A baroclinic, convective mixed-layer was modeled, using water, in a laboratory convection tank identical to that used in the free convection study of Deardorff and Willis (1985). Baroclinicity and mean-flow shearing were achieved by tilting the tank by an angle of 10°. The resulting mechanical-production rate of turbulence kinetic energy was comparable in magnitude to the buoyancy-production rate at mid-levels within the mixed-layer.

Velocities were obtained by taking time-lapse photographs of neutrally-buoyant oil droplets suspended in the mixed-layer fluid. Variances and other statistical descriptors of the turbulence obtained from these velocities are presented in comparison to the free convection results of Deardorff and Willis (1985). The deviation of the present results from those of Deardorff
and Willis (1985) are assumed to be related to the effects of mean-flow shearing and are explained wherever possible with the aid of an appropriate kinetic energy budget (kinetic energy, here, refers to the kinetic energy of the turbulence and is not to be confused with the kinetic energy of the mean-flow).

The results indicate that a maximum in downstream horizontal kinetic energy at mid-levels within the mixed-layer was generated by shear-production and, also, by conversion from vertical kinetic energy. In the lower mixed-layer, vertical kinetic energy was amplified by a mechanical-production term associated with the divergence of the mean vertical velocity. Total turbulence kinetic energy, normalized by the square of the convective velocity scale, was much larger at mid-levels than in Deardorff and Willis (1985) due to mechanical-production which is not accounted for by simple mixed-layer scaling. Horizontal turbulence structure was predominately controlled by convection while vertical turbulence structure was significantly altered by mean-flow shearing.
Turbulence Structure Within an Inclined Laboratory Convection Tank

by

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TURBULENCE STRUCTURE WITHIN AN INCLINED LABORATORY CONVECTION TANK

INTRODUCTION

Laboratory experiments have been used in past investigations to simulate and study the characteristics of an atmospheric boundary layer in a state of free convection (Willis and Deardorff, 1974; Deardorff and Willis, 1985). For this situation - one characterized by the absence of mean-wind shear - a convection tank is positioned such that its bottom plate is normal to the gravity vector. It is then carefully filled with increasingly warm layers of filtered degassed water. After allowing sufficient time for molecular diffusion to damp out any initial disturbances introduced in the filling process, a smooth temperature distribution is attained such that a layer of cool water of uniform temperature underlies a layer of warmer water with a nearly constant positive lapse rate. This fluid system possesses zero available potential energy and is said to be stably stratified. When the bottom plate is heated to a temperature well exceeding that of the adjacent fluid, potential energy is added to the system in the form of buoyancy. As a result of local density differences, the fluid then mixes in a turbulent fashion in
an effort to return the system to a state with zero available potential energy.

Very often in the atmospheric boundary layer wind shear plays an important role in the development and maintenance of turbulence in addition to buoyancy. This is especially true in the surface layer where wind shear is the dominate source of turbulent kinetic energy (TKE). The relevant parameters controlling turbulence structure in the surface layer are the height above the surface $z$, the surface stress $\tau_0$, the surface kinematic heat flux $w'T'S$, and the buoyancy parameter $g/T$ where $g$ is gravity and $T$ is the absolute temperature. The surface layer velocity scale, $u_*$, is given by $u_*^2 = \tau_0/\rho$ where $\rho$ is the density.

The height within the boundary layer above which thermal convection begins to dominate over wind shear as the principal architect of the turbulence is approximately equal to $-L$ (Businger et al., 1971) where $L$ is the Monin-Obukov length given by

$$-L = u_*^3/\left[.4 \ (g/T) \ w'T'S\right].$$

Turbulent structure in this region, called the mixed-layer (ML), loses its dependence on $\tau_0$, but $w'T'S$ and $g/T$ retain their importance, and $z$ is replaced by $h$, the boundary layer depth, as the controlling length scale. The velocity scale arising from these parameters is
\[ w_* = \left[ \frac{(g/T)}{w'T_s h} \right]^{1/3} \]

(Deardorff, 1970; Tennekes, 1970) where \( h \) is the mean depth of the boundary layer. These scales have been used successfully in attempts to provide universal profiles of dimensionless turbulence intensities (variances) as functions of relative height, \( z/h \), within the boundary layer. The results appear to hold well in cases when the surface layer is shallow and little mean wind shear is found within the ML.

Significant wind shear has also been found to exist away from the surface within baroclinic mixed-layers (Pennell and LeMone, 1974; LeMone and Pennell, 1976; Lenschow et al., 1980). Arya and Wyngaard (1975) pointed out, however, that the mean shear equations and the conservation of stress equations form a feedback system which prohibits a magnitude of wind shear in the ML as large as the magnitude of the shear of the geostrophic wind. The system was shown to activate under convective conditions which produce large amounts of variance in the vertical velocity component. This seems to suggest that shear-generated TKE will remain small in magnitude relative to buoyancy-generated TKE (Deardorff and Willis, 1987). Caughey and Wyngaard (1979), however, found that shear production can be large at mid-levels in the ML even in
very convective conditions presumably because of baroclinic effects.

This issue is addressed in the present study which examines a baroclinic boundary layer, produced in the laboratory, in which shear-generated and buoyancy-generated turbulence rival one another for dominance in the ML. Measurements are taken and compared to those of other studies in an attempt to determine the extent to which the effects of baroclinic shear may alter the turbulence structure of the ML.
LABORATORY ANALOGY

The baroclinic boundary layer was modeled in a laboratory convection tank using water as the medium to be studied. The tank used in the experiments was almost identical to the one used in Willis and Deardorff (1974). Its horizontal dimensions were approximately 124 x 124 cm, and the sidewalls were made of transparent plexiglass 1.9 cm thick. The bottom was a 1.25 cm thick aluminum plate overlying a circulating water heat exchanger. The heat exchanger design was that of a thin, watertight chamber encasing a series of parallel running distribution tubes into which preheated water was pumped from a reservoir. Outlet holes along the length of each tube allowed the hot water to be distributed along the underside of the aluminum bottom plate of the tank. Empirically determined optimum spacing of outlet holes and opposing flow in adjacent tubes insured that lower boundary heating was nearly uniform and that horizontal temperature gradients along the lower boundary were minimal.

The baroclinicity was achieved by tilting the convection tank by an angle of 10°. This caused the
horizontal isotherms in the stably stratified region to intersect the ML top thereby giving the desired temperature gradient. Figure 1 demonstrates ideally how this might be accomplished. The horizontal hydrostatic pressure gradient, derived in Appendix I, is

\[
\frac{\partial p(x,z)}{\partial x} = \rho_0 \frac{\partial z_{\text{air}}}{\partial x} - \rho_0 \alpha_0 \int_z^{z_{\text{air}}} \frac{\partial T(x,z)}{\partial x} \, dz \tag{2.1}
\]

where \( z_{\text{air}} \) is the height of the air/water interface or the water level as marked in Figure 1, \( \alpha_0 \) is the coefficient of thermal expansion at temperature \( T_0 \), and \( \rho_0 \), likewise, is
thermal expansion at temperature $T_0$, and $\rho_0$, likewise, is the density at $T_0$. $T_0$ is the mean temperature of the ML.

From (2.1) we can see that there are two terms contributing to the overall pressure gradient. Term II tries to move water upslope in the ML because here the temperature increases horizontally along the upslope direction. Furthermore, the integral sign implies that this effect is greatest near the bottom surface of the tank. In the ideal case pictured in Figure 1 where the horizontal temperature gradient is uniform with height within the ML and identically zero in the stratified-layer, this term reduces to $-\gamma \rho_0 \alpha_0 (h) \frac{\partial T(x)}{\partial x}$.

Term I is a dynamic term related to the slope of the air/water interface. Referring to Figure 1, this term tries to move water to the left if the water surface is positively sloped and to the right if the water surface is negatively sloped. Since term I is independent of $z$ its effect is felt equally at all depths (Pond and Pickard, Appendix I), and a quick calculation using a value of $\partial T/\partial x$ estimated from Figure 3 shows that a very small positive slope, $\partial (z_{air})/\partial x$, on the order of $10^{-4}$ is all that is necessary for term I to balance term II near the bottom surface of the tank.

