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Alexander C. Quinn for the M. S. in Electrical Engineering

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Title MICROWAVE METHODS OF MEASURING RESISTIVITY OF GALLIUM ARSENIDE

Abstract approved

This paper is concerned with the determination of gallium arsenide resistivity by measurement of attenuation of microwave energy at 7500 megacycles transmitted through a slice.

The first section of this paper describes gallium arsenide properties as compared to silicon, germanium, silicon carbide, and diamond. A description is then given of the method used by industry to determine the resistivity of gallium arsenide by the conventional four-point probe method. Various methods of resistivity measurement are discussed in the second section before taking up microwave methods in the third section. A discussion of the various microwave methods of resistivity evaluation then follows.

It is found that it is feasible to measure the resistivity of "N" and "P" type gallium arsenide, of both the single and polycrystalline varieties, by the
method of microwave transmission through a slice of semiconductor inserted transversally in a waveguide.

The results of this investigation show that resistivity evaluation by the microwave method used should be accurate to within plus or minus five percent.

With due consideration given to accuracy of observations and quality of instrumentation, the results of resistivity measurement by microwave means should fall within the five percent accuracy usually attributed to the conventional four-point probe method.

By utilizing one of the microwave methods discussed in this paper it should be possible to cover the range of resistivities encountered in gallium arsenide crystal manufacture.
MICROWAVE METHODS OF MEASURING RESISTIVITY OF GALLIUM ARSENIDE

by

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MICROWAVE METHODS OF MEASURING RESISTIVITY OF GALLIUM ARSENIDE

INTRODUCTION

A basic measurement made on a semiconductor material during its manufacture is its resistivity. This is a measure of its impurity content and determines the suitability of the material for a particular application and the necessary process parameters for subsequent operations on the material during its manufacture. This measurement also determines whether a process step has been performed in a satisfactory manner. Present methods for making this measurement usually are variations of the well known basic voltmeter-ammeter four-point probe circuit using direct-current power supplies and instruments. Such direct-current methods have many causes for error, several of which, surface and point-contact potential effects, difficulty of obtaining an ohmic contact, etc., while known to exist, are difficult to evaluate.

An alternating-current measuring circuit has been developed for the measurement of resistivity retaining the four-point probe, but eliminating or minimizing to a negligible amount the errors inherent in former direct-current systems (1, p. 824). By using an alternating-current bridge arrangement, neither current nor voltage is measured individually, but their ratio is read from an accurate resistance standard.
It is possible to measure resistivity, lifetime, permittivity, as well as the various other electrical properties of semiconductor materials by use of processes involving transmission of microwave energy through a slice of a particular material under investigation (3, p. 938), (23, p. 928), (17, p. 185).

The microwave methods of evaluating the electrical properties of semiconductor materials eliminate the necessity of constructing ohmic contacts on the material; a process that is very difficult to accomplish on some of the newer semiconductors such as high-purity silicon, gallium arsenide, and gallium phosphide.

This paper will be concerned with the applicability of microwave methods to determine the resistivity of gallium arsenide.
GALLIUM ARSENIDE PROPERTIES

As the investigation carried out for this paper involved gallium arsenide, a few of the present facts on its status today are presented (21, p. 47).

Gallium arsenide was first used commercially as a semiconductor material a few years ago in tunnel diodes. Its use was short-lived because of impurities in the material and insufficient knowledge on the users' part which led them to use the diodes in an unsafe region of operation. The apparent failure of gallium arsenide caused it to be discarded as a semiconductor material until recently where it has found use as a coherent and incoherent infrared source in diode form with applications in laser devices and current indicators. Even though gallium arsenide disappeared from the market for a time, its properties were being studied by many research and development people and today finds extensive use in varactor diodes, tunnel diodes, photo diodes, and lasers.

Gallium arsenide crystals are formed from gallium, a mercury-like liquid at 30 degrees centigrade, reacting with arsenic vapor at about 1240 degrees centigrade, to form molten gallium arsenide. As with germanium and silicon, the molten semiconductor is slowly cooled and can be "frozen" to form a single crystal. Once the
crystal is formed, the slicing, dicing and processing steps are similar to those of germanium and silicon.

Gallium arsenide has the following points in its favor:

1. High frequency operation
2. High temperature capabilities
3. Ability to withstand and operate at high voltages
4. High radiation resistance

The electrical characteristics of gallium arsenide are compared with those of a few other well known semiconductor materials in the following compilation:

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>Hole mobility ( u_p (\text{cm}^2/\text{v sec}) )</th>
<th>Electron mobility ( u_e (\text{cm}^2/\text{v sec}) )</th>
<th>Band gap ( eV )</th>
<th>Device temp. ( \text{deg. C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge</td>
<td>1900</td>
<td>3900</td>
<td>0.67</td>
<td>100</td>
</tr>
<tr>
<td>Si</td>
<td>500</td>
<td>1500</td>
<td>1.106</td>
<td>250</td>
</tr>
<tr>
<td>GaAs</td>
<td>450</td>
<td>100</td>
<td>1.4</td>
<td>450</td>
</tr>
<tr>
<td>SiC</td>
<td>20</td>
<td>100</td>
<td>2.8</td>
<td>1000</td>
</tr>
<tr>
<td>C(diamond)</td>
<td>1200</td>
<td>1800</td>
<td>6.7</td>
<td>1000</td>
</tr>
</tbody>
</table>

A further listing of electrical and mechanical characteristics of Ge, Si, and GaAs is to be found in Appendix III.

Other Properties of Gallium Arsenide

Gallium arsenide is a heteropolar III-V semiconductor with a band gap of about 1.4 electron volts at 300
degrees Kelvin. It occupies a position intermediate between the lower band-gap group IV semiconductors germanium and silicon, and the higher band-gap group II-VI photoconductors cadmium sulphide and other chalcogenides of zinc or cadmium (9, p. 1666). When the resistivity of GaAs is low because of the incorporation of suitable imperfections, its electrical properties resemble, qualitatively at least, those of germanium and silicon. When the resistivity is much higher (the intrinsic resistivity at 300° K is of the order $10^8-10^9$ ohm-cm) behavior is found which is very similar to that encountered in II-VI compounds.

An investigation of high-resistivity crystals produced by normal growth processes has indicated the presence of donor levels lying about 0.5, 0.6, and 0.7 electron volts below the bottom of the conduction band (12, p. 315). When these centers are compensated they act as electron traps and can be so detected. The investigation referred to in the previous reference started with a low resistivity n-type GaAs (the donor impurity presumably being silicon), and had for its aim the reproducible compensation of the material to resistivities greater than $10^3$ ohm-cm.

It is possible to prepare gallium arsenide with resistivities at room temperature on the order of $10^6$ ohm-cm or greater. The electrical properties are then
more like the wide band gap II-VI compounds such as cadmium sulphide. This is then semi-insulating gallium arsenide (16, p. 2069).

