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This research addresses the large-scale group fleet replacement problem with multiple vehicle types of multiple units, under budget and demand constraints. Past research in group fleet replacement is limited and has focused on one vehicle type, with multiples units and constraints. Many studies in fleet replacement have also dealt with single replacement, with single units, and with no constraints. The objective of this research is to find the methodology to solve the general large-scale group fleet replacement problem.

A multi-phase methodology based on the grouping concept is presented and is integrated with optimization techniques. The vehicles are grouped according to various technology parameters. A two-level hierarchical replacement process, focusing on inter-group, and intra-group analysis, is developed and the resulting models are solved using integer programming. Finally, a case study using the Oregon Department of Transportation records is examined.

The replacement model presented can incorporate complex variations in the large-scale group fleet replacement problem. It is flexible and can be used in wide variety of replacement problems. Used appropriately, the results from this methodology can result in reduced fleet replacement costs and operation costs.

A METHODOLOGY TO SOLVE LARGE-SCALE GROUP FLEET REPLACEMENT PROBLEMS

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CHAPTER 1
INTRODUCTION

This research is motivated by replacement problems that often occur in transportation organizations. Group replacement planning under budget and demand constraints is a critical problem faced by fleet mangers. In practice, many cases deal with large-scale mixed fleet replacement. However, the research in this area is restricted. Decisions on replacement planning are not simple since the planning of fleet purchase, sale, size, and composition depends on capital budgets, demand, costs, and planning horizon.

Fleet replacement is normally classified as a single or a group replacement problem. Basically, the single replacement problem consists of a single unit in each period, typically with no constraints. The group replacement consists of multiple units with budgetary and demand constraints. Many studies in published literature have focused on the single replacement problem. However, research in group replacement is limited and is usually based on one fleet type with multiple units while incorporating either budget or demand constraints. This research will consider the group replacement problem with multiple types of multiple units under both budget and demand constraints. The general mathematical formulation for the group replacement model is constructed and a methodology to solve large-scale group fleet replacement problems is developed.

PROBLEM STATEMENT

This research involves the study of large-scale group fleet replacement problems under budget and demand constraints over multiple time (planning) periods. The fleet considered can be composed of vehicles of varying types, sizes, age, and costs. A finite planning horizon and deterministic cash flows are assumed. The costs considered in the replacement decision include the purchase costs of new vehicles, operating costs, maintenance costs, and estimated salvage value of current vehicles. The demand for each vehicle type and an overall replacement budget is specified for each period. The topic of interest is to find the optimal replacement plan for the fleet for each planning period. Consequently, questions of interest include what units to replace and when to replace them, thus identifying the most economical sequence of vehicles to meet the demand under a given budget. The fleet replacement decisions are as follows:

- The decision to keep (transfer from one period to next period), purchase, or sell, for each type vehicle at each time period.
- The decision to keep or sell units of each fleet type, considered by age, for a specific time period.

RESEARCH OBJECTIVE

The focus of this research is on fleet replacement and planning of mixed types of vehicles under budget and demand constraints. The replacement problem in this study incorporates planning the size and composition of the fleet over time. Since the problem in this study is complex, the research methodology starts with a generalized mathematical formulation. However, the problem size is the major concern in the development of a solution approach. Hence, the primary interest in this research is to develop an efficient methodology for large-scale replacement problems involving mixed fleets under budget and demand constraints. Data from the Oregon Department of Transportation will be used to illustrate the methodology developed in this research.

RESEARCH CONTRIBUTION

This research contributes to the area of fleet replacement by modeling and solving the large-scale group replacement problem. A solution methodology to help the decision-maker in the analysis of mixed fleet replacement and planning is developed. The solution and analysis of this type of problem will enhance the effectiveness of the decision process and provide the fleet manager insight into the fleet replacement system. Due to the scale of the problem involved, potential cost savings from even modest improvements in vehicle usage and replacement plans are significant.

ORGANIZATION OF DISSERTATION

Chapter 2 presents the state-of-the-art review for fleet replacement problems. The fleet replacement problems are defined and a framework for classifying the replacement problems is developed. In addition, commonly used solution approaches to single and group replacement are described.

In chapter 3, the group fleet replacement problem is presented. The problem involves multiple units of multiple types being evaluated for replacement under multiple constraints over discrete time intervals. General mathematical formulation for group fleet replacement is developed. A methodology to solve the large-scale group fleet replacement is then presented. The methodology involves use of concepts such as Group Technology to reduce the dimensionality of the problem and a two-stage (inter-group and intra-group) integer linear programming models. A case study from Oregon Department of Transportation is used to validate the model.

Chapter 4 concludes this dissertation with an overall summary and direction for future research.

CHAPTER 2
FLEET REPLACEMENT MODELS: A STATE-OF-THE-ART REVIEW

ABSTRACT

Fleet replacement decisions are concerned with the optimum equipment purchase and replacement plans. These are complex decisions, involving a significant amount of capital outlay. Effective solution of replacement problems requires the use of appropriate modeling and solution strategies. This paper defines the fleet replacement problem and presents a framework for classifying fleet replacement problems. Commonly used solution approaches are described and examples of their applications are presented.

Keywords: Fleet replacement, economic life modeling, dynamic programming, integer programming.

INTRODUCTION

Many diverse types of organizations, including public agencies, own fleets of vehicles. A fleet is characterized by various types of vehicles and related equipment which operate at different levels and perform different tasks. The replacement decision is a choice made between keeping the existing fleet, commonly referred to as a defender, or replacing it with a new fleet, or challenger. Management must decide how often to replace the vehicles over some planning horizon to obtain the optimum fleet replacement policy.

Planning and investment in fleet replacements are important and difficult decisions. Essentially, they involve a significant amount of capital, and may have long-lasting effects on operational effectiveness and efficiency. The decision complexity results from characteristics of the replacement problem. Replacement decisions are time dependent; any decisions made at the current time impacts future decisions. Budget requirements for vehicle replacements vary from one time period to the next, and technology enhancements may significantly impact characteristics of challengers. Vehicle-based criteria such as age, mileage and operating and maintenance costs may have an important impact on the decision to keep versus replace. The political process for defining replacement priorities may be particularly important in publicly owned fleets. The number of possible replacement plans increase quickly with increases in planning horizon, size and type of fleet and replacement options.

FLEET REPLACEMENT PROBLEMS

Fleet replacement problems are generally presented as single replacement or group replacement problems (Figures 2.1 and 2.2). Typically, the single replacement problem consists of a single unit in each period. The typical assumptions are deterministic values for purchase costs, operating and maintenance costs and salvage value, with an infinite or finite planning horizon. The objective of single replacement analysis is to find when to replace a single unit. The group replacement problem, as illustrated in Figure 2.2, is basically defined as multiple units of single type, which are grouped by age. Typically, group replacement problems assume deterministic costs with budget constraints for each period. The aim of group replacement analysis is to determine which multiple units to replace and when to replace them over a finite planning horizon.

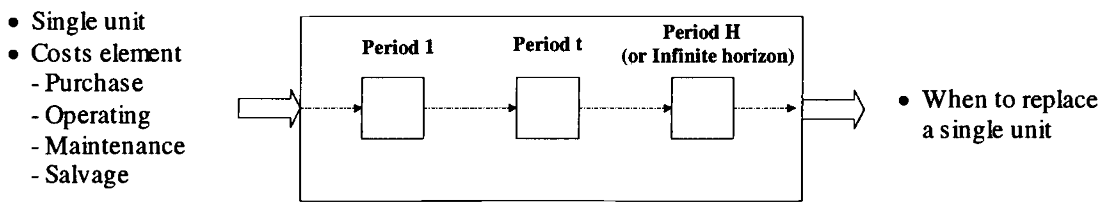

Figure 2.1 Single Replacement Problems

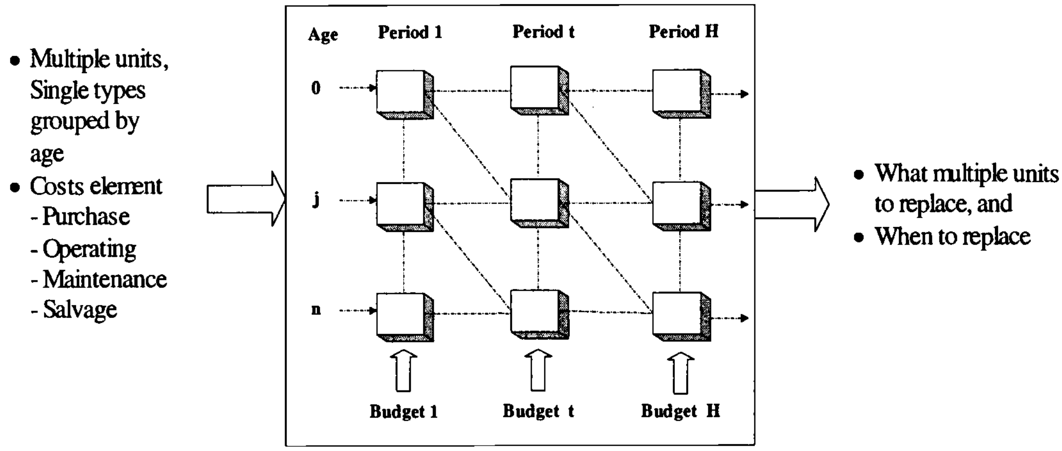

Figure 2.2 Group Replacement Problems

CLASSIFICATION OF FLEET REPLACEMENT PROBLEMS

Research on replacement problems has been conducted since the late 1940's [1]. A variety of replacement problems have been studied and different approaches or models have been developed to address these problems. Replacement models differ in terms of underlying assumptions, scope, flexibility, and practicality, depending on the problem characteristics. The relevant characteristics of replacement models can be found in [1-5]. The replacement problems and models can be categorized in various ways. These include fleet type, replacement alternatives, planning horizon, parameters, and constraints. A classification framework for replacement problems is presented in Figure 2.3. Obviously, there are many possible combinations of these problem characteristics. Consequently, a wide range of models has been developed to represent replacement problems.

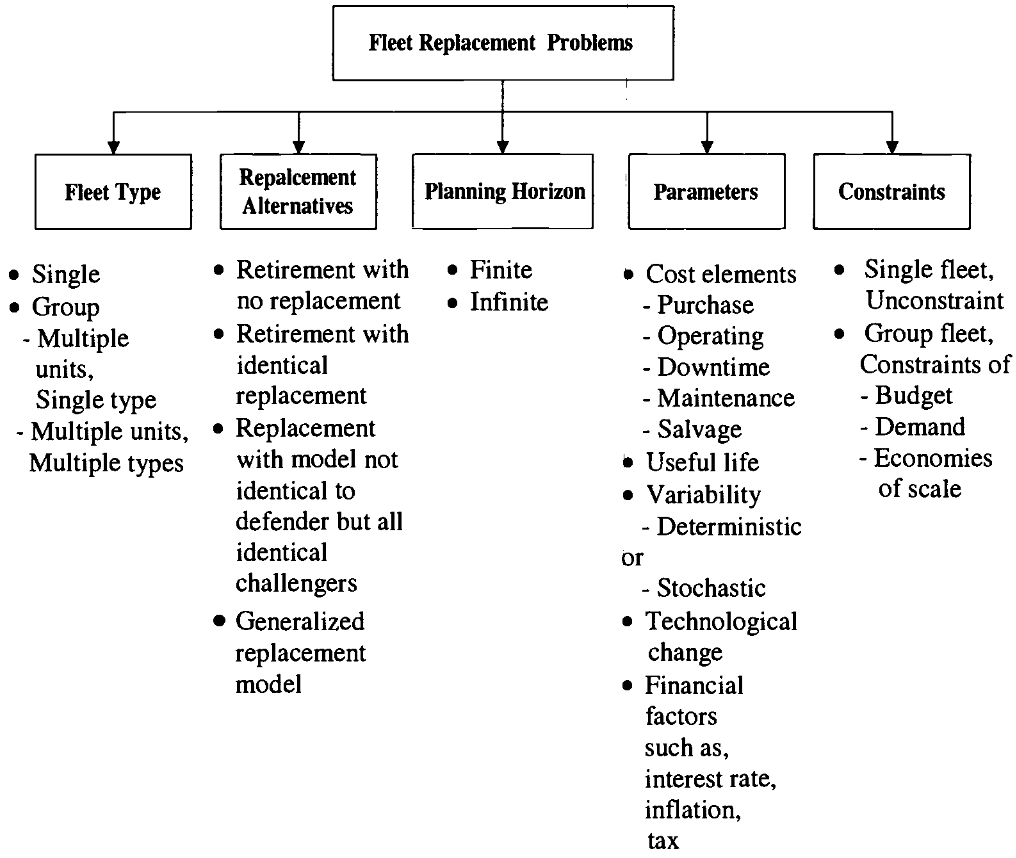

Figure 2.3 Classifications of Fleet Replacement Problems

Fleet Type

Classification based on this factor groups replacement studies into two categories, single replacement and group replacement. These are also referred to as serial replacement and parallel replacement, respectively. In single replacement, choice is between the defender and a single unit of a challenger. Each unit is considered to be economically independent of other units since typically no constraints are included. Group replacement deals with the group of vehicles being replaced by another group. The group replacement may consist of single type or multiple types of defenders. The replacement can be multiple units replaced with a single type of challenger or multiple units replaced with multiple types of challengers. Choices are economically dependent in that the decisions for any vehicle may affect decisions for other vehicles. Basically, budget and/or demand constraints are incorporated in group decision analysis.

Replacement Alternatives

Typically, replacement problems have been categorized into four cases [6]. These are:

- Simple retirement with no replacement, where the alternatives are to keep or retire the defender with no replacement.
- Retirement with identical replacement, where the choices are to either keep or replace the defender with an identical unit.
- Replacement with a unit unlike the defender but all replacements are identical, where the current challenger is unlike the defender, but all future challengers will be identical to the current challenger.
- Generalized replacement model, where the current challenger may be different from the defender and all future challengers may be unlike one another.

Planning Horizon

Planning horizon is the specified period of time for which service is required in the replacement problem. It can be either infinite or finite. An infinite planning horizon is most commonly used in traditional single replacement analysis. It is generally used where many operations are expected to continue for a very long time. However, as forecasts become less precise further into the future, the infinite horizon is rather impractical. The finite-planning horizon is appropriate for projects or operations that have a predictable time frame. The length of the planning horizon may have a strong influence on optimal replacement policies. Thus, an appropriate study period must be selected and all alternatives must be compared over the same planning horizon.

Parameters

Typically, cost elements and life parameters associated with defenders and challengers are employed in replacement analysis. Typical cost elements are purchase cost, operating costs, maintenance costs, and salvage value. Some replacement problems account for downtime costs. Basically, replacement models are either deterministic or stochastic with respect to parameters in replacement problems. Deterministic models assume that pertinent parameters are known with certainty. Stochastic models deal with situations where some of the parameters are uncertain thus necessitating the use of probability distributions for the parameters in question. Replacement problems may take into account technological change necessitating the need for stochastic parameters in the model. In addition, fleet replacement problem may account for factors such as interest rate, inflation rate and taxes.

Constraints

With fleet replacement studies, the system may be unconstrained or constrained. If there are no constraints, each unit is analyzed as single replacement in order to determine the optimal policy for the system. Group replacement decisions normally incorporate constraints. The most common constraint is a budget limitation for each period. Additional constraints may be required to represent demand of vehicles at each period and cost structures resulting due to economies of scale in purchase decisions. The demand is commonly expressed in terms of number of units, and some vehicle utilization factor is incorporated. Demand is normally assumed to be deterministic, i.e., fixed for each period in the planning horizon.

Typical Replacement Scenarios

Replacement problems represent a combination of elements from the different classification elements described above. For example, the replacement type, either single or group, is combined with finite or infinite planning horizon and deterministic or stochastic parameters. The alternatives for replacement, identical or unlike defenders, add to the diversity of problems. Extensions of those studies involve relaxing the deterministic constraint with respect to the model parameters. The one challenger option may be extended to multiple options. Some studies include interest rate and/or taxes. From these combinations, the following research topics have evolved in this area:

- Single replacement, infinite planning horizon and deterministic parameters. Published literature generally presents the single replacement problem with one defender (one piece of equipment).
- Single replacement, finite planning horizon and deterministic parameters.
- Group replacement, finite planning horizon, deterministic parameters, and budget and/or demand constraints. Within group replacement, research problems can involve either one type or multiple types of defenders. However, most reported research has focused primarily on multiple units of a single type.

RESEARCH IN REPLACEMENT PROBLEMS

There has been a great deal of research and case study analysis performed on fleet replacement problems. Hartman [7] provides an excellent survey of literature in replacement analysis. Terbog [8] and Alchin [9] are the pioneers in the area of single replacement policies, and their work has been widely quoted in replacement research. VanderVeen [10] is the pioneer in group replacement with a study on parallel machine replacement in production lines. Most of reported research is in single replacement studies. Recent work focuses on group replacement with constraints. A summary of past research for single replacement and group replacement is presented in Tables 2.1 and 2.2.

As discussed in the previous section, various types of replacement problems have been studied. These are primarily the single replacement problem with deterministic parameters and finite or infinite planning horizon, and deterministic group replacement with budget and/or demand constraints. Extensions to this basic research base include:

- Identifying optimum planning horizon [6, 7, 25, 32, 47, 50].
- Addressing technological change in replacement options [16, 24, 30, 51-55].
- Incorporating utilization into replacement analysis [52, 56, 57].
- Incorporating interest rate and/or tax [53, 55, 58-64].
- Incorporating economics of scale [41, 65, 66]

Table 2.1 Summary of Literature for Single Replacement Problems

Fleet Type	Approach	Parameters		Planning Horizon		Technological change/other options
		Deterministic	Stochastic	Infinite	Finite	
Single Replacement	Economic Life	Armour [11] Bert [12] Chee [13] Degarmo, Sullivan, Bontadelli and Wick [14] Eilon, King and Hutchinson [15] Grinyer [16] Park and Sharpe-Bette [17] Slubicki and Shen [18] Walker and Silas [19] Sussams [20]		Armour [11] Bert [12] Chee [13] Degarmo et al.[14] Eilon, King and Hutchinson [15] Grinyer [16] Slubicki and Shen [18] Walker and Salias [19] Sussams [20]		Grinyer [16]
	Dynamic Programming	Ahmed [21] Bean,Lohmann, and Smith [22][23] Bohner [6] Bylka,Sethi and Sorger [24] Chand and Sethi [25] Fadjar [26] Hearnes [27] Lohmann [28] Oakford,Lohmann and Salazar [29] Richard, Dan, and Harry[30] Sethi and Morton [31] Sethi and Chand [32] Thongthai [33] Waddell [34] Wagner [35]	Bohner [6] Bean,Lohmann, and Smith [22] Hearnes [27] Lohmann [28] Thongthai [33]	Bean,Lohmann, andSmith [22][23] Bohner [6] Chand and Sethi [25] Lohmann [28]	Ahmed [21] Bean,Lohmann, and Smith [22][23] Bohner [6] Fadjar [26] Hearnes [27] Lohmann [28] Oakford,Lohmann and Salazar [29] Richard, Dan, and Harry [30] Sethi and Morton [31] Sethi and Chand [[32] Thongthai [33] Waddell [34] Wagner [35]	Bohner [6] Bylka,Sethi and Sorger [24] Chand and Sethi [25] Oakford,Lohmann and Salazar [29] Richard, Dan, and Harry [30]
	Integer programming	Adil and Gill [36]			Adil and Gill [36]	

Table 2.2 Summary of Literature for Group Replacement Problems

Fleet Type	Approach	Parameters		Planning Horizon		Constraint		Technological change/ other options
		Deterministic	Stochastic	Infinite	Finite	Budget	Demand	
Group Replacement	Economic life & Benefit cost	Appleby [37] Randhawa, Douglas, Somboonwiwat and Budhakul[38]		Appleby [37]	Randhawa, Douglas, Somboonwiwat and Budhakul[38]	Appleby [37] Randhawa, Douglas, Somboonwiwat and Budhakul[38]		
	Dynamic Programming	Simms, Lamarre, Jardine, and Boudreau [39] Lund [40] Jones,Zydiak and Hopp [66]			Simms Lamarre, Jardine, and Boudreau [39] Lund [40] Jones,Zydiak and Hopp [66]	Simms Lamarre, Jardine, and Boudreau [39] Lund [40]	Simms Lamarre, Jardine, and Boudreau [39]	Jones, Zydiak and Hopp [66]
	Linear Programming	Avramovich Cook, Langston, and Sutherland [42] Simms et al. [39] Lund [40] Jones,Zydiak and Hopp [66]			Avramovich et al. [42] Simms et al. [39] Lund [40] Jones,Zydiak and Hopp [66]	Simms et al. [39] Lund [40]	Avramovich et al. [42] Simms et al. [39]	Jones, Zydiak and Hopp [66]
	Integer Programming	Hartman [7, 43] Karabakal [2] Karabakal, Lohman and Bean [44]			Hartman [7, 43] Karabakal [2] Karabakal, Lohman and Bean [44]	Hartman [7, 43] Karabakal [2] Karabakal, Lohman and Bean [44]	Hartman [7, 43]	Hartman [7]
	Network Model	Aggarwal, Oblak and Vemuganti [45] Vemuganti, Oblak and Aggarwal [46]			Aggarwal, Oblak and Vemuganti [45] Vemuganti, Oblak and Aggarwal [46]	Aggarwal, Oblak and Vemuganti [45] Vemuganti, Oblak and Aggarwal [46]	Aggarwal, Oblak and Vemuganti [45] Vemuganti, Oblak and Aggarwal [46]	
	Stochastic Programming	Couillard and Martel [47]	Couillard and Martel [47] Morse [48]		Couillard and Martel [47] Morse [48]	Morse [48]	Couillard and Martel [47]	

APPROACHES TO FLEET REPLACEMENT PROBLEMS

To assist in the replacement decision, a number of models have been developed for general cases as well as specific replacement problems. The economic life model is the primary approach used for single replacement analysis and has been presented in many popular textbooks in engineering economics and in research publications. It is the most general theoretical modeling approach applied to individual (single) machines or vehicles with deterministic cost parameters. However, the assumption of this model, the like-to-like replacement (repeatability), is not applicable in the generalized replacement problem.