Thus the mean flow emerges as follows: the convection is turned on and a horizontal temperature gradient develops as shown in Figure 1. Term II initiates the upslope flow which is strongest near the bottom surface. As the upslope
moving fluid reaches the upslope sidewall of the tank it must rise to make room for more upslope moving fluid. This rising fluid creates a slope in the water surface. Term I quickly reacts by moving water back to the left in an effort to maintain a nearly level water surface. This water is returned to the left against the weakest part of the pressure gradient due to term II in the upper ML. This generates a circulation in which fluid moves upslope to the right at low levels in the ML and downslope to the left at high levels in the ML thus creating a deep layer of mean vertical shearing.

Fig. 2. Laboratory model of baroclinic mixed layer with flow vectors. Dashed line represents mixed layer top. Secondary heat source is adjacent to the upslope sidewall. Front wall and rear wall lie in planes parallel to page.
In an effort to simulate situations more characteristic of the atmosphere with less return flow within the ML and part of it above the ML altogether, an extra heat source was added just inside the upslope sidewall of the tank. The source was a long tube which ran back and forth across the tank in a plane parallel and adjacent to the upslope sidewall (see Figure 2). The tube was heated by pumping warm water through it. Efforts were made to minimize temperature gradients along the tube using an insulating adhesive tape.

Flow vectors shown in Figure 2 attempt to demonstrate the effect of this extra heat source on the mean flow. We can see that the upslope flow near the surface was still quite strong. However, a deep penetrative updraft in the vicinity of the extra heat source allowed some fluid to be drawn out of the boundary layer establishing a secondary return flow in the upper portions of the stratified layer (SL). This effectively weakened the return flow in the upper ML causing a net mean flow in the upslope direction within the ML. Sinking motion was induced over the bulk of the SL as a result of the continual transmigration of boundary layer fluid via the extra heat source. This simulated large scale subsidence of the SL and essentially offset boundary layer growth by entrainment. Estimated mean boundary layer thickness, $h$, normally fluctuated within about plus or minus 20% of 15 cm. This setup may
simulate a daytime slope flow in a conditionally unstable atmosphere, or a sea breeze with strong frontal convergence (Deardorff and Willis, 1987).
DATA COLLECTION AND PROCESSING METHODS

Data Collection Procedure

Velocity measurements were taken using time lapse photography of neutrally-buoyant oil droplets suspended in the ML fluid. The droplets were composed of diethyl phthalate, mesitylene, and a white dye mixed in prescribed proportions to match the density of water at a temperature typical of that found in the laboratory ML during the time at which it was being photographed. Since the thermal expansion coefficient of the droplets differed from that of the water, the droplets underwent small buoyancy accelerations at temperatures other than the temperature of density matching. However, the maximum speed of a droplet of average size (about 1 mm in diameter) relative to the surrounding ML fluid under observed experimental conditions was calculated to be two orders of magnitude smaller than the velocity of a typical Lagrangian fluid parcel (Willis and Deardorff, 1974).

After the tank was filled and the desired initial temperature distribution was reached, the convection was
turned on by activating the two heat sources. It took a few minutes for the convection and the mean circulation to establish themselves. A long, narrow tube attached to a hypodermic syringe was used to inject the oil droplets into the ML shortly before it had reached the temperature of density matching. After allowing sufficient time for the droplets to disperse throughout the ML, photographs were taken to trace the motion of the fluid. The tracers (oil droplets) were illuminated by an 8 cm thick vertical slab of laser light which was perpendicular to the focal axis of the camera. Two things were required with regard to illuminated slab thickness. The slab had to be thick enough to contain several tracers for a full three second time interval, yet it had to be thin enough to disregard tracer displacement discrepancies related to the effects of image parallax. 8 cm was the decided optimal compromise between these two needs.

Photographic slides were taken with a 35 mm camera mounted on a tripod a few feet from the front wall of the tank. The region of the tank which was photographed is found between the two reference lines shown in Figure 2.

There are two cartesian coordinate systems marked in Figure 2. The x-y-z coordinate system is constructed such that the z direction is parallel to the gravity vector and the y direction is perpendicularly into the page and parallel to the focal axis of the camera. When we rotate
this system by 10° about the y axis we arrive at the x"-y-z" coordinate system where z" is perpendicular to the bottom floor of the tank. Tracer velocities were originally measured in x"-z" coordinates and then transformed to x-z coordinates prior to the statistical computations. A more elaborate discussion of this is to follow. Photographs were also taken of illuminated slabs lying in the x"-y plane with the focal axis running parallel to the z" direction.

Mounted in front of the camera lens was a rotating disc with a gap in it through which the camera could see. At the beginning of each photograph the camera shutter was held open behind the gap in the disc causing several initial streaks to appear on the film as the droplets moved about in the water. Then, with the shutter still open, an electric motor was triggered causing the disc to make two complete rotations per second for approximately three seconds after which the shutter was closed completing the photograph. Each photograph therefore contained several sets of an initial streak followed by five or six dots 1/2 second apart. For each of eight experiments, a series of these photographs was taken while alternating the position of the illuminated slab between three different positions along the y-axis. That is, the 8 cm thick vertical slab of laser light was alternately centered at approximately y = 12, y = 31, and y = 50 cm where y = 0 denotes the plane
containing the front wall of the tank. This was done to dampen the effect any slight inhomogeneities along the y-axis might have on the results. The y = 124 cm vertical plane (the rear wall of the tank) was painted black to provide photographic contrast.

Referring again to Figure 2 we note that the study region between the two reference lines did not extend over the full width of the tank. This helped to minimize the effects of the upslope and downslope sidewalls on the study region. A distance of approximately 50 cm between the downslope sidewall and the lefthand reference line was necessary to insure a fully developed mean flow in the study region. With the study region extending for 62 cm along x", a distance of about 10 cm remained between the righthand reference line and the upslope sidewall which was necessary to exclude the immediate effects of the updraft on the mean flow. With these influences far enough removed it was assumed that the turbulence was statistically homogeneous along x".

Once the slides were developed, the data contained on them had to be recorded on digital tape in preparation for the computational analysis. This was done by projecting the photographed image onto a plotting machine. One of the components of the plotter was a light sensor capable of moving freely about in a two-dimensional plane. The exact location of the sensor in a cartesian coordinate plane was electronically monitored such that an operator could easily
record the coordinates of the sensor on floppy disc at any desired time simply by punching a key at the computer terminal. Thus, velocities could be obtained by positioning the sensor beneath the first and fifth dots following each streak and recording their coordinates giving the displacement vectors in image-space of several tiny Lagrangian fluid parcels over a 2.0 second time interval. These displacements were measured in $x''-z''$ coordinates. The usual $u-w$ component velocities ($u$ for the horizontal component and $w$ for the vertical component) could then be determined by converting the Lagrangian displacement components from image-space to real-space, dividing by 2.0 seconds, and finally by doing an $x''-z''$ to $x-z$ coordinate transformation. A velocity obtained in this manner could also be thought of as the average instantaneous velocity of a Lagrangian fluid parcel over a two second time interval.

The two reference lines shown in Figure 2 were marked on the front wall of the tank. They were 20 cm high and 62.2 cm apart. These lines were visible in the photographs and were used to make the transformation from image-space to real-space by recording the coordinates of their endpoints to indicate the equivalent image-space dimensions. However, the reference lines were located at $y = 0$ while the Lagrangian displacements to be measured were located at one of the other three previously mentioned
y-axis positions. Therefore, the first step in the image-space to real-space transformation was to multiply the measured displacements by a parallax correction factor computed as a function of y-axis position and the optical depth of water to obtain displacements as they would appear if projected onto the front wall of the tank. Subsequent multiplication of these corrected Lagrangian displacement components by the appropriate real-space to image-space ratio of reference line dimensions produced the desired real-space displacement components. Dividing by 2.0 seconds gave velocity components in \( x''-z'' \) coordinates which were then used to compute the \( u \) and \( w \) component velocities in \( x-z \) coordinates using simple trigonometric equations.

