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In this paper a table of oriented alternating links with less than 10 crossings is created. Only one representative of the group consisting of the link, its mirror image, its complete reversal and the mirror image of the complete reversal is given. The notation used to encode the links is combinatorial and was based on a similar construction of M. Thistlethwaite for knots. We introduce the code and justify it. Moreover, the algorithms needed to test if the links are prime, non-split and admissable are described.

The main tool used to distinguish different links are the generalized polynomial invariants of links. Therefore a survey of their properties is given. The paper is concluded with two tables: A list of all possible oriented projections with at most 9 crossings of links (up to isotopy in S^2) and a second table that lists all oriented alternating links together with their P-, F-, V- and ∇-polynomials.

A Survey of Combinatorial Link Invariants and Their Application to Link Tabulations

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Table of Contents

I. Introduction	1
I.1. Motivation	1
I.2. Definitions	3
II. Link Projections	7
II.1. Notation	7
II.2. Standard sequences and prime projections	11
II.3. Realizable sequences	16
II.4. From Projections to Diagrams	22
III. Combinatorial Link Invariants	25
III.1. Numerical Link Invariants	25
III.2. Polynomial Link Invariants	28
III.3. Results for Alternating Links	49
Bibliography	51
Appendix	54
How to Read the Tables	54
Table of Projections	57
Table of Invariants	90

List of Figures

1. Example for the sequence 8
2. Checkerboard coloring at a crossing 9
3. Circular chain 12
4. Kink and nugatory crossing 12
5. Obstruction circuit 19
6. Continuation of the obstruction circuit 20
7. Rule for the sign of a new crossing 20
8. Link 6 12 | 2 10 14 4 16 8 21
9. Reidemeister moves 26
10. L_+, L_- and L_0 28
11. P-polynomial for the unlink 30
12. Resolution tree 31
13. Two links with homeomorphic complements 32
14. Conway knot and Kinoshita-Terasaka knot 33
15. P-polynomial of a split link 36
16. L_+, L_-, L_0 and L_∞ 38
17. L_+, L_-, L_0 and L_∞ if L_0 is a fission 39
18. L_+, L_-, L_0 and L_∞ if L_0 is a fusion 40

A Survey of Combinatorial Link Invariants and Their Application to Link Tabulations

I. Introduction

I.1. Motivation

Not only sailors are interested in knots. For mathematicians knot and link theory is the problem of placing one or more copies of a circle in a three dimensional space. This has been researched for more than one hundred years. The interactions with other fields of mathematics are rich and recent applications to chemistry and even theoretical physics have brought the theory into the limelight. This increased interest has also been caused by a discovery in 1984 by Vaughan Jones [Jo] of a new polynomial invariant of links. Since then, several other polynomial invariants have been found via similar methods. These polynomials can be computed in a purely combinatorial way and therefore be easily adapted to computers and applied to large numbers of data.

In this paper we create a table of all oriented alternating links with less than 10 crossings. Efforts to list all links or knots up to a given number of crossings started in the nineteenth century. After years of work Tait [Ta] and Little [Lit] published lists of certain classes of knots up to 11 crossings. The use of computers can increase this number significantly. But an appropriate notation to encode the links must first be found. Conway [Co] partitions every link into basic geometric structures and lists those. This enabled him to get a complete (one error) table of knots and links with 10 or less crossings. On the other hand Dowker and Thistlethwaite [D-T] describe knots combinatorially by sequences of numbers. Using this description, Thistlethwaite compiled a table of all knots with 13 crossings or less. This is the largest table at the moment.

Unlike most currently used tables which deal only with unoriented knots and links, our table will list oriented links. Chapter 2 describes the procedure of creating the table. The notation used to encode the projections of the links is explained and justified and the algorithms that perform necessary combinatorial tests on the code are introduced. Programs following these algorithms were written in Lightspeed Pascal and implemented on a Macintosh SE computer. This provided a complete list of all link projections having less than 10 crossings which was used to produce a list of all alternating diagrams also with less than 10 crossings. In order to remove repetitions from this list various polynomial and numerical invariants were computed for each link on the list. Programs to compute the polynomial invariants were written by Jim Hoste. A brief survey of these invariants is given in Chapter 3. Finally, the table of links and associated invariants is given in the Appendix.

I.2. Definitions

Throughout this paper S^i will denote the unit sphere in $\mathbb{R}^{i+1}$, $S^i = \{(x_1, x_2, \ldots, x_{i+1}) | \|(x_1, x_2, \ldots, x_{i+1})\| = 1\}$. Sometimes it will be helpful to consider the equivalent description $S^i \approx \mathbb{R}^i \cup \{\infty\}$. We shall work entirely in the smooth category.

Definition 1.1. A *link* L with k *components* is the image of a smooth embedding of k disjoint copies of S^1 into S^3. A link of one component is called a *knot*.

Definition 1.2. A link L is *trivial* or an *unlink* if it is the boundary of k disjoint embedded discs. We denote the unlink with k components by U_k.

Two knotted strings can be the same even if they do not look alike. All transformations done without cutting the strings are allowed. In mathematical terms we consider an equivalence relation of links:

Definition 1.3. Two links L_1 and L_2 are *ambient isotopic* if there is an isotopy h_t: $S^3 \to S^3$ such that $h_0 = \mathrm{id}$ and $h_1(L_1) = L_2$, i.e. a map h: $S^3 \times [0, 1] \to S^3$ with $h(L_1, 0) = L_1$ and $h(L_1, 1) = L_2$ and such that for all t, h: $S^3 \times \{t\} \to S^3$ is a diffeomorphism.

From now on by a link L we usually mean its equivalence class under ambient isotopy.

Definition 1.4. An *oriented link* is a link with an orientation assigned to each component. Two oriented links are ambient isotopic if there is an ambient isotopy of S^3 that transforms one to the other and preserves the orientations of the components.

Definition 1.5. A *projection* P of a link L is the image of L under an orthogonal projection $\pi\colon S^3 \to \mathbb{R}^2$. A projection π is called *regular* if for all $x \in P$ $\pi^{-1}(x)$ is either one or two points and if it is a double point, then the projections of the two strands meet transversally at x. If the lower and upper strands are marked at each double point of a projection then we call it a *diagram*. Usually the lower strands are broken at these points.

In the case of oriented links the orientations of the components are indicated by arrowheads on the strands. We can then define the *sign* $\varepsilon(c)$ of each crossing c of the diagram D as follows:

$$\varepsilon(c) = \begin{cases} +1 & \text{if c appears as} \\ -1 & \text{if c appears as} \end{cases}$$

If $\varepsilon(c) = +1$ the crossing is also called positive or right-handed, otherwise negative or left-handed.

Definition 1.6. If L is a link in S^3, the *mirror image* L^* of L is the image of L after reflection through a plane. If L is oriented, the *complete reversal* ρL of L is the result of reversing all orientations of L. A link that is ambient isotopic to its mirror image is called *amphicheiral*. One that is ambient isotopic to its complete reversal is called *invertible*.

A diagram D^* of L^* can be obtained from a diagram D of L by changing all undercrossings to overcrossings and vice versa. Note that L, ρL, L^* and $(\rho L)^* = \rho L^*$ could represent four different links, two pairs of links or a single link.

A special class of links are the alternating links.

Definition 1.7. A link L is *alternating* if there exists an alternating diagram for L, i.e. a diagram D where an undercrossing always follows an overcrossing and vice versa in the course of traversing every component.

Note that the existence of only one alternating diagram of a link is required for it to be alternating. Alternating links have been better explored than most other classes of links. Recently some properties of alternating links have been proven that greatly simplify the generation of the tables in this paper. These properties are described in Chapter III.3.

Definition 1.8. Let L be a link in S^3. If there is an embedded 2-sphere S in S^3 which meets L transversally in two points P and Q, then L is called a *connected sum* of two links L_1 and L_2 obtained as follows. Let $N(S) \cong S \times [-1, 1]$ be a regular neighborhood of S such that $N(S) \cap L = (P \times [-1, 1]) \cup (Q \times [-1, 1])$. Then L_1 and L_2 are the result of replacing $N(S) \cap L$ with $\alpha \times \{-1\}$ and $\alpha \times \{1\}$, where α is a path in S from P to Q. We write $L = L_1 \# L_2$. If L is not the connected sum of two nontrivial links then L is *prime*.

Given two links L_1 and L_2 we can clearly form their connected sum. If additionally L_1 and L_2 are oriented we require that L be oriented and induce the correct orientations on L_1 and L_2. While the connected sum of knots is well defined (i.e. independent of any choices that have to be made), connecting two links can give different links depending on the components that are being connected.

Definition 1.9. A link L is *split* if there is a 2-sphere S that separates L into two non-empty sublinks.

II. Link Projections

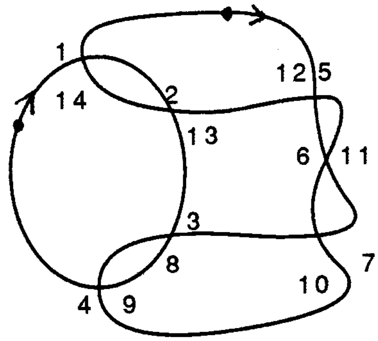
II.1. Notation

In order to generate a table of links we start by listing all possible projections of links, which are then used to produce the diagrams. A notation to encode projections of knots has been described by Thistlethwaite and Dowker in [D-T] and has been used for the generation of knot tables up to 13 crossings [Th 1]. We extend their notation for links as follows.

