AN ABSTRACT OF THE DISSERTATION OF

Porntip Swangrojn for the degree of Doctor of Philosophy in Mathematics Education presented on May 9, 2003.

Title: Solving Algebra Word Problems: Solution Strategies Thai Students Used and Potential Connections with Teachers' Instructional Strategies.

Abstract Approved

Signature redacted for privacy.

Dianne K. Erickson

The main purpose of this study was to investigate and describe strategies that Thai ninth grade students use to solve algebra word problems. The second purpose of this study was to investigate teachers' instruction during word problem solving lessons. Three mathematics teachers and one of each of their classes participated in this study. Word problems were developed for the pretest, the posttest, and the interview from prior research and Thai Standards. The classroom teaching of the three teachers of the 118 participating students was observed during word problem instruction. Six students from each teacher's class were asked to participate in the interview sessions to solve five word problems following instruction.

The results show that overall Thai ninth grade students in this study were somewhat successful at solving word problems. However, they were less successful at solving problems involving two unknown variables requiring different representations. Thai ninth grade students used either algebraic strategies or non-algebraic strategies to solve word problems. Two sub-strategies in the algebraic strategies were found: equations based on comprehensive representations and equations based on poor representations. Five sub-strategies in the non-algebraic strategies were found: verbal or written arithmetic, a drawing or graph, trial and error, a part-and-whole strategy, and a comparison strategy. The results indicate that unsuccessful problem solvers had difficulty translating and representing problem situations into equations by using variables and symbols. Additionally, few students developed a repertoire of strategies to solve the word problems.

The results from the observation indicated that the three teachers had different styles in teaching students to solve word problems. Two teachers used direct instruction with little students' participation. The third teacher used a less directed role and allowed her students to actively participate by asking and answering questions, and participating in collaborative groups. Potential connection between teaching and learning were evidenced by students' performance in a class taught by a teacher who used a less directive role improved much more than student's performance in a class taught by a teacher who used a direct instruction exclusively.

Solving Algebra Word Problems: Solution Strategies Thai Students Used and Potential Connections with Teachers' Instructional Strategies

by

Porntip Swangrojn

A DISSERTATION

submitted to

Oregon State University

in partial fulfillment of
the requirement for the degree of

Doctor of Philosophy

Presented May 9, 2003
Commencement June 2004

Doctor of Philosophy dissertation of Porntip Swangrojn presented on May 9, 2003

APPROVED:

Signature redacted for privacy.

Major Professor, representing Mathematics Education

Signature redacted for privacy.

Chair of Department of Science and Mathematics Education

Signature redacted for privacy.

Dean of Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Signature redacted for privacy.

Porntip Swangrojn, Author

ACKNOWLEDGMENTS

I wish to express my sincere thankfulness and appreciation to Dr. Dianne K. Erickson, my major advisor, for her patience, valuable advice, and guidance in planning and writing of this dissertation from the very beginning to the end. Grateful acknowledgments are also given to Dr. Thomas P. Dick, Dr. Barbara E. Edwards, Dr. Margaret L. Niess, Dr. Lawrence B. Flick, and Dr. Daniel J. Arp for honoring me by serving as committee members and advice in writing of this dissertation. I also wish to thank Dr. Edith S. Gummer and Dr. Larry G. Enochs, for their time and comments in writing of the proposal of this dissertation.

I would like also to give thanks to the Royal Thai Government, Chiang Mai University, Education Opportunities Program (EOP), School Sciences and Mathematics Journal (SSM), and Department of Sciences and Mathematics Education (SED) for their financial support during my study at Oregon State University. Many thanks to the school, teachers, and students participated in this study.

An extraordinary thank to my best friends for their lovingly comfort during working on this thesis. I wish to extend my deepest gratefulness to my father and my mother for their loving support and encouragement in this scheme. My thanks to them cannot be fully articulated in words. A huge thank to my relatives for their support as well.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF FIGURES (Continued)

LIST OF TABLES

LIST OF TABLES (Continued)

DEDICATION

This dissertation is dedicated to the loving memory of my grandparents.

Mr. Pair and Mrs. Poung Swangrojn
Mr. Ta and Mrs. Kumsaj Chaisith

Solving Algebra Word Problems: Solution Strategies Thai Students Used and Potential Connections with Teachers' Instructional Strategies

CHAPTER I
THE PROBLEM

Introduction

Algebra is the branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures, and with operating on those structures (Kieran, 1992). Algebraic competence is important in adult life, both on the job and as preparation for postsecondary education. Competency in algebra is important in today's society for two reasons. First, because successful completion of algebra coursework is often a requirement for high school graduation (Chambers, 1994) and for an undergraduate degree, algebra may serve a gate-keeping function for secondary and postsecondary education. Second, algebra skills are vital in a wide variety of jobs. For example, distribution and communication networks, laws of physics, population models, and statistical results can all be represented in the symbolic language of algebra (National Council of Teachers of Mathematics [NCTM], 2000).

Students' algebraic competency can be developed in several ways. One way is to learn to use algebra knowledge through solving algebra word problems. Therefore, one goal of teaching algebra is to help students learn to solve algebra word problems (Mathews, 1997). Improving student performance on algebra word problems and the fundamental skills associated with solving algebra word problems is considered to be critically important by most mathematics educators (NCTM, 1989), as algebra has become an entry-level skill for most scientific, business, and technical jobs in many cultures. For example, the NCTM standards (NCTM, 1989) state that an "understanding of algebraic representation is a prerequisite to further formal work in virtually all mathematical subjects" (p.1). Furthermore, the NCTM standards (NCTM, 2000) stated that the ability to represent situations with algebraic quantities is a central skill that is a prerequisite to understanding many areas of mathematics. In addition, since science and engineering courses emphasize word problems, achievement in a technical field requires that students be able to translate between verbal representations and algebraic representations (Mestre, Gerace, & Lochhead, 1982). Therefore, being able to use algebraic knowledge to solve algebra word problems is an essential skill for most students.

Students in the United States had considerable difficulty with algebra word problems and that few high school seniors had mastered the fundamentals of algebra. For example, in 1979, a test of all twelfth graders in California public schools revealed that more than one-third were unable to correctly solve simple story problems such as the following (California Assessment Program, 1979):

> "An astronaut requires 2.2 pounds of oxygen per day while in space. How many pounds of oxygen are needed for a team of 3 astronauts for 5 days in space?"(Correct answer is 33 pounds).

Equally troubling results had been reported in national surveys of mathematics problem solving in the United States, such as the Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Lindquist, & Reyes, 1980). For example, 71% of a large national sample of 17-year-olds were unable to solve the following problem:

> "Lemonade costs $0.95 for one 56-ounce bottle. At the school fair, Bob sold cups holding 8 ounces for $0.20 each. How much money did the school make on each bottle?"(Correct answer is $0.45 per each bottle).

Recently, the report from the National Assessment of Educational Progress (NAEP) mathematics assessment in 1996 (Reese, Miller, Mazzeo, & Dossey, 1997) and in 1999 (Campbell, Hombo, & Mazzeo, 2000) showed that the achievement of 17-years old United States students had improved since 1978. However, students did better on numerical operations and beginning problem solving than solving complex word problems and algebra. Students in several countries also showed difficulties in solving algebra word problems. Results from Third International Mathematics and Science Study (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996) showed that students in many countries did well on calculation but did not do well on the algebra word problems. For example, 73% of the United States eighth grade students (79% Thai, 90% Japanese, and 92% Korean eighth grade students) were able to manipulate the following equation:

> "If $3(x + 5) = 30$, then x =?" (Correct answer is $x = 5$)

However, only 43% of the United States eighth grade students (46% Thai, 57% Japanese, and 64% Korean eighth grade students) were able to represent the following algebra word problem with an equation:

"Juan has 5 fewer hats than Maria, and Clarissa has 3 times as many hats as Juan. If Maria has n hats, which of these represents the number of hats that Clarissa has?"(Correct answer is $3(n\text{-}5)$).

The difficulty with algebra word problems is also supported by empirical studies (Carpenter, et al., 1980; Clement, 1982; Clement, Lochhead, & Monk, 1981; Mathews, 1997; Mestre & Gerace, 1986; Mestre, et al., 1982). Many researchers had viewed student's difficulties in the translation and solution of algebra word problems as basically a problem in the student's handling of the verbal structure of the problem. For example, Clement (1982) had shown that, in problems where students were asked to read a sentence stating a relationship between two variables and then wrote an equation expressing that relationship, the students frequently wrote the reverse of what they intended. For example, in the following problem, only 37 % of students in an engineering major answered correctly.

"Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors."(Correct answer is $S = 6P$).

In a sample of 47 non-science majors taking college algebra, only 43% of the equations were correct. In addition, studies done in Thailand indicated that college students in the Faculty of Education in some universities had low achievement in solving algebra word problems (Uthairat & Viamoraphun, 1984) and eleventh grade students had difficulties with translation of word problems to algebraic symbols (Makanong, 1993).

Statement of the Problem

As discussed in the previous section, algebraic competence is important in adult life, both on the job and as preparation for postsecondary education. Learning to solve algebra word problems is one way that helps students develop their algebraic competence. However, several reports and empirical studies had showed that students in several countries tended to have difficulties with algebra word problems (Clement, 1982; Lochhead & Mestre, 1988; Makanong, 1993; Mestre & Gerace, 1986; Niaz, 1989; Uthairat & Viamoraphun, 1984).

From the previous discussion, one might ask "why are algebra word problems so difficult to solve?" In spite of years of training and practice in solving algebra word problems, students were unable to meet the challenge of solving algebra word problems. Lester (1983) identified three factors that were associated with difficulty in solving word problem solving: the structure of problems, individual differences, and teachers' instruction. Varieties of studies had focused on the structure of word problems (e.g., Carpenter, Fennema, & Franke, 1994; De Corte, Greer, & Verschaffel, 1996; De Corte, Verschaffel, & Pauwels, 1990; Koedinger & Nathan, 1998; Koedinger & Tabachneck, 1995; Lewis & Mayer, 1987; Muth, 1992; Riley, Greeno, & Heller, 1983). For example, Riley et al. (1983) found that the problem difficulty is strongly affected by the role (or position) of the unknown quantity within the problem statement.

Varieties of studies had focused on individual differences (e.g., Hall, Kibler, Wenger, & Truxaw, 1989; Hegarty, Mayer, & Monk, 1995; Kieran, 1988, 1992; Koedinger & Nathan, 1998; Malloy & Jones, 1998; Montague, Bos, & Doucette, 1991; Muth, 1992; Petitto, 1979; Silver, Shapiro, & Deutsch, 1993; Tabachneck, Koedinger, & Nathan, 1994; Verschaffel, De Corte, & Pauwels, 1992). For example, comprehension and representation of word problems were the most important factor that differentiated successful word problems solvers from unsuccessful word problem solvers (Hegarty, et al., 1995).

Some studies had focused on teaching methods that could help students learn to better solve word problems (e.g., Cardelle-Elawar, 1992, 1995; Mevarech, 1999; Qin, Johnson, & Johnson, 1995; van Essen & Hamaker, 1990). For example, van Essen and Hamaker (1990) investigated whether encouraging elementary students to generate drawings of arithmetic word problems facilitated problem-solving performance. The results from this study implied that a correct drawing helped students find the answer.

Although the studies on solving algebra word problems had been extensive in the past 30 years in several countries, there is little knowledge about solution strategies used by Thai students in solving algebra word problems. Consequently, the purpose of this present study was to investigate and describe strategies secondary school students in Thailand use to solve algebra word problems. Furthermore, teachers' instruction is one of the factors that might connect to students' word problem solving performance. Accordingly, the second purpose of this present study was to investigate teachers' instruction during word problem solving lessons. This present study sought to answer the following questions:

1. How successful are Thai students in solving algebra word problems?
2. Which strategies are used by Thai students to solve algebra word problems?
3. What are characteristics of classroom instruction during word problem-solving lessons?
4. What are potential connections between classroom instruction and students' word problem-solving performance?

Significance of the Study

In Thailand, studies on algebra problem solving are limited. Most research studies in Thailand were concerned about students' achievement in particular topics in mathematics. Those studies showed that Thai students had low achievement on solving algebra word problems (Makanong, 1993; Uthairath & Viamoraphan, 1984). However, they do not investigate solution strategies of Thai students. Therefore, a detailed study in this area is needed to better understand Thai students' solution strategies in solving algebra word problems. This present study was done in the hope that the results of the study could provide teachers with a better knowledge about how students are able or unable to solve algebra word problems, and understand the factors that make algebra word problems difficult for beginning algebra students. This may in turn enable teachers to assist students with learning to solve algebra word problems. In addition, the results from this present study would help teachers to choose and adapt algebra word problems wisely from instructional materials.

The information from this present study would help mathematics educators understand the cognitive process of Thai students in solving algebra word problems. This information would help mathematics educators in shaping classroom practices, in redeveloping curriculum, and in developing programs for teacher preparation. In addition, mathematics educators in Thailand are currently developing standards for school mathematics. One standard in algebra indicates that students in seventh to ninth grade should be able to describe and represent relationships in problem situations by using symbolic representation (Institute for the Promotion of Teaching Science and Technology [IPTST], Thailand, 2000). However, there is no information about how successful Thai students in developing knowledge and skill in applying to solve algebra word problems.

Thus, the information in this present study would be helpful as a basis for developing the mathematics learning standards as well as mathematics teaching standards in Thailand.

CHAPTER II
REVIEW OF LITERATURE

Introduction

This chapter provides an overall picture of what is known about algebra and algebra word problems, students' successes and difficulties in solving algebra word problems, and solution strategies students' use to solve algebra word problems. This chapter is divided into four sections. The first section reviews algebra and algebra word problems (e.g.,Kaput, 1999; Resnick, Cauzinille-Marmeche, & Mathieu, 1987; Herscovics, 1989; Kieran, 1989). The third section reviews students' successes and difficulties in solving algebra word problems (Clement, et al., 1981; Lochhead & Mestre, 1988; Makanong, 1993; Mestre et al., 1982; Uthairat & Viamoraphun, 1984). The fourth section reviews solution strategies students' use to solve algebra word problems (Bull, 1982; Hall et al., 1989; Kieran, 1988, 1992; Koedinger & Nathan; 1998; Koedinger & Tabachneck, 1994).

Algebra

Algebra is a branch of mathematics in which letters and symbols for operations are used to represent basic arithmetic relationships. As in arithmetic, the basic operations of algebra are addition, subtraction, multiplication, division, and the extraction of roots. Arithmetic, however, cannot generalize mathematical relationships such as the Pythagorean theorem, which states that the sum of the squares of the sides of any right triangle is also a square. Arithmetic can only produce specific instances of these relationships (for example, 3, 4, and 5, where $3^2 + 4^2 = 5^2$). But algebra can make a purely general statement that fulfills the conditions of the theorem: $a^2 + b^2 = c^2$ for any right triangle and c is the hypotenuse. Any number multiplied by itself is termed *squared* and is indicated by a superscript number 2. For example, 3×3 is notated 3^2; similarly, $a \times a$ is equivalent to a^2. Mathematicians developed algebra to be "generalized arithmetic." That is, its formulae are abstractions across a large class of specific arithmetic "sentences." (Kaput, 1999; Resnick, et al., 1987). For example, the expression "$x + y$" is a generalization of specific arithmetic expressions such as "2 + 1,""3 + 2," and "9 + 7." One of the first things that children need to grasp in order to understand formulae is the fact that "x" could stand for the "2," "3" or "9" of the three arithmetic sentences. In fact, "x" could stand for any number.

This notion of a variable is quite difficult for students to understand at first. In particular, research has shown that students seem to progress through several levels of understanding before they master it. At the lowest level, students immediately assign a number to "x" because they fail to grasp the notion of "unknown value." For example, when asked to "write a number that is three more than x," they first put down a value for x (e.g., 5) and then write one that is three more than the value given for x (e.g., 8). At the second level, they think that "x" stands for a specific number that is not yet known. At the third level, "x" is considered to be a generalized number; that is, it can take on more than one value. At the fourth level, "x" is considered to be a variable that not only represents a range of values, but also is linked in a systematic way to a set of values represented by another variable such as "y" (Herscovics, 1989; Kieran, 1989).

If students have trouble understanding the notion of variable, they will also have trouble dealing with algebraic expressions. When given an expression such as "$x + 3 = y$" and asked "What is the value of y?" students initially refuse to put the answer "$x + 3$" because they think that answers have to be a single, determinate value. In addition, students also have trouble combining variables within algebraic expressions. For example, some students do not understand why "$7a$" is the answer to "$2a + 5a$" and others make the error of assuming "z" is the answer to "$2yz - 2y$" (Kieran, 1989). Several explanations are made as to why students make such errors. First, just as "-5" is more abstract than "5," "x" is more abstract than either of the former two numbers (Resnick et al., 1987). In particular, whereas any negative or positive number can be related to a specific concrete array of objects, variables must be related to an array of possible numbers. Second, algebraic operations cannot be easily assimilated to arithmetic operations. For example, although the expression $(3 + 2)^2$ is formally related to the generalization $(a + b)^2$, the "answer" to these two problems would be computed in distinct ways. In particular, few students would solve the former by first forming the expression "$(3 + 2)(3 + 2)$" and then using the "foil" method to generate "$3^2 + (2)(3\times 2) + 2^2$." Most would say, "5 squared is 25."

Third, algebraic operations are usually taught in a non-conceptual, computational way (Wagner & Kieran, 1989).

Given the cognitive complexity of algebra, it may not be surprising to learn that many students performed poorly on the algebra in both the United States and in other countries (e.g, Carpenter et al., 1980; Kieran, 1992; Nathan, Kintsch, & Young, 1992). For example, when given the equation "$W = 17 + 5A$" and asked "According to this formula, for each year older someone gets, how much more should he weigh?" only 64 percent of seventeen-year-olds with two years of algebra gave the right answer of "5 pounds more each year."

Algebra Word Problems

Word problems, also known as story problems, are an essential part of learning to use mathematics effectively. Word problems are often used in the mathematical problem-solving curriculum and constitute an important part of mathematics from elementary school to secondary school level. Word problems are tasks, which require the combination of reading, comprehension, representation, and calculation. In word problems, situations are described in which there are some modifications, exchanges, and combinations of quantities, shapes, or other mathematical applications. According to Reed (1999), there are three different types of word problems. The three types are: (1) elementary problems, (2) multistep problems, and (3) algebra problems. Most word problems, whether from ancient or modern sources follow a three-component compositional structure (Gerofsky, 1996).

1. A "set-up" component, establishing the characters and location of the putative story. (This component is often not essential to the solution of the problem itself.)
2. An "information" component, which gives the information needed to solve the problem.
3. A question.

In terms of the three components typical of word problems stated above, the first component contains the givens and sometimes the operations. The three-component structure of typical word problems seems, then, to be based on the structure of arithmetic algorithms or algebraic problems, rather than on the conventions of oral or written storytelling.

In the case of an algebra word problem, a student is required to write an algebraic equation in terms of a set of variables, which are related to one another in a fixed (or fixable) relationship that can be stated in terms of an equality or inequality. Difficulties with algebra word problems had been found in the past several years and had been studied extensively in many articles and research studies (e.g., Clement, 1982; Clement et al., 1981; Kieran, 1992; Mestre et al., 1982; Nathan et al., 1992). In order to understand why algebra word problems are difficult, we will look at students' mental models of algebra word problems.

Students' Mental Models of Algebra Word Problems

One of the inconsistencies between arithmetic and algebra is in the solving of word problems. In arithmetic, word problems require students to think in terms of the operations they used to solve the problem. In contrast, solving algebra word problems requires students to think in terms of the "forward" operations that represent the structure of the problem. This represents a major cognitive shift for students beginning to learn algebra and loading an ability to interpret and understand the mathematical relationships (Chaiklin, 1989; Kieran, 1992). Solving algebra word problems involves both text processing and mathematical skills (Hall, et al., 1989; Nathan et al., 1992). However attention is often focused on the manipulation of formal mathematical expressions rather than on strategies for comprehending a word problem, even though problem comprehension is largely accountable for students' poor performance (Nathan et al., 1992). The difficulties students experience in representing and solving algebra word problems have been well documented (e.g., Greeno, 1989; Herscovics, 1989; Kieran, 1992; Nathan et al., 1992). A general finding is that students rely on a direct, syntactic approach to solving algebra word problems, that is, students use a "phrase-by-phrase translation of the problem into variables and equations" (Chaiklin, 1989; Hinsley, Hayes, & Simon, 1977). The application of syntactic rules is generally sufficient for identifying the variables and the relations among these. However, the syntactic approach is limited because it does not enable the student to detect irregularities or contradictions in a problem.

A commonly cited case of this limitation is the reversal error students made in solving the "Students-and-Professors" problem, namely, "Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university" (Clement, 1982; Clement, et al., 1981). A significant proportion of adults made a reversal error where, instead of writing $6P = S$, they write the reverse, $6S = P$. One explanation for this error is the literal mapping from the symbols to words where S is read as students and P as professors, rather than S as the number of students and P as the number of professors (Clement, Lochhead, & Soloway, 1979).

Nathan et al. (1992) suggested that problem comprehension is largely accountable for students' poor performance. Nathan et al.(1992) stated that comprehending a problem requires the student to make a connection between the formal equation(s) needed to solve the problem and the student's own informal understanding of the situation described in the problem. An important component here is how the student's mental representation of the problem situation informs and constrains the formal expressions required for solution (Greeno, 1989; Hall et al., 1989). To highlight the important role of the problem-situation model, consider a sample problem similar to that cited by van Dijk and Kintsch (1983) and Nathan et al (1992):

> "A tourist bus leaves Sydney and travels north at 80 kilometers per hour. Two hours later, a second bus leaves Sydney on the same course and travels north at 100 hundred kilometers per hour. How long will it take the second bus to overtake the first?"

An algebra word problem such as this can be viewed from two levels of abstraction: the "quantitative structure" of the related mathematical objects and the "situational structure" of related physical objects within the problem (Hall et al., 1989, p. 227). The quantitative structure refers to the mathematical entities and relationships that are either presented or implied in the problem text. The above problem includes extensive quantities denoting a primary amount (i.e., 80 kilometers and 100 kilometers), intensive elements indicating a constant multiplicative relationship between two quantities (i.e., 80 kilometers per hour and 100 kilometers per hour), and a difference element that compares two extensive (i.e., one time interval is 2 hours longer than the other). From this problem, one can state that the distance, d, traveled by the slower bus is equal to 80 times the time it takes to travel that distance, that is, $d = 80 \times t$. Although students will probably have a schematic knowledge of this relationship, it will be of little assistance to them if they do not consider the situational structure of the problem.

In forming a problem-situation model from this problem text, the student must make some primary inferences. The student must provide this information from his general knowledge, because this problem does not tell them that the buses will meet when they have traveled equal distances. This is where the problem-situation model comes into play. To effectively form this problem-situation model, the student must have sufficient background knowledge to “read between the lines” and to draw any necessary inferences or projections. This helps provide the student with an algebraic interpretation of the text to facilitate solution and also provides situational constraints, against which the student can check the formal constraints. However, this problem-situation model can only assist in the detection and correction of formal problem-schema errors when a clear mapping between it and the mathematical model has been established (Nathan et al., 1992).

In generating a mathematical model for this problem, the student must draw upon his schema that links distance, speed, and time, namely, $d = s \times t$. Two additional, supporting relations then need to be considered, namely, that the buses travel the same distance and that there is a time delay between the two. Hence, for the faster bus, one can write the equation, $d = 100 \times t$. Because the slower bus also travels the same distance, one can use the same variable, d. However the slower bus has a 2-hour head start that must be incorporated in its equation, that is, $d = 80 \times (t + 2)$. The solution-enabling equation, $100 \times t = 80 \times (t + 2)$, can then be formed.

Because the supporting relations have to be inferred from the student's problem-situation model, they can pose a considerable cognitive load. Hence, Nathan et al. (1992) hypothesized that the student will only make inferences when they seem necessary and that poor problem solvers will omit them from their solution to a problem. These poorer problem solvers tend to use a straight translation-based technique of mapping story phrases to equations. In contrast, problem solvers who form a sound problem-situation model will include inference-based equations. Reasoning with this problem-situation model serves as an effective problem-solving strategy when an algebra word problem cannot be solved by simple algebraic substitution (Hall et al., 1989). In conclusion, students frequently treat symbolic representations and real-world situations as unrelated to each other, with the result that they solve symbolic expressions procedurally without any meaningful reasoning.

They are then prone to performing operations on symbolic expressions that no longer map onto the situations to which the intended expressions refer (Greeno, 1989).

Students' Successes and Difficulties in Solving Algebra Word Problems

One of the most important reasons for using word problems in school mathematics is to teach students to use formal mathematical knowledge and to perform problem-solving skills they learn in school to real world situations. Improving students' ability to solve word problems in algebra is considered to be critically important by those who have worked on reforming mathematics education over the past 30 years, as algebra has become an entry level skill for most scientific, business, and technical jobs. However, individual students are different in achieving success or failure in solving algebra word problems. This section presents studies that examine students' successes and difficulties in solving algebra word problems.

Students' Difficulties in Solving Algebra Word Problems

Translating from the languages in algebra word problems to an algebraic equation is particularly difficult (Clement, et al., 1981; Lochhead & Mestre, 1988). Many studies have been carried out concerning the sources of translation errors and difference schemas students' use to translate problem-solving steps through the interview method (Clement, 1982; Clement et al., 1981; Makanong, 1993; Mestre et al., 1982, Uthairat & Viamoraphun, 1984). These studies reported on the robust nature of errors when translating word problems to algebraic equations. This part deals with four studies, which show the difficulties students have in solving algebra word problems. The first two studies deals with students at college level in the United States (Clement, 1982; Mestre et al., 1982). The third study deals with college students in Thailand (Uthairat & Viamoraphun, 1984). The fourth study deals with high school students in Thailand (Makanong, 1993).

Clement (1982) conducted a study with freshman engineering students. Clement described test data showing that a large proportion of science-oriented students were unable to solve very simple kinds of algebra word problems. In this study, a 45-minute written test was given to 150 freshman engineering students. The test was administered during a regularly scheduled class period in the first semester. This same test also was given to 47 non-science majors taking college algebra. Table 1 shows six items used in this study.

Clement (1982) stated that Problems 5 and 6 belong to a class of problems that should be trivial for a scientifically literate person but the results showed that these problems were solved incorrectly by large numbers of these science-oriented students. The contrast between the large number students who correctly solved Problems 1 through 4 and the number who correctly solved Problems 5 and 6 suggested that most errors were due to a difficulty with simple algebraic manipulation skills or with simple ratio reasoning of the type require for Problem 4.

Table 1

Percent correct of six test items

Test question (n = 150)	**Percent Correct**
1.Solve for x: $5x = 50$	99 (n = 150)
2. Solve for x: $6/4 = 30/x$	95 (n = 150)
3. Solve for x in terms of a: $9a = 10x$	91 (n = 34)
4. Jones sometimes goes to visit his friend Lubhoft, driving 60 miles and using 3 gallons of gas. When he visits his friend Schwartz, he drives 90 miles and uses how many gallons of gas. (Assume the same driving conditions in both cases).	93 (n = 150)
5. Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professor at this university." Use S for the number of students and P for the number of professors.	63 (n = 150)
6. Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered.	27 (n = 150)

In addition, the results for 47 non-science majors taking college algebra showed that 43% solved Problem 5 correctly (Clement et al., 1981). At first, Clement thought that the errors on such simple problems must be due primarily to carelessness. However, there was a strong pattern in the errors. In both cases 68% of the errors were reversals: $6S = P$ instead of $S = 6P$ in Problem 5, and $4C = 5S$ instead of $5C = 4S$ in Problem 6. Also, roughly half of the students were given the following hint with both problems: "Be careful, some students put a number in the wrong place in the equation." The percentage correct for the group given the hint was only three points higher on Problem 5 and five points higher on Problem 6.

In order to develop hypotheses concerning the cause of these results, audio taped and videotaped clinical interviews were conducted with 15 freshmen. These freshmen were asked to think aloud as they worked on the Students and Professors problem (Problem 5) and similar problems. Based on these protocols, the researchers developed two conceptual sources of reversal errors; a syntactic word order matching approach, and a semantic static comparison approach. In the word order matching approach, the students simply assumed that the order of key words in the problem statement would map directly into the order of symbols appearing in the equation. This is a syntactic strategy in the sense that it is based on rules for arranging symbols in an expression that do not depend on the meaning of the expression. In a static comparison process, in contrast, the student considered the meaning of the expression. Students who used this approach realized that there were more students than professors but did not know how to express this relation. Therefore, they placed the multiplier (6) next to the symbol associated with the larger group. In the static comparison approach, the students went beyond a syntactic word order matching approach and were using a semantic approach dependent on the meaning of the problem.

Mestre et al. (1982) conducted a study with freshman students to answer two questions. First, in an exam comprised of problems such as the Student and Professor problem (Problem 5 in Table 1), what were the similarities and differences in performance between a group of bilingual Hispanic technical majors and a group of monolingual technical majors? Second, what was the relationship between performance on such an exam and (a) language proficiency and (b) grade point average? The subjects in this study consisted of two groups. The first group was composed of 43 Hispanic bilinguals, of whom 22 were freshmen, 11 were juniors, and the remaining two were seniors. The majority of students were engineering majors, while the rest were majoring in sciences. All but four were balanced bilinguals; that is, they demonstrated nearly equivalent performances in Spanish and English language proficiency exams. The second group, which served as a norm, consisted of 52 monolinguals. Forty-three were freshmen, five were sophomore, and four were juniors. There were 38 monolingual students majoring in engineering, with the remainder majoring in sciences. Both groups, therefore, had approximately equivalent percentages of engineering majors (63% for the bilingual group and 73% for the monolingual group). Both groups were volunteers and were paid to participate in the study.

The four exams administered were designed to measure advanced reading comprehension proficiency and the ability to translate from syntactic to symbolic representation. The language proficiency exams were Test of Reading, Level 5, and Prueba de Lectura, Nivel 5, developed by Guidance Testing Associates in 1962. These tests contained three subsections--covering vocabulary, speed of comprehension, and level of comprehension. The remaining exams, Formula Translation and Traduccion de Formulas, were locally designed and contain 14 questions each. To avoid a redundancy and as a reliability check, in each of the latter exams the second seven questions were equivalent to the first seven questions in both difficulty and content. The time allowed for either the English or Spanish Formula Translations exam was 12 minutes. The Spanish exams were given only to the bilingual group, while the English exams were administered to both groups. All exams were scored on total number correct.

The results from this study showed that performance, as measured by either the mathematical translation tasks or grade point average, was more strongly correlated with language proficiency for the bilingual group. Clinical interviews conducted with samples from each group revealed large differences in the interpretation of the mathematical task between Hispanics and monolingual groups. It was evident that the monolingual group (GPA = 2.74) had an advantage over the bilingual group (GPA = 2.4) in English language proficiency, which may be one of the main causes of the large difference in means between the two groups for the Formula Translation exam. The results also showed that the monolingual group ($M = 9.6$, $SD = 4.5$) performed better than the monolingual group ($M = 4.7$, $SD = 4.4$ in their first language) and in Spanish ($M = 5.1$, $SD = 4.2$), on either the Formula Translation exam or the language proficiency exams.

There was one surprising difference between the bilingual and monolingual groups in the Formula Translation exam. Bilinguals were more prone to make errors different from the variable-reversal type than monolinguals. To study the source of these errors, a sample of nine bilinguals was randomly selected from the group of 43 students for clinical interviews. During the interviews, students were asked to think aloud while solving three problems from the Formula Translation exam. The three problems are presented in Table 2. The interviews were videotaped and subsequently analyzed. Five different types of errors are as follows.

1. The error that was due to language misinterpretation. In Problem 1, the students making this error explained that the phase "as many students as professor" implied an equal number of each, that is, $S = P$. The "six times" in front of the statement was interpreted to mean that each side of $S = P$ should be multiplied by 6. Thus, students wrote $6S = 6P$.

Table 2

Three problems used in the interview

	Problems
1.	Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.
2.	Write an equation using the variables C and P to represent the following statement: "At a certain restaurant, for every four people who ordered cheesecake, there were five who ordered pie." Let C represent the number of cheesecakes ordered and P the number of pies ordered.
3.	Write and equation to represent the following statement: "A certain council has 9 more men than women on it." Use M for the number of men and W for the number of women.

2. In Problem 1, some students wrote $6S + P = T$. The explanation given was that this equation related the number of students, professors, and the total student and professor population, T, in the appropriate proportions. The students who made this type of error could be prompted into realizing that the question asked for a relationship between S and P, and subsequently wrote the variable-reversed equation, $6S = P$. This type of error apparently derived from not carefully ascertaining what the problem was asking.
3. In Problem 2, some students wrote $4C/5P$. When asked if this was an equation relating the number of people eating cheesecake to the number of people

eating pie, these students replied that the fraction above set up a "relationship" to express the appropriate ratios of cheesecakes to pies sold at the restaurant.

4. In Problem 2, some students wrote $4C < 5P$. Those who wrote this inequality claimed that, because of 4 to 5 ratio of cheesecakes to pies, one could never set up an equation relating the two variables. As evidence, the student pointed out that if four people bought cheesecakes, then five people bought pies, if eight people bought cheesecakes, the 10 people bought pies, and so on. Hence the two quantities could not be related by an equation. These students indicated that the only relationship that could possibly be established was the inequality above; which expresses the fact that there will always be fewer cheesecakes than pies served at the restaurant.
5. In Problem 3, some students wrote $9M = W$. This was by far the most common of the non-variable-reversal errors, with five out of the nine students committing it. It stemmed from interpreting the phrase "9 more men than women" to mean "9 times more men than women." When prompted, three of the five students making this error realized that the question called for an equation involving addition rather than multiplication, and subsequently wrote a variant form of the variable-reversal error for this problem, namely, $M + 9 = W$. Two students retained the notion that the phrase "9 more men than women" implied a multiplicative relationship.

For the completeness of this study, similar interviews were carried out with 11 monolingual students from the sample of 52 monolingual students. However, the only type of error made by this group was of the variable-reversal type. The reason for the students committing the five types of errors described above indicated that Types 1, 2, and 5 were linguistic in nature. However, Error Types 3 and 4 appeared to be due more to a mathematical misinterpretation or deficiency. In summary, this study showed the inordinately high occurrence of the variable-reversal error among both bilinguals and monolinguals in the Formula Translation exam.

Not only United States students encountered difficulty in solving algebra word problems, Thai students also encountered this difficulty. Uthairat and Viamoraphun (1984) investigated achievement of senior students majoring in Education in Thailand. The purpose of this study was to compare achievement of senior students majoring in Education from two different universities. The subjects in this study were all seniors.

Four hundred and seventy three students participated in this study, 152 the first university (N1) and 321 from the second university (N2). The instrument used in this study consisted of 66 multiple-choice items. The instrument was divided into three sections. The first section was about fundamental operations (addition, subtraction, multiplication, and division). The second section was about fractions, decimals, ratios, and percents. The last section was about word problem solving. By using Guttman Scalogram Analysis, the researchers obtained a reliability of the instrument (0.83), item of difficulty (0.89), and item discrimination (0.53). The mean scores of the sample in this study were 58.61 (from a total of 66 points) with *SD* = 8.12. This result showed that overall students had high achievement in mathematics (Item Percent Correct = 89.81%). However, most students had a low mean in the third section (Item Percent Correct = 63%). This indicated that senior students majoring in Education still had low achievement in solving word problems.

The last study reviewed in this section was done with high school students, which showed that Thai students also had difficulty in representing word problems into algebraic equations. Makanong (1993) investigated mathematics learning deficiencies of eleventh grade students on exponential and logarithm functions, trigonometry and application, matrix, linear programming, vectors, complex number and statistics. The sample was 21 eleventh grade students. The instruments used in this study composed of exercises from textbooks and lesson series, applied problems and lesson tests constructed by the researcher. The researcher reported no reliability and validity. All the instruments were types of mathematics problem solving, which had to present steps in solving the problems.

Results from this study showed that students had the most mathematics learning deficiencies in using theory, formulas, laws, definitions and properties, followed by computation and problem representation. In the aspect of problem representation, students had the most deficiencies in using data correctly and in translating verbal sentences to symbol sentences. In addition, students had the most deficiencies in using basic concepts about theory, formulas, laws, definitions and properties and in applying the given data and the theory, formulas, laws, definitions and properties. Students also had deficiencies in remembering the theory, formulas, laws, definitions, and properties, and lacked skills in selecting appropriate theory, formulas, laws, definitions and properties to solve problems. In the aspect of computation, students had deficiencies in concluding the answers or concluding the answers from every case. Students were also careless in computation, lacked skills in basic algebra for solving equations and non-equations, misused the correct steps in computation and lacked understanding in fundamental principles of mathematics.

In summary, this part provides evidence that students were unsuccessful in solving algebra word problems. The most common difficulty found was a translation error from word representations to algebraic representations.

Students' Successes in Solving Algebra Word Problems

Although word problems are difficult as identified in many studies above, some studies have identified circumstances where word problems are easy for students to solve. This part deals with three studies which showing the success students had in solving algebra word problems. Since not many studies involved algebra word problems, some studies reviewed in this section involve elementary students solving arithmetic word problems. The first study is about students at elementary levels which showed that they were able to solve word problems in some circumstances but could not perform algorithmic calculation in school. (Carraher, Carraher, & Schlieman, 1985). The second study deals with high school students in solving different types of algebra problems (Koedinger & Nathan, 1998).

Brazilian children who regularly engaged in street trade, for example, solved problems more readily when the problems were presented in a practical context such as a story, an action sequence, or, preferably, as a real-life interaction of the street markets (Carraher et al., 1985). Specifically, contextualized problems presented either as typical word problems or as problems situated in a commercial transaction led to greater levels of performance than symbolically presented problems. Carraher et al. (1985) investigated an everyday use of mathematics by working youngsters in commercial transactions in Brazil. The children in this study were four boys and one girl 9 – 15 years old and ranging in level of schooling from first to eighth grade. One of them had only one year of schooling; two had three years of schooling; one, four years; and one, eight years. All were from very poor backgrounds. Four of the subjects were attending school at the time and one had been out of school for two years. Four of these subjects had received formal instruction on mathematical operations and word problems. The subjects who attended first grade and dropped out of school were unlikely to have learned multiplication and division in school because these operations are usually initiated in second or third grade.

There were two tests used in this study. The first test was an informal test. Test items in the informal test were presented in the course of a normal sales transaction in which the researcher posed as a customer. Purchases were sometimes carried out. In other cases the customer asked the vendor to perform calculations on possible purchases. At the end of the informal test, the children were asked to take part in a second test (a formal test) which was given on a separate occasion, no more than a week later, by the same interviewer. Subjects answered a total of 99 questions on the formal test and 63 questions on the informal test. Since the items of the formal test were based upon questions of the informal test, order of testing was fixed for all subjects.

The results showed that context-embedded problems were much more easily solved than problems without a context. The results showed that 98.2 percent of the 63 problems presented in the informal test were correctly solved. In the formal test word problems, (which provide some descriptive context or the subject), the rate of correct responses was 73.7 percent, which should be contrasted with a 36.8 percent rate of correct responses for mathematical operations with no context. In summary, the results from this study showed that children were much more successful at solving story problems than solving matched problems presented symbolically.

Koedinger and Nathan (1998) investigated the symbolic, situation, and verbal facilitation hypotheses in two studies. The purpose of these studies was to test the claim that comprehension of algebra equations is not trivial for algebra learners and that, as a consequence, algebra story problems can sometimes be easier to solve than matched equations. The symbolic facilitation hypothesis predicts that equations should be easier than matched story problems. The situation facilitation hypothesis predicts that students will make fewer errors on story problems than on situation-free word equations and equations. The verbal facilitation hypothesis predicts that students will make fewer errors on story and word equations than on the more abstract equations. In both studies, students were asked to solve problems selected from a multi-dimensional space of problems. The two factors are problem representation (symbolic, situation, and verbal presentations) and unknown (results and start unknown). In addition, the researchers manipulated the type of the numbers involved (whole number vs. positive decimals) and the final arithmetic that problems required (multiplication and addition vs. subtraction and division).

The subjects in the first studies were 76 students from an urban high school. Fifty-eight students were enrolled in one of three mainstream Algebra 1 classes. Eighteen students were ninth graders enrolled in a Geometry class. These 18 students already took Algebra 1 in eighth grade. Four different teachers taught the classes. The subjects in the second study were 171 students sampled from 24 classrooms at three urban high schools. All students were in a first year Algebra 1 course. Twelve different teachers taught the 24 classroom sections.

The instrument used in this study was a ninety-six problem test using four different cover stories that systematically vary four difficulty factors: three levels of problem presentation (symbolic, situation, and verbal); two levels of unknown positions (results and start unknown); two number types (whole number and positive decimals); and two final arithmetic types (multiplication and addition vs. subtraction and division). The 96 problems were distributed onto sixteen forms with eight problems on each form. The researchers reported no information on reliability and validity. Students in both studies took this test in class during a test day. Students were given 18 minutes to work on the test and were instructed to show their work and put a box around their final answer. Students were not allowed to use calculators because the researcher would like to see students' thinking process in the arithmetic steps they wrote down.

For the analysis, the researchers performed both an item analysis and subject analysis to assess whether the results generalize across both the item and student populations. For the item analysis, the researchers performed a three factor ANOVA with items as the random effect and the three difficulty factors: representation, unknown position, and number type as the fixed effects. The results from the first study showed that students performed better on story problems (66%) and word equations (62%) than equations (43%; $F(2, 108) = 11.5, p < .001$). Students performed better on result-unknown problems (66%) than on start-unknown problems (52%; $F(1, 108) = 10.7, p < .002$). In addition, students performed better on whole number problems (72%) than decimal number problems (46%; $F(1, 108) = 44, p < .001$). A Scheffe's S post-hoc showed a significant difference between story problems and equations ($p < .001$), word equations and equations ($p < .01$), but not story problems and word equations ($p = .80$). However, none of the interactions were statistically significant.

The researchers stated that these results contradicted the symbolic facilitation hypothesis because story problems were not harder than equations. The results supported the verbal facilitation hypothesis because word equations are substantially easier than equations. To confirm that the main effects of the three difficulty factors generalize not only across items but also across students, the researchers performed three separate one factor repeated measures ANOVAs for each of the three difficulty factors: representation, unknown position, and number type. The results from this analysis showed a significant main effect of each factor. The results showed that story and word problems were significantly easier than equations (F (2, 150) = 8.35, $p < .001$). In addition, result-unknown problems were significantly easier than start-unknown problems (F (1, 75) = 19.8, $p < .001$) and whole number problems were significantly easier than decimal number problems (F (1,75) = 42.3, $p < .001$).

Likewise, the results from the second study were the same for the main effects of representation, unknown position and number type. The results showed that students were 67% correct on the whole number problems and only 54% correct on the decimal problems (F (1, 116) = 22, $p < .001$). Students were more often correct on result-unknown problems than start-unknown problems (72% vs. 49%; F (1, 116) = 49, $p < .001$). As in the first study, students performed best on story problems (70%), on word equations (61%). However, students largely performed worse on equations (42%; F (2, 116) = 22, $p < .001$). Unlike the results from the first study, there was a significant interaction between representation and number type (F (2, 116) = 3.7, $p = .03$) in the second study. The results showed that when the number type is whole number, there was no difference between story and word equations (72% vs. 73%). Only when the number type is decimal does the advantage for story problems over word equations appear (68% vs. 48%). As in the first study, a one factor repeated measure ANOVA was conducted for each of the three difficulty factors with student as the random factor. The results from the analysis showed that representation, unknown position, and number difficulty all had statistically significant effects: F (2, 340) = 37, $p < .001$, F (1, 170) = 137, $p < .001$, and F (1,170) = 20, $p < .001$, respectively.

In summary, these two studies showed that in some circumstances students were more successful solving simple algebra word problems than solving mathematically equivalent equations. Since the present study focused on Thai students' solution strategies in solving algebra word problems, the following section presents solution strategies students usually implemented in solving algebra word problems.

