AN ABSTRACT OF THE THESIS OF

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Title: Weibull Diameter Distribution Models for Managed Stands of Douglas-fir in Washington and Oregon

Abstract approve

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The two-parameter Weibull function was used to predict forest stand diameter distributions and growth. Diameter distribution models were developed for even-aged Douglas-fir stands, 20 to 40 years old, in Oregon and Washington.

In order to test if the two-parameter Weibull function can adequately describe the diameter distributions of such stands, maximum likelihood estimated parameters of the two-parameter Weibull function were compared with observed diameter distributions.

Two sets of models were developed. The first set of models predicted the diameter distributions of unthinned forest stands from stand variables. Models were

developed to predict each of the two parameters of the Weibull function from stand variables. The Weibull function with the two estimated parameters became the diameter distribution model for the stand. The second set of models predicted the growth of a stand of trees after thinning. A Weibull diameter distribution model was developed for a forest stand after a growth period. The parameters of the Weibull function were estimated from stand variables at the beginning of the growth period and the length of the growth period.

Results showed that the two-parameter Weibull function can describe the diameter distributions of even-aged stands of Douglas-fir, 20 to 40 years old, in Oregon and Washington. The diameter distribution model for unthinned stands predicted the observed diameter distributions in an independent data set quite well. The diameter distribution model for a thinned stand after a growth period gave a satisfactory prediction for 93 percent of the observed diameter distributions in the independent data set.

The two-parameter Weibull function in this study gave at least as good results as that which has been obtained with the three-parameter Weibull function in previous studies.

Weibull Diameter Distribution Models for Managed Stands of Douglas-fir In Washington and Oregon

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WEIBULL DIAMETER DISTRIBUTION MODELS FOR MANAGED STANDS OF DOUGLAS-FIR IN WASHINGTON AND OREGON

INTRODUCTION

Stand level growth and yield models produce estimates of population parameters at the stand level, such as number of trees per acre, basal area per acre and volume per acre. For the purposes of analysis and decision-making, it would be desirable to be able to disaggregate these stand level characteristics to estimates for diameter classes.

Given not only timber volume per acre, but also the size of trees over which it is distributed, the forest manager can combine this information with anticipated trends in forest products markets, for example, changes in sawtimber versus pulpwood demand, to make better management decisions.

Mathematical probability functions such as the Weibull offer an opportunity to model, for Douglas-fir stands, basal area and number of trees by diameter classes, given only stand level variables. As a result, the stand level model can maintain some of the detail of individual tree models without the necessity for keeping track of individual trees.

The problem, therefore, is to predict forest stand structure by prediction models for the parameters of a probability function that adequately describes the diameter distribution of the stand, and to project stand structure through prediction models for the function parameters after a growth period. Specifically, the objectives of this study are:

1. To determine whether the Weibull probability function can be used to model the diameter distributions of even-aged stands of young-growth Douglas-fir.

2. To develop models for predicting the diameter distributions of unthinned stands given stand level variables. Individual prediction models will be constructed for each parameter of the two-parameter Weibull probability function. The probability function obtained with the two estimated parameters will then act as a prediction model for the diameter distribution of the stand.

3. To model growth of forest stands after thinning by projecting a stand's diameter distribution into the future. The diameter distribution of the stand after a growth period can be modeled by a probability function

whose parameters are predicted from stand variables at the beginning of the growth period. This method of projecting diameter distributions accounts for changes in stand structure as a result of growth.

REVIEW OF THE LITERATURE

The first attempt at modeling forest stand diameter distributions with a mathematical function was probably made by deLiocourt who used the exponential function to describe the diameter distributions of uneven-aged stands in France, in 1898. Meyer (1952) later used this model for uneven-aged stands in North America. Recent studies have concentrated on modeling the diameter distributions of even-aged stands (Feduccia et al., 1979, Dell et al., 1979).

