Economic Optimization of Irrigation

by

M. J. English, A. G. Nelson
B. C. Nakamura, K. C. Nielsen
and G. S. Nuss

Water Resources Research Institute
Oregon State University
Corvallis, Oregon

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Marshall J. English
A. Gene Nelson
Brian C. Nakamura
Kim C. Nielsen
Gary S. Nuss

Departments of Agricultural Engineering
and Agricultural Economics
Oregon State University

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ABSTRACT

The research effort described in this report explores the whole question of optimization of irrigation system design, planning and operations. Optimum irrigation practices, in theory, may involve some degree of deficit irrigation (deliberate under-irrigation of crops), hence the focus of this research was the use of deficit irrigation under practical operating circumstances. Deficit irrigation involves a radical philosophical departure from conventional practice. This technique is based on an axiom of economics; that water use should be reduced to the point at which the marginal cost of irrigation just equals the value of the last increment of yield. The implications of this fundamental axiom are explored, and a case study is also presented in which the full marginal costs of irrigation are examined in detail.

When a decision is made to under-irrigate a crop, the problem of irrigation system design becomes much more complex because there are no longer any hard and fast requirements for system performance. The designer has much more latitude and system performance is more difficult to predict. Design techniques for exploiting the concept of deficit irrigation are explored in a case study, and an algorithm for predicting system performance is outlined.

The question of risk becomes more significant when crops are under-irrigated, in part because it is more difficult to predict yields. This question is discussed, and a case study dealing with risk is presented. Irrigation scheduling will also be a more important consideration under deficit irrigation. It may be necessary to rely on crop stress indicators rather than soil moisture indicators, soil and crop variability may be greater under deficit irrigation, uncertainty will be a greater problem. The problem of scheduling for deficit irrigation is examined, and a new approach to scheduling using mathematical filtering techniques is proposed.

Field experiments were carried out as part of this project to study the relationship between irrigation frequency and crop yields, and to test a variety of models of the process of evapotranspiration under low soil moisture conditions. Both of these subjects were found to be significant considerations in irrigation optimization, and past research on both was found to be inconclusive. Results of these two avenues of field research are presented.

FOREWORD

The Water Resources Research Institute, located on the Oregon State University campus, serves the State of Oregon. The Institute fosters, encourages and facilitates water resources research and education involving all aspects of the quality and quantity of water available for beneficial use. The Institute administers and coordinates statewide and regional programs of multidisciplinary research in water and related land resources. The Institute provides a necessary communications and coordination link between the agencies of local, state and federal government, as well as the private sector, and the broad research community at universities in the state on matters of water-related research. The Institute also coordinates the inter-disciplinary program of graduate education in water resources at Oregon State University.

It is Institute policy to make available the results of significant water-related research conducted in Oregon's universities and colleges. The Institute neither endorses nor rejects the findings of the authors of such research. It does recommend careful consideration of the accumulated facts by those concerned with the solution of water-related problems.

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Summary

This report deals with development of a systematic approach to optimization of irrigation system design, planning and operations. A theoretical framework is developed for evaluating optimum irrigation practices. Utilization of this framework is illustrated by two case studies. The case studies are presented in the appendices. Most of the work presented here involved collecting and assimilating theories, models and field research done by others. These resources were then integrated into an analytical approach for optimization of irrigation practices. Original field experiments were also carried out as part of this project to bridge gaps that were found in the pre-existing body of knowledge.

This research was carried out as a set of stand-alone projects. As noted above, these included:

(i) Development of an analytical approach for selecting optimum irrigation practices.

(ii) A study of the question of designing for deficit irrigation

(iii) A study of risk and the problem of planning for deficit irrigation.

(iv) A study of the problem of scheduling for deficit irrigation.

(v) Field research on the relationship between soil moisture and evapotranspiration under deficit irrigation.

(vi) Field research on the relationship between irrigation frequency and crop yield.

The conclusions and recommendations of each of these sub-projects are presented in the appendices. They are not repeated here, since their interpretation depends upon the context of the respective appendices.

It is widely recognized that partial irrigation of a crop can sometimes yield a greater net economic return than full irrigation. In fact the profit maximizing irrigation practice will always involve using less water than a crop is capable of consuming because the income from the last increment of attainable yield will always be less than the marginal cost of production. This rule would not apply if there were no marginal production costs, but it is impossible to imagine a situation in which the combined total of all marginal capital, operating and opportunity costs would be zero. Nevertheless standard irrigation practice has been to provide for maximum crop water demands, and in so doing to strive for maximum yields. Implicit in this practice is the assumption that the increased income that might be realized through reduced water use does not justify the effort, and perhaps the risk, involved in achieving the profit maximizing level of water use. This implicit assumption may be more widely accepted than it should be, perhaps in part because the marginal costs of production in irrigated agriculture are not fully appreciated by system designers.

In summary then, irrigating for maximum profit implies deficit irrigation, that is, the deliberate under-irrigation of a crop. The research presented in this paper was therefore focused on the practice of deficit irrigation.

Deficit irrigation is already practiced in varying degrees on a wide scale in the United States. However the irrigation industry is founded on the philosophy of meeting the full water requirements of crops. The radical shift in philosophy implied by deficit irrigation requires a completely different economic perspective. For that reason a substantial portion of this report deals with the economics of deficit irrigation. The economic perspectives are developed in Section 2 of the report. Additionally, in the case studies, an effort is made to present as clear a picture as possible of the full marginal costs of irrigating a crop.

Conventional irrigation practice allows very little latitude in system design. The amount of water that must be put on is precisely determined by the water requirements of the crop. The engineer's task is then to design a system that will supply that water. With deficit irrigation on the other hand, the designer has far more latitude because he can choose to design for any level of water use deemed appropriate. The amount of water that will maximize profits is <u>not</u> independant of system design. On the contrary, once the decision is made to under-irrigate, the designer can exploit this decision by designing for reduced system capacity, and perhaps lower irrigation frequency. In doing so there may be substantial reductions in capital costs. Thus the economic optimum amount of water use will depend on the reductions in capital costs that can be achieved in the design process. This approach to system design for deficit irrigation is of fundamental importance.

The complexity of the relationship between system design and crop yield is so great that it becomes impossible to arrive at an optimum design in a single step. Rather, an iterative approach will be required in which a design is prepared, resulting crop yields are estimated, expected net profits are calculated, and then a new design is prepared, and so on. To do this efficiently will require a computer model to relate system design to crop yields. An outline of such a model is presented in detail in Section 3. An important element of that procedure is the use of statistical modeling. Under deficit irrigation the spatial variability of the field and the crop, the variability of weather conditions and variations in operating practices of farmers may cause yields to vary substantially, even within a single field.

The statistical simulation procedures are discussed in Section 3, and the mathematics of the statistical model are presented in detail in Appendix F. The case study presented in Appendix A deals with the problem of designing for deficit irrigation. Design techniques which can take advantage of the deficit irrigation concept are explored.

Another facet of optimum irrigation practice is concerned with scheduling irrigations. Once an irrigation system is in place and a crop is planted, the irrigator must decide on the dates and amounts of water to apply as the season progresses. Deficit irrigation will require more sophisticated techniques for irrigation scheduling than are now practiced. It may be necessary to schedule irrigations on the basis of specified levels of crop stress rather than on the basis of specified levels of soil moisture depletion. However, predicting when a specified stress level will be reached will be very difficult. Suitable indices of crop stress must be developed. Accurate field measurements of soil moisture and crop stress will be made more difficult by virtue of the fact that field conditions may be less uniform than normal under deficit irrigation. This problem of scheduling for deficit irrigation is complex and is beyond the scope of the present project. However, because of its central importance to irrigation optimization the question of scheduling was studied to a limited extent. The results of that study are presented in Appendix C. Significant characteristics of an optimally irrigated field are discussed, and a proposed approach to irrigation scheduling for maximum profit is outlined. That approach involves the use of crop stress indicators in conjunction with data filtering techniques. A general discussion of filtering and its possible application to irrigation scheduling is presented, and an example of a filtering algorithm is demonstrated.

During the course of this project it was found that in at least two respects existing research was not adequate to the task of irrigation optimization. First, it was found that the relationship between crop yield and irrigation frequency is not adequately established. Secondly it was found that there is no commonly accepted model for predicting evapotranspiration under low soil moisture conditions. Although several models of this process have been proposed, none of them has gained general acceptance for any given set of circumstances.

Field experiments were carried out, as part of this project, in order to better define both of these relationships. Obviously it was not possible to arrive at final, universal conclusions on both questions. However it was possible to resolve these questions for one crop in one location. The field research involved irrigated winter wheat in Eastern Oregon, which were the crop and location used in the case studies presented in this report. It was felt that the credibility of the case studies would be in doubt so long as these relationships were not well defined. The results of these field experiments are presented in Appendices D and E.

The appendices of this report are not meant to simply present additional supporting information. Rather they are the essence of this report. The most significant work done under this project is presented there. Several elements of this report have been published or are soon to be published. A summary of the publications follows:

(i) Appendix A has been published in the Proceedings of the American Society of Civil Engineers, ASCE Volume 108 No IR2, June, 1982.

(ii) Appendix B will be presented at the Proceedings of the Western Agricultural Economics Association, meeting in Laramie, Wyoming, July, 1983.

(iii) Appendix C was published in the Proceedings of a Specialty Conference on Irrigation Scheduling, The Palmer House, Chicago, December 1981.

(iv) Appendix D was presented as Paper No. 82-2044, 1982 Summer Meeting, American Society of Agricultural Engineers, University of Wisconsin, Madison, June, 1982.

(v) Appendix E was presented as Paper No. 82-2046, ASAE Summer Meeting, University of Wisconsin, Madison, June, 1982.

(vi) Sections 2 and 3 of the main body of the report, combined with Appendix F, will be presented at a specialty conference of the Irrigation and Drainage Division of the American Society of Civil Engineers, Jackson, Wyoming, July, 1983.

Other publications relating to this research project include:

English, M. J. 1981. The Uncertainty of Crop Models in Irrigation Optimization. Transactions ASAE, Vol. 24, No. 4, July-August.

English, M. J. 1981. "The Potential Advantages of Deficit Irrigation"; In: Irrigation The Hope and the Promise:, The Irrigation Association, Annual Technical Conference, Salt Lake City, Feb. 15-18, 1981.

English, M. J. and G. S. Nuss. 1980. Designing for Deficit Irrigation, In: Proceedings, ASCE Irrigation and Drainage Division Specialty Conference, Irrigation and Drainage, Today's Challenges, Boise, Idaho, July 23-25, 1980.

English, M. J. 1979. Use of Crop Models in Irrigation Optimization, Paper No. 79-4521, ASAE Winter Meeting, New Orleans, LA. Dec. 11-14, 1979.

In addition to the journal publications and professional society meetings listed above, steps have been taken to get the results of this research out to irrigators on a practical level. The concepts, techniques and case studies developed under this project have been presented to a variety of farm related organizations in Oregon. Presentations have been made at meetings of the Oregon Water Resources Congress, the 1983 Annual Farm Fair in Hermiston, and irrigation meetings organized by Umatilla Electric Cooperative and the Extension Service. These presentations have focused on the practical implications of this research and have attracted considerable interest from farmers in attendance. Popularized articles have been published in three Oregon newspapers as well as a magazine (The Furrow) that is published by the John Deere Corporation.

To date one farmer has used the results developed in this research. That farmer practiced deficit irrigation, using recommendations prepared as part of this project, in 1982. A number of other farmers in the vicinity of the Hermiston and Boardman in Eastern Oregon have indicated considerable interest in this research. Extension agents and electrical utilities in the Columbia Basin area have also indicated intense interest in this work because of its relevance to the present strained economic circumstances of farmers in that area. In particular it is felt that the approach developed here should have particular application where energy costs have risen dramatically, as they have in the Columbia Basin.

2.

An Economic Perspective

This section develops an economic perspective on irrigation optimization. The purpose is to outline a set of fundamental concepts relevant to the evaluation of irrigation practices when profit maximization is the goal. Traditionally, the economic philosophy in irrigation system design and operation has been to minimize the cost of meeting the crop's water requirements. However when maximum profit is the goal a different approach is required. This new framework will be based on deficit irrigation; that is, deliberate under-irrigation of crops.

2.1 Optimum Water Use

In general, water use-crop yield relationships have the form shown in Figure 1. At low levels of water use yields are almost linearly related to total water applied. Near maximum yield the function approaches horizontal; relatively more water is required to produce the last increments of yields. If high yields are to be attained it is necessary to compensate for non-uniformity of applications and inhomogeneity of soils by over-irrigating. That is some parts of the field must be over-irrigated to insure that other parts of the field are adequately irrigated. Crop yields may also be increased, in some circumstances, by irrigating more frequently, but this leads to increased evapotranspiration. Beyond the point of maximum yield, increased water use can actually reduce yields, as represented by the falling-off of the curve. In wet soils, gas exchange between soil and atmosphere is low, causing oxygen deficiency. As a result root transpiration and root volume are reduced, resistance to the movement of water and nutrients through the soil increases, and toxic compounds may be formed in the soil as well as in the plants. Reduced nitrification in the soil, low soil temperatures, deterioration of soil

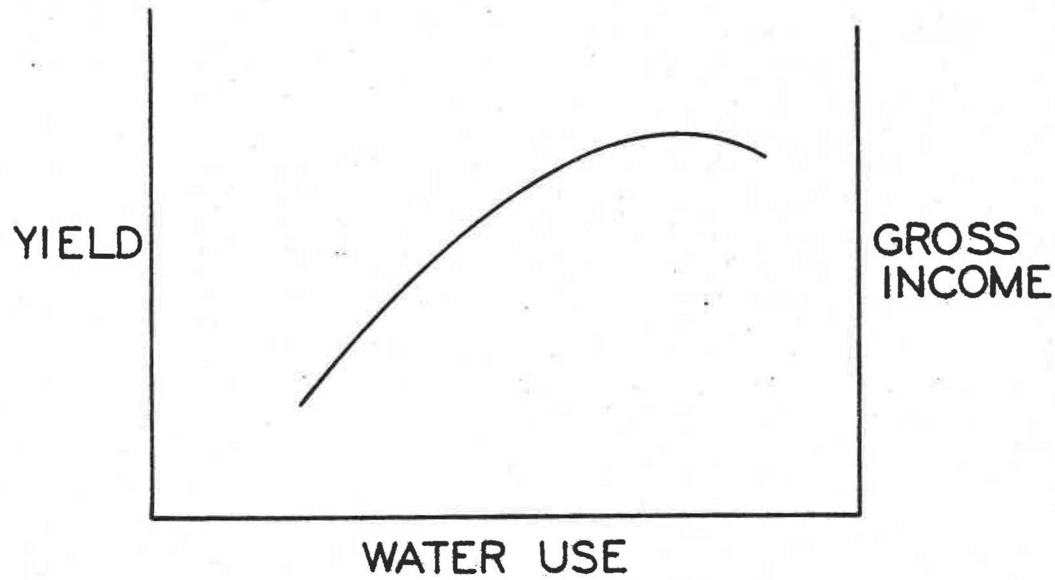

Figure 1. Relationship Between Gross Water Use and Crop Yield or Gross Income

structure and less workable soil adversely affect yields (Feddes, 1981).

If we assume that gross income is a constant multiple of yield (i.e. price times yield) the curve shown in Figure 1 can then also represent the relationship between water applied and gross income. The right hand side of Figure 1 has therefore been labeled gross income.

Figure 2 shows a general representation of irrigation systems costs superimposed on the same gross income curve. The cost curve begins at a point above the origin, representing annualized capital costs and other fixed costs such as taxes, insurance and perhaps an electrical hookup fee. The cost curve then moves upward and to the right, representing the incremental costs associated with energy, labor and maintenance. Eventually the cost curve reaches an end point. The end point represents the capacity of the system; the maximum amount of water than can be delivered to the field by that system.

If we assume that all production costs are included in the cost curve, the net income will be the difference between the two curves. Now let us examine some alternative cost curves. The <u>nominal</u> goal in conventional practice is to design a system to meet the full water requirements of the crop, with the implicit objective of obtaining maximum possible yields. Such a system is represented by the cost curve in Figure 2. Nominally, conventional practice would be to irrigate for maximum yields (point A). However the spread between the two curves (i.e. profits) could be increased by slightly under-irrigating, as represented by point B in Figure 2.

A system with lower capital costs but higher operating costs is shown in Figure 3. Such a system could increase profits even more as water use is reduced to, say, point C. Finally a system with lower capital costs and lower operating costs, but having also a lower system capacity is shown in Figure 4.

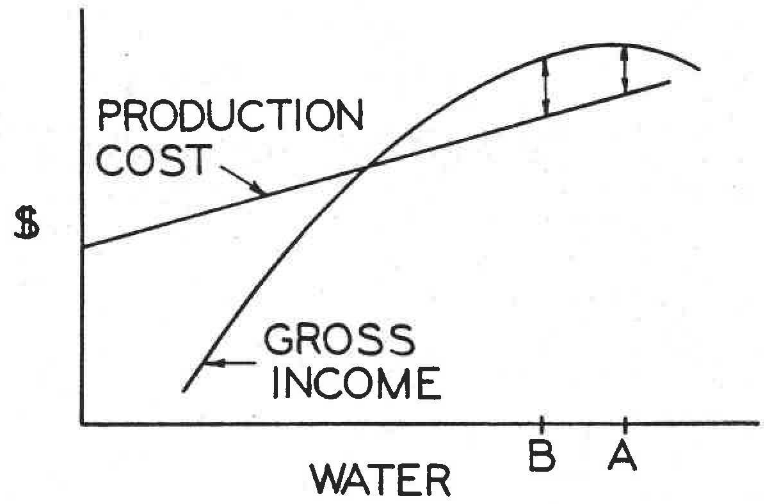

Figure 2. Cost of Production and Gross Income

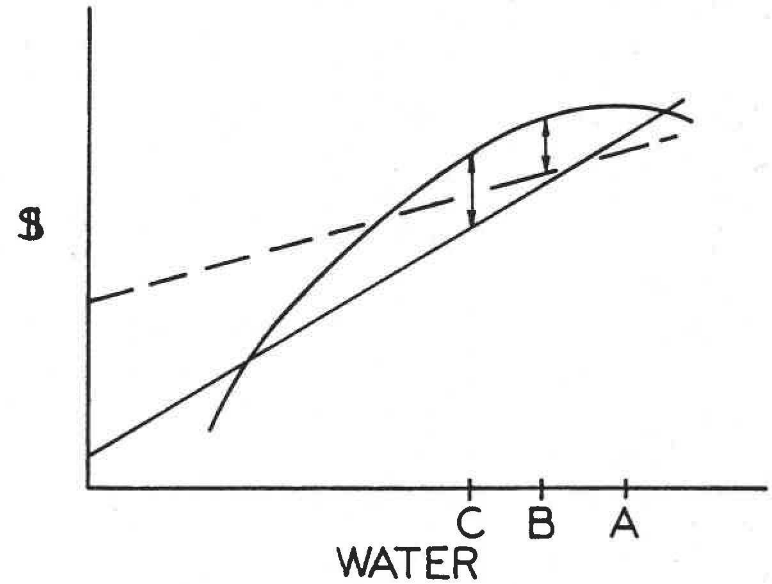

Figure 3. Costs of Two Alternative Systems Designed for Full Capacity

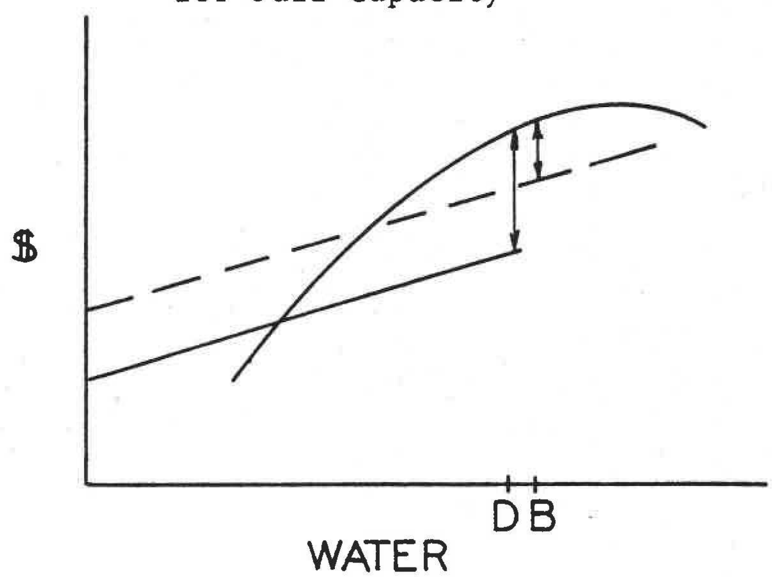

Figure 4. Costs of a Low Capacity System

Rather than simply reducing the amount of water applied by a full capacity system the farmer might elect to use a low capacity system, one that is not capable of fully irrigating the field. Although this means that the farmer has no alternative but to under-irrigate his crop, nevertheless the spread between gross income and irrigation costs may be greatest with this approach. Thus the low capacity system might in fact be the most profitable.

Although standard practice is to irrigate in such a way as to attain maximum yields, these examples indicate that it would be possible to increase income by reducing water use and accepting reduced yields. That increase in income can be even greater if a system designer deliveratety exploits the decision to under-irrigate by designing a system with reduced capacity or slightly lower performance characteristics. This approach to optimum system design is illustrated by a case study presented in Appendix A of this report. In that case study the decision to under-irrigate makes it possible to reduce irrigation adequacy and to irrigate less frequently. These design features result in increased application efficiency, reduced capital costs, reduced labor, maintenance, energy and other production costs, and reduced costs of fertilizer, chemicals and harvesting. The net result is a significant reduction in capital and operating costs, reduced demand for energy and water, and an increase in net farm income.

The illustrations presented above were based on the relationship between total water use and crop yields. A similar argument can be made regarding frequency of irrigation and yields, but the relationship is not so clear cut or well defined. Under full irrigation it may be possible to increase yields by increasing irrigation frequency (Rawlins, 1973). However, under deficit irrigation the relationship between frequency and yields is not well established.

Miller (1979) reported that relatively high yields could be maintained under deficit irrigation by increasing the frequency of irrigations. However Faci and Fereres (1980) were unable to replicate Miller's work and in fact came to the opposite conclusion. An attempt was made to resolve this question for at least one crop, winter wheat, as part of the present study (see Appendix D). That study was conducted similarly to the studies by Miller and Faci and Fereres. It was concluded that high frequency irrigation does not increase yields of winter wheat when deficit irrigation is practiced, except when the intervals between irrigations are extreme (on the order of four weeks).

The relationship between irrigation frequency and system costs can be complex. Rawlins and Raats argued that irrigation costs could be reduced by high frequency irrigation. However their argument was based on the assumption that mainline pipe costs are the dominant factor in system costs. That implies that labor and maintenance costs are not significantly increased as frequency goes up, but such an assumption may not apply to surface systems, wheel lines or hand lines. For example in the case study in Appendix A it is shown that costs of capital, labor maintenance and power could be reduced substantially when wheel lines were used for low frequency irrigation. It seems self-evident that with a furrow system labor savings would be most significant. So, where irrigation frequency is concerned, the relationship between net income and frequency will clearly depend upon the particular circumstances under investigation.

The underlying theme in this discussion has been the use of deficit irrigation to achieve greater income. This concept is founded upon a fundamental axiom of economics; that profits are maximized when marginal costs of production just equal the value of the marginal product. In this case, that means that an irrigator should increase water use until the cost of applying the next increment of water, combined with other production costs, just equals the income from the next increment of yield produced. Thus deficit irrigation is a simple application of a well established principle of economics. Furthermore deficit irrigation should not be thought of an an alternative to full irrigation. Rather it is only a question of what degree of deficit should be attempted. Irrigated fields in the western U.S. are often under-irrigated to some extent. Irrigation systems are commonly designed according to SCS guidelines, which presuppose an irrigation adequacy of about 87.5%. That is, it is assumed that about seven-eighths of the field will be fully irrigated. This implies that one-eighth of the field will be under-irrigated. Individual farmers in some areas of the western U.S. also practice deficit irrigation consciously. The question is not whether or not to adopt this approach to irrigation, but to what degree it should be practiced.

2.2 Risk

Economic analyses of irrigation practices commonly follow a deterministic approach. The yield estimate used in such an analysis will usually be the expected yield based on local experience with fully irrigated fields. The deterministic approach is appropriate when full irrigation is practiced, but if the analyst wishes to consider a reduced level of irrigation it becomes necessary to utilize a crop model that relates water use to crop yield. Such a model introduces a great deal of uncertainty into the analysis.

Figure 5 illustrates the uncertainty of predictions of bean yields (from a study by English, 1981). The variability of estimates of potential yields was simulated for a field in southern Idaho that was to receive 12 inches of water. The term 'potential yield' refers to the yield that would be expected if water were the only limiting factor. (Such things as frosts, hail and soil

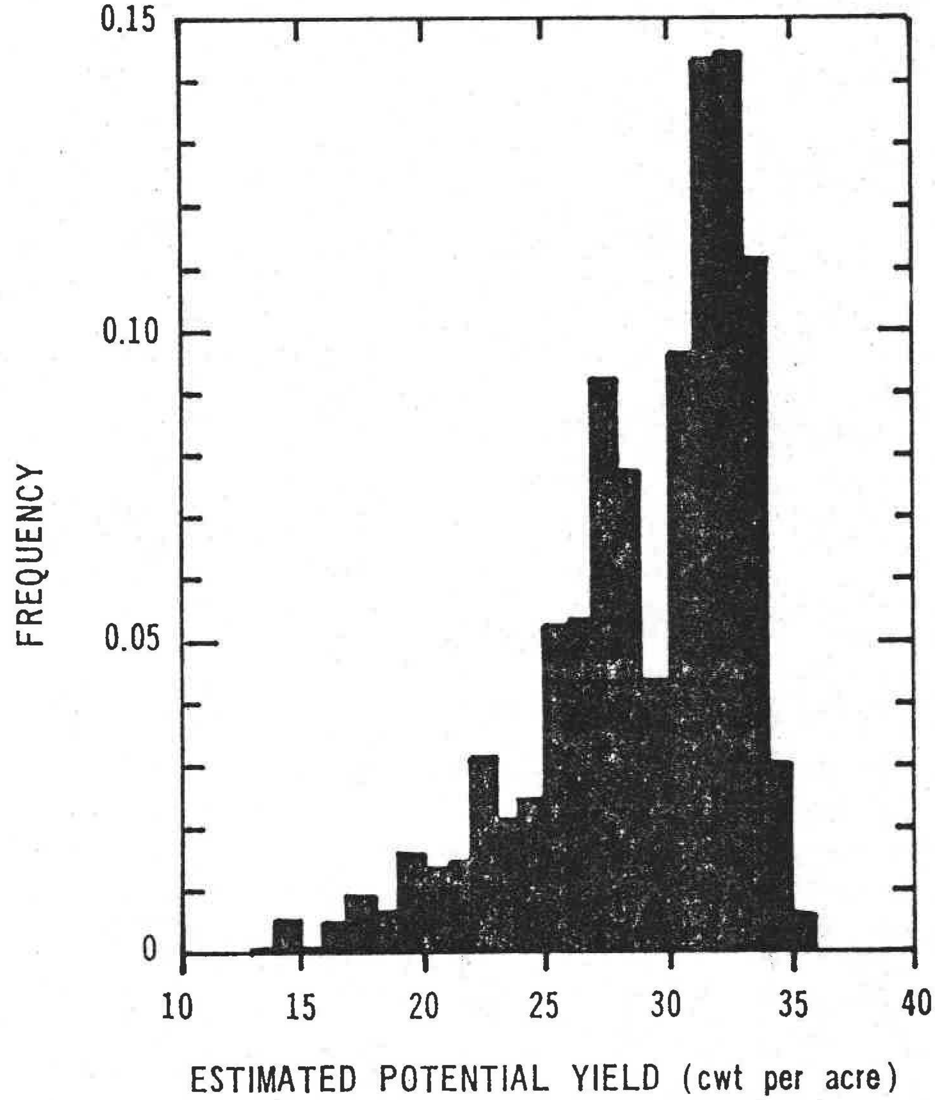

Figure 5 . Histogram of Estimates of Potential Yield for Beans With 12 Inches of Water Alloted.

fertility would add to the uncertainty shown here.) Thus the variability shown in Figure 5 is due to uncertainties associated with the availability of water, including uncertainty of root zone depth and preseason soil moisture, and the uncertainty of the model itself which relates the availability of water to crop yield. As indicated in this example, the statistical distribution of predicted yields ranged from 13 to 36 hundred-weight per acre, with most predictions falling in the range of 25 to 34 cwt/ac.

