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Title: A Linkage Finite Element for Nailed Composite Wood I-Beams and T-Beams

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With current nonlinear analysis computer capabilities, this study was conducted to develop a linkage element that permits analyses of wood composite structures taking into account possible slip in the nailed joints. The finite elements chosen to represent the wood stud, plywood, gypsum and nails are described. The method as applied to T-Beams and I-Beams is verified by comparing the analytical results with the results of previous studies. Results are also compared with static load tests conducted on gypsum-plywood-stud wall panels.
A Linkage Finite Element for Nailed Composite Wood
I-Beams and T-Beams

by

Chyuan-Shen Lee

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**LIST OF SYMBOLS**

- $A_i$: areas of $i$th layer
- $B_j$: regression constants
- $C_0$, $C_1$, $C_2$: constants
- Coh: cohesion
- $C_D(t)$: damping matrix as a function of time
- $E$: elastic modulus
- $E_i$: modulus of elasticity of $i$th layer
- $E_L$, $E_R$, $E_T$: modulus of elasticity in $L$, $R$, $T$ directions, respectively
- $F_i$: axial forces in $i$th layer
- $F_n$: force in the normal direction
- $F_L$: axial force (left-end of beam)
- $F_R$: axial force (right-end of beam)
- $F_t$: force in the tangential direction
- $G$: shear modulus
- $h_1$, $h_2$: depths of layers
- $h$: centroidal location from top of beam
- $h_c$, $h_t$, $h_s$: depths for top flange, bottom flange and stud, respectively
- $I_i$: moment of inertia of $i$th layer
- $I_s$: moment of inertia of equivalent rigidly connected beam
- $K$: slip modulus of fastener
- $K_i$: average inelastic joint stiffness
\( \bar{K} \)  
slip modulus of connector

\( k \)  
curvature

\([K_m]\)  
global stiffness matrix for the mth member

\( K(t) \)  
stiffness matrix as a function of time

\( K_n, K_t \)  
stiffness of linkage element in normal and tangential direction, respectively

\( L \)  
beam span length

\( L_m \)  
length of mth truss element

\( M_i \)  
moments in ith layers

\( M_t \)  
total static moment at any cross-section of beam

\( M(t) \)  
mass matrix as function of time

\( n \)  
number of connectors per row

\( P \)  
load

\( P_0, P_1 \)  
parameters for the foundation load-deflection curve

\( Q \)  
connector force

\( q_{ij} \)  
interlayer shear flow between ith and jth layers

\( [R] \)  
transformation matrix

\( R(t) \)  
forcing matrix as a function of time

\( r_1, r_2, r_3 \)  
distance from beam centroid to centroid of individual layers

\( S_{ij} \)  
relative displacement at interface

\( s \)  
spacing of connectors

\( S_e \)  
percent of early wood

\( s_{ij} \)  
spacing of connectors between ith and jth layers
local stiffness matrix for mth truss element

specific gravity of the stud

specific gravity of the plywood

time

displacement or interlayer slip

total beam shear at any cross section

Wilkinson's slip modulus

deflection of the nail

global coordinate system

local coordinate system

beam deflection coordinate

deflection of equivalent rigidly connected beam

deflection or interlayer slip

interlayer slip between ith and jth layers

time step

stress at yield point

strain at level i for layer j

coefficient of friction

parameter

stress tensor

Poisson's ratio
A LINKAGE FINITE ELEMENT FOR NAILED COMPOSITE WOOD
I-BEAMS AND T-BEAMS

I. INTRODUCTION

1.1. Introduction

In civil engineering practice, composite construction can be found everywhere, such as composite steel-concrete construction, composite wood-concrete construction and composite sandwich and laminated structures. The most important property of composite construction is the interaction at the contact surface of the composite components. For composite wood systems, the degree of composite action depends on the type and the number of connectors used and the characteristics of the wood itself. To understand this phenomenon, consider the two-layer wood T-beam shown in Fig. 1.1a. Considering the beam to be made of homogeneous materials, the behavior of the system is a function of the degree of rigidity of the interlayer connection between the stem and flange of the beam. If a rigid connection system is used, the strains of the T-beam will vary as the distance from the centroid of the cross section as shown in Fig. 1.1b. If no connection is made between the flange and the stem, each layer simply slides over the other, and, assuming the layers remain in contact with each other, the strains in each layer will vary about their own midheight as shown in Fig. 1.1c. For the semi-
rigid joint, the interlayer connection offers a partial restraint to the slippage, an intermediate behavior exists as shown in Fig. 1.1d.

Nails are the primary fastener used to join wood composite structures. Since the nail joints are semirigid, the interlayer slip between the connected elements must be recognized. Goodman (14) has found that the presence of slip has a significant effect on deflection of nailed multilayered beams. Amana and Booth (2) also found that the deflection of a beam with no shear transfer between the components is about 2½ times the value when there is complete interaction. Therefore, the behavior of the nailed joints must be given proper consideration when designing and analyzing wood structures. In general, nails may undergo two types of loading, lateral and withdrawal. In this study, the primary concern is the relative movement at the interface under lateral loading. This relative movement, often termed interlayer slip, usually is a function of the applied load. One means of assessing the interlayer slip is to use the slip modulus. The slip modulus is defined as the ratio of the applied load on the joint to the displacement in the joint at that load. Actually, this ratio is the slope of the load vs. interlayer slip curve.

1.2. Brief Review of Previous Research

Numerous earlier studies have been conducted to describe
Fig. 1.1. Composite Action as Effected by Interlayer Connection
the load-slip behavior of nailed joints. Most of these studies (17, 27, 28) were based on the beam-on-elastic foundation concept, in which the nail was assumed to be supported on an elastic foundation (the wood under the nail shank) so that deflection of the nail is resisted by a pressure proportional to the deflection at any point along the nail shank. This concept leads to a linear relation between load and slip. However, experimental work by Mclain (20) pointed out that the actual nail behavior is quite different because the load-slip characteristic of nails is linear only at small slip. Thus this procedure can be used to predict the initial stiffness of the connector up to a maximum slip value of 0.015 inch only. To estimate the ultimate load, a curvilinear relation must be considered. Since the load-slip curve is non-linear, the slope of the curve is constantly changing. Therefore, approximate methods of obtaining a slip modulus are used. In general, the load-slip curve can be evaluated on the basis of a tangent, secant or chord modulus, the choice of which is dependent on the investigator and the situation. The various methods of defining the slip modulus are shown in Fig. 1.2. For a better approximation of the curve, incremental chord or tangent moduli along the curve can be used. Detailed discussions of these concepts can be found in Reference 22.

Recently, a method based on elastic-plastic analysis was introduced by Foschi (12). The nail shank is considered to be a
Fig. 1.2. Various Definitions of Slip Modulus

- a) Secant Modulus
- b) Initial Tangent Modulus
- c) Tangent Modulus
- d) Chord Modulus
- e) Incremental Chord Modulus
- f) Incremental Tangent Modulus
beam bearing on a nonelastic foundation. The steel of the nail is assumed to be perfectly elastic up to a yield-point stress $\sigma_0$ (Fig. 1.3a.). For the wood, the load-deformation relationship is assumed to have the general form shown in Fig. 1.3b. This curve can be represented in exponential form according to

$$P(w) = (P_0 + P_1w) \left[1.0 - \exp\left(-kw/P_0\right)\right]$$

where the initial modulus $k$ and the constant $P_0$ and $P_1$ are determined by nonlinear least squares fitting of experimental data for any particular nail type and wood species. A finite element elasto-plastic approximation for the nail was also proposed in their study.

![Stress vs. strain for steel and wood](image)

(a) Steel  (b) Wood

Fig. 1.3. Steel and Wood-Bearing Properties
More recently, Polensek and Loferski (18) developed a procedure for predicting the average inelastic joint stiffness on the basis of the material properties of the joint components for Southern pine stud and Douglas-Fir sheathing plywood with 6d box nails. The load-slip trace was divided into three linear sections. The average inelastic joint stiffness (slope) for these line segments can be obtained using the following regression equation

\[ K_i = B_0 + B_1(W) + B_2(1/sgm) + B_3(sgm) + B_4(sgs) + B_5(K_1) + B_6(K_2) + B_7(S_e) + B_8(sgm)^2 \]  

(1.2)

in which

\[ B_j \] = regression constants \((j = 0, 1, 2, 3\ldots8)\) \((i = 1, 2, 3)\)

\[ W \] = Wilkinson's slip modulus

\[ sgm \] = specific gravity of the stud

\[ sgs \] = specific gravity of the plywood

\[ K_1, K_2 \] = stiffness modulus of first (second) elastic section

\[ S_e \] = percent of earlywood

1.3. Objective

Currently, a finite element computer program called SAPWOOD
is being developed at Oregon State University for the static and
dynamic analysis of wood-framed structures. The computer program
SAPWOOD is basically a modification of the program NONSAP (6).
The objective of this investigation and report is to develop a
finite element model that can be used to model the load-slip
behavior of nailed joints, in particular, the nailed joints
between sheathings and studs. Furthermore, this model should
have the capability to model other contact surface problems.
II. MATHEMATICAL MODEL

2.1. Mathematical Model of an I-Beam and Its Closed Form Solution

A closed form solution for the linear mathematical model was developed by Booth (8) using the following basic assumptions:

1) The shear transfer is continuous at all points on the flange-web interface and the fastener stiffnesses are constant at all points along the length.

