Observation of the Third Harmonic Sinusoidal Voltage Response of a Small Incandescent Lightbulb

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Abstract

This thesis looks at the third harmonic voltage response of a small incandescent lightbulb. The third harmonic voltage, or 3 omega (3ω) voltage, arises from applying a 1ω sinusoidal voltage across a resistive material. The 3ω voltage carries with it information about the thermal conductivity of the material. A lock-in-amplifier is used to analyze and measure the 3ω voltage; the abilities of the lock-in amplifier are explored using an RLC series circuit. Our initial findings show the lock-in amplifier is able measure the 3ω signal of a small lightbulb without needing to subtract out the larger first harmonic voltage, the 1ω signal.

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Acknowledgments

First of all I would like to thank my advisor Janet Tate for helping with my project. She was there to help me learn how to work and behave in a lab research setting. She was always willing to help point me in the right direction and answer any questions I had. I would also like the thank Mark Warner for taking the time and educating me about the theory and the apparatus used for making the 3ω measurements. Without his help I would not have the circuit I used to measure the 3ω signal of the lightbulb. I want to thank Kai Zhan for taking time out of his extremely busy schedule to help me better understand the equipment I had to use. He showed me the programs I would use and what I should be looking for in my data to see if I was on the right track. Lastly I would like to thank the other people in Tates lab who helped me in multiple way throughout the project

Chapter 1- Introduction

1.1 The 3ω omega signal

The 3 harmonic or 3ω omega signal is generated as a response to a sinusoidal voltage being applied to a resistive material. We are interested in looking at the 3ω signal as it contains information about the thermal conductivity of the material.

The thermal conductivity of materials has become an important area of study due to the increasing power and decreasing size of today's electronics. As smaller and more powerful electronics are created, new ways of keeping them cool need to be found. One way this is accomplished is by using materials with high thermal conductivity so it can readily dissipate the heat generated.

1.2 How the 3ω signal is generated

An electrical current with magnitude I_0 and frequency ω is passed through a material. The current is given by

$$I = I_0 \cos(\omega t) \tag{1}$$

As the current travels through the material it dissipates power P due to joule heating.

$$P = I^2 R \tag{2}$$

Eq. 2 is the formula for joule heating where R is the resistance of the material and I is the applied current. It can be shown that the power is oscillating by entering Eq.1 into Eq.2.

$$\begin{aligned} P &= (I_0 \cos(\omega t))^2 R_0 \\ &= \tfrac{1}{2} I_0 R_0 + I_0 \mathrm{R} \cos(2\omega t) \end{aligned} \tag{3}$$

Eq.3 shows the power dissipated has a constant component and a component that is oscillating at 2ω.The power dissipated will cause the temperature of the resistor to oscillate. The value of the resistance R changes as a function of temperature T and is given by the equation

$$R = R_0[1 + \alpha(T - T_0)] \tag{4}$$

where R_0 is the initial value of the resistance, T_0 is the initial temperature, and α is the temperature coefficient of resistivity. Eq. 4 can also be written in a form with ΔR and ΔT as the change in the resistance and the temperature.

$$\alpha = \frac{\Delta R}{R_0 \Delta T} \tag{5}$$

Eq. 5 is useful because α can easily be calculated from the slope of a resistance vs. temperature graph. From Eq.4 it can be seen that as the temperature oscillates the resistance of the material will also oscillate. The temperature oscillations will change depending on the thermal mass of the material. A material with a low thermal mass will have larger oscillations in temperature then a material with a high thermal mass. This means the resistance of a material with low thermal mass will have larger resistance fluctuations than a material with a high thermal mass. The thermal mass also causes the oscillations in temperature to lag in time behind the heating. This produces a phase difference between the phase of the applied current and the phase of the resistance oscillations [1].