Dynamic programming, used to relax the repeatability assumption, has been employed in many single replacement problems. However, one of the drawbacks with dynamic programming formulations is the "curse of dimensionality" referring to the difficulties in solving the resulting model due to its size [36]. Linear programming and integer programming are introduced to offset such drawbacks, and to extend the group replacement problem to situations involving system constraints. The group replacement problem is more complex to solve especially when fleet size varies.

In recent years, other approaches have been developed to solve the group fleet replacement problem. Integer programming and network modeling have been applied to the problem; lagrangian relaxation and heuristics are then introduced to solve the resulting mathematical model. In addition, researchers have adopted various approaches for solving replacement problems by relaxing certain assumptions related to the models previously studied. For example, fuzzy logic has been used to study stochastic parameters in single replacement problem [27, 67]. Tables 2.1 and 2.2 summarize the different solution methods presented in reported literature for single replacement and group replacement, respectively. Research is further grouped by type of parameters, planning horizon and constraints. The primary approaches used in developing solution methodologies for the replacement problem are economic life, dynamic programming, linear programming, and integer programming. In the following sections, the different replacement models and the type of problems they address are briefly reviewed.

SINGLE REPLACEMENT

Single replacement involves replacement of a single asset with another asset. The decision can be either to keep the current asset or replace it with one of many asset options. The approaches used to solve the single replacement problem are presented in this section.

Economic Life Modeling

The economic life of a piece of equipment is the optimal period of time, normally in years, that results in the minimum total annual cost of owning and operating the equipment. The economic life of a unit is of critical importance to equipment managers, as it relates to the total stream of costs associated with the unit over time. Basically, there are two cost categories considered in economic models [20]. First, the capital recovery costs, representing the expense of recovering invested capital, are incurred at a decreasing rate with time and/or usage. Second, the operating and maintenance costs for equipment use are incurred at an increasing rate with time and/or usage. In addition, downtime cost, obsolescence costs, and inventory carrying costs can be included as components of operating and maintenance costs. The total average cost is the sum of these two costs as shown in Figure 2.4. Economic life is the period of time (years) that results in the minimum equivalent uniform annual cost of owning and operating an asset.

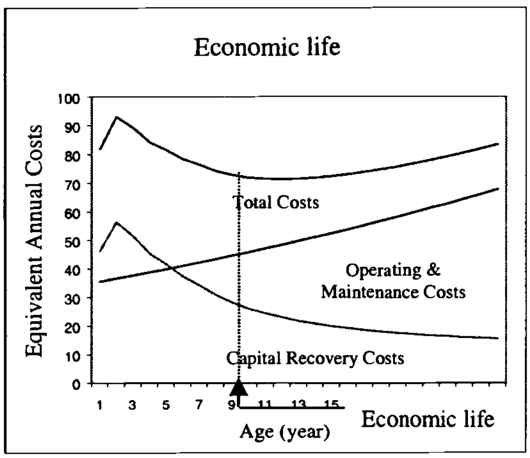

Figure 2.4 Economic Life Model

The economic life model to solve the single equipment replacement problem has been widely adopted both by practitioners and researchers. Various replacement problems using the economic life model have been discussed in [11-13, 15, 16, 18, 19, 37].

In applying economic life model to mixed types of equipment, each type of equipment may be represented individually using mean from the group. Walker and Silas [19] introduced an economic model for the replacement and management of Navy vehicles where the economic life concept has been used to determine the optimal service lives of various vehicle types within the Navy's fleet. Armour [11] used the economic life model to estimate the most optimal replacement age for Seattle Metro's bus fleet upgrading and expansion planning.

The economic life model has been widely used because of its simplicity. However, implementation of this model requires accurate estimates of appropriate costs. Functions for capital recovery costs and operating and maintenance costs are estimated over time. The vehicle's economic life is then obtained from the total cost curve.

Technological change may effect the economic life of capital investment. Grinyer [16] discussed these effects and introduced the relationship between obsolescence and salvage value. The obsolescence may lead to increase in economic life under a realistic range of parameters.

Extensions to the basic cost model are the marginal costs and "repair limit". This marginal cost is used to find the replacement time that minimizes the present worth over a specified planning horizon. The optimality condition is to replace as soon as the marginal cost of keeping the old asset for one additional period is greater than the marginal savings of postponing replacement by one additional period [68]. Matsuo [69] presented the marginal cost or year-by-year cost applied to replacement problem for an existing asset.

Repair limit is defined as the maximum amount economically justified to be spent to repair equipment [61]. Chee [13] addressed the repair limit for fleet replacement by comparing the costs of keeping the current vehicle through its economic life with the costs of repacing a new vehicle. Feldman and Chen [70] discussed an optimal repalcement and repair model. Freitas [61] provided an survey of literature of economic life models and repair limit models. Nosseir and Saad [71] presented a vehicle replacement model where a vehicle is replaced if its expected profit is less than the profit limit obtained for the age considered.

Dynamic Programming

Dynamic programming is a technique used to find the optimal solution to time staged decisions. Application of this technique results in simultaneous optimization for all time periods in terms of which equipment to replace and when to replace them. Generally, one of two optimality criteria are used; maximization of profits, or minimization of costs.

As explained by Howard [72], dynamic programming is used to analyze problems resulting from studies that involve multi-period decisions with multiple options. A sequential decision problem is characterized by a sequence of decisions with each decision affecting future decisions. The dynamic programming method divides the problem into stages with a policy decision required at each stage. Each stage corresponds to a specific time period in the planning horizon. The decision that should be taken at each stage corresponds to the selection between the defender and the challengers.

Dynamic programming has generally been applied to single replacement. It can be used to model group replacement problems [7]. Examples of dynamic programming applications in single replacement are given in [6, 22-25, 29, 31-33, 35]. Dynamic programming with respect to equipment replacement has been presented in a number of textbooks [17, 35, 73]. These models share the same characteristics: deterministic interest rates and cash flow, a finite planning horizon, number of replacements that is equal to or less than the number of periods in the planning horizon, and one challenger for each decision stage.

An example of the replacement plans resulting from dynamic programming is shown in Figures 2.5 and 2.6, and Table 2.3. This method divides the problem into three stages corresponding to three planning periods with a policy decision required at each stage. The decision that should be made at each stage corresponds to the competition between the current fleet and the new fleet. A decision that is made at the current state will transform the current state into a state associated with it. The present worth of various possible alternatives are calculated throughout the planning period. Then, the resulting minimum present worth gives the optimal replacement plan.

Wagner's [35] representation of the equipment replacement network for dynamic programming is shown in Figure 2.5. Nodes 0 to 3 represent the periods; the arc from node 0 to node 3 represents the decision to keep the equipment for three periods with an associated cost of C_{03}. The replacement plan to keep the equipment for two periods and replace in the third period corresponds to the line from node 0 to node 2 with cost C_{02}; at node 2 new equipment will be purchased. Dynamic programming recursion is applied to find the optimal replacement policy that is the minimum cost over the planning horizon.

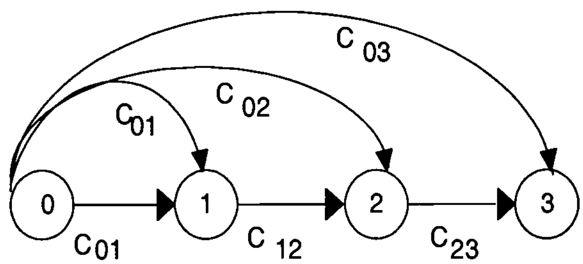

Figure 2.5 Wagner's Network

Park and Sharp-Bette [17] representation of the dynamic programming problem is shown in Figure 2.6. The replacement plan for keeping equipment for three years is the route from D_8 to D_{11}. A forward recursion algorithm is used to solve the problem.

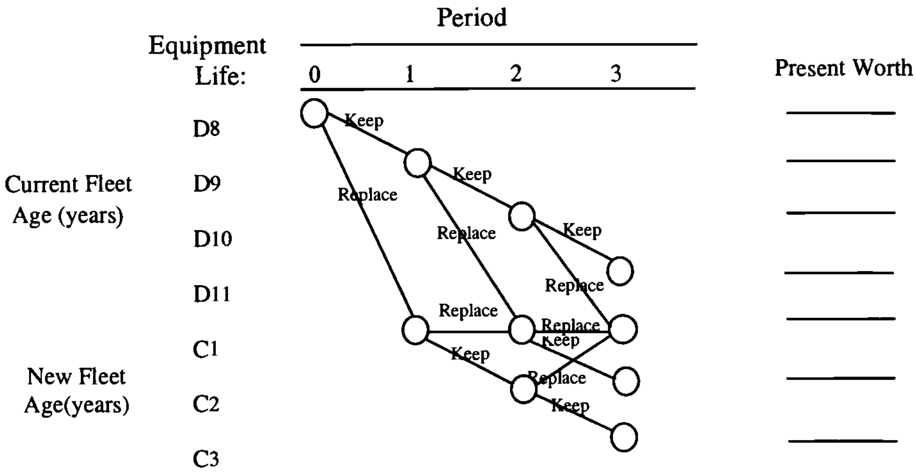

Figure 2.6 Dynamic Programming Applied to Replacement Problem

Fleischer [73] addressed the generalized replacement model using an exhaustive search and an efficient solution algorithm. In general, N-periods of planning horizon result in 2^N possible combinations of defender and replacements lives. For example, with a 3-period planning horizon, eight possible combinations of lives of defender and subsequent challengers are presented in Table 2.3. In the first plan, the defender is retained for all three periods. In the second plan, the defender is retained for 2 periods followed by replacement. The challenger is then retained for the remaining 1 period. In the last plan, the defender is replaced at start of first period and retained for 1 period. Subsequent replacements occur at beginning of second and third periods. Each replacement is retained for 1 period.

Table 2.3 Application of Dynamic Programming to Replacement Problem: Present Worth of Replacement Plans

Replacement Plan	Defender Life	Replacement lives			Present Worth
		First	Second	Third	
1	3	0	0	0	P1
2	2	1	0	0	P2
3	1	2	0	0	P3
4	1	1	1	0	P4
5	0	3	0	0	P5
6	0	2	1	0	P6
7	0	1	2	0	P7
8	0	1	1	1	P8

Oakford, Lohmann and Salazar [29] introduced a generalized version of Wagner's dynamic programming extension to replace one or more challengers. The cash flow of each challenger can vary independently when technological change is considered. The finite planning time, generally used for dynamic programming models, was extended in [22] to involve an infinite planning horizon. Lohmann [28] combined stochastic cash flows and infinite planning time and solved the resulting stochastic replacement model using dynamic programming and Monte Carlo simulation. In addition, the model in [28] accounts for both finite and infinite times. Applications of bus equipment replacement strategies are presented in [23].

The optimal replacement policy for the single replacement problem using dynamic programming model is determined by solving for the minimum total cost. The total costs primarily consist of acquisition costs, operating costs, and salvage value. There is no unique mathematical formulation for the dynamic programming problem. Typically, a search algorithm is required for solving the model for optimum replacement plans. Thongthai [33] presented the "deteriorated" equipment replacement models using an efficient algorithm. The author also attempted to modify the deterministic cash flows to be stochastic. The model combined the Pearson-Turkey technique with four selected measures of effectiveness (expected present worth, variance of present worth, coefficient of variation of present worth, and probability of achieved aspiration level).

Bohner [6] employed exhaustive and efficient search algorithms to solve the dynamic programming model. The forward procedure, backward procedure, and an iterative optimization algorithm were used. The model was extended to change some of the parameters including planning horizon, multiple types of challengers and technological change.

A replacement problem application of dynamic programming analysis a fleet of passenger cars and light trucks at Phillips Petroleum Company are presented in Waddell [34]. Models for the individual trucks and passenger cars were formulated in dynamic programming to optimize the project discounted cash flows. An approach to reduce the computational requirement is also suggested in [34]. Items of similar type can be grouped and the equipment model is then applied to an average item within each group in order to determine when items within the group should be replaced.

Fadjar [26] presented a replacement model for public buses to determine the replacement for a current bus. Dynamic programming formulation for this replacement problem solved the problem by minimizing the present value of total cost of acquiring, operating, and maintaining vehicles throughout a specified planning horizon.

Sethi and Morton [31] proposed the mixed optimization technique for the generalized machine replacement. The Wagner-Whitin formulation was used to incorporate subproblem solutions. Subproblems were the optimum purchase, maintenance, and sale of a given machine between any two time periods. The model can be re-solved at any time if parameters of the problem change.

Integer Programming

Integer programming (IP) can be stated as a special case of the linear programming approach in which the decision variables are restricted to be integers. When all decision variables must be integers, the model is called a pure integer programming model. Most practical IP models restrict the integer variables to two values, 0 or 1, which represent yes or no decisions. Such variables are called binary variables. The IP model that contains only binary variables is called a binary integer programming model [74].

An IP model represents the replacement problem as a discrete time formulation. Examples of IP formulations for replacement problems are given in [7, 43- 46]. Integer programming models consist of three basic components; these are decision variables, objective function, and constraints or feasibility conditions. Basically, the decision variables in single replacement are either to replace or to keep a single unit in each period over a finite planning horizon. When the decision variables involve two possible choices, replace or keep, binary variables (or 0-1 variables) are used. Adil and Gill [36] introduced the binary IP model to the single replacement problem. In group replacement, the choices are either to keep or to replace all units in the same age and type at each period over finite time. Karabakal, Lohmann and Bean [44] presented a binary integer programming model for the group replacement problem. When the number of units at the same age can be relaxed (i.e., all units do not have to be replaced at the same time), the decision variables are the number of units purchased in each period and the number of units sold and available at each vehicle age in each period over a finite period. Examples of this case are given in [7] and [43].

Typically, the objective functions in both single and group replacement models consist of the discounted total costs of acquisition costs, operating and maintenance costs, and salvage value. Generally, minimizing the net present value of cash flows of total costs is used. The constraints in single replacement case involve binary variables that are restricted to one vehicle at any time. In group replacement, budget and/or demand constraints are usually included.

Integer programming models applied to the fleet replacement problem can be solved in different ways. Solutions can be obtained using available operation research software, such as in [7] and [36]. Integer programming models are usually combinatorial in nature and are difficult to solve. Thus, methodologies have been developed to solve the IP model applied to group replacement. Karabakal, Lohmann and Bean [44] developed a branch and bound algorithm based on Lagrangian relaxation to solve the binary IP model. Hartman [43] used the Lagrangian relaxation procedure for solving the pure IP model. Aggarwal, Oblak and Vermuganti [45] used heuristics to solve the network problem of IP model. Adil and Gill [36] reformulated the 0-1 integer programming model for single replacement problem developed in [75]. The binary restrictions were removed and the altered model formulation was solved. Significant improvement resulted from a decrease in the number of variables, constraints and time taken to solve the problem. The assumptions in alternate model formulation included deterministic cash flows, maximum equipment age and a finite planning horizon.

GROUP REPLACEMENT

The group replacement problem involves a set of assets that replace another set of assets. The approaches used to solve the problem are addressed in this section.

Economic life Modeling and Benefit Cost Analysis

Appleby [37] employed the economic life model to study equipment typically used by public agencies (i.e., graders, garbage trucks, one-ton pickups). In this replacement problem, the benefit cost ratio was used to prioritize the fleet to be replaced under budget constraints. Randhawa et al. [38] developed replacement plans for large-scale group fleet replacement problem with budget constraint for the Oregon Department of Transportation. The economic life approach was used to determine recommended replacement life. The results from the economic life model were then adapted, based on managerial and operational considerations, to develop replacement priorities. Benefit-cost analysis was used to identify optimum investment levels.

Dynamic Programming

Simms et al. [39] developed the model to determine the optimal buy, operate and sell policy for a fleet of vehicles by selecting the criteria of minimizing total cost over the finite planning horizon. A two-stage analysis for dynamic programming models is then implemented. The first stage analysis is to determine the utilization policy which will minimize the operating cost. The second stage analysis selects the optimal operating cost given a specific fleet mix found in stage one. The policy required for bus replacement results in a series of fleet mixes for each period over the planning horizon. The authors addressed several factors including the demand of vehicles needed in the fleet, usage in terms of route kilometers to be satisfied by the fleet, and minimum age for a bus to be considered in the sell decision.

Lund [40] proposed the replacement model to determine optimal equipment replacement policies. The objective of this replacement cost model is to evaluate costs in determining the appropriate policy for a non-homogeneous diesel bus fleet via replacement of individual vehicles. The model was applied to minimize the total discounted cost of operations and replacements over the length of the planning horizon of the model for three bus configurations.

Linear Programming

The combination of a dynamic programming model with linear programming or integer programming has been used in group replacement [39,[40]. The problem is structured by the dynamic programming model, and then formulated and solved using linear programming.

Basically, the objective function in the model consists of the discounted total cost of acquiring, operating, and maintaining the fleet and the revenue from selling the fleet at the estimated salvage value. The objective of optimization is to determine the fleet mix that will minimize costs subject to a set of constraints. Constraints may include usage, demand, age limitations, or other operational requirements. Multi-stage optimization models are representations of the replacement problem spanning multiple time periods. There are many factors impact with vehicle replacement problem usage, such as fleet size, demand, and costs that affect the models. Basically, purchase prices, salvage value, operating costs, and maintenance costs are included in the model. Applications of linear programming in fleet planning can be found in [76] where a fleet planning model was developed for a transport fleet. Avramovich et al. [42] presented a linear programming approach used in implementation of a decision support system by the fleet management division at North American Van Lines to plan fleet configuration. The problem dealt with various types of tractors and replacement options. The maximization of profits is the decision criteria of vehicle replacement to obtain optimal fleet replacement scheduling.

Jones, Zydiak, and Hopp [66] stated that increasing maintenance cost motivates replacements, and a fixed replacement cost provides incentive for replacing machines of the same age in clusters. The authors addressed the parallel machine replacement problem and verified a useful "no-splitting" rule. The rule states that it is never optimal to split a cluster of like-aged machines. Dynamic programming was used to formulate this problem and linear programming was used to solve it. Tang and Tang [78] proved the rule that for any period, finite or infinite, an optimal policy is to keep or to replace all the machines regardless of age. This concept is further discussed in [41, 79-82].

Integer Programming Model

Examples of integer programming formulation in group replacement can be found in [[7, 44, 77]. Karabakal, Lohmann and Bean [44] presented parallel (or group) replacement under capital rationing constraints. Single type, multiple unit replacement involves budget constraints, deterministic assumptions, and a finite planning horizon. The problem is formulated as zero-one integer program and a branch-and-bound algorithm based on the Lagrangian dual is developed to solve the problem.

Hartman [7] developed multiple options, buy, lease and rebuild, in parallel replacement under demand and rationing constraints. In the replacement problem, the multiple units of homogeneous fleet are combined with finite horizon and deterministic parameters. An integer programming formulation is then developed and applied to the fleet. The replacement model is applied in a rail car analysis. This research was later extended to larger heterogeneous fleets [43].

