AN ABSTRACT OF THE THESIS OF

ROGER EDWIN SCHOLL for the DOCTOR OF PHILOSOPHY
(Name) (Degree)

in CIVIL ENGINEERING presented on May 13, 1969
(Major) (Date)

Title: DYNAMIC ANALYSIS OF THREE-DIMENSIONAL FRAMES

Redacted for Privacy

Abstract approved: ______________________
Harold I. Laursen, Ph. D.

The response of three-dimensional frames subjected to an arbitrary dynamic force is studied. The method of mode superposition is used to obtain the frame response solution. Two different modeling systems are used to idealize the frame. These are the "discrete mass system" and the "continuous mass system."

For an example frame configuration, the eigenvalues and eigenvectors for four different discrete mass models are investigated. These four models represent varying degrees of discrete mass modeling sophistication for the frame. In addition, the eigenvalues are investigated by considering a continuous mass model of the example frame.

The time-history response for the discrete mass model is determined using both a numerical procedure with a digital computer, and by using an analog computer.

Listings of the digital computer programs are provided. The analog computer circuitry used in the solution is also given.

Dynamic Analysis of Three-dimensional Frames

by

Roger Edwin Scholl

A THESIS

presented to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

June 1969

APPROVED:

Redacted for Privacy

Associate Professor of Civil Engineering

in charge of major

Redacted for Privacy

Head of Department of Civil Engineering

Redacted for Privacy

Dean of Graduate School

Date thesis is presented MAY 7, 1969

Typed by Nancy S. Kerley for Roger Edwin Scholl

ACKNOWLEDGMENT

The writer expresses his gratitude to the following:

the Civil Engineering Faculty at Oregon State University for their important influence over the past four years;

Dr. Harold I. Laursen for his never-failing positive attitude both during the three years of the graduate program and in the formation of this thesis;

the Oregon State University Computer Center for the financial support which enabled the writer to use the CDC-3300 computer;

Dr. John L. Saugen of the Electrical Engineering Department for making the EAI-690 Hybrid computer available, and for his instructional assistance;

and a special gratitude to my wife Peggy for her help in so many tangible and intangible ways.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF FIGURES

LIST OF FIGURES (continued)

APPENDIX FIGURES

LIST OF TABLES

DYNAMIC ANALYSIS OF THREE-DIMENSIONAL FRAMES

CHAPTER 1

INTRODUCTION

Dynamic analysis of simple structures and mechanical devices has been performed for many years (28). Methods for analyzing complex plane frames have been documented by many writers. The methods vary in general by the idealizations that are used in modeling the structure or framework. In constructing such a model it is necessary to consider the structural elements of inertia, stiffness, and damping. The term inertia implies both plain mass and mass moment of inertia. Stiffness implies the simple relationship between force and displacement such as the three member forms--bending, axial, and torsion. Damping implies the dissipation of energy from a vibrating system. Recognizing these factors it is evident that the dynamic analysis of a three-dimensional frame can be achieved by a generalization of the available methods for analyzing plane frames.

1.1 Historical Background

In general, the mass, stiffness, and damping are distributed continuously throughout a frame. Thus, the ultimate in an idealized model is one which would reflect the continuity of the above mentioned parameters.

There are two basic types of models being used for performing dynamic analyses. These are "discrete mass models" and "continuous mass models." These are also referred to as lumped parameter models and distributed mass models respectively. Both of these basic approaches have been documented by a large number of writers, with each of these writers presenting a slightly different method of solution.

Lindsay, who wrote the introduction of (27), gives a very comprehensive historical review of dynamic analysis essentially from the beginning of time through the time of J. W. S. Rayleigh (1842-1919). In 1964, Gillespie and Liaw (14) presented a paper on the dynamic analysis of discrete mass systems, and gave a fairly representative historical review of dynamic analysis for both discrete and continuous mass systems for the period of Rayleigh through the early 1960's. In 1963, Laursen, Shubinski, and Clough (20) presented a paper on the dynamic analysis of continuous mass systems and gave a historical review of continuous mass system methods of analysis.

In addition to the above, the following works are considered to be noteworthy with regard to this study. Nearly synonymous with the concept of discrete mass modeling is the concept of normal-mode superposition. The reason for this is that in linear-elastic dynamic analysis today (1969), most investigators use the method of mode superposition. Bromwich (4, p. 413) attributes Edward J. Routh (1831-1907) and Oliver Heaviside (1850-1925) with having simultaneously developed the normal-mode theory in the 1880's. In 1934, Duncan and Collar (10) described a method of dynamic analysis which was presented using the conciseness of matrix notation. The iterative procedure which they described for determining eigenvalues and eigenvectors is still used today, although the more elegant matrix diagonalization procedure is becoming increasingly more common. The most significant improvement to their method of analysis provided in the intervening 35 years is the procedure for developing framework stiffness matrices.

The majority of the investigators referred to above are concerned only with the determination of frequencies. Few discuss the response. Once the modal coordinate[1/] differential equations of motion have been developed for the discrete mass idealization of a given framework, and if the forcing function is a simple step or harmonic function, a closed form time-history solution can be found in any applied mathematics text. However, if the forcing function is not of simple form, which is usually the case in most real situations, the investigator is left with two possible alternatives. A solution can be

---

[1/] Modal coordinate is sometimes referred to as normal coordinate in the literature.

obtained by using a numerical procedure with a digital computer, or by using a hybrid computer. Some of the pertinent books on numerical procedures are (15, 21, 22). Some current journal publications, representative of other numerical procedures being used, are given by Cakiroglu and Ozmen (5), O'Hara and Conniff (26), and Newmark (24, 25). Since no formal publication is presently available on hybrid computer procedures, the user must rely on manuals provided by the computer manufacturer. However, a comprehensive book on analog computer programming is (17). At the present time, a procedure for obtaining the response analysis for a continuous mass model of a complex framework does not appear to have been developed.

1.2 Scope and Purpose

The purpose of this investigation is to determine the response of a three-dimensional frame disturbed by an arbitrary dynamic forcing function.

For an example frame configuration, the eigenvalues and eigenvectors[2/] for four different discrete mass models were investigated. In addition, the eigenvalues were investigated by considering a continuous mass model of the example frame.

The time-history response for the discrete mass model was

[2/] The terms eigenvalue and angular frequency squared, as well as eigenvector and mode shape, are used synonymously.

determined using both a numerical procedure with a digital computer and by using an analog computer.

CHAPTER 2

METHOD OF ANALYSIS

As mentioned in Chapter 1, there are two types of idealized models for a general framework which can be used in performing a dynamic analysis--the discrete mass system and the continuous mass system. For the example frame of Chapter 3, both methods are used to evaluate the eigenvalues while only the first method is used to evaluate the response. Thus, the continuous mass system method is used to verify the frequencies of the discrete mass system method.

The development of the discrete mass system method of analysis is given in Section 2.1 and the continuous mass system method of analysis is given in Section 2.2.

2.1 Discrete Mass System

To demonstrate the lumped-mass technique let us consider the three-dimensional frame shown in Figure 2.1(a). Figure 2.1(b) shows a system of discrete masses and massless, elastic members which approximate the frame of Figure 2.1(a). The location of the discrete masses will be referred to as nodal points. The reference coordinate system used here and in Chapter 3 is shown in Figure 2.1(c). Figure 2.1(d) shows the corresponding nodal point

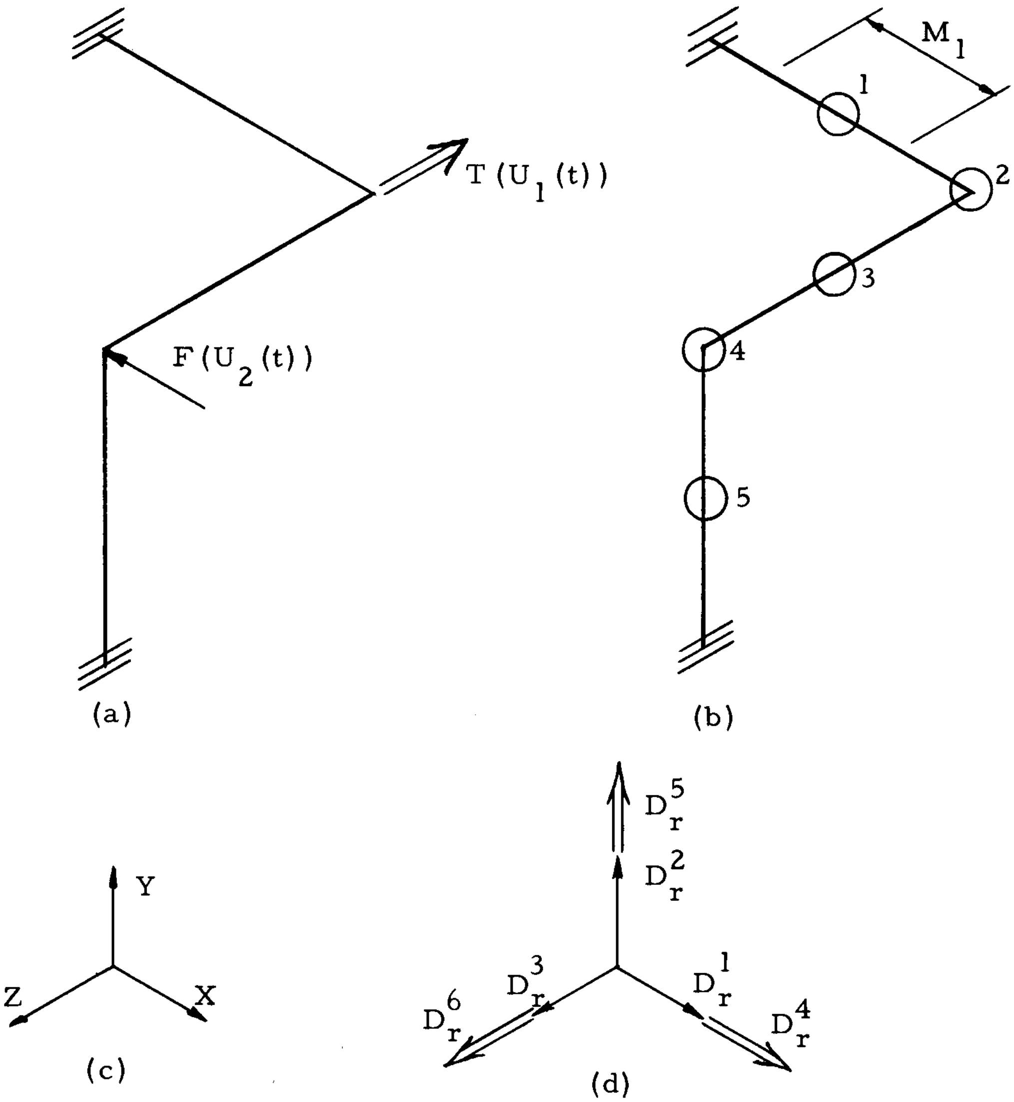

Figure 2.1. Discrete mass modeling convention.

displacement-components. The subscript indicates the nodal point of the frame and the superscript indicates the deflection component. The components D_1^1, D_1^2, and D_1^3 describe the translation of mass 1 while D_1^4, D_1^5, and D_1^6 describe the rotation of mass 1. The latter are denoted in accordance with the right-hand screw rule. Thus the nodal coordinate displacement vector for the discrete mass system of Figure 2.1(b) is:

$$\left\{\begin{matrix} D_1^1 \\ D_1^2 \\ D_1^3 \\ D_1^4 \\ D_1^5 \\ D_1^6 \\ \vdots \\ D_5^1 \\ D_5^2 \\ D_5^3 \\ D_5^4 \\ D_5^5 \\ D_5^6 \end{matrix}\right\} = \text{Nodal Coordinate Displacement Vector}$$

Similarly, the force vector associated with the frame of Figure 2.1 is:

$$\left\{ \begin{array}{c} Q_1^1 \\ Q_1^2 \\ Q_1^3 \\ Q_1^4 \\ Q_1^5 \\ Q_1^6 \\ \vdots \\ Q_5^1 \\ Q_5^2 \\ Q_5^3 \\ Q_5^4 \\ Q_5^5 \\ Q_5^6 \end{array} \right\} = \text{Force Vector}$$

For the given condition of loading all elements of the force vector are zero except

$$Q_2^6 = -T(U_1(t)) \qquad \text{and} \qquad Q_4^1 = -F(U_2(t))$$

2.1.1 Equations of Motion

The nodal coordinate equations of motion for the discrete mass system of Figure 2.1(b) can be expressed in matrix form as

$$\lceil M \rfloor \{\ddot{D}(t)\} + [C]\{\dot{D}(t)\} + [K]\{D(t)\} = \{Q\} \langle U(t) \rangle \qquad (2.1)$$

in which $\lceil M \rfloor$ is the diagonal matrix of discrete masses. $[K]$ is the stiffness matrix of the frame which will be derived in Section

2.1.2, and $[C]$ is the viscous damping matrix. The matrix $\{Q\}$ $\langle U(t) \rangle$ represents the force vector, written as the product of a constant force multiplied by a time varying control function. $\{D(t)\}$ is the nodal coordinate displacement vector. $\dot{D}$ and $\ddot{D}$ represent first and second derivatives with respect to time respectively. The mass matrix may be described in many different ways, including the consistent mass matrix procedure described by Archer (2). The procedure used in this study is to simply lump the mass at the nodal points as shown in Figure 2.1(b).

Since Equation (2.1) is coupled, the dynamic response solution for even a simple frame would entail a large amount of computation. Furthermore, in any dynamic analysis, it is desirable to know the frequency characteristics of the structure. Thus, in this study the dynamic response of the frame is determined using the method of mode-superposition. In order to do this, it is necessary to transform the system of nodal coordinate Equations (2.1) into a system of modal coordinate equations. This development can be found in many references, one of which is Hurty and Rubinstein (16, p. 278). The relationship between nodal coordinates and modal coordinates can be written

$$\{D\} = [\phi]\{S\} \tag{2.2}$$

in which $[\phi]$ is the orthogonal transformation matrix whose columns represent the mode shapes of the frame and $\{S\}$ is the modal coordinate[3/] vector. Substituting Equation (2. 2) into Equation (2. 1) and premultiplying by $[\phi]^T$, the following uncoupled modal equation is obtained

$$\{\ddot{S}(t)\} + \lceil 2c\omega \rfloor \{\dot{S}(t)\} + \lceil \omega^2 \rfloor \{S(t)\} = \{P\} \langle U(t) \rangle \quad (2.3)$$

where

$$[\phi]^T \lceil M \rfloor [\phi] = \lceil I \rfloor \quad (2.4a)$$

$$[\phi]^T [K] [\phi] = \lceil \omega^2 \rfloor \quad (2.4b)$$

$$[\phi]^T \{Q\} = \{P\} \quad (2.4c)$$

and damping is assumed to be of the form (3, p. 332)

$$[\phi]^T [C] [\phi] = \lceil 2c\omega \rfloor \quad (2.4d)$$

In Equation (2. 4d) c is the ratio of desired damping to critical damping for a given mode and ω is the angular frequency. Some discussions of damping in multi-degree freedom structural systems are given by Foss (11), and Caughey and O'Kelly (6, 7, 8). The vector $\{P\}$ defined in Equation (2. 4c) is hereafter referred to as the modal force vector. The procedure for determining the coordinate transformation matrix $[\phi]$, as defined above, is given in Section 2. 1. 3.

Once the solution to the modal coordinate Equations (2. 3) is obtained, the solution for the nodal coordinates can be achieved by applying Equation (2. 2). The solution of Equation (2. 3) for an arbitrary forcing function is obtained in this study by means of a numerical

---

[3/] Sometimes referred to as normal coordinate.

procedure. The numerical method used is the fourth order Runge-Kutta procedure as described by Milne (22, p. 72).

2.1.2 Static Stiffness Matrix

Many procedures for generating stiffness matrices are available. The matrix displacement method described by Laursen (18, p. 50) is used in this study.

Consistent with the notation of this study, the method is as follows: the internal force-displacement relationship for a member is

$$\{q\} = [k_i] \{d\} \tag{2.5}$$

where

$\{q\}$ - member internal force vector

$[k_i]$ - member stiffness matrix

$\{d\}$ - member internal displacement vector

Similarly, the external force-displacement relationship for a structure is

$$\{Q\} = [K] \{D\} \tag{2.6}$$

where

$\{Q\}$ - frame external force vector

$[K]$ - frame stiffness matrix

$\{D\}$ - frame external displacement vector

and

$$[K] = [A]^T [k] [A] \tag{2.7}$$

in which [A] is the displacement transformation matrix, relating internal member displacements to external frame displacements.

Considering the nodal displacement vector { D } as described previously in Section 2. 1, each of the ends of a given member has 6 degrees of freedom, thus each member has 12 degrees of freedom. However, only 6 of these 12 degrees of freedom are independent, therefore the three-dimensional member stiffness matrix $[k_i]$ can be stated as either a 6 x 6 or 12 x 12 matrix. For a member with its axis coincident with the x axis the 6 x 6 member stiffness matrix is

$$\begin{Bmatrix} M^{iy} \\ M^{jy} \\ M^{iz} \\ M^{jz} \\ PL \\ T \end{Bmatrix} = \frac{2EI}{L} \begin{bmatrix} 2 & 1 & & & & \\ 1 & 2 & & & & \\ & & 2 & 1 & & \\ & & 1 & 2 & & \\ & & & & AL^2/2I & \\ & & & & & JG/2EI \end{bmatrix} \begin{Bmatrix} \theta^{iy} \\ \theta^{jy} \\ \theta^{iz} \\ \theta^{jz} \\ e/L \\ \psi \end{Bmatrix} \qquad (2.8)$$

Positive displacements and forces are indicated in Figure 2. 2.

The 12 x 12 member stiffness matrix, in a rearranged form of that presented by Gere and Weaver (13, p. 198), is shown in Equation (2. 9). The vectors of Equation (2. 9) as well as their positive directions are indicated in Figure 2. 3.

Although the 12 x 12 matrix is not actually used in this part of the study, it is referred to in Section 2. 2. 1. Obviously, the form of the displacement transformation matrix [A] will be dependent

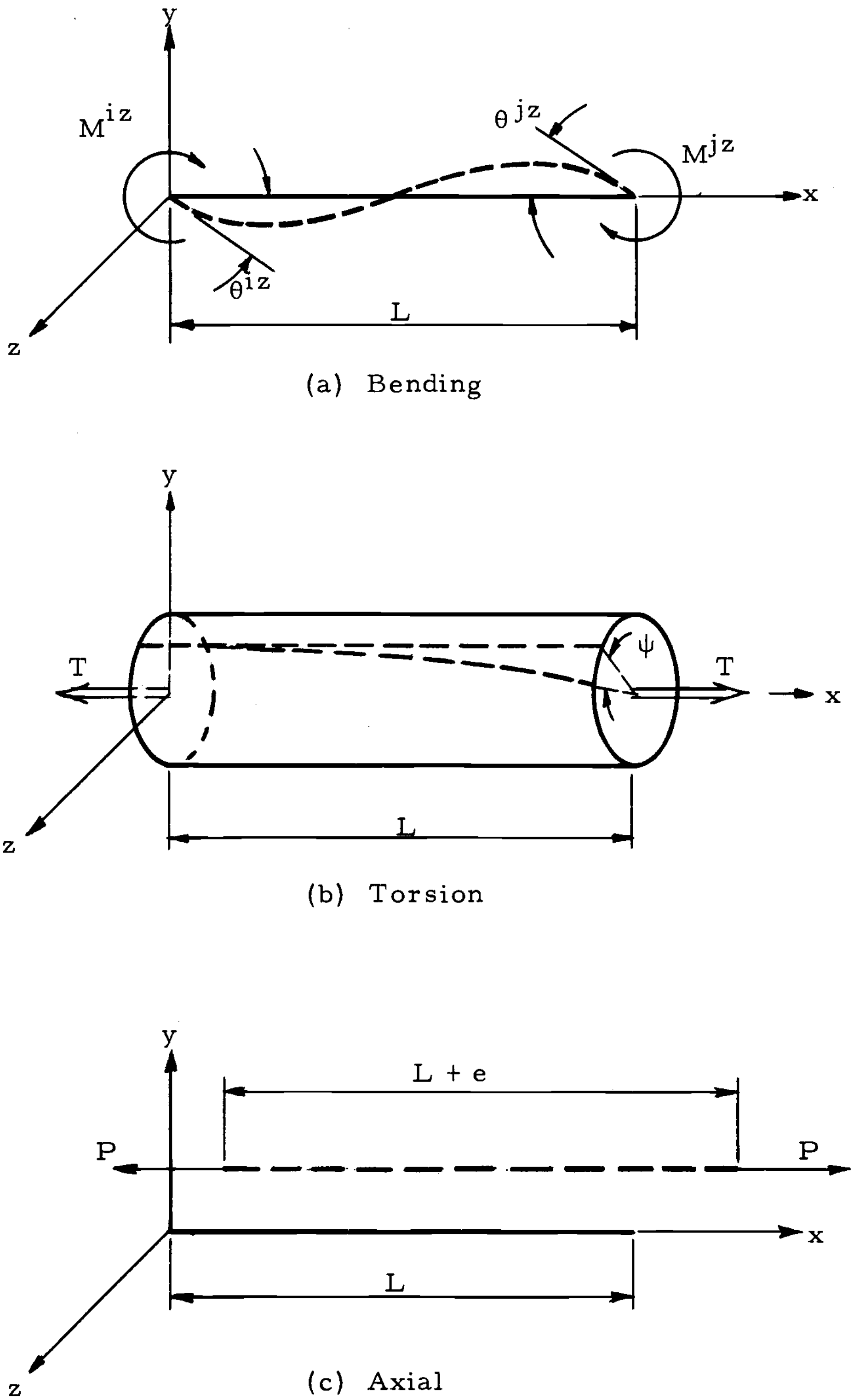

Figure 2. 2. Force-displacement sign convention for 6 x 6 $[k_i]$.