The voltage across the material is given by

$$V = IR \tag{6}$$

In Eq.6 I has a frequency of ω and R has a frequency of 2ω, thus giving the voltage V three different frequency components of 1ω, 2ω, and 3ω. The 1ω component is larger than the 2ω and 3ω components. The ratio of the magnitudes between the 1ω and the 3ω signals depend on the system. For a small lightbulb the 1ω signal is 5 times larger than the 3ω signal and for thin films the 1ω signal can be 1000 to 10000 times larger than the 3ω signal. To view the 3ω component it helps to subtract out the larger 1ω component. The 2ω frequency component does not need to be subtracted out as it is roughly the same size as the 3ω component. The 1ω component is subtracted out by adding a matching resistor in series with the lightbulb. Setting the matching resistor to the same resistance as the lightbulb gives a 1ω frequency across the resistor. The resistor will not have a 3ω component due to its larger thermal mass, meaning the temperature of the resistor is harder to change. The 3ω and 2ω components are generated by the oscillations in the resistance of the material. Since the temperature of the resistor will not fluctuate, the resistance of the resistor will not change.

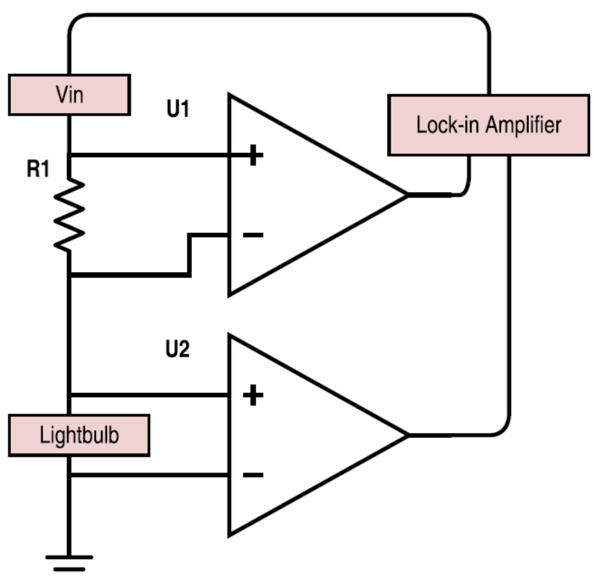

Figure 1: Experimental setup for small lightbulb. The lock-in amplifier is used to generate a sinusoidal 5V signal Vin. R1 is the matching resistor. U1 and U2 are the op-amps used to measure the voltage across the matching resistor and the lightbulb.

In Fig.1 the lock-in amplifier subtracts the voltage across the matching resistor from the voltage across the lightbulb. This leaves the 3ω and 2ω components and a much smaller 1ω component. Now the Lock-in amplifier can measure the 3ω component [2]. However, the lock-in amplifier should be able to read the 3ω component without having to subtract the 1ω component. The 1ω component is subtracted so the lock-in amplifiers can more accurately measure the 3ω component.

1.3 The Lock-In Amplifier

The lock in amplifier measures the 3ω signal by using the orthogonality of sinusoidal functions. When sinusoidal functions of different frequency are multiplied, and integrated for one period of the longest cycle, they give zero. If the frequencies are equal then the result is half of the product of the amplitudes of each sinusoidal function. In this experiment, the lock-in amplifier is set to measure just the 3ω signal meaning it will output only the portions of the voltage that have the 3ω frequency. The Lock-in Amplifier repeats this process with a 90 degree phase shift to give both the in-phase and out-of-phase components. The in-phase component is given by the following equation

$$X = V_{sig}\cos(\theta) \tag{7}$$

where X is the in-phase component, V_{sig} is the amplitude of the output voltage, and θ is the difference between the reference signal and the output signal. The out-of-phase component is similar and is given by the equation

$$Y = V_{sig}\sin(\theta) \tag{8}$$

where Y is the out-of-phase component. Using Eq.4 and Eq.5 it is possible to solve for V_{sig} and θ

$$V_{sig} = \sqrt{X^2 + Y^2} \tag{9}$$

$$\tan(\theta) = \frac{Y}{X} \tag{10}$$

1.4 RLC series circuit

An RLC series circuit is composed of a resistor, inductor, and capacitor in series. This circuit is a harmonic oscillator for the current. As a harmonic oscillator it has a resonate frequency that is given by the equation

