068	4.287	8.084	5.116	2.263	34.027	2.995	33.713	10.960	12.943	33.675	37.830	7.474	3.936
90	5.389	5.410	5.497	10.270	4.752	4.299	31.413	3.552	30.198	11.312	11.409	30.350	33.415	7.174	4.978
91	1.663	1.263	1.681	4.295	5.713	1.249	35.658	2.825	36.270	12.593	15.223	36.325	41.661	6.978	1.732
92	15.698	11.040	12.985	18.559	26.082	10.960	67.335	19.216	66.791	37.310	42.979	68.894	79.730	27.891	11.936
93	8.138	4.726	8.852	13.217	12.773	5.102	49.529	8.468	50.408	19.545	24.483	50.264	55.942	16.574	8.410
94	12.810	10.011	11.505	13.067	23.085	10.530	58.324	16.881	60.298	32.441	38.478	61.425	72.080	24.361	11.535
95	7.327	4.040	5.726	7.435	16.149	4.904	52.151	10.462	53.733	24.972	30.817	53.341	61.924	15.926	5.167
96	6.037	4.048	4.321	9.321	12.883	5.115	48.736	8.292	48.877	24.397	26.894	49.609	56.315	14.483	3.783
97	5.363	8.086	6.740	4.282	2.862	5.094	13.691	2.908	15.038	4.697	5.437	14.878	20.528	4.100	7.419
98	3.092	4.885	2.802	3.705	1.628	3.204	19.263	1.190	19.140	7.251	6.717	18.691	22.735	1.851	2.634
99	7.132	9.921	8.738	9.945	1.999	7.625	15.756	3.077	15.731	5.570	3.899	15.410	17.639	4.801	9.011

100	3.115	3.292	4.333	7.243	3.220	2.733	29.909	1.916	30.168	8.038	9.542	29.555	32.416	5.520	4.342

	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
31	0														
32	7.869	0													
33	23.562	8.631	0												
34	13.799	8.653	4.699	0											
35	10.415	4.188	4.955	2.860	0										
36	11.950	7.123	5.637	1.950	4.931	0									
37	26.260	13.811	4.560	5.681	4.327	11.169	0								
38	60.109	40.102	16.765	20.147	21.905	28.087	8.677	0							
39	8.581	.497	8.706	9.691	4.776	6.751	14.942	40.292	0						
40	8.417	1.056	8.580	9.211	4.419	6.256	15.125	39.451	.308	0					
41	9.740	5.695	23.920	19.783	12.549	19.543	23.965	57.598	7.656	10.033	0				
42	13.711	3.517	16.008	13.931	10.404	12.510	19.953	47.057	5.055	6.950	3.525	0			
43	10.260	10.004	34.256	32.399	24.236	25.204	46.187	88.681	9.645	10.263	11.550	13.772	0		
44	22.629	25.545	60.763	57.791	46.414	47.349	75.214	127.257	24.756	26.006	21.827	27.096	3.990	0	
45	12.421	1.420	6.056	7.543	4.935	4.739	13.585	35.041	1.032	1.037	11.487	5.245	13.970	31.431	0
46	2.715	2.490	14.856	11.511	5.396	9.761	18.281	48.266	2.511	2.421	6.843	9.060	8.039	21.668	5.750
47	14.595	4.475	2.507	2.882	1.177	5.672	3.331	20.437	5.815	5.913	13.913	10.037	26.827	50.591	4.913
48	13.936	3.114	2.745	4.922	1.172	6.438	4.663	22.900	3.649	3.635	12.890	9.675	22.811	44.852	3.489
49	19.871	7.208	2.020	3.616	1.724	6.022	1.951	13.456	7.464	7.199	18.747	13.043	33.245	58.717	5.780
50	1.843	8.875	24.935	18.902	14.161	14.998	31.023	68.397	8.780	8.910	12.020	17.082	7.338	17.972	13.731
51	4.804	9.132	21.290	17.164	14.336	10.589	31.384	64.058	7.436	6.730	18.133	18.619	8.467	20.381	10.424
52	10.356	4.659	5.333	4.089	5.284	1.073	13.608	34.200	4.096	3.510	17.888	11.523	18.475	38.608	2.727
53	27.230	13.502	3.129	4.794	4.627	8.234	.996	7.450	13.620	13.484	26.154	19.436	45.538	74.447	11.192
54	13.588	5.014	2.492	3.376	1.582	2.969	5.640	21.160	4.251	3.652	17.628	12.940	24.934	47.719	3.567
55	2.260	5.998	17.353	13.265	8.004	11.096	21.318	53.177	5.763	5.517	11.366	15.389	10.258	24.303	9.941
56	19.456	12.352	4.581	2.654	5.132	2.330	7.622	17.127	11.261	9.845	28.801	20.698	37.143	63.726	8.471
57	3.429	8.192	20.008	16.354	10.040	14.625	23.331	56.763	8.051	7.886	12.927	18.789	11.714	25.516	13.171