Therefore, crucial to the proper transformation of the data were the coordinates of the endpoints of the Lagrangian displacement vectors, the coordinates of the endpoints of the reference lines, the y-axis position of the illuminated vertical slab, and finally, for scaling purposes, the estimated average height of the ML top, \( h \). \( h \) was determined by estimating the distance between the tank floor and some parallel imaginary straight line drawn such that the variable area of tracer-filled ML fluid above it and the area of tracer-free SL fluid below it were approximately equal.

After all the necessary data had been collected and formatted onto floppy disc, it was then transferred to
digital tape and subsequently processed on the university's Cyber computer.

Data Processing

The Fortran program used to process the data was developed by Dr. James W. Deardorff and performed three primary functions. The first was to take the raw coordinate data and compute the u and w component velocities as described above. Next, these velocities were categorized into one of seven independent sublayers of equal thickness between \( z'' = 0 \) and \( (1.4)h \). A velocity was categorized by examining the midpoint of the \( z'' \) component of its Lagrangian displacement vector. The sublayer into which this midpoint fell was the sublayer to which both the u and w velocity components were assigned. A resolution within the ML of five sublayers was decided upon to minimize the number of Lagrangian displacement vectors which overlapped two or more sublayers and to insure that an adequate number of velocities were assigned to a sublayer so as to generate representative statistical estimates of the parameters necessary for the investigation. These include: the horizontal and vertical mean velocity, \( \bar{u} \) and \( \bar{w} \) respectively; the horizontal and vertical velocity variance, \( u'^2 \) and \( w'^2 \) respectively; the horizontal/vertical velocity covariance \( u'w' \), and the
third-order moments $w'^3$ and $w'u'^2$. Additional statistics concerning the $y$-axis component velocity, $v$, were calculated using data taken from the photographs of the $x''$-$y$ plane.
The mean temperature structure is shown in Figure 3. This was obtained in one experiment using a vertically traversing thermocouple successsively positioned at five different spots along the $y = 62$ cm center-line of the tank. The four non-central soundings were adjusted in reference to the middle sounding to account for the net...
warming rate during the measurements which was approximately 0.5 °C·min⁻¹.

The temperature structure within the ML is similar to what was expected except for the weaker horizontal temperature gradient. In the ideal case where the depth of the ML remains constant along \( x'' \), the expected temperature gradient along \( x'' \) is given by

\[
\frac{\partial T}{\partial x''} = (\tan \theta) \frac{\partial T}{\partial z}
\]  

(4.1)

where \( \theta \) is the slope angle, and the vertical temperature gradient on the right-hand side of (4.1) is measured just above the height of the ML top, \( z_i \), in the lowest part of the SL. In the laboratory model, however, the slope of the locally averaged height of the ML top, \( \bar{z}_i/\partial x \), actually lay intermediate between the horizontal and the slope of the tank floor. The result was an overestimate by (4.1) of \( \partial T/\partial x'' \).

The fact that \( \bar{z}_i/\partial x \) is somewhat less than 10° may be partially related to the enhanced subsidence in the region nearest the deep, penetrative updraft at the upslope sidewall of the tank. This is suggested by the stronger vertical temperature gradients in the SL for increasing values of \( x'' \). \( \bar{z}_i/\partial x \) was also less than 10° during preliminary experiments with the upslope sidewall heating
tube removed, however. So the uneven depth of the ML cannot be completely attributed to the enhanced subsidence at high values of $x$.

The ML depth, as estimated from Figure 3, seems to lie between 9 and 12 cm with a mean ML temperature of around 21.5 °C. Evidently, the soundings were taken while the ML was still young and growing whereas the droplet photographs were taken at a later time when the mean ML temperature was in the neighborhood of 25 °C and the ML depth was closer to 15 cm. It is assumed in the discussions which follow that the mean horizontal temperature gradient within the ML when $h = 15$ cm was the same as that given in Figure 3.

![Vertical profiles of $\bar{u}$ (x data points) and $\bar{w}$ (circle data points).](image-url)
The vertical profiles of $\bar{u}$ and $\bar{w}$ are shown in Figure 4. The data points have been connected with smoothed, solid curves where the data is sound. The dashed curves below $z''/h = 0.13$ indicate that the profiles have been extrapolated.

The upslope flow near the surface, and the return flow aloft within the ML are both evident in the profile for $\bar{u}$. A deep layer of negative shear exists between the positive $\bar{u}$ maximum near $z''/h = 0.1$ and the negative maximum near $z''/h = 0.8$. Strong positive shear is found in the thin surface layer, the small thickness of which could not be determined from the data available. Another region of positive shear falls roughly within the entrainment zone at the top of the ML where $\bar{u}$ falls off from its negative maximum to zero in the lowest part of the SL. The uppermost $\bar{u}$ data point might indicate the presence of the deep-thermal outflow in the upper portion of the SL. However, statistics computed at this level come from a sample of only 18 velocities so that nothing definitive can be said about this region.

Flaws in the data collection procedure resulted in the fictitiously large $\bar{w}$ values in the upper portions of the ML. Deardorff et al. (1980) found that the base of the entrainment zone should approximately coincide with the height where the buoyancy flux first crosses zero. Numerous studies find this height to be near $z''/h = 0.85$. 
(Willis and Deardorff, 1974; Lenschow, 1974; Lenschow et al., 1980; Deardorff and Willis, 1985). Deardorff et al. (1980) also found that within the entrainment zone a significant amount of ML fluid is actually detrained into the SL. When this happens, oil droplets in the detrained fluid will become buoyant and start to rise because their thermal expansion coefficient is then higher than that for water. Therefore, starting at about \( z''/h = 0.85 \), the data becomes increasingly contaminated with buoyant SL velocities as one moves upward. Above the entrainment zone, nearly all of the sampled velocities were invalid, and an oil droplet rise rate of about 0.5 cm·s\(^{-1}\) was reached. The two \( w \) values falling within the entrainment zone proportionally reflect their dependence upon the degree of data contamination.

In the lower ML \( w \) is positive reflecting the upslope nature of the mean flow. However, \( w \) in this region was not as strong as expected. I define two new tilted-coordinate velocities, \( u_{x''} \) and \( w_{z''} \), such that \( u_{x''} \) is the mean tilted-horizontal velocity whose directional vector runs parallel to \( x'' \), and \( w_{z''} \) is the mean tilted-vertical velocity whose directional vector runs parallel to \( z'' \). These are computed from the profiles for \( u \) and \( w \) using

\[
\bar{u}_{x''}(z'') = \bar{u}(z'') \cos \theta + \bar{w}(z'') \sin \theta
\]
\[
\bar{w}_{z''}(z'') = \bar{w}(z'') \cos \theta - \bar{u}(z'') \sin \theta
\]
where $\theta = 10^\circ$.

Fig. 5. Vertical profiles of $\bar{u}_x''$ (x's) and $\bar{w}_z''$ (circles).

It was hoped that $\bar{w}$ could be directly calculated from $\bar{u}$ with

$$\bar{w}(z'') = \bar{u}(z'')\tan\theta. \quad (4.3)$$

This holds in the special case where $\bar{w}_z''$ vanishes at all heights so that $\bar{u}$ and $\bar{w}$ become the true horizontal and vertical velocity components of $\bar{u}_x''$. That is, when (4.3) holds,

$$\bar{u}(z'') = \bar{u}_x''(z'')\cos\theta,$$
\[ \bar{w}(z^\prime) = \bar{u}_{x^\prime}(z^\prime) \sin \theta, \]

and \[ \bar{w}_{z^\prime}(z^\prime) = 0 \text{ for all } z^\prime. \]

But this was not the case as Figure 5 illustrates. Figure 5 shows the profile of \( \bar{w}_{z^\prime} \) obtained from (4.2), and it is indicative of a mean flow that was toward the bottom surface in the lower part of the ML. The mass continuity equation in \( x^\prime-z^\prime \) coordinates

\[
\frac{\partial \bar{u}_{x^\prime}}{\partial x^\prime} = - \frac{\partial \bar{w}_{z^\prime}}{\partial z^\prime}
\]

\[
\frac{\partial \bar{v}}{\partial y} = 0
\]

would then require the mean-flow to be accelerated upslope below \( z^\prime/h = 0.13 \). At very short distances from the bottom plate of the tank \( \bar{w}_{z^\prime} \) approaches zero and (4.3) holds with \( \theta = 10^\circ \). This is why a jump appears in the extrapolated portion of the profile for \( \bar{w} \) in Figure 4.

