Hydrodynamic forces on a cylinder oscillating horizontally underwater with its axis perpendicular to the motion were measured at the Oregon State University Wave Research Facility. The cylinder had an outer diameter of 12 inches and a length of 12 feet. For the data presented herein, the maximum displacement of the motion of the cylinder ranged from 20 feet to 5 feet. The range of proximity of a nearby plane boundary, e/D, was from 0.083 to 6.0. The so-called "drag", and "added mass" coefficients are calculated first by the "maximum value" method, and then by the "least square" method as described herein. The detail and development of the equations for the least-square method are also shown. For the majority of the test runs, the force coefficients obtained by the use of the least-square method are higher than those obtained by the maximum value method. The
drag and added mass coefficients are found to be Reynolds number dependent. There is good agreement on the result of the drag coefficient with other investigators' results; but there is some deviation on the results of the added mass coefficients. The presence of a near-by plane boundary intensifies the wake effect and the potential flow effect and results in higher values of drag coefficients and added mass coefficients. As the amplitude of the cylinder motion decreases (from 20 feet to 5 feet), the value of the force coefficients increase.
Large Displacement, High Reynolds Number
Oscillating Cylinder

by

Yau Wai Chan

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NOMENCLATURE

\[ A/D = \text{Wake parameter; ratio of maximum horizontal displacement to cylinder diameter} \]

\[ C_d = \text{Drag coefficient} \]

\[ C_I = \text{Inertia coefficient} \]

\[ C_L = \text{Lift coefficient} \]

\[ C_m = \text{Added mass coefficient} \]

\[ C_{\text{max}} = \text{Maximum-force coefficient} \]

\[ D = \text{Diameter of the cylinder} \]

\[ e/D = \text{Ratio of distance between plane boundary and cylinder surface to the cylinder diameter} \]

\[ F_a = \text{Activating force needed to accelerate the body in fluid} \]

\[ F_d = \text{Drag force, or the in-line force due to the wake effect} \]

\[ F_f = \text{Fluid dynamic force} \]

\[ F_H = \text{In-line force} \]

\[ F_{\text{Im}} = \text{Measured maximum in-line force} \]

\[ F_L = \text{Theoretical transverse force} \]

\[ F_{\text{Im}} = \text{Measured maximum transverse force} \]

\[ F_m = \text{Added mass force, or the in-line force due to the potential flow effect} \]

\[ F_{\text{max}} = \text{Maximum in-line force experienced by the cylinder} \]

\[ F_V = \text{Measured transverse force} \]

\[ g = \text{Acceleration due to the gravitational force} \]
L = Length of the cylinder
M = The total mass of the test section of the cylinder and the water inside the test section
m = Mass of the body
n = Unit normal vector
P = Fluid pressure around the body
Re = Reynolds number
T = Period of the motion of the cylinder
t = Time
U = Velocity of the cylinder
\dot{U} = Acceleration of the cylinder
\gamma = Volume of the body
U_{\text{max}} = Maximum velocity of the cylinder
\xi = Error between the measured and the predicted value
\Gamma = Circulation around the body
\phi = Phase-lag between the in-line force and the velocity of the cylinder
\rho = Mass density of water
\theta = Angular displacement of the cylinder
\varphi = Angle in radians measured counter-clockwise from the x-axis
I. INTRODUCTION

Increased demand for off-shore structure leads to increased demand for reliable information on structural design. The circular cylinder, with its simple geometrical configuration—symmetrical about its axis, regardless of the flow direction—is a very common structural member, and is particularly used for submerged pipelines. The flow conditions around a cylinder are not as simple as its geometrical configuration.

However, hydrodynamic forces on a submerged cylinder can be predicted by experimental and analytical methods. Experimental methods are necessary when the wake effect is significant. Basically, measurements can be of two types; controlled laboratory measurements and field measurements. Laboratory experiments make it possible to control the variables carefully and to study each variable individually. The controlled laboratory experiment is able to cover a wide and continuous range of conditions and, when properly coordinated with a field experiment, can be ideal in generating a desired model solution.

This thesis describes an experiment which is concerned mainly with the hydrodynamic forces on an oscillating cylin-
der at high Reynolds number and near a plane boundary. The experiment was conducted at the Oregon State University wave Research Facility. A horizontal cylinder was forced to oscillate underwater with its axis perpendicular to the direction of motion (16). The frequency and amplitude of the cylinder were varied individually, so as to study the effects of Reynolds number and the Keulegan-Carpenter period parameter, respectively. Furthermore, the effect of the proximity of a near-by plane boundary was also measured.

The experimental data were recorded on a visicorder and also on digital magnetic tape. The maximum value method, which assumes zero phase-lag between the wake effect and the velocity of the cylinder was used to evaluate the data stored on the visicorder (16). The data on the digital magnetic tape was evaluated by the least-square method (2,13). The least-square method is concerned with the minimization of the square of the total error between the predicted forces and the measured forces. The maximum value method is much simpler to use; but the use of the values of the so-called "drag" and "added mass" coefficients determined by the maximum value method may not be as safe as the use of the values determined by the least-square method.

There are few well-developed theories which predict the wave forces on a submerged cylinder near a plane boundary. Much research has been done in the past to investigate this problem by experimental methods, in the field (3,4), or in
controlled laboratory environments (2, 13, 16). It is hoped that this thesis will contribute to this complex problem.

The emphasis of this thesis is:

1) To present data for the high Reynolds number oscillatory flow;

2) To compare the least-square values of the coefficients for the drag, added mass and lift, against those obtained by the maximum value method that was published earlier (16) and

3) To examine the influence of a near-by plane boundary on the derived coefficients.
II  THEORETICAL DEVELOPMENTS

Hydrodynamic forces on a submerged cylinder in steady flow

According to potential flow theory, a submerged cylinder moving through a still infinite fluid medium of constant density with constant velocity will experience no resistance to motion (11); there will be no boundary layer and no wake will be formed. If the flow is unbounded except for the boundary of the submerged cylinder, there would be no circulation around the cylinder and, no net force would exist perpendicular to the flow. However, the viscosity effect of the fluid changes the flow pattern of the fluid around the cylinder; the fluid separates from the cylinder and a wake is formed. The stagnation pressure on the leading surface of the cylinder, and the low intensity of pressure in the wake region produces a resultant force which opposes the motion of the cylinder. This resultant force is called the drag force, or the wake force.

The proximity of a near-by plane boundary will also alter the flow condition for both the potential flow and wake flow. The potential flow effects from the boundary, as well as the unsymmetrical vortex development will induce transverse forces on the cylinder. This transverse force is sometimes called the lift force. Transverse forces can also result from vortex shedding in an infinite fluid medium.
Hydrodynamic forces on a submerged oscillating cylinder

Fluid forces on a submerged cylinder in incompressible oscillatory flow can sometimes be simulated by forcing the cylinder to oscillate in still fluid. The fluid mechanics of a body accelerating through the still fluid is kinematically the same, relative to the body, as the fluid accelerating past a fixed body with uniform flow, as long as the fluid stays incompressible. The only difference in the force exerted on the body results from the pressure gradient from the fluid accelerating past the fixed body. The pressure gradient force is equal to the product of the displaced fluid mass and the fluid acceleration.

For a body accelerating through a still fluid, according to Newton's second law of motion,

\[ \bar{F} = m\overbrace{d\bar{U}}^{U} \]  

(1)

where \( \bar{F} \) is the total force required to keep the body in motion, \( m \) is the mass of the body, and \( \overbrace{U}^{\bar{U}} \) is the velocity of the body. The overbar indicates vector notation.

The term on the left consists of two kinds of forces: the activating forces needed to accelerate the body through the fluid, \( \bar{F}_a \); and the fluid dynamic forces, \( \bar{F}_f \), exerted on the surface of the body. Thus, equation (1) can be rewritt-
en as:

$$\mathbf{F}_a + \mathbf{F}_f = m\frac{d\mathbf{U}}{dt}$$  \hspace{1cm} (2)

The term $\mathbf{F}_f$ opposes $\mathbf{F}_a$ because the hydrodynamic forces generally act in the opposite direction to the acceleration as shown in Figure 1.