Given a regular projection P of an unoriented link L we will define a function a: $\{1, 2, \ldots, 2n\} \rightarrow \{1, 2, \ldots, 2n\}$, where n is the number of double points of P. To do so, choose an ordering of the components of L and select a basepoint and an orientation for each component. Beginning at the base point of the first component assign consecutive integers, starting with 1, to the double points that one passes while traversing the component in the chosen direction. After completion of one component continue at the basepoint of the next component with the next integer. This procedure assigns two integers i and j to each crossing point of the projection. Let $a(i) = j$ and $a(j) = i$.

The link projection can then be encoded by the sequence of the images $a(1), a(2), \ldots, a(2n)$ together with marks - we will use '|' - between the image of the last number of one component and the image of the first number of the next component. The following example should suffice to illustrate the procedure.

Example:



i:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a(i):	14	13	8	9	12	11	10	3	4	7	6	5	2	1

Figure 1. Example for the sequence

While this procedure automatically produces a parity-reversing involution (images of even numbers are odd and vice versa) in the case of knots, this is not necessarily true for links. However, the following theorem ensures that an enumeration with that property is always possible.

Theorem 2.1. For every projection P of a link L and every choice of orientations and ordering of the components, there exists a choice of basepoints such that the involution a , defined as above, is parity-reversing.

Proof: Begin by coloring the complement of P in a checkerboard fashion, i.e. color each region of $\mathbb{R}^2 \setminus P$ either black or white such that every arc of the projection separates regions of opposite colors. Do this in either of the two possible ways. Now choose a basepoint for each component on an arc which has the black region on its right with respect to the given orientation.

<u>Claim:</u> The labelling with this choice of basepoints has the parity-reversing property.

Assign to each arc of the projection either a '0', if the black region is on the left, or a '1', if the black region is on its right. On every component the arcs are labelled alternately 0 and 1. Also the basepoint always lies on an arc labelled 1. Therefore the integers which label each crossing reduce modulo 2 to the labels of the arcs leading into the crossing. But the black and white regions can surround a crossing only in two ways, as shown below.

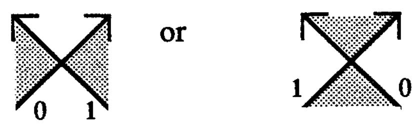


Figure 2. Checkerboard coloring at a crossing

In both cases the labels of the edges entering a crossing have opposite parity and so the integers labelling the crossing must have opposite parity too. □

The sequence a(1), a(2),..., a(2n) of any parity reversing involution a of the set {1, 2,..., 2n} can be recovered from the subsequence of even numbers a(1), a(3),..., a(2n-1), which we will sometimes write as a_1, $a_3, \ldots, a_{2n-1}$. Clearly to every component will correspond an even number of numbers. Therefore the bars, which indicate the components, are always placed in front of the image of an odd number. With the result of Theorem 2.1 we can therefore encode any link projection P with n crossings by a sequence of n even numbers separated by bars indicating the components. Moreover, since we are free to reorder the components, we may assume that the bars divide the sequence into segments of non decreasing length. We call such a sequence a *reduced sequence.*

To encode a given projection we have made choices for the order of the components, their orientations and base points. All of these choices yield possibly different reduced sequences. However we may designate a preferred one, which we call the *standard sequence*, by taking the minimum of all these sequences with respect to lexicographic order. In this order, a reduced sequence $a_1, a_3, \ldots, a_{2n-1}$ is smaller than a sequence $b_1, b_3, \ldots, b_{2n-1}$ if $a_r < b_r$ for the first r where $a_r \neq b_r$. In the case that $a_i = b_i$ for all i, the sequence that has the bars further to the front is smaller. Therefore to each projection of a link is assigned a unique sequence, its standard sequence.

Generating all possible reduced sequences is a trivial, albeit time consuming task. However, not every reduced sequence represents a link projection. Thus we need to identify those sequences which do indeed correspond to projections. Moreover, we are only interested in finding those reduced sequences which correspond to the projections of prime, non-split links.

II.2. Standard sequences and prime projections

Given a sequence S of n even numbers separated by k bars, we would like to determine if it is a sequence that arises from a link projection, if it is the standard sequence for that projection and, moreover, if that projection is of a non-split prime link.

There is only one link with two crossings, the Hopf link 4| 2, and there are no prime links with three crossings. In general, the number of components of a prime non-split link is limited and therefore also the number of bars that can be present in a sequence. In particular, we know the following:

Lemma 2.2. A prime, non-split link with $n > 3$ crossings has at most $n/2$ components.

Proof: First notice that every component has at least 4 crossings with other components. If one component had only two crossings with other components, then there would be two edges, emanating from the two crossings, which separate two non trivial path-components of the projection. Hence it would not be prime. Now for a link with components $K_1, \ldots, K_r$, let s_i be the number of self crossings of K_i and m_i the number of crossings K_i has with other components. Then the total number of crossings is $n = \sum_{i=1}^{r}(s_i + m_i/2)$, since the mixed crossings are counted twice. Hence $n \geq \sum_{i=1}^{r} m_i/2 \geq \frac{1}{2}4r \geq 2r$. □

This inequality is sharp, as the following example of a circular chain with n components shows.

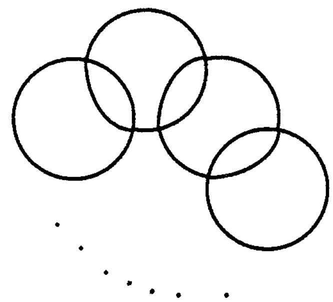


Figure 3. Circular chain

Let S be given as $a_1, a_2, \ldots, a_{2n}$ with k bars inserted after the entries with the indices $b_1, \ldots, b_k$. Let $b_0 = 0$ and $b_{k+1} = 2n$. Since most of the calculations reflect the cyclic order of the numbers in the components, we indicate an addition or subtraction that is performed in this manner by '$+_c$' and '$-_c$'. Specifically, if $i \in [b_{s-1}+1, b_s]$, then

$$i +_c k = b_{s-1} + 1 + ((i - b_{s-1} - 1 + k) \bmod (b_s - b_{s-1}))$$

If a diagram or projection with more than one double points is connected and there is no circle in the plane that cuts it in two points not of the same edge, then it is called *prime*. A projection with more than one double points is therefore prime, if it is still connected after any two edges are removed. This excludes for example

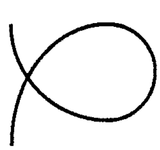
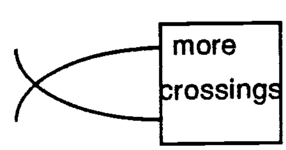
kinks  and crossings like 

Figure 4. Kink and nugatory crossing

Since diagrams of minimal crossing number of a prime link are prime, we are only interested in prime projections. Let $[i, j]$ denote the subsequence $i, i+_c 1, \ldots, j$. The effect of a kink on a sequence is that there exists an i such that $a(i)= i+_c 1$, which can be expressed as $a([i, i+_c 1]) = [i, i+_c 1]$. In general, if a projection can be disconnected by cutting two arcs, then there is a subset I of the numbers $\{1, 2, \ldots, 2n\}$ that is mapped onto itself by a. The subset I has to be of the form

$I=[b_{i_1}+1, b_{i_1+1}] \cup \ldots \cup [b_{i_s}+1, b_{i_s+1}] \cup [r, t]$

with $[r, t] = \emptyset$ or $[r, t] \subset [b_j+1, b_{j+1}]$, i.e. it has to be either all numbers of some of the components or a subset of the numbers of one component possibly together with other complete components.

Thus, reduced sequences which correspond to prime connected link projections must satisfy the following condition.

Condition 2.3. Let S be a reduced sequence with $n \geq 4$ entries and k bars. If S corresponds to a projection of a prime non-split link, then S satisfies

I. $k \leq n/2 - 1$ and

II. there is no subset $I=[b_{i_1}+1, b_{i_1+1}] \cup \ldots \cup [b_{i_s}+1, b_{i_s+1}] \cup [r, t]$ with $[r, t] = \emptyset$ or $[r, t] \subset [b_j+1, b_{j+1}]$ with $a(I) = I$.

Of all the sequences that satisfy Condition 2.3, most are not standard. This means that if there were a projection that yielded this sequence, then a different choice of basepoints, orientations or ordering of the components would produce a sequence that is smaller with respect to lexicographic order.