When the experimental values for \( \bar{u} \) and \( \bar{w} \) at the lowest two levels are used, (4.3) reveals that the upslope flow in the lower ML was tilted only about \( 6^\circ \) off the horizontal. This is very close to \( \tan^{-1}(\partial z_1/\partial x) \) suggesting a dependence of the slope of the mean interface between the ML and SL upon the mean-flow orientation.
VARIANCES

The variance of the downstream horizontal velocity component, $u'^2$, normalized by $w^2$, is shown in Figure 6a.

Fig. 6. Vertical profiles of a) horizontal velocity variance and b) vertical velocity variance normalized by $w^2$. x's and solid curves: present tilted-tank measurements; circles and dashed curves: horizontal convection tank measurements of DW; dash/dot curve: aircraft measurements reported by Lenschow et al., 1980. Asterisks denote values which were computed from the velocities measured in the $x''$-$y$ plane.
\( w_\ast \) is the convective velocity scale defined in the introduction. In the convection tank \( w_\ast \) is given by

\[
    w_\ast = \left( g\alpha_0 \frac{\overline{w'T'}_s}{\overline{u'}_s} \right)^{1/3}
\]

where the atmospheric buoyancy parameter \( g/T \) has been replaced by \( g\alpha_0 \). With \( \overline{h} = 15 \text{ cm}, \alpha_0 = 2.6 \times 10^{-4} \text{ °C}^{-1} \) (corresponding to \( T_0 = 25 \text{ °C} \)), \( w'T'_s = 0.20 \text{ °C cm s}^{-1} \), and \( g = 980 \text{ cm s}^{-2} \), a \( w_\ast \) value of 0.92 cm s\(^{-1}\) is obtained. \( w'T'_s \) is calculated by vertically integrating the simplified thermodynamic equation

\[
    \frac{\partial \overline{T}_{\text{ML}}}{\partial t} + \overline{u''_x(z')} \frac{\partial \overline{T}(x',z)}{\partial x''} = - (\sec\theta) \frac{\partial (\overline{w'T'})}{\partial z''} . \tag{5.1}
\]

This calculation is explicitly carried out in Appendix II.

Shown also in Figure 6a is the \( u'^2 \) profile from Deardorff and Willis (1985) (hereafter called DW) taken from the same convection tank with \( \theta = 0^\circ \). In comparison, it is seen that these are two very different profiles. With \( \theta = 0^\circ \) the data present two maxima in horizontal variance; one in the lower ML near the bottom surface of the tank, and another just below the ML top.

This secondary maximum near \( z = \overline{h} \) represents a partial conversion of vertical kinetic energy to horizontal kinetic energy (kinetic energy, here, refers to the kinetic energy
of the turbulence and is not to be confused with the kinetic energy of the mean-flow) when dominant plumes rise through the ML and bump into the capping inversion. Numerical simulations by Deardorff (1972, 1974) showed that most of the kinetic energy stored in these overshooting plumes is absorbed in forcing entrainment and internal gravity waves rather than in conversion to horizontal kinetic energy. Nevertheless, a strong capping inversion will produce a noticeable peak in the horizontal variance near $z = h$ (Willis and Deardorff, 1974 case S2).

In the case where $\theta = 10^\circ$, the data show a rather broad maximum in the middle portion of the ML where a minimum exists when $\theta = 0^\circ$. Evidently, this is a direct consequence of the presence of mean-flow shearing in the case where $\theta = 10^\circ$ and lack thereof when $\theta = 0^\circ$. An examination of the turbulent kinetic energy budget will help to verify this.

In view of the fact that strong vertical shearing of $\overline{u}$ also existed near the surface, it seems logical that another maximum in $\overline{u'^2}$ would have manifested itself in this region as well. Yet the evidence in support of this is lacking in the data taken from the photographs of the $x$-$z$ plane.

The photographs of the $x''$-$y$ plane, however, tell a different story. Two of the values for $\overline{u'^2}$ extracted from this data set are denoted by the asterisks in Figure 6a.
While the value near $z''/h = 0.5$ agrees well with its counterpart from the $x''-z''$ plane, the value at $z''/h = 0.13$ differs substantially suggesting that another maximum in $u'^2$ was present near the surface. For the reason stated above, it would seem that the $x''-y$ data is theoretically more sound at $z''/h = 0.13$. However, as no clear reason for questioning the validity of either value in favor of the other has been found, the profile line has been drawn in as a compromise between the two (Deardorff and Willis, 1987).

The tranverse velocity variance $v'^2$ was also calculated from the $x''-y$ photographs. Ratios of $v'^2/u'^2$ were found to be 1.02 and 0.95 at $z''/h = 0.13$ and 0.53 respectively. Townsend (1976) reports values between 0.5 and 0.8 for shear-driven, neutral boundary layers. This seems to suggest that convection was most influential in the horizontal structuring of the turbulence. Evidently, vertical shearing must be far more substantial relative to convective instability than obtained here in order to produce significant departures from horizontal isotropy.

The vertical velocity variance, $w'^2$, normalized by $w^2$, is shown in Figure 6b. Shown also in Figure 6b, for comparison, is the profile from DW and the curve presented by Lenschow et al. (1980) for a baroclinic, atmospheric boundary layer. All three profiles have $w'^2$ increasing with height to a maximum at about $z''/h = 0.3$, then decreasing steadily above that to near zero at a low point in the stratified-layer. The tilted-tank data, however,
show magnitudes of $\bar{w'}^2$ far in excess of those seen in the other two studies. This is especially true in the region between $z''/h = 0.3$ and 0.5.

The $\bar{w'}^2$ profile, when examined in conjunction with the profile for $\bar{u'}^2$, indicates the likely presence of convective plumes extending from near the tank floor to about $z''/h = 0.5$ or 0.7 where the shearing is strongest. It is within this layer that $\bar{w'}^2$ decreases most rapidly and $\bar{u'}^2$ reaches its peak suggesting a conversion from vertical kinetic energy to horizontal kinetic energy. If so, then this occurs at a smaller height than what would normally be expected. The horizontal and vertical variance profiles of Willis and Deardorff (1974) and DW suggest the presence of plumes which extend almost throughout the entire depth of the ML. This may serve to illustrate the extent to which vertical shearing of the mean horizontal wind is capable of disrupting the vertical convective structure of the flow.
The vertical profile of the downstream, horizontal/vertical covariance, \( u'w' \) (called the stress or the momentum flux), non-dimensionalized by \( w_*^2 \) is shown in Figure 7. In comparison with the mean wind profile in Figure 4, we see that momentum is transferred with the gradient of mean horizontal wind over most of the ML. The

Fig. 7. Normalized stress profile. Dashed curve is an hypothesized extrapolation.
only visible exception to this occurs roughly in the
entrainment zone where the momentum flux is
countergradient. Below $z''/h = 0.13$, $u'w'$ presumably goes
negative in response to the reversing wind shear.

At $z''/h = 0.5$, $K_m/(w_*h) = 0.06$ where

$$K_m = -\frac{u'w'}{\partial u/\partial z}$$

is the "eddy viscosity". This is in accordance with the
finding of Wyngaard (1983) that $K_m$ has a magnitude on the
order of $0.05w_*h$ in the middle portion of a convective,
baroclinic boundary layer. Wyngaard demonstrated how this
finding leads to linear differential equations which can be
solved to show that under very convective conditions the ML
has wind shear values which are much smaller than the
geostrophic wind shear.

In most atmospheric studies, stress is scaled by the
square of the friction velocity $u_*$. Unfortunately, an
accurate direct measurement of $u_*$ could not be performed.
Moreover, given that $\partial u/\partial z$ changes sign twice within the
ML, the influence of $u_*$ on the stress profile could hardly
be considered exclusive. Other important influences
include the magnitude of the baroclinicity (responsible for
the low-level wind maximum and, therefore, the negative
shearing) and the entrainment flux of momentum in the upper
Thus, the efficacy of scaling $u'w'$ by $u_\ast^2$ is diminished in all but the lowest portions of the baroclinic ML. Indeed, as was pointed out in the introduction, the ML is defined as the region within the boundary layer which is not influenced by $\tau_0 = \rho u_\ast^2$. Therefore, $u'w'$ here was instead scaled by $w_\ast^2$ as it appears in the non-dimensional equation for turbulence kinetic energy.