58	1.650	11.602	25.986	17.620	14.491	15.038	30.644	67.268	12.035	11.843	15.241	20.806	11.988	24.528	16.828
59	4.287	5.721	12.969	9.556	6.576	7.591	19.383	47.335	5.522	4.217	16.493	16.417	12.598	29.562	7.311
60	2.860	2.642	11.882	8.890	5.058	7.069	17.053	46.346	2.830	2.593	9.556	10.369	9.498	24.975	5.195
61	13.046	6.041	2.830	2.787	3.916	1.509	9.427	26.932	5.669	4.830	20.934	14.586	24.106	47.183	4.013
62	4.760	6.399	10.184	3.738	2.359	4.753	10.716	32.839	6.952	5.989	14.806	14.656	21.274	41.828	8.088
63	26.955	13.779	1.659	4.320	5.834	5.829	3.342	10.278	13.200	12.650	30.098	21.813	43.963	73.088	10.197
64	2.361	6.347	16.166	11.603	8.193	8.934	21.915	52.834	6.052	5.374	14.137	16.288	10.831	25.822	9.187
65	4.206	10.499	25.842	22.214	15.864	17.665	33.095	70.533	9.664	9.366	15.835	21.387	7.825	18.381	15.111
66	16.445	8.130	3.214	2.782	2.280	3.106	5.566	17.466	7.326	6.040	22.837	16.534	30.938	55.870	5.536
67	12.976	8.369	9.482	8.469	3.028	11.796	5.115	24.950	9.046	9.695	12.744	15.677	29.105	50.931	11.893
68	2.073	8.068	21.621	16.560	11.450	13.293	27.116	61.646	7.670	7.262	13.424	17.922	8.871	21.297	12.235
69	10.511	16.732	38.396	37.080	27.300	30.535	47.971	92.768	15.616	16.068	18.928	27.322	6.390	11.769	23.166
70	16.137	20.705	43.078	42.215	36.075	31.296	58.728	105.053	18.774	20.001	23.760	28.073	6.733	9.669	24.867
71	3.065	5.096	20.253	17.108	9.357	14.886	23.556	57.723	5.047	5.368	7.266	12.499	7.177	18.677	10.059
72	11.704	4.532	7.437	11.430	6.889	8.576	16.059	41.089	3.517	2.895	17.691	15.572	14.313	32.392	4.735
73	7.171	4.474	9.658	9.842	6.537	6.655	17.954	44.480	3.586	2.653	16.697	15.130	12.171	29.304	4.972
74	7.283	2.789	6.421	5.367	4.139	2.536	13.767	37.054	2.216	1.675	14.195	10.314	14.022	32.367	2.242
75	22.304	16.997	8.722	2.261	7.770	3.835	9.154	16.668	17.171	15.885	31.495	22.200	45.376	73.689	13.137
76	19.918	9.023	2.227	3.406	4.956	2.014	7.457	18.716	7.875	7.144	24.871	16.081	32.176	57.193	5.072
77	2.555	11.632	29.899	23.110	17.422	19.653	35.631	75.855	11.918	11.982	13.651	20.544	7.862	17.448	17.737
78	13.327	6.371	3.980	2.955	3.685	1.022	9.768	25.225	5.349	4.271	21.009	13.992	24.622	47.139	3.503
79	8.554	5.589	5.807	4.921	3.162	4.108	10.464	31.714	5.114	4.131	18.312	16.318	20.285	41.500	5.772
80	11.671	6.641	4.912	3.687	5.163	1.021	12.401	30.821	5.716	4.759	21.468	15.175	22.427	44.187	4.309
81	4.220	4.405	10.071	6.267	3.705	4.502	14.561	38.975	3.886	2.849	14.382	13.514	14.591	32.548	5.277
82	4.888	5.459	9.679	6.030	4.192	4.305	14.519	38.587	4.918	3.812	16.426	15.428	16.312	35.125	6.191
83	12.797	6.074	2.489	2.521	2.917	2.105	7.626	24.812	5.831	5.006	20.432	15.165	25.157	48.763	4.630
84	9.585	4.476	5.284	6.602	3.299	5.564	9.989	32.158	3.877	3.421	16.048	14.808	18.706	39.020	5.147
85	11.127	6.665	5.232	2.451	4.818	.332	11.817	29.910	6.181	5.369	20.383	13.813	23.344	45.345	4.451
86	17.016	7.743	1.726	2.875	3.190	2.389	6.000	19.080	6.891	6.025	23.079	16.399	30.016	54.927	5.055
87	20.502	20.815	53.978	52.815	39.971	45.047	65.398	116.049	20.870	22.525	15.841	22.451	3.386	1.386	27.893
88	5.577	1.354	15.468	12.540	6.331	10.408	19.177	48.097	1.607	2.040	3.877	3.817	7.323	19.708	3.787
89	6.944	2.369	13.382	13.560	8.232	9.125	22.414	51.563	1.455	.965	11.307	9.457	5.931	19.049	3.060
90	8.307	1.169	11.966	12.395	6.688	8.920	19.424	46.586	.617	.414	8.813	6.179	7.185	20.795	1.697