2) The interface load-slip characteristic is linear and no vertical separation occurs between the flange and the web so that under load the deflections of the flange and the web are the same. This implies that the flanges and the web have the same curvature.

3) Plane sections of both the flanges and the webs remain plane after bending; this implies a linear variation of strain over the depth of each layer (flanges and web).

4) The relationship between stress and strain (Hooke's law) is linear in both the flanges and the web over the whole range of the applied loads.

Practical joints will not satisfy all these conditions. The first assumption is true only for glued joints. For nailed joints, it is approximately true only if all the nails are equally spaced, the nails have equal capacity and the conditions of nailing are the same for every nail. The second assumption
can be satisfied over the range of working load without much error, and within this range no vertical separation is to be expected. If the analysis is under ultimate load condition, a non-linear equation would be required. In this study, a stepwise process was used to take into account the effect of the nonlinearity of the slip curve. In general, assumptions (3) and (4) are satisfied, because the wall analysis is structured to follow these assumptions (a section of wall with stud is considered as an I-beam as shown in Fig. 2.1a and b). Another assumption that has been imposed is that the elastic and geometric properties are constant along the length of the I-beam. Previous studies have verified that using the average effective stud properties based on ASTM standards and average properties of wall coverings introduces only negligible errors (2, 3).

When the composite beam is subjected to a static load, the moment M, axial force F and shear V along the beam are denoted as M(x), F(x) and V(x). The effect of shear deflection can be shown to be small and is neglected.

From the third and fourth assumptions that the strain distribution through each layer of the cross-section is linear, it follows that the strains in each level of the cross-section may be evaluated by considering the effects of the moment and axial forces (see Fig. 2.1c):
\[ e_{11} = \frac{F_1}{E_1 A_1} - \frac{M_1 h_c}{E_1 I_1} \]
\[ e_{12} = \frac{F_1}{E_1 A_1} + \frac{M_1 h_c}{E_1 I_1} \]
\[ e_{22} = \frac{F_2}{E_2 A_2} - \frac{M_2 h_s}{E_2 I_2} \]
\[ e_{23} = \frac{F_2}{E_2 A_2} + \frac{M_2 h_s}{E_2 I_2} \]
\[ e_{33} = \frac{F_3}{E_3 A_3} - \frac{M_3 h_t}{E_3 I_3} \]
\[ e_{34} = \frac{F_3}{E_3 A_3} + \frac{M_3 h_t}{E_3 I_3} \]

(2.1)

The interlayer shear flow is evaluated by considering horizontal equilibrium for the top and bottom flange respectively which gives

\[ q_{12} = \frac{dF_1}{dx} \quad q_{23} = \frac{dF_3}{dx} \]

(2.2)

Since there is no axial force on either end of the beam, the equilibrium condition (Fig. 2.1d) for the left side of element \( dx \) gives

\[ F_1 + F_2 + F_3 = 0 \]

(2.3)
Fig. 2.1. Typical Layered I-Beam System
Fig. 2.1. Typical Layered I-Beam System (continued)
\[ M_t = M_1 + M_2 + M_3 - F_1r_1 + F_2r_2 + F_3r_3 \]  
\[(2.4)\]

where
- \( M_t \) = external moment about z axis
- \( M_i \) = resisting moment of flanges and web
- \( F_i \) = axial force acting at the centroid of flanges and web
- \( r_i \) = distance between the centroid of flanges and web and the centroid of composite beam section

From the second assumption that there is no vertical separation of the flange and the web, each deflects an equal amount \( y \) at all points along their length and has the same curvature (the sign convention is shown in Fig. 2.1.):

\[- \frac{d^2y}{dx^2} = \frac{M_1}{E_1I_1} = \frac{M_2}{E_2I_2} = \frac{M_3}{E_3I_3} = \frac{M_t + F_1r_1 - F_2r_2 - F_3r_3}{\sum_{i=1}^{3} E_iI_i} \]  
\[(2.5)\]

where
- \( E_iI_i \) = flexural rigidity of the flanges and web

The relative slip between the top flange and the web over a length \( x \) is given by
Similarly, the relative slip between the bottom flange and web is given by

\[ S_{23} = \int_{0}^{x} \varepsilon_{33} \, dx - \int_{0}^{x} \varepsilon_{23} \, dx \]  

(2.6-2)

For mechanical connectors, the displacement modulus of the fastener is assumed to be predicted by

\[ S_{ij} = \frac{S_{ij} q_{ij}}{K_{ij}} = \left( \frac{S_{ij}}{\bar{K}n} \right)_{ij} q_{ij} \]  

(2.7)

where

- \( S_{ij} \) = relative displacement at interface (L)
- \( K_{ij} \) = \( n\bar{R} \), displacement modulus of fastener (F/L)
- \( n \) = number of connectors per row (I)
- \( \bar{R} \) = slip modulus of connector (F/L)
- \( q_{ij} \) = interlayer shear flow (F/L)
- \( s_{ij} \) = spacing of connectors between ith and jth layers (L)

For a glued system, the displacement modulus \( K \) is modified to account for the area on which the glue acts. Combining Eq. 2.2, Eq. 2.6 and Eq. 2.7 leads to
\[ \varepsilon_{12} - \varepsilon_{22} = \frac{ds_{12}}{dx} = \frac{s_{12}}{K_{12}} \frac{d^2F_1}{dx^2} \]

\[ \varepsilon_{33} - \varepsilon_{23} = \frac{ds_{23}}{dx} = \frac{s_{23}}{K_{23}} \frac{d^2F_3}{dx^2} \]  

(2.8)

Substituting for \( \varepsilon \) from Eq. 2.1, Eq. 2.8 becomes

\[ \frac{s_{12}}{K_{12}} \frac{d^2F_1}{dx^2} = \frac{F_1}{E_1A_1} - \frac{F_2}{E_2A_2} + \frac{M_1h_c}{E_1I_1} + \frac{M_2h_s}{E_2I_2} \]

\[ \frac{s_{23}}{K_{23}} \frac{d^2F_3}{dx^2} = \frac{F_3}{E_3A_3} - \frac{F_2}{E_2A_2} - \frac{M_3h_t}{E_3I_3} - \frac{M_2h_s}{E_2I_2} \]  

(2.9)

Using Eq. 2.5 and rewriting Eq. 2.9 gives

\[ \frac{s_{12}}{K_{12}} \frac{d^2F_1}{dx^2} = \frac{F_1}{E_1A_1} - \frac{F_2}{E_2A_2} + \frac{(M_t+F_1r_1-F_2r_2-F_3r_3)(h_c+h_s)}{\Sigma EI} \]

(2.10-1)

and

\[ \frac{s_{23}}{K_{23}} \frac{d^2F_3}{dx^2} = \frac{F_3}{E_3A_3} - \frac{F_2}{E_2A_2} - \frac{(M_t+F_1r_1-F_2r_2-F_3r_3)(h_t+h_s)}{\Sigma EI} \]

(2.10-2)

or
\[
\frac{s_{12}}{K_{12}} \frac{d^2 F_1}{dx^2} = \frac{F_1}{E_1 A_1} - \frac{F_2}{E_2 A_2} - (h_c + h_s) \frac{d^2 y}{dx^2} \quad (2.10-3)
\]

and

\[
\frac{s_{23}}{K_{23}} \frac{d^2 F_3}{dx^2} = \frac{F_3}{E_3 A_3} - \frac{F_2}{E_2 A_2} + (h_t + h_s) \frac{d^2 y}{dx^2} \quad (2.10-4)
\]

Differentiating Eq. 2.3 and substituting Eq. 2.10-1 and Eq. 2.10-2, we get

\[
\frac{d^2 F_2}{dx^2} + \frac{K_{12}}{s_{12}} \left[ \frac{F_1}{E_1 A_1} - \frac{F_2}{E_2 A_2} + \frac{(M_t + F_1 r_1 - F_2 r_2 - F_3 r_3)(h_c + h_s)}{\Sigma EI} \right] \\
+ \frac{K_{23}}{s_{23}} \left[ \frac{F_3}{E_3 A_3} - \frac{F_2}{E_2 A_2} - \frac{(M_t + F_1 r_1 - F_2 r_2 - F_3 r_3)(h_t + h_s)}{\Sigma EI} \right] \\
= 0 \quad (2.11)
\]

For simplicity, consider the usual case of a symmetrical cross-section, such that

\begin{align*}
E_1 &= E_3, & A_1 &= A_3, & r_1 = r_3 = r = h_c + h_s, \\
h_c &= h_t = h_f, & r_2 &= 0, & K_{12} = K_{23} = K \text{ and} \\
s_{12} &= s_{23} = s.
\end{align*}

If we substitute these conditions into Eq. 2.11, we obtain
\[ \frac{d^2F_2}{dx^2} + \frac{K}{s} \left[ \frac{F_1 + F_3}{E_1 A_1} - \frac{2F_2}{E_2 A_2} \right] = 0 \]

Since \( F_1 + F_2 + F_3 = 0 \), we find

\[ \frac{d^2F_2}{dx^2} - \frac{K}{s} \left[ \frac{2}{E_2 A_2} + \frac{1}{E_1 A_1} \right] F_2 = 0 \]

Under the assumed loading condition, the boundary condition at the end of the beam (\( x=0, L \)) is \( F_2 = 0 \). Then we can verify that the solution of Eq. 2.12 is

\[ F_2 = 0 \quad (2.13) \]

Thus for a symmetrical section the web is subjected to the moment \( M_2 \) only.