Christer and Scarf [83] present a robust replacement model with applications to medical equipment. Scarf and Bouamra [84] described the replacement decision for a mixed fleet. A single subfleet replacement is assumed instead of making replacement to the whole fleet simultaneously. The concept of penalty cost for unavailability is considered and the minimization of equivalent rent is employed as the decision criteria. . Scarf and Christer [85] introduced the capital replacement models with the finite planning horizons; roles of penalty cost and variable planning horizon are also discussed.

Network Models

Vemuganti, Oblak and Aggarwal [46] addressed a network-based minimum cost flow model to determine the optimal replacement policy. Various models are presented for different policies. These are: (1) replacement for a single vehicle assuming a fleet of fixed size consisting of a single type of vehicles with various ages, (2) a fleet of vehicles of various types and ages with no constraints, (3) a fleet of vehicles of various types and ages incorporating budget constraints over a finite planning horizon, and (4) fleet size variations. The model formulation assumed that the vehicles are homogeneous.

Aggarwal, Oblak and Vemuganti [45] presented a heuristic method for multicommodity integer flows along with an application to the group replacement problem. The model includes multiple types of equipment with budget for all types in each period and the number of units of equipment required for each type in each period.

Stochastic Programming

Couillard and Martel [47] developed a model to determine the size and composition of a fleet of trailers. A model and algorithm were developed to generate economically optimal vehicle purchase, replacement, sale, and rental plans in a transportation network. The demand was a function of trips required in a day, and it was modeled as a stochastic process with seasonal fluctuations. The objective of the model was minimum expected cost over the planning horizon under demand, purchase and budget constraints.

Morse [48] addressed the multiple assets problem combined with stochastic considerations. A nonhomogeneous Markov decision process linked by side constraints is the mathematical model used in this research. Simulated annealing was used to solve the model.

IMPLICATIONS FOR USE

Identifying problem characteristics and parameters and selecting an appropriate modeling technique are the more quantitative steps in the decision making process. The ultimate success in obtaining and using effective results depends on a number of other considerations including the involvement and acceptance of the process by the users and the availability and quality of input data.

Factors that should be considered in selecting appropriate modeling and solution strategies for the problem at hand include:

1. Size of the vehicle fleet, as this may impact the size of the resulting model and consequently, the efficiency of the solution approach.
2. Quantity and accuracy of input data and its impact on results.
3. Simplicity of applying the model and communicating it to the users.

4. Robustness of the model to accommodate different users, different purposes, and different work environments. Replacement decisions are recurring and the quantity and composition of fleet often changes over time. The modeling approach should be able to accommodate such time-based changes.

Replacement plans obtained from a model may have to be adjusted to incorporate tradeoffs associated with multiple user groups. It is therefore important that the users be involved in all phases of the study, including problem definition, model formulation and evaluation of results. Like any complex decision making environment, replacement decisions involve many individuals and user groups, each with their own priorities for replacing vehicles. The decision making process is inherently iterative in nature. The decision makers and users perceptions of the problem, their beliefs about the likelihood of various uncertain events, and preferences for outcomes mature as the decision making process unfolds. The approach should provide a structured way of thinking about replacement problems.

Data Requirements

Data provides the information required in a model for effective managerial decision making. Model results depend significantly on the input data. If correct conclusions are to be inferred from the model, the input data must include all pertinent costs that would affect the replacement decision. The literature is consistent in stating that input data must be accurate if the right decision is to be reached [19]. Model results will improve as more data with a higher degree of consistency and accuracy becomes available over time.

In the fleet replacement problem, data required for analysis include:

- Purchase costs, by model and year
- Operating costs, by age and model
- Maintenance costs, by age and model
- Downtime, by age and model
- Salvage values, by retirement age and model
- Usage (mileage), by age and model
- Interest rate
- Inflation rate
- Depreciation schedules

Acquisition costs, operating and maintenance costs, salvage values, and usage are common requirement for the modeling techniques discussed earlier. Elements included in these cost categories may differ. For example, maintenance costs may or may not include estimates of downtime. Besides these common cost parameters, use of additional data depends on the particular application. For example, tax and inflation are incorporated in some applications. Simms, et al. [39] detail elements of maintenance costs in replacement modeling. The cost of fuel, tires, lubricant, spare parts and labor were separated, and different inflation rates were used for each category.

The data required for analysis can come from two sources, internal (in-house) or external. For example, information on current fleet such as usage and costs is usually obtained from internal records. Performance of existing fleet may also be used to approximate replacement units if there are no significant differences between challengers and defenders. On the other hand, advances in technology may significantly impact the design and operation of new vehicles. External sources, including vehicle and equipment manufacturers and distributors and other users of like equipment, would be likely sources for cost and usage estimates.

Many organizations assign the collection and organization of data to a department or group separate from the users of data [86]. It is important to index historical data by age and model year. The data required implies a data base system that tracks each model year by age. This must be an on going effort and attention should be given to data obsolescence, where past history is not an accurate representation of current operations or a production of the future. Examples of data requirements and management can be found in [87, 88]. Historical and current cost data are frequently used to estimate future cost. To use appropriate statistical tools for projecting past patterns, data must exist for a sufficient number of time periods, with preferably the same or similar equipment. Examples of replacement model analyzed for individual equipment can be found in [19, 39]. Waddell [34] introduced the grouping of vehicles. The vehicles are grouped according to age, odometer mileage, and function. The average vehicle in each group was used to determine the replacement policy. Christer and Goodbody [89] presented the analysis of data collected and developed a model of the operating costs of a truck. Jaafari and Mateffy [90] illustrated a realistic economic model of the cost components used in construction equipment replacement.

Data collection and analysis is an important element in obtaining accurate replacement results. Output quality depends upon the quality of available data. If the input data is lacking or is inconsistent and inaccurate, an appropriate model that could provide reliable solutions becomes ineffective.

CONCLUSIONS

Generally, the objective of fleet replacement policy is to optimize the economic consequences of owning and operating a fleet such that it minimizes total costs or maximizes total net benefits. The pertinent literature in this area indicates that much of the research work in fleet replacement is with a single vehicle type fleet assuming independent deterministic parameters. More recent work extends this basic framework to incorporate technological changes and /or stochastic parameters. Some studies incorporate fleet utilization into replacement analysis. Recent work deals with the group replacement problem involving multiple units with one type of vehicle, deterministic parameters, and budget constraints. There is little work associated with multiple units with multiple types and multiples constraints.

Several research approaches have been developed and applied to the fleet size problems of relatively small size. The use of economic life replacement models and dynamic programming structures are the most common techniques. Integer programming has been applied to solve more complex replacement problems, including group replacement. Heuristics have been introduced to solve complex mathematical models resulting from integer programming or network formulations.

Effective applications of replacement methodologies require both the use of an appropriate modeling and solution strategy and an accurate database of information for estimating model parameters. Forecasting future costs for challengers is as important as the ability to obtain a fair assessment of the condition of current fleet. The modeling methodology must be appropriate for the needs of the decision making agency, and it must be adaptable to address changing needs over time, and must be understood by the users. Fleet replacement models provide recommendations for replacement; where implementation is often a political process. The success in effective replacement decisions involves engaging the right people at the right time in the replacement process.

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CHAPTER 3
A METHODLOGHY TO SOLVE LARGE-SCALE GROUP FLEET REPLACEMENT PROBLEMS

ABSTRACT

A methodology is presented for solving the large-scale group fleet replacement problems involving multiple types of vehicles, with multiple units, under budget and demand constraints. A multi-phased methodology based on a grouping concept is developed. Integer programming models are formulated for inter-group replacement and intra-group replacement problems. The methodology is illustrated using fleet operations at Oregon Department of Transportation.

INTRODUCTION

A generalized complex group fleet replacement problem is addressed in this chapter. The system consists of various types and units of vehicles with different ages in situations that have demand and budget constraints. Decisions are made concerning what types and units to replace and when to replace them in equipment and time dependent situations.

Mathematical programming has been used to develop models for the group fleet replacement. Exact and heuristic algorithms are then used to solve the resulting models. Available operation research software is employed to provide exact algorithmic solutions, as in [1] and [2]. Basically, the structure of the replacement problem is combinatorial in nature, so the model is not simple to solve. This is particularly true, when the size of the problem becomes large. In recent studies, heuristic algorithms have been developed to solve the group replacement model [3,4]. Obviously, an exact solution to the integer programming model would be preferable, if such can be found. This research develops a methodology to provide an optimum solution to the large-scale group fleet replacement problem. With group replacement problems, the computation time is not as critical as the quality of the solution. Decision making in fleet replacement problems occurs infrequently, perhaps once a year. However, the decisions have long-term consequences for an organization, both in terms of cost and performance.

This chapter is organized as follow: First, the system of study is presented. This is followed by the development of the generalized model for group fleet replacement. The methodology to solve the large-scale group fleet replacements is then described, including a multi-phase grouping concept process to simplify the problem. The integration of the grouping concept and the optimization integer model is developed. Integer programming models are formulated for inter-group and intra-group fleet replacement problems. Finally, the case study is presented.

SYSTEM OF STUDY

The group replacement problem is composed of different types of vehicles and multiple units of varied ages within each type. The problem incorporates budget constraints for each time period and demand constraints for each vehicle type and each time period. In practice, large-scale group replacement occurs in many organizations. The primary objective of this research is to develop a methodology for solving the large-scale group replacement problem of mixed fleets under budget and demand constraints. The decision criteria used will be to minimize the total costs of replacement.

Figure 3.1 delineates the group fleet replacement system. There are n types of vehicles, of age j for each type. The budget for all types is specified for each period, for a total of H periods. The fleet replacement problem becomes more complex when the age of vehicles within a fleet is also considered in replacement and planning decisions. Within each type, multiple units are grouped by age j and vehicles are subject to demand constraints for each period t. Each type i is assumed to be replaced by an identical or similar model. The cost elements, consisting of purchase costs, operating and maintenance costs, and salvage values, are associated with vehicle type i, period t, and age j.

The replacement plans of interest are presented in a hierarchy of decisions. At the first level, the replacement decision considers types of vehicles and periods. Since age is the significant factor in fleet operation, the replacement decision in the second level includes age of vehicle for each type and period. These two levels collectively serve to define the managerial and operation aspects of a replacement plan. Consequently, the questions of interest for these two levels are as follows:

- For vehicle type *i*, what multiple units should be sold, purchased or kept (transferred from period *t* to period *t+1*) in each planning period *t*.

- For each vehicle type *i* with age *j*, what multiple units should be sold, purchased or kept (transferred from period *t* to period *t+1*) in each planning period *t*.

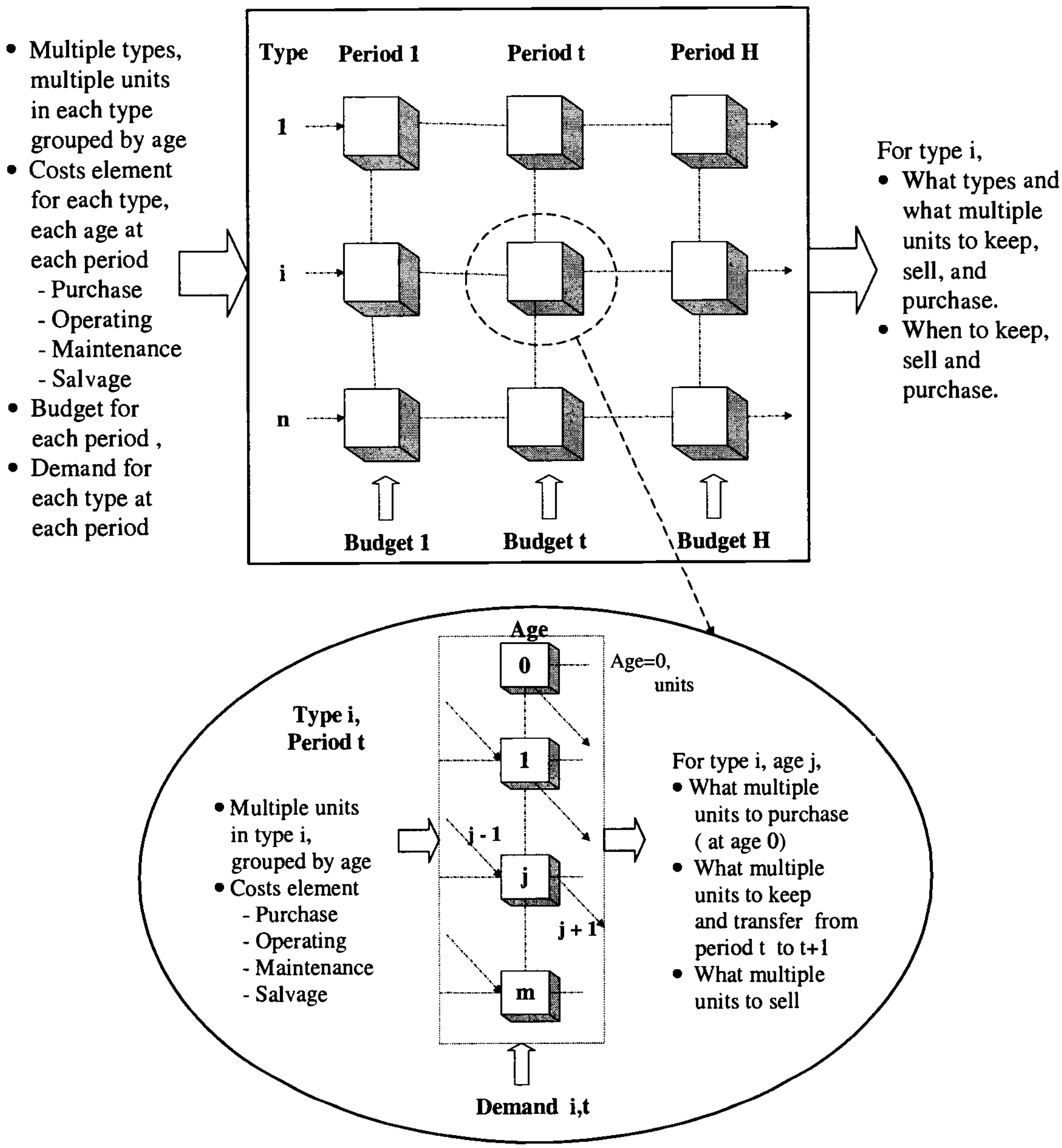

Figure 3.1 Group Fleet Replacement System

The model developed in this research represents time-dependent mixed fleet replacement under budget and demand constraints. Multi-period linear programming is used to construct a series of decision models. The replacement model is concerned with making the choice between keeping the current unit for the next period or selling it and purchasing a new unit. Given budget and demand constraints, the model will find the replacement plan with the minimum total cost over the planning horizon. This cost is the net present worth, and incorporates the interest rate. The general mathematical formulation along with assumption and notations used in the development of the mathematical model are presented in Appendix A.

Computational complexity in solving large-scale optimization problems is usually directly dependent upon the size of the problem. However, size is not the only factor that contributes toward complexity. Structural complexity, diverse sources and different kinds of data can also cause a model to be complex. Simon [5] defines a complex system as one made of a large number of parts that interact in non-simple ways. In addition, the time dimension may also add to model complexity.

For group replacement problems, the size of the formulation depends on the number of vehicle types, the number of age groups within each type, and the number of periods within a planning horizon. The fleet size also changes according to the demand constraints for each period. The vehicles to be replaced need to be identified within the budget constraint for each period and the remaining vehicles need to be transferred to the next budget year or sold according to the solution results in order to satisfy the demand for each vehicle type in each period.

For the general mathematical model developed for this study and shown in Appendix A1, the total number of decision variables and constraints are (4nH + 2nHm) and (3n + H + 5nH+ 2nHm), respectively, where n = number of vehicle types, m = number of age groups, and H = number of planning horizons. For example, the case study presented in this research has 90 vehicle types, 12 age groups, and 5 periods. This results in 12,600 decision variables and 13,325 constraints, a level of complexity that is difficult to handle with regular software systems available to solve liner-integer programming models.

MULTI-PHASE METHODOLOGY BASED ON GROUPING CONCEPT

Since the problem dimensions result in a high degree of solution complexity, a multi-phase grouping methodology is developed. The grouping is based on the group technology (GT) concept. Group Technology approach is used to integrate data for grouping vehicles. The term "group", as used here, represents a virtual group of vehicles involving multiple types and multiple units based on some pre-defined criteria. Thus, several vehicle types can be classified in a group. The generalized group technology concept is given in [6]. Additional GT literature review is presented in Appendix B.1.

Figure 3.2 represents the methodology to solve the large-scale group fleet replacement problem. The methodology represents an integration of management and technical dimensions of the problem. Fleet organization is used to reduce the dimension of the problem. The existing vehicle types are organized into *N* groups based on vehicle functions and costs. The problem is then solved hierarchically. First, the inter-group optimization replacement model is formulated for all groups. The solution of this optimization model yields replacement plan for each group. This translates into the number of units to keep, sell, or purchase, and the timing of these decisions for each group. Budget is allocated on a group basis rather than on type basis.

In the second stage, the intra-group optimization replacement model is formulated for a group. The solution to this model results in replacement plans for individual vehicle types within a group. It should be pointed out that the result is one integer programming model for the inter-group problem and *N* models for the series of intra-group problems.

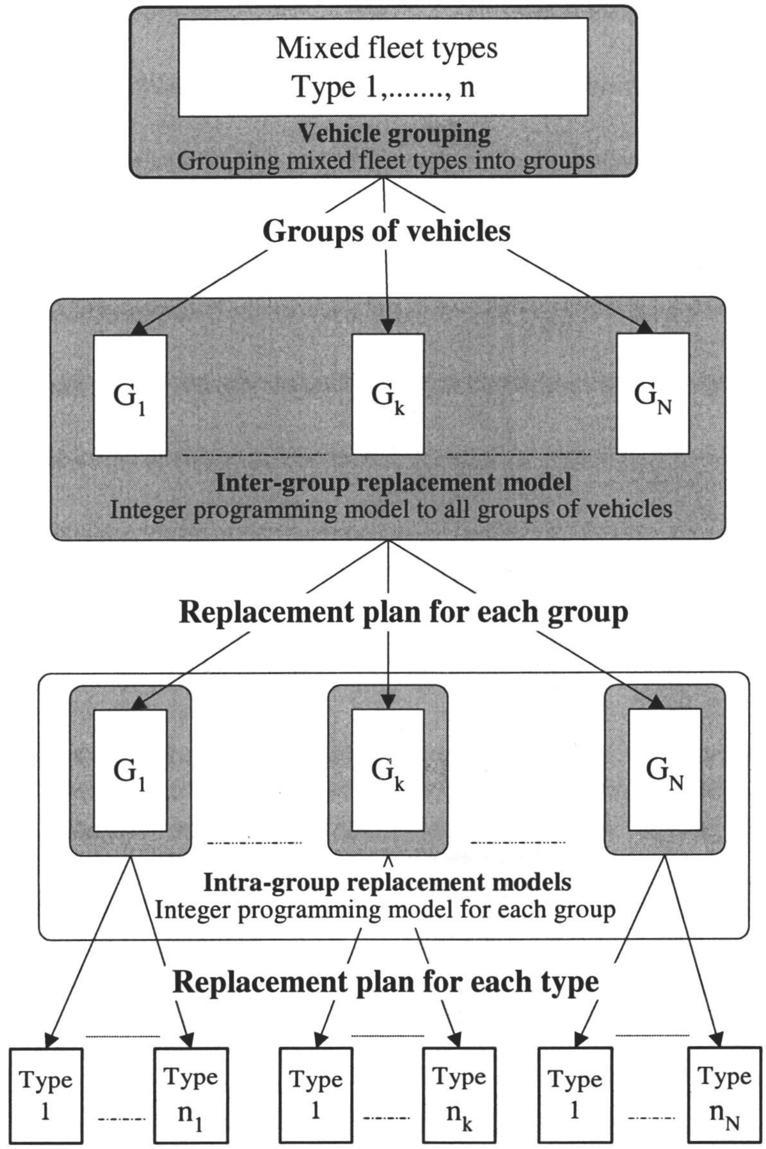

Figure 3.2 A Methodology for Large-scale Group Fleet Replacement Problems

VEHICLE GROUPING

Vehicle data are composed of various types of vehicles with different ages. Basically, each vehicle type is based on a vehicle model. A vehicle type belongs to one category and a category can have many vehicle types (Figure 3.3).