A quantitative description of the influence of baroclinicity on the stress profile is derived from the equation of motion for the mean horizontal wind. Neglecting the effects of viscosity and coriolis accelerations this takes the form

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{w} \frac{\partial \bar{u}}{\partial z} = - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \frac{\partial (\bar{u}' \bar{w}')}{\partial z} \tag{6.1}$$

where $\bar{v} = 0$. Assuming steady state flow and neglecting advection by the mean wind reduces (6.1) to a balance between the pressure gradient force and the shear of the Reynold's stress giving

$$\frac{\partial (\bar{u}' \bar{w}')}{\partial z} = - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} \tag{6.2}$$

Differentiating both sides with respect to $z$ while using the hydrostatic approximation, $\bar{\rho}/\partial z = -\bar{\rho}g$, and the equation of state (Pedlosky, 1979, eqn. 1.4.20),
\[
\bar{\rho} = \rho_0 [1 + \alpha_o (T_0 - \bar{T})],
\]

changes (6.2) to

\[
\frac{\partial^2 (u'w')}{\partial z^2} = - g \alpha_o \frac{\partial \bar{T}}{\partial x}. 
\tag{6.3}
\]

This equation relates the curvature of the stress profile to the horizontal temperature gradient within the ML. Using values given in Appendix II, the right-hand side of (6.3) gives a stress profile curvature of about \(-4 \times 10^{-3} \, \text{s}^{-2}\) which is very close to what actually occurs in the region between \(z''/h = 0.4\) and \(0.7\). Below \(z''/h = 0.4\), however, the magnitude of the curvature increases significantly by a factor of 3 or more. This discrepancy is thought to extend from the neglected advection terms in (6.1) which after differentiation with respect to \(z\) would appear on the right-hand side of (6.3) as \((\bar{u}) \partial^2 \bar{w}/\partial z^2\) and \(- (\bar{w}) \partial^2 \bar{u}/\partial z^2\) (Deardorff and Willis, 1987). Here the incompressible continuity equation

\[
\frac{\partial \bar{u}}{\partial x} = - \frac{\partial \bar{w}}{\partial z},
\tag{6.4}
\]

\[
\frac{\partial \bar{v}}{\partial y} = 0
\]
has been used to change the appearance of the first of these terms and to bring about the cancellation of two others. Since the mean wind profile below $z''/h = 0.13$ is unknown these differentiated advection terms (note that after differentiating (6.1) with respect to $z$ these terms represent, in part, the advection of mean vorticity in the $x$-$z$ plane) are impossible to determine accurately. A reasonable estimate, however, indicates that each is capable of exceeding the baroclinic term by an order of magnitude or more below $z''/h = 0.13$ where mean wind curvatures and the mean wind itself were at peak magnitudes. The implication is that stress profiles are additionally influenced by the mean vertical motion induced within slope flows.

In the tilted-coordinate system with $w_{z''} = 0$ for all $z''$, these vorticity advection terms would vanish. However, since the mean flow did not remain parallel to the tank floor (ie., $w_{z''}$ became nonzero at short distances above the lower boundary), the tilted-coordinate advection terms retain their importance. This may be of general significance for slope flows when one considers that the dynamic adjustment of any initially static fluid of nonuniform density is primarily controlled by the gravity vector. For a temperature distribution like that shown in Figure 1, the fluid is mixed in such a manner as to produce a final state with zero available potential energy. This
means the isotherms will be perpendicular to $\vec{g}$ and parallel to $x$ with the temperature increasing upward. The influence of the sloping lower boundary on the motion of the fluid decreases rapidly with increasing distance from it, while the influence of $\vec{g}$ is felt equally everywhere. With this in mind it should come as no surprise that the mean flow did not align itself perfectly with $x''$. This may be remedied to some unknown extent if the length of the slope is increased relative to $h$. But if in general for well developed convective slope flows the mean wind does not run parallel to the sloping surface, then it may also be said that an intrinsic feature of such flows is the additional influence of mean vorticity advection on the stress profile at small $z''/h$. 
TURBULENCE KINETIC ENERGY

The equation for turbulence kinetic energy (TKE) is given by

\[
\frac{\partial \overline{e}}{\partial t} = g \alpha (w'T') - u'w' \frac{\partial \overline{u}}{\partial z} + (u'^2 - \overline{w'^2}) \frac{\partial \overline{w}}{\partial z} \\
\text{I} \quad \text{II} \quad \text{III} \\
- \frac{\partial (\overline{w'e})}{\partial z} - \frac{1}{\rho_o} \frac{\partial (\overline{w'p'})}{\partial z} - \varepsilon \\
\text{IV} \quad \text{V} \quad \text{VI}
\]

where \( e = (1/2) (u'^2 + v'^2 + w'^2) \) is the mean TKE per unit mass and \( \varepsilon = \nu \overline{(\partial u'_i/\partial x_j)^2} \) (\( \nu \) is the kinematic viscosity).

(7.1) is made dimensionless after multiplication by \( h/w_x^3 \).

The left-hand side of (7.1) represents the time rate of change of the TKE. This is assumed to be zero. Advection also is assumed to be insignificant and thus is omitted. Term I is buoyancy production. It is a source in the ML and a sink in the entrainment zone. Term II represents shear production. It is a source of TKE except when the momentum flux is countergradient. Term III is a mechanical
production term significant to this particular type of flow because of the induced vertical motion field. Term IV is called turbulent-transport. This term redistributes TKE without creating or destroying it. Term V is another redistribution term called pressure-transport. Finally, term VI represents the viscous dissipation of TKE.

The fact that terms II, III, and IV were the only ones to be measured directly gives this budget a sketchy quality. The buoyancy term was assumed to have a shape identical to that found in the laboratory by DW for a convective ML with no mean shearing. Similar profiles have been obtained repeatedly in other laboratory studies (Willis and Deardorff, 1974; Deardorff et al., 1980; DW) and also in several atmospheric studies (Lenschow, 1974; Pennell and LeMone, 1974; Caughey and Palmer, 1979; Lenschow et al., 1980). The pressure-transport was assumed to cancel 40% of the turbulent-transport as proposed by Zeman and Lumley (1976). Experimental justification for this assumption may be loosely drawn from Lenschow et al. (1980), Deardorff (1980), and DW. Dissipation was then taken to be the residual.

For some parts of the TKE budget discussion it will be helpful to break down equation (7.1) into two component equations representing the horizontal and vertical kinetic energy budgets. These are given as
\[
\frac{\partial}{\partial t}\left[ \frac{u'^2}{2} + \frac{v'^2}{2} \right] = -u'w' \frac{\partial u}{\partial z} + u'^2 \frac{\partial w}{\partial z} - \frac{\partial (w'u'^2)}{\partial z} + \frac{p'}{\rho_0} \left[ \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right] - \varepsilon_H + \frac{p'}{\rho_0} \left[ \frac{\partial w'}{\partial z} \right] - \frac{1}{\rho_0} \frac{\partial (w'p')}{\partial z} - \varepsilon_V
\]

(7.2)

where (7.2) is the horizontal kinetic energy equation and (7.3) is the equation for vertical kinetic energy. With regard to the terms found in (7.1), (7.2), and (7.3) the following rules apply:

\[\text{III}_H + \text{III}_V = \text{III}\]

\[\text{IV}_H + \text{IV}_V = \text{IV}\]

\[\text{VI}_H + \text{VI}_V = \text{VI}\]

(7.4)

\[\text{VII}_H + \text{VII}_V = 0.\]

The assumption concerning term \(\text{IV}_H\) that \(w'u'^2 + w'v'^2 = 2w'u'^2\) is considered reasonable since the previously
reported values of the ratio $\bar{\nu'^2}/\bar{u'^2}$ were very close to 1.0. Terms VII$_H$ and VII$_V$ are known as return-to-isotropy terms. They represent the intercomponent transference of energy and are not found in (7.1) because their sum is zero.

