AN ABSTRACT OF THE THESIS OF

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David W. Hann

Crown profile was modeled using a system of equations with three components. The first equation predicts the maximum crown width of an open grown tree based on the tree's diameter at breast height. The second equation modifies maximum crown width to represent the largest width of the crown in stand grown trees. The third component of the system is a group of equations which model the relative position of a stand grown tree's largest crown width and modify the largest width of the crown to represent the width of the crown at all heights throughout the crown. Existing maximum crown width equations were scaled to match the geographic region of the Western Willamette Valley. Independent datasets and non-linear regression techniques were used to fit the remaining models.

Equations for modeling crown widths at heights at and above the height of the largest crown width were highly predictive. Unfortunately, equations for modeling crown widths at lower heights than the largest crown width were considerably less accurate. Verification of the overall system indicated the presence of a small amount of bias, but the system produced a high level of accuracy even when the bias was included in predictions.

Modeling Crown Profile of Douglas-Fir in the Western Willamette Valley

by

Joseph T. Mortzheim

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Joseph T. Mortzheim, Author

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LIST OF VARIABLE NAMES

ANG = a measure of either VA or VA* (differentiated by I)

BA = basal area

BD = branch diameter

BD_{base} = BD of the lowest measured branch in a tree

BL = branch length

BL_{prd} = predicted BL

BL* = branch length as measured along the curved length of the branch

BL_b* = BL* as measured on branches with broken tips

BT = DOB of a branch at the break point when the branch tip is broken

CA_h = cross sectional area of the crown at height "h"

CL = crown length

CR = crown ratio

CW_h = crown width at height "h"

CV = total crown volume

DACB = distance above crown base where LCW occurs

DBH = diameter outside bark at breast height

DBT_{hb} = double bark thickness at HB

$DFTPA_8$ = number of greater than 8" DBH Douglas-fir trees per acre

DIB_b = diameter inside bark at H_b

DIB_{hb} = diameter inside bark at HB

DOB_b = diameter outside bark at H_b

LIST OF VARIABLE NAMES (Continued)

DOB_{hb} = diameter outside bark at HB

DOB_{h} = diameter outside bark at height "h"

H = total tree height

H_{b} = total height of the tree to the point where the top is broken off

h = height from ground to arbitrary point in the crown

HB = height from ground to the base of a branch

HCB = height to crown base

$HLCW_{f}$ = height from ground to LCW_{f}

I = an indicator variable

= when using VA, I = 1

= when using VA*, I = 0

L = length along the tree bole from tree tip to height "h"

= H - h

LCW_{f} = largest crown width of a tree, as measured while the tree is felled

LCW_{s} = largest crown width of a tree, as measured while the tree is standing

LCW_{s}* = geometric mean of two LCW_{s} measurements from the same tree

MCW = maximum crown width of an open grown tree with equivalent DBH

QMD = quadratic mean diameter of all trees in the stand

QMD_{8} = quadratic mean diameter of trees greater than 8" in DBH

RC_{h} = radius of the crown from center of bole at height "h"

= $CW_{h} / 2$

LIST OF VARIABLE NAMES (Continued)

RC^*_h = radius of the crown from edge of bole at height "h"

= $(CW_h - DOB_h) / 2$

RCW_h = relative CW_h modifier above LCW

= CW_h / LCW

$RCWX_h$ = relative CW_h modifier below LCW

RH_h = relative height of "h" above LCW

RHX_h = relative height of "h" below LCW

RLCW = relative LCW modifier

= LCW/MCW

RP = relative position of a branch within the crown

TPA = trees per acre

VA = vertical branch angle

VA_{adj} = VA_{prd} adjusted to reflect within tree variation

VA_{prd} = predicted VA

VA* = vertical branch angle as measured while the bole is prone

VA^*_{msr} = measured VA* value

VA^*_{prd} = predicted VA* value

vd = vertical deflection of branch tip from branch base

= $BL \cdot \cos(VA)$

Modeling Crown Profile of Douglas-fir in the Western Willamette Valley

Introduction

The ORGANON model predicts the growth and yield of stands within the geographic region of southwest Oregon and the western Willamette Valley. The model can estimate the development of 14 different species of trees in southwest Oregon and 5 different species in the western Willamette Valley of Oregon. (Hann et. al., 1994) However, in both ecological and managerial terms, Douglas-fir (Pseudotsuga menziesii) is by far the most important tree species within the forests of this region. Because any improvement in the ability to model Douglas-fir crown attributes will lead to a corresponding improvement in the predictive ability of the ORGANON model, and because Douglas-fir is the species of greatest concern in this region, this study deals exclusively with Douglas-fir.

The general purpose of the study is to develop equations which can describe the crown profile of individual Douglas-fir trees. These equations must accept as independent variables, data about individual trees which are readily available either as initial input into the ORGANON model or as a result of internal model calculations. The equations must provide as dependent variable predictions, the radius, width, or horizontal cross-section of the crown for all heights within the tree's crown.

Description of System

Tree crowns are composed of a bole, which provides the main structural support; the leaves or needles, which perform photosynthesis; and the branches, which support the leaves or needles. The physical dimensions of a given tree's crown can be described using several attributes: the total tree height, the height to the base of the crown, the crown ratio, the diameter of the bole, the height to each branch, the lengths and angles of branches, and the crown's position within the canopy. The width of a tree's crown at all heights, from the base of the crown to the top of the tree, can be used to characterize the shape of the space occupied by the crown.

A tree may extend portions of its crown into space above the crown of another tree. This reduces the photosynthetic production as well as the growth and vitality of the affected tree. The growth rate of a tree is an essential factor in all growth and yield models.

Measures of a tree's position in relation to other trees in the canopy are important components of the growth rate of a tree, particularly in terms of height growth. (Spurr and Barnes, 1980), (Ritchie and Hann, 1986), (Hann and Ritchie, 1988), (Ritchie and Hann, 1989) For individual trees, the ORGANON model calculates the ratio of space available for growth at the height of the terminal bud to the space occupied by the crowns of other trees. The trees on the ORGANON input tree list are used to calculate the crown dimensions of "competitor" trees. This ratio of available space to occupied space is used as a modifier to the growth in height of the individual tree. It also affects the tree's probability of death. (Hann and Ritchie, 1988), (Ritchie and Hann, 1989) This calculation is made for all trees on the tree list. Thus, crown width at any height within the crown must be estimated for all trees on the ORGANON tree list, and these estimated crown properties are used to model the growth and yield behavior of all trees during projection.

To date, very little data on crown profile of Douglas-fir has been collected in the Pacific Northwest. Ritchie and Hann (1985) used Brown's (1978) felled tree data for 43 Douglas-fir trees in northern Idaho and western Montana to estimate crown profile. Biging and Wensel (1990) collected 800 crown radii measurements on 115 felled Douglas-fir trees in northern California to model crown volume and associated area at any point in the crown. Finally, Roeh (1993) used 295 crown radii measurements on 53 standing trees in western Washington to model crown profile of small Douglas-fir.

Such small data sets would not be a problem if the data were applicable across the entire geographic region. Unfortunately, analysis of available crown data indicates that crown attributes change across the range of Douglas-fir. Paine and Hann (1982) found that maximum crown width (MCW) of open grown trees varied by geographic location within southwest Oregon. A comparison of the Arney (1973) MCW equation for northwest Oregon to the Paine and Hann (1982) equation indicates that MCW's in northwest Oregon exceed those in southwest Oregon for DBH's up to 45.7-inches in size. An examination of the height-to-crown-base (HCB) equations of Ritchie and Hann (1985) for southwest Oregon and the Zumrawi and Hann (1989) equations for northwest Oregon indicates that, for the same tree and stand conditions, Douglas-fir in southwest Oregon exhibited substantially longer crown lengths (CL) than Douglas-fir in northwest Oregon.

Finally, St. Clair (1994) found significant variation in relative crown width and in crown ratio among families of Douglas-fir.

Study Objectives

Given the paucity of existing data, the time and expense of collecting crown profile data, and the possibility of geographic variation in crown attributes, the objectives of this study are:

1. Derive a modeling approach for characterizing crown profile in Douglas-fir that can be effectively applied to existing trees measured on sample plots or points, and that can be easily calibrated/extrapolated to other geographic areas;
2. Apply the method to data collected from the McDonald-Dunn Research Forest.

Literature Review

Previous models of crown profile have used the one of the following two general approaches:

1. Indirect Prediction. (e.g., Roeh 1993)
2. Direct Prediction.

The direct method can be further divided into two approaches:

2a. Predict crown width at any given height (h) above the ground (i.e. CW_h) directly as a function of tree attributes and coefficients. (e.g., Honer 1971), (e.g., Biging and Wensel, 1990)

2b. Predict CW_h as a product of multipliers upon maximum crown width (MCW) of an open grown tree with the same diameter at breast height (DBH). (e.g., Ritchie and Hann 1985), (e.g., Dubrasich et al. 1996)

Method for Indirect Prediction of Crown Profile

Roeh (1993) used a system of equations to model crown profile of Douglas-fir. First, he constructed a series of individual regressions to predict branch diameter, the angle that the branch takes as it leaves the bole, and the branch length. Then, the equations were aggregated and considered as a system in order to model crown profile:

$$BD = a_1(L) + a_2(L)^2 + a_3(DBH \cdot L^2) + \varepsilon_{BD}$$

$$VA = a_4 \; [1 - e^{a_5(L)}]^{a_6(BD)} + \varepsilon_{VA}$$

$$BL = e^{a_7} \cdot L \cdot e^{a_8(L)} \cdot BD^{a_9} + \varepsilon_{BL}$$

$$RC^*_{h+vd} = BL \cdot \sin(VA)$$

Where:

BD = branch diameter

BL = branch length

DBH = diameter outside bark at breast height

L = length along the tree bole from tree tip to height "h"

= H - h

H = total tree height

h = height from ground to arbitrary point in the crown (in this application, to an arbitrary branch location in the crown)

RC^*_{h+vd} = radius of the crown from edge of bole at height "h+vd"

VA = vertical branch angle

vd = vertical deflection of branch tip from branch base

= BL·cos(VA)

ϵ = the error associated with each prediction

Roeh (1993) noted during model construction that the disturbance terms in this system were contemporaneously correlated across equations. Therefore, he used a systems of equations parameter estimation technique that incorporated the cross-equation error covariance to overcome this difficulty. Resulting parameter values are shown in Appendix C.

Roeh (1993) does not explicitly describe the effect of the bole's diameter outside bark at the height where crown radius is estimated (DOB_{h+vd}). Thus, crown radius (RC^{*}_{h+vd}) is the distance from the bole to the edge of the crown. A true crown radius estimate would have to include $DOB_{h+vd} / 2$, which can be predicted from existing taper equations such as Walters and Hann (1986).

The advantage of the indirect approach to modeling crown profile is that it gives a more functional and detailed picture of the crown which can be useful for assessing wood quality and characterizing the crown in tree and stand visualization programs such as VIZ4ST. (Hanus, 1995) There are two disadvantages to this approach. First, application to existing trees would require the difficult and time consuming measurement of whorl positions throughout the crown. Second it would be difficult to calibrate the resulting equations to other geographic areas or tree sizes without additional costly and time consuming field measurements.

Methods for Direct Prediction of Crown Profile as a Function of Tree Attributes

Honer (1971) developed the following equation which describes crown profile of balsam fir and black spruce:

$$RC_h = a_1(L) + a_2(L \cdot H) + a_3(\frac{L^2}{H}) + a_4(L^2) + \varepsilon$$

Where,

RC_h = radius of the crown from center of bole at height "h"

Parameter values for this model are shown in Appendix D.

Application of Honer's (1971) equation was limited to where L was above the point of largest crown width (LCW). This sidesteps the issue of where the largest crown width occurs on a given tree. By comparison, Roeh (1993) assumed that LCW peaks within the crown. Honer (1971) reported a high predictive value and suggested that the model form may be applicable to other tree species. Unfortunately, he did not test the model form on any species other than balsam fir and black spruce, and there is no discussion of how well the assumptions of regression were met during model building.

Biging and Wensel (1990) developed models to describe the crown volume and the width of the crown at varying heights in the crown for six conifer species in northern California. Biging and Wensel (1990) noted that "...crown width or radius were not always monotonically decreasing from crown base to the tip of the tree." Thus, Biging and Wensel (1990) rejected all model forms where crown width is a function of height above crown base. Rather, they developed a geometric crown volume model and derived crown surface areas from it.

Biging and Wensel (1990) used the following equation to describe the cross sectional area of the crown at h (CA_h):

$$CA_h = \left(\frac{k \cdot CV}{H-HCB}\right)\left(\frac{H-h}{H-HCB}\right)^{(k-1)} + \varepsilon$$

Where:

CA_h = cross sectional area of the crown at height "h"

CV = total crown volume

= $b_1 (DBH^{b2} \cdot H^{b3} \cdot CR^{b4})$

CR = crown ratio

HCB = height to crown base

Crown width at any arbitrary height "h" (CW_h) can then be determined by:

$$CW_h = \sqrt{\frac{4}{\pi} \cdot CA_h}$$

In this formulation, LCW occurs at HCB and the shape of the profile is entirely dependent on the value of the k parameter. Biging and Wensel (1990) noted that fits for the true firs (Abies spp.) and incense cedar (Calocedrus decurrens) were better than those for pines (Pinaceae spp.) and Douglas-fir. They suggested that this is due to the shade tolerance of the firs, which permits a denser stand. Presumably more uniform crowns result from more densely grown trees. However, the poor fits might also be due to the restriction that LCW occur at HCB.