Any reordering of components can be done by repeated exchanges of components. The new sequence $\{a_i'\}$ after exchanging the components i and j which have the same length is the following:

$$a'(k) = \begin{cases} a(k') & , \text{if } a(k') \notin [b_{i-1}+1, b_i] \cup [b_{j-1}+1, b_j] \\ b_{j-1} + a(k') - b_{i-1}, & \text{if } a(k') \in [b_{i-1}+1, b_i] \\ b_{i-1} + a(k') - b_{j-1}, & \text{if } a(k') \in [b_{j-1}+1, b_j] \end{cases}$$

where

$$k' = \begin{cases} k & , \text{if } k \notin [b_{i-1}+1, b_i] \cup [b_{j-1}+1, b_j] \\ b_{j-1} + k - b_{i-1}, & \text{if } k \in [b_{i-1}+1, b_i] \\ b_{i-1} + k - b_{j-1}, & \text{if } k \in [b_{j-1}+1, b_j] \end{cases}$$

Since moving a single basepoint forward past an odd number of crossings would destroy the parity reversing property, possible moves of the basepoints are those where the basepoints of all components are moved by numbers of the same parity. Then the change in the sequence can be computed by treating each component separately in the following way: The new sequence $\{a_j'\}$ is

$$a'(j) = \begin{cases} a(\rho(j)) & , \text{if } j \in [b_{i-1}+1, b_i] \text{ and } a(\rho(j)) \notin [b_{i-1}+1, b_i] \\ \rho^{-1}(a(\rho(j))) & , \text{if } j \in [b_{i-1}+1, b_i] \text{ and } a(\rho(j)) \in [b_{i-1}+1, b_i] \\ \rho^{-1}(a(j)) & , \text{if } j \notin [b_{i-1}+1, b_i] \text{ and } a(j) \in [b_{i-1}+1, b_i] \\ a(j) & , \text{otherwise} \end{cases}$$

where $\rho(r) = r +_c k$, if the basepoint of the i-th component is moved k crossings forward, and $\rho(r) = (b_i + 1) -_c (r - b_{i-1} + k)$, if the orientation of the i-th component is reversed and the basepoint is then moved k crossings forward.

If none of these procedures yields a smaller sequence, then S is standard.

II.3. Realizable sequences

In order to generate a table of links from the set of all reduced sequences we must be able to eliminate those sequences which do not arise from link projections. For example the reduced sequence 4 8 10 | 6 12 2 cannot correspond to any projection of a link in the plane. (However, it does correspond to a link projection on the torus.)

Again let S be a sequence $S = \{a_1, a_2, \ldots, a_{2n}\}$ with k bars $b_1, b_2, \ldots b_k$. We define $b_0 = 0$, $b_{k+1} = 2n$. Now S represents an abstract graph G which we obtain from k + 1 disjoint intervals $I_1, I_2, \ldots, I_{k+1}$ with $I_j = [b_{j-1}, b_j]$ by identification of b_{j-1} with b_j in I_j and of i in I_r, $b_{r-1} < i \leq b_r$, with a_i in I_m, $b_{m-1} < a_i \leq b_m$. At each vertex of G this identification produces two pairs of opposite edges, so the four edges are cyclicly ordered.

If S satisfies Condition 2.3, then the graph G has the following properties:

(i) G is a four-valent graph with n double labelled vertices;

(ii) every edge joins two different vertices;

(iii) if any two of its edges are cut, G remains connected and

(iv) if any vertex is removed, G remains connected.

G can always be embedded in $\mathbb{R}^3$, but not necessarily in $\mathbb{R}^2$.

Definition 2.4. A sequence S is *realizable*, if the graph G obtained from S in the way described above can be smoothly embedded in $\mathbb{R}^2$, such that the unoriented cyclic order of the edges at each vertex is preserved. We call the embedding a *realization* of S.

The embedding can be viewed as the projection of a link. The two opposite edges at a vertex are on the same strand at the crossing. We regard an embedding η as lying in $S^2 = \mathbb{R}^2 \cup \{\infty\}$ and define:

Definition 2.5. Two embeddings η_1 and η_2 in $\mathbb{R}^2$ of a graph G are *equivalent*, if there is a homeomorphism h: $S^2 \rightarrow S^2$, such that $h \circ \eta_1 = \eta_2$.

We have the following theorem:

Theorem 2.6. Any two realizations of a graph G obtained from a reduced sequence S that satisfies Condition 2.3 are equivalent.

Proof: The theorem is a consequence of Lemma 1 in [D-T]. □

Corollary 2.7. Projections of unoriented links up to isotopy in S^2 are in one-to-one correspondence with realizable standard sequences.

Thus, we need to be able to determine if a sequence S is realizable or not.

Dowker and Thistlethwaite develop a criterion for the realizability of sequences representing knot projections in [D-T]. Rather than extending their algorithm to links, we give a different algorithm that applies equally well to knots or links.

Since the fact that a sequence is realizable does not depend on it being standard, we may assume in these arguments, that $a(b_s +1) < b_s +1$

for s=1, 2,..., k. This assures that a component is not disconnected from the previous ones. For i=1,..., 2n let H_i denote the subgraph of G corresponding to $I_1 \cup I_2 \cup \ldots \cup I_r \cup [b_r + 1, i + 1/2)$ where $b_r < i \leq b_{r+1}$. Hence, in the case that G can be embedded $\eta(H_1) \subset \ldots \subset \eta(H_{2n})$ are increasingly more complete embeddings of G in $\mathbb{R}^2$. Our goal will therefore be to start with an embedding of H_1 and to extend it step by step through embeddings of the H_i until finally an embedding of $G = H_{2n}$ is reached.

There is no obstruction to doing this before a whole loop is completed, i.e. before the first crossing is completely embedded. Also the embedding of the first crossing of a new component, which is just its connection to a previously embedded component, is always possible. So S fails to be realizable, if there exists an i such that H_i can be embedded but H_{i+1} cannot. In particular, if there is an i such that

1. H_i can be embedded and
2. $a(i +_c 1) < i$ and
3. for all possible embeddings η of H_i, $\eta(a(i +_c 1))$ is separated from $\eta(i)$ by a closed loop.

Definition 2.8. Assume H_i has been embedded in $\mathbb{R}^2$. The i-th *obstruction circuit* ϑ_i is the boundary of the component of $S^2 \setminus \eta(H_i)$, that contains $\eta(i + 1/2)$.

Figure 5 illustrates H_9 and ϑ_9 for the sequence 4 8 10 | 6 12 2. The embedding cannot be extended to H_{10}, because ϑ_9 separates $\eta(9)$ from $\eta(a_{10}) = \eta(5)$.

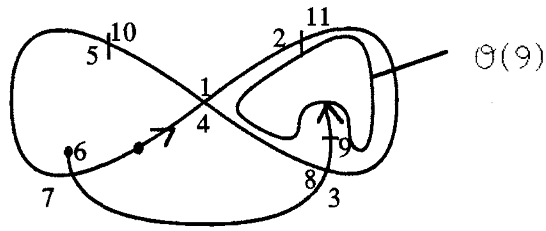


Figure 5. Obstruction circuit

Clearly, ϑ_i depends on $\eta(H_i)$ and we can extend the embedding of H_i to H_{i+1} if $\eta(a(i +_c 1))$ lies on ϑ_i. So we need to know which images of integers lie on ϑ_i. To obtain ϑ_i from $\eta(H_i)$ imagine $\eta(H_i)$ as a descending diagram, i.e. at any crossing point $\eta(j)=\eta(a_j)$ of $\eta(H_i)$, let $\eta(j)$ be on the overpass if $j< a_j$ and vice versa. In this diagram each crossing has a sign according to the usual rule: has sign +1 and has sign -1. After the sign of the first crossing is chosen as +1 all others are uniquely determined as a consequence of Theorem 2.6. We find ϑ_i by backing up from $\eta(i + 1/2)$ and turning right at every crossing of $\eta(H_i)$. The obstruction circuit will be completed when $\eta(i + 1/2)$ is reached again. There are 8 possibilities for ϑ_i at a crossing $\{\eta(j), \eta(a_j)\}$ of $\eta(H_i)$, relating to the possibilities that $\eta(j)$ is on the overcrossing or undercrossing, that the direction of ϑ_i is opposite to the direction of $\eta(H_i)$ at $\eta(j)$ or not, and that the sign of the crossing is +1 or -1.

To proceed in ϑ_i from $\eta(j)$ we have to jump to the other strand and continue to $\eta(a_j + 1)$ in four of these cases and to $\eta(a_j - 1)$ in the other four cases.

Example:

In this case $\varepsilon = -1$, $j < a_j$ and the direction of ϑ_j is opposite to the direction of the strand at j. So the next point is $a_j - 1$.

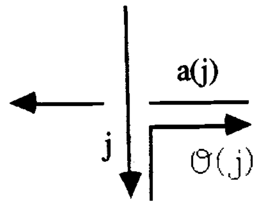


Figure 6. Continuation of the obstruction circuit

If $\eta(a_{i+1}) \notin \vartheta_i$, then $\eta(H_{i+1})$ cannot be completed as an extension of $\eta(H_i)$. If $\eta(a_{i+1}) \in \vartheta_i$, then we can define the embedding of H_{i+1}, though not necessarily uniquely. We know that $\eta(i + 1)$ is on the underpass since $i + 1 > a_{i+1}$. To obtain ϑ_i we always turned right, so the arc from $\eta(i)$ to $\eta(a_{i+1})$ has to intersect ϑ_i from the right. In the case that ϑ_i meets $\eta(a_{i+1})$ only once, there is a unique extension to H_{i+1}. The sign of the new crossing is defined as follows. If ϑ_i has the same direction as the arc through $\eta(a_{i+1})$, then the induced sign is +1, if it is opposite, then the sign is -1.

