DEVELOPING A PREDATOR-PREY MODEL FOR THE HAKE AND BLUE WHITING SPANISH FISHERIES

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ABSTRACT

The aim of this work is to develop a predator-prey model for two species of commercial importance captured by the Spanish fishing fleet in the National Fishing Ground (ICES areas VIIIc and IXa). In this model, the Southern hake (*Merluccius merluccius*) represents the predator, and the blue whiting (*Micromesistius poutassou*) is the prey. Blue whiting is the hake's main prey in the study area, and it represents about 40% of the Southern hake diet. Both the predator and prey population dynamics follow the Lotka-Volterra formulation, and population dynamics are assumed as logistic. It is also assumed a linear interaction between predator and prey populations, with two interaction coefficients: α is the effect of a unit change in the prey on the percent growth rate of the predator and β is the attack rate or searching efficiency of the predator. Logistic predator-prey equations were applied to the Southern hake and blue whiting stocks, including biomass, intrinsic rates of growth, carrying capacity and capture for both species. The goal is to maximize the present value of profit, forming the current value Hamiltonian for the maximization problem. Capture costs and prices of hake and blue whiting and discount rate were introduced at this point. Landings and SSB (Spawning Stock Biomass) data from both stocks over the period 1988-2010 were used for an econometric estimation by means of the Ordinary Least Squares method, to determine the form taken by the predator-prey growth functions.

INTRODUCTION

The effects of the fishing activity are not only reflected in the exploded species but also in the whole ecosystem where they inhabit. Therefore, the fisheries research needs to find alternative approaches beyond the mono-specífic vision historically adopted by the fishing management to evaluate and predict the population dynamics of the exploited species. The *Ecosystem-Based Fisheries Management* was adopted as a fundamental principle of the natural resources management during the Second meeting of the *Convention on Biological Diversity* (Yakarta, November 1995). FAO proposed an international framework for a fisheries management based on the ecosystem through the *Code of Conduct for Responsible Fisheries* [1]. The *Reykjavik Conference on Responsible Fisheries in the Marine Ecosystem* (2001) [2] led to the adoption by the international organizations and States of the *ecosystem approach to fisheries* as part of the implementation of the Code. At the present, the research directions in the field of the ecosystem-based fisheries management are: the development of multi-specific models that provide a greater ecological knowledge of the trophic relationships between the organisms, the study of the interaction between the environment and the marine resources, the establishment of marine protected areas in certain habitats, or the reduction of the *bycatch* and discards in fisheries.

This work proposes a practical case of ecosystem-based fisheries management, through the application of a multiespecífic model, in particular a predator-prey model with capture, to two commercial species captured by the Spanish fleet. The species selection criteria were economical (two species of commercial importance), geographical (species captured in the National Fishing

Ground by the Spanish fishing fleet) and biological (species with a significant trophic interaction). Based on previous studies, the European hake was selected as the predator species and the blue whiting was selected as the prey species. The geographical scope is the Iberian waters, (ICES areas VIIIc and IXa). The data of biomass and captures correspond to the 1988-2010 period.

The European hake is an important and commercially valuable resource, and it is the subject of numerous targeted fisheries. Diverse studies of hake diet in European areas categorize hake as a large piscivorous predator of many commercial species, like blue whiting, horse mackerel, mackerel, pilchard and anchovy. The ecological position of European hake, at the top of the foodweb, probably plays an important role in its ecosystem, and therefore in the dynamics of other economically important species. That point, coupled with its commercial importance, makes particularly necessary its study from a multi-specific perspective.

DESCRIPTION OF THE SPECIES

In Iberian waters, the hake *Merluccius merluccius* (Linnaeus, 1758), and the blue whiting *Micromesistius poutassou* (Risso, 1827), are common commercial species, mainly distributed along the continental shelf, where spawning takes place during the winter months. There is a clear predator-prey relationship between the two species, representing the blue whiting the most important prey of the hake.