There is no mention of a weighting factor for the non-linear estimation of equations. Nor is there any discussion of possible autocorrelation invoked by multiple crown measurements on the same tree.

As with the indirect method, the disadvantage of predicting CW_h directly from tree measurements is that it is difficult to calibrate the resulting equations to other geographic areas or tree sizes without additional costly and time consuming field measurements.

Methods for Direct Prediction of Crown Profile as a Function of Maximum Crown Width

Ritchie and Hann (1985) predicted CW_h as a product of an estimated MCW and two modifiers:

$$CW_h = M\hat{C}W \cdot RL\hat{C}W \cdot R\hat{C}W_h + \epsilon$$

Where,

$$\begin{aligned} M\hat{C}W &= \text{An unbiased, least-squares estimator of } MCW \\ RL\hat{C}W &= \text{An unbiased, least-squares estimator of } \frac{LCW}{M\hat{C}W} \\ R\hat{C}W_h &= \text{An unbiased, least-squares estimator of } \frac{CW_h}{M\hat{C}W \cdot RL\hat{C}W} \end{aligned}$$

This approach to modeling crown width is analogous to the "potential and modifier" approach taken by Arney (1972, 1985), Burkhart et. al. (1987), Daniels and Burkhart (1975), Ek and Monserud (1975), Hann and Ritchie (1988), Hegyi (1974), Krumland and Wensel (1981), Ritchie and Hann (1986), and Wensel and Koheler (1985) to model growth rates of trees. In the current application, "potential" is characterized by the MCW of open grown trees and is modified by two multipliers whose values range from zero to one. The RLCW modifier adjusts MCW to reflect the largest crown width (LCW) found on stand grown trees. It should be formulated such that RLCW equals one (i.e., LCW = MCW) when CR is one.

The RCW_h modifier adjusts LCW to predict crown profile and resulting widths (CW_h) throughout the crown. It should be formulated in a manner such that RCW_h equals zero at the tip of the tree and equals one (i.e., CW_h = LCW) at the value of "h" where LCW occurs.

The advantage of this approach is that it can be more easily calibrated to other locations. For example, if MCW differs from area to area, then one can easily use different MCW equations if one is willing to assume the relative value (LCW / MCW) and the relative value (CW_h / LCW) are constant across areas. If one is not willing to make that assumption, then it is relatively easy to collect LCW data and develop a RLCW equation for the area. One would then only have to assume that relative profile (i.e. CW_h / LCW) is constant across areas. CW_h data is very expensive to collect. Thus, it is a strong advantage to be able to calibrate individual MCW and/or LCW equations rather than to create a new CW_h equation.

The specific model forms used by Ritchie and Hann (1985) were:

$$MCW = a_1 + a_2(DBH) + a_3(DBH^2)$$

$$RLCW = CR^{a_4}$$

$$RCW_h = \left(\frac{H - h}{H - HCB}\right)^{a_4}$$

For the western Willamette Valley version of ORGANON, the parameter estimates for the MCW equation came from Arney (1973). For the southwest Oregon version of ORGANON, the parameter estimates for the MCW equation came from Paine and Hann (1982). Ritchie and Hann (1985) assumed that the parameter for RLCW and RCW_h had the same value (i.e., a_4) and that LCW occurred at HCB. They estimated the value for a_4 at 0.76528 using Brown's (1978) Douglas-fir data from northern Idaho and western Montana.

The Biging and Wensel (1990) equations can be manipulated mathematically to also predict CW_h as a product of an estimator of MCW and two modifiers. The resulting model forms are:

$$\begin{aligned} M\hat{C}W &= a_1(DBH)^{a_2} \\ RL\hat{C}W &= CR^{a_3} \\ R\hat{C}W_h &= \left(\frac{H - h}{H - HCB}\right)^{a_4} \end{aligned}$$

The parameter estimates for Douglas-fir in the Biging and Wensel (1990) model were:

$a_1 = 6.10844$

$a_2 = 0.488225$

$a_3 = 0.231365$

$a_4 = 0.40251$

Note that in this study the parameters for RLCW (i.e., a_3) and RCW_h (i.e., a_4) took on different values.

Both Ritchie and Hann (1985) and Biging and Wensel (1990) assumed that the power on CR (associated with the RLCW equation) and the power on [(H-h)/(H-HCB)] (associated with the RCW_h equation) were both invariant with tree attributes. However, Dubrasich et al. (1996) found that the power on the RLCW equation did vary by tree attributes. Using a sample of LCW measurements from 152 standing Douglas-fir trees in five stands in southwest Oregon, they developed the following model form:

$$RLCW = CR^{[a_1(CL) + a_2(\frac{DBH}{H})]}$$

Where,

CL = crown length, feet

$a_1 = 0.00568664$

$a_2 = 0.546499$

Dubrasich et al. (1996) estimated the parameters using nonlinear regression and reported that the resulting variance about RLCW was homogeneous. To estimate CW_h, Dubrasich et al. (1996) used the estimated RCW_h (from the modifier equation) and parameter estimates derived from the Biging and Wensel (1990) equations.

Data Collection and Completion

Two independent datasets were collected on the McDonald-Dunn Research Forest. Independent datasets were used to model relative largest crown width (RLCW) and relative crown width at a given height (RCW_h) respectively. This was done to avoid possible problems with correlated error terms between the two equations.

Construction of a Dataset for Modeling Relative Largest Crown Width

During the field seasons of 1994 and 1995, LCW was measured on a subsample of standing Douglas-fir trees from 64 stands as a part of the permanent inventory of the McDonald-Dunn Research Forest. The McDonald-Dunn Research Forest is located near Corvallis, Oregon. The inventory system is based on a grid of permanent sample points located throughout each stand. Centered at each sample point is a sample unit consisting of three nested subplots: a 7.78' fixed radius subplot for trees between 6" tall and 4.0" DBH, a 15.56' fixed radius subplot for trees 4.1" - 8.0" DBH, and a BAF 20 subplot for trees larger than 8.0" DBH. DBH, total height (H), height to crown base (HCB), and largest crown width (LCW_s) along two perpendicular axes were recorded on each sample tree. LCW_s was recorded along two axes; the first axis originated at plot center and passed through the tree's bole, the second axis was perpendicular to the first. Trees were rejected if their two LCW_s measurements differed by more than 55 percent. The RLCW dataset consists of these measurements for 894 sampled trees with a DBH greater than or equal to 2.0 inches. The geometric mean of the two LCW_s measures recorded from each tree (LCW^*_s) was used in modeling RLCW. A summary of the RLCW dataset is shown in table 1.

Table 1. RLCW dataset and its component variables

Measurement	N	Min.	Max.	Mean	Standard Deviation
DBH (inches)	894	2.8	81.0	26.5	11.5
H (feet)	894	19.5	210.7	127.7	34.0
HCB (feet)	894	4.0	140.8	70.6	25.2
LCW_s* (feet)	894	5.9	66.0	32.6	10.8

Construction of a Dataset for Modeling Crown Profile

This dataset consists of a sample of 108 Douglas-fir trees selected from the McDonald-Dunn Research Forest located near Corvallis, Oregon. Six steps were used in conducting the field work:

1. Twenty-seven stands were randomly selected a stratified list of stands on the property.
2. Within each stand, four trees were chosen for a destructive sample.
3. Measurements were taken on each tree prior to falling.
4. After the trees were felled, measurements were taken on a branch at each of 10 equally spaced whorls.

5. A subsample of branches (and associated whorls) were cut from the bole and stood upright for additional measurements.
6. The resulting data were completed and adjusted in preparation for data analysis.

Stand Selection

Based upon previous work, is was expected that CW_h would vary by tree size and crown length or ratio. Therefore, a data collection procedure was designed to sample across a large range of both of these attributes. This procedure used the McDonald-Dunn permanent forest inventory system to locate stands for measurement. Suitable stands were divided into 9 classifications or "cells" based on the combined characteristics of average tree size and average crown ratio of the dominant trees in the stand. Three stands were randomly chosen from each cell, resulting in the selection of a total of 27 stands.

The classification of average tree size was complicated by the presence of multi-storied stands on the property. To allow the inclusion of two-storied stands with a substantial understory, it was necessary to use both the stand's quadratic mean diameter of trees greater than 8" in DBH (QMD_8) and the number of Douglas-fir trees greater than 8" in DBH per acre ($DFTPA_8$) in developing the following key for classifying average tree size:

Table 2. Key for classifying average tree size of stands.

$DFTPA_8$	QMD_8		
	0" - 10.0"	10.1" - 20"	20.1"+
0 - 10	Small	Small	Small
11 - 50	Small	Eliminated	Eliminated
51+	Small	Medium	Large

Unsuitable stands were eliminated from the classification for the following reasons:

1. Proportion of the stand's basal area in Douglas-fir was less than 80%.
2. The stand had been administratively withdrawn from destructive sampling.
3. No crown ratio measurements had been included in the McDonald-Dunn inventory data.
4. The stand had fewer than 20 trees per acre (TPA).
5. The quadratic mean diameter (QMD) of the stand was under two inches, indicating that the stand was too young.

Tree Selection

Four Douglas-fir were chosen for felling in each selected stand. Trees were chosen so that one Douglas-fir was selected in each quartile of the range of Douglas-fir DBH's. Field reconnaissance of the entire stand was used to locate the trees selected for felling.

All selected trees had relatively undamaged stems and crowns, and were representative of the stand as a whole. Trees with damaged or highly asymmetrical crowns, mistletoe brooms, broken tops, or evidence of such conditions in the past were deemed unsuitable for sampling, as were remnants of prior stand structures. Suppressed or intermediate trees were selected for felling if they were otherwise acceptable. Finally, selected trees were spatially distributed over the stand as widely as possible in order to minimized disturbance to the stand and interdependence between the individual trees.

Standing Tree Measurements

The following measurements were taken prior to felling each tree: DBH in inches, total height in feet, height to crown base in feet, crown radius in feet along four directions, and crown class. The "pole tangent" method was used to measure both total height and height to crown base. (Curtis and Bruce, 1968) For trees of uneven crown length, crown base was determined by ocularly transferring lower branches on the longer side to fill holes in the upper portion of the shorter side to generate a full, even crown (epicormic and very short branches were ignored in this process). (Hann, 1992) While this ocular "adjustment" is somewhat subjective, trees selected for this study had relatively uniform shapes and the amount of ocular "adjustment" required was very limited. For trees of an even crown length, there was no need for an ocular "adjustment" and crown base was defined as the lowest whorl which had live branches at least three quarters of the way around the circumference of the stem. (Curtis and Reukema, 1970) Two crown radii were measured on a randomly chosen axis and two crown radii were measured on the axis perpendicular to the randomly chosen one.

Felled Tree Measurements

After the tree was cut down in a manner which minimized damage to the crown's branches, total height and height to crown base were remeasured along the bole of the felled tree. Ten whorls were selected at equal spacings between crown base and tip of the terminal leader. The length of live crown (CL) in feet was calculated using the difference between total tree height (H) and height to crown base (HCB). Whenever possible, the first whorl of the live crown was the first whorl measured, the next whorl was chosen by moving up the crown a distance of one tenth of the live crown length and measuring the whorl nearest to that point. On each measured whorl, the largest undamaged branch was selected for measurement of branch angle (VA*) in degrees, branch height (BH) in feet, branch diameter outside bark (BD) in inches, and length of the branch along the curved path of the branch itself (BL*) in feet.

Additional Measurements on a Subsample of Branches

The branch angle of a prone tree can differ from the branch angle of a standing tree. Further, branches usually exhibit some degree of curvature causing the length of the branch itself to be longer than the straight line distance from the base of the branch to the branch tip.

These problems can be minimized if a bole segment incorporating the whorl is cut from the bole (with the branch intact) and stood upright in order to mimic the branch's position before the tree was cut. (Honer, 1971), (Remphrey and Powell, 1984) However, cutting and standing bole segments incorporating the whorls takes considerably more time and effort than measuring the whorl's branch on the ground. Therefore, only a subset of measured whorls were cut from the bole and stood horizontally. For branches in this subset, branch angle (VA) in degrees was remeasured as the angle between the bole, branch base, and the branch tip. Branch length (BL) in feet was also remeasured as the straight line distance between branch base and branch tip.

This data facilitated establishing a relationship between the angles and distances measured on the branch while the bole segment is horizontal and while the bole segment is upright. The relationship can then be used to determine appropriate standing values for branches which were measured only while in the horizontal position.

Data Completion and Adjustment

Generally trees were felled with a minimum of damage. However, a certain amount of damage was sustained on all trees and, in several instances, broken limbs or broken boles had to be pieced together from fragments. In rare instances the appropriate pieces of branches or the bole could not be located. In these cases, data was collected on the crown components that could be located and regression methods were applied to complete the gaps in the data set.