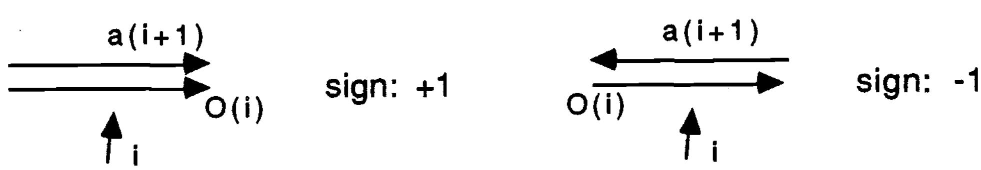
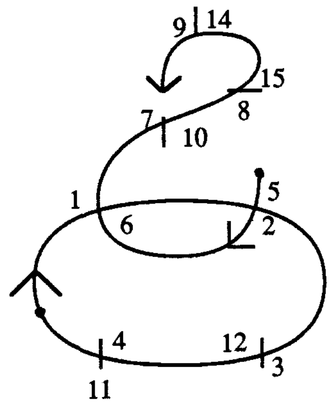


Figure 7. Rule for the sign of a new crossing

However, if ϑ_i meets $\eta(a_{i+1})$ twice as in the example below, H_{i+1} can be embedded in two possible ways corresponding to the choice of a positive or negative sign for the new crossing. But only one of the two possible embeddings can possibly be extended up to H_{2n}. A similar choice can be made for the sign of the first crossing of each component.

Example:



6 12 | 2 10 14 4 16 8

After H_9 has been embedded, the sign at the crossing {10, 7} may be chosen, according to the two possible choices of $\eta(H_{10})$. If the choice is -1, then H_{14} cannot be embedded. So one has to back up and choose the sign at this crossing as +1. With this choice, the embedding can be extended all the way to H_{16}. Hence, the sequence is realizable.

Figure 8. Link 6 12 | 2 10 14 4 16 8

In order to complete the r-th component, not only must $\eta(b_{r-1} + 1)$ be in $\mathcal{O}_{b(r)}$ but the sign induced by this procedure must be the same as the sign already chosen for this crossing.

As the above example illustrates, if there is an obstruction in the step from H_i to H_{i+1}, then one has to go back to previous stages where choices were made and try again. The sequence S is not realizable if it is not possible to complete H_{2n} for any set of possible choices. Clearly, this algorithm can be applied to knots as well as to links.

II.4. From Projections to Diagrams

With the methods and the justification that has been developed up to this point, a complete list of projections with a given number of crossings can be produced. We simply start with all reduced sequences and then eliminate those which are not realizable or standard. Only one representative sequence per isotopy class of unoriented projections remains. Now there are two major steps to be done in order to list all oriented alternating links without repetitions.

1. All oriented alternating diagrams have to be found and
2. Those diagrams that represent equivalent links have to be determined and all but one of them eliminated from the list.

Furthermore, we shall only list one of L, L*, ρL and ρL* even if all four of these links are pairwise non ambient isotopic. Thus the table of links given in the Appendix lists, without repetitions, all oriented prime alternating links with less than 10 crossings up to ambient isotopy, mirror image and complete reversal. Recall that the standard sequence of an oriented projection was found by taking the minimum of all the reduced sequences generated by all possible choices of ordering, basepoints and orientations. However, to oriented projections we may associate an *oriented standard sequence.* This is the minimum reduced sequence of the projection taken over all possible choices of ordering, basepoints and complete reversal of the given orientations. Having found all oriented projections we now want to pass to diagrams. Therefore at each crossing an overpass and an underpass has to be determined. Hence one projection yields 2^n diagrams. But we are only interested in alternating diagrams. In this case there can be at most two diagrams of one projection, since the choice at one crossing determines all others uniquely. The following proposition ensures the existence of alternating diagrams:

Proposition 2.9. Each projection P of a link admits an alternating diagram.

Proof: Choose an orientation for each component of P. Shade the regions of $S^2 \setminus P$ in checkerboard fashion. At each double point the two opposite corners are shaded. Of the two incoming strands, one has the dark region to its right, the other to its left. If the region to the right is dark, then let this strand be the overcrossing at the next double point. Otherwise let it be the undercrossing. The resulting diagram is alternating, since the shading to the right of a component changes at each double point. The two possible diagrams correspond to the two possible checkerboard colorings. □

The two alternating diagrams associated to a projection are mirror images of each other. Since we have chosen to not list both a link and its mirror image, we may choose to put the odd numbers of the sequence on the overpasses. During the test for realizability the sign at each crossing was determined under the assumption of creating a descending diagram. Comparing the descending diagram with the alternating diagram, the signs of the crossing $\{j, a_j\}$ in the two diagrams are opposite if j is odd and $a_j < j$. Thus our algorithm to test a sequence for realizability can be used to determine the signs of the crossings in an alternating diagram associated to a realizable sequence. Finally, we have established a one-to-one correspondence between oriented standard sequences and reduced alternating diagrams of links modulo isotopy in S^2, mirror images and reversing all orientations.

After listing all oriented alternating diagrams with less than 10 crossings we must still eliminate all but one link from each equivalence class. To do this we computed the polynomial invariants described in Chapter III for each link. For those sets of links not distinguished by the polynomials we showed by hand that in every case but three the links were indeed equivalent. The cases of inequivalent links with identical polynomial invariants are the pairs 6 10 | 2 14 | 4 16 | 12 8 and 6 10 | 2 14 | 16 4 | 12 8, 6 10| 2 14|16 4 18 8 12 and 6 10| 4 14| 2 16 8 18 12, 6 10| 2 14 |4 16 18 8 12 and 6 10| 4 16| 14 2 18 8 12. However careful examination of their linking numbers proves that they are different.

The following table shows the number of links in comparison with the number of processed data.

Crossings	Reduced Sequences	Standard[1] Sequences	Unoriented Projections	Unoriented Links	Oriented Projections	Oriented Links
1	0	0	0	0	0	0
2	2	1	1	1	1	1
3	6	0	0	0	0	0
4	24	1	1	1	2	2
5	120	1	1	1	1	1
6	2,160	11	6	5	10	8
7	15,120	29	8	7	10	11
8	214,920	210	36	21	74	43
9	2,540,160	1462	97	55	202	103

[1]Only standard sequences that pass Condition 2.3.II

III. Combinatorial Link Invariants

III.1. Numerical Link Invariants

Since one of the main tasks in link theory is to classify links, it is necessary to find properties that distinguish links that are not ambient isotopic. A *link invariant* is a map that associates to each ambient isotopy class of links an element of a numerical or algebraic structure. If the links are oriented we need to use invariants of oriented ambient isotopy which are defined for classes of oriented links. The best possible invariant would tell any two different links apart. Such an invariant is called *complete*. At the moment there is no such invariant that can easily be computed.

Many combinatorial invariants can be calculated from the diagram of the link. However, the diagram by itself is obviously not an invariant. But the following well known theorem by Reidemeister tells us exactly which diagrams represent the same link.

Theorem 3.1.[Re]: Any two diagrams D_1 and D_2 of ambient isotopic links L_1 and L_2 are related by a sequence of *Reidemeister moves*, which are as follows:

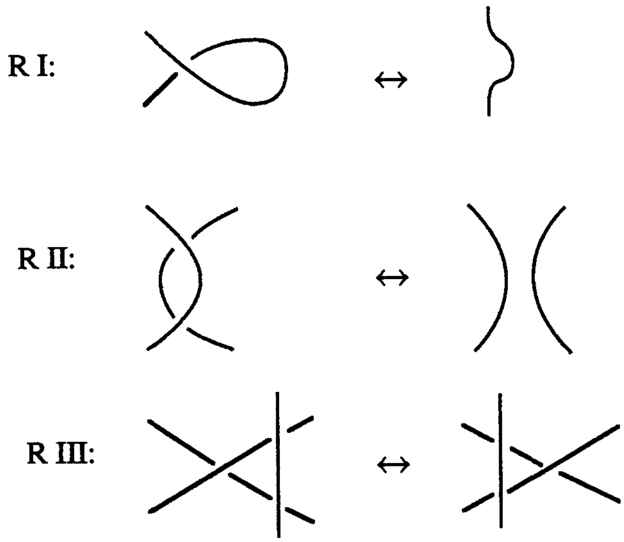


Figure 9. Reidemeister moves

This result is especially useful in proving that a function defined on diagrams is an invariant of links. Only invariance under the three Reidemeister moves must be shown.

In the remainder of this chapter we describe several important combinatorial link invariants which may be computed directly from a link diagram. While all of these invariants may, in theory, be computed by hand, for some a computer is a practical necessity. We have not aimed for a complete description of combinatorial link invariants. Indeed such a survey is far beyond the scope of this thesis. Instead we concentrate on those invariants which were employed to produce the oriented link table given in the Appendix. We describe several properties of these invariants, especially those which describe the form these invariants can take.

We begin by describing several classical numerical invariants of diagrams and links. Let D be a diagram of the oriented link L. Denote by D_i the component of D representing the component L_i of L.

Definition 3.2.