$$
\begin{Bmatrix} M^{iy} \\ V^{iz}L \\ M^{jy} \\ V^{jz}L \\ M^{iz} \\ V^{iy}L \\ M^{jz} \\ V^{jy}L \\ P^{i}L \\ P^{j}L \\ T^{i} \\ T^{j} \end{Bmatrix}
= \frac{2EI}{L}
\begin{bmatrix}
2 & -3 & 1 & 3 & & & & & & & & \\
-3 & 6 & -3 & -6 & & & & & & & & \\
1 & -3 & 2 & 3 & & & & & & & & \\
3 & -6 & 3 & 6 & & & & & & & & \\
 & & & & 2 & 3 & 1 & -3 & & & & \\
 & & & & 3 & 6 & 3 & -6 & & & & \\
 & & & & 1 & 3 & 2 & -3 & & & & \\
 & & & & -3 & -6 & -3 & 6 & & & & \\
 & & & & & & & & * & -* & & \\
 & & & & & & & & -* & * & & \\
 & & & & & & & & & & ** & -** \\
 & & & & & & & & & & -** & **
\end{bmatrix}
\begin{Bmatrix} \theta^{iy} \\ \eta^{iz}/L \\ \theta^{jy} \\ \eta^{jz}/L \\ \theta^{iz} \\ \eta^{iy}/L \\ \theta^{jz} \\ \eta^{jy}/L \\ e^{i}/L \\ e^{j}/L \\ \psi^{i} \\ \psi^{j} \end{Bmatrix}
\qquad (2.9)
$$

where: $* = \dfrac{L^2 A}{2I}$ $\qquad ** = \dfrac{JG}{2EI}$

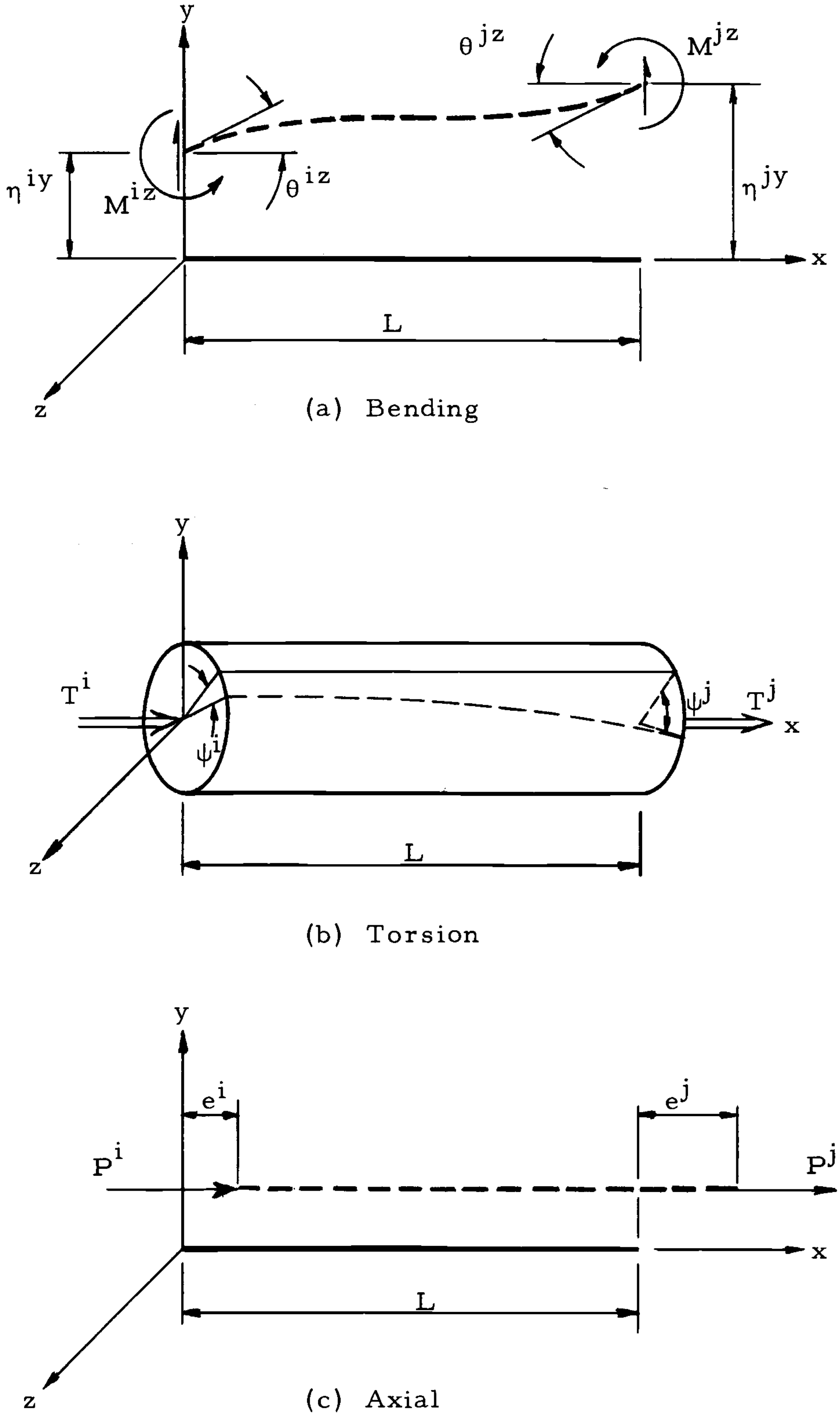

Figure 2.3. Force-displacement sign convention for 12×12 $[k_i]$.

upon which of the above two member stiffness matrices is used.

Both the matrices given in Equations (2. 8) and (2. 9) are derived with the assumption that shear deformation is negligible. However, the effect of shear can be included in the development of the member stiffness matrix (19, p. 237).

2. 1. 3 Eigenvalues and Eigenvectors

The characteristic equation for the discrete mass system of Figure 2. 1(b) can be expressed as

$$[K]\{D\} = \omega^2 \lceil M \rfloor \{D\} \tag{2.10}$$

or

$$[K - \omega^2 M]\{D\} = \{0\}$$

The eigenvalues and eigenvectors for the frame are determined in this study by means of the Jacobi matrix diagonalization procedure described by Crandall (9, p. 118).

The eigenvector matrix $[\phi]$ is then normalized as defined in Equation (2. 4). Other methods for diagonalizing matrices are described by Franklin (12) and Wilkinson (29).

2. 2 Continuous Mass System

Consider again the frame shown in Figure 2. 1(a). The reference axis and nodal coordinate system shown in Figures 2. 1(c) and 2. 1(d) will apply to this method. However, the model for this method will differ from the one shown in Figure 2. 1(b). If it is assumed that the frame of Figure 2.1(a) has a uniform mass distribution, then the model to be used in this method of analysis would consist of three members and two nodal points as shown in Figure 2.4.

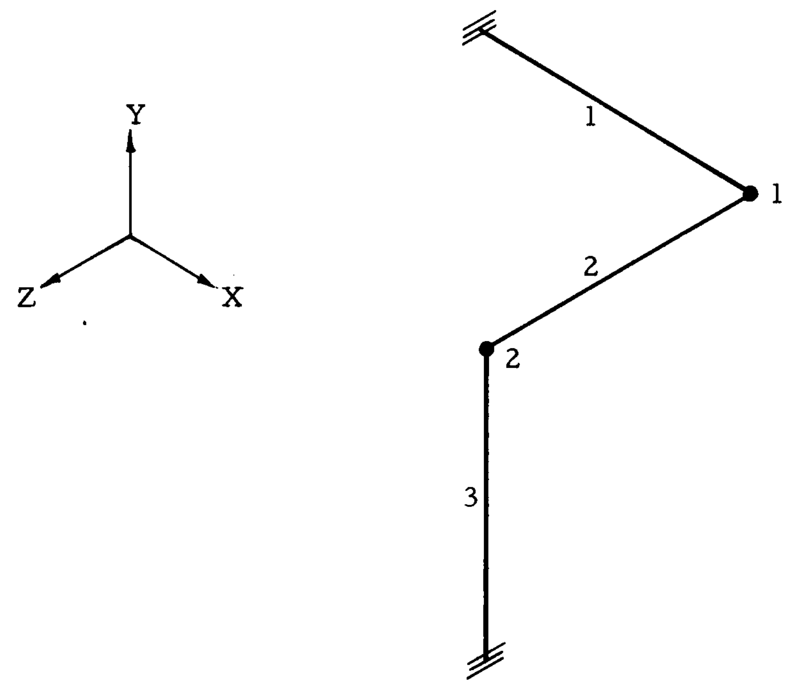

Figure 2.4. Continuous mass modeling convention.

2.2.1 <u>Member Dynamic Stiffness Matrix Development</u>

This study is an extension into three-dimensions of the development given by Laursen (19, p. 449) for vibrations in a single plane. The member dynamic stiffness matrix for vibrations in a single plane is given in the expression

$$\begin{Bmatrix} M^i \\ V^i L \\ M^j \\ V^j L \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} k1 & -k2 & k3 & k4 \\ -k2 & k5 & -k4 & -k6 \\ k3 & -k4 & k1 & k2 \\ k4 & -k6 & k2 & k5 \end{bmatrix} \begin{Bmatrix} \theta^i \\ \eta^i/L \\ \theta^j \\ \eta^j/L \end{Bmatrix} \qquad (2.11)$$

where

$$k1 = \frac{Cs - cS}{1 - cC}\lambda \qquad s = \sin\lambda$$

$$k2 = \frac{sS}{1 - cC}\lambda \qquad c = \cos\lambda$$

$$k3 = \frac{S - s}{1 - cC}\lambda \qquad S = \sinh\lambda$$

$$k4 = \frac{C - c}{1 - cC}\lambda^2 \qquad C = \cosh\lambda$$

$$k5 = \frac{sC + cS}{1 - cC}\lambda^3 \qquad \lambda = aL$$

$$k6 = \frac{S + s}{1 - cC}\lambda^3 \qquad a^4 = \frac{m\omega^2}{EI}$$

In the above, m is the member mass per unit length which is assumed to be constant. It should be noted that Equation (2.11) is derived from the "space part" differential equation for lateral (bending) vibration of a uniform slender beam.

$$\frac{d^4 y}{dx^4} - \frac{\omega^2 m y}{EI} = 0 \qquad (2.12)$$

in which it is assumed that shear deformation and rotational inertia effects are negligible.

From Hurty and Rubinstein (16, p. 194), the "space part" differential equation for torsional vibration of a uniform slender beam as shown in Figure 2.5 is

$$\frac{d^2\psi}{dx^2} + \frac{\omega^2 I_m}{JG} = 0 \qquad (2.13)$$

in which

- x - beam axis coordinate
- ψ - amplitude of torsional vibration
- JG - torsional stiffness
- I_m - mass moment of inertia per unit length
- ω - natural frequency of vibration

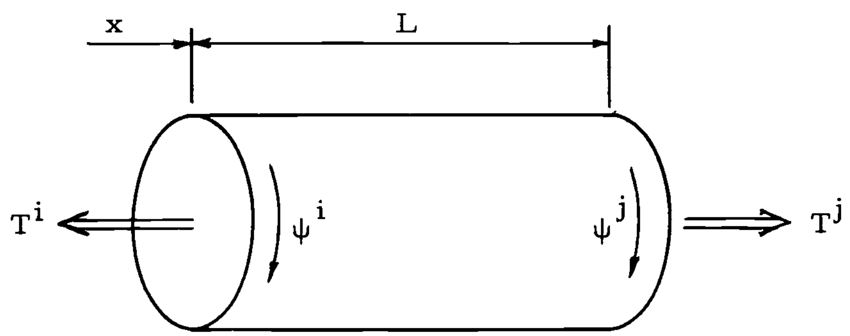

Figure 2.5. Torsional member component for $[k_i(\omega)]$.

The solution to Equation (2.13) can be expressed in terms of two undetermined coefficients in the form

$$\psi(x) = C_1 \cos cx + C_2 \sin cx \qquad (2.14)$$

where

$$c^2 = \frac{\omega^2 I_m}{JG}$$

Using the notation of Figure 2.3, the boundary conditions are

$$\begin{aligned} \psi^i &= \psi(0) = C_1 \\ \psi^j &= \psi(L) = C_1 \cos cL + C_2 \sin cL \end{aligned} \qquad (2.15)$$

$$\begin{aligned} T^i &= -JG\psi'(0) = -JGC_2 c \\ T^j &= JG\psi'(L) = JG(-C_1 c \sin cL + C_2 c \cos cL) \end{aligned} \qquad (2.16)$$

Equation (2. 15) can be written in matrix form as

$$\begin{Bmatrix} \psi^i \\ \psi^j \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \cos cL & \sin cL \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} \qquad (2.17)$$

or symbolically,

$$\{r\} = [W]\{v\} \qquad (2.18)$$

Equation (2. 16) can be expressed

$$\begin{Bmatrix} T^i \\ T^j \end{Bmatrix} = JG \begin{bmatrix} 0 & -c \\ -c \sin cL & c \cos cL \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} \qquad (2.19)$$

or symbolically,

$$\{R\} = [U]\{v\} \qquad (2.20)$$

Equation (2. 18) can be rewritten as

$$\{v\} = [W]^{-1}\{r\}$$

which upon substitution into Equation (2. 20) yields

$$\{R\} = [U][W]^{-1}\{r\} \qquad (2.21)$$

The product $[U][W]^{-1}$ relates forces at the ends of the member to displacements at the ends, thus it represents the torsional component of the member dynamic stiffness matrix. In expanded form, Equation (2. 21) is

$$\begin{Bmatrix} T^i \\ T^j \end{Bmatrix} = \frac{J\,G\,c}{\sin c\,L} \begin{bmatrix} \cos c\,L & -1 \\ -1 & \cos c\,L \end{bmatrix} \begin{Bmatrix} \psi^i \\ \psi^j \end{Bmatrix} \qquad (2.\,22)$$

The axial (longitudinal) component of the member dynamic stiffness matrix can be derived using the same procedure as that used in deriving the torsional component. The resulting dynamic stiffness matrix relating axial force to axial displacement is

$$\begin{Bmatrix} P^i \\ P^j \end{Bmatrix} = \frac{A\,E\,b}{\sin b\,L} \begin{bmatrix} \cos b\,L & -1 \\ -1 & \cos b\,L \end{bmatrix} \begin{Bmatrix} e^i \\ e^j \end{Bmatrix} \qquad (2.\,23)$$

where

$$b^2 = \frac{\omega^2 m}{A\,E}$$

$A\,E$ = longitudinal stiffness

m = mass per unit length

The dynamic load-deflection relations presented in the preceding paragraphs can be summarized in a single matrix expression. Thus, for a member subjected to bending, axial force, and twist, the member deformations are related to the member forces by a dynamic stiffness matrix in the form shown in Equation (2. 24). Symbolically, Equation (2. 24) can be written

$$\{q\} = [\,k_i(\omega)\,]\,\{d\} \qquad (2.\,25)$$

It can be shown that in the limiting condition of $\omega \rightarrow 0$, the dynamic member stiffness matrix of Equation (2. 24) reverts back to the static member stiffness matrix of Equation (2. 9).

$$
\begin{Bmatrix} M^{ix} \\ V^{iy} L \\ M^{jx} \\ V^{jy} L \\ M^{iy} \\ V^{ix} L \\ M^{jy} \\ V^{jx} L \\ P^{i} L \\ P^{j} L \\ T^{i} \\ T^{j} \end{Bmatrix}
= \frac{EI}{L}
\begin{bmatrix}
k1 & -k2 & k3 & k4 & & & & & & & & \\
-k2 & k5 & -k4 & -k6 & & & & & & & & \\
k3 & -k4 & k1 & k2 & & & & & & & & \\
k4 & -k6 & k2 & k5 & & & & & & & & \\
& & & & k1 & k2 & k3 & -k4 & & & & \\
& & & & k2 & k5 & k4 & -k6 & & & & \\
& & & & k3 & k4 & k1 & -k2 & & & & \\
& & & & -k4 & -k6 & -k2 & k5 & & & & \\
& & & & & & & & k7 & -k8 & & \\
& & & & & & & & -k8 & k7 & & \\
& & & & & & & & & & k9 & -k10 \\
& & & & & & & & & & -k10 & k9
\end{bmatrix}
\begin{Bmatrix} \theta^{ix} \\ \eta^{iy}/L \\ \theta^{jx} \\ \eta^{jy}/L \\ \theta^{iy} \\ \eta^{ix}/L \\ \theta^{jy} \\ \eta^{jx}/L \\ e^{i}/L \\ e^{j}/L \\ \psi^{i} \\ \psi^{j} \end{Bmatrix}
\qquad (2.24)
$$

where:

$$k1 = \frac{sC - cS}{1 - cC}(\lambda A) \qquad k5 = \frac{sC + cS}{1 - cC}(\lambda A)^3 \qquad k7 = \beta \frac{\cos bL}{\sin bL}(\lambda B) \qquad k9 = \gamma \frac{\cos cL}{\sin cL}(\lambda C)$$

$$k2 = \frac{sS}{1 - cC}(\lambda A)^2 \qquad k6 = \frac{S + s}{1 - cC}(\lambda A)^3 \qquad k8 = \beta \frac{(\lambda B)}{\sin bL} \qquad k10 = \gamma \frac{(\lambda C)}{\sin cL}$$

$$k3 = \frac{S - s}{1 - cC}(\lambda A) \qquad \begin{aligned} s &= \sin(\lambda A) \\ c &= \cos(\lambda A) \end{aligned} \qquad b = \sqrt{\frac{\omega^2 m}{AE}} \qquad c = \sqrt{\frac{\omega^2 I_m}{JG}}$$

$$k4 = \frac{C - c}{1 - cC}(\lambda A)^2 \qquad \begin{aligned} S &= \sinh(\lambda A) \\ C &= \cosh(\lambda A) \end{aligned} \qquad \beta = \frac{AEL^2}{EI} \qquad \gamma = \frac{JG}{EI}$$

$$(\lambda A) = aL \qquad a = \sqrt[4]{\frac{\omega^2 m}{EI}} \qquad (\lambda B) = bL \qquad (\lambda C) = cL$$

2.2.2 Frame Dynamic Stiffness Matrix

The frame dynamic stiffness matrix $[K(\omega)]$ can be determined using the same procedure as was used in Section 2.1.2 to obtain the static frame stiffness matrix. Thus, the frame dynamic stiffness matrix is defined by the relationship

$$[K(\omega)] = [A]^T [k(\omega)] [A] \qquad (2.26)$$

where

$[K(\omega)]$ - frame dynamic stiffness matrix

$[k(\omega)]$ - member dynamic stiffness matrix

$[A]$ - displacement transformation matrix

2.2.3 Eigenvalues of Continuous Mass System

The characteristic equation for the continuous mass system can be expressed as

$$[K(\omega)]\{D\} = \{0\} \qquad (2.27)$$

for which $\{D\}$ will have non-zero values only when the determinantal equation

$$\text{Det}\left|K(\omega)\right| = 0 \qquad (2.28)$$

is satisfied.

It is important to note that the elements of the frame dynamic stiffness matrix $[K(\omega)]$ are not constants. That is, there exists a different matrix $[K(\omega)]$ for every value of ω. The solution of

Equation (2.28) for the eigenvalues ω^2 is accomplished by a trial and error procedure.

CHAPTER 3

EXAMPLE FRAME ANALYSIS

The frame chosen to demonstrate the methods described in Chapter 2 is shown below in Figure 3. 1. The members are each 12-inch double-extra-strong steel pipe which according to the AISC Manual of Steel Construction (1) has the properties shown in Figure 3. 1.

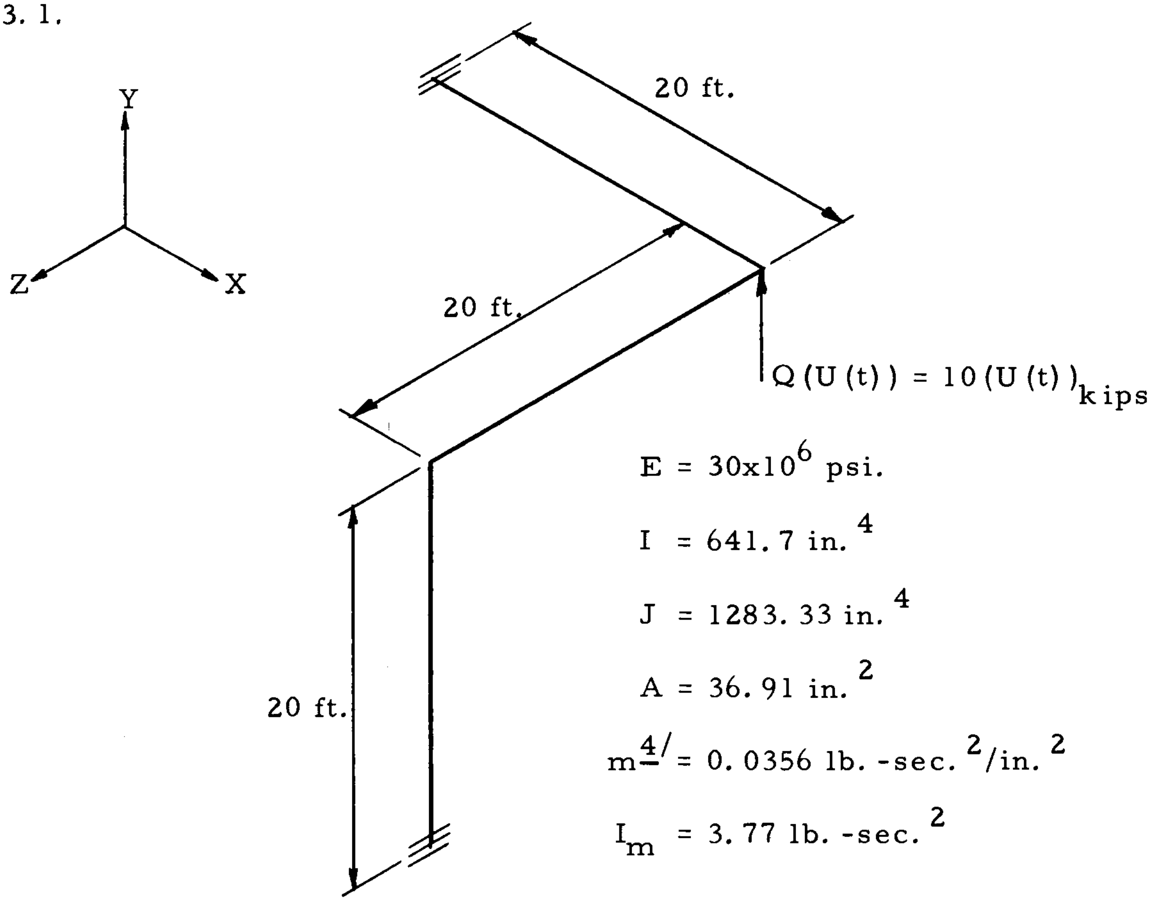

Figure 3. 1. Example frame.