Estimating Total Tree Height for Trees with Broken Tops

The extreme upper portion of some trees was damaged during felling. In most instances, the entire tree length could be reconstructed from the broken pieces of bole. Occasionally it was impossible to find a length of the upper stem and, as a result, it was necessary to derive total tree height from measurements of the remaining crown and stem.

DBH, crown ratio (as measured while the tree was standing), height of the tree at the break (H_b), and the diameter outside bark of the stem at the break (DOB_b) were used in these calculations. The diameter inside bark of the bole at the break (DIB_b) was determined using the equation of Larsen and Hann (1985). The taper equation of Walters and Hann (1986) was then used iteratively to determine the total height needed to predict the DIB_b at the point of the break on the tree.

In all, 11 trees had broken tops with DOB_b ranging from 0.52" to 2.24". Iterative calculation yielded total heights which were 1.7' to 6.4' taller than the portion of the tree which was physically reconstructed in the field.

Estimating Curved Branch Length for Branches with Broken Tips

When trees are felled, the branches within the crown often receive considerable damage. In nearly all cases, complete branches could be reconstructed from broken pieces. Whenever a given branch could not be completely reconstructed, another branch in the whorl was selected for measurement. Occasionally in severely damaged trees, no surrogate branch was present but only a small portion of the selected branch was missing. For the 55 instances where this situation was encountered, the diameter outside bark of the branch at the break (BT) in inches and the branch length to the break (BL_b*) in feet were measured.

Least squares regression was used to estimate the branch length as measured along the curved length of the branch (BL*) of a branch where the tip was missing. Branches from all whorls within the dataset were pooled to develop a relationship between BL* and the diameter of the broken tip (BT). For complete branches, BL_b* was defined as equal to BL* and BT was defined as being zero. The following relationship was derived from simple, known geometric relationships:

$$BL^* = a\ (BD^n - BT^n)$$

Where n is equal to 2/3, 1, or 2, if the relationship is neilodic, conic, or parabolic respectively. This relationship was fit to the data using linear regression and various values for n ranging from 0.25 to 2 in increments of 0.25. The value n = 1.25 provided the best performance in terms of mean squared error and the distribution of residuals. In order to account for tree to tree variation, individual regressions were constructed for all 29 trees where a branch with a missing tip was encountered. A f-test comparing the individual regressions to the pooled regression indicated that individual regressions were superior (p-value = $5.42 \cdot 10^{-20}$).

Appendices A and B show the coefficient values associated with the regression lines and the S+ algorithm used to construct the regression lines. (Venables and Ripley, 1994), (S-plus, 1995) Table 3 summarizes the mean squared error terms and associated summary statistics resulting from the regressions.

Table 3. Summary statistics of BL* regressions for 29 trees.

Squared Error of the Regressions			
Mean	Median	Min.	Max.
21.1	14.54	2.786	61.24

Relationship Between Curved Branch Length and Straight Branch Length

In order to estimate crown width for each whorl in the dataset, BL must be known or estimated for each sampled branch. In order to determine BL on whorls which were not cut from the bole and stood horizontally, a relationship between BL and BL* was established using the subset of 251 whorls containing both measurements.

While BL* alone could be used to represent a good deal of the variation in BL a variable to represent relative position (RP) was incorporated into the final model form. This increased the model's predictive power slightly. In this case, RP was calculated as the relative position of a branch within the crown; where RP = 1.0 at crown base and RP = 0.0 at the tree tip.

The presence of skewness and kurtosis in initial non-linear model forms, indicated that non-linear regression was needed to homogenize variance about the regression. The following model form produced homogenous variance and yielded an adjusted R^2 of 0.9713:

$$BL = BL^* \cdot e^{(a_1 \cdot BL*)^{(a_2 + a_3 \cdot RP)}}$$

Table 4 shows summary statistics of the parameter estimates.

Table 4. Summary statistics of the BL to BL* regression.

Parameter	Value	Standard Error	t-value
a_1	-0.035820	0.00759086	-4.71884
a_2	0.794281	0.14059800	5.64929
a_3	-0.399676	0.11289800	-3.54016

Estimating Vertical Angle

During construction of the dataset, vertical branch angle was measured on all undamaged whorls while the tree's bole was prone (VA*). On a subset of undamaged whorls, vertical branch angle was remeasured while a portion of the bole was stood upright (VA). In a few instances, the largest branch at a whorl was broken at the bole in a manner making it impossible to measure VA*.

The crown width produced by a given whorl can be estimated from branch attributes only if vertical angle (VA) is known or estimated. Therefore, the following equation was developed to estimate VA* on damaged branches and VA when it was not measured:

$$ANG = 90 - \frac{a_1}{BD^2_{base}} - (a_2 + a_3(I))\cdot((1 - RP)^{(a_4 + a_5(I))}$$

Where,

ANG = VA if I = 1

= VA* if I = 0

I = 1 if predicting VA

= 0 if predicting VA*

BD_{base} = the branch diameter (BD) of the lowest measured branch in a tree

RP = the relative position of a branch within the crown

= 1 if HB = HCB

= 0 if HB = H

Table 5 shows summary statistics of the parameter estimates.

Table 5. Summary statistics of the VA model.

Parameter	Value	Standard Error	t-value
a_1	5.03203	0.420720	11.96050
a_2	40.35350	1.532320	26.33490
a_3	12.97840	2.141140	6.06145
a_4	1.91709	0.120268	15.94020
a_5	-1.36030	0.132365	-10.27690

This approach uses pooled information about VA and VA* to determine a general model form. The resulting parameter estimates are presented in table 5. Examination of residuals indicated homogeneous variance and no trends across tree or branch attributes. The adjusted R^2 was 0.602 for the equation.

However, predictions of VA based on this equation are not affected by within tree variations in VA*. In order to account for within tree variation, VA* was used as a calibration factor on VA predictions. Information about VA* as measured (VA^*_{msr}) and predictions of VA and VA* (VA_{prd} and VA^*_{prd} respectively) were combined to produce an *adjusted* predicted VA value (VA_{adj}):

$$VA_{adj} = VA^*_{msr} - VA^*_{prd} - VA_{prd}$$

This calibration technique provided a modest increase in predictive power while retaining model characteristics. All subsequent crown widths were calculated using VA_{adj} for branches without a measured VA.

Summary of Crown Profile Dataset

Table 6 shows a brief summary of the crown profile dataset and its component variables.

Table 6. Dataset and its component variables for modeling crown profile.

Measurement	N	Min.	Max.	Median	Mean	Standard Deviation
DBH (inches)	108	2.0	47.5	15.7	17.58	9.614522
$H_{(standing)}$ (feet)	108	15.1	177.2	105.4	99.89	39.73987
$H_{(felled)}$ (feet)	108	15.4	179.3	103.8	99.16	39.21045
$HCB_{(standing)}$ (feet)	108	0.0	121.4	59.25	53.51	30.96011
$HCB_{(felled)}$ (feet)	108	0.0	142.5	59.85	55.71	34.52311
$CR_{(standing)}$	108	0.09	1.0	0.46	0.507	0.202324
$CR_{(felled)}$	108	0.118	1.0	0.452	0.4903	0.2149334
BD (inches)	1069	0.1	4.76	1.21	1.421	.8302264
BL (feet)	251	0.8	18.5	5.2	5.631	3.171638
BL* (feet)	1069	0.8	31.4	8.9	9.823	5.505552
VA_{msr}	990	31	116	75	73.58	14.998
VA^*_{msr}	251	7	99	44	46.33	17.25571
BH (feet)	1069	0.0	178.0	75.5	75.34	38.40017
LCW_s* (feet)	107	9.644	56.66	24.25	26.55	11.18325
DBH/H	101	0.08527	0.2703	0.1665	0.1703	0.043285

Calculation of h and Crown Width at h

Given VA (either measured or estimated), BL (either measured or estimated) and height above ground to the base of the branch (HB), height (h) of the associated crown width is determined from (Figure 1):

$$h = HB + \cos(VA) \cdot BL$$

Crown width at that height (CW_h) is determined from (Figure 1):

$$CW_h = 2 \cdot (\sin(VA) \cdot BL) + DOB_{hb}$$

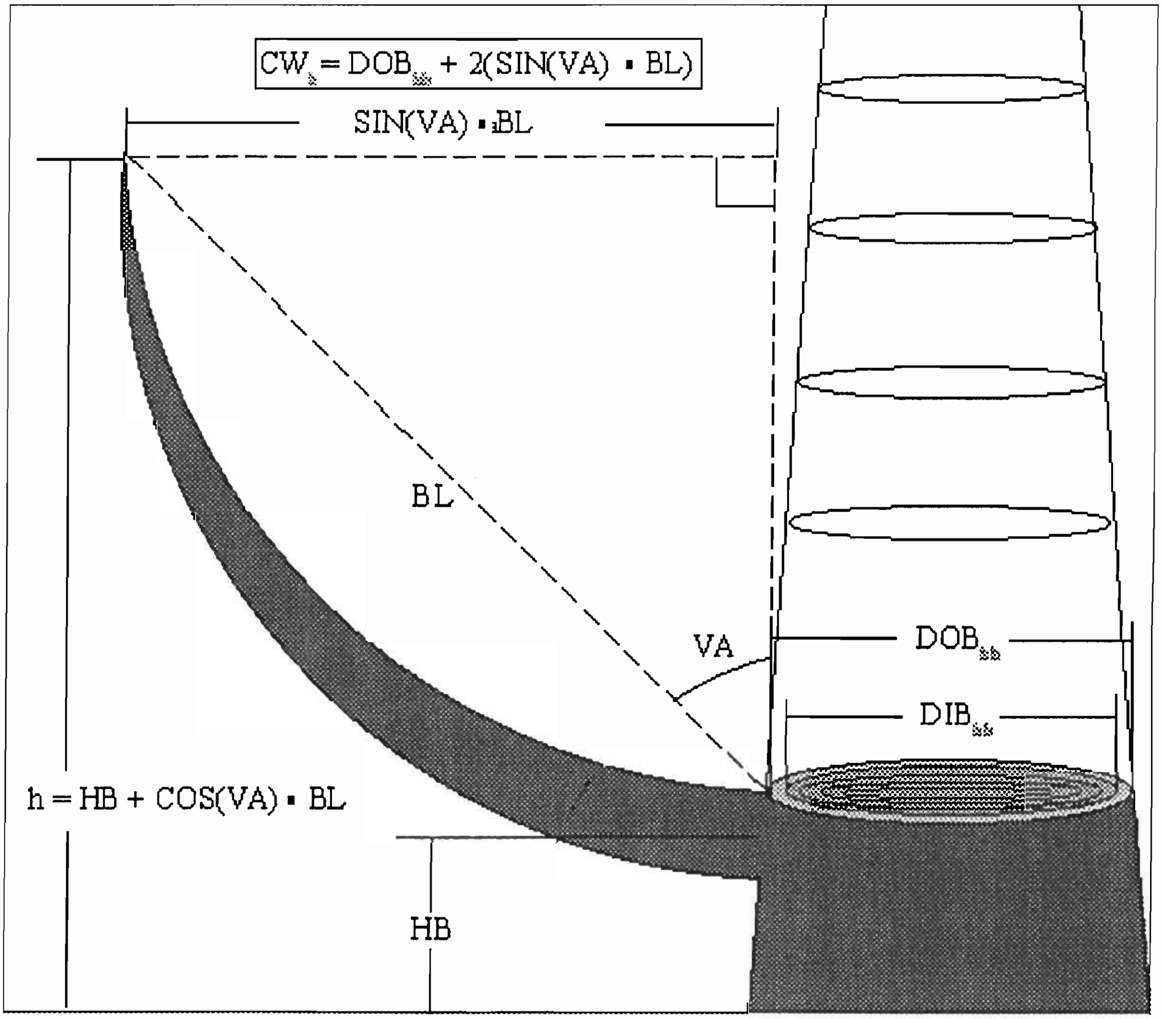
Where,

$DOB_{hb} = DIB_{hb} + DBT_{hb}$

DIB_{hb} = Diameter inside bark of the bole at HB predicted from the Walters and Hann (1986) bole taper equation for Douglas-fir.

DBT_{hb} = Double bark thickness of the bole at HB predicted from the Maguire and Hann (1990) bark thickness taper equation for Douglas-fir.

Figure 1. Geometry involved in crown width and crown height calculations.



<u>Location of Largest Crown Width Within the Crown</u>

Exploration of the CW_h dataset indicated that the largest width of the crown as measured while the tree is felled (LCW_f) does not necessarily occur at the base of the crown (HCB). Therefore, a new value was calculated to represent the distance above crown base (DACB) where LCW_f occurs.

$$DACB = HLCW_f - HCB$$

Where,

DACB = Distance above crown base where LCW_f occurs

$HLCW_f$ = Height above ground where LCW_f occurs

When LCW_f occurs at the crown base, DACB equals 0.0. When LCW_f occurs above crown base, DACB is greater than 0.0. By definition, DACB is always less than crown length (CL). In 52 out of 101 trees, DACB was 0.0. For the remaining trees, DACB exceeded 0.0.