The governing equation for \( F_3 \) may be established by combining Eqs. 2.3, 2.13, 2.10-2 and 2.10-3

\[ \frac{s_{23}}{K_{23}} \frac{d^2F_3}{dx^2} = \frac{F_3}{E_3 A_3} - \frac{(M_t - 2F_3 r) r}{\Sigma EI} \quad (2.14) \]

or
\[
\frac{d^2 F_3}{dx^2} - \frac{K_{23}}{s_{23}} \left[ \frac{1}{E_3 A_3} + \frac{2r^3}{\Sigma EI} \right] F_3 = -\frac{K_{23}}{s_{23}} \frac{M_t r}{\Sigma EI} 
\]  
(2.15)

For simplicity let

\[
C_0 = \left[ \frac{s_{23}}{K_{23}} \right] \left[ \frac{1}{L^2} \right] \left[ \frac{E_3 A_3 \Sigma EI}{EI} \right] 
\]  
(2.16)

where \( C_0 \) is a non-dimensional parameter.

\[
\bar{EI} = \Sigma EI + 2r^2 E_3 A_3 
\]

Eq. 2.15 then becomes

\[
\frac{d^2 F_3}{dx^2} - \frac{1}{C_0 L^2} F_3 = -\frac{1}{C_0 L^2} \frac{E_3 A_3 r}{EI} M_t 
\]  
(2.17)

This equation can be solved if the distribution of \( M_t \) along the beam is known together with the appropriate boundary conditions. After \( F_3 \) is determined, the other three unknowns, \( F_1 \), \( M_1 \) and \( M_3 \) can be found from Eq. 2.3 and Eq. 2.5.

Consider the condition of a static concentrated load \( P \) is applied at a distance "a" from the left support of the simply supported beam. If the coordinate \( x \) is measured from the left
support, the bending moment along the member is given by

\[ M_t(x) = P \left(1 - \frac{a}{L}\right)x \quad 0 < x < a \]  
\[ M_t(x) = Pa \left(1 - \frac{x}{L}\right) \quad a < x < L \]  

(2.18)

The governing differential Eq. 2.17 now becomes

\[
\frac{d^2 F_{3L}}{dx^2} - \frac{1}{C_0 L^2} F_{3L} = -\frac{1}{C_0 L^2} \frac{E_3 A_3 r}{E_I} P \left(1 - \frac{a}{L}\right)x \quad 0 < x < a
\]

\[
\frac{d^3 F_{3R}}{dx^2} - \frac{1}{C_0 L^2} F_{3R} = -\frac{1}{C_0 L^2} \frac{E_3 A_3 r}{E_I} Pa \left(1 - \frac{x}{L}\right) \quad a < x < L. \]  

(2.19)

The solutions to these equations are

\[ F_{3L}(x) = A_1 \sinh \frac{x}{\sqrt{C_0 L}} + A_2 \cosh \frac{x}{\sqrt{C_0 L}} - \frac{E_3 A_3 r}{E_I} P \left(1 - \frac{a}{L}\right)x \quad 0 < x < a \]

\[ F_{3R}(x) = B_1 \sinh \frac{x}{\sqrt{C_0 L}} + B_2 \cosh \frac{x}{\sqrt{C_0 L}} - \frac{E_3 A_3 r}{E_I} Pa \left(1 - \frac{x}{L}\right) \quad a < x < L \]  

(2.20)

where \( A_1, A_2, B_1, B_2 \) are integral constants. The boundary conditions are
\[ F_{3L}(0) = 0 \quad F_{3R}(L) = 0 \]
\[ F_{3L}(a) = F_{3R}(a) \quad \frac{dF_{3R}(a)}{dx} = \frac{dF_{3L}(a)}{dx} \quad (2.21) \]

Hence \( F_3 \) is given by

\[
F_{3L}(x) = -\frac{E_3A_3r}{EI} \frac{P\sinh \frac{a}{\sqrt{C_0L}}}{\tanh \frac{1}{\sqrt{C_0}}} \left[ 1 - \frac{1}{\tanh \frac{a}{\sqrt{C_0}}} \right] \sinh \frac{x}{\sqrt{C_0L}} - \frac{E_3A_3r}{EI} P(1 - \frac{a}{L})x \quad 0 < x < a
\]

\[
F_{3R}(x) = -\frac{E_3A_3r}{EI} \frac{P\sinh \frac{a}{\sqrt{C_0L}}}{\tanh \frac{1}{\sqrt{C_0}}} \sinh \frac{x}{\sqrt{C_0L}} - \frac{E_3A_3r}{EI} P_0 a \left( 1 - \frac{x}{L} \right) + \frac{E_3A_3r}{EI} P\sinh \frac{a}{\sqrt{C_0L}} \cosh \frac{x}{\sqrt{C_0L}} \quad a < x < L \quad (2.22)
\]

The curvature of the beam is given by Eq. 2.5, in which we can substitute for \( F_3 \) from Eq. 2.17. Hence

\[
-\frac{d^2y}{dx^2} = \frac{Mt - 2F_3r}{\Sigma EI} = \frac{Mt}{EI} - 2\frac{s}{k} \frac{E_3A_3r}{EI} \frac{d^2F_3}{dx} \quad (2.23)
\]
Integrating twice gives

\[ y = - \int \int \frac{M_t}{EI} \, dx \, dx + 2\frac{E}{K} \frac{A_3^r}{EI} \, F_3 + C_5 x + C_6 \quad (2.24) \]

where \( C_5 \) and \( C_6 \) are integration constants. The boundary conditions are

\[ y(0) = 0 \quad F_{3L}(0) = 0 \quad \int \int \frac{M_t}{EI} \, dx \, dx = 0 \text{ at } x = 0 \]

\[ y(L) = 0 \quad F_{3R}(L) = 0 \quad \int \int \frac{M_t}{EI} \, dx \, dx = 0 \text{ at } x = L \]

Hence \( C_5 = C_6 = 0 \)

Thus the deflection \( y \) is given by

\[ y_L(x) = -\int \int \frac{M_t}{EI} \, dx \, dx + 2\frac{E}{K} \frac{A_3^r}{EI} \, F_{3L}(x) \]

and

\[ y_R(x) = -\int \int \frac{M_t}{EI} \, dx \, dx + 2\frac{E}{K} \frac{A_3^r}{EI} \, F_{3R}(x) \quad (2.25) \]

Note that the first term in Eq. 2.25 represents the deflection of the equivalent solid beam and the second term is due to the flange-web joint displacement.
2.2. Mathematical Model of a T-Beam and Its Closed Form Solution

Goodman and Popov (15) have presented their theoretical studies of multilayered beam systems with interlayer slip based on the same assumptions as that of Booth (8) except that a non-linear slip curve was introduced. Goodman's work was extended by Kuo to the special cases of two-layered T-Beam systems. A typical two-layered T-Beam to be considered is shown in Fig. 2.2. Applying the equations of equilibrium and the assumptions pointed out in Sec. 2.1., the governing equations for the deflection of the member and the axial force $F$ are:

\[
\frac{d^2y}{dx^2} = \frac{-Mt + C_{12}F}{E_1I_1 + E_2I_2} \tag{2.26}
\]

and

\[
\frac{s}{Kn} \frac{d^2F}{dx^2} = \left[ \frac{1}{E_1A_1} + \frac{1}{E_2A_2} \right] F + C_{12} \frac{d^2y}{dx^2} \tag{2.27}
\]

where

- $E_i$ = modulus of elasticity for reference layer
- $I_i$ = moment of inertia of the $i$th individual layer
- $A_i$ = area of individual layer
- $s$ = spacing of connectors
\( \bar{R} \) = slip modulus of connector

\( n \) = number of connectors per row

\( M_t \) = total static moment at any cross section of beam

\[
C_{12} = \frac{h_1 + h_2}{2} \quad \text{one-half the sum of depths of the layers}
\]

\( F, F_1, F_2 \) = axial forces in layers; \( F = -F_1 = F_2 \)

Deflections of any beam for any loading may be computed by solving Eqs. 2.26 and 2.27 subject to the appropriate boundary conditions.