(a) The crossing number of D is the number of crossings or double points in D. The *crossing number* of L is the minimum crossing number of any diagram representing L.

(b) The unknotting number of the diagram D is the smallest number of crossings of D that have to be changed so that it is a diagram of an unlink. The *unknotting number* of the link L is the minimum over the unknotting numbers of all diagrams of L.

(c) The *writhe* of the diagram, w(D), is the sum of the signs of all crossings in D. Clearly the writhe is not an invariant of links since it changes under Type I Reidemeister moves. However it is preserved by Type II and III moves.

(d) The *linking number*, $lk(L_i, L_j)$, between the components L_i and L_j of L is one half the sum of the signs of all crossings between D_i and D_j. It is trivial to show that this is preserved by the Reidemeister moves and hence an invariant of the link.

(e) The *number of components* of L, c(L), is the number of copies of S^1 in the preimage of L.

III.2. Polynomial Link Invariants

A new era of link invariants started with the discovery of the Jones-polynomial [Jo] in 1984. The similarity between the Jones-polynomial and the Alexander (or Conway) polynomial led to two generalizations. The idea is to define a polynomial invariant of the link inductively by giving the polynomial of the unknot plus a relation between the polynomials associated to links that are the same except in the neighborhood of one point. The calculation itself is purely combinatorial and uses only a diagram of the link. Therefore these polynomials are easily computed with the aid of a computer.

We will use the standard notation L_+, L_- and L_0 for links with diagrams that are identical except at one crossing where they appear as follows:

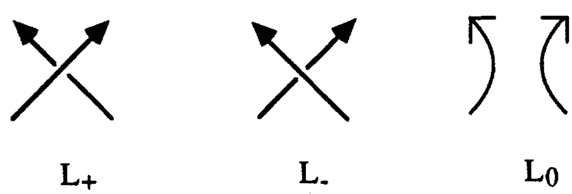


Figure 10. L_+, L_- and L_0

We say L_0 results by *smoothing* the crossing. There are two cases: If L_0 has more components than L_+, the smoothing is called a *fission*, if it has less components, it is called a *fusion*. There are two major independent polynomials based on the concept of changing and smoothing crossings.

1. The Homfly or oriented polynomial

This invariant was discovered in 1984 independently by several authors [FHLMOY] and [P-T] after the discovery of the Jones polynomial [Jo]. The oriented polynomial is described in the following theorem:

Theorem 3.3. [FHLMOY],[P-T] To each oriented link L there is a unique Laurent polynomial $P(L) \in \mathbb{Z}[z^{\pm 1}, v^{\pm 1}]$ which is invariant under ambient isotopy such that

(a) $P(L) = 1$ if L is the unknot,

(b) $v^{-1}P(L_+) - vP(L_-) = zP(L_0)$ where L_+, L_- and L_0 are as described above.

An easy application of the defining relation is the calculation of the polynomial of the unlink:

Proposition 3.4. If U_k is the unlink with k components, then $P(U_k) = q_k = \left(\frac{v^{-1} - v}{z}\right)^{k-1}$.

Proof: We shall induct on the number of components k. For k=1, U_1 is the unknot and $P(U_1) = 1$. Assume inductively that it is true for every unlink with less than k components.

The unlink U_k is related to U_{k-1} via the following smoothing.

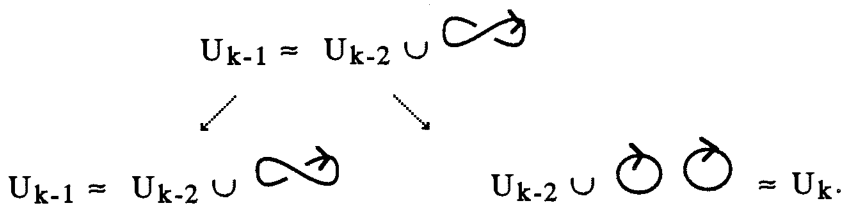
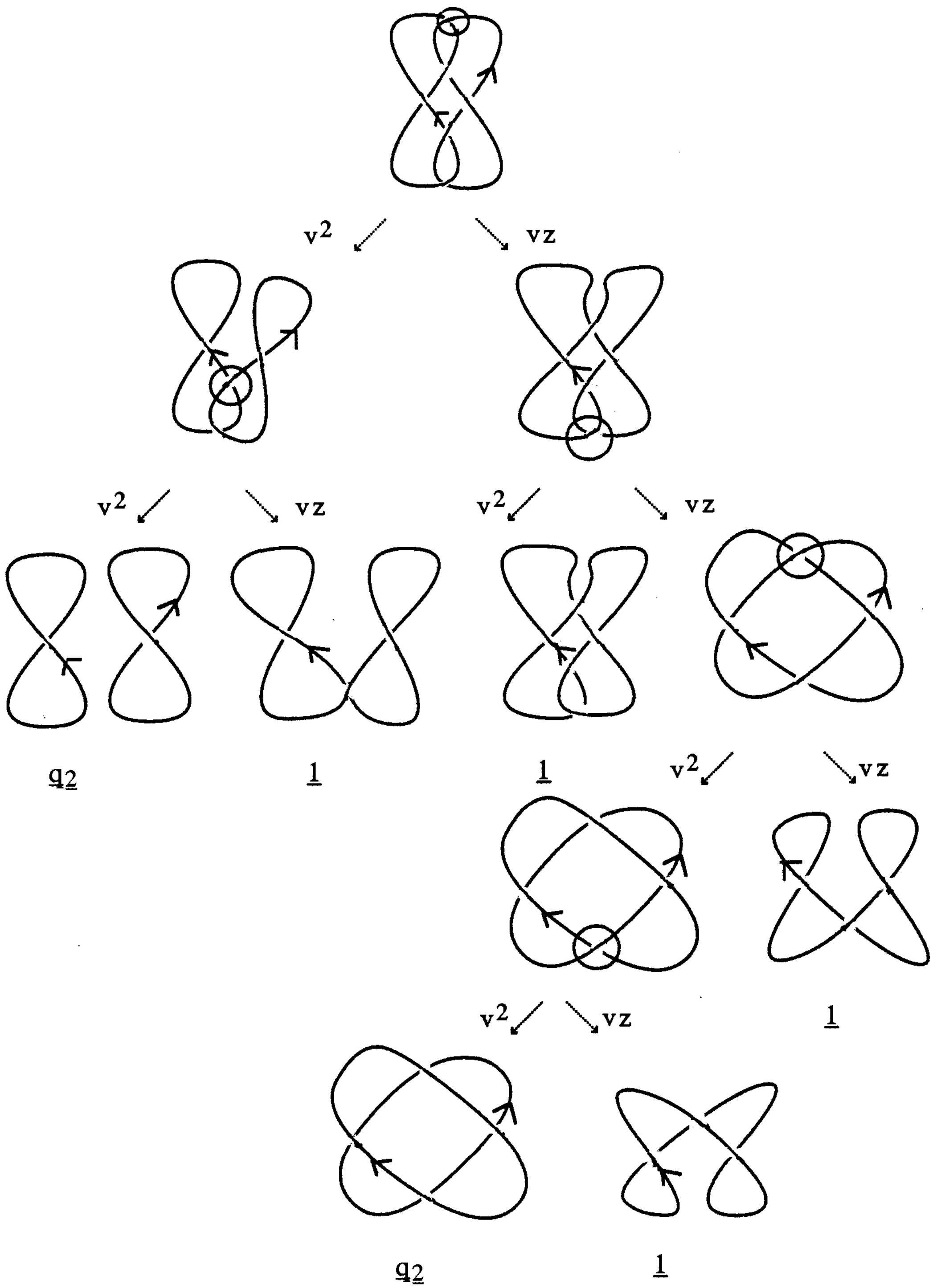


Figure 11. P-polynomial for the unlink

The defining relation for this tripel is $v^{-1}P(U_{k-1}) - vP(U_{k-1}) = zP(U_k)$. Hence $q_k = \left(\frac{v^{-1} - v}{z}\right) q_{k-1} = \left(\frac{v^{-1} - v}{z}\right)^{k-1}$. □

To illustrate that properties (a) and (b) of Theorem 3.3 suffice to calculate the polynomial of any link let us give an example.

Example: The P-polynomial of the link L below. We start with a diagram of the link. By smoothing and changing crossings we want to reduce the diagram to that of an unlink. This is demonstrated by the tree of diagrams we build. The original diagram is at the top and two branches descend from it to the diagrams that result from changing and smoothing a selected crossing. We always draw the changing branch to the left. From each of the new diagrams two more branches descend. This procedure is continued until all the outermost diagrams represent unlinks. Their polynomials are known from Proposition 3.4. Hence, the polynomial of L is found by tracing back the paths from the final vertices of the tree to the top. Each path yields one term of the polynomial, found by multiplication of the correct q_i with one factor for each traversed edge. To find this factor solve the defining relation for the diagram at the upper end of the branch. We call these factors *branching factors*. The tree with the original diagram at the root and the unlink diagrams as final stages is called a *resolution tree* of the diagram. The tree of our example is shown below. The changing and smoothing edges are labelled with the branching factors.