When the surface shear stresses are negligible, the fluid dynamic forces can be represented by the equation:

$$\mathbf{F}_f = -\int P\mathbf{n}dS$$  \hspace{1cm} (3)

where $P$ is the fluid pressure exerted on the surface of the body, and $\mathbf{n}$ is the unit normal vector, directed away from the surface. In the case of a cylinder, one can express the fluid dynamic force in the $x$-direction in cylindrical coordinates as:

$$F_{fx} = -\int_0^{2\pi} \int_0^D P\cos\varphi\,d\varphi\,dz$$  \hspace{1cm} (4)

where $\varphi$ is the angle measured counter-clockwise from the $x$-axis, and $D$ is the diameter of the cylinder, as shown in Figure 2.

For potential flow, one can solve for the pressure distribution that constitutes $F_f$ (6). The resultant force for a body accelerating in a still fluid is:

$$F_f = C_m \frac{\mathbf{U}}{\mathbf{U}}$$  \hspace{1cm} (5)

where $V$ is the volume occupied by the body, $C_m$ is the added
Fig. 1 Forces on a cylinder

Fig. 2 Coordinate system of the cylinder
mass coefficient, and \( \gamma \) is the density of the fluid.

In the case of a cylinder, \( C_m \) has a value of 1.0; but for a body with a different shape, \( C_m \) might have a different value. For a sphere, the value of \( C_m \) is 0.5.

Thus, the potential flow force on a single cylinder accelerating through an otherwise still fluid infinite in extent, is

\[
F_I = C_m \frac{\gamma D^2}{4} \frac{d\bar{U}}{dt}
\]  

(6)

where \( L \) is the length of the cylinder.

When the fluid accelerates past a fixed body, the added mass coefficient in equation (6) is replaced by an inertia coefficient which is defined as \( C_i = 1 + C_m \), where the 1 accounts for the pressure gradient.

Replacing the fluid dynamic force term in equation (2) with equation (6), one will get:

\[
\bar{F}_a = \gamma C_m \frac{d\bar{U}}{dt} + m \frac{d\bar{U}}{dt}
\]  

or,

\[
\bar{F}_a = (\gamma C_m \gamma + m) \frac{d\bar{U}}{dt}
\]  

(8)

The term \((\gamma C_m \gamma + m)\) is sometimes called the virtual mass as it is this apparent "total mass" that the activating force will encounter.

As pointed out by Yamamoto et. al. (15), if the ratio of the amplitude of the water particle displacement to the
cylinder diameter is larger than 1.5, the boundary layer will separate from the cylinder surface and a significant wake is formed. The wake due to vortex shedding alters the flow field from predominantly potential flow to predominantly wake flow and results in an in-line force opposing the motion of the cylinder. For a pronounced wake condition, potential flow theory can no longer completely predict the hydrodynamic forces on the cylinder, and the drag and added mass coefficients must be determined experimentally.

The force on a cylinder is due to the sum of the wake effect and the potential flow effect. Morison et al. were the first to derive their widely used equation to predict the forces on a cylinder for design engineers. The in-line hydrodynamic force on an oscillating horizontal cylinder is thus given as:

\[
F_H = F_m + F_d \quad (9)
\]

or,

\[
F_H = C_m \rho \pi D^2 \frac{dU}{dt} + \frac{1}{2} C_d \rho U |U| \quad (10)
\]

where \( F_d \) is the force from the wake effect, \( F_m \) is the force from the potential flow effects, and \( C_d \) is the drag coefficient.

Part of the wake effect is from the vortex formation behind the cylinder which induces a force perpendicular to the direction of the ambient flow. That which is due to
vortex shedding may be given by the Kutta-Joukowski theorem (7) as:

\[ F_L = \oint \Gamma UL \]  \hspace{1cm} (11)

where \( F_L \) is the transverse force, and \( \Gamma \) is the circulation around the cylinder.

Transverse forces can also be caused by potential flow effects due to the proximity of a near-by boundary (9,15) or from unsymmetrical wake formation, also due to the near-by boundary (9). Usually, the lift coefficient is given as:

\[ C_L = \frac{F_L}{\frac{1}{2} \rho U^2 D_L} \]  \hspace{1cm} (12)

where \( C_L \) is the lift coefficient.

**Least Square Method**

Other researchers have applied the least square method to the Morison equation to solve for the drag, and added mass (or inertia) coefficients (2,13). It is intended to show here the development of the equations for the least-square method.

Usually the acceleration, and the horizontal and vertical forces of the test body are measured separately (2,13, 16). The measured total horizontal hydrodynamic force is the sum of the wake effect and the potential flow effect; and the error, \( \varepsilon \), between the measurements and the theory,

\[ F_H = F_d + F_m + \varepsilon \]  \hspace{1cm} (13)
The least-square method for determining the force coefficients consists of the minimization of the square of the error between the measured and the predicted forces for at least one complete cycle of motion. Rearranging the previous equation and squaring both sides, one would obtain:

$$
\varepsilon^2 = (F_H - F_d - F_m)^2
$$

(14)

or

$$
\varepsilon^2 = F_H^2 + F_d^2 + F_m^2 - 2F_H F_m - 2F_H F_d + 2F_m F_d
$$

(15)

One can obtain a total squared error by integrating equation (15) with respect to time, over one complete cycle of the motion of the cylinder. The equation becomes:

$$
\int_0^T \varepsilon^2 dt = \int_0^T F_H^2 dt + \int_0^T F_d^2 dt + \int_0^T F_m^2 dt - 2 \int_0^T F_H F_m dt - 2 \int_0^T F_H F_d dt + 2 \int_0^T F_d F_m dt
$$

(16)

This equation is a function of $C_d$ and $C_m$:

$$
\int_0^T \varepsilon^2 dt = f(C_d, C_m)
$$

(17)

Taking the derivative of equation (16) with respect to $C_d$, and setting the result to zero, one will minimize the square of the total error between the predicted and the measured forces:

$$
\frac{d}{dC_d} f(C_d, C_m) = \frac{d}{dC_d} \int_0^T \varepsilon^2 dt = 0
$$

(18)
Substituting Equations 10 and 16 into Equation 18:

\[
\frac{d}{dC_o} \int_0^T \left( \frac{\dot{C}_1}{t} \right)^2 dt = \frac{1}{2} \rho \frac{D^2 L^2}{\pi^2} \int_0^T \left( \frac{\dot{C}_m}{t} \right)^2 dt - \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \dot{U} \cdot \ddot{U} dt + \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \dot{U} \cdot \ddot{U} dt
\]

\[
\frac{d}{dC_o} \int_0^T \left( \frac{\dot{C}_1}{t} \right)^2 dt = 0
\]  

(19)

Using the same procedure but taking the derivative with respect to \(C_m\), one obtains:

\[
\frac{d}{dC_m} \int_0^T \left( \frac{\dot{C}_m}{t} \right)^2 dt = \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \left( \frac{\dot{C}_m}{t} \right)^2 dt - \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \dot{U} \cdot \ddot{U} dt
\]

\[
+ \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \dot{U} \cdot \ddot{U} dt
\]

\[
= 0
\]  

(20)

Thus, one obtains two simultaneous equations with \(C_d\) and \(C_m\) as the two unknown variables;

\[
\begin{bmatrix}
\frac{1}{2} \rho D^2 L \int_0^T U^2 dt & \frac{1}{2} \frac{D^2 L}{\rho} \int_0^T \dot{U} \cdot \ddot{U} dt
\end{bmatrix}
\begin{bmatrix}
C_d
\end{bmatrix}
= \begin{bmatrix}
\int_0^T \dot{U} \cdot \ddot{U} dt
\end{bmatrix}
\]

(21)

The instantaneous value of the velocity of the body can be obtained by integrating the acceleration of the body with respect to time:

\[
U = \int_0^T \dot{U} dt
\]

(22)

where \(U\) is zero at \(t=0\). Finally, one solves for \(C_d\) and \(C_m\);

\[
C_d = \frac{2 \int_0^T F_H U |U| dt \int_0^T \dot{U} \cdot \ddot{U} dt - 2 \left| \int_0^T U |U| \dot{U} dt \right| F_H \dot{U} dt}{\rho D L \left[ \int_0^T U^2 dt \right] \left( \int_0^T \dot{U}^2 dt \right) - \left( \int_0^T U |U| \dot{U} dt \right)^2}
\]

(23)
and
\[ C_m = \frac{\int_0^T F_H U dt \int_0^T U^4 dt - \int_0^T F_H U|U| dt \int_0^T U|U| U dt}{\frac{4}{3} \rho D^2 L^2 \int_0^T U^2 dt \int_0^T U^4 dt - (\int_0^T U|U| U dt)^2} \] (24)