As an example, consider the simply supported composite T-beam to be subjected to a concentrated load at location "a" from the left support as shown as Fig. 2.2. Proper substitution and double integration, with appropriate boundary conditions, give the following formulas:

\[
y_L(x) = y_s(x)_L + \frac{C_{12}}{E_1 I_1 + E_2 I_2} \frac{1}{C_1} F_L(x) \quad (2.28)
\]

\[
y_R(x) = y_s(x)_R + \frac{C_{12}}{E_1 I_1 + E_2 I_2} \frac{1}{C_1} F_R(x) \quad (2.29)
\]

where
\[ F_L(x) = -\frac{C_2}{C_1} \frac{P}{\sqrt{C_1} \sinh(\sqrt{C_1}(L-a))} \sinh(\sqrt{C_1}x) \]
\[ + \frac{C_2}{C_1} \frac{P}{\sqrt{C_1}} (1 - \frac{a}{L})x \quad 0 \leq x \leq a \quad (2.30) \]

\[ F_R(x) = -\frac{C_2}{C_1} \frac{P}{\sqrt{C_1}} \sinh(\sqrt{C_1}a) \cosh(\sqrt{C_1}x) \]
\[ + \frac{C_2}{C_1} \frac{P}{\sqrt{C_1}} \frac{\sinh(\sqrt{C_1}a)}{\tanh(\sqrt{C_1}L)} \sinh(\sqrt{C_1}x) \]
\[ + \frac{C_2}{C_1} \frac{P}{\sqrt{C_1}} a (1 - \frac{x}{L}) \quad a \leq x \leq L \quad (2.31) \]

\[ C_1 = \frac{Kn}{s} \left[ \frac{1}{E_1 A_1} + \frac{1}{E_2 A_2} \right] \frac{I_S}{I_1 + I_2} \]

\[ C_2 = \frac{Kn}{s} \left[ \frac{C_{12}}{E_1 I_1 + E_2 I_2} \right] \]

\[ F_R \text{ or } F_L(x) = \text{ axial layer force} \]

\[ I_S = \text{ moment of inertia if the cross-section in Fig. 2.2. acts as a solid beam} \]

\[ y_S(x)_R \text{ or } L = \text{ deflection of an equivalent solid (rigidly-connected) beam at location "x" from the left end support either to the right or left of the concentrated load} \]
(a) Beam with Sign Convention

(b) Cross-Section

(c) Strain Distribution

(d) Force on the Composite Beam Element

Fig. 2.2. Two Layered T-Beam System
Fig. 2.2. Two Layered T-Beam System (continued)
\( y(x)_R \) or \( L \) = deflection of the layered beam at location "x" from the left end support either to the right or left of the concentrated load

The interlayer shear flow is obtained by considering horizontal equilibrium for the top layer which gives

\[
q_L(x) = \frac{dF_L(x)}{dx} = - \frac{C_2}{C_1} p \frac{\sinh(\sqrt{C_1}(L - a))}{\sinh(\sqrt{C_1}L)} \cosh(\sqrt{C_1}x)
\]

\[
+ \frac{C_2}{C_1} p (1 - \frac{a}{L}) \quad 0 \leq x \leq a
\] (2.32)

\[
q_R(x) = \frac{dF_R(x)}{dx} = - \frac{C_2}{C_1} p \sinh(\sqrt{C_1}a) \sinh(\sqrt{C_1}x)
\]

\[
+ \frac{C_2}{C_1} p \frac{\sinh(\sqrt{C_1}a)}{\tanh(\sqrt{C_1}L)} \cosh(\sqrt{C_1}x)
\]

\[
- \frac{C_2}{C_1} p \frac{a}{L} \quad a \leq x \leq L
\] (2.33)

The connector force \( Q \), can be obtained by

\[
Q = \frac{Q_S}{n}
\] (2.34)
III. FINITE ELEMENT MODEL

3.1. Introduction

A completely rational analysis of wood composite beams for internal stresses and displacements is made difficult by several factors. These include: (a) the local variations in elastic and geometric properties of studs and wall coverings along the length of each composite beam and (b) the nonlinear load-slip characteristics of connectors.

In Sec. II, a mathematical model was presented based on the assumption of constant slip modulus, properties of composite components and connector spacing. To remove the restrictions of constant slip modulus and connector spacing, a finite element procedure which allows for variation in physical properties at any location along the beam offers a more flexible solution. The finite element method of analysis has been presented in detail elsewhere (10, 31) and will not be redeveloped here. One of the objects of this study is to include a linkage element in an existing finite element model that is capable of predicting the behavior of contact surfaces subjected to loads up to their ultimate carrying capacities. The proposed model will be added to the nonlinear structural analysis program SAPWOOD.
3.2. Linkage Element Model

The physical behavior of an interface between two continua can be classified under four cases:

1) Open condition-- there exists a gap at the interface.
2) Complete interaction-- in this case, both the tangential component and the normal component of the interface stress do not exceed the "shear strength" and "tensile strength" of the discontinuity. Neither slip nor separation occurs.
3) Incomplete interaction with slippage but without separation-- when the tangential component of the interface-stress exceeds the "shear strength" of the discontinuity, tangential slip occurs. However, the normal component of the interface-stress remains less than the allowable normal stress, thus no separation occurs.
4) Incomplete interaction with separation but without slippage-- in this case, the tangential component of the interface-stress is smaller than the allowable tangential stress, but the normal component is larger than the allowable normal stress.

In general, the allowable shear strength and tensile strength can be calculated by some "sliding friction" law such as Coulomb's law. Experiments have shown that the interface slippage and
separation can significantly affect the overall behavior of the structure and its behavior is complex and inherently nonlinear. Since slip, separation and recontact may all occur during a given stress-time history in one system, the solution to the problem is possible only if the loading sequence is prescribed.

Several finite element models (4, 9, 16, 25) have been developed to investigate joint behavior in systems such as composite steel-concrete beams, layered wood construction connected with nails, rock joints, rafted and shell foundations and block or brick-mortar interfaces. Since the primary object is to develop a special element for modeling nailed joint behavior, a technique that has been used for modeling bond behavior in reinforced concrete was selected because of its similarity to nailed joints. This model was first proposed by Ngo and Scordelis in 1967 (21). The linkage element shown as Fig. 3.1. was used to simulate bond-slip behavior between concrete and the reinforcement. The development begins by placing a series of double-node pairs along the interface at the same initial geometric location, one on each side of the interface, and to link them with a pair of nonlinear springs normal and tangent to the contact surface. The bond of reinforcement to concrete can be modeled by the load-slip relationship observed in a "pull-out" test by selecting the nonlinear stiffness characteristics of the tangential bond springs.

One of the principal advantages of this model is the simplicity of the bond stress calculations and the straightforward
manner in which the slippage is introduced. The other advantage is that the linkage element has no physical dimensions at all, only its mechanical properties are of importance, and the element can be placed anywhere along the contact surface without disturbing the structure geometry. The difficulty is that the bond links are a compatibility model and thus require stiffness information. Typical examples are shown in Fig. 3.2.

To incorporate the linkage element into the NONSAP program, the element stiffness matrix of the original truss element in NONSAP can be modified. Consider the mth member of a space truss as shown in Fig. 3.3a. The possible displacements at the i and j ends are indicated in Fig. 3.3a. for element local coordinates $X_i'$ and in Fig. 3.3b. for the global coordinates $X_i$. The member stiffness matrix with respect to the local coordinates is computed in the usual manner as

\[
[SM_m] = \frac{E_m A_m}{L_m} \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(3.1)

where $\frac{E_m A_m}{L_m}$ is the stiffness of element m.

In order to transform the member stiffness matrix from local
Fig. 3.1. Linkage Element

Fig. 3.2. (a) Typical Model of Reinforced Concrete Member
(b) Detail of Bond Linkage
coordinates to global coordinates, the transformation matrix $R$ for a space truss member is required. This matrix is given by the transformation

$$[R] = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} \quad (3.2)$$

where $a_{ij} = \cos(x'_i, x'_j)$. The total transformation matrix for the $m$th member is found to be

$$[RT_m] = \begin{bmatrix}
R & 0 \\
0 & R
\end{bmatrix} \quad (3.3)$$

The stiffness matrix for global coordinates Fig. 3.3b. can then be obtained from:

$$[K_m] = [RT_m]^T [S_m] [RT_m]$$
Fig. 3.3. Space Truss Element
where

\[
C_x = \frac{x_j - x_i}{L} \quad C_y = \frac{y_j - y_i}{L} \quad C_z = \frac{z_j - z_i}{L}
\]

and

\[
L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}
\]

For a plane truss in the Y - Z plane, substituting \( C_x = 0 \) into Eq. 3.4 gives
As pointed out before, the linkage element consists of two nonlinear springs parallel to the set of orthogonal axes H and V in Fig. 3.1. Substituting the spring stiffness $K_h$ and $K_V$ for the truss element stiffness $\frac{E_m A_m}{L_m}$ and superposing the two truss element stiffness matrices, the stiffness matrix for the linkage element becomes

\[
[K_m] = \frac{E_m A_m}{L_m} \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_Y^2 & C_Z C_Y & 0 & -C_Y^2 & -C_Z C_Y \\
0 & C_Y C_Z & C_Z^2 & 0 & -C_Y C_Z & -C_Z^2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -C_Y^2 & -C_Z C_Y & 0 & C_Y^2 & C_Z C_Y \\
0 & -C_Y C_Z & -C_Z^2 & 0 & C_Y C_Z & C_Z^2
\end{bmatrix}
\]

(3.5)

\[
[K]_{\text{Linkage}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_h C^2 + K_V S^2 & K_h S^2 - K_V C^2 & 0 & -K_h C^2 - K_V S^2 & -K_h S^2 + K_V C^2 \\
0 & K_h S^2 - K_V S^2 & K_h S^2 + K_V C^2 & 0 & -K_h S^2 + K_V S^2 & -K_h C^2 - K_V C^2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -K_h C^2 - K_V S^2 & -K_h S^2 + K_V S^2 & 0 & K_h C^2 + K_V S^2 & K_h S^2 - K_V C^2 \\
0 & -K_h S^2 + K_V S^2 & -K_h S^2 - K_V C^2 & 0 & K_h S^2 - K_V S^2 & K_h C^2 - K_V S^2
\end{bmatrix}
\]

(3.6)
where
\[ C = \cos \theta \]
\[ S = \sin \theta \]
\[ K_h, K_v = \text{the stiffness of the spring in the H and V directions respectively} \]

The above formulas can be used to modify the NONSAP program. The remaining problem is to determine the two spring stiffnesses in the linkage elements. In general, the spring constants \( K_h \) and \( K_v \) are established experimentally. For the layered beam system, \( K_h \) and \( K_v \) correspond to the slip modulus and withdrawal modulus respectively. In this study, it appears reasonable to believe that the vertical relative displacement at the nail locations is very small, thus a very large value of \( K_v \) was arbitrarily assigned to each linkage element and a corresponding load-withdrawal curve was input.