4/ Includes a full load of water in the pipe.

The dynamic force applied to the frame is as shown in Figures 3. 1 and 3. 2.

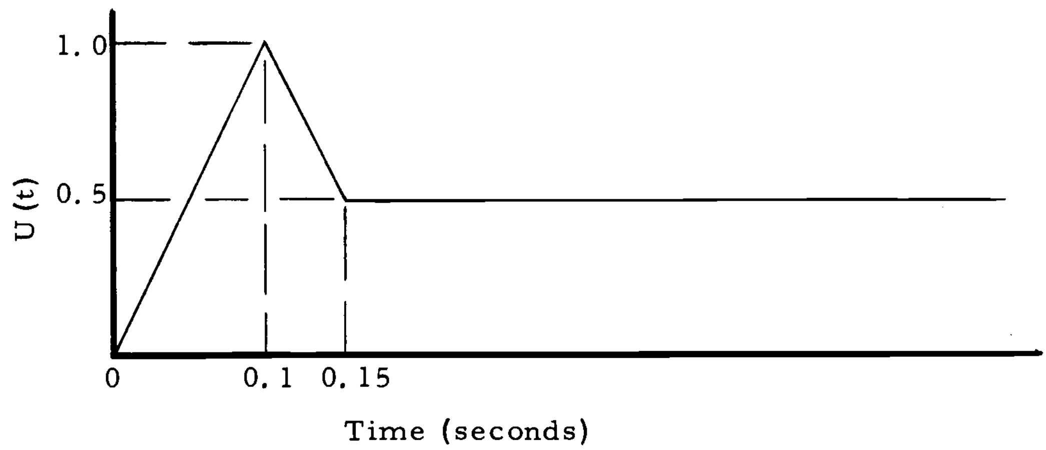

Figure 3. 2. Example frame control function.

In order that a comparison of the discrete mass and continuous mass analyses can be made, it is necessary that the assumptions made in each be the same. As was stated in Chapter 2, the member dynamic stiffness matrix, Equation (2. 24), excludes the effect of bending rotational inertia. However, Equation (2. 24) includes torsional rotational inertia. For the example frame, the effect of torsional rotational inertia is assumed negligible. Thus, for this investigation, the effect of both torsional and bending rotational inertia is neglected in the discrete mass analysis. The effect of shear deformation was excluded in the development of both the static and dynamic member stiffness matrices of Chapter 2. Hurty and Rubinstein (16, p. 177) show that the effect of these exclusions is small.

3.1 Discrete Mass System Analysis

A review of the literature provided no real assistance in determining the number of discrete masses needed to adequately represent the frame of Figure 3.1 for analysis purposes. Thus, four different models were analyzed. These models are shown in Figure 3.3. The investigation of four different models does not in itself reflect any information regarding the approximation of the real frame. However, by comparing these results with the results of the continuous mass analysis of Section 3.2, some conclusions can be made.

3.1.1 Example Frame Static Stiffness Matrix

For this investigation, the frame stiffness matrices were determined according to Equation (2.7). To show the matrices for each of the models of Figure 3.3 would result in numerous pages of superfluous information. Thus, only the [A] and [k] matrices for the two mass model are presented. These matrices for the other three models are similar but larger. The size of the [A] and [k] matrices for each of the models is

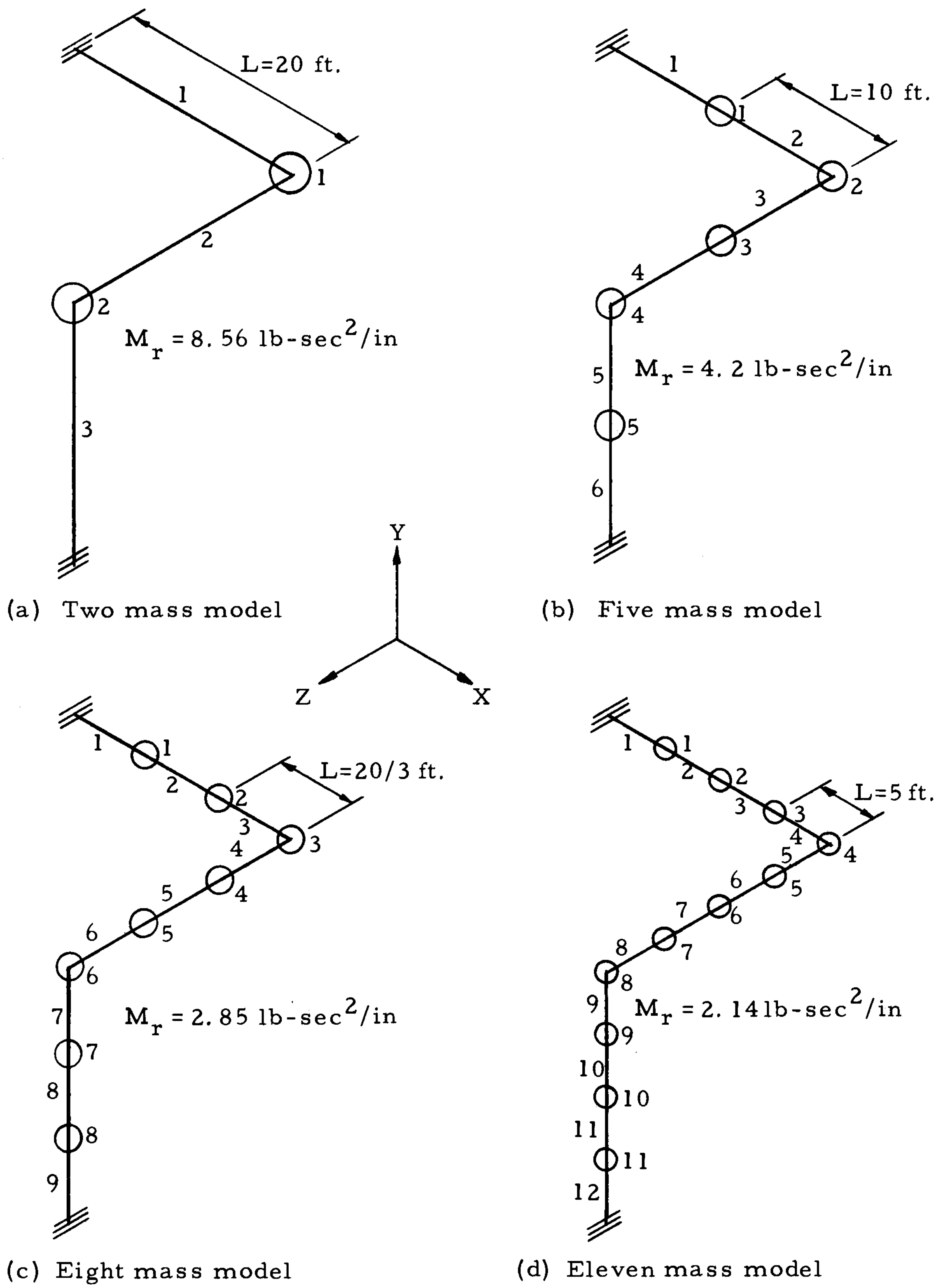

Figure 3.3. Example frame discrete mass models.

	[A]	[k]
Two mass model	[18 x 12]	[18 x 18]
Five mass model	[36 x 30]	[36 x 36]
Eight mass model	[54 x 48]	[54 x 54]
Eleven mass model	[72 x 66]	[72 x 72]

The [A] and [k] matrices for the two mass model are shown in Figure 3. 4 and 3. 5 respectively.

The computer program STIFF, shown in Appendix A, was used to calculate the frame stiffness matrix [K]. As presented, STIFF calculates the matrix tripleproduct $[A]^T [k] [A]$ and then, by assuming the rotational inertia forces to be zero, it reduces the size of [K] from order 6r to order 3r where r is the number of discrete mass elements for a given frame.

3. 1. 2 Example Frame Eigenvalues and Eigenvectors

The eigenvalues and eigenvectors for each of the four discrete mass models were determined using the computer program JEIGEN shown in Appendix A. To present all of the mode shape and frequency information which was generated would require on the order of 100 pages. Insight concerning the dynamic characteristics of the frame can be obtained with significantly less information than is available. Thus, only selected mode shape and frequency information is presented here.

$$
\begin{Bmatrix}
\theta_1^{iy} \\
\theta_1^{jy} \\
\theta_1^{iz} \\
\theta_1^{jz} \\
e_1/L \\
\psi_1 \\
\theta_2^{ix} \\
\theta_2^{jx} \\
\theta_2^{iy} \\
\theta_2^{jy} \\
e_2/L \\
\psi_2 \\
\theta_3^{ix} \\
\theta_3^{jx} \\
\theta_3^{iz} \\
\theta_3^{jz} \\
e_3/L \\
\psi_3
\end{Bmatrix}
=
\begin{bmatrix}
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{Bmatrix}
D_1^1/L \\
D_1^2/L \\
D_1^3/L \\
D_2^1/L \\
D_2^2/L \\
D_2^3/L \\
D_1^4 \\
D_1^5 \\
D_1^6 \\
D_2^4 \\
D_2^5 \\
D_2^6
\end{Bmatrix}
$$

where $L = 240$ inches

Figure 3.4. [A] matrix for two mass model.

$$
\begin{Bmatrix} M_1^{iy} \\ M_1^{jy} \\ M_1^{iz} \\ M_1^{jz} \\ P_1 L \\ T_1 \\ M_2^{ix} \\ M_2^{jx} \\ M_2^{iy} \\ M_2^{jy} \\ P_2 L \\ T_2 \\ M_3^{ix} \\ M_3^{jx} \\ M_3^{iz} \\ M_3^{jz} \\ P_3 L \\ T_3 \end{Bmatrix} = \frac{2EI}{L}
\begin{bmatrix}
2 & 1 \\
1 & 2 \\
& & 2 & 1 \\
& & 1 & 2 \\
& & & & \ast \\
& & & & & \ast\ast \\
& & & & & & 2 & 1 \\
& & & & & & 1 & 2 \\
& & & & & & & & 2 & 1 \\
& & & & & & & & 1 & 2 \\
& & & & & & & & & & \ast \\
& & & & & & & & & & & \ast\ast \\
& & & & & & & & & & & & 2 & 1 \\
& & & & & & & & & & & & 1 & 2 \\
& & & & & & & & & & & & & & 2 & 1 \\
& & & & & & & & & & & & & & 1 & 2 \\
& & & & & & & & & & & & & & & & \ast \\
& & & & & & & & & & & & & & & & & \ast\ast
\end{bmatrix}
\begin{Bmatrix} \theta_1^{iy} \\ \theta_1^{jy} \\ \theta_1^{iz} \\ \theta_1^{jz} \\ e_1/L \\ \psi_1 \\ \theta_2^{ix} \\ \theta_2^{jx} \\ \theta_2^{iy} \\ \theta_2^{jy} \\ e_2/L \\ \psi_2 \\ \theta_3^{ix} \\ \theta_3^{jx} \\ \theta_3^{ix} \\ \theta_3^{jz} \\ e_3/L \\ \psi_3 \end{Bmatrix}
$$

where $L = 240$ inches

$\ast = 1656.55$

$\ast\ast = 0.4$

Figure 3.5. [k] matrix for two mass model.

Table 3. 1 shows the frequency information for the first five modes of the two, five, and eight mass models, and for the first twenty modes of the eleven mass model. Table 3. 2 is a listing of the first five mode shapes for the two mass model, and Table 3. 3 is a listing of the first five mode shapes for the eleven mass model. Figures 3. 6 through 3. 10 are graphical representations of the first five modes for the two mass model, and Figures 3. 11 through 3. 15 are the mode shape plots for the eleven mass model.

3. 1. 3 Example Frame Response

Only the eleven mass model, shown in Figure 3. 3(d), of the example frame is used in this study to evaluate the dynamic response.

Based upon the recommendation of Newmark (23), damping for the example frame is assumed to be two percent. For this magnitude of damping, the difference between the damped and undamped natural frequency is 0. 02 percent. Thus, the difference is neglected in this study.

The most nebulous aspect of evaluating dynamic response using model superposition is that of determining the number of modes which must be considered. Clearly, any pronouncement regarding a general dynamics problem is virtually impossible. However, once the dynamic characteristics for a frame have been established, and given the type and location of the forcing function, some statements

Table 3.1. Frequencies for discrete mass models.

	n	Eigenvalue (Rad/sec)^2	n	Frequency (cps)	n	Period (sec)
Two Mass Model	1	5.52362E 02	1	3.74048E 00	1	2.67345E-01
	2	1.35979E 03	2	5.86883E 00	2	1.70392E-01
	3	1.52440E 03	3	6.21391E 00	3	1.60929E-01
	4	5.39444E 05	4	1.16893E 02	4	8.55483E-03
	5	5.39445E 05	5	1.16893E 02	5	8.55483E-03
Five Mass Model	1	9.47576E 02	1	4.89922E 00	1	2.04114E-01
	2	1.44475E 03	2	6.04945E 00	2	1.65304E-01
	3	2.44837E 03	3	7.87515E 00	3	1.26982E-01
	4	2.57847E 04	4	2.55565E 01	4	3.91290E-02
	5	2.58836E 04	5	2.56055E 01	5	3.90542E-02
Eight Mass Model	1	7.65040E 02	1	4.40212E 00	1	2.27163E-01
	2	2.16607E 03	2	7.40724E 00	2	1.35003E-01
	3	2.20548E 03	3	7.47432E 00	3	1.33791E-01
	4	2.79609E 04	4	2.66131E 01	4	3.75755E-02
	5	2.79630E 04	5	2.66141E 01	5	3.75741E-02
Eleven Mass Model	1	7.96014E 02	1	4.49035E 00	1	2.22700E-01
	2	2.21625E 03	2	7.49255E 00	2	1.33466E-01
	3	2.26019E 03	3	7.56646E 00	3	1.32162E-01
	4	2.83986E 04	4	2.68206E 01	4	3.72848E-02
	5	2.84242E 04	5	2.68327E 01	5	3.72680E-02
	6	5.46205E 04	6	3.71961E 01	6	2.68845E-02
	7	5.87957E 04	7	3.85916E 01	7	2.59124E-02
	8	7.92787E 04	8	4.48124E 01	8	2.23152E-02
	9	8.46206E 04	9	4.62976E 01	9	2.15994E-02
	10	3.13010E 05	10	8.90429E 01	10	1.12305E-02
	11	3.13114E 05	11	8.90577E 01	11	1.12287E-02
	12	4.32282E 05	12	1.04641E 02	12	9.55644E-03
	13	4.56425E 05	13	1.07524E 02	13	9.30026E-03
	14	5.03545E 05	14	1.12938E 02	14	8.85443E-03
	15	5.36603E 05	15	1.16586E 02	15	8.57735E-03
	16	1.00671E 06	16	1.59688E 02	16	6.26222E-03
	17	1.00691E 06	17	1.59704E 02	17	6.26158E-03
	18	1.30362E 06	18	1.81717E 02	18	5.50306E-03
	19	1.30471E 06	19	1.81793E 02	19	5.50077E-03
	20	1.38348E 06	20	1.87200E 02	20	5.34187E-03

n indicates the mode number

Table 3.2. Mode shapes for two mass model.

		Mode				
r	k	I	II	III	IV	V
1	1	2.23498E-05	-2.03823E-04	-2.94923E-04	-2.41686E-01	-2.41682E-01
	2	-1.87128E-01	-2.41684E-01	1.52952E-01	2.03727E-04	-2.03951E-04
	3	-1.52952E-01	1.06809E-04	-1.87128E-01	-2.13663E-04	2.14207E-04
2	1	1.87128E-01	-2.41684E-01	-1.52952E-01	2.03731E-04	2.03947E-04
	2	-2.23498E-05	-2.03823E-04	2.94923E-04	-2.41682E-01	2.41686E-01
	3	-1.52952E-01	-1.06809E-04	-1.87128E-01	2.13667E-04	2.14204E-04

r indicates the frame nodal point
k indicates the mode shape deflection component

Table 3.3. Mode shapes for eleven mass model.

r	k	Mode I	Mode II	Mode III	Mode IV	Mode V
1	1	1.49098E-05	1.14438E-04	-1.16382E-04	-1.09437E-03	-1.08372E-03
	2	-1.82471E-02	3.55558E-02	2.33861E-02	-6.84994E-02	5.69448E-02
	3	-1.92490E-02	8.76503E-03	-2.91438E-02	-5.42382E-02	-6.55681E-02
2	1	2.98182E-05	2.28847E-04	-2.32734E-04	-2.18513E-03	-2.16386E-03
	2	-6.53149E-02	1.18800E-01	8.14007E-02	-1.67284E-01	1.41145E-01
	3	-6.78974E-02	2.31086E-02	-9.12915E-02	-1.24255E-01	-1.52673E-01
3	1	4.47238E-05	3.43196E-04	-3.49024E-04	-3.26870E-03	-3.23687E-03
	2	-1.30105E-01	2.16756E-01	1.57306E-01	-1.82546E-01	1.61377E-01
	3	-1.32728E-01	2.55805E-02	-1.50266E-01	-1.06698E-01	-1.40386E-01
4	1	5.96253E-05	4.57458E-04	-4.65223E-04	-4.34150E-03	-4.29922E-03
	2	-2.02824E-01	3.02898E-01	2.38944E-01	-1.05048E-01	1.15471E-01
	3	-2.01878E-01	-1.44839E-04	-1.74816E-01	2.70070E-02	-6.73767E-04
5	1	6.46358E-02	6.31562E-02	-2.73830E-02	-1.91312E-01	-1.93585E-01
	2	-1.66552E-01	2.30023E-01	1.66106E-01	1.38678E-01	-1.32646E-01
	3	-2.01901E-01	-7.24270E-05	-1.74884E-01	2.71412E-02	-3.37447E-04
6	1	1.20690E-01	1.46191E-01	-8.99246E-02	-2.62949E-01	-2.62073E-01
	2	-1.20690E-01	1.46191E-01	8.99246E-02	2.62949E-01	-2.62074E-01
	3	-2.01910E-01	3.89430E-09	-1.74907E-01	2.71859E-02	-1.38667E-08

Table 3.3. (continued)

r	k	I	II	III	IV	V
7	1	1.66552E-01	2.30023E-01	-1.66106E-01	-1.38678E-01	-1.32646E-01
	2	-6.46358E-02	6.31562E-02	2.73830E-02	1.91312E-01	-1.93585E-01
	3	-2.01901E-01	7.24348E-05	-1.74884E-01	2.71412E-02	3.37419E-04
8	1	2.02824E-01	3.02898E-01	-2.38944E-01	1.05048E-01	1.15471E-01
	2	-5.96253E-05	4.57458E-04	4.65223E-04	4.34149E-03	-4.29922E-03
	3	-2.01873E-01	1.44847E-04	-1.74816E-01	2.70070E-02	6.73739E-04
9	1	1.30105E-01	2.16756E-01	-1.57306E-01	1.82546E-01	1.61377E-01
	2	-4.47238E-05	3.43196E-04	3.49024E-04	3.26869E-03	-3.23688E-03
	3	-1.32728E-01	-2.55805E-02	-1.50266E-01	-1.06698E-01	1.40386E-01
10	1	6.53149E-02	1.18800E-01	-8.14007E-02	1.67284E-01	1.41145E-01
	2	-2.98182E-05	2.28847E-04	2.32734E-04	2.18513E-03	-2.16386E-03
	3	-6.78974E-02	-2.31086E-02	-9.12915E-02	-1.24255E-01	1.52673E-01
11	1	1.82471E-02	3.55558E-02	-2.33861E-02	6.84995E-02	5.69447E-02
	2	-1.49098E-05	1.14438E-04	1.16382E-04	1.09437E-03	-1.08372E-03
	3	-1.92490E-02	-8.76503E-03	-2.91438E-02	-5.42381E-02	6.55682E-02

r indicates the frame nodal point
k indicates the mode shape deflection component

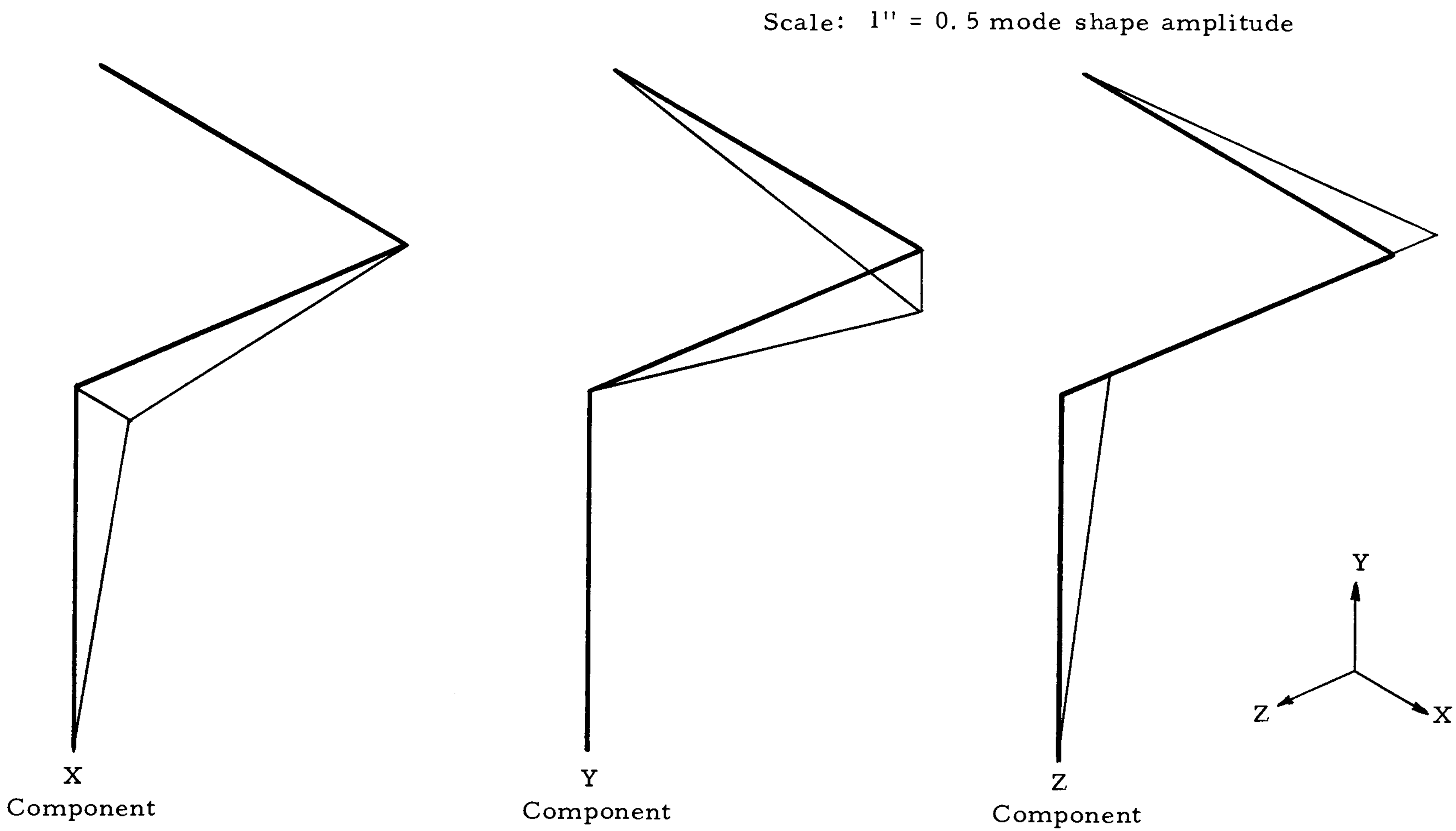

Figure 3.6. Mode shape plot for mode I - two mass model.