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{11}$$

Eq.11 gives the resonance frequency in radians/sec. The circuit acts as a harmonic oscillator because the inductor and capacitor both have complex impedance. The impedance is made of a real component R and an imaginary component called reactance. The reactance of an inductor given by the equation

$$X_L = \omega L \tag{12}$$

The reactance of a capacitor is inversely proportional to ω and is given by

$$X_C = \frac{1}{\omega C} \tag{13}$$

The total impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{14}$$

Eq.14 gives the complex resistance for the RLC series circuit. Because Z is the complex resistance of the circuit, the current flowing through the circuit can be found using the equation

$$\begin{aligned} I &= \frac{V}{Z} \\ &= \frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)}} \end{aligned} \tag{15}$$

Eq. 15 is ohms law with a complex resistance. The phase of the current is found by using a phasor diagram of the impedances. Doing this yields the following equation

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \\ &= \tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right) \end{aligned} \tag{16}$$

Eq.15 and Eq.16 are used to theoretically predict the behavior of the current's magnitude and phase.

Chapter 2 – Methods

2.1 RLC Circuit Measurements

In order to measure the current amplitude and phase of the RLC circuit, a lock-in amplifier is used. The circuit is set up is shown in Fig.2

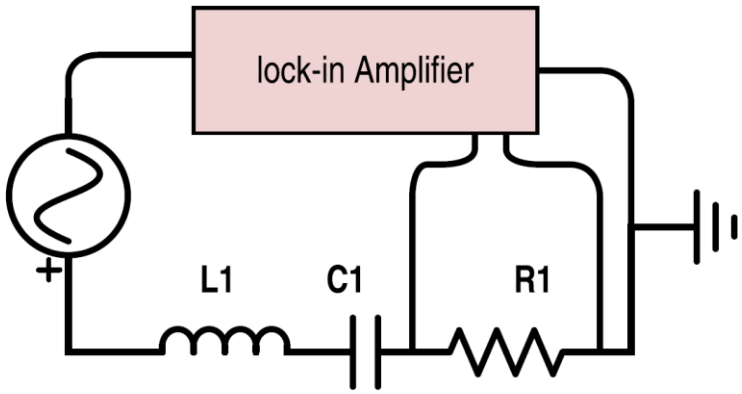

Figure 2: Diagram of the RLC circuit. L1 is the inductor, C1 is the capacitor, and R1 is the resistor. A lock-in amplifier is used to generate a sinusoidal voltage signal.

In Fig.2 the lock-in amplifier is used to both generate a sinusoidal wave signal and measure the voltage across the resistor. The lock-in amplifier set to output at the resonate frequency in Hz. As the frequency is changed from low to high values the in-phase and out-of-phase components of the voltage are measured. Using equations 7, 8, and 9 gives the voltage across the resistor V_R. Then using Ohm's law

$$I = \frac{V}{R} \tag{17}$$

gives the current through the resistor as a function of frequency. To get the phase of the current, Eq.10 is used.

$$\theta = \arctan\left(\frac{Y}{X}\right) \tag{18}$$

Eq.18 is the phase of the current through the resistor. The current and phase are then both plotted as a function of frequency.

2.2 Finding the 3ω Voltage

Using the circuit in Fig.1 the lock-in amplifier is set to generate a sinusoidal signal. The frequency and magnitude of the signal depend on the sample. For a small lightbulb a 5V signal at 1Hz works well. The waveform across the sample should be a sinewave. At this point it is not possible to see the 2ω and 3ω components. To measure the 3ω the lock-in amplifier needs to be properly set up. This is accomplished through the steps located in the appendix A. These steps tell the lock-in amplifier the frequency and the phase of the 3ω component so it knows what signal to measure. A LAB-view file is used to automate the measuring process. The program measures the in-phase and out-of-phase components of the 3ω signal at different frequencies of ω. For the lightbulb ω was varied from 0.1Hz to 10Hz. The in-phase and out-of-phase components of the 3ω signal are then plotted on the same graph as a function of frequency.