Copper is the most thoroughly studied acceptor in gallium arsenide semiconductor mixes.

Normal GaAs crystals with normal copper acceptor levels have 0.14 electron volts ionization energy. After diffusion at 650 degrees centigrade, the crystal becomes "p" type with a hole concentration of $9 \times 10^{15}$ cm$^{-3}$ and a hole mobility of 270 cm$^2$/volt-second at 300 degrees centigrade.

**Determination of the Electrical Properties of GaAs**

The evaluation of the electrical properties of gallium arsenide carried out in the manufacturing process is time-consuming and accurate only as concerned with the particular slice of crystal being evaluated. The electrical properties such as resistivity and mobility can usually be determined to within five and seven percent respectively. As an example of what preparation is involved in order to measure Hall effect and resistivity of a sample of a gallium arsenide crystal, consider the following preparations as carried out by the Monsanto Chemical Company (20, p. 28).

A slice of gallium arsenide crystal between one and two millimeters thick is cut with a diamond saw. The
surfaces of the slice are lapped on wet silicon carbide as necessary until the surfaces are smooth and parallel. A rectangular specimen is then cut from this slice and the edges lapped in a manner similar to that used for the other surfaces. Final dimensions of the sample are kept in the ratio $1 : 3 : 9$ for the thickness, width and length respectively, with an average sample being $1 \times 3 \times 9$ millimeters.

In order to prepare the sample for electrical contacts (Indium has been used successfully in making electrical contacts to gallium arsenide), the sample is first etched in a solution of one part fifty percent hydrofluoric acid and two parts distilled water by volume. The sample is etched until the surfaces appear bright and shiny, usually in a few seconds.

The indium solder is applied on a freshly etched sample without the use of flux. The ends of the sample are usually entirely covered with indium and two contacts are applied to both sides of the sample roughly one-third of the distance from the ends.

To insure a bond between the indium and the gallium arsenide which will provide adequate electrical contact, the specimen is subjected to a heat treatment in a hydrogen or argon atmosphere. After placing the sample in a controlled atmosphere furnace and purging the system with hydrogen or argon, the furnace temperature is
raised to 350 degrees centigrade and maintained there for 15 minutes. Finally the furnace is allowed to cool and the specimen removed. Suitable wires are then attached to the indium spots and measurement procedures carried out.

With all the preparation described in the preceding paragraphs it is obvious that a simpler method of electrical property evaluation would save time and allow more measurements to be made during a particular run of crystals and perhaps yield more precise results. One of the microwave methods of electrical property evaluation may be the answer.

The precision in resistivity measurements by the probe methods is limited by the stability of the circuits involved and by the accuracy of the distance measurement between probes. The size of the solder spots is usually such that the distance between probes is uncertain to about five percent. With very small samples the uncertainty of the distance between probes is very much greater than five percent.

When using the four-point probe method, the measurements made of resistivity represent an evaluation of only a small area of the specimen and if the crystal is non-uniform, the quality of the material may change markedly in a relatively short distance.

Although the four-point probe method is used in the
industry, there are disadvantages in using this method as will be shown in following discussions on methods of measuring electrical properties.

METHODS OF RESISTIVITY MEASUREMENT

The most-used method of measuring resistivity of semiconductor materials is by the well-known four-point probe method.

Direct-current methods such as the four-point probe method have several disadvantages. Foremost among these are the influence of thermal emf's and stray pickup. The latter arises primarily because some of the probes have a reverse bias so that partial rectification of stray induced currents, particularly from high frequency sources, can cause quite large spurious voltages.

The thermal effects can be eliminated by reversing the current and taking the average of the two voltage drops found. This procedure also minimizes the error due to pickup, but only careful shielding can eliminate such error.

The problem of probe spacing introduces a source of error when the resistivity of several slices of the same material is being measured, such as for a particular production run of semiconductor material.

Another method of measuring resistivity is by means of the two-point probe method which is generally
considered more accurate than the four-point method but requires careful measurement of the cross-section of the material being investigated. It is not suitable for slice measurements. Although the two-point probe procedure is more accurate in its applications there are some difficulties found in using this method. In a study done on all the problems encountered when using contacts on semiconductor materials it was found that the major difficulty arose because of contact resistances which could be larger than the bulk resistance of the sample being measured (18, p. 2198). Some samples are too small to use the four-point probe techniques and the two-point method is substituted. The contacts on the material may be separated from intimate contact with the surface by barriers consisting of surface states, atmospheric contaminants, worked semiconductor surface, oxide layers, chemical films, etc. All these problems would seem to build a good argument for a reliable microwave technique of measuring semiconductor electrical properties. Since either direct-current method will have at least one probe with a reverse bias applied, the voltage measurement must be made with a low level, high-input impedance instrument. This is usually done with a potentiometer circuit and a chopper amplifier null indicator. The combination of current reversal for the removal of error voltages and the requirement for a
balance adjustment makes these measurements tedious and not particularly suited for high-speed production use.

Contactless Resistivity Meters for Semiconductors

If an alternating-current system were used to eliminate thermal emf's, it would still have the problem of wave form distortion by contact rectification and would be useable only on low resistivity materials such as germanium. Some of these problems are eliminated by an alternating-current, four-point probe resistivity meter described in the following paragraphs (1, p. 824).

The alternating-current resistivity meter described has all probes biased in the forward direction by a small current, thus reducing contact resistance to a very small value. The resistivity can then be measured by a superimposed alternating-current signal. The bias current is small enough so that the area between the probes is not flooded with excess carriers. The peak a-c current cannot be larger than the direct-current bias or rectification would occur during some parts of the a-c cycle.

Another contactless resistivity meter described in the literature makes use of the principle of induced eddy currents in a sample of semiconductor being measured (10, p. 307). The semiconductor sample is exposed to an inductor which produces eddy currents in the
sample. As the inductor is part of a tuned circuit of an oscillator, the degree of eddy current amplitude changes the oscillator's amplitude thus changing the plate current flow in the oscillator circuit. A plot of output voltage of the oscillator circuit versus resistivity that was previously made up for samples by the four-point probe method, is used to compare and evaluate unknown semiconductor samples. This device has been applied successfully to samples of germanium, silicon and gallium arsenide to accuracies of approximately ten percent.

A non-contact resistivity meter that makes use of comparison of two induced voltages between two sets of coils terminating in a differential amplifier and zero center meter circuit is another device being used today (11, p. 472). A slice of semiconductor is inserted between two coils in one leg of the differential amplifier circuit and the other set of coils' induced voltage is attenuated until a balance is had with the leg containing the slice of semiconductor. The loss in signal caused by inserting the semiconductor slice or any conducting sheet was evaluated by Maxwell in 1872 using image methods. By suitable electrical expressions the resistivity can be related to frequency from

\[ \rho = kft \]

where "f" is the frequency, \( t \) is the sample thickness,
and \( k \) is a constant equal to \( \frac{2\pi^2d}{3 \times 10^9 G} \)

where "d" is the distance separating the coils and "G" is a constant relating coil radius to coil separation.