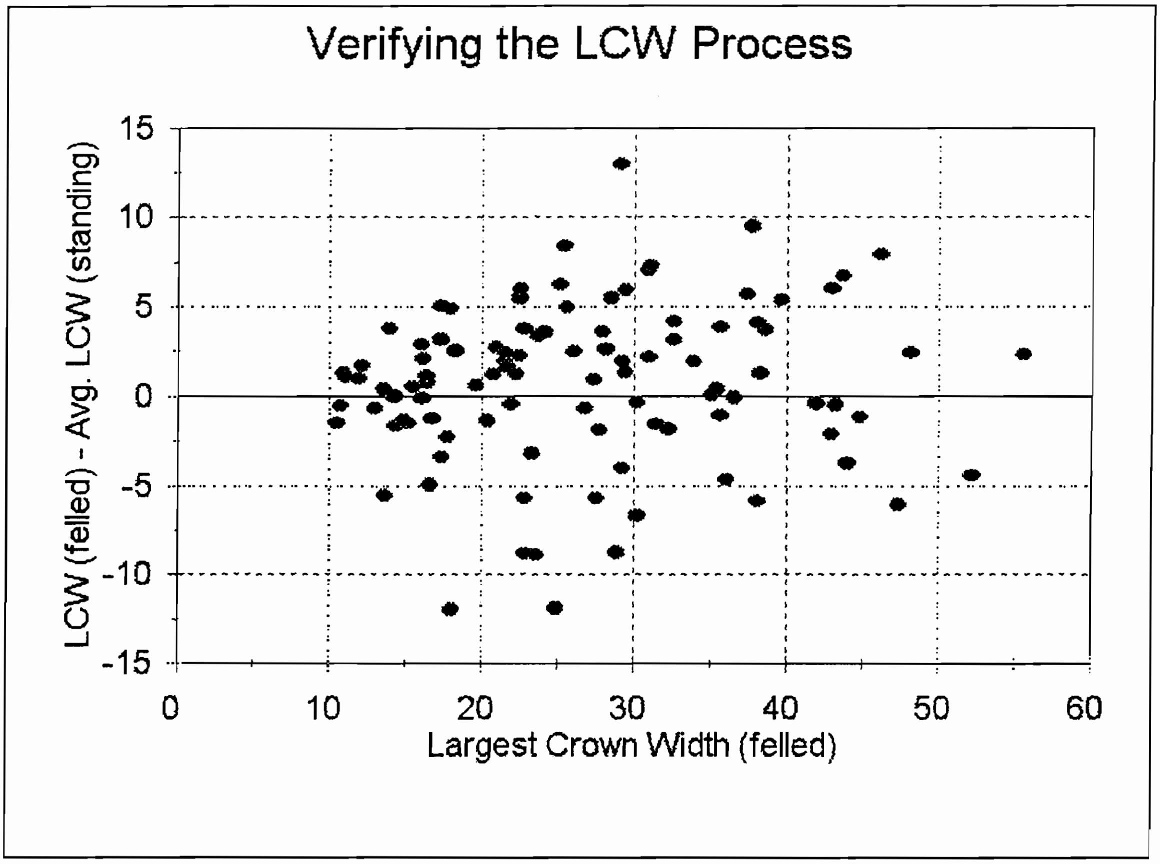
Trees with an Uncertain Largest Crown Width

When trees were felled for measurement, some branches within the crown were damaged or destroyed. Generally, one branch from the lowest whorl in the crown (i.e. crown base) was undamaged and was therefore available to be measured. When this was not the case, the lowest undamaged branch on the tree was measured. In 6 out of 108 trees, the lowest undamaged branch measured was high enough into the crown such that the whorl which produces LCW_f may have occurred lower in the crown. For these instances, the actual position and size of LCW_f could not be ascertained from branch attributes. Measurements from these trees were used in modeling BL and VA, but they were eliminated from consideration when validating the RLCW model, and constructing the RCW_h, probability DACB > 0, determining DACB, and $RCWX_h$ models.

Verifying Largest Crown Width

By definition, LCW_f and LCW^*_s should be independent estimates of the same tree attribute . To verify this, the difference $LCW_f - LCW^*_s$ was determined for each tree and the mean and variance of this difference were computed across the 101 trees containing both measurements. A t-test was then used to determine if the mean was significantly different from zero. The resulting values (mean = 0.727923, variance = 19.5904, t-statistic = 1.6528) indicated that the process used to estimate LCW_f produced an unbiased estimator of LCW^*_s. Finally, a graph of the differences across LCW_f (Figure 2) showed no trend further confirming unbiasedness.

Figure 2. Verifying unbiasedness of LCW_f process.



Data Analysis

Three basic models are needed to characterize CW_h; a model to represent MCW, a model to represent RLCW, and a model to represent RCW_h. This study adjusted existing equations for characterizing MCW and developed new equations for characterizing RLCW and RCW_h. Three additional models were developed to estimate the probability of LCW_f occurring above HCB (probability DACB > 0), the height of LCW_f when it did not occur at HCB, and crown width below LCW_f ($RCWX_h$).

Adjustment of Existing Maximum Crown Width Equations

There are two existing studies in Oregon which have modeled maximum crown width (MCW) as a function of DBH. Arney (1973) used a sample of trees from western Oregon and the following model form:

$$MCW = a_1 + a_2(DBH) + a_3(DBH)^2$$

Paine and Hann (1982) used the same model form for a sample of 206 Douglas-fir in Southwest Oregon. Values for the constants, as determined by the two studies are shown in table 7.

Table 7. Parameter values in MCW Equations.

Study	a_1	a_2	a_3
Arney (1973)	4.707100	2.016800	-0.018600
Paine and Hann (1982)	4.636600	1.607800	-0.009625

Because of the use of the quadratic model form, both prediction equations have a point where the MCW of trees will begin to decrease as the DBH increases. Paine and Hann (1982) list the DBH where MCW peaks as 83.5 inches. The Arney (1973) equation has a peak MCW at 54.2 inches. Clearly, an equation should not be used for a tree when its DBH is at, or near, this maximum MCW point.

Arney (1973) reported that his western Oregon dataset has a mean DBH of 11.9 inches and Paine and Hann (1982) reported that their dataset had a range of 0 inches to 69.5 inches. The Paine and Hann (1982) model has a scope of inference which approximates the range of DBH's measured for the RLCW and RCW_h datasets, but it is based on measurements from a region somewhat distant from McDonald-Dunn Research Forest. Conversely, the Arney (1973) equation is based on data from western Oregon but is accurate over a smaller range of DBH's.

A single equation which represents the MCW of Douglas-fir in Western Oregon and possesses the wider scope of inference of Paine and Hann's (1982) study is needed. In order to accomplish this, the Paine and Hann (1982) equation was modified to better match Arney (1973) over the tree sizes he sampled.

Since the intercept term was nearly identical for both models, the Arney (1973) value (4.707100) was retained. The a_2 and a_3 from Paine and Hann (1982) were then algebraically adjusted to pass through Arney's (1973) reported mean MCW at his reported mean DBH and mean DBH^2. The resulting adjusted model was:

$$MCW = b_1 + b_2 \cdot (DBH) + b_3 \cdot (DBH^2) \qquad [1]$$

Where,

b_1 = 4.7071

b_2 = 1.8917

b_3 = -0.011324

Modeling Relative Largest Crown Width

RLCW has usually been modeled as a non-linear function of CR. (Ritchie and Hann, 1985) (Biging and Wensel, 1990) (Dubrasich et. al., 1996) Both Ritchie and Hann (1985) and Biging and Wensel (1990) used a model form where CR is raised to a constant power, while Dubrasich et. al. (1996) characterized the power on CR as a function of CL and DBH/H. Therefore, the following two alternative model forms were fitted to the RLCW dataset using non-linear regression:

$$RL\hat{C}W = CR^{a_1}$$

$$RL\hat{C}W = CR^{(a_1 \cdot CL + a_2 \cdot (\frac{DBH}{H}))}$$

The model based on Dubrasich et. al. (1996) had a lower mean squared residual. As a result, it was explored more thoroughly by graphing residuals over predicted RLCW, CL, DBH, H, DBH/H, and CR. This examination indicated that the model was underestimating RLCW for small vales of CR because the model form forces RLCW to be zero for a zero CR. Therefore, an intercept value was added in the following fashion:

$$RL\hat{C}W = b_4 + (1 - b_4) \cdot CR^{[b_5 \cdot CL + b_6 \cdot (\frac{DBH}{H})]} \qquad [2]$$

In this formulation, RLCW is still constrained to a value of one for a CR of one. Examination of residuals from this model form indicated that the trend across CR was eliminated.

Modeling Height to Largest Crown Width

Height to LCW is modeled using two component equations; an equation to determine whether LCW_f occurs above HCB, and an equation to determine the position of LCW_f if it is above HCB.

Determining when Distance Above Crown Base to $LCW_f > 0$

In order to determine when DACB exceeds 0.0, tree attributes (i.e., DBH, H, CL, CR) were subdivided into classes and the probability of DACB exceeding 0.0 was calculated for each class. The only variable that suggested the possibility of a trend was

DBH. Therefore, the RISK program (Hamilton, 1974) was used to estimate the parameters for the following non-linear logistic model to predict the probability that DACB is greater than zero:

$$Probability\ DACB > 0 = \frac{1}{1 + e^{(a_1 + a_2 \cdot DBH)}}$$

All measures of goodness of fit indicated that b_2 was not significantly different from zero. As a result the probability of DACB > 0 is best characterized by a simple mean.

Determining the Size of Distance Above Crown Base, Given that it is Greater than Zero

For all trees where DACB > 0, DACB was graphed over the three tree size attributes of DBH, H, and CL. The plots indicated that DACB increases with tree size. The variable DACB/CL was examined in more detail because the fact that DACB/CL is bounded between 0.0 and 1.0 can be useful in choosing a well behaved model form.

Graphs of DACB/CL over DBH, H, DBH/H, and CR did not indicate any trends. Further, no significant linear relationships could be established over any of these variables. Therefore DACB/CL is best estimated using mean DACB/CL.

Modeling Crown Profile

Once the LCW_f of a tree has been determined, all other widths in the crown can be expressed as a ratio between crown width at a given height (CW_h) and LCW_f:

$$RCW_h = \frac{CW_h}{LCW_f}$$

Thus, the remainder of this modeling effort is centered on estimating RCW_h throughout the crown.

Modeling Crown Profile Above Height to Largest Crown Width

RCW_h has been modeled as a non-linear function of the relative position of h in the crown (RH_h). (Richie and Hann, 1985) (Biging and Wensel, 1990) This suggests that the following model form should be used:

$$R\hat{C}W_h = RH_h^{a_1}$$

Where,

$$RH_h = \frac{H - h}{H - HLCW}$$

Individual fits of this equation were made to each of the 101 trees non-linear regression. Plots of a_1 over tree variables indicated a simple linear trend over the inverse of DBH/H. Thus, H/DBH was incorporated into the model form as follows:

$$R\hat{C}W_h = RH_h^{(b_7 + b_8 \cdot (\frac{H}{DBH}))} \qquad [3]$$

No further trends were observed over tree variables.

Modeling Crown Profile Below Height to Largest Crown Width

The data available for modeling crown profile below HLCW ($RCWX_h$) was weak. Of the 49 trees in which DACB was greater than zero, LCW_f occurred at the first measurement above HCB in 33 trees, at the second measurement in 13 trees and at the third measurement in the remaining 3 trees. This provided a total of 68 branches for modeling CW_h below HLCW and, of these, 49 branches occurred at crown base.

$RCWX_h$ is calculated just as RCW_h is:

$$RCWX_h = \frac{CW_h}{LCW}$$

In the model for RCW_h, RH_h was used to represent the relative position of "h". Since RH_h was only applicable when "h" was above HLCW, a new relative height variable was formed for characterizing $RCWX_h$:

$$RHX_h = \frac{h - HCB}{DACB}$$

RHX_h is equal to 1.0 when "h" is at HLCW and is equal to 0.0 when "h" is at the lowest branch measured. At HLCW, RCWh and RCWXh are both equal to one.

To develop an appropriate model form, $RCWX_h$ data for the 49 branches at crown base were graphed across DBH, H, CR, CR DBH/H and DACB. These graphs revealed the following relationship with DACB:

$$RCWX_{HCB} = b_9 + (1 - b_9) \cdot (1 - e^{(b_{10} \cdot DACB)}) \qquad [4]$$

A graph of $RCWX_h$ across DACB for the remaining 19 branches indicated that the data was highly variable with no strong trends. Therefore, the following linear relationship was used to characterize $RCWX_h$ between HCB and HLCW:

$$RCWX_h = RHX_h + (1 - RHX_h) \cdot RCWX_{HCB} \qquad [5]$$

Statistical Procedures

Parameter estimates, their standard errors and associated fit statistics were estimated using the linear or nonlinear regression routines found in S+. (Venables and Ripley, 1994) (S-plus, 1995) Homogeneity of variance for each model was examined by plotting the residuals across the predicted dependent variable and evaluating it for trends. Weighted regression was used if trends were found.

The dataset used to model RCW_h is composed of multiple branches measured on individual trees. As a result, autocorrelation may occur between the individual branches on a tree. The possible presence of autocorrelation was ignored in this study because: (1) presence of autocorrelation does not bias the parameter estimates, and (2) the sample of 1069 branches was divided among 108-individual trees such that no more than 10 branch measurements were taken from a given tree which should reduce the impact of autocorrelation on the variance estimates.