91	4.913	5.604	16.959	15.733	9.748	11.241	24.533	55.718	4.228	3.584	13.975	15.715	7.603	20.492	7.734
92	19.048	20.313	39.529	43.608	32.738	34.168	53.071	95.868	17.421	17.793	26.440	32.571	9.135	14.094	24.553
93	12.131	8.627	22.468	24.958	17.977	18.375	36.182	70.726	7.114	6.067	20.045	19.069	5.123	15.496	9.608

94	18.327	23.317	36.262	38.993	29.926	31.409	47.456	86.224	20.271	19.725	33.117	40.186	17.246	27.003	27.387
95	8.876	13.957	29.600	28.817	21.671	23.151	38.724	79.232	12.906	13.360	18.703	25.412	9.132	18.388	19.305
96	9.327	9.681	25.929	27.420	17.382	22.132	34.248	71.468	8.173	8.136	15.184	20.454	6.067	15.095	13.962
97	12.600	7.112	4.800	6.380	4.533	4.445	9.968	28.890	5.709	5.109	20.738	17.991	23.535	45.307	6.328
98	6.445	1.663	7.130	6.436	2.512	5.315	10.249	34.777	1.602	2.065	8.046	7.756	14.122	31.646	3.440
99	13.474	2.881	3.708	5.584	2.328	4.646	8.702	26.418	2.332	1.590	15.239	9.746	19.533	40.142	1.371
100	6.350	2.247	11.328	10.148	5.933	6.865	19.068	45.652	1.626	.735	12.161	9.490	8.612	23.685	2.477

	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
46	0														
47	8.340	0													
48	6.328	.636	0												
49	11.732	1.354	1.492	0											
50	3.260	17.764	15.776	23.736	0										
51	4.874	18.119	15.509	21.795	2.477	0									
52	7.022	5.549	5.258	6.937	11.331	6.979	0								
53	18.827	4.002	4.932	1.222	31.879	29.286	10.900	0							
54	7.161	2.409	1.621	1.688	15.360	12.180	2.758	4.014	0						
55	1.307	11.297	9.357	15.554	1.251	3.129	8.095	22.268	9.070	0					
56	14.861	6.371	6.941	4.108	23.377	17.392	4.335	4.370	2.448	16.316	0				
57	2.451	13.403	11.213	18.242	1.579	4.602	11.108	25.144	11.460	.344	19.798	0			
58	4.945	18.203	17.228	24.706	.759	3.902	12.208	32.128	16.419	1.865	22.905	2.015	0		
59	2.678	8.857	7.495	12.566	4.254	3.484	4.479	19.206	6.572	1.976	11.326	3.171	4.327	0	
60	.780	6.915	5.584	10.847	2.844	3.606	4.297	17.367	5.835	1.005	12.200	2.199	3.759	1.059	0
61	8.579	3.658	3.675	4.517	14.248	10.167	.758	7.381	1.510	9.351	2.642	12.000	14.485	5.092	5.458
62	4.021	5.001	5.314	7.275	8.147	8.642	4.876	11.151	4.176	4.149	6.581	5.785	6.965	2.816	2.864
63	18.990	4.845	5.535	2.261	29.922	25.106	7.685	1.183	3.040	21.031	2.268	24.010	30.035	16.644	16.116
64	2.083	11.146	9.619	15.450	1.567	2.016	5.987	22.110	8.470	.502	14.135	1.287	1.729	.815	.846
65	4.057	19.708	16.642	25.252	.765	2.442	12.794	33.810	16.002	1.452	24.886	1.277	1.968	4.219	3.652
66	10.342	3.481	3.248	1.796	19.861	15.513	3.865	3.396	.721	12.600	.979	15.515	20.178	8.412	8.754
67	7.620	4.551	4.099	5.601	14.921	18.445	12.170	7.515	5.523	8.496	11.757	8.897	15.132	11.071	7.955
68	2.337	15.202	13.028	20.209	.472	2.162	9.696	27.842	12.300	.391	19.677	.673	1.037	2.407	1.870
69	9.482	31.201	26.464	38.312	4.130	7.291	23.210	49.280	27.369	6.721	40.570	5.711	7.092	11.976	10.084
70	16.208	39.026	34.361	46.149	8.338	8.124	23.999	57.190	33.074	13.535	44.807	13.916	12.555	18.460	15.612
71	.886	12.731	10.089	17.216	1.937	5.180	11.245	25.023	11.426	.988	21.164	1.220	3.804	4.431	1.992
72	4.923	7.118	4.545	9.348	9.014	6.629	4.267	15.175	4.372	5.117	11.011	6.216	11.129	3.213	3.152
73	3.221	7.942	5.886	10.717	5.597	3.353	3.022	16.894	4.723	2.856	10.046	4.194	6.846	.937	1.420
74	3.662	5.110	4.066	7.004	7.628	4.738	.627	11.952	2.487	4.609	6.044	6.951	8.851	2.293	1.850
75	20.394	8.829	11.440	6.928	29.607	24.585	8.108	6.200	7.016	22.684	2.090	27.014	27.729	16.963	17.460