Solution Strategies to Algebra Word Problems

There are several studies on students' solution strategies to algebra word problems (e.g., Bull, 1982; Hall et al., 1989; Kieran, 1988; Koedinger & Nathan; 1998; Koedinger & Tabachneck, 1994). These studies showed that there are several solution strategies students used in solving algebra word problems. This part deals with five studies on students' solution strategies to algebra word problems. The first article deals with middle school students (Kieran, 1988). The other four articles deal with college students (Bull, 1982; Hall et al., 1989; Koedinger & Nathan; 1998; Koedinger & Tabachneck, 1994).

Kieran (1988) investigated solution strategies among six thirteen-year-old seventh graders in solving algebra problems. These students participated in a three-months teaching experiment. These students had not yet begun to take algebra in their mathematics classes. They had followed a standard arithmetic program during their years in elementary school. They were all of average mathematical ability. Three types of questions were used in this study: questions on (1) different parts of an equation, such as the equal sign and unknown term; (2) equation solving; and (3) equivalence of equations. Each student was interviewed. Students' responses suggested that algebra learners could be divided into two groups.

The first group focused on the given operations. They were called the arithmetic group. The second group focused on the inverses of the given operations. They were called the algebraic group. These two groups not only had different preferred methods of solving equations but also different views on the meaning and significance of the various parts of an equation. Students in this study were asked what the letter means in $5 + a = 12$ and $2c + 15 = 29$. There were two distinct types of answers. Those whose answers referred to the inverse operations necessary to find the value of the letter were placed in the "algebra group," and those who did not mention inverse operations but rather stated that the letter was a number were placed in the "arithmetic group."

For the algebra group, the letter seemed to have meaning only when its value was found. A typical answer to the question, "What does the letter mean in $5 + a = 12$?" was "An answer -12 minus 5 is 7." For them, an equation had to be reformulated for the letter to have some meaning, as in, for example, $5 + a = 12$ being transformed to $a = 12 - 5$, or $2c + 15 = 29$ to $c = (29 - 15)/2$. The arithmetic group, however, seemed to view the letter as standing for some unknown number. In that sense, the letter was part of the numerical relationship of the equation. These students expressed the unknown in the equation using the given operations. For example, a typical answer of this group to the question, "What does the letter mean in $5 + a = 12$?" was "It means a whole number that's going to be added to 5 and it's going to equal 12." The results from this study showed that even though students had not yet begun to take algebra, they were able to solve some simple equations. The procedures that they used correspond with the way they viewed the letter in equations. The arithmetic group used the given operations to solve equations, substituting different numbers for the letter until they found one that balanced the left side and right side of the equation. The algebra group used the inverses of the given operations and solved by transposing terms to the other side.

According to Bull (1982), it was hypothesized that solutions to word problems could be divided into three general categories which were assumed to be based on three different types of problem representations.

1. Non-algebraic strategies. This term roughly combined several different categories of processes, which had been described in the literature—"successive approximation," "trial and error," and "deduction" as used by Kilpatrick (1967) and "mean-ends analysis" as used by information processing theorists (e.g., Newell & Simon, 1972). It was hypothesized that when subjects were unable to find a specific solution plan, they would approach the task through random trial and error or by performing a series of operations on the numbers given in the problem text. Even though subjects might not have been able to recall formulas and other information necessary to write an equation, they often had enough mathematical or real-world knowledge to allow them to solve a problem in a non-algebraic manner. Those subjects who built a partial representation into a complete one could use this strategy successfully. However, those subjects whose representation remained partial or fragmentary throughout the solution attempt used it unsuccessfully.
2. Equations based on incomplete problem representation. Some of the attempts to write equations which turned out to be incorrect seemed to be derived from a partial representation of a problem. A partial representation could result from an incomplete or incorrect understanding of the problem text and/or a representation, which does not include all of the background information necessary to make inferences about the problem. Simple one-step problems,

like the Student and Professor problem (Clement et al., 1981), in which an equation is derived by translation directly from the verbal problem statement, can lead to solution strategies which are based on an incomplete problem representation.

3. Equations based on well-integrated representations. If a subject is able to map a problem statement onto a well-integrated representation, writing and solving an equation is expected to be a rather trivial task. A well-integrated representation was hypothesized to lead to an equation based upon recall of a problem category or a formula or upon a well-integrated physical or symbolic representation of the problem text. While it was recognized that good problem solvers might also use non-algebraic strategies for easy problems, they were expected to be able to write a correct equation for more difficult problems.

Hall et al., 1989 investigated solution strategies on representative algebra story problems. Participants in this study were 85 undergraduate computer science majors in their junior and senior years. Students were asked to solve the four algebra story problems. Participants were allowed eight minutes to solve each problem, and all worked through the problems at the same time. Before solving any problems, students were asked to "show all of your work" in a written form, to "work from top to bottom, writing new material below previous material," and not to erase after making a mistake. Instead, students were asked to mark through any mistake with a single line. Finally, students were instructed to "Draw a box around your answer." After solving all four problems, the students were given 20 minutes to explain their solutions in writing on facing pages of the test booklet without changing their original work.

Results from this study showed that most students used algebra in their solution attempts (63.5% to 85.9% across problems). Although direct algebraic problem solving is sometimes effective, results suggested that the algebraic formalism might be of little help in comprehending the quantitative constraints posed in a problem text. Instead, problem solvers often reason within the situational context presented by a story problem, using various forms of model-based reasoning to identify, pursue, and verify quantitative constraints required for solution.

Koedinger and Nathan (1998) investigated the solution strategies in two studies. The subjects in the first studies were 76 students from an urban high school. Fifty-eight students were enrolled in one of three mainstream Algebra 1 classes. Eighteen students were ninth graders enrolled in a Geometry class. These 18 students already had taken Algebra 1 in eighth grade. Four different teachers taught the classes. The subjects in the second study were 171 students sampled from 24 classrooms at three urban high schools. All students were in a first year Algebra 1 course. Twelve different teachers taught the 24 classroom sections.

The instrument used in this study was a ninety-six problems using four different cover stories that systematically varied four difficulty factors: three levels of problem presentation (symbolic, situation, and verbal); two levels of unknown positions (results and start unknown); two number types (whole number and positive decimals); and two final arithmetic types (multiplication and addition vs. subtraction and division). The 96 problems were distributed onto sixteen forms with eight problems on each form. The researchers reported no information on reliability and validity. Students in both studies took this test form in class during a test day. Students were given 18 minutes to work on the test and were instructed to show their work and put a box around their final answer. Students were not allow to use calculators so that because they researcher would like to see students' thinking process in the arithmetic steps they wrote down.

The researchers coded student solutions for the strategies apparent in their written solutions. The strategy analysis focused on the early algebra start-unknown problems. The results from the analysis showed that students used both formal and informal strategies in solving algebra word problems. The formal strategy was the symbolic manipulation approach. The informal strategies were guess-and-test, and unwind strategy. In the guess-and-test strategy, students guessed at the unknown value and then followed the arithmetic operators as described in the problem. They compared the outcome with the desired result from the problem statement and if different, tried again. The second strategy is the unwind strategy. In this strategy, the student reversed the process described in the problem to find the unknown start value. The student addressed the last operation first and inverted each operation to work backward to obtain the start value. In addition to investigating differences in strategy selection, the researchers analyzed the effectiveness of these strategies. The results showed that the informal strategies; unwind and guess-and-test, showed a higher likelihood of success (69% and 71% respectively) than use of the symbol manipulation approach (51%). So, it appeared one reason these algebra students did better on story and word problems than equations was they selected more effective strategies more often.

Koedinger and Tabachneck (1994) analyzed a verbal protocol in the domain of mathematical word problem solving of twelve Carnegie Mellon undergraduates (six male and six female). Subjects were asked to solve two algebra word problems. The task was presented as a Hypercard stack on a Macintosh IIx computer. Subjects were identified as using four different kinds of strategies. The following strategies were identified.

1. Algebra (ALG): The verbal problem statement was translated to algebraic assignments and equations. The equations were transformed to find a solution (solve for the unknown).
2. Model-based reasoning, heuristic (MH): The verbal problem statement was translated into arithmetic constraints, represented either verbally or as written arithmetic. A value for an unknown was guessed at and that value was propagated through the constraints. Resulting values were checked against given values to determine whether or not a contradiction was reached. If so, further guesses were made until a consistent value was found.
3. Verbal-math (VM): The verbal problem statement was transformed into alternative verbal forms. There are two types of transformations: (a) verbal recoding intended to facilitate translation or (b) qualitative operations to estimate unknown values. Included in this strategy were translations to "verbal algebra" where equations were described verbally and transformations were performed that are analogous to written algebra transformations.
4. Diagrammatic (DG): The verbal problem statement was translated into a diagrammatic representation. Transformations were performed on the diagram, including annotations and diagram supported inferences.

The key result of this study was that students were more effective when they used multiple strategies in solving a problem than when they stuck with (or got stuck with) a single strategy. Of the 19 solutions in which more than one strategy was used, 15 of them or 79% were correct. All 19 solutions involved at least one switch from a schooled to an unschooled strategy (or vice versa). Of the 17 solutions involving a single strategy, seven of them or 41% were correct. In other words, multiple strategy use was about twice as effective as single strategy use. Other candidate features of successful performance (algebra use, algebra effectiveness, other strategy use and/or effectiveness, interactions with aptitude and time spent) did not distinguish problem solving success from failure. For example, subjects were neither more nor less effective when using algebra than they were when using other strategies. 54% (7/13) final uses of algebra led to a correct solution while 65% (15/23) final uses of other strategies led to a correct solution.

In summary, from reviewing research studies about solution strategies to algebra word problems, three different solution strategies were identified. First - non-algebra strategies- this strategy included the use of arithmetic approach or used other strategies such as trial and error, deduction, or verbal math (as indicated in Koedinger and Tabachneck, 1994). Second - algebra strategies - this strategy include the use of variables to represent verbal structures of word problems. Third - diagrammatic strategies - this strategy include the use of diagrams to represent word problems.

Conclusion

This chapter provides an overall picture of what is known about algebra and algebra word problems, students' successes and difficulties in solving algebra word problems, and solution strategies students' use to algebra word problems. Algebra is a branch of mathematics in which letters are used to represent basic arithmetic relations. Word problems are an essential part of learning to use mathematics effectively. However, algebra word problems are difficult. Difficulties with algebra word problems had been found in past several years and had been studied extensively in many articles and research studies (e.g., Clement, 1982; Clement et al., 1981; Mestre et al., 1982). These studies had reported on the robust nature of errors when translating word problems to algebraic equations. Not only United States students encountered difficulty in solving algebra word problems, Thai students also encountered this difficulty (Makanong, 1993; Uthairat & Viamoraphun, 1984).

Since this present study focused on Thai students' solution strategies in solving algebra word problems, solution strategies students usually implemented in solving algebra word problems are also reviewed. From reviewing research studies about solution strategies to algebra word problems, three different solution strategies were identified: non-algebra strategies; algebra strategies; and diagrammatic strategies. The methodology of this present study is presented in the next chapter (Chapter III).

CHAPTER III
METHOD

Introduction

Few studies about solution strategies used by Thai students in solving algebra word problems have been done. Since little is known about Thai students' solution strategies and difficulties in solving algebra word problems, a qualitative research was designed to study this important mathematical skill. This present study sought to answer the following questions:

1. How successful are Thai students in solving algebra word problems?
2. Which strategies are used by Thai students to solve algebra word problems?
3. What are characteristics of classroom instruction during word problem-solving lessons?
4. What are potential connections between classroom instruction and students' word problem-solving performance?

This chapter is divided into four sections. The first section describes the participants in this study. The second section describes the data collection. The third section describes the data analysis. The fourth section profiles the researcher who did this study.

Participants

This section provides information about the school and people who participated in this study. The school selected for this study is a secondary school located in a rural district in northern Thailand. The school educates students from the seventh to twelfth grades. The students in this school were grouped by their ability using their previous grade point average. The population of interest was the ninth grade students and their mathematics teachers. Three classes were selected for this study by the chair of the mathematics department of that school. One each of low, medium, and high achieving classes were chosen. This provided three mathematics teachers and their students in one class as the participants in this study. The details about the ninth grade mathematics teachers and the students follow.

The Teachers

Three mathematics teachers participated in this study. The three mathematics teachers were: (1) Mr. Jack, (2) Ms. Rose, and (3) Mr. Bond. Mr. Jack had taught mathematics since 1980. He taught at the 10^{th} grade level for 20 years. He had been teaching at the ninth grade level for two years. This semester, Mr. Jack was assigned to teach the eighth and ninth grade classes. Two of the ninth grade classes that he taught were low achievers. The chair of the mathematics department assigned the researcher a class of low achievers.

Ms. Rose had been teaching mathematics for 18 years. She had taught several grade-levels during her career: the seventh through the ninth grade and also the tenth grade level. She had been doing her teaching across grade levels from time to time. This semester, Ms. Rose was assigned to teach three ninth-grade classes. The class that the researcher was assigned by the school was a class of medium achievers.

Mr. Bond had been teaching mathematics at the ninth grade level for 31 years. Mr. Bond was assigned to teach three ninth grade classes this semester. One was a class of high achievers. The other two were classes of medium and low achievers. A class of high achievers was assigned to the researcher by the chair of mathematics department. Prior to any data collection all three teachers completed a consent form that indicated their willingness to participate in the study.

The Students

The students who agreed to participate in this study were 126 ninth-grade students from the classes of the three mathematics teachers indicated above. However, eight students were absent on the day of testing either on the pretest or posttest. They were automatically dropped from the study. This results in 118 ninth grade students who participated in this study (see Table 3).

Table 3

Summary of participants in this study (N=118)

	Mr. Jack	**Ms. Rose**	**Mr. Bond**	**Overall**
Boys	22	11	15	48
Girls	16	31	23	70
Overall	**38**	**42**	**38**	**118**

The three classes were taught by Mr. Jack, Ms. Rose, and Mr. Bond. These students had already been taught fundamental concepts of equations in the seventh and eighth grades.

Mr. Jack

Mr. Jack's mathematics class consisted of 43 students (25 boys and 18 girls). Students in this class were classified as having low achievement. The results were analyzed from 38 students. Five students were absent on the day of testing and were automatically dropped from the study.

Ms. Rose

There were 44 students in Ms. Rose's class (11 boys and 33 girls). Students in this class were classified as having medium to high achievement. The results were analyzed from 42 students. Two students were absent on the day of testing so they were automatically dropped from the study.

Mr. Bond

There were 39 students in Mr. Bond's class (15 boys and 24 girls). Students in this class were classified as having high achievement. The results were analyzed from 38 students. One student was absent on the day of testing so he was automatically dropped from the study.

Selecting the ninth grade students was purposeful for a variety of reasons. First, students at this grade level had already been taught about mathematical languages and symbols, and had been taught how to solve algebraic equations and simple word problems. Therefore, it was pertinent to see how they used their prior knowledge about equations to solve algebra word problems before they actually engaged in their formal instruction. In addition, it was a good opportunity to see how instruction affected students' solution strategies. All students received permission from their parents to participate in the study. Additionally, students were asked to sign a permission form.

Data Collection

This section provides a description of each phase of the data collection. Phase I describes the development of the pretest and posttest used in this study. Phase II presents data collection of the pretest. Phase III describes classroom observations. Part IV presents data collection of the posttest. Phase V details an interview with 18 students, six from each three teachers' class.

Phase I: Problem Development (Pretest and Posttest)

Ten problems were developed for this study. These problems were used to determine students' successes and difficulties in solving equations and algebra word problems, as well as the strategies students' use. These ten problems were developed based on the appropriate Thai mathematical standards for this grade level (IPTST, 2000). The appropriate standards that guided the development of the test are shown in Table 4. The ten problems were: (1) one equation problem; (2) four translation problems; and (3) five algebra word problems. Appendix A presents the ten problems used in this study. The problem set was mixed with both difficult and easy problems. The reason for having both easy and difficult problems was to probe students' thinking strategies at a variety of difficulty levels. It was desirable to have a variety of problems so that all students could solve some problems, but also be challenged by others. The ten problems were written in Thai language so Thai students could solve them. To determine the validity of the ten problems, five people reviewed these problems in order to determine validity. The five people were two ninth grade mathematics teachers and three mathematics educators. These people read the problems to determine the match of the problems to the objectives of the mathematics curriculum as well as the readability and difficulty of the problems. This test then was revised to match the curriculum, objectives, and the ninth grade students in this study.

Table 4

Thai Mathematical Standards

Standards (Grade 7 – 9)		Expectations
4.2) Algebra: Symbols, Equality, Inequality, and Other Representation	**a)**	Students should be able to solve equations.
	b)	Students should be able to write equations.
	c)	Students should be able to represent a problem situation, solve a problem, and determine the reasonableness of the problem.
6.1) Mathematical Ability: Problem Solving	**a)**	Students should be able to use a variety of strategies in solving problems in mathematics.
	b)	Students should be able to use knowledge of mathematics in solving a variety of problems.
6.3) Mathematical Ability: Communication and Representation	**a)**	Students should be able to use mathematical language and symbols to represent a problem situation.

The ten problems then were tried in a pilot test with 35 ninth grade students from a school not otherwise participating in this study. The purpose of the pilot test was to further refine the questions and to determine the reliability of the test. Thirty-five students were given these ten problems in the Thai language. The test took place in the classroom during the mathematics period. The initial time set for this test was 45 minutes. However, 45 minutes was not sufficient for students to complete the test. Therefore, 10 more minutes were given for students to finish the test. The ten problems were similar both on the pretest and the posttest. Only the numbers and characters in the problems were changed for the posttest.

The results from the test with 35 students indicated that students in this study were generally successful in solving equation, translating word problems, and solving word problems. The majority of students could solve Problems 1, 3, 6, 7 and 9. Problems 2, 4, 5, 8, and 10 were difficult for students to solve. However, Problems 2, 4, 5, 8, and 10 were not eliminated from this study because the researcher was curious about how students in this study solved these two difficult problems.

Item difficulty is simply the ratio of students taking the test who answered the problem correctly. By calculating the difficulties of each item, the results confirm that Problems 1, 3, 6, 7 and 9 were easy but Problems 2 (student-professor problem), 8, and 10 was difficult (see Table 5). Discrimination shows the measure of how well an item differentiates between high achievers and low achievers. The higher the discrimination, the better the problem because such a value indicates that the problem discriminates in favor of the upper group. Table 5 shows that all ten problems had high discrimination value, above 0.20. It would explain that the ten problems used in this study could differentiate between high achievers and low achievers.

Table 5

Item of difficulty and item of discrimination (N = 35)

Problem	Item Difficulty	Item Discrimination
1. An Equation Problem	0.51	1.00
Translation Problems		
2. Student-Professor	0.66	0.60
3. Number	0.89	0.80
4. Rectangular	0.51	1.00
5. Number	0.60	1.00
Word Problems		
6. Salary	0.86	1.00
7. Time-Distance	0.83	0.60
8. Student	0.40	1.00
9. Earrings	0.57	1.00
10. Fruit	0.40	1.00

Phase II: Pretest

This test took place one week before the formal instruction on equations began. A hundred and eighteen students participated on the pretest. This pretest was given to the students in each classroom during their mathematics period. The researcher provided direction to students about taking the test. Ninth grade mathematics teachers in charge of that period were asked to help the researcher administer and collect the test. However, teachers did not have access to the instrument before or after the test to avoid teachers' bias. Students were asked to put their student identification number and their class number on the test. Students who finished early were asked to leave the test on a table and allowed to wait outside the classroom.

Phase III: Observation with Three Teachers

Observation of teaching was designed to collect information about word problem-solving instruction and classroom environments in Thailand. The classroom observations were done after the pretest. Three mathematics teachers from three different classes were observed during mathematics period. Each mathematics teacher was observed during their instruction on the topic of equations and inequalities. The reason to observe the whole topic was to see if the lessons observed were typical for each teacher and to desensitize students to the presence of the researcher.

The same researcher conducted all the observations. The researcher sat in the back of the class to avoid interfering in classroom activities. During the observation, the researcher took notes of what happened in the classes. Each observation was tape recorded for later study. The recordings were then translated into English by the researcher. Additionally, classroom artifacts of teachers such as worksheets were collected.

In addition, during the classroom observation, 18 students were selected. Six high achieving students from Mr. Bond's class, six medium achieving students from Ms. Rose's class, and six low achievers from Mr. Jack's class were selected. These students were selected by their mathematical achievement as recommended by their mathematics teachers. In particular, students who communicated well were selected. The researcher did an informal interview with these 18 students once a week. The purpose of the informal interview was to build a rapport with these students before the formal interview.

Phase IV: Posttest

The posttest was given to the same 118 students one week after instruction. The same test used in the pretest was also used in the posttest, only the numbers and characters in the problems were changed. The researcher provided direction to students about taking the test. A ninth grade mathematics teacher in charge of that period was asked to help the researcher to administer and to collect the test. However, teachers did not have access to the instrument before or after the test to avoid teachers' bias. Students were asked to put their student identification number and their classroom number on the test. Students who finished early were asked to leave the test on a table and allowed to wait outside the classroom.

Phase V: Interview with Students

The purpose of the interview was to identify strategies that Thai ninth-grade students used to solve algebra word problems. A formal interview with selected students took place one week after the posttest. Eighteen students selected during the observation participated in the interview. Five algebra word problems used in the interview were selected from outside the students' textbook. The problems were the same for all 18 students. The interview problems can be seen in Appendix B.

Students were interviewed by the researcher in the Thai language during school mathematics periods in a quiet room. Each student was told that the interviewer was interested in how ninth grade students solved algebra word problems. Each student was asked to read and to solve five algebra word problems, one problem at a time. Each interview took 55 – 60 minutes. The interviewer and the student sat on the same side of a table. On the table, paper and pencils were available to the student. In order to put the student at ease prior to the interview, the interviewer talked with the students about their lives. The interviewer explained to each student what she would do during the interview and told the student that she would use a tape recorder in order to help her remember everything that was said. The tape recorder was in plain view of the students.

The students were asked to think aloud during solving the word problems. When the student was not talking, the interviewer asked a question to encourage students to talk. The prepared question was such as what were you thinking now? However, some students did not want the interviewer to ask questions during their thinking processes because they could not think with the interruption. Therefore, for some students, the interviewer asked the question after the students finished solving each problem. The prepared questions used in the interview were:

- What did the problem ask you to do?
- Can you describe the solution to the problem you've solved? How?
- Could you solve the problem another way?
- Have you thought about using other strategies?
- What are your ideas about where to go from here?

Other questions were asked to clarify individual students' problem solving processes. The questions began with the following: "Could you describe your solution to the problem and how you found it?" or "What did the problems ask you to find?" "Tell me how you knew that." Depending on the student's response to the initial question, the interviewer used several further questions like, "Tell me how you assumed the variable." When the student indicated the use of a mental strategy, the interviewer would ask, "Tell me out loud what you did in your mind." When the student got stuck for a long time, the interview would ask, "Where are you having difficulty?" After the student gave the final answer, the interviewer would ask "Are you sure this is the correct answer to the problem? Why?"

The following is an example of an interview protocol from one of the 18 interviewed students. In this protocol, the student was asked to solve, "The number of girls is $\frac{2}{3}$ of the number of boys in one class. If the total number of the students in this class is 45, find the number of girls in this class."

S: "Girls are $\frac{2}{3}$ of boy so boys are x."

I: "What is x?"

S: "x is the number of boys. The number of girls is $\frac{2}{3}$ of the number of boys. So I times $\frac{2}{3}$ with x. So the number of girls is $\frac{2}{3}x$."

I: "Why did you times $\frac{2}{3}$ with x?"

S: "Because the word 'of' ".

I: " 'Of' is multiply. So, you multiply $\frac{2}{3}$ with boys?"

S: "Yes."

I: "What are you going to do next?"

S: "Umm. I don't know."

I: "What is the total numbers of students?"

S: "45."

I: "How many are girls now?"

S: "$\frac{2}{3}x$."

I: "And how many are boys?"

S: "$\frac{1}{3}$."

I: "Umm. What did you assume for boys?"

S: "Oh. x."

I: "And what are you going to do next?"

S: "$\frac{2}{3}x$ equals 45."

I: " How come? $\frac{2}{3}x$ are the number of girls but 45 are total students. So overall students should have?"

S: "Both boys and girls."

I: "How many are girls?"

S: "$\frac{2}{3}x$."

I: "And how about are boys."

S: "$\frac{x}{3}$. Is it right?"

I: "What did you define for boys?"

S: "x."

I: "Ok, what is $\frac{2}{3}x$?"

S: "The number of girls."

I: "And what is x?"

S: "The number of boys."

I: "And what are you going to do to get 45?"

S: "Add boys and girls."

I: "What did you add?"

S: "$\frac{2}{3}x$ and x."

I: "Are you sure?"

S: "Yes"

I: "What is x?"

S: "The number of boys."

S: "27."

I: "What is 27?"

S: "The number of boys?"

I: "And how many are girls?"

S: "$\frac{2}{3}$ times 27 is 18."

I: "Are you sure now?"

S: "Yes. I checked the answer already."

During student's problem solving processes, the interviewer also checked the correctness of computations and errors. However, the interviewer did not give feedback about the correctness of any response. The interviewer continued questioning until it was clear whether or not the students understood the problem and what strategy the student was using. The student was also asked to describe their mathematics capability and what helped or hindered them in solving algebra word problems. While the student was solving each problem, the interviewer observed and took notes about what the student was doing.

The interviewer coded responses as the students solved each problem on the notepad. When a solution strategy that the student used was obvious or the student could not explain how he or she completed a problem, the interviewer went on to the next problem. Students' written work on each problem was also collected. Student's verbatim responses were recorded in Thai language using a tape recorder. The recordings were then translated into English by the researcher.

Researchers' Journal

Since the researcher was the primary data collection instrument, she might be a potential major threat to the reliability of the data analysis. Therefore, it was important to establish possible sources of biases or misinterpretations. Therefore, a daily journal was kept. The journal contained the researcher's reflections such as thoughts, questions, reactions, interpretations, and insights that were made during the data collection. The journal discouraged the researcher from relying on personal interpretations of the behaviors of the teacher and students.

Data Analysis

This section describes the methods and procedures that were used to analyze the data collected from this study. The researcher was the person to analyze the data for each phase of the study. The data were analyzed in both qualitative and quantitative ways. The data then were analyzed in order to answer the following five research questions.

How Successful are Thai Students in Solving Algebra Word Problems?

To answer this research question, individuals' work on ten problems on the pretest and posttest were graded. The student was classified as successful if he or she used an effective solution strategy that led to a correct answer and the student implemented the strategy without any misconceptions or errors in calculations. The student was classified as partially successful if he or she used an effective solution strategy that led to a correct answer, but an error had occurred. The student who could not solve the problem or did not attempt to solve the problem was classified as unsuccessful on that problem. The findings follow in CHAPTER IV.

Which Strategies are Used by Thai Students to Solve Algebra Word Problems?

Work from 118 students during the posttest was analyzed to determine solution strategies. In addition, verbal interaction from the individual interviews were transcribed and analyzed to determine solution strategies. The researcher noted solution strategies that students used. Based on previous literature (e.g., Bull, 1982; Koedinger & Tabachneck, 1994), students' solution strategies on word problems were categorized into algebraic strategies and non-algebraic strategies. Additional strategies were also recorded. Results were reported using numerical and descriptive methods. The presentation of these findings follows in CHAPTER IV.

What are Characteristics of Classroom Instruction during Word Problem-Solving Lessons?

Verbal transcription from a tape recorder of observations was analyzed. Notes taking during the observation were also analyzed. Data were analyzed to see what types of mathematics instructional strategy were used in each classroom as well as teachers' instruction and assessment, and students' participation. Demonstrations, explanations, questions, and responses between teachers and students were analyzed by searching for patterns. Next, the researcher compared data from each of the three teachers to see differences and/or similarities among the three teachers. No initial criterion was established. Results are reported using descriptive methods. The discussion of these findings follows in CHAPTER IV.

What are Potential Connections between Classroom Instruction and Students' Word Problem-Solving Performance?

The researcher compared data from teachers' observation and student's performance to examine potential connections between teachers' instruction and students' performance in solving algebra word problems. No initial criterion was established.

Results were reported using descriptive methods. The discussion of these findings follows in CHAPTER IV.

The Researcher

The primary instrument for data collection and analysis in this study was the researcher. The researcher was the person who developed instruments, administered the instruments, interviewed students, and observed the classroom. The researcher earned a Bachelor's degree in Mathematics Education in 1995 at Chiang Mai University in northern Thailand. She had taught secondary school mathematics for eight months; six months as a student teacher and two months as an in-service teacher. Following this brief experience as a teacher, the researcher came to Oregon State University in 1996 as a scholar from Thailand. She earned her Master's degree in Mathematics Education at Oregon State University in March 1999.

Her masters' thesis involved word problem solving at the elementary level. Her interest in word problem solving appeared when she had a chance to visit a second grade classroom several times in a small town in Oregon. The researcher observed elementary students solving a variety of word problems. She was surprised that the children had different ways to approach and to solve problems and had the ability to explain their own reasoning and thinking. Since she had done word problem solving study with elementary students for her masters' degree, she became interested in studying the same topic in algebra with secondary school students.

CHAPTER IV
RESULTS

Introduction

This chapter reports results on Thai students' solution strategies to algebra word problems and explores potential connections to teachers' instructional strategies. The results are presented in order to answer the research questions: (1) How successful are Thai students in solving algebra word problems? (2) Which strategies are used by Thai students to solve algebra word problems? (3) What are characteristics of classroom instruction during word problem-solving lessons? and (4) What are the potential connections between classroom instruction and students' word problem-solving performance?

This chapter is divided into four sections. The first section reports the success and difficulties of Thai students at solving equations and algebra word problems, which is related to the first research question. The second section describes strategies Thai students used to solve algebra word problems, which is related to the second research question. The third section describes teachers' instruction on solving algebra word problems, which is related to the third research question. The fourth section explains teacher's instruction in connection to students' performance in solving algebra word problems, which is related to the fourth research question.

Section one: Success and Difficulties of Thai Students

This section report results on students' success and difficulties in solving an equation and in translation from mathematical situations into equations. Finally, students' success and difficulties at solving algebra word problems will be reported.

Solving Equations and Translating Situations into Equations

This part presents results of Thai ninth grade students' success and difficulties in solving an equation (Problem 1) and in translating situations into equations (Problems 2 – 5). Please refer to Appendix A to find a complete set of these five problems. The results in this part are presented in order to show Thai students' ability to solve equations and their ability to translate situations into equations.

Problem 1

Pretest	Posttest
Solve for x: $\frac{4}{3}(x+8) = \frac{2}{5}(2x+1)$	Solve for x: $\frac{1}{2}(3x+6) = \frac{3}{4}(x+8)$

This problem asked students to solve an equation. In Table 6, the results indicate that before instruction, 42 of 118 students (35.6%) successfully solved this equation, 12 students (10.2%) partially solved this equation, and 64 students (54.2%) could not solve this equation. After instruction, more students were able to solve this problem. The results in Table 6 show that 71 of 118 students (60.2%) successfully solved this problem. Sixteen students (13.6%) partially solved this problem and 31 students (26.7%) could not solve this problem. For those who could not solve this problem both before and after instruction, many had misconceptions about adding terms in polynomials and fractions (see Figure 1). In Figure 1, the student incorrectly added x to 8 and got $8x$, and added $2x$ to 1 and got $3x$.

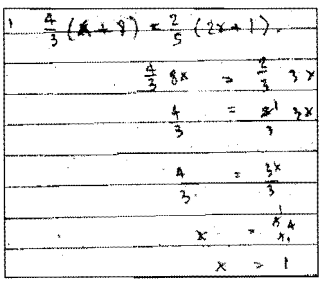


Figure 1. Misconception about combining terms.

Table 6

The number of students solving each problem by classroom before and after instruction (Problems 1 – 5)

Problem/Classroom	Mr. Jack (N = 38) Low Achieving Students		Ms. Rose (N = 42) Medium Achieving Students		Mr. Bond (N = 38) High Achieving Students		**Overall (N = 118)**	
	Number of Students: Successfully Solved (Partially Solved) [Not Solved]		Number of Students: Successfully Solved (Partially Solved) [Not Solved]		Number of Students: Successfully Solved (Partially Solved) [Not Solved]		Number of Students: Successfully Solved (Partially Solved) [Not Solved]	
	Before	After	Before	After	Before	After	Before	After
1.) Solve for x: $\frac{1}{2}(3x+6)=\frac{3}{4}(x+8)$ $(x = 4)$	0 (2) [36]	11 (3) [24]	9 (5) [28]	27 (4) [11]	33 (5) [0]	33 (2) [3]	**42** **(12)** **[64]**	**71** **(16)** **[31]**
2.) The ratio of professors and students in one college is 1:7. If S represent the number of students and P represent the number of professors, write the equation for the number of students. $(S = 7P)$	4 (0) [34]	1 (0) [37]	15 (0) [27]	15 (0) [27]	31 (0) [7]	6 (0) [32]	**50** **(0)** **[68]**	**22** **(0)** **[96]**
3.) Let X represent a number, please write an equation to represent the following statement. "Three times a number and six is 24. $(3x + 6 = 24)$	11 (0) [27]	16 (0) [22]	35 (0) [7]	38 (0) [4]	35 (0) [3]	32 (0) [6]	**81** **(0)** **[37]**	**86** **(0)** **[32]**
4.) Let W represent the width and L represent the length of a tennis court, please write an equation to represent the following statement. " The length of the tennis court is six meters more than twice the width" $(L = 2W + 6)$	3 (0) [35]	0 (0) [38]	6 (0) [36]	20 (0) [22]	19 (0) [19]	21 (0) [17]	**28** **(0)** **[90]**	**41** **(0)** **[77]**
5.) Let Y represent a number, please write an equation to represent the following statement. "Two times the sum of a number and three is 24." $[2(y + 3) = 24]$	2 (0) [36]	4 (0) [34]	30 (0) [12]	32 (0) [10]	21 (0) [17]	25 (0) [13]	**53** **(0)** **[65]**	**61** **(0)** **[57]**

In addition, some students did not understand the concept of equality or the distributive property (see Figure 2). In Figure 2, the student simply removed the bracket without applying the distributive property when solving $4(x+8)/3 = 2(2x + 1)/5$.

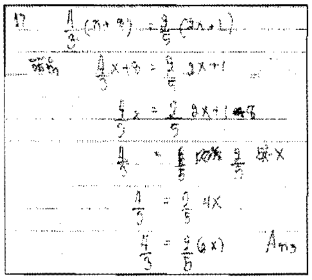


Figure 2. Incorrect use of the distributive property.

The results in Table 6 show that few students in Mr. Jack's class were successful at solving this problem either before or after instruction. Only 14 students (36.8%) were successful at solving this problem (including students who had errors in calculation) after instruction. Unlike Mr. Jack's class, the majority of students in Ms. Rose's class (64.3%) were successful at solving this equation after instruction. Only 11 students (26.2%) were unsuccessful at solving this problem. For Mr. Bond, the majority of his students (86.8%) were successful at solving this problem both before and after instruction.

Problem 2

Pretest

The ratio of teachers and students in one college is 1:6. If *S* represent the number of students and *T* represent the number of teachers, write the equation for the number of students.

Posttest

The ratio of professors and students in one college is 1:7. If *S* represent the number of students and *P* represent the number of professors, write the equation for the number of students.

This problem asked students to translate verbal statements into equations. The results indicate that few students in this study were successful at translating this problem into equations either before or after instruction. The results in Table 6 reveal that before instruction, only 50 of 118 students (42.4%) successfully translated this problem into equations and 68 students (57.6%) could not translate this problem.

After instruction, few students were able to translate this problem. The results in Table 6 indicate that 22 of 118 students (18.7%) successfully translated this problem and 96 students (81.3%) could not translate this problem. The results in Table 6 show that few students in Mr. Jack's and Ms. Rose's classes were successful at translating this problem either before or after instruction. For Mr. Bond, the results in Table 6 show that the majority of his students were successful at translating this problem before instruction. However, the students in Mr. Bond's class were unsuccessful in translating this problem after instruction. All of them left the space blank, so it is difficult to explain why this happened.

Problem 3

Pretest

Let *X* represent a number, please write an equation to represent the following statement. "Two times a number and three is 21."

Posttest

Let *X* represent a number, please write an equation to represent the following statement. "Three times a number and six is 24."

Like Problem 2, this problem asked students to translate verbal statements into equations. The results indicate that students in this study were successful at translating this problem into equations both before and after instruction. The results in Table 6 indicate that before instruction, 81 of 118 students (68.6%) successfully translated this problem and 37 students (31.4%) could not translate this problem successfully. For those who could not translate this problem successfully, five of them had errors as shown in Figure 3.

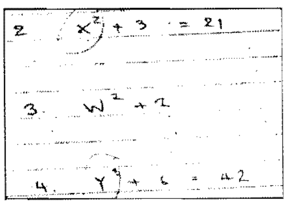


Figure 3. Errors in translation Problems 2, 3, and 4 in the pretest.

In Figure 3, the student translated "twice a number and three is 21" as $x^2 + 3 = 21$ rather than giving the correct equation ($2x + 3 = 21$). This was found across all three classes indicating that the student could not differentiate between symbol representing multiplication and exponents.

The results in Table 6 reveal that after instruction, 86 of 118 students (72.9%) successfully translated this problem and 32 students (27.1%) could not translate this problem. For those who could not translate this problem successfully, most of them had errors such as in Figure 3. All of the students who made this error on the posttest were from the low achieving class. When looking at each class, the results in Table 6 illustrate that few students in Mr. Jack's class were successful at translating this problem either before or after instruction. For Ms. Rose and Mr. Bond, the majority of their students were successful at translating this problem both before and after instruction.

Problem 4

Pretest	Posttest
Let W represent the width of a tennis court, please write an equation to represent the following statement. "The length of the tennis court is two meters more than twice the width."	Let W represent the width and L represent the length of a tennis court, please write an equation to represent the following statement. "The length of the tennis court is six meters more than twice the width."

This problem also asked students to translate verbal statements into equations. The results indicate that few students in this study were successful at translating this problem either before or after instruction. The results in Table 6 show that before instruction, 28 of 118 students (23.7%) successfully translated this problem and 90 students (76.3%) could not translate this problem successfully. For those who could not translate this problem successfully, most of them had errors such as in Figure 3. In Figure 3, the student translated "the length is two more twice the width" to "$w^2 + 2$". This result indicates that the student could not differentiate between symbol representing multiplication and exponents.

After instruction, the results in Table 6 indicate that 41 of 118 students (34.7%) successfully translated this problem and 77 students (65.3%) could not translate this problem successfully. The results in Table 6 demonstrate that few students in Mr. Jack's class were successful at translating this problem either before or after instruction. For Ms. Rose, almost half of her students did better after instruction. Mr. Bond's students were more successful than students in the other two classes in translating this problem both before and after instruction.

Problem 5

Pretest	Posttest
Let Y represent a number, please write an equation to represent the following statement. "Three times the sum of a number and six is 42."	Let Y represent a number, please write an equation to represent the following statement. "Two times the sum of a number and three is 24."

This problem also asked students to translate verbal statements into equations. The results in Table 6 illustrate that before instruction, 53 of 118 students (44.9%) successfully translated this problem and 65 students (55.1%) could not translate this problem successfully.

After instruction, the results in Table 6 show that 61 of 118 students (51.7%) successfully translated this problem and 57 students (48.3%) could not translate this problem successfully. Those students, who could not translate this problem successfully, wrote the equation as if it were the same situation as in Problem 3. That is, the students wrote "$2y + 3 = 24$" rather than "$2(y + 3) = 24$". The results in Table 6 show that few students in Mr. Jack's class were successful in translating this problem either before or after instruction. For Ms. Rose and Ms. Bond, their students were more successful at translating this problem both before and after instruction, though little improvement from before instruction to after instruction was evident.

Conclusion

This part presents the overall picture of Thai ninth grade students' success and difficulties in solving an equation (Problem 1) and in translating situations into equations (Problems 2 – 5). The results show that Thai ninth grade students in this study have moderate knowledge and skill in solving equations and in translating situations into equations. In solving an equation (Problem 1), the results indicate that many of the students in this study (60.2%) were able to solve equations, even though some errors still existed on the posttest. Most errors were due to misconceptions about adding terms in a polynomial expression, and about the equality concept or distributive property. The results from this study show that Thai ninth grade students were able to solve equations as indicated in Thai Mathematical Standards (IPTST, 2000) and in Principles and Standards for School Mathematics (NCTM, 2000). The NCTM Standards 2000 also indicate that students at this grade level should be able to understand equality concepts and the distributive property. However, the results from this study show that some of Thai ninth grade students still had difficulty in using the distributive property and did not understand the concept of equality as recommended in the NCTM Standards 2000.

The results demonstrate that 72.9 % of the students in this study were successful at translating Problem 3 and 51.7 % of the students in this study were successful at translating Problem 5 into equations. However, the results show that many students had difficult in translating Problems 2 and 4 into equations even after the students had gone through formal instruction. Only 18.7% and 34.7% of the students in this study were successful at translating Problems 2 and 4 into equations respectively.

Although no data were collected to attempt to explain this situation, one could conjecture that structure of the problem situations might affect student's ability to translate Problems 2 and 4 successfully. The difference between Problems 2 and 4, and Problems 3 and 5 is unknown variables presented in the problem situation. Problems 2 and 4 contain more than one unknown variable and students need to relate those variables in order to form an equation. In contrast, Problems 3 and 5 contain one unknown variable and students need to use that variable to form an equation. Even though Problem 2 was adapted from the students' eighth grade textbook, many students were still unable to translate this problem situation into equations. Thus, it is possible that many students in this study have difficulty in translating the problem contained more than one unknown variable, which were related.

The results from the four translating problems show that Thai ninth grade students were sometimes able to use mathematical language and symbols to represent a problem situation as indicated in both Thai Mathematical Standards (IPTST, 2000) and in NCTM Standards 2000. However, many students in this study were unsuccessful at using mathematical language and symbols to represent a problem situation, which contained more than one unknown variable.

In conclusion, many Thai ninth grade students in this study had some skills at solving equations. However, their skills in translating verbal statements into equations were limited to the structure of the problems (e.g., unknown variables presented in the problem). As one would expect, the results indicate that few low achieving students in Mr. Jack's class were successful at solving equations and translating problem situations into equations. Their performance improved little after instruction. In contrast, medium and high achieving students from Ms. Rose's and Mr. Bond's classes were more successful at solving an equation and translating situations into equations. Their performance improved much after instruction, especially Ms. Rose's class. Next, students' successes and difficulties in solving algebra word problems will be discussed.

Solving Algebra Word Problems

This section reports success and difficulties of Thai ninth grade students in solving algebra word problems. This part focuses mainly on students' success and difficulties on Problems 6 to 10, which are algebra word problems. The complete set of Problems 6 – 10 used before and after instruction can be seen in Appendix A. The results in this part are presented separately by each of the three teachers' classes. The results from the pretest and posttest indicated that 118 students used either algebraic strategies or non-algebraic strategies to solve algebra word problems. In the algebraic strategies, the students used variables and symbols to form an equation. Then, the students solved the equation to find an answer to the problem. In the non-algebraic strategies, the students used their arithmetic knowledge to solve algebra word problems. More discussion about solution strategies that students in this study used are discussed in section "Thai Students' Solution Strategies" of CHAPTER IV.

Problem 6

Pretest	Posttest
Wallace works as a part-time typist. Last month, he worked 3 extra hours. When he added the amount he earned for working extra hours and his monthly wage of 500 Baht, he found that he earned 665 Baht. How much per hour did Wallace get? (Note: Baht is Thai currency, 41 Baht = One US Dollar)	Kobe works as a part-time typist. Last month, he worked 5 extra hours. When he added the amount he earned for working extra hours and his monthly wage of 6,500 Baht, he found that he earned 7,945 Baht. How much per hour did Kobe get?