Setting up the equations the same way as before but using the transverse force measurements of the cylinder:
\[ \xi = F_V - F_L \] (25)
where \( F_V \) is the measured transverse force and \( F_L \) is the theoretical transverse force. Thus, one can solve for the lift coefficient;
\[ \xi^2 = F_V^2 + F_L^2 - 2F_V F_L \] (26)
\[ \int_0^T \xi^2 dt = f(C_L) = \int_0^T F_V^2 dt + \int_0^T F_L^2 dt - 2 \int_0^T F_V F_L dt \]
\[ = \frac{1}{2} C_L \rho D^2 L^2 \int_0^T U^4 dt - \frac{1}{2} C_L \rho D L \int_0^T F_V U|U| dt \] (27)
\[ \frac{d}{dC_L} f(C_L) = \frac{1}{2} C_L \rho D^2 L^2 \int_0^T U^4 dt - \rho D L \int_0^T F_V U|U| dt = 0 \] (28)
\[ C_L = \frac{2 \int_0^T F_V U^2 dt}{\int_0^T F_V U|U| dt} \] (29)

**Maximum Value Method**

The maximum value method applied to the oscillating cylinder experiment utilizes the following assumptions: the drag force is zero when the velocity of the cylinder is
zero and it is maximum when the velocity of the cylinder is maximum. At that time, the acceleration of the cylinder is zero, and the added mass force is maximum when the velocity of the cylinder is zero. Thus, for harmonic motion, it is assumed that the drag force is maximum when the added mass force is zero and the drag force is zero when the added mass force is maximum.

The maximum value method is simple to use. One needs to know only the maximum peak value of the velocity and acceleration of the cylinder, and the corresponding horizontal and transverse forces of the cylinder at that instant. The coefficients are given as:

\[ C_d = \frac{F_H(t_1)}{\frac{1}{2} \rho D U_{\max}^2} \]  \hspace{1cm} (30)

\[ C_m = \frac{F_H(t_2)}{\frac{1}{2} \rho D U_{\max}^2} \]  \hspace{1cm} (31)

where \( t_1 \) is the time at which the velocity of the cylinder is at the maximum, \( U_{\max} \); and \( t_2 \) is the time at which the acceleration of the cylinder is at its maximum, \( U_{\max} \). The lift coefficient, \( C_L \), is defined from the maximum transverse force of the cylinder as:

\[ C_L = \frac{F_{Lm}}{\frac{1}{2} \rho D U_{\max}^2} \]  \hspace{1cm} (32)

where \( F_{Lm} \) is the maximum transverse force.
As mentioned before, the transverse force depends on the circulation around the cylinder from the vortex shedding, and the complicated interaction between the potential flow effect and wake asymmetry when the fluid flow is influenced by a plane boundary near the oscillating cylinder. The maximum transverse force does not necessarily occur at the instant when the horizontal velocity of the cylinder is maximum. There may be a small phase shift between the transverse force and the horizontal velocity of the cylinder.

The added mass force is always in phase with the acceleration of the cylinder. Thus, when the acceleration of the cylinder is zero, the added mass force is also zero. At this instant, the velocity of the cylinder will be maximum for harmonic motion; there will be only drag force and no temporal inertia component acting on the cylinder.

There is a phase-lag between the in-line wake force and the horizontal velocity of the cylinder. Thus, when the potential flow effect is at its maximum at the maximum acceleration of the cylinder, the wake component still possesses some value, instead of zero as the theory assumes. Though it is possible, there is no easy way to separate the drag component from the inertia component (10). This phase-lag may cause error in the evaluation of the added mass coefficient.

**The Maximum Force Coefficient Method**

In most experiments, the required force coefficients
are evaluated from the Morison equation. The Morison equation predicts the force, as a function of time, due to the wake effect and the potential flow effect. For a wake effect predominant case, the force, as a function of time, predicted by the Morison equation is a function of the square of the ambient velocity. For a potential flow effect predominant case, the force, as a function of time, predicted by the Morison equation is a function of the acceleration of the ambient. In general, the maximum hydrodynamic forces on the submerged cylinder, $F_{\text{Hm}}$, is a vector sum of both the wake effect and the potential flow effect as shown in Figure 3. This maximum hydrodynamic force does not occur at the instant of maximum velocity of the cylinder or maximum acceleration of the cylinder. In most cases, the maximum hydrodynamic forces experienced by the cylinder is considerably higher than either the wake force or the potential flow force alone.

Grace, et.al., tried to predict the maximum hydrodynamic force on the submerged cylinder with a different approach (4). Instead of separating the wake effect and the potential flow effect and evaluating each coefficient individually, Grace, et.al., used a different equation to evaluate the force coefficient:

$$C_{\text{max}} = \frac{F_{\text{Hm}}}{\frac{1}{2} \rho D U_{\text{max}} |U_{\text{max}}|}$$  (33)
Fig. 3 In-line forces on an oscillating cylinder under the assumption of Morrison equation
where $C_{\text{max}}$ is the maximum-force coefficient and $F_{\text{Hm}}$ is the maximum in-line hydrodynamic force experienced by the cylinder as shown in Figure 4. This equation will not separate the wake effect and the potential flow effect, but it will predict the maximum hydrodynamic force experienced by the cylinder. Equation (33) gives only the value for the maximum force on the cylinder. It does not say when the maximum force will occur and what kind of phase-relation the maximum force has, with respect to the ambient flow. In general, there is a phase-lag between the maximum in-line hydrodynamic force and the maximum ambient velocity, except when the force is due to the potential flow effect alone. Although the theoretical force predicted by Equation (33) can be periodic, it can never be harmonic; even when the wake effect may be negligible.

**Comparison Between the Three Methods**

For a sinusoidal motion, where

$$U(t) = U_{\text{max}} \cos \theta$$  \hspace{1cm} (34)

Equations (23) and (24) can be greatly simplified (12):

$$C_d = \frac{\int_0^{2\pi} \cos \theta d\theta}{\frac{3}{2} \pi D L U_{\text{max}}^2}$$  \hspace{1cm} (35)
Cylinder displacement, $X(t)$

In-line force $F_H(t)$, for fully wake dependent conditions

Fig. 4  In-line force on an oscillating cylinder under Grace's assumption
\[ C_m = \frac{2\pi U_{\text{max}}}{\pi D L} \int_{0}^{2\pi} F_H \sin \theta \, d\theta \]  \hspace{1cm} (36)

where \( \theta = \frac{2\pi t}{T} \) \hspace{1cm} (37)

and \( T \) is the period of motion of the cylinder.

For a wake effect predominant case, the in-line force acting on the cylinder can be approximated by:

\[ F_H(\theta) = F_{Hm} \cos(\theta - \phi) \cos(\theta - \phi) \] \hspace{1cm} (38)

where \( \phi \) is the phase-lag between the in-line forces and the ambient velocity and it is a constant with respect to time; \( F_{Hm} \) is the maximum in-line forces (Fig.3). Thus, Equation (35) can be rewritten as:

\[ C_d = \frac{-32F_{Hm} \int_{0}^{\pi} \cos^2(\theta - \phi)\cos\theta|\cos\theta| \, d\theta}{3\pi D L U_{\text{max}}^2} \] \hspace{1cm} (39)

Utilizing the addition rule for trigonometric functions, Equation (39) becomes:

\[ C_d = \frac{-32F_{Hm} \int_{0}^{\pi} (\cos\theta \cos\phi + \sin\theta \sin\phi)^2 \cos\theta|\cos\theta| \, d\theta}{3\pi D L U_{\text{max}}^2} \] \hspace{1cm} (40)

The value of drag coefficient thus evaluated will be a function of \( \phi \), the phase-lag. For the wake effect predominant case and assuming there is no phase-lag, Equation (40)
can be simplified to:

\[ C_d = \frac{-32F_{Hm}}{3\pi^2DLU^2_{\text{max}}} \int_0^{\pi} \cos^2 \theta \cos \theta | \cos \theta | d\theta \] (41)

Integrated over one complete cycle of motion of the cylinder, Equation (41) becomes:

\[ C_d = \frac{-32F_{Hm}}{3\pi^2DLU^2_{\text{max}}} \left[ \left( \frac{3\theta}{8} + \frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{32} \right) \right]_0^{\frac{\pi}{2}} \] (42)

Thus, the drag coefficient can be resolved as:

\[ C_d = \frac{-1.0F_{Hm}}{\frac{3\pi}{2}DLU^2_{\text{max}}} \] (43)

The negative sign is to indicate that the wake effect is acting in the opposite direction to the velocity of the cylinder. Comparing Equation (43) to Equation (30), one will find that both equations give the same result. Thus, for the wake effect predominant case, and assuming no phase lag and the motion of the cylinder is harmonic, the value of \( C_d \) evaluated by the least-square method is the same as that evaluated by the maximum value method.