3.3. Finite Element Model of Layered I-Beam and T-Beam System

Figs. 3.4a and b show finite element models for a layered I-beam and a T-beam system. In the model, the flanges and web are modeled as 4 to 8 nodes two-dimensional elements which can be linear isotropic, linear orthotropic, or a two-line non-linear approximation using the von Mises two-dimensional plane stress finite element of NONSAP dependent on the material properties of the components. Since the components of test wall
Fig. 3.4a. Finite Element Model for Composite I-Beam
Fig. 3.4b. Finite Element Model for Composite T-Beam
panels are plywood, wood and gypsum wallboard, linear orthotropic plane stress elements will be used for the layered I-beam system.

Two methods are currently available for modeling the nail connections. The first method uses truss elements originally contained in NONSAP. The nail connections are modeled as truss elements with two truss elements for every nail. The two truss elements are perpendicular to each other, one modeling the load-withdrawal stiffness of the nail and the other parallel to the wood surfaces modeling the load-slip behavior. The second method for modeling the nail connector is by using the linkage element package developed herein. The disadvantage of using truss elements is that additional nodes are required. This greatly increases the degrees of freedom for the system. The linkage elements needs only two nodes. Fig. 3.4c. shows a typical section of finite element mesh with nodal point used in this study.

3.4. Other Applications of Linkage Element

If the load-slip and load-withdrawal relations of the contact surface behavior are appropriately applied, the linkage element developed in this study can be used to solve two-dimensional (2-D) friction gap/closure interface problems.

Consider two bodies A and B as shown in Fig. 3.5. The bodies are divided into finite elements by the standard methods except
Fig. 3.4c. Typical Section of Finite Element Mesh with Nodal Point
for the contact surface where pairs of nodes for linkage elements are chosen as shown in Fig. 3.5. The springs of the linkage element connect the parallel surface of substructure A and substructure B at node i and node j respectively (Fig. 3.6.). The relative displacements parallel to the contact surface and normal to the contact surface between node i and j are denoted as $\Delta V(t)$ and $\Delta U(t)$. In general, for contact surface problems, the element may resist compressive loads only in the normal direction and shear in the tangent direction.

When the normal force $F_n$ is negative (compression), the interface remains in contact and the two surfaces do not separate; thus, $\Delta V = 0$. The normal displacement and force relation can be simulated by assuming a large stiffness $K_n (K_n = \infty)$ for the fictitious springs.

When the normal force $F_n$ is positive (tension), the contact is broken. The change in displacement in the normal direction is

$$\Delta V = V_j - V_i + \text{GAP} \quad (3.7)$$

where

GAP is a gap opening (if GAP > 0) or preload (if GAP < 0).

The slope of the load deflection curve shown as Fig. 3.7a can be used to model the spring stiffness $K_n$ (19). Fig. 3.7b and 3.7c show the load-deflection curve for positive and negative gaps respectively.
Fig. 3.5. Finite Element Model for 2-D Contact Problems

Fig. 3.6. Relative Displacement at Interface
Fig. 3.7a. Load-Deflection Curve for $K_n$

$$-F_n = K_n V$$

Fig. 3.7b. Load-Deflection Curve Positive GAP (Opening)

Fig. 3.7c. Load-Deflection Curve Negative GAP (Preload)
In the tangential direction, the load deflection behavior is governed by the friction law of the interface. For example, if one assumes Coulomb's friction law for the interface, the maximum bond stress is

\[(F_t)_{\text{max}} = \text{Coh} - \mu |F_n|\]

in which Coh and \(\mu\) are constants. In the absence of cohesion, force \(F_t\) is limited by the friction force, i.e. \((F_t)_{\text{max}} = \mu |F_n|\). When the absolute value of tangent force \(F_t\) is less than \((F_t)_{\text{max}}\), slippage does not occur and the interface responds as a linear spring. If the tangent force \(F_t\) exceeds the slip condition, the slip nodes move relative to each other in the tangential direction while still maintaining compatibility of displacement for the normal direction.

The relative displacement in the tangent direction is

\[\Delta U = U_{Lj} - U_{Li} - U_{\text{slip}}\]  \hspace{1cm} (3.8)

where \(U_{\text{slip}}\) is the accumulated amount of sliding in the tangential direction. The load-deflection curve for \(K_t\) is shown as Fig. 3.8.
3.5. Nonlinear Analysis

The main source of the nonlinear response in a nailed wood composite system is the nonlinear behavior of joints and stud. The curvilinear relation between lateral load and relative slip has been previously studied (12, 13). Recently, Polensek (23, 24) has developed what is referred to hereafter as a "Rational Design Procedure" for wood stud walls. The nonlinear behavior was accounted for by linear step-by-step analysis. This approach calls for a multilinear representation of material stiffness, each associated with a linear modulus. A large number of lines representing the curve results in increased
accuracy but also in increased computational work and cost. Extensive physical testing and rational design parameter study has shown that a three-line approximation offers the optimum solution in terms of accuracy and computational costs. Thus, the load-slip traces were divided into three linear sections $K_1$, $K_2$ and $K_3$ as shown in Fig. 3.9.

![Fig. 3.9. Secants Representing a Typical Load-Slip Curve](image)

The slopes of these line segments are the slip moduli that define the inelastic joint stiffness in predetermined slip intervals. A recent study (18) indicates that the slip intervals of zero to 0.0013, 0.0013 to 0.026 and 0.026 to 0.1 inch achieve the best trilinear representation of the curve for joints constructed with 6d box nails.
The stepwise linear solution used in this study is started by describing the load-slip curve by a series of discrete points. The initial modulus is the slope corresponding to the first two points. At the end of each interval, the slip modulus for every linkage element along the beam length is re-evaluated using the slip corresponding to the load-slip curve from the preceding interval. Iteration can be performed at any time during the solution. A new joint stiffness value is assigned to the corresponding linkage element and a new stiffness matrix is formed. A flow diagram is shown in Fig. 3.10.

To consider nonlinear characteristic of the stud, the stud can be modeled by the von Mises two-dimensional plane stress finite elements in which the nonlinear stress-strain relationship is presented by two straight lines shown as Fig. 3.11. Zakic (30) presented a stress analysis for the bending of beams with a rectangular cross section using an inelastic theory. In his study, the analytical procedure is based on the assumption that the parabola is the mathematical approximation of the compression part and the straight line of the tension part of stress-strain line, was satisfactorily proven by the tests of poplar wood. He also concluded that the ratio between the ultimate inelastic and elastic bending moments obtained by the appropriate theories is 1.76 for this species. In this study the stud was materially linear in the model as the experimental data was kept in the linear range.
Fig. 3.10. Flow Diagram for Linear Step-by-Step Analysis Including Iteration
Fig. 3.11. Typical Stress-Strain Curve for von Mises Elastic-Plastic Material

\( E_t \): Tangent modulus of deformation

- \( \sigma_y \)
- \( \sigma_p \)

\( E \)
3.6. Solution Technique for Nonlinear Dynamic Analysis

The basic problem in a general nonlinear analysis is to find the state of equilibrium of a body corresponding to the applied loads. Assuming that the externally applied loads are described as a function of time, the governing equilibrium equation is

$$M(t) \ddot{U} + C_D(t) \dot{U} + K(t) U = R(t)$$  \hspace{1cm} (3.9)

where $M(t)$, $C_D(t)$ and $K(t)$ are the mass, damping and stiffness matrices at time $t$; $R(t)$ is the external load vector; and $U$, $\dot{U}$ and $\ddot{U}$ are the displacement, velocity and acceleration vectors of the finite element assemblage.

Currently only the stiffness matrix and load vector are formulated as functions of time, but the numerical solution technique can allow for the mass and damping matrices to be functions of time also. The damping matrix used in the program NONSAP has two types of damping available; Rayleigh damping and concentrated nodal dampers. Rayleigh damping was developed to approximate the overall energy dissipation of the structure, the damping matrix for this type is formed from contributions of the stiffness and mass matrices. Concentrated nodal dampers can be specified for each degree of freedom at each node point. Detailed discussions can be found in Reference 11.