76	13.462	4.952	4.900	3.039	22.301	16.208	2.872	4.000	1.499	15.697	1.009	19.211	23.243	11.217	10.917
77	4.725	21.257	19.169	28.367	.417	4.311	15.314	37.432	19.576	2.212	28.701	2.004	.879	5.701	4.475
78	8.679	4.732	4.371	4.083	15.634	10.402	1.071	6.717	1.133	10.406	1.399	13.633	16.327	6.224	6.508
79	4.755	4.393	3.555	6.067	8.805	6.932	2.506	9.876	1.973	4.256	5.174	5.675	8.777	2.002	2.565
80	8.256	5.845	5.568	6.489	12.665	7.735	.379	9.580	2.175	8.612	2.743	11.459	13.005	4.683	5.245
81	2.153	6.455	5.307	8.560	5.339	3.983	2.907	13.762	3.572	2.236	7.225	3.927	5.615	.843	1.089
82	3.089	6.681	5.732	8.917	5.631	4.052	2.738	13.733	3.641	2.473	6.799	4.035	5.515	.813	1.432
83	8.081	2.811	2.818	3.623	14.080	10.834	1.469	6.215	1.045	8.667	2.578	10.952	14.094	4.899	5.195
84	4.251	4.104	2.649	5.744	8.719	7.431	3.347	9.707	1.923	4.057	6.896	5.146	9.594	2.967	2.535
85	8.630	5.460	5.830	6.276	13.235	8.762	.443	9.112	2.552	9.363	2.489	12.505	13.221	5.422	5.646
86	10.749	3.346	3.129	2.274	18.879	14.262	2.609	3.709	.553	12.197	1.179	14.937	19.380	8.226	8.280
87	17.744	43.073	37.650	51.355	16.102	20.755	36.393	66.370	42.649	20.834	60.140	21.373	22.492	26.734	21.353
88	1.636	8.747	6.799	11.454	7.516	8.636	8.118	18.906	8.468	5.619	16.508	7.831	10.530	6.796	3.383
89	2.166	10.110	7.234	12.844	5.953	3.932	4.909	20.830	7.203	4.225	14.486	6.202	8.994	3.145	2.119
90	2.467	8.238	5.624	10.317	8.630	7.000	5.384	17.935	6.453	6.119	13.890	8.508	12.098	5.066	3.119
91	1.883	13.060	9.853	16.221	2.619	1.694	7.026	23.772	8.712	1.317	16.151	2.164	4.652	2.066	1.677
92	14.475	36.313	29.800	41.529	9.884	9.585	25.507	52.378	29.334	11.842	43.075	11.036	14.742	16.620	14.937
93	6.910	19.773	15.432	24.013	8.057	5.317	11.169	34.487	15.616	7.672	24.959	8.935	11.745	5.937	6.337
94	15.092	34.044	28.382	38.708	9.402	8.861	23.899	47.377	26.153	9.760	37.092	8.224	11.878	13.569	14.082
95	7.842	24.549	20.837	30.973	2.705	4.915	17.029	39.778	20.730	4.184	31.667	3.317	4.660	8.458	7.013
96	4.818	20.744	15.930	25.219	4.337	5.512	15.620	34.717	16.743	3.979	28.614	3.651	7.778	7.215	5.715
97	7.173	5.650	4.230	5.874	11.632	8.360	3.119	8.453	1.578	6.489	4.647	7.971	12.400	5.019	4.951
98	2.029	3.725	2.493	5.777	6.792	7.184	3.830	10.246	2.734	3.297	8.871	4.773	8.391	4.213	1.660
99	5.944	2.810	1.416	2.652	15.297	12.041	3.082	6.944	1.193	9.465	5.209	12.168	17.420	6.103	5.290
100	2.098	7.894	5.737	10.220	6.834	4.606	3.559	17.513	5.400	4.337	11.113	6.592	9.044	2.165	1.786

61 62 63 64 65 66 67 68 69 70 71 72 73 74 75

61	0														
62	4.386	0													
63	4.441	10.841	0												
64	7.321	3.501	19.753	0											
65	15.233	9.916	31.116	1.866	0										
66	2.077	4.726	2.394	11.325	20.726	0									
67	9.906	5.735	9.947	10.598	16.078	7.956	0								
68	11.692	6.000	25.845	.579	.544	16.075	12.470	0							
69	27.023	21.250	46.528	8.233	2.430	34.472	25.600	4.977	0						
70	30.258	29.567	52.006	13.805	7.727	40.970	36.995	10.861	4.323	0					
71	13.126	6.976	25.178	2.533	2.218	15.909	9.089	1.546	5.404	12.614	0				
72	4.059	7.320	12.160	4.541	7.598	7.357	10.085	6.647	13.818	18.916	6.675	0			
73	3.587	4.630	13.697	1.814	4.933	7.173	10.700	3.660	11.853	16.461	5.009	.877	0		
74	1.460	3.505	9.431	3.315	8.484	4.272	9.560	6.019	17.223	19.902	6.801	2.297	1.179	0	
75	6.524	8.762	5.244	19.972	33.418	4.304	15.981	26.381	52.011	55.976	28.113	19.716	17.285	10.958	0