Mr. Jack's class. The results in Table 7 indicate that before instruction, ten students (26.3%) successfully solved this problem by using a non-algebraic strategy. After instruction, the results in Table 7 indicate that seven of these ten successful students attempted to solve the problem while the other three students did not attempt to solve the problem because they left the paper blank. Of the seven students who attempted to solve the problem, four students still used a non-algebraic strategy to solve the problem (see Figure 4). In Figure 4, the student first subtracted 6500 from 7945 and got 1445. Then the student divided 1445 by 5 to get the amount per hour, which is 289 Baht per hour. The results in Table 7 show that among the four students who used a non-algebraic strategy, two of them had errors in computation.

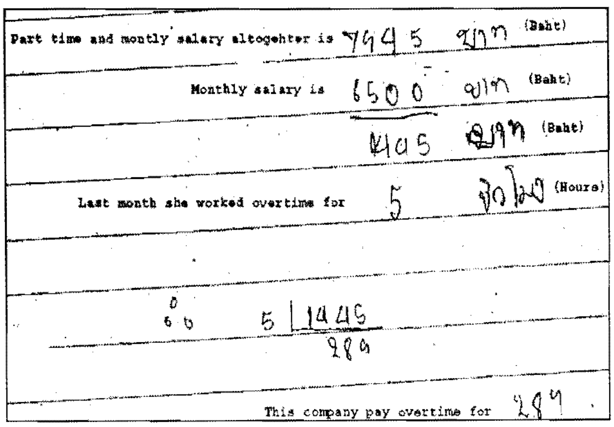

Part time and montly salary altogehter is 7945 (Baht)
Monthly salary is 6500 (Baht)
1445 (Baht)
Last month she worked overtime for 5 (Hours)
5 | 1445
289
This company pay overtime for 289 .

Figure 4. Example of a non-algebraic strategy used by the student in Mr. Jack's class to solve Problem 6 successfully.

Table 7

The number of students solving Problem 6 separated by strategies used before and after instruction

Class	Before Instruction		After Instruction: Attempted	After Instruction: Not Attempted
Mr. Jack	Successful (n = 10)	Non-algebra + (10)	Non-algebra + (2) Non-algebra – (2) Algebra + (1) Random Calculation - (2)	3
	Partial Successful (n = 3)	Non-algebra – (3)	Non-algebra + (1) Non-algebra – (2)	0
	Unsuccessful (n = 25)	Paper Blank (25)	Non-algebra + (1) Non-algebra – (1) Algebra + (3) Random Calculation - (4)	16
Ms. Rose	Successful (n =27)	Non-algebra + (24)	Non-algebra + (2) Non-algebra – (1) Algebra + (19) Algebra – (2)	0
		Algebra + (3)	Algebra + (2) Algebra – (1)	
	Partial Successful (n = 1)	Non-algebra – (1)	Algebra + (1)	0
	Unsuccessful (n = 14)	Paper Blank (14)	Algebra + (12) Algebra – (1) Non-algebra + (1)	0
Mr. Bond	Successful (n = 36)	Non-algebra + (22)	Non-algebra + (8) Non-algebra – (1) Algebra + (13)	0
		Algebra + (14)	Algebra + (10) Non-algebra + (4)	
	Unsuccessful (n = 2)	Paper Blank (2)	Non-algebra + (1) Algebra + (1)	0

Note: + indicates that students successfully solved the problem.
- indicates that students unsuccessfully solved the problem.

The results in Table 7 demonstrate that another student changed to use an algebraic strategy to solve the problem after instruction (see Figure 5). In Figure 5, the student defined x for amount per hour, and the equation formed was $5x = 7{,}945 - 6{,}500$. After that, the student solved for the variable x to get the amount per hour.

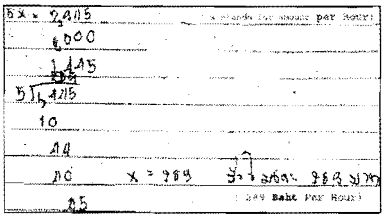


Figure 5. Example of an algebraic strategy used by the student in Mr. Jack's class to solve Problem 6 successfully.

The results in Table 7 demonstrate that before instruction, three students (7.9%) partially solved this problem by using a non-algebraic strategy with errors in calculation (e.g. wrongly subtracted or divided numbers). After instruction, the results in Table 7 show that all three of these students attempted to solve the problem. Among these three students, only one student successfully solved the problem by using a non-algebraic strategy similar to Figure 4. The other two students also used a non-algebraic strategy but still had errors in computations.

Before instruction, the results in Table 7 demonstrate that 25 students (65.8%) were unsuccessful at solving this problem because they left the paper blank. After instruction, only nine students attempted to solve the problem while the other 16 students did not attempt to solve the problem. The results in Table 7 indicate that of these nine students who attempted to solve the problem, two students used a non-algebraic strategy (see Figure 4) to solve the problem. Between the two students who used a non-algebraic strategy, one had errors in computations. The other three students used an algebraic strategy to solve the problem successfully (see Figure 5) and four other students did some random calculations with the numbers in the problems.

In summary, the results show that few students in Mr. Jack's class were successful at solving this problem either before or after instruction. Before instruction, ten students (26.3%) successfully solved this problem by using a non-algebraic strategy. Three students

(7.9%) partially solved this problem by using a non-algebraic strategy with errors in calculations (e.g. wrongly subtracted or divided numbers). Twenty-five students (65.8%) did not attempt to solve the problem and left the paper blank.

After instruction, eight students (21%) successfully solved this problem. Of the eight successful students, four students used a non-algebraic strategy. The other four successful students used an algebraic strategy. Five students (13.2%) partially solved this problem, four of them used a non-algebraic strategy and one student used an algebraic strategy. However, they had errors in calculations. Twenty-five students (65.8%) could not solve this problem. Of these 25 students, 19 students left the paper blank and six students did some calculations with the numbers in the problem. These results indicate that students' performance in Mr. Jack's class improved little after instruction. Half of the students in this class did not attempt to solve the problem and left the paper blank after instruction.

Ms. Rose's class. The results in Table 7 indicate that before instruction, 27 students (64.3%) successfully solved this problem. Of these 27 students, 24 students used a non-algebraic strategy and the other three students used an algebraic strategy to solve the problem successfully. After instruction, all 27 successful students attempted to solve the problem. The results in Table 7 show that of the 24 who used a non-algebraic strategy before instruction, three students continued to use this same strategy to solve the problem successfully (see Figure 6).

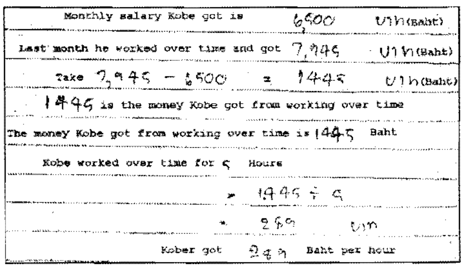
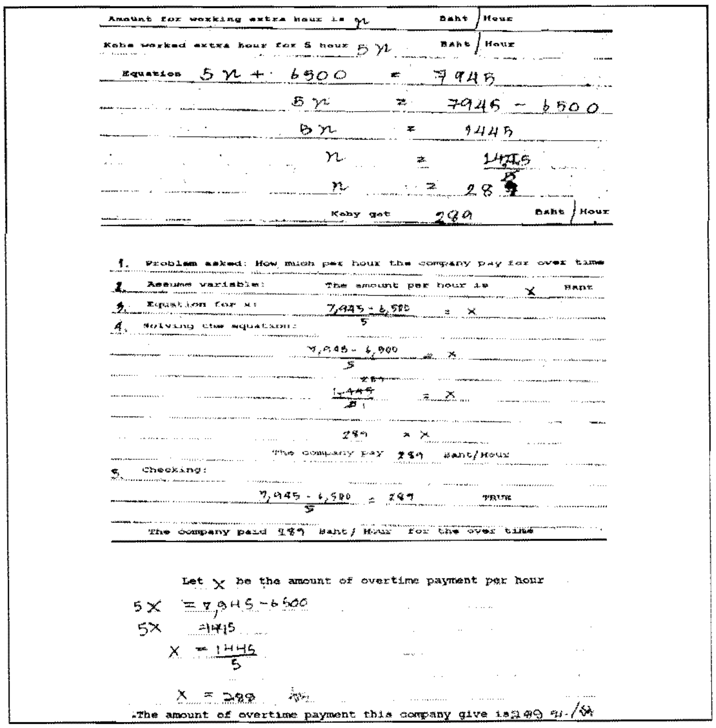

Monthly salary Kobe got is 6,500 บาท (Baht)
Last month he worked over time and got 7,945 บาท (Baht)
Take 7,945 − 6,500 = 1445 บาท (Baht)
1445 is the money Kobe got from working over time
The money Kobe got from working over time is 1445 Baht
Kobe worked over time for 5 Hours
= 1445 ÷ 5
= 289 บาท
Kober got 289 Baht per hour

Figure 6. This is an example of a non-algebraic strategy used by the student in Ms. Rose's class to solve Problem 6 successfully.

In Figure 6, the student subtracted 6,500 from 7,945 and got 1445. Then the student divided 145 by 5 to get the amount per hour, which is 289 Baht. Among the three students who used a non-algebraic strategy, two students successfully solved this problem while one student had errors in computations. The results in Table 7 show that the other 21 students who used a non-algebraic strategy before instruction changed to use an algebraic strategy to solve the problem after instruction (see Figure 7). In Figure 7 the student defined x as the amount per hour and then formed the equation. However, their equations were varied. Then, the student solved for x to get the amount per hour, which is 289 Baht. Among the 21 students who used an algebraic strategy, 19 students successfully solved this problem while the other two students had errors in computation. The results in Table 7 demonstrate that three students who used an algebraic strategy before instruction continued to use an algebraic strategy after instruction. Of these three students, two students successfully solved the problem while one student had errors in computation.



Amount for working extra hour is n Baht / Hour

Kobe worked extra hour for 5 hour $5n$ Baht / Hour

Equation $5n + 6500 = 7945$

$5n = 7945 - 6500$

$5n = 1445$

$n = \frac{1445}{5}$

$n = 289$

Koby got 289 Baht / Hour

1. Problem asked: How much per hour the company pay for over time
2. Assume variable: The amount per hour is x Baht
3. Equation for x: $\frac{7,945 - 6,500}{5} = x$
4. Solving the equation:

$\frac{7,945 - 6,500}{5} = x$

$\frac{1,445}{5} = x$

$289 = x$

The company pay 289 Baht/Hour

5. Checking:

$\frac{7,945 - 6,500}{5} = 289$ TRUE

The company paid 289 Baht / Hour for the over time

Let x be the amount of overtime payment per hour

$5x = 7,945 - 6500$

$5x = 1445$

$x = \frac{1445}{5}$

$x = 289$

The amount of overtime payment this company give is 289

Figure 7. Variety of equations used by the student in Ms. Rose's and Mr. Bond's classes to solve Problem 6 successfully.

Before instruction, the results in Table 7 show that one student (2.4%) partially solved this problem by using a non-algebraic strategy but had an error in computation. After instruction, this student attempted to solve the problem successfully by using an algebraic strategy. Before instruction, the results in Table 7 indicate that 14 students (33.3%) were unsuccessful at solving this problem because they left the paper blank. After instruction, all 14 students attempted to solve the problem. Of these 14 students, 13 students used an algebraic strategy to solve the problem. Of the 13 students who used an algebraic strategy, 12 students successfully solved the problem while one student had an error in computation. One student used a non-algebraic strategy to solve the problem successfully.

In summary, the results show that many students in Ms. Rose's class were successful at solving Problem 6 before instruction. The results indicate that, before instruction, 27 students (64.3%) successfully solved this problem. Of these 27 students, 24 students used a non-algebraic strategy and three students used an algebraic strategy. One student (2.4%) partially solved this problem by using a non-algebraic strategy, but had an error in computation. Fourteen students (33.3%) did not attempt to solve this problem and left the paper blank. After instruction, the results indicate that all students in Ms. Rose's class attempted to solve this problem. Thirty-seven students (85.7%) successfully solved this problem. Of the 37 successful students, 34 students used an algebraic strategy and three students used a non-algebraic strategy. Five students (14.3%) partially solved this problem. Of these five students, four students used an algebraic strategy and one student used a non-algebraic strategy. However, all of them had an error in calculation. After instruction, the results show that only five students (4.2%) in Ms. Rose's class checked their answer by substituting the answer into the equation. These results indicate that students' performance in Ms. Rose's class improved much after instruction. All students attempted to solve the problem after instruction and used more algebraic strategies to solve this problem.

Mr. Bond's class. The results in Table 7 show that before instruction, 36 students (94.7%) successfully solved this problem. Of these 36 students, 22 students used a non-algebraic strategy and the other 14 students used an algebraic strategy. After instruction, all 36 successful students attempted to solve the problem. The results in Table 7 show that of the 22 students who used a non-algebraic strategy before instruction, nine students continued to use this same strategy to solve the problem after instruction. Among the nine students who still used a non-algebraic strategy, eight students successfully solved the problem while one student had errors in computation. The other 13 students who used a non-algebraic strategy before instruction changed to use an algebraic strategy (see Figure 7). Of the 14 students who used an algebraic strategy before instruction, ten of them continued to use an algebraic strategy while the other four students changed to use a non-algebraic strategy to solve the problem successfully.

The results in Table 7 indicate that before instruction, two students (5.3%) did not solve this problem and left the paper blank. After instruction, these two students attempted to solve the problem. The results in Table 7 show that between these two students, one student successfully solved this problem by using a non-algebraic strategy and another student successfully solved this problem by using an algebraic strategy.

In summary, the results show that students in Mr. Bond's class were successful at solving this problem both before and after instruction. Before instruction, 36 students (94.7%) successfully solved this problem. Of these 36 students, 22 students used a non-algebraic strategy and 14 students used an algebraic strategy. Two students (5.3%) did not attempt to solve this problem and left the paper blank. After instruction, the results demonstrate that 37 students (97.4%) successfully solved this problem. Of 37 successful students, 24 students used an algebraic strategy and 13 students used a non-algebraic strategy. One student (2.6%) partially solved this problem because he used a non-algebraic strategy with errors in calculation. These results indicate that students' performance in Mr. Bond's class improved after instruction. The results show that all the students in Mr. Bond's class attempted to solve the problem and used more algebraic strategies to solve this problem after instruction.

Summary. The results indicate that overall many students in this study were successful at solving Problem 6 both before and after instruction. The results from this study show that before instruction, 73 of 118 students (61.9%) successfully solved this problem. Four students (3.4%) partially solved this problem. Forty-one students (34.7%) did not solve this problem and left the paper blank. The strategy students used before engaging in formal instruction was generally a non-algebraic strategy. The results indicate that, regardless of any errors the students made, 67 students (56.8%) used a non-algebraic strategy and 17 students (14.4%) used an algebraic strategy before instruction. After instruction, the results indicate that 82 of 118 students (69.5%) successfully solved this problem after instruction. Eleventh students (9.3%) partially solved this problem and 25 students (21.2%) could not solve the problem. The strategy students used after instruction was more algebraic-based. The results indicate that, regardless of any errors the students made, 67 students (56.8%) used an algebraic strategy and 26 students (22%) used a non-algebraic strategy after instruction.

Problem 7

Pretest

Kim begins to bike at 9:00 am at the rate 5 kilometer per hour. Two hours later, at the same starting point, Tim begins to bike to the same direction as Kim at the rate 10 kilometer per hour. At what time Kim and Tim will meet?

Posttest

John begins to bike at 10:00 am at the rate 10 kilometer per hour. At 12:00 pm, at the same starting point, Jordan begins to bike to the same direction as John at the rate 20 kilometer per hour. At what time John and Jordan will meet?

Mr. Jack's class. The results in Table 8 indicate that before instruction, one student (2.6%) successfully solved this problem by using a non-algebraic strategy. After instruction, this student attempted to solve the problem by using a non-algebraic strategy (see Figure 8). In Figure 8, the student drew two lines. The first line represents distances and times of John. The second line represents distances and times of Jordan. When the distance of both people was equal, the student stopped comparing and gave the times that John and Jordan meet, which is 2 p.m.

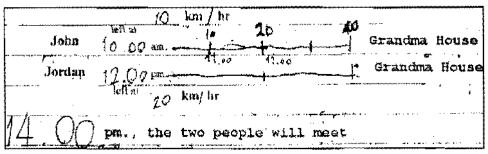


Figure 8. A non-algebraic strategy used by the student in Mr. Jack's class to solve Problem 7 successfully.

Table 8

The number of students solving Problem 7 separated by strategies used before and after instruction

Class	Before Instruction		After Instruction: Attempted	After Instruction: Not Attempted
Mr. Jack	Successful (n = 1)	Non-algebra + (1)	Non-algebra + (1)	0
	Partial Successful (n = 1)	Only answer given +(1)	0	1
	Unsuccessful (n = 36)	Paper Blank (31) Random Calculation - (5)	Non-algebra + (3) Non-algebra - (1) Random Calculation - (7) Only answer given - (3)	22
Ms. Rose	Partial Successful (n = 4)	Non-algebra – (2)	Non-algebra + (1) Algebra + (1)	0
		Only answer given +(2)	Only answer is given - (2)	
	Unsuccessful (n = 38)	Paper Blank (34) Random Calculation - (4)	Algebra + (17) Algebra – (2) Non-algebra + (4) Non-algebra – (3) Only answer is given - (2)	10
Mr. Bond	Successful (n = 18)	Non-algebra + (18)	Non-algebra + (9) Algebra + (6) Algebra - (1) Only answer is given - (2)	0
	Partial Successful (n = 4)	Non-algebra – (1)	Non-algebra + (1)	0
		Only answer is given + (3)	Non-algebra + (2) Only answer is given - (1)	
	Unsuccessful (n = 16)	Paper Blank (16)	Algebra + (1) Algebra - (1) Non-algebra + (2) Only answer is given - (3)	9

Note: + indicates that students successfully solved the problem

- indicates that students unsuccessfully solved the problem.

The results in Table 8 show that one student (2.6%) partially solved this problem before instruction. This student gave the correct answer without a clear explanation. After instruction, the results in Table 8 indicate that this student did not attempt to solve the problem because the paper was blank.

Before instruction, the results in Table 8 show that 36 students (94.8%) were unable to solve this problem. Of these 36 students who were unable to solve this problem, 31 students left the paper blank and the other five students did something to combine the numbers in the problem without using any apparent logic. After instruction, 14 students attempted to solve the problem while the other 22 students did not attempt to solve the problem because they left the paper blank. The results in Table 8 indicate that of the 14 students who attempted to solve the problem, four students used a non-algebraic strategy. Among the four students who used a non-algebraic strategy, three students successfully solved this problem while one student had errors in computations. He had errors in counting of time and distance. For example:

At 10:00 a.m. John went 5 km.

At 11:00 a.m. John went 10 km.

At 12:00 p.m. John went 15 km., Jordan began to bike and went 10 km.

At 13:00 p.m. John went 20 km., and Jordan also went 20 km.

Therefore, the answer is 13 o'clock that John and Jordan met, which is an incorrect answer. The student implemented an effective strategy but had errors in counting the interval of times and distance. The results in Table 8 show that the other seven students did some random calculation with the numbers given in the problem and the other three students provided the correct answer without a clear explanation.

In summary, the results show that few students in Mr. Jack's class were successful at solving Problem 7 either before or after instruction. The results indicate that before instruction, one student (2.6%) successfully solved this problem by using a non-algebraic strategy. One student (2.6%) partially solved this problem. This student gave the correct answer without a clear explanation. Thirty-six students (94.8%) were unable to solve this problem. Of these 36 students, 31 students left the paper blank and five students did something to combine the numbers in the problem apparently without using any apparent logic. After instruction, few students in this class were successful at solving this problem. The results show that four students (10.5%) correctly solved this problem by using a non-algebraic strategy. Four students (10.5%) partially solved this problem. Of the four students who partially solved this problem, one student used an algebraic strategy, but she had errors in counting of time and distance. Thirty students (80%) could not solve this problem. Among these 30 students, 23 students left the paper blank and seven students did some calculations with the numbers in the problem, which did not correspond to the situation or the question asked in the problem. These results indicate that students' performance in Mr. Jack's class improved little after instruction. The results show that many students (60.5%) in this class still could not solve this problem and none of the students in this class used an algebraic strategy to solve the problem after instruction.

Ms. Rose's class. The results in Table 8 indicate that before instruction, no students in Ms. Rose's class successfully solved this problem. Four students (9.5%) partially solved this problem. Of these four students, two students used a non-algebraic strategy but made similar errors to those in Mr. Jack's class. The other two students produced a correct answer but their solution strategy was not clearly explained. After instruction, the results indicate that these four students attempted to solve the problem. The results in Table 8 indicate that of the two students who had errors before instruction, one student still used a non-algebraic strategy to solve this problem successfully (see Figure 9). In Figure 9, the students compared distances of John and Jordan hour by hour. When the distance of both people was equal, the students stopped comparing and gave the times that John and Jordan met. The results in Table 8 show that another student who used a non-algebraic strategy successfully before instruction changed to use an algebraic strategy successfully after instruction (see Figure 10).

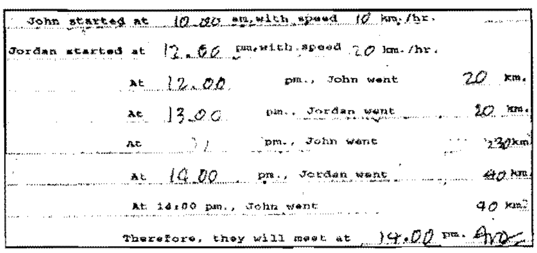


Figure 9. Example of a non-algebraic strategy that the student in Ms. Rose's class used to solve Problem 7 successfully.

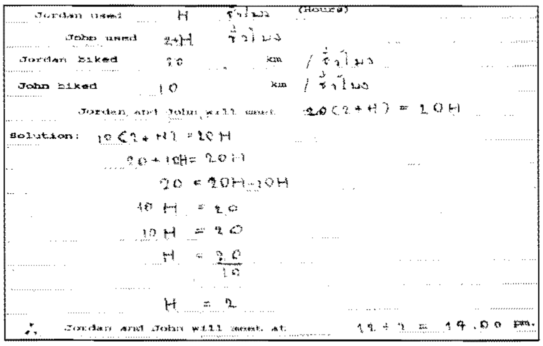


Figure 10. Example of an algebraic strategy used by the student in Ms. Rose's and Mr. Bond's classes to solve Problem 7 successfully.

In Figure 10, *H* stood for times in hours that Jordan used to bike from the beginning until he met John. The expressions *2 + H* stood for times in hours John used to bike from the beginning until he met Jordan. Expressions 20(*H*) and 10(2 + *H*) stood for the distance John and Jordan attain, which must be equal. Therefore, 10(2 + *H*) = 20(*H*) is the equation student formed. The student, then, solved for *H*, which is equal to 2 hours. Finally, the student added 2 hours to 12:00 p.m., which is the time Jordan began to bike. The answer then, is 14:00 p.m. when John and Jordan would meet. The results in Table 8 indicate that the other two students who produced a correct answer without a clear explanation before instruction still produced a correct answer without a clear explanation after instruction.

The results in Table 8 indicate that 38 students (90.5%) were unsuccessful at solving this problem before instruction. Of these 38 students, 34 students left the paper blank and four students did some random calculations with the number in the problems, which did not correspond to the situation or the question asked in the problem. After instruction, 28 students attempted to solve this problem while the other ten students still did not attempt to solve the problem because they left the paper blank. The results in Table 8 show that, of the 28 students who attempted to solve this problem, 19 students used an algebraic strategy (see Figure 10). Among these 19 students, 17 students successfully solved this problem while two students had errors in computation. The other seven students used a non-algebraic strategy (see Figure 9). Among these seven students, four students successfully solved this problem while the other three students had errors in computation. Two students produced a correct answer without a clear explanation.

In summary, the results show that none of students in Ms. Rose's class were successful at solving this problem before instruction. Four students (9.5%) partially solved this problem. Of these four students, two students used a non-algebraic strategy but they made errors. The other two students produced a correct answer but their solution strategy was not clearly explained. Of the 38 students (90.5%) who could not solve this problem, 34 students left the paper blank and four students did some calculations with the number in the problems, which did not correspond to the situation or the question asked in the problem. After instruction, the results indicate that more students attempted to solve this problem. The results show that 23 students (54.8%) were successful at solving this problem. Of the 23 successful students, 18 students used an algebraic strategy. The other five successful students used a non-algebraic strategy. Nine students (21.4%) partially solved this problem. Of these nine students, three students used a non-algebraic strategy but had errors, two students used an algebraic strategy with an error in calculation, and four students produced the correct answer without a clear explanation. After instruction, ten students (23.8%) did not attempt to solve this problem and left the paper blank. After instruction, the results show that two students (4.8%) in Ms. Rose's class checked their answer by substituting the answer into the equation. These results indicate that students' performance in Ms. Rose's class was much improved after instruction. The results show that more students attempted to solve this problem and used more algebraic strategies to solve the problem after instruction.

Mr. Bond's class. The results in Table 8 show that before instruction, 18 students (47.4%) successfully solved this problem. All of them used a non-algebraic strategy. After instruction, all 18 successful students attempted to solve this problem. The results indicate that nine students who used a non-algebraic strategy before instruction continued to use this strategy to solve the problem successfully (see Figure 11). In Figure 11, the student compared times and distances by drawing two rows. The first row represented the time and distances of John. The second row represented the time and distance of Jordan. Finally, the student matched the equal distance and found the time as the answer.

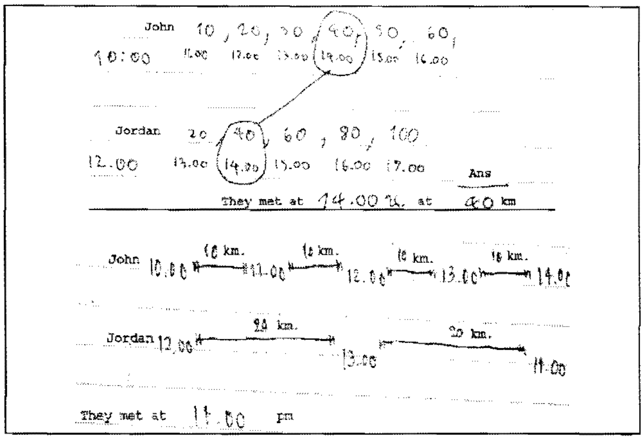


Figure11. A non-algebraic strategy used by the student in Mr. Bond's class to solve Problem 7 successfully.

The results in Table 8 indicate that the other seven students who used a non-algebraic strategy before instruction changed to use an algebraic strategy. Of these seven students, six students successfully solved this problem while another student had errors in computation. The other two students who used a non-algebraic strategy before instruction provided a correct answer without a clear explanation after instruction.

Before instruction, the results in Table 8 indicate that four students (10.5%) partially solved this problem. Among these four students, one student used a non-algebraic strategy but had an error, and the other three students produced the correct answer without a clear explanation. After instruction, all four students attempted to solve the problem. The results in Table 8 show that one student who had errors before instruction solved the problem successfully by using a non-algebraic strategy after instruction. For the three students who provided only a correct answer before instruction, two of them changed to use a non-algebraic strategy to solve the problem successfully and one student still provided a correct answer without a clear explanation.

The results in Table 8 show that before instruction, 16 students (42.1%) were unable to solve this problem because they left the paper blank. After instruction, the results show that only seven students attempted to solve the problem while the other nine students could not solve this problem because they left the paper blank. The results in Table 8 indicate that of the seven students who attempted to solve the problem, two students used an algebraic strategy. Between these two students, one student successfully solved this problem while one student had an error in computation. The other two students used a non-algebraic strategy and solved this problem successfully. The other three students provided a correct answer without a clear explanation.

In summary, the results show that 18 students in Mr. Bond's class (47.4%) successfully solved this problem before instruction. All of them used a non-algebraic strategy. Four students (10.5%) partially solved this problem, one student used a non-algebraic strategy but had an error and three students produced the correct answer without a clear explanation. Sixteen students (42.1%) did not attempt to solve this problem because they left the paper blank. After instruction, the results indicate that more students in Mr. Bond's class attempted to solve this problem. The results show that 21 students (55.3%) successfully solved this problem. Of these 21 students, 14 students used a non-algebraic strategy. The other seven students used an algebraic strategy. Eight students (21.1%) partially solved this problem, two students used an algebraic strategy but had an error in calculation and six students gave a correct answer without clear explanation. Nine students (23.6%) did not attempt this problem. All of them left the paper blank. These results indicate that students' performance in Mr. Bond's class improved little when compared to their performance before instruction. The results show that more students attempted to solve this problem and used more algebraic strategies to solve the problem after instruction. However, some of them still used a non-algebraic strategy to solve this problem.

Summary. The results show that overall few students in this study were successful at solving the time-rate-distance problems before instruction. The results show that before instruction, only 19 of 118 students (16.1%) successfully solved this problem by using a non-algebraic strategy. Nine students (7.6%) partially solved this problem. Ninety students (76.3%) could not solve this problem. The results show that, regardless of any errors the students made, 22 students (50.8%) used a non-algebraic strategy. None of the students in this study used an algebraic strategy before instruction. After instruction, the results show that few students in this study were successful at solving this problem. Only 49 of 118 students (41.5%) successfully solved this problem. Twenty students (17%) partially solved this problem. Forty-nine students (41.5%) could not solve this problem. After instruction, the students were still using a non-algebraic strategy to solve the problem rather than using an algebraic strategy. The results show that, regardless of any errors the students made, 27 students (22.9%) still used a non-algebraic strategy while 29 students (24.6%) used an algebraic strategy before instruction.

Problem 8

Pretest	Posttest
In 12 years, the ratio of father and son's ages will be 3:1. If the father is 30 years older than the son at the present time, find the age of the father.	In 12 years, the ratio of mother and son's ages will be 3:1. If the mother is 34 years older than the son at the present time, find the age of the mother.

Mr. Jack's class. The results in Table 9 indicate that before instruction, none of the students in this class successfully solved this problem. Of the 38 students (100%) who were unable to solve this problem, 34 students left the paper blank and the other four students did some random calculations with the numbers in the problem. After instruction, the results indicate that only nine students attempted to solve this problem while the other 29 students did not attempt to solve this problem. The results in Table 9 show that of the nine students who attempted to solve this problem, three students used an algebraic strategy (see Figure 12).

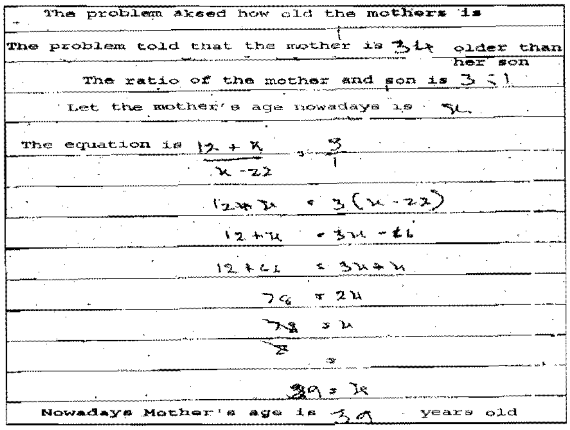

The problem asked how old the mothers is
The problem told that the mother is 34 older than her son
The ratio of the mother and son is 3 : 1
Let the mother's age nowadays is x.
The equation is $\frac{12 + x}{x - 22} = \frac{3}{1}$
$12 + x = 3(x - 22)$
$12 + x = 3x - 66$
$12 + 66 = 3x - x$
$78 = 2x$
$\frac{78}{2} = x$
$39 = x$
Nowadays Mother's age is 39 years old

Figure 12. An algebraic strategy that the student in all three classes used to solve Problem 8 successfully.

In Figure 12, the student assumed x for the age of the mother. In 12 years, the mother would be $12 + x$ and the son would be $x - 22$. Since the ratio of the mother and the son's ages would be 3:1 in the next 12 years, so that the student formed the proportional equation as $(12 + x)/(x - 22) = 3/1$. Then, the student solved for x, which was the mother's age. Among the three students who used an algebraic strategy, two students successfully solved this problem while another student had errors in computation. The results in Table 9 show that the other six students did some random calculation with the numbers given in the problem.

Table 9

The number of students solving Problem 8 separated by strategies used before and after instruction

Class	Before Instruction		After Instruction: Attempted	After Instruction: Not Attempted
Mr. Jack	Unsuccessful (n = 38)	Paper Blank (34) Random Calculation - (4)	Algebra + (2) Algebra – (1) Random Calculation - (6)	29
Ms. Rose	Unsuccessful (n = 42)	Paper Blank (42)	Algebra + (19) Algebra – (15) Only answer is given - (1)	7
Mr. Bond	Successful (n = 2)	Non-algebra (2)	Algebra + (2)	0
	Partial Successful (n = 3)	Algebra – (1)	Algebra + (1)	0
		Only answer is given + (2)	Algebra - (1) Only answer is given - (1)	
	Unsuccessful (n = 33)	Paper Blank (33)	Algebra + (14) Only answer is given - (4)	15

Note: + indicates that students successfully solved the problem.
- indicates that students unsuccessfully solved the problem.

In summary, the results indicate that few students in Mr. Jack's class were successful at solving this problem either before or after instruction. The results show that none of the students successfully solved this problem before instruction. All of the 38 students (100%) in this class could not solve this problem. Among these 38 students, 34 students left the paper blank and the other four students did some random calculations with the numbers in the problem. After instruction, the results indicate that only nine students attempted to solve this problem. Of these nine students, two students (5.3%) were successful at solving this problem using an algebraic strategy. One student (2.6%) who partially solved this problem used an algebraic strategy, but did not finish solving the equation. There were 35 students (92.1%) who could not solve this problem. Among these 35 students, 29 students did not attempt to solve the problem and left the paper blank. The other six students did some random calculations with the number in the problem. These results indicate that students' performance in Mr. Jack's class improved little after instruction. The results show that many students (70.3%) in this class still did not attempt to solve this problem after instruction.

Ms. Rose's class. Before instruction, the results in Table 9 demonstrate that none of the students in this class were able to solve this problem because they left the paper blank. After instruction, the results in Table 9 indicate that 35 students attempted to solve this problem while the other seven students left the paper blank. The results in Table 9 indicate that of the 35 students who attempted to solve this problem, 34 students used an algebraic strategy. All of them solved this problem algebraically as in Figure 12. In addition, some students in Ms. Rose's class defined x as the age of the son rather than the age of the mother (see Figure 13), which is different from what is seen in Figure 12.

In Figure 13, the student assumed x for the age of the son. The expression $x + 34$ represented the age of the mother. In 12 years the son would be $12 + x$ and the mother would be $12 + x + 34$. Since the ratio of the mother and son's ages would be 3:1 in 12 years, the student formed the equation as $(x + 46)/3 = (12 + x)/1$. Then, the student solved for x, which gave the son's age. The student added the number 5 to 34 to get the mother's age. Among 34 students who used an algebraic strategy, 19 students successfully solved the problem while the other 15 students had errors in computation or wrongly copied the number from the problem. The results in Table 9 show that one student provided a correct answer without a clear explanation.

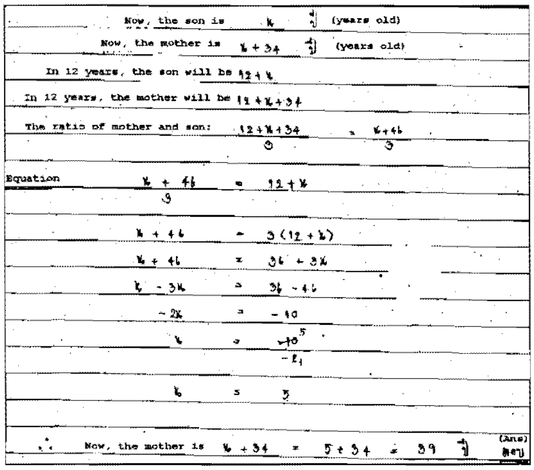


Figure 13. Another algebraic strategy that the student in Ms. Rose's and Mr. Bond's classes used to solve Problem 8 successfully.

In summary, the results show that no students in Ms. Rose's class were successful at solving this problem before instruction. After instruction, more students attempted to solve this problem. The results indicate that after instruction, 19 students (45.2%) successfully solved this problem. All of them solved this problem algebraically. The results demonstrate that 16 students (38.1%) partially solved this problem. Of these 16 students, 15 students used an algebraic strategy to solve this problem but had errors in calculation or wrongly copied numbers to the problem. Another student produced the correct answer without a clear explanation. Seven students (16.7%) did not attempt to solve this problem and left the paper blank. After instruction, the results show that only two students (4.8%) in Ms. Rose's class checked their answer by substituting the answer into the equation. These results indicate that students' performance in Ms. Rose's class improved much after instruction. The results show that more students attempted to solve this problem and used an algebraic strategy to solve the problem after instruction.

Mr. Bond's class. The results in Table 9 indicate that before instruction, only two students (5.3%) successfully solved this problem by using a non-algebraic strategy. For example, the student multiplied 15 to 3:1 so it became 45:15. Then the student subtracted 12 from 45 and 15 to get the father and son's age nowadays (Father is 33 and son is 3). Next, they subtracted the son's age from the father's age (33 – 3 = 30). The student got a difference of 30 as indicated in the problem. Therefore, the father is 33 years old. After instruction, these two successful students attempted to solve the problem by using an algebraic strategy as shown in Figures 12 and 13. They were successful at solving this problem after instruction.

Before instruction, the results in Table 9 show that three students (7.9%) partially successful at solving this problem. Of these three students, one student used an algebraic strategy but had an error in calculation. The other two students gave the correct answer without a clear explanation. After instruction, the student who had errors attempted to solve this problem by using an algebraic strategy and solved this problem successfully. For the two students who gave only the correct answer, one of them changed to use an algebraic strategy with errors in computation and another student still gave the correct answer without a clear explanation.

The results in Table 9 indicate that before instruction, 33 students (86.8%) were unable to solve this problem and left the paper blank. After instruction, 18 students attempted to solve the problem while the other 15 students left the paper blank. The results in Table 9 show that of the 18 students who attempted the problem, 14 students used an algebraic strategy to solve the problem successfully. The other four students provided a correct answer without a clear explanation.

In summary, the results show that few students in Mr. Bond's class were successful at solving this problem before instruction. Only two students (5.3%) successfully solved this problem by using a non-algebraic strategy. Three students (7.9%) partially solved this problem. Of these three students, one student used an algebraic strategy but had an error in calculation. The other two students gave the correct answer without a clear explanation. In addition, 33 students (86.8%) did not attempt to solve this problem and left the paper blank. After instruction, the results show that few students in this class were successful at solving the problem. Seventeen students (44.7%) successfully solved this problem by using an algebraic strategy. For six students (15.8%) who partially solved this problem, one student used an algebraic strategy with an error in calculation and five students produced the correct answer without a clear explanation. Fifteen students (39.5%) did not solve this problem and left the paper blank. These results indicate that students' performance in Mr. Bond's class improved little after instruction. The students used more algebraic strategies to solve the problem. The results show that some students (19.5%) still did not attempt to solve this problem.

Summary. The results show that overall few students in this study were successful at solving Problem 8 either before or after instruction. The results indicate that before instruction, only two of 118 students (1.7%) successfully solved this problem by using an algebraic strategy. Three students (2.5%) partially solved this problem. One hundred and thirteen students (95.8%) could not solve this problem. The results show that, regardless of any errors the students made, three students (2.5%) used an algebraic strategy before instruction. After instruction, the results indicate that few students were successful at solving this problem. Thirty-eight of 118 students (32.2%) successfully solved this problem. Twenty-three students (19.5%) partially solved this problem. Fifty-seven students (48.3%) could not solve this problem. The results show that, regardless of any errors the students made, 56 students (46.6%) used algebraic strategies after instruction.

Problem 9

Pretest	Posttest
There are silver and gold earrings in one box. The numbers of silver earrings are twice the number of gold earrings. The total of both earrings in that box is 36. How many silver and gold earrings are in this box?	The number of girls is twice the number of boys in the classroom. If there are 45 students in this classroom, find the number of girls in this classroom.

Mr. Jack's class. The results in Table 10 indicate that before instruction, none of the students in this class were successful at solving this problem. One student (2.6%) was partially successful at solving this problem. This student used a non-algebraic strategy but copied the number wrong.

Table 10

The number of students solving Problem 9 separated by strategies used before and after instruction

Class	Before Instruction		After Instruction: Attempted	After Instruction: Not Attempted
Mr. Jack	Partial Successful (n = 1)	Non-algebra - (1)	Non-algebra + (1)	0
	Unsuccessful (n = 37)	Paper Blank (32) Random Calculation - (5)	Algebra + (5) Random Calculation - (10) Only answer given - (1)	21
Ms. Rose	Successful (n = 3)	Non-algebra + (3)	Algebra + (3)	0
	Unsuccessful (n = 39)	Paper Blank (39)	Algebra + (20) Algebra – (15) Non-algebra + (1) Only answer is given - (1)	2
Mr. Bond	Successful (n = 11)	Non-algebra + (6)	Non-algebra + (1) Algebra + (5)	0
		Algebra + (5)	Algebra + (3) Algebra – (1) Non-algebra + (1)	
	Partial Successful (n = 3)	Algebra – (1)	Algebra + (1)	0
		Only answer is given - (2)	Algebra - (1)	1
	Unsuccessful (n = 24)	Paper Blank (24)	Algebra + (12) Algebra - (1)	11

Note: + indicates that students successfully solved the problem.
- indicates that students unsuccessfully solved the problem.

After instruction, this student attempted to solve the problem by using a non-algebraic strategy and solved it successfully (see Figure 14). In Figure 14, the student thought that since the number of girls was twice the number of boys, boys were one part then the girls are two parts. Therefore, the total of students was three parts. So they divided 45 by 3 and got 15. Since girls are two parts of the total, the student added 15 and 15 to get the number of girls.

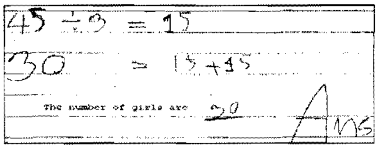


Figure 14. A non-algebraic strategy that the student in this study used to solve Problem 9 successfully.

The results in Table 10 show that before instruction, 37 students (97.4%) were unable to solve this problem. Of these 37 students, 32 students left the paper blank and the other five students did some random calculations with the numbers in the problem. After instruction, only 16 students attempted to solve the problem while the other 21 students did not attempt to solve the problem because they left the paper blank. The results in Table 10 indicate of the 16 students who attempted to solve the problem, five students used an algebraic strategy to solve the problem successfully (see Figure 15).

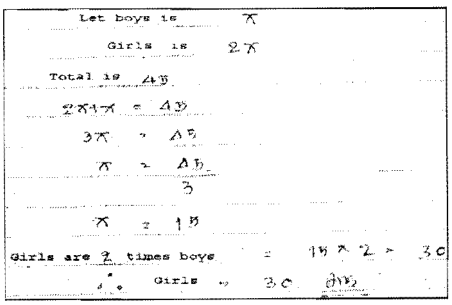


Figure 15. An algebraic strategy the student in this study used to solve Problem 9 successfully.

In Figure 15, the student assumed x as the number of boys and $2x$ as the number of girls. The student then formed the equation by adding the number of girls and boys equal to the total students, which is $2x + x = 45$. Then, the student solved for x, which is the number of boys. The student got $x = 15$ then they times 15 by 2 to get the number of girls, which is 30. The results in Table 10 demonstrate that the other ten students did some random calculations with the numbers given in the problem and one student gave a correct answer without a clear explanation.

In summary, the results show that few students in Mr. Jack's class were successful at solving this problem either before or after instruction. Before instruction, one student (2.6%) partially solved this problem. This student used a non-algebraic strategy but copied the wrong number. Thirty-seven students (97.4%) could not solve this problem. Of these 37 students, 32 students left the paper blank and five students did some random calculations with the numbers in the problem. After instruction, the results show that six students (15.8%) successfully solved this problem while the majority of students in this class still were unable to solve this problem. Of the six successful students, five students used an algebraic strategy. Another successful student solved this problem by using a non-algebraic strategy (see Figure 15). For the one student (2.6%) who partially solved this problem, she produced the correct answer but her solution strategy was not clearly explained. Of the 31 students (81.6%) who could not solve this problem, 21 students left the paper blank and ten students did some random calculations with the numbers in the problems, which did not correspond to the situation or question asked in the problem. These results indicate that students' performance in Mr. Jack's class improved little after instruction. Many students still did not attempt to solve this problem because they left the paper blank.