Anyone who works with agriculture has at least a general awareness of the intrinsic uncertainties of crop yields. But how does this uncertainty influence the economic evaluation of an irrigation system? Uncertainty implies risk. If the risk is great enough a farmer may prefer to follow an irrigation practice that reduces expected net income, if by doing so the risk is also reduced. In a study of this question, English (1981) found that the uncertainty of crop yield predictions is great enough to significantly alter farmers' irrigation practices. To some extent this uncertainty can be mitigated by judiciously combining different irrigation practices in a portfolio of alternative irrigation strategies.

In summary, then, where crop production models are involved, the analyst should be aware that a deterministic analysis may lead to recommendations that are inconsistent with the farmer's preferences. Thus, crop models should be used not only to predict most probable yields, but also to quantify the uncertainty of yield predictions. This could be accomplished either by direct analysis of the statistical characteristics of prediction errors or by employing a deterministic model in a statistical simulation procedure; (i.e. Monte Carlo simulation or stratified sampling).

2.3 Dealing With the Time Value of Money

Investment in an irrigation system involves an initial capital cost followed by a continuous outflow of cash for operations and maintenance, all of which are offset by an income stream from the crop produced. Eventually various system components will need to be replaced, at which time a new capital outlay may occur, offset perhaps by salvage of the old equipment. Algorithms commonly used for economic analyses of this type are the net present value, Equivalent Annual Cost and Internal Rate of Return methods. These methods are all fundamentally sound but there are a few questions concerning the application of these methods that deserve mention.

First is the question of which method to select. All three are mathematically identical; that is, if all inputs and assumptions are identical the three methods will yield identical results. However the net present value method has an important advantage in that all assumptions must be stated unambiguously at the beginning of the analysis, whereas there are implicit assumptions embodied in the other two which, if accepted inadvertantly, can lead to error. In a word, it is easier to make an error in the formulation of the problem when using the equivalent annual cost and internal rate of return methods. In this respect the net present value analysis is most reliable. Nevertheless, the other methods may have special appeal for some individuals; e.g. a farmer who is attuned to annual cash flows may prefer to see an annualized cash flow comparison, and corporations often have a preference for internal rate of return comparisons.

A second question concerns the discount rate that should be used to relate future worth to the present. Charles Howe, an economist at the University of Colorado who has studied the matter of discount rates, refers to it as a 'baffling' question. There are two schools of thought. One school follows an opportunity cost approach; the discount rate should be the rate of interest that could be earned from an alternative investment. This choice of discount rate is commonly used for analysis of private investment decisions. The other school argues that the discount rate is a subjective thing which reflects individual feelings about providing for the future. A person who feels strongly about planning for a secure future or for the well being of the next generation might use a low discount rate. Another person with an itch to buy something today might be unwilling to invest money except at a very high interest rate. That person would want to use a high discount rate. Howe (1971, p. 67) has written a very readable commentary on the two schools of thought.

Another view of discount rates was proposed by English (1965). He suggested that if the discount rate reflects an individual's attitude about the future, it should somehow reflect his perception of time. The argument will not be presented here, but in summary English proposed a variable discount rate that increases with time. Income received ten years in the future might be discounted at twelve percent (for example) while next year's income might be discounted at eight percent.

With respect to irrigation systems in particular, Keller (1983) has argued for an opportunity cost approach with a substantial risk factor included. Specifically, he feels that "the time value of unsecured money to the developer should be used as the appropriate interest rate....This is normally higher than bank interest rates due to the higher risks involved. Returns from agricultural developments should be in the neighborhood of 10% higher than interest rates on high grade, tax-free, long-term securities unless there are some special tax benefits involved."

The opportunity cost approach is appealing because the discount rate is then relatively easy to establish. But, having worked with farmers, how many would believe that opportunity costs are a dominant factor in their decisions. For example, farmers as a group are strongly motivated to own land (Lin, 1974) and that feeling may take precedence over the opportunity to make more money. All of this is to suggest that the discount rate should be chosen with care, and an effort should be made to relate it to the attitudes of individual decision makers.

In some cases it will be possible to avoid the question of discount rates and the choice of a method of analysis by simply presenting a summary of the projected time series of costs and incomes to the decision maker without attempting to calculate any single measure of the worth of the project. Where only a few alternatives are considered this might be a practical approach. The decision maker can then assign to these numbers whatever subjective or emotional weights he chooses.

Another problem area is the way in which inflation is handled. At any given time different commodities will be inflating at different rates. The obvious example is the rapid relative increase in the cost of energy during the past decade. An economic analysis that projects into the future could account for this by using two or three or perhaps several inflation rates. Some analysts have used this approach. For example the Agnet PUMP program allows for a separate inflation rate for energy, and Allen (1982) has incorporated several inflation rates into an analysis of irrigation systems. There is a potential trap here however. We must keep in mind that a constant increase in relative price cannot proceed indefinitely. Sooner or later it must come to some new equilibrium. So if any different inflation rates are to be used for different commodities in an anlysis with a planning horizon more than a few years distant, it would be appropriate to use rates that vary with time.

Tax management may play a significant role in the choice of technology and design for irrigation systems. Researchers often discover that they can better explain or predict farmers' actions using after-tax rather than before-tax net income. Furthermore, changes in tax rules such as the major changes in U.S. tax code made by the Economic Recovery Tax Act of 1981 may significantly alter farm decision making and investment behavior (Boehlje and Carman, 1982).

The Economic Recovery Tax Act replaced the traditional depreciation of assets with the accelerated cost recovery system (ACRS) for assets placed in service after 1980. The ACRS permits more rapid capital cost recovery and substantially increases the present value of income tax deductions when compared to traditional straight-line or accelerated methods of depreciation. These changes will encourage greater investments in irrigation systems by reducing the after-tax cost of ownership. This could promote the adoption of energy and water-saving irrigation system designs and the substitution of capital for labor in the selection of irrigation technology.

The 1981 Tax Act made lease financing more attractive. This means that irrigators without large taxable incomes can also benefit from new tax legislation. In effect, it allows the leasor to pass along the ACRS benefits to the leasee. While the Tax Act of 1982 made some major modifications in leasing provisions of the 1981 Act, those that apply to farmers and ranchers were basically unchanged.

3.

An Algorithm for Optimum Irrigation System Design

Conventional practices for design of irrigation systems involve determining in advance the required specifications of system performance (e.g. gross water to be applied, irrigation frequency, application rate, etc.), then designing a system to meet those specifications. But a design that is truly optimal in the economic sense implies no such *a priori* specifications of system performance. The designer must consider a wide range of alternative performance characteristics. The costs of alternative systems and the crop yields that will eventuate must be estimated. Thus the complexity of the system design problem is increased by an order or magnitude.

The case study in Appendix A demonstrates that, in the search for an optimum design, it is necessary to explicitly account for irrigation adequacy, uniformity and the spatial variability of applied water. Relatively sophisticated models of the interrelationships between irrigation frequency, soil moisture, evapotranspiration and crop yields are required. All costs of capital, operations, maintenance, labor and other field operations must be accounted for.

This section summarizes an algorithm to facilitate the comprehensive evaluation of alternative irrigation systems. The procedure is designed to be flexible, permitting the analyst to exercise judgment or introduce constraints as he deems appropriate. The algorithm is a computer program to evaluate sprinkler system performance; that is, to relate total water use to crop yields for a given application system.

3.1 Irrigation System Performance

Braodly speaking, system performance can be characterized by the relationship between water use and crop yield. This relationship, illustrated by the yield curves in Figures 2, 3 and 4, is relatively difficult to derive. The precise shape of this curve will depend upon the variability of crop, soil and weather and will be influenced by the farmer's irrigation schedule as well.

To accurately relate gross water use to yields it is first necessary to relate gross water use to application efficiency, but application efficiency is a function of system design (e.g. nozzle size, pressure, rate of movement), ambient conditions (weather, crop, soils), and the way the system is operated. Hence one must model the complete soil-water-crop-atmosphere system to evaluate its performance. If an irrigation system with perfect uniformity were used to irrigate a perfectly homogeneous soil, and if all parts of the field were irrigated at the same time, yield would increase more or less linearly with water applied until the maximum yield point was reached. On the other hand, if the uniformity coefficient were less than 100 percent, and if the normal spatial variability of the soil were taken into account, the water use-yield relationship would look more like Figure 1. At low levels of water use the relationship might still be linear. At levels of water use, parts of the field would be fully irrigated while other parts owuld be under-irrigated. To fully irrigate all parts of the field would require that most of the field be over-irrigated. Because of this and other factors, most systems will become progressively less efficient as the level of irrigation approaches the yield maximizing point.

System performance will be defined in this paper as the relationship between gross water use and crop yield, as illustrated by the yield curve in Figure 1. To relate water use to yields it is necessary to evaluate application efficiency, but the application efficiency of an irrigation system will change as circumstances of weather, crop and soils change, and will be effected by the way the system is managed. Most systems will become progressively less efficient as the level of irrigation approaches the yield maximizing point. Hence one must model the performance of the hardware in conjunction with the soil-water-crop-atmosphere system to evaluate its performance.

An algorithm has been developed which models this relationship. Specifically the algorithm models the yield-water use curve between the horizontal axis (zero water use) and the peak of the curve (maximum yield). This algorithm has been incorporated into a computer program. The program is designed to be general; that is, it was not developed with any particular system type in mind. Rather, it utilizes parameters that can characterize any system; these include the statistical distribution of applied water, the frequency and duration of irrigations and the relationship between soil moisture, evapotranspiration and crop yields.

The program begins by setting up a season calendar, including dates of planting, emergence (or end of dormancy), full cover and harvest, along with daily weather data for the season, including reference ET, wind and humidity.

The program then sets up a simulated field with variable soil characteristics. The field is divided into sectors (the number of sectors is specified by the program operator). For each sector a moisture holding capacity and root depth are generated by Monte Carlo simulation. (Monte Carlo simulation is used rather than stratified sampling because it was felt that the number of combinations of random variables involved in this algorithm would be too great to permit exhaustive enumeration at a reasonable cost.) The user may specify a coefficient of correlation between these characteristics as well. In that case the program generates the moisture holding capacity first, then generates a correlated, random root depth. Details of the statistical simulation procedure are presented in Appendix K.

The result is a computer model of a field divided into a number of hypothetical sectors, with randomly varying root depth and holding capacity in each sector, in which the means, variances and correlation of these characteristics are specified by the program user. If an irrigation cycle takes more than one day to complete the program divides the field into sets of sectors, each set being irrigated on a different day.

When an irrigation takes place the program estimates the amount of water lost as wind drift and evaporation based on wind and humidity conditions on the day of irrigation. These losses are assumed to be expressed as a linear function of the form

$$L = a_o + a_1 W + a_2 W^2 + a_3 D + a_4 D^2 \qquad (1)$$

where L = decimal fraction of losses

W = wind speed

D = vapor pressure deficit

a_o, a_1, a_2 = coefficients.

The coefficients in Equation 1 might be derived from Frost and Schwalen (1955) (or some other suitable source) by linear regression, for a specific nozzle diameter and operating pressure. For example a center pivot system with 9/32 inch diameter nozzles and operated at 70 psi would have the characteristics shown in Table 1. (from Frost and Schwalen):

Table 1. Wind and Evaporation Losses for 9/32" nozzle at 75 psi.

Vapor Pressure Deficit (psi)	Wind Velocity (miles/hr)	Losses (%)
0	0	1.2
0	4	2.2
0	10	4.0
0	15	4.8
0.2	0	2.3
0.2	4	4.2
0.2	10	7.0
0.2	15	9.0
0.6	0	4.8
0.6	4	9.0
0.6	10	14.0
0.6	15	18.0
1.0	0	7.0
1.0	4	13.0
1.0	10	20.0
1.0	15	26.0

As an example, the following model was derived by stepwise linear regression based on these data ($R^2 = 0.986$):

$$\text{Percent loss} = 4.78 + 0.377\ W + 0.917\ W^2 + 8.15\ D - 5.58\ D^2 \qquad (2)$$

(where W is in units of miles/hr and D is in units of psi).

To calculate percent loss the above coefficients are read into the computer at the beginning of a simulated irrigation season. On the day an irrigation is to take place the program reads in average temperature, relative humidity and total wind run. Vapor pressure deficit is then calculated by Equation 3.

$$D = e_s(T) \cdot (1.0 - \frac{RH}{100}) \ \ \text{lbs/in}^2 \qquad (3)$$

where RH = relative humidity

$e_s(T)$ = saturation vapor pressure

T = temperature; °C

and

$$e_s(T) = 33.8639 \left[(0.00738T + 0.8072)^8 - 0.000019 \cdot |1.8T + 48| + 0.001316\right] \tag{4}$$

(Equation 4 is from Bosen (1960)).

Since the statistical distribution of applied water (i.e. the uniformity expressed as a statistical distribution) is determined in part by wind conditions, the uniformity coefficient is treated as a polynomial function of wind of the form:

$$U = b_0 + b_1 W + b_2 W^2 \tag{2}$$

where U = uniformity coefficient.

It was assumed that the uniformity coefficient will be defined according to the Hawaiian Sugar Planter's Association definition (Hart and Reynolds, 1969) in which uniformity coefficient is related to the standard deviation of applied water by the equation:

$$S = 1.253\ \bar{x}\ (1.0-U) \tag{3}$$

where S = standard deviation of applied water

$\bar{x}$ = average depth of application

It is often assumed that applied water has an approximately normal distribution. Alternatively a uniform distribution may be used. Hart and Heerman (1976) have argued that a normal distribution is appropriate. Seginer (1978) has recommended a uniform distribution. If a uniform distribution is used the mean and standard deviation are related to the upper and lower bounds of the distribution by the equations:

$$\bar{x} = \frac{a + b}{2} \tag{4}$$

$$S = \frac{a - b}{\sqrt{12}} \tag{5}$$

where a and b represent the upper and lower bounds repsectively. The standard deviation can be determined from the uniformity coefficient using Equation 3. Then a (or b) can be determined from S and $\bar{x}$;

$$a = S \cdot \sqrt{12} + b$$

$$\bar{x} = \frac{S \cdot \sqrt{12} + 2b}{2} = S \cdot \sqrt{3} + b$$

$$b = \bar{x} - S \cdot \sqrt{3} \tag{6a}$$

$$a = \bar{x} + S \cdot \sqrt{3} \tag{6b}$$

The procedure for simulating an irrigation is as follows: The computer program specifies an average application ($\bar{x}$). The program then uses wind data from the weather data file to compute the uniformity coefficient, which is used in turn to compute the standard deviation of applied water by Equation 3. If the statistical distribution is normal it is completely defined by $\bar{x}$ and S. If a uniform distribution is assumed, the parameters a and b define the distribution and must therefore be calculated using equations 6(a) and 6(b). The program then uses Monte Carlo simulation to generate random depths of water applied to each field sector during an irrigation event.

At this point, the program has simulated nonuniform applications of water on an inhomogeneous soil. The effects of weather on spray losses have been preserved as well. Serial correlation of the spatial distribution of applied water can also be simulated if desired. That is, the relative amount of water received in one sector in a given irrigation can be correlated to the relative amounts received during prior irrigations of that sector.

Evapotranspiration (ET) is modelled separately in each field sector. Reference daily ET is multiplied by a crop coefficient for that day to arrive at an estimate of maximum crop ET. This maximum is then adjusted for wet woil surface or dry soil profile conditions using the following simple model suggested by Jensen, et al (1969): The potential rate (PET) is computed by a Penman equation. Actual ET is then

$$ET = k_c \cdot PET \tag{7}$$

in which

$k_c = k_{co} \cdot K_a + K_s$,

k_c = crop coefficient (a time dependent coefficient),

$K_a = \ell n(M_A + 1)/\ell n(101)$,

M_A = percent of available moisture remaining at the time of the estimate,

$K_s = (0.9 - k_{co} \cdot K_a) \cdot 0.8$ on first day after water is applied

$= (0.9 - k_{co} \cdot K_a) \cdot 0.5$ on second day after water is applied

$= (0.9 - k_{co} \cdot K_a) \cdot 0.3$ on third day after water is applied

$= 0$ at all other times

k_c is limited to a maximum value of 1.0, K_a is an adjustment term for low soil moisture conditions, and K_s adjusts for wet soil surface conditions. The parameter k_{co} is usually defined shomewhat ambiguously as the ratio of the actual ET rate to the potential rate for a reference crop under well watered conditions. The adjustments for wet or dry soil are calculated separately for each field sector.

Some ambiguities in the definition of k_{co} arise from the meaning of the potential rate and the determinations of the actual rates from which the crop coefficients were derived. PET may be either the evapotranspiration rate as measured for a reference crop in a lysimeter or the rate calculated by one of several available models (e.g., Penman or Jensen-Haise). The actual ET rates used to derive k_{co} may or may not include evaporation from wet soil surfaces during the period immediately following an irrigation or rainfall. For this study k_{co} was defined as the ratio of actual ET, for a well watered crop with dry surface conditions, to the potential rate for alfalfa calculated by the Penman equation with a modified wind function and with the vapor saturation deficit calculated as the daily mean of the maximum and minimum deficits.

It should be noted that there is no general agreement on a model for simulating ET under low soil moisture conditions. The above expression for K_a represents only one of several expressions that have been proposed for this relationship. In doing the case study in Appendix A this adjustment for low soil moisture conditions was found to be of critical importance. Field experiments were therefore carried out to select an appropriate model for the particular circumstances of the case study (winter wheat in Eastern Oregon). Three models were tried, Jensen's logarithmic model described above, a 'power' model proposed by Boonyathorokul and Walker (1979) and a combination model used by various individuals (see Feddes, et al, 1980). The combination method is described and used in the case study (Appendix A).

The results of the field experiments indicated that the logarithmic and combination models worked reasonably well. However it was concluded that there is a great deal of room for improvement in this area. The full details of this field research are presented in Appendix D.

Finally, the program must simulate irrigation scheduling decisions. Two alternatives for selecting the date of the next irrigation are avialable. One option is to irrigate on pre-arranged dates. The other option is to irrigate at specified soil moisture depletion levels. In the second option the program user stipulates a field sector that is to be used as an indicator for purposes of scheduling irrigations. When calculated soil moisture in that sector reaches a specified critical level the program carries out a simulated irrigation. The sector is chosen according to moisture holding capacity; for example if the 80th percentile is specified the sector in which the total moisture holding capacity is closest to the 80th percentile value of all sectors is chosen.

The program also allows for two ways of determining the amount of water to be applied. The simplest way is to irrigate with fixed amounts. The alternative method allows the user to specify a percentage of soil moisture depletion which will be replaced.

Crop Yields and Gross Income

Crop yields are assumed to be a polynomial function of season total ET. This assumption has seen common use in crop production modeling J. Doorenbos, et al (1979). At the present state of development of the model there are no provisions for dealing with time-critical stages of growth. As noted by Doorenbos and Kassam, if crop stresses are distributed evenly throughout the season a single production function can be used as a reasonable approximation of the relationship between ET and yield.

The assumption that a single crop production function can be used implies that irrigation frequency will not be a significant consideration. The field experiments discussed in the appendices of this report indicated that there is nothing to be gained by high frequency irrigation of winter wheat. On the other hand, if the farmer irrigates infrequently, either by choice or because the irrigation system cannot get across the field quickly enough, it will be difficult to estimate yields because irrigation timing and the critical stages of growth would need to be accounted for. But consideration of the ways in which irrigation would be practiced in the real world leads to assumptions that make the analysis simpler and more manageable. If low frequency irrigation were a matter of choice, one can assume that the farmer would attempt to irrigate at favorable times. In that case, yields should at least not be excessively reduced by low irrigation frequencies. However, it would be unlikely that the farmer would be able to irrigate the entire field at exactly the right time for maximum yields. In consequence, crop yields would represent some sort of average between the possible extremes. Alternatively, if a low irrigation frequency were dictated by system limitations, some parts of the field would probably be irrigated at favorable times while other parts would be irrigated at unfavorable times. In that case, yields would again fall somewhere between the extremes. So even though yields can be significantly affected by the timing of irrigations, one might reasonably assume that in practice the average yields will be close to the median of potential yields.

Season total ET is calculated separately for each sector of the field. The resulting yield is then calculated for each sector and, finally, a field-wide average yield is calculated. Gross income is then a simple multiple (crop price) of the average yield.

Summary

In principle, the profits to be realized from an irrigated field may be increased by under-irrigating the crop to some extent. This is due to the fact that operating expenses will generally exceed revenues from the last increment of attainable yield.

If the system is deliberately undersized, fixed costs can also be reduced, and net income will then be further increased. However, selecting an optimum design for such a system will be difficult. The design must account for the complex interactions that ultimately link system performance to crop yield.

An algorithm has been proposed which can be used to generate a yield-water use curve such as was illustrated in Figure 1. The algorithm accounts for the spatial variability of soil characteristics, the irrigation system perpormance characteristics, the variability of seasonal weather and the irrigaton management plan. By incorporating these factors into the analysis the algorithm can estimate the spatial distribution of yields and ultimately average yield, for a given level of water use and specified management plan.

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APPENDIX A

DESIGNING FOR DEFICIT IRRIGATION

Designing for Deficit Irrigation

By M. J. English and G. S. Nuss[1/]

Introduction

This paper is concerned with optimum irrigation practices. In theory, optimum practice implies some degree of deficit irrigation (deliberate underirrigation of a crop). This is because the marginal income from the last increment of attainable yield produced by full irrigation will generally not be as great as the last increment of production costs.

Nevertheless, the standard practice has been to irrigate for maximum crop water requirements, and in doing so to strive for maximum yields. Implicit in this practice is the assumption that the increased income that might be realized through deficit irrigation does not justify the required effort or the risk involved. This implicit assumption may be more widely accepted than it should be, perhaps in part because the marginal costs involved in irrigation agriculture are not fully appreciated by system designers. Deficit irrigation implies reduced expenditures for water, and perhaps for energy as well. But it is also possible to reduce marginal capital costs, production costs and opportunity costs by designing irrigation systems specifically for deficit irrigation. In some situations these design-related savings can be substantially greater than the savings in water and energy costs, as will be demonstrated in a case study in this paper.

1/ The authors are Associate Professor and past Graduate Research Assistant, respectively, Department of Agricultural Engineering, Oregon State University, Corvallis, Oregon.

This paper presents an approach to irrigation system design that is tailored explicitly for deficit irrigation. An example is included which demonstrates the design concepts and provides a perspective on the economics of deficit irrigation. In the particular circumstances of the example, deficit irrigation would allos the farmer to greatly reduce his use of energy and water without loss of income, or alternatively, to increase his income without significantly increasing his use of energy and water.

The key features of the design presented in this paper are the following: 1. A lateral system capable of being moved over an arbitrarily large area was specified. A side-roll system was chosen for this purpose. Such a system makes it possible to reduce capital costs per unit of land area by increasing the area irrigated with each lateral. 2. The return interval (the minimum time required for the moveable system to return to a designated part of the field) was extended well beyond the interval time that would normally be allowed under full irrigation. By extending the return interval the designer can realize the potential capital savings mentioned and can reduce labor and maintenance costs as well. 3. The soil moisture uniformity specified in the design was well below that which would normally be specified for full irrigation. By allowing a lower soil moisture uniformity the designer can achieve a higher application efficiency, with consequent reductions in all categories of costs.

These departures from standard design practices become feasible when deficit irrigation is practiced. The decision to underirrigate a crop implies a willingness to accept some degree of crop stress. Long intervals between irrigations and reduced soil moisture uniformity will both result in increased crop stress. However, these practices also imply increased flexibility in system design. The designer can exploit this flexibility to gain the fullest possible advantage from the practice of deficit irrigation.

The design procedure presented in this paper will not be appropriate in all circumstances. For example, in a situation in which a center pivot system is required the advantages of long intervals between irrigations would be lost. Nevertheless, this analysis should be of general interest because it demonstrates that the economic merits of deficit irrigation may be largely dependent on innovative system designs.

Assumptions and Relationships

Key assumptions and relationships used in the case study are outlined and reviewed in this section. The emphasis is on estimation of evapotranspiration and crop yields under stress conditions, the effect of irrigation interval on yields, and the relationship between uniformity, adequacy and application efficiency.

Irrigation Interval, Evapotranspiration and Crop Yield. Estimated evapotranspiration (ET) was used in the analysis both to determine irrigation requirements and to estimate crop yields. The relationship between the actual ET experienced by the crop and the maximum possible ET varies with climatic conditions, crop development, and soil moisture. Actual evapotranspiration will proceed at approximately the maximum rate until perhaps half of the available soil moisture has been depleted. Beyond that point actual evapotranspiration will fall progressively father behind the maximum rate. For purposes of this analysis it was assumed that the ratio of actual evapotranspiration (ET_A) to maximum possible evapotranspiration (ET_M) will equal 1.0 until 50% of the maximum available soil moisture has been depleted. Then it will decline linearly from 1.0 to 0 as available moisture approaches exhaustion.

Based on this simplified model a relationship between time and soil moisture depletion was derived for that period of the irrigation season when the highest ET demand occurs. If ET_M is assumed constant for a period of several days, the elapsed time until 50% depletion (T_{50}) occurs will be

$$T_{50} = \frac{M_o}{2 \times ET_M} \tag{1}$$

in which M_o = initial available soil moisture. After the 50% depletion level has been reached the relationship between soil moisture depletion and ET_A will be defined by the equations

$$\frac{dM}{dT} = -ET_A \tag{2a}$$

$$ET_A = ET_M\left(\frac{M}{0.5M_o}\right) \tag{2b}$$

in which M = available soil moisture at time T. Cumulative ET_A can therefore be expressed as the following function of time ($T>T_{50}$):

$$\int^T ET_A \, dT = M_o - (1/2)M_o e^{-k(T)} \qquad (T>T_{50}) \tag{3}$$

$$\text{in which } k(T) = \frac{2ET_M}{M_o}(T-T_{50}) \qquad (T>T_{50})$$

Estimated cumulative ET can be used in turn to estimate yields. A close relationship between crop yields and evapotranspiration has been established by a number of researchers. Fig. 1 presents an ET-yield relationship in which actual yield, Y_A, expressed as a percentage of maximum yield, Y_M, is related to ET_A, and is expressed as a percentage of ET_M. Fig. 1 is also derived from a general relationship for winter wheat that was developed by Doorenbos and Kassam (1) from worldwide research on wheat yields. This functional relationship assumes an irrigation schedule in which ET deficits are uniformly distributed throughout the season.

Irrigation Frequency/Return Interval. The terms irrigation frequency and return interval are used interchangeably in this section to refer to the minimum time between irrigations. As mentioned earlier, return interval may be closely related to irrigation costs. Rawlins and Raats (6) pointed out certain benefits to be gained by high frequency irrigation with pressurized systems. They observed that the capital costs of such systems depend largely upon pipe size, which in turn depends upon water delivery rate. Delivery rate, and therefore capital costs, can be minimized by designing the system for continuous operation. Furthermore, they pointed, out, uniform, frequent irrigations may optimize the root environment while reducing water use. Thus, maximum attainable yields could be realized with a minimum of water.