Chapter 3 – Results

3.1 RLC Current Calculations

In the RLC circuit, the resistor had a measured value of $R = 814\,\Omega$ and the capacitor had a measured value of $C = 1 * 105\,pF$. The given value for the inductor was $L = 111.6\,mH$. For the theoretical calculations the measured values for R and C were used. In order to fit the theoretical prediction to the experimental values, L was allowed to very. An 814Ω resistor was used because the input resistance of the lock-in amplifier is roughly 50Ω. If a small R value is chosen then the overall resistance of the circuit changes and becomes much larger than the value of the resistor.

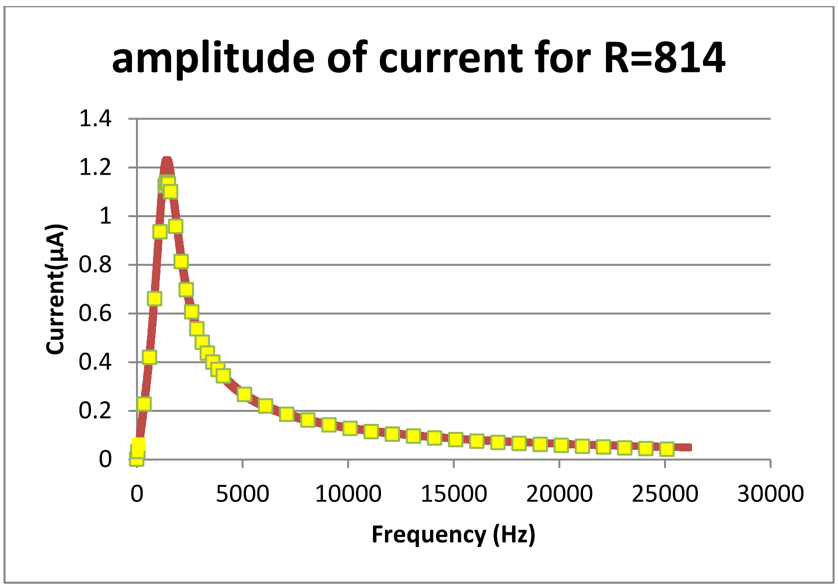

Figure 3: Current amplitude as a function of frequency. The red line represents the theoretical prediction and the yellow squares are the experimental values.

In Fig.3 the theoretical line was made using Eq.15 and $L = 125mH$. The steps in the frequency were 100Hz near the resonate frequency and changed to 1000Hz farther away from resonate frequency. The error for the given value of the inductor is 15%. This means the theoretical value for L is within reason. The peak in the graph is at the resonate frequency. Theory predicts a resonate frequency of 1506.6 Hz while the actual resonate frequency is 1438.6 Hz. The discrepancy between the two may have multiple causes. Frist the actual value of the inductor is unknown. This means the theoretical resonate frequency may not represent the circuit used. Second, the measured values of the in-phase and out-of-phase voltage components would change for a given frequency. After sweeping through a range of frequencies and then repeating the same sweep, a constant offset was measured. This offset was about 1mV to 5mV. The offset will cause the data to be shifted to the left or to the right thus changing the resonate frequency.