MICROWAVE METHODS OF MEASURING RESISTIVITY

There are various methods of utilizing the techniques of microwave transmission through semiconductor material to evaluate resistivity, dielectric constant, surface recombination, lifetime and Hall-effect. Much work has been done investigating Hall-effects at microwave frequencies but as this paper is mainly concerned with resistivity measurements no further mention will be made of Hall-effect studies (5, p. 231), (6, p. 2177), (7, p. 1073), (8, p. 286), (29, p. 790), (31, p. 27).

Some of the methods of semiconductor electrical property evaluation investigated by workers in the field include the following:

1. Transmission through a slice of semiconductor material using a distributed line approach.

2. Standing wave measurement of microwave energy impinging on a slice of semiconductor terminating a waveguide.

3. Resonant cavity methods where the semiconductor material becomes part of the resonant cavity (22, p. 131).

4. Comparison methods.

In the following paragraphs will be found a brief
discussion of the various methods of measuring resistivity. The methods used to determine resistivity can also be used to evaluate bulk lifetime (14, p. 1604), (25, p. 229), (26, p. 124), (30, p. 1054). In conjunction with conductivity measurements, permittivity can also be evaluated with the same configuration of test equipment (28, p. 100).

Transmission of Microwave Energy Through a Slice of Semiconductor

This method of measuring resistivity utilizes the analogy of a transmission line with distributed constants where the waveguide acts as a distributed line. Figure 1 below shows the insertion of the semiconductor slab in the waveguide and the parameters to be evaluated are also shown (24, p. 928).

The slice of semiconductor material is inserted in the waveguide transversally with the large surface oriented perpendicularly to the direction of propagation of the microwave energy.

As the power transmitted through the semiconductor slice can be measured and the power on the generator side of the slice known, the theory applying in this case will involve the ratio $E_o/E_{in}$, where $E_o$ is the electric field out of the waveguide after transmission through the semiconductor slab and $E_{in}$ is the incident field on the
Figure 1

Equivalent Circuit for a Slab of Semiconductor in a Waveguide

Waveguide wall

Air filled waveguide

Gallium arsenide plug

Incident Wave

Waveguide side view

Electrical Parameters

Transmission line analog
generator side of the slab. The ratio of $E_0/E_{in}$ is given by equation 1 below and is derived in Appendix I (22, p. 932).

$$\frac{E_0}{E_{in}} = r_t \left( \cosh K_2 l_2 - \frac{Z_{02}}{Z_{ab}} \sinh K_2 l_2 \right) \quad \text{eq (1)}$$

where

$$r_t = \frac{2Z_{ab}}{Z_{ab} + Z_{01}} \quad \text{eq (2)}$$

$$Z_{ab} = \frac{Z_{02}}{Z_{01} + \tanh K_2 l_2} \left( \frac{Z_{01} + Z_{02} \tanh K_2 l_2}{Z_{01} + Z_{02} \tanh K_2 l_2} \right) \quad \text{eq (3)}$$

The values in equations 1 - 3 are as follows:

- $Z_{01}$ = the impedance of the waveguide in air and is equal to $Z_{03}$
- $Z_{02}$ = the impedance of the waveguide filled with gallium arsenide or other material being investigated
- $Z_{ab}$ = the impedance of the material at the surface looking toward the generator
- $K_1$ = the propagation constant in air in the waveguide
- $K_2$ = the propagation constant in the semiconductor material in the waveguide
- $l_2$ = the thickness of the semiconductor slice in meters

**SWR Method of Measuring Semiconductor Resistivity**

In the discussion on the previous pages it is seen that the microwave energy is transmitted through a slice of semiconductor and the ratio of transmitted to impinging power noted. The method described in the following paragraphs makes use of the reflection of energy from a semiconductor terminating a waveguide (27, p. 377).
The slice of semiconductor is sandwiched between the end of the waveguide and a metal backing. Figure 2 below shows the physical arrangement.

Figure 2

Semiconductor With Low-Resistivity, Metal Plate Backing

<table>
<thead>
<tr>
<th>Waveguide</th>
<th>Crystal with</th>
<th>Low-resistivity reflecting metal plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident wave</td>
<td>Low-resistivity reflecting metal plate</td>
<td></td>
</tr>
<tr>
<td>reflected wave</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

Waves propagating in a homogeneous semiconducting medium in one dimension are described by

\[
\frac{\partial^2 E_x(z)}{\partial z^2} = \frac{\mu_0}{\varepsilon_0} \frac{\partial E_x(z)}{\partial t} + \varepsilon_0 \varepsilon_r \frac{\partial^2 E_x(z)}{\partial t^2} \quad \text{eq (1)}
\]

which is obtained directly from Maxwell's equations.

The electromagnetic wave propagates in the z-direction with an electric field component $E_x$ and a magnetic field component $H_y$. In the above equation

- $\mu_0$ = permeability of free space; the relative permeability is assumed to be unity
- $\rho$ = resistivity of the semiconductor
- $\varepsilon_0$ = permittivity of free space
- $\varepsilon_r$ = relative dielectric constant

As a sinusoidal field is being propagated in the "z" direction in the waveguide, the field can be assumed to be periodic everywhere in the guide, and
\( E_x(z,t) = E(z) \exp(j\omega t) \). Inserting this expression into equation 1 gives,

\[
\frac{\partial^2 E}{\partial z^2} = \left( \frac{u_0}{j\omega \varepsilon} - \frac{\omega^2 \varepsilon_0 \mu_0}{\varepsilon} \right) E = a^2 E \quad \text{eq (2)}
\]

The solution of the above expression takes the form

\[
E(z) = E_0 e^{-az} + E_{02} e^{az} \quad \text{eq (3)}
\]

The constants in the above equation are determined by the physical arrangement of the semiconductor in the waveguide as shown in Figure 2 on the previous page.

As shown in Figure 2, the waveguide is terminated by a semiconductor which is backed by a low resistivity metal. The thickness of the slice of semiconductor is much greater than the depth of penetration of the microwaves into the low resistivity backing metal. For a backing of copper the equivalent depth of penetration is given by,

\[
S = 0.0664 \sqrt{\frac{f}{f}}
\]

and for a frequency of 7500 megacycles used in measurements for this paper the equivalent depth of penetration would be approximately \( 0.8 \times 10^{-6} \) meters.