These conclusions were predicated on several implicit assumptions: that the crop is to be fully irrigated, that pipe cost is the dominant component of system cost, and that labor and maintenance costs will not increase significantly with high frequency irrigation. For a center pivot or permanent system these assumptions would be appropriate. However, there are other situations in which these

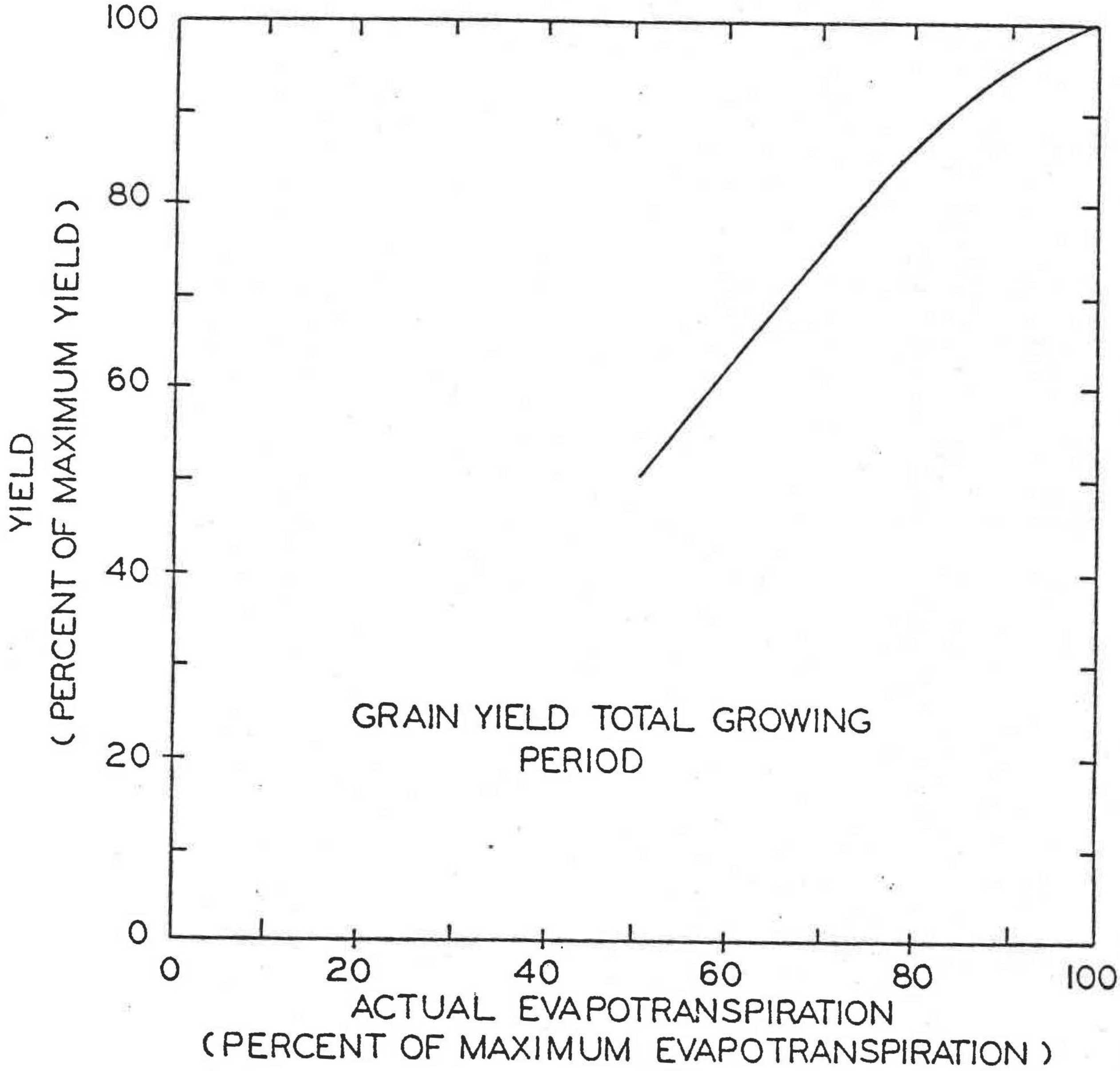

Figure 1. Relationship between season total evapotranspiration and crop yield, for winter wheat (adapted from Doorenbos and Kassam, 1979).

assumptions do not apply. Irrigation with side roll laterals is an example. At high irrigation frequencies more laterals may be required to cover a field, implying higher capital and maintenance costs, while more labor will be needed to move the laterals. In that case, reduced capital, maintenance and labor costs might be possible by designing the system for long intervals between irrigations. Such a system would also be operated continuously. Thus, mainline pipe costs would be minimized.

As noted by Rawlins and Raats, high frequency irrigation may have a beneficial effect on yields under full irrigation. However, when deficit irrigation is practiced it is not at all certain how the length of the intervals between irrigations will effect yields. With high frequency deficit irrigation, water will be applied frequently but in amounts too low to prevent the decline of soil moisture. Soil moisture therefore will fall to a level at which the crop will experience moderate stress more or less continuously. With low-frequency deficit irrigation on the other hand, soil moisture will fluctuate within a wider range. A heavy irrigation will be followed by a long period of extraction, during which the stress experienced by the crop will range from none at all to severe. A subsequent full irrigation will then refill the profile and the pattern will be repeated.

Whether yields will be significantly different under these two regimes is an essential question. Unfortunately research on this question has been inconclusive. Miller (5) reported relatively good yields of sugar beets, wheat and beans in a high-frequency deficit irrigation experiment. According to Miller the high irrigation frequencies appeared to mitigate the effects of water deficits. However, other researchers have arrived at an opposite conclusion. Fereres et al. (3) found that yields produced under deficit high-frequency irrigation were the same as, or lower than, yields produced under normal frequencies with the same levels of deficit. The lower yields reported under the high-frequency regime were attributed to a lower application efficiency associated with that regime.

For the moment, the effect of irrigation frequency on crop yields under deficit irrigation must be regarded as uncertain. A research program addressing this question has begun at Oregon State University. Similar to the work done by Fereres, the research involves side-by-side deficit irrigation experiments conducted at several different irrigation frequencies. For purposes of the analysis presented in this paper it was assumed that a moderately low irrigation frequency would at least not compound the adverse effects of water deficits on crop yields.

Soil Moisture Uniformity. The efficieincy with which an irrigation system stores water in the root zone (water storage efficiency) is largely dependent on two design parameters. One of these is the uniformity of the pattern of applied water, which is usually represented quantitatively by the uniformity coefficient. The other is the percentage of the total field area in which soil moisture is refilled to field capacity during an irrigation which is represented quantitatively as irrigation adequacy.

Shearer (7) has analyzed the relationship between uniformity, irrigation adequacy and water storage efficiency. As he pointed out, if irrigation adequacy is reduced application efficiency can be increased. Shearer's argument is shown by Fig. 2, which represents the spatial distribution of applied water for a situation in which the net water requirement is 10 cm. A normal distribution is assumed for the pattern of applied water, the uniformity coefficient is 85% and the adequacy of irrigation is either 87.5% (Fig. 2a) or 50% (Fig. 2b). The steps represent 10% increments of field area. The horizontal axis indicates the percentage of the field area receiving at least the depth of water indicated. The shaded area above the 10 cm line denotes water applied in excess of the holding capacity of the soil. The shaded area below the 10 cm line denotes soil moisture deficit.

In the example shown in Fig. 2, water storage efficiency can be increased from approximately 77% to 92% by reducing the adequacy from 87.5% to 50%. This increase would be due to reduced deep percolation. Tabulated values of deep percolation for different uniformity coefficients and irrigation adequacies are presented in Table 1 (from Shearer).

If some part of a field is repeatedly underirrigated, no leaching will occur and a salinity problem could develop. This could be avoided with a side roll system by using lateral offsets to shift the pattern of applied water with each irrigation.

It is common practice for engineers to use Soil Conservation Service (SCS) recommended application efficiencies in designing irrigation systems. SCS recommendations are based on the assumption that the average depth of water applied to that quarter of a field receiving the least water will just equal the average soil

TABLE 1. Relationship of Uniformity and Adequacy of Irrigation to Deep Percolation Losses (From Shearer, 1978)

Uniformity coefficient (%)	Deep Percolation as Percent of Total Infiltration		
	50% adequacy	75% adequacy	87.5% adequacy
60	20	41	62
70	15	31	46
80	10	21	31
90	5	10	15

moisture deficit. This is roughly equivalent to stipulating an adequacy of 87.5% (i.e., 87.5% of the field will received enough water to refill the profile). The SCS also recommends a minimum uniformity coefficient of 85% for design purposes. Thus an irrigation adequacy of 87.5% and a uniformity coefficient of 85% can be regarded as typical design practice.

Case Study

The setting for the case study was a farm in eastern Oregon which encompasses approximately 500 acres (200 ha) of fully irrigated land and 2,400 acres (1,960 ha) of dryland wheat and barley. It has been estimated that the output of wells on the farm could be increased by 800 gallons per minute (3,000 l/min). The additional water is to be used to irrigate winter wheat in a field that is 2,800 ft long and 1,415 ft wide (92 acres). This will involve lifting the water from a depth of 200 ft (61 m) and moving the water approximately 0.5 mi (0.41 km) overland. All circumstances of this analysis were real except the well depth. The actual well depth of the farm in question is about 23 ft (7 m); but a deeper well was postulated in order to examine the significance of energy costs in deficit irrigation.

Two alternative systems were designed to irrigate the 92 acres (37 ha) field. One system was designed for full irrigation, the other for deficit irrigation.

Design for Full Irrigation. The configuration of the full irrigation system was based on standard design procedures. It was specified that available soil moisture in the root zone would not be depleted by more than 50% between irrigations during a period of maximum evapotranspiration demand. The soil in the study area is a loamy fine sand. The nominal available water when the soil is at field capacity is 1.5 in per ft (12.5 cm per m). The nominal effective root depth is 24 in (61 cm). Field capacity and root depth estimates were taken from SCS publications for the study area. The authors did not attempt to refine these estimates. Average daily ET demand for winter wheat during a six day interval of peak demand was estimated to be 0.28 in per day (0.70 cm per day). At this rate soil moisture depletion would reach 50% (1.5 in) in six days. The system was therefore designed to irrigate the entire field to a depth of 1.5 in (3.8 cm) within a six day period. A uniformity coefficient of 85% and an irrigation adequacy of 87.5% were stipulated. Using these figures and taking local winds into consideration an application efficiency of 65% was estimated for design purposes. This efficiency is recommended by the SCS for sprinkler systems in the study area. The gross water requirement would therefore be 2.3 in (5.8 cm) per irrigation.

The farmer prefers not to move laterals more than twice daily. The system was therefore designed for 11 hour sets, allowing one hour per set for lateral movement and system maintenance. The soil intake rate is 0.8 in/hr (2.0 cm/hr), so the required application rate (2.3 in in 11 hr) would be feasible.

Seasonal evapotranspiration for winter wheat in the study area averages 26 in (66 cm). Approximately 5 in (13 cm) is supplied by winter and spring rains (assuming 60% effective rainfall), leaving a net requirement of 21 in (53 cm). Using the 65% application efficiency, a gross seasonal water requirement of 32 in (81.5 cm) was calculated.

In summary, the irrigation schedule specified for the full irrigation system was:

1. Two sets per day (11 hours each set).
2. 2.3 in (5.8 cm) gross water applied per set.
3. 32 in (81.5 cm) gross water applied per season.
4. A six day minimum interval between irrigations.

The system was designed with four aluminum laterlas, each 1,400 ft (427 m) long and 4 in (10.2 cm) in diameter, operating at 50 psi (345 kPa). Lateral wheels 72 in (1.82 m) in diameter were selected to provide adequate clearance of the crop without encountering undue wind forces on the wheel line. A springkler spacing of 40 ft (12.2 m) was selected, the sprinklers having a nozzle diameter of 0.172 in (4.36 mm), and a capacity of 5.7 gpm (21.5 l/min).

The mainline was designed with 1,854 ft (565 m) of 8 in (20.3 cm) diameter PVC pipe, 361 ft (110 m) of 6 in (15.2 cm) diameter PVC pipe, and 361 ft (110 m) of 5 in (12.7 cm) diameter PVC pipe. Twenty-four openers, spaced 60 ft (18.3 m) apart on the mainline, would provide connections for the laterals. The flow rate calculated for full irrigation was 795 gpm (3,010 l/min). Total dynamic head required at the pump would be 370 ft (113 m). A 125 hp (94 Kw) motor was selected for the system. A 28 stage vertical turbine pump was designed, with an 8 in (20.32 cm) pipe, a 2 in (5.08 cm) tube and a 1.2 in (2.95 cm) shaft. A 200 ft (61 m) bowl depth was specified.

The configuration and operating characteristics of the full irrigation system are summarized in the first column of Table 2.

Design for Deficit Irrigation. The procedure followed in designing the alternative system departed from standard procedures in two ways, as outlined earlier in this paper. First, the specified

TABLE 2. System Configuration and Operating Characteristics

	Full Irrigation System	Deficit System
Number of laterals	4	2
Mainline spacing of risers	60 ft (18.3 m)	60 ft (18.3 m)
Lateral dimensions	4 in x 1,400 ft (10.2 cm x 427 m)	4 in x 1,400 ft (10.2 cm x 427 m)
Number of sprinklers per lateral	35	35
Sprinkler head spacing	40 ft (12.2 m)	40 ft (12.2 m)
Sprinkler nozzle diameter	0.172 in (4.36 mm)	0.187 in (4.76 mm)
Nozzle flow rate	5.7 gpm (21.5 (ℓ/min)	7.2 gpm (27.2 ℓ/min)
Acres irrigated	92 ac (37 ha)	92 ac (37 ha)
Average pumping lift	200 ft (61.0 m)	200 ft (61.0 m)
Mainline head loss	49.5 ft (15.1 m)	25.9 ft (7.9 m)
Total dynamic head	370.7 ft (113 m)	367.5 ft (112 m)
Pump motor rating	125 Hp (94 Kw)	75 Hp (56 Kw)
Pumping rate	795 gpm (3,010 ℓ/min)	506 gpm (1,914 ℓ/min)
Annual hours of pumping	1,690	1,670
Annual water use	245 ac-ft (302,000 m^3)	155 ac-ft (191,000 m^3)
Annual energy use (Kwh x 10^3)	140.9	85.4

minimum interval between irrigations was increased from six days to 12 days. Soil moisture depletion would therefore exceed 50% of available water during times of peak demand. Secondly, an irrigation adequacy of 50% was stipulated, rather than the 87.5% adequacy used in standard design practice, which would result in less uniform soil moisture conditions.

As shown in the development of Eq 3, the fact that soil moisture depeletion will exceed 50% implies that actual ET will be less than maximum ET. Based on the specified root depth and moisture holding capacity, M_o will be 3 in (7.5 cm). The estimated maximum Et rate for a six day interval (0.28 in/day) was not used in designing for 12 day intervals because the longer interval between irrigations will result in a somewhat lower average ET for the peak period. Estimates of potential ET based on weather data in the study area indicate the maximum 12-day average ET rate will be approximately 6% lower than the six-day rate, or approximately 0.26 in/day (0.66 cm/day) for wheat. Thus, T_{50} will be approixmately six days. Using these numbers in Eq. 3, the expression for ET_A becomes:

$$\int^{T} ET_A = 3.0 - 1.1\, e^{-0.173(T-6)} \qquad (T > T_{50}) \qquad (4)$$

Based on Eq. 4, the available moisture remaining at the end of a 12-day interval during a period of peak demand will be 0.5 in (1.35 cm). Total depletion therefore would be 2.5 in (6.25 cm), which represents a 21% ET deficit. It was postulated that the system would be managed in such a way that this deficit level would be reached before each irrigation. Thus, allowing for the 21% ET deficit, seasonal water use would be 20 in (52 cm), of which 5 in (13 cm) would be supplied as precipitation.

Designing the system for 50% adequacy would result in a higher application efficiency as shown by Fig. 2a and 2b. After allowing for wind and evaporative losses it was estimated that a water storage efficieincy of 78% would be achieved, contrasting the 65% efficiency anticipated for the full irrigation system with an adequacy of 87.5%. Gross irrigation water requirement would therefore be 20 in (50.2 cm). In summary, the irrigation regime specified for the deficit system was:

1. Two sets per day (11 hrs each set).
2. 3.15 in (8.0 cm) gross water applied per set.
3. 20 in (50.2 cm) gross water applied per season.
4. A 4.12 day minimum intervals between irrigations.

Because the minimum irrigation interval is 12 days rather than six, the field could be irrigated with two moving laterlas rather than the four required for full irrigation. The two laterls were designed to operate at 45 psi (310 kPa) with 0.187 in (4.76 mm) diameter nozzles, and a 7.2 gpm (27.2 l/min) capacity. Sprinkler spacing of 40 ft (12.2 m) was also specified.

The mainline was designed with 2,215 ft (675 m) of 8 in (20.3 cm) diameter PVC pipe, and 718 ft (219 m) of 5 in (12.7 cm) diameter PVC pipe, with 24 lateral risers spaced 60 ft (18.3 m) apart. Lateral offset lines of 25 ft (7.6 m) were specified to insure that excessive soil moisture deficits would not develop in any part of the field. Total dynamic head at the pump then was calculated to be 367 ft (112 m). The total flow for the deficit irrigation system would be 506 gpm (1,915 l/min), rather than the 795 gpm (3,010 l/min) required for full irrigation. Accordingly, the specified motor rating was 75 hp (56 Kw). A 21 stage pump, with 6 in (15.24 cm) pipe, 1.5 in (3.81 cm) tube, and a one-inch (2.54 cm) shaft was designed. The bowl depth was assumed to be the same as that for the fully irrigated case, which is 200 ft (61 m) long.

The configuration and operating characteristics of the deficit irrigation system are summarized in the second column of Table 2.

Estimation of Crop Yields. Maximum attainable yield was assumed to be 80 bu/a (5,360 kg/ha), the yield expected from fully irrigated winter wheat on the farm involved in the study. Yields from the deficit system were calculated from the anticipated ET deficit using the relationship in Fig. 1.

Design of the deficit irrigation system was based on an estimated ET deficit of 21%. If this deficit were evenly distributed with respect to time, and uniformly distributed over the field, there would be a corresponding 16% deficit in yields, according to Fig. 1. However, the deficit system design was also based on 50% irrigation adequacy, which implies a high degree of spatial variability in ET. This would be mitigated by the use of lateral offsets, but some spatial variability of ET will remain. The offset which this variability could have on yields was estimated as follows. The curve shown in Fig. 1 was approximated by a quaradtic function of the ET ratio of the form

$$Y = g(R) = a_0 + a_1 R + a_2 R^2 \quad \text{(bu/ac)} \tag{5}$$

in which R representes the ration ET_A/ET_M. If R is assumed to be a random variable, the expected yield will be

$$E(Y) = \int_{-\infty}^{\infty} g(R)\ f(R)\ dR \quad \text{(bu/ac)} \tag{6}$$

in which f(R) = the probability density function of R. If g(R) is approximated by a Taylor series expansion and the expression is integrated term-by-term the expected yield becomes

$$E(Y) = g(\bar{R}) + a_2 \, Var(R) \qquad (bu/ac) \qquad (7)$$

in which $\bar{R}$ and Var(R) represent the expected value and variance of R, respectively.

Assuming a maximum yield of 80 bu/ac (5,360 kg/ha), the following quadratic function was fit to the curve in Fig. 1 for the interval 0.6 is less than or equal to R is less than or equal to 1.0.

$$E(Y) = -115.5 + 370.3R - 174.7 \; R^2 \quad (bu/ac) \qquad (8)$$

Combining Eq. 7 and 8, the expression for espected yield is

$$E(Y) = -115.5 + 370.3 \; R - 174.7 \; R^2 + Var \; (R) \quad (bu/ac) \qquad (9)$$

The average yield for the entire field can therefore be estimated from the mean and variance of the ET ratio.

If lateral offsets are used it is reasonable to assume that the pattern of applied water would be reversed with each irrigation. Hart and Hermann (4) have presented an analysis of patterns of applied water, in which a normal statistical distribution was assumed. Following their analysis, allowing for the effects of the lateral offsets and using Eq. 3 to estimate soil water uptake, the mean and variance of R for the deficit system would be 0.758 and 0.0040. Predicted yield, from Eq. 9, would then be 64.1 bu/ac (4,311 kg/ha).

Economic Comparison. Annual system costs were calculated in 1979 dollars using the PUMP program of the AGNET computer system at the University of Nebraska. The electrical energy cost used in the analysis was 3.0 cents per Kwh. An electrical connection charge of $3.73 per year per Kw was postulated. The inflation rate was assumed to be 15% for electrical energy and 10% for all other factors. Capital investment on 12% interest was anticipated by the farmer for this investment. Fuel for the lateral drive units was priced at $1.90 per gallon ($0.50 per liter). A pump efficiency of 70% and an 88% motor efficiency were assumed. Labor costs were set at $4.50 per hour for field labor and $10.00 per hour for repair and maintenance of the system.

Estimated capital investment for each system is summarized in Table 3. Total annual costs, including capital and operating costs, are summarized in Table 4.

Costs of land preparation, harvest, and other operations are closely linked to anticipated yields. Estimates of these costs for irrigated winter wheat in the study area have been prepared by the Oregon State University Extension Service for yields of 45, 60, and 85 bu/ac. These are summarized in Table 5. The fitures in Table 5 were adjusted to account for inflation from 1978 to 1979, and production costs of $130 and $112 per acre were then estimated by interpolation for the full irrigation and deficit irrigation schemes, respectively. The market price of wheat was assumed to be $4.10 per bushel (0.15 per kg).

An economic summary of the full irrigation and deficit irrigation systems is presented in Table 6. As indicated, gross income with the deficit system would be $6,035 less than the full irrigation system and total production costs would be $6,210 less. Using deficit irrigation the farmer would therefore realize and increase of $175 in net income above what he would expect from full irrigation.

TABLE 3. Capital Investment

Item	Configuration	Cost
I. Full irrigation system		
Pump	125 Hp motor, 28 stage turbine pump, 200 ft column	$23,887
Mainline	PVC pipe, 24 lateral risers, PVC check valve, gate valves, pressure relief valves, system drains, suction tube, thrust blocks	14,390
Laterals	4 laterals, 1,400 ft x 4 in aluminum, 35 sprinkler heads on each lateral, 76 in wheels	18,120
Installation	Excavation, laying and backfilling of mainline	2,830
Total		$59,227
II. Deficit irrigation system		
Pump	75 Hp motor, 21 stage turbine pump, 200 ft column	$16,516
Mainline	PVC pipe, 24 risers, gate valves, pressure relief valve; system drains; concrete thrust reinforcement	15,880
Laterals	2 - 1,400 ft x 4 in aluminum wheel lines, 35 sprinkler heads on each, 76 in wheels	9,060
Installation	Excavation, laying and backfilling of mainline	2,830
Total		$44,286

TABLE 4. Summary of Annual Irrigation Costs

	Full irrigation system	Deficit irrigation system
1. Equivalent annual capital cost		
Pump column assembly	$ 2,761.34	$ 1,909.25
Mainline	1,582.90	1,747.24
Laterals	2,295.20	1,147.60
Installation	311.30	311.30
Total capital costs	$ 6,950.74	$ 5,115.39
2. Taxes and insurance		
1 percent of investment	$ 592.27	$ 442.86
3. Total equivalent annual fixed costs	$ 7,543.01	5,558.25
4. Annual operating costs		
Pumping energy	$ 4,227.00	$ 2,562.00
Pump maintenance	507.58	437.42
Laterals maintenance	621.92	307.28
System operations labor	538.00	269.10
Electrical connection charge	625.00	375.00
Total operating costs	$ 6,519.50	$ 3,950.80
5. Total annual system costs	$14,062.51	$ 9,509.05

TABLE 5. Cultural, Harvest and Miscellaneous Costs of Wheat Production (Dollars per Acre)

	45 bu/ac	60 bu/ac	85 bu/ac
I. Irrigated wheat (1978 prices)			
1. Cultural practices			
Chisel	3.27	3.27	3.27
Disk	3.79	3.79	3.79
Fertilize	15.96	23.96	35.36
Disk and pack	4.52	4.52	4.52
Drill	9.81	10.71	13.14
Herbicide	4.32	4.32	5.50
2. Harvest			
Combine	12.95	17.27	21.58
Haul	3.60	4.80	6.80
3. Other charges			
Truck and other machinery	7.60	7.60	7.60
Pick-up	6.52	6.52	6.52
Crop insurance	1.80	2.40	3.40
Int. on op. cap.	2.13	2.50	3.60
Management	4.50	5.30	6.80
Overhead	3.40	4.00	5.10
Total	82.04	100.96	123.38

TABLE 6. Economic Comparison of Systems

	Full irrigation system	Deficit Irrigation system
Annual irrigation costs		
1. Fixed annual costs	$ 7,543	$ 5,558
2. Operating expenses	6,520	3,951
Other production costs	11,960	10,304
Total cost of production	$26,023	$19,813
Gross income	30,176	24,141
	$ 4,153	$ 4,328

Differences in system costs are presented in detail in Table 7. The third column in Table 7 shows the savings (or added costs) of the deficit system. The lower pumping energy used by this system would save $1,665, which represents 27% of the $6,209 in total savings. Reduced fixed costs (capital investment, taxes, insurance, and electrical hookup charge) would account for a greater share of the savings (36%). Reduced cultural maintenance and labor cost reductions would account for the largest share of overall savings (37%). Table 7 also demonstrates the importance of considering all marginal costs when evaluating deficit irrigation strategies.

Opportunity Cost of Water. The water saved through deficit irrigation of the 37 ha field could be used to put additional land into production. The foregoing analysis was repeated based on this possibility. It was estimated that the irrigated area could be increased from 92 ac to 143 ac (37 ha to 58 ha). Net farm income therefore would be increased 42% as a result.

Capital costs for the deficit system would increase substantially with the increased acreage, due in part to increased mainline costs for pipe to reach the additional land, and in part to the added cost of two more laterals. Energy costs would be 3.7% greater than for the full irrigation case because of added head losses in the longer lines. Table 8 provides an irrigation cost comparison for full irrigation of 92 acres (37 ha) and deficit irrigation of 143 acres (58 ha).

The cost of land preparation, fertilizer use, and other costs enumerated in Table 5 would also increase. As the area of irrigated land is expanded and total wheat production is increased these costs would rise from $11,960 for 92 acres (37 ha) to $16,016 for

TABLE 7. Production Costs

	Full irrigation system	Deficit irrigation system	Difference
1. Equivalent annual capital costs (see Table 4)	$ 6,950.74	$ 5,115.39	$ 1,835.35
2. Taxes and insurance	592.27	442.86	149.41
3. Electrical connection charge	625.00	375.00	250.00
4. Total fixed costs	$ 8,168.01	$ 5,933.25	$ 2,234.76
5. Annual operating costs			
Pumping energy	4,227.00	2,562.00	1,665.00
Pump maintenance	507.58	437.42	70.16
Lateral maintenance	621.92	307.28	314.64
System operating labor	538.00	269.10	268.90
Total	$ 5,894.50	$ 3,575.80	$ 2,318.70
6. Total annual irrigation costs	$14,062.51	$ 9,509.05	$ 4,553.46
7. Other production costs	$11,960.00	$10,304.00	$ 1,656.00
8. Total production costs	$26,022.51	$19,813.05	$ 6,209.46

TABLE 8. Costs of Irrigation for 92 ac and 143 ac Irrigated Fields

	Full irrigation of 92 ac	Deficit irrigation of 143 ac
1. Capital costs		
Pump	$23,887	$23,887
Mainline	14,390	24,175
Laterals	18,120	18,120
Installation	2,830	4,664
Total	$50,227	$70,846
2. Annualized fixed costs		
Pump	$ 2,761.34	$ 2,761.34
Mainline	1,581.46	2,656.83
Laterals	2,295.80	2,295.80
Installation	311.30	513.04
Taxes and insurance	592.27	708.46
Total	$ 7,542.17	$ 8.935.47
3. Annual operating costs		
Pumping energy	$ 4,227.00	$ 4,382.00
Pump maintenance	507.58	516.58
Laterals maintenance	621.92	621.92
System operations labor	538.00	538.00
Electrical connection charge	625.00	625.00
Total	$ 6,519.50	$ 6,683.50

143 acres (58 ha). Total wheat production would increase approximately 25% because the increased land under production would more than compensate for the reduced yield per acre. Gross income would increase to $37,524. Estimated net farm income would therefore be increased from $4,153 per year for the full irrigation system to $5,889 for the deficit system, an increase of 42%.

Discussion

A case study has been presented in which a low-frequency, low adequacy, deficit irrigation scheme is seen to have advantages over a full irrigation scheme. In this case deficit irrigation would make it possible to reduce energy use by 40% and consumptive use of water by 24% without reducing farm income. Alternatively it would be possible to increase farm income 42% without significantly increasing energy or water use, by increasing the irrigated acreage.

The lower costs of the deficit irrigation scheme are partly attributable to reduced water use and lower yields, which result in reduced pumping costs, lower electrical hookup charge, lower pump price, and lower fertilizer and harvest costs. The balance of the savings derive from the low frequency operation (i.e., 12-day intervals). Eliminating two of the laterals would reduce capital investment by $9,060. This reduces annualized capital costs by $1,148, taxes and insurance by $99, and labor and maintenance costs by $682. The annual savings of $1,929, 32% of the total, would therefore be attributable to the low frequency operation.