Ms. Rose's class. The results in Table 10 show that before instruction, three students (7.1%) successfully solved Problem 9 by using a non-algebraic strategy. After instruction, these three successful students attempted to solve the problem and changed to use an algebraic strategy as explained in Figure 15. In addition, some of the students in Ms. Rose's class defined x as the number of girls rather than the number of boys (see Figure 16). In Figure 16, the student assumed x as the number of girls (as we can see at the bottom of the Figure 17) and $x/2$ as the number of boys. The student then formed the equation by adding the number of girls and boys equal to the total students, which is $(x/2) + x = 45$. Then, the student solved for x, which is the number of girls. The student got 30 as the answer.

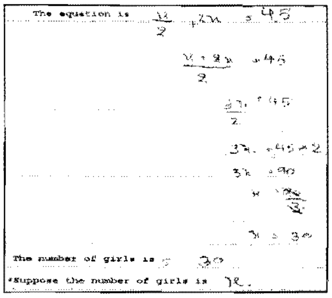


Figure 16. Another algebraic strategy the student in Ms. Rose's and Mr. Bond's classes used to solve Problem 9 successfully.

The results in Table 10 indicate that 39 students (92.9%) were unable to solve this problem and left the paper blank before instruction. After instruction, 37 students attempted to solve the problem while the other two students did not attempt to solve the problem and left the paper blank. The results in Table 10 show that of the 37 students who attempted to solve the problem, 35 students used an algebraic strategy. Of the 35 students who used an algebraic strategy, 20 students successfully solved the problem while the other 15 students had errors in computation or copied the wrong numbers from the problem. One student used a non-algebraic strategy as explained in Mr. Jack's class to solve the problem successfully. One student gave a correct answer without a clear explanation.

In summary, the results show that few students in Ms. Rose's class were successful at solving this problem before instruction. Only three students (7.1%) successfully solved this problem by using a non-algebraic strategy while 39 students (92.9%) did not attempt to solve this problem and left the paper blank. After instruction, the results indicate that many more students attempted to solve this problem. Twenty-four students (57.1%) successfully solved this problem. Of these 24 students, 23 students used an algebraic strategy and one student used a non-algebraic strategy. For the 16 students (38.1%) who partially solved this problem, 15 students used an algebraic strategy but they had errors in calculation. Another student produced the correct answer, but her solution strategy was not clearly explained. Two students (4.8%) did not solve this problem and left the paper blank. After instruction, the results show that only three students (7.1%) in Ms. Rose's class checked their answer by substituting the answer into the equation. These results indicate that students' performance in Ms. Rose's class improved much after instruction. The results show that more students attempted to solve this problem and used an algebraic strategy to solve the problem after instruction.

Mr. Bond's class. The results in Table 10 indicate that before instruction, 11 students (28.9%) solved this problem successfully. Of the 11 successful students, six students used a non-algebraic strategy and the other five students used an algebraic strategy. After instruction, these 11 students attempted to solve the problem. Of the six students who used a non-algebraic strategy (see Figure 14) before instruction, one student continued to use this strategy after instruction while the other five students changed to use an algebraic strategy (see Figure 16). For five students who used an algebraic strategy before instruction, four of them still used this strategy. Of these four students, three students solved this problem successfully while another student had errors in computation. Another student who used an algebraic strategy before instruction changed to use a non-algebraic strategy after instruction.

Before instruction, the results in Table 10 show that three students (7.9%) partially solved this problem. Among these three students, one student used an algebraic strategy but had errors in calculation. The other two students produced a correct answer without a clear explanation. After instruction, the students who used an algebraic strategy but had errors in calculation solved this problem successfully by using an algebraic strategy. Of the two students who produced a correct answer without a clear explanation, one of them changed to use an algebraic strategy but had errors in computation. Another student did not attempt to solve the problem and left the paper blank.

In summary, the results show that few students in Mr. Bond's class were successful at solving this problem before instruction. Eleven students (28.9%) solved this problem successfully. Of the 11 successful students, six students used a non-algebraic strategy and five students used an algebraic strategy. Three students (7.9%) partially solved this problem. One of them used an algebraic strategy but had errors in calculation. The other two students produced a correct answer without a clear explanation. Twenty-four students (63.2%) did not attempt to solve this problem and left the paper blank. After instruction, several additional students attempted to solve this problem. The results show that 23 students (60.5%) in Mr. Bond's class successfully solved this problem. Of these 23 students, 21 students used an algebraic strategy. The other two successful students solved this problem by using a non-algebraic strategy. Three students (7.9%) who partially solved the problem used an algebraic strategy with an error in calculation. Twelve students (31.6%) did not attempt to solve this problem and left the paper blank. These results indicate that students' performance in Mr. Bond's class improved after instruction. The results show that more students attempted to solve this problem and used more algebraic strategies to solve the problem after instruction.

Summary. The results indicate that overall few students in this study were successful at solving Problem 9 before instruction. The results indicate that 14 out of 118 students (11.9%) successfully solved this problem. Four students (3.4%) partially solved this problem. One hundred students (84.7%) could not solve this problem. The results show that, regardless of any errors the students made, ten students (8.5%) used a non-algebraic strategy and six students (5.1%) used an algebraic strategy before instruction.

After instruction, the results demonstrate that more students in this study were successful at solving this problem and the strategies they used were more algebra-based. The results indicate that 53 out of 118 students (45%) successfully solved this problem. Twenty students (17%) partially solved this problem. Forty-five students (38%) could not solve this problem. The results show that, regardless of any errors the students made, four students (3.4%) used a non-algebraic strategy and 67 students (56.8%) used an algebraic strategy after instruction.

Problem 10

Pretest

Lisa and Dan picked 252 oranges altogether. Lisa picked 9 oranges per box and Dan picked 6 oranges per box. There are 34 boxes altogether. Find the number of oranges Lisa and Dan each picked.

Posttest

Jack and Jill picked 252 apples altogether. Jack picked 9 apples per box and Jill picked 6 apples per box. There are 34 boxes altogether. Find the number of apples Jack and Jill each picked.

Mr. Jack's class. Before instruction the results in Table 11 show that none of the 38 students (100%) were able to solve this problem successfully and left the paper blank. After instruction, only six students attempted to solve the problem while the other 32 students were unable to solve the problem. The six students who attempted to solve the problem used an algebraic strategy (see Figure 17). In Figure 17, the student defined x as the number of oranges Jack picked. The expression $252 - x$ represent the number of oranges Jill picked. Then, the student defined $x/9$ as the number of boxes Jack had and $(252 - x)/6$ as the number of boxes Jill had. The student formed the equation $(x/9) + (252 - x)/6 = 34$. Then, the student solved for x, which is the number of oranges Jack picked ($x = 144$). Finally the student subtracted 144 from 252 to get the number of oranges Jill picked. Among these six students who used an algebraic strategy, five students successfully solved this problem while one student had errors in computation.

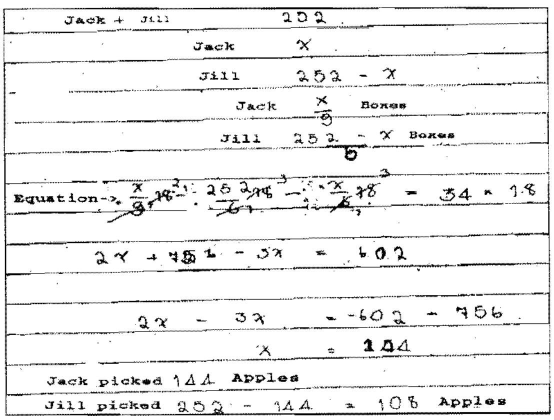


Figure 17. An algebraic strategy the student in this study used to solve Problem 10 successfully.

Table 11

The number of students solving Problem 10 separated by strategies used before and after instruction

Class	Before Instruction		After Instruction: Attempted	After Instruction: Not Attempted
Mr. Jack	Unsuccessful (n = 38)	Paper Blank (38)	Algebra + (5) Algebra – (1)	32
Ms. Rose	Unsuccessful (n = 42)	Paper Blank (42)	Algebra + (12) Algebra – (11) Only answer is given - (3) Non-algebra + (1)	15
Mr. Bond	Successful (n = 1)	Algebra + (1)	Algebra – (1)	0
	Partial Successful (n = 11)	Algebra – (5)	Algebra + (4) Only answer is given -(1)	0
		Only answer is given (6)	Algebra + (6)	
	Unsuccessful (n = 26)	Paper Blank (26)	Algebra + (9) Algebra - (1) Only answer is given - (1)	15

Note: + indicates that students successfully solved the problem.
- indicates that students unsuccessfully solved the problem.

In summary, the results show that none of the students in Mr. Jack's class were successful at solving this problem before instruction. Before instruction, none of the 38 students (100%) were able to solve this problem successfully and left the paper blank. After instruction, the results indicate five students (13.2%) successfully solved this problem. All of them used an algebraic strategy to solve this problem. One student (2.6%) who partially solved this problem used an algebraic strategy but she had errors in calculations. The 32 students (84.2%) who did not attempt to solve this problem left the paper blank. These results indicate that students' performance in Mr. Jack's class improved little after instruction. Many students still did not attempt to solve this problem after instruction.

Ms. Rose's class. The results in Table 11 indicate that before instruction all 42 students (100%) in Ms. Rose's class were unable to solve this problem. They left the paper blank. After instruction, 27 students attempted to solve the problem while the other 15 students were unable to solve the problem. The results in Table 11 show that of the 27 students who attempted to solve the problem, 23 students used an algebraic strategy as shown in Figure 16. In addition, some students in Ms. Rose's class defined x as the number of boxes Jack had (see Figure 18).

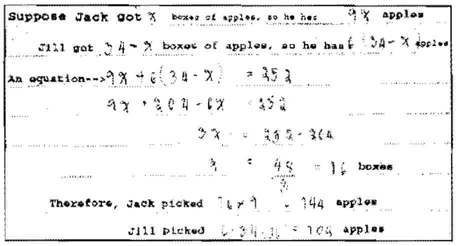


Figure 18. An algebraic strategy the student in Ms. Rose's and Mr. Bond's classes used to solve Problem 10 successfully.

In Figure 18, the student defined x as the number of boxes Jack had. The expression $9x$ represent the number of oranges Jack would pick. Then, student defined $34 - x$ as the number of boxes Jill had and $6(34 - x)$ as the number of oranges Jill would pick. The student formed the equation $9x + 6(34 - x) = 252$. Then, the student solved for x, which is the number of boxes Jack had ($x = 16$). The student then multiplied 16 by 9 to get the number of oranges Jack picked (which is 144 oranges). Finally, the student multiplied 6 by (34 – 16) to get the number of oranges Jill picked (which is 108 oranges). Among the 23 students who used an algebraic strategy, 11 students successfully solved this problem while the other 11 students had errors in computation.

The other three students who attempted to solve this problem after instruction provided a correct answer without a clear explanation and one student used a non-algebraic strategy to solve the problem (see Figure 19). In Figure 19, this student randomly multiplied two pairs of numbers and added the two multiplied numbers together. The student used a non-algebraic strategy until he got the correct answer, which is

(16 × 9 = 144) and (18 × 6 = 108), and 144 + 108 equals to the number of oranges Jack and Jill each picked respectively.

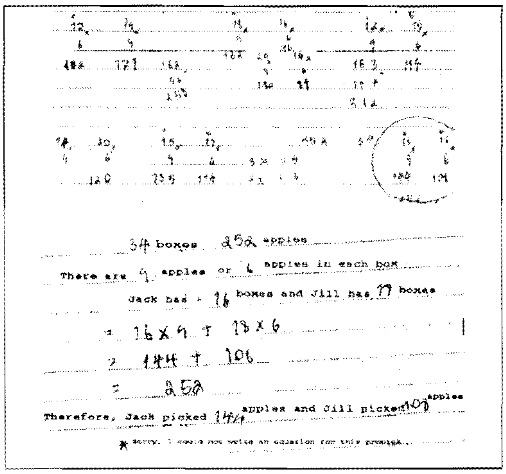


Figure 19. A non-algebraic strategy that one student in this study (from Ms. Rose's class) used to solve Problem 10 successfully.

In summary, the results show that all 42 students (100%) in Ms. Rose's class were unable to solve this problem before instruction. They left the paper blank. After instruction, the results demonstrate that 13 students (31%) successfully solved this problem while the majority of the students in this class still were unsuccessful at solving this problem. Of the13 successful students, 12 students in Ms. Rose's class solved this problem algebraically. Another successful student solved this problem by using a non-algebraic strategy to get an answer. For the 14 students (33.3%) who partially solved this problem, 11 students used an algebraic strategy but had errors in calculations. The other three students produced the correct answer, but their solution strategies were not clearly explained. The 15 students (35.7%) who did not attempt to solve this problem left the paper blank. These results indicate that students' performance in Ms. Rose's class improved little after instruction. The results show that more students attempted to solve this problem and used algebraic strategies to solve the problem after instruction.

Mr. Bond's class. The results in Table 11 indicate that before instruction, one student (2.6%) successfully solved this problem by using an algebraic strategy. After instruction, the results indicate that this student continued to use an algebraic strategy (see Figures 17 and 18) but had errors in computation.

Before instruction, the results in Table 11 show that 11 students (29%) partially solved this problem. Of these 11 students, five students used an algebraic strategy but had errors in calculation. The other six students produced the correct answer without a clear explanation. After instruction, these 11 students attempted to solve the problem. The results in Table 11 indicate that of the five students who used an algebraic strategy but had errors in calculation before instruction, four of them used the same strategy and solved the problem successfully. Another student gave a correct answer without a clear explanation. The six students who produced the correct answer without a clear explanation changed to use an algebraic strategy and solved the problem successfully. The results in Table 11 indicate that before instruction, 26 students (68.4%) were unable to solve this problem and left the paper blank. After instruction, 11 students attempted to solve the problem while the other 15 students were unable to solve the problem after instruction. Of these 11 students who attempted to solve the problem, ten students used an algebraic strategy. Among these ten students, nine students successfully solved this problem while another student had an error in computation. Another student provided a correct answer without a clear explanation.

In summary, the results show that few students in Mr. Bond's class were successful at solving this problem before instruction. One student (2.6%) successfully solved this problem by using an algebraic strategy. Eleven students (29%) partially solved this problem. Of these 11 students, five students used an algebraic strategy but had errors in calculation. The other six students produced the correct answer without a clear explanation. Twenty-six students (68.4%) could not solve this problem and left the paper blank. After instruction, the results indicate that more students attempted to solve this problem. Nineteen students (50%) successfully solved this problem. All successful students solved this problem algebraically. Four students (10.5%) partially solved the problem. Of these four students, two of them used an algebraic strategy but had errors in calculation. Another two students produced a correct answer without a clear explanation. The 15 students (39.5%) who could not solve this problem left the paper blank. These results indicate that students' performance in Mr. Bond's class improved much after instruction. The results show that more students attempted to solve this problem and used more algebraic strategies to solve the problem after instruction.

Summary. The results indicate that overall few students in this study were successful at solving Problem 10 either before or after instruction. The results show that before instruction, one of 118 students (0.9%) successfully solved this problem. Eleven students (9.3%) partially solved this problem. One hundred and six students (89.8%) could not solve this problem. The results show that, regardless of any errors the students made, only six students (5.1%) used an algebraic strategy to solve this problem before instruction. After instruction, 37 of 118 students (31.3%) successfully solved this problem. Nineteen students (16.1%) partially solved this problem. Sixty-two students (52.5%) could not solve this problem. The results show that, regardless of any errors the students made, 50 students (42.4%) used an algebraic strategy to solve this problem while only one student (0.9%) used a non-algebraic strategy to solve this problem after instruction.

Conclusion

The results from this study show that overall Thai ninth grade students were successful at solving some algebra word problems and unsuccessful at other problems on the pretest and posttest. The results indicate that the majority of the students were successful at solving Problem 6. However, the results indicate that the majority of Thai ninth grade students were unsuccessful at solving Problems 7, 8, 9 and 10.

Although no data were collected to attempt to explain this situation, one could conjecture that the structure of the word problem might affect student's ability to solve algebra word problems successfully. Previously, we conjectured that students might have difficulty with the problem involving more than one unknown variable. We will use this same conjecture to explain why many students were more successful at solving Problem 6 than any other problems. The difference between Problem 6 and other four problems is the unknown variable presented in the problem. Problem 6 contains one unknown variable and students need to use that variable in order to find the solution to the problem. In contrast, the other four problems contain more than one unknown variable. The students need to find the relationship between those variables in order to form an equation to represent the situation in the problem. However, the students in this study were taught to use only one variable in order to solve the problem. Thus, perhaps when the students faced with the word problems containing more than one unknown variable, they were unsuccessful at solving them.

In conclusion, many Thai ninth grade students in this study were successful at solving some algebra word problems and were unsuccessful at the others. Before instruction, the students used strategies that were more informal (a non-algebraic strategy) such as comparing times and distances. After instruction, the students who were successful used more algebraic knowledge to solve the word problems. However, some informal strategies were used by some of these students. As one would expect, the results indicate that low achieving students in Mr. Jack's class were unsuccessful at solving algebra word problems. Their performance improved little after instruction even though they had gone through formal instruction. In contrast, medium and high achieving students from Ms. Rose's and Mr. Bond's classes were more successful at solving algebra word problems. Their performance improved much after instruction, especially in Ms. Rose's class.

The results from this study show that Thai ninth grade students were able to solve a word problem and used knowledge of mathematics to solve word problems as recommended in Thai Mathematical Standards (IPTST, 2000) and in NCTM Standards 2000. However, they were unsuccessful at solving a variety of problems, because the majority of the students were unable to solve a problem involving more than one unknown variable. As indicated in the NCTM Standards 2000 that students at this grade level should be able to determine the reasonableness of the answer to the problem. However, the results from this study show that few Thai ninth grade students checked their answers or solution processes in order to determine the reasonableness of the answer to the problem.

This section presents results of Thai students' success and difficulties in solving equations and translating mathematical problems into equations in general. Next section will explore a few of these students' solution strategies, their solution processes, and difficulties in solving algebra word problems.

Section Two: Solution Strategies

This section presents performance of 18 interviewed students on five algebra word problems. These five problems were different from what students had done on either the pretest or posttest. Please refer to Appendix B for more details on these five algebra word problems. This section, then, reports solution strategies Thai ninth grade students used to solve algebra word problems. Table 12 shows the number of problems that each of the 18 students could solve successfully on the pretest and posttest, and the number of word problems that each student could solve successfully during the interview sessions. The characteristics of each of the 18 students who participated in the interview sessions can be seen in Appendix C.

Table 12

Number of problems that each student correctly solved during the pretest, posttest, and interviews. (N= 18)

	Pretest (10 problems)	**Posttest (10 problems)**	**Interview (5 problems)**	**Tutoring Center (Yes or No)**
High Achievement (All from Mr. Bond's Class)				
William (BH1)	9	10	5	No
Phil (BH2)	9	10	5	Yes
Nat (BH3)	9	10	5	Yes
Ann (GH1)	6	9	4	Yes
Patty (GH2)	6	7	2	No
Nancy (GH3)	6	8	1	Yes
Medium Achievement (All from Ms. Rose's Class)				
Billy (BM1)	2	4	3	Yes
Tom (BM2)	4	8	3	No
Sean (BM3)	4	7	4	No
Sara (GM1)	4	9	3	Yes
Rita (GM2)	4	8	3	Yes
Jenny (GM3)	4	6	2	No
Low Achievement (All from Mr. Jack's Class)				
Lee (BL1)	4	3	2	No
Sam (BL2)	0	2	0	No
Andy (BL3)	1	4	2	Yes
Jill (GL1)	1	4	1	Yes
June (GL2)	0	4	2	Yes
Wilma (GL3)	1	6	2	Yes

Note: B = boy, G=Girl, H=High Achievement, M=Medium Achievement, and L=Low Achievement. For example, BH1 means "the first boy with high achievement".

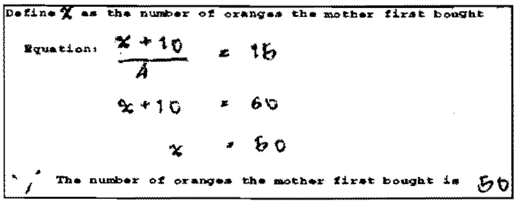
Thai Student's Performance on Five Algebra Word Problems During Interviews

In this section, the 18 interviewed students' performance to five algebra word problems are reported. The students were categorized into three groups. Students in Group1 were defined as students who used an effective solution strategy that would result in a correct answer and applied the strategy without any errors or misconceptions (Successful students). In Group 2, students used an effective solution strategy that would result in a correct answer but errors occurred (e.g., computation errors or translation errors) or they could not solve for the unknown (Partially successful students). The last group, Group 3, students could not solve the problems (Unsuccessful students). Furthermore, the results from the interview indicated that 18 students used either algebraic strategies or non-algebraic strategies to solve algebra word problems. In the algebraic strategies, the students used variables and symbols to form an equation. Then, the students solved the equation to find an answer to the problem. In the non-algebraic strategies, the students used their arithmetic knowledge to solve algebra word problems.

Problem 1: Orange Problem

> "At first, a mother bought some oranges. However, there were not enough oranges to equally divide the oranges among 15 people. Therefore, she went to buy 10 more oranges so each person could get four oranges. How many oranges did the mother buy the first time?"

Group 1: Successful students. Fifteen students were successful at solving the orange problem. Of the 15 successful students, 11 students (BH1, BH2, BH3, GH1, GH2, BM1, BM2, BM3, GM2, GM3and GL3) used an algebraic strategy to solve the problem (see Figure 20). The student formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem.



Define x as the number of oranges the mother first bought

Equation: $\frac{x+10}{4} = 15$

$x+10 = 60$

$x = 50$

$\therefore$ The number of oranges the mother first bought is 50

Figure 20. Example showing an algebraic strategy the student used to solve the orange problem successfully.

In Figure 20, the student defined x as the number of oranges mother first bought. Then she added the number 10 to x ($x + 10$) to indicate the total number of oranges mother bought. Next, she divided $x + 10$ by the number 4 and made the equation equal to 15 (the number children who got four oranges each.

The other four successful students (GM1, BL1, BL3, and GL2) used a non-algebraic strategy to solve the problem (see Figure 21). Figure 21 is an example of how students solved the orange problem verbally by using simple arithmetic.

Interviewer: "What did the problem ask you to find?"
Lee: "Oranges mother first bought and it is 60."
Interviewer: "How did you know?"
Lee: "Multiply 4 and 15, each person got 4 and times 15."
Interviewer: "And why did you multiply by 15?"
Lee: "Well, the total orange. We need to find the overall orange first and then minus 10, which is the additional orange."
Lee: "This problem is like our daily life so it's easy."

Figure 21. Example of the conversation shows how the student successfully solved the orange problem (Problem 1) verbally by using simple arithmetic.

Group 2: Partially successful students. The results show that one student was placed into this group. Jill (GL1) employed a non-algebraic strategy to solve the problem. However, she got stuck at 15×4 because she did not know what to do next.

Group 3: Unsuccessful students. The results indicate that two students, one girl (GH3) and one boy (BL2) were placed into this group. Nancy (GH3) used an algebraic strategy. However, she formed an incorrect algebraic equation to represent the situation in the problem and incorrectly solved the problem. She got a correct answer, but she got it by performing the wrong order of operations. This student matched word-for-word from the situation described in the problem rather than trying to make sense of the problem. Therefore, she got an incorrect equation. Her equation was "$x/15$ (a number of oranges first bought divided by 15 people) plus 10 (bought 10 more) equal to 4 (each people got four oranges)." From the equation $(x/15) + 10 = 4$, she wrongly multiplied 4 to 15 to get 60 and then she subtracted the number 10 from the number 60. She got a correct answer based on an incorrect equation and incorrect calculations. Therefore, she was classified into this group.

Sam (BL2) failed to solve this problem. He tried to solve the problem but he could not solve the problem. He sat quietly, wrote the numbers from the problem onto the given paper, and went back to read the problem. He did this over and over again. When asked what the problem asked for, he answered that the problem asked for the number of oranges that mother first bought. When asked why he got stuck, he said that he did not know what to do with the numbers in the problem.

Summary. The results in Table 13 indicate that of the 18 students, 15 students succeeded at solving this problem. Of these 15 students, 11 students used an algebraic strategy to solve the problem and the other four students used a non-algebraic strategy. One student from medium achieving group was partially successful at solving this problem. This student used a non-algebraic strategy but did not finish solving the problem. Two students (one high and one low achiever) were unsuccessful at solving this problem. One student used an algebraic strategy but she formed an incorrect equation to represent the situation in the problem and one student was unable to solve the problem. The results indicate that the majority of high and medium achievers used an algebraic strategy to solve the problem while the majority of low achievers used a non-algebraic strategy to solve the problem.

Table 13

Summary of strategies students used to solve the orange problem (N = 18)

Group / Problem	Problem 1		
	High	**Medium**	**Low**
Group 1: Successful students who:			
- Used an algebraic strategy and formed a correct equation.	5	5	1
- Used a non-algebraic strategy.	0	1	3
Group 2: Partially successful students who:			
- Used a non-algebraic strategy but did not finish solving the problem.	0	0	1
Group 3: Unsuccessful students who:			
- Used an algebraic strategy but formed an incorrect equation.	1	0	0
- Failed to solve the problem (No work done).	0	0	1

Problem 2: Student Problem

"The number of girls is 2/3of the number of boys in one class. If the total number of the students in this class is 45, find the number of girls in this class."

Group 1: Successful students. The results demonstrate that ten students were successful at solving this problem. Of these ten students, nine students (BH1, BH2, BH3, GH1, BM1, BM2, BM3, GM1, and GM2) used an algebraic strategy to solve the problem. The students formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem (see Figure 22).

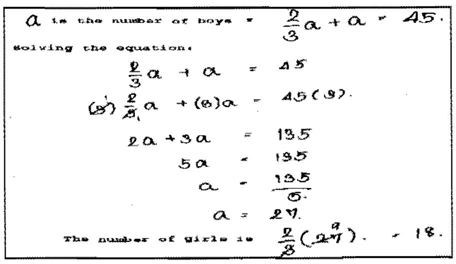


Figure 22. Example showing an algebraic strategy the student used to solve the student problem successfully.

In Figure 22, the student defined *a* as the number of boys and $2a/3$ as the number of girls. Then, the student added $2a/3$ and *a* to get the total of the students in the class (which was $2a/3 + a = 45$). One student (GH3) used a non-algebraic strategy to solve the problem. Nancy (GH3) thought that the total of students should be divided into five parts so that she divided 45 by 5. She then got 9 as the number of students in each part. The number of girls should be two out of five parts. Therefore, 18 would be the number of girls. The strategy she used is illustrated in the following interview session.

Nancy: "30."

Interviewer: "What is 30?"

Nancy: "The number of girls."

Interviewer: "How did you get 30, could you please explain to me?"

Nancy: "The number of girls is two third's and overall it should be three parts. So I divided 45 by 3. Oh wait, it's wrong."

She calculated on the piece of paper again and after a while she told the interviewer the following:

Nancy: "The number of girls is 18."

Interviewer: "Why did you change and divide by 5 instead of 3?"

Nancy: "Well, because girls were two thirds of boys and the total should be five parts. So I divided 45 by 5 and I got 9. Girls is 2 parts so the number of girls is 18."

It seems that Nancy found the whole and then found the part, which gave her the number of girls, which is two parts. Therefore, the number of girls was 18.

Group 2: Partially successful students. The results show that only one student was categorized into this group. Wilma (GL3) used an algebraic strategy to solve the problem. This student formed a correct algebraic equation to represent the situation in the problem but she could not finish solving the problem. She almost got it but after a long time working on the problem, she decided not to try it anymore. She said it was difficult and she did not like solving word problems very much. She already got the correct equation but she could not solve for the variable *x*. The following is the excerpt between Wilma and the interviewer. The interviewer tried to help Wilma to form the equation. She got an equation but she did not want to continue solving the problem.

Interviewer: "What is the problem asking for?"

Wilma: "It asks for the number of girls and the total students are 45 and the number of girls is 2/3 of..."

Interviewer: "Of what?"

Wilma: "Umm."

Interviewer: "What is x?"

Wilma: "x is the number of boys and $2x/3$ is the number of girls."

Interviewer: "And what's next?"

Interviewer: "What are you going to do with x and $2x/3$?"

Wilma: "Multiply."

Interviewer: "Are you sure?"

Wilma: "Add."

Interviewer: "So what is the equation?"

Wilma: "$2x/3$ plus y."

Interviewer: "How so?"

Wilma: "Oh, plus x."

Interviewer: "And what is an equation?"

Wilma: "$2x/3$ plus x equal 45." [$(2x/3) + x = 45$]

Interviewer: "Ok, are you sure?"

Wilma: "Well, kind of.

Wilma attempted to solve the above equation for the variable x but she could not solve it so she requested to stop solving the equation.

Group 3: Unsuccessful students. The results indicate that seven students were unsuccessful at solving this problem. Of these seven students, two students were high and medium achievers (GH2 and GM3) and five students were low achievers (BL1, BL2, BL3, GL1, and GL2). Jenny (GM3) used an algebraic strategy to solve the problem but she wrote an incorrect equation to represent the problem situation. Therefore, she got an incorrect answer. First Jenny defined x as the number of boys and she defined $2x/3$ as the number of girls. However, Jenny struggled to find the equation. Later on, the equation she determined was $2x/3 = 45$. She then solved for x but she got a decimal answer. She realized that the number of people should not be decimal. Therefore, she tried again by defining x as the number of girls (see Figure 23). Jenny thought that the number of girls were two out of three of the total students. Therefore, $2x/3$ is equal to $x/45$, which is the proportional equation of two ratios. She ended up getting 30 as the number of girls. As seen in Figure 23, at her first attempt to solve this problem, she got x and $2x/3$ to represent the number of boys and girls but she could not write a correct equation. This might be because she did not realize the fact that if she add x and $2x/3$, it would be equal to 45 (the total students).

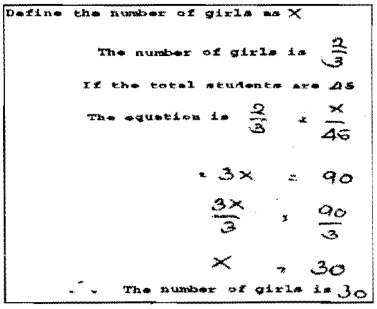


Figure 23. Jenny's strategy to solve the student problem (Problem 2) unsuccessfully.

Lee (BL1) and June (GL2) tried to do random operations with the numbers in the problem. Patty (GH2) and Jill (GL1) got stuck while solving this problem. Patty was confused about whether the number of girls was two-thirds of boys or two-thirds of all students. The following presents the narration between the interviewer and Patty while solving this problem.

> Interviewer: "What are you thinking?"
>
> Patty: "I'm thinking about finding an equation."
>
> Interviewer: "What's the problem asking for?"
>
> Patty: "The number of girls"
>
> Interviewer: "And what did you define here?"
>
> Patty: "The number of girls"

Patty continued to attempt to find the equation, but it took a long time so the interviewer prompted her to explain her thinking.

Interviewer: "What did you get stuck on?"

Patty: "Umm. It told us that girls is 2/3 of boys. But they did not tell that 2/3 of the total students."

Interviewer: "What did you define here?"

Patty: "The number of girls. Umm."

Patty: "I thought it was 2/3 of the overall students but it said 2/3 of girls. So I am confused."

Jill (GL1) tried to do this problem but she got confused so she did not finish solving this problem.

Interviewer: "What are you doing? Please tell me."

Jill: "Finding an equation. The teacher at the tutoring center told me to read the problem sentence by sentence and then write it out. That's it. Well, I assume boys. I think it will be ok. But when I look here, it is girl. I want to try. The teacher told me that if you can attack the problem you could do it. I will try to solve it."

Interviewer: "At first, you assumed boys and why didn't you use it?"

Jill: "Boys, right. I look at it again and the problem asked for girls. If I assume boys, I cannot think about it."

Jill: "This problem sounds familiar. Wait."

Interviewer: "What is the problem giving us?"

Jill: "The total students, the number of girls is 2/3 of the number of boys, and the problem asks for the number of girls."

Interviewer: "And what did you get stuck on?"

Jill: "Can I skip to the next problem?"

Interviewer: "Sure, and what did you get stuck on?"

Jill: "I don't know how to find it. I assumed x for the number of girls and the number of girls is 2/3 boys and where to put x for girls. It confused me."

The difficulty was that she assumed x for the number of girls and the problem said the number of girls is two third of the number of boys. So, she did not know what to do with the variable x that she assumed. The possible explanation for Jill's confusion is that she assigned a variable to what the problem asked for (which is the number of girls). Then, when she went back to read the problem again and found out that the problem gave "the number of girls is two-thirds the number of boys", she could not find a relationship between the variable she defined and the situation in the problem.

Sam (BL2) and Andy (BL3) failed to solve this problem. As in Problem 1, Sam (BL2) read the problem over and over again and said that he could not solve this problem because he did not know what to do with the numbers in the problem. No work was shown on the given paper. The following is the conversation between interviewer and Sam for this problem.

Interviewer: "How are you going solve this problem?"

Sam: "I don't know. I cannot do it."

Interviewer: "What are you doing now?"

Sam: "Think about what to do with all these numbers."

Interviewer: "What are you thinking?"

Sam: "Thinking about what to do."

Sam: "I cannot do it."

The interviewer further asked Sam what he would do first when solving word problems. He mentioned that he would read through the problem and then find the number in the problem to do random operations. He mentioned that his mathematics teacher taught him to write an equation to represent the problem but he could not do that. Therefore, he would try to do random operations with the numbers in the problem.

Andy (BL3) said that this problem was difficult and he did not like fractions so he asked if he could skip this problem. At the end of the interview, the interviewer asked Andy to solve this problem again by changing "the number of girls is 2/3 of the number of boys" to "the number of girls is twice the number of boys". The reason for doing this was to see whether Andy could solve this problem if changing fractions to integers. However, Andy still could not solve this problem.

Summary. The results in Table 14 indicate that of the 18 students interviewed in this study, 10 students were successful at solving this problem. Of these 10 students, nine students used a non-algebraic strategy to solve the problem and another student used a non-algebraic strategy. One student was partially successful at solving this problem. The student used a non-algebraic strategy but did not finish solving the problem. Seven students were unsuccessful at solving this problem. Of these seven students, one student used an algebraic strategy but formed an incorrect equation. The other two students tried to do random operations with the number in the problem. The other two students got stuck while solving the problem. The other two students left the paper blank. The results indicate that the majority of high and medium achievers used an algebraic strategy to solve the problem while the majority of low achievers were unsuccessful at solving this problem.

Table 14

Summary of strategies students used to solve the student problem (N = 18)

Group / Problem	Problem 2		
	High	**Medium**	**Low**
Group 1: Successful students who:			
- Used an algebraic strategy and formed a correct equation.	4	5	0
- Used a non-algebraic strategy.	1	0	0
Group 2: Partially successful students who:			
- Used an algebraic strategy but did not finish solving the problem	0	0	1
Group 3: Unsuccessful students who:			
- Used an algebraic strategy but formed an incorrect equation.	0	1	0
- Tried to do random operations with the numbers in the problem.	0	0	2
- Could not think through and got stuck at some points during solving the problem.	1	0	1
- Failed to solve the problem (No work done).	0	0	2

Problem 3: Age Problem

> "Six years ago Jennifer's age was twice as old as Jonathan's age. Nowadays, if Jennifer is six years older than Jonathan, how old is each now?"

Group 1: Successful students. The results indicate that five students were successful at solving this problem. Of these five students, four students (BH2, BH3, GH1, and BM3) used an algebraic strategy to solve the problem (see Figure 24).

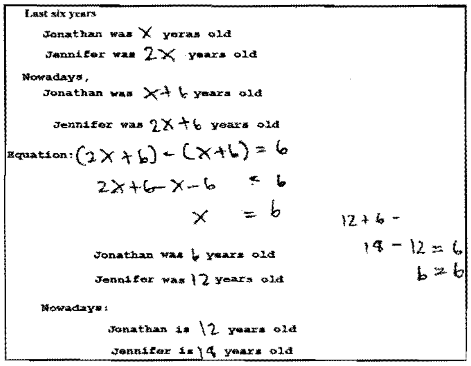


Figure 24. Example of an algebraic strategy the student used to solve the age problem (Problem 3) successfully.

The student formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem. In Figure 24, the students assumed x as the age of Jonathan six years ago. Then, they formed the equation based on the situation described in the problem. There were two students (GH1 and BM3) who checked the answer by substituting the value of x into their equation.

Another successful student, William (BH1), drew a graph to get the answer. William could not form an equation but he thought of something else. William came up with drawing a graph (see Figure 25).

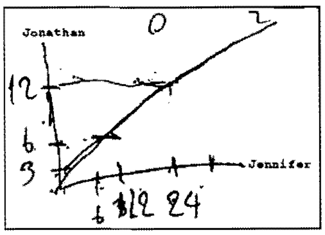


Figure 25. Example showing that William used a graph to solve the age problem successfully.

Figure 25 is an example showing that William used a graph to solve Problem 3. The x-axis was the age of Jennifer six years ago. The y-axis was the age of Jonathan six years ago. He dragged the middle line and tried to find two pairs of numbers that matched the condition given in the problem. He said he remembered this strategy somewhere but he could not recall where. The following is the description given by William to explain his graphing idea:

Interviewer: "What were you thinking?"

William: "Thinking about age."

Interviewer: "How?"

William: "It's like a graph."

Interviewer: "Please show me."

William: "It has two steps. Like this. This one is Jonathan's age and this one is Jennifer's age. Jennifer is 12 years old. Jonathan is...Umm."

William: "And the age difference is 6. Last 6 years it should be 12 years old. This is difficult."

William: "Umm. When you drag it up the age will increase."

Interviewer: "And the age you wrote here is the age in the past or nowadays?"

William: "It is the assuming age, because if Jennifer is 3 years old, Jonathan should be 6 years old. But you can't use this answer because the age of Jennifer will be only 3 years greater than the age of Jonathan nowadays. So going up. When add the Jonathan's age with 6, it will be 12. It should be here. But I can't find the equation."

Interviewer: "It's ok if you cannot find the equation. Did you get an answer?"

William: "Yes. 3 and 6 is not working because it's not 6 years difference for nowadays. So Jonathan is 12 years old and Jennifer is 18 years old because it's 6 years differences here."

Interviewer: "How did you figure that?"

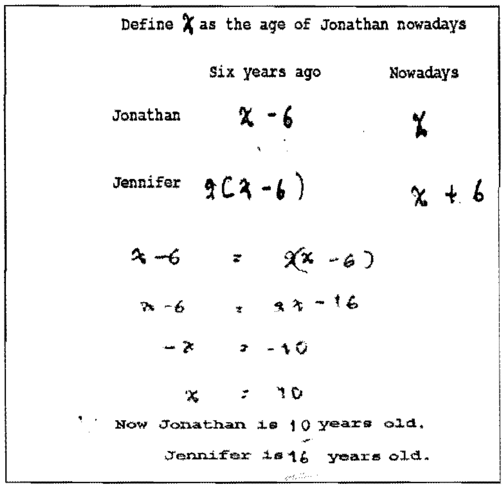
William: "Because the graph shows last 6 years. But nowadays, we have to add 6 more and it is 12. The graph is just the age of two people in the last 6 years."

Interviewer: "Ok, are you sure?"

William: "Yes."

Group 2: Partially successful students. The results show that only one student was placed into this group. Sara (GM1) used an algebraic strategy to solve the problem. The student formed a correct algebraic equation to represent the situation in the problem. However, she incorrectly solved the problem (see Figure 26). In Figure 26, Sara defined x

as Jonathan's age nowadays. However, she had an error in her calculation and got an incorrect answer. Instead of getting $2x - 12$, she got $2x - 16$. Therefore, she ended up with an incorrect answer. This result suggests that if Sara checked her procedures, she might not get it wrong. This implies that checking an answer would be another important factor in solving word problems.


Define x as the age of Jonathan nowadays

	Six years ago	Nowadays
Jonathan	$x-6$	x
Jennifer	$2(x-6)$	$x+6$

$x-6 = 2(x-6)$

$x-6 = 2x-16$

$-x = -10$

$x = 10$

Now Jonathan is 10 years old.

Jennifer is 16 years old.

Figure 26. Sara's strategy to solve the age problem (Problem 3) unsuccessfully.

Group 3: Unsuccessful students. The results show that 12 students were unsuccessful at solving this problem. Of these 12 students, two students were high achievers (GH2 and GH3), four students were medium achievers, and six students were low achievers. Of these 12 unsuccessful students, three students (GH2, GM3, and GL2) used an algebraic strategy. However, they formed an incorrect algebraic equation to represent the situation in the problem. For example, Jenny (GM3) mentioned that she did not know how to find the relationship between the age of the two people six years ago and the age nowadays (see Figure 27).

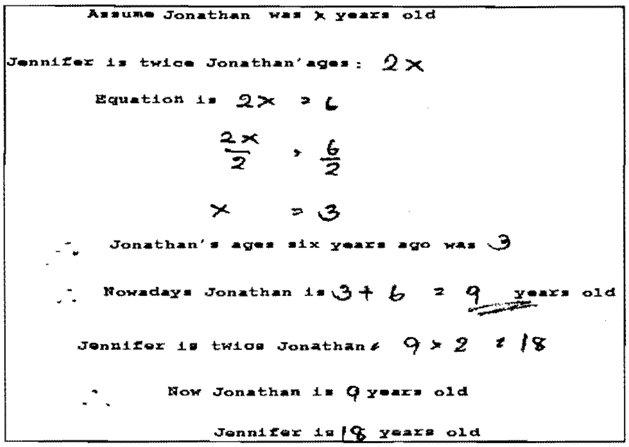


Figure 27. Jenny's strategy to solve the age problem (Problem 3) unsuccessfully.

In Figure 27, Jenny defined x as Jonathan's age and $2x$ as Jennifer's age and she wrote $2x = 6$ as her equation. She said that $2x$ stands for Jonathan's age and the number 6 was six years ago. So the equation $2x = 6$ stands for the age of Jonathan six years ago, which had no meaning to the situation described in the problem.

The other three unsuccessful students (GH3, GM2, and GL1) got stuck while solving this problem. They got the expression for the age of Jennifer and Jonathan six years ago and nowadays. However, they could not continue to form an equation. The following is an example from the interview with Rita (GM2), which showed that she could not find an equation for this problem.

Rita: "This problem is confusing."

Interviewer: "And what do you define?"

Rita: "Jonathan is x, and Jennifer is $2x$."

Interviewer: "And what is plus 6 here?"

Rita: "x and $2x$ are the age last six years so we have to added 6 to get the age nowadays."

Interviewer: "So $x + 6$ and $2x + 6$ are what?"

Rita: "The age nowadays. Jonathan is $x + 6$ and Jennifer is $2x + 6$ years old."

Interviewer: "And what are you going to do next?"

Rita: "Umm. I don't know."

Interviewer: "You got both Jonathan's and Jennifer's age nowadays. And, from the problem, you know that Jennifer is six years older than Jonathan nowadays. What are you going to do with $x + 6$ and $2x + 6$ you assumed to get the six years difference?"

Rita: "Umm."

Interviewer: "Let's look at this way. If you were 15 years old and your brother was 10 years old. What is the difference between you and your brother's age?"

Rita: "5 years difference."

Interviewer: "How did you find that?"

Rita: "Subtracting 10 from 15."

Interviewer: "Ok. Using the same idea. You know that Jonathan is $x + 6$ years old and Jennifer is $2x + 6$ years old nowadays. And, Jennifer is six years older than Jonathan nowadays. Using the same idea above. How can you form an equation to represent this situation?"