The electric field at \( z = d \) will then be zero, \( E(d) = 0 \). Applying this boundary condition to equation 3, the electric field is found in terms of \( E_{01} \). As all components of the transmitted field into the semiconductor would have to be evaluated, it is found to be
simpler to find the sum of the reflected waves. The standing-wave ratio in the waveguide is given by

\[
\text{SWR} = \frac{1 + \left| \frac{E_r}{E_{i}} \right|}{1 - \left| \frac{E_r}{E_{i}} \right|} \quad \text{eq (4)}
\]

where \( E_r \) is the total reflected wave and \( E_i \) is the incident wave.

The reflected wave components can be collected by referring to Figure 3 below.

Figure 3
Reflected Waves of Semiconductor With Metal Backing in a Waveguide

In the above figure of reflected field components the reflection coefficient at \( z = 0 \) for waves moving to the right is \( K \), and to the left is \(-K\). At \( z = d \) the reflection coefficient is \(-1\). At \( z = 0 \) then

\[
E_{r1} = KE_i \quad \text{eq (5)}.
\]

The electric field is continuous at \( z = 0 \) and the incident plus the reflected wave equals the transmitted wave. The transmission coefficient is then \( 1+K \) and the first transmitted wave is given by
This wave propagates to \( z = d \) where it is a factor \( \exp(-ad) \) smaller, according to equation (2). Since the field must be zero at this point, an equal but opposite wave must start in the reverse direction. When this wave arrives at \( z = 0 \), its magnitude has also been reduced by a factor \( \exp(-ad) \). The reflected wave that arrives at \( z = 0 \) is given by

\[
E'_{r1} = -E_{t1} e^{-2ad} = -E_{t1} (1+K)e^{-2ad}.
\]

The reflection coefficient for the return wave \( E'_{r1} \) is \(-K\) and

\[
E_{t2} = E_{i}K(1+K)e^{-2ad},
\]

eq (8)

and the second reflection component is then

\[
E_{r2} = -E_{t}(1 - K)(1 + K)e^{-2ad}.
\]

eq (9)

Repeating the above cycle gives the recursion formula

\[
\frac{E_{r}}{E_{i}} = \frac{\sum_{n=1}^{\infty} E_{rn}}{E_{i}} = \frac{K - K^2}{K} \sum_{n=1}^{\infty} e^{-2adn}K^n
\]

\[
= K - \frac{K^2}{K} \left[ \sum_{n=0}^{\infty} (Ke^{-2ad})^n -1 \right].
\]

eq (10)

The summation is a power series and as the absolute value of the argument is less than 1, equation (10) can be written

\[
\sum_{n=0}^{\infty} (Ke^{-2ad})^n = \frac{1}{1-Ke^{-2ad}}.
\]

eq (11)

Using equation (11) the ratio \( E_{r}/E_{i} \) becomes

\[
\frac{E_{r}}{E_{i}} = \frac{K - e^{-2ad}}{1 - Ke^{-2ad}}.
\]

eq (12)
Equation (12) can be regarded as the effective reflection coefficient. In equation (12) the factor $K$ is given by

$$K = \frac{Z_{SC} - Z_g}{Z_{SC} + Z_g} \quad \text{eq (13)}$$

where $Z_{SC}$ and $Z_g$ are the characteristic impedances of the semiconductor and waveguide, respectively. The impedance for a material with resistivity $\rho$ and dielectric constant $\varepsilon_r$ is given by

$$Z_{SC} = \left( \frac{j\omega \rho}{1 + j\omega \varepsilon_r \rho} \right)^{1/2} \quad \text{eq (14)}.$$

The impedance of free space follows from equation (14) if $\varepsilon_r$ is equal to one and $\rho \to \infty$. The characteristic impedance of free space is then $(\omega / \varepsilon)^{1/2}$. The impedance of the waveguide is always larger than that of free space and is

$$Z_g = b \left( \frac{\omega}{\varepsilon} \right)^{1/2} \quad \text{eq (15)}$$

where $b$ is determined by the guide geometry and mode of operation and is always greater than 1.

Combining equations (12), (13), (14), (15), and the definition of "a" from (2), the final form for the standing-wave ratio becomes

$$\text{SWR} = \frac{1 + \left| \frac{[1-bA]/(1+bA)}{1-[(1-bA)/(1+bA)]} \exp(-j\pi 2A/c) \right|}{1 - \left| \frac{[1-bA]/(1+bA)}{1-[(1-bA)/(1+bA)]} \exp(-j\pi 2A/c) \right|}$$

where $c = (\varepsilon_0 \omega)^{1/2}$ is the speed of light, and $A =$
\[ \left( \frac{1}{e_{\tau} - j\omega e_{\circ}p} \right)^{1/2}. \]

In the above equation the mode of operation is the \( \text{TE}_{10} \) and the constant \( "b" = \)

\[ \frac{1}{\left(1 - \frac{f_{h}^2}{f^2}\right)^{1/2}} = \frac{1}{\left(1 - \frac{c^2}{4f^2 h^2}\right)^{1/2}} \]

where \( f_{h} \) is the frequency of cutoff of the guide used and "h" is the wide dimension of the guide.

A less involved method of applying the SWR method of determining resistivity is described in an article by J. N. Bhar (4, p. 1623).

**Reflection Coefficient Method of Measuring Resistivity**

The volume resistivity of a semiconductor sample can be measured by a method utilizing the reflection coefficient of microwave energy from a sample inserted in a waveguide (2, p. 176).

This method involves placing the semiconductor sample across the open end of a waveguide, thus terminating the waveguide. The microwave signal generator supplies energy to the waveguide sending end through directional couplers that allow separated paths for transmitted and reflected waves. The incident wave detector's signal is compared with the reflected waves' detected signal and by means of attenuators, adjusted until the two signals are of equal amplitude. The amount of attenuation inserted to bring equality to the two signals is a measure of the reflection coefficient.
Return loss decibels = 20 log(reflection coefficient).

A plot can be made of known resistivities of semiconductor samples versus return loss decibels and the curves then used to determine the resistivities of unknown samples. This is strictly a comparison method and would be applicable to a particular run of semiconductor crystals after resistivities in a particular range had been determined and plotted using the four-point probe method to determine resistivities.

DISCUSSION OF RELATIVE MERITS OF VARIOUS MICROWAVE METHODS OF RESISTIVITY MEASUREMENT

Transmission of Microwave Energy Through a Thin Slice of Semiconductor Material

This method utilizes the analogy of a transmission line with distributed constants taking the place of the waveguide in which is inserted the slice of semiconductor. The slice of semiconductor completely fills a cross-section of the guide.

One of the limiting factors of this method is to be found in obtaining a slice of semiconductor large enough to fill completely the waveguide on hand. The necessity of shaping the semiconductor slice to the dimensions of the cross-section of waveguide also presents a problem when brittle materials are being investigated. These
factors would present few problems in the industry where facilities are available to shape the semiconductor slices and to utilize higher microwave frequencies to allow reduction in the size of the slices.