The European hake is widely distributed throughout the north-east Atlantic. It is a demersal and benthopelagic species, found mainly between 70 and 370 m depth; however, it also occurs in inshore waters (30 m) and down to depths of 1 000 m. It lives in shoals of fish, next to the coast in summer and far away in winter. Reproduction takes place between 100 and 300 meters deep. Spawning occurs from January to May in the Bay of Biscay and from May to July in the Celtic Sea. Juvenile live on muddy beds on the continental shelf, whereas large adult individuals are found on the shelf slope, on rough bottoms. The maximum age is 12 years (140 centimeters). The European hake is a predator on the top of the trophic pyramid of the Northeast Atlantic demersal community, and its preys are the anchovy (*Engraulis encrasicholus*), the sardine (*Sardina pilchardus*), the blue whiting (*Micromesistius poutassou*), the horse mackerel (*Trachurus trachurus*) or the mackerel (*Scomber scombrus*).

The blue whiting is common in the North-East Atlantic, from the south of the Barents Sea and the eastern Norway Sea to the Cape Bojador, in the African coast. It is a demersal species of the gadidae family. The highest concentrations are found along the edge of the continental shelf in areas west of the British Isles and on the Rockall Bank plateau where it occurs in large schools at depths ranging between 150 and more than 1 000 meters, although it is more common between 300 and 400 meters. It migrates in the summer, after the spawning, towards the North (Feroe Islands, east of Iceland and Norway) and it returns to the spawning areas between January and February. Adults reach its first maturation at 3 years old. Most of the spawning takes place between March and April, along the shelf edge and banks west of the British Isles. The growth is fast, and females are usually larger than males. The maximum age is 20 years (45 centimeters). The blue whiting diet shows a marked seasonality and it is composed mainly of crustaceans, being its main preys copepods, euphausids, decapods larvae and the decapod *Pasiphaea sivado*.

The ecological interdependence of the hake and blue whiting populations in Iberian waters has been reported by different authors. Velasco and Olaso (1998) studied the feeding of the European hake in the Cantabrian Sea (VIIIc Division), analyzing its seasonal, bathymetric and length variations, based on the stomach contents of 5 828 sampled individuals. They highlight the importance of the blue whiting as the main prey of the hake at depths larger than 100 m, while the horse mackerel and the clupeids play a dominant role in the shallowest depth strata (<100 m), because at this depth, blue whiting's abundance is very low in the Cantabrian Sea [3,4]. Blue whiting becomes the fundamental basis of the hake diet from the hake size of 40 centimeters, practically disappearing the consumption of horse mackerel and other species. The regression analysis of the predator-prey sizes relationship confirms the existence of a significant relationship between the size of the hake and the size of the blue whiting, already reported by Gonzalez *et al*. (1985) and Guichet (1995). However, hake predators reach a size at which, although they continue to grow, they cannot not find larger blue whiting, since such individuals do not exist in the population. This limit is reached approximately in hakes measuring 40 cm, which consume blue whiting of up to 33 cm. The authors observe that, in the Cantabrian, the hake has a much greater dependence on the blue whiting than in the northern Bay of Biscay. Mahe (2007) confirmed these results, demonstrating that the blue whiting is the most important prey of the hake in the Cantabrian Sea, while it is a prey of moderate importance in the northern Bay of Biscay and the Celtic Sea. Cabral and Murta (2002) studied the diet of blue whiting, hake, horse mackerel and mackerel in the Portuguese waters (IXa Division), analyzing the stomach contents of individuals of these species collected along the Portuguese coast, at 20 m to 750 m depths. The results obtained for the hake showed that the blue whiting is the most important prey in number, occurrence and weight.

THE SPANISH FISHERIES OF HAKE AND BLUE WHITING

The hake in the Spanish National Fishing Ground (VIIIc and IXa Divisions) is caught by a multi-gear fleet: otter trawlers, pairtrawlers, gillnetters, longliners, and artisanal. Gillnetters fleet is the most dependent on the hake. Hake is caught by the trawl fleet in mixed fisheries together with megrim, anglerfish, blue whiting, horse mackerel, mackerel, and crustaceans. Discards occur mainly in the trawl fisheries which targets smaller fish than gillnetters and longliners. In 2007, hake landings made by the trawl fleet represented 75% of the Spanish total landings of hake in the National Fishing Ground (ICES, 2008). The catches of hake made by the Spanish fleet in the National Fishing Ground show a growing tendency during the last years. Landings usually exceed the assigned annual quota due to the further negotiations and exchanges with other countries with quota assigned in the divisions VIIIc and IXa (Francia and Portugal). Since 2006 a *Southern hake and Norway lobster recovery plan* (EC 2166/2005), has been implemented, aimed at recovering the SSB (spawning-stock biomass) above Bpa (precautionary approach biomass), that is, above 35 000 tonnes, and reducing F (fishing mortality) to 0.27. This regulation includes measures relative to the TAC and also to the effort management. Since 2006 an annual reduction of 10% of the fishing days at sea was applied to all fleets except in the Gulf of Cadiz area. The positive results of the recovery plan have allowed a 15% elevation of the Southern hake TAC of 2011 (6 844 tonnes) in comparison with the TAC of 2010 (5 952 tonnes).