It was shown in reference (10), that the phase-lag, \( \phi \), might be as large as \( \frac{\pi}{4} \) for some cases. Evaluating \( C_d \) for such a case with Equation (40):
\[
C_d = \frac{-32F_{Hm} \int_0^{\pi/2} \left( \cos^4 \phi \cos^2 \theta + \sin^2 \phi \cos^2 \theta \sin^2 \phi + 2 \cos^3 \theta \sin \theta \cos \phi \sin \phi \right) d\theta}{3 \pi^2 DL U_{\text{max}}^2}
\]

\[
= \frac{-32F_{Hm}}{3 \pi^2 DL U_{\text{max}}^2} \left[ \left( \frac{3 \theta}{4} + \frac{\sin 4 \theta}{32} \right) \cos^2 \phi + \left( \frac{\theta}{8} - \frac{\sin 4 \theta}{32} \right) \sin^2 \phi - \left( \frac{2 \cos 4 \theta}{4} \right) \cos \phi \sin \phi \right]
\]

\[
C_d = \frac{-32F_{Hm}}{3 \pi^2 DL U_{\text{max}}^2} \left( \frac{3 \pi}{16} \cos^2 \phi + \frac{\pi}{16} \sin^2 \phi + \frac{1}{2} \cos \phi \sin \phi \right) \quad (44)
\]

Taking \( \phi \) equal to \( \frac{\pi}{4} \), Equation (43) becomes:

\[
C_d = \frac{-32F_{Hm}}{3 \pi^2 DL U_{\text{max}}^2} \left( \frac{3 \pi}{32} + \frac{\pi}{32} + \frac{1}{4} \right)
\]

\[
C_d = \frac{-4.091F_{Hm}}{\frac{1}{2} \pi^2 DL U_{\text{max}}^2} \left( \cos^2 \text{ function, } \phi = \frac{\pi}{4} \right) \quad (47)
\]

Thus, the value of \( C_d \) evaluated by the least-square method can depend on the value of the phase-lag between the in-line forces and the velocity of the ambient flow. When comparing Equation (47) to Equation (30), one can see that the least-square method can give a higher value of drag coefficient than the maximum value method.

The least-square method takes into account the instantaneous dynamic response of the cylinder; the maximum value method only considers the peak value of the in-line force acting on the cylinder. The value of the drag coefficient
evaluated by the maximum value method stays the same despite any change in the dynamic response of the cylinder, providing the magnitude of the peak value stays the same.

In order to estimate the difference in effect a forcing function has on the results of the least-square method, three different functions were used to substitute for the one described by Equation (38). If the forcing function of the in-line force approaches a sinusoid function (Fig. 5) instead of that described by Equation (38);

\[ F_H(\theta) = F_{Hm} \cos(\theta - \phi) \quad (48) \]

the value of \( C_d \) evaluated by the least-square method will be:

\[ C_d = \frac{-1.13 F_{Hm}}{\frac{1}{2} \rho D L U_{\text{max}}^2} \quad \text{(cos function, } \phi = 0) \quad (49) \]

For a triangular wave function (Fig. 5),

\[ F_H(\theta) = \begin{cases} F_{Hm}(1 - \frac{2}{\pi}\theta) & 0 < \theta < \pi \\ F_{Hm} \left(\frac{2}{\pi}\theta - 3\right) & \pi < \theta < 2\pi \end{cases} \quad (50) \]

\[ (51) \]

the value of \( C_d \) evaluated by the least-square method is:

\[ C_d = \frac{-32 \int_0^{\pi} F_{Hm}(1 - \frac{2}{\pi}\theta) \cos \theta | \cos \theta | d\theta}{3\pi \rho D L U_{\text{max}}^2} \quad (52) \]
Fig. 5 Different kinds of dynamic responses used in the comparison
\[ C_d = \frac{0.968F_{Hm}}{\frac{1}{2} \rho DL U_{\text{max}}^2} \] (triangular function) \hspace{1cm} (53)

For a square-wave function, (Fig. 5)

\[ F_H(0) = \begin{cases} F_{Hm} & 0 < \theta < \frac{\pi}{2}, \text{ and } \frac{3\pi}{2} < \theta < 2\pi \ \text{ (54)} \\
-F_{Hm} & \frac{\pi}{2} < \theta < \frac{3\pi}{2} \ \text{ (55)} \end{cases} \]

the value of \( C_d \) thus evaluated by the least-square method is:

\[ C_d = \frac{-32}{3} \int_0^{\pi} F_{Hm} \cos \theta |\cos \theta| d\theta 
= \frac{1.333F_{Hm}}{\frac{1}{2} \rho DL U_{\text{max}}^2} \] (square function) \hspace{1cm} (57)

Thus, the value of the drag coefficient evaluated by the least-square method depends on the forcing function of the in-line force and how closely the forcing function follows the harmonic assumptions within the Morison equation. Comparing Equations (30) and (47), one can observe that, under the assumptions of the Morison equation, the value of \( C_d \) evaluated by the least-square method can be higher than that evaluated by the maximum value method. Comparing Equation (30) and Equation (43), one can observe that the assumption of a constant drag coefficient through a complete cycle of motion of the cylinder can be applied without serious error.
The value given by the least-square method is a weighted average value pivoted at the maximum velocity of the cylinder. The error due to the very low velocity at the beginning and the end of the cycle is small as shown in Equations (53) and (57).

In the case where the wake effect is predominant, Equation (33) will give a force coefficient that would approach the value of $C_d$ given by the Morison equation, since both coefficients are function of the square of the ambient velocity.

One can also examine the difference between the least-square method and the maximum value method for the added mass coefficient. With the assumption of sinusoidal motion of the cylinder and no phase-lag, Equation (24) can be simplified and becomes:

$$C_m = \frac{\int_0^{\pi/2} F_H \sin \theta d\theta \int_0^{\pi/2} \cos^4 \theta d\theta - \int_0^{\pi/2} F_H \cos^2 \theta d\theta \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta}{\frac{1}{4}\pi \rho D^2 L \max \left[ \int_0^{\pi/2} \sin^2 \theta d\theta \int_0^{\pi/2} \cos^4 \theta d\theta - \left( \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta \right)^2 \right]}$$

Under the assumption of the Morison equation, for a potential flow effect predominant case;

$$F_H(\theta) = F_{Hm} \sin \theta$$

Equation (52) can be rewritten as:

$$C_m = \frac{\int_0^{\pi/2} F_{Hm}^2 \sin^2 \theta d\theta \int_0^{\pi/2} \cos^4 \theta d\theta - \int_0^{\pi/2} F_{Hm}^2 \cos^2 \theta \sin \theta d\theta \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta}{\frac{1}{4}\pi \rho D^2 L \max \left[ \int_0^{\pi/2} \sin^2 \theta d\theta \int_0^{\pi/2} \cos^4 \theta d\theta - \left( \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta \right)^2 \right]}$$
Thus, one can see that the value of $C_m$ evaluated by the least-square method is the same as that evaluated by the maximum value method for a potential flow effect predominant case with the assumptions of a sinusoidal motion of the cylinder and no phase-lag between the potential flow force and the acceleration of the cylinder. If the forcing function of the added mass force is not sinusoidal, the value of $C_m$ thus evaluated by the least-square method will also be different.

For a potential flow effect predominant case, the force predicted by the Morison equation disagrees with that predicted by Equation (33). The in-line force due to the potential flow effect is a function of the acceleration of the ambient flow but the value of $C_{m_{\text{max}}}$ predicted by Equation (33) varies with the square of the ambient velocity.

**Summary**

It is clear that the least-square method can give a higher value of force coefficient than that given by the maximum value method. It is more conservative, for the design purpose, to use the value given by the least-square method. Equation (33) gives a statistical analysis of the
force from a simple calculation. The Morison equation is more complicated to use but it can give a more descriptive analysis of the force. When used together, they can provide a good system for the analysis of the force on a cylinder. The fact that Equations (30) and (43) give the same value, under the ideal conditions, for the drag and added mass coefficients shows that the assumption of a constant force coefficient throughout a complete cycle of cylinder motion does not lead to serious error.