Detection of small differences in magnitudes of transmitted microwave energy through the semiconductor slice is a problem that makes the resistivity evaluation difficult for materials having high resistivity values. A power ratio $\frac{E_o^2}{E_{in}}$ of approximately 50 db can be detected with commercially available test equipment but for ratios of more than 50 db more elaborate test facilities are needed.

The range of attenuation measurement thus depends on the sensitivity of the detector. A video (square law) detector such as a crystal is able to detect radio frequency power down to about a -50 dbm. If a microwave superheterodyne receiver is used to detect the energy transmitted through the semiconductor slice under test, the attenuation can be measured to approximately a -90 dbm. This would be the attenuation found for a semiconductor resistivity of approximately 10 ohm-cm.

Even though problems in attenuation measurement may appear, this method of resistivity evaluation seems to be most adaptable to a quick and fairly accurate determination of semiconductor resistivity.
SWR Method of Measuring Semiconductor Resistivity

The SWR method of resistivity evaluation is most suited for semiconductors of high resistivity as the difficulty in detecting standing wave ratio increases with decreasing resistivity samples where the SWR becomes too high.

This method has been applied successfully to semiconductors having resistivities down to 10 ohm-cm. Attempts were made to measure resistivity by this method of semiconductor samples with a resistivity of 2 and 1 ohm-cm but without noticeable success.

The availability of a computer is another factor to be considered in using this method as the computations used in evaluating resistivity are quite lengthy.

Added to the above limitations is another factor to consider that might lead to erroneous results if its existence were not known. This error might be introduced in the measuring of the SWR if the tuning of the slotted line probe was not carried out in a precise manner.

It is the usual procedure to tune a slotted line probe by first finding the maximum energy point in the line and tuning the probe at this position. An analysis of this procedure has shown that it will lead to distortion of the detected standing wave pattern (13, p. 788).

A more accurate procedure of tuning the probe is
as follows:

1. Always use a short-circuit termination of the slotted line.
2. Locate two consecutive nulls.
3. Move the probe to the center point between the two nulls.
4. Tune the probe for a maximum output at this point.

Reflection Coefficient Method of Measuring Resistivity

In the previous discussion of the reflection coefficient method of measuring resistivity it was stated that this method was strictly a comparison method. This is the case if resistivity values have been previously determined and further samples are to be evaluated by a comparison of reflection coefficients of the known resistivities with the unknown sample resistivities.

The reflection coefficient method can also be used by determining the reflection coefficient of an unknown sample resistivity and computing the conductivity from the following relations:

\[ R = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \]  

where \( Z_{01} = \frac{j\omega}{K_1} \) and \( Z_{02} = \frac{j\omega}{K_2} \) are the waveguide impedance, waveguide propagation constant, semiconductor impedance, and semiconductor propagation constant respectively.
where $\lambda_1$ is the wavelength of the microwave signal impinging on the semiconductor slab inserted transversally in a waveguide and $\lambda_2$ is the wavelength of the signal in a lossless semiconductor. The cutoff wavelength of the guide is $\lambda_0$. The term $\lambda_2 = \frac{\lambda_1}{\sqrt{\varepsilon_r}}$.

The accuracy of this method depends on the waveguide circuitry, a two-way directional coupler and a ratio meter forming a reflectometer, and the accuracy of the reflectivity coefficient determined by the reflection coefficient meter.

Some of the disadvantages of this method are:

1. The sample has to be at least as large as the waveguide opening. At frequencies below 10,000 megacycles this sample size stipulation might cause some difficulties.

2. The return losses are low for resistivities less than 0.05 ohm-cm and measurements would not be very accurate.

3. Measurements on materials having resistivities above 50 ohm-cm would also be inaccurate as it is difficult to distinguish between resistivities in that range.
A particular disadvantage is that the spread between low and high resistivities in the range suitable for this method is on the order of only four decibels when calculating return loss by $20 \log(\text{reflection coefficient})$. This would indicate that it would be difficult to get an accurate indication of the reflection coefficient for small ranges of resistivity values.

Resonant Cavity Method

The method of determining semiconductor resistivity by making the slab of semiconductor to be evaluated a part of a resonant cavity and determining its effect on the "Q" and resonant frequency of the cavity requires a knowledge of the relations between the cavity "Q" and resonant frequency with and without the semiconductor insertion. These relations are quite involved and would entail much more effort than that required by some of the other methods discussed. This method is not suited for resistivities outside the range of 1.0 to 10 ohm-cm as less measurement precision results from this method compared to other methods.
RESISTIVITY MEASUREMENT PROCEDURE FOR TRANSMISSION
OF MICROWAVE ENERGY THROUGH A SEMICONDUCTOR SLICE

In making the resistivity measurements on the slices of gallium arsenide for this investigation, the source of microwave power was obtained from a Hewlett-Packard Dy-5636 H-Band test set.

The microwave signal path from the klystron oscillator in the test set to the slice of gallium arsenide mounted in the waveguide can be followed by referring to Figure 4, page 30. It is seen that there is incorporated in the test set an 18 db ferrite isolator which isolates the klystron from any reflections caused by terminating the test set output in anything but a matched load, thus insuring that the klystron power output remains constant with the semiconductor slice in and out of the waveguide.

The microwave energy from the test set emanated from an open waveguide and it was at this point the slice of semiconductor was placed in the waveguide transversally to the direction of propagation of the microwave energy. The energy transmitted through the slice was transferred to a coaxial cavity wavemeter by means of a waveguide to coaxial coupler, and detected in the cavity by a 1N23 diode. The rectified direct-current flowing through a resistor furnished a potential that was applied to a potentiometer.
Figure 4
Experimental Circuit Block Diagram

- Klystron Oscillator
- 18 db Ferrite Isolator
- Power Set Attenuator
- 100 db Attenuator

- Frequency Meter
- Detector

- RF Output
- Waveguide to Coax Coupler
- Potentiometer
- Absorption Cavity Wavemeter
A frequency of 7500 megacycles was used and to insure that the slice of gallium arsenide was being viewed homogeneously, a calculation of the depth of penetration was made as shown in Appendix IV. The frequency of 7500 megacycles was chosen to make some of the calculations simpler, as the wavelength at this frequency is 0.04 meters, and also the limit of the coaxial cavity wavemeter used for a power detector had an upper frequency limit of 7500 megacycles.

The attenuator of the test set was set to zero decibels and the semiconductor slice inserted in the waveguide. The potentiometer was balanced and the slice then removed from the waveguide. The attenuator of the test set was then set to a decibel value that allowed the potentiometer to be balanced at the same potential as before when the slice was in the waveguide. The attenuation of the semiconductor slice was the difference in the two attenuator settings in decibels.