Scale: 1" = 0.5 mode shape amplitude

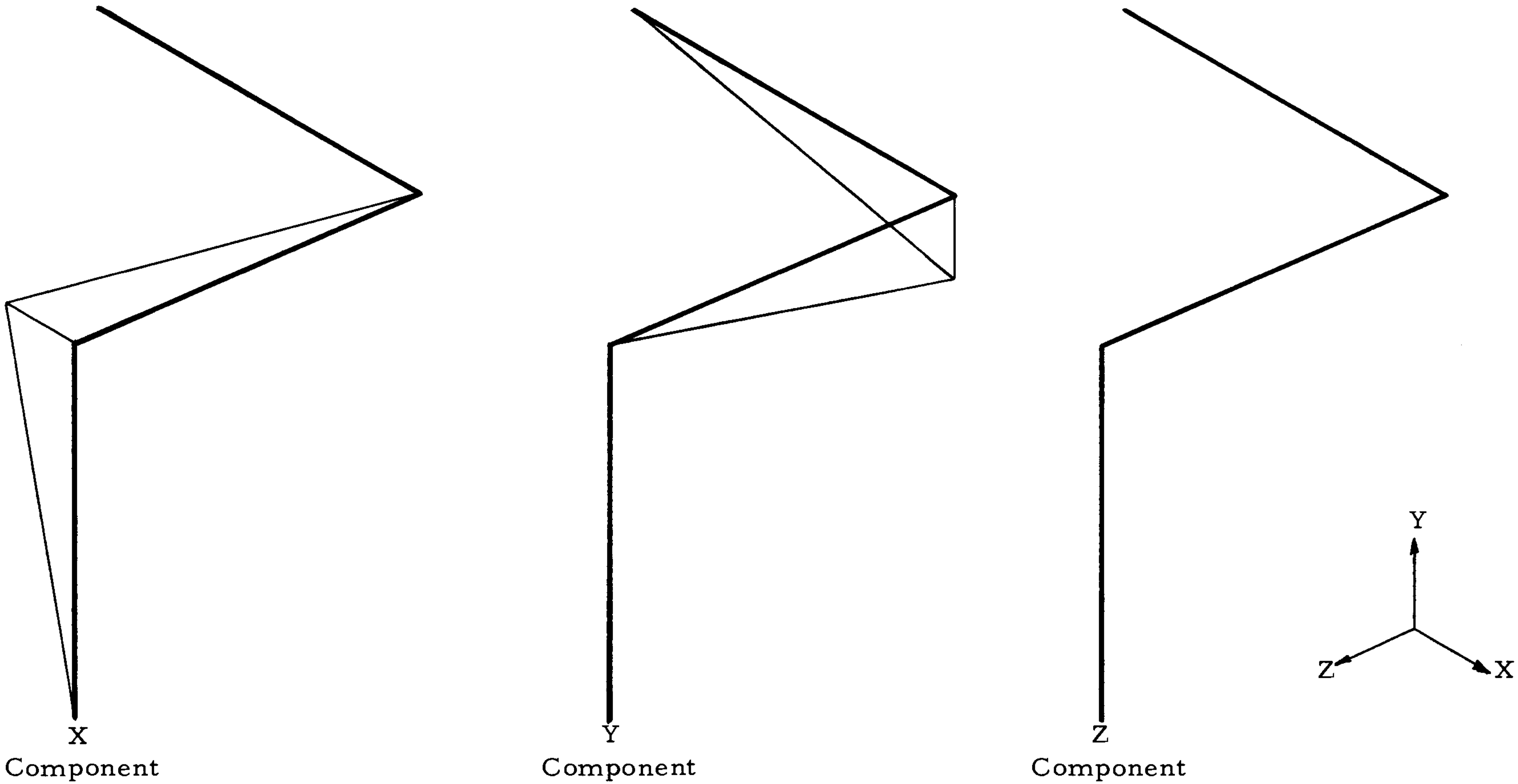

Figure 3.7. Mode shape plot for mode II - two mass model.

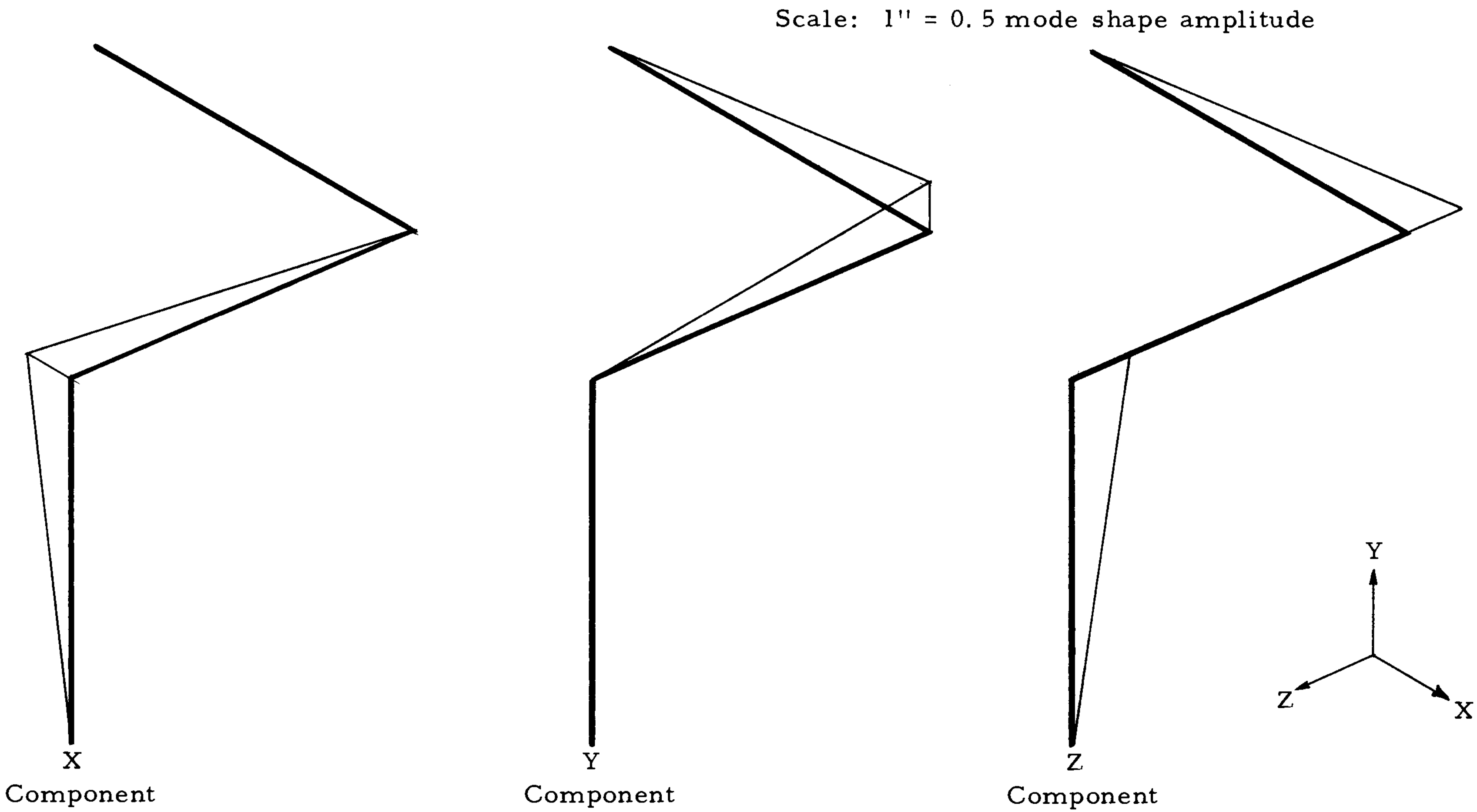

Figure 3. 8. Mode shape plot for mode III - two mass model.

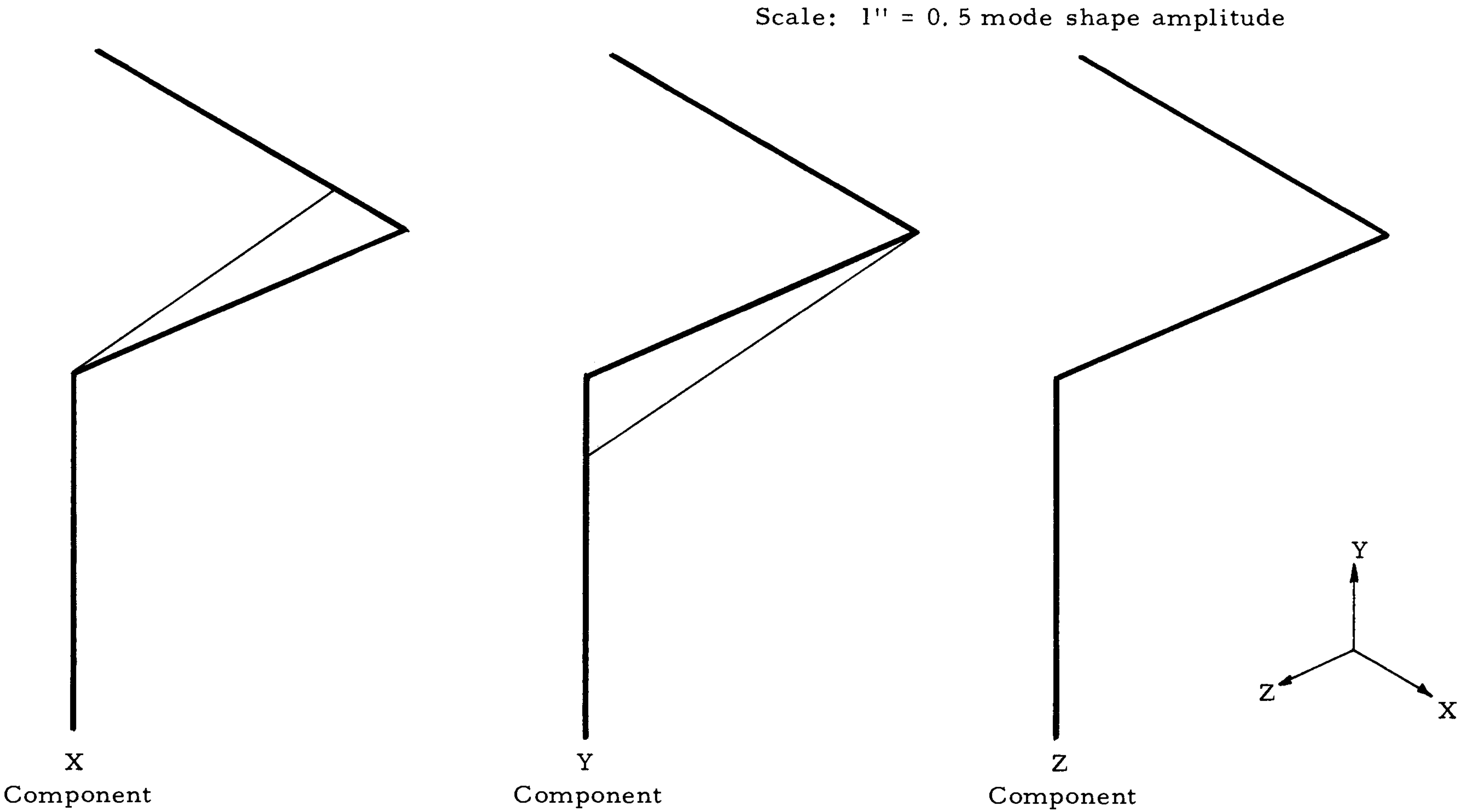

Figure 3. 9. Mode shape plot for mode IV - two mass model.

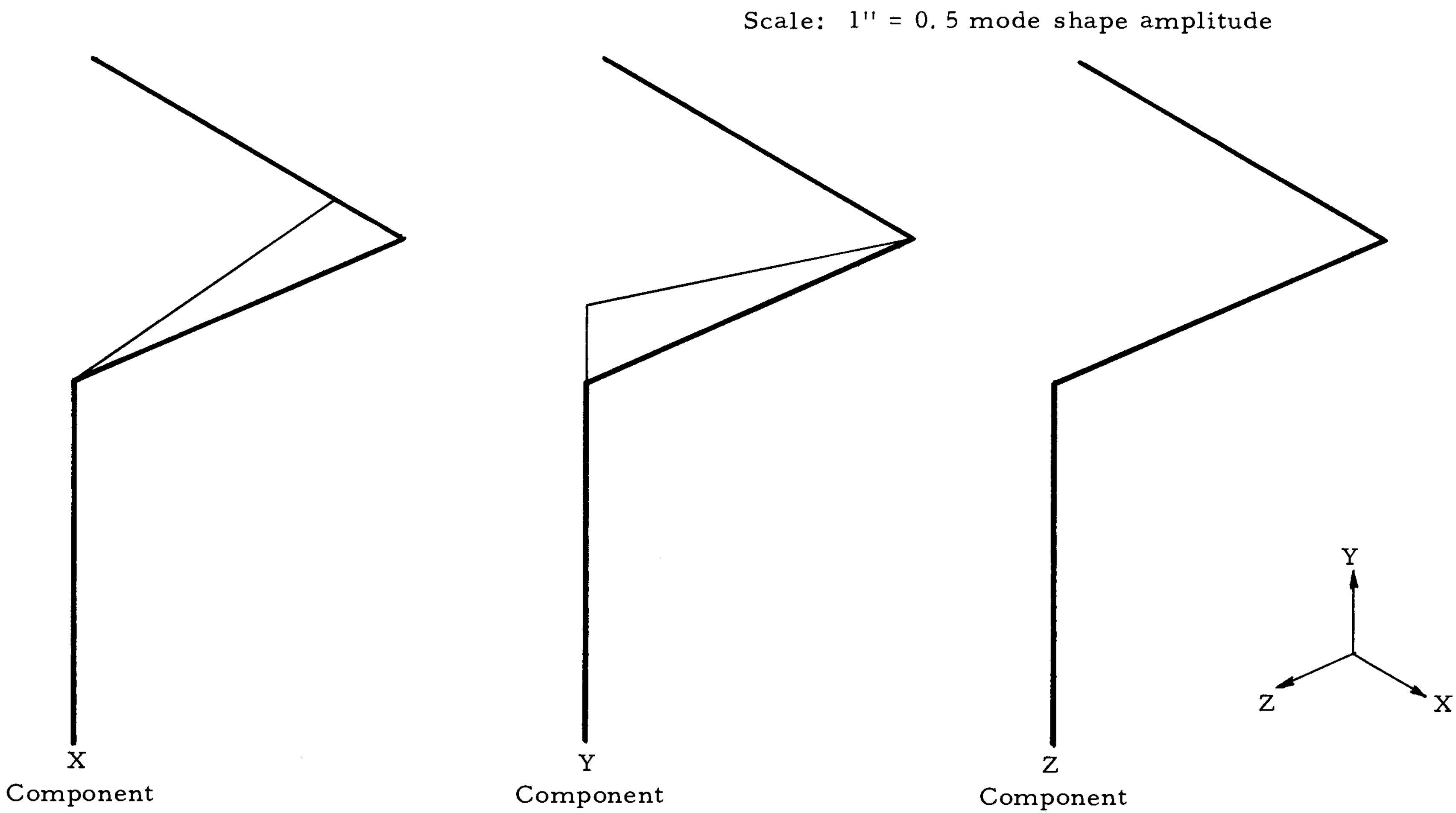

Figure 3.10. Mode shape plot for mode V - two mass model.

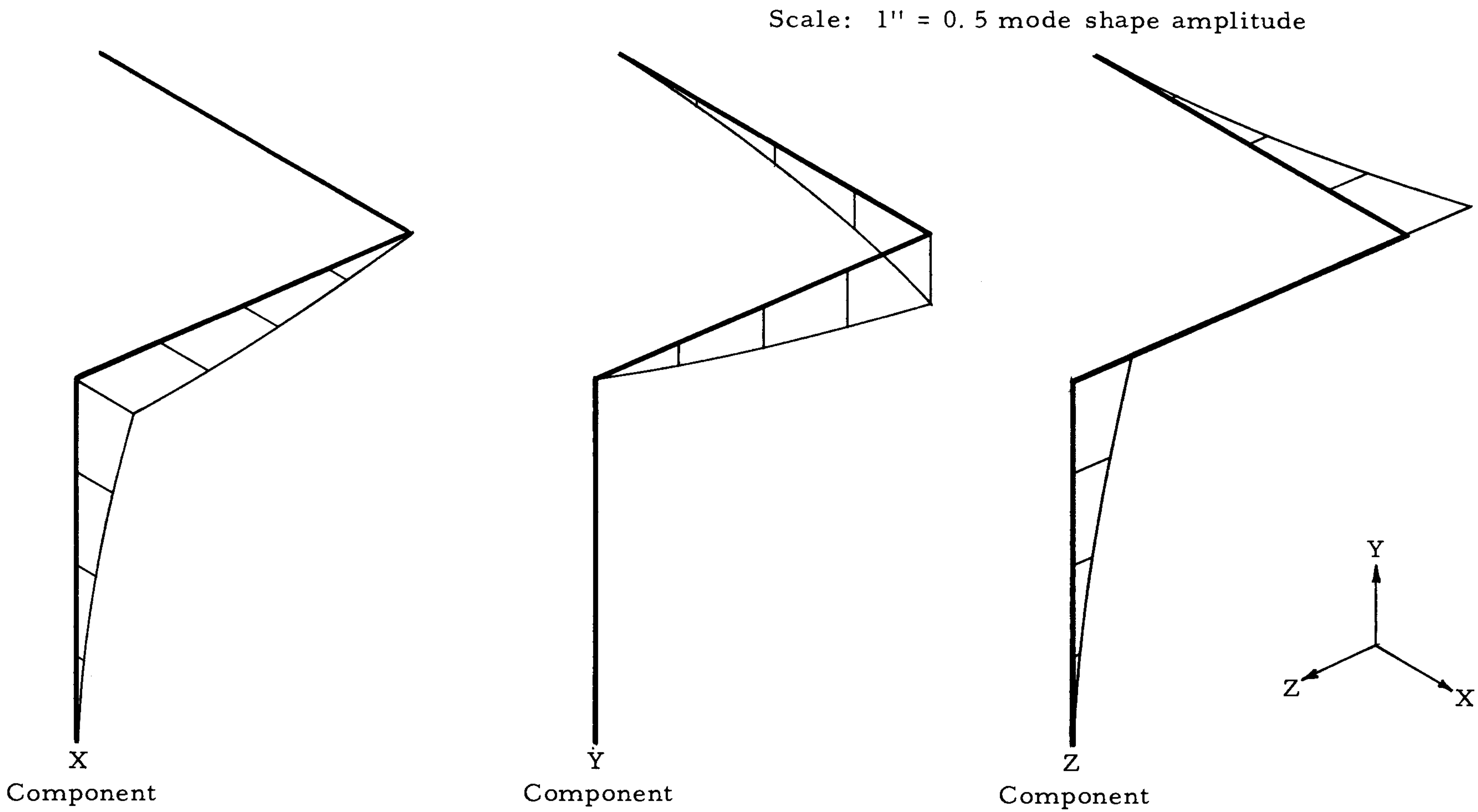

Figure 3. 11. Mode shape plot for mode I - eleven mass model.