Normality of the residuals for each model was evaluated using the skewness and kurtosis statistics calculated in S+ using *rms.curv* from Venables and Ripley (1994).

Results

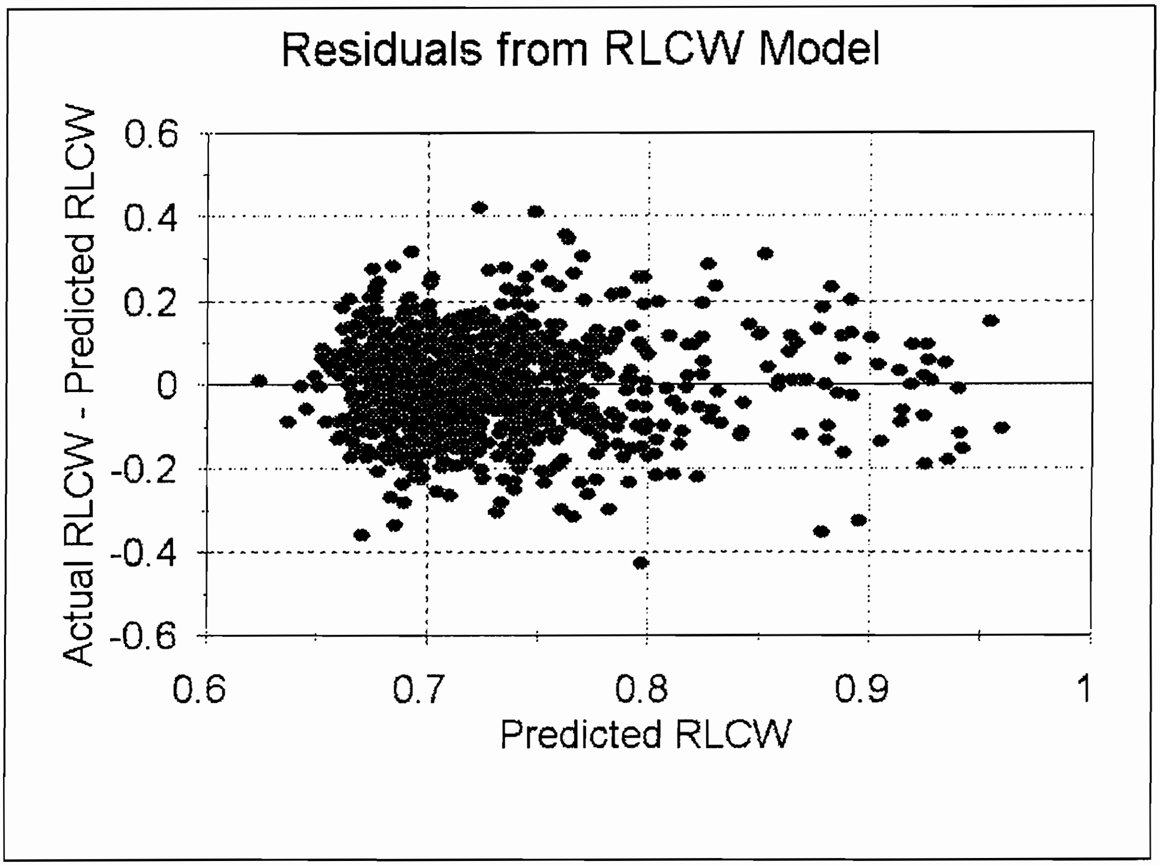
Modeling Relative Largest Crown Width

Equation [2] was fit to the 894 observations available for modeling RLCW using non-linear, least squares regression. The resulting parameter estimates and associated standard errors are presented in Table 8. The mean squared error (MSE) for equation [2] was 0.0138546, and the adjusted coefficient of determination ($\overline{R}^2$) was 0.1778. A graph of residuals over predicted RLCW indicated that the variance of the residuals was homogeneous (Figure 3). Finally, the skewness and kurtosis statistics indicated that the residuals were normally distributed.

Table 8. Summary statistics for the parameters in the RLCW equation [2].

Parameter	Value	Standard Error	t-value
b_4	0.572274	0.0406289	14.09
b_5	0.0145806	0.00288395	5.06
b_6	2.51611	0.728231	3.46

Figure 3. Residuals for the RLCW model.

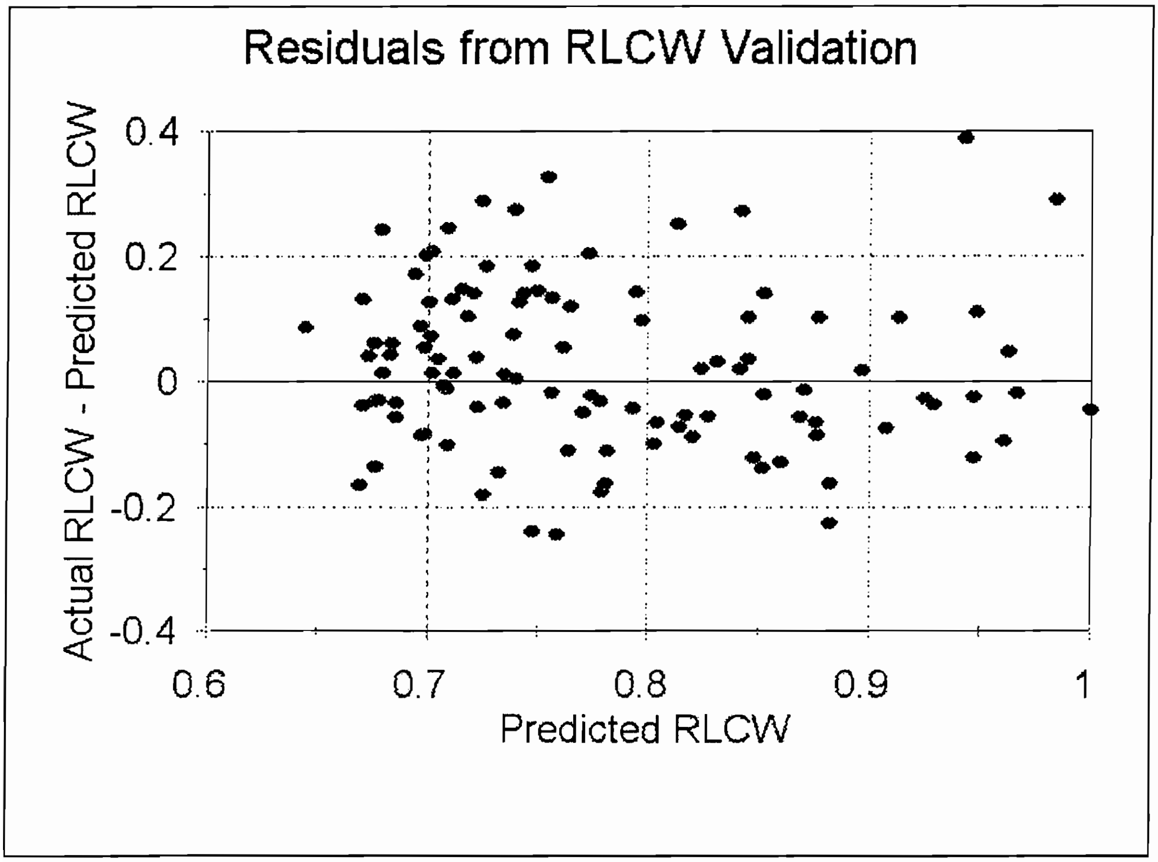


Validation of the Relative Largest Crown Width Model

Of the 108 trees in the crown profile dataset, 107 had been measured for LCW_{s1} and LCW_{s2} while still standing. Because these trees were measured independently from the trees in the RLCW modeling dataset, they could be used to validate the unbiasedness of the RLCW equation. Residuals were formed by subtracting observed RLCW from predicted RLCW and the mean and variance of the residuals were computed. A t-test was then used to test for bias (i.e., that the mean residual was zero).

The resulting mean residual was 0.02446, which was not significantly different from zero at the 95% confidence level (t-statistic = 1.939 with 106 degrees of freedom). A plot of the residuals across predicted RLCW showed no evident trend (Figure 4). Therefore, the RLCW model is judged to be an unbiased estimator for RLCW in the independent crown profile dataset.

Figure 4. Residuals for RLCW validation dataset.



Modeling Height to Largest Crown Width

$HLCW_f$ is estimated using two equations. The first estimates the probability that LCW_f does not occur at HCB. The second estimates DACB, for trees where LCW_f does not occur at HCB.

Modeling the Probability that Distance Above Crown Base is Greater than Zero

Of the 102 trees in which LCW could be defined, 49 trees had a DACB greater than zero. The resulting mean estimator of the *Probability DACB > 0* is:

$$Probability\ DACB > 0 = 0.480392$$

The standard error of the mean is 0.049469 for this estimator.

Modeling DACB, Given that DACB > 0

Using the 49 trees with a DACB greater than zero, the mean relative position of DACB within the crown was computed resulting in the following estimator:

$$\frac{DACB}{CL} = 0.18414$$

The standard error of the mean was 0.015438 for this estimator.

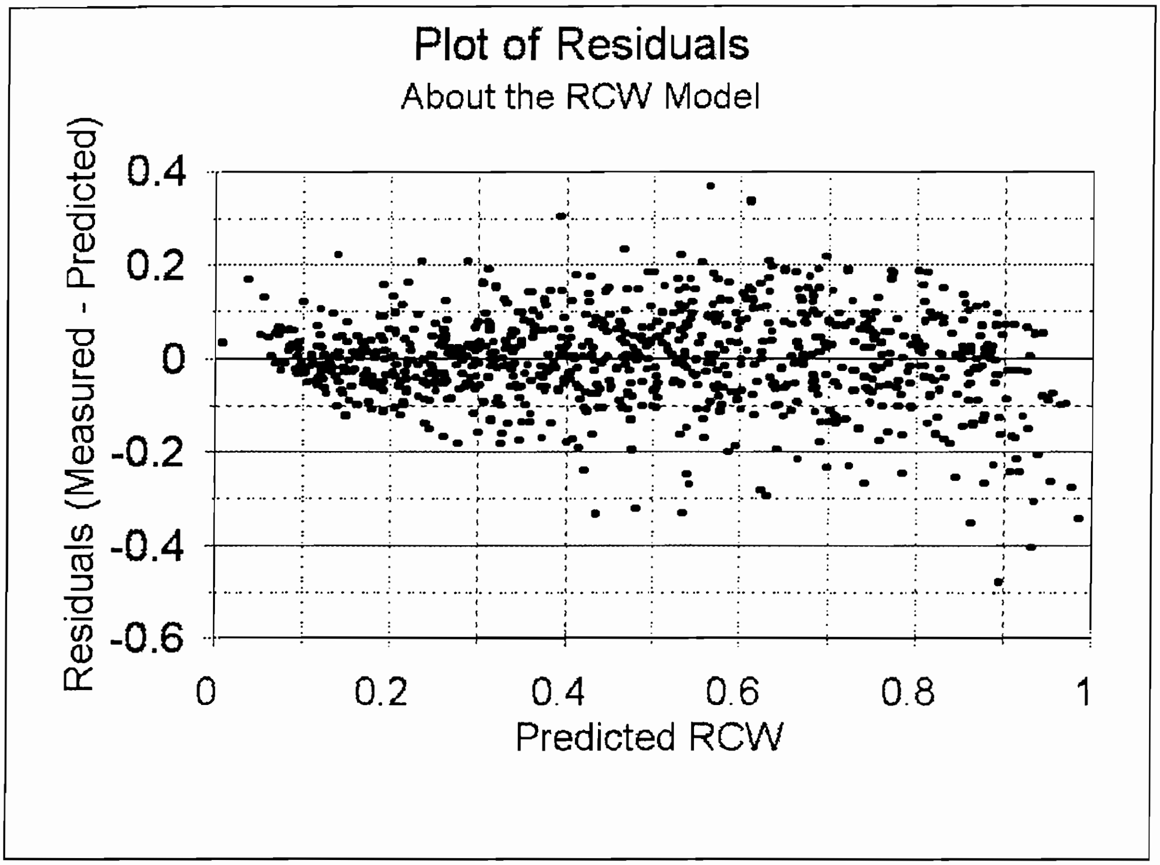
Modeling Crown Profile

Equation [3] was fit to the 839 branch measurements above $HLCW_f$ that were available for modeling RCW_h using non-linear, least squares regression. The resulting parameter estimates and associated standard errors are shown in table 9. The MSE for equation [3] was 0.009947, and the $\overline{R}^2$ was 0.84408. A graph of residuals over predicted RCW_h (Figure 5) indicated homogeneous variance. Further, calculations of the skewness and kurtosis statistics indicate that the residuals are normally distributed.

Table 9. Summary statistics for the parameters in the RCW_h equation [3].

Parameter	Value	Standard Error	t-value
b_7	0.9472800	0.03431870	27.60240
b_8	-0.0280132	0.00512314	-5.46798

Figure 5. Residuals for the RCW_h model.