Rita: "Umm. I don't know. This is difficult. May I stop and do the next problem?"

Interviewer: "Ok."

From this example, one could see that the ability to use the fact about how we find the age difference between two people in real life is necessary in solving this problem. Rita might not be thinking about this fact while she tried to write the equation to this problem. Thus, she was unable to write an equation to show the relationship. The other six unsuccessful students (BM1, BM2, BL1, BL2, BL3, and GL3) failed to solve this problem. These students showed no work.

Summary. The results in Table 15 indicate that of the 18 students, five students were successful at solving this problem. Of these five successful students, four students used an algebraic strategy and formed a correct equation and another student drew a graph to find a solution to the problem. One student was partially successful at solving this problem. This student used an algebraic strategy. She formed a correct equation but she had errors in calculation. The results in Table 15 show that 12 students were unsuccessful at solving this problem. Of these 12 unsuccessful students, three students used an algebraic strategy but they formed an incorrect equation. The other three students got stuck while solving the problem and the other six students failed to solve the problem. The results in Table 15 indicate that the majority of high achievers were successful at solving this problem by using an algebraic strategy or a drawing while the majority of medium and low achievers were unsuccessful at solving this problem.

Table 15

Summary of strategies students used to solve the age problem (N = 18)

Group / Problem	Problem 3		
	High	Medium	Low
Group 1: Successful students who:			
- Used an algebraic strategy and formed a correct equation.	3	1	0
- Used a drawing strategy.	1	0	0
Group 2: Partially successful students who:			
- Used an algebraic strategy and formed a correct equation but had error in calculation.	0	1	0
Group 3: Unsuccessful students who:			
- Used an algebraic strategy but formed an incorrect equation.	1	1	1
- Could not think through and got stuck at some points during solving the problem.	1	1	1
- Failed to solve the problem (No work done).	0	2	4

Problem 4: Car Wash Problem

> "Natasha, Gibson, Jim, and Robinson had a car wash on Sunday. Natasha washed twice as many cars as Gibson. Gibson washed one fewer than Jim. Jim washed six more than Robinson. Robinson washed six cars. How many cars did each person wash? (Adapted From Malloy and Jones, 1998)"

Group 1: Successful students. Sixteen students were successful at solving this problem. The results indicate that, of the 16 successful students, five students (BH1, BH2, BH3, BM2, and GM1) used an algebraic strategy to solve the problem. The student formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem. William (BH1), Phil (BH2), Nat (BH3), and Tom (BM2) assumed the variable for the number of cars Robinson washed and wrote expressions for Jim, Gibson, and Natasha respectively based on that variable. For example, in Figure 28, Tom (BM2) assumed "*a*" for the number of cars Robinson washed. Then he formed an equation for Jim, Gibson, and Natasha based on the variable "*a*". After that, he substituted $a = 6$ to every equation he formed to get the number of cars washed by Jim, Gibson, and Natasha respectively.

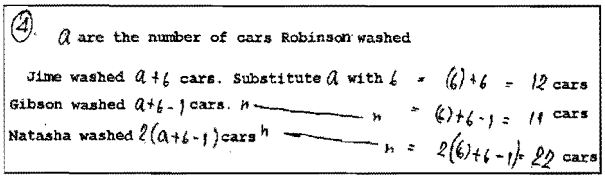


Figure 28. Tom's strategy to solve the car wash problem (Problem 4) successfully.

Sara (GM1) also used an algebraic strategy to solve the problem. She formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem. Sara did this problem forward (see Figure 29). In Figure 29, Sara assumed x for the number of cars Gibson washed and went from there to get other equations. When she got the equation of cars washed by Robinson, she took that equation to equal six and solved for x. To get the other number of car washed, she then substituted x into each person's equation.

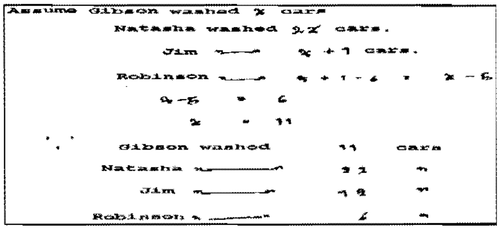


Figure 29. Sara's strategy in solving the car wash problem (Problem 4) successfully.

The other 11 successful students (GH1, GH2, BM1, BM3, GM2, GM3, BL1, BL3, GL1, GL2, and GL3) tried to find an equation to describe the situation in the problem. After a while, they changed to do the problem by working backwards by arithmetically adding up the number of cars washed by Robinson to get the number of cars washed by Jim, Gibson, and Natasha respectively. For example, Robinson washed six cars and Jim washed six cars more than Robinson so Jim washed 6 + 6 = 12 cars. Then Gibson washed one cars fewer than Jim so Gibson washed $12 - 1 = 11$ cars. Finally, Natasha washed twice as many cars as Gibson washed so Natasha washed $11 \times 2 = 22$ cars.

Group 2: Partially successful students. The results indicate that one student (GH3) was placed into this group. Nancy (GH3) used an algebraic strategy to solve this problem. Like Sara, Nancy solved this problem going forward. However, she misrepresented the situation for Robinson. Instead of writing $(x + 1) - 6$ as the equation for the number of cars Robinson washed, Nancy wrote $x - 6$ instead and she got an incorrect answer. The following is her description of this problem.

Interviewer: "What did the problem ask you to find?"

Nancy: "Number of cars each person washed."

Interviewer: "And what did the problem gave us?"

Nancy: <read the problem>

Interviewer: "How did you find it?"

Nancy: "Well, Natasha washed twice Gibson, so Gibson is x. Natasha, then, is $2x$. Gibson washed one car less than Jim. So Jim is $x + 1$ and Jim is 6 cars greater than Robinson so Robinson is $x - 6$."

Interviewer: "And how many cars did Robinson wash?"

Nancy: "Six cars and x is 12 and substitute in the equation and Gibson washed 12 cars, Natasha washed 24 cars and Jim washed 13."

Interviewer: "Are you sure?"

Nancy: "Yes."

This is another example showing that checking the answer in the equation or problems is necessary in solving word problems. If Nancy would check her procedure, she might find her error and not get the incorrect answer.

Group 3: Unsuccessful students. The results indicate that one student (BL2) failed to solve this problem. Sam (BL2) tried a little to solve the problem. However, he seemed not to be thinking during the entire interview session. He could not solve the problem. He did not know what to do with the number in the problem. The following is the excerpt after the interviewer let him read and thought through the problem for a while.

Interviewer: "Well, what do you think?"

Sam: "I can't do it."

Interviewer: "Why?"

Sam: "I cannot find the number to calculate."

Interviewer: "Did you read everything in the problem?"

Sam: "Yes."

Interviewer: "What did the problem tell you?"

Sam: ... <read the problem again >...

Interviewer: "And did you know how many cars Robinson washed?"

Sam: "No I didn't."

Summary. The results in Table 16 indicate that of the 18 students interviewed in this study, 16 students were successful at solving this problem. Of these 16 successful students, five students used an algebraic strategy to solve the problem. The student formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem. The other 11 successful students used a non-algebraic strategy to solve this problem.

Table 16

Summary of strategies students used to solve the car wash problem (N = 18).

Group / Problem	Problem 4 High	Medium	Low
Group 1: Successful students who:			
- Used an algebraic strategy and formed a correct equation.	3	2	0
- Used a non-algebraic strategy.	2	4	5
Group 2: Partially successful students who:			
- Used an algebraic strategy but had errors.	1	0	0
Group 3: Unsuccessful students who:			
- Failed to solve the problem (No work done).	0	0	1

The results in Table 16 show that one student was partially successful at solving this problem because the student had error in representing situation. Another student was unsuccessful at solving this problem. The results in Table 16 indicate that the majority of high achievers were successful at solving this problem by using an algebraic strategy while the majority of medium and low achievers were successful at solving this problem by using a non-algebraic strategy.

Problem 5: Distance Problem

> "Simon and Henry decided to bike to his uncle's house from their house. Henry left at 10:00 am and biked at a rate 20 kilometer per hour. Simon, at the same starting point, left at 10:45 am and biked at a rate 30 kilometer per hour. They reached their uncle's house at the exact same time. What is the distance from their house to their uncle's house?"

Group 1: Successful students. The results indicate that only three students were successful at solving this problem (BH1, BH2, and BH3). Of these three students, two students (BH2 and BH3) used an algebraic strategy to solve the problem. The student formed a correct algebraic equation to represent the situation in the problem and correctly solved the problem. The only difference between these two students was they assumed a different value for the variable.

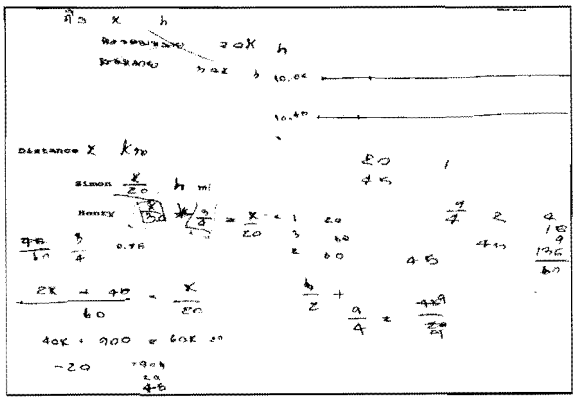


Figure 30. Phil's strategy in solving the distance problem (Problem 5) successfully.

In Figure 30, Phil (BH2) assumed x to be the distance and wrote the equation. His equation came up as $(x/30) + (3/4) = x/20$, in which $x/30$ represented the time Simon used to bike to his uncle's house and $x/20$ represented the time Henry used to bike to his uncle's house. He made the travel time of Simon to equal the travel time of Henry by adding 3/4 hours differences to Simon's travel time. In Figure 30, the equation Phil had written is in the middle left. Everything else around his equation was part of his thinking and finding the time that both Simon and Henry met at his uncle house. After solving the equation, Phil got the value of x as the distance right away. The following is the conversation between the interviewer and Phil during the solution to this problem.

Interviewer: "What is the problem asking for?"

Phil: "Distance."

Interviewer: "And what were you thinking now?"

Phil: <...draw picture...>

Phil: "Well, it's 10.45. That's hard"

Phil: "45 minutes later, Simon began to bike. He left later so we have to add to be equal."

Interviewer: "Add what?"

Phil: "Add time."

Interviewer: "What is this?"(The interviewer point at the equation he made in Figure 30)

Phil: "$(x/30) + (3/4)$ is the minutes that takes Henry to reach his uncle's house (left equation in Figure 30) and $x/20$ this is the minutes that Simon reaches his uncle's house (right equation in Figure 30).

Interviewer: "What is the distance?"

Phil: "45 kilometer."

Interviewer: "How did you make sure that it was correct?"

Phil: "Substitute into the equation."

Phil: "Umm. It's equal, I think."

Interviewer: "And what time would they meet?"

Phil: "If 20 kilometers it took one hour. 45 km got 45/20 or 9/4. So it is 2 hours and 15 minutes"

Interviewer: "And that would be?"

Phil: "12:15 pm and Simon is the same."

Phil: "This is the most difficult problem. A distance problem is difficult, but if you could solve it, it's fun."

Unlike Phil, Nat (BH3), assumed x as the length of time Simon used for biking (see Figure 31). Nat also used a table to help him formed an equation to represent the situation in the problem.

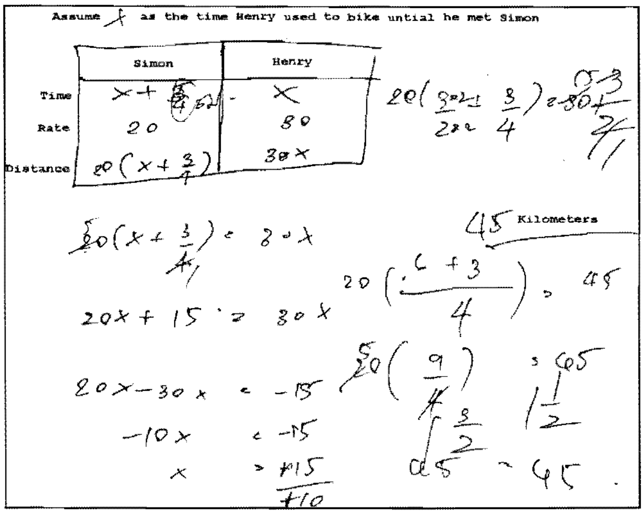


Figure 31. Nat's strategy in solving the distance problem (Problem 5) successfully.

In Figure 31, Nat added 3/4 hours difference to Henry's travel time in order to make the equation equal. So his equation came up as $20\,[x + (3/4)] = 30x$. After solving the equation, he got x as the length of time Simon used to bike. He, then, calculated the distance by comparing rate and time. Nat mentioned that he had not done this kind of problem either at the class or at a tutoring center. He remembered this strategy from some mathematics books he had studied. However, the strategy in that book involving time in hours such as 10:00 and 11:00, not 10:45 like this. So he just applied the strategy to this problem by changing the 45 minutes difference into a hour difference.

Interviewer: "What did you do?"

Nat: "I remember from the manual that I read. It has to be a table."

Nat: "Assume Simon bikes in x hours."

Nat: "This is time, rate, and distance." (Nat wrote into table.)

Nat: “If the time Simon used to bike is x, the time Henry used to bike is x + (3/4).

Interviewer: “Why 3/4?”

Nat: “Because Henry left 45 minutes before and 45 minutes is 3/4 hour. So we have to add 3/4 to x. Distance is length of time multiply by rate so the distance for Henry is 20 [x + (3/4)] and distance for Simon is 30x and the equation is 20 [x + (3/4)] = 30x.”

Interviewer: “Why are they equal?”

Nat: “Because they will meet at the same time and the same distance.”

Nat: “And x will be 3/2.”

Nat: “Simon will use 3/2 hours to reach his uncle. One hour is 30 kilometer and a half hour is 15 kilometer. So the distance is 45 kilometer.”

Nat: “So the distance from Simon’s house to his uncle’s house is 45 kilometer.”

Interviewer: “Are you sure this is correct?”

Nat: “Sure, because I put x in Henry’s equation and the distance is the same.”

Interviewer: “Are you sure?”

Nat: “Absolutely.”

From the above results, Phil and Nat used an algebraic strategy to solve the distance problem. William (BH1) could not solve this problem by using an algebraic strategy. He used a non-algebraic strategy to solve this problem. He first tried to use the graph that he used in Problem 3 (Age problem) but it did not work well. He even tried an algebraic strategy, but it did not go well either. Therefore, he compared rate and time to get the distance (Figure 32).

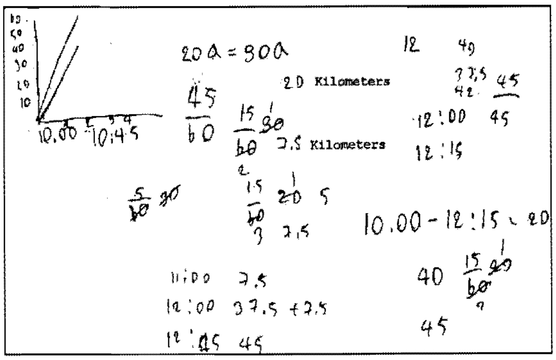


Figure 32. William’s strategy in solving the distance problem (Problem 5) successfully.

The following is the conversation between the interviewer and William during solving this problem.

Interviewer: "What is the problem asking for?"

William: "Distance from Simon's house to his uncle's house."

Interviewer: "What were you doing?"

Interviewer: "What is 7.5?"

William: "From 10:45 to 11:00, Simon bike 7.5 kilometer."

Interviewer: "What are you thinking now?"

William: "How to make their distances equal."

William: "I know that at 11 am. Henry went 20 kilometer and Simon went 7.5 kilometer. I add 30 and 20 minutes intervals but they should not meet at 12:00. It should be like 12: 15, something like that."

Interviewer: "Ok."

William: "At 12 pm, Henry went another 20 kilometer."

William: "Time used is 2 hours and 45 minutes."

Interviewer: "What was that?"

William: "Wait."

William: "The distance is 45 kilometer."

Interviewer: "How did you get that?"

William: "I'm not sure, I forgot."

Interviewer: "Umm. How did you know that they met at 12:15?"

William: "Because at 12:15 the distance of two people will be equal."

William: "When Henry biked at 20 kilometer per hour, he would go 5 kilometer from 12 to 12:15. At 12 Henry already went 40 kilometer and 12: 15 went another 5 kilometer so it was 45 kilometer."

William: "For Simon, from 10:45 to 11:00 he went 7.5 kilometer and at 12 pm he went 37.5 and plus another 15 more minutes he went 7.5 kilometer more. Add it together it will be 45 kilometer. So, the time is equal and distance is equal.

Interviewer: "So you compare minutes by minutes."

William: "Right."

Interviewer: "Where did you get this strategy?"

William: "I invented it myself."

Group 3: Unsuccessful students. The results indicate that 15 students were unsuccessful at solving this problem. Of these 15 students, three of them were from the high achieving group: Ann (GH1), Patty (GH2), and Rita (GH2). The other twelve students were medium and low achievers.

Ann (GH1) used an algebraic strategy to solve this problem. However, she misrepresented the situation of the problem for time as shown in (see Figure 33). In Figure 33, Ann assumed x as the time that Simon and Henry met at their uncles' house. However, the time she assumed was the exact time that Simon and Henry would meet (e.g., 13:00 or 14:00) rather the time interval (e.g. 1 hours or 2 hours). Thus, Ann represented the equation based on an incorrect concept of time.

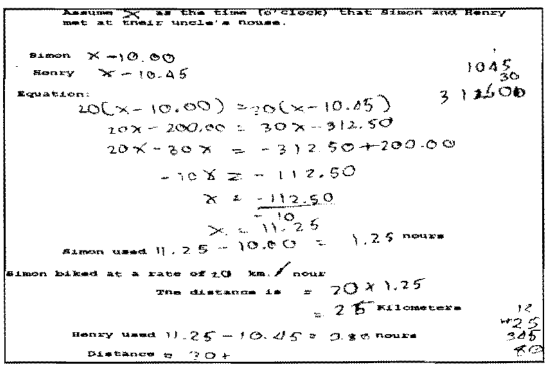


Figure 33. Ann's strategy in solving the distance problem (Problem 5) unsuccessfully.

The following is her description of her work on this problem.

Interviewer: "Could you explain to me?"

Ann: "I assume x be the time Simon and Henry reach their uncle's house."

Interviewer: "What is the scale for x?"

Ann: "Hour."

Interviewer: "It is time, right?"

Ann: "Yes, it's time like 13:00, 14:00."

Ann: "Then minus 10:00 from x for Henry. For Simon, minus 10:45 from x."

Interviewer: "What's next?"

Ann: "And then write an equation. So, multiply 20 to (x – 10:00). This gives you distance and equal to 30 multiply (x – 10:45) and then solving the equation for x."

Ann: "And I got x as 11:25 is the time they were at uncle's house."

Ann: "And then we would like to know the distance, right, so using Henry times'. Subtract 10:00 from 11:25 so it is 1:25 hours that Henry used and then, multiply 20 with 1.25 and get 25 kilometer."

Interviewer: "This is the distance of what?"

Ann: "From Simon's house to his Uncle's house."

June (GL2) tried to do a variety of random operations to get the answer for this problem. Sean (BM3) could not think the problem through and he got stuck while solving this problem. He tried to find the times and distance Simon and Henry would meet but he could not find them. He got an equal distance but the time was not equal. The following is a description of his strategy.

Sean: "The rate of Henry is 20 km/hr. In one hour, Henry went 20 kilometer. Oh no! For Simon, in one hour, he went 30 kilometer. If they meet at the same time so what time they will meet?"

Interviewer: "Have you ever seen this problem before?"

Sean: "No."

Sean: "So what is going to be, huh?"

Sean: "Henry left first. So at 11 am, Henry went 20 km. And at 10:45 Simon began to bike. So at 11:45 am, Simon went 30 km. At 12:45, Simon went 60 km. At one pm, Henry went 60 km."

Sean: " Umm. Should the time be the same?"

Interviewer: "Is one pm and 12:45 pm the same times?"

Sean: "Huh. No. How can we do this?"

Interviewer: "What do you think?"

Sean: "In 3 hours Henry went 60 kilometers and it's 1:00 pm. In 2 hours Simon went 60 kilometers and it's 12:45."

Interviewer: "Is the time equals?"

Sean: "Umm. No."

Sean: "How did we make the time equal? Can we form an equation for this problem?"

Interviewer: "Not necessary. You can use any strategies."

Sean: "Henry left at 10 am. Simon left at 10:45."

Sean: "The time interval is 45 and bike with.... Umm."

Sean: "I got the same distance but the time is not equal. What do I have to do with this 15 minutes difference?"

Sean: "Oh, I cannot do it. I would like to stop."

The other 12 unsuccessful students failed to solve the problem. They tried to solve it but after a while they insisted they could not solve it. There were several reasons such as:

Billy (BM1): "Umm. It's difficult."

Interviewer: "How?"

Billy: "Here, two people will meet at the same time but their rate and times is not the same. It's difficult."

Tom (BM2): "If finding the time, I think I can do it such as what time they will meet but when it asked for distance, it's confusing."

Interviewer: "Did Simon and Henry meet at the same time?"

Tom: "Yes."

Interviewer: "Can you do it?"

Tom: "No because the rates are different and the time is not in hours, it's 10:45. Usually it's like 10:00, 10:30, something like that."

Summary. This was the most difficult problem for the 18 interviewed students. The results in Table 17 indicate that, of the 18 students interviewed in this study, three students were successful at solving this problem. The results in Table 17 show that two successful students used an algebraic strategy and formed a correct equation and another student used a non-algebraic strategy. Fifteen of 18 interviewed students were unable to solve this problem. One student tried to use an algebraic strategy but she misrepresented the situation in the problem. Another student tried to do random operations with the number given in the problem and one student got stuck while solving this problem. There were 12 students who were unable to solve this problem because no work was shown. The results in Table 17 indicate that half of high achievers were successful at solving this problem by using either an algebraic strategy or a non-algebraic strategy while all medium and low achievers were unsuccessful at solving this problem.

Table 17

Summary of strategies students used to solve the distance problem (N = 18)

Group / Problem	Problem 5		
	High	Medium	Low
Group 1: Successful students who:			
- Used an algebraic strategy and formed a correct equation.	2	0	0
- Used a non-algebraic strategy.	1	0	0
Group 3: Unsuccessful students who:			
- Used an algebraic strategy but formed an incorrect equation	1	0	0
- Tried to do random operations with the numbers in the problem.	0	0	1
- Could not think through and got stuck at some points during solving the problem.	0	1	0
- Failed to solve the problem (No work done).	2	5	5

Conclusion

The results in Table 18 indicate that the majority of high achieving students were successful at solving most of the five problems given during an interview. The strategy they used were mostly algebraic-based. The resulst in Table 18 show that only a few students used a non-algebraic strategy to solve the problems given during the interview. The results in Table 18 show that the majority of medium achieving students were successful at solving Problems 1, 2, and 4. Few students were successful at solving Problem 3 and none of the students were successful at solving Problem 5. The strategy students used were both algebraic and non-algebraic strategies. The results in Table 18 show that low achieving students were successful at solving Problems 1 and 4, but they were unsuccessful at solving problems 2, 3, and 5(see Table 18). The strategy that low achieving students used were mostly non-algebraic strategies. Only some students used an algebraic strategy. Some of the students in this group solved the problem by using random operations. However, the results from the interview show that students did not check their answer or solution processes after solving a problem.

Table 18

Strategies each interviewed student used to solve five word problems (N = 18)

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
High Achievement (Mr. Bond)					
William (BH1)	Algebra +	Algebra +	Graph	Algebra +	Trial and error+
Phil (BH2)	Algebra +	Algebra +	Algebra +	Algebra +	Algebra +
Nat (BH3)	Algebra +	Algebra +	Algebra +	Algebra +	Algebra +
Ann (GH1)	Algebra +	Algebra +	Algebra +	Non-algebra +	Algebra -
Patty (GH2)	Algebra +	Got stuck	Algebra -	Non-algebra +	Failed
Nancy (GH3)	Algebra - (error)	Non-algebra+	Stuck	Algebra - (error)	Failed
Medium Achievement (Ms. Rose)					
Billy (BM1)	Algebra +	Algebra +	Failed	Non-algebra +	Failed
Tom (BM2)	Algebra +	Algebra +	Failed	Algebra +	Failed
Sean (BM3)	Algebra +	Algebra +	Algebra +	Non-algebra +	Stuck
Sara (GM1)	Non-algebra +	Algebra +	Algebra - (error)	Algebra +	Failed
Rita (GM2)	Algebra +	Algebra +	Stuck	Non-algebra +	Failed
Jenny (GM3)	Algebra +	Algebra -	Algebra -	Non-algebra +	Failed
Low Achievement (Mr. Jack)					
Lee (BL1)	Non-algebra c+	Random Operations-	Failed	Non-algebra +	Failed
Sam (BL2)	Failed	Failed	Failed	Failed	Failed
Andy (BL3)	Non-algebra +	Failed	Failed	Non-algebra +	Failed
Jill (GL1)	Non-algebra - (error)	Stuck	Stuck	Non-algebra +	Failed
June (GL2)	Non-algebra +	Random Operations-	Algebra -	Non-algebra +	Random Operations-
Wilma (GL3)	Algebra+	Algebra - (not finished)	Failed	Non-algebra +	Failed

Note: + indicates that students correctly solved the problem.
- indicates that students incorrectly solved the problem.
(error) = Students had misunderstanding or computational errors.

As mentioned in Thai mathematical standards (IPTST, 2000) and in NCTM Standards 2000 that students at this grade level should be able to use a variety of strategies such as symbolic, table, or graph to solve algebra word problems. The results from this present study, however, show that each of Thai ninth grade in this study did not use a variety of strategies to solve algebra word problems. For example, in Table 18, the results show that each of the 18 interviewed students use mostly the algebraic strategy to solve algebra word problems. The results from the interview indicate that if the students could not use algebraic strategies to solve the problem, they would use either non-algebraic strategies to solve the problem or did not attempt to solve the problem at all. The results in Table 18 show that there was only one student, William, who consistently used a variety of strategies to solve the five word problems giving during the interview session.

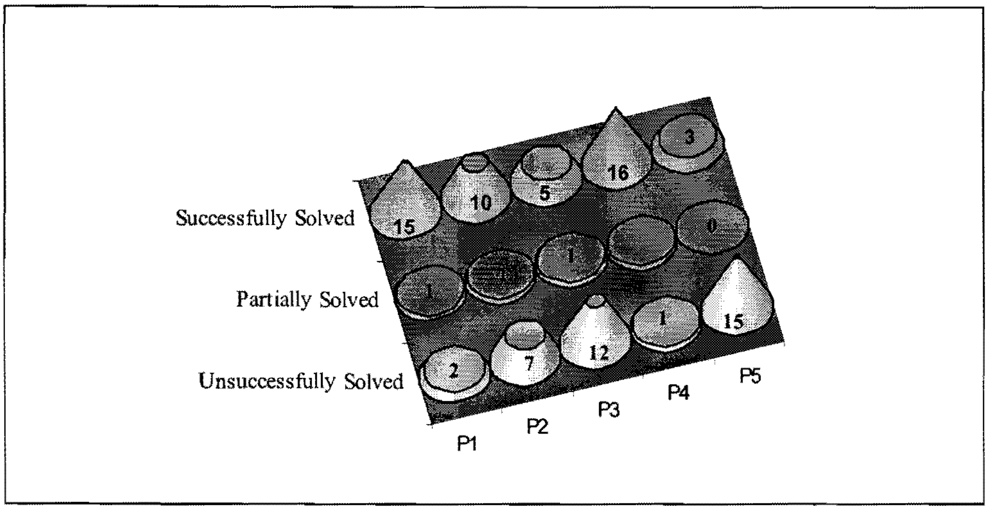


Figure 34. Number of students who solved the five interview problems (N = 18)

The results in Figure 34 demonstrate that the majority of the 18 interviewed students were successful at solving Problems 1, 2, and 4. However, they were unsuccessful at solving Problems 3 and 5. Only 5 out of 18 students (27.8%) were successful at solving Problem 3 and only three out of 18 students (16.7%) were successful at solving Problem 5.
Thus, Problems 1, 2 and 4 were easy for these 18 interviewed students to solve while Problems 3 and 5 was the most difficult word problems for them to solve.

Previously in the first section, we conjectured that the students in this study were unsuccessful at solving word problems involving more than one unknown variable. This conjecture might not hold for the results from the interview because Problems 2 and 4 also contained more than one unknown variable but the majority of students were successful at solving them. However, the results in Table 18 tells us that only the high and medium achieving students were sucessful at solving Problems 2 while the low achieving students were unsuccessful at solving them. Thus, another conjecture about students' ability and their thinking in solving word problems is of concern.

Mayer (1985, 1987) suggested that four types of processes are required to solve mathematics word problems: translation, integration, planning and monitoring, and solution execution. Based on Mayer's four steps in solving problems and the results from the interview session, successful word problem solvers were able to translate the problem statement into a mental model of the situation described in the problem. Then, they could integrate all the information they had into an equation. Unlike successful word problem solvers, unsuccessful word problem solvers struggled in the translation and integration processes during the solution of word problems. Thus, they were more likely to focus on computing using numbers given in the problem. Unsuccessful students were sometimes able to solve algebra word problems because the problems could be solved by using arithmetic knowledge. However, they were unable to solve other problems because using their arithmetic knowledge could not easily solve those problems.

In summary, the results from this section indicate that not only a structure of word problems affect students' performance at solving word problems, but students' ability and their thinking as well. Student's ability and their thinking is further disscussed later in CHAPTER V.

Thai Students' Solution Strategies

The focus of this study was to examine strategies Thai ninth grade students used to solve algebra word problems. The solution strategies reported in this section were analyzed from 118 students participated in the posttest and the 18 students participating in the interview sessions. The results indicate two strategies students used to solve algebra word problems. Those two strategies were algebraic strategies and non-algebraic strategies

Algebraic Strategies

This is the most formal strategy employed by the 18 interviewed students and other 100 non-interviewed students in this study. In this strategy, the situation in the problem was translated to algebraic assignments of variables and symbols. Then, the student formed an equation by using those variables and symbols. The equation then was solved to find the answer to the problem. There were two sub-strategies in the algebraic strategies (see Table 19).

Table 19

Strategies students used to solve algebra word problems

Algebraic Strategies	Non – Algebraic Strategies
▪ An equation based on comprehensive representations (a successful strategy) ▪ An equation based on poor representations (an unsuccessful strategy)	▪ A verbal/written arithmetic strategy ▪ Drawing/Graph ▪ Trial and error ▪ A comparison strategy ▪ A part-and-whole strategy

Equations based on comprehensive representations (a successful strategy). In this strategy, students assigned variables for the unknown in the word problems and they wrote correct algebraic equations to represent the situation in the problem by using variables and symbols. They were also able to then solve the problem and were able to get the correct answer to the word problems. Examples of this strategy can be seen in Figures 5 and 7.

Equations based on poor representations (an unsuccessful strategy). In this strategy, students assigned variables for the unknown in word problems. However, they could not successfully write the correct algebraic equation to represent the situation in the problem. Thus, they ended up getting an incorrect answer. Examples of this strategy can be seen in Figure 27.

Non -Algebraic Strategies

This is the strategy many students in this study (both interviewed and non-interviewed students) employed. There are five sub-strategies emerged in the non-algebraic strategies. Those strategies are a verbal/written arithmetic, a drawing, a trial and error, a comparison strategy, and a part-and-whole strategy (see Table 19).

A verbal/written arithmetic strategy. In this strategy, the situation in the problem was translated into either verbal or written arithmetic. The answer then was found by using basic arithmetic thinking or writing. The verbal arithmetic strategy can be seen in Figure 21. Figure 21 is an example of how students solved the orange problem (Problem 1) verbally by using simple arithmetic. The written arithmetic strategy can be seen in Figures 5 and 10.

Drawing/Graph strategy. This was another non-algebraic strategy employed by only one student (William) during the interview sessions. William drew a graph in order to find the answer to the problem. Figure 25 is an example showing that this student used a graph to solve the problem.

Trial and error strategy. This was also a non-algebraic strategy employed by one student (William) during the interview session and by another student from the posttest. In this strategy, students tried to find the answer that best matched the situation and question asked in the problem by using a trial and error method. Figure 32 is an example showing that the student used a trial and error strategy to find a distance.

A part-and-whole strategy. This was another strategy the students in this study used often in solving Problem 8 given on the pretest and posttest. Please see Appendix A for more details about Problem 8. In this strategy, students find the whole and then give the part as an answer to the problem. For example, in solving Problem 8, the students thought that since the number of girls was twice the number of boys, if boys are one part then the girls are two parts. Therefore, the total of students is three parts. So they divided 45 by 3 and got 15. Since girls are two parts of the total, the students added 15 two times to get the number of girls.

A comparison strategy. This was another informal strategy the students in this study used in solving Problem 7 (Time and rate problem) given on the pretest and posttest. Please refer to Appendix A for more detail about Problem 7. There was only one student (William) during the interview used the comparison strategy to solve Problem 5. Please refer to Appendix B for more details about Problem 5. In this strategy, the students compared distances of two people biking or walking hour by hour. Until the distance of both people is equal, the students stop comparing and gave the times that they would meet.

Conclusion

The results from this study indicate that Thai ninth grade students used both algebraic strategies and non-algebraic strategies to solve algebra word problems. There were two sub-strategies in the algebraic strategies: equations based on comprehensive representations (Successful strategy) and equations based on poor representations (Unsuccessful strategy). There were five sub-strategies in the non-algebraic strategies: verbal or written arithmetic, a drawing or graph, trial and error, a part-and-whole strategy, and a comparison strategy. The results from this study indicate that high and medium achievers used both algebraic and non-algebraic strategies to solve most of the problems. Most low achievers used non-algebraic strategies or were unable to solve most of the word problems.

Most students began solving the problem by thinking about forming an equation. For successful students, if they could not solve the problem by using an equation, they would use other strategies. That is, the successful students were more flexible in their strategy use and they attempted to use variety of strategies to solve algebra word problems. In addition, the successful students read the problem and found the unknown to assume the variable. After that, they went back to read the problem again in order to form the equation based on the situations and relationship in the problem. For unsuccessful students, they also read the problem, but they had difficulty translating the problem and finding a relationship between the situations in the problem. The unsuccessful students depended solely on finding an equation to the problem. In other words, they were not flexible in their own thinking. More discussion about the differences between successful and unsuccessful problem solvers will be found in CHAPTER V. The results from this study show that few students checked their answers when solving word problems either on the pretest, posttest, or during the interview sessions. Even the results demonstrate the possibility that if students had checked their solution, they might be able to adjust and get a correct answer.

This section examines solution strategies students in this study used to solve algebra word problems. In the next two sections, teacher's instruction and potential connections between teaching and students' performance will be explored.

Section Three: Teacher's Instruction on Solving Algebra Word Problems

This section provides results about three teachers' instruction on solving algebra word problems. The results were analyzed from three classes taught by these mathematics teachers: Mr. Jack, Ms. Rose, and Mr. Bond. The three classrooms looked similar. There were rows of students' desks, posters of cultural events on the wall, the teachers' desk and a chalkboard in front. Students always dress in school uniforms: white shirts and dark blue skirts for girls, and white shirts and light brown shorts for boys. Their names were written on the top left of their white shirts, and students' identification numbers and their schools' initial were written at the top right of their white shirts.

The lessons described in this section are about the same length, 45-50 minutes. Ninth grade students who participated in this study took mathematics five days a week. The lesson observed for this study was equations and inequalities. The lesson, which followed the book chapter, contained three topics: solving equations, solving word problems, and solving inequalities. The researcher decided to observe all the lessons in the unit because the researcher wanted to see the consistency of each teacher's teaching style even though the topic of interest was on solving word problems. Next, the each teacher's teaching style is described.

Mr. Jack's Teaching Style

The students in Mr. Jack's class were classified as low achievers. Mr. Jack mentioned that there were only two or three students in his class who were doing well in mathematics. Mr. Jack's teaching style was categorized as direct instruction and was procedurally based. Every day of instruction was focused solely on memorizing rules and practicing routine procedures. In many lessons, Mr. Jack led the students through the development of procedures for solving equations, word problems, and inequalities. There was no attempt to develop new content conceptually. The procedure for solving the problems was stressed. Mr. Jack followed every detail from the students' textbook. No outside workbooks, worksheets, or activities were used. Every example he used was from the students' textbook. For example from Day 2 of Mr. Jack's instruction, the lesson was on solving equations. Mr. Jack began by asking the students about the exercise assigned the previous day. Mr. Jack then wrote the following: "$1.5x + 2 + 2.5x - 0.5 = 4x - 1.5 - 3x$" on the chalkboard and then began to question the students throughout the process of solving for x. While Mr. Jack was writing on the chalkboard, the students were very noisy and did not pay much attention to his explanation. Mr. Jack brought back students' attention by asking:

Mr. Jack: "OK what are we going to do next?"

Students†: "Changing sides."

Mr. Jack: "OK moving variables to one side and numbers to the other side"

Mr. Jack also wrote the following equation on the chalkboard while he was explaining, "$1.5x + 2.5x - 4x + 3x = -1.5 - 2 + 0.5$".

Mr. Jack: "And what do we get?"

Students: "$3x$ equal negative 3." ($3x = -3$)

Mr. Jack: "OK, so x is equal to $-3/3$, which is -1."

After this problem was solved, Mr. Jack asked the students to solve the next problem in the students' textbook. Mr. Jack walked around the room to monitor students' work and then he came back and sat at his desk. Five minutes later, Mr. Jack asked the students to give the answer. The lesson continued like this until the class was dismissed. If the students could not solve some of the problems, Mr. Jack would explain the procedure to them on the chalkboard. For example, the students could not solve: $(x/2) + (x/3) = 18 - (x/6)$.

Mr. Jack: "First, we have to find LCD, what is LCD of 2, 3, and 6?

Students: "Umm. Six."

Mr. Jack: "Then we will multiply six to every term of the equation and it becomes $6(x/2) + 6(x/3) = (18 \times 6) - 6(x/6)$."

Mr. Jack also wrote on the chalkboard while he was explaining.

Mr. Jack: "And we got $3x + 2x = 108 - x$."

Mr. Jack: "And then what will we get? OK, we get $6x = 108$ and then $x = 108/6$ so $x = 18$"

As usual, Mr. Jack also wrote the answer on the chalkboard. In general, Mr. Jack gave the lecture to the whole class. Sometimes, he asked his students to answer but the students

† Students in the excerpt participating were approximately half of the students who answered teachers' questions chorally.

rarely replied. So, he answered his own question. He did not call on students by name. None of his explanations or questions attempted to develop the reasoning behind the procedure he taught. Mr. Jack explained the routine procedure while he was writing on the chalkboard and students wrote whatever was on the chalkboard into their notebook. Next, specific information about Mr. Jack's instruction on how to solve algebra word problems is reported.

Mr. Jack's Instruction on Solving Algebra Word Problems

Mr. Jack spent two teaching days on solving algebra word problems (see Table 20). His teaching style was similar to how he had taught on the first two days. That is, Mr. Jack used examples from the students' textbook and he directed students to solve algebra word problems by following his work at the chalkboard.

Table 20

Days Mr. Jack taught Chapter 6

Topic \ Day	1	2	3	4	5	6	7
Equations	✓						
Solving Equations	✓	✓					
Solving Word problems			✓	✓			
Inequalities					✓	✓	✓

Day 3. Five minutes after an electronic bell sounded, Mr. Jack entered the classroom. At a signal from the head student, "Ready Bow", all students put their two hands together and bowed their head toward their hands while saying "Good morning teacher." Mr. Jack greeted the students "Good morning" and the lesson was officially under way. Mr. Jack walked around the room to check the exercise given during the past week. After five minutes of checking, Mr. Jack began the new lesson.

> Mr. Jack: "Today we will begin solving word problems. Let's open to page 200 in your textbook and copy the problem number one, exercise 6.2, into your notebook."
>
> Students: <… the students were copying the problem into their notebook…>

Mr. Jack: "Please read the problem out loud for me."

Students: "There is a number such that twice the number plus 3 is equal to 21. Find the number."

Mr. Jack: "How are you going to solve this problem? Why don't we form an equation?"

Mr. Jack wrote on the chalkboard, while, at the same time, he explained how to form the equation.

Mr. Jack: "Let a number be x. Twice a number is $2x$, and then plus 3 is equal to 2. So the equation is $2x + 3 = 21$."

Mr. Jack: "What is x?"

Mr. Jack and his students then solved for x. Mr. Jack asked the whole class each step of solving for x and wrote the steps on the chalkboard. Mr. Jack did not tell or ask the students to check their answers. In addition, Mr. Jack did not check whether his students understood how he arrived at that equation. After students finished with this problem, Mr. Jack asked the students to read and solve the problem number two by themselves with the help of the teacher. The problem number two was – three times a sum of a number and 7 is 33. Find the number. The class was quiet at this moment because the students were doing their work. After about five minutes, Mr. Jack asked for the answer.

Mr. Jack: "Did you get the answer?"

Some students in the front: "Four."

Mr. Jack did not ask the students how they solved the problem or ask them to explain their solution. He instead explained and wrote the solution on the chalkboard.

Mr. Jack: "So let x be a number. Three times the sum of a number and 7. So it's $3(x + 7)$. And it is equal to 33, so the equation is $3(x + 7) = 33$. Next, we will solve for x."

After getting the equation, Mr. Jack then solved the equation by questioning the students all the way until they finished. After that, Mr. Jack asked the students to do the problem number three by themselves. The problem number three was – find three consecutive integers whose sum is 108.

Mr. Jack: "Please read the problem several times to understand the problem OK, read it several times."

Three minutes later, Mr. Jack read the problem to the students so the whole class could form the equation.

Mr. Jack: "If we let x be the first number, what is the second number?

Students: "y."

At this point, Mr. Jack corrected the students that they could not assume "y" because they can assume only one variable. Therefore, he asked the students to read the problem again but not out loud. After the students had read the problem, some students still told Mr. Jack to assume "y" as the second number. At this time, Mr. Jack did not say anything. Instead, he continued to explain and wrote on the chalkboard.

Mr. Jack: "If we let x be the first number, what are we going to assume for the second number."

S1(only one student in the class replied): "x plus one: $(x + 1)$."

Mr. Jack: "How about the third number?"

S1: "x plus two: $(x + 2)$."

Mr. Jack: "OK, if the first number be x, the second number should be $x + 1$, the next number should be $x + 2$."

Mr. Jack: "Now you've got all three assumed numbers, why don't you write an equation by yourself."

Only one student in this class answered Mr. Jack's question. Mr. Jack did not check students' understanding of his explanation or the students did not ask where x, $x + 1$, and $x + 2$ came from. All they did was copy what Mr. Jack wrote on the chalkboard into their notebook. After about five minutes, the students wrote the equation and solved for x. Mr. Jack then asked the students for the answer. Likewise, Mr. Jack did not ask the students to check their answer and did not check students' understanding. Mr. Jack, instead, asked the students to do the problem number four by themselves. He, then, walked around the room to check whether the students were on task. The problem number four was – the sum of two consecutive even integers is 46. What are the numbers?

Mr. Jack: "OK 2, 4, 6, and 8 are even numbers. If we let the first even number be x, what is the next number? It's twice apart from the first number."

Students: "x and $x + 2$."

At this point, Mr. Jack asked the students to write the equation by themselves and he walked around the room to check whether the students were on task or not. For this problem, Mr. Jack did not ask for the answer or for an explanation. He, instead, asked the students to do the problem number five. The problem number five was – when subtracting 1/6 of a number from 1/2 of that number, the result is three less than the sum of 1/4 of that number and 1/8 of that number. Find the number. Mr. Jack reminded the students to read the problem carefully and read it several times. The students then read the problem out loud. After that, Mr. Jack began his explanations.

Mr. Jack: "What are we going to assume for that number?

Students: "*x*."

Mr. Jack: "What's next?"