An effective method for the solution of Eq. 3.9 is direct
numerical integration where a step-by-step solution scheme is employed. In the step-by-step solution procedure the load is applied in several small steps ($\Delta t$), in this study ten steps were used, and the structure is assumed to respond linearly within each step. The basic idea for direct integration is that instead of trying to satisfy Eq. 3.9 at any time $t$, the intent is to satisfy Eq. 3.9 only at discrete time intervals $\Delta t$ apart and that the variation of displacements, velocities and accelerations within each time interval $\Delta t$ is assumed. Two of the most commonly used direct integration techniques, the Wilson $\theta$-method and the Newmark method are available in program NONSAP. The Wilson $\theta$-method is essentially an extension of the linear acceleration method, the acceleration is assumed to be linear from time $t$ to time $t+\theta \Delta t$, where $\theta \geq 1.4$, i.e.

$$t+\tau \dot{U} = t \ddot{U} + \frac{\tau}{\theta \Delta t} (t+\theta \Delta t \dddot{U} - t \dddot{U}) \quad 0 \leq \tau \leq \Delta t \quad (3.10)$$

In this expression it is assumed that the solutions of Eq. 3.9 at times $0, \Delta t, 2\Delta t \ldots, t$ are known and that the solution at time $t + \Delta t$ is required next. With the Wilson $\theta$ operator, the equations of motion reduce to the following form

$$\ddot{U}_{t+\theta \Delta t} = t+\theta \Delta t \dddot{R} \quad (3.11)$$
where $\hat{K}$ is the effective stiffness matrix given by

$$\hat{K} = a_0 M + a_1 C_D + K_t \tag{3.12}$$

in which

$K_t$ is the tangent stiffness matrix at time $t$

and

$$T + \theta \Delta t \hat{R} = \text{the effective load at time } t + \theta \Delta t$$

$$= t_R + \theta (t + \Delta t R - t_R) + M(a_0 t_U + a_2 t\dot{U} + 2t\ddot{U})$$

$$+ C_D(a_1 t_U + 2t\dot{U} + a_3 t\ddot{U}) \tag{3.13}$$

in which

$$a_0 = \frac{6}{(\theta \Delta t)^2} \quad a_1 = \frac{3}{\theta \Delta t} \tag{3.14}$$

$$a_2 = 2a_1 \quad a_3 = \frac{\theta \Delta t}{2}$$

After solving Eq. 3.11, $T + \Delta t U$, $T + \Delta t \dot{U}$, and $T + \Delta t \ddot{U}$ can be calculated as follows

$$T + \Delta t \ddot{U} = a_4 (T + \theta \Delta t U - t_U) + a_5 \dot{U} + a_6 t\ddot{U}$$

$$T + \Delta t \dot{U} = t\dot{U} + a_7 (T + \Delta t \ddot{U} + t\ddot{U})$$

$$T + \Delta t U = t_U + \Delta t \dot{U} + a_8 (T + \Delta t \ddot{U} + 2t\ddot{U}) \tag{3.15}$$
In the Newmark method, it is assumed that the increments in velocity and acceleration are related to the increment in displacement and the state of motion at time $t$. The following assumptions are used:

\[
\begin{align*}
    t^+\Delta t \dot{U} &= \dot{U} + [(1 - \delta)\ddot{U} + \delta t^+\Delta \ddot{U}] \Delta t \\
    t^+\Delta t \ddot{U} &= \ddot{U} + \dot{U} \Delta t + [(1/2 - \alpha)\dddot{U} + \alpha t^+\Delta \dddot{U}] \Delta t^2
\end{align*}
\]

(3.16)

where $\alpha$ and $\delta$ are parameters that can be determined to obtain integration accuracy and stability. With the Newmark operator, the equations of motion reduce to the following form:

\[
\begin{align*}
    \hat{K} t^+\Delta t \dot{U} &= \hat{t} \Delta t \hat{R} \\
    \hat{K} &= \hat{t} \hat{K} + a_0 \hat{M} + a_1 \hat{C}_D
\end{align*}
\]

(3.17)  (3.18)

and

\[
\begin{align*}
    t^+\Delta \hat{R} &= \hat{t} \Delta \hat{R} + M(a_0 t^+U + a_2 t^+\dot{U} + a_3 t^+\ddot{U}) \\
    &+ C_D(a_1 t^+U + a_4 t^+\dot{U} + a_5 t^+\ddot{U})
\end{align*}
\]

(3.19)

In which

\[
\begin{align*}
    a_4 &= \frac{a_0}{\delta} \\
    a_5 &= \frac{-a_2}{\delta} \\
    a_6 &= 1 - \frac{3}{\delta} \\
    a_7 &= \frac{\Delta t}{2} \\
    a_8 &= \frac{\Delta t^2}{6}
\end{align*}
\]
A detailed derivation of Eqs. 3.9 - 3.19 can be found elsewhere (5). As pointed out in Reference 5, the solution of Eq. 3.9 by the direct integrate method, in general, results in approximate displacements $U$. To improve the solution accuracy, equilibrium iterations may or may not be necessary in each time step depending on the degree of non-linearity. A widely used iteration procedure is the modified Newton-Raphson iteration. In this method the discrete incremental equations of motion are

$$t+\Delta t_R - t+\Delta t_F - M^{t+\Delta t}U - C_D^{t+\Delta t}U = 0$$  \hspace{1cm} (3.20)

where

- $t+\Delta t_R$ = the externally applied nodal loads
- $t+\Delta t_F$ = the vector of nodal point forces that are equivalent to the element stress

If we define a vector function $f$ such that

$$f(U, R) = R - [M\ddot{U} + C_D\dot{U} + F]$$  \hspace{1cm} (3.21)

then the displacement $t+\Delta t U$ is the solution of the nonlinear equations $f(t+\Delta t_U, t+\Delta t_R) = 0$. Computational formulae for step-by-step iterative procedures can be obtained by considering a first-order Taylor series expansion of $f(t+\Delta t_U, t+\Delta t_R)$ about the known displacements $U$ and assuming
\[ t^{+ \Delta t} U = t U + \Delta U \quad (3.22) \]

then

\[ t^{+ \Delta t} F = t_F + t_{K\Delta U} \quad (3.23) \]

Substituting Eq. 3.23 into Eq. 3.20, the desired incremental form of the equations of motion is obtained:

\[ M t^{+ \Delta t} \ddot{U} + C_D t^{+ \Delta t} \dot{U} + t_{K\Delta U} = t^{+ \Delta t} R - t_F \quad (3.24) \]

Combined with the Wilson method or Newmark method, Eq. 3.24 can be simplified as

\[ t^{\hat{K}\Delta U} = t^{+ \hat{T}_R} \quad (3.25) \]

in which

\[ t^{\hat{K}} = K + a_0 M + a_1 C_D \quad (3.26) \]

\[ t^{+ \hat{T}_R} = t_R + \theta(t^{+ \Delta t} R - t_R) + M(a_2 t \ddot{U} + a_3 t \dot{U}) + C_D(a_4 t \dot{U} + a_5 t U) - t_F \quad (3.27) \]
The solution of Eq. 3.25 yields the incremental displacement vector $\Delta U$ for the time increment $\tau$. Then the displacement, velocity and acceleration are obtained by the direct integration method. The principal advantage of the modified Newton iteration is that the tangent stiffness matrix is not formed and decomposed at every iteration. Hence, this procedure is computationally attractive for structures with a large number of degrees-of-freedom. However, the procedure can be expected to converge more slowly than Newton-Raphson iteration, and the use of a scheme to accelerate convergence may be desirable. An "alpha-constant" acceleration scheme has been found to be reliable for structures with "softening" load-displacement response. Since the program NONSAP does not include an acceleration scheme, it will not be discussed in this study.
IV. VERIFICATION OF COMPUTER MODEL

4.1. T-Beam Section

Verification of the developed computer model was based in part on comparisons between values obtained from the computer output of this study and the results of a theoretical T-Beam analysis by Vanderbilt; Goodman and Criswell (26). The dimensions of the T-Beam used in the theoretical study were a 16" x 3/4" flange and a 1.5" x 7.25" web as shown as Fig. 4.1a. The moduli of elasticity for the flange and the web were $5 \times 10^5$ psi and $2 \times 10^6$ psi, respectively. Three typical slip moduli ($K = 10,000,000 \text{ lb/in}$, $K = 30,000 \text{ lb/in}$, $K = \text{zero}$) were used to model the fully composite action case, the incomplete composite action case and the no interaction between two layers case. Data used in their study were used as input data to the computer model. Slip moduli are represented by the springs parallel to the contact surface between flange and web. A comparison between the results of the computer model developed herein with the results of the theoretical study by Vanderbilt, etc., are shown in Figs. 4.1 to 4.4. The agreement seems to be very good for both deflection and shear force. This good comparison offers assurance that the computer model developed in this study can be used with confidence.
1000 lb
60"
60"

1

60"

2

16".

0.75"

7.25"

E_1 = 5 \times 10^5 \text{ psi} \quad E_2 = 2 \times 10^6 \text{ psi} \quad S = 8 \text{ in}

(a) Beam and Loading

(b) Deflections, in \times 10^{-1}

(c) Variation in Shear Flow

Fig. 4.1. Results of T-Beam Analysis (26)
Fig. 4.2. Deflection of Composite T-Beam under 1000 lb Concentrated Load at Midspan
Fig. 4.3. Variation in Shear Flow of Composite T-Beam under 1000 lb Concentrated Load at Midspan
Fig. 4.4. Variation of Slip along the Interface of a Composite T-Beam
4.2. I-Beam Section

4.2.1 Development of Basic Material and Connector Properties

One of the two most important material properties influencing the composite behavior of a layered I-Beam system of given dimensions is the MOE (modulus of elasticity) values for the material in each layer. Since wood is a natural material and a tree is subject to numerous, constantly changing influences (such as moisture, soil conditions and growing space), the MOE values of wood may vary considerably even in the same piece of lumber. Thus MOE values are usually evaluated by testing in a particular manner. In the past, a large number of studies have been conducted to develop methods for evaluating the MOE values of the wood. The MOE values of wood studs and sheathings used in this study were determined at the Forestry Research Laboratory at Oregon State University according to ASTM specification (1). The static bending concept was used in the tests with the studs and sheathing panel acting as a simply supported beam. Deflections are produced by a concentrated load or a line load at midspan for studs and sheathing respectively and recorded using a LVDT.