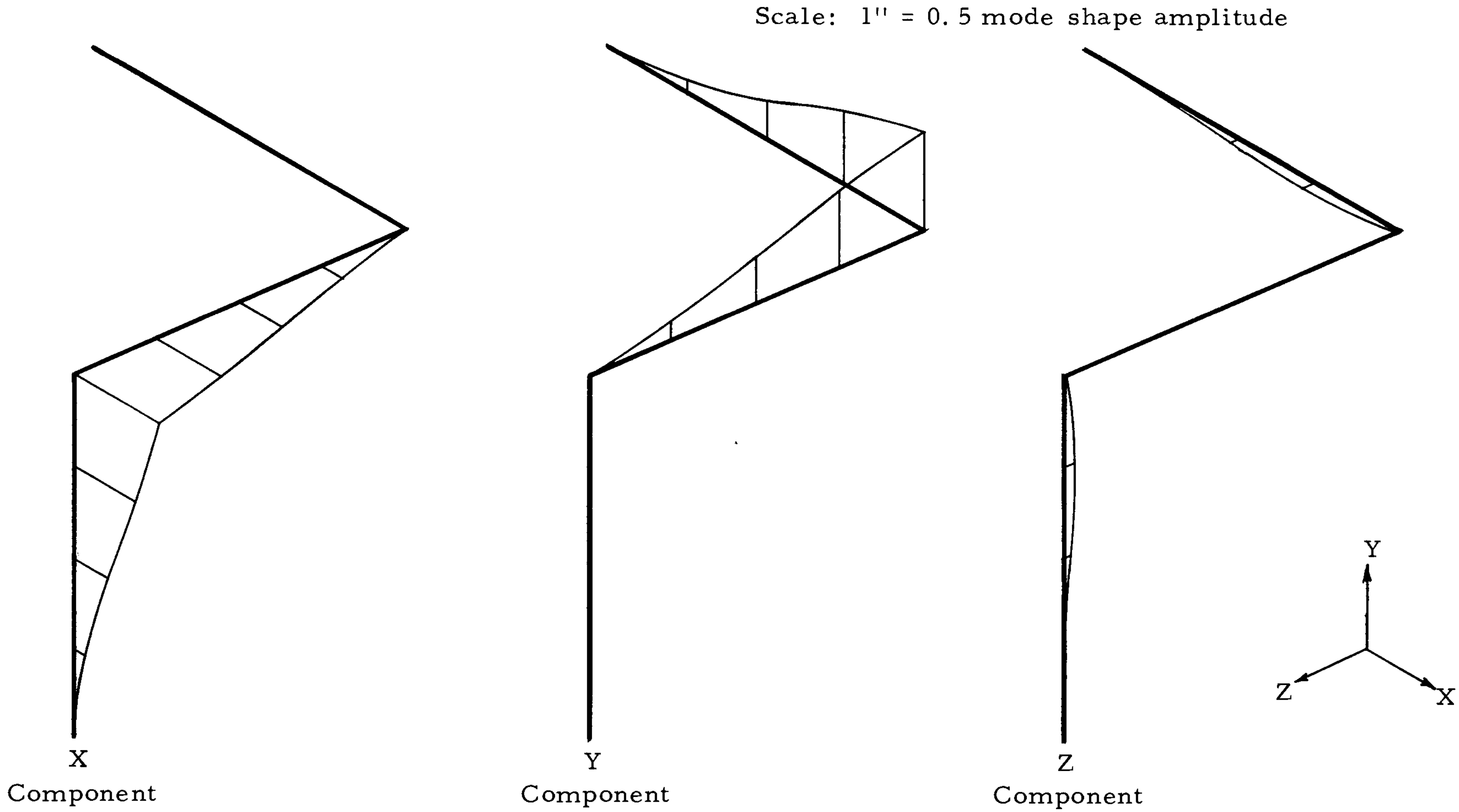

Figure 3. 12. Mode shape plot for mode II - eleven mass model.

Scale: 1" = 0. 5 mode shape amplitude

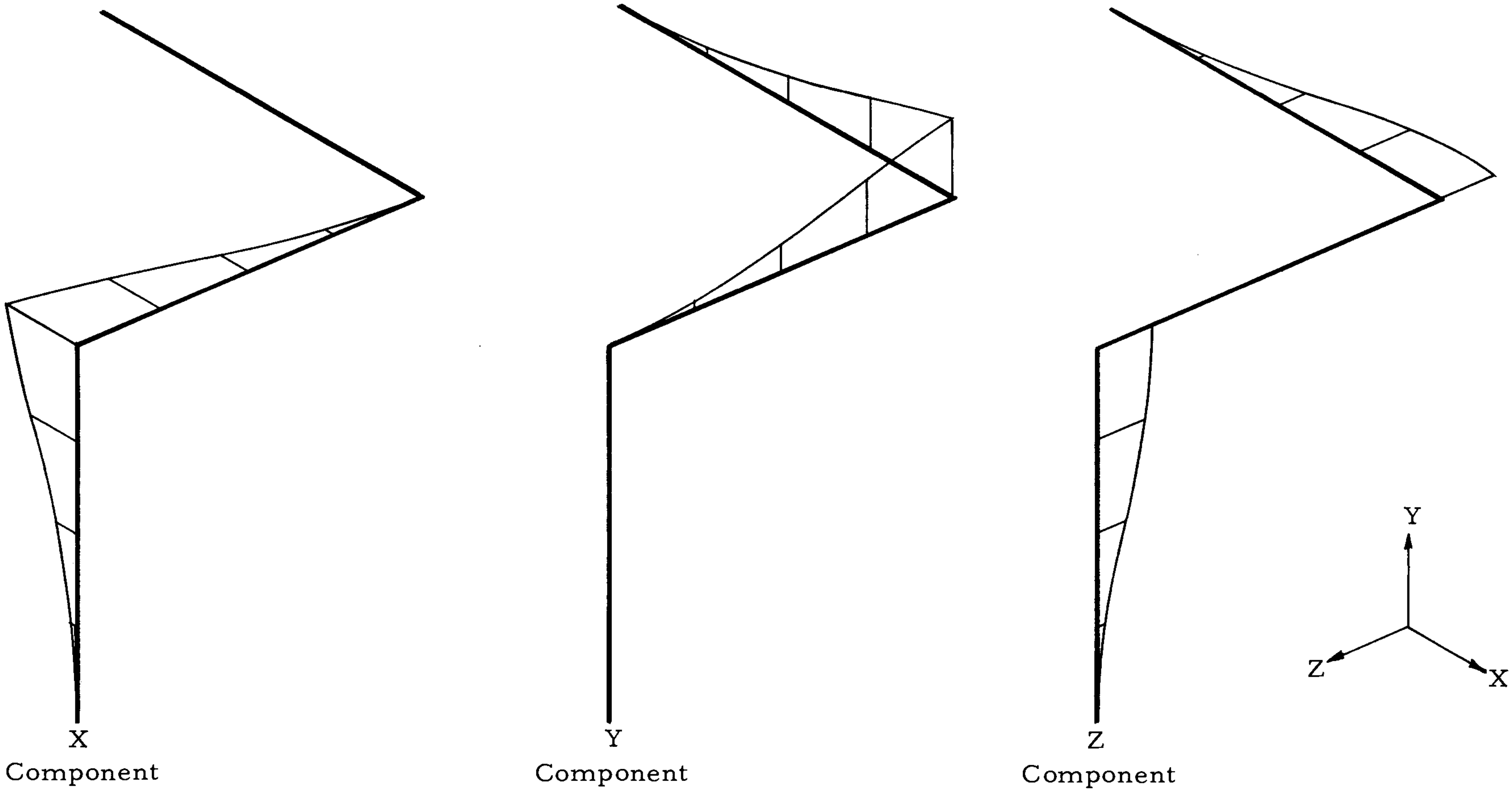

Figure 3. 13. Mode shape plot for mode III - eleven mass model.

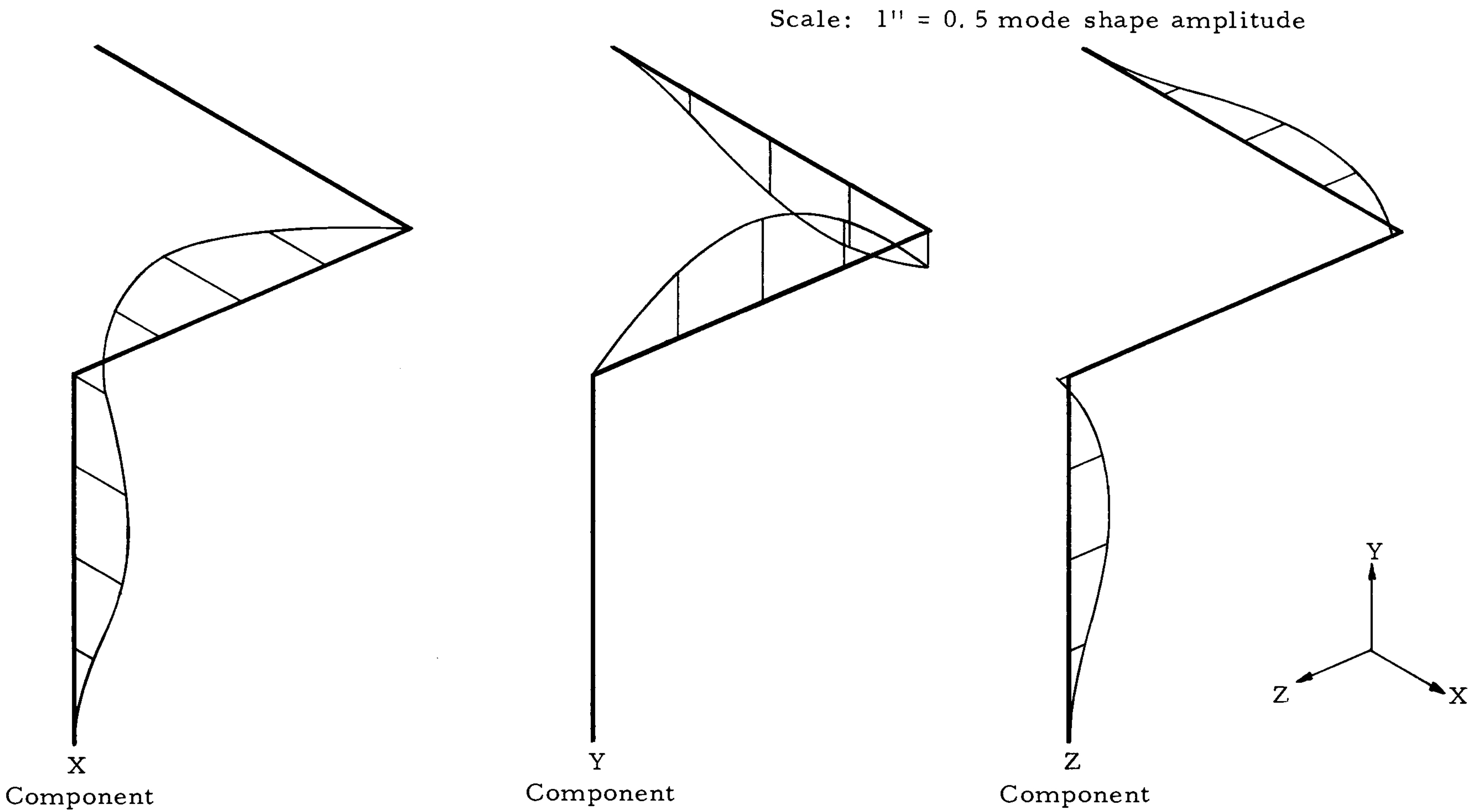

Figure 3. 14. Mode shape plot for mode IV - eleven mass model.

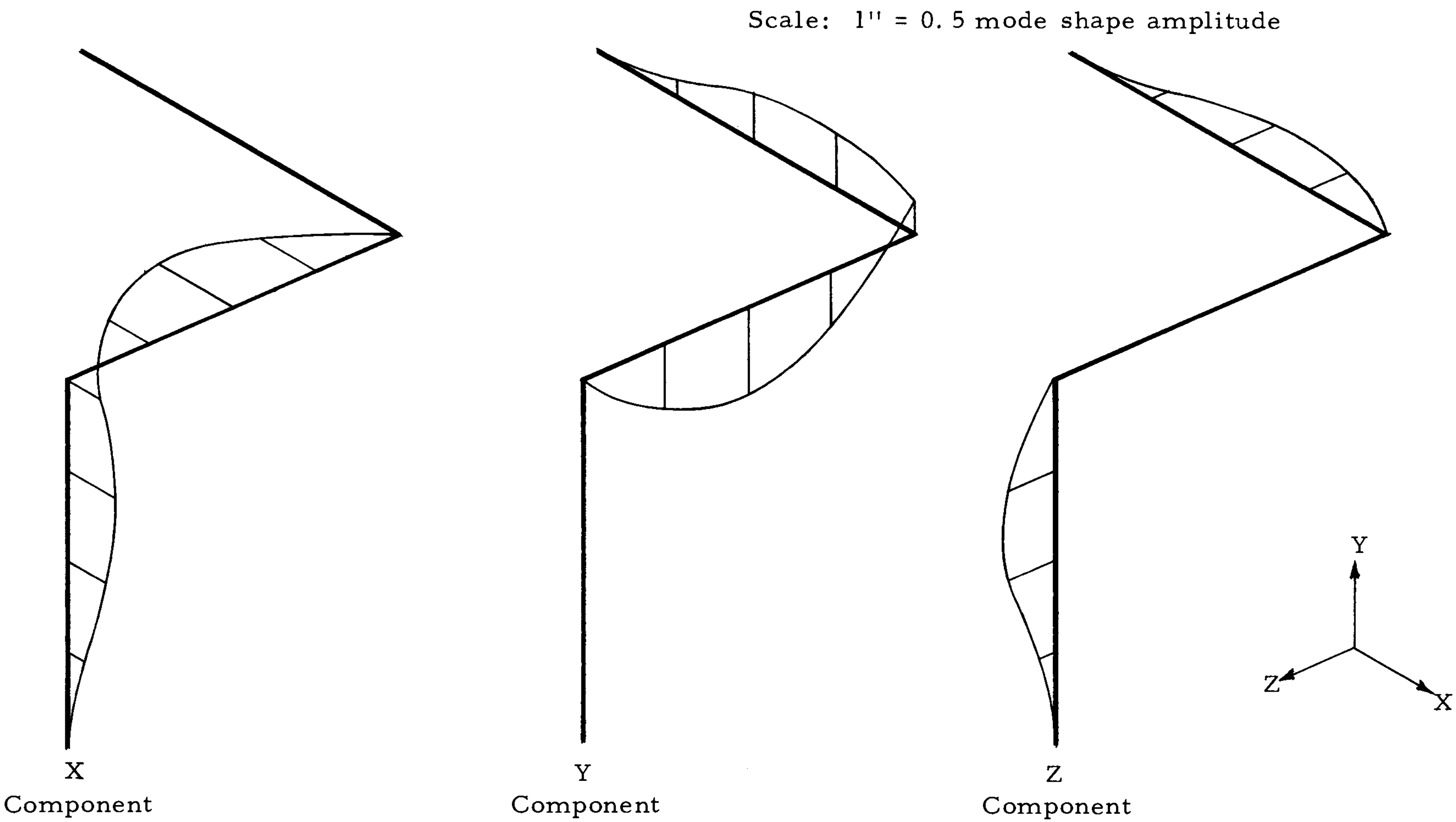

Figure 3. 15. Mode shape plot for mode V - eleven mass model.

can be made regarding the number of modes required. By type of force, reference is being made to the three basic types of dynamic forces namely; impulse, step, and harmonic.

The control function of this example, shown in Figure 3. 2, can be approximated by the superposition of a series of step functions. Furthermore, it can be seen that an upper bound of the control function is that of a step function having U(0) = 1. 0. Using this upper bound with the concept of modal response described by Biggs (3, p. 120), insight can be gained concerning the number of modes which must be considered. Damping is neglected in the following upper bound discussion.

Maximum modal response can be described by

$$S_n(t) = S_{n\,st}(DLF)_n \tag{3.1}$$

in which the modal static response is given by

$$S_{n\,st} = \frac{P_n}{\omega_n^2} \tag{3.2}$$

The modal force P_n is defined in Equation (2. 4c). The dynamic load factor $(DLF)_{max}$ for a step function is 2. 0, from Biggs (3, p. 46).

For the example frame, the modal static responses for the first ten modes are given in Table 3. 4. In addition, the percentage of the absolute magnitude of modal static response for the first ten modes and the accumulative percentage is given. As the value of $S_{10\,st}$ indicates, the modal static response for modes 11 through 33 is negligible.

Table 3.4. Modal static response for first ten modes.

Mode	$S_{n\,st}$ (inches)	Percentage of $\sum_{n=1}^{10} \lvert S_{n\,st} \rvert$	Accumulative Percentage
1	-2.5480	49.36	49.36
2	1.3666	26.48	75.83
3	1.0571	20.48	96.31
4	-0.0367	.71	97.02
5	0.0406	.79	97.81
6	0.0408	.79	98.60
7	-0.0195	.38	98.98
8	0.0307	.59	99.57
9	-0.0184	.36	99.93
10	0.0034	.06	100.00

Table 3.4 shows only the order of magnitude of response for the individual modes. In order to gain insight concerning the response of individual nodal points, it is necessary to consider the mode shapes as given by Equation (2.2). In this manner, it can be shown that for D_4^2 approximately 97 percent of the response is obtained by considering the first three modes. An exact percentage cannot be computed because the (D L F)'s for the individual modes cannot be predicted

precisely. However, since the range is $0 < (DLF)_n < 2$, and because of the large order of magnitude difference in $S_{n\,st}$ between the first three and the remaining modes, the percentage will not vary significantly.

It was decided to limit the response solution to the first three modes for the following reasons. First, Table 3.4 shows that in order to improve on the percentage significantly, it is necessary to consider several more modes. The increase in computational cost resulting from including these additional modes is disproportionately high when weighed against the percentage improvement of the solution. Secondly, it was thought desirable to obtain a response solution using an analog computer. Because the pots on the available analog computer can be set to only four places, and because of the large order of magnitude difference in response between the first three and the remaining modes, it is impractical to use more than three modes. Because of the physical size of the available analog computer, five modes is the maximum that can be considered. However, because of the four place pot setting limitation, the error in the response of the fourth and fifth modes would be 10 to 20 percent.

3.1.3.1 Numerical Response Solution

The numerical response solution for the example frame is obtained using the computer program RESPON, shown in Appendix A.

As stated in Chapter 2, the fourth order Runge-Kutta method is the numerical process used to solve the differential equations.

To verify that the Runge-Kutta process yielded an accurate solution for the example frame it was decided to solve the problem using an analog computer and compare the two solutions. In addition, the response solution for a step force was computed exactly and the results were compared with the solution obtained using RESPON. The effect of varying the time increment, used in the numerical solution, was also considered in this comparison. The results of this comparison showed that the difference between the exact and the numerical solutions was 0.005 percent and 0.05 percent for time increments of 0.005 and 0.01 seconds, respectively. Using only three modes, these time increments represent ratios of approximately $T_{min}/30$ and $T_{min}/15$. Where T_{min} is the shortest natural period of vibration used in the solution. The effect of varying the time increment for the forcing function of the example frame was investigated and the results showed that there was a difference of approximately one percent for RESPON solutions using time increments of 0.005 and 0.01 seconds.

Since conclusions regarding the accuracy of the numerical process can be made by considering a single nodal coordinate and since solutions for many nodal coordinates using the analog computer are very tedious and time consuming, the comparison here is limited to a single nodal coordinate. For the eleven mass model of Figure 3.3(d), the nodal coordinate chosen is that of translation in the Y direction of mass four, i.e. D_4^2. The first second of the numerical time-history response solution for nodal coordinate D_4^2 is shown in Figure 3.16.

Table 3.5 is a listing of the maximum nodal coordinate responses and their occurrence times for the example frame.

3.1.3.2 Analog Computer Response Solution

The analog computer circuitry used to obtain the response solution for a single nodal coordinate of the example frame, considering three modes, is shown in Appendix C. The first second of the analog computer time-history response solution for nodal coordinate D_4^2 of the example frame is shown superimposed on the numerical solution in Figure 3.16.

3.2 Continuous Mass System Analysis

The model used to perform the continuous mass analysis is shown below in Figure 3.17. The physical properties used in the analysis are given in Figure 3.1

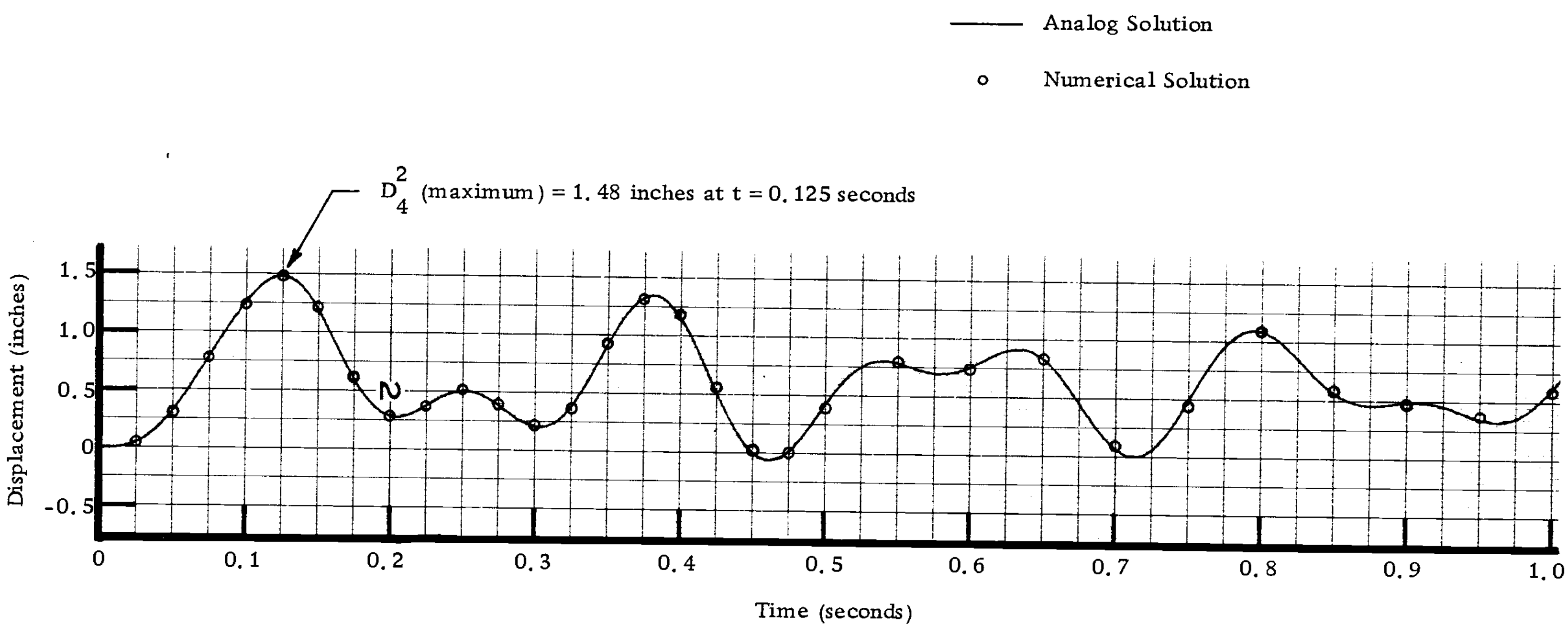

Figure 3.16. Example frame time-history response for nodal coordinate D_4^2.