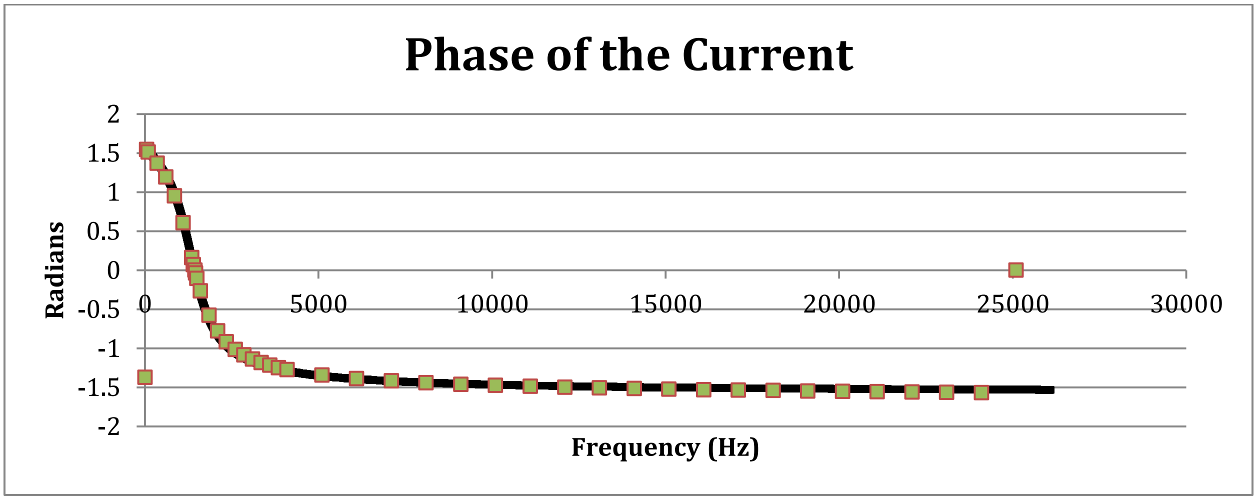

Figure 4: The phase of the current as a function of frequency. The black line represents the theoretical prediction and the green squares are the experimental values.

The theoretical line in Fig.4 was made using Eq.16. Figures 3 and 4 both go to high frequencies 25000Hz compared to the resonance frequency of1438.6Hz. This was done to show the sensitivity of the lock-in amplifier to detect very small differences between the in-phase and the out-of-phase voltages. Near the high and low ends of the frequency range the voltage components were 0.01mV to 3mV. The data point near 25000Hz does not lie on the theoretical data because the measured in-phase voltage at this point was zero. Thus when calculating the phase using Eq.18, it gave an error due to dividing by zero. I believe the program then treated the error as zero and plotted it as such. The problem seems to be due to the sensitivity setting of the lock-in amplifier. The in-phase voltage was smaller than the lock-in amplifier could measure so it measured it as zero, thus producing the error. If this experiment was repeated and at that point the sensitivity increased, a non-zero value for the in-phase voltage should be measured.