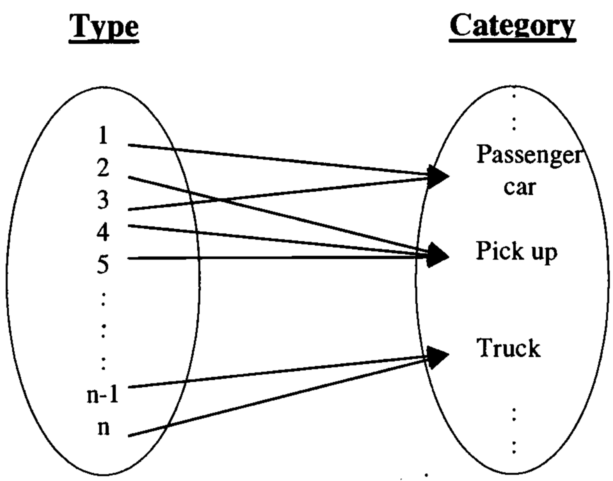

Figure 3.3 Type and Category Relationship

Grouping Procedure

The grouping procedure is displayed in Figure 3.4. Fleet data are composed of multiple types of vehicles. In many situations, types are analogous to models. In the first step of grouping, data are sorted by category. Within each category, vehicles are sorted by functions (e.g., equipment transport, passenger transport, etc.). Finally, each functional subgroup is further sorted and grouped on a cost per mile or cost per hour basis. The costs include the cost of capital and operating and maintenance costs.

The objective of grouping is to reduce the number of decision variables in the optimization model to a level where the model can be solved within the constraints of available commercial software. The three levels of grouping in Figure 3.4 are used for the case study presented later, but may not be required for all systems.

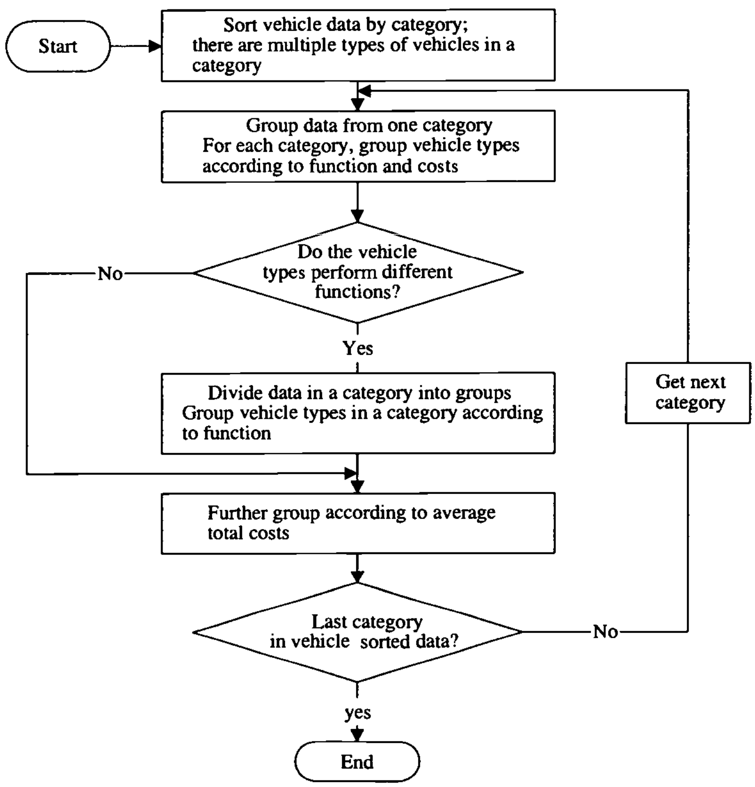

Figure 3.4 Grouping Methodology Flowchart

MATHEMATICAL FORMULATION

A symbolic notation to characterize the elements of mathematical formulation for inter-and intra-group problems is based on the general replacement model given in Appendix A. Each vehicle group is represented by the subscript k ($k = 1,2, \ldots N$). τ ($\tau = 1,2,\ldots n_k$) denotes τ th vehicle type in a group. The vehicles in the group are classified according to their age, represented by the subscript j ($j = 1,2, \ldots m$). The age of the vehicles will vary with time period t ($t = 1,2\ldots H$). The age of a vehicle increases by 1 time period (from j to $j+1$) if it is retained from period t to $t+1$. The indices, data requirements, and decision variables for the model are given in Table 3.1.

Table 3.1 Definitions of Notations and Decision Variables

Symbol	Definition
Indices	
k	Index for vehicle group ; $k = 1,\ldots\ldots,N$
τ	Index for vehicle τth type in group k ; $\tau = 1,\ldots\ldots,n_k$
j	Index for vehicle age ; $j = 1,\ldots\ldots,m$
t	Index for replacement period ; $t = 1,\ldots\ldots,H$
Ψ	Index for discount period ; $\Psi = 1,\ldots\ldots,t$
Data Requirements	
B_t	Budget available for period t
D_{kt}	Demand for group k, period t

Table 3.1 Definitions of Notations and Decision Variables (Continued)

Symbol	Definition
M_{ktj}	Maintenance cost of vehicle group *k* of age *j* in period *t*
O_{ktj}	Operating cost of vehicle group *k* of age *j* in period *t*
P_{kt}	Acquisition cost of vehicle group *k* when purchase at the end of period *t*
R_{ktj}	Resale value (salvage value) of vehicle group *k* of age *j* at the end of period t
V_{kHj}	Value (Book value) of vehicle group *k* of age *j* kept at the end of planning horizon *H*
H	Number of periods in the planning horizon
L_k	Minimum age of vehicle group *k* for it to be sold
I_{ko}	Initial number of vehicles for group *k*
I_{koj}	Initial number of vehicles for group *k* of age *j* $(j = 1,..,m-1)$
IB_{ko}	Initial number of vehicles buys for group *k* at the beginning of the planning horizon
γ_t	Discount rate in the t^{th} period
Decision Variables	
b_{kt}	Number of vehicles of group *k* purchased at end of period *t*
s_{ktj}	Number of vehicles of group *k* sold, age *j* at end of period *t*
s_{kt}	Number of vehicles of group *k* sold at end of period *t*
y_{ktj}	Number of vehicles of group *k*, age *j* kept at end of period *t*

Table 3.1 Definitions of Notations and Decision Variables (Continued)

Symbol	Definition
Decision Variables	
x_{ktj}	Number of vehicles of group *k*, age *j* available at the beginning of period *t*, or equivalent to the number of vehicles of group *k*, age *j* at the beginning of period *t* transferred from age *j-1* at the end of period *t-1*
x_{kt}	Number of vehicles of group *k* available at the beginning of period, or the number of vehicles of group *k* transferred from the end of period *t-1*

Inter-group Replacement Model

The model is solved to determine units to sell, purchase, or keep for each period, each group, and age.

$$\text{Minimize Total Cost} = \sum_{k=1}^{N}\sum_{t=1}^{H}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_{\psi})}P_{kt}b_{kt} \qquad \text{Acquisition costs}$$

$$+ \sum_{k=1}^{N}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_{\psi})}M_{ktj}x_{ktj} \qquad \text{Maintenance costs}$$

$$+ \sum_{k=1}^{N}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_{\psi})}O_{ktj}x_{ktj} \qquad \text{Operating costs}$$

$$- \sum_{k=1}^{N}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_{\psi})}R_{ktj}s_{ktj}$$ Sales of current fleet

$$- \sum_{k=1}^{N}\sum_{j=1}^{m}\prod_{\psi=1}^{H}\frac{1}{(1+\gamma_{\psi})}V_{kHj}(x_{kHj}-s_{kHj})$$ Value of current fleet kept at the end of period H

b_{kt}, s_{ktj}, s_{kt}, x_{ktj}, x_{kt} are integers for $\forall_k, t, j$

Subject to:

$$\sum_{j=1}^{m} x_{ktj} \geq D_{kt}$$ for $\forall k, t$ (1)

$$\sum_{k=1}^{N} P_{kt} b_{kt} \leq B_t$$ for $\forall t$ (2)

$$x_{k(t-1)(j-1)} - x_{ktj} = 0$$ $t>1,\ 1<j<L_k,\ \forall k, t$ (3)

$$x_{k(t-1)(j-1)} - x_{ktj} \geq 0$$ $t>1,\ j \geq L_k, \forall k, t$ (4)

$$x_{kt} = \sum_{j=1}^{m} x_{ktj}$$ for $\forall k, t$ (5)

$$s_{kt} = \sum_{j=1}^{m} s_{ktj}$$ for $\forall k, t$ (6)

$$x_{kt} = IB_{ko} + \sum_{j=1}^{m-1} I_{koj}$$ for $t=1, \forall k$ (7)

$$x_{kt} = x_{k(t-1)} + b_{k(t-1)} - s_{k(t-1)}$$ for $\forall k, t\ ;\ t>1$ (8)

$$x_{ktj} - I_{k0(j-1)} = 0$$ for $\forall k, j\ ,\ t=1, j>1$ (9)

$$x_{ktj} - IB_{k0} = 0$$ for $\forall k\ ,\ t=1, j=1$ (10)

$$x_{ktj} - b_{k(t-1)0} = 0 \qquad \text{for } \forall k,t \ , \ j = 1, t > 1 \qquad (11)$$

$$x_{ktj} - x_{k(t-1)(j-1)} + s_{k(t-1(j-1)} = 0 \qquad \text{for } \forall k,t,j \ , t > 1 \ , 1 < j \qquad (12)$$

$$x_{ktJ} - y_{k(t-1)J} - x_{kt-1(J-1)} + s_{ktJ} = 0 \qquad \text{for } \forall k,t \ , t > 1 \ , j = J \qquad (13)$$

$$y_{ktJ} - x_{ktJ} + s_{ktJ} = 0 \qquad \text{for } \forall k,t \ , t > 1 \ , j = J \qquad (14)$$

$$IB_{ko} + I_{ko} + \sum_{t=1}^{H} b_{kt} - \sum_{t=1}^{H} s_{kt} - (x_{kH} + b_{kH} - s_{kH}) = 0 \qquad \text{for } \forall k \qquad (15)$$

$$IB_{ko} + \sum_{j=1}^{m-1} I_{koj} + \sum_{t=1}^{H} b_{kt} - \sum_{t=1}^{H}\sum_{j=1}^{m} s_{ktj} - \sum_{j=1}^{m} (x_{kHj} + b_{kH} - s_{kHj}) = 0 \quad \text{for } \forall k \qquad (16)$$

The objective function is to minimize total costs of all groups throughout the planning horizon. The costs include acquisition costs, maintenance costs, operating costs, and salvage values from sale including value of fleet at the end of the planning horizon. The fleet can be sold or kept at the end of each period. The costs are discounted and summed over the planning period. The model is able to incorporate different discount factors for each group in each period. At the end of any period there are two sets of decisions that are made. The first set of decision variables is the number of units to sell, buy, or keep, for each group in each period. The second set is that, in any given period, for each group, how many units associated by age to sell, buy, or keep.

Constraint set (1) ensures that for each group and each period, the total number of all vehicles available at the beginning of each period has to satisfy demand for each vehicle group. Constraint set (2) ensures that for each period, the total acquisition costs for all vehicle groups should not exceed the budget.

Constraint sets (3) and (4) state that the age of replacement should be greater than minimum age L for each vehicle group at each period. Constraint sets (5) and (6) serve to ensure that the number of vehicles available and sold for each vehicle group at each time period are equal to sum of the number of vehicles from different age groups for that vehicle group in that time period. Constraint set (7) states that the number of vehicles available at the beginning of the first period is equal to the initial number of vehicles, by vehicle group. Constraint set (8) states that the number of vehicles of each group available at the beginning of the period t is equal to the difference between the number of vehicles of each group available at the beginning of period t-1 and the number of vehicles sold at end of that period. Constraint sets (9) to (12) serve to ensure conservation of vehicles by group and age from one period to the next. Constraints sets (13) to (14) serve to ensure the balance of units in the end of period t at the last age group. Constraints sets (15) and (16) serve to ensure the flow balance; the sum of the initial number of vehicles plus the number of the vehicles bought throughout the planning horizon be equal to the number of vehicles sold throughout the planning horizon plus the number of vehicles kept at the end of the planning horizon.

Intra-group Replacement Model

The intra-group model is constructed for each individual group. In group k, the model determines units for purchase, sale and transfer for each τth vehicle type for n_k types. The objective function is to minimize total costs of n_k types in a group throughout the planning horizon. The definition of cost and notation are same as the inter-group formulation, but the subscript is changed to type τ instead of group k.

Minimize Total Cost = $\sum_{\tau=1}^{n_k}\sum_{t=1}^{H}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}P_{\tau t}b_{\tau t}$ Acquisition costs

$+\sum_{\tau=1}^{n_k}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}M_{\tau tj}x_{\tau tj}$ Maintenance costs

$+\sum_{\tau=1}^{n_k}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}O_{\tau tj}x_{\tau tj}$ Operating costs

$-\sum_{\tau=1}^{n_k}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}R_{\tau tj}s_{\tau tj}$ Sales of current fleet

$-\sum_{\tau=1}^{n_k}\sum_{j=1}^{m}\prod_{\psi=1}^{H}\frac{1}{(1+\gamma_\psi)}V_{\tau Hj}(x_{\tau Hj}-s_{\tau Hj})$ Value of current fleet kept at the end of period H

$b_{\tau t}, s_{\tau tj}, s_{\tau t}, x_{\tau tj}, x_{\tau t}$ are integers for $\forall\tau, t, j$

Subject to:

$$\sum_{j=1}^{m}x_{\tau tj}\geq D_{\tau t} \qquad \text{for } \forall\tau, t \qquad (1)$$

$$\sum_{\tau=1}^{n_k}x_{\tau tj} = X_{ktj} \qquad \text{for } \forall t, j \qquad (2)$$

$$\sum_{\tau=1}^{n_k} s_{\tau tj} = S_{ktj} \qquad \text{for } \forall t, j \qquad (3)$$

$$\sum_{\tau=1}^{n_k} b_{\tau t} = B_{kt} \qquad \text{for } \forall t \qquad (4)$$

$$x_{\tau(t-1)(j-1)} - x_{\tau tj} = 0 \qquad t>1,\ 1<j<L_\tau,\ \forall \tau, t \qquad (5)$$

$$x_{\tau(t-1)(j-1)} - x_{\tau tj} \geq 0 \qquad t>1,\quad j \geq L_\tau, \forall\ \tau, t \qquad (6)$$

$$x_{\tau t} = \sum_{j=1}^{m} x_{\tau tj} \qquad \text{for } \forall \tau, t \qquad (7)$$

$$s_{\tau t} = \sum_{j=1}^{m} s_{\tau tj} \qquad \text{for } \forall \tau, t \qquad (8)$$

$$x_{\tau t} = IB_{\tau 0} + \sum_{j=1}^{m-1} I_{\tau 0j} \qquad \text{for } t=1, \forall \tau \qquad (9)$$

$$x_{\tau t} = x_{\tau(t-1)} + b_{\tau(t-1)} - s_{\tau(t-1)} \qquad \text{for } \forall \tau, t\ ; t > 1 \qquad (10)$$

$$x_{\tau tj} - I_{\tau 0(j-1)} = 0 \qquad \text{for } \forall \tau, j\ , t = 1, j > 1 \qquad (11)$$

$$x_{\tau tj} - IB_{\tau 0} = 0 \qquad \text{for } \forall \tau\ , t = 1, j = 1 \qquad (12)$$

$$x_{\tau tj} - b_{\tau(t-1)0} = 0 \qquad \text{for } \forall \tau, t\ , j = 1, t > 1 \qquad (13)$$

$$x_{\tau tj} - x_{\tau(t-1)(j-1)} + s_{\tau(t-1(j-1)} = 0 \qquad \text{for } \forall \tau, t, j\ , t > 1\ , 1 < j < J \qquad (14)$$

$$x_{\tau tJ} - y_{\tau(t-1)J} - x_{\tau(t-1)(J-1)} + s_{k(t-1)(J-1)} = 0 \qquad \text{for } \forall \tau, t\ , t > 1\ , j = J \qquad (15)$$

$$y_{\tau tJ} - x_{\tau tJ} + s_{\tau tJ} = 0 \qquad \text{for } \forall \tau, t\ , t > 1\ , j = J \qquad (16)$$

$$IB_{\tau 0} + I_{\tau 0} + \sum_{t=1}^{H} b_{\tau t} - \sum_{t=1}^{H} s_{\tau t} - (x_{\tau H} + b_{\tau H} - s_{\tau H}) \; = \; 0 \qquad \text{for } \forall \tau \qquad (17)$$

$$IB_{\tau 0} + \sum_{j=1}^{m-1} I_{\tau 0 j} + \sum_{t=1}^{N} b_{\tau t} - \sum_{t=1}^{H}\sum_{j=1}^{m} s_{\tau t j} - \sum_{j=1}^{m} (x_{\tau H j} - b_{\tau H} - s_{\tau H j}) \; = \; 0 \qquad \text{for } \forall \tau \quad (18)$$

As stated earlier, the integer programming model for intra-group replacement is similar to the inter-group model, except that it is focused on type τ of n_k instead of group k. The decision variables are applied to each type in a group instead of each group from all vehicle groups. The results of inter-group are the input of intra-group model. A few constraints are different for intra-group model. Constraint set (2) serves to ensure that the number of units available in a group, resulting from an inter-group analysis, is equal to sum of the number of units available from all types in that group. Similar to constraint (2), constraints set (3) and (4) serve to ensure the number of vehicles sold and bought for a group is equal to the total number of units sold and bought from all types in that group. Others constraints are presented in the same format as inter-group formulation, except that there is no budget constraint.

CASE STUDY

The case study analyzed here represents the operations at the Oregon Department of Transportation (ODOT). The ODOT fleet is a critical resource and a major capital investment requiring legislative approval for funding equipment replacement needs. The fleet is characterized by various types of vehicles and related equipment which perform different tasks and have varying usage patterns. A replacement decision is a choice between the present asset and the currently available replacement alternative.

The need for an equipment replacement policy is indicated by the following factors: (1) availability of equipment to provide high quality and timely service to public; (2) limited budget allocation that generally falls short of the replacement needs of the agency; and (3) tradeoffs among the agency's divisions and branches to ensure that the highest priority needs are funded.

Replacement decisions are critically important to ODOT as a modern, well-maintained and reliable fleet contributes to the performance of the entire department. ODOT requires a fleet replacement plan to improve the fleet replacement practices and projection. For this purpose, two scenarios were analyzed:

Scenario 1: No budget constraint, representing a base scenario

Scenario 2: With budget constraint, representing the actual practice at ODOT, and the most likely scenario in practice.

Data

Data collection is an important element for replacement analysis. In 1995, ODOT began to consolidate equipment cost information in the Transportation Equipment Accounting Management System accounting system by equipment number. The data used in the analysis is based on this database. Before employing the group fleet replacement methodology, the current fleet environment was studied to:

- Identify and understand the current equipment replacement policies, information and cost structures and the technical requirements of the problem.
- Identify and understand current and future customer needs.
- Identify fleet replacement models and current management practices current practice.

According to available costs data at ODOT, the vehicles base consists of 90 vehicle types for a total of 2,791 units. The vehicles in each type are grouped by age into 12 age groups. Typical replacement planning is for five periods. Primary costs included in the model are purchase costs, salvage value, operating costs, and maintenance costs. Maintenance costs include estimates of downtime. Downtime costs reflect lost productivity due to equipment breakdown. Estimates for downtime were developed using expert input on frequency of breakdowns, average time required to reassign personnel, and the number of people in the working crew affected due to unavailability of equipment.

Fleet data from ODOT consists of the beginning units for each vehicle type with associated vehicle age. Additionally, there is demand for each type of vehicle for each period, and an overall budget for each period. Due to data limitations and ODOT requirements, the demand is assumed constant for the planning period. The maintenance costs and operating costs are given for each vehicle type and are associated with vehicle age. The minimum acceptable rate of return (MARR) used in this analysis is 5% per year. Inflation rates for purchase costs, salvage value, operating costs and maintenance costs, obtained from the Consumer Price Index [7], equal 0.73% (1994-2000), 1.53% (1998-2000), 14% (2000), and 2.7% (1998-2000), respectively. Salvage value at the end of ODOT standard life is assumed to be 5 percent of the acquisition cost. The MACRS (Modified Accelerated Cost Recovery System) depreciation method is used in computing the salvage value for each vehicle type and age.