Table 3. 5. Example frame maximum deflections and occurrence times.

r	k	Time (sec)	Deflection (inches)
1	1	.260	5.40975E-05
1	2	.125	1.49380E-01
1	3	.160	6.24781E-02
2	1	.260	1.08184E-04
2	2	.125	5.17723E-01
2	3	.160	2.20779E-01
3	1	.260	1.62249E-04
3	2	.125	9.92157E-01
3	3	.160	4.32322E-01
4	1	.260	2.16283E-04
4	2	.125	1.48067E-00
4	3	.160	6.58048E-01
5	1	.160	-2.10159E-01
5	2	.125	1.14664E-00
5	3	.160	6.58160E-01
6	1	.160	-3.92746E-01
6	2	.130	7.58860E-01
6	3	.160	6.58221E-01
7	1	.160	-5.43562E-01
7	2	.130	3.57694E-01
7	3	.160	6.58236E-01
8	1	.160	-6.64308E-01
8	2	.115	1.58434E-03
8	3	.160	6.58199E-01
9	1	.165	-4.16171E-01
9	2	.115	1.18859E-03
9	3	.165	4.13572E-01
10	1	.165	-2.06235E-01
10	2	.115	7.92554E-04
10	3	.255	-2.08636E-01
11	1	.165	-5.69951E-02
11	2	.115	3.96324E-04
11	3	.255	-6.61320E-02

r indicates the frame nodal point
k indicates the deflection component

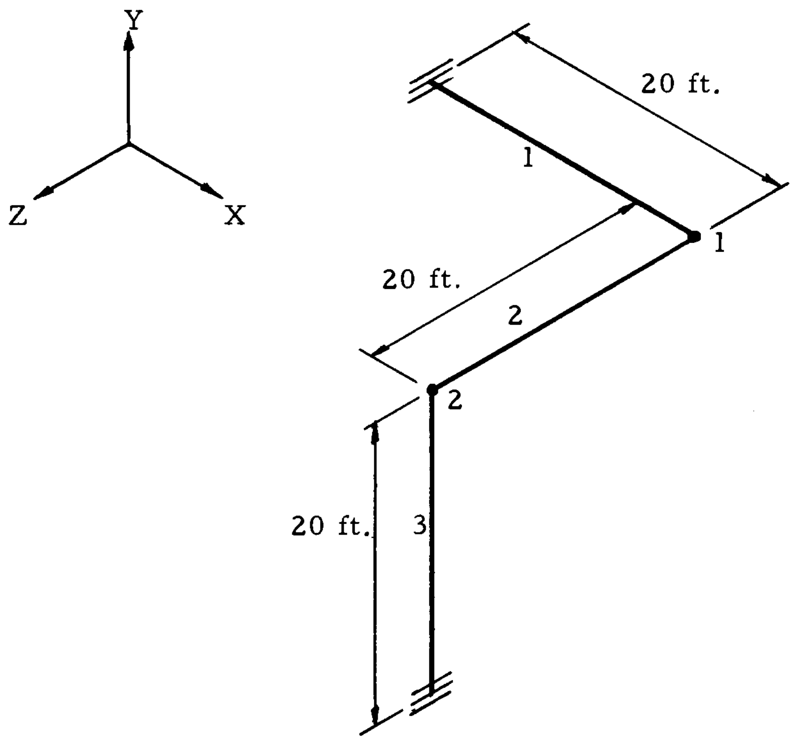

Figure 3. 17. Example frame continuous mass model.

The frame dynamic stiffness matrix is computed in essentially the same way that the frame static stiffness matrix was computed. The size of the [A] matrix is [36 x 12] and is shown in Figure 3. 18 in the relationship $\{d\} = [A]\{D\}$. The $[k(\omega)]$ matrix for the frame is similar to that shown in Figure 3. 5. The difference is that here each of the individual member stiffness matrices is a [12 x 12] and is a function of ω as shown in Equation (2. 24). Thus, the size of the frame member dynamic stiffness matrix is [36 x 36].

$$
\begin{Bmatrix}
\theta_1^{iy} \\ \eta_1^{iz}/L \\ \theta_1^{jy} \\ \eta_1^{jz}/L \\ \theta_1^{iz} \\ \eta_1^{iy}/L \\ \theta_1^{jz} \\ \eta_1^{jy}/L \\ e_1^{i}/L \\ e_1^{j}/L \\ \psi_1^{i} \\ \psi_1^{j} \\
\theta_2^{ix} \\ \eta_2^{iy}/L \\ \theta_2^{jx} \\ \eta_2^{jy}/L \\ \theta_2^{iy} \\ \eta_2^{ix}/L \\ \theta_2^{jy} \\ \eta_2^{jx}/L \\ e_2^{i}/L \\ e_2^{j}/L \\ \psi_2^{i} \\ \psi_2^{j} \\
\theta_3^{iz} \\ \eta_3^{ix}/L \\ \theta_3^{jz} \\ \eta_3^{jx}/L \\ \theta_3^{ix} \\ \eta_3^{iz}/L \\ \theta_3^{jx} \\ \eta_3^{jz}/L \\ e_3^{i}/L \\ e_3^{j}/L \\ \psi_3^{i} \\ \psi_3^{j}
\end{Bmatrix}
=
\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&1&0&0&0&0 \\
0&0&1&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&1&0&0&0 \\
0&1&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&1&0&0&0&0&0 \\
0&0&0&0&0&0&1&0&0&0&0&0 \\
0&1&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&0&0 \\
0&0&0&0&1&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&1&0&0&0&0 \\
1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&1&0 \\
0&0&0&1&0&0&0&0&0&0&0&0 \\
0&0&1&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&1&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&1 \\
0&0&0&1&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&0&0 \\
0&0&0&0&0&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&1&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&1&0
\end{bmatrix}
\begin{Bmatrix}
D_1^1/L \\ D_1^2/L \\ D_1^3/L \\ D_2^1/L \\ D_2^2/L \\ D_2^3/L \\ D_1^4 \\ D_1^5 \\ D_1^6 \\ D_2^4 \\ D_2^5 \\ D_2^6
\end{Bmatrix}
$$

Figure 3. 18. [A] matrix for continuous mass model.

The eigenvalues for the example frame were evaluated using the computer program DEIGEN, shown in Appendix B. The input to the program consists of the physical parameters of a frame, an [A] matrix, and a trial eigenvalue. DEIGEN first computes the member dynamic stiffness matrix for the frame and then computes the frame dynamic stiffness matrix. Finally, the program evaluates the determinant of the frame dynamic stiffness matrix.

Figure 3.19 is a plot of the determinant of $[K(\omega)]$ versus ω^2 for the example frame. The first three eigenvalues are noted in the figure. The fourth eigenvalue was investigated and found to be approximately 26,100. This compares reasonably well with the fourth eigenvalue for the eleven mass discrete solution - 28,399. Since it was decided prior to this that the fourth and higher modes would not be used in the response solution, and in order to conserve on computer time, the extraction of eigenvalues was not pursued further.

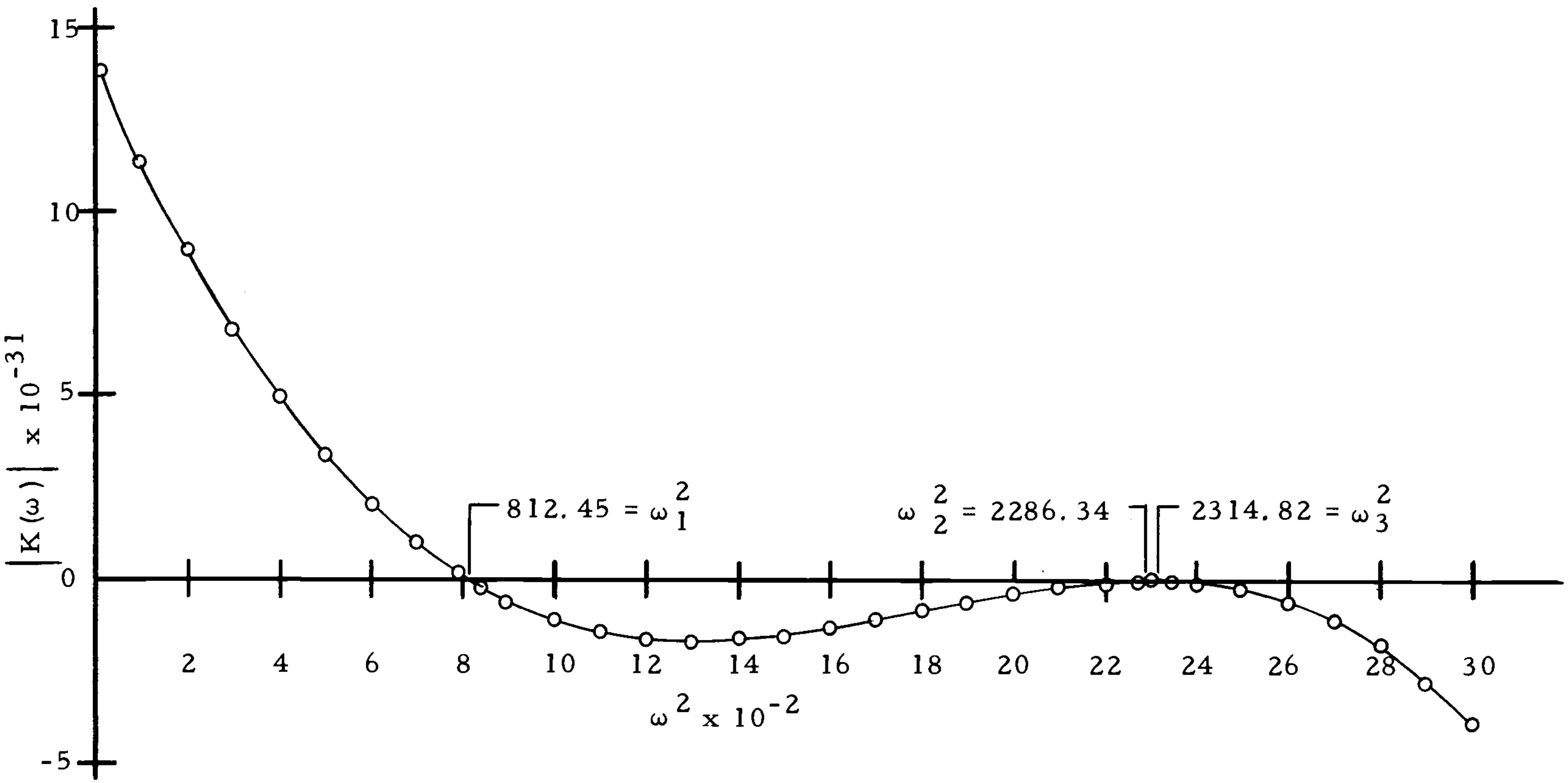

Figure 3.19. Eigenvalues for continuous mass model.

Chapter 4

DISCUSSION AND CONCLUSIONS

Table 4. 1 shows a comparison of the first three frequencies for the four discrete mass models and for the continuous mass model. The percent differences shown, are based on the frequencies of the continuous mass model. Two observations of consequence can be made from Table 4. 1. First, as discrete mass model sophistication increases, the frequencies approach the exact solution. Second, for each of the three modes, the frequencies for the eleven mass model are within approximately one percent of the exact frequencies.

A comparison of nodal coordinate maximums was made for RESPON solutions using three and five modes, and a difference of approximately 0. 7 percent was found. This essentially verifies the discussion of Section 3. 1. 3 regarding the number of modes which must be considered.

As stated in Section 3. 1. 3. 1, the error incurred in the numerical integration procedure when compared with an exact solution is very small. Restating it here, the comparison showed that for time increments of 0. 01 and 0. 005 seconds, the error was 0. 05 percent and 0. 005 percent, respectively. Also, it was stated that these time increments represent ratios of $T_{min}/15$ and $T_{min}/30$, respectively.

Table 4.1. Comparison of discrete mass and continuous mass frequencies.

Model	MODE 1		MODE 2		MODE 3	
	Frequency (CPS)	Percent Difference	Frequency (CPS)	Percent Difference	Frequency (CPS)	Percent Difference
Discrete						
Two Mass	3.74	17.6	5.87	22.8	6.21	18.9
Five Mass	4.89	7.7	6.05	20.5	7.88	2.9
Eight Mass	4.40	3.1	7.41	2.6	7.47	2.5
Eleven Mass	4.49	1.1	7.49	1.4	7.57	1.2
Continuous Mass	4.54	---	7.61	---	7.66	---

Most investigators have stated that ratios of $T_{min}/5$ to $T_{min}/10$ are adequate. Although these recommended ratios were not used, the results of this investigation indicate that the time increment could have been increased and that perhaps the recommended ratios are adequate for a practical structural analysis.

Figure 3. 16 shows that the difference between the analog computer and numerical response solutions is very small. Since the analog solution is obtained from a strip-chart recorder, it is impossible to make any statement concerning the percentage difference between the two solutions.

To the writer's knowledge, the analog computer has never been used to analyze a multi-degree-freedom structure in this way. That is, by mode superposition. Initially, it was intended to solve the modal equations of motion, Equation (2. 3), on the analog; then obtain a digital read-out of these modal time-history solutions. The nodal coordinate time-history solutions could then be obtained by reading the digitized modal coordinate time-history solutions into the CDC-3300 and using Equation (2. 2). The analog computer used, an EAI-680, is part of an EAI-690 Hybrid computer; the digital part is an EAI-640. Thus, the capability for performing the above described objective exists at Oregon State University. The obstacle which prevented completing the objective is the incompatability of the EAI-690 and the CDC-3300. That is, there is no available means of transferring data from one to the other without reproducing it manually. Based on the results of this investigation, the above described objective appears worth pursuing.

BIBLIOGRAPHY

1. American Institute of Steel Construction. Manual of steel construction. 6th ed. New York, 1966. Various paging.

2. Archer, J. S. Consistent mass matrix for distributed mass systems. Proceedings of the American Society of Civil Engineers, Journal of the Structural Division 89:161-178. Aug., 1963.

3. Biggs, John M. Introduction to structural dynamics. New York, McGraw-Hill, 1964. 341 p.

4. Bromwich, T. J. I'A. Normal coordinates in dynamical systems. Proceedings of the London Mathematical Society, ser. 2, 15:401-448. 1916.

5. Cakiroglu, Adan and Gunay Ozmen. Numerical integration of forced-vibration equations. Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division 94:711-729. June, 1968.

6. Coughy, T. K. Classical normal modes in damped linear dynamic systems. Transactions of the American Society of Mechanical Engineers, ser. E, Journal of Applied Mechanics 82:269-271. 1960.

7. Coughy, T. K. and M. E. O'Kelly. Classical normal modes in damped linear dynamic systems. Transactions of the American Society of Mechanical Engineers, ser. E, Journal of Applied Mechanics 87:583-588. 1965.

8. ________________ Effect of damping on the natural frequencies of linear dynamic systems. The Journal of the Acoustical Society of America 33:1458-1461. 1961.

9. Crandall, Stephen H. Engineering analysis. New York, McGraw-Hill, 1956. 417 p.

10. Duncan, W. J. and A. R. Collar. A method for the solution of oscillation problems by matrices. Philosophical Magazine and Journal of Science 17:865-909. 1934.

11. Foss, K. A. Co-ordinates which uncouple the equations of motion of damped linear dynamic systems. Transactions of the American Society of Mechanical Engineers, ser. E, Journal of Applied Mechanics 79:361-364. 1958.

12. Franklin, Joel N. Matrix theory. Englewood Cliffs, Prentice-Hall, 1968. 292 p.

13. Gere, James M. and William Weaver, Jr. Analysis of framed structures. Princeton, New Jersey, Van Nostrand, 1965. 475 p.

14. Gillespie, James W. and Boer-Dar Liaw. Frequency analysis of beams by flexibility method. Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division 90:23-46. Feb., 1964.

15. Hildebrand, F. B. Introduction to numerical analysis. New York, McGraw-Hill, 1956. 511 p.

16. Hurty, Walter C. and Moshe F. Rubinstein. Dynamics of structures. Englewood Cliffs, Prentice Hall, 1964. 455 p.

17. James, M. L., G. M. Smith and J. C. Wolford. Analog and digital computer methods in engineering analysis. Scranton, International, 1964. 457 p.

18. Laursen, Harold I. Matrix analysis of structures. New York, McGraw-Hill, 1966. 159 p.

19. ________________ Structural analysis. New York, McGraw-Hill, 1969. 486 p.

20. Laursen, H. I., R. P. Shubinski and R. W. Clough. Dynamic matrix analysis of framed structures. In: Proceedings of the Fourth U. S. National Congress of Applied Mechanics, Berkeley, 1962. Vol. 1. New York, American Society of Mechanical Engineers, 1962. p. 99-105.

21. McCormick, John M. and Mario G. Salvadari. Numerical methods in fortran. Englewood Cliffs, Prentice-Hall, 1964. 324 p.

22. Milne, William Edmund. Numerical solution of differential equations. New York, Wiley, 1953. 275 p.

23. Newmark, N. M. Design criteria for nuclear reactors subjected to earthquake hazards. Paper presented at the Tokyo World Earthquake Conference, June, 1967.

24. ________________ A method of computation for structural dynamics. Transactions of the American Society of Civil Engineers 127, part 1: 1406-1435. 1962.

25. Newmark, N. M. A method of computation for structural dynamics. Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division 85: 67-94. July, 1959.

26. O'Hara, G. J. and Patrick F. Cunniff. Numerical method for structural shock response. Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division 90:51-82. April, 1964.

27. Rayleigh, J. W. S. The theory of sound. 2d ed. Vol. 1. New York, Dover, 1945. 480 p.

28. Timoshenko, S. and D. H. Young. Vibration problems in engineering. Princeton, New Jersey, Van Nostrand, 1956. 468 p.

29. Wilkinson, James Hardy. The algebraic eigenvalue problem. Oxford, Clarenden, 1965. 662 p.

APPENDICES

APPENDIX A

DISCRETE MASS SYSTEM COMPUTER PROGRAMS

Three individual computer programs were written to perform the complete dynamic analysis for the discrete mass system. These programs are named STIFF, JEIGEN, and RESPON. As is somewhat indicated by the program names, STIFF generates the frame stiffness matrix, JEIGEN evaluates the eigenvalues and eigenvectors, and RESPON provides the time-history solution for the governing differential equations. Since some of the subroutines are used in more than one program, they are all listed separately after the main programs.