3.2 Lightbulb 3ω Voltage

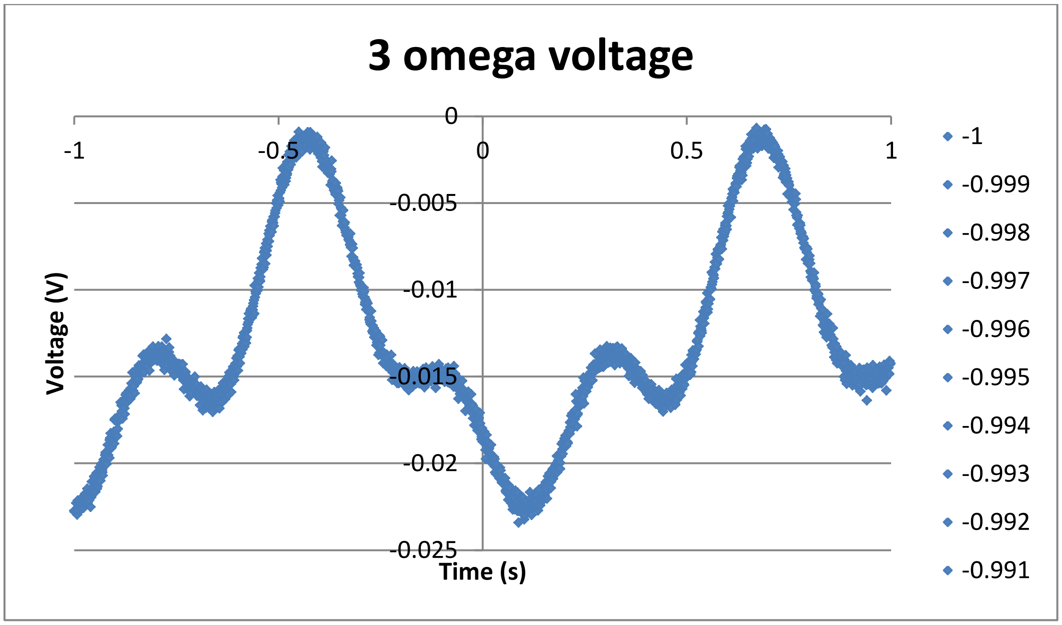

Figure 5: 3ω voltage signal for the small lightbulb. For this data the matching resistor was set to 22.6Ω

In Fig.5 an oscilloscope is used to show the voltage obtained by subtracting the voltage across the matching resistor from the voltage across the lightbulb. The voltage still contains the 3ω and 2ω components. In order to view just the 3ω signal the lock-in amplifier is used.
Using the lock-in amplifier rather than an oscilloscope will show the in-phase and out-of-phase voltage components.

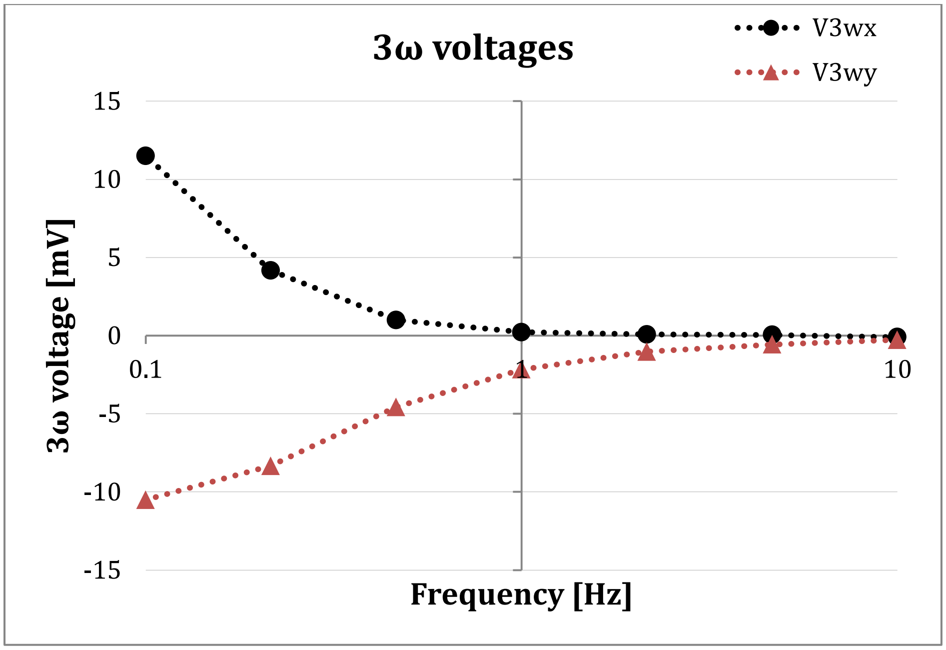

Figure 6: In-phase (V3wx) and the out-of-phase (V3wy) components of the 3ω voltage.

In Fig.6 it is unknown if the data is correct or not. This data was taken by switching the leads of the op amp measuring the voltage across the lightbulb. This means that the negative of the voltage across the lightbulb was measured. The reason for this is Fig.6 resembles the expected result for a thin film. But for a small lightbulb all of the approximations made for thin films no longer hold true. This means the data could change as a result. Thus it is unknown if our data is correct or incorrect and just happens to resemble a thin film result.