In a study done for ODOT by Oregon State University researchers in 1998 [7], the raw data collected by ODOT was transferred so that it would be more useful for economic-based and other modeling approaches. The first step in this process was to estimate usage and costs within each type using regression analysis. To facilitate regression analysis, particularly where data is limited, multiple vehicle types for each category were formed into a class, with the vehicles in the same class assumed to have similar usage and cost patterns. Based on the usage and cost projections, cost functions were developed for each class-age combination. Grouping by age within a class was used to provide homogeneous groups of vehicles in terms of their usage and cost patterns. To minimize generalizations, every effort was made to use the analysis for each class-age combination. Four statistical models, associated with each class of equipment, were formulated using regression analysis:

- Usage model: Usage (i.e., miles driven) was the dependent variable with age and equipment class the independent variables. This model was formulated to estimate yearly usage of equipment to be used as an independent variable for subsequent models.
- Operating Costs: The regression model used age, usage and class as independent variables to estimate operating costs, the dependent variable.
- Overhead and Insurance Costs: The regression model had overhead and insurance costs as a dependent variable, and age and class as independent variables. These costs are included as operating costs in the replacement model.
- Repair-Maintenance Costs: Age, usage and class were the independent variables used to estimate repair and maintenance costs, the dependent variable.

The statistical system, SAS [9], was used to fit the mathematical models. Two primary data sources at ODOT were used to obtain pertinent information for estimating the usage and cost models. The Equipment Management System (EMS) at ODOT was used to obtain information on fleet characteristics. Specific information used in the analysis included: Vehicle Category and Type; vehicle number, make, model, and year; crew using the vehicle; and replacement information such as the original acquisition cost and the cost of replacement with new equipment of the same functionality. The second database used in the analysis was the Transportation Equipment Accounting Management System (TEAMS). This information base provides information on operating costs, repair and maintenance costs, and overhead and insurance costs, normally tracked by equipment type and equipment number on an annual basis.

The costs associated with a group of vehicles consist of the operating costs, maintenance costs, salvage value, and purchase costs of all vehicle types in that group. The number of units in a group is the sum of units for all types in that group. An Excel-based spreadsheet system was developed to perform the cost computations that provide input into the replacement model. LINGO 4 [10] is the optimization software used to solve the linear-integer programming model.

Results

The vehicle grouping is presented in appendix B.2. First, all types are grouped by category (eg., passenger car, pick up, truck etc). Within each category, vehicles are grouped by functions (a total of 18 functions) and ODOT cost base for charge per hour or charge per mile. Examples of groups include transport passengers, transport passengers and equipment, roadway measurement, grading, and compaction. Table 3.2, the result of vehicle grouping, shows vehicle types and units for each group. The grouping process groups vehicles from 90 types into 25 groups. Consequently, the inter-group mathematical model applies to a problem size of 25 instead of 90.

Table 3.2 Vehicle grouping

Group	Vehicle Type	Available Units	Group	Vehicle Type	Available Units
G1	1,2	207	G14	59,60	23
G2	3,4,5,6,7,8,9	274	G15	61,62,63,64,65,66	72
G3	10	185	G16	67,68,69,70	67
G4	11,12,13,14,15,16	372	G17	71,72	3
G5	17,18	2	G18	73	12
G6	19,20,21,22,23	326	G19	74,75	5
G7	24,25,26,27,28,29	227	G20	76	3
G8	30,31,32,33,34,35,36,37,38,39,40	452	G21	77,78	42
G9	41,42	79	G22	79,80,81,82	31
G10	43,44,45	57	G23	83,84	7
G11	46,47,48,49,50,51,52,53	264	G24	85,86,87,88,89	21
G12	54,55,56,57	37	G25	90	1
G13	58	22			

Two scenarios, no budget constraint and with budget constraint, are analyzed. The scenario with no budget constraints provides a baseline for comparison. The scenario with budget constraints can be used to find the optimal replacement plan in practice. This scenario provides the fleet replacement plan that fleet manager can use to plan the budget for fleet replacement in future years.

The solutions of inter-group and intra-group replacement problems for the two scenarios are presented in Appendix C1 and C2, respectively. Appendix C1 shows the inter-group replacement plans for the two scenarios. Table C1-1 shows the replacement plans for Scenario1 with units purchased and their acquisition costs for each vehicle group in each period of a 5-period planning horizon. This is followed by the replacement plan for Scenario2 (Table C1-2). Tables C1-4 through C1-7 present examples of the inter-group replacement plan by age for scenario2. The replacement plans are presented in two levels. The first level presents units that should be sold, purchased, or kept in each period of a 5-period planning horizon for each group. The second level presents similar information but by vehicle age.

Appendix C2 presents an intra-group replacement plan for scenario2. Table C2-1 shows the replacement plans with units purchased and their acquisition costs for each vehicle type in each period of a 5-period planning horizon, instead of vehicle group. Table C2-2 shows the units that should be sold, purchased, or kept in each period of a 5-period planning horizon for each vehicle type. Tables C2-3 through C2-10 present examples of similar information for each vehicle type by age.

Table 3.3 shows the summary of results of inter-group replacement for the two scenarios. The table shows the total units bought and the acquisition costs for a 5-period planning horizon. With no budget constraint, the model determined the replacement need to be approximately $71.3M for 836 units over the five year period. Realistic budget constraints resulted in replacement of 626 units for $43M.

It should be noted that the units purchased in period 5 is equal to zero because the units purchased occur at end of period for the last period of the planning horizon.

Table 3.3 Inter-group Replacement: Purchased Units and Acquisition Costs

Period	Vehicles Replaced		Acquisition Cost of Replacement	
	No Budget Constraints	With Budget Constraints	No Budget Constraints	With Budget Constraints
1	535	227	$48,691,378	$10,508,365
2	102	98	$10,870,029	$10,504,014
3	49	122	$3,680,789	$11,041,548
4	150	179	$8,083,284	$10,949,307
5	0	0	$0	$0
	836	626	$71,325,481	$43,003,234

Solution summaries for inter-group and intra-group replacement analyses are presented in tables 3.4 and 3.5. These tables show the number of decision variables and constraints, as well as the total net present value, the total units replaced, and total acquisition costs for a 5-period planning horizon.

Table 3.4 Summary of Inter-group Replacement Analysis

Scenario	Decision Variables	Constraints	Objective Function (Present Value)	Total Units to be Replaced	Total Acquisition Costs
No budget constraint	3500	1976	1.27×10^8	836	$71,325,481
With budget constraint	3500	1981	1.29×10^8	626	$43,003,234

Table 3.5 Summary of Intra-group Replacement Analysis

Group	Scenario	Decision Variables	Constraints	Objective Function (Present Value)	Total Units Replaced	Total Acquisition Cost
G1	No budget	280	274	3.1960×10^{6}	118	$1,691,902
	With budget	280	274	3.1901×10^{6}	121	$1,727,920
G4	No budget	840	570	1.1418×10^{7}	147	$3,832,095
	With budget	840	570	1.1418×10^{7}	147	$3,832,095
G7	No budget	840	570	1.4351×10^{7}	172	$15,564,889
	With budget	840	570	1.2748×10^{7}	172	$15,139,787
G8	No budget	1540	940	3.4553×10^{7}	293	$30,007,192
	With budget	1540	940	2.9088×10^{7}	102	$10,124,334
G10	No budget	420	348	4.6255×10^{4}	4	$187,752
	With budget	-	-	0	0	0
G11	No budget	1120	718	5.2545×10^{6}	38	$3,977,529
	With budget	1120	718	5.2545×10^{6}	38	$3,977,529
G13*	No budget	-	-	-	11	$284,467
	With budget	-	-	0	0	0
G14	No budget	280	274	3.1693×10^{5}	3	$93,810
	With budget	280	274	3.1693×10^{5}	3	$93,810
G22	No budget	560	422	7.4186×10^{4}	3	$51,317
	With budget	560	422	7.4222×10^{4}	3	$44,156
G23	No budget	280	274	1.4293×10^{5}	3	$249,950
	With budget	280	274	1.4293×10^{5}	3	$249,950
G24	No budget	700	496	8.3132×10^{5}	39	$3,741,800
	With budget	700	496	8.2749×10^{5}	35	$3,028,277

*There is one type in a group.

Sensitivity Analysis

The primary objective of group fleet replacement is to minimize the total cost of the replacement decisions over a finite planning horizon. The minimum cost, however, is subject to input estimates for cost parameters. Some of these estimates are based on best estimates of experienced personnel or minimal data. The cost estimation errors may affect the replacement costs and the number of units replaced. Thus, it is important to evaluate the sensitivity of results to estimate the input variables.

The sensitivity of results was evaluated to changes in estimates of salvage value, operating costs, and maintenance costs. Figures 3.5 and 3.6 present the sensitivity analysis for the case study. The present worth of total costs, units purchased and acquisition costs for replacement did not change with ±20 percent variation in estimates for operation and maintenance costs, indicating the results to be fairly robust to estimation errors in input data. However, the results are sensitive to change in salvage value. For example, for Scenario1 a 10% increase in salvage would decrease present worth by 19% and increase purchased units from 836 to 6,519.

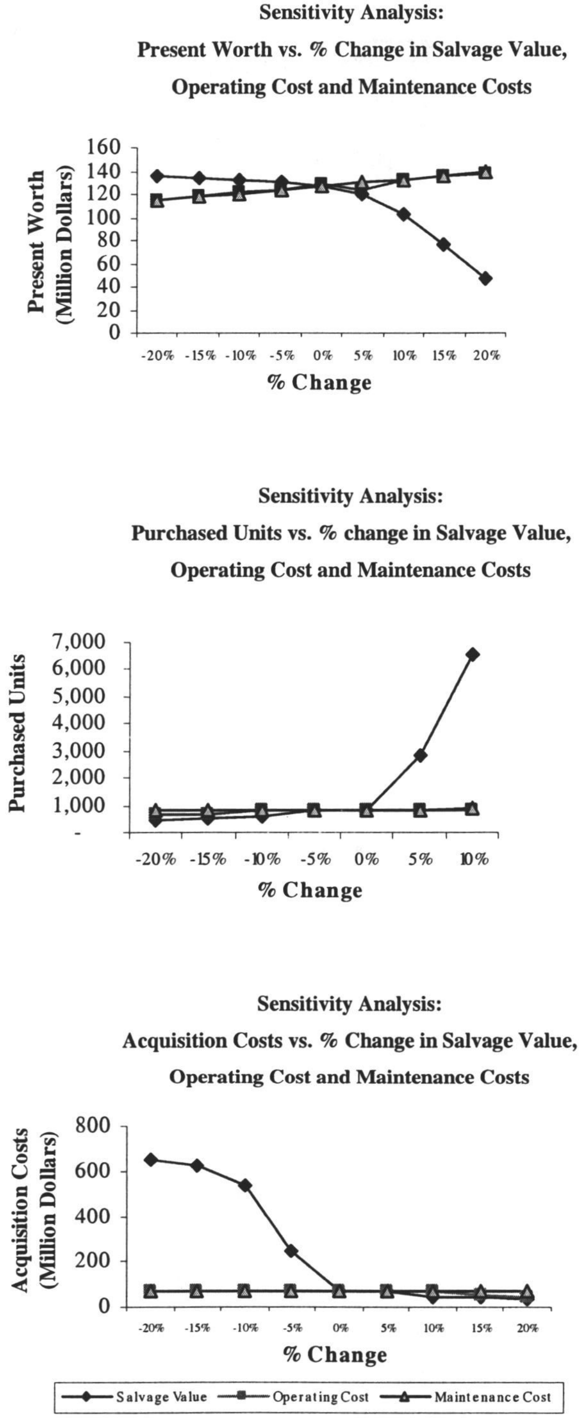

Figure 3.5 Sensitivity Analysis of Inter-group Replacement Plans Scenario 1 (No Budget Constraint)

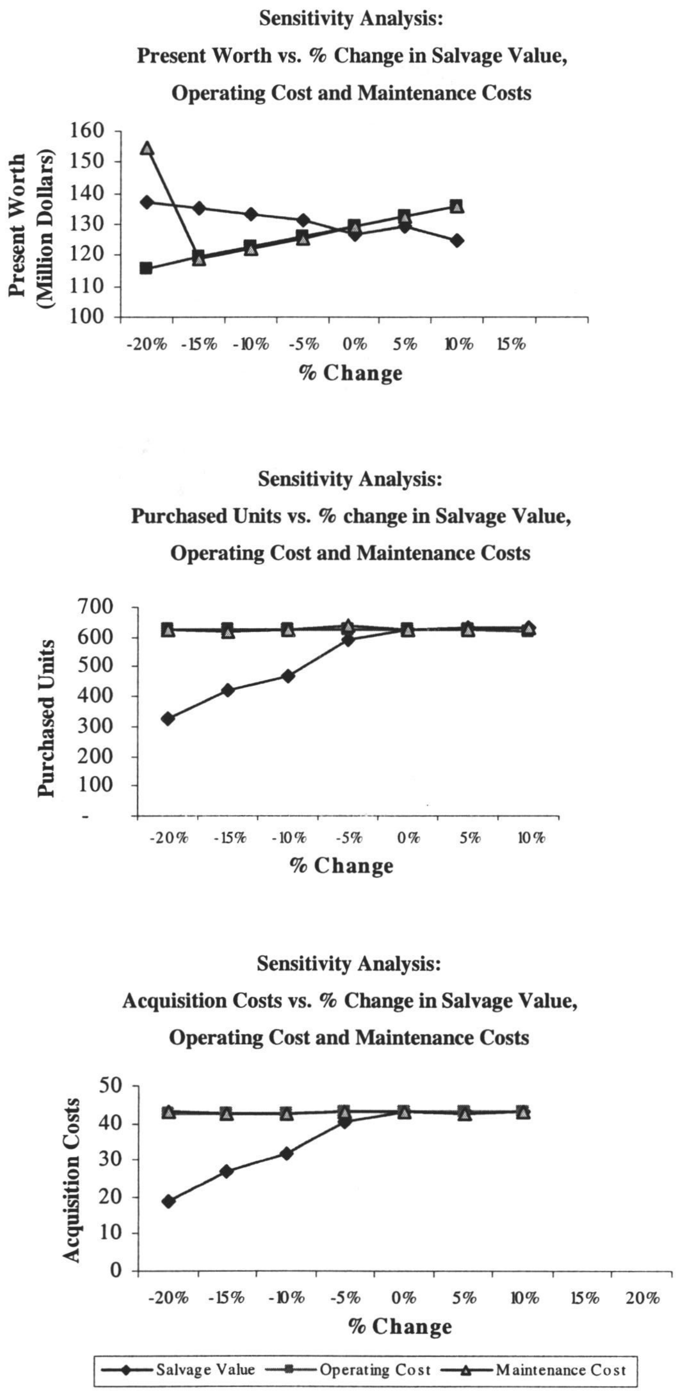

Figure 3.6 Sensitivity Analysis of Inter-group Replacement Plans Scenario 2 (With Budget Constraint)

Discussion and Implications for Use

The use of this methodology would enable the fleet managers to determine the size, mix, and value of the replacement fleet, and to improve the reliability of future fleet replacement projections. The scenario with no budget constraints provides a baseline for comparison. The scenario with budget constraints can be used to find the optimal replacement plan in practice. Both scenarios may be used in conjunction to determine the replacement plan that would work best for management.

The methodology presented can be applied to a range of fleet replacement problems. The generalized model is recommended when the size of problem study is small. The grouping technique is suggested when study deals with large-scale problem size. The grouping factors may be different application domains. However, the concept of the grouping methodology can be applied to incorporate other factors, as appropriate.

The model in this research can be adapted to various input scenarios so that the user is able to effectively use it in real practice. Different cost elements can use different inflation rates and interest rates in each period of the planning horizon. Thus, it is possible to incorporate factors or constraints that affect the value of other parameters. For example, vehicle type i can be replaced with a similar model that has different cost data input for each period. Another example is to change the method of estimating salvage values for some types of vehicle.

Since data is specific to each organization, data elements and assumptions may vary from those used in this model. Data collection and data implementation described in the previous section can be used as guidelines. The cost estimates that are input to the model depend on available data. If there is sufficient data, the cost model should be used for individual vehicle types instead of grouping like vehicle types in the cost estimation process.

Basically, the results from a replacement study provide guidelines for developing and implementing the actual replacement plan. Practical factors may need to be included in applying the results obtained from the solution of replacement models. For example, the replacement plan resulting from the model may need to be adjusted to incorporate trade-offs associated with various types of fleet and user groups. Major factors in the success in using results depend on the involvement and acceptance of the process by users and availability and quality of input data. Users' experiences can provide realistic constraints that should be included in the use of results from a replacement model. Furthermore, results should be revised and adjusted over time, since both needs and available resources change over time. The results of fleet replacement in previous periods can be fed back to the model to improve the replacement plan in the following periods.

SUMMARY

This chapter defined and developed a mathematical model for the generalized group fleet replacement problem. The methodology for solving the large-scale replacement problem is presented. The multi-phase methodology based on the grouping concept is integrated with optimization techniques. This methodology is distinguished by its ability to represent real world applications. The case study from ODOT is presented as an application of the methodology.

The replacement model can incorporate complex formulations in the large-scale group fleet replacement problem. It is flexible and can be used in a wide variety of replacement problems. The methodology addressed in this chapter can be used to obtain the optimal fleet replacement plan. The results from this methodology, used appropriately, can result in reduction in fleet replacement costs and operation costs.

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CHAPTER 4
CONCLUSIONS

The problem studied in this research is concerned with the large-scale group fleet replacement of multiple types of vehicles and multiple units within types under budget constraints. This study is motivated by practical needs in this area. Although, fleet replacement has been the subject of much study, very little attention has been given to the large-scale group fleet replacement problem. This research presents a state-of-the-art review of the fleet replacement problem. In this review, different types of fleet replacement problems are presented as a framework for classifying the fleet replacement problem and commonly used solution approaches.

A multi-phase methodology based on a grouping concept is developed to solve the large-scale group fleet replacement problem. The vehicles are grouped according to various technology parameters. Then, two levels of replacement models are introduced. These are designated as inter-group replacement and the intra-group replacement models. The inter-group replacement model is applied for each group of vehicles. The intra-group replacement model is used to find the replacement plan for each type in a particular group. These two levels are formulated as integer programming models and solved hierarchically.

To show that this methodology is capable of solving real world replacement problems, a case study using the Oregon Department of Transportation records is examined. Additionally, sensitivity analysis is carried out to model uncertainty in input parameters. In addition, the implications and use of the final results are discussed.

The topics for future research from the work presented in this dissertation include:

- The effectiveness of results depends among other factors on estimation of cost parameters. The use of techniques such as fuzzy theory should be investigated to describe the relationship between costs and factors such as technology changes or vehicle utilization.
- Extension of the model to accommodate multiple choices for each vehicle type, particularly the impact of such choices on the dimensionality of the problem as it would impact the solution methodology.
- Extension of the model to joint dependency relationship among variables.

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APPENDICES

APPENDIX A

A GENERAL MATHEMATICAL FORMULATION FOR MIXED FLEET REPLACEMENT MODEL

1. Assumptions

(1) All parameters are deterministic.

(2) The vehicles can be purchased and sold in any period.

(3) A unit purchase has age 0 at the end of the period in which it is purchased.

2. Mixed fleet management decision options

Different models may be developed by altering certain constraints in order to provide different fleet management decision options.

Model 1. (Base line). Constraints included: budget for all vehicle types in each period, demand for each vehicle type in each period, and minimum age replacement. Replacement assumes replacement with an identical model.

Model 2. There is no minimum replacement age constraint.

Model 3. No budget constraints are included, but minimum replacement age constraint is included.

Model 4. Both budget and minimum replacement age constraints are not included.

Model 1 is presented in this chapter.

3. Notations

A symbolic notation is introduced to characterize the elements of mathematical formulation for the mixed fleet replacement model.

<u>Time Periods</u>

Each time period will be represented by the letter "*t*." The planning horizon is *H* periods. The mutiperiod model over the planning horizon *H* be developed with decisions made in each period *t*.

<u>Mixed Fleet</u>

Each vehicle type is represented by the subscript *i* (*i* = *1,2, ...n*). Within each type *i*, vehicles are grouped according to their age, represented by the subscript *j* (*j* = *1,2, ...m*). The age of the vehicles will vary with time period *t*. The age of a vehicle increases by 1 time period of the decision is made to retain it from period *t* to *t+1*.