Yang, Kozak and Smith (1978) gave three criteria by which to select a distribution function for modeling diameter distributions: 1) the function should be capable of depicting the full range of unimodal shapes that diameter distributions can take on. 2) Parameters of the function should be easily related to shape and location features of the distribution and should vary in a manner consistent with stand characteristics. 3) The function should be integrable in closed form so that numerical integration can be avoided.

Hafley and Schreuder (1977) compared the flexibility of curve shape of several functions in terms of the range of skewness and kurtosis they cover. They concluded that Johnson's (Johnson,1949) S_b and the beta function are more flexible than the Weibull function, while the Weibull is more flexible than the gamma, normal, lognormal and expontial function.

The Johnson's S_b function has not yet been used for modeling forest stand diameter distributions. Cao et al. (1982) claim that the main drawback of the beta function is that its cumulative distribution function does not exist in closed form. As a result, the proportion of trees in each diameter class must be obtained by numerical integration techniques. Zohrer (1972) admits that "the computations for deriving a beta distribution are rather time consuming", but still recommends it due to its superior flexibility. Paivinen (1980) used the beta function as part of a volume yield prediction system. He concluded that the system was not satisfactorily accurate.

Bailey (1980) noted that choosing a particular distribution implies a particular diameter growth relationship, and that this could be used as a guide in the choice of a distribution function. It is implicitly assumed that if X_1 is the diameter at age A_1, and X_2 is the diameter at age A_2, then the growth equation that relates X_1 to X_2 places the distribution function of X_2 in the same family as the distribution function of X_1. He showed that for the Weibull, lognormal and generalized gamma function, the non-linear diameter growth equation

$$X_2 = B0 + B1(X_1 - B3)^{B2}$$

will keep the distribution function within the appropriate family, while for the S_b and beta function it is necessary that the diameter growth relationship be linear for the functions to regenerate themselves.

Schreuder and Swank (1974) compared the performance of the Weibull, normal, lognormal and gamma functions for describing diameter distributions of loblolly pine. Maximum likelihood estimators were calculated for all distributions, and the observed and expected fit were compared, using the likelihood criterion. In six out of seven cases, the Weibull function gave the best fit to the observed diameter distributions. They concluded that "the consistent superiority of the Weibull function is remarkable".

Two types of response variables have generally been used for modeling forest stand diameter distributions with the Weibull function. The first type is the parameters of the function or transformations of these. Indexing parameters are calculated plot by plot, usually by maximum likelihood estimation, and regression equations are fit to predict the parameters from stand variables.

Mann (1967) noted that maximum likelihood estimated parameters are consistent and asymptotically efficient, unbiased and normally distributed. They are particularly good estimators for large samples. They are consistent for small samples, and correction for bias is possible.

The second type of response variable is percentile estimators. The 100p percentile is that value X_p of X such that a randomly chosen observation has a probability p of being less than or equal to X_p. Sample percentiles can be tallied for all plots, and regressed against stand variables. Once satisfactory prediction equations have been found for the percentiles, they can be converted to parameter estimates by solving for X_p in the definition of the Weibull cumulative distribution function.

Dubey (1967) showed that the 24th and 93rd sample percentiles gave the most efficient parameter estimates for the two-parameter Weibull function. Abernethy (1981) used the 24th, 63rd and 93rd percentiles based on Dubey's results and the following relationship to obtain the third percentile:

$$F(a+b)=1-\exp[-((a+b-a)/b^c)]=1-\exp(-1)=0.63.$$

Krumland and Wensel (1979) found that the 89th, 60th and 32nd percentiles were most efficient in their study.

The percentile estimators will have to be transformed back to parameter estimates, and percentile estimators are less than 100 percent efficient when compared with maximum likelihood estimators. Dubey (1967) showed that percentile estimators are about 41 percent asymptotically efficient when compared with maximum likelihood estimators.