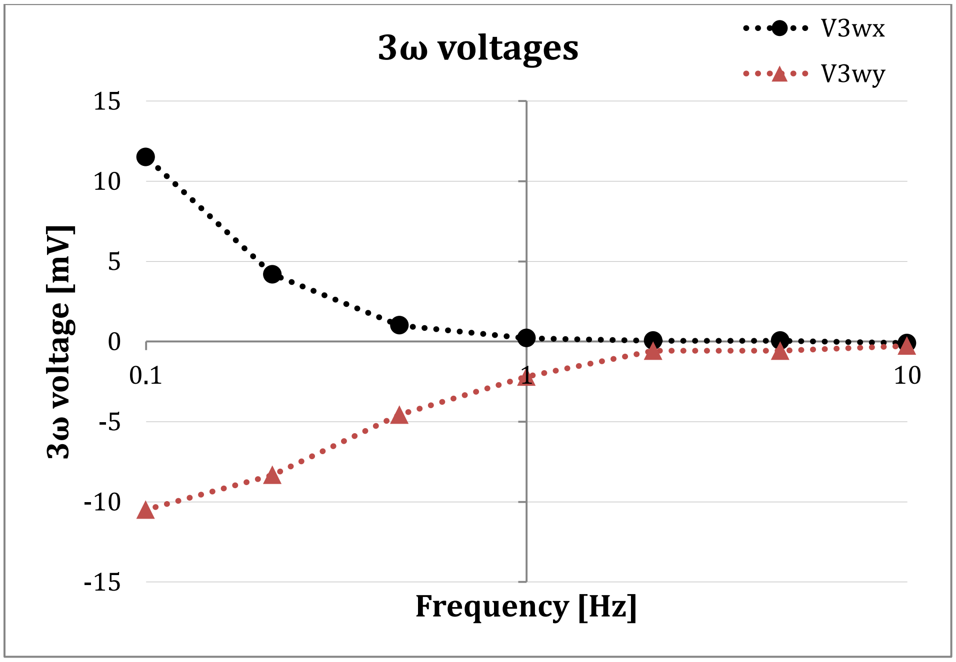

Figure 7: In-phase (V3wx) and the out-of-phase (V3wy) components of the 3ω voltage. For this data set the leads for both the lightbulb and matching resistor op-amps were switched.

Fig.7 shows a result that looks very similar to that of Fig.6. By switching the leads on both op amp for the lightbulb and matching resistor, a negative sign was introduced for both voltages. The leads of the lightbulbs op amp were switched so that a result similar to that of a thin film could be achieved. It is possible that the original set up was incorrect and that switching the leads of the op amps made it correct. But it is unknown if this is the case. As stated above for Fig.6, it is unknown if the results for a thin film can be applied to a small lightbulb.

Chapter 4 – Discussion

4.1 RLC Circuit

It was found that the lock-in amplifier does indeed have the capability to measure the in-phase and out-phase components of signals. On the lowest sensitivity accurate measurements up to 25 kHz were made. In the 25 kHz range the in-phase and out-phase voltages were just a few millivolts. The lock-in amplifier has the capability to measure microvolts but doing so changes the dynamic range. This means that the range of measurable values changes depending on what you want to measure. On the highest sensitivity the dynamic range is 100-1000 microvolts. On the lowest sensitivity setting the dynamic range is 0.01mV to 5V. The lowest sensitivity was used because the voltage was larger than 0.01mV near the resonate frequency.

4.2 Small Lightbulb

The data for figures 6 and 7 are very similar. The difference between the two measurements is Fig.7 had the leads of the op amp for the matching resistor switched whereas Fig.6 did not. Changing the leads of the matching resistor does not change the result; this is because the matching resistor is not needed. The lightbulb is where the 3ω signal originates. If the sign of the lightbulbs voltage is changed by switching the leads of the op amp that is measuring it, then the sign of the 3ω signal changes. The results seem to show that the matching resistor voltage is not needed to find the 3ω voltage in our lightbulb case. The lock-in amplifier can detect a very specific frequency from a mix of frequencies. The matching resistor is there just to diminish the 1ω voltage signal. But for this case it is not needed as the 3ω signal is rather large, only 5 times smaller than the 1ω signal.