This method, known as the radio frequency substitution method, depends on substituting an rf attenuator of known characteristics for an unknown attenuation. As the detector is always operated at the same level, detector characteristics and associated errors introduced by operating outside the detector's square law range is no problem.
EXPERIMENTAL RESULTS

Numerous measurements were taken of the attenuation through various gallium arsenide slices, and when the problems of standing waves in the waveguide on the detector side of the slices were eliminated and instrumentation found to measure accurately a few microamperes with repeatability, the results were positive.

The measurements were taken at room temperature, 290° Kelvin, and the semiconductor slices kept as close to this temperature as possible to eliminate the possibility of an increase in transmitted power due to heating of the gallium arsenide slices.

The frequency of the test set remained constant, as was indicated by a double check on its accuracy from the test set frequency meter and the coaxial cavity absorption wavemeter and detector.

The attenuator of the test set was calibrated at several points to insure that it was accurate in the ranges required.

Measurements of attenuation fell in the ranges shown in Figures 5 and 6, indicating that the microwave method is feasible for resistivity evaluation of both polycrystalline and single crystal varieties of gallium arsenide as long as an accurate measurement of attenuation can be obtained. The test set used in this investigation had
Figure 5

Attenuation of 7500 Megacycles versus Resistivity of Gallium Arsenide
Slices Inserted Transversally in a Waveguide - Sample 1

Average Value and Range of 21 Attenuation Measurements of Sample No. 1

- 50 mil thickness
- 20 mil
- 10 mil
Figure 6

Attenuation at 7500 Megacycles versus
Resistivity of Gallium Arsenide Slices
Inserted Transversally in a Waveguide -
Samples 2, 3 and 4

- Sample 2
- Sample 3
- Sample 4

Average and range of attenuation for 4 measurements -
Sample 3

Average and range for 11 measurements of attenuation -
Sample 2

Average and range for 11 measurements of attenuation -
Sample 4

Attenuation in Decibels

Resistivity in Ohm-Centimeters
an attenuator calibrated so as to make decibel attenuation readings accurate to 0.1 decibels, which corresponds to 1.8% variation in resistivity. As the coaxial cavity wavemeter was tuned for a maximum indication of detected power and the potentiometer balanced for each reading, the accuracy of the decibel attenuation was given a double-check.

The results obtained, and the repeatability of the measurements within a reasonable range of the sample's resistivity, indicate that there should be no insurmountable problems involved in resistivity measurement of any sample having attenuations up to about fifty decibels, the limit of the crystal detector and potentiometer.

The curves in Figure 5 for the 20 mil and 10 mil slices were calculated to show the relative ranges of attenuation and resistivity values to be expected for other slice thicknesses of gallium arsenide. Measurements were made on four slices of gallium arsenide; two single-crystal N-type slices, one N-type polycrystalline slice, and a single-crystal P-type slice. The results of these measurements are shown as points in Figures 5 and 6.

The average value of attenuation found for each sample, except sample 3, corresponded to a resistivity that was within plus or minus five percent of the correct value. The fact that sample 3 did not yield as
accurate a result as the other samples was due to the limited number of measurements taken before the slice was accidentally shattered.

The experimental data indicate that the range of resistivities that can be determined by the microwave method used is from 0.0001 to 10 ohm-cm for gallium arsenide. The limitation at the higher resistivities is due to lack of measurable attenuation, and at the lower ranges of resistivity, attenuation values are too high to be measured by a crystal detector.

CONCLUSIONS

It has already been proven by many workers in the field of microwave semiconductor research that the microwave method of resistivity evaluation is a practical and reliable means of resistivity measurement for silicon and germanium.

The results of this investigation have indicated that the microwave method can also be utilized for gallium arsenide.

Depending on the requirements of resistivity evaluation for a particular situation and the availability of test equipment to fulfill the requirements, the method used for this paper or one of the other microwave methods described should be able to cover resistivity measurements over a considerable range.
Production line testing of semiconductor samples would be speeded up considerably by this method over the four-point probe method and accuracy and repeatability would be more accurate than that obtained by the four-point probe method once the microwave equipment had been set up and calibrated, especially if calibrated on the basis of known resistivity samples for a particular production run.

Although the cost of microwave equipment is much greater than that needed for the conventional four-point probe method, the saving in labor costs would more than offset the difference, particularly if many measurements were required.

The microwave technique would also be applicable to those materials where the problem of ohmic contacts arose. Some of the newer materials, such as gallium arsenide, exhibit rectification at the contact probes at low temperatures. To evaluate the resistivity at low temperatures by conventional means would be difficult if not impossible.
BIBLIOGRAPHY


Appendix I

Microwave Transmission Through a Semiconductor Slice;
Transmission Line analogy

\[ \frac{E_0}{E_{\text{in}}} = r_t \left( \cosh K_2 l_2 - \frac{Z_{02}}{Z_{ab}} \sinh K_2 l_2 \right) \] (1)

and

\[ r_t = \frac{2Z_{ab}}{Z_{ab} + Z_{01}} \] (2)

where

\[ Z_{ab} = Z_{02} \left( \frac{Z_{01} + Z_{02} \tanh K_2 l_2}{Z_{02} + Z_{01} \tanh K_2 l_2} \right) \] (3)

In the above equations (1)-(3)

\[ Z_{01} \] = the impedance of the waveguide in air and is equal to \( Z_{03} \).

\[ Z_{02} \] = the impedance of the waveguide filled with the semiconductor material and is a function of conductivity and wavelength.

\[ Z_{ab} \] = the impedance of the semiconductor slab at the front surface on the generator side.

\[ K_1 \] = the propagation constant in air in the waveguide.

\[ K_2 \] = the propagation constant in the semiconductor material in the waveguide.

\[ l_2 \] = the thickness of the slab of semiconductor.

In equation (3) above the impedance of the line ab in Figure 1, page 15 to the left is given by \( Z_{ab} \). The field just adjacent to ab and \( Z_{02} \) is designated \( E_T \).