Students: <...silent...>

Mr. Jack: "What is the problem asking for? It's asking what that number is right. Let's do it."

Mr. Jack let the students work on the problem for a while and he walked around the room. It took about five minutes for the students to solve the problem. However, no one seemed to be able to solve it so that Mr. Jack went to the chalkboard and began to explain. He wrote on the chalkboard while he was questioning the students.

Mr. Jack: "OK, let's do it."

Mr. Jack: "Let's x be that number. Take 1/6 of that number from half of that number so we will get."

Students: <...silent....>

Mr. Jack: "Well, x is that number. 1/6 of that number is $x/6$ and 1/2 of that number is $x/2$. Take 1/6 of that number from half of that number, so it is $(x/2) - (x/6)$."

Mr. Jack: "Then the sum of 1/4 of that number and 1/8 of that number. So 1/4 of that number is $x/4$ and 1/8 of that number is $x/8$ and the sum will be $(x/4) + (x/8)$."

Mr. Jack: "So the equation will be $(x/4) +(x/8)= [(x/2) - (x/6)] + 3$. It is plus three because the sum of $x/4$ and $x/8$ is three more that the difference of $x/2$ and $x/6$."

Mr. Jack: "Now, you can solve for x."

The students were solving for x at their own seat. After that, Mr. Jack asked for the answer.

Mr. Jack: "Did you get the answer?"

Students: "Yes. 24."

Mr. Jack: "Why don't you check the answer?"

From this excerpt, Mr. Jack explained everything to the students. The students did not ask any questions and Mr. Jack did not check for students' understanding. Even checking the answer, he just asked some students to check the answer. It should be noted that the students gave the wrong answer to Mr. Jack (24 instead of 72), but Mr. Jack did not realize that mistake or did not check an accuracy of the answer the students gave. At the end of the class, Mr. Jack gave two problems (problem numbers six and seven) as homework and reminded the students of the upcoming test. Then, the class was dismissed. Again, at the signal from the student head "Ready Bow" all students bowed to the teacher while saying "Thank you teacher".

Day 4. The class today was in the afternoon (period 7). As a custom in most Thai schools, Mr. Jack and his students exchanged a bow at the beginning of the class. Mr. Jack began the lesson by asking about the homework given on the Day 3 and then let the student do the problem number eight from their textbook. Mr. Jack asked the students to read the problem. The problem number eight was – in 12 years, the ratio of father and son's ages will be 3:1. If the father is 30 years older than the son at the present time, find the age of the father.

Mr. Jack: "What is the problem telling us?"

Instead of answering the question, the students read the problem and copied the problem into their notebook. Mr. Jack, then, walked around the room and went back to ask the same question.

Mr. Jack: "OK. What is the problem telling us?"

Students: <...read the problem out loud...>

After the students finished reading the problem, Mr. Jack wrote on the chalkboard while he was explaining.

Mr. Jack: "In 12 years the ratio of father and son will be 3 to 1."(Mr. Jack wrote Father: Son = 3:1 on the chalkboard.)

Mr. Jack: "Let x be for the son's age, how about the father?"

Mr. Jack: "Father is 30 years older than the son, right. If the son is x years old, his father is 30 years older, what is the father's age?

Students:< ...silent...>

Mr. Jack: "The father is $x + 30$ years old. This is the father's age nowadays."

Mr. Jack: "How old is the son in 12 years?"

Students: <...silent...>

Mr. Jack: "The son will be $x + 12$ years old."

As the lesson continued, there were only two or three students in the front row answered Mr. Jack's question.

Mr. Jack: "How about the father's age in 12 years?"

Students: <...silent...>

Mr. Jack: "Well, the father will be $x + 30 + 12$ years old."

Mr. Jack: "The problem said the ratio of father's and son's age is 3 to 1. Father to son is $x + 42$ divide by $x + 12$ equal 3 over 1, $(x + 42)/(x + 12) = 3/1$."

Mr. Jack: "Well, the ratio of the father to son's age is 3 to 1 so we put the father's age on top and divide by the son's age equal three divided by one."

Mr. Jack: "Therefore, the equation is $x + 42 = 3(x + 12)$"

As before, Mr. Jack did not check students' understanding along the way. In addition, the students did not ask any questions, except some students in the front row asked some questions. After getting the equation above, Mr. Jack let the students solve the equation for x. After two minutes, Mr. Jack explained briefly how to solve that equation without writing on the chalkboard.

Mr. Jack: "What do you get?"

Mr. Jack: "x equal 3. Therefore, what is the son's age?"

Students: "Three years old."

Mr. Jack: "What is the father's age?"

Students: "33 years old."

Next, Mr. Jack asked the students to do the problem number nine by letting the students read and solve the problem by themselves. The problem nine was – the first man (A) begins to walk at 10:00 am at the rate 5 kilometer per hour. Two hours later, the second man (B) began to walk from the same starting point to the same direction as the first person at the rate 10 kilometer per hour. At what time will the two people meet? After five minutes, Mr. Jack began to explain and wrote on the chalkboard.

Mr. Jack: "A and B. What is the distance?"

Students: "It's equal."

Mr. Jack: "How about time?"

Students: "It's not equal."

Mr. Jack: "Time is not equal but the distance will be...?"

Together: "Equal."

Mr. Jack: "The second person (B) use...?"

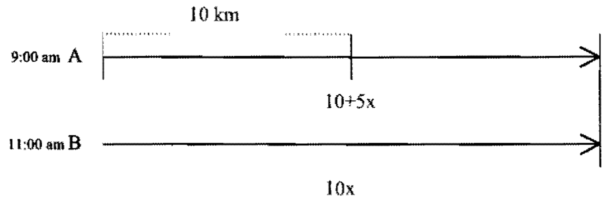
Students: "More time."

It should be noted that the students made a mistake but Mr. Jack did not correct them right away.

Mr. Jack: "Who left home first?"

Students: "A."

Mr. Jack began to explain by drawing on the chalkboard. He drew two lines to represent two people's distance. Based on the drawing, Mr. Jack explained how to form the equation by using the following drawing.



Mr. Jack: "In x hours, how far will A go and how far will B go?"

Mr. Jack: "A left two hours before B, so A went 10 km and let the time that they will meet is x hour."

Mr. Jack: "In one hour, A went 5 km. So in x hour, A went $5x$ km. In one hour, B went 10 km. So in x hour, B went $10x$ km."

Mr. Jack: "And the distance of two people is…?"

Students: "Equal."

Mr. Jack: "So what is the equation?"

Students: <…silent…>

Mr. Jack: "The equation is $10 + 5x = 10x$."

After getting the equation, Mr. Jack and the students solved the equation for x, which took about five minutes. As before, Mr. Jack did not check for students' understanding.

Mr. Jack: "What do you get?"

Students: "They will meet at 13:00."

Mr. Jack: "So in two hours, they will meet. What time is it?"

At this point, some students shouted 13:00, but Mr. Jack did not hear it so he kept asking the students. Since the students did not answer, Mr. Jack explain on the chalkboard as the following:

Mr. Jack: "We got $10 + 5x = 10x$. So $-10x + 5x = -10$ and then $-5x = -10$. And we get $x = 2$."

Mr. Jack: "Therefore, they will meet within two hours."

Mr. Jack: "So they will meet at 13:00 o'clock."

Mr. Jack: "Let's do the next problem."

As noted earlier, Mr. Jack did not check for students' understanding and did not ask the students to explain their thinking. Some students could answer Mr. Jack's questions but he did not respond to it. After this, Mr. Jack asked the students to do the problem number ten. Mr. Jack read the problem to the students and then explained on the chalkboard. However, there was not enough time to finish this problem. The class ended without finishing this problem. Therefore, Mr. Jack told the students to do this problem on their own. Again, at the signal from the student head "Ready Bow" all students bowed to the teacher and said, "Thank you teacher." For the next day (Day 5), Mr. Jack did not get back to the problem number ten, which was not finished on Day 4. Instead, Mr. Jack began a new lesson, which was about inequalities.

Summary

Mr. Jack taught students to solve word problems by having the students solve word problems from the students' textbook and by following his lead at the chalkboard and then he gave an explanation of the procedure afterward. No outside problems were used during those two days. Mr. Jack mostly directed the students to solve the problem. Even if Mr. Jack asked questions, few students answered. The question Mr. Jack used in his class was more a question for answer (e.g., What is x? or Did you get the answer?) rather than a question to initiate students' thinking. When the students did not answer, Mr. Jack did not encourage the students to think. Instead, he gave an explanation to the students. From the transcription, it is obvious that the students did not answer Mr. Jack's question or did not ask any questions and Mr. Jack did not check students' understanding. Mr. Jack asked students to solve a problem by themselves in almost every example. However, he did not encourage students to think, to give explanations, or to work cooperatively. Sometimes students answered his questions but he did not use that answer nor did he respond to the students' answer. In addition, Mr. Jack rarely emphasized the importance of checking the answer.

In summary, Mr. Jack was the person who did the most thinking during instruction and the students did little thinking, just copied what was at the chalkboard into their notebook. There were a lot of times the students were silent and Mr. Jack did not encourage the students to think through the problem. There were few interactions between the teacher and the students in this mathematics class.

Ms. Rose's Teaching Style

The students in Ms. Rose's class were medium achievers. Ms. Rose mentioned that she had been teaching several grade-levels and she knew what students at each grade-level needed to know in order to step up to the next grade-level. Therefore, her intention in teaching was to assist in raising each student's understanding to the same point before going to a new topic. Ms. Rose took a more interactive teaching role in some lessons than her teaching peers and allowed her students to be actively involved in their own learning. Ms. Rose was concerned about students' understanding of the concept behind the procedure that she taught. Ms. Rose led students through the development of procedures for solving equations, word problems, and inequalities. She regularly emphasized the reasons behind those procedures. Ms. Rose did not usually call on individual students. Mostly, all students replied chorally to her questions. For example:

Ms. Rose: "Open to page 176 in your blue workbook, look at problem one; $4x - 3 = 2x + 1$, what are we going to do first?"

Students: "Changing terms."

Ms. Rose: "We will not talk about changing terms, rather, which properties are we going to use?"

Students: "Addition property of equality."

Ms. Rose: "Add what?"

Students: "Add $-2x$."

Ms. Rose: "That means we applied the property of equality to both sides. From the problem, $4x - 3 = 2x + 1$, we want to get rid of $2x$." (Ms. Rose points at $2x$ on the chalkboard).

Together: "So we get $4x - 2x - 3 = 2x - 2x + 1$."

Ms. Rose: "What is next?"

Students: "We got $2x - 3 = 1$."

Ms. Rose: "Is there anything changing for the number 1?"

Students: "No."

Ms. Rose: "OK, what is next?"

Students: "$2x - 3 + 3 = 1 + 3$."

Ms. Rose: "Which properties do you use here?

Students: "Addition property of equality with the number 3."

Ms. Rose: "To how many sides?"

Students: "To both sides." (Ms. Rose also wrote on the chalkboard)

Ms. Rose: "What do you have next?"

Students: "$2x = 4$."

Ms. Rose: "Which properties do you use next?"

Students: "Multiplication property of equality with the number1/2."

Ms. Rose: "And what do we get?"

Students: "$2x/2 = 4/2$."

Ms. Rose: "And we will write beside here that 'multiplication by 1/2."

The above example show that Ms. Rose practiced, then required the students to state the properties of equality to solve an equation. In each step for solving the equation, she always asked the students to provide an explanation of their procedure because she wanted the students to know why they had to add or multiply both sides by the same numbers. She told the students to write down their reasoning for each step of the solution to an equation that they solved (see Figure 35) for the homework.

$4x - 3 = 2x + 1$	
$4x - 3 - 2x = 2x - 2x + 1$	Addition by $-2x$
$2x - 3 = 1$	Results from above
$2x - 3 + 3 = 1 + 3$	Addition by 3
$2x = 4$	Results from above
$2x/2 = 4/2$	Multiplication by 1/2
$x = 2$	Results from above

Figure 35. Procedures Ms. Rose wrote on the chalkboard.

Mr. Rose rarely used the students' textbook in teaching. She always used extra worksheets and asked the students to do extra problems from the blue workbook. In addition, Ms. Rose used group work. She put the students together in a group to help each other master the concept. For example, on Day 4, Ms. Rose let the students work in a group to solve equation problems. The objective of this activity was to practice student's reasoning behind the procedure they used in solving equations. There were 12 equation problems (Figure 36).

1	$-7 + x = -3$
2	$2x - 5 = 5$
3	$3x - x + 4 = 5x + 1$
4	$0 = (8 - 9a) + (17a - 20)$
5	$\frac{2+x}{10} = \frac{4-2x}{9}$
6	$1 - \frac{x}{3} = \frac{x}{6} - \frac{1}{2}$
7	$3x - 2 = -(2-x)$
8	$6 - 3[x - 2\{1 + 2(5 - x)\} - 2x] = 3(2 - x)$
9	$2.5x - 2 - 1.5x = 3x - 2.5 - 4x$
10	$\frac{z}{8} + \frac{z}{4} - \frac{z}{5} = \frac{z}{2} - 1$
11	$2(3y - 1) - (5 + y) = 7 - (3y - 4)$
12	$\frac{7x-4}{15} + \frac{x-1}{3} = \frac{3x-11}{5} - \frac{2+x}{10}$

nstruction: Write the steps for solving the above equations by using the following symbols.

♣	stands for Distributive property (Remove bracket)
♦	stands for Using LCD
♥	stands for Combining like terms
ω	stands for Applying equality property of addition
Ω	stands for Applying equality property of multiplication
□	stands for Using symmetry property

Figure 36. Ms. Rose's worksheet for group work in class on Day 2.

These 12 problems were grouped into four sets. Each set was {1, 5, 9}; {2, 6, 10}; {3, 7, 11}; and {4, 8, 12}. The students were divided into 11 groups of four people. Each person in the group worked on one of the four sets. The students needed to think about how to solve each equation step by step without any written solution. The students in each group were allowed to help each other. The students needed to put a sign to indicate the procedure they would use in each step. While the students were working, Ms. Rose walked around the room to give suggestions and to answer students' questions. Figure 37 shows an example of how to do this activity.

Example:

Solve

$$\frac{3}{5}(x - 1) + \frac{1}{2}(3 - x) = \frac{1}{4}(5x - 6) - \frac{1}{10}(5 - 3x)$$

Steps for solving the equation above

1 Eliminate the denominator by using LCD ♦

2 Apply distributive property ♣

3 Combine like terms ♥

4 Apply equality property of addition ω

5 Apply equality property of addition ω

6 Apply equality property of multiplicaiton Ω

Figure 37. Example of how to do the activity

After the students were finished working in four, Ms. Rose asked the students who got the same set of problems to sit together in order to check their work before presenting to the class. There was some chaos because some students had not finished their work. Ms. Rose told the class that she would select a low achiever from each group to be a presenter. She mentioned that each group needed to make sure the presenter knew and understood every step of their solution; otherwise the group would get a low score. Unfortunately, the class time came to an end. The students did not have a chance to present their work that day. Ms. Rose continued this activity on the next day. However, only one group had a chance to present their work because Ms. Rose did something else on the following day.

In general, Ms. Rose played more of a facilitator role in her mathematics class than her teacher peers and her students participated more in the teaching and learning process. Although she required her students to practice routine procedures, she emphasized the reasoning behind those procedures. Ms. Rose also asked the students to work in groups and in pairs. However, often times she did not finish the activity, possibly because of time constraints. She had a set amount of time to complete the chapter.

Ms. Rose's Instruction on Solving Algebra Word Problems

Ms. Rose spent two separate days teaching the solution of algebra word problems (see Table 21). The reason was that she wanted the students to be comfortable with solving both equations and inequalities before she went on to solving word problems. Ms. Rose also taught the students to solve inequality word problems, which was not taught in the other two classes.

Table 21

Days Ms. Rose taught Chapter 6

Topic \ Day	1	2	3	4	5	6	7	8	9
Equations	✓	✓							
Solving Equations		✓	✓	✓	✓				
Solving Word problems					✓			✓	
Inequalities						✓	✓		✓

Day 5. When the bell sounded, Ms. Rose had not entered the classroom yet. The class was filled with students' talking and joking with each other. Some students worked quietly at their desks. Ten minutes later, Ms. Rose made her appearance. As is a custom in most Thai schools, the students and the teacher exchanged a bow at the beginning of the class. Ms. Rose began the day's lesson by clarifying the difficulty she found from the students' work in solving equation problems involved with fractions. Ms. Rose asked the students to check the answer to every problem that the students had already completed. If the students did not check, Ms. Rose would not give a full score. This took about 25 minutes of the class time. After that, Ms. Rose switched to solving word problems. Ms. Rose distributed a new worksheet (Figure 38) about solving algebra word problems.

> Ms. Rose: "Put your pen down and listen. I'm going to begin a new topic. Now, each pair of you got one new worksheet. Each of you will work on each set of the problems." (Figure 38)
>
> Ms. Rose: "This topic is difficult. If you don't understand, it will be hard for every one. The steps are the following."

Problem Set 1: Please show your five steps in solving the following word problems.

1. Five times a number and 10 is 50. Find the number
2. If $\frac{2}{3}$ of a number minus 11 equal to $\frac{1}{6}$ of that number plus 5. Find the number.
3. $\frac{2}{3}$ of the sum of a number and 2 equals to $\frac{3}{4}$ of the difference between that number and 5. Find the number.
4. $\frac{3}{4}$ of the sum of a number and 5 is 12 greater than $\frac{2}{3}$ of the diffrence between that number and 8. Find the number.
5. Find three consecutive integer number, whose sume is 53

Problem Set 2: Please show your five steps in solving the following word problems

1. Five times a number and 15 is 65. Find the number
2. If $\frac{2}{3}$ of a number minus 8 equal to $\frac{1}{6}$ of that number plus 4. find the number.
3. $\frac{2}{3}$ of the sum of a number and 3 equals to $\frac{3}{4}$ of the difference between that number and 7. Find the number.
4. $\frac{3}{4}$ of the sum of a number and 5 is 12 greater than $\frac{2}{3}$ of the diffrence between that number and 8. Find the number.
5. Find three consecutive even integer numbers, whose sume is 66.

Figure 38. Ms. Rose's worksheet on Day 5.

Ms. Rose: "A word problem is like a sentence, a story. When you see word problems, please read the problem first. Some students do not read the problem first. They just look for numbers to calculate. Please read the problem first. After reading it, find out what we know and what we don't know from the problem. Then, we will assume the variable. We will look at an example from the eighth grade textbook."

Mr. Rose wrote the following word problem on the chalkboard: "A barbed wire 36 meters long will be used to fence a rectangular area. If the width is 4 meters shorter than the length. Find the length and the width."

Ms. Rose: "A rectangle has four sides: width, length, width, length (Ms. Rose points to a drawing on the chalkboard while explaining). Therefore, we have to read the problem and analyze it."

Ms. Rose: "What is the length of the wire?"

Students: "36 meters."

Ms. Rose: "And how many the widths are there in the rectangle?"

Students: "Two."

Ms. Rose: "How about the length?"

Students: "Two."

Ms. Rose: "We will write $2W$ and $2L$, right?"

Students: "Yeah."

Ms. Rose: "What is the length of the wire?"

Students: "36 meters."

Ms. Rose: "OK, so we got $2W + 2L = 36$."

Ms. Rose: "Are you following me here?"

Students: "Yeah."

Ms. Rose: "The problem asked us to find the length and the width. Therefore, from reading the problem, if we know the width, we will know length or if we know the length, we will know the width."

Ms. Rose: "What is this problem asking for?"

Students: "The length and the width."

Ms. Rose: "In the second step, you have to assume only one variable for the unknown or what the problem asks for. The problem wants to know the length and the width of the square. So what we are going to assume a for, length or width?"

Students: "Width."

Ms. Rose: "OK so the width is a. The width of this rectangle is a meter. Then we will get $a + 4$ for the length."

Ms. Rose: "And how can we form an equation? It will come from here (Ms. Rose points at the equation $2W + 2L = 36$ on the chalkboard). $2W$ is $2a$ and $2L$ is $2(a + 4)$. So we will get the equation, which is the third step, forming an equation."

Ms. Rose: 'Therefore, the equation is $2a + 2(a + 4)$ equal to...?"

Students: "36."

Ms. Rose: "This is the equation. [$2a + 2(a + 4) = 36$]"

Ms. Rose: "What are we going to do next?"

Students: "Solve the equation."

Ms. Rose: "Yes, I know that you can solve the equation. Therefore, I want you to analyze and work these three steps on the five word problems on the new worksheet (Figure 38). The three steps are: reading the problem and analyze the problem to see the known and unknown value and what the problem asked for, and then assume one variable and then form the equation. I need only these three steps. Can you do this?"

Students: "Yes."

Ms. Rose: "You can talk to your partner."

Ms. Rose walked around the room to answer students' questions until the class ended. Ms. Rose also left a board explaining steps in solving word problems in front of the room so that the students could come up and read. Again, at the end of the class period, the teacher and the students exchanged a bow. For the Days 6 and 7, Ms. Rose taught students about solving inequalities. The lesson on solving word problems continued again on Day 8, which is as follows.

Day 8. Today, Ms. Rose began the lesson right away. Ms. Rose did not refer to the activity that the students did on Day 5. The following is her lesson on the Day 8.

Ms. Rose: "How many steps are there in solving word problems?"

Students: "Five steps."

Ms. Rose: "What is the first step?"

Students: "Read and analyze the problem."

Ms. Rose: "Analyzing the problem and what's next?"

Students: "Separate into parts."

Ms. Rose: "How many parts?"

Students: "Two parts."

Ms. Rose: "What are they?"

Students: "What is the problem giving us and what is the problem asking for?"

Ms. Rose: "Read the problem from page 180 in the blue workbook for me please."

Students: "Four times a number minus seven is five. Find a number."

Ms. Rose: "Well, separate into two parts, what is the first part?"

Ms. Rose: "The first part is what the problem is asking for. That number is? Ok. What is the problem telling us?

Students: "Fours times a number minus seven is five."

Ms. Rose: "Therefore, who works on the activity should be able to separate the parts."

Ms. Rose: "What is the next step?"

Students: "Assuming the variable."

Ms. Rose: "How many variables?

Students: "One variable."

Ms. Rose: "Therefore one variable stands for "a number." Therefore, which variables you are going to use for a number?"

Students: "x."

Ms. Rose: "So assuming x for a number." (Ms. Rose wrote on the chalkboard.)

Ms. Rose: "Next, find the relationship and then write an equation."

Ms. Rose: "So when we find a relationship, read this." (Ms. Rose pointed on the problem.)

Students: "Four times a number."

Ms. Rose: "Four times a number. That number is x. Therefore four times a number can be written as…?"

Students: "$4x$."

Ms. Rose: "This is how we find the relationship. Now we have (Ms. Rose wrote on the chalkboard.) four times a number is $4x$."

Ms. Rose: "Ok. What's next?"

Students: "Minus 7."

Ms. Rose: "Minus 7, it will be…?"

Students: "$4x$ minus 7. " (Ms. Rose wrote on the chalkboard $4x - 7$)

Ms. Rose: "What is the result?"

Students: "5."

Students: "We will get $4x$ minus 7 equal to 5" (Ms. Rose wrote on the chalkboard $4x - 7 = 5$)

Ms. Rose: "What is it?"

Students: "An equation."

Ms. Rose: "What is this step?"

Students: "Step three."

Ms. Rose: "OK, step three. We got an equation $4x$ minus 7 equals 5: ($4x - 7 = 5$)."

Ms. Rose: "Can you do it?

Students: "Yes."

Ms. Rose: "What is the fourth step?"

Students: "Solving the equation."

After this, the students and Ms. Rose solved for the unknown. At this point, Ms. Rose did not require the students to write steps in solving the equation. She wanted the students to focus on the first three steps of solving equation word problems.

Ms. Rose: "For solving word problems, you don't have to write the step in solving an equation for me; write only three steps of forming the equation for the word problems. Do you understand?

Students: "Yes."

Ms. Rose: "Good, so what is x?"

Students: "3."

Ms. Rose: "OK, x equals 3. x stands for a number, therefore we will write as 'the number is three' as our answer."

Ms. Rose: "What is the last step?"

Students: "Checking the answer."

Ms. Rose: "Checking the answer. When checking the answer, we check from the problem."

Then, Ms. Rose put the number 3 into the equation in order to check the answer. At this point, Ms. Rose told the students that we would check the answer by looking at the problem. However, what she showed was to put the number x, which is the number 3, into the equation formed in the third step.

Ms. Rose: "Therefore, we have five steps in solving word problems but I need details on steps one, two, and three. You have to write as many details as you can. You have to remember that this is only one variable."

Ms. Rose: "Again, I need only steps one through three."

After this, Ms. Rose talked about the worksheet for five minutes and then she wrote the next example on the chalkboard (see Figure 39). It is a coin problem. When the students saw the problem, they complained that it was too long like an article. After Ms. Rose finished writing on the chalkboard, she asked the students to read the problem out loud.

Ms. Rose: "What is the problem asking for?"

Students: "The numbers of each coin."

Ms. Rose: "Well, we have to use one variable. We have to read it carefully and assume only one variable."

Ms. Rose: "Read and think carefully, and tell me what you want to assume."

Students: “A ten-Baht coin.”

Ms. Rose: “What does the problem ask for?”

At this point, Ms. Rose and the students read through the problem again and look for what the problem asked for and she wrote that on the chalkboard (see Figure 39).

In one savings bank, there are one-Baht coins, five-Baht coins, ten-Baht coins, and 50-Stang coins. These coins are worth 200 Baht totally. The numbers of one-Baht coins are twice the numbers of ten-Baht coins. The numbers of one-Baht coins are 20 greater than the number of five-Baht coins. The numbers of 50-Stang coins are twice the number of one-Bath coins. Find the numbers of each coin.

Step 1: The problem asked:
1) How many one-Baht coins
2) How many five-Baht coins
3) How many ten-Baht coins
4) How many 50-Stang coins

Step 2: Assume B as the number of ten-Baht coins

Step 3: Find the relationship and form an equation

1) The numbers of one-Baht coins are $2B$ and it is worth $2B$ Baht
2) The numbers of five-Baht coins are $2B - 20$ and it is worth $5(2B - 20)$ Baht
3) The numbers of ten-Baht coins are B and it is worth $10B$ Baht
4) The numbers of 50-Stang coins are $4B$ and it is worth $4B/2= 2B$ Baht

Figure 39. A coin problem and the explanation Ms. Rose wrote on chalkboard on Day 8.

Ms. Rose: “So, how many questions?”

Students: “Four questions.” (see step one in Figure 39)

Ms. Rose: “So, let’s try to separate the problem.”

Ms. Rose: “Which sentence can we use to create an equation?”

At this point, Ms. Rose underlined the sentences that would be used to create the equation (see Figure 39).

Ms. Rose: “What do you want to assume?”

Students: “A one-Baht coin as x.”

Ms. Rose: “Can you change x to something else. We always assumes x.”

Students: “How about B?”

Ms. Rose: “OK. Assuming we have B one-Baht coin, and what’s next?”

Ms. Rose: “How many of the ten-Baht coins?

The students were stuck because they did not want to assume B for the one-Baht coin. The problem said, "The numbers of one-Baht coins are twice the numbers of ten-Baht coins". The students said that if they assumed B for the one-Baht coin, it was difficult to find the numbers of ten-Baht coins in terms of B.

Students: "Umm. Change to assume a ten-Baht coin. It's better."

Ms. Rose: "Why can't we assume a one-Baht coin?"

Students: "You can but I cannot find the relationship."

At this point, the students are involved in the thinking and learning process. Ms. Rose also asked the students to explain their reasoning. In addition, Ms. Rose used student's thinking to solve the problems, rather than to direct them.

Ms. Rose: "Umm. Some of your friends want to change to a ten-Baht coin, wanna change?"

Students: "OK."

Ms. Rose: "OK, change to assume a ten-Baht coin. So assume that we have B ten-Baht coins."

Ms. Rose: "The number of one-Baht coin is twice the number ten-Baht coins. So the numbers of one-Baht coins will be…?"

Students: "$2B$."

Ms. Rose: "The number of one-Baht coins is 20 greater than the number of five-Baht coins. So the numbers of five-Baht coins are?"

Students: "$2B - 20$."

Ms. Rose: "Why?"

Students: "Because the number of one-Baht coins is 20 greater than the number of five-Baht coins."

Ms. Rose: "The number of 50-Stang coins is twice the number of one-Baht coin. What is it?"

Students: "$4B$."

Ms. Rose: "These are the numbers of each coin that we have found, but what is left?"

Students: "Amount of the money."

Ms. Rose: "We have to change to Baht right?"

Ms. Rose: "OK, how many Baht in one-Baht coins?

Students: "Umm."

Ms. Rose: "A one-Baht coin is worth one Baht, so $2B$ one-Baht coins are worth $2B$ Baht, right?"

Students: "Yes."

Ms. Rose: "A five-Baht coin is worth five Baht. Two five-Baht coins are worth ten Baht and 50 five-Baht coins are worth 250 Baht. If we have $2B - 20$ five-Baht coins, it will be worth?"

Students: "$5(2B - 20)$ Baht."

Ms. Rose: "Yes, 5 times $(2B - 20)$."

Ms. Rose: "Do you understand?"

Ms. Rose: "A ten-Baht coin is worth ten Baht. Two of them are worth 20 Baht. So if B ten-Baht coins, we multiply B right, so it will worth…"

Students: "$10B$ Baht."

Ms. Rose: "How many 50-Stang[‡] coins is one Baht?

Students: "Two coins."

Ms. Rose: "Two 50-Stang coins is how much?

Students: "One Baht."

Ms. Rose: "If we have ten 50-Stang coins, we will have how much?

Students: "Five Baht."

Ms. Rose: "If we have 18 50-Stang coins, we will have…"

Students: "Nine Baht."

Ms. Rose: "OK, we divided by?"

Students: "Two."

Ms. Rose: "So, as we assume here, how much will we have?"

Ms. Rose: "We have $4B$ 50-stang coins, so we divided $4B$ by

Students: "Two."

Ms. Rose: "How much?"

Student: "$2B$ Baht."

Ms. Rose: "Well, we haven't used the 200 Baht yet."

Students: "Add all together."

Ms. Rose: "And that is the equation, right?"

Students: "Yes."

[‡] A 50-Stang is worth half a Baht. That is, two 50-Stang coins = a one-Baht coin.

Ms. Rose: "OK, before we begin anything, we have to separate the problem into?"

Students: "Three parts."

Ms. Rose: "The important thing is to assume a reasonable variable. For this example, when we assumed a one-Baht coin, it was difficult to find a relationship so we changed to assume a ten-Baht coin. Can we find a relationship?"

Students: "Yes."

Ms. Rose: "OK, can you write an equation for me?"

Students: "Yes."

It was almost the end of this period. Ms. Rose asked the students to copy one word problem onto their notebook for working in pairs. Ms. Rose then wrote the problem on the chalkboard and the students complained that the problem was too long. It was time for the next period. Therefore, Ms. Rose reminded the students to do this problem in pairs and give it to her at the end of the day.

Summary

Ms. Rose taught solving word problems in two separate days (Days 5 and 8). On the first day, Ms. Rose gave an activity for students to do. The activity was about forming an equation. Ms. Rose explained the activity by using an example of a word problem that the students had seen from the eighth grade. In the explanation, Ms. Rose explained five steps in solving word problems. Three days later, Ms. Rose taught solving word problems again. However, she did not refer to the activity that she gave the students three days ago. Therefore, Ms. Rose did not conclude the activity. Ms. Rose used examples from outside the textbook. She did not make the students do the problems on their own. Instead, Ms. Rose and the students did the word problem together on the chalkboard. She asked questions to initiate student's thinking. When the students did not answer, Ms. Rose encouraged the students to think, but she did not jump to explain to the students. Ms. Rose always checked students' understanding. She asked the students to check their answer but not consistently.

In summary, both Ms. Rose and the students were actively involved in the teaching and learning process. Students in this class worked more cooperatively with peers and worked more problem than in the other two classes. Ms. Rose encouraged the students to think through the problem solving process. There were interactions between the teacher and the students in this class. However, the checking process happened only occasionally in this class.

Mr. Bond's Teaching Style

The students in Mr. Bond's class were classified as having high achievement. Mr. Bond mentioned that students, no matter which generation, always had problems with solving word problems. In particular, when students encountered a complicated problem, most students avoided solving it. Some were not even able to read the problem. This is a problem for many decades, especially for low achievers. Like Mr. Jack, Mr. Bond was using a lecture style. Mr. Bond presented definitions of terms and demonstrated the procedure for solving specific problems from the students' textbook. Students were then asked to practice the procedure. There was no attempt to develop conceptual understanding behind the procedure taught. Mr. Bond followed each detail from the students' textbook. The following transcript is from Mr. Bond's teaching on Day 1.

Mr. Bond: "Look at page 189 in your textbook."

Mr. Bond: "Number one – three times one number and three equal to ten. Is this language or symbol sentence?"

Some students in the front: "Language sentence."

Mr. Bond continued to question students from the exercise in the textbook. Some students answered and some students did not. If the students were quiet, Mr. Bond gave the answer to the students instead. After the set of similar questions above, Mr. Bond went on to explain equivalent by using an example in the students' textbook.

Mr. Bond: "From the textbook, consider $x - 3 = 8$ and $x = 11$."

Mr. Bond: "What is the value of x in $x - 3 = 8$?" (Pause for two seconds)

Mr. Bond: "What is the value of x in $x - 3 = 8$?"

Mr. Bond: "The value of x is equal to $8 + 3$ so x is?"

Some students: "11."

Mr. Bond: "In this case, we said that two equations are equivalent because x has the same value."

Mr. Bond: "Consider $x - 3 = 8$ and $x^2 = 121$."

Mr. Bond: "In this case, what is the value of x in $x - 3 = 8$. It is the same as above, right, 11."

Mr. Bond: "So, what is the value of x in $x^2 = 121$?"

The teachers paused for two second. No student answered so teachers went on.

Mr. Bond: "x equals to square root 121. So x is both positive and negative 11."

Mr. Bond: "Are these two equations equivalent?"

Some students: "No."

Mr. Bond: "If they are equivalent, the value of x in both equations should be the same."

Mr. Bond: "In the second case, are the equations equivalent?

Students: "No."

Mr. Bond: "Right, if they are equivalent, the value of x in both equations should be the same."

At this point, a few students answered Mr. Bond's question. However, Mr. Bond did not ask the students to explain their reasoning. He, instead, stated the reason. The teacher read the definition of equivalent in the students' textbook to the students. Then, the teachers and the students did the exercise about equivalence in the textbook together. Mostly, Mr. Bond wrote equations on the chalkboard and asked students to answer. Mr. Bond did not ask the students to explain their answers. Mr. Bond always taught using the same technique. He did not make use of students' answers nor did he let students' explain their thinking. Students in Mr. Bond's class did not work together or individually. Mr. Bond always read the definitions to the students, but did not clearly explain them. Mr. Bond told the students what to do to check whether their answer was correct by saying, "If you want to check the answer, just put x into the original equation". Mr. Bond reminded the students to check the answer but he did not encourage students to do it.

In general, Mr. Bond used a lecture style in his class and his students were also active in answering Mr. Bond's questions. Most of the time Mr. Bond explained the routine procedure while writing on the chalkboard and students wrote whatever was on the chalkboard into their notebook. Sometimes, he asked his students to answer, but students rarely replied and Mr. Bond did not encourage the students to think or explain. Mr. Bond rarely corrected students' misconceptions or errors.

Mr. Bond's Instruction on Solving Algebra Word Problems

Mr. Bond spent two days in teaching solving algebra word problems (see Table 22). Mr. Bond used problems and examples in the textbook and he led the students to solve algebra word problems. Next, specific information about Mr. Bond's instruction on how to solve algebra word problems is reported.

Table 22

Days Mr. Bond taught Chapter 6

Topic \ Day	1	2	3	4	5	6
Equations	✓					
Solving Equations	✓	✓				
Solving Word problems			✓	✓		
Inequalities					✓	✓

Day 3. Eight minutes after the electronic bell sounded, Mr. Bond entered the classroom. As is a custom in Thai school, the students and the teacher exchanged a bow at the beginning of the class. Mr. Bond began the lesson by writing five steps in solving word problems on the chalkboard.

Mr. Bond: "Today we will begin the lesson on solving word problems."

Mr. Bond: "There are five steps in solving word problems. OK, the first step is that we have to read the problem and then assume the variable for the value that we want to know. Next, we will form an equation for an unknown and other known variables. After that, we will solve the equation. And finally, we will check an answer by using the equation and the answer from the previous step."

Mr. Bond: "Ok, let's look at the problem number one. The problem is that there is a number such that twice the number plus 3 is equal to 21. Find the number."

Mr. Bond: "Carefully read the problem and then try to understand the problem."

At this moment, the students did not pay much attention because they said to the teacher that they did not like solving word problems. Mr. Bond then let the students solved the problem by themselves and then he questioned the students throughout the process. Mr. Bond also wrote on the chalkboard while he was explaining.

Mr. Bond: "What do you assume...?"

Mr. Bond: "What is it?"

Mr. Bond: "So, we let x be that number, OK. What is the equation?"

Students: "$2x$ plus three equals to 21: ($2x + 3 = 21$)."

Mr. Bond: "OK, then solve for x. What is the value of x?"

The students were solving the equation. After they finished, Mr. Bond asked for the answer. After that, Mr. Bond asked the students to do the problem number two. The problem number two was – three times a sum of a number and 7 is 33. Find the number. Mr. Bond read the problem to the students and asked students to find an equation. About five minutes later, Mr. Bond questioned students.

Mr. Bond: "What do you assume?"

Students: "x be the number."

Mr. Bond: "What is the equation?"

Students: "Three, in the parenthesis, x plus seven equal to 33: [$3(x + 7) = 33$]."

Mr. Bond also wrote down what the students said on the chalkboard and as before he asked the students to solve the equation for x. Mr. Bond did not check students' understanding or let them explain how they got that equation. At the beginning Mr. Bond mentioned the checking answer step. However, he did not ask the students to check the answer of the two examples above. Mr. Bond continued by asking the students to solve the problem numbers 3, 4, and 5 from exercise 6.2 in the textbook. Mr. Bond walked around while the students were solving the problems. For the problems 3 and 4, Mr. Bond told the students to write the equations by themselves. What Mr. Bond did was ask for the equation. Then, the students and Mr. Bond solved the equation together on the chalkboard. For the problem 5, many students could not solve it, so Mr. Bond instructed them to solve this problem step by step. Mr. Bond wrote the problem 5 on the chalkboard and then gave the students three minutes to read the problems. After that, Mr. Bond taught the students to analyze each sentence. At the end of the class, Mr. Bond asked the students to do the problem numbers six and seven (exercise 6.2) as homework.

Day 4. As a custom in most Thai schools, Mr. Bond and the students exchanged a bow at the beginning of the class. Mr. Bond began that morning by asking the students about the homework assigned on Day 3.

Mr. Bond: "Could you do the problem number six?"

Mr. Bond: "What is the answer, Nat?

Nat: "36 meters."

Mr. Bond: "How about the problem number seven, what is the answer?"

Students: "16 years old."

Mr. Bond: "OK, on the problem number seven, what did you assume?

Students: "x."

Mr. Bond: "How many brothers did Dang have?"

Mr. Bond read the problem number seven to the students again. The problem number seven was – Dang has two brothers. The first brother is three years older than Dang. The second brother is four years older than Dang. Together, the age of the three people is 43 years old. How old is the first brother?

Mr. Bond: "If the age of Dang is x, what is the age of the first brother?"

Together: "$x + 3$ years old."

Mr. Bond: "The age of the second brother is…?"

Together: "$x + 4$ years old."

Mr. Bond: "So the equation is…?"

Students: "x plus x plus 3 plus x plus 4: $(x + x + 3 + x + 4)$."

Mr. Bond: "And it equals to…?"

Together: "43: $(x + x + 3 + x + 4 = 43)$."

Mr. Bond: "Ok, now let's look at the problem number eight on exercise 6.2"

At this time, the students took out their textbook and made noise while Mr. Bond was writing the problem number eight on the chalkboard. The problem number eight was – in 12 years, the ratio of father and son's ages will be 3:1. Nowadays, if the father is 30 years older than the son, find the age of the father. After writing the problem on the chalkboard, Mr. Bond began to lead the students through the problem solving process.

Mr. Bond: "What do you assume?"

Students: <… silent….>

Since the students did not answer the question, Mr. Bond read the problem to the students again.

Mr. Bond: "What do we assume for the son's age?"

Together: "x years old."

Mr. Bond: "What is the age of the father?"

Mr. Bond: "What is the age of the father?" Mr. Bond repeated.

Students: "$x + 30$ years old."

Mr. Bond: "In 12 years, how old is the father?"

Mr. Bond: "Ah, the father should be $x + 30 + 12$ years old, which is $x + 42$ years old."

Mr. Bond: "How about the son?"

Mr. Bond: "Well, how about the son, anyone?"

Students: (speak softly) "$x + 12$ years old."

Mr. Bond: "x plus what?"

Students: "$x + 12$ years old."

Mr. Bond: "OK, what is the equation of this problem?"

Students: <...silent...>

Mr. Bond: "The proportion of father and son is $x + 42$ over $x + 12$ equal to?

Students: "Three over one (3/1)"

Mr. Bond, then, wrote down the equation for this problem on the chalkboard: $(x + 42)/(x + 12) = 3/1$.

Mr. Bond: "So, what is next?"

Students: "Cross multiply."

After this, Mr. Bond showed how to solve the equation for x. Mr. Bond did not say anything about checking the answer for this problem. Next, Mr. Bond presented another example. Mr. Bond read the problem to the students and also wrote the problem on the chalkboard and he let the students solved the problem.

Mr. Bond: "At 9:00 the first man began walking at the rate of 5 kilometer per hour. Two hours later, at the same starting point, the second man began walking at the rate of 10 kilometer per hour. At what time did these two people meet?

Mr. Bond paused for two seconds and waited for students to answer. When no students answered, Mr. Bond guided students as follows:

Mr. Bond: "The first person began to walk at 9:00 so the second person started waking at what time?"

Students: "11 o'clock."

Mr. Bond: "And what time did these two people meet, you all?"

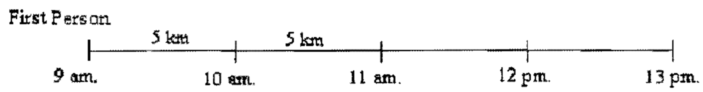
Students: <… silent….>

Mr. Bond: "We have to assume it to be x. OK, assume x."

It was not clear what Mr. Bond assumed x for. At this point, Mr. Bond drew a picture of two lines on the chalkboard comparing the time and distance of the two people. Mr. Bond explained along while he was drawing the picture.

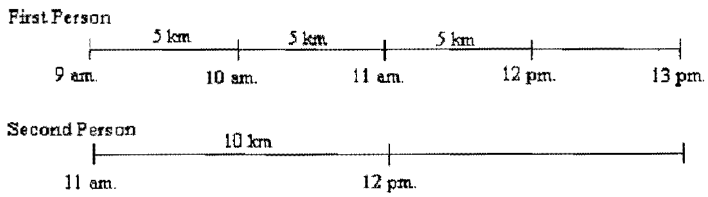
Mr. Bond: "This is the first person, OK, how far did this person go. Five kilometer."

Mr. Bond: "At 10:00, the first person went five kilometer. One hour from 10:00 he went another five kilometer, and what time is it? It's 11:00."



Mr. Bond: "At 11:00, the second person also began to walk to here… to 12:00, what distance did he go?"

Students: "Ten."



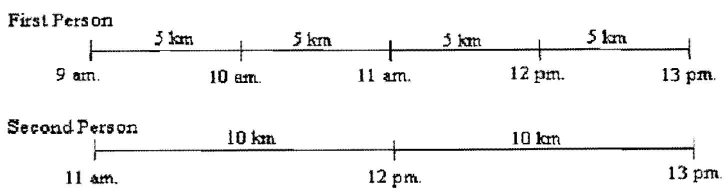
Mr. Bond: "While the second person was walking, is the first person still walking?"