Bailey (1972) found that percentile estimators fit the observed data less well than maximum likelihood estimators. Bailey and Dell (1973) recommended maximum likelihood estimators. Zarnoch and Dell (1985) claimed that maximum likelihood and percentile estimators can model loblolly pine plantations equally well.

Most researchers have used the three-parameter form of the Weibull function for modeling diameter distributions, presumably upon Bailey's (1972) recommendation. The third parameter, the so-called location parameter, can be interpreted as the smallest tree diameter in the stand.

Schreuder et al. (1979) and Lohrey and Bailey (1979) used the two-parameter form of the Weibull function for diameter distribution modeling. Their models compared well to three-parameter Weibull diameter distribution models.

METHODS

I used the two-parameter Weibull function to model the diameter distributions in this study. Most investigators who have used the three-parameter Weibull function have had problems obtaining consistent and reliable estimates for the third parameter, the location parameter (Bailey, 1972, Abernethy, 1981, Rustagi,1978). In many cases, the estimates of the location parameter were inconsistent. Negative location parameter values often occurred. This result was attributed mainly to erratic mortality in smaller diameter classses (Rustagi, 1978).

In some instances, it was found that the location parameter actually decreased over time, an impossible result in a growing forest stand. This problem was solved by imposing a constraint on the location parameter: if the parameter estimate in any period fell below that of the previous period, it was set equal to the estimate from the previous period, or zero (Bailey and Dell, 1973, Bailey and Abernethy, 1982). Instead of using this rather empirical approach, I decided that the two-parameter Weibull function would be more suitable for diameter distribution modeling of young-growth Douglas-fir stands.

Since this function always has zero as its minimum value, it may not be optimal for modeling non-symmetric diameter distributions, which could result from repeated thinnings from below or above. None of the diameter distributions in this study were severely skewed. The thinnings were "neutral"; an equal proportion of trees was attempted removed from each diameter class. Also, most stands tend to grow towards a symmetric diameter distribution after a disturbance.

The data for this study came from six installations of the Levels-Of-Growing-Stock (LOGS) cooperative study in Douglas-fir, located in western Oregon and Washington (fig. one). The study was designed to examine growth-growing stock ratios as affected by eight different thinning regimes. The majority of the plots are pure Douglas-fir stands, except those at Skykomish. The plots here are a mixture of Douglas-fir and western hemlock (Williamson and Staebler, 1971).

The same experimental design is applied to each installation. Eight treatments, consisting of different thinning intensities, and a control (no thinning) are tested (table one). Three plots per treatment are arranged in a completely randomized design for a total of twenty-seven 1/5-acre plots (Williamson and Staebler, 1971). Thinnings are made whenever average tree height

Fig. 1: The nine installations of the levels-of-growing-stock study. Each installation is marked by a triangle. Installations providing data for this study are marked with black triangles.

TABLE 1: Levels-of-growing-stock study treatment schedule, showing percent of gross basal area increment of control plot to be retained in growing stock.

	Treatment							
Thinning	1	2	3	4	5	6	7	8
First	10	10	30	30	50	50	70	70
Second	10	20	30	40	50	40	70	60
Third	10	30	30	50	50	30	70	50
Fourth	10	40	30	60	50	20	70	40
Fifth	10	50	30	70	50	10	70	30

growth since the previous thinning exceeds 10 feet. Periodic remeasurements exist for four to six thinning periods for each installation. The thinning type is the same for all treatments, a "neutral" thinning where the same proportion of trees is attempted removed from each diameter class.

The original data set was divided into two parts. The first part, containing the control plot data, was used to develop prediction models for diameter distributions of unthinned stands. The second part, containing the data from all the thinned plots, at the beginning and end of each growth period, was used to develop growth projection models for stands after thinning.

The two-parameter Weibull function was fitted to the diameter distribution of all plots, before and after thinning, and to the control plots, for all treatment periods. I calculated maximum likelihood estimates of the Weibull parameters from the observed diameter distributions (Zutter, 1982). I judged maximum likelihood estimation to be the best parameter estimation method (Bailey and Dell, 1973). Zarnoch and Dell (1985) found that maximum likelihood estimators of the Weibull parameters had smaller bias and mean square error than percentile estimators.