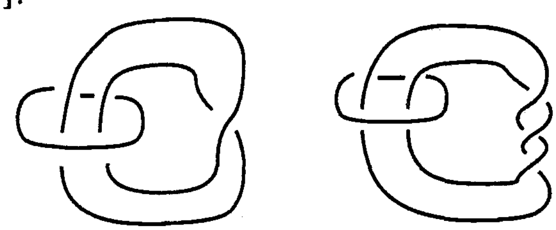
$$P(v, z) = z^{-1}(v^3-v^5)+z(2v^3+v^5-v^7)+z^3(v^3+v^5)$$

Figure 12. Resolution tree

This definition of a polynomial associated to links is based purely on the diagram of the link. Since its discovery several relations to topological properties of the link have been found, mainly for evaluations at special values of v and z. But one of the main problems is still to find a general geometric or topological definition of the polynomial.

We know that the P-polynomial cannot depend on the complement of the link: The links in the following example have homeomorphic complements but their P-polynomials are different.

Example [Ro, p.49]:

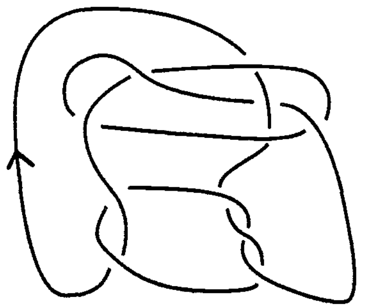


6 8| 2 10 4   6 10| 2 14 12 4 8

Figure 13. Two links with homeomorphic complements

As this example shows two non equivalent links can have homeomorphic complements! This is impossible for knots as has been recently shown by Gordon and Luecke [G-L].

However the P-polynomial is not the omnipotent invariant we would like to find. For example the Conway knot and the Kinoshita-Terasaka knot are different knots with the same P-polynomial.



Conway Knot

Kinoshita-Terasaka Knot

Figure 14. Conway knot and Kinoshita-Terasaka knot

These two knots are related by a *mutation,* i.e. a disc, whose boundary cuts the knot in four points, was cut out and replaced differently so that the four points on the boundary still match up. Mutation is a change which the P-polynomial does not detect at all:

Theorem 3.5. [Ho],[L-M 2] If L is a link and m(L) a link obtained from L by a mutation, then P(L) = P(m(L)).

In this project we are mainly interested in the format of the polynomial and the way it changes when we take mirror images or complete reversals of links.

Theorem 3.6. [L-M 2] P(L) can be written as
$P(L)(v,z) = z^{1-c(L)} \{ p_0(v) + z^2 p_1(v) + \ldots + z^{2r} p_r(v) \}$
where p_i is an odd (resp. even) polynomial in v, if 1-c(L) is odd (resp. even).

Proof: We shall induct on the crossing number of the link. If there are no crossings, L is an unlink and $q_i = \left(\frac{v^{-1} - v}{z}\right)^{i-1} = z^{1-i}(v^{-1} + v)^{i-1}$ satisfies the theorem.

Assume inductively that the theorem is true for all links with less than k crossings. Let L be a link with k crossings. Let D be a diagram of minimum crossing number of L and T be a resolution tree for D.

Case 1: L has one component.

Let R be the subtree of T that consists of the changing branches on the extreme left of T and the immediately descending smoothing branches. Name the final diagrams of R $D_1, D_2, \ldots, D_s$ with D_s at the end of the changing branch. Then D_i, $i = 1, \ldots, s-1$, is the diagram of a link L_i which has two components and less than k crossings. So $P(L_i) = z^{-1}P'(L_i)$ and $P'(L_i)$ has only even powers of z and odd powers of v. Also D_s is an unlink with polynomial equal to one. Therefore $P(L) = \sum_{r=1}^{s-1} v^{2t_r}(\pm z v^{\pm 1})P(L_r) + v^{2t_s}1 = \sum_{r=1}^{s-1} v^{2t_r \pm 1} P'(L_r) + v^{2t_s}$, which is a polynomial with non negative even powers of z and even powers of v. ($2t_r$ is the product of the factors on the changing branches that lead to D_r).

Case 2: L has $w > 1$ components.

Select one component of L. Let R be a tree of changing branches with the immediately descending smoothing branches where only those crossings of D are changed at which the selected component crosses under other components. Name the end diagrams $D_1, \ldots, Ds$ as before and the links $L_1, \ldots, L_s$. Then $D_1, \ldots, D_{s-1}$ have $w-1$ components, since all smoothings have to be fusions. Therefore, for $i=1, \ldots, s-1$, $P(L_i)=z^{2-w} P'(L_i)$, where $P'(L_i)$ has non negative even powers of z and the powers of v have the parity of 2-w. Also L_s is a split link with less crossings than L but still w components. So $P(L_s)=z^{1-w}P'(L_s)$ and $P'(L_s)$ has the correct form. So

$$\begin{aligned} P(L) &= \sum_{r=1}^{s-1} v^{2t_r}(\pm z v^{\pm 1})P(L_r) + v^{2t_s}P(L_s) \\ &= \sum_{r=1}^{s-1} \pm z^{3-w} v^{2t_r \pm 1} P'(L_r) + z^{1-w} v^{2t_s} P'(L_s) \\ &= z^{1-w}\Big(\sum_{r=1}^{s-1} \pm v^{2t_r \pm 1} z^2 P'(L_r) + v^{2t_s} P'(L_s)\Big) \end{aligned}$$

The terms in parentheses have the required properties. □

Corollary 3.7. $P(L)(-v,-z) = P(L)(v, z)$

Theorem 3.8. [Ho] Let L be a link, L^* its mirror image and ρL the complete reversal. Then

(i) $P(L^*)(v, z) = P(L)(-v^{-1}, z)$ and

(ii) $P(\rho L) = P(L)$.

Proof: (i) If T is a resolution tree for L, then a resolution tree T^* for L^* is obtained from T by changing all crossings in all diagrams of T. The polynomials at the outermost diagrams, which are unlinks, are q_i and do not change. But all diagrams labelled L_+ have to be changed to L_- and vice versa. So when going up in the tree the factor of every changing edge needs to be changed from v^2 to v^{-2} and vice versa and on every smoothing edge we must trade vz with $-v^{-1}z$. This is accomplished by replacing v with $-v^{-1}$.

(ii) Any resolution tree for L is also a resolution tree for ρL. Also the unlink and its complete reversal are equivalent. So the algorithm produces the same result. □

This theorem is of special importance to our link tabulation since we do not distinguish links from their mirror image or complete reversal. So two links could be equivalent if their polynomials are different but related by the substitution $v' := -v^{-1}$ and $z' := z$. In addition out table only contains prime non-split links, hence no polynomials for composite links or the union of links are listed. However they can be easily calculated by the following theorem.

Theorem 3.9. [Ho] Let R and S be oriented links.
(i) $P(R \cup S) = \frac{v^{-1} - v}{z} P(R) \cdot P(S)$, where $R \cup S$ is the split union of two separate links R and S.
(ii) $P(R \# S) = P(R) \cdot P(S)$, where $R \# S$ is the connected sum of R and S.

<u>Proof:</u> (i) Let T_R and T_S be resolution trees of the subdiagrams of R and S in a diagram of $R \cup S$. Add the diagram of S to each diagram in T_R. The outermost diagrams are therefore unlinks plus the diagram of S. Attach a copy of T_S at each of these vertices and carry the unlinks at these vertices through T_S to get a resolution tree of the diagram of $R \cup S$. The final diagrams of this tree all have one factor $(v^{-1}-v)/z$ more than the final stages of T_R and T_S. This factors through and yields the result.

(ii) Use Figure 15

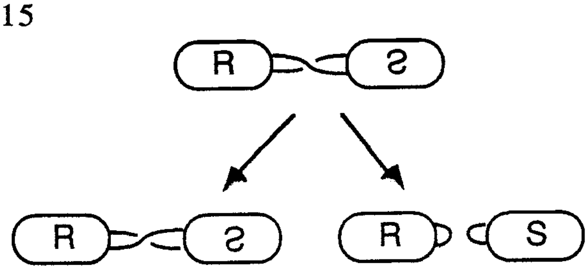


Figure 15. P-polynomial of a split link

and the defining relation $v^{-1}P(R \# S) - vP(R \# S) = zP(R \cup S)$.

So $P(R \# S) = z \dfrac{P(R \cup S)}{v^{-1} - v} = P(R)P(S)$ by (i). □

Part (i) of this theorem shows a way to construct different links with the same polynomial: Although the polynomial of the connected sum of two links is determined by their polynomials, the connected sum is not unique.

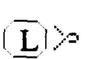
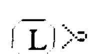
2. The F-polynomial or semi-oriented polynomial or Kauffman-polynomial

This polynomial is the second generalization of the Jones polynomial mentioned at the beginning of this chapter. Although it was discovered after the Q-polynomial, which is introduced on page 47, the Kauffman-polynomial contains the Q-polynomial as a special case. This polynomial is defined in two steps. First a polynomial Λ of unoriented diagrams is constructed which is invariant under Reidemeister moves II and III. By multiplication with a balancing factor it is then expanded to a link invariant of oriented type.