76	1.720	8.071	1.737	13.858	23.580	1.246	11.360	19.139	37.165	39.471	19.384	8.201	8.478	4.474	4.413
77	18.284	10.372	35.680	2.763	.977	24.446	17.529	1.049	3.419	9.030	2.634	11.554	7.816	10.851	35.202
78	.757	4.584	4.306	8.495	16.788	1.073	10.565	12.860	29.192	32.104	13.854	5.648	4.738	1.842	4.930
79	1.564	2.104	7.561	3.243	8.823	3.278	6.412	6.149	18.830	25.330	7.467	2.262	1.465	1.433	10.559
80	.406	4.470	6.061	6.376	13.678	2.745	11.572	10.403	25.185	26.992	12.604	4.512	3.271	1.138	6.760
81	3.406	1.308	12.043	1.422	5.931	4.974	7.840	3.376	14.926	20.888	4.560	3.514	1.348	1.229	11.950
82	2.942	1.357	11.461	1.399	6.089	4.979	8.104	3.571	15.472	21.295	5.437	3.409	1.254	1.309	11.614
83	.185	3.604	3.809	7.041	14.883	1.642	7.799	11.276	26.753	31.557	12.339	3.851	3.595	1.791	6.600
84	2.403	3.583	7.798	3.784	8.344	4.184	5.081	6.234	16.802	23.256	6.289	1.259	1.497	1.769	13.524
85	.645	4.048	6.087	7.046	15.102	2.837	11.654	11.255	27.360	28.896	13.369	6.243	4.511	1.530	5.093
86	.936	5.589	1.687	10.730	19.596	.605	8.236	15.515	32.532	37.155	15.823	5.758	6.095	3.368	5.254
87	43.477	37.480	66.580	23.307	16.396	51.084	42.243	18.940	9.940	11.142	14.532	28.067	26.363	29.607	70.124
88	10.632	7.348	20.240	6.585	9.418	11.644	10.464	7.116	14.637	19.336	3.671	8.038	6.699	5.226	20.854
89	7.149	7.672	18.886	3.815	5.839	10.312	13.778	4.824	10.607	13.750	4.056	2.860	1.964	2.444	21.709
90	7.478	8.039	17.312	6.113	9.157	9.320	12.548	7.444	14.471	18.051	5.100	4.154	3.724	3.061	20.142
91	8.865	6.715	21.309	1.352	1.714	12.264	12.221	1.403	5.984	10.807	2.044	3.150	1.570	3.700	24.511
92	29.437	28.511	47.922	13.231	6.387	37.020	31.334	10.166	2.494	4.216	10.403	14.011	13.785	19.756	57.991
93	14.004	15.532	30.488	6.798	6.039	19.715	24.115	6.929	7.833	10.619	7.504	5.114	4.427	7.713	35.412
94	25.725	23.729	41.976	10.539	5.188	32.470	27.025	8.347	4.486	9.369	11.081	12.429	11.704	18.577	51.712
95	19.669	16.000	36.285	5.081	1.271	27.132	19.653	3.084	1.266	5.083	4.531	9.431	7.839	12.332	42.250
96	17.968	14.704	32.700	5.397	2.184	22.433	17.067	3.654	2.148	8.040	2.725	6.571	6.068	10.350	40.002
97	2.013	4.877	5.690	5.788	11.175	3.309	6.539	8.837	20.945	25.829	9.744	2.624	2.832	2.410	11.316
98	4.218	3.493	10.056	3.888	7.927	5.714	3.887	5.528	15.247	20.419	3.804	3.349	3.065	1.946	13.985
99	2.588	5.641	6.260	8.812	15.741	2.136	8.300	12.388	25.629	31.199	10.527	3.635	4.239	2.326	10.264
100	5.212	5.249	15.831	3.518	7.123	7.457	12.387	5.232	13.852	18.056	4.892	3.004	1.681	1.581	16.987

	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
76	0														
77	27.843	0													
78	.927	20.196	0												
79	4.943	11.582	2.571	0											
80	2.081	16.799	.597	1.841	0										
81	7.336	7.674	3.454	1.124	2.857	0									
82	7.094	7.990	3.379	.725	2.415	.145	0								
83	1.942	17.903	1.061	1.042	.872	3.143	2.663	0							
84	5.532	11.517	3.624	.511	2.958	1.982	1.727	1.724	0						
85	2.139	17.457	.678	2.572	.252	3.236	2.881	1.176	4.013	0					
86	.530	23.607	.817	2.624	1.624	5.161	4.812	.708	3.105	1.951	0				
87	53.334	15.306	44.234	37.193	41.838	29.575	32.166	44.260	33.931	43.017	50.246	0			
88	13.869	9.744	9.712	8.703	10.413	5.236	6.894	10.620	7.799	10.213	12.449	15.926	0		
89	11.550	8.204	7.147	5.377	6.511	3.232	4.123	7.600	4.767	7.501	9.896	16.759	2.720	0	