If a satisfactory goodness of fit was obtained, these maximum likelihood estimates of the Weibull parameters were to be used as response variables in fitting the prediction models and growth projection models for the Weibull parameters. Since the validity of all subsequent models rested on the goodness of these estimates, I decided to pool the data for the three plots assigned to each treatment in order to obtain as good parameter estimates as possible. I reckoned the advantage of having the best possible maximum likelihood estimates of the parameters would outweigh the disadvantage of having the data base reduced to one-third its original size.

The predictor variables in the diameter distribution and growth projection models were: number of trees per acre (TPA), basal area per acre (BA), height of the 40 largest trees per acre (HT40), average stand height (HEIGHT), stand age (AGE) and length of the growth period (TIME). The height of the 40 largest trees per acre was computed from the plot data as described by Marshall and Bell (1982).

I restricted the predictor variables used in the models to be strictly stand level variables. Most of the utility of these models lies in the fact that accurate diameter-class information can be obtained from stand variables, which can be measured fast and inexpensively (number of trees per acre, basal area per acre, stand age). Other studies have used variables which require actual diameter measurements on individual trees, such as average, minimum and maximum stand diameter (Little, 1983, Rustagi, 1978). In such cases, the usefulness of these models becomes questionable.

Both the control data set and the growth data set were divided into an estimation data set and a validation data set. The estimation data sets were used to develop the regression models. The validation data sets were used to test the goodness-of-fit of the regression models, and to provide a testing data set for selecting the best models.

FITTING THE WEIBULL FUNCTION TO DIAMETER DISTRIBUTIONS

The Kolmogorov-Smirnov (K-S) test for goodness of fit (Steel and Torrie, 1980) was applied to the diameter distribution of all plots in order to determine whether the fitted two-parameter Weibull diameter distribution was the underlying population model for the observed diameter distribution. I compared the cumulative diameter distribution defined by the maximum likelihood estimated parameters, with the observed cumulative diameter distributions.

The test statistic of the K-S test is $d=|F(X)-F_n(X)|$, where F(X) is the estimated cumulative diameter distribution and $F_n(X)$ is the observed cumulative diameter distribution. None of the fitted diameter distributions were rejected as being significantly different from the observed diameter distributions at the 0.05 level of significance. Sample sizes ranged from 51 to 1727 observations.

Based on these tests, I concluded that the Weibull function can adequately describe diameter distributions of managed stands of 20 to 40 years old Douglas-fir in Oregon and Washington.

DIAMETER DISTRIBUTION MODELS FOR UNTHINNED STANDS

I regressed the maximum likelihood estimated parameters of the Weibull function against stand variables, using ordinary least squares multiple regression techniques. The most important predictor variable was $\sqrt{BA/TPA}$. This is logical, since this variable, when multiplied by a constant, gives the quadratic mean diameter (QMD) of the stand. QMD is a very useful stand descriptor in that it gives direct information about the diameter distribution of the stand while at the same time it is calculated from stand variables. It is clear that the scale parameter is more highly correlated with stand variables than the shape parameter (table two). This is also verified in previous studies (Rustagi, 1978), and it is reflected in the fit of the equations in table three and four in this study.

The parameter estimates produced by the regression models for each individual parameter were combined in the two-parameter Weibull distribution function to form a diameter distribution model. The best models for the two parameters, as measured in terms of their coefficient of determination and standard error of estimate, may not, when combined, give the best fitting diameter distribution model.

TABLE 2: Coefficients of correlation of the Weibull scale and shape parameter with stand variables.