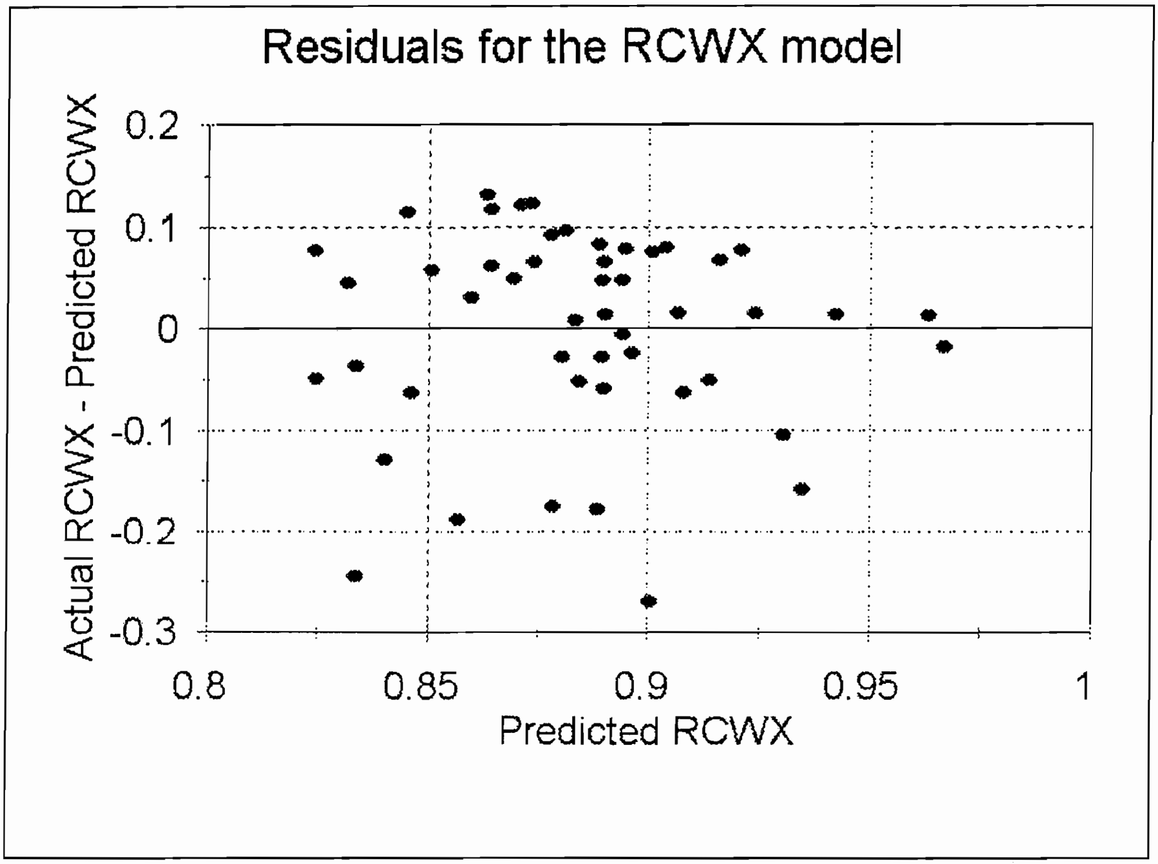
Modeling $RCWX_h$

Equation [4] was fit to the 49 observations measured a crown base using non-linear, least squares regression. The resulting parameter estimates and associated standard errors are presented in table 10. The MSE for equation [4] was 0.0098858, and the $\overline{R}^2$ was 0.0662. A graph of residuals over predicted $RCWX_{HCB}$ indicated that the variance of the residuals was homogeneous (Figure 6). Finally, the skewness and kurtosis statistics indicated that the residuals were normally distributed.

Table 10. Summary statistics for the parameters in the RCWX$_h$ model.

Parameter	Value	Standard Error	t-value
b_9	0.8187870	0.0379018	21.6029
b_{10}	-0.0644642	0.0323258	-1.9942

Figure 6. Residuals for the RCWX$_{HCB}$ model.



Discussion

Crown profile can be modeled as the maximum crown width of an open grown tree (MCW) and two modifiers. One modifier reduces maximum crown width to represent the largest crown width of a stand grown tree (RLCW), and the other modifier reduces the largest crown width of the tree to represent crown widths throughout the crown (RCW_h). This produces the following equation form for predicting crown width at any point in the crown:

For h greater than or equal to HLCW,

$$CW_h = M\hat{C}W \cdot RL\hat{C}W \cdot R\hat{C}W_h + \epsilon \qquad [6a]$$

For h less than HLCW,

$$CW_h = M\hat{C}W \cdot RL\hat{C}W \cdot RC\hat{W}X_h + \epsilon \qquad [6b]$$

This "potential and modifier" approach is advantageous for two reasons. First, individual components of the model can be calibrated to other geographic regions. For example; existing MCW equations can be substituted for the one developed in this study, or a new MCW equation can be constructed. A different RLCW equation could also be substituted or developed. It is considerably less expensive to develop new MCW or RLCW equations than it is to create an entire new CW_h model. Second, the independent variables required to implement these models (i.e., DBH, H and CR) are inexpensively measured, are readily found in many current forest inventories, and are available on the array of tree attributes currently inputted into the ORGANON model. Approaches which require other, more difficult or expensive to obtain, data would have considerably less utility. Examples of such data include the need to locate individual whorls on a tree or the need to spatially locate trees within a stand.

The ORGANON model is a distance independent, individual tree, growth and yield model and, therefore, it does not require data on the placement of individual trees. As a result, a crown profile model requiring tree coordinates for placement in the stand would be unusable in ORGANON. Further, recording the location of individual trees is expensive, time consuming, and data on tree placement is unlikely to be a part of most forest inventories.

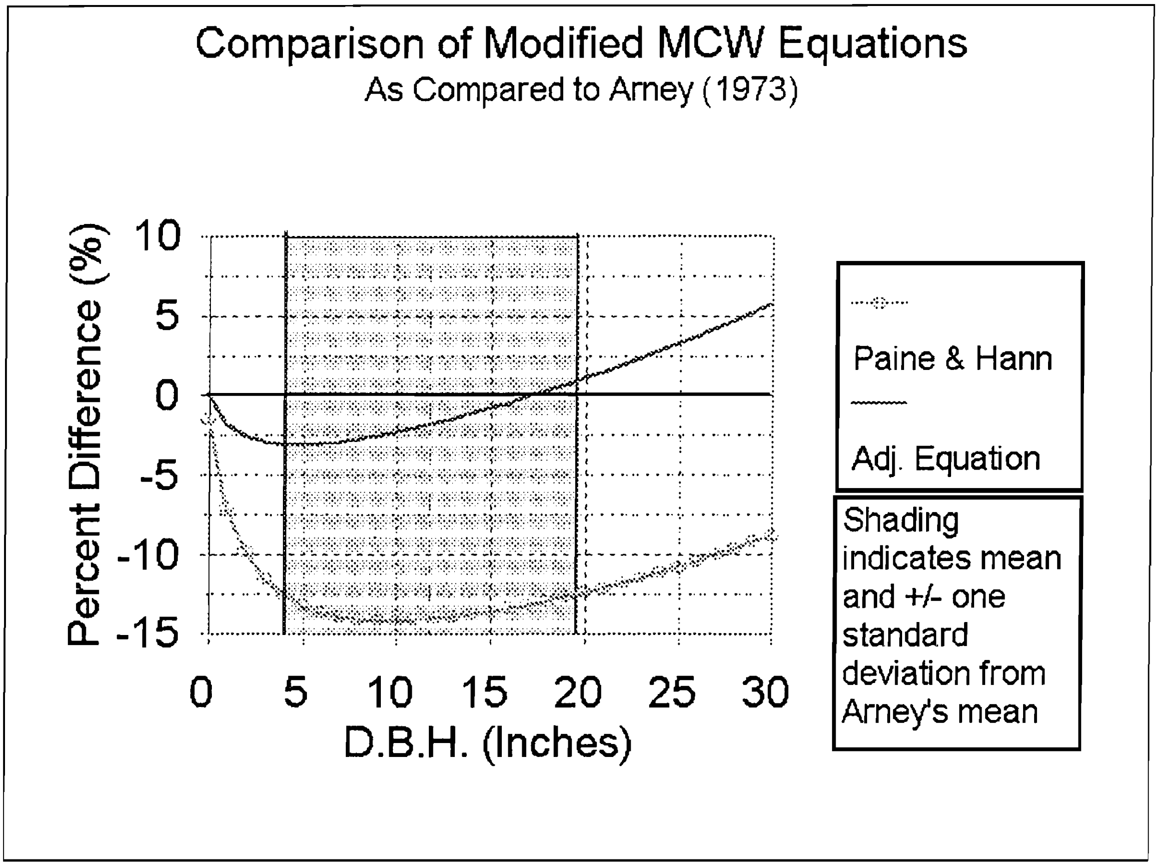
It has long been known that stand density and site quality can affect the crown of a tree (Larson 1963), and, as a result, it has been suggested that these attributes should be included in a crown profile model. However, the tree specific combination of DBH, H, and CR can be considered to represent the effects of stand and site characteristics. (Roeh, 1993) Curtis and Reukema (1970) noted that "trees of given DBH or total height but in different spacings are quite similar in other stem and crown dimensions." Finally, Smith and Bailey (1964) explained as much as 87% of variation in the largest crown widths (LCW) of Douglas-fir in interior British Columbia, and as much as 90% of variation in LCW of Douglas-fir in coastal British Columbia, using only DBH and height as independent variables. This suggest that incorporating stand and site variables into crown profile models is unnecessary.

Adjusted Maximum Crown Width Equation

All published MCW models for Douglas-fir have used a quadratic model form. This creates a situation where MCW peaks and begins to decrease with increasing DBH. Therefore, one measure of the utility of the adjusted MCW equation [1] is the point where this peak occurs. Equation [1] produces a peak MCW at a DBH of 83.5 inches, which is an improvement over the Arney (1973) model which peaks at a DBH of 54.2 inches.

Another way to examine the effectiveness of equation [1] is in terms of how closely it matches the Arney (1973) model over the range of tree sizes in his dataset. Figure 7 shows the percent difference between equation [1] and the Arney (1973) equation. The mean DBH and ± one standard deviation for the Arney (1973) dataset is shown to illustrate the range tree sizes used in fitting the Arney (1973) model. The maximum difference between equation [1] and Arney's (1973) model was -3.1% over this range of DBH's, and the maximum difference at 30 inches DBH was +5.8%. Therefore equation [1] retains much of the predictive power of Arney's (1973) model over Arney's (1973) scope of inference.

Figure 7. Comparison of maximum crown width equations.



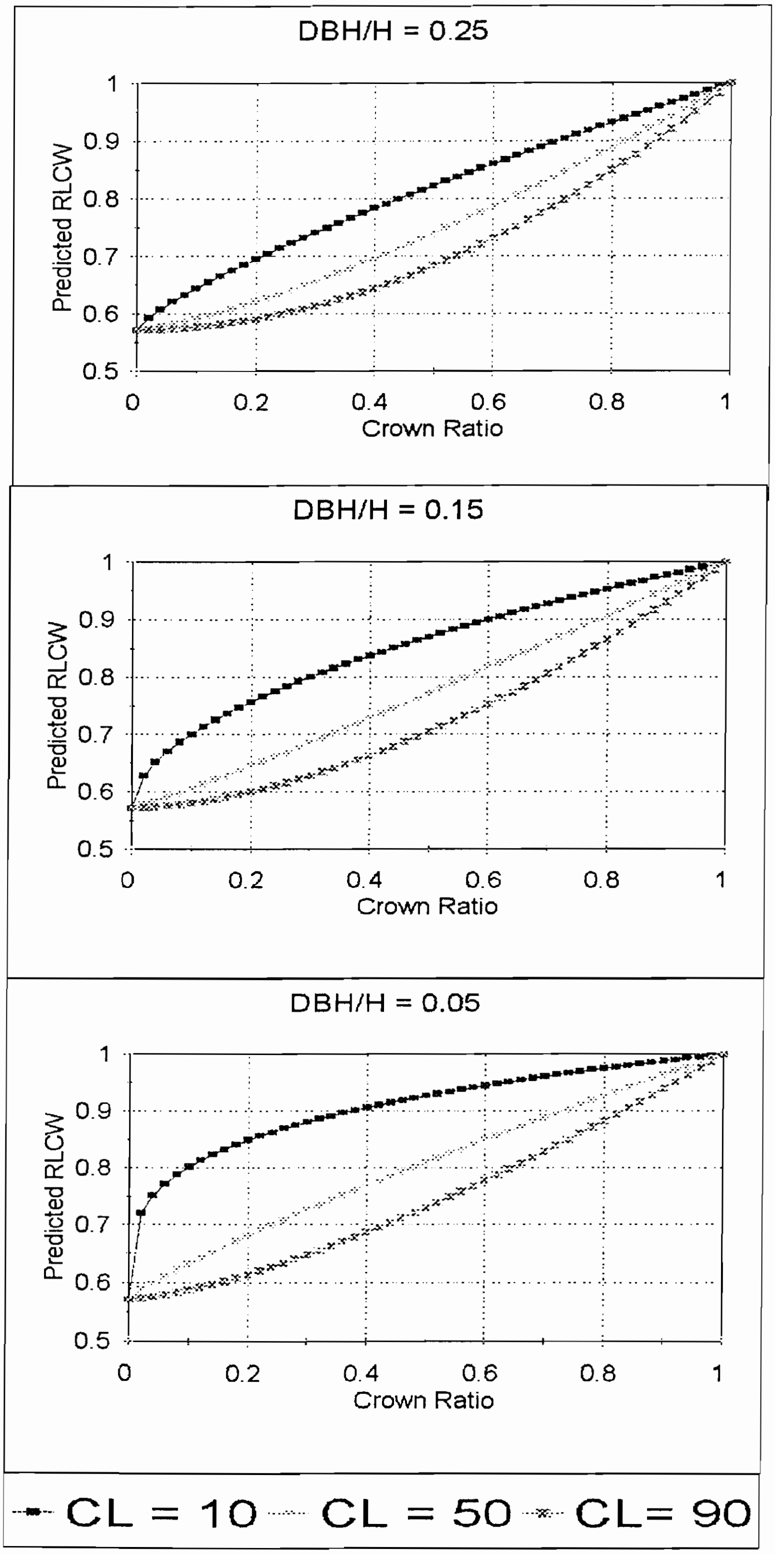
Modeling Relative Largest Crown Width

The RLCW model form developed by both Ritchie and Hann (1985) and Biging and Wensel (1990) used a simple constant as the power term on CR. Dubrasich et. al. (1996), on the other hand, included the tree attributes of CL and DBH/H in their power term. Both model forms were tested, and the inclusion of tree attributes were found significant. The final equation [2] was then successfully validated on an independent data set.