Chapter 5 – Conclusion and Future Research

5.1 Conclusion

The RLC circuit is well-known and its properties are well documented. The data from the RLC agrees with the theoretical prediction. This shows we accurately and correctly used the lock-in amplifier to measure the in-phase and out-of-phase components of a voltage.

The 3ω data shows that the lightbulb generated a 3ω response but the matching resistor did not. This is because the lightbulb has a smaller thermal mass then the matching resistor with a much larger thermal mass. We tried, but failed, to measure the 3ω of thing films. It is currently unknown as to what was causing problems when trying to measure the thin films compared to the lightbulb. The thermal conductivity of the thin films was also investigated but without any 3ω measurements it was impossible to determine.

5.2 Future Research

Looking into the 3ω signal of thin films and seeing if the matching resistor is needed. The matching resistor will help increase the capabilities of the lock-in amplifier by diminishing the 1ω signal. But the lock-in amplifier should be powerful enough to measure the 3ω signal even if the 1ω signal is not subtracted out. Furthering this research would involve using the 3ω measurements to calculate the thermal conductivity of thin films.

Works Cited

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2.) Zhan, K, M.S. thesis, Dr. Tate lab at Oregon State University (Oct. 20, 2014), Thermal conductivity measurements of amorphous thin films on silicon. (Unpublished)

3.) Cahill, D. G. "Thermal Conductivity Measurement from 30 to 750 K: The 3ω Method." *Review of Scientific Instruments*. **61**, pages 802-808 (1990).
Erratum in: Rev. Sci. Instrum. **73**, 3701 (2002)

For more information on the 3ω signal

1) Kaul, P.B, Day, K.A, Abramson, A.R. "Application of the three omega method for the thermal conductivity measurement of polyaniline." *Journal of Applied Physics.* **101**, 083507 (2007)

Appendix A

1. The sample is properly connected to the measurement circuit.

2. In the “REF PHASE” menu on the lock-in amplifier, “Sine Output” is set to the input voltage for the measurement. This is typically 5 V.

3. In the “INPUT FILTERS” menu, “Coupling” is set to “DC.” AC coupling causes significant phase errors in the measurement below 100 Hz.

4. The resistance of the matching resistor bank is set to the maximum value below the sample resistance. The gain of amplifier B is then adjusted to cancel out the 1ω voltage all the way.

5. In the “REF PHASE” menu, “Harmonic” is set to “1.”

6. In the “INPUT FILTERS” menu, “Source” is set to “A.”

7. In the “GAIN TC” menu, “Sensitivity” is set to its minimum value (maximum full scale voltage).

8. In the “GAIN TC” menu, the “Time Constant” is set to an appropriate value. For measurements above 100 Hz, 3 s is sufficient. For lower frequency measurements, larger time constants must be used to achieve proper stabilization.39

9. The phase of the internal reference is “zeroed.” This is accomplished by pressing the “AUTO PHASE” button on the lock-in until the out-of-phase voltage (Y) is zero. This means that the phase of the driving voltage is defined as zero phase.

10. Because we really want to lock-on to the 3ω signal in time rather than in phase (a phase shift for a 1ω signal is not the same shift in time as for a 3ω signal), we must adjust the reference phase from its autophased value. In the “REF PHASE” menu, “Ref. Phase” is set to three times its displayed value. Typically, this value is small unless and external current source is used.

11. “Harmonic” is set to “3.” The lock-in is now properly locked-on to the 3ω signal.

12. “Source” is set to “A − B” if amplifier C is the differential input of the lock-in. Otherwise, this is left on “A.”

13. “Sensitivity” is set to its the maximum value such that the full scale voltage is still greater than the maximum voltage that will be measured.

14. In the “GAIN TC” menu, “Reserve” is set to an appropriate value. Typically the “MAX” setting works well.

15. In the “GAIN TC” menu, “ Filter” is set to “24 dB/oct.”. The synchronous filter does not seem to affect the measurement.

16. The system is now ready to take automated measurements.