STAND VARIABLE	MODEL PARAMETER	
	b	c
Trees per acre	-0.5749	-0.6148
Basal area per acre	0.6241	-0.2695
Height of the 40 largest trees per acre	0.9356	0.0753
Average stand height	0.9539	0.1176
Age	0.8019	-0.0681

where b is the scale parameter

c is the shape parameter

In order to find the best diameter distribution model, I developed three models for each parameter, each with a high coefficient of determination (R^2) and low standard error of estimate (SEE). All possible combinations of these models were tested with the validation data set using the K-S test for goodness-of-fit. The set of these best models are displayed in table three.

The best diameter distribution model, which had the K-S d statistic with the smallest mean and variance when tested on the validation data set, consisted of equations 3) and 4) in table three. None of the predicted distributions produced by this best diameter distribution model were significantly different from the observed diameter distributions, at the 0.05 level of significance.

None of the nine combinations of the equations in table three showed any evident lack of fit. Regression diagnostics for the best model for the scale and shape parameter revealed no exceptionally influential data points.

TABLE 3: Parameter prediction models for unthinned stands. The scale and shape parameter of the two-parameter Weibull function are predicted from stand variables.

Scale Parameter:

1) $B=-0.145+14.196(\sqrt{BA/TPA})$

$R^2=0.984 \qquad SEE=0.2017$

2) $B=5.945+890.870(1/TPA)+0.113(HEIGHT)-2.237(LN(AGE))$

$R^2=0.983 \qquad SEE=0.2133$

3) $B=-0.296-0.027(HEIGHT)+18.037(\sqrt{BA/TPA})$

$R^2=0.987 \qquad SEE=0.1800$

Shape Parameter:

4) $C=2.925+812.756(1/TPA)+0.161E-3(HEIGHT^2)-0.065(AGE)$

$R^2=0.746 \qquad SEE=0.2317$

5) $1/C=0.574-115.435(1/TPA)+0.176E-3(AGE^2)-0.442(\sqrt{BA/TPA})$

$R^2=0.746 \qquad SEE=0.03742$

6) $C=7.584+0.43E-3(HEIGHT^2)-1.161(LN(BA))-0.223E-4(AGE^3)$

$R^2=0.717 \qquad SEE=0.2446$

where

LN denotes the natural logarithm

BA is basal area per acre in square feet

TPA is number of trees per acre

HEIGHT is average stand height in feet

AGE is the age of the stand, in years

GROWTH PROJECTION

The concept of modeling stand growth by modeling the changes in the fitted Weibull parameters over time is not new. Schreuder and Swank (1974) suggested that only the parameter values of the Weibull function need to be changed in order to model diameter distributions over time, and that the changes in these parameters may be a good way to characterize and interpret changes in stands over time.

In order to predict the growth of a stand of trees after thinning, I developed a two-parameter Weibull distribution model for the diameter distribution of the stand at the end of a growth period. I regressed the scale and shape parameter of this distribution against stand variables at the beginning of the period and length of the growth period. All combinations of the regression models 1) through 6) for the scale and shape parameters in table four were tested on the validation data set. Residual plots for the equations in table four showed no evident lack of fit.

Abernethy (1981) developed similar models. The parameter prediction equations in that study had a better fit than the above equations, probably largely due to the fact that the predicted parameter from the previous

TABLE 4: Growth projection models for stands after thinning. The scale and shape parameter of the two-parameter Weibull function are predicted from stand variables at the beginning of the growth period and the length of the growth period.

Scale Parameter:

1) $B=0.7758+0.6283(LN(TIME))+14.23(\sqrt{BA/TPA})$

$R^2=0.987$ SEE=0.3336

2) $B=7.607-0.8036(LN(TPA))-2.601(1/TIME)+12.64(\sqrt{BA/TPA})$

$R^2=0.992$ SEE=0.2594

3) $B=2.399-0.008566(BA)-2.431(1/TIME)+15.11(\sqrt{BA/TPA})$

$R^2=0.992$ SEE=0.2618

Shape Parameter:

4) $C=16.38-2.730(LN(TPA))+1.439(LN(BA))-0.003791(AGE^2)$
$+0.006191(TIME^3)$

$R^2=0.711$ SEE=0.4393

5) $C=8.954-1.008(LN(TPA))+5.360(\sqrt{BA/TPA})-0.8386E-4(AGE^3)$
$+0.007499(TIME^3)$

$R^2=0.726$ SEE=0.4166

6) $C=3.157-0.004224(AGE^2)+0.006780(TIME^3)+8.238(\sqrt{BA/TPA})$

$R^2=0.705$ SEE=0.4458

TABLE 4, continued:

Using the previous parameter estimates as predictors:

7) $B=7.698+0.9875(B_s)-0.04932(AGE)-3.919(1/TIME)$
$-0.6639(LN(TPA))$

$R^2=0.996$ SSE=0.1916

8) $C=4.075+0.9247(C_s)-0.04576(AGE)-0.4387(LN(TPA))$
$+0.001669(TIME^3)$

$R^2=0.975$ SSE=0.1946

where

LN denotes the natural logarithm

TIME is the length of the growth period in years

TPA is number of trees per acre

BA is basal area per acre in square feet

AGE is stand age in years

B_s is the value of the B (scale) parameter at the beginning of the growth period

C_s is the value of the C (shape) parameter at the beginning of the growth period

period was used as a predictor for the parameter at the end of the current period. This requires the assumption that the Weibull parameters characterizing the diameter distribution before and after thinning are the same. This is a reasonable assumption in the LOGS study, where all thinnings are neutral.

Equations 7) and 8) in table four use the response variables from the previous period as predictor variables. Although the diameter distribution model generated by these two equations performed the best of all the models when tested with the validation data set, I decided not to use this model. Using the response variable from one prediction as a predictor variable in a subsequent prediction gives an artificially inflated goodness of fit. The goodness of fit of all the predictions will rest solely on how good is the first prediction in the series.

The next best model which includes equations 2) and 6) enables the user of the equations to predict growth directly from stand variables, without having to predict a set of function parameters first. When I tested this model on the validation data set using the K-S test at a 0.05 level of significance, seven percent of the plots were rejected as not having come from a Weibull distribution with the specified parameters. Eighteen percent of the plots were rejected at the 0.10 level of significance.

All the plots which were rejected at the 0.05 level of significance and 63 percent of the plots which were rejected at the 0.10 level came from the Skykomish installation (fig. 1). This study area contained approximately 50 percent western hemlock after the first thinning. The fact that these were mixed species plots could be a part of the reason why they were rejected.

It should be noted that the length of the growth periods in the data set from which the models were developed ranged from two to five years. Predictions beyond this range must be considered extrapolations of the model.

The growth model obtains size-class information from stand characteristics. It is an approximation to a stand diameter distribution and should not be expected to give the same level of accuracy as an individual tree growth model.

CONCLUSIONS

Diameter distributions of even-aged stands of young-growth Douglas-fir in Washington and Oregon can be adequately described with the two-parameter Weibull function. Maximum likelihood estimates of the function parameters fit the observed distributions well.

A diameter distribution model is given for unthinned stands of Douglas-fir. The model uses only overall stand characteristics as predictor variables, thus facilitating calculation of diameter-class information from stand level variables only. The model predicted diameter distributions very well when tested on an independent data set.

A diameter distribution model for predicting growth of stands after thinning is given. Satisfactory results were obtained when testing it on an independent data set. The range of growth periods over which the model is valid is fairly limited (two to five years), so the model has limited usefulness for non-intensive management regimes. It does not perform as well for mixed stands of western hemlock and Douglas-fir as for pure stands of Douglas-fir.

The two-parameter Weibull function performed very well for predicting diameter distributions of unthinned stands. The results obtained with the two-parameter Weibull function in this study were equivalent to or better than those of comparable studies in which the three-parameter Weibull function was used (Smalley and Bailey, 1974, Clutter and Belcher, 1978, Feduccia et al., 1979).

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