Definitions of Notations and Decision Variables

Symbol	Definition
<u>Indices</u>	
i	Index for vehicle type ; $i = 1,.....,n$
j	Index for vehicle age ; $j = 1,.....,m$
t	Index for replacement period ; $t = 1,.....,H$
Ψ	Index for discount period ; $\Psi = 1,.....,t$
<u>Data Requirements</u>	
B_t	Budget available for period t
D_{it}	Demand for type i, period t
M_{itj}	Maintenance cost of vehicle type i of age j in period t
O_{itj}	Operating cost of vehicle type i of age j in period t
P_{it}	Acquisition cost of vehicle type i when purchase at the end of period t
R_{itj}	Resale value (salvage value) of vehicle type i of age j at the end of period t

Definitions of Notations and Decision Variables (Continued)

Symbol	Definition
Data Requirements (Continued)	
V_{iHj}	Value (Book value) of vehicle type *i* of age *j* kept at the end of planning horizon *H*
H	Number of periods in the planning horizon
L_i	Minimum age of vehicle type *i* for it to be sold
I_{io}	Initial number of vehicles for type *i*
I_{ioj}	Initial number of vehicles for type *i* of age *j* $(j = 1,\ldots\ldots,m\text{-}1)$
IB_{io}	Initial number of vehicles buys for type i at the beginning of planning horizon
γ_t	Discount rate in the t^{th} period
Decision Variables	
b_{it}	Number of vehicles of type *i* purchased at end of period *t*
s_{itj}	Number of vehicles of type *i* sold, age j at end of period *t*
s_{it}	Number of vehicles of type *i* sold at end of period *t*
y_{itj}	Number of vehicles of type *i*, age *j* kept at end of period *t*
x_{itj}	Number of vehicles of type *i*, age *j* available at the beginning of period *t*, or equivalent to the number of vehicles of type *i*, age *j* at the beginning of period t transferred from age *j-1* at the end of period *t-1*
x_{it}	Number of vehicles of type *i* available at the beginning of period, or the number of vehicles of type *i* transferred from the end of period *t-1*

4. Mathematical Model

The objective function is to minimize total costs throughout the planning horizon. Minimize Total Cost = $\sum_{i=1}^{n}\sum_{t=1}^{H}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}P_{it}b_{it}$ Acquisition costs

$+ \sum_{i=1}^{n}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}M_{itj}x_{itj}$ Maintenance costs

$+ \sum_{i=1}^{n}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}O_{itj}x_{itj}$ Operating costs

$- \sum_{i=1}^{n}\sum_{t=1}^{H}\sum_{j=1}^{m}\prod_{\psi=1}^{t}\frac{1}{(1+\gamma_\psi)}R_{itj}s_{itj}$ Sales of current fleet

$- \sum_{i=1}^{n}\sum_{j=1}^{m}\prod_{\psi=1}^{H}\frac{1}{(1+\gamma_\psi)}V_{iHj}(x_{iHj}-s_{iHj})$ Value of current fleet kept at the end of period H

$b_{it}, s_{itj}, s_{it}, x_{itj}, x_{it}$ are integers for $\forall i, t, j$

Subject to:

Demand constraint

For each type and each period, the total number of all vehicles available at the beginning of the period should satisfy the demand for each vehicle type.

$$\sum_{j=1}^{m} x_{itj} \geq D_{it} \qquad ,\text{for } \forall i, t \qquad (1)$$

Budget constraint

For each period, the total acquisition costs for all vehicle types should not exceed the budget.

$$\sum_{i=1}^{n} P_{it}b_{it} \leq B_t \qquad ,\text{for } \forall t \qquad (2)$$

Minimum age considered for sale

The number of vehicles available of type i age j depend on the minimum age considered for sale.

Vehicle type i is not eligible for sale when the age is less than L_i

$$x_{i(t-1)(j-1)} - x_{itj} = 0 \qquad ;\ t>1,\ 1<j<L_i\ ,\ \forall\ i, t \qquad (3)$$

Vehicle type i is eligible for sale when the age is equal to or greater than L_i

$$x_{i(t-1)(j-1)} - x_{itj} \geq 0 \qquad ;\ t>1,\quad j \geq L_i\ ,\forall i, t \qquad (4)$$

Balance constraints:

(1) All vehicles of type i by period t

$$x_{it} = \sum_{j=1}^{m} x_{itj} \qquad ,\text{for } \forall i, t \qquad (5)$$

$$s_{it} = \sum_{j=1}^{m} s_{itj} \qquad ,\text{for } \forall i, t \qquad (6)$$

(2) Vehicles available at the beginning of the period by type

$$x_{it} = \begin{cases} IB_{io} + I_{io} & ;\ t = 1, \forall i \\ x_{i(t-1)} + b_{i(t-1)} - s_{i(t-1)} & ;\ \text{Otherwise} \end{cases}$$

The number of vehicles available at the beginning of the period 1 for type i is equal to the initial number of vehicles in the same type.

$$x_{it} = IB_{io} + I_{io} \qquad ,\text{for } t = 1, \forall i$$

$$x_{it} = IB_{io} + \sum_{j=1}^{m-1} I_{ioj} \qquad ,\text{for } t = 1, \forall i \qquad (7)$$

The number of vehicles available at the beginning of the period t > 1 for type i is equal to the different between the number of vehicles available at the beginning of period t-1 and the number of vehicles sold of type i at ending of the same period.

$$x_{it} = x_{i(t-1)} + b_{i(t-1)} - s_{i(t-1)} \qquad ,\text{for } \forall i,t \ ; t > 1 \qquad (8)$$

(3) Vehicles available at the beginning of the period by type and age

$$x_{itj} = \begin{cases} I_{i0(j-1)} & ; \ t = 1,\ j > 1 \\ IB_{i0} & ; \ t = 1,\ j = 1 \\ b_{i(t-1)0} & ; \ j = 1,\ t > 1 \\ x_{i(t-1)(j-1)} - s_{i(t-1)(j-1)} & ; \ \text{Otherwise} \end{cases}$$

The number of vehicles available at the beginning of the period 1 for type i of age j is equal to the initial number of vehicles for type i of age j-1.

$$x_{itj} - I_{i0(j-1)} = 0 \qquad ,\text{for } \forall i,j \ , t = 1, j > 1 \qquad (9)$$

$$x_{itj} - IB_{i0} = 0 \qquad ,\text{for } \forall i \ , t = 1, j = 1 \qquad (10)$$

The number of vehicles available at the beginning of the period t > 1 for type i age 1 is equal to the number of vehicles purchased of type i at ending of period t-1.

$$x_{itj} - b_{i(t-1)0} = 0 \qquad ,\text{for } \forall i,t \ , j = 1, t > 1 \qquad (11)$$

The number of vehicles available at the beginning of the period t > 1 or age 1<j< J for type i is equal to the different between the number of vehicles available at the beginning of period t-1 and the number of vehicles sold of type i at ending of the same period.

$$x_{itj} - x_{i(t-1)(j-1)} + s_{i(t-1(j-1)} = 0 \qquad ,\text{for } \forall \text{i,t,j} \quad , \text{t} > 1 \text{ , 1<j<J} \qquad (12)$$

The number of vehicles available at the beginning of the period t > 1 or j = J for type i is equal to the different between sum of the number of vehicles available at the beginning of period t-1 with the number kept at the end of period t-1 and the number of vehicles sold of type i at ending of the same period.

$$x_{itJ} - y_{i(t-1)J} - x_{it-1(J-1)} + s_{itJ} = 0 \qquad \text{for } \forall i,t \quad , t > 1 \text{ , } j = J \qquad (13)$$

The number of vehicles at the end of the period t > 1 or age j =J for type i is equal to the different between the number of vehicles available at the beginning of period t and the number of vehicles sold of type i at ending of the same period.

$$y_{itJ} - x_{itJ} + s_{itJ} = 0 \qquad \text{for } \forall i,t \quad , t > 1 \text{ , } j = J \qquad (14)$$

Flow balance

The sum of the initial number of vehicles plus the number of the vehicles bought throughout the planning horizon is equal to the number of vehicles sold throughout the planning horizon and the number of vehicles keep in the end of planning horizon.

$$IB_{io} + I_{io} + \sum_{t=1}^{H} b_{it} - \sum_{t=1}^{H} s_{it} - (x_{iH} + b_{iH} - s_{iH}) \quad = \quad 0 \qquad ,\text{for } \forall i \qquad (15)$$

$$IB_{io} + \sum_{j=1}^{m-1} I_{ioj} + \sum_{t=1}^{H} b_{it} - \sum_{t=1}^{H}\sum_{j=1}^{m} s_{itj} - \sum_{j=1}^{m}(x_{iHj} + b_{it} - s_{iHj}) \quad = \quad 0 \qquad ,\text{for } \forall i \qquad (16)$$

APPENDIX B

B.1 Group Technology (GT)

Group technology (GT) is a management philosophy using the advantage of similarities among entities in order to reduce the complexity of a problem. GT principles can be applied to diverse range of entities (manufactured parts, capital equipment, decision processes, and human characteristics) [1].

The GT concept was introduced by Mitrofanov (1966) and Birbridge (1975) [2]. Several books on the GT concept and GT manufacturing have been published [3-6]. There has been a great deal of research on GT. Kusiak [7] presented the generallized group technology concept. Birbridge [8] described the first step in planning group technology. Selim et al. [9] addressed the classification of cell formation in group technology. Song [10] presented classification schemes based on the conceptual procedures of forming parts and families in GT. Catamessa and Turroni [11] addressed approaches to GT in cellular manufacturing systems. Offodile et al. [13] developed the taxonomic review framework of the cellular manufacturing in GT.

In order to solve the grouping problem in GT, many approaches have been proposed. Examples of these approaches include inspections based on part family analysis and geometric analogy, coding and classification analysis, and process plan analysis [11]. In addition, cluster analysis is one of the applied mathematical tools in GT [7].

In this research, the concepts of classification and cluster analysis are studied. Classification and coding (CC) is the approach that organizes similar entities into groups and assigns symbolic code to these entities. The methodology of classification and coding system can found in [3, 5, 6]. The empirical study of classification and coding systems is presented by Takikonda and Wemmerlov [1].

Cluster analysis is composed of several different techniques for recognizing structure in a complex data set. This tool is used to group objects or entities or their attributes into clusters such that individual elements within a cluster have a high degree of association among themselves and there is very little association between clusters [9]. Two basics formulations of the clustering models are matrix formulation and integer programming [7].

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B.2 Vehicle Grouping Implementation

Type	Description	Category				Function				Rate	Group
		Passeneger car	Pickup	Special Vehicle	Truck	Transport Passenegers	Transport Passengers and Equipment	Roadway measurement	Size Light		
1	SEDAN	1	0	0	0	1	0	0	0	3.97	G1
2	STA WGN	1	0	0	0	1	0	0	0	3.97	
3	STATION BUS	1	0	0	0	0	1	0	0	5.06	
4	CARRYALL	1	0	0	0	0	1	0	0	5.06	
5	CARRYALL	1	0	0	0	0	1	0	0	5.06	
6	UTILITY VEHICLE	1	0	0	0	0	1	0	0	5.06	G2
7	MINI VAN	1	0	0	0	0	1	0	0	5.06	
8	STATION BUS	1	0	0	0	0	1	0	0	5.06	
9	STN BUS EXT BODY	1	0	0	0	0	1	0	0	5.06	
10	PICKUP	0	1	0	0	0	1	0	0	2.70	G3
11	PICKUP	0	1	0	0	0	1	0	0	4.96	
12	PICKUP	0	1	0	0	0	1	0	0	4.96	
13	PICKUP	0	1	0	0	0	1	0	0	4.96	G4
14	PICKUP	0	1	0	0	0	1	0	0	4.96	
15	PICKUP	0	1	0	0	0	1	0	0	4.96	
16	PICKUP	0	1	0	0	0	1	0	0	4.96	
17	DEFLECTOMETER	0	0	1	0	0	0	1	0	20.05	G5
18	PROFILOGRAPH	0	0	1	0	0	0	1	0	20.05	
19	TRUCK	0	0	0	1	0	0	0	1	6.29	
20	TRUCK	0	0	0	1	0	0	0	1	6.29	
21	TRUCK	0	0	0	1	0	0	0	1	6.29	G6
22	TRUCK	0	0	0	1	0	0	0	1	6.29	
23	TRUCK	0	0	0	1	0	0	0	1	6.29	

B.2 Vehicle grouping Implementation (Continued)

Type	Description	Category				Function					Rate	Group
		Truck	Graders	Rollers	Loaders	Size Medium	Size Heavy	Grading	Compaction	Loading material		
24	TRUCK	1	0	0	0	1	0	0	0	0	14.82	
25	TRUCK	1	0	0	0	1	0	0	0	0	14.82	
26	TRUCK	1	0	0	0	1	0	0	0	0	14.82	G7
27	TRUCK	1	0	0	0	1	0	0	0	0	14.82	
28	TRUCK	1	0	0	0	1	0	0	0	0	14.82	
29	TRUCK	1	0	0	0	1	0	0	0	0	14.82	
30	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
31	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
32	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
33	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
34	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
35	TRUCK	1	0	0	0	0	1	0	0	0	18.40	G8
36	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
37	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
38	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
39	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
40	TRUCK	1	0	0	0	0	1	0	0	0	18.40	
41	GRADER	0	1	0	0	0	0	1	0	0	28.41	G9
42	GRADER	0	1	0	0	0	0	1	0	0	28.41	
43	ROLLER	0	0	1	0	0	0	0	1	0	13.29	
44	ROLLER	0	0	1	0	0	0	0	1	0	13.29	G10
45	ROLLER	0	0	1	0	0	0	0	1	0	13.29	
46	LOADER	0	0	0	1	0	0	0	0	1	15.22	
47	LOADER	0	0	0	1	0	0	0	0	1	15.22	
48	LOADER	0	0	0	1	0	0	0	0	1	15.22	
49	LOADER	0	0	0	1	0	0	0	0	1	15.22	G11
50	LOADER	0	0	0	1	0	0	0	0	1	15.22	
51	LOADER	0	0	0	1	0	0	0	0	1	15.22	
52	LOADER	0	0	0	1	0	0	0	0	1	15.22	
53	LOADER	0	0	0	1	0	0	0	0	1	15.22	

B.2 Vehicle grouping Implementation (Continued)

Type	Description	Category						Function						Rate	Group
		Sweepers	Asphalt	Mowers/ Tractors	Attachments	Snow Removal Equipment	Boats w/ Trailers	Sweeping	Oiling roads	Cutting Equipment	Chips and Brush	Snow and remove	Water related bridge Activities		
54	SWEEPER	1	0	0	0	0	0	1	0	0	0	0	0	32.04	
55	SWEEPER	1	0	0	0	0	0	1	0	0	0	0	0	32.04	G12
56	SWEEPER	1	0	0	0	0	0	1	0	0	0	0	0	32.04	
57	SWEEPER	1	0	0	0	0	0	1	0	0	0	0	0	32.04	
58	SWEEPER	1	0	0	0	0	0	1	0	0	0	0	0	18.87	G13
59	OILER	0	1	0	0	0	0	0	1	0	0	0	0	11.78	G14
60	OILER	0	1	0	0	0	0	0	1	0	0	0	0	11.78	
61	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	14.57	
62	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	14.57	
63	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	14.57	G15
64	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	14.57	
65	TRACTOR	0	0	1	0	0	0	0	0	1	0	0	0	14.57	
66	TRACTOR	0	0	1	0	0	0	0	0	1	0	0	0	14.57	
67	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	19.66	
68	BRUSH CUTTER	0	0	1	0	0	0	0	0	1	0	0	0	19.66	G16
69	TRACTOR	0	0	1	0	0	0	0	0	1	0	0	0	19.66	
70	TRACTOR	0	0	1	0	0	0	0	0	1	0	0	0	19.66	
71	MOWER	0	0	1	0	0	0	0	0	1	0	0	0	33.70	G17
72	MOWER W/FLAIL	0	0	1	0	0	0	0	0	1	0	0	0	33.70	
73	BRUSH CHIPPER	0	0	0	1	0	0	0	0	0	1	0	0	21.61	G18
74	TRACTOR	0	0	0	0	1	0	0	0	0	0	1	0	55.36	G19
75	SNOW TRACK	0	0	0	0	1	0	0	0	0	0	1	0	55.36	
76	BOAT	0	0	0	0	0	1	0	0	0	0	0	1	36.36	G20

B.2 Vehicle grouping Implementation (Continued)

Type	Description	Category				Function			Rate	Group
		Arrow-boards	Electrical	Drill Equipment	Miscelleaneous	Directing Traffic	Running Equipment	Drill Equipment		
77	BOARD	1	0	0	0	1	0	0	12.38	G21
78	MESSG. SIGN	1	0	0	0	1	0	0	12.38	
79	COMPRESSOR	0	1	0	0	0	1	0	8.41	G22
80	COMPRESSOR	0	1	0	0	0	1	0	8.41	
81	GENERATOR	0	1	0	0	0	1	0	8.41	
82	LIGHTING SYSTEM	0	1	0	0	0	1	0	8.41	
83	DRILL	0	0	1	0	0	0	1	29.82	G23
84	DRILL	0	0	1	0	0	0	1	29.82	
85	PAVING	0	0	0	1	0	0	0	38.39	G24
86	EPOXY DISP	0	0	0	1	0	0	0	38.39	
87	GRINDER	0	0	0	1	0	0	0	38.39	
88	GROUTING MACHINE	0	0	0	1	0	0	0	38.39	
89	JET RODDER	0	0	0	1	0	0	0	38.39	
90	SKID TEST	0	0	0	1	0	0	0	81.96	G25

APPENDIX C

Table C1-1 Inter–group Replacement Plan (No Budget Constraint)

Number of Units Purchased and Acquisition Costs in each Period for 5-period Planning Horizon

Group	Period 1		Period 2		Period 3		Period 4		Period 5	
	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)
G1	60	955,809	0	0	9	133,243	55	794,615	0	0
G2	0	0	0	0	0	0	0	0	0	0
G3	0	0	0	0	0	0	0	0	0	0
G4	99	2,598,832	10	249,994	0	0	38	904,691	0	0
G5	0	0	0	0	0	0	0	0	0	0
G6	0	0	0	0	0	0	0	0	0	0
G7	146	14,785,888	0	0	21	1,976,488	5	459,239	0	0
G8	200	28,233,187	51	6,856,255	9	1,180,736	33	4,224,912	0	0
G9	0	0	0	0	0	0	0	0	0	0
G10	0	0	0	0	4	199,782	0	0	0	0
G11	0	0	20	1,761,487	1	85,949	17	1,425,888	0	0
G12	0	0	0	0	0	0	0	0	0	0
G13	11	284,467	0	0	0	0	0	0	0	0
G14	0	0	0	0	2	61,526	1	30,021	0	0
G15	0	0	0	0	0	0	0	0	0	0
G16	0	0	0	0	0	0	0	0	0	0
G17	0	0	0	0	0	0	0	0	0	0
G18	0	0	0	0	0	0	0	0	0	0
G19	0	0	0	0	0	0	0	0	0	0
G20	1	31,033	0	0	0	0	0	0	0	0
G21	0	0	0	0	0	0	0	0	0	0
G22	0	0	0	0	3	43,065	0	0	0	0
G23	0	0	0	0	0	0	1	243,919	0	0
G24	18	1,802,162	21	2,002,293	0	0	0	0	0	0
G25	0	0	0	0	0	0	0	0	0	0
Total	535	48,691,378	102	10,870,029	49	3,680,789	150	8,083,284	0	0

Table C1-2 Inter–group Replacement Plan (With Budget Constraint)

Number of Units Purchased and Acquisition Costs in each Period for 5-period Planning Horizon

Group	Period 1		Period 2		Period 3		Period 4		Period 5	
	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)
G1	60	955,809	0	0	6	88,828	55	794,615	0	0
G2	0	0	0	0	0	0	0	0	0	0
G3	0	0	0	0	0	0	0	0	0	0
G4	99	2,598,832	10	249,994	0	0	38	904,691	0	0
G5	0	0	0	0	0	0	0	0	0	0
G6	0	0	0	0	0	0	0	0	0	0
G7	46	4,658,568	14	1,350,235	98	9,223,611	14	1,285,868	0	0
G8	6	846,996	51	6,856,255	9	1,180,736	39	4,993,078	0	0
G9	0	0	0	0	0	0	0	0	0	0
G10	0	0	0	0	0	0	0	0	0	0
G11	0	0	20	1,761,487	1	85,949	17	1,425,888	0	0
G12	0	0	0	0	0	0	0	0	0	0
G13	0	0	0	0	0	0	0	0	0	0
G14	0	0	0	0	2	61,526	1	30,021	0	0
G15	0	0	0	0	0	0	0	0	0	0
G16	0	0	0	0	0	0	0	0	0	0
G17	0	0	0	0	0	0	0	0	0	0
G18	0	0	0	0	0	0	0	0	0	0
G19	0	0	0	0	0	0	0	0	0	0
G20	1	31,033	0	0	0	0	0	0	0	0
G21	0	0	0	0	0	0	0	0	0	0
G22	1	15,446	0	0	2	28,710	0	0	0	0
G23	0	0	0	0	0	0	1	243,919	0	0
G24	14	1,401,682	3	286,042	4	372,187	14	1,271,227	0	0
G25	0	0	0	0	0	0	0	0	0	0
Total	227	10,508,365	98	10,504,014	122	11,041,548	179	10,949,307	0	0
Budget		10,509,766		10,509,766		11,047,481		11,047,481		11,686,722