This has been a limited and highly specific assessment of the marits of deficit irrigation. The situation analyzed here obviously must be considered unique, involving only one of the virtually countless possible combinations of circumstances of weather, crop, soil, prices, or pumping lift, etc. Under other circumstances deficit irrigation might offer the farmer very little. For example, when land is limited, water is abundant and the various costs of irrigation are low. Likewise, circumstances more favorable to deficit irrigation can easily be imagined. If, for example, the pumping lift in the present case study had been 500 ft (152 m) rather than 200 ft (61 m), pump and energy costs alone would make the deficit scheme preferable. Despite the uncertainty of crop model predictions, rainfall and other facotrs may be so great as to substantially alter the conclusions of a deterministic analysis such as the foregoing study (2).

In spite of the limitations of this study three important conclusions can be drawn from it:

1. Under some circumstances deficit irrigation can offer significant benefits, particularly in circumstances of constrained resources;
2. the benefits to be realized from deficit irrigation may be largely dependent upon system design; and
3. an accurate assessment of these benefits require a complete economic analysis, which must include all marginal production costs, capital, labor and maintenance costs, and opportunity costs.

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APPENDIX B

CONSIDERING RETURNS AND RISK IN THE DESIGN OF SPRINKLER IRRIGATION SYSTEMS

CONSIDERING RETURNS AND RISK
IN THE DESIGN OF SPRINKLER IRRIGATION SYSTEMS

Introduction

Typically, irrigation systems have been designed to meet the maximum water demand even during extremely dry growing seasons [Jensen and King; English and Nuss.] Thus, the irrigation systems have excess capacity to meet the extreme evapotranspiration rates that may occur. This practice assumes that the goal of irrigators is to maximize crop yield, but yield maximization may not be compatible with maximizing net returns.

However, net returns is not the only consideration when designing irrigation systems. Farmers are also faced with risk due to weather and random variables over which they have little control. Designing an irrigation system to apply less water may result in operating and capital cost reductions [English and Nuss] with a corresponding increase in net returns. But as the capacity of the irrigation system is reduced, it will be more constrained during years of higher-than-normal water demand. As a result, yields and net returns might be expected to fluctuate more than with a higher capacity system. Thus, the risk associated with the increase in net returns should also be considered in irrigation system design.

This study involves the development of a simulation model to estimate wheat yields, net returns and risk associated with alternative irrigation system designs and operating rules in the production of winter wheat. The alternative designs are for a side-roll sprinkler system with water delivered by an electrically powered pump from a well. The operating rules are the irrigation set time and the soil moisture level at which irrigation is initiated. The net returns, considering sprinkler system investment costs and operating costs, are an important factor to the profit-maximizing irrigator. In addition, this analysis evaluates the risk associated with each irrigation alternative, measured as the standard deviation of net returns.

The Simulation Model

The model to simulate irrigated winter wheat production consists of a grain yield component, a soil moisture component, and a risk/returns analysis component. Controllable and uncontrollable driving variables are entered into the simulation model. Precipitation, evaporation and temperature are the uncontrollable variables and interact with the soil moisture and yield components. The controllable variables are the design of the irrigation and the system's operating rules. The operating rules relate to the irrigation set time and the soil moisture threshold at which irrigation is initiated. Design and operation of the system interact to determine the rate of daily water application. The soil moisture component determines the soil moisture level which is an input to the yield component. The yield and water application affect the total returns, costs and risk.

The Yield Component

According to Martin et al. [p. 90], the cumulative growth of most plants over time will have a shape similar to that of a sigmoid curve. This curve depicts two phases of plant growth. During the first phase the rate of plant growth is increasing over time. At the inflection point this phase ends and the rate of growth declines over the second period. A combination of two functional forms can be used to describe this type of a curve. An exponential form is used during the first phase and a Spillman function is used during the second phase.

Nielsen derived daily yield response functions from these two functional forms. For the first phase of growth the daily response function is:

(1) $$\Delta Y1_t = Y1_{t-1} \cdot k, \text{ for } t \leq t'$$

where $\Delta Y1_t$ is the daily yield response, Y1 is the cumulative growth, k is the daily growth coefficient, t is the number of days since the end of winter dormancy and t' is the number of days in the first growth period. The daily response for the second phase of growth is:

(2) $$\Delta Y2_t = mp \cdot c^{t-t'-1}, \text{for } t > t'$$

where $\Delta Y2_t$ is the daily yield response, mp is the initial daily growth increment and c is the base for the daily growth coefficient.

Equations 1 and 2 simulate the maximum potential growth on a daily basis, and must be modified to account for moisture and temperature stress. When temperature and/or soil water is less than optimal, the actual daily growth will be less than the maximum potential growth. The function used to explain the relationship between soil moisture and yield is that used by Rickman et al.

(3) $$KA = \ln(100 \cdot ASM/SMFC + 1)/\ln(101)$$

where KA is the normalized response of growth to soil moisture, and ASM is the available soil moisture, SMFC is the available soil moisture at field capacity and ln is the natural logarithm.

The effect of temperature is represented in the model as response to hot or cold extremes. The relationship is:

$$\text{(4)} \qquad TX = 0, \text{ for } 37^{0}\text{ F} \geq TA \geq 104^{0}\text{ F}$$

$$\text{(5)} \qquad TX = 1, \text{ for } 37^{0}\text{ F} \leq TA \leq 104^{0}\text{ F}$$

where TX is the normalized response of yield to temperature and TA is the average temperature.

The effects of temperature and moisture are incorporated into the daily response function by multiplying the response function by the minimum of TX and KA. This is based on the assumption that growth is limited to the smallest amount of any necessary input. Equations (1) and (2) become respectively:

$$\text{(6)} \qquad \Delta Y1_t = Y1_{t-1} \cdot k \cdot \min(KA, TX), \text{ for } t \leq t'$$

$$\text{(7)} \qquad \Delta Y2_t = mp \cdot c^{\,t-t'-1} \cdot \min(KA, TX), \text{ for } t \geq t'$$

where k, c, and mp are coefficients to be estimated. The value t' is also an estimated value.

Soil Moisture Component

The model also calculates available soil moisture (ASM) and evapor-transpiration (ETA). The calculation of ASM is simply a bookkeeping technique. Soil moisture inflows are added to ASM and the outflows are subtracted.

$$\text{(8)} \qquad ASM_t = ASM_{t-1} + b \cdot IRR_{t-1} - ETA_{t-1} - RNF_{t-1}$$

where ASM is available soil moisture, b is irrigation efficiency, IRR is irrigation water applied, PRCP is rainfall, ETA is actual evaportranspiration and RNF is water runoff from the soil surface. PRCP is acquired from historical data, IRR is a model variable and RNF is considered negligible. ETA is calculated using pan evaporation [Bates, et al.] and adjusting it for soil moisture level [Nielsen].

Risk/Returns Analysis Component

Net returns above irrigation costs (other crop production costs were not subtracted) are calculated for each year by the model. The average net return and standard deviation of net returns are calculated for each system design alternative over the simulation period (19 years). The utility of each alternative is measured as:

$$U = NR - R \cdot SD \qquad (9)$$

where U is utility, NR is the average net return for a particular strategy, SD is the standard deviation of NR and R is a coefficient to weight SD according to the risk aversion of a manager. R will vary from zero to two for the risk neutral manager and most risk averse manager respectively [Brink and McCarl].

Model Estimation and Validation

During the 1981 crop season, the Department of Agricultural Engineering at Oregon State University conducted field trips near Hermiston, Oregon to determine the effects of deficit irrigation on evaportranspiration and wheat yield. The data acquired from the field trials were used to estimate the coefficients for the plant growth and soil moisture models [Nielsen].

The simulation model was validated to check the accuracy of the logic of the relationships and the consistency of the model predictions with the actual wheat production. Testing the model logic is a subjective process and involved examining intermediate results and the flow of data among the various components of the overall model.

While the 1981 field trial data are insufficient to make strong conclusions regarding the performance of the model, when compared to other similar models [Rasmussen and Hanks] its accuracy based on the mean of the squared deviations is similar. Soil moisture and crop yield data from other years and locations would be helpful in improving the model. This is particularly true for determining the role that temperature plays in yield prediction.

System Design Variables

The production unit modelled was a 160-acre winter wheat field near Hermiston, Oregon, irrigated with a side-roll sprinkler irrigation system. The spacing of the risers and nozzles is 60 feet and 40 feet respectively. The field dimensions are 2,640 feet by 2,640 feet. The mainline runs through the middle of the field so each lateral is 1,320 feet long. The riser spacing is 60 feet. This gives a total of 88 sets with 44 on each side of the mainline. A 250-foot well is locatd next to the field with an electrically powered pump.

The alternative sprinkler irrigation systems for this 160 acres of wheat were designed using a computer model developed by Marshall English, Department of Agricultural Engineering, Oregon State University. The first aspect of system design is the number of laterals. Systems with 10, 8, 4 and 2 laterals were compared. The next design consideration is the rate of water flow. The systems could either deliver water at a full irrigation level or at a lower level.

The operating rules relate to the set time and the level of soil moisture when irrigation is started. With 8 and 10 laterals, set times of 23 hours and 11 hours are considered. The alternative soil moisture levels for initiating irrigation are 5.0 inches and 3.0 inches. With 2 and 4 laterals, set times considered are 11 hours and 7 hours and moisture levels of 5.0 inches, 3.0 inches and 2.0 inches are used as alternatives for initiating irrigation.

The set time, rate of water flow, soil moisture depletion and the uniformity of water distribution combine to give a measure of irrigation adequacy. An irrigations adequacy of approximately 87 percent is considered full irrigation by the Soil Conservation Service. This means that 87 percent of the area irrigated receives at least enough water to fill the soil profile. The remaining 13 percent of the area will receive less water than soil moisture depletion. Two pumping rates are considered which would achieve approximately 87 and 50 percent adequacy given the maximum daily ETA and the sprinkler system's capacity.

Results of Simulation Analysis

The production of winter wheat was simulated for 19 years of weather data from the Hermiston Agricultural Experiment Station, Hermiston, Oregon, to evaluate the alternative irrigation system designs. The wheat yields, water use, average net returns, and variability of net returns were estimated. The utility for each alternative was calculated from the average net return and standard deviation of net returns.

The prices used in the analysis were for 1982. The assumed wheat price was \$3.80 per bushel. Irrigation labor and interest rates were \$4.50 per hour and 12.5 percent, respectively. Capital costs were annualized by amortizing the total investment cost minus the discounted salvage value. The salvage value was calculated as 10 percent of the investment in pumps and laterals. The amortization period was 15 years.

Water Use and Yields

A total of 45 different combinations of system design and operating rules were simulated. Table 1 describes the alternative system designs and operating rule assumptions for 15 selected alternatives. Table 1 also reports the number of irrigations, water use, and average yields resulting from the 19-year simulation runs. The alternatives selected for discussion here are those that had either the highest yields or highest utility levels.

The highest average yield was obtained with alternative 3. The yield is 124.7 bushels per acre. The highest 5 yields were for alternatives 3, 7, 11, 15 and 20. All of these alternatives assumed that irrigation begins at a soil moisture level of 5 inches. The highest yields are associated with design alternatives including 10, 8 and 4 laterals. Set times for these 5 high-yield alternatives were either 11 or 7 hours--none of the 23-hour alternatives were in these 5.

The relationship between wheat yield and water use is generally expected to be positive. The simulated results tend to show a positive relationship between water use and yield but there are exceptions. The results indicate that it is possible to reduce water application and not appreciably affect yield. Alternatives 3 and 7 used 32.8 inches and 22.4 inches of water respectively. The yields for 3 and 7 were 124.7 bushels and 122.5 bushels respectively. Thus, achieving a water savings of 10.4 inches results in an average yield reduction of 2.2 bushels. Alternatives 3 and 7 are the same except that alternative 7 is under-designed in terms of irrigation adequacy. Alternative 3 had an adequacy of 87 percent and alternative 7 had an adequacy of 50 percent.

Utility and Net Returns

Table 2 shows the utility for each alternative ranked in descending order. Only the 10 alternatives with the highest utilities are included. Alternative 26 had the maximum utility and average net return. The utility and net return are $337.12 and $362.60 respectively. The risk aversion coefficient used for calculating the utilities in Table 2 is 2.0. The results indicate that risk does not seem to increase with the higher average net returns. The ranking of the alternatives is basically the same with risk aversion coefficients of 0 or 2.

Higher utility tends to be associated with a lower capital investment. The three design alternatives with the highest utilities and highest average net returns had either 4 or 2 laterals, compared to 10 for the yield maximizing design. The three utility/average net return alternatives were designed for an irrigation adequacy of 50 percent and assumed 7- or 11-hour sets. The average yield for the maximum-utility alternative (no. 26) is 121.4 bushels compared to 124.7 bushels for the maximum-yield alternative (no. 3), but water use is 7.3 inches less. The average net return per acre is estimated to be $44.60 greater for alternative 26 than for alternative 3.

The results show that for the alternatives considered, irrigating when soil moisture reaches 5 inches is the best operating decision rule. Initiating irrigation at 5 inches is probably consistent with current practices. Farmers tend to irrigate early in the season before a high water demand exists. This practice builds up a reserve of moisture in the soil profile so the wheat will not be as stressed later in the season when there is a higher water demand and the system may not have sufficient capacity to meet the water demand.

Sensitivity of Results

The cost of labor was changed from $4.50 to $10 per hour to analyze the sensitivity of the results. As might be expected, the strategies with relatively fewer irrigations tended to become more appealing. Table 3 shows the ranked results with labor at $10. While this may be a high wage rate, it could be considered a realistic opportunity cost of the manager's labor. Alternative 26 with a set time of 7 hours and 11 irrigations dropped from first to third in the rankings. Alternatives 23 and 35 with 11-hour sets moved up. Alternatives 23 and 35 had 8 and 6 irrigations respectively.

The results with the higher labor costs tend to be consistent with the irrigation practices followed by farmers in the Hermiston area. They tend to prefer systems that require fewer moves of the laterals. Most use 60-foot riser spacings for this reason. Their reasons seemed to be related more to irrigation management than to labor costs. The higher wage rate could reflect this management cost also.

The effects of other assumptions were also analyzed.

1. Increasing energy costs by 50 percent or adding a water charge of $10 per acre-foot did not change the rankings of the first four irrigation system design alternatives in Table 2.
2. The results of the initial simulation runs indicated that initiating irrigation at a soil moisture threshold of 5 inches was preferable to a 3-inch threshold. When alternatives initiating irrigation at 4 inches were simulated, the results showed that a system similar to 23 with a utility of $327.67 which would place it third in relation to the alternatives listed in Table 2.

3. When the soil moisture at field capacity was reduced from 6 inches, which was assumed for the initial simulation runs, to 4 inches, the results indicated that risk aversion may be more important for soils with lower capacities for holding moisture.

Summary

The results of this analysis of wheat production at Hermiston, Oregon, indicate that designing sprinkler irrigation systems for maximum yields do not result in the maximum utility or maximum average net return. The value of yield reduction was more than offset by the reduction in irrigation costs. The higher level of net returns was achieved without significant increases in the level of risk facing the irrigator. The level of risk aversion made very little difference in the rankings of the different irrigation designs. The optimal irrigation systems were those with relatively lower capital investments and that initiated irrigation at a low level of soil moisture depletion.

The results were sensitive to labor costs--as the costs for irrigation labor increased the rankings of the systems changed, favoring those with fewer irrigations. It also appears that if the moisture holding capacity of the soil were reduced, the level of risk aversion would be a more important consideration in the design of sprinkler irrigation systems for wheat production in this area.

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Table 1. System Design Factors, Operating Rules, Number of Irrigations, Water Use and Average Yield for Slected Alternatives

Alternative	Number of Laterals	Irrigation Adequacy Index[a]	Set time	Soil Moisture threshold[b]	Number of Irrigations	Water use[c]	Average wheat yield
		(%)	(hrs.)	(inches)		(inches)	(bu./acre)
3	10	87	11	5	16	32.8	124.7
5	10	50	23	5	10	26.9	120.1
7	10	50	11	5	17	22.4	122.5
11	8	87	11	5	14	37.6	123.7
13	8	50	23	5	8	27.5	117.8
15	8	50	11	5	14	23.4	122.3
17	4	87	11	5	8	40.5	116.9
18	4	87	11	3	5	25.8	108.5
20	4	87	7	5	12	39.1	121.7
23	4	50	11	5	8	26.3	117.6
25	4	50	11	2	4	13.6	91.0
26	4	50	7	5	11	25.5	121.4
32	2	50	11	5	4	21.4	104.8
35	2	50	7	5	6	25.5	113.4
36	2	50	7	3	4	18.6	101.7

a/ SCS measurement of irrigation, 87 percent is considered full irrigation.

b/ The level of soil moisture that initiates irrigation.

c/ Average gross water application in acre inches.

Table 2. Alternatives Ranked According to Utility, With Average Net Returns and Costs (Labor at $4.50 Per Hour) for Each Alternative in Dollars Per Acre[a]

Alternative	Utility[b]	Total Revenue	O & M[c] Costs	Labor Costs	Capital[d] Costs	Ave. Net Return	SD Net Return	Min Net Return
26	337.12	487.90	40.47	28.13	56.71	362.60	12.74	335.07
23	332.87	472.88	39.53	19.67	54.39	359.29	13.21	333.03
35	324.58	455.69	43.69	14.96	44.20	352.83	14.13	325.09
15	319.89	491.70	35.80	35.06	74.31	346.53	13.32	316.85
20	315.59	489.12	67.33	28.73	58.10	334.96	9.69	313.91
13	315.20	473.71	39.48	19.71	74.31	340.22	12.51	315.65
5	310.29	482.72	38.86	24.04	84.49	335.32	12.51	309.68
17	307.61	470.03	66.89	19.78	58.26	325.10	8.75	307.38
7	302.15	492.27	37.35	41.92	82.63	330.38	14.12	299.31
11	299.81	497.37	61.26	35.17	78.47	322.47	11.33	295.70

Table 3. Alternatives Ranked According to Utility, With Average Net Returns and Costs (Labor at $10.00 Per Hour) for Each Alternative in Dollars Per Acre[c]

Alternative	Utility[b]	Total Revenue	O & M[c] Costs	Labor Costs	Capital[d] Costs	Ave. Net Return	SD Net Return	Min Net Return
23	307.28	472.88	39.53	43.71	54.39	335.25	13.98	308.28
35	305.48	455.69	43.69	33.25	44.20	334.55	14.53	306.53
26	298.60	487.90	40.47	62.50	56.71	328.23	14.81	297.95
13	289.60	473.71	39.48	43.79	74.31	316.14	13.27	290.90
17	281.61	470.03	66.89	43.95	58.26	300.93	9.66	282.63
32	279.65	421.08	35.52	22.20	46.04	317.32	18.84	281.40
5	277.98	482.72	38.86	53.43	84.49	305.93	13.98	278.74
20	276.44	489.12	67.33	63.84	58.10	299.85	11.70	276.78
18	273.20	435.96	51.75	27.95	58.26	298.01	12.40	274.91
36	271.90	408.95	38.88	24.23	42.34	303.50	15.80	278.46

[a] 1982 prices.

[b] Risk aversion coefficient equal to 2.0.

[c] Operation and Maintenance Costs.

[d] Annualized at 12.5 percent over 15 years.

APPENDIX C

SCHEDULING FOR OPTIMUM WATER USE

SCHEDULING FOR OPTIMUM WATER USE

by Marshall English, Michael Glenn and John VanSickle*

Current irrigation scheduling practice is generally designed to maximize application efficiencies without adversely affecting crop yields. However with the high cost of energy and increasing competition for water, irrigation scheduling may be gradually reoriented toward a new goal, economically optimum water use. This paper outlines a proposed approach to optimum scheduling. The approach involves the use of a crop stress indicator as an index of water requirements, combined with the use of mathematical filtering techniques to detect critical stress values.

Irrigation scheduling, as it is now generally practiced, involves irrigating before soil moisture reaches a critical point at which crop yields will be adversely affected. The goal has been to use water as efficiently as possible to achieve maximum yields. However, the approach usually taken in practice is to schedule irrigations conservatively. Farmers often apply water earlier and in greater quantities than recommended (English et al., 1980; Brost, 1977). Scheduling services tend to be conservative in their assessments of allowable depletion. For example, irrigations may be scheduled in such a way as to refill the profile in those areas of a field in which water-use is the highest (Gear et al., 1976). While this practice results in

*Associate Professor, Department of Agricultural Engineering; Assistant Professor, Department of Crop Science; and Assistant Professor, Department of Electrical and Computer Engineering, respectively, Oregon State University.

overestimation of field-average moisture depletion it is consistent with the fundamental constraint of maintaining full yields.

Optimal scheduling will need to be more precise. Rather than calling for an irrigation when moisture depletion is approaching a critical point, the goal will be to recognize when it has reached a critical point. In addition the critical point may not be that point at which yields begin to be affected, but rather some later point at which yields are reduced by a specified amount.

Optimum Irrigation Practices

Economically optimum irrigation practice involves reducing water use from the yield maximizing level to the level at which marginal cost equals the value of the marginal product. Such a strategy of deliberate under-irrigation is sometimes referred to as deficit irrigation. Broadly speaking, there are two ways to practice deficit irrigation. One is to apply water frequently but in amounts insufficient to refill the soil profile (high frequency deficit irrigation). The other is to prolong the intervals between irrigations until actual evapotranspiration has fallen well behind the maximum rate, and then apply sufficient water to refill the soil profile (low frequency deficit irrigation). It has been claimed that the first of these techniques may result in somewhat higher yields (Miller, 1977), but that claim is still in doubt (Ferreres et al., 1978). Under certain circumstances the first approach may reduce capital costs (Rawlins and Raats, 1975), while the second approach may reduce capital, maintenance and labor.

It is useful to consider the relevant characteristics of an optimally irrigated field. The most obvious characteristics will be a dry soil profile at the time of irrigation. There will be no appreciable reduction in water use until soil moisture declines to a point at which evapotranspiration is significantly inhibited. That point varies with crop, soil type and weather conditions, but as a rough rule of thumb it can be said to occur at a soil matric potential of between -0.5 and -1.0 bars (Feddas, et al., 1978, p. 19).

Soil moisture may also be more variable under deficit irrigation. Application efficiency may be increased as irrigation adequacy is reduced (Shearer, 1978). So deliberately operating an irrigation system to achieve low adequacy can be to the farmer's advantage (English and Nuss, 1980). Soil moisture variability may be aggravated if irrigation adequacy is reduced.

Crop growth and development may also be less uniform in an optimally irrigated field. If the distribution system has been designed for low frequency deficit irrigation the return intervals between irrigations will be long (English and Nuss, 1980) and some parts of the field will receive water at more favorable times than others. The result may be more variable crop conditions.

Based on these premises, one can speculate on the directions in which irrigation scheduling is likely to develop. The discussion which follows deals first with various indicators which might be used to determine irrigation requirements. Secondly, the use of filtering to enhance the effectiveness of these indicators is proposed.

1. Indicators of Crop Water Requirements

An indicator of crop water requirements should be related to crop yields, since yield reduction will be the basis for optimum irrigation decision-making. It should also be one which can be monitored quickly and cheaply at numerous points in a field, since the spatial variability of soil moisture and crop conditions may be significant.

Direct measurement of crop stress is a more likely indicator of irrigation requirements than soil moisture measurements because yields are directly related to stress and only indirectly related to moisture conditions (Kramer, 1969). Of course, soil moisture measurements must be made as well. The plant can only signal its need for water while the soil must indicate how much to apply (Teare et al., 1974; Stegman et al., 1976; Stanley, 1981).

Soil water tension is not, at present, a suitable indicator. Currently available tensiometers are only effective above about -0.8 bars tension (ASAE, 1980, p. 797). Thus a tensiometer would be operating at the limit of effectiveness when soil moisture is just approaching the range of greatest interest. Development of an inexpensive method of in situ measurement of low matric potentials appears to be a likely research problem in this respect.

Direct measurements of plant-water status (total plant water, matric, turgor and osmotic potential, and stomatal resistance) are accurate indicators of water stress since they integrate the effects of crop species, age, soil moisture, evaporative demand and pest damage. Turner (1981) provides a detailed review of these parameters and their research limitations. A disadvantage of direct measurement of plant-water status is that it is generally expensive due to the sampling frequency, number of samples required, sampling timing, and the time and conditions required to evaluate the samples.

Attempts have been made to correlate plant-water status with more easily measurable variables (radiation, air temperature) with some success (Stegman et al., 1976; Stanley et al., 1981; Smart and Barre, 1973). However, these statistical approaches are generally area specific and require calibration for new crops or plant types.

The calculated ratio of actual evapotranspiration (ET_A) to maximum evapotranspiration (ET_M), termed relative ET, could serve as a quantitative measure of plant stress, since stress will result in reduced ET_A through partial stomatal closure. The departure of ET_A/ET_M from 1.0 occurs at varying soil moisture levels depending upon root density and ET_M levels (Meyer and Green, 1980; Stegman et al., 1976; Mogensen, 1980). Mogensen (1980) demonstrated in a lysimeter study that relative ET began to decrease before leaf water potential in barley. Relative ET was a more sensitive indicator of drought stress than leaf water potential due to the physiological control mechanisms associated with critical leaf water potentials (Denmead and Millar, 1976). Meyer and Green (7980) suggest that there is a reduction in growth prior to the departure of ET_A/ET_M from 1.0. The disadvantage of this indicator is that measurement of relative ET under field conditions is likely to be quite inaccurate.

The use of infrared thermometry presents an indirect method of measuring plant water status. Water stress results in partial stomatal closure and reduction in transpiration, causing canopy temperature to rise above ambient air temperature (Jackson et al., 1977). Thus, the difference between canopy temperature (T_c) and the temperature of the surrounding air (T_a), indicates plant water status. Several researchers have proposed the use of infrared thermometry as a relative measure of plant water stress (Bartholic et al., 1972; Reicosky et al., 1980; Brown, 1974). However, the quantitative relationship between (T_c - T_a) and measured plant water stress has not been fully established. A major advantage of canopy temperature as an index of crop stress is that it is adaptable to remote sensing (Idso et al., 1975). If this technique advances to the point that it is possible to reliably monitor conditions in a heterogeneous field at low cost, it could become a valuable tool for optimum scheduling.

2. Dealing with Uncertainty: Filtering

The indicators discussed above must be determined from field measurements if they are to be used for irrigation scheduling. Such measurements are characterized by substantial and largely irreducible uncertainty. The uncertainty can be mitigated by scheduling conservatively, but this tends to reduce expected income. Data filtering is an alternative way of dealing with uncertainty without necessarily reducing expected income. The concept of filtering is briefly outlined below.

Let $x(t)$ represent true values of a state variable x at time t. In the parlance of filtering $x(t)$ is a *signal*. Let $y(t_1)$, $y(t_2)$,... $y(t_n)$ represent *data*, i.e., measured values of $x(t)$ at times t_1, t_2, ...t_n. These variables are related by

$$y(t_i) = x(t_i) + v(t_i) \tag{1}$$

Where $v(t_i)$ is measurement error (*noise*).

Suppose we wish to use the data to estimate x at some time t^*. If $t^* < t_n$ this is a *data* smoothing problem. if $t^* > t_n$ it is a *prediction* problem. If $t^* = t_n$ it is a *filtering* problem (Papoulis, 1965). Thus filtering is concerned with real-time estimation of a state variable using measurements that are noisy.

Under certain circumstances, real-time estimation can be accomplished by a recursive filtering procedure known as Kalman filtering, which can be represented by

$$x(t_n) = A(t_n)\,\hat{x}(t_{n-1}) + \beta(t_n)\, y\,(t_n) \qquad (2)$$

where x(t) is a filtered estimate of x(t) and $A(t_n)$ and $B(t_n)$ are weights derived at each time step.

Kalman filtering produces estimates that are optimum in the sense that the expected value of the square of the estimation error is minimized (Papoulis, 1965). When the special circumstances required for Kalman filtering are not present, other filtering algorithms may be devised which will still yield *improved* estimates, though it is often not possible to prove that these estimates are optimal. In general, filtering can be said to yield estimates which are better than the measurements from which they are derived, according to some appropriate criterion.

The improved estimates are arrived at by systematic analysis of the data in conjunction with other information such as models of the physical relationship involved or statistical characteristics of the data. Filters range in sophistication from simple equations, conceived intuitively and developed empirically, to complex and sophisticated algorithms based on the principals of Kalman filtering. Filtering has been used in a variety of applications in which real-time measurements of a state variable are not sufficiently precise. Examples include estimation of the orbital position of satellites, estimation of plankton populations in an aquatic environment (Moore, 1971), real-time determination of atmospheric pollution levels (Soeda and Ishihara, 1974), and a variety of applications in hydrology (AGU, 1978).