Most research on oscillating forces on submerged cylinder has been done in the lower Reynolds number range and at a smaller scale. However, in practical applications, the Reynolds number may be beyond $10^5$. It is generally believed that large scale experiments can produce more realistic data because large scale experiments can approximate actual conditions more closely. Large scale in-field experiments may produce good results, but they may not be economical to conduct and there is no control over the incoming wave. The random nature of waves in the ocean makes reduction of data into systematic information a difficult task.

Usually, the experiments of oscillating the fluid around the cylinder are limited to the lower Reynolds number range by the size of the oscillation generating system. For waves, vertical velocity gradients within the fluid medium will result in a non-uniform flow. In using a water tunnel for oscillatory fluid experiments, the fluid is confined in
the tunnel, and the blockage effect may affect the results. In oscillating the cylinder, one can reach for much higher values of Reynolds number and can also minimize most of the deficits in the other methods. A problem here, however, is to obtain smoothness of operation of the equipments.

As shown in references (13) and (14), the force coefficients for a circular cylinder depend on at least four factors: The Reynolds number, the Keulegan-Carpenter period parameter, the proximity of a plane boundary and the free water surface. In this experiment, three of those four important factors—the Reynolds number, the Keulegan-Carpenter period parameter and the proximity of a plane boundary—were investigated. The water depth was kept constant throughout the experiment (10). In reference (9), it was shown that, for a small amplitude of the oscillation of the cylinder, if the distance from the free water surface to the cylinder is six times the cylinder diameter, the effect of the free water surface can be neglected. In the present experiment, this ratio was kept at 6. The range of Reynolds number covered was from $10^5$ to $1.4 \times 10^6$. The proximity ratio covered was from 0.083 to 6.0.
III EXPERIMENTATION

Apparatus

Wave Research Facility

Experiments for this study were conducted at the Oregon State University Wave Research Facility from February 1976 to April 1976. The wave basin is 342 feet long, 12 feet wide and is 15 feet deep at the test section. The basin is equipped to conduct large scale investigations. At one end of the basin is a flap-type wave board hinged at the bottom; the board is activated by a 150 hp pump with an hydraulic servomechanism (Fig.6). For this study, the wave board was used to oscillate the test cylinder through a system of cables, pulleys, and sheaves (16) (Fig.7).

At the test section, four 12 feet square concrete slabs were placed to provide the required false bottom configuration and the proximity ratio. Each slab was rigidly secured in place by heavy steel-angles bolted to the side walls of the basin. The gaps between the four slabs were carefully sealed with plywood and silicone rubber to prevent the up-lift currents from below the false bottom to influence the flow conditions.

Test Cylinder

The test cylinder (10), consisting of three sections of
Fig. 6 OSU Wave Research Facility
Fig. 7 Close-up of the sheaves

Fig. 8 Close-up of the end-plate
aluminum tubes, (Fig. 9) each twelve inches in diameter and a quarter inch thick, was forced to oscillate in otherwise still water with its axis perpendicular to the direction of the motion. The middle section, the test section, was 31 inches in length. Adjacent to each end of the test section, was a dummy section, each was 48 inches in length. This configuration was designed to best approximate two dimensional flow. Spacing between the test section and the dummy section were kept to a minimum. A six inches outer-diameter and quarter-inch thick galvanized steel tube was placed inside the cylinder along the center axis as rigid supports for the dummy sections. An aluminum plate of half inch thick and twelve inches by fifty-two inches was bolted to each end of the cylinder. Two pillow blocks, each 44 inches apart, were mounted at the mid-height position on the outer side of each end-plate (Fig. 8). The pillow blocks were to keep the cylinder moving smoothly along the steel guide-rails which were mounted on the side-walls of the basin, six feet above the bottom.

**Force Dynamometer**

Two one-inch-diameter, 48 inches long aluminum rod were machined to accommodate 16 Micro-measurements CEA-13-189-UW-120 strain gages (Fig. 10). The gages sense the deflections of the test section of the cylinder as horizontal and vertical displacements. Four pairs of gages were attached to the
Fig. 9 Test cylinder
Fig. 10 Force dynamometer
machined rod to sense the horizontal component of the displacement, while the other four pairs sensed the vertical component. Every two pairs of horizontal or vertical gages were connected to form one complete Wheatstone circuit (Fig. 11,12). Thus, all the gages together formed two complete horizontal and two complete vertical circuits. The strain gages were water-proofed after the completion of the circuits. Only one of each kind was used to detect the force, while the remaining pairs were to act as back-up circuits in case the first two pairs failed to function properly. At the middle of the test section and at the adjacent ends of the dummy section, rigidly constructed boxes of half-inch aluminum plate were fastened against the inside of the cylinder wall (Fig.13,14). The boxes inside the dummy section were also bolted to the galvanized steel tube. The two machined rods were fastened rigidly to the boxes such that the weight of the test section could act on the rods freely.

Output from the strain gages were conditioned and balanced through a Honeywell Accudata 117 DC amplifier, and was recorded both on the Visicorder and on the magnetic tape through a PDP-11 mini-computer.

Accelerometer

A setra System model 114 linear acceleration transducer was mounted on one of the end-plates of the cylinder. The transducer produced a d.c. output signal proportional to the
Fig. 11  Horizontal force dynamometer circuit

Fig. 12  Vertical force dynamometer circuit
Fig. 13  Anchorage at the test section

Fig. 14  Anchorage at the dummy section
sensed acceleration.

The transducer was of variable-capacitance sensor type, consisted of a one-piece thin stiff metal disc and flexures, assembled between two fixed insulated metal electrode plates. The motion of the seismic disc is proportional to the acceleration vector perpendicular to the plates. The transducer also had built-in electronics, utilizing a special switching type integrated circuit; it converted the changes of capacitance due to the acceleration variations into a high level d.c. output signal.

The transducer operated on 6V DC excitation voltages. The output also recorded on the visicorder and on the magnetic tape through the PDP-11 computer. A special waterproof housing was custom-designed to contain the accelerometer. The housing was a cylindrical box made of quarter inch plastic-glass, two inches in diameter and three inches long.

**Experiment Set-up**

The motion of the wave board was transmitted to the test cylinder and amplified through a system of cables, pulleys, and sheaves. A dam was constructed to isolate the body of water from the wave board so that the motion of the wave board would not be transmitted to the water (Fig. 15). Four cables—two on each side—with one end of the cables anchored to the sheaves, ran from the smaller sheaves, th-
Fig. 15 Experiment set-up
rough a system of pulleys and anchored to the end-plates of the cylinder. The cables were anchored to the sheaves in order to avoid any slippages. The cables were pre-tensioned by a system of springs and pulleys. The frequency and amplitude of the motion of the cylinder was controlled through the control system of the wave board. The maximum displacement of the motion of the cylinder was 20 feet. The cylinder slid along on two guide rails, each 24 feet in length. Those two rails were rigidly mounted on the side walls of the wave basin, to provide the rigid support and path of motion for the test cylinder. The Reynolds number of the test runs was calculated using the maximum velocity of the cylinder at that run. The motion of the cylinder approached that of a sinusoid function. Water depth, which was 12 feet, and the vertical position of the cylinder remained unchanged throughout the whole experiment. The proximity ratio was varied by the lifting the three concrete slabs to the predetermined height (Fig.16,17).

**Calibration**

The force dynamometer was calibrated both in air and in water. Scaled weights were applied to the test section of the cylinder through a low friction pulley system in the horizontal and vertical directions.

Only the calibrations in water were used for the calculation of the forces. The accelerometer was pre-calibrat-
Fig. 16 Test cylinder with $e/D = 0.25$

Fig. 17 Sheaves and the transmission system
ed in factory at the 0.5g acceleration scale; where g is the acceleration due to gravity. The dynamometer constants were

\[ F_H = 240 \text{ lb/calibration unit} \]

\[ F_V = 185 \text{ lb/calibration unit} \]

**Testing Procedure**

Five amplitudes, in combination with eight different frequencies were tested in this study. For those runs with large displacements, e.g., a displacement of 20 feet, the frequencies tested ranged from 0.0159 Hz to 0.223 Hz. For those runs with smaller cylinder displacement, only the lower frequencies were tested due to the high dynamic forces involved in the motion generating system. Thus, there were a combination of 29 runs as shown in Table 1 for each proximity ratio.