Together: "Yes, he is."

Mr. Bond: "So, the first person walked another hour so he went…?"

Students: "Another five kilometer."

Mr. Bond: "So, when will they meet? (Mr. Bond pointed on the picture) Right here at 13:00, what is the distance? Five kilometer."



Mr. Bond: "They meet at 13:00 so what should the equation be?"

Students: <...silent...>

Mr. Bond: "The equation should be $5x = 10x - 10$. Solving the equation, we got $5x - 10x = -10$. Then $-5x = -10$. So that x equals to two ($x = 2$)."

Mr. Bond: "What is the number 2 here?"

Jimmy: "Two hours."

Mr. Bond: "We will add these two hours to 11:00. Therefore, the two people will meet at 13:00."

It was not clear how Mr. Bond got the equation for this problem because he first drew the picture. After that, he formed the equation without telling the students how he got it. After this problem, Mr. Bond walked around the room to see whether the students all wrote down details in solving this problem. He also explained to some students who still did not understand this problem. Next, Mr. Bond presented the last problem of today's lesson. The problem was – two sisters picked 252 oranges altogether. The older sister picked nine oranges per pile and the younger sister picked six oranges per pile. There are 34 piles altogether. Find the number of oranges that each of them picked. Mr. Bond read the problem to the students and showed the students how to solve this problem on the chalkboard.

Mr. Bond: "You have to pick an important point in the problem. If two sisters picked 252 oranges, you can assume the variable either for the older or the younger"

Nat: "Let's assume x for the older sister."

Mr. Bond: "The older sister picked x oranges. So, the younger sister picked...?"

Students: "$252 - x$ oranges."

Mr. Bond: "The older sister put nine oranges into each pile, so how many piles does she have?"

Students: "x over nine ($x/9$)."

Mr. Bond: "And how many piles does the younger sister have...?"

Mr. Bond: "($252 - x$) over $6(252 - x)$. And altogether it will be."

Students: "$x/9 + [(252 - x)/6] = 34$."

Mr. Bond: "And what do we get?"

At this moment, Mr. Bond and some students at the front row solved the equation together. No explanation was provided to the whole class except what Mr. Bond was writing on the chalkboard.

Mr. Bond: "So x will be…?"

Students: "144."

Mr. Bond: "And this is for the older or the younger sister"

Students: "The older one."

Mr. Bond: "So the older sister picked 144 oranges so the younger sister picked…? So we subtracted 144 from 252 and it is 108 oranges.

Mr. Bond: "Is it 252 oranges altogether?"

Students: "Yes."

After this, Mr. Bond walked around the room to check whether the students copied the work on the chalkboard and answered students' question. No homework was assigned for today.

Summary

Mr. Bond taught word problem solving by first explaining five steps in solving word problems to the students. After that Mr. Bond asked students to solve word problems by using the five steps as he explained. The examples that Mr. Bond used were from the students' textbook. Like Mr. Jack, Mr. Bond mostly directed students to solve the problem by doing the explanation at the chalkboard. However, the students in Mr. Bond's class interacted with the teacher much more than the students in Mr. Jack's class. Mr. Bond did not always check students' understanding. Like Mr. Jack, Mr. Bond did not always encourage students to make sense of what they were doing. In addition, Mr. Bond did not make use of students' answers when the students gave them. Mr. Bond sometimes reminded students to check the answer, but it was just verbally. In fact, there was not much checking of answers in his teaching.

Conclusion

In conclusion, this section reports teaching styles of three mathematics teachers who participated in this study. The three teachers were Mr. Jack, Ms. Rose, and Mr. Bond. The results from this section indicated that the three teachers had different styles in teaching students to solve word problems. Two teachers, Mr. Jack and Mr. Bond, directed the whole lesson in a lecture style. However, students in Mr. Bond's class interacted much more with the teacher than the students in Mr. Jack's class. Unlike these two teachers, Ms. Rose took a more facilitator role in some lessons and allowed her students to actively participate in the teaching and learning. There were many interactions between the teacher and the students in this class and among student peers. However, the results from this study showed that the three teachers rarely encouraged the students to check their answers or to check the processes in solving word problems. In addition, the three teachers in this study did not encourage students to use a variety of strategies to solve algebra word problems. They only taught student to use variables and symbols to represent the situation in the problem.

The next section will explore potential connections between teacher's instruction and students' performance in solving algebra word problems.

Section IV: Potential Connections between Teacher's Instruction and Students' Performance in Solving Algebra Word Problems

This section provides potential connections between teacher's instruction and students' performance in solving algebra word problems. The explanation will be based on the instruction of three participating mathematics teachers and their students in this study. The three mathematics teachers are Mr. Jack, Ms. Rose, and Mr. Bond. Since the students in each of the three teachers had different abilities, the explanation in this section will be reported by each of the three classes.

Mr. Jack's Class

Mr. Jack's teaching was classified as directed instruction. The results from the observation show that Mr. Jack taught students to solve word problems by having the students solve word problems from the students' textbook and the teacher giving an explanation. Mr. Jack mostly directed the students to solve word problems by providing an explanation on the chalkboard. The students participated in class by copying what was on the chalkboard into their notebook. Mr. Jack did not provide opportunities nor did he encourage the students to think or to make sense of what they were learning. Instead, he gave explanations to the students without checking for understanding. In addition, when the students made mistakes during the instruction, Mr. Jack did not correct them right away. Fewer interactions between the teacher and the students in this class took place than in the other two classes. Mr. Jack always used the students' textbook as a source of teaching. No activities, worksheets, or outside problems were evident.

Students in Mr. Jack's class had low achievement in mathematics. The results from previous sections indicate that the students in this class did not attempt to solve algebra word problems and were unsuccessful at solving algebra word problems before instruction (see Table 23). In Table 23, none of the students used an algebraic strategy to solve algebra word problems on the pretest (before instruction). The results in Table 23 indicate that after instruction, the majority of the students in this class still did not attempt to solve algebra word problems and were unsuccessful at solving the five word problems on the posttest.

Some students, but very few, were using more algebra to find the solutions to algebra word problems on the posttest.

Table 23

Strategies students in Mr. Jack's class used to solve the five algebra word problems (N = 38)

Strategy/Problem		Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Arithmetic Strategy	No Error	10	4	-	-	-	-	-	-	-	-
	Error	3	4	-	-	-	-	-	-	-	-
Algebraic Strategy	No Error	-	4	-	-	-	2	-	5	-	5
	Error	-	1	-	-	-	1	-	-	-	1
Comparison Strategy	No Error	-	-	1	3	-	-	-	-	-	-
	Error	-	-	-	2	-	-	-	-	-	-
Part-and-Whole Strategy	No Error	-	-	-	-	-	-	-	1	-	-
	Error	-	-	-	-	-	-	1	-	-	-
Paper Blank		25	19	31	23	34	29	32	24	37	32
Do something with the number		-	6	5	7	4	6	5	7	1	-
Only the answer is given		-	-	1	3	-	-	-	1	-	-

Problems 7, 8, and 10 were adapted from the student's textbook. The results from the observations indicate that Mr. Jack used these three problems as examples in his instruction of solving word problems. However, the results in Table 23 show that the majority of the students in Mr. Jack's class were still unsuccessful at solving these three problems after instruction. For Problem 7, the results in Table 23 show that the students still used a comparison strategy to solve this problem rather than using an algebraic strategy that Mr. Jack taught in class. The results in Table 23 also show that the students had errors in calculation or copying errors. The results from the observations indicate that Mr. Jack rarely emphasized the importance of checking the process of solving equations or checking the accuracy of the answer with the situations in the problems. This might be one reason for such errors.

In addition, the results from the interview sessions (see Table 18) indicate that students in Mr. Jack's class were unsuccessful at solving five algebra word problems (except Problem 4 where five of the six students solved the problem). During the interview sessions, the results in Table 18 indicate only two interviewed students (June and Wilma) from Mr. Jack's class used an algebraic strategy to solve the problem. June used an algebraic strategy to solve Problems 1 and 2, and Wilma used an algebraic strategy to solve Problem 3.

In summary, Mr. Jack's teaching emphasized practicing procedures and the students learned to copy what was on the chalkboard. Mr. Jack mentioned that students in his class were low achieving, so no matter how much he taught the topic did not make much sense to his students. Therefore, he did not do much to help his students to understand more about solving word problems. Thus, it is possible that Mr. Jack's beliefs about students' learning affected his instructional decisions. The results indicate that the performance of students in Mr. Jack's class at solving algebra word problems improved little when compared to the pretest. One might predict that since the students had low ability in mathematics, therefore, their performance was still poor. However, it is also possible that Mr. Jack's instruction and his beliefs about students' learning had affected students' performance to algebra word problems.

Ms. Rose's Class

Ms. Rose used an active teaching strategy including question and answers to initiate student's thinking and ability to make sense of the mathematics. Both Ms. Rose and the students were actively involved in the teaching and learning process. Ms. Rose encouraged the students to think and to reason through the problem solving process. There were interactions between the teacher and the students in this class, and also among student peers. Ms. Rose emphasized on both practicing procedures and underlying rationale. Ms. Rose did not use the students' textbook as a source of teaching. Instead, Ms. Rose used an outside textbook, activities, and worksheets in her class.

Ms Rose's class consisted of medium achievers. The results from previous sections indicate that the students in this class were unsuccessful at solving algebra word problems on the pretest but many were successful on the posttest (see Table 24). The results in Table 24 show that the students in Ms. Rose' class did not attempt to solve the problem on the pretest and did not often use an algebraic strategy to solve the five algebra word problems before they engaged in the formal instruction. After instruction, the results on the posttest (see Table 24) show that the students attempted more problems and used more algebraic strategies to solve algebra word problems. As mentioned earlier that Problems 7, 8, and 10 were adapted from the student's textbook. The results from the observations indicate that Ms. Rose did not use these three problems as examples in her instruction of solving word problems.

Table 24

Strategies students in Ms. Rose's class used to solve the five algebra word problems (N = 42)

Strategy/Problem		**Problem 6**		**Problem 7**		**Problem 8**		**Problem 9**		**Problem 10**	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Arithmetic Strategy	No Error	24	3	-	-	-	-	-	-	-	-
	Error	1	1	-	-	-	-	-	-	-	-
Algebraic Strategy	No Error	3	33	-	14	-	19	-	28	-	12
	Error	-	5	-	6	-	15	-	10	-	11
Comparison Strategy	No Error	-	-	-	5	-	-	-	-	-	-
	Error	-	-	2	3	-	-	-	-	-	-
Part-and-Whole Strategy	No Error	-	-	-	-	-	-	3	1	-	-
	Error	-	-	-	-	-	-	-	-	-	-
Trial and Error		-	-	-	-	-	-	-	-	-	1
Paper Blank		14	-	34	11	42	7	39	2	42	15
Do something with the number		-	-	4	-	-	-	-	-	-	-
Only the answer is given		-	-	2	3	-	1	-	1	-	3

However, the results in Table 24 show that the majority of students in Ms. Rose's class were successful at solving these three problems after instruction. These results suggest that it is possible that Ms. Rose's instruction might be a variable affecting student's choice of solution strategies and student's performance. In addition, the results in Table 24 indicate that even though the students used an algebraic strategy, they still had errors in calculation or copied the wrong number from the problem. From the observation, the checking process happened occasionally in Ms. Rose's class. Therefore, if Ms. Rose emphasized the checking process more, errors the students made might possibly decrease.

Furthermore, the results from the interview sessions (see Table 18) indicate that the students in Ms. Rose's class were successful at solving Problems 1, 2, and 4 but were unsuccessful at solving Problems 3 and 5. During the interview sessions, the results in Table 18 indicate all six students used an algebraic strategy to solve two or more problems given during the interview.

In summary, Ms. Rose used questions to initiate students' thinking during her instruction. Ms. Rose believed that students should understand concepts and procedures in order to do well in mathematics. So, most of her teaching focused on developing students' concepts and procedures. For solving word problems, Ms. Rose emphasized the procedure of translating the situation in the problem into equations. In addition, her students did some thinking throughout the teaching and learning. Thus, it is possible that Ms. Rose's beliefs about students' learning affected her instructional decisions. Regardless of any errors the students made, the results in Table 24 show that students' performance at solving algebra word problems improved much after instruction. One might expect that since the students had medium ability in mathematics, therefore, their performance would be much improved. However, Ms. Rose's instruction and her beliefs about students' learning might also account for such improvement.

Mr. Bond's Class

Like Mr. Jack, Mr. Bond mostly directed students to solve the problem by doing explanations at the chalkboard. The results from the observation show that Mr. Bond taught word problem solving by explaining five steps in solving word problems to the students. After that Mr. Bond asked the students to solve word problems by using the five steps he explained. The examples that Mr. Bond used were from the students' textbook. Mr. Bond did not always check students' understanding and did not encourage students to think about the mathematics they were learning. In addition, Mr. Bond did not make use of students' answers in his instruction. Mr. Bond always used the students' textbook as a source of teaching. No activities, worksheets, or outside problems were used in Mr. Bond's class.

Mr. Bond's class consisted of high achieving students. The results from previous sections indicate that students in this class were unsuccessful at solving algebra word problems on the pretest (except Problem 6) but many were successful at solving algebra word problems on the posttest (see Table 25). The results in Table 25 show that on the posttest, students in Mr. Bond's class used more algebraic strategies to solve algebra word problems than on the pretest.

Table 25

Strategies students in Mr. Bond's class used to solve the five algebra word problems (N = 38)

Strategy/Problem		Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Arithmetic Strategy	No Error	22	13	-	-	-	-	-	-	-	-
	Error	-	1	-	-	-	-	-	-	-	-
Algebraic Strategy	No Error	-	24	-	7	-	16	5	21	1	17
	Error	14	-	-	2	1	2	1	3	5	4
Comparison Strategy	No Error	-	-	18	14	2	-	-	-	-	-
	Error	-	-	1	-	-	-	-	-	-	-
Part-and-Whole Strategy	No Error	-	-	-	-	-	-	6	2	-	-
Paper Blank		2	-	16	9	33	15	24	12	26	15
Only the answer is given		-	-	3	3	2	5	2	-	6	2

As mentioned earlier that Problems 7, 8, and 10 were adapted from the student's textbook. The results from the observations indicate that Mr. Bond used these three problems as examples in his instruction of solving word problems. The results in Table 25 show that the majority of students in Mr. Bond's class were successful at solving these three problems after instruction. However, for Problem 7, the results in Table 25 show that the students still used a comparison strategy to solve this problem rather than using an algebraic strategy that Mr. Bond taught in class. The results suggest that it is possible that Mr. Bond's instruction might affect student's solution strategies.

Furthermore, the results in Table 25 show that even though students used an algebraic strategy to solve algebra word problems, they still had errors in calculation. From the observation, Mr. Bond sometimes reminded students to check their answers but just verbally. In fact, checking the answer was not emphasized much in his teaching. Thus, if Mr. Bond emphasized the checking process, errors the students mad might possibly decrease.

In addition, the results from the interview session (see Table 18) indicate that students in Mr. Jack's class were successful at solving almost every problem giving during the interview. During the interview sessions, the results in Table 18 indicate all six students used an algebraic strategy to solve almost all word problems given during the interview.

In summary, like Mr. Jack, Mr. Bond's teaching emphasized practicing procedures and students learn to copy what was on the chalkboard. Mr. Bond mentioned that he did not emphasize this topic because the students had already studied it in the eighth grade. Also, Mr. Bond indicated that teaching students to solve word problems took time. Sometimes, the students had their own thinking and their own solution strategies, and that is why he did not emphasize solution of word problems and did not worry much about his students. Thus, it is possible that Mr. Bond's beliefs about students' learning affected his instructional decision. Regardless of any errors the students had, the results in Table 25 show that the students' performance at solving algebra word problems improved much after instruction. However, their performance improved little as one would expect for students who were categorized as high achievers. One might expect that since the students had high ability in mathematics, therefore, their performance would be much improved. However, Mr. Bond's instruction and his beliefs about students' learning might also account for such improvement. It is possible that Mr. Bond's instruction and his beliefs about students' learning might connect to student's performance in solving word problems.

Summary and Discussion

Two similar potential connections between teachers' instruction and students' performance from the three teachers participating in this study follow. First, teachers' instruction might affect students' choice of solution strategies. For example, many students' choice of solution strategies had changed from non-algebraic base to algebraic base. Even though some non-algebraic base strategies were still used by some students, they were used less than on the pretest. In addition, the results from the observation show that the three teachers in this study did not encourage students to use a variety of strategies to solve algebra word problems. The teachers encouraged students to use only variables and symbols to solve the problems. Thus, the results from this study show that few students used other strategies such as graphs or tables to solve algebra word problems.

Second, three teachers participating in this study rarely emphasized the importance of checking answers. As a result, the students made errors in the process of solving word problems. Of the three teachers, Ms. Rose emphasized the importance of checking answers on solving equations the most. However, she did not emphasize the importance of checking answers against the situation in word problems.

Besides the two possibilities stated above another alternative explanation such as teachers' beliefs about students' learning and their decision making about instruction are also possible. Mr. Jack believed that students in his class were low achieving, so no matter how much he taught, it did not make much sense to his students. Therefore, he did not do much to help his students to understand more in solving word problems. He only demonstrated for his students how to solve word problems, but his students were not encouraged to develop thinking skills in the class. This, as a result, might affect his students' performance in solving algebra word problems. Ms. Rose believed that students should understand concepts and procedures in order to do well in mathematics. So, most of her teaching focused on developing students' concepts and procedures. For solving word problems, Ms. Rose emphasized the procedure of translating the situation in the problem into equations. In addition, her students did some thinking throughout the teaching and learning. This, as a result, might affect her students' performance in solving algebra word problems. Mr. Bond believed that students in his class were high achieving and the students had already learned solving algebra word problems from the eighth grade, so he did not emphasize the solution of solving word problems much and was not worried much about his students. In addition, Mr. Bond did not ask students to solve the difficult problem. This, as a result, might affect his students' performance in solving algebra word problems especially the difficult problems. However, more research on teachers' beliefs about students' learning should be done in Thailand.

Table 26

Summary of teaching and learning situations in each of the three classes

	Mr. Jack (Low Achievers)	**Ms. Rose (Medium Achievers)**	**Mr. Bond (High Achievers)**
Nature of Content	• Emphasis on practicing procedures	• Emphasis on practicing procedures • Emphasis on underlying rationale	• Emphasis of learning terms and practicing procedures.
Who does the work?	• Teacher	• Both teacher and students	• Teacher. The students did the work on some occasions.
Source of Instruction	• Students' textbook	• Outside Workbook • Worksheets	• Students' textbook
Activities	• None	• Some activities • Group Work	• None
Teaching Style	• Directed-instruction and explanation	• Mostly questions to initiate students' thinking	• Directed-instruction and explanation
Student's participations	• Rarely participated.	• Always participated.	• Occasionally participated.
Encouraging students to think	• Barely	• Often	• Barely
Check student's understanding	• Barely	• Often	• Barely
Using examples similar to problems on the test (Problems 7, 8 and 10)	• Yes	• No	• Yes
Student's performance after instruction	• Not improved when compared to the pretest	• Much improved when compared to the pretest.	• Somewhat improved when compared to the pretest

Furthermore, the results in Table 26 show some potential connections between teaching styles and student's performance. The results in Table 26 indicate that Mr. Jack and Mr. Bond used the same teaching style, which was a directed-instruction. These two teachers did most of the work and students rarely participated actively. Both Mr. Jack and Mr. Bond emphasized practicing procedures without underlying rationale. In addition, both teachers rarely used questions to initiate student's thinking. The results show that student's performance in these two classes improved little much when compared to the pretest, even though these two teachers showed examples similar to problems on the posttest during instruction. Unlike Mr. Jack and Mr. Bond, Ms. Rose often used question and answer techniques to initiate students' thinking and the students in her class often participated in the learning processes. In addition, Ms. Rose emphasized both practicing procedures and underlying rationale. The results show that student's performance in this class was much improved when compared to their performance before instruction even though Ms. Rose did not show examples similar to problems on the posttest.

In summary, the results from this study show some potential connections between teacher's instruction and students' performance in solving algebra word problems. Not only teachers' instruction at school might affect students' performance, but also instruction at a tutoring center might affect students' performance as well. As a result, going to the tutoring center might be a factor affecting students' performance and student's solution strategies in solving algebra word problems as well. Other factors affecting students' performance might be individual student's ability, perceptions, and beliefs and thought about themselves and mathematics.

This chapter has provided the results on students' success at solving algebra word problems, and students' solution strategies to algebra word problems. Also, this chapter also had identified potential connections between teacher's instruction and students' performance in solving algebra word problems. In the next chapter, CHAPTER V, summary and discussion, implications for learning and instruction, recommendations for future research, and limitations of this study will be discussed.

CHAPTER V
DISCUSSION AND CONCLUSION

Introduction

The primary focus of this study was to investigate success and difficulties of Thai ninth grade students in solving algebra word problems as well as their solution strategies used for solving algebra word problems. This study examined the following questions: (1) How successful are Thai students in solving algebra word problems? (2) Which strategies are used by Thai students to solve algebra word problems? (3) What are characteristics of classroom instruction during word problem-solving lessons? (4) What are potential connections between classroom instruction and students' word problem-solving performance? This section contains a summary and discussion of the main findings and how the findings compare with other research. Finally, this chapter presents implications for learning and instruction, recommendations for future research, and limitations of the study.

Summary and Discussion of the Main Findings

The summary and discussion are organized to answer the four research questions stated above. The first section summarizes success of Thai ninth grade students in solving algebra word problems including their difficulties and errors. The second section presents a summary of strategies Thai students used to solve algebra word problems. The third section summarizes characteristics of classroom intruction of three teachers who participated in this study. The last section summarizes potential connections between teachers' instruction and students' performance in solving algebra word problems.

Success of Thai Students in Solving Algebra Word Problems

Since students' success and difficulties in solving algebra word problems depend on their ability to solve equations and translations, their solutions to equations and translations of mathematical situations to equations will be reported. In general, the results from this study show that some Thai ninth grade students in this study were able to solve equations as indicated in Thai Mathematical Standards (IPTST, 2000) and in Principal and Standards for School Mathematics (NCTM, 2000). The NCTM Standards 2000 also indicated that students at this grade level should be able to understand equality concepts and distributive properties. However, the results from this study show that some of Thai ninth grade students still had difficulty in using distributive property and did not understand the concept of equality as indicated in the NCTM Standards 2000. The results from the translating problems show that Thai ninth grade students were able to use mathematical language and symbols to represent a problem situation as indicated in both Thai Mathematical Standards (IPTST, 2000) and in NCTM Standards 2000. However, many students in this study were unsuccessful at being able to use mathematical language and symbols to represent a problem situation, which required representing more than one unknown variable. Next, the success of Thai students in solving algebra word problems is discussed.

The results from this study show that Thai ninth grade students were able to solve some word problems and use their knowledge of mathematics in solving word problems as indicated in Thai Mathematical Standards (IPTST, 2000) and in NCTM Standards 2000. However, they were less successful at solving a variety of problems because the majority of the students were unable to solve a problem involving more than one unknown variable. As indicated in the NCTM Standards 2000, students at this grade level should be able to determine the reasonableness of the answer to the problem. However, the results from this study show that few Thai ninth grade students checked their answers or solution processes in order to determine the reasonableness of the answer to the problem.

Before instruction, the students in this study used strategies that were more informal and arithmetic based, such as a comparison strategy or a part-and-whole strategy to solve algebra word problems. After instruction, the students who were successful used more algebraic knowledge to solve the algebra word problems. However, some informal strategies (e.g., arithmetic strategies) were still used by some of both successful and unsuccessful students. The results from the pretest, the posttest, and the interview session show that the low achieving students were unsuccessful at solving algebra word problems even though they had gone through formal instruction. In contrast, the medium and high achieving students were successful at solving some of the algebra word problems. Next, a discussion about the factors that contributed to Thai students' success and failures in solving algebra word problems, and students' difficulties and errors in solving algebra word problems will be reported.

Solution Processes in Solving Word Problems

Mayer (1985, 1987) suggested that four types of processes are required to solve mathematics word problems: translation, integration, planning and monitoring, and solution execution. Translation involves converting each sentence in the problem into an internal mental representation. Integration involves combining the information into a coherent structure. Planning and monitoring involves developing and keeping track of a plan to solve the problem. Solution execution involves carrying out the plan by using procedural knowledge to apply the rules of arithmetic and algebra accurately and efficiently while carrying out the calculations. Factors that might contribute to these Thai students' success and failure in solving algebra word problem are discussed below using Mayer's model.

Translation. In the first step, students must employ reading and language comprehension to translate each statement of the problem into an internal mental representation. The language used in word problems can make easy problems difficult (Bruning, Schraw, & Ronning, 1995). This indicates that the structure of word problems might contribute to students' success and failures in solving mathematics word problems. For example, the results from this study indicate that a majority of students could not solve Problems 7, 8, 9, and 10 on the posttest. Please refer to Appendix A for more detials on Problems 7, 8, 9, and 10. As mentioned in CHAPTER IV, we have conjectured that the structure of word problems such as the unknown variable presented in the problem might effect students' performance in solving word problems. For example, the students in this study were unsuccessful at solving Problems 7, 8, 9, and 10 because the problems involved more than one unknown variable. The students in this study were more successful at solving Problem 6 on the posttest because Problem 6 contained one unknown variable and required little interpretation in writing the equation. However, the results from the interview indicate that even though Problems 2 and 4 contianed more than one variable, the majority of interview students were successful at solving them. Therefore, it is possible that students' mathematical ability and their thinking might account for their success at solving word problems.

The results from this study were similar to other empirical studies which indicated that students had great difficulty in translating problem statements into equations (Clement, et al., 1981; Lochhead & Mestre, 1988). Although translating word problems were difficult, the results from the interview session show that some students were successful at translating the problems. One possible explanation for why some students were successful at translating word problems and some were unsuccessful is that successful word problem solvers were able to understand the situation in the problems and were able to translate a problem from its linguistic representation into equations. This explanation is supported by the results during the interview sessions from this study. The results show that most high, medium, and low achieving students successfully solved Problem 1 given during the interview sessions (see Appendix B). The results indicate that all high and medium achieving students applied algebraic knowledge to solve this problem. In contrast, low achieving students applied arithmetic knowledge to solve this problem. This example still did not show much difference between successful and unsuccessful word problem solvers because most of the students could solve this problem.

The difference is clearly presented when students were asked to solve interview Problems 2, 3, and 5 (see Appendix B). For these three problems, the results in Table 18 show that low achieving students were unsuccessful at solving them. It is possible that the low achieving students could solve Problem 1 because they still were able to apply arithmetic knowledge to solve the problem. However, solving the later three problems could no longer rely on the arithmetic approaches used in solving Problem 1. Since the interview Problems 2, 3, and 5 could rarely be accomplished by carrying out some sequence of arithmetic operations on the given numbers, the majority of low achieving students were unable to solve Problems 2, 3 and 5. In contrast, the results from Table 18 show that high achieving students were successful at solving interviewed Problem 1 as well as Problems 2, 3, and 5. Regardless of any errors, the high achieving students were able to apply their algebraic knowledge to translate the problem situation by using variables and symbols, and then solved the problems. This suggests that unsuccessful problem solvers had difficulties in translating the situation described in the problem into symbolic representations. Therefore, they were more likely to focus on computing a quantitative answer to a problem. Unlike unsuccessful problem solvers, successful problem solvers were likely to apply algebraic knowledge to translate the situation described in the problem by using variables and symbols. In short, the ability to translate linguistic problems into equations is one possible factor that contributes to students' success and failure at this step. However, even if successful problem solvers were able to translate the problem, they often failed to represent the problem correctly and they were as likely to perform correct calculations on incorrect representations. The ability to represent problems is discussed next.

Integration. In this step, students would combine problem statements into a coherent representation. In order to integrate the information in problems, students need the schematic knowledge to recognize problem types. Researchers found that successful word problem solvers possess more useful schemata than unsuccessful word problem solvers (e.g., Chi & Bassock, 1991; Chi, Feltovich, & Glaser, 1981; de Jong & Ferguson-Hessler, 1991; Larkin, 1983; Silver, 1981, 1987). Schematic knowledge helps guide students' attention so they can distinguish relevant from irrelevant information. For example, the results from this study show that one of the successful students could distinguish what the question asked from other information given in the problem. In one problem in this study, the question asked for the number of girls. The problem also provided that the number of girls is 2/3 the number of boys. The student defined x as the number of girls and she then defined the number of boys as 45 - x (which is the total number of students minus the number of girls). Then, she went back to read the problem again and combined the information she had to form the equation as $x = 2(45 - x)/3$ which correctly represented the problem situation. On the other hand, one of the unsuccessful students could not distinguish what the question asked from other information given in the problem. She defined x as the number of girls. However, when she went back to retrieve the information given in the problem, she became confused because the problem also said that the number of girls is 2/3 of the number of boys. Therefore, she struggled to solve this problem because she could not distinguish the variable she had defined from the information given in the problem. From this example, it could be concluded that the ability to integrate the information in the problem into a coherent representation is also involved in students' success and failure when solving word problems.

Planning and monitoring. In this step, problem solving requires knowledge of strategies (strategic knowledge) that focus on how to solve problems. The plan involves breaking the problem into sub-problems and establishing a sequence for the solution (e.g., what operation algorithm should be employed first, second, etc.). Student's success and failure in solving a word problem often depends on the planning and monitoring in the problem solution (Mayer, 1982; Mayer, Larkin, & Kadane, 1988). The results from this present study also supported this claim. In solving one of the problems from this study, one girl used an equation based on a comprehensive representation but she could not finish solving the problem. She already had determined the correct equation but she could not solve for the variable x. This example indicates that this student could not establish a sequence for her solution. Another example is taken from another girl. To solve the interview Problem 3 (see Appendix B), she used an algebraic strategy to attempt to solve this problem. However, she misrepresented the situation for Robinson. This example shows that if this student would monitor and keep track of her plan to solve the problem, she might not get an incorrect answer. From these two examples, it could be concluded that the ability to plan and monitor the sequence of solution are also involved in students' success and failure when solving word problems.

Solution execution. In this step, problem solving requires students to use procedural knowledge to apply the rules of arithmetic and algebra accurately and efficiently. Computational skill is used in this step. Knifong and Holtan (1976; 1977) suggested that improving computational skills could have eliminated nearly half of the word problem errors. For example in this study, in solving the interview Problem 3, one student defined x as Jonathan's age nowadays. However, she had an error in her calculation and got an incorrect answer. To calculate $2(x - 6)$, she got $2x - 16$ instead of $2x - 12$.

Therefore, she ended up with an incorrect answer. This example indicates that the student had an error in her calculation and thus she got an incorrect answer. If this student would check her sequence of calculation, she possibly would not get an incorrect answer for this problem. The conclusion was that solution execution might also be involved in students' success and failure in solving word problems. However, if students would check the sequence of their computations, they could have eliminated many word problem errors.

Summary. From the above discussion, two main important factors associated with students' successes and failures in solving word problems emerged. Those two factors were the structure of the problem and the differences in thinking and solution's processes between successful and unsuccessful word problem solvers. Differences in solution processes between successful and unsuccessful word problem solvers are discussed next. Based on Mayer's four steps in solving problems and the results from this study, it could be concluded that successful word problem solvers were able to translate the problem and combine all the information they had into an equation.

Unlike successful word problem solvers, unsuccessful word problem solvers struggled in the translation and integration processes during the solution of word problems. Thus, they were more likely to focus on computing using numbers given in the problem. Unsuccessful students were sometimes able to solve algebra word problems because the problems could be solved by using arithmetic knowledge. However, they were unable to solve other problems because using their arithmetic knowledge could not easily solve those problems.

The results above suggest that unsuccessful word problem solvers had difficulty understanding how to apply algebra to solve word problems. In other words, the unsuccessful students in this study might have difficulty making transition from arithmetic to algebra. This argument is supported by Tall (1989), who suggested that in arithmetic, the goal in solving word problems is to find the answer, and this goal is usually accomplished by carrying out some sequence of arithmetic operations on the given numbers in the problem. In contrast to arithmetic, this approach is often inapplicable in algebra word problem solving. Adjustment to be made in learning algebra is to deal with the structure of algebra, in particular, the symbolic representation of numerical relationships, which concerns the translations of problem situations into equations. Algebraic equations are structural representations that involve a non-arithmetic perspective on both the use of the equal sign and the nature of the operations that are depicted. As a result, students who had difficulty making adjustment in learning algebra could have difficulty in solving algebra word problems.

The above discussion suggests that successful and unsuccessful algebra word problem solvers differed in their translation and integration processes. In addition, the ability to plan, monitor, and execute algebraic solutions might also affect students' ability to solve word problems successfully. This suggests that if students would monitor their solution plans and check the sequence of their computations, it could have eliminated many word problem errors.

In summary, unsuccessful students differed from successful students in the quality of problem translation and representation strategies. This finding is particularly important given the critical role that problem translation and problem representation play in mathematical word problem solving. This finding was also similar to previous studies (e.g., Janvier, 1987; Montague et al., 1991). It may be that unsuccessful students could read and compute when solving word problems, but they lack information necessary for representing problems algebraically and developing well-organized and logical solution paths. The results of this present study suggest the main difference between unsuccessful problems and successful problem solvers may be the ability to translate and represent problems.

Students' Difficulties and Errors

The results from this study show that Thai ninth grade students had great difficulty in translating from situations into equations. Many studies carried out in the United States also showed that translating from the language in algebra word problems to an algebraic equation is particularly difficult. These studies were carried out concerning the sources of translation errors and the different schemas students' use to translate problem-solving steps. (Clement, 1982; Clement et al., 1981; Makanong, 1993; Lochhead & Mestre, 1988; Mestre et al., 1982, Uthairat & Viamoraphun, 1984). These studies and the current study showed the robust nature of errors when translating word problems into algebraic equations.

Furthermore, the students in the current study, particularly the low achieving students, had difficulty in solving equation problems. The reason was that the students did not understand the distributive property and the concept of equality. In addition, the misconception leading to an error that was often found with the Thai ninth graders was operations with polynomials and operations with fractions. For example, students added $2x + 1 = 3x$, $3x + 6= 9x$, or $x + 8 = 8x$. This error happened because the students could not distinguish between numbers that are and are not coefficients (Ashlock, 2002). Another error found was $x^2 + x = x^3$. This happened because students could not differentiate between symbol representing multiplication and exponents. Other errors such as copying errors, wrong computation, or changing signs of the numbers when applying the equality properties were also seen in student's work. However, these latter errors would not be as likely to occur if students learned to check and monitor their processes of solving problems.

Strategies Thai Students' Used

This second section presents a discussion of strategies Thai ninth grade students used to solve algebra word problems. The results show that Thai ninth grade students used many similar solution strategies as addressed in Chapter II of this study. In the present study, students used both algebraic and non-algebraic strategies to solve algebra word problems.

Algebraic Strategies

Algebraic strategies are the most formal strategies employed by students in this study. The situation in the problem was translated to algebraic assignments of variables and symbols. Next, the students formed an equation by using those variables and symbols. The equation then was solved to find the answer to the problem. The results from this study were similar to previous studies conducted by Bull (1982), Hall et al. (1989), Kieran (1988), Koedinger and Nathan (1998), and Koedinger and Tabachneck (1994). Those studies showed that the most formal strategy students used to solve algebra word problems were an algebraic strategy. For example, Koedinger and Tabachneck (1994) suggested that in an algebraic strategy, the verbal problem statement was translated to algebraic assignments and equations. The equations were transformed to find a solution (solve for the unknown). In addition, Koedinger and Nathan (1998) indicated that the formal strategy students used was the symbolic manipulation approach. Based on the results from this present study, there were two sub-strategies in the algebraic strategies.

Equations based on comprehensive representations (a successful strategy). In this strategy, students assigned variables to the unknown in the word problems and they wrote correct algebraic equations to represent the situation in the problem. They were also able to then solve the problem and would be able to get the correct answer to the word problem. This sub-strategy was similar to the strategy Bull (1982) had categorized. According to Bull, if a student was able to map a problem statement onto a well-integrated representation, writing and solving an equation was expected to be a rather trivial task.

Equations based on poor representations (an unsuccessful strategy). In this strategy, students assigned variables for the unknown in word problems. However, they could not successfully write the correct algebraic equation to represent the situation in the problems. In particular, some students in this study had an inability to represent relationships when more than one unknown variable presented in the problem. Thus, they ended up getting the wrong answer. This sub-strategy was also similar to the strategy Bull (1982) had categorized. According to Bull, some of the attempts to write equations that turned out to be incorrect seemed to be derived from a partial representation of a problem. A partial representation could result from an incomplete or incorrect understanding of the problem text and/or a representation, which does not include all of the background information necessary to make inferences about the problem.

Non-Algebraic Strategies

In this strategy, the situation in the problem was translated into verbal arithmetic, a drawing, or using trial and error. Then, the answer was checked against the given values and the situation in the problems to determine the correctness of the answer. Consistent with previous research, non-algebraic strategies were also used by students in the United States to solve algebra word problems (e.g., Bull, 1982; Koedinger and Nathan, 1998; and Koedinger and Tabachneck, 1994). Based on the results from the current study, there were five sub-strategies in the non-algebraic strategies.

Verbal/Written arithmetic strategy. In this strategy, the situation in the problem was translated into verbal or written arithmetic. The answer then was acquired by using basic arithmetic thinking. This result was similar but not identical to findings from Koedinger and Tabachneck (1994), which indicated that students also used a verbal-math strategy to solve algebra word problems. In this strategy, the verbal problem statement was transformed into alternative verbal forms. There are two types of transformations: (a) verbal recoding intended to facilitate translation or (b) qualitative operations to estimate unknown values. Included in this strategy were translations to "verbal algebra" where equations were described verbally and transformations were performed that are analogous to written algebra transformations.

Drawing/Graph strategy. This was an informal strategy employed by only one student during the interview sessions. This student drew a graph in order to find the answer to the problem. In addition, this strategy was also found quite often among 118 students in this study when they were asked to solve Problem 7 (time and rate problem) on the posttest (see Appendix A). This sub-strategy was also similar to the strategy Koedinger and Tabachneck (1994) had identified. Koedinger and Tabachneck indicated that in the diagrammatic strategy, the verbal problem statement was translated into a diagrammatic representation. Transformations then were performed on the diagram, including annotations and diagram supported inferences.

Trial and error strategy. This was also an informal strategy employed by two students who participated in this study (one from the posttest and one from the interview). In this strategy, the student solved the word problems by trying to find the answer that best matched the situation and the question asked in the problem by using a trial and error method. This was similar to the strategy hypothesized by Bull (1982). Bull hypothesized that when students were unable to find a specific solution plan, they would approach the task through random trial and error or by performing a series of operations on the numbers given in the problem text. Even though students might not have been able to recall formulas and other information necessary to write an equation, they often had enough mathematical or real-world knowledge to allow them to solve a problem in a non-algebraic manner. Furthermore, the trial-and-error strategy found in this present study is also similar to what Koedinger and Nathan (1998) found. They found that besides the symbolic manipulation approach that students used to solve algebra word problems, the informal, such as guess-and-test, and unwind strategy, were also used by the students in their study. In the guest-and-test strategy, students guessed at the unknown value and then followed the arithmetic operators as described in the problem. They compared the outcome with the desired result from the problem statement and if different, tried again. The guess-and-test strategy was somewhat similar to a trial and error strategy Thai students' in this study used.

The second strategy was an unwind strategy. In this strategy, the student reversed the process described in the problem to find the unknown start value. The student addressed the last operation first and inverted each operation to work backward to obtain the start value. However, the unwind strategy was not found in this study.

A part-and-whole strategy. This was another strategy the students in this study often used in solving Problem 8 given on the pretest and posttest. Please refer to Appendix A for more details on Problem 8. In this strategy, students found the whole and then gave the part as an answer to the problem. For example, in solving Problem 8, the students thought that since the number of girls was twice the number of boys, if boys were one part then the girls were two parts. Therefore, the total number of students is in three parts. So they divided 45 by 3 and got 15. Since girls are two parts of the total, the students took 15 two times to get the number of girls.

A comparison strategy. This was another informal strategy some students in this study used most in solving Problem 7 (time and rate problem) given on the pretest and posttest. In this strategy, the students compared the distances of two people biking or walking hour by hour until the distance of both people is equal. Then students stopped comparing and gave the times that the two people would meet.

Summary.

The results from this study show that Thai ninth grade students used either algebraic strategies or non-algebraic strategies to solve algebra word problems. There were two sub-strategies in the algebraic strategies: (1) equations based on comprehensive representations; and (2) equations based on poor representations. These strategies were also found with the students in the United States (e.g., Bull, 1982; Hall et al., 1989; Kieran, 1988; Koedinger and Nathan 1998; and Koedinger and Tabachneck 1994). In the non-algebraic strategy, there were five sub-strategies: (1) a verbal/written arithmetic strategy; (2) a drawing or graph; (3) trial and error strategy; (4) a part-and-whole strategy; and (5) a comparison strategy. These strategies were also found with the students in the United States (e.g., Bull, 1982; Koedinger and Nathan 1998; and Koedinger and Tabachneck 1994). However, the unwind strategy found in the study by Koedinger and Nathan (1998) was not found in this present study. The results from this study show that few students in this study were able to develop a repertoire of strategies to solve the word problem. As mentioned in Thai Mathematical Standards (IPTST, 2000) and in NCTM Standards 2000 that students at this grade level should be able to use a variety of strategies such as symbolic, table, or graph to solve algebra word problems. The results from this present study, however, show that Thai ninth grade student in this study did not use a variety of strategies to solve algebra word problems.

Characteristics of Classroom Instruction During Word Problem Solving Lessons

Three teachers participated in this study. The students in each of the three teachers' classes were grouped by their ability: low, medium, and high achieving. The results from this section show that the three teachers had different styles in teaching students to solve word problems. Two teachers lectured and directed the whole lesson with little active participation of their students. Unlike these two teachers, the third teacher took a less directed role in some lessons and allowed students to actively participate in the teaching and learning. The first teacher taught low achieving students by using a directed-instruction approach with independent seatwork exclusively. He emphasized practicing procedures without underlying rationale or attempted to develop students' conceptual understanding. He used students' textbook as the lone source for instruction. The teacher mostly lectured while the students copied what were on chalkboard to their notebook. The teacher rarely used questions to initiate students' thinking. The results from the posttest suggested that the low achieving students in this class were unsuccessful at solving algebra word problems even when compared to their performance on the pretest.

The second teacher taught medium achieving students. This teacher took a less directed role in class. The teacher emphasized practicing procedures and underlying rationale for the concepts. She did not always use the students' textbook as a source of instruction. Instead, she used outside textbooks and worksheets in her class. This teacher often required that students work in groups or in pairs. She often asked questions to initiate students' thinking. The interaction in this class was more evident than the other two classes described above. The results from the posttest indicated that the performance of students in this class improved a great deal when compared to the pretest.

The third teacher taught high achieving students by using the same strategy, a directed-instruction, as the first teacher. He emphasized learning terms and practicing procedures without underlying rationale. He also used the students' textbook as the lone source for instruction. The teacher mostly lectured, but the interaction in this class was much higher than in the first teachers' class. The results from the posttest suggested that the performance of high achieving students in this class were improved but not much when compared with their performance on the pretest.

The results from this current study show that students were placed into the same group ability. The results also show that achievement of students who were placed into the low ability group was still low after instruction. This indicates that grouping the students with low ability might not be effective. This finding was consistent with previous research which showed that the achievement of students assigned to higher ability grouping are better than those who had been placed in lower ability groupings (e.g., Fuligni, 1995; Hoffer, 1992; Kerckhoff, 1986; Oakes, 1982; Reuman, 1989; Slavin, 1990). In addition, some studies indicated that that low-ability settings lead to low-quality teaching. Low-quality teaching is characterized by teachers' low expectations; a low-status, nonacademic curriculum; valuable class time spent on managing students' behavior; and most class time devoted to paperwork, drill, and practice. (e.g., Davidson & Kroll, 1991; Goldring & Eddi, 1989; Willie, 1990; Wortman & Bryant, 1985). This was also found in the current study. The first teacher, who taught the low ability class, expected that no matter how much he taught the algebra word problems, it would not make much sense to the low achieving students. This leads to a low quality of teaching of the first teacher because most of the class time was devoted to drill and practice and the teacher did not do much to help his students understand more about solving word problems.