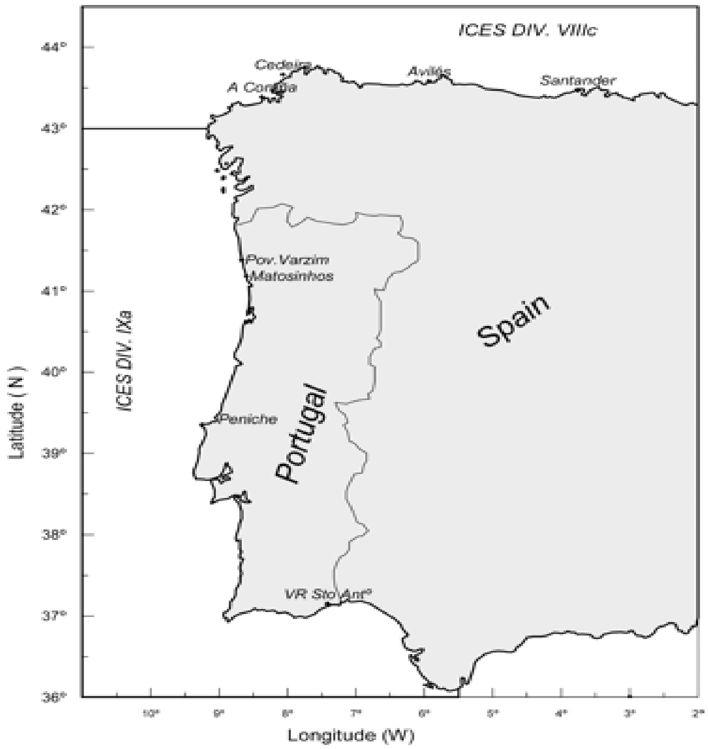

Figure 1: ICES divisions VIIIc and IXa. Source: ICES

The Spanish blue whiting fishery is carried out mainly by bottom pair trawlers in a directed fishery (approx. one third of the fleet) and by single bottom otter trawlers in a by-catch fishery (approx. two thirds of the fleet). This species represents, for the bottom pair trawlers fleet, 64% of the total captures in the whole National Fishing Ground, and up to 82% in the IXa area. The fleet operates throughout the year. Small quantities are also caught by longliners. These coastal fisheries have trip durations of 1 or 2 days and catches are for human consumption. Thus, coastal landings are driven mainly by market forces, and are rather stable. The fleet operates only in Spanish waters and does not follow any blue whiting migration. The Spanish fleet has decreased from 279 vessels in the early 1990s to 135 vessels in 2008. Spanish landings decreased in 2010 having a total landing of 12 900 tonnes [9]. It is a species commercially important and traditionally landed in great volume, although its market price is low. The blue whiting plays an important role in the quota exchanges with other States, to obtain quota for more valuable species. For example, the Spanish blue whiting quota exchanges with Norway to obtain more cod quota. Because of this, only a part of the initially assigned annual quota is captured, and final landings are sometimes notably lower than the quota.

Table I: Southern hake and blue whiting data of Spanish landings and spawning-stock biomass in Iberian waters (VIIIc y IXa Divisions) (1988-2010).