Irrigation scheduling has the attributes of a filtering problem. The objective is real-time evaluation of a signal (soil moisture, canopy temperature, etc.). Measurements of these signals tend to be quite noisy due to such things as spatial variability and intrinsic measurement errors. The problem is one of separating the signal from the noise without reducing the clarity of the signal. This effort requires systematic evaluation of the characteristics of both the signal and the noise.

The use of filtering will be illustrated with an example. The example was artificially contrived solely to illustrate the concept and should not be construed as a demonstration of an actual application of filtering for irrigation scheduling.

Relative ET is used as the crop stress indicator in the example because it is particularly sensitive (as noted earlier). On the other hand field measurements of this indicator will be quite noisy, hence filtering might prove useful.

The following relationship between relative ET and soil moisture was assumed

$$\frac{ETA(n)}{ETA(n)} = 1.0 - e^{-c \cdot MR(n)} \qquad (3)$$

where $ETA(n)$ = actual daily ET on the n^{th} day

$ETM(n)$ = maximum daily ET on the n^{th} day

$MR(n)$ = relative soil moisture at the beginning of the n^{th} day

$$= \frac{MO - \sum_{i=1}^{m-1} ETA(i)}{MO}$$

where MO = available soil moisture at nominal field capacity

C = a constant

This model was derived and calibrated using paired lysimeter data provided by Pruitt (1981). Figure 1 shows this function for variious values of C.

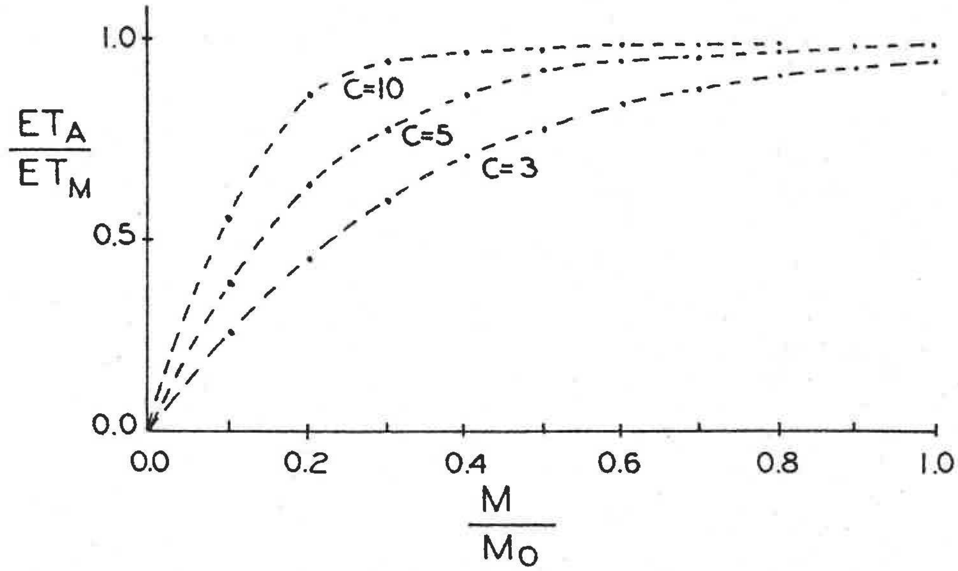

Figure 1. Ratio of actual ET to maximum ET $(\frac{ET_a}{ET_m})$

The irrigation decision will be based on cmuulative relative ET, defined as

$$CETR(n) \quad \frac{\sum_{i=1}^{n} ETA(i)}{\sum_{i=1}^{n} ETM(i)} = \frac{D(n)}{\sum_{i=1}^{n} ETM(i)} \tag{4}$$

Where $CETR(n)$ = cumulative relative ET between the last irrigation and the n^{th} day.

$D(n)$ = soil moisture depletion (from a full profile) between the last irrigation and the n^{th} day.

In this example, the decision rule will be to irrigate a field when $CETR(n)$ reaches 0.9.

Measurement of $CETR(n)$ requires measuring $D(n)$ and $\sum_{i=1}^{n} ETM(i)$ separately. It is assumed that $D(n)$ is measured with a neutron probe. Direct measurement of $\sum_{i=1}^{n} ETM(i)$ is impractical under ordinary field conditions, so daily estimates of ETM based on Penman model will be used in lieu of direct measurements. Thus from Equation 4 field measurements of $CETR(n)$ can be calculated by

$$\overline{CETR}(n) = \frac{\overline{D}(n)}{\sum_{i=1}^{m} \overline{ETM}(i)} \tag{5}$$

Where the bar denotes measured values of the variables.

This model is demonstrated using a sequence of hypothetical values of $\overline{CETR}$. These data were synthesized as follows. A 50 day sequence of ETM data from Pruitt's experiment were used. These were assumed to be true values of ETM. Using Equation (3) with $c = 5.0$ and $MO = 7.0$ inches a sequence of daily values of ETA were calculated. These were assumed to be true values of ETA. Successive sums of the true values of ETA were computed and these were used as true values of D. Then hypothetical measurements, $\overline{D}(n)$ and $\sum_{i=1}^{n} \overline{ETM}(i)$, were generated by adding simulated random measurement errors to the true values of

$\sum_{i=1}^{n}$ ETM and D. Statistical models of these errors were derived from theoretical statistical characteristics of neutron probe measurement errors and observed errors in Penman estimates of evapotranspiration. Finally, hypothetical values of $\overline{CETR}(n)$ were calculated from Equation (5).

These variables are tabulated in Table 1. Figure 2 shows plotted values of $\overline{CETR}$. If these $\overline{CETR}$ data were viewed sequentially, one measurement each day, it would be difficult to detect which day the critical level, 0.9, was reached. The problem then is to use a filter to reduce the uncertainty illustrated by the data in Figure 2.

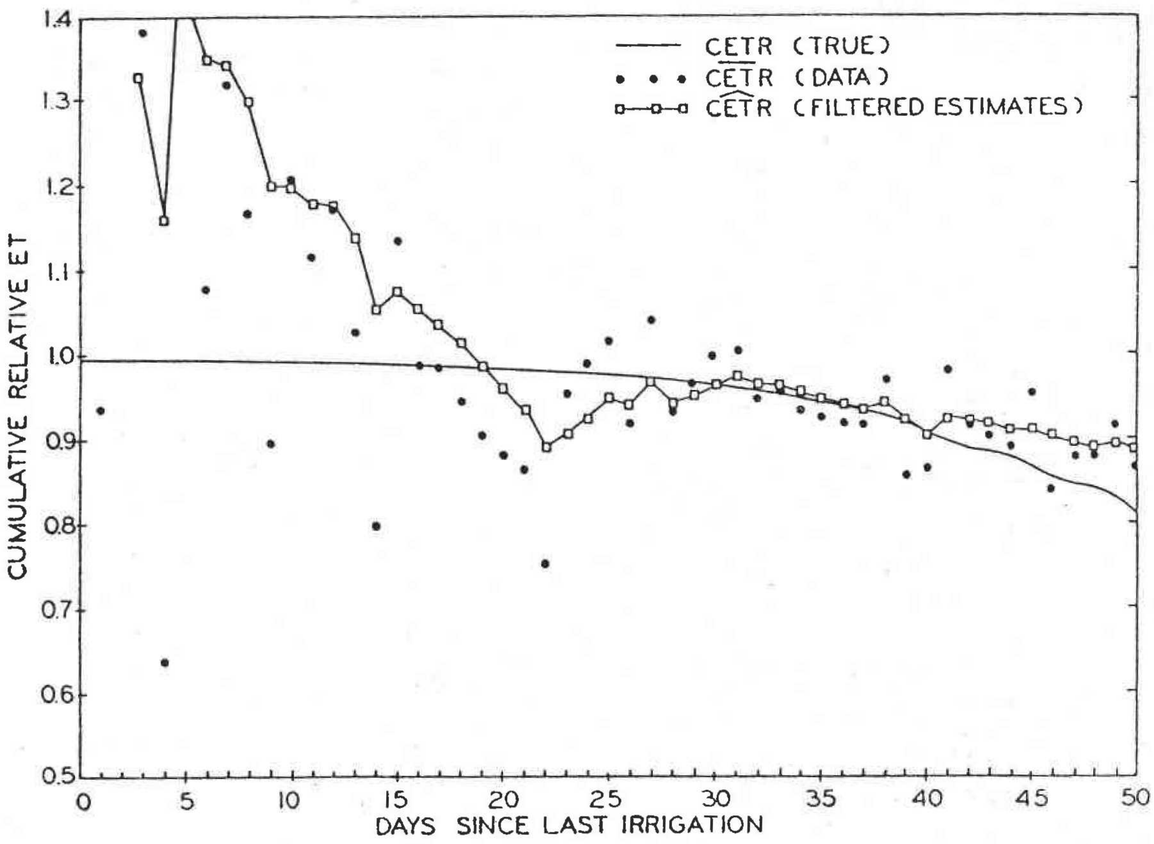

Figure 2. Synthesized true and measured values of CETR and estimates generated by low-pass filter.

Table 1. Synthesized True and Measured Values of Variables in the Filtering Example

	Synthetic True Values					Synthetic Measured Values			Filtered Estimates
Day	ETM (in.)	ETA (in.)	ΣETM (in.)	D (in.)	CETR	$\Sigma\overline{ETM}$ (in.)	$\overline{D}$ (in.)	$\overline{CETR}$	$\widehat{CETR}$
1	.07	.070	.070	.070	.993	.105	.098	.934	
2	.13	.129	.200	.199	.993	.209	.517	2.474	
3	.15	.149	.350	.347	.993	.312	.431	1.381	1.335
4	.09	.089	.440	.437	.992	.380	.242	.637	1.160
5	.14	.139	.580	.575	.992	.529	1.207	2.281	1.440
6	.13	.129	.710	.704	.992	.635	.684	1.077	1.350
7	.11	.109	.820	.813	.991	.764	1.008	1.319	1.342
8	.07	.069	.890	.882	.991	.840	.980	1.166	1.298
9	.16	.158	1.050	1.040	.990	1.042	.934	.896	1.197
10	.12	.118	1.170	1.158	.990	1.143	1.382	1.209	1.200
11	.08	.079	1.250	1.237	.990	1.211	1.351	1.116	1.179
12	.17	.167	1.420	1.404	.989	1.396	1.630	1.168	1.176
13	.11	.108	1.530	1.512	.988	1.506	1.541	1.023	1.138
14	.13	.127	1.660	1.640	.988	1.615	1.288	.797	1.053
15	.18	.176	1.840	1.816	.987	1.867	2.111	1.131	1.072
16	.17	.166	2.100	1.982	.986	2.001	1.971	.985	1.051
17	.16	.156	2.170	2.137	.985	2.188	2.152	.984	1.034
18	.20	.194	2.370	2.331	.984	2.454	2.317	.944	1.011
19	.11	.106	2.480	2.437	.983	2.566	2.321	.905	0.985
20	.11	.106	2.590	2.543	.982	2.674	2.348	.878	0.958
21	.19	.182	2.780	2.725	.980	2.879	2.491	.865	0.935
22	.13	.124	2.910	2.849	.979	2.963	2.230	.753	0.889
23	.13	.123	3.040	2.972	.978	3.100	2.948	.951	0.905
24	.20	.189	3.240	3.161	.976	3.289	3.240	.985	0.925
25	.10	.094	3.340	3.254	.974	3.415	3.464	1.014	0.947
26	.11	.102	3.450	3.357	.973	3.529	3.240	.918	0.940
27	.09	.083	3.540	3.440	.972	3.642	3.785	1.039	0.965
28	.17	.157	3.710	3.597	.969	3.862	3.397	.880	0.943
29	.15	.137	3.860	3.734	.967	3.924	3.785	.964	0.949
30	.25	.226	4.110	3.959	.963	4.195	4.173	.995	0.960
31	.22	.195	4.330	4.154	.959	4.407	4.421	1.003	0.971
32	.23	.200	4.560	4.354	.955	4.592	4.336	.944	0.964
33	.21	.178	4.770	4.532	.950	4.852	4.633	.955	0.962
34	.16	.133	4.930	4.665	.946	4.985	4.640	.931	0.954
35	.08	.065	5.010	4.730	.944	5.046	4.666	.925	0.947
36	.18	.144	5.190	4.874	.939	5.169	4.738	.917	0.939
37	.20	.156	5.390	5.030	.933	5.285	4.839	.915	0.933
38	.20	.151	5.590	5.181	.927	5.370	5.198	.968	0.942
39	.32	.233	5.910	5.414	.916	5.796	4.949	.854	0.920
40	.25	.169	6.160	5.584	.906	5.984	5.156	.862	0.905
41	.22	.140	6.380	5.724	.897	6.098	5.967	.979	0.924
42	.22	.132	6.600	5.855	.887	6.358	5.829	.917	0.922
43	.10	.056	6.700	5.911	.882	6.465	5.819	.900	0.917
44	.10	.054	6.800	5.965	.877	6.537	5.789	.886	0.909
45	.24	.125	7.040	6.091	.865	6.813	6.501	.954	0.920
46	.18	.086	7.220	6.177	.855	6.925	5.808	.839	0.900
47	.16	.071	7.380	6.248	.847	7.001	6.147	.878	0.894
48	.10	.042	7.480	6.289	.841	7.063	6.195	.877	0.890
49	.24	.096	7.720	6.385	.827	7.315	6.689	.914	0.896
50	.28	.100	8.000	6.484	.811	7.583	6.583	.868	0.889

Since the signal, CETR, is not stationary this problem is not well suited to Kalman filtering. A filter was therefore developed intuitively. Initially a filter defined by Equation (6) was tried.

$$\hat{CETR}(n) \quad = \quad \alpha \cdot \hat{CETR}(n-1) \neq \beta \cdot \overline{CETR}(n) \qquad (6)$$

in which $\hat{CETR}$ represents a filtered estimate of the signal, and where $0 \leq \alpha \leq 1.0$ and $\beta = 1.0 - \alpha$. If α is greater than β this is referred to as a low-pass filter because a low frequency signal (the slowly changing value of CETR) will pass through the filter with relatively little change while high frequency inputs (the data noise) will be severely attenuated.

After some experimentation, α was set equal to 0.75. The results are tabulated in Table 1 and displayed in Figure 2. The filter eliminated much of the measurement error, though it gave an erroneous indication of the critical level on day 22. Also it did not produce a good estimate during the crucial period between days 30 and 50 when CETR did in fact fall below 0.9.

One problem with this filter is the persistence of prior estimates. In Equation (6) $\hat{CETR}(n-1)$ is, in effect, a preliminary estimate of $\hat{CETR}(n)$, with a weight determined by α. The filter will therefore tend to reproduce prior estimates, and as a result trends in $\hat{CETR}$ will lag behind trends in CETR. If α is reduced the filter will respond more rapidly to changes. But then the interference from noise will be increased and erroneous indications such as occurred on day 22 will be more likely. This illustrates a basic dilemma in filtering, the tradeoff between signal tracking and noise suppression.

Based on our understanding of relative ET we know the signal, CETR, will be monotonically decreasing. But the above model does not take this knowledge into account. To take advantage of this knowledge a filter could be developed which explicitly predicts the decrease in CETR that will take place during the next time step. By predicting the next value of CETR with a model rather than simply using the last estimated value as a prediction, the problem of persistence can be overcome. Such a filter might be defined by

$$\hat{CETR}(n) = \alpha \cdot \hat{\hat{CETR}}(n) + \beta\, \overline{CETR}(n) \tag{7}$$

where $\hat{\hat{CETR}}(n)$ is a preliminary estimate of CETR(n) based on the estimate of CETR(n-1) and the current estimate of ETM. This analysis is illustrated by Figure 3. From Equations (3) and (4) it is evident that CETR(n) is a function of $\sum_{i=1}^{n} ETM(i)$. The derivative at time n-1 is indicated by the arrow in Figure 3. A preliminary estimate for time n could therefore be written

$$\hat{\hat{CETR}}(n) = \hat{CETR}(n-1) + \left.\frac{dCETR}{dETM}\right|_{n-1} \cdot ETM(n) \tag{8}$$

(Such a filter was developed by the authors, but space limitations preclude a detailed derivation of the filter and presentation of results.)

This formulation (Equations 7 and 8) illustrates another facet of filtering. The estimates produced by the filter would be generated from a combination of field data and a model of the underlying physical relationships. Thus additional information, in the form of the model, is brought to bear on the problem.

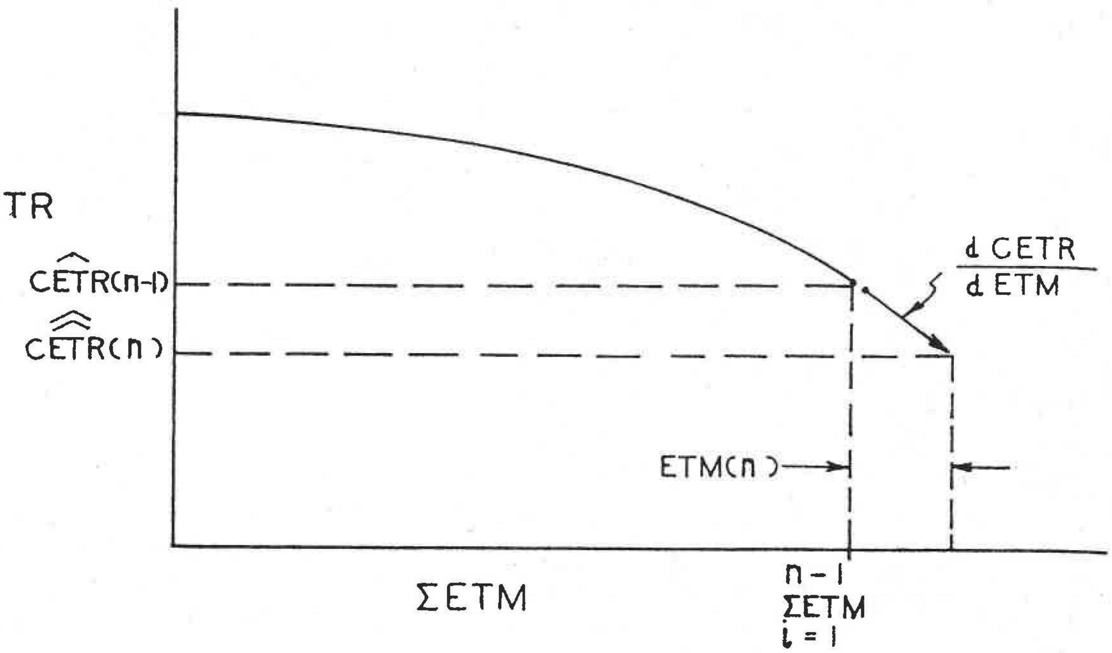

Figure 3. Preliminary Estimation of CETR(n) from CÊTR(n-1) and ETM(n)

It should be noted that the model would probably be biased, since it would have been developed and calibrated under other circumstances. By introducing a biased model the results will be biased as well. If the purpose of the analysis were scientific inquiry it would be best to forego the additional information rather than bias the results. But the purpose of irrigation scheduling is decision-making, and in that context the additional information may be welcome (English and Orlob, 1978, p. 5).

Summary

It is proposed that the use of crop stress indicators in combination with data filtering techniques will be a basis for successful irrigation scheduling when economic optimization is the goal. A general discussion of the problem of optimum irrigation scheduling has been presented. An optimally irrigated field may be characterized by (a) soil water matric potentials below the level of -0.5 to -1.0 bar at the time of irrigation, (b) more variable soil moisture conditions than found in fully irrigated fields, and (c) more variable crop development. The optimum time to irrigate a crop will be when potential yield has begun to decline. Bearing these facts in mind, it appears that optimal irrigation scheduling will entail the use of crop stress indicators to determine the best timing of irrigations. Ideally such indicators should be sampled quickly and cheaply at several points in a field.

Uncertainty will be an important consideration in scheduling for optimum water use. Filtering techniques might reduce this uncertainty to a manageable level. Two examples of filters were presented to illustrate basic concepts of filtering. The first filter, developed intuitively, demonstrated the process of noise suppression and the dilemma of compromising between noise suppression and signal tracking. The proposed second filter would be based on underlying physical relationships. Field data would be combined with knowledge of the relationships involved to produce refined real-time estimates of the signal.

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APPENDIX D

EVAPOTRANSPIRATION OF WINTER WHEAT

UNDER DEFICIT IRRIGATION

EVAPOTRANSPIRATION OF WINTER WHEAT UNDER DEFICIT IRRIGATION

by

G. S. Nuss and M. J. English

INTRODUCTION

Agriculture in the western United States is faced with a water problem of increasing magnitude. Water use and development are becoming limited due to availability and quality of supplies. Increased demands by industrial, municipal, and recreational users have added additional constraints. The promised development of synthetic fuel production in the west will require enormous quantities of water (Stansbury and Patten, 1981). Ground-water reservoirs are being depleted at alarming rates in some areas, and the effects of overexploitation are felt in others. Irrigated agriculture is being scrutinized by the public with regard to its water management performance. Many farmers are faced with uncertain supply and increased costs of water.

The applicability of deficit irrigation is currently being investigated within this western water scenario. Deficit irrigation is the practice of deliberately underirrigating a crop. This method of irrigation would be employed to maximize the net economic return from irrigation water use and/or conserve limited supplies of energy and capital.

The effect of low soil moisture on evapotranspiration (ET) is investigated here. Three models of this effect were analyzed. These were the logarithmic (Jensen et al., 1971), combination (Slabbers, 1980), and power models (Boonyatharokul and Walker, 1979). Irrigation intervals and depths were varied to impose low, moderate, and severe levels of moisture stress on the crop.

Daily ET was measured using a water balance calculation with neutron probe data. Predicted values of ET were compared to measured data using cumulative double mass curve analysis.

The experiment dealt with a specific crop (winter wheat) in a specific location (the Butter Creek area near Hermiston, Oregon). The objective of this research was to evaluate methods for predicting crop ET under deficit irrigation.

DEFICIT IRRIGATION

As pointed out by English and Nuss (1980):

> "It is widely recognized that partial irrigation of a crop can sometimes yield a greater net economic return than full irrigation. In fact, optimum irrigation practice will always be to apply less water than a crop is capable of utilizing."

Currently, two basic approaches to deficit irrigation are encountered: (1) high irrigation frequency with reduced application depth, and (2) low irrigation frequency with a full application depth. The fundamental difference between these two approaches is the length of irrigation interval.

A study into the effects of high frequency deficit irrigation was conducted for several crops by Miller (1977). The results for winter wheat showed no significant difference between various water treatments. Irrigation levels were based on Class A evaporation pan data. The results from Miller's study tend to favor high frequency deficit irrigation. However, efforts at duplicating his results at Davis, California, were unsuccessful (Faci and Fereres, 1980).

Systems meeting the requirement for high frequency irrigation range from solid-set or traveling sprinklers to drip or trickle to small basins periodically filled with water (Rawlins and Raats, 1975). The application of some of these systems requires considerable capital investment. This capital investment cost, however, is not the only economic factor in deficit irrigation. "Optimal irrigation management will generally consider cost for purchasing and delivering water to the land where irrigation is needed, and the economic losses suffered due to inadequate or overirrigation" (Wu and Liang, 1972).

A case study into the merits of low frequency deficit irrigation was conducted for winter wheat grown in the Columbia River Basin (English and Nuss, 1980). It focused on the economic feasibility of adapting a low frequency approach for irrigation management on an individual farm. The results of this study indicated that it was possible to reduce energy use by 40 percent and consumptive use by 24 percent without reducing farm income.

When viewed from the standpoint of economics, low frequency irrigation appears to offer significant advantages over high frequency irrigation. Wu and Liang (1972) point out that lower labor costs per unit of water delivered will be accomplished by irrigating less frequently. One important conclusion was derived from the case study of English and Nuss (1980), "a complete economic analysis, including marginal production costs, capital, labor, maintenance, and opportunity costs, is required to accurately assess deficit irrigation benefits."

MODELS ANALYZED

Three models were selected to analyze the ET response to depleting soil water. These were the logarithmic, power, and combination models. Complete development of these models is available elsewhere (Jensen et al., 1973; Boonyatharokul and Walker, 1979; Slabbers, 1980).

Equation 1 is the logarithmic model for crop response to limiting soil moisture.

$$K_1 = \frac{ETa}{ETm} = \frac{\log (AW + 1)}{\log 101} \qquad 1$$

where

K_1 = coefficient to account for the effects of limiting soil moisture

AW = percent of available water remaining within the profile

ETa = actual evapotranspiration

ETm = maximum evapotranspiration

The power model is outlined in Equation 2.

$$Kp = \frac{ETa}{ETm} = \left[1.0 - \left(\frac{\bar{D}p}{\bar{D}t}\right)^{\bar{m}}\right]^{n} \qquad 2$$

where

Kp = coefficient to account for the effects of limiting soil moisture

$\bar{D}p$ = depleted available soil moisture from the root zone

$\bar{D}t$ = total available soil moisture from the root zone

m, n = empirical constants related to climate, soil, and crop factors

The exponents m and n above were shown to be correlated with root zone parameters, saturated hydraulic conductivity, and reference evapotranspiration (Boonyatharokul and Walker, 1979). Regression equations were developed for these exponents reflecting these correlations. However, since saturated hydraulic conductivity is a highly variable property, a sensitivity analysis was performed on the exponent regression equations. Results of this analysis showed that the exponents were not sensitive to changes in saturated hydraulic conductivity or reference evapotranspiration over the range of expected values for this experiment.

The assumed water uptake distribution for irrigated winter wheat reflected a relative extraction pattern of 40, 30, 20, and 10 percent in each quarter layer of root depth. This extraction pattern led to the calculated exponent values of 2.43 and 1.73 (m and n, respectively).

Equations 3 and 4 represent the combination model.

$$Ks = \frac{ETa}{ETm} = 1.0 \quad \text{for } ASMt \geq f \cdot ASMo \qquad 3$$

$$Ks = \frac{ETa}{ETm} = \frac{ASMt}{f \cdot ASMo} \quad \text{for } ASMt < f \cdot ASMo$$

where

Ks = coefficient to account for the effects of limiting soil moisture

ASMo = maximum available soil moisture

ASMt = actual available soil moisture

f = fraction of available soil moisture at which the reduction in ET begins (threshold level)

This model reflects the threshold level theory of ET response to limiting soil moisture. It is believed that a certain level of available soil moisture exists above which the crop ET proceeds at its maximum rate. Below the threshold level the crop ET is at some decreased rate (Meyer and Green, 1979; Ritchie et al., 1972; Feddes et al., 1980). A threshold level of 50 percent available water during the peak ET period was assumed for this research.

Figure 1 illustrates the relative differences among the three models.

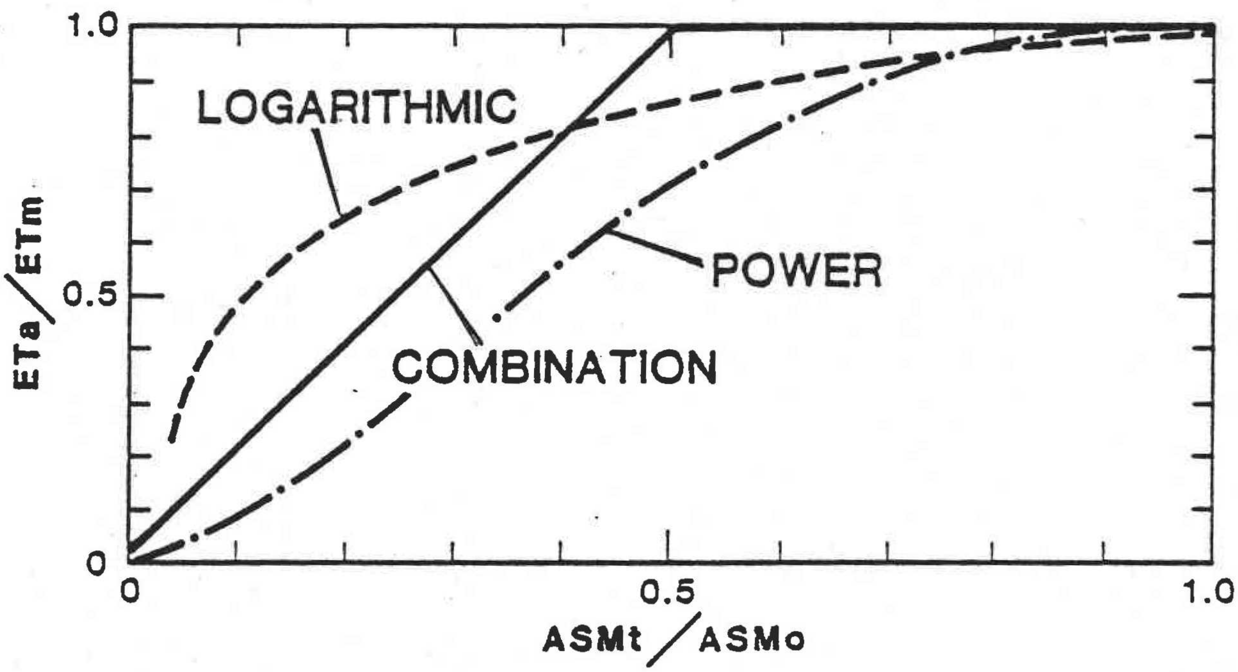

Figure 1

RELATIVE DIFFERENCES IN MODELS FOR CROP RESPONSE TO LIMITED SOIL MOISTURE

EXPERIMENTAL METHODS

Site Characteristics

The experimental plots were located approximately 10 miles south of Hermiston, Oregon, in what is locally known as the Butter Creek area. A cooperating landowner donated three acres of farmland for this research.