The programs are written to be run on the OS-3 (Oregon State Open Shop Operating System) system which is centered around a CDC-3300 computer. Thus, the programs are subject to the peculiarities of the system.

```
      PROGRAM STIFF
C     THIS PROGRAM GENERATES THE STIFFNESS MATRIX FOR A GENERAL
C       FRAME USING THE DISPLACEMENT METHOD.
      REAL LØ
      REAL 1  A,AK,F,AK1,AK2,AK3,AK4,BKZ,BKZZ,FK
      COMMON A,AK,F,AK1,AK2,AK3,AK4,BKZ,BKZZ,FK,B
      DIMENSION A(72,72), AK(72,72), F(72,72), AK1(36,36)
      DIMENSION AK2(36,36), AK3(36,36), AK4(36,36)
      DIMENSION BKZ(36,36), BKZZ(36,36) , FK(36,36)
      DIMENSION B(36,1)
      M = FFIN(1)
      N = FFIN(1)
      NN = FFIN(1)
      NA = FFIN(1)
      NK = FFIN(1)
      E = FFIN(1)
      AI = FFIN(1)
      LØ = FFIN(1)
      WRITE (61,2) N,NN,E,AI,LØ
    2 FORMAT('1TOTAL NUMBER OF DEGREES OF FREEDOM (N) = ',I2/
     1' NUMBER OF TRANSLATIONAL DEGREES OF FREEDOM (NN) = ',I2/
     2' MODULUS OF ELASTICITY (E) = ',E13.5 '  LB./IN.SQ.'/
     3' AREA MOMENT OF INERTIA (AI)  = ',E13.5 '  IN.↑4'/
     4' REFERENCE MEMBER LENGTH (LØ) = ' E13.5 '  INCHES'////)
      DO 8 I = 1,M
      DO 8 J = 1,N
    8 A(I,J) < Ø.Ø
      DO 1Ø L = 1,NA
      I = FFIN(1)
      J = FFIN(1)
   1Ø A(I,J) = FFIN(1)
      WRITE(61,11)
   11 FORMAT('1          DISPLACEMENT TRANSFORMATION  MATRIX'/)
      CALL OUTPUT72(M,N,A)
      DO 18 I = 1,M
      DO 18 J = 1,M
   18 AK(I,J) = Ø.Ø
      DO 2Ø L = 1,NK
      I = FFIN(1)
      J = FFIN(1)
   2Ø AK(I,J) = FFIN(1)
      WRITE(61,22)
   22 FORMAT('1               MEMBER STIFFNESS MATRIX'/)
      CALL OUTPUT72(M,M,AK)
C     COMPUTE BIG K
      DO 3Ø I = 1,N
      DO 3Ø J = 1,M
      F(I,J) = Ø.Ø
      DO 3Ø K < 1,M
   3Ø F(I,J) = F(I,J) +A(K,I) * AK(K,J)
      DO 4Ø I = 1,N
      DO 4Ø J = 1,N
      AK(I,J) = Ø.Ø
      DO 4Ø K = 1,M
   4Ø AK(I,J) = AK(I,J) + F(I,K) * A(K,J)
      WRITE(61,41)
   41 FORMAT('1                STRUCTURE STIFFNESS MATRIX')
      CALL OUTPUT72(N,N,AK)
C     DIVIDE BIG K INTO FOUR QUADRANTS
      DO 5Ø I = 1,NN
      DO 5Ø J = 1,NN
   5Ø AK1(I,J) = AK(I,J)
      K = NN+1
      DO 52 I = K,N
      DO 52 J = 1,NN
      KK = I-NN
   52 AK2(KK,J) = AK(I,J)
      DO 54 I = 1,NN
      DO 54 J = K,N
      KKK = J-NN
   54 AK3(I,KKK) = AK(I,J)
      DO 56 I = K,N
      DO 56 J = K,N
      KK = I-NN
      KKK = J-NN
   56 AK4(KK,KKK) =AK(I,J)
C     COMPUTE TRANSLATIONAL K MATRIX
      L = N - NN
      CALL MATINV(AK4,L,B,Ø,DETERM)
      DO 6Ø I = 1,L
      DO 6Ø J = 1,NN
      BKZ(I,J) = Ø.Ø
      DO 6Ø K = 1,L
   6Ø BKZ(I,J) = BKZ(I,J) - AK4(I,K) * AK2(K,J)
      DO 62 I = 1,NN
      DO 62 J = 1,NN
      BKZZ(I,J) = Ø.Ø
      DO 62 K = 1,L
   62 BKZZ(I,J) = BKZZ(I,J) + AK3(I,K) * BKZ(K,J)
      DO 64 I = 1,NN
      DO 64 J = 1,NN
      FK(I,J) = Ø.Ø
   64 FK(I,J) = FK(I,J) + AK1(I,J) + BKZZ(I,J)
      DO 66 I = 1,NN
      DO 66 J = 1,NN
   66 FK(I,J) = (2*E*AI/LØ**3) * FK(I,J)
      WRITE(61,7Ø)
   7Ø FORMAT(///'              STRUCTURE TRANSLATION STIFFNESS MATRIX')
      CALL EQUIP (3,5HFILE )
      DO 72 I = 1,NN
   72 WRITE(3,1Ø1) (FK(I,J), J = 1,NN)
  1Ø1 FORMAT(1X,5E13.5)
      CALL OUTPUT36(NN,NN,FK)
      RETURN
      END
```

```
      PROGRAM RESPON
C     THIS PROGRAM SOLVES THE DIFFERENTIAL EQUATION:
C       (D↑2 + [2WC]D + [W↑2])S = P*<U(T)>
C       USING 4TH ORDER RUNGE-KUTTA PROCEDURE.
C     NM - THE TOTAL NUMBER OF MODES.
C     NMU - THE NUMBER OF MODES TO BE USED IN THE SOLUTION.
C     NT - THE NUMBER OF TIME PERIODS FOR WHICH THE SOLUTION IS
C       TO BE DETERMINED.
C     DT - THE NUMERICAL SOLUTION INTERVAL SPACING.
C     C - THE DESIRED DAMPING RATIO.
C     U(I) - THE VALUES OF THE CONTROL VARIABLE AT THE VARIOUS
C       NT PERIODS OF TIME.  THE TOTAL NUMBER MUST BE (NT+1),
C       STARTING WITH U(0).
C     FORCE(I) - THE NODAL COORIDNATE FORCE VECTOR.
C     WW(I) - THE EIGENVALUES - W**2.
C     PHI(I,J) - THE COMPLETE EIGENVECTOR MATRIX.
      DIMENSION S(33,125), D(33,125), U(125), FORCE(33)
      DIMENSION WW(33), W(33), PHI(33,33), P(33)
      DIMENSION DS(33,125), YMAX(33), JCOLUMN(33)
      DIMENSION TIME(33), UU(125)
      COMMON S,DS,D,U,FORCE,WW,W,PHI,P,YMAX,JCOLUMN
   10 FORMAT(2X,I3,1X,5E13.5)
      NM=FFIN(1)
      NMU=FFIN(1)
      NT=FFIN(1)
      DT = FFIN(1)
      C=FFIN(1)
      CC=C*100
      WRITE(61,19) DT,DT,CC,NMU,NM
   19 FORMAT('1          INPUT FORCE INTERVAL = ',F7.4 ' SECS'/
     1'         OUTPUT RESPONSE INTERVAL = ',F7.4 ' SECS'/
     2'         DAMPING = ',F7.4 ' PERCENT'/
     3'         NUMBER OF MODES CONSIDERED IS: ',I3 /
     4'         TOTAL NUMBER OF DEGREES OF FREEDOM IS: ',I3/)
      NTT = NT + 1
      DO 20 I=1,NTT
   20 U(I) = FFIN(1)
      WRITE(61,24)
   24 FORMAT('1         INPUT COEFFICIENT FORCING FUNCTION,U(T), IS:'/)
      WRITE(61,25) (U(I), I=1,NT)
   25 FORMAT(5X,5E13.5)
      DO 30 I=1,NM
   30 FORCE(I) = FFIN(1)
      WRITE(61,32)
   32 FORMAT('1      MAXIMUM FORCE AT EACH MASS IS:'/
     1'     MASS NO.        FORCE'/)
      DO 34 I=1,NM
   34 WRITE(61,35) I,FORCE(I)
   35 FORMAT(6X,I3,4X,E13.5)
      DO 40 I=1,NM
   40 WW(I) = FFIN(1)
      DO 42 I=1,NM
   42 W(I) = SQRT(WW(I))
      DO 46 I=1,NM
      DO 46 J=1,NM
   46 PHI(I,J) = FFIN(1)
      DO 50 I=1,NM
      P(I) = 0.0
      DO 50 J=1,NM
   50 P(I) = P(I) + PHI(J,I) * FORCE(J)
      WRITE(61,52)
   52 FORMAT('1    MODAL FORCE'/)
      DO 54 I=1,NM
   54 WRITE(61,10) I, P(I)
      DO 80 I=1,NMU
      X=0.0
      Y=0.0
      DO 80 J=1,NT
      JJ = J+1
      UU(J) = (U(J) + U(JJ))/2.
      DELF1 < DT*Y
      DELG1 = DT*(-(2.*W(I)*C*Y) - (WW(I)*X) + (P(I)*UU(J)))
      DELF2 = DT*(Y+DELG1/2.)
      DELG2 = DT*(-(2.*W(I)*C*(Y+DELG1/2.))-(WW(I)*(X+DELF1/2.))
     1+(P(I)*UU(J)))
      DELF3 = DT*(Y+DELG2/2.)
      DELG3 = DT*(-(2.*W(I)*C*(Y+DELG2/2.))-(WW(I)*(X+DELF2/2.))
     1+(P(I)*UU(J)))
      DELF4 = DT*(Y+DELG3)
      DELG4 = DT*(-(2.*W(I)*C*(Y+DELG3))-(WW(I)*(X+DELF3))
     1+(P(I)*UU(J)))
      DELF = (DELF1 + 2.0*DELF2 + 2.0*DELF3 + DELF4)/6.0
      DELG = (DELG1 + 2.0*DELG2 + 2.0*DELG3 + DELG4)/6.0
      X = X+DELF
      Y = Y+DELG
      DS(I,J) = Y
   80 S(I,J) = X
      DO 90 I=1,NM
      DO 90 J=1,NT
      D(I,J) = 0.0
      DO 90 K=1,NMU
   90 D(I,J) = D(I,J) + PHI(I,K)*S(K,J)
      WRITE(61,56)
   56 FORMAT('1                DISPLACEMENT TIME HISTORY'/
     1'  TIME PRINTED ABOVE THE COLUMNS OF OUTPUT IS IN SECONDS'//)
      CALL OUTPUT(NM,NT,D,DT)
      CALL ENVEL(NM,NT,D,YMAX,JCOLUMN)
      WRITE(61,120)
  120 FORMAT('1 ENVELOPE OF DISPLACEMENTS'/
     1'    MASS  TIME(SEC)      D MAX.'/)
      DO 130 I=1,NM
      TIME(I) = JCOLUMN(I)*DT
  130 WRITE(61,131) I, TIME(I), YMAX(I)
  131 FORMAT(3X,I3,4X,F5.3,2X,E13.5)
      CALL ENVEL(NM,NT,DS,YMAX,JCOLUMN)
      WRITE(61,140)
  140 FORMAT('1 ENVELOPE OF MODAL VELOCITY (DS)'/
     1'    MODE  TIME(SEC)      DS MAX.'/)
      DO 100 I=1,NMU
      TIME(I) = JCOLUMN(I)*DT
  100 WRITE(61,131) I,TIME(I), YMAX(I)
      CALL ENVEL(NM,NT,S,YMAX,JCOLUMN)
      WRITE(61,160)
  160 FORMAT('1 ENVELOPE OF MODAL DISPLACEMENT (S)'/
     1'    MODE  TIME(SEC)      S MAX.'/)
      DO 170 I=1,NMU
      TIME(I) = JCOLUMN(I)*DT
  170 WRITE(61,131) I,TIME(I), YMAX(I)
      RETURN
      END
```

```
      PROGRAM JEIGEN
C     THE PROGRAM COMPUTES THE EIGENVALUES AND ORTHONORMAL
C       EIGENVECTORS FOR A SYSTEM GOVERNED THE THE
C       CHARACTERISTIC EQUATION: [K](D) = W↑2[M](D).
C     THE JACOBI DIAGONALIZATION PROCEDURE IS USED.
C     N - THE ORDER OF THE K AND M ARRAYS.
C     AK(I,J) - THE ELEMENTS OF THE K ARRAY.
C     AM(I,I) - THE DIAGONAL ELEMENTS OF THE M ARRAY.
      DIMENSION AK(40,40), AM(40,40), DYN(40,40), FN(40), T(40)
      DIMENSION PHI(40,40),AMASS(40,40), GM(40), ORTHOG(40,40)
      COMMON AK, AM, DYN, PHI, ORTHOG
   10 FORMAT(2X,I3,E13.5,2X,I3,E13.5,2X,I3,E13.5,2X,I3,E13.5)
   11 FORMAT(5E13.5)
      N = FFIN(1)
      DO 20 I = 1,N
      DO 20 J = 1,N
   20 AK(I,J) < FFIN(1)
      WRITE(61,22)
   22 FORMAT( '1                                  STIFFNESS MATRIX')
      CALL OUTPUT(N,N,AK)
      DO 30 I = 1,N
   30 AM(I,I) = FFIN(1)
      WRITE(61,32)
   32 FORMAT( '1                                    MASS MATRIX')
      CALL OUTPUT(N,N,AM)
      DO 40 I=1,N
   40 AMASS(I,I) = 1./SQRT(AM(I,I))
      DO 44 I=1,N
      DO 44 J=1,N
   44 DYN(I,J) = AMASS(I,I) * AK(I,J) * AMASS(J,J)
      CALL MATINV(DYN,N,0,0,DETERM)
      CALL EIGENJ(DYN,N,0,PHI,NR)
      DO 48 I=1,N
      DO 48 J=1,N
   48 PHI(I,J) = AMASS(I,I) * PHI(I,J)
      DO 50 I=1,N
   50 DYN(I,I) = 1.0/DYN(I,I)
      CALL NORM(PHI,AM,GM,ORTHOG,N)
      DO 60 I=1,N
      FN(I) = SQRT(DYN(I,I))/(6.2831853)
   60 T(I) = 1.0/FN(I)
      WRITE(61,70)
   70 FORMAT('1,     EIGENVALUE          FREQUENCY(CPS)         PERIOD(SEC)   '
     1' GENERALIZED MASS'/)
      DO 80 I=1,N
   80 WRITE(61,10) I,DYN(I,I),I,FN(I), I, T(I), I, GM(I)
      WRITE(61,90)
   90 FORMAT('1                          EIGENVECTORS PRINTED AS COLUMN VECTORS'/
     1'                     NORMALIZED SUCH THAT M*PHI**2=1')
      CALL OUTPUT(N,N,PHI)
      WRITE(61,100)
  100 FORMAT('1                          ORTHOGONALITY CHECK')
      CALL OUTPUT(N,N,ORTHOG)
      CALL EQUIP(4,5HFILE )
      WRITE(4,11)(DYN(I,I), I=1,N)
      DO 110 I=1,N
  110 WRITE(4,11) (PHI(I,J), J=1,N)
      RETURN
      END
```

```
      SUBROUTINE EIGENJ(H,N,IG,U,NR)
C     DIAGONALIZATION OF A REAL SYMMETRIC MATRIX BY THE
C     JACOBI METHOD.
C     THE CALLING SEQUENCE FOR DIAGONALIZATION IS:
C     CALL EIGENJ(H,N,IG,U,NR)
C     WHERE H IS THE ARRAY TO BE DIAGONALIZED
C     N IS THE ORDER OF THE MATRIX H
C     IG IS THE INDICATOR USED TO SPECIFY WHETHER OR
C     NOT THE EIGENVECTORS ARE TO BE COMPUTED
C         IF IG=0, EIGENVECTORS AND EIGENVALUES ARE COMPUTED.
C         IF IG=1, ONLY THE EIGENVALUES ARE COMPUTED.
C     U IS THE UNITARY MATRIX USED FOR FORMATION OF THE
C     EIGENVECTORS. EIGENVECTORS ARE STORED IN COLUMNS.
C     NR INDICATES THE NUMBER OF ROTATIONS WHICH
C     WERE REQUIRED TO DIAGONALIZE THE MATRIX.
C     THE SUBROUTINE OPERATES ONLY ON THE ELEMENTS OF H THAT
C     ARE TO THE RIGHT OF THE MAIN DIAGONAL, THUS, ONLY A
C     TRIANGULAR SECTION NEED BE STORED IN THE ARRAY H.
      DIMENSION H(40,40) , U(40,40), X(40), IQ(40)
      IF (IG) 15,10,15
   10 DO 14 I=1,N
      DO 14 J=1,N
      IF(I-J) 12,11,12
   11 U(I,J) = 1.0
      GO TO 14
   12 U(I,J) = 0.0
   14 CONTINUE
   15 NR =0
      IF (N-1) 1000,1000,17
C     SCAN FOR LARGEST OFF DIAGONAL ELEMENT IN EACH ROW
C     X(I) CONTAINS LARGEST ELEMENT IN ITH ROW
C     IQ(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
   17 NMI1=N-1
      DO 30 I=1,NMI1
      X(I) = 0.0
      IPL1=I+1
      DO 30 J=IPL1,N
      IF(X(I)-ABS(H(I,J))) 20,20,30
   20 X(I) = ABS(H(I,J))
      IQ(I) = J
   30 CONTINUE
C     SET INDICATOR FOR SHUT-OFF. RAP=2**-27,NR=NO.OF ROTATIONS
      RAP=0.745058059E-08
      HDTEST=1.0E38
C     FIND MAXIMUM OF X(I)'S FOR PIVOT ELEMENT AND
C     TEST FOR END OF PROBLEM
   40 DO 70 I=1,NMI1
      IF(I-1) 60,60,45
   45 IF(XMAX-X(I)) 60,70,70
   60 XMAX = X(I)
      IPIV=I
      JPIV=IQ(I)
   70 CONTINUE
C     IS MAX. X(I) EQUAL TO ZERO, IF LESS THAN HDTEST,REVISE HDTEST
      IF(XMAX) 1000,1000,80
   80 IF(HDTEST) 90,90,85
   85 IF(XMAX-HDTEST) 90,90,148
   90 HDMIN=ABS(H(1,1))
      DO 110 I=2,N
      IF(HDMIN-ABS(H(I,I))) 110,110,100
  100 HDMIN=ABS(H(I,I))
  110 CONTINUE
      HDTEST = HDMIN*RAP
C     RETURN IF MAX.H(I,J) LESS THAN (2**-27) * ABS(H(K,K)-MIN)
      IF (HDTEST-XMAX) 148,1000,1000
  148 NR =NR+1
C     COMPUTE TANGENT,SINE AND COSINE, H(I,I), H(J,J) -
      XDIF=H(IPIV,IPIV)-H(JPIV,JPIV)
      XO=SIGN(2.0,XDIF)*H(IPIV,JPIV)
      XS=XDIF**2 + 4.0*H(IPIV,JPIV)**2
  150 TANG=XO/(ABS(XDIF) + SQRT(XS))
```

```
      COSINE=1.0/SQRT(1.0+TANG**2)
      SINE=TANG*COSINE
      HII=H(IPIV,IPIV)
      H(IPIV,IPIV) = COSINE**2*(HII+TANG*(2.0*H(IPIV,JPIV)+TANG*H(JPIV,J
     1PIV)))
      H(JPIV,JPIV)=COSINE**2*(H(JPIV,JPIV)-TANG*(2.0*H(IPIV,JPIV)-TANG*H
     1II))
      H(IPIV,JPIV) = 0.0
C     PSEUDO RANK THE EIGENVALUES
C     ADUUST SINE AND COS FOR COMPUTATION OF H(IK) AND U(IK)
      IF(H(IPIV,IPIV) - H(JPIV,JPIV)) 152,153,153
  152 HTEMP = H(IPIV,IPIV)
      H(IPIV,IPIV) = H(JPIV,JPIV)
      H(JPIV,JPIV) = HTEMP
C     RECOMPUTE SINE AND COS
      HTEMP = SIGN(1.0, -SINE) * COSINE
      COSINE = ABS(SINE)
      SINE = HTEMP
  153 CONTINUE
C     INSPECT THE IQS BETWEEN I+1 AND N-1 TO DETERMINE WHETHER A NEW
C     MAXIMUM VALUE SHOULD BE COMPUTED SINCE THE PRESENT MAXIMUM
C     IS IN THE I OR J ROW.
      DO 350 I=1,NMI1
      IF(I-IPIV) 210,350,200
  200 IF(I-JPIV) 210,350,210
  210 IF(IQ(I)-IPIV) 230,240,230
  230 IF(IQ(I)-JPIV) 350,240,350
  240 K=IQ(I)
  250 HTEMP=H(I,K)
      H(I,K) = 0.0
      IPL1=I+1
      X(I) = 0.0
C     SEARCH IN DEPLETED ROW FOR NEW MAXIMUM
      DO 320 J = IPL1,N
      IF(X(I)-ABS(H(I,J))) 300,300,320
  300 X(I) = ABS(H(I,J))
      IQ(I) = J
  320 CONTINUE
      H(I,K) = HTEMP
  350 CONTINUE
      X(IPIV) = 0.0
      X(JPIV) = 0.0
C     CHANGE THE ORDER ELEMENTS OF H
      DO 530 I=1,N
      IF(I-IPIV) 370,530,420
  370 HTEMP = H(I,IPIV)
      H(I,IPIV) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF(X(I) - ABS(H(I,IPIV))) 380,390,390
  380 X(I) = ABS(H(I,IPIV))
      IQ(I) = IPIV
  390 H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,JPIV)
      IF(X(I) - ABS(H(I,JPIV))) 400,530,530
  400 X(I) = ABS(H(I,JPIV))
      IQ(I) = JPIV
      GO TO 530
  420 IF(I-JPIV) 430,530,480
  430 HTEMP = H(IPIV,I)
      H(IPIV,I) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF( X(IPIV) - ABS(H(IPIV,I))) 440,450,450
  440 X(IPIV) = ABS(H(IPIV,I))
      IQ(IPIV) = I
  450 H(I,JPIV) = - SINE*HTEMP + COSINE*H(I,JPIV)
      IF (X(I) - ABS( H(I,JPIV))) 400,530,530
  480 HTEMP = H(IPIV,I)
      H(IPIV,I) = COSINE*HTEMP + SINE*H(JPIV,I)
      IF( X(IPIV) -ABS(H(IPIV,I))) 490,500,500
  490 X(IPIV) = ABS( H(IPIV,I))
      IQ(IPIV) = I
  500 H(JPIV,I) = -SINE*HTEMP + COSINE*H(JPIV,I)
      IF(X(JPIV) - ABS(H(JPIV,I))) 510,530,530
  510 X(JPIV) = ABS(H(JPIV,I))
      IQ(JPIV) = I
  530 CONTINUE
C     TEST FOR COMPUTATION OF EIGENVECTORS
      IF(IG) 40,540,40
  540 DO 550 I=1,N
      HTEMP = U(I,IPIV)
      U(I,IPIV) = COSINE*HTEMP+SINE*U(I,JPIV)
  550 U(I,JPIV) = -SINE*HTEMP+COSINE*U(I,JPIV)
      GO TO 40
 1000 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NORM(PHI,AM,GM,ORTHOG,N)
C     THIS SUBROUTINE STARTS WITH THE ORTHOGONAL EIGENVECTOR
C     MATRIX, PHI, AND REDUCES IT TO AN ORTHONORMAL MATRIX.
C     THE ORTHONORMAL EIGENVECTORS ARE STORED IN PHI.
C     AM IS THE ARRAY OF MASS ELEMENTS.
C     OGTHOG IS THE COMPUTED MATRIX PRODUCT:
C        [PHI]' * [AM] * [PHI]
C     WHICH SHOULD EQUAL [I].
C     GM = SUM(AM*PHI)↑2
      DIMENSION PHI(40,40), AM(40,40), ORTHOG(40,40), GM(40)
      DIMENSION DUM(40,40)
      DO 30 J=1,N
      DUM(J,J) = 0.0
      DO 30 I=1,N
   30 DUM(J,J) = DUM(J,J) + (PHI(I,J)**2)*AM(I,I)
      DO 40 J=1,N
      DO 40 I=1,N
   40 PHI(I,J) = PHI(I,J)/SQRT(DUM(J,J))
      DO 50 J=1,N
      GM(J) = 0.0
      DO 50 I=1,N
   50 GM(J) = GM(J) + PHI(I,J) * AM(I,I)
      DO 60 I=1,N
   60 GM(I) = GM(I)**2
      DO 70 I=1,N
      DO 70 J=1,N
      DUM(I,J) = 0.0
      DO 70 K=1,N
   70 DUM(I,J) = DUM(I,J) + PHI(K,I) * AM(K,J)
      DO 80 I=1,N
      DO 80 J=1,N
      ORTHOG(I,J) = 0.0
      DO 80 K=1,N
   80 ORTHOG(I,J) = ORTHOG(I,J) + DUM(I,K) * PHI(K,J)
      RETURN
      END
```

```
      SUBROUTINE MATINV(A,N,B,M,DETERM)
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR
C      EQUATIONS OF THE FORM AX = B.  JORDON'S METHOD
C     A IS THE ARRAY TO BE INVERTED.
C     B IS THE COLUMN OF CONSTANTS FOR LINEAR EQUATION SOLUTION.
C     N IS THE ORDER OF A
C     M IS THE INDICATOR FOR SPECIFYING INVERSION OR SOLUTION
C      OF LINEAR EQUATIONS.
C          M=0, INVERSION IS PERFORMED.
C          M=1, SOLUTION OF LINEAR EQUATIONS IS PERFORMED.
C     AT THE RETURN TO THE COLLING PROGRAM, A INVERSE
C      IS STORED AT A AND X AT B.
C     NOTE.. IF USED SOLELY FOR INVERSION, THE CALL SATTEMENT
C      MUST STILL CONTAIN AN ENTRY CORRESPONDING TO B.
C     DETERM IS THE LOCATION IN WHICH THE DETERMINANT IS STORED.
      DIMENSION IPIVOT(36), A(36,36), B(36,1), INDEX(36,2), PIVOT(36)
      DETERM=1.0
      DO 20 J=1,N
   20 IPIVOT(J)=0
      DO 550 I=1,N
C     SEARCH FOR PIVOT ELEMENT
      AMAX=0.0
      DO 105 J=1,N
      IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF (IPIVOT(K)-1) 80, 100, 740
   80 IF (ABSF(AMAX)-ABSF(A(J,K))) 85, 100, 100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
      IF (IROW-ICOLUM) 140, 260, 140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 260, 260, 210
  210 DO 250 L=1, M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
      A(ICOLUM,ICOLUM)=1.0
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C     REDUCE NON-PIVOT ROWS
  380 DO 550 L1=1,N
      IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
      IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C     INTERCHANGE COLUMNS
      DO 710 I=1,N
      L=N+1-I
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