Each frequency was repeated for a different proximity ratio, e/D. A total of five proximity values were investigated; they ranged from 6.0, 1.0, 0.5, 0.25, to 0.083. So there were a combination of 145 runs.

The depth of water was 12 feet. The cylinder was 6 feet away from the bottom floor of the wave basin. The desired proximity ratio was obtained by lifting the four concrete slabs close to or away from the cylinder. The maximum cylinder displacement was obtained through the control system
Table 1  Sample of scheduled run number for e/D = 0.5
of the wave board. The frequency of the cylinder motion was equal to the frequency of the wave board.

With all the equipments and apparatus in position, the wave generator was set to the pre-determined frequency and span. The motion of the wave board was amplified and transmitted to the cylinder. The cylinder oscillated horizontally along the two guide rails with its axis perpendicular to the motion. The forces acting on the cylinder were sensed by the strain gages as variations in resistance. The acceleration of the cylinder was sensed by the accelerometer as the variation in capacitance. The generated data then were recorded on the visicorder and on the magnetic tape. Due to a time limitations, only the data from the proximity ratio of 6, 1, and 0.083 were recorded on the magnetic tape.
IV DATA ANALYSIS

The measurements of the in-line and transverse forces and the acceleration of the cylinder were calibrated into voltage, then recorded in digital form and stored on digital magnetic tape. Measurements were made every 0.01 second during each test run, except for those with low frequency and large displacement of the cylinder. For those runs, measurements were made every 0.02 second. The measurements were recorded in four channels, namely channels 1, 2, 3, and 4, for the position, in-line forces, transverse force, and the acceleration of the cylinder, respectively. The average temperature of the water during the whole investigation was about 10°C., with a deviation of ± 2°C.

The experimental data were extracted from the magnetic tape through the PDP-11 mini-computer. The print-out was in digital form, and was in voltage. The print-out was first calibrated back into its corresponding physical unit. The calibration constants used throughout the experiment were (Fig. 18):

\[ F_V = 185 \text{ lb/} 0.74 \text{ volts} \]
\[ F_H = 240 \text{ lb/} 0.34 \text{ volts} \]
\[ U = \frac{32.2 \text{ ft/sec}^2}{0.45 \text{ volts}} \]
Fig. 18 Calibration curves for $F_H$ and $F_V$. 

- Horizontal forces
- Vertical forces
to filter out the noise-signal of the instruments utilizing the moving average method. The moving average method replaces the values at the point \( t \) by an average value of \( n \) points with point \( t \) as the pivot point (Fig. 19, 20) (6);

\[
F(t) = \frac{F(t-n) + \cdots + F(t) + \cdots + F(t-n)}{2n + 1} \quad (59)
\]

For this investigation, \( n \) was arbitrarily chosen to be 7.

After the smoothening process, Equations (23) and (24) were used to evaluate the drag coefficient, \( C_d \), and the added mass coefficient, \( C_m \), respectively. The calculations were broken down into several steps. First, the instantaneous velocity of the cylinder was obtained by integrating the instantaneous value of the acceleration of the cylinder. The trapezoidal rule was used to perform all the numerical integration. The trapezoidal rule states that the area under a curve can be approximated by fitting a series of infinitesimally small trapezoids under that curve, so long as one knows the measurements of each individual trapezoid;

\[
F(t) = \sum_{0}^{n} \left[ \frac{F(t) + F(t+\Delta t)}{2} \right] \Delta t
\]

where \( n = T/\Delta t \), and \( T \) is the period of the motion of the cylinder.

Applying the trapezoidal rule to Equation (22);
Fig. 19  A sample of data before smoothening
Fig. 20 A sample of data after smoothening
\[ U(t) = \sum_{n} R_n \left[ \Delta t \left( \frac{\dot{U}(t) + \dot{U}(t+\Delta t)}{2} \right) + C \right] \quad (64) \]

where \( C \) is a constant. Then, Equation (23) and Equation (24) were broken down into five parts:

\[ AA = \int_{0}^{T} F_H \dot{U} dt \quad (65) \]
\[ BB = \int_{0}^{T} U^4 dt \quad (66) \]
\[ CC = \int_{0}^{T} |U| \dot{U} dt \quad (67) \]
\[ DD = \int_{0}^{T} F_H |U| \dot{U} dt \quad (68) \]
\[ EE = \int_{0}^{T} \dot{U}^2 dt \quad (69) \]

Thus, Equations (23) and (24) can be rewritten as:

\[ C_d = \frac{2 \left[ (DD)(EE) - (CC)(AA) \right]}{\int_{0}^{T} \left[ (EE)(BB) - (CC)^2 \right]} \quad (70) \]
\[ C_m = \frac{4 \left[ (AA)(BB) - (CC)(DD) \right]}{\int_{0}^{T} \left[ (EE)(BB) - (CC)^2 \right]} \quad (71) \]

The results were tabulated and plotted on graph and will be discussed in the next Chapter.
RESULTS AND DISCUSSION

Wake effect and the correlating parameters

The drag coefficients of the cylinder in this oscillating cylinder experiment are evaluated using Equation (23) and the trapezoidal rule. The range of wake parameter, A/D, covered is from 20 to 5. The results for the free-stream condition, e/D = 6, are tabulated in Table 2. The results of the drag coefficients are plotted in Figure 21. In general, for the free-stream condition, the results for the drag coefficients agree pretty well with other investigators' (2,13). Within the range of the Reynolds number tested, the value of the drag coefficients, C_d, decreases to a minimum at the Reynolds number of 3X10^5 and then starts to increase slowly with the Reynolds number.

There is a definite phase-lag between the drag force and the cylinder velocity; but no measurements were made to evaluate this phase-lag. As the maximum displacement of the cylinder or the wake parameter, A/D, decreases, the value of C_d increases. The general trend of the curves stays the same despite the change of the maximum displacement of the cylinder.

The presence of a near-by plane boundary does not change the general trend of the curves, but there is clear indication that the presence of a near-by plane boundary will in-
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Table 2: Results for the free-stream condition
Fig. 21 Drag coefficient for free-stream condition
tensify the wake effect (Fig. 22). This effect becomes significant when the proximity ratio, \( e/D \), is less than 1; the value of the drag coefficient increases dramatically as the proximity ratio approaches zero. The experiment results indicate that the drag coefficient is a function of both the wake parameter and the proximity of a plane boundary.

In reference (2), the author indicated that the drag coefficient decreases with a decreasing value of the wake parameter. The results of this experiment shows just the opposite trend, which agrees with reference (13), the drag coefficient decreases with increasing value of wake parameter.

**Potential flow effect and correlating parameter**

The added mass force experienced by the cylinder oscillating in still fluid is due to the potential flow effect and the inertia force of the total mass of the cylinder. Thus, a modified version of the Equation (71) is needed to evaluate the added mass coefficient of the oscillating cylinder. Rewriting Equation (71) to eliminate the inertia force due to the total mass of the test section of the cylinder and the mass of water occupied by the test section:

\[
C_m = \frac{4[(AA)(BB) - (CC)(DD)]}{\frac{2}{3} \pi D^2 L [(EE)(BB) - (CC)^2]} - \frac{M}{\frac{1}{2} \rho \pi D^2 L} \quad (72)
\]

where \( M \) is the total mass of the test section of the cylin-
Fig. 22 Drag coefficient for $A/D = 20$

(least-square method)
der and the mass of the water displaced by the test section.

The added mass coefficient of the cylinder in this study is evaluated using Equation (71) and the trapezoidal rule. The range of wake parameter, \( A/D \), covered in this study is from 20 to 5. The results for the free-stream condition, \( e/D = 6 \) are plotted in Figure (23). The results of this study are different from those of other investigators, (2, 13). Results from references (2) and (13) showed that the added mass coefficients approach an upper limit and then level off as the Reynolds number increases. The added mass coefficient for this study does not exhibit that trend. The data show that the added mass coefficient, \( C_m \), remained rather constant until the Reynolds number reached about \( 4 \times 10^5 \), the added mass coefficient then increased dramatically with the Reynolds number.

This difference in the results of the added mass coefficients may be from the sensitivity of the instruments. The instruments were set to detect the total in-line force. If the added mass force was only of a small percentage of the total force, it could be mis-interpretted during the evaluation process and may lead to the error in evaluating the added mass coefficient.