Table C1-3 Inter–group Replacement Plan (With Budget Constraint)

Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G1	1,2	Purchase	0	60	0	6	55	0
		Available	207	207	207	207	207	207
		Sell		60	0	6	55	0
		Transfer	207	207	207	207	207	207
		Demand		207	207	207	207	207
G2	3,4,5,6,7,8,9	Purchase	0	0	0	0	0	0
		Available	274	274	274	274	274	274
		Sell		0	0	0	0	1
		Transfer	274	274	274	274	274	273
		Demand		274	274	274	274	274
G3	10	Purchase	0	0	0	0	0	0
		Available	185	185	185	185	185	185
		Sell		0	0	0	0	0
		Transfer	185	185	185	185	185	185
		Demand		185	185	185	185	185
G4	11,12,13,14,15,16	Purchase	0	99	10	0	38	0
		Available	372	372	372	372	372	372
		Sell		99	10	0	38	2
		Transfer	372	372	372	372	372	370
		Demand		372	372	372	372	372
G5	17,18	Purchase	0	0	0	0	0	0
		Available	2	2	2	2	2	2
		Sell		0	0	0	0	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2
G6	19,20,21,22,23	Purchase	0	0	0	0	0	0
		Available	326	326	326	326	326	326
		Sell		0	0	0	0	22
		Transfer	326	326	326	326	326	304
		Demand		326	326	326	326	326

Table C1-3 Inter–group Replacement Plan (With Budget Constraint) (Continued)

Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
		Purchase	0	46	14	98	14	0
		Available	227	227	227	227	227	227
G7	24,25,26,27,28,29	Sell		46	14	98	14	10
		Transfer	227	227	227	227	227	217
		Demand		227	227	227	227	227
		Purchase	0	6	51	9	39	0
		Available	452	452	452	452	452	452
G8	30,31,32,33,34,35,36,37,38,39,40	Sell		6	51	9	39	1
		Transfer	452	452	452	452	452	451
		Demand		452	452	452	452	452
		Purchase	0	0	0	0	0	0
		Available	79	79	79	79	79	79
G9	41,42	Sell		0	0	0	0	0
		Transfer	79	79	79	79	79	79
		Demand		79	79	79	79	79
		Purchase	0	0	0	0	0	0
		Available	57	57	57	57	57	57
G10	43,44,45	Sell		0	0	0	0	1
		Transfer	57	57	57	57	57	56
		Demand		57	57	57	57	57
		Purchase	0	0	20	1	17	0
		Available	264	264	264	264	264	264
G11	46,47,48,49,50,51,52,53	Sell		0	20	1	17	0
		Transfer	264	264	264	264	264	264
		Demand		264	264	264	264	264
		Purchase	0	0	0	0	0	0
		Available	37	37	37	37	37	37
G12	54,55,56,57	Sell		0	0	0	0	0
		Transfer	37	37	37	37	37	37
		Demand		37	37	37	37	37

Table C1-3 Inter–group Replacement Plan (With Budget Constraint) (Continued)

Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G13	58	Purchase	0	0	0	0	0	0
		Available	22	22	22	22	22	22
		Sell		0	0	0	0	0
		Transfer	22	22	22	22	22	22
		Demand		22	22	22	22	22
G14	59,60	Purchase	0	0	0	2	1	0
		Available	23	23	23	23	23	23
		Sell		0	0	2	1	0
		Transfer	23	23	23	23	23	23
		Demand		23	23	23	23	23
G15	61,62,63,64,65,66	Purchase	0	0	0	0	0	0
		Available	72	72	72	72	72	72
		Sell		0	0	0	0	4
		Transfer	72	72	72	72	72	68
		Demand		72	72	72	72	72
G16	67,68,69,70	Purchase	0	0	0	0	0	0
		Available	67	67	67	67	67	67
		Sell		0	0	0	0	0
		Transfer	67	67	67	67	67	67
		Demand		67	67	67	67	67
G17	71,72	Purchase	0	0	0	0	0	0
		Available	3	3	3	3	3	3
		Sell		0	0	0	0	0
		Transfer	3	3	3	3	3	3
		Demand		3	3	3	3	3
G18	73	Purchase	0	0	0	0	0	0
		Available	12	12	12	12	12	12
		Sell		0	0	0	0	0
		Transfer	12	12	12	12	12	12
		Demand		12	12	12	12	12

Table C1-3 Inter–group Replacement Plan (With Budget Constraint) (Continued)

Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G19	74,75	Purchase	0	0	0	0	0	0
		Available	5	5	5	5	5	5
		Sell		0	0	0	0	0
		Transfer	5	5	5	5	5	5
		Demand		5	5	5	5	5
G20	76	Purchase	0	1	0	0	0	0
		Available	3	3	3	3	3	3
		Sell		1	0	0	0	1
		Transfer	3	3	3	3	3	2
		Demand		3	3	3	3	3
G21	77,78	Purchase	0	0	0	0	0	0
		Available	42	42	42	42	42	42
		Sell		0	0	0	0	1
		Transfer	42	42	42	42	42	41
		Demand		42	42	42	42	42
G22	79,80,81,82	Purchase	0	1	0	2	0	0
		Available	31	31	31	31	31	31
		Sell		1	0	2	0	0
		Transfer	31	31	31	31	31	31
		Demand		31	31	31	31	31
G23	83,84	Purchase	0	0	0	0	1	0
		Available	7	7	7	7	7	7
		Sell		0	0	0	1	1
		Transfer	7	7	7	7	7	6
		Demand		7	7	7	7	7
G24	85,86,87,88,89	Purchase	0	14	3	4	14	0
		Available	21	21	21	21	21	21
		Sell		14	3	4	14	0
		Transfer	21	21	21	21	21	21
		Demand		21	21	21	21	21

Table C1-3 Inter–group Replacement Plan (With Budget Constraint) (Continued)

Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G25	90	Purchase	0	0	0	0	0	0
		Available	1	1	1	1	1	1
		Sell		0	0	0	0	0
		Transfer	1	1	1	1	1	1
		Demand		1	1	1	1	1
	Total	Purchase	0	227	98	122	179	0
		Available	2,791	2,791	2,791	2,791	2,791	2,791
		Sell	0	227	98	122	179	44
		Transfer	2,791	2,791	2,791	2,791	2,791	2,747

Table C1-4 Inter-group Replacement Plan for a Group by Age

GROUP 1

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	60	0	6	55	0
	Available	0	0	60	0	6	55
1	Sell		0	0	0	6	0
	Transfer	0	0	60	0	0	55
	Available	29	0	0	60	0	0
2	Sell		0	0	0	0	0
	Transfer	29	0	0	60	0	0
	Available	15	29	0	0	60	0
3	Sell		0	0	0	0	0
	Transfer	15	29	0	0	60	0
	Available	23	15	29	0	0	60
4	Sell		0	0	0	0	0
	Transfer	23	15	29	0	0	60
	Available	25	23	15	29	0	0
5	Sell		0	0	0	0	0
	Transfer	25	23	15	29	0	0
	Available	0	25	23	15	29	0
6	Sell		0	0	0	0	0
	Transfer	0	25	23	15	29	0
	Available	46	0	25	23	15	29
7	Sell		0	0	0	0	0
	Transfer	46	0	25	23	15	29
	Available	9	46	0	25	23	15
8	Sell		0	0	0	0	0
	Transfer	9	46	0	25	23	15
	Available	0	9	46	0	25	23
9	Sell		0	0	0	0	0
	Transfer	0	9	46	0	25	23
	Available	38	0	9	46	0	25
10	Sell		0	0	0	0	0
	Transfer	38	0	9	46	0	25
	Available	22	38	0	9	46	0
11	Sell		38	0	6	46	0
	Transfer	22	0	0	3	0	0
	Available	0	22	0	0	3	0
12	Sell		22	0	0	3	0
	Transfer	0	0	0	0	0	0
	Available	207	207	207	207	207	207
Total	Sell	0	60	0	6	55	0
	Transfer	207	207	207	207	207	207

Purchase : Number of units purchased at the end of period t

Available : Number of units available, age j at the end of period t

Equal to units transferred from age j-1 at the end of period t-1

Sell : Number of units sold, age j at the end of period t

Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period

Last age group included vehicle age > 12

Table C1-5 Inter-group Replacement Plan for a Group by Age

GROUP 4

Age		Period 0	1	2	3	4	5
0	Purchase	0	99	10	0	38	0
1	Available	0	0	99	10	0	38
	Sell		0	0	0	0	0
	Transfer	0	0	99	10	0	38
2	Available	57	0	0	99	10	0
	Sell		0	0	0	0	0
	Transfer	57	0	0	99	10	0
3	Available	88	57	0	0	99	10
	Sell		0	0	0	0	0
	Transfer	88	57	0	0	99	10
4	Available	32	88	57	0	0	99
	Sell		0	0	0	0	0
	Transfer	32	88	57	0	0	99
5	Available	46	32	88	57	0	0
	Sell		0	0	0	0	0
	Transfer	46	32	88	57	0	0
6	Available	2	46	32	88	57	0
	Sell		0	0	0	0	0
	Transfer	2	46	32	88	57	0
7	Available	38	2	46	32	88	57
	Sell		0	0	0	0	0
	Transfer	38	2	46	32	88	57
8	Available	0	38	2	46	32	88
	Sell		0	0	0	0	0
	Transfer	0	38	2	46	32	88
9	Available	10	0	38	2	46	32
	Sell		0	0	0	0	0
	Transfer	10	0	38	2	46	32
10	Available	49	10	0	38	2	46
	Sell		0	0	0	0	0
	Transfer	49	10	0	38	2	46
11	Available	50	49	10	0	38	2
	Sell		49	10	0	38	2
	Transfer	50	0	0	0	0	0
12	Available	0	50	0	0	0	0
	Sell		50	0	0	0	0
	Transfer	0	0	0	0	0	0
Total	Available	377	372	372	372	372	372
	Sell	0	99	10	0	38	2
	Transfer	377	372	372	372	372	370

Purchase : Number of units purchased at the end of period t

Available : Number of units available, age j at the end of period t

Equal to units transferred from age j-1 at the end of period t-1

Sell : Number of units sold, age j at the end of period t

Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period

Last age group included vehicle age > 12.

Table C1-6 Inter-group Replacement Plan for a Group by Age

GROUP 7

Age		Period 0	1	2	3	4	5
0	Purchase	0	46	14	98	14	0
1	Available	0	0	46	14	98	14
	Sell		0	0	0	0	0
	Transfer	0	0	46	14	98	14
2	Available	39	0	0	46	14	98
	Sell		0	0	0	0	0
	Transfer	39	0	0	46	14	98
3	Available	0	39	0	0	46	14
	Sell		0	0	0	0	0
	Transfer	0	39	0	0	46	14
4	Available	6	0	39	0	0	46
	Sell		0	0	0	0	0
	Transfer	6	0	39	0	0	46
5	Available	0	6	0	39	0	0
	Sell		0	0	0	0	0
	Transfer	0	6	0	39	0	0
6	Available	10	0	6	0	39	0
	Sell		0	0	0	0	0
	Transfer	10	0	6	0	39	0
7	Available	5	10	0	6	0	39
	Sell		0	0	0	0	0
	Transfer	5	10	0	6	0	39
8	Available	21	5	10	0	6	0
	Sell		0	0	0	0	0
	Transfer	21	5	10	0	6	0
9	Available	0	21	5	10	0	6
	Sell		0	0	0	0	0
	Transfer	0	21	5	10	0	6
10	Available	47	0	21	5	10	0
	Sell		0	0	0	0	0
	Transfer	47	0	21	5	10	0
11	Available	99	47	0	21	5	10
	Sell		46	0	21	5	10
	Transfer	99	1	0	0	0	0
12	Available	0	99	100	86	9	0
	Sell		0	14	77	9	0
	Transfer	0	99	86	9	0	0
Total	Available	227	227	227	227	227	227
	Sell	0	46	14	98	14	10
	Transfer	227	227	227	227	227	217

Purchase : Number of units purchased at the end of period t

Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1

Sell : Number of units sold, age j at the end of period t

Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period

Last age group included vehicle age > 12.

Table C1-7 Inter-group Replacement Plan for a Group by Age

GROUP 8

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	6	51	9	39	0
1	Available	0	0	6	51	9	39
	Sell		0	0	0	0	0
	Transfer	0	0	6	51	9	39
2	Available	64	0	0	6	51	9
	Sell		0	0	0	0	0
	Transfer	64	0	0	6	51	9
3	Available	7	64	0	0	6	51
	Sell		0	0	0	0	0
	Transfer	7	64	0	0	6	51
4	Available	86	7	64	0	0	6
	Sell		0	0	0	0	0
	Transfer	86	7	64	0	0	6
5	Available	1	86	7	64	0	0
	Sell		0	0	0	0	0
	Transfer	1	86	7	64	0	0
6	Available	1	1	86	7	64	0
	Sell		0	0	0	0	0
	Transfer	1	1	86	7	64	0
7	Available	33	1	1	86	7	64
	Sell		0	0	0	0	0
	Transfer	33	1	1	86	7	64
8	Available	9	33	1	1	86	7
	Sell		0	0	0	0	0
	Transfer	9	33	1	1	86	7
9	Available	51	9	33	1	1	86
	Sell		0	0	0	0	0
	Transfer	51	9	33	1	1	86
10	Available	39	51	9	33	1	1
	Sell		0	0	0	0	0
	Transfer	39	51	9	33	1	1
11	Available	161	39	51	9	33	1
	Sell		6	51	9	33	1
	Transfer	161	33	0	0	0	0
12	Available	0	161	194	194	194	188
	Sell		0	0	0	6	0
	Transfer	0	161	194	194	188	188
Total	Available	452	452	452	452	452	452
	Sell	0	6	51	9	39	1
	Transfer	452	452	452	452	452	451

Purchase : Number of units purchased at the end of period t

Available : Number of units available, age j at the end of period t

Equal to units transferred from age j-1 at the end of period t-1

Sell : Number of units sold, age j at the end of period t

Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period

Last age group included vehicle age > 12.

Table C2-1 Intra–group Replacement Plan (With Budget Constraint)
Number of Units Purchased and Acquisition Costs in each Period for 5-period Planning Horizon

Group	Type	Period 1		Period 2		Period 3		Period 4		Period 5	
		# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)
G1	1	55	796,508	0	0	6	82,750	47	632,566	0	0
	2	5	86,892	0	0	0	0	8	129,205	0	0
G4	1	2	36,826	1	17,969	0	0	0	0	0	0
	2	25	657,641	7	179,697	0	0	23	562,286	0	0
	3	16	420,937	0	0	0	0	11	268,949	0	0
	4	35	899,692	0	0	0	0	2	47,779	0	0
	5	21	624,535	1	29,022	0	0	1	27,639	0	0
	6	0	0	1	30,284	0	0	1	28,840	0	0
G7	1	0	0	1	46,743	0	0	0	0	0	0
	2	0	0	1	83,471	0	0	2	158,983	0	0
	3	43	3,677,976	12	1,001,648	94	7,656,939	0	0	0	0
	4	3	465,492	0	0	0	0	0	0	0	0
	5	0	0	0	0	3	410,541	11	1,469,000	0	0
	6	0	0	0	0	1	85,529	1	83,466	0	0
G8	1	6	574,791	51	4,767,850	0	0	36	3,205,102	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	3	325,012	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	1	147,767	0	0	0	0
	6	0	0	0	0	1	187,172	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	1	148,251	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	3	768,388	0	0	0	0
	11	0	0	0	0	0	0	3	259,563	0	0

Table C2-1 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Acquisition Costs in each Period for 5-period Planning Horizon

Group	Type	Period 1		Period 2		Period 3		Period 4		Period 5	
		# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)	# units	Acq. Costs($)
G11	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	1	146,373	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	1	23,948	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
	6	0	0	2	178,474	0	0	3	254,949	0	0
	7	0	0	7	736,206	0	0	8	801,269	0	0
	8	0	0	10	1,168,584	0	0	6	667,725	0	0
G14	1	0	0	0	0	2	63,047	0	0	0	0
	2	0	0	0	0	0	0	1	30,763	0	0
G20	1	1	31,033	0	0	0	0	0	0	0	0
G22	1	0	0	0	0	0	0	0	0	0	0
	2	1	15,446	0	0	2	28,710	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
G23	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	1	249,950	0	0
G24	1	2	310,328	0	0	0	0	2	288,404	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	10	299,369	0	0	1	28,510	10	278,219	0	0
	4	0	0	1	30,284	0	0	0	0	0	0
	5	2	413,770	2	403,787	3	591,068	2	384,538	0	0

Table C2-2 Intra–group Replacement Plan (With Budget Constraint)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G1	1	Purchase	0	55	0	6	47	0
		Available	185	185	185	185	185	185
		Sell	0	55	0	6	47	0
		Transfer	185	185	185	185	185	185
		Demand		185	185	185	185	185
G1	2	Purchase	0	5	0	0	8	0
		Available	22	22	22	22	22	22
		Sell	0	5	0	0	8	0
		Transfer	22	22	22	22	22	22
		Demand		22	22	22	22	22
G4	1	Purchase	0	2	1	0	0	0
		Available	20	20	20	20	20	20
		Sell	0	2	1	0	0	0
		Transfer	20	20	20	20	20	20
		Demand		20	20	20	20	20
G4	2	Purchase	0	25	7	0	23	0
		Available	132	132	132	132	132	132
		Sell	0	25	7	0	23	0
		Transfer	132	132	132	132	132	132
		Demand		132	132	132	132	132
G4	3	Purchase	0	16	0	0	11	0
		Available	113	113	113	113	113	113
		Sell	0	16	0	0	11	1
		Transfer	113	113	113	113	113	112
		Demand		113	113	113	113	113
G4	4	Purchase	0	35	0	0	2	0
		Available	44	44	44	44	44	44
		Sell	0	35	0	0	2	1
		Transfer	44	44	44	44	44	43
		Demand		44	44	44	44	44

Table C2-2 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period 0	1	2	3	4	5
G4	5	Purchase	0	21	1	0	1	0
		Available	53	52	52	52	52	52
		Sell	1	21	1	0	1	0
		Transfer	52	52	52	52	52	52
		Demand		53	53	53	53	53
G4	6	Purchase	0	0	1	0	1	0
		Available	10	10	10	10	10	10
		Sell	0	0	1	0	1	0
		Transfer	10	10	10	10	10	10
		Demand		10	10	10	10	10
G7	1	Purchase	0	0	1	0	0	0
		Available	4	4	4	4	4	4
		Sell	0	0	1	0	0	1
		Transfer	4	4	4	4	4	3
		Demand		4	4	4	4	4
G7	2	Purchase	0	0	1	0	2	0
		Available	3	3	3	3	3	3
		Sell	0	0	1	0	2	0
		Transfer	3	3	3	3	3	3
		Demand		3	3	3	3	3
G7	3	Purchase	0	43	12	94	0	0
		Available	193	193	193	193	193	193
		Sell	0	43	12	94	0	8
		Transfer	193	193	193	193	193	185
		Demand		193	193	193	193	193
G7	4	Purchase	0	3	0	0	0	0
		Available	3	3	3	3	3	3
		Sell	0	3	0	0	0	0
		Transfer	3	3	3	3	3	3
		Demand		3	3	3	3	3

Table C2-2 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period 0	1	2	3	4	5
G7	5	Purchase	0	0	0	3	11	0
		Available	22	22	22	22	22	22
		Sell	0	0	0	3	11	1
		Transfer	22	22	22	22	22	21
		Demand		22	22	22	22	22
G7	6	Purchase	0	0	0	1	1	0
		Available	2	2	2	2	2	2
		Sell	0	0	0	1	1	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2
G8	1	Purchase	0	6	51	0	36	0
		Available	377	377	377	377	377	377
		Sell	0	6	51	0	36	0
		Transfer	377	377	377	377	377	377
		Demand		377	377	377	377	377
G8	2	Purchase	0	0	0	0	0	0
		Available	5	5	5	5	5	5
		Sell	0	0	0	0	0	0
		Transfer	5	5	5	5	5	5
		Demand		5	5	5	5	5
G8	3	Purchase	0	0	0	3	0	0
		Available	38	38	38	38	38	38
		Sell	0	0	0	3	0	0
		Transfer	38	38	38	38	38	38
		Demand		38	38	38	38	38
G8	4	Purchase	0	0	0	0	0	0
		Available	2	2	2	2	2	2
		Sell	0	0	0	0	0	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2