A plot of the normalized TKE balance is given in Figure 8. The assumed standard buoyancy profile is seen to decrease linearly with height up to its zero cross-over point at $z''/h = 0.83$. The heat flux is expected to remain

![Normalized budget of turbulence kinetic energy versus $z''/h$.](image)

Fig. 8. Normalized budget of turbulence kinetic energy versus $z''/h$. Curve I (right-hand solid) is buoyant-production; curve II (long-dash curve drawn through boxes) is shear-production; curve III (short-dash) is mechanical production from term III; curve IV (long-dash/dot curve drawn through triangles) is turbulent-transport; curve V (short-dash/dot) is pressure-transport; curve VI (left-hand solid curve drawn through circles) is the residual (dissipation).
negative within the entrainment zone as warmer SL water is entrained into the cooler ML water.

The shear-production of TKE, term II, was calculated in a straightforward manner using Figures 4 and 7. Since \( \frac{\partial u}{\partial z} \) changes sign twice within the ML, term II theoretically falls to zero at these heights which are located approximately at \( z''/h = 0.1 \) and 0.8. Between these two heights is a region of strong shear-production of TKE with a maximum near \( z''/h = 0.5 \). This should largely explain the mid-level maximum in \( u'^2 \) seen in Figure 6, and

![Normalized budget of horizontal kinetic energy versus z''/h.](image-url)

Fig. 9. Normalized budget of horizontal kinetic energy versus \( z''/h \). Curve II (long-dash) is shear-production; curve III (short-dash) is mechanical production; curve IV (long-dash/dot curve drawn through triangles) represents turbulent-transport of horizontal kinetic energy; curve VI (solid) is dissipation; curve VII (dash/double-dot curve drawn through diamonds) is intercomponent energy transfer.
it raises questions concerning the validity of the earlier hypothesis that mean-flow shearing in the ML may have aided in the conversion of vertical kinetic energy to horizontal kinetic at mid-levels thereby contributing to the $u'^2$ maximum in that region.

In an effort to assess the relative importance of these two horizontal kinetic energy generating mechanisms, the budgets of horizontal and vertical kinetic energy given by (7.2) and (7.3) are presented in Figures 9 and 10 respectively. These budgets are also very sketchy due to

![Normalized budget of vertical kinetic energy versus $z''/\bar{h}$](image_url)

**Fig. 10.** Normalized budget of vertical kinetic energy versus $z''/\bar{h}$. Curve I (right-hand solid) is buoyancy-production; curve III$_V$ (short-dash) is mechanical production; curve IV$_V$ (long-dash/dot curve drawn through triangles) represents turbulent-transport of vertical kinetic energy; curve V (short-dash/dot) is pressure-transport; curve VI$_V$ (left-hand solid) is dissipation; curve VII$_V$ (dash/double-dot curve drawn through diamonds) is intercomponent energy transfer.
the assumptions concerning the numerous unmeasured terms. In the vertical kinetic energy balance, terms $III_V$ and $IV_V$ are the only terms to be measured directly. The buoyancy and pressure-transport terms are assumed as previously described, while the dissipation, $\varepsilon_V$, is estimated assuming isotropy on small scales by $\varepsilon_V = \varepsilon/3$. In the horizontal kinetic energy balance terms $II$, $III_H$, and $IV_H$ were measured while the dissipation, $\varepsilon_H$, was estimated with $\varepsilon_H = 2\varepsilon/3$. The return-to-isotropy terms, $VII_H$ and $VII_V$, were subsequently estimated from (7.2) and (7.3) respectively assuming steady state.

In spite of the crudity of these budgets, Figures 9 and 10 convincingly illustrate that kinetic energy was indeed transferred from the vertical component to the horizontal components at mid-levels as hypothesized with the height of maximum intercomponent transfer being at $z''/h = 0.5$. A question which now arises concerns the extent to which this conversion is related to the presence of mean-flow shearing. Part of it should be unrelated since we would not expect $u'^2$ to disappear as term II approaches zero. Rather, we would expect term VII to become the sole source of $u'^2$ in the absence of mean shearing.

Insight into this question may be gained by comparing the return-to-isotropy terms in the present study to those calculated from the data given in DW. Horizontal and vertical kinetic energy budgets were calculated from
Figures 14 and 15 in DW using (7.2) and (7.3) as before except with terms II and III removed. The residual return-to-isotropy terms are shown in Figure 11 along with those from the present study.

Fig. 11. Vertical profiles of intercomponent energy transfer for the present tilted-tank study (dash/double-dot curves) and the free convection study of DW (solid curves drawn through circles). Below \( z''/\bar{h} = 0.7 \), the left-hand side curves represent energy loss in the vertical component while the right-hand side curves represent energy gain in the horizontal components.

Figure 11 shows a sharp contrast in the behavior of term VII when mean-flow shearing is absent. Both studies show that kinetic energy is predominately transferred from the vertical component to the horizontal. The DW free convection profiles, however, indicate that without the
presence of mean-flow shearing in the ML, term VII has maxima in the lower and upper ML and a minimum at mid-levels. Once again, this is suggestive of convective plumes which extend almost throughout the entire depth of the ML. The maximum in vertical to horizontal energy transference in the upper ML arises as the upward progress of rising thermals is impeded by the temperature inversion which caps the ML. On the other hand, when strong mean-flow shearing is present in the ML, term VII shows a sharp maximum at mid-levels and falls off rapidly above and below this region. These profiles are more suggestive of convective plumes which extend roughly between the surface and $z''/h = 0.6$. In this case, the upward progress of rising thermals is apparently impeded by the mean horizontal wind shear which is strongest at $z''/h = 0.5$. It therefore appears that while vertical shearing of the mean horizontal wind at mid-levels in the ML was an effective producer of horizontal kinetic energy via term II, it also was very effective at converting buoyancy-generated vertical kinetic energy to horizontal kinetic energy via term VII. Furthermore, since Figure 9 suggests that these two horizontal kinetic energy generating mechanisms were comparable in magnitude, one is lead to conclude that the mid-level maximum in $u''^2$ is best explained by the sum of their effects. DW found a mid-level minimum in $u''^2$ in the absence of local mean shearing.
Referring back to Figure 9, at $z''/\bar{h} = 0.5$ term II offset about 55% of the dissipation rate, while in an earlier study Caughey and Wyngaard (1979) found three runs from the 1973 Minnesota atmospheric boundary layer experiment for which the shear production term averaged about 30-50% of mid-layer dissipation. These runs also were characterized by convective conditions and the presence of baroclinicity.

Above $z''/\bar{h} = 0.8$ the momentum flux, $\bar{u}'w'$, is countergradient resulting in negative shear-production. This phenomenon has been previously observed and reported on in the atmosphere (Pennell and LeMone, 1974) and involves a process by which TKE is converted into kinetic energy of the mean flow.

Term III proved to be a significant though weak source of TKE near $z''/\bar{h} = 0.3$. Figure 9 indicates that $\text{III}_h$ was a sink for horizontal kinetic energy, while Figure 10 shows that $\text{III}_v$ was a more substantial source of vertical kinetic energy. This may help to explain the disproportionately large magnitude of $\bar{w}'^2$ relative to $\bar{u}'^2$ between $z''/\bar{h} = 0.3$ and 0.5. Above $z''/\bar{h} = 0.7$ the significance of term III is unknown due to uncertainty in $\bar{w}$.

The turbulent-transport, term IV, was obtained by differentiating the curve for $\bar{w}'e = (2\bar{w}'u' + \bar{w}'^3)/2$ shown in Figure 12. The individual contributions from $\bar{w}'^3$ and $2\bar{w}'u'^2$ are also shown and indicate that the magnitude of
Fig. 12. Vertical profiles of: $\frac{w'e}{w_3^3}$ (solid curve); $\frac{w'^3}{w_3^3}$ (dashed curve drawn through circles); and $2w'u'^2/w_3^3$ (dash/dot curve drawn through x's). Curves are extrapolated below $z''/h = 0.13$.

$w'^3$ is somewhat more substantial than the magnitude of $2w'u'^2$ for the bulk of the ML. This is to be expected and is consistent with the findings of other studies (Lenshow et al., 1980; DW).