The climate of the experimental site is mild and semiarid with an average annual precipitation of 9.4 inches (24 centimeters). The seasonal pan evaporation (Class A pan) averages 50 inches (127 centimeters) annually.

The soil at the experimental site is Koehler loamy fine sand. The mineral material from which these soils have developed originally consisted largely of wind-laid deposits. A water-holding capacity of 1.5 inches/foot (12.5 centimeters/meter) with an effective rooting depth of 2 feet (0.6 meter) is reported for this soil and crop (SCS, 1973). Neutron probe measurements during the experiment confirmed the water-holding capacity; however, a rooting depth of 4 feet (1.2 meters) was indicated.

Plot Layout

During the fall of 1980, approximately 3 acres (1.2 hectares) of winter wheat (*triticum aestevium* var Stephens) was planted at the research site. A preirrigation of approximately 4 inches (10.2 centimeters) followed by 300 pounds (136.2 kilograms) of 16-16-16 fertilizer and 40 pounds (18.2 kilograms) of sulfur were applied prior to planting. The tract was also sprayed with Roundup herbicide prior to planting to eliminate volunteer barley.

Three stress levels were designed to observe the effects of irrigation interval on crop evapotranspiration. They are qualitatively described as low (T1), moderate (T2), and severe (T3). One treatment level was maintained for typical irrigation under production conditions in the project area. This treatment (W1) was irrigated weekly.

Stress was imposed on the crop by increasing the irrigation interval. A weekly interval represents a traditional design for full irrigation. Each irrigation was designed to refill the soil reservoir to a depth of 4 feet. Irrigation depths were scheduled by using a neutron probe to estimate the available soil moisture. Approximate irrigation intervals for each treatment are given in Table 1.

Table 1
APPROXIMATE IRRIGATION INTERVALS
FOR EACH TREATMENT LEVEL

Treatment	Irrigation Interval
W1	Weekly
T1	Two weeks
T2	Three weeks
T3	Four weeks

Solid set sprinkler lines were used to apply water to the experimental plots. The plots were separated by sufficient buffer area to prevent inadvertent water applications between adjacent plots. Applied water was monitored through catch cans located at canopy height.

Each treatment plot was equipped with six neutron probe access tubes. The tubes were installed by a hydraulically driven auger to a depth of 6 to 10 feet (1.8 to 3.1 meters). The tubes were located axially between the two sprinkler lines.

Figure 2 illustrates a typical treatment plot. A more detailed description of the plot layout is available elsewhere (Nuss, 1981).

Data Collected

Weather data was monitored at the Hermiston Agricultural Experiment Station. Daily measurements of maximum, minimum, and dew point temperature, wind run, solar radiation, and pan evaporation (Class A pan) were recorded. Daily Class A pan evaporation was also recorded at the treatment plots. These data were compared to the Hermiston station data to determine whether significant localized differences in climate existed. This comparison showed no discernable difference between the two locations. Rainfall was measured at the experiment site.

Neutron probe data was monitored every other day in two selected tubes within each treatment. All six tubes were read one day prior to a scheduled irrigation within a treatment plot, to determine the required irrigation depth. Each probe reading was begun at 6 inches (15.0 centimeters) below the surface and proceeded in 1 foot (30-centimeter) increments to a depth of at least 6 feet (1.8 meters). The readings were continued to 8 feet (2.4 meters) when allowable. Throughout the season, noticeable depletion was limited to the top 4 feet (1.2 meters) of the soil profile.

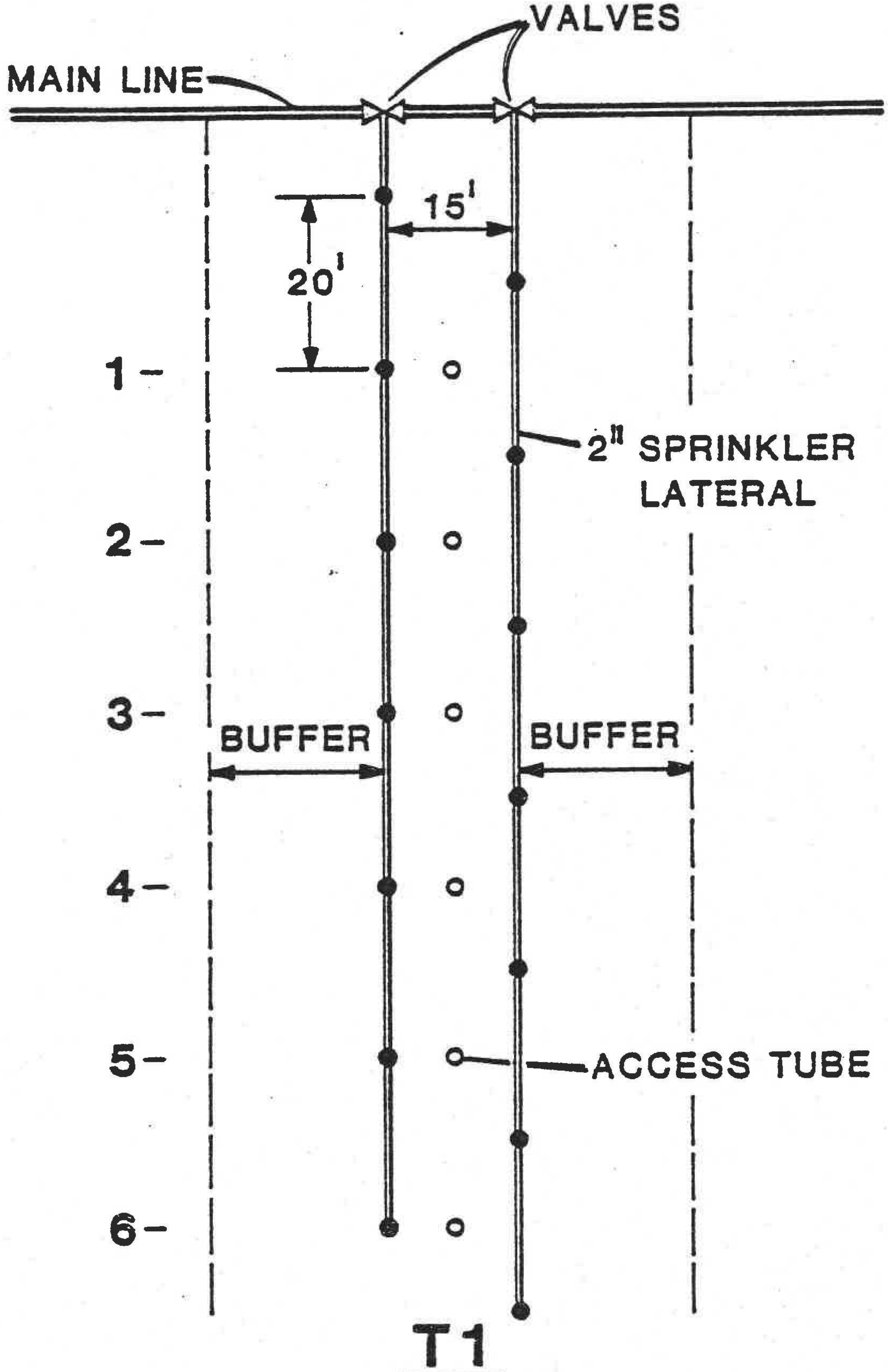

Figure 2

TYPICAL TREATMENT LAYOUT

Maximum Crop ET

Maximum crop ET was computed using the method outlined by Wright (1982). The reference crop was alfalfa. Meterological data collected at the Hermiston Experiment Station was used as input to the reference crop calculation. The crop coefficient under nonlimiting soil conditions was obtained for winter wheat in the Columbia River Basin with an alfalfa reference crop.[1] The computed value of maximum crop ET was used as an estimate of ET uncorrected for soil moisture effects.

Measured Crop ET

Soil moisture data were obtained from the neutron probe measurements. Since experiment logistics did not allow for daily monitoring of soil moisture, estimates were generated for the days when no measurements were made. These estimates were calculated by assuming a linear decrease (or increase) in soil moisture between dates of measurement.

The daily ET was calculated by using the water balance equation as shown in Equation 4.

$$ETn = (SMn - SM_{n+1}) + In + Rn \qquad 4$$

where

ETn = calculated ET for day n, inches/day

SMn = measured or estimated soil moisture on day n, inches

SM_{n+1} = measured or estimated soil moisture on day n + 1, inches

In = irrigation water measured in catch can on day n, inches

Rn = rainfall measured on day n, inches

An assumption of negligible drainage beyond the probe monitoring depth is implicit in Equation 4. This assumption was affirmed by inspection of the incremental soil moisture estimates at various depths.

The initial estimates of daily ET showed an extreme amount of variability, with severe positive/negative fluctuations. This variation can be in part attributed to a time lag in distribution of water through the profile. A daily water balance calculation

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[1] Communication from Dr. James Wright, Snake River Conservation Research Center, Kimberly, Idaho.

will also cause fluctuation in evapotranspiration data as a result of small errors in soil moisture measurements. Measurements with the neutron probe will reflect some amount of intrinsic variability due to the random nature of neutron emissions and other sources of measurement noise. To mitigate this daily variability, a three-day moving mean smoothing technique was applied to the measured soil moisture data. The water balance equation was again used to calculate daily ET data. A lesser degree of fluctuation was obtained. Daily variations were of no particular interest to this research. The smoothing technique was applied to the calculated ET data and the best estimates of daily crop ET were obtained.

ANALYSIS

Analysis of each model included the following procedures:

1. Model estimates of actual crop ET for each data site (W1-3, W1-4, T1-3, T1-5, T2A-3, T2A-5, T3A-3, and T3A-4) were generated.

2. The ratio of predicted values of crop ET to measured values were calculated and cumulative mass curves were plotted for measured ET, predicted ET, and maximum ET.

3. A linear function was derived for each cumulative mass curve by linear regression.

4. The linear regression curve for each model was compared to the uncorrected ET estimate at that site.

5. A qualitative review of each mass curve, focusing on the effects of data anomalies, was done.

The cumulative mass curve approach was taken because of the variability in measured ET data. The ratio of daily measured ET to predicted ET exhibited extreme fluctuation. The cumulative mass curve of predicted versus measured ET allows investigation of the seasonal performance of a model. Linear regression analysis on these curves had two primary functions, (1) to give an unbiased estimate of the absolute deviation at the end of the season, and (2) to compare the model to uncorrected data at the individual sites. Indirect "goodness of fit" can be inferred from the intercept and slope of regression curves (theoretically 0 and 1:1). The correlation coefficient (R^2) cannot be used as an indicator for a fit to measured data. It is presented only to illustrate the scatter of data points about the derived regression line.

RESULTS

Results of linear regression analysis for the three models and each treatment level are shown in Table 2.

Table 2
RESULTS OF LINEAR REGRESSION ANALYSIS
ON CUMULATIVE MASS CURVES
(y = a + bx) R^2 is correlation coefficient

Treatment	Logarithmic			Combination			Power		
	a	b	R^2	a	b	R^2	a	b	R^2
W1-3	0.929	0.983	0.992	0.895	1.051	0.992	1.886	0.869	0.998
W1-4	1.631	0.956	0.993	1.657	1.021	0.987	2.617	0.837	0.982
T1-3	3.457	0.835	0.986	3.725	0.873	0.988	4.322	0.709	0.979
T1-5	2.739	0.835	0.991	2.968	0.864	0.988	3.211	0.732	0.985
T2A-3	2.460	0.972	0.991	2.797	0.986	0.990	3.064	0.763	0.986
T2A-5	1.915	0.983	0.994	2.282	0.984	0.991	2.905	0.763	0.984
T3A-3	0.825	1.118	0.989	1.091	1.061	0.974	0.821	0.869	0.961
T3A-4	1.613	1.124	0.990	1.791	1.126	0.986	2.024	0.972	0.984

All three models performed similarly with the data from treatment WI during the early part of the season. The logarithmic model did the best job of approximating the measured ET data at both sites (W1-3 and W1-4) throughout the season. The combination model tends to overpredict during the later portion of the season, while the power model underpredicted in this interval. Generally speaking, each model could have been replaced with the uncorrected data (maximum ET) for this treatment. The better performance by the logarithmic model in this treatment is not surprising, since this model was developed under conditions similar to treatment level W1.

The slight water stress treatment (T1) once again indicated a similar response for all three models in the early part of the season. During the later portion of the season, all three models underpredicted the ET to a certain degree. The combination model does the best job in modeling the ET response during the late season, followed closely by the logarithmic model. The power model underpredicts the most severely, and does a generally poor job in this treatment. Uncorrected data could be substituted for the models without incurring significantly greater errors. A good example of this is illustrated on Figure 3. The close approximation of measured data by the uncorrected data indicates that this treatment level (T1) did not receive the designated level of stress.

The moderate water stress treatment (T2A) indicated no significant difference between the logarithmic and combination models throughout the irrigation season. Both models do a good job of predicting the ET in this treatment. Figure 4 illustrates the logarithmic model response in this treatment. The power model response was quite similar to the other two in the early season, but it begins to significantly underpredict after midseason.

The severe water stress treatment (T3A) did not reflect overly encouraging results for any of the models. All the models are similar in the early season and each overpredicts, to some degree, from the middle to the end of the season. The power model produced the worst estimates during this interval (as illustrated on Figure 5), while the logarithmic and combination models were similar throughout. The uncharacteristic overprediction indicates that this treatment could have received too much water stress, thus inhibiting ET.

Instances of negative ET measurements can be observed in the data (refer to Figure 3). The occurrence of these measurements is not restricted to a particular data site or group of sites, but is generally distributed at each location of data measurement.

The occurrence of data anomalies is closely linked to dates of water applications. These anomalies have been attributed to the noninstantaneous distribution of applied water within the root zone and variations in time of soil moisture measurement with respect to applied water. These factors are difficult to avoid

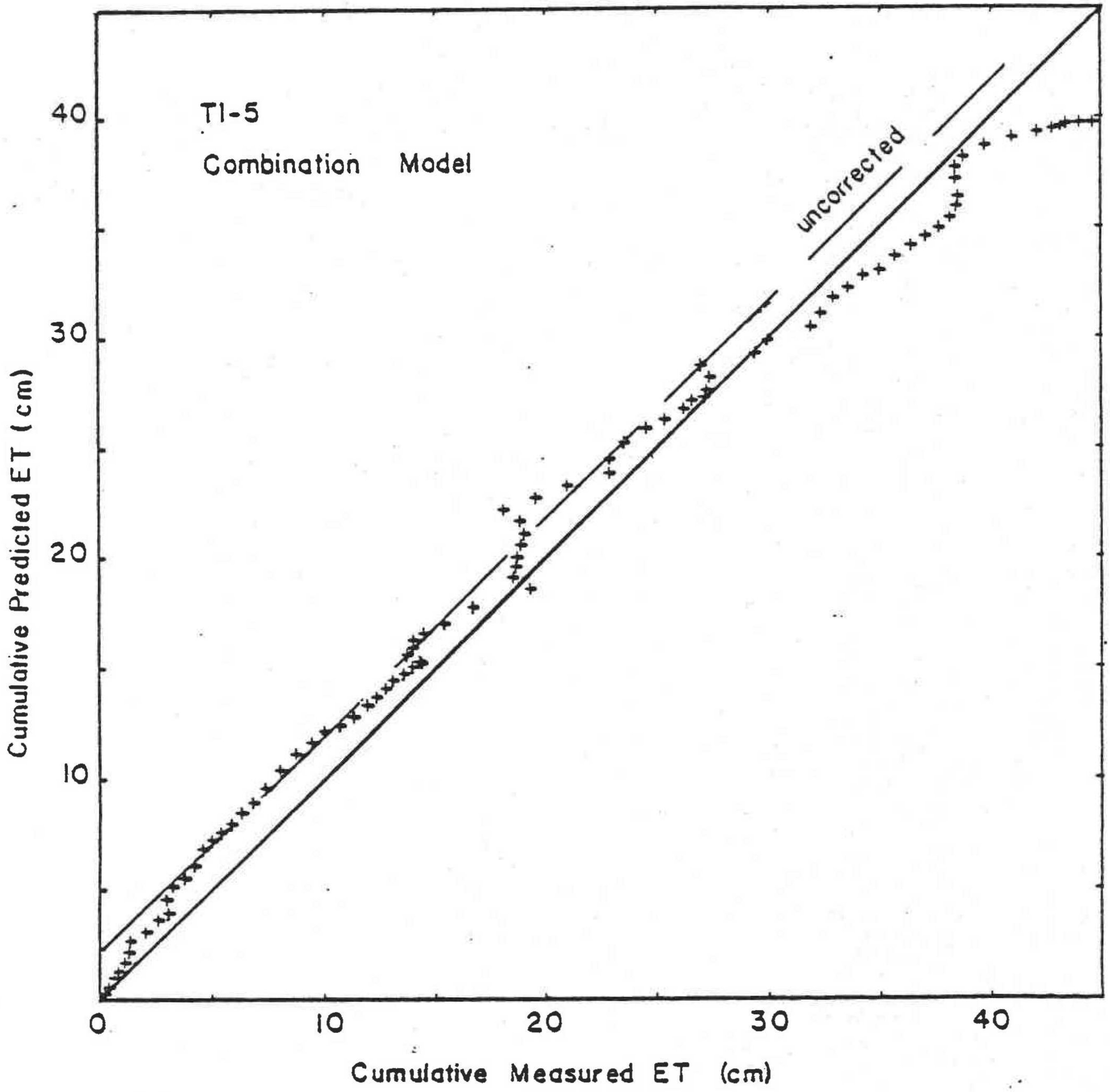

Figure 3
CUMULATIVE DOUBLE MASS CURVE FOR
SITE T1-5 USING THE COMBINATION MODEL

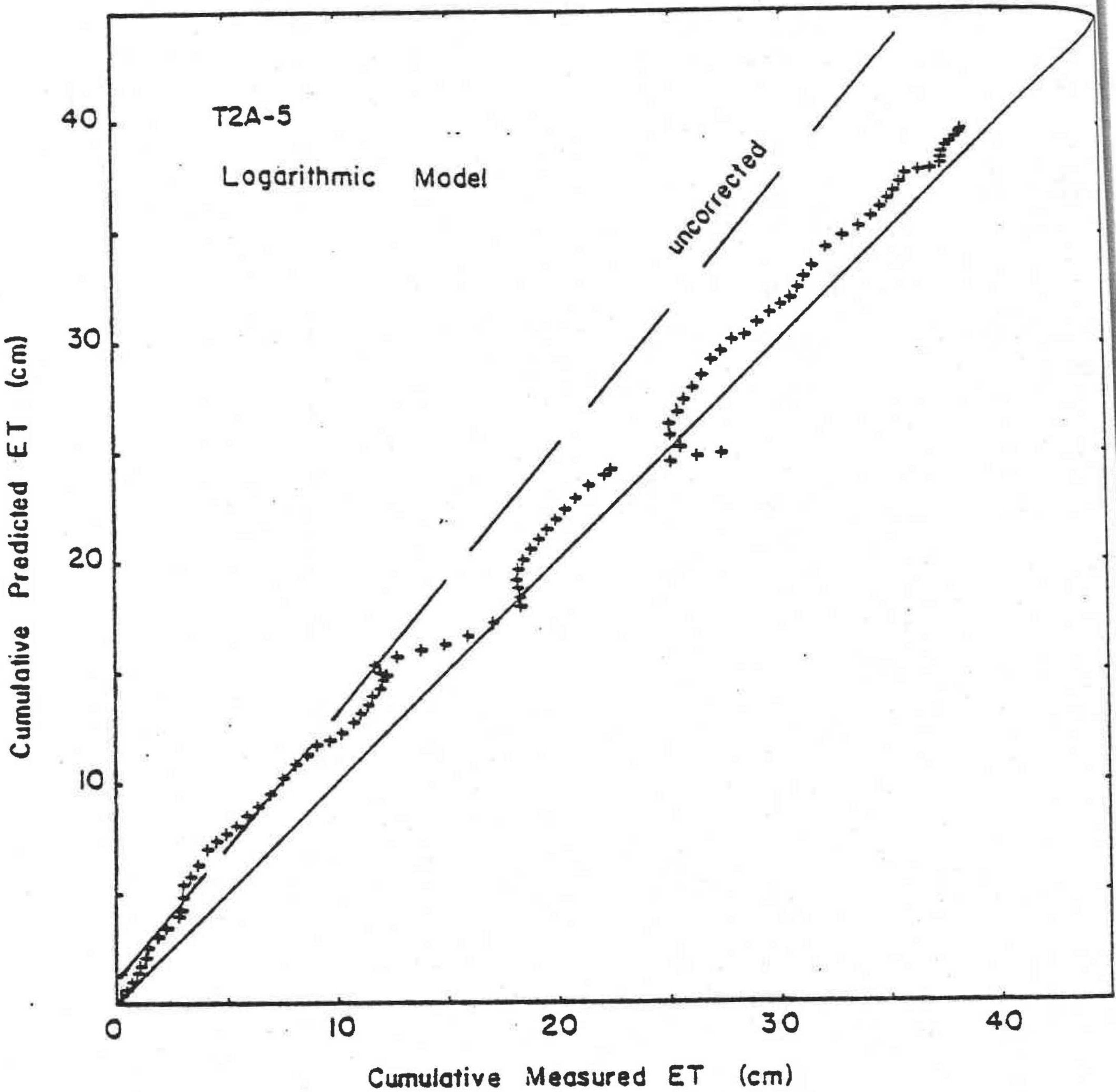

Figure 4
CUMULATIVE DOUBLE MASS CURVE FOR
SITE T2A-5 USING THE LOGARITHMIC MODEL

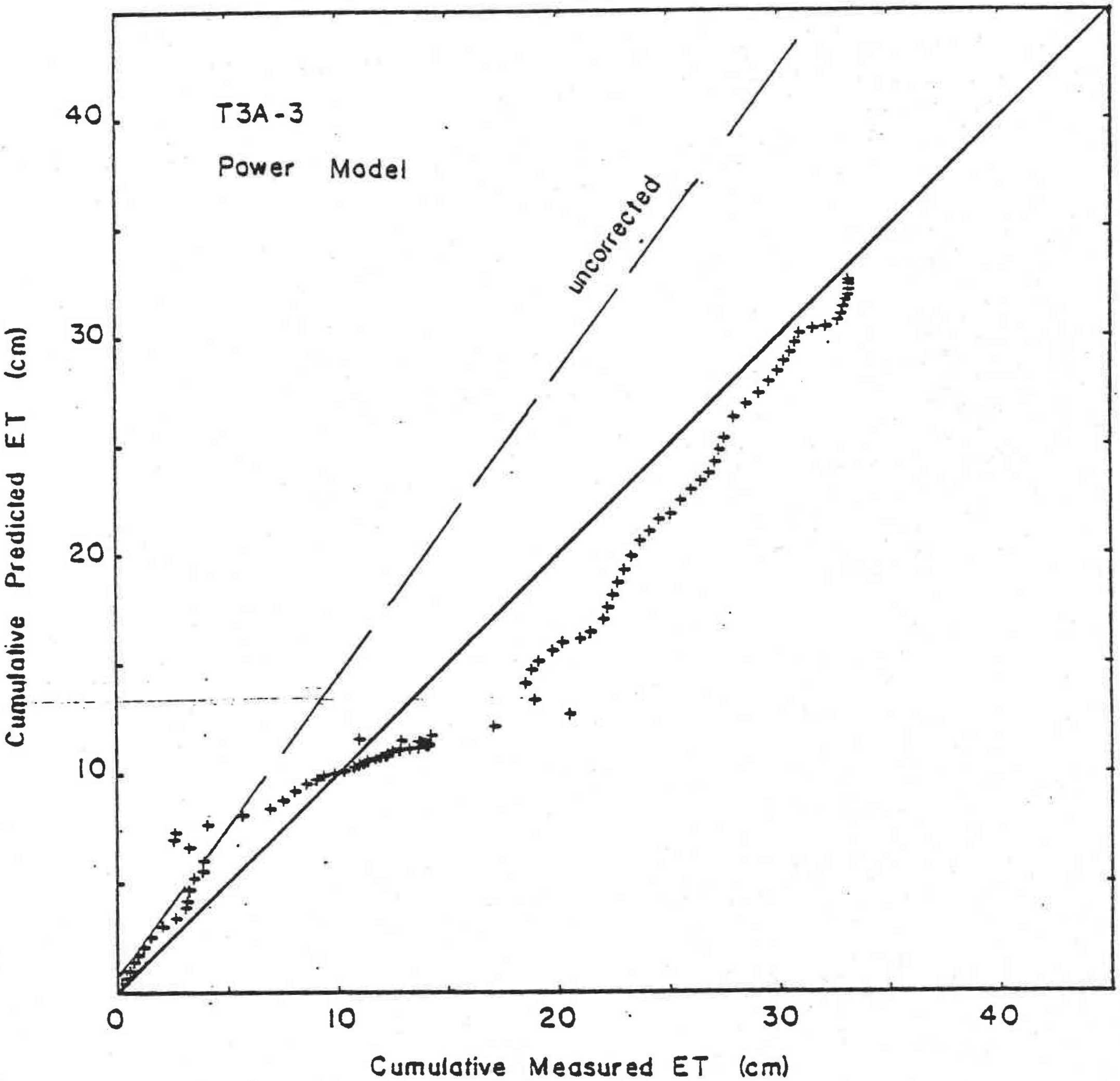

Figure 5
CUMULATIVE DOUBLE MASS CURVE FOR
SITE T3A-3 USING THE POWER MODEL

with the instrumentation available to the research team for soil moisture determination.

Each model at all the data sites overpredicted crop ET early in the season. This translated the cumulative mass curve upward for that site. The overprediction made comparison of the models difficult. In many cases, this translation caused the regression analysis to indicate a less than favorable performance of the model. Overprediction in the early season can be attributed primarily to the uncertainty of the crop coefficient when the canopy is not fully developed.

CONCLUSIONS

Three stress levels were designed to observe the effects of irrigation interval on crop evapotranspiration. Unfortunately, it is difficult to ascertain whether these stress levels were reached. The occurrence of rainfall throughout the irrigation season was evenly distributed, and 67 percent greater than normal. Although this precipitation did not always measurably increase soil water (as evidenced by neutron probe measurements), the effects on the overall stress of the plant cannot be determined. Intuition and past research do tell us that the crop will not be as severely stressed under this precipitation regime. Partly because of this precipitation, farms in the region experienced an overall bumper crop of winter wheat, especially in dryland fields.

The results of this research indicate that the logarithmic and combination models could represent a valid ET correction for low soil moisture conditions. The power model did not adequately predict crop ET during the irrigation season.

It is evident that a great deal of uncertainty exists in this area of irrigation. The design engineer would be well advised to exercise caution in regard to the design of a deficit irrigation system.

These models should only be viewed as an approximation to crop ET under low soil moisture. The instrumentation and methodology used in this experiment did not lend itself to a more exact analysis of ET models. This experiment was highly site and crop specific, and the results should be viewed as such.