	Southern hake		**Blue whiting**	
Year	**Landings (tonnes)**	**SSB (tonnes)**	**Landings (tonnes)**	**SSB (tonnes)**

1988	10 600	26 500	24 847	98 299
1989	9 200	19 500	30 108	91 506
1990	9 800	15 800	29 490	88 809
1991	8 900	16 000	29 180	171 225
1992	8 840	15 200	23 794	160 007
1993	7 840	12 800	31 020	172 586
1994	7 090	9 200	28 118	160 229
1995	9 530	7 600	25 379	110 787
1996	7 910	9 000	21 538	84 162
1997	7 170	6 900	27 683	100 658
1998	5 570	6 400	27 490	83 859
1999	4 350	8 200	23 777	81 973
2000	5 320	9 700	22 622	73 698
2001	4 770	10 000	23 218	65 050
2002	4 280	10 400	17 506	69 660
2003	5 110	10 300	13 825	49 544
2004	5 020	10 400	15 612	57 567
2005	6 580	10 900	17 643	73 839
2006	11 350	12 400	15 173	67 662
2007	13 990	14 700	13 557	58 646
2008	16 050	15 100	14 342	62 640
2009	17 780	17 200	20 637	115 825
2010	14 000	18 700	12 891	83 666

MULTI-SPECÍFIC MODEL AND ECONOMETRIC ESTIMATION

The model used in this work is based on the Lotka-Volterra predator–prey mathematic model (Volterra, 1926; Lotka, 1932), and the predator-prey logistic equations used by Brown *et al.* (2005) for the populations of Nile perch (predator), and dagaa (prey) in the lake Victoria (Central-eastern Africa). The equations in this model are:

$$\frac{dX}{dt} = r_m X\left[1 - \frac{X}{\bar{X}}\right] - h_m + \alpha XY \qquad \text{(Eq. 1)}$$

$$\frac{dY}{dt} = r_L Y\left[1 - \frac{Y}{\bar{Y}}\right] - h_L + \beta XY \qquad \text{(Eq. 2)}$$

Where X is the stock of hake at time *t* and Y is the stock of blue whiting; r_m is the intrinsic rate of growth for the hake and r_L is the same for the blue whiting; X and Y are the carrying capacities for both species; h_m is the harvest of hake and h_L is the harvest of blue whiting; and α and β are the interaction coefficients of the species: α is the effect of a unit change in blue whiting on the percent growth rate of hake, and β is a measure of the attack rate or searching efficiency of the hake. The goal is to maximize the present value of profit:

$$\Pi=\int_0^\infty e^{-\rho t}[P_m h_m+P_L h_L]dt-e^{-\rho t}[C_m h_m+C_L h_L]dt \qquad \text{(Eq. 3)}$$

$$\Pi=\int_0^\infty [(P_m h_m+P_L h_L)-(C_m h_m+C_L h_L)]e^{-\rho t}dt \qquad \text{(Eq. 4)}$$

$$max\int_0^\infty [(P_m h_m+P_L h_L)-(C_m h_m+C_L h_L)]e^{-\rho t}dt \qquad \text{(Eq. 5)}$$

s.t.

$$\frac{dX}{dt}=f(X)-h_m \qquad \text{(Eq. 6)}$$

$$\frac{dY}{dt}=g(Y)-h_L \qquad \text{(Eq. 7)}$$

Where C_m and C_L are the capture costs of the hake and the blue whiting, and P_m and P_L are the respective prices. The Hamiltonian function is:

$$H=P_m h_m+P_L h_L-C_m h_m-C_L h_L+\lambda_m[f(X)-h_m+\alpha XY]+\lambda_L[g(Y)-h_L+\beta YX] \qquad \text{(Eq. 8)}$$

Where λ_m and λ_L are the respective shadow prices, *f(X)* represents the growth of the hake stock and *g(Y)*, the growth of the blue whiting stock. Both growth functions were estimated by the ordinary least squares (OLS) method.

The Spanish landings and spawning-stock biomass data of hake and blue whiting in Iberian waters over the period 1988-2010 (shown in Table I) were used for an OLS econometric estimation, in order to determine the form adopted by the growth functions of both species. That form may be quadratic, exponential or potential.