90	10.631	11.280	6.938	6.489	7.304	4.279	5.562	7.846	5.608	7.901	9.425	17.652	1.401	.617	0
91	14.456	4.135	9.278	4.602	7.688	2.522	2.926	8.742	4.089	9.119	11.643	18.285	5.369	1.757	3.823
92	37.928	10.210	31.148	22.093	27.214	19.295	19.745	29.575	18.981	30.566	34.076	13.088	19.322	12.194	16.434
93	21.137	9.272	15.071	10.812	13.015	8.472	9.161	14.759	9.946	15.152	18.526	14.318	9.053	2.446	4.775
94	34.481	9.578	28.078	17.321	23.686	16.312	15.766	25.143	15.346	27.256	29.608	25.349	23.563	14.640	20.112
95	28.789	2.764	22.354	12.784	18.185	10.748	10.632	19.315	11.123	20.212	24.505	16.043	13.859	9.071	13.052
96	25.246	4.781	19.153	11.361	16.968	8.849	9.569	17.497	9.234	19.253	21.293	12.020	8.691	4.865	7.332
97	3.638	15.319	2.643	1.022	2.082	3.087	2.540	1.512	.755	3.208	1.945	41.007	10.975	7.072	8.127
98	6.859	9.616	4.692	2.519	4.519	2.315	2.743	3.683	1.561	4.804	5.030	26.634	3.366	3.736	3.465
99	3.252	19.162	2.075	3.297	3.438	3.948	4.564	2.468	3.022	3.915	2.372	35.313	5.713	4.426	3.198

100	9.206	9.234	4.985	3.896	4.775	1.946	2.751	5.563	4.020	5.411	7.617	21.199	2.760	.452	.800

	91	92	93	94	95	96	97	98	99	100
91	0									
92	8.031	0								
93	3.366	7.562	0							
94	7.520	2.979	10.480	0						
95	4.125	3.572	7.774	3.017	0					
96	2.174	3.082	3.864	4.617	2.370	0				
97	6.014	21.791	13.081	16.975	14.000	12.595	0			
98	4.064	18.793	10.709	18.192	10.873	8.629	2.965	0		
99	7.840	27.277	11.360	26.876	20.796	14.749	3.891	3.217	0	
100	2.445	16.529	3.997	17.701	11.621	7.204	6.119	3.395	2.968	0

APPENDIX G

APPENDIX G

Listing of the McKeon Cluster Analysis Program

The listing is included on the following pages of this appendix.

```
      PROGRAM MCKEON
C
C  THIS PROGRAM OBTAINED FROM NATIONAL INSTITUTES OF HEALTH
C     COMPUTER CENTER IN BETHESDA, MARYLAND.  MODIFIED TO RUN
C     UNDER OS3, AND CERTAIN CONVENIENCE CHANGES MADE
C     JUNE 1974 BY K. BYRAM, US ENVIRONMENTAL PROTECTION AGENCY
C
C     MCKEON CLUSTER ANALYSIS      VERSION  I
      DIMENSION B(150, 150), LV(150, 150),
     1  KV(150), KC(150), C(150), FMT(180)
     2 ,R(150),H(150),IOUT(150)
      INTEGER C,FMT,HARDWARE
      REAL1 B
      EQUIVALENCE (B,LV),(R,C),(H,KV),(FMT,IOUT)
C
C  PROGRAM READS UNIT 60
C           WRITE UNIT 61
C
      DO  1 LUN=1,63
      IF(.NOT.HARDWARE(LUN))GO TO 3
    1 CONTINUE
      STOP 1
    3 CALL EQUIP(LUN,5HFILE )
C*************************************************************
C                          INPUT FROM USER
C*************************************************************
C....SPECIFICATIONS INPUT
    2 READ(60,2020)N,KOP,NV,KRIT1,KRIT2,MMCOL,MATOP
      IF(EOF(60))GO TO 216
 2020 FORMAT (10I4)
      IF(N.LE.0)STOP
      NNCOL=20*MMCOL
      READ(60,1199) (FMT(II),II=1,NNCOL)
 1199 FORMAT (20A4)
      WRITE(61,1198)N,KOP,NV,KRIT1,KRIT2,MMCOL,MATOP
      MATOP=1+MATOP
 1198 FORMAT("1MCKEON CLUSTER ANALYSIS VERSION I.1"/
     *  "0CONTROL INPUT:",10I4)
      WRITE(61,1197)(FMT(II),II=1,NNCOL)
 1197 FORMAT(14X,1H:,20A4)
      WRITE(61,1196)
 1196 FORMAT("0FIRST ROW OF DATA FOR VERIFICATION-")
      NM1=N-1
      FN=N
      GO TO (5,18,7,7),KOP
C
```

```
      B(N,N)=1.
      IF(NV.NE.1)GO TO 25
C
C  REFLECTION IF REQUESTED
C
      WRITE(61,802)
  802 FORMAT(1H  ,19HVARIABLES REFLECTED//)
      DO 804 I=1,N
      DO 804 J=I,N
  804 B(J,I)=B(I,J)
      WRITE(LUN)  ((B(I,J),J=1,N),I=1,N)
      REWIND LUN
      DO 856 J=1,N
      B(J,J)=0.