Table C2-2 Intra-group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
		Purchase	0	0	0	1	0	0
		Available	4	4	4	4	4	4
G8	5	Sell	0	0	0	1	0	0
		Transfer	4	4	4	4	4	4
		Demand		4	4	4	4	4
		Purchase	0	0	0	1	0	0
		Available	5	5	5	5	5	5
G8	6	Sell	0	0	0	1	0	1
		Transfer	5	5	5	5	5	4
		Demand		5	5	5	5	5
		Purchase	0	0	0	0	0	0
		Available	4	4	4	4	4	4
G8	7	Sell	0	0	0	0	0	0
		Transfer	4	4	4	4	4	4
		Demand		4	4	4	4	4
		Purchase	0	0	0	1	0	0
		Available	1	1	1	1	1	1
G8	8	Sell	0	0	0	1	0	0
		Transfer	1	1	1	1	1	1
		Demand		1	1	1	1	1
		Purchase	0	0	0	0	0	0
		Available	2	2	2	2	2	2
G8	9	Sell	0	0	0	0	0	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2
		Purchase	0	0	0	3	0	0
		Available	11	11	11	11	11	11
G8	10	Sell	0	0	0	3	0	0
		Transfer	11	11	11	11	11	11
		Demand		11	11	11	11	11

Table C2-2 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period 0	1	2	3	4	5
G8	11	Purchase	0	0	0	0	3	0
		Available	3	3	3	3	3	6
		Sell	0	0	0	0	0	0
		Transfer	3	3	3	3	6	6
		Demand		3	3	3	3	3
G11	1	Purchase	0	0	0	0	0	0
		Available	122	122	122	122	122	122
		Sell	0	0	0	0	0	0
		Transfer	122	122	122	122	122	122
		Demand		122	122	122	122	122
G11	2	Purchase	0	0	1	0	0	0
		Available	14	14	14	14	14	14
		Sell	0	0	1	0	0	0
		Transfer	14	14	14	14	14	14
		Demand		14	14	14	14	14
G11	3	Purchase	0	0	0	0	0	0
		Available	13	13	13	13	13	13
		Sell	0	0	0	0	0	0
		Transfer	13	13	13	13	13	13
		Demand		13	13	13	13	13
G11	4	Purchase	0	0	0	1	0	0
		Available	10	10	10	10	10	10
		Sell	0	0	0	1	0	0
		Transfer	10	10	10	10	10	10
		Demand		10	10	10	10	10
G11	5	Purchase	0	0	0	0	0	0
		Available	13	12	12	12	12	12
		Sell	1	0	0	0	0	0
		Transfer	12	12	12	12	12	12
		Demand		13	13	13	13	13

Table C2-2 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period 0	1	2	3	4	5
G11	6	Purchase	0	0	2	0	3	0
		Available	40	40	40	40	40	40
		Sell	0	0	2	0	3	0
		Transfer	40	40	40	40	40	40
		Demand		40	40	40	40	40
G11	7	Purchase	0	0	7	0	8	0
		Available	15	15	15	15	15	15
		Sell	0	0	7	0	8	0
		Transfer	15	15	15	15	15	15
		Demand		15	15	15	15	15
G11	8	Purchase	0	0	10	0	6	0
		Available	37	37	37	37	37	37
		Sell	0	0	10	0	6	0
		Transfer	37	37	37	37	37	37
		Demand		37	37	37	37	37
G14	1	Purchase	0	0	0	2	0	0
		Available	18	18	18	18	18	18
		Sell	0	0	0	2	0	0
		Transfer	18	18	18	18	18	18
		Demand		18	18	18	18	18
G14	2	Purchase	0	0	0	0	1	0
		Available	5	5	5	5	5	5
		Sell	0	0	0	0	1	0
		Transfer	5	5	5	5	5	5
		Demand		5	5	5	5	5
G20	1	Purchase	0	1	0	0	0	0
		Available	3	3	3	3	3	3
		Sell	0	1	0	0	0	1
		Transfer	3	3	3	3	3	2
		Demand	0	3	3	3	3	3

Table C2-2 Intra-group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period 0	1	2	3	4	5
G22	1	Purchase	0	0	0	0	0	0
		Available	9	9	9	9	9	9
		Sell	0	0	0	0	0	0
		Transfer	9	9	9	9	9	9
		Demand		9	9	9	9	9
G22	2	Purchase	0	1	0	2	0	0
		Available	12	12	12	12	12	12
		Sell	0	1	0	2	0	0
		Transfer	12	12	12	12	12	12
		Demand		12	12	12	12	12
G22	3	Purchase	0	0	0	0	0	0
		Available	8	8	8	8	8	8
		Sell	0	0	0	0	0	0
		Transfer	8	8	8	8	8	8
		Demand		8	8	8	8	8
G22	4	Purchase	0	0	0	0	0	0
		Available	2	2	2	2	2	2
		Sell	0	0	0	0	0	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2
G23	1	Purchase	0	0	0	0	0	0
		Available	6	6	6	6	6	6
		Sell	0	0	0	0	0	0
		Transfer	6	6	6	6	6	6
		Demand		6	6	6	6	6
G23	2	Purchase	0	0	0	0	1	0
		Available	1	1	1	1	1	1
		Sell	0	0	0	0	1	0
		Transfer	1	1	1	1	1	1
		Demand		1	1	1	1	1

Table C2-2 Intra–group Replacement Plan (With Budget Constraint) (Continued)
Number of Units Purchased and Sold and Available in each Period for 5-period Planning Horizon

Group	Type		Period					
			0	1	2	3	4	5
G24	1	Purchase	0	2	0	0	2	0
		Available	2	2	2	2	2	2
		Sell	0	2	0	0	2	0
		Transfer	2	2	2	2	2	2
		Demand		2	2	2	2	2
G24	2	Purchase	0	0	0	0	0	0
		Available	0	0	0	0	0	0
		Sell	0	0	0	0	0	0
		Transfer	0	0	0	0	0	0
		Demand		0	0	0	0	0
G24	3	Purchase	0	10	0	1	10	0
		Available	11	11	11	11	11	11
		Sell	0	10	0	1	10	0
		Transfer	11	11	11	11	11	11
		Demand		11	11	11	11	11
G24	4	Purchase	0	0	1	0	0	0
		Available	1	1	1	1	1	1
		Sell	0	0	1	0	0	0
		Transfer	1	1	1	1	1	1
		Demand		1	1	1	1	1
G24	5	Purchase	0	2	2	3	2	0
		Available	7	7	7	7	7	7
		Sell	0	2	2	3	2	0
		Transfer	7	7	7	7	7	7
		Demand		7	7	7	7	7

Table C2-3 Intra-group Replacement Plan for a Type by Age

TYPE 1 GROUP 1

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	55	0	6	47	0
	Available	0	0	55	0	6	47
1	Sell		0	0	0	6	0
	Transfer	0	0	55	0	0	47
	Available	26	0	0	55	0	0
2	Sell		0	0	0	0	0
	Transfer	26	0	0	55	0	0
	Available	10	26	0	0	55	0
3	Sell		0	0	0	0	0
	Transfer	10	26	0	0	55	0
	Available	23	10	26	0	0	55
4	Sell		0	0	0	0	0
	Transfer	23	10	26	0	0	55
	Available	24	23	10	26	0	0
5	Sell		0	0	0	0	0
	Transfer	24	23	10	26	0	0
	Available	0	24	23	10	26	0
6	Sell		0	0	0	0	0
	Transfer	0	24	23	10	26	0
	Available	38	0	24	23	10	26
7	Sell		0	0	0	0	0
	Transfer	38	0	24	23	10	26
	Available	9	38	0	24	23	10
8	Sell		0	0	0	0	0
	Transfer	9	38	0	24	23	10
	Available	0	9	38	0	24	23
9	Sell		0	0	0	0	0
	Transfer	0	9	38	0	24	23
	Available	35	0	9	38	0	24
10	Sell		0	0	0	0	0
	Transfer	35	0	9	38	0	24
	Available	20	35	0	9	38	0
11	Sell		35	0	6	38	0
	Transfer	20	0	0	3	0	0
	Available	0	20	0	0	3	0
12	Sell		20	0	0	3	0
	Transfer	0	0	0	0	0	0
	Available	185	185	185	185	185	185
Total	Sell	0	55	0	6	47	0
	Transfer	185	185	185	185	185	185

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period
Last age group includes vehicle age > 12.

Table C2-4 Intra-group Replacement Plan for a Type by Age

TYPE 2 GROUP 1

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	5	0	0	8	0
1	Available	0	0	5	0	0	8
	Sell		0	0	0	0	0
	Transfer	0	0	5	0	0	8
2	Available	3	0	0	5	0	0
	Sell		0	0	0	0	0
	Transfer	3	0	0	5	0	0
3	Available	5	3	0	0	5	0
	Sell		0	0	0	0	0
	Transfer	5	3	0	0	5	0
4	Available	0	5	3	0	0	5
	Sell		0	0	0	0	0
	Transfer	0	5	3	0	0	5
5	Available	1	0	5	3	0	0
	Sell		0	0	0	0	0
	Transfer	1	0	5	3	0	0
6	Available	0	1	0	5	3	0
	Sell		0	0	0	0	0
	Transfer	0	1	0	5	3	0
7	Available	8	0	1	0	5	3
	Sell		0	0	0	0	0
	Transfer	8	0	1	0	5	3
8	Available	0	8	0	1	0	5
	Sell		0	0	0	0	0
	Transfer	0	8	0	1	0	5
9	Available	0	0	8	0	1	0
	Sell		0	0	0	0	0
	Transfer	0	0	8	0	1	0
10	Available	3	0	0	8	0	1
	Sell		0	0	0	0	0
	Transfer	3	0	0	8	0	1
11	Available	2	3	0	0	8	0
	Sell		3	0	0	8	0
	Transfer	2	0	0	0	0	0
12	Available	0	2	0	0	0	0
	Sell		2	0	0	0	0
	Transfer	0	0	0	0	0	0
Total	Available	22	22	22	22	22	22
	Sell	0	5	0	0	8	0
	Transfer	22	22	22	22	22	22

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-5 Intra-group Replacement Plan for a Type by Age

TYPE 1 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	5	0	0	8	0
	Available	0	0	5	0	0	8
1	Sell		0	0	0	0	0
	Transfer	0	0	5	0	0	8
	Available	3	0	0	5	0	0
2	Sell		0	0	0	0	0
	Transfer	3	0	0	5	0	0
	Available	5	3	0	0	5	0
3	Sell		0	0	0	0	0
	Transfer	5	3	0	0	5	0
	Available	0	5	3	0	0	5
4	Sell		0	0	0	0	0
	Transfer	0	5	3	0	0	5
	Available	1	0	5	3	0	0
5	Sell		0	0	0	0	0
	Transfer	1	0	5	3	0	0
	Available	0	1	0	5	3	0
6	Sell		0	0	0	0	0
	Transfer	0	1	0	5	3	0
	Available	8	0	1	0	5	3
7	Sell		0	0	0	0	0
	Transfer	8	0	1	0	5	3
	Available	0	8	0	1	0	5
8	Sell		0	0	0	0	0
	Transfer	0	8	0	1	0	5
	Available	0	0	8	0	1	0
9	Sell		0	0	0	0	0
	Transfer	0	0	8	0	1	0
	Available	3	0	0	8	0	1
10	Sell		0	0	0	0	0
	Transfer	3	0	0	8	0	1
	Available	2	3	0	0	8	0
11	Sell		3	0	0	8	0
	Transfer	2	0	0	0	0	0
	Available	0	2	0	0	0	0
12	Sell		2	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	22	22	22	22	22	22
Total	Sell	0	5	0	0	8	0
	Transfer	22	22	22	22	22	22

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-6 Intra-group Replacement Plan for a Type by Age

TYPE 2 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	25	7	0	23	0
	Available	0	0	25	7	0	23
1	Sell		0	0	0	0	0
	Transfer	0	0	25	7	0	23
	Available	16	0	0	25	7	0
2	Sell		0	0	0	0	0
	Transfer	16	0	0	25	7	0
	Available	35	16	0	0	25	7
3	Sell		0	0	0	0	0
	Transfer	35	16	0	0	25	7
	Available	10	35	16	0	0	25
4	Sell		0	0	0	0	0
	Transfer	10	35	16	0	0	25
	Available	16	10	35	16	0	0
5	Sell		0	0	0	0	0
	Transfer	16	10	35	16	0	0
	Available	0	16	10	35	16	0
6	Sell		0	0	0	0	0
	Transfer	0	16	10	35	16	0
	Available	23	0	16	10	35	16
7	Sell		0	0	0	0	0
	Transfer	23	0	16	10	35	16
	Available	0	23	0	16	10	35
8	Sell		0	0	0	0	0
	Transfer	0	23	0	16	10	35
	Available	7	0	23	0	16	10
9	Sell		0	0	0	0	0
	Transfer	7	0	23	0	16	10
	Available	0	7	0	23	0	16
10	Sell		0	0	0	0	0
	Transfer	0	7	0	23	0	16
	Available	25	0	7	0	23	0
11	Sell		0	7	0	23	0
	Transfer	25	0	0	0	0	0
	Available	0	25	0	0	0	0
12	Sell		25	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	134	132	132	132	132	132
Total	Sell	0	25	7	0	23	0
	Transfer	134	132	132	132	132	132

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-7 Intra-group Replacement Plan for a Type by Age

TYPE 3 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	16	0	0	11	0
	Available	0	0	16	0	0	11
1	Sell		0	0	0	0	0
	Transfer	0	0	16	0	0	11
	Available	24	0	0	16	0	0
2	Sell		0	0	0	0	0
	Transfer	24	0	0	16	0	0
	Available	31	24	0	0	16	0
3	Sell		0	0	0	0	0
	Transfer	31	24	0	0	16	0
	Available	8	31	24	0	0	16
4	Sell		0	0	0	0	0
	Transfer	8	31	24	0	0	16
	Available	22	8	31	24	0	0
5	Sell		0	0	0	0	0
	Transfer	22	8	31	24	0	0
	Available	1	22	8	31	24	0
6	Sell		0	0	0	0	0
	Transfer	1	22	8	31	24	0
	Available	11	1	22	8	31	24
7	Sell		0	0	0	0	0
	Transfer	11	1	22	8	31	24
	Available	0	11	1	22	8	31
8	Sell		0	0	0	0	0
	Transfer	0	11	1	22	8	31
	Available	0	0	11	1	22	8
9	Sell		0	0	0	0	0
	Transfer	0	0	11	1	22	8
	Available	2	0	0	11	1	22
10	Sell		0	0	0	0	0
	Transfer	2	0	0	11	1	22
	Available	14	2	0	0	11	1
11	Sell		2	0	0	11	1
	Transfer	14	0	0	0	0	0
	Available	0	14	0	0	0	0
12	Sell		14	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	118	113	113	113	113	113
Total	Sell	0	16	0	0	11	1
	Transfer	118	113	113	113	113	112

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-8 Intra-group Replacement Plan for a Type by Age

TYPE 4 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	35	0	0	2	0
	Available	0	0	35	0	0	2
1	Sell		0	0	0	0	0
	Transfer	0	0	35	0	0	2
	Available	3	0	0	35	0	0
2	Sell		0	0	0	0	0
	Transfer	3	0	0	35	0	0
	Available	2	3	0	0	35	0
3	Sell		0	0	0	0	0
	Transfer	2	3	0	0	35	0
	Available	0	2	3	0	0	35
4	Sell		0	0	0	0	0
	Transfer	0	2	3	0	0	35
	Available	1	0	2	3	0	0
5	Sell		0	0	0	0	0
	Transfer	1	0	2	3	0	0
	Available	1	1	0	2	3	0
6	Sell		0	0	0	0	0
	Transfer	1	1	0	2	3	0
	Available	2	1	1	0	2	3
7	Sell		0	0	0	0	0
	Transfer	2	1	1	0	2	3
	Available	0	2	1	1	0	2
8	Sell		0	0	0	0	0
	Transfer	0	2	1	1	0	2
	Available	0	0	2	1	1	0
9	Sell		0	0	0	0	0
	Transfer	0	0	2	1	1	0
	Available	35	0	0	2	1	1
10	Sell		0	0	0	0	0
	Transfer	35	0	0	2	1	1
	Available	0	35	0	0	2	1
11	Sell		35	0	0	2	1
	Transfer	0	0	0	0	0	0
	Available	0	0	0	0	0	0
12	Sell		0	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	44	44	44	44	44	44
Total	Sell	0	35	0	0	2	1
	Transfer	44	44	44	44	44	43

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-9 Intra-group Replacement Plan for a Type by Age

TYPE 5 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	21	1	0	1	0
	Available	0	0	21	1	0	1
1	Sell		0	0	0	0	0
	Transfer	0	0	21	1	0	1
	Available	12	0	0	21	1	0
2	Sell		0	0	0	0	0
	Transfer	12	0	0	21	1	0
	Available	13	12	0	0	21	1
3	Sell		0	0	0	0	0
	Transfer	13	12	0	0	21	1
	Available	2	13	12	0	0	21
4	Sell		0	0	0	0	0
	Transfer	2	13	12	0	0	21
	Available	3	2	13	12	0	0
5	Sell		0	0	0	0	0
	Transfer	3	2	13	12	0	0
	Available	0	3	2	13	12	0
6	Sell		0	0	0	0	0
	Transfer	0	3	2	13	12	0
	Available	1	0	3	2	13	12
7	Sell		0	0	0	0	0
	Transfer	1	0	3	2	13	12
	Available	0	1	0	3	2	13
8	Sell		0	0	0	0	0
	Transfer	0	1	0	3	2	13
	Available	1	0	1	0	3	2
9	Sell		0	0	0	0	0
	Transfer	1	0	1	0	3	2
	Available	10	1	0	1	0	3
10	Sell		0	0	0	0	0
	Transfer	10	1	0	1	0	3
	Available	11	10	1	0	1	0
11	Sell		10	1	0	1	0
	Transfer	11	0	0	0	0	0
	Available	0	11	0	0	0	0
12	Sell		11	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	54	53	53	53	53	53
Total	Sell	0	21	1	0	1	0
	Transfer	54	53	53	53	53	53

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.

Table C2-10 Intra-group Replacement Plan for a Type by Age

TYPE 6 GROUP 4

Age		Period					
		0	1	2	3	4	5
0	Purchase	0	0	1	0	1	0
	Available	0	0	0	1	0	1
1	Sell		0	0	0	0	0
	Transfer	0	0	0	1	0	1
	Available	0	0	0	0	1	0
2	Sell		0	0	0	0	0
	Transfer	0	0	0	0	1	0
	Available	1	0	0	0	0	1
3	Sell		0	0	0	0	0
	Transfer	1	0	0	0	0	1
	Available	5	1	0	0	0	0
4	Sell		0	0	0	0	0
	Transfer	5	1	0	0	0	0
	Available	2	5	1	0	0	0
5	Sell		0	0	0	0	0
	Transfer	2	5	1	0	0	0
	Available	0	2	5	1	0	0
6	Sell		0	0	0	0	0
	Transfer	0	2	5	1	0	0
	Available	1	0	2	5	1	0
7	Sell		0	0	0	0	0
	Transfer	1	0	2	5	1	0
	Available	0	1	0	2	5	1
8	Sell		0	0	0	0	0
	Transfer	0	1	0	2	5	1
	Available	1	0	1	0	2	5
9	Sell		0	0	0	0	0
	Transfer	1	0	1	0	2	5
	Available	0	1	0	1	0	2
10	Sell		0	0	0	0	0
	Transfer	0	1	0	1	0	2
	Available	0	0	1	0	1	0
11	Sell		0	1	0	1	0
	Transfer	0	0	0	0	0	0
	Available	0	0	0	0	0	0
12	Sell		0	0	0	0	0
	Transfer	0	0	0	0	0	0
	Available	10	10	10	10	10	10
Total	Sell	0	0	1	0	1	0
	Transfer	10	10	10	10	10	10

Purchase : Number of units purchased at the end of period t
Available : Number of units available, age j at the end of period t
Equal to units transferred from age j-1 at the end of period t-1
Sell : Number of units sold, age j at the end of period t
Transfer : Number of units transferred, age j-1 at the end of period t-1 to age j at the beginning of period t
Last age group included vehicle age > 12.