A graph of equation [2] indicates that RLCW increases as crown ratio increases, and that crown length and DBH/H modify the shape of the RLCW curve (Figure 8). All values for CL and DBH/H produce a predicted RLCW of 1.0 for a tree with a crown ratio on 1.0 (open grown).

Equation [2] predicts that a large CL results in a smaller RLCW than a small CL. For a given CR, a larger CL indicates a taller tree and, therefore, a greater chance for crown damage due to abrasion caused by wind whipping. (Oliver and Larson 1996) Equation [2] also predicts that a small DBH/H results in a larger RLCW than a large DBH/H. This ratio (or its reciprocal) has often been used in the European literature as a measure of form. (Assmann, 1970) Trees under stress from competition for sunlight would be relatively taller for a given DBH, thus DBH/H would be smaller. Therefore, trees with a small DBH/H ratio are often suppressed, understory trees and trees with a large DBH/H are dominant, overstory trees. Douglas-fir generally has a high degree of epinastic control. (Oliver and Larsen, 1996) As a tree with strong epinastic control receives shade from overtopping trees (i.e., as it becomes suppressed and its DBH/H becomes smaller), the tree loses its epinastic control and develops a broad, flat-topped or "umbrella like" appearance. (Oliver and Larsen, 1996).

Figure 8. Behavior of predicted RLCW with changes in CR, CL, and DBH/H.



Modeling Height to Largest Crown Width

Contrary to previous findings or assumptions, the LCW_f in a tree does not necessarily occur at crown base. In this study, LCW_f occurred above crown base in approximately 48-percent of the trees. The point in the crown that LCW_f occurred was, on average, approximately 18-percent above crown base.

Modeling Crown Profile

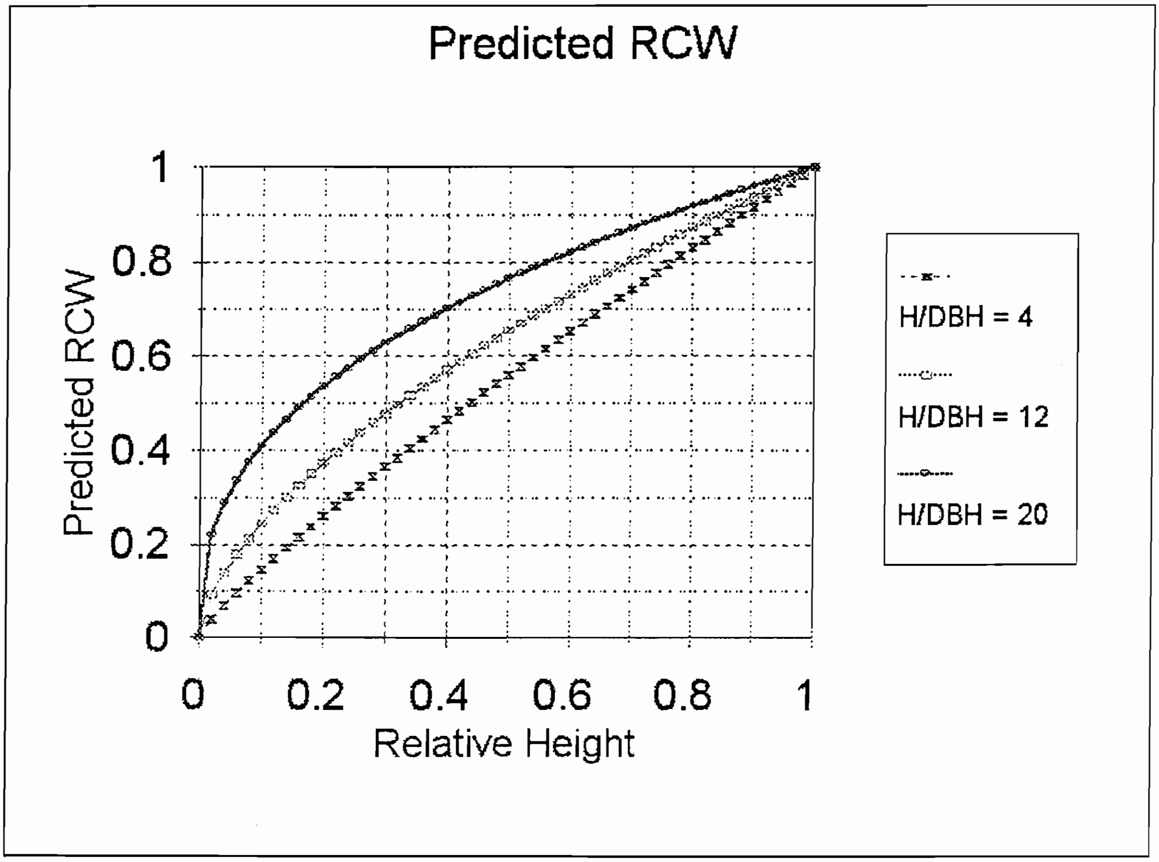
In order to model crown profile, a series of crown widths at heights throughout the crown were calculated using branch attributes. Branch length (BL) was readily estimated, but vertical angle (VA) was considerably more difficult to model. While VA may approach 90 degrees for branches near the crown base, it becomes more acute with increasing height. A simple constant value for VA is inadequate for estimating crown width from branch attributes in the upper crown. Further, VA is surprisingly influential in determining the crown width produced by a given branch.

Crown profile throughout the tree was represented as the ratio between crown width at height "h" (CW_h) and the largest crown width (LCW) of the tree (i.e. RCW_h). Further, RCW_h was modeled as two populations, crown profile above LCW (RCW_h) and crown profile below LCW ($RCWX_h$).

Modeling Crown Profile Above Height to Largest Crown Width

A graph of equation [3] over RH_h for three values of H/DBH is shown in figure 9. As expected, RCW_h is equal to 1.0 when RH_h is equal to 1.0, and that RCW_h decreases to 0.0 as RH_h approaches 0.0. Trees with a large H/DBH are more "rounded" in shape than trees with a small H/DBH.

Figure 9. Behavior of predicted RCW_h with exchanges in relative height and H/DBH.



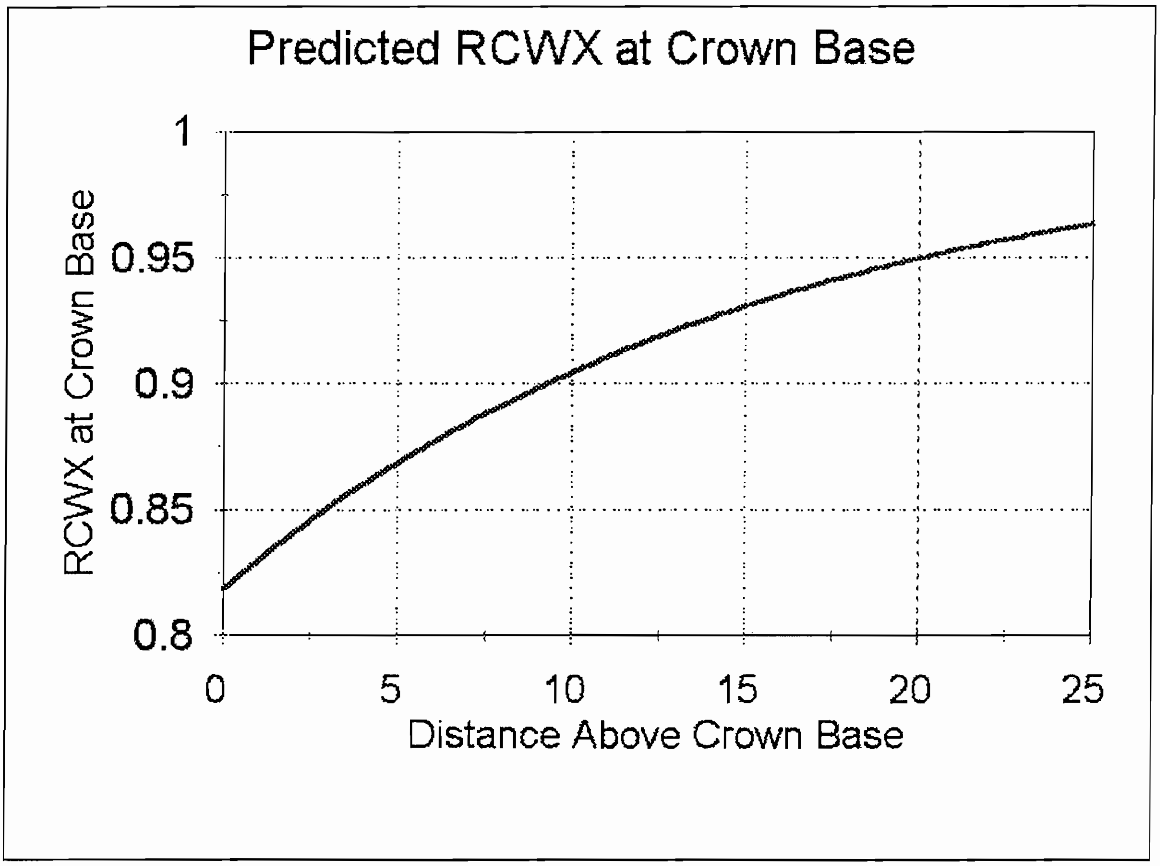
The H/DBH (or the reciprocal of DBH/H) in the power term produces model behavior where trees with a large H/DBH have a relatively larger crown width at the tree tip as compared to trees with a smaller H/DBH. Again, this probably reflects the effect of competition within the canopy upon epinastic control (i.e., trees with a large H/DBH will be more "umbrella" shaped).

Modeling Crown Profile Below Height to Largest Crown Width

Unfortunately, the sample of branches available for modeling contained few whorls below HLCW and, of those few, most were located at HCB. As a result, equation [4] was developed to predict $RCWX_{HCB}$ and intermediate values are predicted using linear interpolation (equation [5]).

A graph of equation [4] indicates that the difference between RLCW and $RCWX_{HCB}$ becomes smaller as DACB increases (Figure 10).

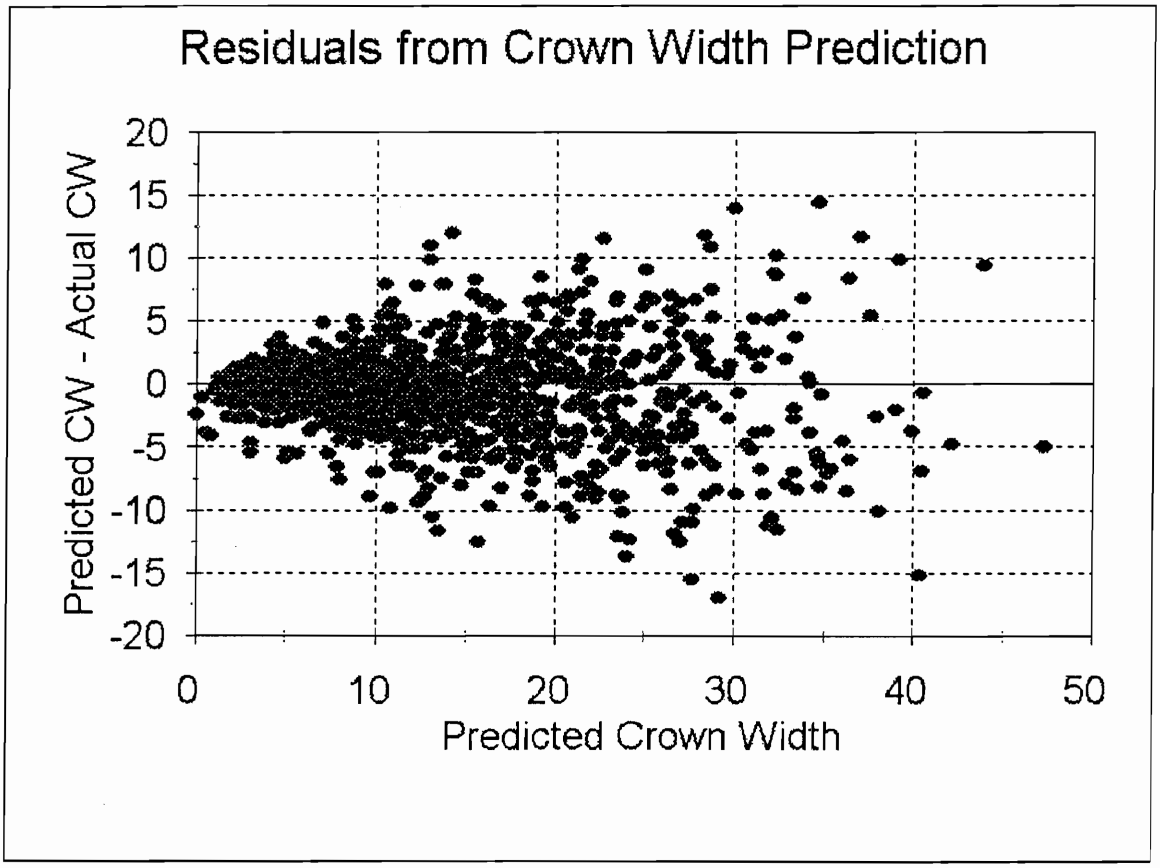
Figure 10. Behavior of predicted $RCWX_{HCB}$ with a change in DACB.