      KC(J)=1.
      BM=0.
      DO 854 I=1,N
      TEMP=ABS (B(I,J))
      IF(TEMP.LE.BM)GO TO 854
      BM=TEMP
      H(J)=B(I,J)
      IM=I
  854 B(I,J)=0.
  856 B(IM,J)=H(J)
      DO 810 I=1,N
      R(I)=0.
      DO 810 J=1,N
  810 R(I)=R(I)+B(I,J)
      ITR=0
  811 RM=FN
      DO 816 I=1,N
      TEMP=R(I)+H(I)
      IF(TEMP.GE.RM)GO TO 816
      RM=TEMP
      IM=I
  816 CONTINUE
      IF(RM.GE.0.)GO TO 890
      DO 822 J=1,N
      B(IM,J)=-B(IM,J)
  822 B(J,IM)=-B(J,IM)
      DO 824 J=1,N
      H(J)=H(J)+2.*B(IM,J)
  824 R(J)=R(J)+2.*B(J,IM)
      H(IM)=-H(IM)
      R(IM)=-R(IM)
      KC(IM)=-KC(IM)
      WRITE(61,830) IM
  830 FORMAT (1H ,6X,I8)
      ITR=ITR+1
      IF(ITR.LT.N)GO TO 811
  890 CONTINUE
      READ(LUN)       ((B(I,J),J=1,N),I=1,N)
      REWIND LUN
      DO 870 I=1,N
      IF(KC(I).GE.0)GO TO 870
      DO 864 J=1,N
```

```
      B(I,J)=-B(I,J)
  864 B(J,I)=-B(J,I)
  870 CONTINUE
      GO TO 25
C
C....KOP=2...INPUT DATA AS COVARIANCE MATRIX IN FULL MATRIX FORM
C
   18 DO 20 I=1,N
      READ(60,FMT) (B(I,J),J=I,N)
      IF(I.EQ.1)WRITE(61,1195)(B(I,J),J=1,NV)
   20 CONTINUE
      GO TO 25
C
C....KOP=3 OR 4...INPUT DATA AS RAW NUMBERS
C
    7 DO 8 I=1,N
      READ(60,FMT) (B(I,J),J=1,NV)
      IF(I.EQ.1)WRITE(61,1195)(B(I,J),J=1,NV)
    8 CONTINUE
      IF(KOP.EQ.4)GO TO 602
C...KOP=3...STANDARDIZE OVER COLUMNS
      DO 10 J=1,NV
      SUM=0.
      SS=0.
      DO 9 I=1,N
      SUM=SUM+B(I,J)
    9 SS=SS+B(I,J)**2
      FMEAN=SUM/FN
      FMSQ=SS/FN
      SD=SQRT (FMSQ-FMEAN**2)
      DO 10 I=1,N
   10 B(I,J)=(B(I,J)-FMEAN)/SD
C....COMPUTE DISTANCES....USING
  602 IF(N.GT.75)GO TO 14
C.........TOP OF MATRIX SINCE N LT 75
      DO 12 I=1,N
      DO 12 J=I,N
      B(I+75,J)=0.
      DO 12 K=1,NV
   12 B(I+75,J)=B(I+75,J)+B(I,K)*B(J,K)
      DO 13 I=1,N
      DO 13 J=I,N
   13 B(I,J)=B(I+75,J)
      GO TO 25
C.........USING EXTERNAL SCRATCH FILE SINCE N GT 75
   14 DO 15 J=1,NV
   15 WRITE(LUN)      (B(I,J), I=1,N)
      REWIND LUN
      DO 16 I=1,N
      DO 16 J=I,N
   16 B(I,J)=0.
      DO 17 K=1,NV
      READ(LUN)      (R(I),I=1,N)
      DO 17 I=1,N
      DO 17 J=I,N
   17 B(I,J)=B(I,J)+R(I)*R(J)
```

```
C*****************************************************************
C                 INPUT COMPLETE...INITIALIZE
C*****************************************************************
   25 TB=0.
      DO 26 I=1,NM1
      IP1=I+1
      DO 26 J=IP1,N
      B(I,J)=B(I,I)+B(J,J)-2.*B(I,J)
   26 TB=TB+B(I,J)
      DO 27 I=1,N
   27 B(I,I)=0.
      DO 28 I=1,NM1
      IP1=I+1
      DO 28 J=IP1,N
   28 B(J,I)=B(I,J)
      NRC=N
      KSH=0
      LOC=1
      IF(KRIT1.EQ.N.AND.MATOP.EQ.1)GO TO 1021
      WRITE(61,2000)
 2000 FORMAT("-SQUARED DISTANCES BETWEEN POINTS BEFORE CLUSTERING")
      GO TO 1000
 1021 CONTINUE
      IF(MATOP.EQ.3)WRITE(61,2014)
      NC=N
      NCM1=NC-1
      BMIN=0.