Theorem 3.10. [Ka 4] For every unoriented link diagram D there exists a unique Laurent polynomial $\Lambda(D) \in \mathbb{Z}[a^{\pm 1}, x^{\pm 1}]$ that satisfies

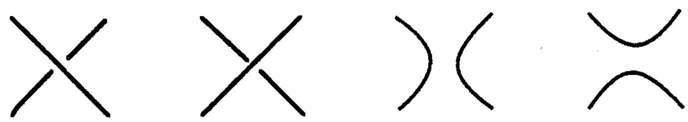
(i) $\Lambda(O) = 1$,

(ii) Λ is invariant under Reidemeister moves II and III,

(iii) $\Lambda($  $) = a\Lambda(L)$ and $\Lambda($  $) = a^{-1}\Lambda(L)$ and

(iv) $\Lambda(L_+) + \Lambda(L_-) = x\,(\Lambda(L_0) + \Lambda(L_\infty))$

where L_+, L_-, L_0, L_∞ are unoriented diagrams that are the same except near a point where they appear as follows:



L_+ L_- L_0 L_∞

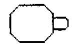
Figure 16. L_+, L_-, L_0 and L_∞

Note: This is only a polynomial defined on diagrams. Therefore (i) does not mean that Λ(unknot) = 1 but only that $\Lambda=1$ for a diagram that has one component and no crossings. Moreover it is not possible to differentiate L_+ and L_- or L_0 and L_∞ so only the symmetry of (iv) allows this definition. The proof of the existence of Λ is similar to the proof of Theorem 3.3.

From (iii) it is clear that the Λ-polynomial is not a link invariant. However, if L is an oriented link note that the right case in (iii) corresponds to a positive crossing and the left case to a negative crossing for both possible orientations of the kink. Recall that the writhe has a behavior similar to that of the Λ-polynomial: The writhe does not change under Reidemeister moves II and III but a Type I move adds or subtracts one to the writhe. Combining writhe and Λ we construct an invariant of oriented links.

Theorem 3.11. If L is an oriented link, a unique Laurent polynomial $F(L) \in \mathbb{Z}[a^{\pm 1}, x^{\pm 1}]$ is defined by

$$F(L)(a, x) = a^{-w(D)} \cdot \Lambda(D)(a, x) \text{ where } D \text{ is any diagram of } L.$$

Proof: Only invariance under Reidemeister move I needs to be verified. For example in the case of a positive kink, let D be the diagram  and D' the altered diagram  . Then
$a^{-w(D)}\Lambda(D) = a^{-(w(D)-1)}\, a^{-1}\Lambda(D) = a^{-w(D')}\Lambda(D')$. □

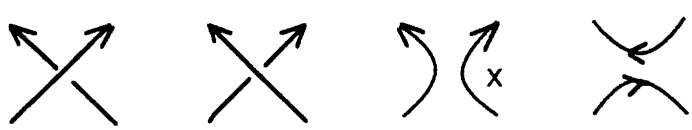
Kauffman calls a function on diagrams that is invariant under the Reidemeister moves II and III an invariant of *regular isotopy*. This idea of creating an invariant of ambient isotopy of links from an invariant of regular isotopy of diagrams is an important idea that can often be made to work in general.

Before the description of some of the properties of this polynomial, one should note a different definition of the polynomial:

Theorem 3.12. For every link L the polynomial F(L) satisfies

(i) F(L) = 1 if L is the unknot and

(ii) (a) if in the quadrupel L_+, L_-, L_0, L_∞ the L_0-smoothing is a fission, orient L_∞ in the following way



L_+ L_- L_0 L_∞

Figure 17. L_+, L_-, L_0 and L_∞ if L_0 is a fission

and use the formula

$aF(L_+) + a^{-1}F(L_-) = x\ (F(L_0) + a^{-4\lambda}F(L_\infty)$ where λ is the sum of the linking numbers of component x with all other components in L_0.

(b) if the smoothing is a fusion, orient L_∞ in the following way

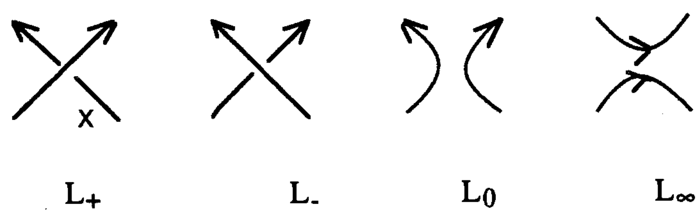


Figure 18. L_+, L_-, L_0 and L_∞ if L_0 is a fusion

and use the formula

$aF(L_+) + a^{-1}F(L_-) = x\ (F(L_0) + a^{-4\mu+2}F(L_\infty)$ where μ is the sum of the linking numbers of the component x with all other components in L_+.

Theorem 3.12 was used for the computation of the F-polynomial in this project. The computation time increased sharply compared to the P-polynomial, since a ternary resolution tree needs to be built rather than a binary tree. Theorem 3.12 can also be used to derive properties of the F-polynomial similar to those satisfied by the P-polynomial.

Here are the properties of the F-polynomial corresponding to Theorems 3.8 and 3.9:

Theorem 3.13. [L-M 4] Let L, L_1, L_2 be oriented links. Then

(i) $F(L_1 \# L_2) = F(L_1) \cdot F(L_2)$

(ii) $F(L_1 \cup L_2) = ((a^{-1} + a)(x^{-1} - 1)) \cdot F(L_1) \cdot F(L_2)$

(iii) $F(L^*)(a, x) = F(L)(a^{-1}, x)$

(iv) $F(\rho L) = F(L)$

(v) F is invariant under mutation

The proofs are similar to those for the P-polynomial. Unlike the P-polynomial, this polynomial behaves regularly when the orientation of one component is reversed. (Hence it is sometimes referred to as the semi-oriented polynomial.) This should be expected, since the polynomial was created from a polynomial of unoriented diagrams by multiplication with a power of a. There is no similar rule for the P-polynomial.

Theorem 3.14. [Li 2] If L' is the link obtained from the link L by reversing the orientation of the component K, then $F(L') = a^{4\lambda} F(L)$ where λ is the sum of the linking numbers of K with all other components of L.

Now, for the notation in the table we will use knowledge about the form of the terms of the F-polynomial.

Proposition 3.15. In every term of the F-polynomial, $F(L)(a,x) = \sum c_{ij} a^i x^j$, the sum of the exponents of a and x, $i + j$, is even.

Proof: This is clearly true for the unknot. Also the branching factors of all possible 3 branchings satisfy the property. For example consider the branching of a positive crossings where L_0 is a fusion:

$$\begin{tikzcd} & L_+ \arrow[dl, "-a^{-2}"'] \arrow[d, "a^{-1}x"] \arrow[dr, "a^{1+4\mu}x"] & \\ L_- & L_0 & L_\infty \end{tikzcd}$$

Since each term of the polynomial is a product of those factors and the unlink values, it is true for the polynomial. □

As for the P-polynomial some special evaluations have a significant value. They are summarized in the following theorem.

Theorem 3.16. [L-M 1] For any oriented Link L

(i) $F(L)(1, -2) = (-2)^{c(L)-1}$

(ii) $F(L)(a, -(a+a^{-1})) = \frac{1}{2}(-1)^{c(L)-1} E(L)$ where $E(L) = \sum_{S \subset L} a^{-4lk(S, L-S)}$

Initially, it was not clear if the F-polynomial was a new polynomial or just a special case of the P-polynomial. But there are examples that show that they are in fact independent. For example, 4 10 12 14| 2 18 16 8 6 and 8 12 10| 14 6 16 18 4 2. These are the only pair of links in our table with the same P-polynomials but different F-polynomials. One open question is, if both P and F are special cases of some master-invariant .

The last two polynomials we mention were found chronologically before the P- and F-polynomials. They are special cases of P and F. However understanding them has led to some of the most important recent results in knot theory.

3. The Jones-polynomial or V-polynomial

The discovery of this polynomial in 1984 by Vaughan Jones was responsible for the later discoveries of P and F.

Theorem 3.17. [Jo] To every oriented link L there exists a unique Laurent polynomial $V(L) \in \mathbb{Z}[t^{\pm 1/2}]$ that satisfies

(i) $V(L) = 1$ if L is the unknot, and

(ii) $t^{-1}V(L_+) - tV(L_-) = (t^{1/2} - t^{-1/2})V(L_0)$ where L_0, L_-, L_+ are defined as before.

The Jones polynomial can be derived from both P and F by

Theorem 3.18. [Li 1] For any oriented link L,

$$V(L)(t) = F(L)(-t^{-3/4}, t^{-1/4} + t^{1/4}) \text{ and}$$

$$V(L)(t) = P(L)(t, (t^{1/2} - t^{-1/2}))$$

Different from the proofs for the existence of the P- and F-polynomials there is an almost trivial proof for the existence of the V-polynomial found by Kauffman [Ka 3]: Again he defines an invariant of regular isotopy of diagrams and combines it with the writhe to get an invariant of links.