Because the expected scatter is larger for statistical estimates of third order moments like $w'u'^2$ and $w'^3$ than it is for second order moment and mean estimates from the same samples (Lumley and Panofsky, 1964), some thought was given to assuming a zero value for $2w'u'^2$ throughout the ML. The very slight amount of scatter found in the data for the covariance $u'w'$ (which ordinarily shows considerable
scatter in atmospheric studies; for example, Lenschow et al., 1980), however, suggests that the sample sizes are adequately large to ensure good overall reliability in the statistical estimates. For this reason, and also since the ratio $\frac{\bar{\omega}'u'^2}{\bar{\omega}^3}$ below $\frac{z''}{h} = 1.0$ varied roughly from -1.0 to +0.5, it was decided that the profile for $\bar{\omega}'u'^2$ should be treated as significant and therefore included in the calculation of $\bar{\omega}'e$.

The resulting profile for $\frac{\bar{\omega}'e}{\bar{\omega}^3}$ differs somewhat from profiles previously presented by Lenschow et al. (1980) and DW (see Figure 13). Their profiles are similar and very smooth with maxima of 0.13 at $\frac{z''}{h} = 0.35$ and 0.15 at $\frac{z''}{h} = 0.4$ respectively. The current profile has a

Fig. 13. Vertical profiles of $\frac{\bar{\omega}'e}{\bar{\omega}^3}$ taken from the present study (solid curve), DW (dashed curve), and Lenschow et al. (1980) (dash/dot curve).
maximum of 0.19 at $z''/h = 0.55$. This is significant not only because the larger maximum implies more transport of TKE, but also because the greater relative height at which the maximum occurs mandates an upward shift of the transport curve at midlevels. Figure 9 reveals that this upward shift is mostly due to the vertical diffusion of shear-generated TKE at mid-levels.

The pressure-transport, term V, is also drawn in Figure 8. The curve presented is a smooth fit to the data points obtained from

$$\frac{1}{\rho_0} \frac{\partial \overline{w'p'}}{\partial z} = -0.4 \frac{\partial \overline{w'e}}{\partial z}. \quad (7.5)$$

The normalized values for $\frac{\partial \overline{w'e}}{\partial z}$ used in (7.5) are plotted as triangles in Figure 8.

After making the assumption (7.5), the residual was calculated by summing terms I through V and subtracting this from zero. The result, shown in Figure 8 by the circles, represents dissipation, $\mathcal{E}$, plus the sum of neglected terms, computational errors, and errors of assumption such as those concerning terms I and V.

Numerous studies lend support to the notion that the normalized dissipation rate remains nearly constant throughout the ML with a slightly decreasing upward trend (Deardorff, 1974; Lenschow, 1974; Kaimal et al., 1976;
Caughey and Palmer, 1979; Caughey and Wyngaard, 1979; Lenschow et al., 1980). These studies also indicate that maximum normalized dissipation rates in the ML occur at small $z/h$. If so, then the residual in Figure 8 most resembles a typical dissipation profile at mid-levels between $z''/h = 0.3$ and 0.7 while displaying uncharacteristic drops in magnitude at high and low $z''/h$. It is important to consider, however, that conditions in the above-mentioned studies were not characterized by the type of shear-production found in the present study at mid-levels.

The results of atmospheric studies by Wyngaard and Coté (1971), McBean and Elliot (1975), and Caughey and Wyngaard (1979) imply that shear-produced TKE is dissipated locally in the surface layer. In the present study between $z''/h = 0.3$ and 0.7, Figure 9 shows that about half of the shear-production was offset by term $IV_H$. The horizontal kinetic energy produced in this region by term II which was not exported by term $IV_H$ remained to be dissipated locally thereby producing the dissipation maximum in this region. So a mid-level dissipation maximum could be the natural result of substantial shear-production at mid-levels. However, a very small value close to zero for $\varepsilon$ at $z''/h = 0.9$, and a 50% reduction in $\varepsilon$ from $z''/h = 0.3$ to 0.2 point to possible errors in the other terms in (7.1) used to calculate $\varepsilon$. 
One possibility is that the turbulent-transport term may be significantly inaccurate. Given the relatively high expected scatter in statistical estimates of $w'^3$ and $w'u'^2$, this would not seem at all improbable.

Perhaps a more likely source of error, however, stems from the pressure-transport assumption (7.5). McBean and Elliot (1975) studied the atmospheric surface layer at Suffield, Alberta in July of 1971 and discovered that $w'p'$ becomes increasingly negative with increasing instability. This implies that term V can be an energy source at small $z''/h$. They also found that pressure-transport can be larger (but opposite in sign) than turbulent-transport in this region—a finding which is additionally supported by the results of other studies (Wyngaard and Coté, 1971; Caughey and Wyngaard, 1979). In the present case, if this were true as high as $z''/h = 0.2$ the local minimum in the dissipation residual here would completely vanish, and the dissipation value at $z''/h = 0.13$ would be increased to a higher value more consistent with its proximity to the surface layer.

In the upper ML, Deardorff's (1980) three-dimensional numerical simulations of a stratocumulus capped ML found that at and just below cloud base the pressure-transport term actually complements the turbulent-transport term and contributes more toward the maintenance of TKE in that region. This also agrees with the results of Lenschow
et al. (1980) whose pressure-transport term (inferred from the budget imbalance) became positive above $z/h = 0.7$. If this were additionally the case in the present study, the dissipation residual would be boosted to a more realistic value at high $z''/h$. 
In this study, mixed layer scaling was used to investigate the turbulence within an inclined laboratory convection tank. Tilting the tank by an angle of $10^\circ$ resulted in a ML which was baroclinic with a low-level upslope mean wind maximum and an upper-level downslope return flow giving a thick layer of mean-flow shearing at midlevels. This shearing was strong enough to cause the rate of shear-production of TKE to exceed the rate of buoyancy-production between $z''/h = 0.5$ and 0.7. This occurred in spite of the Arya and Wyngaard (1975) prediction that strong convection should limit mean-flow shearing in the ML.

Vertical profiles of normalized velocity variance in the present, tilted-tank, baroclinic study differed significantly from those of the earlier, horizontal-tank, free convection study of DW. A maximum in the longitudinal velocity variance, $\overline{u'^2}$, at mid-levels was directly related to the presence of mean-flow shearing in this region. Two mechanisms are offered to explain the mid-level maximum in $\overline{u'^2}$:
1) horizontal kinetic energy was created directly through the interaction of $\frac{\partial u}{\partial z}$ and $u'w'$ (term II in (7.1) and (7.3)) in this region; and

2) strong mean-flow shearing at mid-levels amplified the rate of conversion from vertical kinetic energy to horizontal kinetic energy (terms $VII_H$ and $VII_V$ in (7.2) and (7.3) respectively) in this region.

Figure 9 shows that these two horizontal kinetic energy generating mechanisms were comparable in magnitude at mid-levels.

Mean-flow shearing caused the maximum in vertical to horizontal kinetic energy transfer to occur at a lower height than in DW. Figures 6 and 11 suggest that in the free convection case convective plumes extended almost throughout the entire depth of the ML, while in the baroclinic case, the indication is that most convective plumes had their tops sheared off near $z''/h = 0.6$. This serves to illustrate the large extent to which mean-flow shearing is capable of shaping the vertical structure of the convective, baroclinic boundary layer.

Horizontal structure was predominately controlled by convection, however. This was indicated by a lateral to longitudinal velocity variance ratio, $\frac{\bar{v}'^2}{\bar{u}'^2}$, which was very close to unity at $z''/h = 0.53$ where mean-flow shearing was relatively strong.
The vertical profile of vertical velocity variance, $w'^2$, normalized by $w^*^2$ was consistently larger than in the profiles of DW and Lenschow et al. (1980). This was especially true in the region from $z''/h = 0.3$ to 0.5. Here also it was found that $w'^2$ was up to 2.1 times larger than $u'^2$. This may be partially explained by the mechanical production term $-(w'^2)\partial w/\partial z$ which was a substantial source of $w'^2/2$ in this layer. Conversely, $(u'^2)\partial w/\partial z$ was a significant sink of $u'^2/2$. These terms are absent in studies where $w = 0$ for all $z$.