Predicting Crown Widths

In order to assess how equations 6a and 6b perform, we used the component equations to predict CW_h at all 1070 whorls measured on all 108 trees. Residuals were calculated by subtracting actual CW_h from predicted CW_h. Figure 11 shows a plot of the resulting residuals over predicted CW_h.

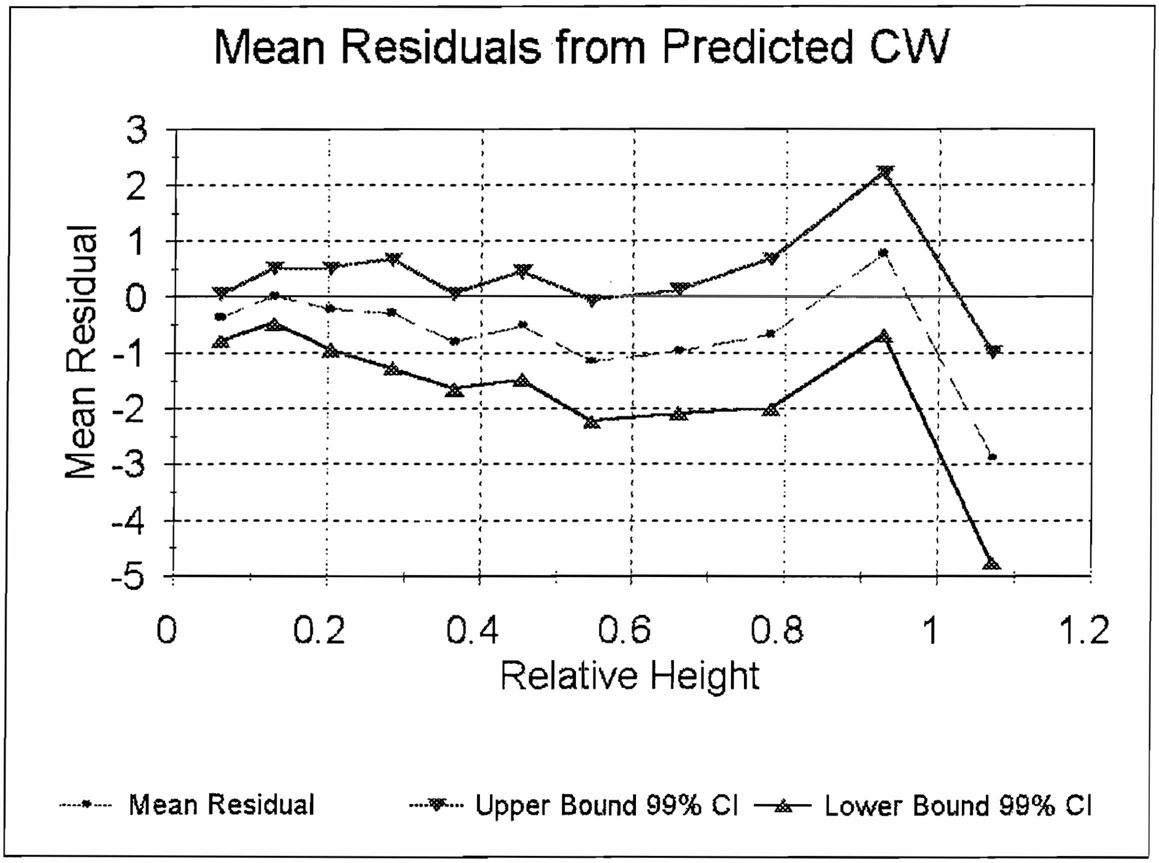
Figure 11. Residuals from predicted CW_h.



The overall mean residual underestimates CW_h by 0.58 feet. This represents 3.92% of the mean CW_h of 14.77 feet. Even with this bias, models 6a and 6b explain 84% of the variation in CW_h more than a simple mean (i.e. an $\overline{R}^2$ of 0.84). As expected, residuals increase as predicted CW_h increases.

To examine the fit of the model more closely, we divided the residuals into cells based on RHh and computed a mean residual and approximate upper and lower bounds to a 99% confidence interval about the mean for each cell. Figure 12 shows a plot of the resulting mean residuals and confidence intervals. There appears to be a tendency to underestimate CW_h in the middle of the crown. The base of the crown also has the problem of underestimation.

Figure 12. Mean residuals and confidence intervals for predicted CW_h.



Conclusion

It is apparent that CW_h is modeled more accurately at heights at and above HLCW than at heights below HLCW. However, the overall system of equations is highly predictive (i.e. an $\overline{R}^2$ of 0.84) indicating that the influence of relatively poorer fits below HLCW is limited. In ORGANON, height growth calculations are not affected by CW_h values for heights below HLCW. This means that ORGANON predictions are not influenced by crown profile predictions based on $RCWX_h$. Thus, the relatively poorer accuracy of CW_h predictions below HLCW is insignificant in ORGANON calculations.

Under certain circumstances, it may be necessary to predict crown profile for purposes which are more sensitive to CW_h below HLCW. For example, CW_h below HLCW might be more important in calculating canopy porosity than crown closure. In future studies, equations which predict CW_h below HLCW might be more thoroughly explored. A simple change of the measurement procedure used in this study which might assist in this effort would be to sample whorls at the base of the crown more heavily than they were measured in this study. In this study, whorls were sampled at equally spaced intervals throughout the crown. A similar number of trees might provide better data for modeling CW_h below HLCW if all whorls in the lower portion of the crown (for example; the lower 25% of the crown) were sampled.

The structure of the equations in this study was designed to facilitate developing crown profile models which apply to other geographic areas. Since the ORGANON model has a version designed for use in southwestern Oregon, future studies might expand the crown profile equations in this study to apply to southwestern Oregon.

A final recommendation based on the experiences of this study is to attempt to avoid the problem of modeling the relationship between VA and VA*. Vertical angles as measured while the tree is felled (VA*) are difficult to accurately measure. Unfortunately, removing whorls from a tree's bole and standing them vertical is also difficult. Therefore, at least two persons should work together to get VA measurements. For nearly all whorls where the bole has a large DOB, two people may find standing the whorl vertically to be impossible. In these situations, special equipment such as winches or even power equipment with grappling arms or a simple crane may be required. Standing each whorl vertically might require more time or expense, but the increased accuracy may outweigh the expense or limitation in sample size.

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Appendices

Appendix A. Coefficient Values for Regressions Modeling BL*

Regression No.	Sum of Squared Error	Degrees of Freedom	f-statistic	P-value	Regression Coefficient
1	29.24	9.00	0.49	0.12	4.92
2	58.53	7.00	1.26	0.73	4.59
3	12.52	9.00	0.21	0.01	6.33
4	61.24	9.00	1.03	0.58	3.83
5	14.09	7.00	0.30	0.05	3.84
6	31.29	9.00	0.52	0.14	5.15
7	57.06	9.00	0.96	0.52	4.12
8	30.61	9.00	0.51	0.13	6.16
9	25.98	7.00	0.56	0.21	4.37
10	35.58	9.00	0.60	0.20	5.02
11	41.20	9.00	0.69	0.28	6.41
12	2.79	6.00	0.07	0.00	5.32
13	13.47	9.00	0.23	0.01	5.37
14	11.15	9.00	0.19	0.00	5.86
15	3.97	9.00	0.07	0.00	6.09
16	6.63	9.00	0.11	0.00	5.38
17	14.54	9.00	0.24	0.01	4.50
18	3.60	9.00	0.06	0.00	5.20
19	27.73	9.00	0.46	0.10	4.22
20	19.63	9.00	0.33	0.03	5.07

Regression No.	Sum of Squared Error	Degrees of Freedom	f-statistic	P-value	Regression Coefficient
21	6.84	9.00	0.11	0.00	7.27
22	7.87	9.00	0.13	0.00	5.35
23	17.45	9.00	0.29	0.02	5.31
24	10.17	9.00	0.17	0.00	6.14
25	4.77	9.00	0.08	0.00	8.42
26	14.63	9.00	0.25	0.01	4.88
27	28.76	9.00	0.48	0.11	6.17
28	10.63	9.00	0.18	0.00	5.69
29	9.91	9.00	0.17	0.00	5.12

Appendix B. S+ Code for Constructing BL* for Broken Tip Branches

```
## MAKE 29 INDIVIDUAL REGRESSIONS
## This reads raw data on all 1070 whorls from "DATA_2.DAT"
## Then reads a list of 29 combinations of STAND and WHORL from "LIST.DAT"
## Regressions are calculated for each of the 29 combinations....
##                ...using only data from the individual tree in question

DataSet <- read.table("DATA_2.DAT",header=T)
List    <- read.table("LIST.DAT",  header=F)
attach (DataSet, 2)
bigreg  <- matrix(nrow=29,ncol=5)
n       <- 1.25

for (index in 1:29)
        {

        cat("----------------------------------------------------","\n")
        cat("Stand number is:",List[index,1],"\n")
        cat("Tree number  is:",List[index,2],"\n")

      tip <- DataSet$TIP[(DataSet$STAND == List[index,1])&(
                  DataSet$TREE == List[index,2])]
      dia <- DataSet$DIA[(DataSet$STAND == List[index,1])&(
                  DataSet$TREE == List[index,2])]
      bent <- DataSet$BENT[(DataSet$STAND == List[index,1])&(
                  DataSet$TREE == List[index,2])]

        Results <- lm(bent ~ I((dia^n)-(tip^n)) - 1)

        print(Results)

        bigreg[index,1] <- deviance(Results)
        bigreg[index,2] <- Results$df.residual
        bigreg[index,5] <- coefficients(Results)
        }

        Results <- lm(BENT ~ I((DIA^n) - (TIP^n)) - 1)

        my.df <- Results$df.residual
        my.MS <- deviance(Results) / my.df

      for(i in 1:29) {
```

```
    bigreg[i,3] <- (bigreg[i,1] / bigreg[i,2])/my.MS
    bigreg[i,4] <- pf(bigreg[i,3],bigreg[i,2],my.df)
  }

##     Now the matrix bigreg has the following variables
##     [,1] <- deviance(Results) <- Sum of Squares for the individual regression
##     [,2] <- df.residual <- Degrees of freedom  from the individual regression
##     [,3] <- F-stat                that applies to the individual regression
##     [,4] <- P-value    derived from the F-stat for the individual regression
##              ...a p-value < 0.05 indicates that the individual model is
##              ...significantly different
##              ...(i.e. the individual model explains more variance)
##     [,5] <- the regression coefficient for the individual regressions
##              ...note that these are all forced through zero:
##              ...calculating several coeffs. may not fit into the matrix properly


##     f.stat <- the F-stat that compares the total of the 29 regressions to
##              ...the regression using all 1070 whorls and making no allowances
##              ...for tree to tree variation

##     p.value <- the p-value associated with f,stat
##              ...a p.value < 0.05 indicates:
##              ...the two models are significantly different
##              ...(i.e. there is more variance in the big 1070 point model
##                     ....than the 29 small ones

  f.stat <- (sum(bigreg[,1]) / sum(bigreg[,2]))/my.MS
  p.value <- pf(f.stat,sum(bigreg[,2]),my.df)

    cat("---------------------------------------------------","\n")
    cat("The overall p-value is :",p.value,"\n")

    detach(2)
```

Appendix C. Parameter Estimates Used in Roeh (1993).

Dependent Variable	Parameter	Value
BD	a_1	8.5795
BD	a_2	-1.5846
BD	a_3	0.0677
VA	a_4	84.3782
VA	a_5	-0.1666
VA	a_6	0.0230
BL	a_7	3.3543
BL	a_8	0.5935
BL	a_9	-0.0745

Appendix D. Parameter Estimates Used in Honer (1971).

Parameter	Value	
	balsam fir	black spruce
a_1	0.58939	0.57332
a_2	-0.00592	-0.00414
a_3	-0.38653	-0.32997
a_4	0.00658	0.00356