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APPENDIX E

WHEAT PRODUCTION FUNCTIONS UNDER DIFFERENT IRRIGATION INTERVALS

WHEAT PRODUCTION FUNCTIONS UNDER DIFFERENT IRRIGATION INTERVALS

by

B. C. Nakamura[1] and M. J. English[2]

INTRODUCTION

With the increased costs of irrigation and constraints on energy, water, and capital, it has become imperative to maximize the economic efficiency of irrigation practices. An approach to irrigation that has recently received attention is the combination of reduced water application with either short interval (high frequency) or extended interval (low frequency) irrigation. The effects of reduced water use are well defined on many crops. There is limited information available on the combined effects of reduced water use and irrigation intervals. Up-to-date planning of new and existing irrigation systems will require more information on how crops respond to irrigation strategies with varying frequencies.

As part of a project concerned with irrigation optimization, a model of crop yields at different irrigation intervals was required. A series of field experiments were conducted in the 1980-81 growing season to quantify some of these relationships between crop yield, water use, and irrigation interval. The objective was not to develop a broader, general model, but to obtain relationships that would allow use of the model to make reliable predictions of winter wheat yields.

PRODUCTION FUNCTIONS

Production functions are mathematical expressions that relate crop water use to crop yield. Such information is essential in planning for optimum water use. Crop water use is a combination of several parameters including precipitation, soil moisture depletion, and water applied through irrigations. Another term used for crop water use is evapotranspiration; the amount of water transpired by the crop plus evaporation from the soil and plant surfaces. Evapotranspiration cannot be readily measured in the field but can be estimated from measurements of the other parameters. Crop yields are usually expressed in terms of marketable yield, such a grain weight in cereal crops.

A crop production function for wheat, developed by Doorenbos and Kassam (1979) is reproduced in modified form in Figure 1. Yield is expressed as a percentage of the maximum attainable yield on the vertical axis and evapotranspiration (ET) is expressed as a percentage of the maximum potential ET on the horizontal axis.

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[1]Research Assistant, Agricultural Engineering Department, Oregon State University, Corvallis, Oregon.

[2]Assistant Professor, Agricultural Engineering Department, Oregon State University, Corvallis, Oregon.

It has been difficult to apply production functions from one site for one crop and one year to other situations. Most production functions relate crop water use to crop yield using data from several growing seasons, and sometimes several sites, but usually for only a single irrigation interval. The relationship shown in Figure 1 is for general use in many areas, but applies to water use reductions distributed more or less uniformly throughout the irrigation season.

A common technique for developing production functionsis the line source experiment conceived by Hanks, et. al in 1976. This type of experiment utilizes a single sprinkler line that distributes water in a pattern that decreases lineraly as a function of distance from the sprinkler line. The amount of water applied at varying distances from the line are measured with catch cans. The amount applied ranges from something in excess of crop water requirements close to the line to no water at all at the limit of reach of the sprinkler. Yield samples are taken in different sections as a function of the distance from the line. Crop yields over the full range of water use are therefore determined by a relatively simple and inexpensive experimental configuration.

One drawback to this system is that with a single line only a single irrigation interval or frequency can be used. But in practice, if water use is reduced, it may also be desirable to alter irrigation frequency (English and Nuss, 1980 and Miller, 1977). A single line source experiment does not provide information on how crops will respond to limited water supplied at varying irrigation intervals.

GROWTH STAGES

As crops mature, they pass through several different stages of growth. These levels of plant development are often separated by physiological changes in the plant. A common system used to identify growth stages in wheat is the Feeke's scale. This identification system was used to delineate the different stages of growth of the wheat crop used in this experiment.

The influence of crop water stress on yields varies with the type of crop and the timing and magnitude of the water deficit. Cereals are characterized by the presence of certain stages of plant growth that are particularly sensitive to water deficits. An example of the sensitivity of wheat in different growth stages is shown in Figure 2 (Doorenbos and Kassam, 1979).

The sensitivity to stress may depend upon the availability of water earlier in the season. For example, when wheat is stressed early in the season the plant will limit shoot growth before it limits root growth. The plant will extend its roots to try to locate water deeper in the soil profile. If the root system has been stimulated by stress early in the season, the plant will generally not be as susceptible to stress later in the season.

Severe reduction in yield may occur if stress occurs during the jointing stage when the plant stem is extending. The number of spikelets or potential grain forming portions of the wheat plant can be reduced as well as general plant vigor and height. To maximize wheat yields, when water supplies are limited, it has been recommended that soil moisture deficits be avoided during this period (Ehlig and LeMert, 1976). Other reports have stated that the flowering period is the most sensitive to water deficits (Doorenbos and Kassam, 1979). When wheat plants are stressed during this period they will mature earlier and have lighter seeds.

As wheat matures the susceptability to stress is reduced. Once past the flowering stage, water deficits do not have drastic effects on yields. Water deficits occurring during the grain filling period will not have a very large effect on yields. Though, if water is available this late in the growing season it will still contribute to a boost in yields if applied in this period.

Production functions have been developed for wheat stressed evenly over a growing season, that is, irrigated at regular intervals at a reduced rate. Comparisons of wheat yields irrigated at different intervals is not presently available.

IRRIGATION INTERVAL

Conventional irrigation practice involves a cycle of short, intense applications of water followed by long periods of soil moisture depletion. The length of the irrigation interval is determined by the amount of easily extractable water the soil can store at one time. Recently, several investigators have looked into the benefits of irrigating at intervals much shorter or longer than in conventional practices.

It may be possible, in cases where irrigation costs are high or water is limited, to increase net income by increasing the irrigation interval while deliberately under-irrigating the crop (English and Nuss, 1980). Conversely it may be possible to increase income by adopting short interval applications of water, particularly when pipe costs for delivering water to the field are high (Rawlins and Raats, 1975 and Miller, 1977). The relatively small amount of research done in this area has not demonstrated definitively the effects which different irrigation intervals have on crop yields. Variation in results from different approaches demonstrate the need to further investigate the effects of irrigation intervals.

Extending the irrigation interval will generally force the crop to use water deeper in the soil profile compared to more frequent irrigations. The ability of different crops to extract water from these deeper portions of the profile will depend on the extent of the root system of the crops. One advantage of short interval or high frequency irrigation is that water is supplied to the plant as it is needed; hence there is little need to store water deeper in the soil profile. Under this irrigation regime, the root system will not tend to develop in the lower part of the soil profile, so the crop will be more vulnerable to stress should a serious water deficit occur.

Crop yields under short interval irrigations have been found to be maximum when the full evapotranspiration requirements of the crop are met. One recent study of crop yields under high frequency irrigation reports that yields could be maintained at high levels even when ET was reduced as much as 25 percent below the maximum rate (Miller, 1977). This report utilizes a soil profile that was maintained at the optimum level until after full cover was established. Differential irrigation treatments were started after this period. There has been some doubt that the actual ET reduction was as large as stated. A similar experiment was designed specifically to test Miller's conclusions. Faci and Fereres (1980) experimented with grain sorghum under high frequency irrigations. They were unable to duplicate Miller's results, and in fact found that yields were reduced proportionately as ET was reduced below the maximum rate.

Differences in these reports may be partially explained by variances in the experimental procedure. Miller started his irrigation treatments with a full soil profile, while Faci and Fereres began with a partially depleted soil profile. The soils used also had different textures and moisture retention characteristics.

An experiment to compare crop yields under high frequency deficit irrigation with yields from low frequency deficit irrigation was initiated in 1981 near the Oregon State University Agricultural Experiment Station at Hermiston. That experiment is the subject of this paper.

EXPERIMENTAL METHODS

Crop production functions were derived for winter wheat under three different irrigation regimes. To make comparisons of the different production functions a modified line source type of irrigation distribution system was used. Sixteen irrigated plots were used (Figure 3). Each plot was divided into five separate replicate subplots. Two dryland plots were included to provide information on unirrigated yield potential of the wheat. Water use and yield data were collected independently for each subplot and used to derive the three production functions.

All the plots were irrigated with 10 cm of water in the Fall of 1980. In the Fall, 290 kilograms per hectare of 16-16-16 fertilizer were spread uniformly over the field, along with 38.7 kilograms per hectare of sulphur. The plots received about 8 cm of effective precipitation over the winter and a 6.4 cm pre-irrigation in March, 1981. A top dressing of nitrogen fertilizer was applied the last week of March. Differential irrigation treatments were started in April.

One set of five plots was irrigated daily and another set of five plots was irrigated weekly. Different amounts of water were applied to each set of five plots ranging from 20 percent to 100 percent of the estimated crop water requirements. The last six plots were irrigated on extended intervals of either two weeks, three weeks, or four weeks (Table 1).

A set of six parallel sprinkler lines, spaced 6.1 meters apart, was used to irrigate the five daily plots. Sprinklers were spaced 6.1 meters apart along each lateral line. Rainbird 14VH, 5° sprinklers were selected for their uniform pattern of water distribution at close spacing. The range of applied water was introduced to the plots by operating the lines at varying durations.

Figure 4 shows the pattern of water distribution of the six-line system, with a typical line source distribution pattern shown for comparison.

The five weekly plots were irrigated with a set of six lateral lines identical to the daily system. The extended interval plots were watered with sets of two lateral lines spaced 4.6 meters apart, with a sprinkler spacing of 6.1 meters. Each irrigation in the extended interval plots was designed to return the soil profile to field capacity.

Data from the daily irrigated plots were used to derive a production function for a deficit irrigation regime, on a daily interval. The data from the weekly plots was used to derive a deficit irrigation production function for the weekly interval. The weekly interval was used as the "normal" interval, since weekly irrigation was recommended for winter wheat in the area. The third production function combined the results of the extended interval plots with the results from the fully irrigated daily and weekly plots. Water use could then be compared to yield on the basis of differences in irrigation interval.

Soil moisture depletion was measured throughout the growing season with a neutron probe calibrated for the Koehler loamy fine sand in the experimental field. Initial measurements of soil moisture were made in March. Final soil moisture data were gathered prior to harvest over a three-day period in July.

There were six neutron probe access tubes installed in each of the irrigated plots. The probe tubes were set along the center line of the plots, spaced equal distance from the two sprinkler lines covering each plot, at a 6.1 meter interval down the length of the plots. Measurements of applied water were obtained at the site of each probe tube using a catch can mounted on top of the tube, at canopy height, during each irrigation. The efficiency of the applied water was estimated from catch can measurements and flow meter measurements, comparing the quantity of water pumped to the amount caught in the cans. Due to high winds in the area, losses were estimated at 15 percent, even though the sprinkler spacing was extremely close.

A weather station was established at the experimental field site to measure minimum and maximum temperature, humidity, wind run, evaporation and precipitation. The weather data were intended to be used to make estimates of daily ET using a modified Penman equation. Partial equipment failure early in the irrigation season made it impossible to make these estimates on a daily basis, so evaporation pan data were used with a crop coefficient for daily estimates of evapotranspiration. Further checks were made with weather data from the Hermiston Agricultural Experiment Station (16 kilometer distance).

Most irrigationswere performed in the morning to reduce evaporation losses. The daily plots were covered fairly uniformly throughout most of the irrigation season, mainly due to the short duration required for each daily irrigation and the relative absence of wind early in the morning. The wind shifted the amount of applied water within some of the weekly and extended interval plots. The replicates further downwind (parallel to the sprinkler line) received more water during the periods of high winds.

Irrigations were completed the second week of June. Harvest operations began following the completion of the final soil moisture measurements. An approximate three meter square section of wheat plants was removed from the areas between the six probe tubes along the centerline of each plot. These five replicates from each plot were individually cut, threshed, and packaged in the field. The plots were measured for width and length after the threshing equipment was removed from the field. The grain samples were cleaned, weighed, and reweighed the first week of August. Standard grain weights (grain density) measurements were taken from each grain yield sample the last week of August.

RESULTS

Yields of winter wheat in the Hermiston were exceptionally high in 1981. High yields could probably be attributed to mild weather conditions that prevailed throughout the irrigation season. Daily temperatures were consistently 1°C to 2°C cooler than normal. Approximately seven centimeters of rainfall fell following the start of irrigations in April. The average rainfall for the same period at the Hermiston Agricultural Experiment Station is four centimeters. Lower temperatures and abundant precipitation prevented crop stress during the critical periods of crop growth. These nearly ideal growing conditions produced near optimum yields in the experimental plots and the surrounding area.

The relationship between grain yield and water use over the irrigation season is shown in Figure 5. Three general relationships derived from regression analysis are shown in this figure for the daily, weekly, and fully irrigated plots at extended, daily, and weekly intervals. Figure 6 shows the relationship between water use and yield for 24 data pairs from the daily plots. One yield sample was damaged during harvest operations from the D5 (20 percent ET) plot and was not included in this analysis. The line fitted to these data pairs by regression analysis has a fairly low correlation coefficient ($r^2 = 0.60$). The plots irrigated weekly produced data that resulted in the regression line with the shallowest slope (Figure 7). For each increment of water use, the increase in yield was not as great as in the daily or different irrigation interval treatments. The set of replicates from the W2 (80 percent ET) plot consistently had lower yields than anticipated by comparison with the other four weekly plots. These five replicates contributed both to the shallow slope and the low coefficient of correlation ($r^2 = 0.43$).

The relationship between yield and water use for the full irrigated, different interval plots is shown in Figure 8. The fitted line for 40 data pairs also has a low degree of correlation ($r^2 = 0.45$).

Water use for the irrigation season was calculated from measurements of soil moisture depletion, applied water, and precipitation. The precipitation total over the irrigation season was about seven centimeters as stated before. The amount of applied water ranged from 3.5 cm to 31 cm. Soil moisture depletion was limited to the top 1.2 meters of the soil profile in most of the daily and weekly plots. Some of the extended interval plots used water from deeper in the profile, down to 1.6 meters.

The nonuniformity of water application in the weekly and extended interval plots did not affect the total water use levels to a very large degree. The replicates receiving smaller levels of applied water in the same plot compensated by depleting the soil profile to a larger degree. Total water use ranged from 23 cm to 51 cm in the daily plots; 23 cm to 48 cm in the weekly plots; and 30 cm to 50 cm in the extended interval plots. (See Table 2).

Grain yields were very good in all irrigation treatments (Table 2). As a comparison, the dryland plots harvested next to the irrigated plots were on the order of 2600 kg/ha (45 bu/ac) where normally 1450 kg/ha (25 bu/ac) would be expected. Similar high levels of yields were observed in the extended interval plots that were supposed to be severely stressed.

One interesting result of this experiment was the comparison of standard weights from the yield samples (Table 2). The standard weight is a measurement of the density of the grain. The extended plots had a higher average grain density, higher than both the average of the daily and weekly plots. The average values of the daily, weekly, and extended plots were 706 kg/m^3, 716 kg/m^3, and 752 kg/m^3 respectively.

DISCUSSION

Crop water use was somewhat lower than normal in the 1980-81 crop season in the Hermiston area. The mild weather conditions prevented the development of crop water stress throughout the irrigation season. The lower temperatures reduced the occurrence of daily water deficits. The abundant rainfall was distributed so that long periods of water deficits were avoided. The high level of wheat yields observed reflected these conditions.

The amount of precipitation is important as when the storm occurs. Figure 9 shows the timing and magnitude of precipitation events from April to the middle of July. Each storm event is represented by a bar; the height of the bar expresses the total rainfall of the storm and the width of the bar corresponds to the duration of the storm. A major storm during the third week of May lasted six days and totaled about 2.5 cm of rain. Water deficits were held to a minimum during the flowering period because of this storm. The importance of providing water to wheat in this growth stage is evident from the consistently high yields in all treatments.

These mild weather conditions were a boon to local farmers, but dampened the anticipated yield reduction for the field experiment. Data from the 1980-81 growing season does not provide the information needed to model water use of wheat under different irrigation regimes. The variability of crop response to environmental factors has been demonstrated by the vagueness of the experimental results.

The effect of reduced water applications and extended irrigation intervals was not quantified in the first year of field experiments. A second year of data will be available following completion of the 1982 growing season. The weather has been more cooperative for the field work, staying fairly dry after April, 1982. A new set of production functions will be developed from the second year of data and used to construct the model of yields needed for the use in the study of alternative irrigation practices for wheat in Eastern Oregon.

DISCLAIMER

Trade names are used for identification purposes only and do not imply preference for this item by Oregon State University.

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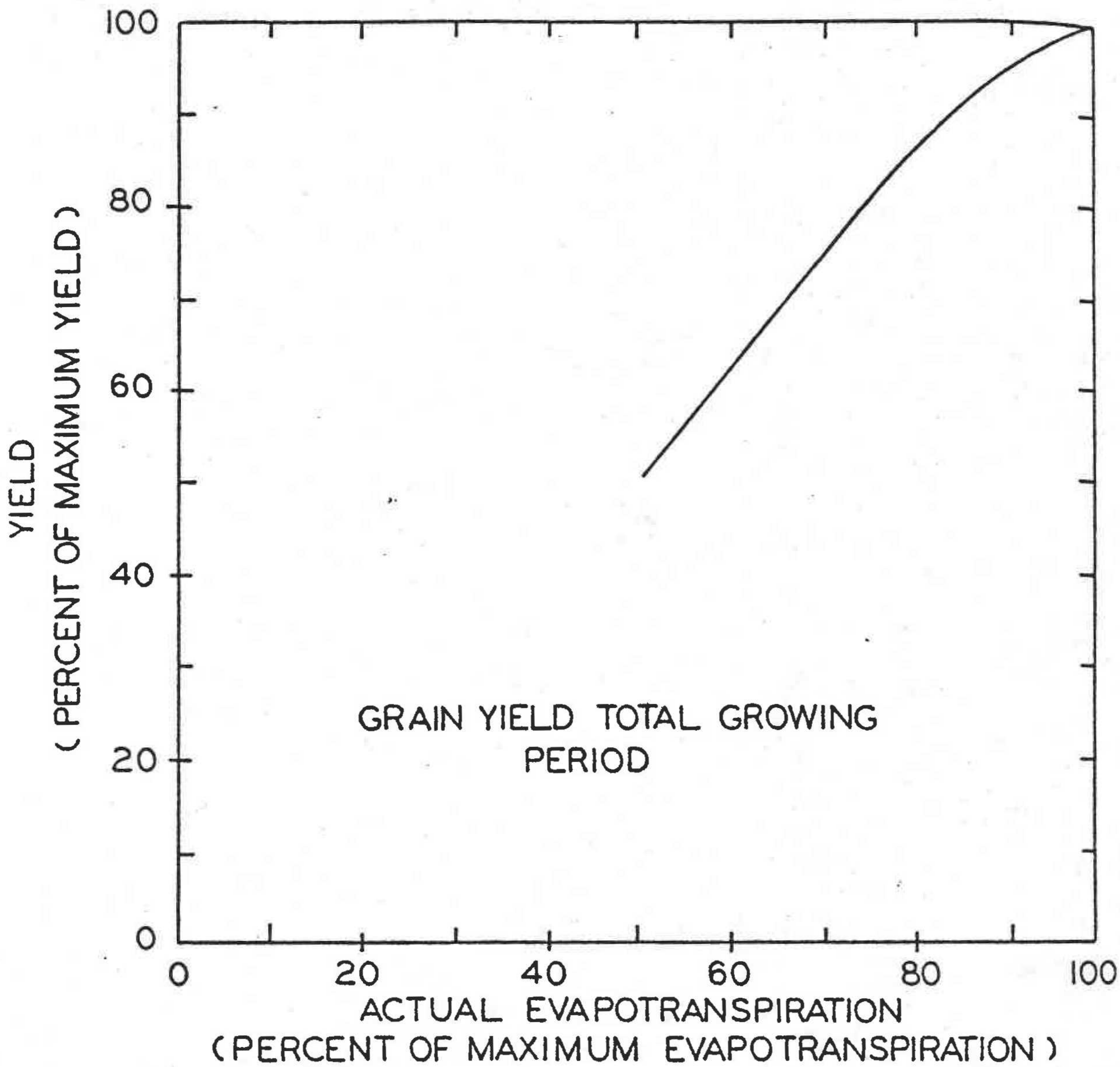

Figure 1. Relationship between season total evapotranspiration and crop yield for winter wheat (adapted from Doorenbos and Kassam, 1979).

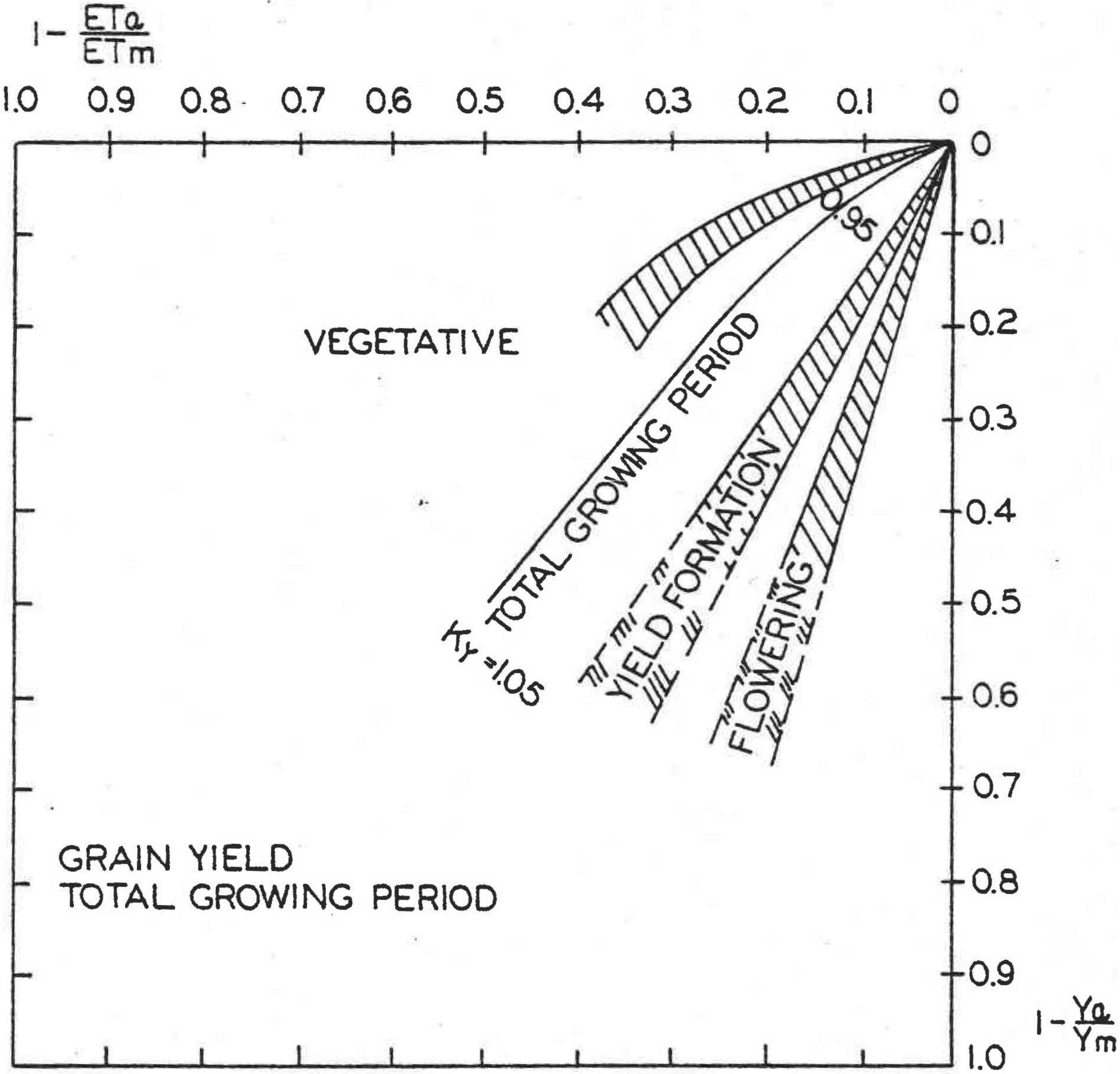

Figure 2. Relationship between relative yield decrease and relative evapotranspiration for winter wheat (Doorenbos and Kassam, 1979).

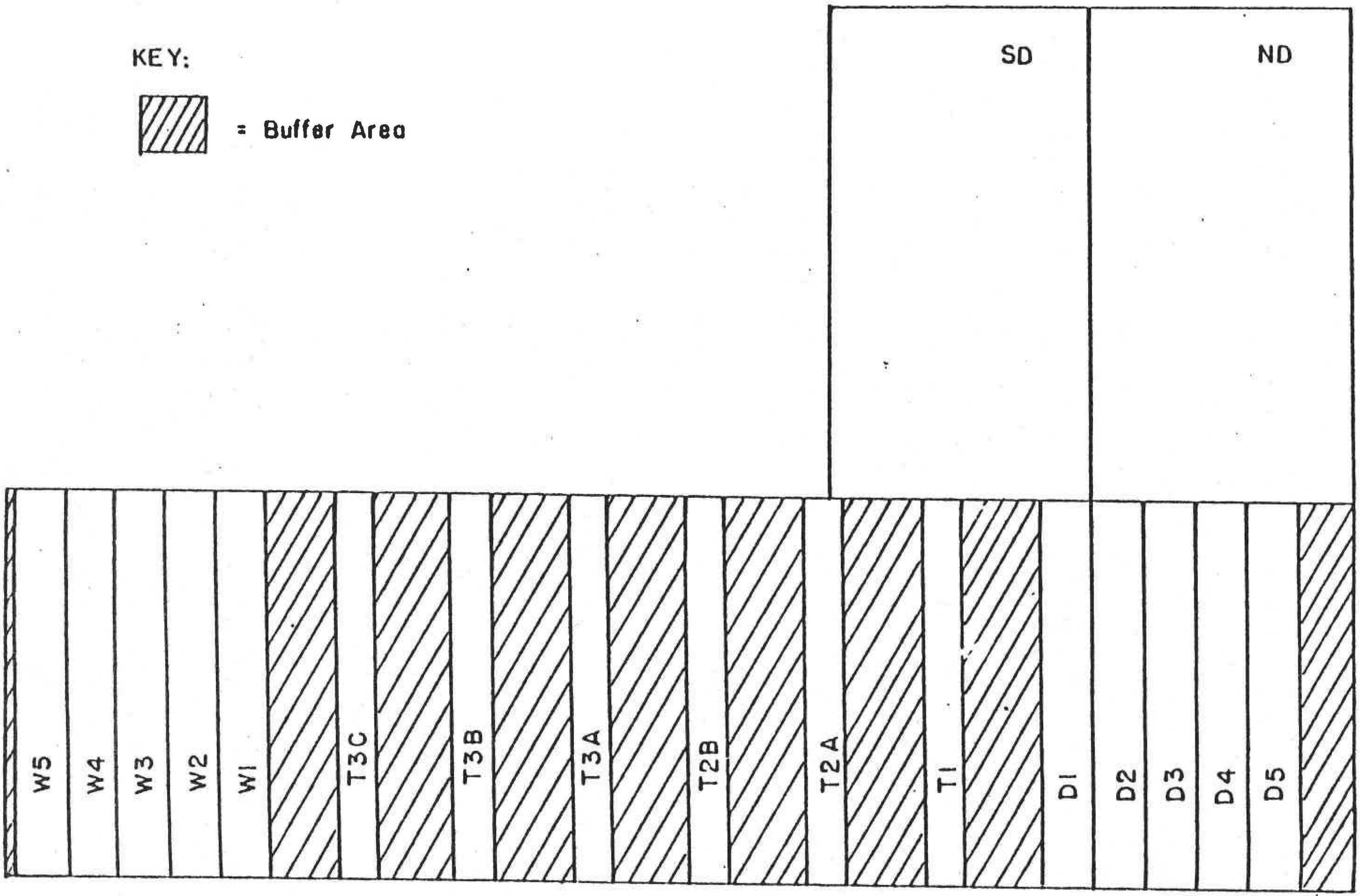

Figure 3. Layout of plots in experiment.