```
      SUBROUTINE OUTPUT(M,N,A)
C     THIS SUBROUTINE IS USED TO WRITE OUT THE ARRAY A IN
C     AN ORDERLY ROW-COLUMN FORM.
C     M : NUMBER OR ROWS IN A
C     N : NUMBER OF COLUMNS IN A
      DIMENSION A(40,40)
  810 FORMAT(//12X,I3,10X,I3,10X,I3,10X,I3,10X,I3)
  811 FORMAT(2X,I3,1X,5E13.5)
      K = -4
      L = 0
      DO 830 IJK = 1,N,5
      K = K + 5
      L = L + 5
      IF(L .GT. N) 818,819
  818 L=N
  819 ID1 = K
      ID2 = K+1
      ID3 = K+2
      ID4 = K+3
      ID5 = K+4
      WRITE(61,810) ID1, ID2, ID3, ID4, ID5
      DO 820 I = 1,M
  820 WRITE(61,811) I, (A(I,J), J = K,L)
  830 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ENVEL(M,N,A,AMAX,JCOLUMN)
C     THIS SUBROUTINE SEARCHES THRU THE ROWS OF THE
C     ARRAY, A, TO FIND THE MAXIMUM OF EACH ROW.
C     THE MAXIMUM VALUE IN EACH ROW IS STORED IN AMAX,
C     AND THE ASSOCIATED COLUMN NUMBER IS STORED IN JCOLUMN.
      DIMENSION A(40,150), AMAX(40), JCOLUMN(40)
      NN = N
      DO 100 I=1,M
      DUM1 = 0.0
      DO 100 J=1,N
      IF (ABS(A(I,J))-ABS(DUM1)) 55,55,50
   50 DUM1 = A(I,J)
      IROW = I
      JCOL=J
   55 IF(N-NN) 100,60,100
   60 AMAX(I) = DUM1
      JCOLUMN(I) = JCOL
  100 CONTINUE
      RETURN
      END
```

APPENDIX B

CONTINUOUS MASS SYSTEM COMPUTER PROGRAM

```
      PROGRAM DEIGEN
C     THE PROGRAM GENERATES THE DYNAMIC MATRIX FOR
C     THREE DIMENSIONAL FRAMES AND EVALUATES THE DETERMINANT OF
C     THE SAME MATRIX.
C     ALL INPUT VARIABLES ARE DEFINED BELOW EXCEPT FOR:
C     (W2) - TRIAL EIGENVALUE, I.E. OMEGA↑2
C     (NPRINT) - SPECIFIES  INFORMATION  TO BE PRINTED OUT
C         IF Ø, ONLY THE TRIAL EIGENVALUE AND DET. ARE PRINTED
C         IF 1, INPUT DATA, A AND K MATRICES ARE ALSO PRINTED.
      COMMON AK,A,FK,AK1,AK2,AK3,AK4, BKZ,BKZZ
      DIMENSION AK(4Ø,4Ø),A(4Ø,4Ø), FK(4Ø,4Ø)
      DIMENSION AK1(1Ø,1Ø), AK2(1Ø,1Ø), AK3(1Ø,1Ø),AK4(1Ø,1Ø)
      DIMENSION B(1Ø,1), BKZ(1Ø,1Ø), BKZZ(1Ø,1Ø), DETMAT(1Ø,1Ø)
      DIMENSION OMEGA(1Ø), OMEGA2(1Ø), DETER(1Ø)
      NPRINT = FFIN(1)
      M = FFIN(1)
      N=FFIN(1)
      NN=FFIN(1)
      E=FFIN(1)
      AI=FFIN(1)
      AJ=FFIN(1)
      AM=FFIN(1)
      AIM=FFIN(1)
      AREA=FFIN(1)
      AL=FFIN(1)
      G=Ø.4*E
      AE=AREA*E
      GJ=G*AJ
      EI=E*AI
      IF(NPRINT) 19,25
   19 WRITE(61,2Ø) N,NN,AREA,AI,AJ
   2Ø FORMAT(' TOTAL NUMBER OF JOINT DEGREES OF FREEDOM = ',I2/
     1' NUMBER OF JOINT TRANSLATIONAL DEGREES OF FREEDOM = ',I2/
     2' CROSS SECTIONAL AREA (AREA) = ',E13.5 '  SQ. IN.'/
     3' AREA MOMENT OF INERTIA (AI) < ',E13.5'  IN.↑4'/
     4' POLAR AREA MOMENT OF INERTIA (AJ) = ',E13.5'  IN.↑4'/)
      WRITE(61,21) AIM,E,G,AL,AM
   21 FORMAT(' MASS MOMENT OF INERTIA (AIM) = ',E13.5'  IN.↑4'/
     1' MODULUS OF ELASTICITY (E) = ' ,E13.5'  LB./IN.↑2'/
     2' MODULUS OF RIGIDITY  = ' ,E13.5'  LB./IN↑2'/
     3' REFERENCE MEMBER LENGTH (AL) = ',E13.5'  IN.'/
     4' MASS PER UNIT LENGTH (AM) = ',E13.4 ' LB-SEC↑2/IN↑2'/)
   25 NA=FFIN(1)
      DO 28 I=1,M
      DO 28 J=1,N
   28 A(I,J) = Ø.Ø
      DO 3Ø L=1,NA
      I=FFIN(1)
      J=FFIN(1)
   3Ø A(I,J) = FFIN(1)
      IF(NPRINT) 31,33
   31 WRITE(61,32)
   32 FORMAT('1          DISPLACEMENT TRANSFORMATION MATRIX'/)
      CALL OUTPUT(M,N,A)
   33 NW2 = FFIN(1)
      KIT = Ø
   34 W2 = FFIN(1)
      W=SQRT(W2)
      KIT = KIT + 1
      CALL SMALLK(AK,M,W2,AM,EI,AE,AIM,GJ,AL)
      IF(NPRINT) 37,39
   37 WRITE(61,38)
   38 FORMAT('1          STRUCTURE MEMBER STIFFNESS MATRIX'/)
      CALL OUTPUT(M,M,AK)
C     AK IS SMALL K
   39 DO 4Ø I=1,N
      DO 4Ø J=1,M
      FK(I,J) = Ø.Ø
      DO 4Ø K=1,M
   4Ø FK(I,J) = FK(I,J) + A(K,I)*AK(K,J)
      DO 43 I=1,N
      DO 43 J=1,N
      AK(I,J) = Ø.Ø
      DO 43 K=1,M
   43 AK(I,J) = AK(I,J) + FK(I,K)*A(K,J)
C     AK IS BIG K
      IF(NPRINT) 44,49
   44 WRITE(61,46)
   46 FORMAT('1          STRUCTURE STIFFNESS MATRIX'/)
      CALL OUTPUT(N,N,AK)
C     DIVIDE BIG K INTO FOUR QUADRANTS
   49 DO 5Ø I = 1,NN
      DO 5Ø J = 1,NN
   5Ø AK1(I,J) = AK(I,J)
      K = NN+1
      DO 52 I = K,N
      DO 52 J = 1,NN
      KK = I-NN
   52 AK2(KK,J) = AK(I,J)
      DO 54 I = 1,NN
      DO 54 J = K,N
      KKK = J-NN
   54 AK3(I,KKK) = AK(I,J)
      DO 56 I = K,N
      DO 56 J = K,N
      KK = I-NN
      KKK = J-NN
   56 AK4(KK,KKK) =AK(I,J)
C     COMPUTE TRANSLATIONAL K MATRIX
      L = N - NN
      CALL MATINV(AK4,L,B,Ø,DETERM)
      DO 6Ø I = 1,L
      DO 6Ø J = 1,NN
      BKZ(I,J) = Ø.Ø
      DO 6Ø K = 1,L
   6Ø BKZ(I,J) = BKZ(I,J) - AK4(I,K) * AK2(K,J)
      DO 62 I = 1,NN
      DO 62 J = 1,NN
```

```
      BKZZ(I,J) = 0.0
      DO 62 K = 1,L
   62 BKZZ(I,J) = BKZZ(I,J) + AK3(I,K) * BKZ(K,J)
      DO 64 I = 1,NN
      DO 64 J = 1,NN
      FK(I,J) = 0.0
   64 FK(I,J) = FK(I,J) + AK1(I,J) + BKZZ(I,J)
      DO 66 I=1,NN
      DO 66 J=1,NN
   66 FK(I,J) = (EI/AL**3)*FK(I,J)
      IF(NPRINT) 67,69
   67 WRITE(61,68)
   68 FORMAT('1          STRUCTURE TRANSLATION STIFFNESS MATRIX'/)
      CALL OUTPUT(NN,NN,FK)
   69 DO 70 I=1,NN
      DO 70 J=1,NN
   70 DETMAT(I,J) = FK(I,J)
      CALL MATINV(DETMAT,NN,B,0,DET)
      OMEGA(KIT) = W
      OMEGA2(KIT) = W2
      DETER(KIT) = DET
      IF(NW2-KIT) 34,100,34
  100 WRITE(61,110)
  110 FORMAT('1',7X,'  OMEGA      OMEGA ↑2      DETERMINANT'/)
      DO 120 I=1,NW2
  120 WRITE(61,125) I,OMEGA(I), OMEGA2(I), DETER(I)
  125 FORMAT(2X,I3,3E13.5/)
      RETURN
      END
```

```
      SUBROUTINE SMALLK(SMK,M,W2,AM,EI,AE,AIM,GJ,AL)
      DIMENSION SMK(40,40)
      AT=SQRT((W2*AM)/EI)
      AT=SQRT(AT)
      BT=SQRT((W2*AM)/AE)
      CT=SQRT((W2*AIM)/GJ)
      ALAM=AT*AL
      DEN=(1.0-COS(ALAM)*COSH(ALAM))
      S=SIN(ALAM)
      C=COS(ALAM)
      SH=SINH(ALAM)
      CH=COSH(ALAM)
      DUM1=(S*CH)-(C*SH)
      EK1=(ALAM*DUM1)/DEN
      DUM2=S*SH
      EK2=((ALAM**2)*DUM2)/DEN
      DUM3=SH-S
      EK3=(ALAM*DUM3)/DEN
      DUM4=CH-C
      EK4=((ALAM**2)*DUM4)/DEN
      DUM5=(S*CH)+(C*SH)
      EK5=((ALAM**3)*DUM5)/DEN
      DUM6=SH+S
      EK6=((ALAM**3)*DUM6)/DEN
      BLAM=BT*AL
      DEN=SIN(BLAM)
      DUM7=BLAM*COS(BLAM)
      EK7=DUM7/DEN
      EK8=BLAM/DEN
      CLAM=CT*AL
      DEN=SIN(CLAM)
      DUM9=CLAM*COS(CLAM)
      EK9=DUM9/DEN
      EK10=CLAM/DEN
      DO 20 I=1,M
      DO 20 J=1,M
   20 SMK(I,J) = 0.0
      SMK(1,1) = EK1
      SMK(1,2) = -EK2
      SMK(1,3) = EK3
      SMK(1,4) = EK4
      SMK(2,1) = -EK2
      SMK(2,2) = EK5
      SMK(2,3) = -EK4
      SMK(2,4) = -EK6
      SMK(3,1) = EK3
      SMK(3,2) = -EK4
      SMK(3,3) = EK1
      SMK(3,4) = EK2
      SMK(4,1) = EK4
      SMK(4,2) = -EK6
      SMK(4,3) = EK2
      SMK(4,4) = EK5
      DO 30 I=1,4
      ID = I+4
      DO 30 J=1,4
      JD=J+4
   30 SMK(ID,JD) = ABS(SMK(I,J))
      SMK(5,8) = -SMK(5,8)
      SMK(8,5) = -SMK(8,5)
      SMK(6,8) = -SMK(6,8)
      SMK(8,6) = -SMK(8,6)
      SMK(7,8) = -SMK(7,8)
      SMK(8,7) = -SMK(8,7)
      BK=(AE*(AL**2))/EI
      SMK(9,9) = BK*EK7
      SMK(9,10) = -BK*EK8
      SMK(10,9) = -BK*EK8
      SMK(10,10) = BK*EK7
      CK=GJ/EI
      SMK(11,11) = CK*EK9
      SMK(11,12) = -CK*EK10
      SMK(12,11) = -CK*EK10
      SMK(12,12) = CK*EK9
      DO 50 K=13,M,12
      IJ=K-1
      DO 40 I=1,12
      ID= I+IJ
      DO 40 J=1,12
      JD = J+IJ
   40 SMK(ID,JD) = SMK(I,J)
   50 CONTINUE
      RETURN
      END
```

NOTE: See Appendix A for listing of subroutines OUTPUT and MATINV.

APPENDIX C

ANALOG COMPUTER SIMULATION CIRCUITRY

The computer used is an EAI-690 hybrid consisting of an EAI-680 analog and an EAI-640 digital.

In order to perform an analog simulation it is most convenient to write the pertinent equations in "state variable" form. This is particularly advantageous with regard to "scaling." Thus, making the substitution

$$\{x\} = \{S\}$$

and (C. 1)

$$\{y\} = \{\dot{S}\}$$

in Equation (2. 3), the state variable form of the modal equations of motion is

$$\begin{aligned} \{\dot{x}\} &= \{y\} \\ \{\dot{y}\} &= -[2\omega c]\{y\} - [\omega^2]\{x\} + \{P\}\langle U(t)\rangle \end{aligned} \qquad \text{(C. 2)}$$

Figure C. 1 shows the analog circuitry for the first three modal equations and for the solution of nodal coordinate D_4^2 - eleven mass model. Figure C. 2 shows the circuitry used to generate the control function, U (t), as shown in Figure 3. 2.

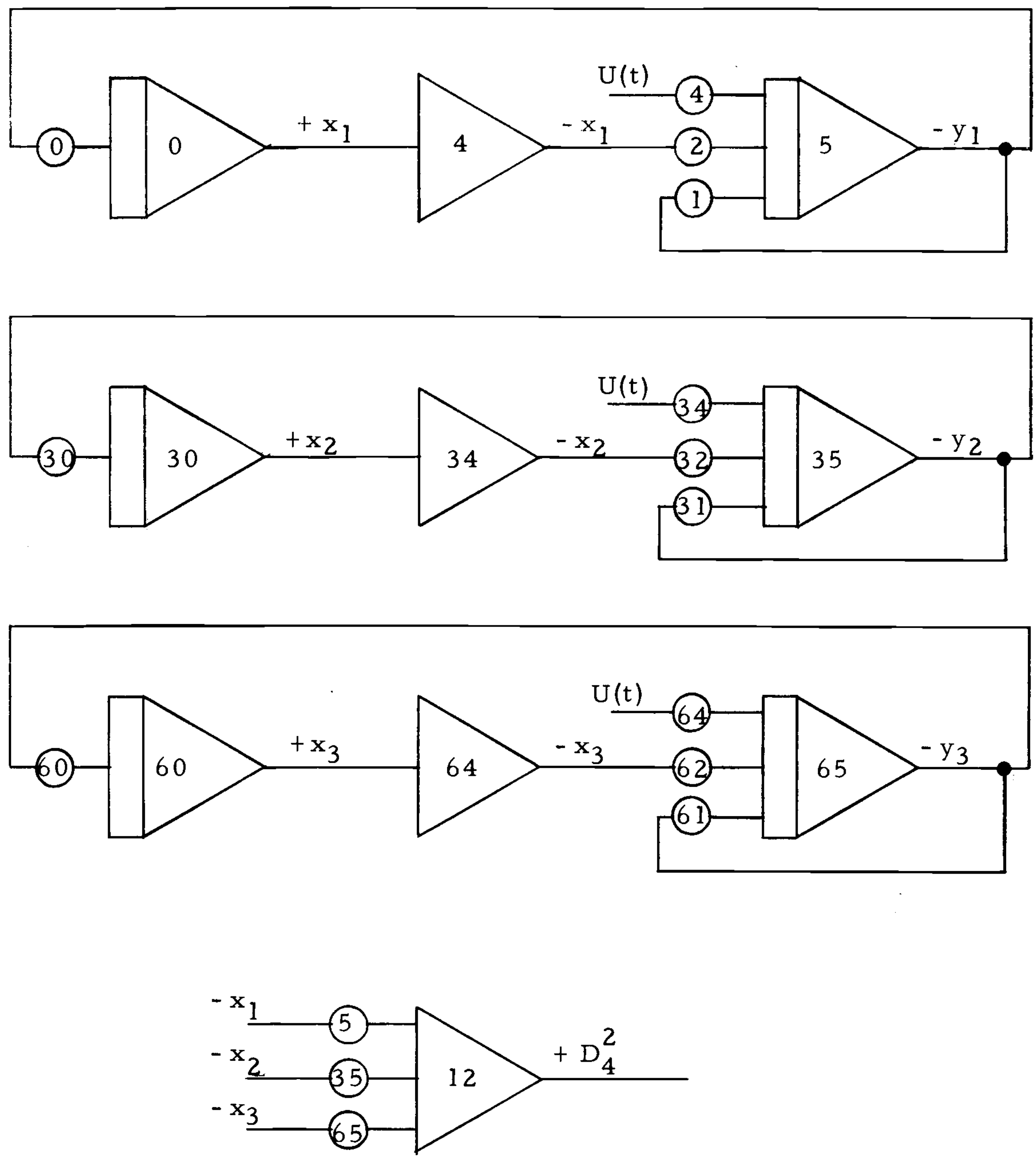

Figure C. 1. Example frame simulation circuit for nodal coordinate D_4^2 - eleven mass model.

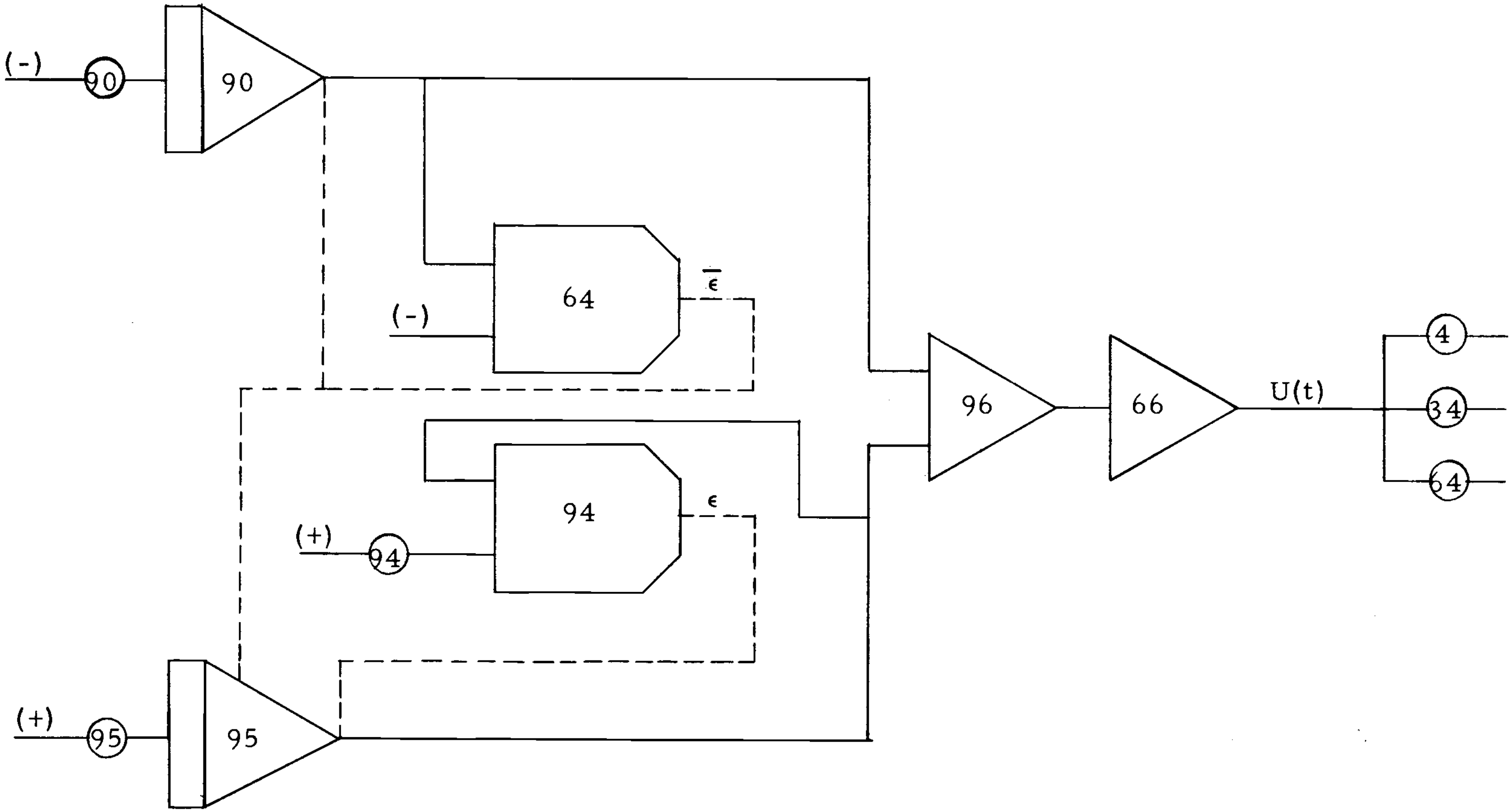

Figure C. 2. Example frame simulation circuit for control function - U(t).