In general, wood can be described as an orthotropic material. The compliance form of the Hooke's law for wood is
\[
\begin{pmatrix}
\varepsilon_{LL} \\
\varepsilon_{RR} \\
\varepsilon_{TT} \\
2\varepsilon_{RT} \\
2\varepsilon_{LT} \\
2\varepsilon_{LR}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{E_L} & -\frac{\nu_{RL}}{E_R} & -\frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\
-\frac{\nu_{LR}}{E_L} & \frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 & 0 & 0 \\
-\frac{\nu_{LT}}{E_L} & -\frac{\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{RT}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LR}}
\end{pmatrix}
\begin{pmatrix}
\tau_{LL} \\
\tau_{RR} \\
\tau_{TT} \\
\tau_{RT} \\
\tau_{LT} \\
\tau_{LR}
\end{pmatrix}
\]

where

\( L, R, T \) = the three mutually perpendicular symmetric axes of wood assumed as the longitudinal, radial and tangential directions which are parallel to the fiber (grain), normal to the growth rings and perpendicular to the grain but tangent to the growth rings, respectively.

\( \varepsilon_{ij} \) = strain tensor

\( \tau_{ij} \) = stress tensor

\( E_i \) = Young's modulus along the \( i \) axes of symmetry.

\( G_{ij} \) = shear modulus in the \( ij \) planes.
\[ \nu_{ij} = \text{Poisson's ratio (for a uniform normal stress in the } i \text{ axes direction)} \]
\[ = \frac{\text{strain on the } j \text{ axes}}{\text{strain on the } i \text{ axes}} \]

Twelve elastic parameters are used to relate the six stresses and strains. However, due to the strain energy of symmetry consideration, not all 12 elastic parameters are independent, and the number of independent elastic constants are reduced to 9. These are \( E_L, E_R, E_T, G_{LR}, G_{LT}, G_{RT}, \nu_{LR}, \nu_{LT}, \nu_{RL} \). The other three may be calculated from the following relationships:

\[
\frac{\nu_{LR}}{E_L} = \frac{\nu_{RL}}{E_R}
\]
\[
\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T} \quad (4.2)
\]
\[
\frac{\nu_{RT}}{E_R} = \frac{\nu_{TR}}{E_T}
\]

Inverting the compliance matrix of Eq. 4.1 provides the stiffness form of Hooke's law (7) i.e.
\[
\begin{pmatrix}
\tau_{LL} \\
\tau_{RR} \\
\tau_{TT} \\
\tau_{RT} \\
\tau_{LT} \\
\tau_{LR}
\end{pmatrix} =
\begin{pmatrix}
C_{LL} & C_{LR} & C_{LT} & 0 & 0 & 0 \\
C_{RL} & C_{RR} & C_{RT} & 0 & 0 & 0 \\
C_{TL} & C_{TR} & C_{TT} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{RTRT} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{LTLT} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{LRLR}
\end{pmatrix}
\begin{pmatrix}
e_{LL} \\
e_{RR} \\
e_{TT} \\
2e_{RT} \\
2e_{LT} \\
2e_{LR}
\end{pmatrix}
\]

(4.3)

where

\[
C_{LL} = \frac{E_L (1 - \nu_{TR} \nu_{RT})}{1 - 2\nu_{LR} \nu_{RT} \nu_{TL} - \nu_{LT} \nu_{TL} - \nu_{LR} \nu_{RL} - \nu_{RT} \nu_{TR}}
\]

\[
C_{LR} = C_{RL} = \frac{E_L (\nu_{RL} + \nu_{RT} \nu_{TL})}{1 - 2\nu_{LR} \nu_{RT} \nu_{TL} - \nu_{LT} \nu_{TL} - \nu_{LR} \nu_{RL} - \nu_{RT} \nu_{TR}}
\]

\[
C_{LT} = C_{TL} = \frac{E_L (\nu_{TL} + \nu_{RL} \nu_{TR})}{1 - 2\nu_{LR} \nu_{RT} \nu_{TL} - \nu_{LT} \nu_{TL} - \nu_{LR} \nu_{RL} - \nu_{RT} \nu_{TR}}
\]

\[
C_{RR} = \frac{E_R (1 - \nu_{LT} \nu_{TL})}{1 - 2\nu_{LR} \nu_{RT} \nu_{TL} - \nu_{LT} \nu_{TL} - \nu_{LR} \nu_{RL} - \nu_{RT} \nu_{TR}}
\]

\[
C_{TR} = C_{RT} = \frac{E_T (\nu_{RT} + \nu_{RL} \nu_{LT})}{1 - 2\nu_{LR} \nu_{RT} \nu_{TL} - \nu_{LT} \nu_{TL} - \nu_{LR} \nu_{RL} - \nu_{RT} \nu_{TR}}
\]
Consider the 2-D finite element as shown in Fig. 4.5, where the inplane orthotropic material axes are "L" and "R". The third orthogonal material direction is "T" and is perpendicular to the plane defined by "L" and "R". The compliance matrix is

\[
\begin{bmatrix}
\frac{1}{E_L} & -\frac{\nu_{LR}}{E_R} & -\frac{\nu_{LT}}{E_T} & 0 \\
-\frac{\nu_{LR}}{E_L} & \frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 \\
-\frac{\nu_{LT}}{E_L} & -\frac{\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 \\
0 & 0 & 0 & \frac{1}{G_{LR}}
\end{bmatrix}
\]

\[ (4.4) \]
(a) Wood Beam in an Orthogonal Coordinate System

(b) Principal In-Plane Material Axes Orientation

Fig. 4.5. 2-D Linear Orthotropic Material Model
Since the stiffness matrix is obtained from the inversion of the compliance matrix, thus requires the following determinant to be positive

\[
\begin{vmatrix}
\frac{1}{E_L} & -\frac{\nu_{RL}}{E_R} & -\frac{\nu_{TL}}{E_T} & 0 \\
\frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 & 0 \\
\text{sym} & \frac{1}{E_T} & 0 & 0 \\
\frac{1}{G_{LR}} & & & \\
\end{vmatrix} > 0 \quad (4.5)
\]

The input data for a 2-D linear orthotropic element which models the stud and sheathings consists of the Young's modulus, Poisson's ratio in three mutually perpendicular symmetric axes and the in-plane shear modulus of each layer. For the stud, consider the case when the long axes of the beam coincides with the long axes of the fiber. Then the modulus of elasticity along the grain direction, \( E_L \), can be obtained from a bending test. Handbook values (29) were used to determine the remaining elastic properties.

For plywood sheathing, due to the orthotropic nature of wood and the orthotropic orientation of the adjacent plies of plywood,
the mechanical properties in the two principal directions of plywood are quite different. The MOE values determined by a bending test based on the moment of inertia of the nominal thicknesses of plywood are valid for bending only. These values are different from those that are valid for use with transformed sections. To determine the effective axial MOE values, the MOE values in the directions of the three mutually perpendicular symmetric axes of the veneer must be determined first.

The MOE value in the face grain direction of the veneer can be determined as follows:

\[ E_L = \frac{E_{\text{Leff}} I_{\text{gross}}}{I_{\text{tr}}} \]  

(4.6)

where

- \( E_{\text{Leff}} \) = the effective modulus of elasticity from bending test
- \( E_L \) = modulus of elasticity in the longitudinal direction of the face veneer
- \( I_{\text{tr}} \) = moment of inertia based on the transformed section
- \( I_{\text{gross}} \) = the moment of inertia of the material based on gross dimension

Handbook values (29) can be used to determine the MOE values in the other two directions of the veneer denoted as \( E_R \) and \( E_T \). Calculations of the elastic properties of plywood are based on
the law of mixtures. Each composite parameter consists of elastic parameters which are weighted by the ratios of veneer thickness to the plywood thickness. For three-layer plywood, the entire set of equations takes the following form (7):

\[
E_L = \frac{1}{t} (2E_L^f t^f + E_L^c t^c)
\]

\[
E_T = \frac{1}{t} (2E_T^f t^f + E_T^c t^c)
\]

\[
E_R = \frac{E_R^f E_R^c t}{2E_R^c t^f + E_R^f t^c}
\]

\[
G_{LT} = \frac{1}{t} (2G_{LT}^f t^f + G_{LT}^c t^c)
\]

\[
G_{LR} = \frac{G_{LR}^f G_{LR}^c t}{2G_{LR}^c t^f + G_{LR}^f t^c}
\]

\[
G_{TR} = \frac{G_{TR}^f G_{TR}^c t}{2G_{TR}^c t^f + G_{TR}^f t^c}
\]

\[
\nu_{LT} = \frac{1}{t} (2\nu_{LT}^f t^f + \nu_{LT}^c t^c)
\]

\[
\nu_{LR} = \frac{1}{t} (2\nu_{LR}^f t^f + \nu_{LR}^c t^c)
\]

\[
\nu_{TR} = \frac{1}{t} (2\nu_{TR}^f t^f + \nu_{TR}^c t^c)
\]
where

\[ E_j^i = \text{MOE value of } j\text{th layer in } i \text{ direction of coordinate} \]

\[ t_f(t^c) = \text{thickness of face (core) veneer} \]

For gypsum wallboard, little information is available. The elastic properties as found during the development of the Rational Design Procedure (23, 24) will be used in this study.