Table 1

Summary of Test Plots

Plot Designation	
D1	Daily irrigation at 100% of ET demand
D2	Daily irrigation at 80% of ET demand
D3	Daily irrigation at 60% of ET demand
D4	Daily irrigation at 40% of ET demand
D5	Daily irrigation at 20% of ET demand
W1	Weekly irrigation at 100% of ET demand
W2	Weekly irrigation at 80% of ET demand
W3	Weekly irrigation at 60% of ET demand
W4	Weekly irrigation at 40% of ET demand
W5	Weekly irrigation at 20% of ET demand
T1	Approximately two weeks interval between irrigations; 100% of depletion applied
T2A, T2B	Approximately 3 weeks interval between irrigations; 100% of depletion applied; staggered
T3A, T3B, T3C	Approximately 4 weeks interval between irrigations; 100% of depletion applied; staggered
DRY/DRY	Pre-planting irrigation in Fall, no further irrigation
DRY/Pre-irrigated	Pre-planting irrigation in Fall, pre-irrigation in March, no further irrigation

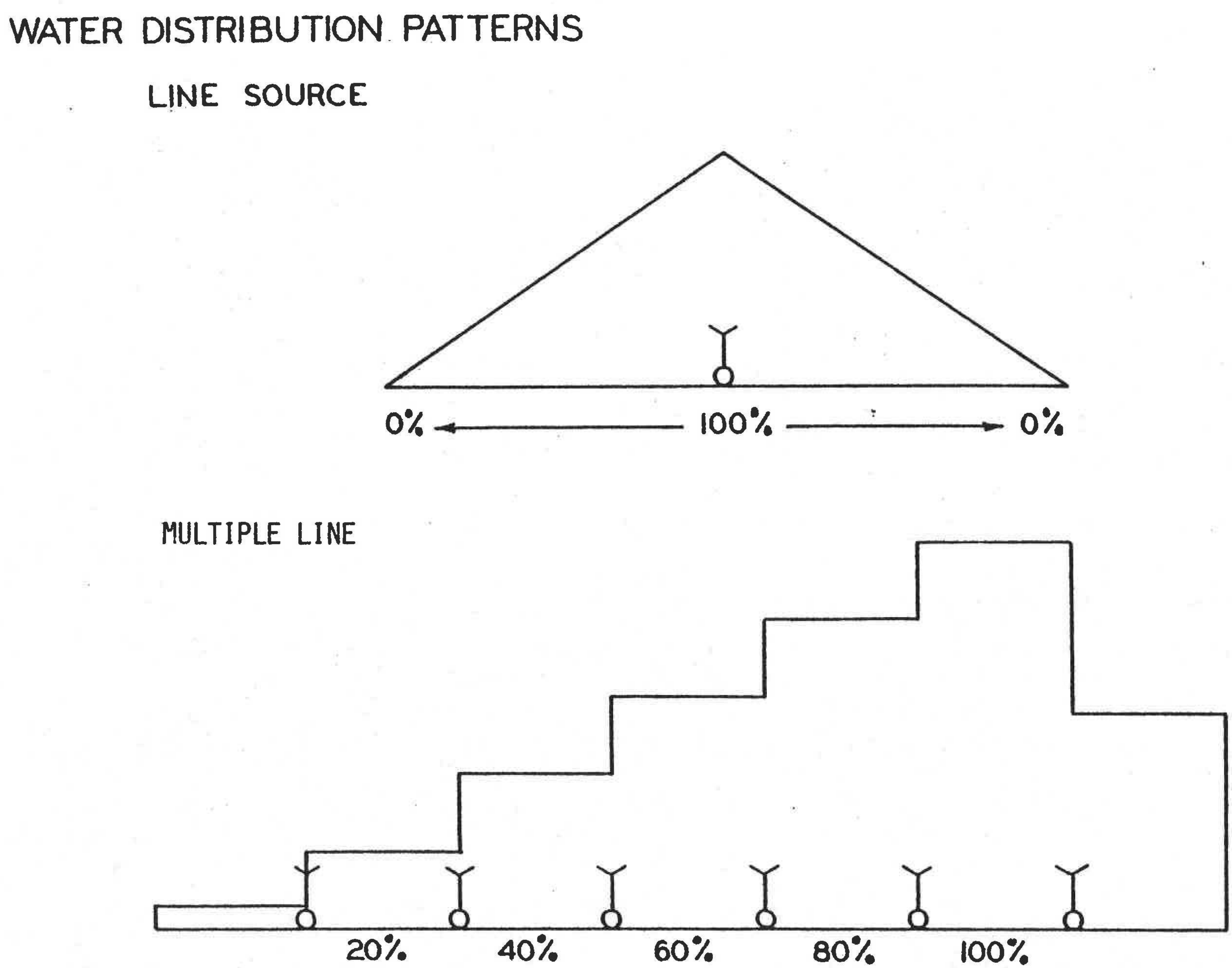

Figure 4. Comparison between water distribution pattern of a line source system and the six-line system used in the experiment.

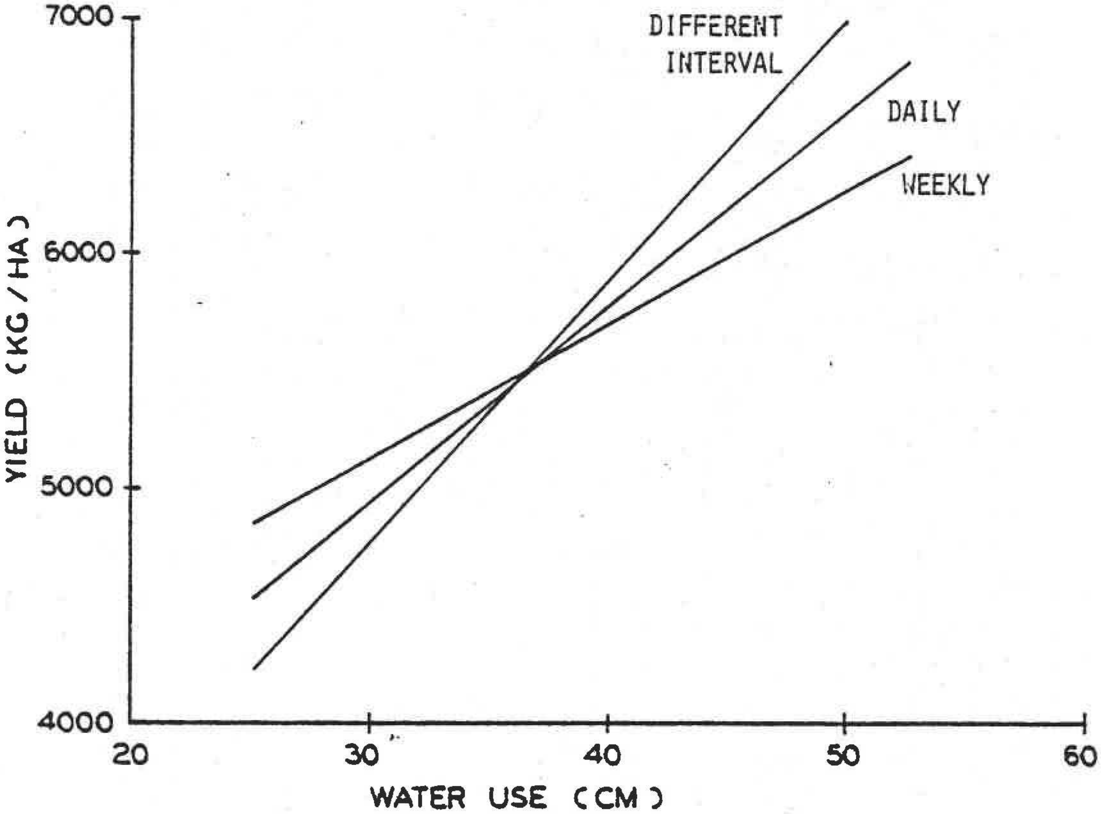

Figure 5. Relationship between season total evapotranspiration and crop yield; daily, weekly, and different intervals.

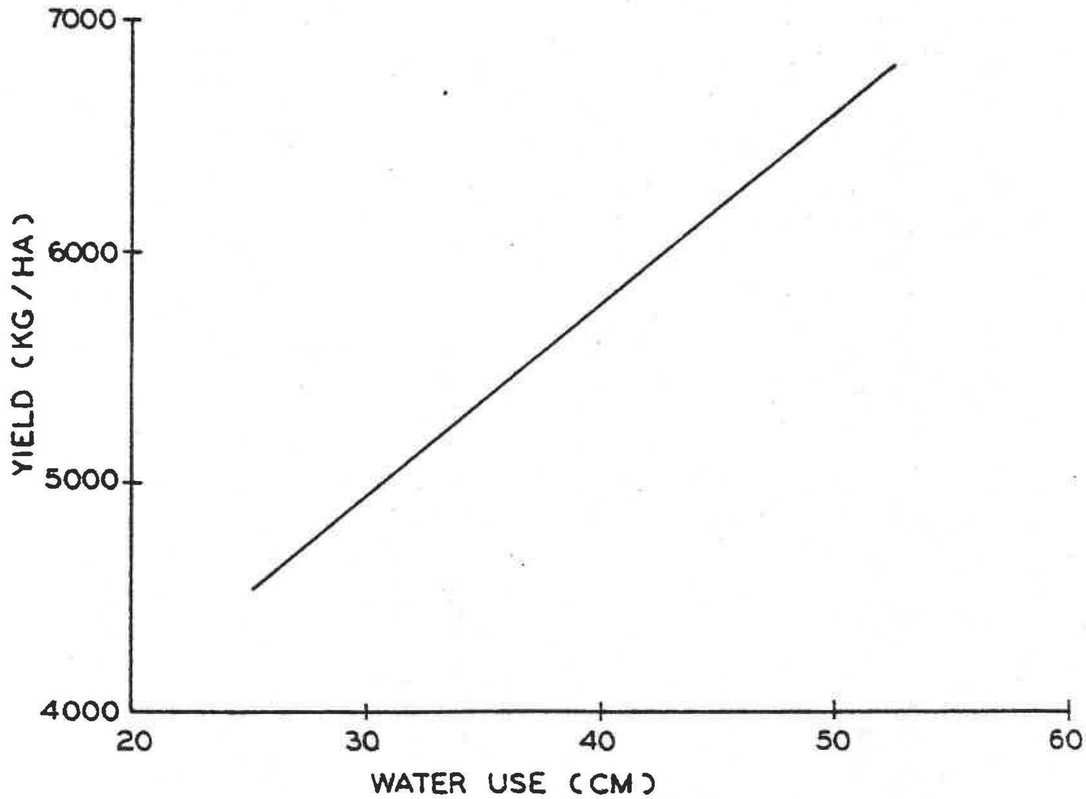

Figure 6. Relationship between season total evapotranspiration and crop yield; daily irrigation interval.

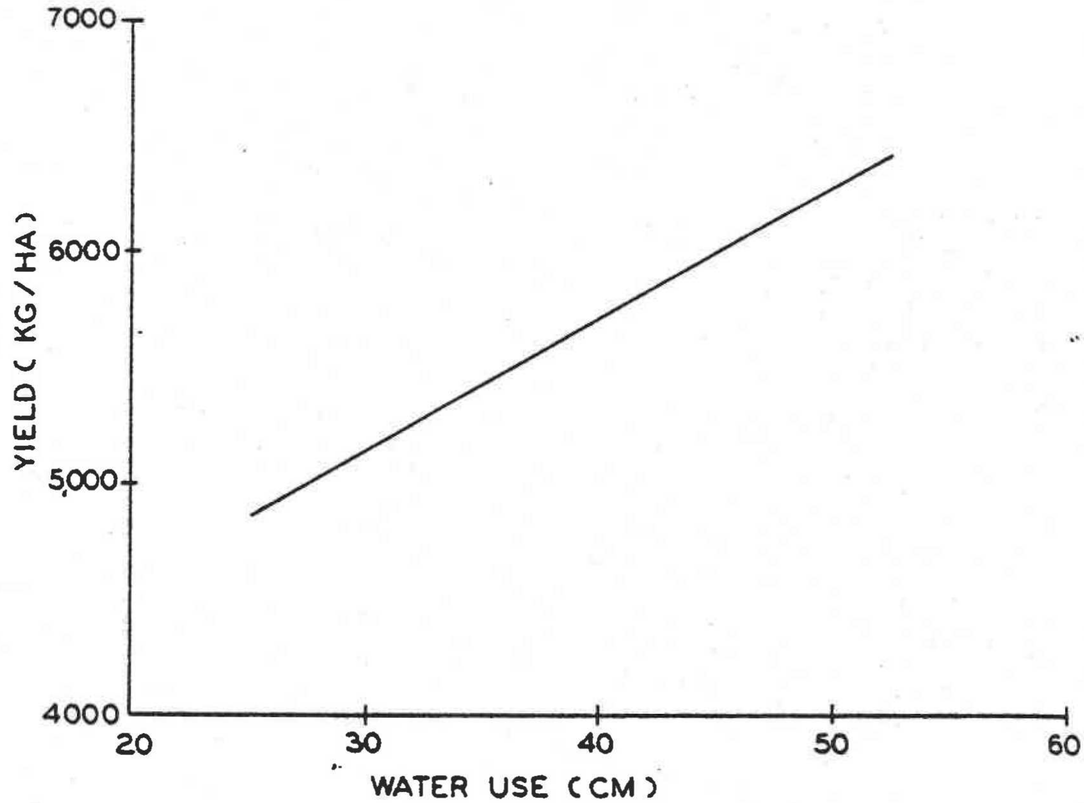

Figure 7. Relationship between season total evapotranspiration and crop yield; weekly irrigation interval.

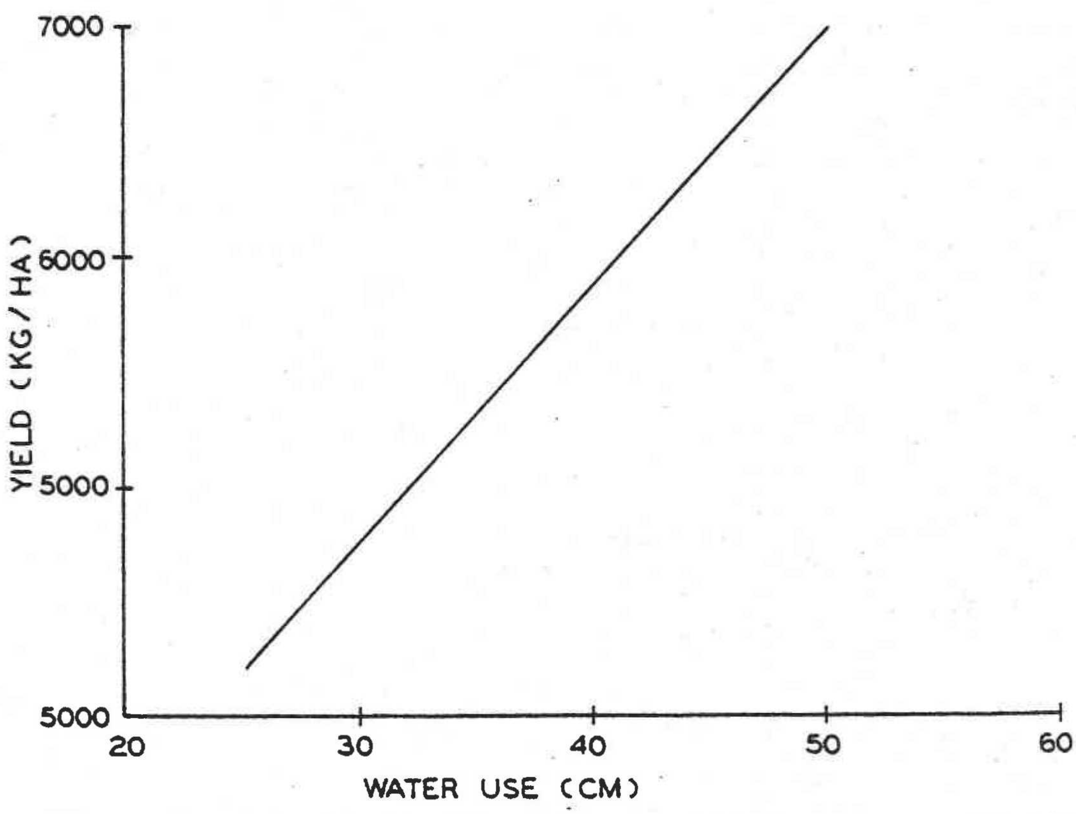

Figure 8. Relationship between season total evapotranspiration and crop yield; different fully irrigated intervals.

Table 2. Water use, yields, and standard weights.

Treatment Plot	Treatment Replicate	Water Use (cm)	Grain Yield (kg/ha)	Standard Weight (kg/m^3)
D1	1,2	47.6	5625	707
	2,3	48.6	6260	715
	3,4	49.6	5310	715
	4,5	51.1	6280	721
	5,6	48.7	6345	703
D2	1,2	42.1	5695	700
	2,3	41.1	5310	699
	3,4	42.0	5260	697
	4,5	43.8	6240	710
	5,6	40.2	6530	713
D3	1,2	33.6	5120	731
	2,3	32.7	5015	700
	3,4	33.8	4750	657
	4,5	35.6	4540	664
	5,6	34.4	4460	655
D4	1,2	29.3	5210	725
	2,3	29.3	5210	735
	3,4	29.3	5350	720
	4,5	29.9	5200	720
	5,6	29.5	4375	671
D5	1,2	25.3	3430	727
	2,3	25.7	4270	724
	3,4	26.9	4285	737
	4,5	29.9	3580	710
	5,6	--	--	671
W1	1,2	44.4	5910	709
	2,3	43.6	4945	723
	3,4	43.5	6490	715
	4,5	43.0	5760	707
	5,6	43.2	6270	713
W2	1,2	41.7	5620	727
	2,3	42.6	4485	714
	3,4	45.3	5100	715
	4,5	48.2	5555	717
	5,6	46.2	5060	700
W3	1,2	37.2	4862	714
	2,3	37.5	5500	722
	3,4	39.6	5370	698
	4,5	43.2	5430	723
	5,6	45.7	5750	714

Table 2. Water use, yields, and standard weights (continued).

Treatment Plot	Treatment Replicate	Water Use (cm)	Grain Yield (kg/ha)	Standard Weight (kg/m^3)
W4	1,2	31.9	5335	705
	2,3	31.5	4480	710
	3,4	32.6	4910	730
	4,5	32.6	5800	728
	5,6	33.8	5240	717
W5	1,2	25.2	4130	735
	2,3	24.0	3950	727
	3,4	26.1	4885	698
	4,5	26.3	4520	731
	5,6	27.1	4875	720
T1	1,2	49.6	5340	745
	2,3	48.7	6690	750
	3,4	48.7	6410	747
	4,5	49.9	5623	766
	5,6	45.1	6905	745
T2A	1,2	38.8	5160	755
	2,3	38.8	4780	735
	3,4	38.3	5895	766
	4,5	38.1	5580	758
	5,6	33.2	5845	764
T2B	1,2	40.3	5675	757
	2,3	39.5	6140	750
	3,4	40.8	6370	764
	4,5	43.8	5870	760
	5,6	40.6	6295	754
T3A	1,2	31.7	4170	747
	2,3	33.6	4270	747
	3,4	35.9	4510	748
	4,5	36.2	4640	745
	5,6	33.8	4940	754
T3B	1,2	30.5	3970	725
	2,3	32.9	4640	735
	3,4	35.5	5170	760
	4,5	36.6	5440	758
	5,6	35.4	4925	753
T3C	1,2	41.5	5730	755
	2,3	43.7	5310	749
	3,4	43.1	6255	752
	4,5	40.0	5585	750
	5,6	39.0	5390	758

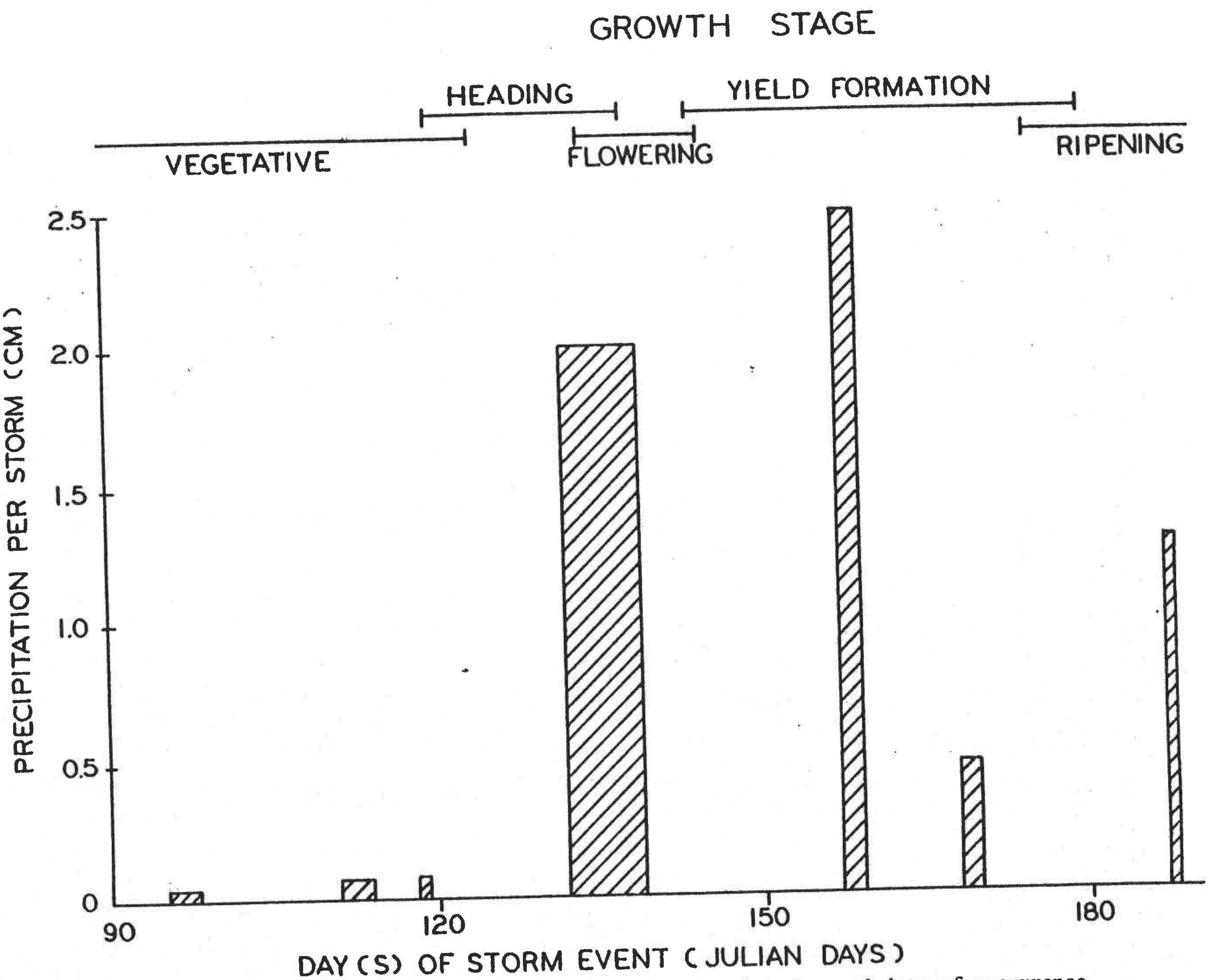

Figure 9. Schedule of storm events; magnitude, duration and date of occurrence.

APPENDIX F

STATISTICAL SIMULATION PROCEDURES

Appendix F

Statistical Simulation Procedures

1. Generating normal random numbers with given mean (μ) and standard deviation (σ):

Let Z_i = u(0,1) a uniformly distributed random number between zero and 1.0.

Then $E(Z_i) = \frac{1}{2}$ A-1

$\sigma(Z_i) = \frac{1}{12}$ A-2

Let

$$\overline{Z} = \frac{1}{N} \sum_{i=1}^{N} Z_i \qquad \text{A-3}$$

Then $N \cdot \overline{Z}$ will also be normally distributed, with a mean of $\frac{N}{Z}$ and a standard deviation of $\frac{N}{12}$.

Then, a random variable X, defined by the equation

$$\frac{\sum_{i=1}^{N} Z_i - \frac{N}{Z}}{\frac{N}{12}} = X \qquad \text{A-4}$$

will be normally distributed, with mean zero and variance one.

Equation A-4 can be used to generate normally distributed random numbers with a specified mean (μ_x) and standard deviation (σ_x) as follows:

(i) a set of u(0,1) random numbers are generated,

(ii) using equation A-4 the set of u(0,1) numbers are converted to a single simulated sample of a normally distributed random variable, t.i.e., let A-5

$$t_i = \frac{\sum_{i=1}^{N} Z_i - \frac{N}{2}}{\frac{N}{12}}$$

Then $t_i = N(0,1)$, a normally distributed random variable with mean zero and variance 1.

(iii) A new variable, χ_i , can be calculated from t_i:

$$\chi_i = \mu_x + t_i \cdot \sigma_x \qquad \text{A-6}$$

Then χ_i will be normally distributed with mean μ_x and variance σ_x^2.

Most computer systems include a uniform random number generator. (For those that do not, algorithms for generating uniform random numbers are widely available. See, for example, Law and Kelton, 1982, p. 219.) Where a normal distribution is required, the algorithm outlined above can be used in conjunction with the uniform random number generator. Where a uniform random number is needed the random variate Z_i can be used directly.

2. Generating correlated random variables:

The correlation coefficient for two random variables (χ_1 and χ_2) is

$$\rho_{x_1 x_2} = \frac{E\ \{(x_1 - \mu_1)(\chi_2 - \mu_2)\}}{\sigma_{x_1}\ \sigma_{x_2}} \qquad \text{A-7}$$

This coefficient is, in effect, a measure of the predictability of one variable by another. It is instructive to relate this coefficient to a general linear model. Suppose it is assumed that χ_1 can be predicted from χ_2 by the equation

$$\chi_1 = a_o + a_1 \cdot \chi_2 + \varepsilon \qquad \text{A-8}$$

Where ε is a prediction error and a_o and a_1 are coefficients. a_o and a_1 can be derived by linear regression. Their values will be

$$a_1 = \frac{(\chi_2 - \overline{\chi}_2)(\chi_1 - \overline{\chi}_1}{(\chi_2 - \overline{\chi}_2)^2} \qquad \text{A-9}$$

$$a_o = \chi_1 - a_1 \overline{\chi}_2 \qquad \text{A-10}$$

It can be shown that

$$a_1 = \frac{\sigma_{x_1}}{\sigma_{x_2}} \; \frac{E\{(\chi_2 - \overline{\chi}_2)(\chi_1 - \overline{\chi}_1)\}}{\sigma_{x_1} \; \sigma_{x_2}} \qquad \text{A-11}$$

or

$$a_1 = \frac{\sigma_{x_1}}{\sigma_{x_2}} \; \rho_{x_1 x_2} \qquad \text{A-12}$$

The linear prediction model then becomes:

$$\chi_1 = a_o + a_1 \chi_2 + \varepsilon$$

$$= \chi_1 + a_1 (\chi_2 - \overline{\chi}_2) + \varepsilon$$

$$\chi_1 = \overline{\chi}_1 + \frac{\chi_1}{\chi_2} (\chi_2 - \overline{\chi}_2) + \varepsilon \qquad \text{A-13}$$

If it is assumed that ε is independent of χ_2 it can be shown that the mean and standard deviation of ε are

$$\mu_\varepsilon = o \qquad \text{A-14}$$

$$\sigma_\varepsilon = \sigma_{x_1} \sqrt{1 - \rho_{x_1 x_2}^{\;2}} \qquad \text{A-15}$$

Then, if one wishes to generate correlated pairs of random values of x_1 and x_2 the procedure is:

(i) Estimate μ and σ for each of the variables χ_1 and χ_2 using available data.

(ii) Estimate $\rho_{x_1 x_2}$ from the data.

(iii) Generate random values of χ_2, using μ_{x_2}, σ_{x_2}

(iv) Generate random values of ε using a mean of zero and a standard deviation of $\sigma_{x_1} \sqrt{1 - \rho_{x_1 x_2}^{\;2}}$.

(v) Calculate estimates of χ_1 using Equation A-13.

3. Eliminating the bias:

In Monte Carlo simulation a set of numbers generated at random using the expression

$$\chi = \mu + \sigma \cdot t$$

(where χ is the random number to be generated, μ is the nominal mean and σ is the nominal standard deviation) will not have the same statistical characteristics as the nominal characteristics. For example the mean of the simulated sample will be different from μ. However, one goal of the algorithm described here is to simulate systems with specified characteristics (specified means, for example). The program, therefore, utilizes a subrouting that adjusts the random variables to produce a desired mean value. After a set of random numbers has been generated their mean value is calculated. Then each number in the set is multiplied by their ratio; that is

$$\chi_i' = \chi_i \cdot \frac{\mu}{\bar{\chi}} \text{ for each } i$$

where χ_i = initial random number

χ_i' = revised random numbers

$\bar{\chi}$ = sample mean

The result is a set of numbers generated at random, with the desired mean μ.