\( E_{\text{in}} \) is the incident field next to ab and \( Z_{01} \), and \( r_t \) is
the ratio of the two fields. At the line cd the impedance is \( Z_{01} \) and at ab by transmission line theory,

\[
E_T = E_0 \left( \cosh K_{21}l_2 + \frac{Z_{02}}{Z_{01}} \sinh K_{21}l_2 \right) \tag{4}
\]

At the line cd going in the reverse direction,

\[
E_0 = E_T \left( \cosh K_{21}l_2 - \frac{Z_{02}}{Z_{ab}} \sinh K_{21}l_2 \right) \tag{5}
\]

Substituting equation \( E_T = E_{in} r_t \) into equation (5) gives

\[
\frac{E_0}{E_{in}} = r_t \left( \cosh K_{21}l_2 - \frac{Z_{02}}{Z_{ab}} \sinh K_{21}l_2 \right) \tag{6}
\]

The term \( r_t \) can be obtained by analogous procedures. For a connection between two dissimilar lines,

\[
r_t = \frac{2Z_{02}}{Z_{01}+Z_{02}} \tag{7}
\]

In the three dissimilar line case pertaining to the waveguide line, in place of \( Z_{02} \) use \( Z_{ab} \), and on substituting

\[
r_t = \frac{2Z_{ab}}{Z_{01}+Z_{ab}} \tag{8}
\]

In appendix II further development of the equation (6) will be found.
Appendix II

Relation of Decibel Attenuation to Conductivity for Microwave Transmission Through a Semiconductor Slice

The equation for the field ratio pertaining to the waveguide of Figure 1, page 15 is given by equation (6) of the previous page. Combining equations 3, 6, and 7 from the previous page give,

\[
\frac{E_0}{E_{in}} = \frac{1}{\cosh K_2 l_2 + \frac{1}{2} \left( \frac{Z}{Z_0 + Z_{02}} \right) \sinh K_2 l_2}
\]

where \( Z \) is the impedance of the waveguide in air.

Substituting \( K_2 = \frac{a}{\lambda} + j \beta \) in equation (1) the ratio of transmitted to incident power is obtained.

\[
\left| \frac{E_0}{E_{in}} \right|^2 = \left\{ \frac{\cosh \alpha \cos \beta l + A_1 \sinh \alpha \cos \beta l - A_2 \cosh \alpha \sin \beta l}{A_2 \cosh \alpha \sin \beta l} \right\}^2
\]

where \( A_1 = \frac{X(x^2 + y^2 + z^2)}{2Z(x^2 + y^2)} \) and \( A_2 = \frac{Y(x^2 + y^2 - z^2)}{2Z(x^2 + y^2)} \)

and \( \alpha = \frac{1}{\sqrt{2}} \left\{ + \left( \frac{\pi}{a} \right)^2 - \omega^2 \mu \epsilon + \left[ \left( \omega^2 \mu \epsilon - \frac{\pi}{a} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \)

\( \beta = \frac{1}{\sqrt{2}} \left\{ - \left( \frac{\pi}{a} \right)^2 + \omega^2 \mu \epsilon + \left[ \left( \omega^2 \mu \epsilon - \frac{\pi}{a} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \)

and \( X = \frac{\omega \mu B}{\lambda^2 + \beta^2}, \quad Y = \frac{\omega \mu a}{\lambda^2 + \beta^2} \)
It is assumed that \( u = u_0 = 4 \pi \times 10^{-7} \) henry per meter and \( \varepsilon = \varepsilon_r \varepsilon_0 = 11.1 \ (8.854 \times 10^{-12}) \) farads per meter. It has been found in previous studies that the relationship \( \frac{E_0}{E_{in}} = 2 \) and \( \sigma \) can be considered to vary linearly (23, p. 583).

Assumptions that can be made when "L", the thickness of the slice of semiconductor whose resistivity is being measured, is a very small fraction of a wavelength as is the case in this investigation where "L" is a small fraction of the wavelength of a frequency of 7500 megacycles are given below.

Wavelength in the sample of gallium arsenide is denoted by the symbol \( \lambda_m \) and is equal to

\[
\lambda_m = \frac{\lambda_o}{\sqrt{\mu_r \varepsilon_r}} \quad \text{where} \quad \lambda_o = \text{wavelength in air},
\]

\( \mu_r = \text{relative permittivity of a medium and is equal to} \),

\( \frac{\lambda}{\lambda_o} = 1. \) The term \( \varepsilon_r \) is equal to \( \frac{\varepsilon}{\varepsilon_o} = 11.1 \) for gallium arsenide.

When \( BL \ll 1 \) and \( L \) is a small fraction of a wavelength, the following approximations can be used:

\( \cos BL = \cosh \alpha L = 1, \sin BL = BL, \) and \( \sinh \alpha L = \alpha L. \) (Eq. 1)

Substituting equation 1 into equation 6 of page 42 gives,
\[
\left| \frac{E_o}{E_{in}} \right|^2 \ll \lambda \quad \text{where} 
\]
\[
\left| \frac{E_o}{E_{in}} \right|^2 = \frac{1}{\left[ 1 + (BL)^2 \left( \frac{\lambda_g}{\lambda} + \frac{\lambda g}{\lambda} \right) \right]^2} 
\]

(Eq. 3)

or using \( A_1 = \frac{1}{2} \left( \frac{\lambda}{\lambda_g} + \frac{\lambda_g}{\lambda} \right) \) and \( A_2 = \frac{\lambda}{2B} \left( \frac{\lambda}{\lambda_g} - \frac{\lambda_g}{\lambda} \right) \) where \( \lambda_g \) is the guide wavelength in an air-filled guide and \( \lambda \) is the wavelength in an infinite lossless semiconductor.

Equation 3 above can then be written,

\[
\left| \frac{E_o}{E_{in}} \right|^2 \ll \lambda 
\]

(Eq. 4)

The term \( \left| \frac{E_o}{E_{in}} \right|^2 \) in equation (4) can be found from the relationship, \( 10 \log_{10} \left| \frac{E_o}{E_{in}} \right|^2 = \text{db of attenuation} \) where the decibels of attenuation refer to the attenuation of the slice of gallium arsenide inserted in the waveguide.
Appendix III

Electrical and Mechanical Characteristics of
Germanium, Silicon and Gallium Arsenide

<table>
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Appendix IV

Resistivity Calculations for a Fifty Mil Slice of "N" Type Polycrystalline Gallium Arsenide

When the thickness of the semiconductor slice is a small fraction of a wavelength of the frequency being used to determine resistivity, the following equation can be utilized.

\[
\left( \frac{E_0}{E_{in}} \right)^2 = \frac{1}{\left[ \frac{1}{1 + (BLK_1)^2} \right]} \times \frac{1}{\left[ 1 + \frac{2BL(K_1 \frac{\phi}{B} - K_2)}{1 + (BLK_1)^2} \right]} \quad \text{eq (1)}
\]