Total TKE was somewhat larger for the present study than for DW - especially at mid-levels. Since mixed-layer scaling should account for variations in lower surface heating, the difference was due primarily to shear-production of TKE and, to a lesser extent, the mechanical-production of TKE by term III in (7.1). A more elaborate scaling scheme which accounts for these influences is required for the proper comparison of any future studies of this sort. A comprehensive scaling scheme would also account for the effects of entrainment. In the present study, mixed-layer scaling was used so that the effects of mean-flow shearing could be isolated when the present results were compared to those of DW.

One of the more troublesome aspects of this study was the discovery that the mean flow did not remain parallel to the lower surface. Instead, it was tilted by an angle of
about 6° along with \( \frac{\partial z_i}{\partial x} \). In the future, this might be alleviated somewhat by extending the length of the lower boundary in proportion to \( h \) or by decreasing the surface slope. Complete eradication of this problem may be impossible, however, due to the omnipresent influence of gravity which controls the dynamic adjustment of any fluid to a density field which is non-uniform. The influence of a sloping surface on a baroclinic, convective mean-flow is expected to diminish rapidly at increasing distances from it thereby increasing the relative importance of the influence of gravity. In the present case, the result was a mean-flow orientation which was intermediate between the slope angle and the horizontal except at very short distances from the sloping surface. This lead to advection of mean vorticity at small \( z''/h \) which strongly influenced the shape of the stress profile (Deardorff and Willis, 1987).
BIBLIOGRAPHY


Deardorff, J. W., 1970: Convective velocity and temperature scales for the unstable planetary boundary


APPENDICES
Appendix I

The Horizontal Hydrostatic Pressure Gradient

The equation of state for an incompressible or shallow fluid - in this case water - is (Pedlosky, 1979, eqn. 1.4.20)

\[ \bar{\rho}(x, z) = \rho_0 \{1 + \alpha_0 [T_0 - \bar{T}(x, z)]\} \]  \hspace{1cm} (A1.1)

where \( \rho_0 \) is the density of water at reference temperature \( T_0 \), and \( \alpha_0 \) is the coefficient of thermal expansion for water at \( T_0 \).

Assume the mean-flow follows the hydrostatic assumption

\[ \frac{\partial \bar{\rho}(x, z)}{\partial z} = - g \bar{\rho}(x, z). \]  \hspace{1cm} (A1.2)

To find the pressure at height \( z \), integrate (A1.2) from the air/water interface, \( z_{air} \), down to \( z \). This gives the weight of a unit cross-section column of water overlying a given point at height \( z \).

\[ \int_{z}^{z_{air}} \frac{\partial \bar{\rho}(x, z)}{\partial z} \, dz = - g \int_{z}^{z_{air}} \bar{\rho}(x, z) \, dz \]
\[ \bar{p}(x, z) = g \int_{z}^{z_{\text{air}}} \bar{p}(x, z) \, dz + \bar{p}(x, z_{\text{air}}) \quad (A1.3) \]

Now differentiate with respect to \( x \) using Leibnitz' Rule to obtain the horizontal hydrostatic pressure gradient. Assume \( \bar{p}(x, z_{\text{air}}) \) is constant.

\[ \frac{\partial \bar{p}(x, z)}{\partial x} = \left( \frac{\partial}{\partial x} \right) \int_{z}^{z_{\text{air}}} g \bar{p}(x, z) \, dz \]

\[ = g \bar{p}(x, z_{\text{air}}) \frac{\partial z_{\text{air}}}{\partial x} - g \bar{p}(x, z) \frac{\partial z}{\partial x} \quad (A1.4) \]

\[ + \int_{z}^{z_{\text{air}}} g \frac{\partial \bar{p}(x, z)}{\partial x} \, dz \]

Note that \( z \) and \( x \) are orthogonal coordinates so that \( \partial z / \partial x = 0 \). Also assume that \( \bar{p}(x, z_{\text{air}}) = \rho_0 \). Differentiating (A1.1) with respect to \( x \) gives

\[ \frac{\partial \bar{p}(x, z)}{\partial x} = - \rho_0 \alpha_0 \frac{\partial T(x, z)}{\partial x} \quad . \quad (A1.5) \]

With these simplifications (A1.4) becomes

\[ \frac{\partial \bar{p}(x, z)}{\partial x} = \rho_0 \frac{\partial z_{\text{air}}}{\partial x} - \rho_0 \alpha_0 \int_{z}^{z_{\text{air}}} \frac{\partial T(x, z)}{\partial x} \, dz. \]
Appendix II

Convective Velocity Scale

The convective velocity scale, \( w^* \), is given by

\[
w^* = \left( g \alpha_o \overline{w'T_s' h} \right)^{1/3}
\]

The surface heat flux, \( \overline{w'T_s'} \), is calculated by vertically integrating a simplified thermodynamic equation. When the surface slope angle \( \theta = 0^\circ \) this would take the form

\[
\frac{\partial T_{ML}}{\partial t} + u_x \left( \frac{\partial T(x, z)}{\partial x} \right) = - \frac{\partial (\overline{w'T'})}{\partial z} \tag{A2.1}
\]

where \( T_{ML} \) is the mean temperature in the ML. In tilted coordinates with \( \theta = 10^\circ \) we have

\[
\frac{\partial (\overline{w'T'})}{\partial z''} = (\cos \theta) \frac{\partial (\overline{w'T'})}{\partial z} 
\]

and (A2.1) becomes

\[
\frac{\partial T_{ML}}{\partial t} + u_x'' \left( \frac{\partial T(x, z)}{\partial x''} \right) = - (\sec \theta) \frac{\partial (\overline{w'T'})}{\partial z''} \tag{A2.2}
\]
where $\overline{u_x}$ is the mean velocity parallel to the tank floor given by (4.2).

Assume that $w'T' \rightarrow 0$ as $z''/h \rightarrow 1.2$. Integrate (A2.2) term by term from $z'' = (1.2)h$ down to $z'' = 0$.

$$\int_{z''=0}^{z''=1.2h} \frac{\partial T_{ML}}{\partial t} \, dz'' = 1.2h \frac{\partial T_{ML}}{\partial t}$$

$$\int_{z''=0}^{z''=1.2h} u'' \frac{\partial T(x,z)}{\partial x''} \, dz'' = \frac{\partial T(x,z)}{\partial x''} (5.25 \text{ cm}^2 \text{ s}^{-1})$$

$$\int_{z''=0}^{z''=1.2h} - (\sec \theta) \frac{\partial (w'T')}{\partial z''} \, dz'' = - (\sec \theta) [w'T'_{z''=1.2h} - w'T'_{z''=0}]$$

Here, $5.25 \text{ cm}^2 \text{ s}^{-1} = \int_{z''=0}^{z''=1.2h} u'' \, dz''$, and $w'T'_{z''=1.2h} = 0$.

This gives

$$\frac{w'T'}{s} = \frac{1.2h \frac{\partial T_{ML}}{\partial t} + \frac{\partial T(x,z)}{\partial x''} (5.25 \text{ cm}^2 \text{ s}^{-1})}{1.2(\sec \theta)}$$

where 1.2 rather than 1.0 has been substituted into the denominator to compensate for the entrainment warming contribution (Deardorff et al., 1980). With $h = 15 \text{ cm}$,
\[ \frac{\partial T_{ML}}{\partial t} = 8.3 \times 10^{-3} \, \text{°C} \cdot \text{s}^{-1}, \text{ and } \frac{\partial T(x,z)}{\partial x} = 1.7 \times 10^{-2} \, \text{°C} \cdot \text{cm}^{-1} \text{ from Figure 3, a value of } 0.20 \, \text{°C} \cdot \text{cm} \cdot \text{s}^{-1} \text{ for } w' T'_s \text{ is the result.} \]

Using \( \alpha_o = 2.6 \times 10^{-4} \, \text{°C}^{-1} \) (corresponding to \( T_0 = 25 \, \text{°C} \)), \( w' T'_s = 0.20 \, \text{°C} \cdot \text{cm} \cdot \text{s}^{-1} \), and \( g = 980 \, \text{cm} \cdot \text{s}^{-2} \), a \( w_s \) value of 0.92 \( \text{cm} \cdot \text{s}^{-1} \) is obtained.