The other important material property influencing the composite behavior of a layered I-beam system is the slip modulus of the connectors. This is even more complicated than the MOE properties. Table 4.1 summarizes the major variables which affect the load-slip behavior of nailed joints. Since numerous earlier studies have been devoted to this area, no further investigation of load-slip behavior is presented in this study. The joint moduli as recommended in the Rational Design Procedure for one-nail joints between Douglas-Fir studs and wall coverings with gypsum and plywood as presented in Table 4.2, will be used. The remaining data which are needed are the values of Poisson's ratio. Forbes (11) has pointed out that some realistic values of the Poisson's ratio do not satisfy Eq. 4.5. He also verified that a change in Poisson's ratio does not substantially change the results. Therefore, reasonable values of Poisson's ratio that satisfy the requirement of Eq. 4.5 can be used.
| Wood Characteristics | Specific gravity  
|                      | Species  
|                      | Physical, chemical and mechanical properties (including creep)  
|                      | Moisture content at fabrication and seasoning after fabrication  
|                      | Defects (including straightness of grain)  
|                      | Dimensions  
| Nail Characteristics  | Diameter  
|                      | Nail head  
|                      | Length  
|                      | Type of nail shank  
|                      | Penetration  
|                      | Surface condition  
|                      | Mechanical Properties  
| Joint Configuration   | Prebored lead holes  
|                      | Nail spacing  
|                      | Number of nails  
|                      | Friction between contacting surfaces  
|                      | Interlayer gap between members  
|                      | Angle of connector to surface  
|                      | Direction of nailing (into radial, tangential or end grain)  
|                      | Type of specimen (two or three members)  
|                      | Type of shear in the connector (single double, or multiple)  
| Loading               | Direction (perpendicular, parallel or inclined to grain)  
|                      | Duration of load  
|                      | Rate of loading  
|                      | Tension, compression or bending  
|                      | Axial component on nail  

Table 4.1. Factors Affecting the Load-Slip Characteristics of a Laterally Loaded Nailed Joint (3)
Table 4.2. Shear Load-Slip Properties for I-Beam Shown in Figure 3.4a
4.2.2 Description of Test Panel and Test Method

In order to verify that the computer model developed herein is valid for the layered I-beam system, results are compared with five panels tested under static loading at the Forest Research Laboratory. Each panel consisted of two 2" x 4" Douglas-Fir studs, 3/8-inch plywood of sheathing grade with C and D faces, and 1/2-inch gypsum board. Thirty studs were used in the series of tests. Every stud was graded according to the deflection in a bending test and the stud weight. Panels were constructed using the same grade of stud to match their stiffnesses with the exception of panel #4 where studs were purposely mismatched for stiffness and weight. The 8-foot direction of the board was parallel to the span direction. The face grain of the plywood was oriented parallel to the grain of the stud, since this is the normal orientation used in a typical wall system. The specimen were connected with 6d box nails between the plywood and studs and with 4d sheetrock nails between the gypsum board and studs. Fig. 4.6 shows the location of the nails on the test panel.

The load was applied through an MTS hydraulic actuator. The actuator was positioned at midheight (POS 0) on the test panel. Seven displacement transducers were used to record the deflections (POS 1 - POS 2) and slip (POS 3 - POS 7) at various position shown as Fig. 4.7 during the test. Load was applied by the head of a testing moving at 1.2 inch per minute up to the maximum 1.5
Fig. 4.6. Nail Location for Test Panel
Fig. 4.7. Load and LVDT Transducers Position
inch deflection recorded, then a reverse load was applied. One cycle of load was applied in each test. The configuration and properties of panels 3 to 5 are shown as Fig. 4.8 - 4.10.

4.2.3 Comparison of Computer Model and Experiment Results

Because panels 1 and 2 were used primarily for equipment checks, only the results of panels 3 to 5 are discussed in this section. Table 4.3 lists the data that was used for input to the computer model. For panels 3 and 5, the two studs are assumed to have the same stiffness, therefore, it was assumed that the load has applied equally to the two studs. Fig. 4.11 to 4.12 and Table 4.4 present comparisons of the experimental and computer model values of deflection at positions 1 and 2. These results show that the solutions from the computer model closely predict the static behavior of the experimental models.

Figs. 4.13 to 4.22 and Table 4.5 present comparisons of the experimental and computer model slip values at various locations along the span.

For panel 4, because the two studs are purposely mismatched, the applied load was assumed distributed according to the effective stiffness of the studs. Unfortunately, this assumption can predict the deflection of the stud #11 which is the stiffer one of the two studs only as shown in Fig. 4.23. Therefore, further investigations are required.
Description of panel 3

Top Flange: 3/8" CD plywood
\[ E_{\text{bending}} = 1.914 \times 10^6 \text{ psi} \]

Web:
- stud #25 2" x 4" Douglas Fir
  \[ E_{\text{bending}} = 2.114 \times 10^6 \text{ psi} \]
- stud #3 2" x 4" Douglas Fir
  \[ E_{\text{bending}} = 2.074 \times 10^6 \text{ psi} \]

Bottom Flange: 1/2" gypsum wallboard
\[ E_{\text{bending}} = 3.479 \times 10^6 \text{ psi} \]

Measured Thickness = 0.380 in

Measured Dimension = 1.499 in x 3.387 in

Measured Dimension = 1.492 in x 3.399 in

Measured Thickness = 0.485 in

Fig. 4.8. Configuration and Properties of Panel 3
Description of panel 4

Top Flange: 3/8" CD plywood
E\text{bending} = 1.888 \times 10^6 \text{ psi}

Web:
- Stud #11 2" x 4" Douglas Fir
  E\text{bending} = 2.856 \times 10^6 \text{ psi}
- Stud #15 2" x 4" Douglas Fir
  E\text{bending} = 1.647 \times 10^6 \text{ psi}

Bottom Flange: 1/2" gypsum wallboard
E\text{bending} = 4.048 \times 10^5 \text{ psi}

Measured Dimension = 1.466 in x 3.348 in
Measured Thickness = 0.405 in
Measured Dimension = 1.515 in x 3.399 in
Measured Thickness = 0.491 in

Fig. 4.9. Configuration and Properties of Panel 4
Description of panel 5

Top Flange: 3/8" CD plywood
\[ E_{\text{bending}} = 2.036 \times 10^6 \text{ psi} \]

Web:
- stud #20 2" x 4" Douglas Fir
\[ E_{\text{bending}} = 2.017 \times 10^6 \text{ psi} \]
- stud #29 2" x 4" Douglas Fir
\[ E_{\text{bending}} = 1.894 \times 10^6 \text{ psi} \]

Bottom Flange: 1/2" gypsum wallboard
\[ E_{\text{bending}} = 4.852 \times 10^5 \text{ psi} \]

Fig. 4.10. Configuration and Properties of Panel 5
<table>
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<tr>
<th>Panel #</th>
<th>Component</th>
<th>$E_x$ (psi)</th>
<th>$E_y$ (psi)</th>
<th>$E_z$ (psi)</th>
<th>$G$ (psi)</th>
<th>$\nu_{yz}$</th>
<th>$\nu_{yx}$</th>
<th>$\nu_{zx}$</th>
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Table 4.3. Material Properties used for Computer Model
Fig. 4.11. Deflection as Function of Load at 1.5 in from Midheight (Panel 3)
Fig. 4.12. Deflection as Function of Load at 1.5 in from Midheight (Panel 5)
Table 4.4. Comparison of Experimental and Theoretical Deflection at 1.5 in from Midheight
Fig. 4.13. Slip vs. Load at Position 3 (Panel 3)
Fig. 4.14. Slip vs. Load at Position 4 (Panel 3)
Fig. 4.15. Slip vs. Load at Position 5 (Panel 3)
Fig. 4.16. Slip vs. Load at Position 6 (Panel 3)
Fig. 4.17. Slip vs. Load at Position 7 (Panel 3)
Fig. 4.18. Slip vs. Load at Position 3 (Panel 5)
Fig. 4.19. Slip vs. Load at Position 4 (Panel 5)
Fig. 4.20. Slip vs. Load at Position 5 (Panel 5)
Fig. 4.21. Slip vs. Load at Position 6 (Panel 5)
Fig. 4.22. Slip vs. Load at Position 7 (Panel 5)
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<td>Experiment in x 10^{-1}</td>
<td>Difference (%)</td>
<td>Theoretical in x 10^{-1}</td>
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Table 4.5. Comparison of Experimental and Theoretical Slip at Position 3
Fig. 4.23. Deflection as Function of Load at 1.5 in from Midheight (Panel 4)
V. DISCUSSION AND CONCLUSION

From the results presented for the finite element analyses and the comparison to test results, it is evident that the linkage element developed herein closely predicts the deflections and the load-slip behavior for the T-beam and I-beam systems.

Current research being conducted by Polensek and Laursen involves the dynamic response of low rise wood-framed structures. One of the aspects of the research is to develop a computer program with different finite elements to model various components. This paper has developed a finite element that can be used to model connectors used in various arrangements. With appropriate load-slip relations, the element developed in this study should have the capability to model other contact surface problems.
BIBLIOGRAPHY