The quadratic expressions of the hake and blue whiting growth functions are:

$$X_{t+1} = \alpha X_t - \beta X_t^2 + \gamma X_t Y_t - h_m \quad \text{(Eq. 9)}$$

$$Y_{t+1} = \varphi Y_t - \mu Y_t^2 + \omega X_t Y_t - h_L \quad \text{(Eq. 10)}$$

According to the exponential form, they should be (form1):

$$X_{t+1} = \alpha e^{\beta X_t} + \gamma X_t Y_t - h_m \quad \text{(Eq. 11)}$$

$$Y_{t+1} = \varphi e^{\mu Y_t} + \omega X_t Y_t - h_L \quad \text{(Eq. 12)}$$

And (form 2):

$$X_{t+1} = \alpha e^{\beta X_t + \gamma X_t Y_t} - h_m \quad \text{(Eq. 13)}$$

$$Y_{t+1} = \varphi e^{\mu Y_t + w X_t Y_t} - h_L \quad \text{(Eq. 14)}$$

And the potential functions are (form1):

$$X_{t+1} = \alpha X_t^{\beta} + \gamma X_t Y_t - h_m \quad \text{(Eq. 15)}$$

$$Y_{t+1} = \varphi Y_t^{\mu} + w X_t Y_t - h_L \quad \text{(Eq. 16)}$$

And (form 2):

$$X_{t+1} = \alpha X_t^{\beta X_t Y_t} - h_m \quad \text{(Eq. 17)}$$

$$Y_{t+1} = \varphi Y_t^{\mu X_t Y_t} - h_L \quad \text{(Eq. 18)}$$

These six possible expressions were estimated through the OLS method, selecting for each case a dependent variable and several independent variables. In the case of the quadratic form, the dependent variable selected for the hake growth function was “endomerlu” ($X_{t+1}+h_m$) and the independent variables were “xmer” (X_t), “sq_xmer” (X_t^2) and “xy” (X_tY_t). For the blue whiting, the dependent variable was “endolirio” ($Y_{t+1}+h_L$) and the independent variables were “ylirio” (Y_t), “sq_ylirio” (Y_t^2) and “xy” (X_tY_t).

In the case of the exponential form 1, the dependent variable selected for the hake growth function was “l_endomerlu” ($\ln(X_{t+1}+h_m)$) and the independent variables were “xmer” (X_t), and “l_xy” ($\ln X_tY_t$). For the blue whiting, the dependent variable was “l_endolirio” ($\ln(Y_{t+1}+h_L)$) and the independent variables were "ylirio" (Y_t), and "l_xy" ($\ln X_t Y_t$). For the form 2, the independent variables were "xmer" (X_t), "ylirio" (Y_t) and "xy" ($X_t Y_t$).

In the case of the potential form 1, the dependent variable selected for the hake growth function was "l_endomerlu" ($\ln(X_{t+1}+h_m)$) and the independent variables were "l_xmer" ($\ln X_t$), and "l_xy" ($\ln X_t Y_t$). For the blue whiting, the dependent variable was "l_endolirio" ($\ln(Y_{t+1}+h_L)$) and the independent variables were "l_ylirio" ($\ln Y_t$), and "l_xy" ($\ln X_t Y_t$). For the form 2, the independent variables were "xy l_xmer" ($X_t Y_t \ln X_t$) and "xy l_ylirio" ($X_t Y_t \ln Y_t$).

RESULTS

Quadratic functions: hake

Model 1: OLS, using the observations 1988-2009 (T = 22). Dependent variable: endomerlu

	Coefficient	Standard deviation	t-ratio	p-value
const	-6966,63	6369,97	-1,094	0,2885
xmer	2,34514	0,256401	9,1463	0,0000 **
sq_xmer	-3,42881e-05	1,15099e-05	-2,9790	0,0077 **
xy	-1,98703e-06	1,83231e-06	-1,0844	0,2917

Mean of dependent variable	20.611,36	S.D. of dep. vble.	7.011,487
Sum of squared errors	3,27e+08	S.D. of regression	4.148,211
R-square	0,968498	R-square adjusted	0,965182
F(3, 19)	194,7129	p value (F)	1,93e-14
Log-likelihood	-212,8735	Akaike criterion	431,7471
Schwarz criterion	435,0202	Hannan-Quinn criterion	432,5181
rho	0,832863	Durbin-Watson	0,524206

Quadratic functions: blue whiting

Model 2: OLS, using the observations 1988-2009 (T = 22). Dependent variable: endolirio

	Coefficient	Standard deviation	t-ratio	p-value
const	50276,3	57486,3	0,8746	0,3933
ylirio	1,38892	0,231632	5,9962	0,0000
sq_ylirio	-3,15675e-06	1,39002e-06	-2,2710	0,0350
xy	1,42684e-05	1,27969e-05	1,1150	0,2788