For the free-stream condition, there is evidence that the wake parameter has some effect on the added mass coefficient, \( C_m \); as the value of the added mass coefficient increases with a decreasing value of wake parameter. But as
Fig. 23 Added mass coefficient for free-stream condition
the plane boundary becomes closer to the cylinder, where $e/D$ is close to zero, the influence of the wake parameter is not as clear.

Results also show that the presence of a near-by plane boundary will also intensify the potential flow effect. The effect becomes significant when the value of the proximity ratio, $e/D$, is less than 1 (Fig. 24). The added mass coefficient, $C_m$, increases with a decreasing value of proximity ratio, $e/D$. The added mass coefficient also is a dependent on the proximity of a near-by plane boundary and the wake parameter.

**Comparison between least-square method and maximum value method**

As shown in Chapter II, the least-square method can give a value of drag and added mass or inertia coefficient higher than that given by the maximum value method. The experimental results also verified this point. Since this study is concerned with the experimental data, and there is no theoretical value to compare with; it is hard to conclude which method of evaluation will give a more realistic result. It is more conservative for the design purpose to use the values of the force coefficients evaluated by the least-square method.

The maximum value method only utilize of the maximum horizontal velocity, the maximum horizontal
Fig. 24  Added mass coefficient for $A/D = 20$
acceleration and the corresponding measurements for the in-line forces of the cylinder. The least-square method takes into account the instantaneous dynamic response of the cylinder and gives an weighted average value corresponding to a complete cycle of motion of the cylinder (Fig. 25, 26). The maximum value method is much simpler to use; it does not require the aid of a computer. The least-square method is more time consuming and the evaluation of the experimental data seems impossible without the aid of a computer.

**Transverse force and correlating parameter**

The least-square method works well for the evaluation of the drag coefficient and the added mass or inertia coefficient, but it need some modification on Equation (32) before it can be applied to the transverse force record to evaluate the lift coefficient of the cylinder. The lift coefficient of the cylinder, $C_L$, as given by Equation (12) is a function of the square of the horizontal velocity of the cylinder; but in the actual case, the transverse force is more complex than as stated by the equation.

The transverse force acting on the cylinder depends on the wake formation, vortex shedding and the proximity of a plane boundary. It does not have a smooth curve like those of the in-line force of the cylinder, (Fig. 27). It has more than one peak in one cycle of motion of the cylinder. Ex-
Fig. 25 Comparison of $C_d$ between the least-square method and maximum value method, for $e/D = 6$
Fig. 26 Comparison of $C_m$ between the least-square method and maximum value method, for $e/D = 6$
Fig. 27  Transverse force on the cylinder
experiments show that the frequency of the peaks depends on
the frequency of the vortex shedding. In most researches,
the lift coefficient, $C_L$, is determined by using the posi-
tive maximum transverse force measurement and the negative
maximum transverse force measurement of the cylinder in one
complete cycle of motion. Thus, there are two values for
the lift coefficient; a positive value and a negative value
were obtained for each run. The least-square method, with
Equation (29), will give a value that is equal to an average
value of the positive and the negative values. For the
free-stream condition, the least-square method gives a value
close to zero. Thus, some modification is needed on Equa-
tion (29) for the evaluation of the lift coefficient.

Instead of using one complete cycle of motion and ob-
tain an average value for the whole cycle, the measurements
of the transverse force record can be divided into two por-
tions, the positive and the negative values, and then Equa-
tion (29) can be used to evaluate each portion correspond-
ingly to obtain a positive and a negative value of $C_L$ for
the test. Another way to modify Equation (29) is to use
the absolute value of the transverse force measurement to
evaluate the lift coefficient. This method gives only one
value for each run instead of the conventional positive and
negative values.

The transverse force records in the magnetic tape
does not have a reference reading for the establishment of
a zero reading for the transverse force. Thus, the lift coefficient, $C_L$, was evaluated by the conventional maximum value method only. The results for the lift coefficient, $C_L$, from the maximum value method have been presented in reference (16); herein, only a brief discussion is given.

The value of lift coefficient, $C_L$, increases as the wake parameter, $A/D$, decreases. For a given wake parameter, $A/D$, the lift coefficient decreases gradually as the Reynolds number is increased. The positive lift force increases dramatically as the proximity ratio, $e/D$, decreases. As the proximity ratio approaches zero, the transverse force becomes predominantly positive or away from the plane boundary.

For the free-stream condition, the positive transverse force is about the same in magnitude as the negative transverse force. In this case, the transverse force is due to the vortex shedding. The lift frequency is equal to the frequency of the vortex shedding. As the proximity ratio approaches zero, the lift occurs at twice the frequency of the cylinder motion. In this case, the transverse force is due to the unsymmetric flow around the cylinder.
VI CONCLUSIONS

Experimental Results

The experimental results clearly indicated that the in-line and transverse hydrodynamic forces are dependent upon the proximity of the plane boundary, $e/D$, the wake parameter $A/D$, and the Reynolds number.

1) This thesis considers data from tests with $5 \leq A/D \leq 20$, wherein the wake effect significantly altered the flow pattern around the cylinder and the potential flow theory can not be used to predict the hydrodynamic forces acting on the cylinder.

2) The drag coefficient, $C_d$, decreases with an increasing value of wake parameter, $A/D$.

3) The effect of the wake parameter, $A/D$, on the added mass coefficient, $C_m$, is not as clear but there are indications showing that the wake parameter can affect the added mass coefficient, $C_m$.

4) The presence of a near-by plane boundary will intensify the wake and the potential flow effect and results in an increase in the value of the drag, added mass or inertia, and lift coefficients as the proximity ratio, $e/D$, approaches zero.

5) For the free-stream condition, the transverse force is due to the vortex shedding; but as the cylinder approaches
the plane boundary, or as the value of e/D approaches zero, the transverse force is due to the unsymmetric flow around the cylinder and the lift has a frequency twice that of the motion of the cylinder.

6) Within the range of Reynolds number tested, the drag coefficient, $C_d$, decreases to a minimum at the Reynolds number of $3 \times 10^5$ and increases gradually with an increasing Reynolds number.

7) Within the range of Reynolds number tested, the added mass coefficient, $C_m$, first increases gradually with the Reynolds number and then rapidly as the Reynolds number passes the value of $6 \times 10^5$. This result is different from the results of other investigators (2, 13).

8) The value of the drag and added mass, or inertia coefficients evaluated by the least-square method is more conservative than those evaluated by the maximum value method for these data.

9) The assumption of a constant force coefficient throughout a complete cycle of motion of the cylinder will not, at least for the ideal condition, lead to serious error in the evaluation of the drag and added mass coefficients.

**Recommendations for further research**

Further tests are necessary to determine the effect of the Reynolds number on the added mass coefficients, $C_m$. The pillow blocks at the ends of the cylinder provided a
good support and path of motion for the cylinder, but a system with lower friction would reduce the noise signal and produce better experimental data. A reference reading at the beginning of every test run should be established before the recording of the test data to help evaluating the experimental data. A modified procedure for the least-square method should be used to evaluate the transverse force. Further studies should be conducted to find out which method of modification works better for the evaluation of the lift coefficient, $C_L$. Also, further studies should be conducted to investigate the difference in results, for the added mass coefficient, between this study and other investigators.
REFERENCES


4. Grace, R.A. and Zee, T.Y., Further tests on ocean wave forces on a sphere, Department of Ocean Engineering, Technical Report, University of Hawaii, Honolulu, Hawaii, April 1977


12. Sarpkaya, T., Vortex shedding and resistance in harmonic flow about smooth and rough circular
cylinders at higher Reynolds number, Naval Postgraduate School, Monterey, California, Feb 1976


15. Yamamoto, T., Nath, J.H., and Slotta, L., Yet another report on cylinder drag or wave forces on horizontal submerged cylinders, Engineering Experiment Station Bulletin No. 47, Oregon State University, Corvallis, Oregon, April 1973

APPENDICES
Appendix A.-- scheduled run numbers for the experiment

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Appendix C. Computer Program