      TEMP=SQRT (FN)
      IF(KRIT1.LE.0)KRIT1=2.*TEMP+.5
      IF(KRIT2.LE.0)KRIT2=2
      MCL=0
      MIS=N
      DO 120 I=1,N
      C(I)=1
      KV(I)=I
  120 KC(I)=I
C*****************************************************************
C          MAIN LOOP
C*****************************************************************
C...FIND MINIMUM ENTRY IN OFF DIAGONAL ELEMENTS
  130 BPR=BMIN
      BMIN=1.E30
      DO 134 I=1,NCM1
      IP1=I+1
      DO 134 J=IP1,NC
      IF(B(I,J).GE.BMIN)GO TO 134
      BMIN =B(I,J)
      IM=I
      JM=J
  134 CONTINUE
      DEL=BMIN-BPR
      IF(NC.GT.KRIT1)GO TO 139
      GO TO (149,1022,135),MATOP
  135 WRITE(61,420) BMIN,IM,JM,DEL,MIS,MCL
  420 FORMAT (1H ,8X,F8.3,9X,I3,1H,,I3,8X,F10.3,18X,I8,15X,I10)
  139 CN=C(IM)+C(JM)
```

```
      DO 136 K=1,NC
      B(IM,K)=AMAX1(B(IM,K),B(JM,K))
  136 B(K,IM)=B(IM,K)
      IF(JM.GE.NC)GO TO 141
C...CLOSE UP MATRIX NOW THAT WE ELIMINATED A CLUSTER
      DO 138 I=1,NC
      DO 138 J=JM,NCM1
  138 B(I,J)=B(I,J+1)
      DO 140 J=1,NCM1
      DO 140 I=JM,NCM1
  140 B(I,J)=B(I+1,J)
C...ADJUST KC TO SHOW NEW ARRANGEMENT
  141 DO 146 I=1,N
      IF(KV(I).EQ.KC(JM))KV(I)=KC(IM)
  146 CONTINUE
      KTR=1
C...ADJUST NUMBER OF ISOLATED POINTS
      IF(C(IM).LE.1)KTR=KTR+1
      IF(C(JM).LE.1)KTR=KTR+1
      GO TO (311,314,313), KTR
  311 MCL=MCL-1
      GO TO 314
  313 MCL=MCL+1
  314 MIS=NCM1-MCL
      IF(JM.GE.NC)GO TO 321
C...CLOSE UP COUNTER ARRAYS
      DO 148 J=JM,NCM1
      C(J)=C(J+1)
  148 KC(J)=KC(J+1)
  321 C(IM)=CN
      NC=NCM1
      NCM1=NC-1
      GO TO 130
C*****************************************************************
C     OUTPUT SECTION
C*****************************************************************
  149 WRITE(61,2006)
 2006 FORMAT(1H1 ,44HMAXIMUM SQUARED DISTANCE BETWEEN CLUSTERS    ,
     1 53HMAXIMUM SQUARED DISTANCE WITHIN CLUSTERS IN DIAGONALS)
      NRC=NC
      KSH=75
      LOC= 2
C      LOWER TRIANGULAR MATRIX PRINTOUT
 1000 DO 1002 JL=1,NRC,15
      JU=JL+14
      IF(JU.GT.NRC)JU=NRC
      WRITE(61,1010)(J,J=JL,JU)
 1010 FORMAT(1H0,2X,15I8)
      DO 1014 I=JL,NRC
      JUU=JU
      IF(JUU.GT.I)JUU=I
 1014 WRITE(61,1016)I,(B(I,J),J=JL,JUU)
 1002 CONTINUE
 1016 FORMAT(1H ,I4, 15F8.3)
      GO TO (1021,1022), LOC
 1022 CONTINUE

      WRITE(61,2008)
 2008 FORMAT(1H-,22X,"PTS INCLUDED IN CLUSTER",
     *  " (ORIGINAL NUMBERING OF DATA)")
      DO 156 I=1,NC
      K=0
      DO 152 J=1,N
      IF(KV(J).NE.KC(I))GO TO 152
      K=K+1
      IOUT(K)=J
  152 CONTINUE
  156 WRITE(61,160)I,C(I),(IOUT(J),J=1,K)
  160 FORMAT(" CLUSTER",I4,1H:,I4," PTS",20I5/(21X,20I5/))
      WRITE(61,2014)
 2014 FORMAT(1H0 ,7X, 22HNEXT CLUSTER.  MAXIMUM ,11X,
     1 17HCHANGE IN MAXIMUM ,11X ,9HNUMBER OF
     2 14X,17HCLUSTERS WITH TWO/1H ,7X,26HSQUARED DISTANCE, LOCATION,7X,
     3 16HSQUARED DISTANCE,12X ,15HISOLATED POINTS,8X,
     4 14HOR MORE POINTS//)
      IF(NC.GT.KRIT2)GO TO 135
      WRITE(61,420)          BMIN, IM, JM, DEL,MIS,MCL
C....RETURN TO GET ANOTHER CASE OR EXIT
      GO TO 2
  216 CALL UNEQUIP(LUN)
      END
```