Mean of dependent variable	117.279,9	S.D. of dep. vble.	41.810,57
Sum of squared errors	1,44e+10	S.D. of regression	27.500,22
R-square	0,957652	R-square adjusted	0,953195
F(3, 19)	143,2229	p value (F)	3,19e-13
Log-likelihood	-254,4869	Akaike criterion	514,9738
Schwarz criterion	518,2469	Hannan-Quinn criterion	515,7448
rho	-0,169058	Durbin-Watson	2,118618

Exponential functions 1: hake

Model 3: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endomerlu

	Coefficient	Standard deviation	t-ratio	p-value	
const	9,76387	2,36413	4,130	0,0006	***
xmer	5,62829e-05	1,39339e-05	4,039	0,0007	***
l_xy	-0,0280500	0,119961	-0,2338	0,8176	

Mean of dependent variable	9,882880	S.D. of dep. vble.	0,319333
Sum of squared errors	0,766008	S.D. of regression	0,200789
R-square	0,642293	R-square adjusted	0,604640
F(2, 19)	17,05806	p value (F)	0,000057
Log-likelihood	5,717010	Akaike criterion	-5,434019
Schwarz criterion	-2,160892	Hannan-Quinn criterion	-4,662969
rho	0,694340	Durbin-Watson	0,535532

Exponential functions 1: blue whiting

Model 4: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endolirio

	Coefficient	Standard deviation	t-ratio	p-value	
const	7,63501	2,58700	2,951	0,0082	***
ylirio	4,88918e-06	1,89762e-06	2,576	0,0185	**
l_xy	0,169312	0,131132	1,291	0,2121	

Mean of dependent variable	11,61721	S.D. of dep. vble.	0,333084
Sum of squared errors	0,855276	S.D. of regression	0,212166
R-square	0,632905	R-square adjusted	0,594263
F(2, 19)	16,37886	p value (F)	0,000073
Log-likelihood	4,504458	Akaike criterion	-3,008916
Schwarz criterion	0,264211	Hannan-Quinn criterion	-2,237866
rho	-0,091106	Durbin-Watson	2,062818

Exponential functions 2: hake

Model 5: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endomerlu

	Coefficient	Standard dev.	t-ratio	p-value	
const	5884,46	2799,49	2,102	0,0491	**
xmer	1,36456	0,311315	4,383	0,0003	***
xy	-1,87251e-06	2,01361e-06	-0,9299	0,3641	

Mean of dependent variable	20611,36	S.D. of dep. vble.	7011,487
Sum of squared errors	3,89e+08	S.D. of regression	4525,709
R-square	0,623047	R-square adjusted	0,583368
F(2, 19)	15,70208	p value (F)	0,000094
Log-likelihood	-214,7897	Akaike criterion	435,5793
Schwarz criterion	438,8524	Hannan-Quinn criterion	436,3504
rho	0,825954	Durbin-Watson	0,417144

Exponential functions 2: blue whiting

Model 6: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endolirio

	Coefficient	Standard deviation	t -ratio	p-value	
const	39617,1	15998,9	2,476	0,0229	**
ylirio	0,620382	0,242398	2,559	0,0192	**
xy	1,51858e-05	1,25845e-05	1,207	0,2424	

Mean of dependent variable	117279,9	S.D. of dep. vble.	41810,54
Sum of squared errors	1,38e+10	S.D. of regression	26962,09
R-square	0,623756	R-square adjusted	0,584151
F(2, 19)	15,74956	p value (F)	0,000093
Log-likelihood	-254,0521	Akaike criterion	514,1042
Schwarz criterion	517,3774	Hannan-Quinn criterion	514,8753
rho	-0,120168	Durbin-Watson	2,073544

Potential functions 1: hake

Model 7: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endomerlu

	Coefficient	Standard deviation	t-ratio	p-value	
l_xmer	0,792478	0,188442	4,205	0,0004	***
l_xy	0,118289	0,0850432	1,391	0,1795	