Potential Connections between Teachers' Instruction and Students' Performance

This part summarizes some potential connections between teachers' instruction and students' performance at solving algebra word problems found in this study. The results from this study show that teachers' instruction was different in each class and might connect to students' performance at solving algebra word problems. First, teachers' instruction affected students' choice of solution strategies. For example, many students' choice of solution strategies had changed from non-algebraic base to algebraic base after instruction. In addition, the findings from the current study indicate that students' performance in the class taught by using an exclusively directed-instruction approach was improved little compared to their performance before instruction. This indicates that using directed-instruction approach exclusively might not be effective in teaching word problem solving.

In contrast, the results from the current study show that students' performance of the teacher who took a less directed role, used more questions to initiate students' thinking in her class, and allowed for collaborative practice improved much compared to their performance before instruction. In addition, more students of this teacher attempted to solve problems after instruction. This suggests that taking less directed role in class and using more questions to initiate students' thinking might have a positive connection to students' performance. This finding was supported by previous studies indicating that asking more questions in class had increased students' academic tasks (Baker, Gersten, & Lee, 2002; Flanders, 1970; Ostergard & Dwight, 1995).

Second, the three teachers, who participated in this study, rarely emphasized the importance of checking answers. As a result, the students in this study rarely checked their answers and they often made errors in the process of solving word problems. Finally, besides the two possibilities stated above, teachers' beliefs about students' learning might affect teachers' decisions about instruction and that might also affect students' performance. Anning (1988) found that the teachers held various common sense theories about students' learning that influenced how they structured their instruction. Teachers with different beliefs about students' learning tended to provide different types of classroom instruction. As reported in this current study, the first teacher believed that students in his class were low achieving, so no matter how much he taught it did not make much sense to his students. Therefore, he did not do much to help his students to understand more about solving word problems. He only demonstrated for his students how to solve word problems, but his students did less thinking in the class. This, as a result, might affect his students' performance in solving algebra word problems.

The second teacher believed that students should understand concepts and procedures in order to do well in mathematics. So, most of the teaching focused on developing students' concepts and procedures. For solving word problems, this teacher emphasized the procedure of translating the situation in the problem into equations. In addition, her students were required to do some thinking about the solution processes throughout the teaching and learning. This, as a result, might affect her students' performance in solving algebra word problems.

The third teacher believed that students in his class were high achieving and the students had already learned solving algebra word problems from eighth grade, so he did not emphasize solving word problems much and was not worried about his students. This, as a result, might affect his students' performance in solving algebra word problems.

Implications for Learning and Instruction

The preceding discussion of the study's main findings provided a number of implications for mathematics instruction regarding teaching and learning mathematics and curriculum development. First, the results from this study show that solving algebra word problems is difficult for many students. This suggests that teachers need to find a way that could help students minimize their difficulties and find a way to encourage students to think. The results from this study show that some students had difficulty in solving word problems because they could not represent linguistic situations into equations. Furthermore, during the interview, all students mentioned that translating word problems into equations was the most difficult part in solving word problems. This suggests a need for students to receive more training in skills of problem representation, since most students' failure in solving word problems were due to the difficulty in translation and integration rather than in solution execution phase. Also, it may be desirable to assess a problems' difficulty in terms of the translation and representation processes for arriving at a solution equation rather than in terms of the execution phase of solving word problems. Balacheff (1990) also suggested that examining students' word problem solving for types of errors is a possible way to describe word-problem-solving performance. Balacheff mentioned that assessing student difficulties in solving mathematical word problems must look further than just whether problems are correctly or incorrectly solved, because errors do not necessarily reveal failures. It appears that an examination of students' erroneous approaches serves better to identify and subsequently help students overcome their word problem solving difficulties (Babbitt, 1990; Drucker, McBride, & Wilbur, 1987).

Second, the results from this study show that unsuccessful problem solvers did not switch to a more algebraic-based strategy. Unfortunately, an arithmetic-based strategy may be effective for many of the word problems they were asked to solve within the context of school mathematics so that these students never developed the algebraic-based strategy in school and persist in using the arithmetic-based approach as adults. Thus, the first step in instruction is to present students with problems that help them see that arithmetic-based strategy does not work well for some problems. The second step is to provide instruction in a method that emphasized understanding the situation described in the problem. Previous studies have suggested that students could be taught to represent word problems via concrete manipulative and pictorial displays. For example, when participants successfully represented the problem situation using models (i.e., concrete or pictorial), fewer errors were made in generating problem solution (Maccini & Hughes, 2000).

Third, the results from this study show that some students had misconceptions and systematic errors. Teachers, together with their colleagues, should discuss this problem and find ways to help students overcome such errors. One error found in this study was in solving equations. This happened because students did not understand the concept of equality. In order to alleviate this problem, teachers might emphasize the concepts of equality, and also other properties of numbers. Another error that was often found with the students in this study was operations with polynomials and fractions. Teachers should check their students' work and help student's overcome this problem. Teacher can use algebra tiles to help student learn to add polynomials and fractions (Ashlock, 2002). In addition, some errors were due to carelessness. Even though some students checked their answer by substituting an answer into the equation, this strategy was not adequate for checking the answers when solving word problems. This suggests that teachers should train students to monitor their solution plan and to check their answer against situations given in the problems, not just only substituting an answer into the equation, which could be inadequate. Previous studies suggested that training students to monitor their processes in solving problems helped students perform better in mathematical problem solving (Schurter, 2002). In addition, teachers should look at students' work in order to determine the students' difficulties and errors students might have or interview students sometimes during the term to get insight the information about their students so that teachers could use such information to help students correct their errors. An action research conducted by Buschman (2001) supported this recommendation that using student interview increased teachers' focus on meeting the needs of individual students. In addition, teachers more aware of what individual students knew and what tasks they could perform with their knowledge about their students. Students interview also helped teachers understand how students learn mathematics. The interviews gave teachers the opportunity to observe students' attempts at solving problems in ways that made sense to the student.

Fourth, the results from this study show that one teacher planned the lesson based on students' prior knowledge and thus the students of that teacher made fewer errors and had the most improvement after instruction. This suggests that teachers should build knowledge from what students already know and put more variety of activities in the classroom. According to Carpenter, Fennema, Peterson, & Carey (1988), the results show that teachers' knowledge of their students was significantly correlated with student achievement. In addition, correlational analyses showed significant positive between teachers' knowledge of students' knowledge and students' mathematics problem-solving achievement (Peterson, Fennema, & Carpenter, 1989).

For the other two teachers, the results from this current study show that mathematics instruction relied mostly on teacher directed-instruction and individual seatwork. Students rarely initiated questions or discussion with teachers or their friends. As a result, students' performance in these two classes less improved after instruction. This suggests that teachers should consider a less directive role because previous studies indicate that teachers who took a less directive instruction could help students engaged in the mathematical work, maintained their focused involvement, and helped them take advantage of instruction to learn (Baker et. al, 2002). Furthermore, Strigth and Supplee (2002) suggested that when teachers look less directive instruction, students were more self-regulated and were more likely to monitor their progress. In addition, teachers should used questions and ask students individually to explain their thinking on a regular basis because asking more questions in class had increased students' ability to complete academic tasks (Flanders, 1970; Ostergard & Dwight, 1995). Furthermore, teachers should let students do group work such as, cooperative or collaborative learning, to discuss and exchange their ideas or communicate their thinking and reasoning when working together. Previous research (Quin et al., 1995) suggested that cooperative groups would be better able to deal with problem solving than working alone. Therefore, teachers should consider implementation of cooperative or collaborative learning in their classes to help students solve more complex word problems.

Fifth, the results from the observations suggested that two teachers in this study used students' textbook as the only factor in determining their method of instruction. This suggests that if teachers tend to teach what is in the students' textbooks, then it would be good to modify the way word problem solving is presented in the students' textbook. Furthermore, the current curriculum in Thailand was developed by qualified people and experts in various fields. This suggests that teachers from different schools in different areas should be part of curricula development, not only experts in various fields but also teacher educators and mathematics teachers who might know a lot about their students. To change the curriculum, the processes of student's cognitive development should be considered. In addition, social context and the national situation must also be considered. The curriculum should emphasize thinking and reasoning skills, problem solving rather than solving routine exercises and word problems.

Another factor that has a bearing on how teachers interpret and adapt the material in a textbook is their understanding of both their students' cognition and the roles of students' behavior. Therefore, educators should conduct programs for teachers to develop knowledge about students' cognition and students' problem solving process. Programs such as Cognitively Guided Instruction program (CGI) might be useful to introduce teachers about student's cognition and problem solving. From the study by Carpenter, Fennema, Peterson, Chiang, & Loef (1989), the CGI program provided teachers access to explicit knowledge derived from research on children's thinking that influenced their instruction and their students' achievement. The results of introducing the CGI program for teachers in primary schools (1-6) in Thailand indicated that CGI program could be implemented effectively to some degree in a primary school in Thailand (Komalabutr, 1995). If teachers understand student's thinking, perhaps they then can assist student's development of word problem solving skills. In addition, action research could be introduced to teachers because teachers can do a small research with their colleagues in the same field. Teachers could do action research on student's thinking and understanding of algebra. A small research study might help teachers improve their teaching and thus giving an idea in developing mathematics curriculum for their students (Buschman, 2001; Sunthornprasert, 2002).

Finally, Thai Mathematical Standards (IPTST, 2000) and NCTM Standards 2000 recommended that students at the ninth grade level should be able to use variety of solution strategies to solve word problems. However, the results from this present study indicate that the three teachers did not encourage students to use a variety of solution strategies or use multiple strategies to solve algebra word problems. The teachers only emphasized the use of algebraic strategies to solve algebra word problems. As a result, when some students in this study could not form an equation by using variables and symbols, they stopped solving the problems. This suggests that teachers should teach students to use variety of strategies in solving algebra word problems. Also, teachers should teach students to use multiple strategies to solve algebra word problems because previous study suggested that students were more effective when they used multiple strategies in solving a problem than when they stuck with (or got stuck with) a single strategy (Koedinger & Tabachneck, 1994).

Recommendations for Future Research

The preceding discussion of this study provides a number of recommendations for future research in mathematics education and teacher education. First, the results from this study show that some students had difficulties in solving word problems because they could not represent linguistic situations into equations. Thus, future studies should focus on student's representation of linguistic information into symbolic representations. Second, the results from this study show that students had several errors in algebra. Thus, Thai student's errors in algebra should be investigated. In addition, future study might also focus on students' misconceptions in algebra. This result might help teachers to minimize those errors and misconceptions.

Third, the results from this study suggest that students who had difficulty in solving algebra word problems might have difficulties making the transition from arithmetic to algebra. Thus, it would be important for future research to study the connection between arithmetic and algebra in order to help students build a bridge from arithmetic to algebra. Fourth, the results from this study indicate possible relationships between teachers' beliefs about students' learning and their decisions in instruction, thus, educators must understand the nature of teachers' beliefs about students' learning and the roles these beliefs play in the decisions teachers make as they present the material to their students. Thus, more research on teachers' beliefs about students' learning should be done in Thailand.

Fifth, the results from this study indicate that students' performance of two teachers who mostly directed the students to solve algebra word problems improved little after instruction. In contrast, students' performance of the teacher who used questions to initiate students' thinking improved much after instruction. Thus, it would be interesting to conduct a study about teachers' encouragement of students to think through more the process of solving word problems rather than directing them. Would their performance improved be? Sixth, the results from this study indicate that all three teachers rarely emphasized the checking process, it would be interesting to conduct a study that if teachers emphasized checking the process of solving word problems more and checking the accuracy of an answer, would student's errors in calculations or copying errors decrease?

Seventh, the results from this present study indicate that students were grouped by their ability. Grouping students with the same ability had both positive and negative results (e.g., Fuligni, 1995; Hoffer, 1992; and Reuman, 1989). Those studies showed that ability grouping helped the advanced and sometimes harmed or had no correlation with the slower students. However, there are few studies about ability grouping in Thailand. Thus, more research on ability grouping should be done in Thailand. Eighth, the results from this study show that most students went to a tutoring center. Not only students in this school but also many students in Thailand went to a tutoring center. Thus, it is interesting for future research to investigate why students need to go to the tutoring center and what are differences between teaching in the regular class and in the tutoring center.

Ninth, the interview with some of the students from the low achieving class gave value information. The students said, "I don't understand in class because the teacher always told us what to do. If the teacher let us think by ourselves, it would be better." Thus, it is interesting to see if teachers encouraged the low achieving students to think more through the process of solving word problems rather than directed them, would their performance be improved? Finally, the results from this study and from previous research show that translating words into equations is difficult. However, fewer research studies have investigated ways to help students better learn how to solve word problems, since solving word problems is important. Word problems provide students a first glimpse into how mathematics is used in the real word.

Limitations of the Study

The findings of this study are limited in the research design and methods. The observations of teachers' instruction were done mostly in one school. Thus, we could not conclude that mathematics instruction in Thailand is not concerned about developing students' word problem solving abilities. Some schools might emphasize word problem solving and some schools might not emphasize word problem solving. Again, all teachers at all grade levels in other schools may not use neither teacher-centered nor student-centered approach in teaching. Indeed, some teachers in different grade levels or in different schools might use a variety of teaching strategies.

A small sample of problems was used in this study, thus, the problem set might not be enough to test student's knowledge of solving word problems and would reveal more variety of strategies. The set of problems used in this study should be modified and tested before being used in future studies. Another limitation of this study is that the students in this study were not used to having somebody interview or look at them while they were solving word problems. Thus, some explanations given by students in this study were not clear. In addition, the researcher inexperience in using think-aloud techniques might also affect the results from the interview. Thus, more training in using thinking aloud technique is required for future studies.

The sample size of this study was small. Since the number of students participating in this study was small and from only one school in one region of Thailand, we cannot conclude that all ninth grade students in Thailand solved algebra word problems in the way indicated in this study. The results from this study showed that individual students used different of strategies in solving algebra word problems. However, we should not conclude that students had learned how to solve algebra word problems only from their ninth grade mathematics teachers. The students might have learned strategies for solving algebra word problems from their parents, their previous eighth grade teachers, their tutors from a tutoring center, or from other mathematics books.

Bibliography

Anning, A. (1988). Teachers' theories about children's learning. In J. Calderhead (Ed.), *Teachers' professional learning* (pp. 128 – 145). London: Falmer Press.

Ashlock, R. B. (2002). *Error patterns in computation: using error patterns to improve instruction* (8th ed.). New Jersey: Merrill Prentice Hall.

Babbitt, B. C. (1990). *Error patterns in problem solving.* Paper presented at the International Conference of the Council for Learning Disabilities, Austin, TX. (ERIC Document Reproduction Service No. ED 338 500).

Baker, S., Gersten, R. Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal 103*(1), 51 – 73.

Balacheff, N. (1990). Toward a problematique for research on mathematics teaching. *Journal for Research in Mathematics Education, 21*(4), 258 – 272.

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1996). *Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study (TIMSS).* Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.

Bruning, R. H., Schraw, G. J., & Ronning, R. R. (1995). *Cognitive psychology and instruction* (2nd ed.). Upper Saddle River, NJ: Merrill.

Bull, E. K. (1982). *Problem representations and solution procedures used in solving algebra word problems.* Unpublished doctoral dissertation, University of Colorado.

Buschman, L. (2001). Using student interviews to guide classroom instruction: An action research project. *Teaching Children Mathematics, 8*(4), 222 – 227.

California Assessment Program (1979). *Student achievement in California schools: 1978 – 1979 annual report.* Sacramento, Calif.: State Department of Education.

Campbell, J. R., Hombo, C. M., & Mazzeo, J. (2000). *NAEP 1999 trends in academic progress*: Three decades of students' performance, NCES 2000 – 469. Washington, DC: U.S. Department of Education.

Cardelle-Elawar, M. (1992). Effects of teaching metacognitive skills to students with low mathematics ability. *Teaching & Teacher Education, 9*, 109 – 121.

Cardelle-Elawar, M. (1995). Effects of metacognitive instruction on low achievers in mathematics problems. *Teaching & Teacher Education, 11*(1), 81 – 95.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1994). *Children's thinking about whole numbers*. Madison: University of Wisconsin-Madison. Wisconsin Center for Education Research

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*, 385 – 401.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematical thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499 – 532.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reyes, R. E. (1980). A perspective of mathematics achievement in the United States. In R. Karplus (Ed.), *Proceedings of the Fourth International Conference for the Psychology of Mathematics Education.* Berkeley, Calif.: International Group for the Psychology of Mathematics Education.

Carraher, T. N., Carraher, D. W., & Schlieman, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology, 3*, 21-29.

Chaiklin, S. (1989). Cognitive studies of algebra problem solving and learning. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra (pp. 93 – 114)*. Reston, VA: National Council of Teachers of Mathematics.

Chambers, D. L. (1994). The right algebra for all. *Educational Leadership, 51*, 85-86.

Chi, M.T.H. & Bassock, M. (1991). Learning from examples vs. self explanations. In. L.B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 251-282). Hillsdale, NJ: Lawrence Erlbaum Associates.

Chi, M.T.H., Feltovich, P.J., & Glaser,R. (1981). Categorization and representation of physics problems y experts and novices. *Cognitive Science, 5*,121-152.

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*(1), 16 – 30.

Clement J., Lochhead, J., & Monk, G. S. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly, 88*, 286 – 290.

Clement, J., Lochhead, J., & Soloway, E. (1979). *Translating between symbol systems: Isolating a common difficulty in solving algebra word problems.* Unpublished manuscript, University of Massachusetts at Amherst, Department of Physics and Astronomy, Cognitive Development Project.

Davidson, N., & Kroll, D. L. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education, 22*, 362–365.

De Corte, E., Greer, B., & Verschaffel, L. (1996). Mathematics Teaching and Learning. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology.* New York: Macmillan.

De Corte, E., Verschaffel, L., & Pauwels, A. (1990). Influence of the semantic structure of word problems on second graders' eye movements. *Journal of Educational Psychology, 82*, 359 – 365.

de Jong , T. & Ferguson-Hessler, M.G.M. (1991). Knowledge of problem situations in physics: a comparison of good and poor novice problem solvers. *Learning and Instruction, 1*, 289-302.

Drucker, H., McBride, S., & Wilbur, C. (1987). Using a computer-based error analysis approach to improve basic subtraction skills in the third grade. *The Journal of Educational Research, 80*(6), 363-365.

Flanders, N. (1970). *Analyzing teaching behavior.* Reading, Mass.:Addison-Wesley.

Fuligni, A. J. (1995). The long-term effects of seventh-grade ability grouping in mathematics. *Journal of Early Adolescence, 15*(1), 58-89.

Gerofsky, S. (1996). A linguistic and Narrative View of Word Problems in Mathematic Education. *For the Learning of Mathematics, 16*(2), 36 – 45.

Goldring, E. B., & Eddi, A. (1989). Using meta-analysis to study policy issues: The ethnic composition of the classroom and academic achievement in Israel. *Studies in Educational Evaluation, 15*, 231–246.

Greeno, J. G. (1989). A Perspective on Thinking. *American Psychologist, 44*(2), 134-41.

Hall, R., Kibler, D., Wenger, E., & Truxaw, C. (1989). Exploring the episodic structure of algebra story problem solving. *Cognition and Instruction, 6*, 223-283.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 60 – 86). Reston, VA: National Council of Teachers of Mathematics.

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology, 87*(1), 18 – 31.

Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations: Meaning and representation in algebra word problems. In P. A. Carpenter & M. A., Just (Eds). *Cognitive process in comprehension* (pp. 89 – 106). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hoffer, T. B. (1992). Middle school ability grouping and student achievement in science and mathematics. *Educational Evaluation and Policy Analysis, 14*(3), 205-27.

Institute for the Promotion of Teaching Science and Technology (IPTST, 2000). *Mathematical Standards for Secondary School Students*. Thailand.

Javier, C. (1987). *Problems of representation in the teaching and learning of mathematics*. Hillsdale, NJ: Erlbaum.

Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding (*pp. 133 – 156). Mahwah, NJ: Erlbaum.

Kerckhoff, A. C. (1986). Effects of ability grouping in British secondary schools. *American Sociological Review,* 51, 842–858.

Kieran, C. (1988). Two different approaches among algebra learners. In A. Coxford (Ed.), *The ideas of algebra, K – 12* (1988 Yearbook, pp. 91 – 96). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (1989). The early learning of algebra: A structural prospective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33 – 56). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 155 – 556). New York: Macmillan.

Kilpatrick, J. (1967). *Analyzing the Solution of Word Problems in Mathematics: An Exploratory Study*. Eric Documents. Accession No: ED027182

Knifong, J. D., & Holtan, B. D. (1976). An analysis of children's written solutions to word problems. *Journal for Research in Mathematics Education, 7*(2), 106-112.

Knifong, J. D., & Holtan, B. D. (1977). A search for reading difficulties among erred word problems. *Journal for Research in Mathematics Education, 8*(3), 227 – 230.

Koedinger, K. R., & Nathan, M. J. (1998). *The real story behind story problems: Effects of representation on quantitative reasoning.* Manuscript Under Review.

Koedinger, K. R., & Tabachneck, H. J. M. (1994, April). *Two strategies are better than one: Multiple strategy use in word problem solving.* Paper presented at the annual meeting of the American Education Research Association, New Orleans, LA.

Koedinger, K. R., & Tabachneck, H. J. M. (1995, April). *Verbal reasoning as a critical component in early algebra.* Paper presented at the annual meeting of the American Education Research Association, San Francisco.

Komalabutr, F (1995). Improving mathematics instruction and teachers' decision making: A case study in Thailand. *Dissertation Abstract*, ERIC DOC: AAI9604374.

Larkin, J.H. (1983). The role of problem representation in physics. In D. Gentner & A.L. Stevens (Eds.). *Mental models* (pp. 75-98). Hillsdale, NJ: Lawrence Erlbaum Associates.

Lewis, A. B., & Mayer, R. E. (1987). Students miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology, 79*, 363 – 371.

Lester, F. K. (1983). Trends and issues in mathematical problem solving research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and process*. New York: Academic Press.

Lochhead, J., & Mestre, J. P. (1988). From words to algebra: Mending misconceptions. In A. F. Coxford (Ed.), *The ideas of algebra, K – 12* (pp. 127 – 135). Reston, VA: National Council of Teachers of Mathematics.

Maccini, P. & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary school students with learning disabilities. *Learning Disabilities Research & Practice, 15*(1), 10 – 21.

Makanong, A. (1993). *A diagnosis of mathematics learning deficiency of eleventh-grade students of Chulalongkorn University Demonstration School*. Unpublished Research. Chulalongkorn University, Thailand.

Malloy, C. E., & Jones, M. G. (1998). An investigation of African American students' mathematical problem solving. *Journal for Research in Mathematics Education, 29*(2), 143 – 163

Mathews, S. M. (1997). The effect of using two variables when there are two unknowns in solving algebraic word problems. *Mathematics Education Research Journal, 9*(2), 122 – 135.

Mayer, R. E. (1982). Memory for algebra story problems. *Journal of Educational Psychology, 74*, 199 – 216.

Mayer, R. E. (1985). Mathematical ability. In R.J. Sternberg (Ed.), *Human abilities: Information processing approach* (pp. 127 – 150). San Francisco: Freeman

Mayer, R. E. (1987). *Educational psychology*. Boston: Little Brown.

Mayer, R. E., Larkin, J. H., & Kadane, J. B (1988). A cognitive analysis of mathematical problem solving. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (pp. 231 – 273). Hillsdale, NJ: Lawrence Erlbaum Associates.

Mestre, J. P., & Gerace, W. J. (1986). The interplay of linguistic factors in mathematical tasks. *Focus on Learning in Mathematics, 8*(1), 58 – 72.

Mestre, J. P., Gerace, W. J., & Lochhead, J. (1982). The interdependence of language and translational math skills among bilingual Hispanic engineering students. *Journal of Research in Science Teaching, 19* (5), 399 – 410.

Mevarech, Z. R. (1999). Effects of metacognitive training embedded in cooperative settings on mathematical problem solving. *The Journal of Educational Research, 92*(4), 195 - 205

Montague, M., Bos, C., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eighth-grade mathematical problem solvers. *Learning Disabilities Research & Practices, 6*, 145 – 152.

Muth, K. D. (1992). Brief research report: Extraneous information and extra steps in arithmetic word problems. *Contemporary Education Psychology, 17*, 278 - 285.

Nathan, M. J., Kintsch, W., & Young, E. (1992). A theory of algebra word problem comprehension and its implications for the design of computer learning environments. *Cognition and Instruction, 9*(4). 329-389.

National Council of Teachers of Mathematics (NCTM, 1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM, 2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Newell, A., & Simon, H. A. (1972). *Human Problem Solving*. Englewood Cliffs, N.J.: Prentice Hall.

Niaz, M. (1989). Translation of algebraic equations and its relation to formal operational reasoning. *Journal of Research in Science Teaching, 26*(9), 785 – 793.

Oakes, J. (1982). The reproduction of inequality: The content of secondary school tracking. *The Urban Review, 14*, 107–120.

Ostergard, S. , & Dwight D. (1995). *Eisenhower Mathematics and Science Education State Grant Program*. Nevada Higher Education Programs. University and Community College System of Nevada. Project TEACh (Teaching Effectively All Children). Grant # 1345-113-0376.

Petitto, A. (1979). The role of formal and non-formal thinking in doing algebra. *Journal of Children's Mathematical Behavior, 2*(2), 69 – 82.

Peterson, P. L., Fennema, E., & Carpenter, T. P. (1989). Using knowledge of how students think about mathematics, *Educational Leadership, 46*(4), p42.

Qin, Z., Johnson, D. W., & Johnson, R. T. (1995). Cooperative versus competitive efforts and problem solving. *Review of Educational Research, 65*, 129 – 144.

Reed, S. K. (1999). *Word problems: research and curriculum reform*. NJ. Erlbaum.

Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*: Washington, DC: National Center for Education Statistics.

Resnick, L. B., Cauzinille-Marmeche, E., & Mathieu, J. (1987). Understanding algebra. In J. A. Sloboda & D. Rogers (Eds.), *Cognitive processes in mathematics*. Oxford: Oxford University Press.

Reuman, D. A. (1989). *Effects of between-classroom ability grouping in mathematics at the transition to junior high school.* U.S.; Connecticut.

Riley, M. S., & Greeno, J. G., & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153 – 196). San Diego, CA: Academic Press.

Schurter, W. A. (2002). Comprehension Monitoring: An aid to mathematical problem solving. *Journal of Developmental Education, 26*(2), 22 – 33.

Silver, E. A. (1981). Recall of mathematical problem information: Solving related problems. *Journal for Research in Mathematics Education, 12*, 54 – 64.

Silver, E. A. (1987). Foundations of cognitive theory and research for mathematics problem solving instruction. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 111 – 131). Hillsdale, NJ: Erlbaum.

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education, 24*(2), 117 – 135.

Slavin, R. E. (1990). Achievement effects of ability grouping in secondary schools: A best-evidence synthesis. *Review of Educational Research, 60*, 471–499.

Strigth, A. D., & Supplee, L. H. (2002). Children's self-regulated behaviors during teacher-directed, seat-work, and small-group instructional contexts. The Journal of Educational Research, 95(4), 235 – 244.

Sunthornprasert, S. (2002). Simple research in a classroom. *Education Research Journal, 2*(1), 1 – 5. This article was written in Thai language. Thailand.

Tabachneck, H. J. M., Koedinger, K. R., & Nathan, M. J. (1994). Toward a theoretical account of strategy use and sense-making in mathematics problem solving. In *Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society*. Hillsdale, NJ: Erlbaum.

Tall, D. (1989). *Concept image, computers, and curriculum change.* Invited address presented at the research presession of the annual meeting of the National Council of Teachers of Mathematics, Orlando, FL.

Uthairat, S., & Viamoraphun, S. (1984). *Mathematics achievement of seniors students majoring in education in Thailand.* Unpublished Research, Chulalongkorn University, Thailand.

van Dijk, T., & Kintsch, W. (1983). *Strategies of discourse comprehension*. San Diego, CA: Academic Press.

Verschaffel, L., De Corte, E., & Pauwels, A. (1992). Solving compare problems: An eye movement test of Lewis and Mayer's consistency hypothesis. *Journal of Educational Psychology, 84*, 85 – 94.

vanEssen, G., & Hamaker, C. (1990). Using self-generated drawings to solve arithmetic word problems. *Journal of Educational Research, 83*(6), 301 – 312.

Wagner, S., & Kieran, C. (1989). *Research issues in the learning and teaching of algebra.* Reston, VA: National Council of Teachers of Mathematics.

Willie, C. V. (1990). Diversity, school improvement, and choice: Research agenda items for the 1990s. *Education and Urban Society, 23* (1), 73–79.

Wortman, P. M., & Bryant, F. B. (1985). School desegregation and Black achievement. *Sociological Methods & Research, 13,* 289–324.

Appendices

Appendix A
Pretest and Posttest

Ten problems used in the pretest

Problem 1: Solve for x: $\frac{4}{3}(x + 8) = \frac{2}{5}(2x + 1)$

Problem 2: The ratio of teachers and students in one college is 1:6. If S represent the number of student and T represent the number of teachers, write the equation for the number of students.

Problem 3: Let X represent a number, please write an equation to represent the following statement. "Two times a number and three is 21."

Problem 4: Let W represent the width of a tennis court, please write an equation to represent the following statement. "The length of the tennis court is two meters more than twice the width."

Problem 5: Let Y represent a number, please write an equation to represent the following statement. "Three times the sum of a number and six is 42."

Problem 6: Wallace works as a part-time typist. Last month, he worked 3 extra hours. When he added the amount he earned for working extra hours and his monthly wage of 500 Baht, he found that he earned 665 Baht. How much per hour did Wallace get? (Note: Baht is Thai currency, 41 Baht = One US Dollars)

Problem 7: Kim begins to bike at 9:00 am at the rate 5 kilometer per hour. Two hours later, at the same starting point, Tim begins to bike to the same direction as Kim at the rate 10 kilometer per hour. At what time Kim and Tim will meet?

Problem 8: In 12 years, the ratio of father and son's ages will be 3:1. If the father is 30 years older than the son at the present time, find the age of the father.

Problem 9: There are silver and gold earrings in one box. The numbers of silver earrings are twice the number of gold earrings. The total of both earrings in that box is 36. How many silver and gold earrings are in this box?

Problem 10: Lisa and Dan picked 252 oranges altogether. Lisa picked 9 oranges per box and Dan picked 6 oranges per box. There are 34 boxes altogether. Find the number of oranges Lisa and Dan each picked.

Ten problems used in the posttest

Problem 1: Solve for x: $\frac{1}{2}(3x + 6) = \frac{3}{4}(x + 8)$

Problem 2: The ratio of professors and students in one college is 1:7. If *S* represent the number of student and *P* represent the number of professors, write the equation for the number of students.

Problem 3: Let *X* represent a number, please write an equation to represent the following statement. "Three times a number and six is 24."

Problem 4: Let *W* represent the width and *L* represent the length of a tennis court, please write an equation to represent the following statement. "The length of the tennis court is six meters more than twice the width."

Problem 5: Let *Y* represent a number, please write an equation to represent the following statement. "Two times the sum of a number and three is 24."

Problem 6: Kobe works as a part-time typist. Last month, he worked 5 extra hours. When he added the amount he earned for working extra hours and his monthly wage of 6,500 Baht, he found that he earned 7,945 Baht. How much per hour did Kobe get? (Note: Baht is Thai currency, 41 Baht = One US Dollars)

Problem 7: John begins to bike at 10:00 am at the rate 10 kilometer per hour. At 12:00 pm, at the same starting point, Jordan begins to bike to the same direction as John at the rate 20 kilometer per hour. At what time John and Jordan will meet?

Problem 8: In 12 years, the ratio of mother and son's ages will be 3:1. If the mother is 34 years older than the son at the present time, find the age of the mother.

Problem 9: The number of girls is twice of the number of boys in the classroom. If there are 45 students in this classroom, find the number of girls in this classroom.

Problem 10: Jack and Jill picked 252 apples altogether. Jack picked 9 apples per box and Jill picked 6 apples per box. There are 34 boxes altogether. Find the number of apples Jack and Jill each picked.

Appendix B
Set of Algebra Word Problems Used During the Interview

Problem 1: Orange Problem

"At first, a mother bought some oranges. However, there were not enough oranges to equally divide the oranges among 15 people. Therefore, she went to buy 10 more oranges so each person could get four oranges. How many oranges did the mother buy the first time?"

Problem 2: Student Problem

"The number of girls is 2/3 of the number of boys in one class. If the total number of the students in this class is 45, find the number of girls in this class."

Problem 3: Age Problem

"Six years ago Jennifer's age was twice as old as Jonathan's age. Nowadays, if Jennifer is six years older than Jonathan, how old is each now?"

Problem 4: Car Wash Problem

"Natasha, Gibson, Jim, and Robinson had a car wash on Sunday. Natasha washed twice as many cars as Gibson. Gibson washed one fewer than Jim. Jim washed six more than Robinson. Robinson washed six cars. How many cars did each person wash? (Adapted From Malloy and Jones, 1998)"

Problem 5: Distance Problem

"Simon and Henry decided to bike to his uncle's house from their house. Henry left at 10:00 am and biked at a rate 20 kilometer per hour. Simon, at the same starting point, left at 10:45 am and biked at a rate 30 kilometer per hour. They reached their uncle's house at the exact same time. What is the distance from their house to their uncle's house?"

Appendix C

Characteristics of Each Student Participated in the Interview Sessions

This appendix presents characteristics of each of the 18 students who participated in the interview sessions. The 18 students had different achievement levels: low; medium; and high. In addition, these students were a mix of boys and girls.

High Achieving Students

All high achieving students were from Mr. Bond's class. Three male and three female students were selected.

William (BH1): William did not like mathematics because he was not getting along with his mathematics teacher. He said that he might like mathematics more if he and his mathematics teacher got along well. He mentioned that he did not pay much attention in class, because of the teacher. "I played with my friends all the time during instruction." Since William was not paying any attention in class, most of his knowledge was from independent study. William said that he always studied from the class mathematics textbook. He did not go to a tutoring center[1]. The results show that William successfully solved nine problem on the pretest and ten problems on the posttest. Also, he solved all five problems correctly during the interview.

Phil (BH2): Phil liked mathematics because he thought it was fun and had nothing to do with memorizing. He believed that if we understood mathematics, we could do mathematics without memorizing any rules or formulas. Phil liked to solve word problems because he thought it was challenging. He learned how to solve word problems both from his mathematics class and from a tutoring center. He also did different kinds of problems from books borrowed from the school library. When he had problems at solving word problems, he would open his book first. If he still did not understand, he would call the

[1] A tutoring center is the place students go to study mathematics outside of the school (either on weekday evenings or on the weekend). It is not a school requirement for students to go to this place. It is up to each individual student. A teacher who teaches in the center either is a science or mathematics teacher from the students' school or from other institutions. Tutoring takes place at either the teacher's house or some rented building in town. It is illegal to use a classroom in any schools to do tutoring. At a tutoring center, students need to pay money either per hour or per month. At some places, students need to pay per term. Students from the same school may not go to the same tutoring center.

pre-service teacher who used to teach him in the eighth grade. He also asked his mathematics teacher at school, a teacher at the tutoring center, and friends. The results indicate that Phil successfully solved nine problems on the pretest and ten problems on the posttest. He could solve all five problems during the interview.

Nat (BH3): Nat liked mathematics because he thought it was a good subject. He liked solving word problems. His knowledge of solving word problems came from both his mathematics class and from a tutoring center. He learned more techniques in solving word problems, however, from the tutoring center. He usually studied by himself. Sometimes, he asked friends. However, he rarely asked his parents. The results indicate that Nat successfully solved nine problems on the pretest and ten problems on the posttest. He could solve all five problems during the interview.

Ann (GH1): Ann liked mathematics because it had patterns and formulas. She also liked solving word problems, even though the problems were sometimes difficult. She mentioned that learning in class helped her to solve word problems better because her mathematics teacher taught a step-by-step method in solving word problems. In addition, she got more information from other books she borrowed from the library. Ann went to a tutoring center as well. When she had problems, she first asked friends. If that did not help, she went to her mathematics teacher. However, she asked the teacher at the tutoring center more often than her mathematics teacher at school. She never asked her parents. The results show that Ann successfully solved six problems on the pretest and nine problems on the posttest. She correctly solved four out of five problems during the interview.

Patty (GH2): Patty liked mathematics even though she mentioned that it was difficult. She mentioned that solving word problems were difficult because it was hard to write an equation. She tried to understand mathematics on her own, but if she did not, she would ask her mathematics teacher. She also asked friends to give her suggestions. She also read other books borrowed from the school library to do more problems and exercises. At home, she always asked her aunt about math. She did not go to a tutoring center. The results show that Patty successfully solved six problems on the pretest and seven problems on the posttest. She could solve two out of five problems during the interview.

Nancy (GH3): Nancy liked mathematics because it helped her do calculations. She mentioned that solving for **x** or **y** was easy but solving word problems was difficult, because she did not know what to do after reading the problems. She did not know what she was going to do with the problems. She said that learning in class helped her. She also went to a tutoring center. When she did not understand, she asked her mathematics teacher or friends. She never asked her parents. Even if she got stuck at home, she stopped doing the homework and waited to ask her friends later. The results show that Nancy successfully solved six problems on the pretest and eight problems on the posttest. She could solve one out of five problems during the interview.

Medium Achieving Students

All medium achieving students were from Ms. Rose's class. Three male and three female students were selected.

Billy (BM1): Billy thought that mathematics was a good subject and could be used in daily life. Billy thought the topic of equations would be useful in higher education. He liked solving word problems only when the problem was easy. The difficulty he had was defining variables and forming equations. He understood better how to solve word problems after being taught in his mathematics class. When he did not understand, he always asked his mathematics teachers. He also went to a tutoring center. At the tutoring center, he learned more techniques and gained skill. In addition, he asked friends for help but he rarely asked his parents to help. The results indicate that Billy successfully solved two problems on the pretest and four problems on the posttest. He could solve three out of five problems during the interview.

Tom (BM2): Tom thought mathematics was useful and fun, and could be used in daily life. He liked mathematics, especially polynomials. He thought solving word problems was difficult because he had to define variables and form equations. However, he liked solving word problems. He mentioned that learning in class helped him solve word problems better. He did not go to a tutoring center. When he had trouble with mathematics, he always asked his mathematics teacher. Sometimes, he asked friends, but he never asked his parents. The results show that Tom successfully solved four problems on the pretest and eight problems on the posttest. He could solve three out of five problems during the interview.

Sean (BM3): Sean thought that mathematics was a boring subject and difficult because he had to memorize and follow rules and formulas. He said "if you are not sure about those rules and formulas, you are dead". He believed that there were several ways to solve word problems. However, there would be only one answer. For Sean, solving word problems was difficult. He thought that learning in class would help him do better in mathematics. However, he always felt sleepy during class time so he did not understand much about mathematics. He did not always ask his mathematics teacher for help. Instead, he asked friends. He did not go to a tutoring center. The results indicate that Sean successfully solved four problems on the pretest and seven problems on the posttest. He could solve four out of five problems during the interview.

Sara (GM1): Sara thought mathematics was useful for daily life such as buying things. She liked mathematics, especially the circle topic. She did not do well at solving word problems. However, she liked it because she could practice her thinking skills while solving word problems. She mentioned that breaking down the problem into cases was the most difficult part in solving word problems. She learned best in her mathematics class. When she did not understand, she always asked friends first and then her mathematics teachers. She also went to a tutoring center and asked the teacher there as well. She rarely asked her parents. The results indicate that Sara successfully solved four problems on the pretest and nine problems on the posttest. She could solve three out of five problems during the interview.

Rita (GM2): Rita thought mathematics was fun. If she could do the mathematics, she would be very proud of herself. She liked to solve difficult problems because she said, "if I could do it, I would be so proud." She thought mathematics was important for going to a market, for example. About word problems, she mentioned, "if we could analyze word problems, we can solve it". Learning in class helped her to solve word problems better. She also went to a tutoring center and it also helped her. At the tutoring center, she learned new knowledge and new techniques. When she did not understand, she always asked her mathematics teacher first, and if she still did not understand, she would ask friends. The results show that Rita successfully solved four problems on the pretest and eight problems on the posttest. She could solve three out of five problems during the interview.

Jenny (GM3): Jenny thought mathematics was fun and used a lot of thinking. She thought solving word problems was difficult and complicated. The difficulty was that a word problem had lots of words and it was hard to translate those words into variables. Learning in class helped her better solve word problems. She did not go to a tutoring center. When she had trouble, she always asked her friends. However, if she still could not do it, she would ask her mathematics teachers. She rarely asked her parents. The results demonstrate that Jenny successfully solved four problems on the pretest and six on the posttest. She could solve two out of five problems during the interview.

Low Achieving Students

All high achieving students were from Ms. Rose's class. Three male and three female students were selected.

Lee (BL1): Lee liked mathematics because of the influence of his father. He thought solving word problems was exciting. Also, it helped him improve his calculation and thinking. However, sometimes he could not do it. When he had trouble, he always asked his mathematics teacher, his friends, his mother, and his brothers. He did not go to a tutoring center. He always studied by himself. The results indicate that Lee successfully solved four problems on the pretest and three problems on the posttest. He could solve two out of five problems during the interview.

Sam (BL2): Sam mentioned that if mathematics was easy, he liked it. He thought that mathematics was important in computation because it could be used in our life. He liked solving word problems because it was challenging. However, he could not solve many problems because he did not know what to do with the numbers in the word problems. When he did not understand, he usually asked his friends. He was afraid of asking his mathematics teacher. The results indicate that Sam could not solve any problems on the pretest but he successfully solved two problems on the posttest. He could do none of the five problems during the interview.

Andy (BL3): In general, Andy liked mathematics because he got good grades. He liked solving equations. He liked solving word problems only when he could do it. The difficulty in solving word problems was that he could not write equations. He mentioned that learning from class helped him a bit. He also went to a tutoring center. When he did not understand, he asked his mathematics teachers and the teacher at the tutoring center, but not often. Mostly, he did it himself. But if he could not do it, he would ask his friends and did it with them. The results indicate that Andy successfully solved one problem on the pretest and four problems on the posttest. He could solve two out of five problems during the interview.

Jill (GL1): In fact, Jill did not like mathematics, because there was a lot to think about in mathematics. However, she liked solving equations because she could practice her thinking. She did not like solving word problems because it was difficult and she could not solve them. The difficulty was her inability to translate the problem into equations. She gained her knowledge both in her mathematics class and from a tutoring center. However, she asked the teacher at the tutoring center more often than her mathematics teacher in class. When she got into trouble, she asked friends, but she never asked her parents. The results show that Jill successfully solved one problem on the pretest and four problems on the posttest. She could solve one out of five problems during the interview.

June (GL2): Even though June sometimes stressed out and could not do mathematics, she still liked mathematics because it was fun. Learning in class helped her understand better. She also went to a tutoring center. At the tutoring center, she did extra exercises outside the textbook. When she did not understand, she always asked teachers both at school and at the tutoring center. She did not ask her friends or parents very often. The results indicate that June could not solve any problems on the pretest but she successfully solved four problems on the posttest. She could solve two out of five problems during the interview.

Wilma (GL3): Wilma thought mathematics was necessary because it could be used in daily life. However, she did not like solving word problems because she did not like to analyze and translate the problems. Wilma gained knowledge about solving word problems from her mathematics class and from a tutoring center. She did not ask her parents when she did not understand. She, instead, always asked friends or the teacher at the tutoring center. The results show that Wilma successfully solved one problem on the pretest and six problems on the posttest. She could solve two out of five problems during the interview.