PROGRAM OSCYL
C THIS PROGRAM USES LEAST-SQUARE METHOD TO EVALUATE THE
C TRAP AND ADDITIONAL COEFFICIENTS FOR THE OSCILLATORY FORCE
C ON A CYLINDER
DIMENSION INPUT(256,8), JOBUF(2048), F(2048), FCH(2048), VEL(2048)
C INPUT DESIRED RUN NUMBER
C 100 FORMAT(' INPUT RUN NUMBER ', I1)
CALL ASSIGN(-1, FILE)
C DEFINE FILE 1 (0.256, U-IBLK)
C INPUT THE DESIRED CHANNEL, 1=ACCEL., 24=HORIZONTAL FORCE
C 110 FORMAT(THESELECTED CHANNEL, I)
C VERTICAL FORCE=
C 120 FORMAT('INPUT SPEED OF RECORDING', F)
C TYPE 110
C 110 FORMAT(' TIME ? ', F)
C ACCEPT 120, TIM
C 120 FORMAT('FREQUENCY OF THIS RUN', F)
C TYPE 120
C 120 FORMAT(' FREQUENCY ? ', F)
C ACCEPT 140, F80
C NP73=2040
NPLK=NPTS/256
C COMPUTER START READING DATA NOW
DO 150 I=1, NPLK
READ (FILE) (INPUT(J,I), J=1,256)
IBLK=IBLK+1
150 CONTINUE
C CHOOSE PROCEDURE, 0=PRINT OUT DATA NOW
C 180 MOVING AVERAGE
C TYPE 150
C 180 FORMAT(' CML IS ', I1)
C ACCEPT 170, N
C 170 FORMAT('II)
IF (N EQ. 1) GO TO 120
DO 120 II=1, NPLK
120 VEL(II)=FLOAT(JBUF(I))
IF (N EQ. 0) GO TO 440
C INPUT NUMBER OF POINT FOR MOVING AVERAGE
C 190 TYPE 200
C 190 FORMAT(' I IS ', I1)
C ACCEPT 210, L
C 210 FORMAT('II)
DO 220 II=1, NPLK
VEL(II)=FLOAT(JBUF(I))
NML=NPLK+1-L
DO 220 II=1, NPLK
220 IF (L) GO TO 0
DO 250 II=1, NML

DC 240 K=1, L
IF(K.EQ.1)II=I
II=II+1
CONTINUE
C 0=NOT TO PRINT, 1=PRINT OUT DATA NOW
TYPE 260
ACCEPT 270, N
270 FORMAT(I1)
280 CON=0.0
DO 280 I=1, NPTS
290 CON=CON+FL0AT(NPTS)+CON
TYPE 200,CON
200 FORMAT( I0.5) TYPE 210
210 FORMAT( AVG IS ''''I, !X ACCEPT 220, AVG
220 FORMAT(F10.2)
DO 230 I=1, NPTS
230 VEL(I)=AVG-VEL(I)/230.3
IF(N.EQ.0) GO TO 250
TYPE 240, (F10.5)
240 FORMAT(F10.2)
250 BEG=0.0
THE DATA WILL BE INTEGRATED TO GET VELOCITY
DO 250 I=1, NPTS-1
VEL(I)=(TIM(I-1)+F(I))/250.0
BEG=VEL(I)
250 CONTINUE
AVG=0.0
DO 260 I=1, NPTS
260 AVG=VEL(I)/Fl0AT(NPTS)+AVG
DO 270 I=1, NPTS-1
270 VEL(I)=VEL(I)-AVG
VMIN=0.0
DO 280 I=1, NPTS-15
IF(VMIN.GT. VEL(I)) VMIN=VEL(I)
280 IF (VEL(I).GT. VMAX) VMAX=VEL(I)
TYPE 400. VMAX
400 FORMAT( MAX = ''''F10.5) TYPE 410. VMIN
410 FORMAT MIN = ''''F10.5
TYPE 420
420 FORMAT( PRINT OUT ??''''I ACCEPT 420, M
420 FORMAT(I1)
IF (M.EQ.0) GO TO 460
IF (M.EQ.2) GO TO 440
440 TYPE 450, (VEL(I)), II=1, NPTS, 5
450 FORMAT(8F10.2)
460 IBLK=2
C THIS IS TO WORK ON THE HORIZONTAL FORCE TO FIND DRAG AND ADDED
C MASS COEFFICIENTS.
DO 500 I=1,NELKS
READ(3,IBLK)INPUT(J,I),J=1,256
IBLK=IBLK+1
500 CONTINUE
DO 510 II=1,NPTS
FOCH(I)=0.0
DO 520 II=1,NML
DO 520 K=1,L
IF(II.EQ.I)II=II
FOCH(I)=FOCH(I)+FLOAT(JBUF(I))/FLOAT(L)
520 II=II+1
510 CONTINUE
AVG=0.0
DO 540 II=1,NPTS
540 AVG=AVG+FOCH(II)
DO 550 I=1,NPTS

II=I
550 FOCH(I)=240.0*(FOCH(I)-AVG)/432.0
560 FORMAT(' PRINT OUT ?',*)
ACCEPT 570,M
570 FORMAT(12)
IF(M.EQ.0 .GO TO 550)
TYPE 580,(FOCH(I),I=1,NPTS,5)
580 FORMAT(9F10.5)
590 RAN=1.0/100+FPO+TIN
TYPE 600,RAN
600 FORMAT(' RANGE = ',F10.5)
TYPE 610
610 FORMAT(' RANGE ? ',*)
ACCEPT 620,LL
620 FORMAT(15)
TYPE 630
630 FORMAT(' START ? ',*)
ACCEPT 640,HH
640 FORMAT(14)
H=HH
640 HH=H
EE=0.0
CC=0.0
DD=0.0
EE=0.0
II=KK, (KK+LL)
II=II
HH=HH+FCH(I)*TIN+AA
EE=VEL(I)**4+TIM+EE
CC=VEL(I)**2+ABS(VEL(I1)**2)+F(I1)+VEL(I1)+VEL(I1+1)+F(I1+1)
%+TIM/2.0+CC
DD=VEL(I1)**2+ABS(VEL(I1)||**2)+FCH(I)+FCH(I+1)+VEL(I1)
%+ABE(VEL(I1)**2)+TIM/2.0+DD
EE=EE+F(I1)**2+F(I1+1)**2+TIM/2.0+EE
CC=CC+1.0*(EE+CC-CC)/1.38+1.0*(EE+CC-CC)**2
TYPE 660,CD
EEO FORMAT(COD =',F10.5)
C01=12.0*40*(AA*BB-CC*DD)*(.31416*1.984:1.04,(EE*EE-CC**2
TYPE 670, COI
FORMAT(C CI =',F10.5)
THIS IS TO WORK ON THE VERTICAL FORCES TO FIND LIFT
IBLK=2
CC080:ycled:NLK
READ (,'IBLK')(INPUT(I,J,1),J=1,1256)
IBLK=IBLK+1
CONTINUE
DO 650 I=1, NPTS
F(I)=0.0
DO 710 II=1, NML
DO 700 K=1, L
IF(K.EQ.1717=II
F(II)=F(II)+FLOAT(JEU(I))/FLOAT(L)
700 CONTINUE
CONTINUE
TYPE 720
FORMAT('AVERAGE IS ','$'
ACCEPT 730, AVG
750 FORMAT(F10.5)
DO 740 I=1, NPTS
740 F(II)=F(I)+AVG)/561.0
750 FORMAT('PRINT VERTICAL ', "$'
760 CONTINUE
770 FORMAT(CF10.
7-90
780 IF (FOCH(I).LT.0.0) FOC(I)=0.0
790 FF=1.0
DO 810 II=KK, (KK+LL)
810 FF=VEL(I)+VEL(I)+FOCH(I)+VEL(II+1)+VEL(I)+VEL(II+1)
K=FOCH(1)+II/TIM/2.0+FF
COL=12.0*2.0+FF/1.98+1.0*BB
820 CONTINUE
830 FORMAT(COL =',F10.5)
DO 850 I=1, NPTS
850 FOC(I)=185.0*(F(I)-AVG)/561.0
860 CONTINUE
STOP
END
Appendix D.--Results for least-square method

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<th>freq. (Hz)</th>
<th>vel. (ft/sec)</th>
<th>$C_d$</th>
<th>$C_{m'}$</th>
<th>$\frac{C_{m'}}{C_m}$ = 1.13</th>
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Fig. 28 Drag coefficient for e/D = 6
Fig. 29 Drag coefficient for $e/D = 0.5$
Fig. 30 Drag coefficients for e/D = 0.083
Fig. 31  Added mass coefficient for e/D = 6
Fig. 32 added mass coefficient for e/D = 0.5
Fig. 33  Added mass coefficients for e/D = 0.083