Mean of dependent variable	4,292080	S.D. of dep. vble.	0,138684
Sum of squared errors	0,155198	S.D. of regression	0,088090
R-square	0,999617	R-square adjusted	0,999598
F(1, 20)	26129,97	p value (F)	6,71e-35
Log-likelihood	23,27841	Akaike criterion	-42,55683
Schwarz criterion	-40,37474	Hannan-Quinn criterion	-42,04279
rho	0,568458	Durbin-Watson	0,745628

Potential functions 1: blue whiting

Model 8: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endolirio

	Coefficient	Standard deviation	t-ratio	p-value	
l_ylirio	0,537643	0,200121	2,687	0,0142	**
l_xy	0,264271	0,109856	2,406	0,0259	**

Mean of dependent variable	5,045292	S.D. of dep. vble.	0,144657
Sum of squared errors	0,175031	S.D. of regression	0,093550
R-square	0,999688	R-square adjusted	0,999672
F(1, 20)	32010,00	p value (F)	8,83e-36
Log-likelihood	21,95554	Akaike criterion	-39,91108
Schwarz criterion	-37,72900	Hannan-Quinn criterion	-39,39705
rho	-0,298922	Durbin-Watson	2,293859

Potential functions 2: hake

Model 9: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endomerlu

	Coefficient	Standard deviation	t-ratio	p-value
const	9,60765	0,110848	86,67	3,07e-027 ***
xy l_xmer	2,37421e-011	8,12721e-012	2,921	0,0084 ***

Mean of dependent variable	9,882880	S.D. of dep. vble.	0,319333
Sum of squared errors	1,500970	S.D. of regression	0,273950
R-square	0,299084	R-square adjusted	0,264038
F(1, 20)	8,534088	p value (F)	0,008442
Log-likelihood	-1,682408	Akaike criterion	7,364816
Schwarz criterion	9,546901	Hannan-Quinn criterion	7,878850
rho	0,856495	Durbin-Watson	0,373413

Potential functions 2: blue whiting

Model 10: OLS, using the observations 1988-2009 (T = 22). Dependent variable: l_endolirio

	Coefficient	Standard deviation	t-ratio	p-value
const	11,2435	0,0978514	114,9	1,10e-029 ***
xy l_ylirio	2,65446e-011	5,91586e-012	4,487	0,0002 ***

Mean of dependent variable	11,61721	S.D. of dep. vble.	0,333084
Sum of squared errors	1,161055	S.D. of regression	0,240941
R-square	0,501661	R-square adjusted	0,476744
F(1, 20)	20,13334	p value (F)	0,000225
Log-likelihood	1,142203	Akaike criterion	1,715595
Schwarz criterion	3,897680	Hannan-Quinn criterion	2,229628
rho	0,212414	Durbin-Watson	1,408578

DISCUSSION

The OLS regression analysis results show that potential forms are the most suitable for the hake and blue whiting growth functions, followed by the exponential ones. On the other hand, the quadratic form is clearly the less suitable and can be rejected. In order to determine which of the potential and exponential functions offered the most robust results, Akaike and Schwarz criteria values were compared (see table below). According to these criteria, given a set of candidate models for the data, the preferred model is the one with the minimum values. In this case, both the hake and the blue whiting growth functions show the minimum values for the "potential 1" form, so these must be the chosen functions to develop a predator-prey model for these two species. The final expressions of the growth functions are:

$$X_{t+1} = X_t^{0,792478} + 0,118289\, X_t Y_t - h_m \qquad \text{(Eq. 19)}$$

$$Y_{t+1} = Y_t^{0,537643} + 0,264271\, X_t Y_t - h_L \qquad \text{(Eq. 20)}$$

This is the starting point for further steps of this work, in particular, the introduction of the growth functions in the preceding bioeconomic model to solve it.

Table II. Akaike and Schwarz criteria values for the hake and blue whiting growth functions.

Criteria	Hake				
	Quadratic	**Exponential 1**	**Exponential 2**	**Potential 1**	**Potential 2**
Akaike criterion	431,7471	-5,434019	435,5793	-42,55683	7,364816
Schwarz criterion	435,0202	-2,160892	438,8524	-40,37474	9,546901
	Blue whiting				
Akaike criterion	514,9738	-3,008916	514,1042	-39,91108	1,715595
Schwarz criterion	518,2469	0,264211	517,3774	-37,72900	3,897680

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