The definitions of the terms in the above equation are as follows:

\[
\left( \frac{E_0}{E_{in}} \right)^2 = \text{ratio of power transmitted through the slice of gallium arsenide to the power incident on the slice on the generator side of the slice}
\]

\[
B = \omega \left( \mu_0 \varepsilon \right)^\frac{1}{2} \text{ where } \omega = 2\pi (7500 \times 10^6) \text{ where "f" = 7500 mc}
\]

\[
\phi = 4\pi \times 10^{-7} \text{ henrys per meter and } \varepsilon = \varepsilon_0 \varepsilon_r
\]

\[
\varepsilon = (8.854 \times 10^{-12}) \times (11.1) = 98.279 \times 10^{-12}
\]

\[
B = 523.547
\]

\[
BL = (523.547) (0.00127) \text{ where "L"= 0.00127 meters = 50 mil slice of gallium arsenide}
\]

\[
= 0.6649
\]

\[
K_1 = \frac{1}{2} \left( \frac{\lambda}{\lambda g} + \frac{\lambda g}{\lambda} \right) \text{ and } K_2 = \frac{\phi}{2B} \left( \frac{\lambda}{\lambda g} - \frac{\lambda g}{\lambda} \right)
\]

also \( \lambda = \text{wavelength in an infinite lossless semiconductor} \)

\[
\lambda = \frac{\lambda_o}{\left( \mu_r \varepsilon_r \right)^\frac{1}{2}} \text{ where } \lambda_o = \text{wavelength in free space}
\]

\[
= 0.04 \text{ meters for 7500 mc.}
\]
then \( \lambda_g = \frac{0.04}{\sqrt{11.1}} = 0.012 \) meters

\( \lambda_g = \) wavelength in the waveguide

\[ \frac{\lambda_c}{\lambda_0} \left( 1 - \left( \frac{\lambda_0}{\lambda_c} \right)^2 \right)^{1/2} \]

where \( \lambda_c = \) cutoff wavelength of waveguide being used = 0.057 meters

\( \lambda_g = \frac{0.04}{\sqrt{11.1 - \left( \frac{0.04}{0.057} \right)^2}} = 0.0123 \) meters

\( \lambda = \frac{0.012}{0.0123} = 0.976 \) and \( \lambda_g = \frac{0.0123}{0.012} = 1.025 \)

then \( \frac{\lambda}{\lambda_g} = 1.025 + 0.976 = 2.0 \)

also \( K_1 = \frac{1}{2} \left( \frac{A}{\lambda} + \frac{\lambda_g}{\lambda} \right) = 1.0 \)

\[ K_2 = \frac{1}{2} \left( \frac{\sigma}{2\omega\varepsilon} \right) \left( \frac{\lambda}{\lambda_g} - \frac{\lambda_g}{\lambda} \right) \]

where \( \omega = 4.63 \)

\[ K_2 = \frac{\sigma}{18.52} \left( -0.05 \right) = -0.0027 \sigma \]

Inserting numerical values in equation (1) gives

the result:

\[ \left( BK_1 \right)^2 = (0.6649)^2 = 0.4421 \]

and \( 2BL(B_1 - K_2) = 1.3298 \left[ \frac{\sigma}{9.26} + 0.0027 \sigma \right] = 0.1472 \sigma \)

where \( \frac{\sigma}{B} = \frac{\sigma}{2\omega\varepsilon} \)

then \( \frac{E_0}{E_{in}} \left( \frac{1}{1.4421 + 0.1472 \sigma} \right) \)
as \(10 \log \left| \frac{E_o}{E_{in}} \right|^2 = -\text{db}\) it follows that

\[
\text{db} = -10 \log \frac{1}{c + K\sigma}
\]

where \(c = 1 + (BLK_1)^2\)

and \(K = 2BL(K_1\sigma - K_2)\).

As \(\sigma\) for the gallium arsenide used in this investigation is 909.09 where 909.09 is the reciprocal of the resistivity 0.11 ohm-cm, the power ratio \(\left| \frac{E_o}{E_{in}} \right|^2\) should be equal to 0.00740 and putting the power ratio in terms of decibels attenuation measured through the slice of gallium arsenide gives:

\[
10 \log \left| \frac{E_o}{E_{in}} \right|^2 = \text{decibels attenuation}
\]

and \(\log \left| \frac{E_o}{E_{in}} \right|^2 = \log 0.00740\)

\[
= 7.86923-10 \text{ or } 10.00000-10
\]

\[
= \frac{7.86923-10}{2.13077}
\]

\[
= 10 (2.13) = 21.3 \text{ db attenuation.}
\]

A simpler form that could be used to calculate \(\sigma\) could be found from the following:

As \(10 \log \left| \frac{E_o}{E_{in}} \right|^2 = \text{db}\) it follows that

\[
\text{db} = -10 \log \frac{1}{c + K\sigma}
\]

where \(c = 1 + (BLK_1)^2\)

and \(K = 2BL(K_1\sigma - K_2)\)

then \(\sigma = \frac{K}{\frac{\text{db}}{10} - c}\)
Decibel Attenuation at 7500 Megacycles for Samples of Gallium Arsenide, Measured and Calculated Values for Various Resistivities

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>1</th>
<th>2</th>
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<th>4</th>
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<tr>
<td>Type</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>P</td>
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<tr>
<td>Resistivity in ohm-cm</td>
<td>0.11</td>
<td>0.088</td>
<td>0.014</td>
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<td>Thickness in mils</td>
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<td>Attenuation db(calculated)</td>
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<td>Attenuation db(measured)</td>
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<td>19.40</td>
<td>23.3</td>
<td>5.4</td>
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<tr>
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<td>21.8</td>
<td>19.00</td>
<td>24.7</td>
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<td>21.75</td>
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<td>20.80</td>
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<tr>
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<td>Attenuation Sample Standard Deviation &quot;s&quot;</td>
<td>0.304</td>
<td>0.333</td>
<td>1.21</td>
<td>0.56</td>
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Depth of Penetration of a Microwave Signal into a Fifty Mil Slice of "N" Type, Polycrystalline Gallium Arsenide for a Frequency of 7500 Megacycles

\[
\delta = \frac{1}{\sqrt{\frac{\mu \varepsilon}{2} \left( \sqrt{1 + \frac{\delta^2}{\omega^2 \varepsilon^2}} - 1 \right)}} \quad \text{eq} \ (1)
\]

where \( \delta \) = depth of penetration in meters

\( \omega = 2\pi f = 2\pi (7500 \times 10^6) = 4.71 \times 10^{10} \)

\( \omega^2 = 22.2 \times 10^{20} \)

\( \varepsilon = \varepsilon_o \varepsilon_r = (8.854 \times 10^{-12})(11.1) = 98.28 \times 10^{-12} \)

\( \varepsilon^2 = 9.66 \times 10^{-21} \)

\( \mu \varepsilon = (4\pi \times 10^{-7})(98.28 \times 10^{-12}) = 12.35 \times 10^{-17} \)

\( \mu \varepsilon^2 = 6.175 \times 10^{-17} \)

\( \omega^2 \varepsilon^2 = 21.5 \)

Substituting the above values in equation (1) gives the depth of penetration to be 0.001936 meters.

As the semiconductor slice used for resistivity measurement was 0.00127 meters thick, the depth of penetration assured that the slice was being viewed homogeneously by the microwave energy.