Definition 3.19. The *unreduced bracket polynomial*, $\langle\ \rangle$, of unoriented link diagrams is defined by:

(i) $\langle \bigcirc \rangle = 1$,

(ii) $\langle \bigcirc \cup D \rangle = d \langle D \rangle$ if D is non empty and

(iii) $\langle \times \rangle = A \langle \asymp \rangle + B \langle)(\rangle$

Calculation of the effects of the Reidemeister Type II and III moves leads to the substitutions $B := A^{-1}$ and $d := -(A^2 + A^{-2})$ to get an invariant of regular isotopy. This regular isotopy invariant is called the *reduced bracket* or more simply the bracket polynomial and is still denoted by $\langle D \rangle$. Since $\langle \rangle\!\!\circ \rangle = -A^3 \langle \rangle \rangle$ and $\langle \rangle\!\!\circ \rangle = -A^{-3} \langle \rangle \rangle$, the polynomial $f(K) = (-A)^{-3w(D)} \langle D \rangle$ is an invariant of oriented links if D is any diagram of K. This polynomial is related to the V-polynomial by

Proposition 3.20. $V(K)(t) = f(K)(t^{-1/4})$

<u>Proof:</u> We have

$$\langle \times \rangle = A \langle \asymp \rangle + A^{-1} \langle)(\rangle \text{ and}$$
$$\langle \times \rangle = A^{-1} \langle \asymp \rangle + A \langle)(\rangle.$$

Multiplication with $-A^{-1}$ and A respectively and addition yields

$$A \langle \times \rangle - A^{-1} \langle \times \rangle = (A^2 - A^{-2}) \langle)(\rangle.$$

In oriented diagrams $w(\times) = w()() - 1$ and $w(\times) = w()() + 1$.

Therefore multiplication with $(-A)^{-3w()()}$ yields

$$A \cdot (-A^3)\, f(\times) - A^{-1} \cdot (-A^{-3})\, f(\times) = (A^2 - A^{-2})\, f()(),$$

which equals the defining equation for V after the substitution $A := t^{-1/4}$. □

More results about the Jones polynomial are summarized in the following theorem:

Theorem 3.21. If L is an oriented link, then

(i) $V(L^*)(t) = V(L)(-t^{-1})$

(ii) $V(\rho L) = V(L)$

(iii) Reversing result: Let L' be the link obtained from L by reversing the orientation of one component K, and λ be the sum of all linking numbers of K with other components of L. Then $V(L') = t^{-3\lambda}V(L)$.

Corollary 3.22. (to part i) If L is an amphicheiral link, then the span of V(L) is even.

The *span* of V(L) is the difference between the maximal and the minimal degree of the polynomial.

Besides these properties of V, the Jones-polynomial yielded some of the best results about the crossing numbers of links. One can be stated as follows.

Theorem 3.23. [Th 2] If D is a connected diagram with n crossings of an oriented link L, then $n \geq \text{span}(V(L))$. If D is prime and non alternating, then $n > \text{span}(V(L))$, if it is prime and alternating then $n = \text{span}(V(L))$.

4. The Conway-polynomial

Chronologically the Conway polynomial was the first polynomial using the technique of changing and smoothing crossings. The axioms it satisfies are :

Theorem 3.24. [Ka 1] To each oriented link L there is associated a unique polynomial $\nabla(L)$, the Conway polynomial, which is an invariant of ambient isotopy and satisfies

(i) $\nabla(L) = 1$ if L is the unknot and

(ii) $\nabla(L_+) - \nabla(L_-) = z\nabla(L_0)$.

These defining relations easily give the following property:

Proposition 3.25. If L is a split link, then $\nabla(L) = 0$.

From equation (ii) it is clear that $\nabla(L)$ is a polynomial that can be gotten from P(L) by a substitution.

Theorem 3.26. For every oriented link L

$\nabla(L)(z) = P(L)(1, z)$.

The Conway polynomial is a normalized form of the classical Alexander polynomial. Therefore it can be also defined in terms of algebraic topology and covering space theory.

5. The Q-polynomial or unoriented polynomial

Although the Q-polynomial is not listed in the Appendix we want to give a short description of it and some of its properties. It can be viewed as the evaluation of the semi-oriented polynomial, $Q(L)(x) = F(L)(1, x)$, but it was found before the F-polynomial [B-L-M], [H]. Its original definition is worth noting.

Theorem 3.27. [B-L-M],[H] To each unoriented link L there exists a unique polynomial $Q(L) \in \mathbb{Z}[x^{\pm 1/2}]$ such that

(i) $Q(L) = 1$, if L is the unknot

(ii) $Q(L_+) + Q(L_-) = x(Q(L_0) + Q(L_\infty))$.

Again (ii) is only well defined because of its symmetry.

Summary of the properties of Q:

Proposition 3.28. [B-L-M] Let L, L_1, L_2 be unoriented links.

(i) $Q(L_1 \# L_2) = Q(L_1) \cdot Q(L_2)$

(ii) $Q(L_1 \cup L_2) = \mu \cdot P(L_1) \cdot P(L_2)$, $\mu = 2x^{-1} - 1$

(iii) $Q(U^k) = \mu^{k-1}$,

(iv) $Q(L^*) = Q(L)$,

(v) Q does not change under mutation,

(vi) the lowest power of Q is $c(L)-1$,

(vii) $\deg(Q(L)) < c(L)$.

Proposition 3.29. [B-L-M] $Q(L) - 1$ is divisible by $2(x - 1)$.

Corollary 3.30. $Q(L)(1) = 1$

Proofs can be found in [B-L-M].

Summary

We have introduced five polynomials that can be all defined from a given diagram of the link through smoothing and changing crossings. Two of them, the P-polynomial and the F-polynomial, are (currently) the most general polynomials that can be defined in this way. All together their relation is described by the following diagram:

$$\begin{tikzcd}
& F(L) \arrow[dl] \arrow[dr] & & P(L) \arrow[dl] \arrow[dr] & \\
Q(L) & & V(L) & & \nabla(L)
\end{tikzcd}$$

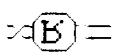
Since their nature is combinatorial, they are especially useful for applications involving computers. Both P and F were computed for the links listed in the Appendix and served as our main tool in distinguishing links.

III.3. Results for Alternating Links

In [Ta] Tait raises several conjectures concerning alternating links:

(T1) Any two alternating prime diagrams of a link have the same number of crossings.

(T2) For every alternating link there is an alternating diagram with the minimum number of crossings.

(T3) Any two prime alternating diagrams of a link L are related by means of flypes. (A *flype* is a transformation of a diagram described by  $\leftrightarrow$  .)

(T4) If D is a prime alternating diagram of a link L and $w(D) \neq 0$, then L is cheiral (i.e. $L \neq L^*$).

New light on these almost 90 years old conjectures has been shed by the Jones-polynomial: T1, T2 and T4 have been answered positively by [Ka 2], [Th 2], [Mu]. The flyping conjecture, T3, is still unknown.

Two corollaries to Theorem 3.23 solve the first two conjectures.

Corollary 3.31. [Ka 2], [Th 2], [Mu] (answers T1)
The number of crossings in a prime alternating diagram of a link is a topological invariant of the link.

Corollary 3.32. (answers T2)
Any diagram with the minimum crossing number of an alternating link is an alternating diagram.

According to Kauffman [Ka 2], T4 has been proven by Thistlethwaite. Thistlethwaite [Th 2] and Murasugi [Mu] also proved

Theorem 3.33. The writhe is an invariant of prime alternating diagrams of a link.

This would also be a consequence of Tait's flyping conjecture: Flypes do not change the writhe, so if all alternating diagrams of a link were related by flypes they would all have the same writhe. However the full conjecture has not yet been proven. In searching for counter examples the unreduced bracket polynomial could be helpful. It is invariant under flypes, so two prime alternating diagrams of a link with different unreduced bracket polynomials would contradict the conjecture.

Besides these new results about crossing numbers and the writhe of alternating diagrams of links Thistlethwaite [Th 2] also proves a theorem about the form of the Jones-polynomial of alternating links.

Theorem 3.34. [Th 2] The Jones-polynomial of alternating links is alternating.

This can be observed in the tables at the end of this paper.

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APPENDIX

Appendix

How to Read the Tables

In the following pages we list all oriented alternating links with less than 10 crossings up to mirror image or complete reversal. Let the *oriented standard sequence of a link* be the minimum over all oriented projections of their oriented standard sequences. The *(unoriented) standard sequence of the link* is defined similarly.

The data are split in two sections. Part 1 contains all oriented projections and oriented standard sequences of links. In the second table the links are listed with some of their polynomial and numerical invariants. The links in both tables are in groups of links that are equivalent in the unoriented category and sorted by the lexicographic order of their unoriented standard sequences.

Detailed description: Table 1 contains all possible projections of oriented links. Since we only consider alternating links up to mirror images, each projection determines a diagram uniquely. In the left column we show the oriented standard sequences of the links and their induced projections. An underlined sequence indicates that this is also the (unoriented) standard sequence of the following group of oriented links. To the right of each projection in the left column are all other oriented projections and sequences of the same oriented link. So given the projection of a link you need to find its projection in the list and determine its oriented standard sequence directly to the left. Since the information in both tables is sorted by the order of the unoriented standard sequence, one has to use the next underlined sequence above the oriented standard sequence as reference for the other table.

This second table lists four polynomials and two numerical invariants for each oriented link. Again, the links are ordered by unoriented standard sequences. Below those underlined sequences are all oriented links that correspond to it. The polynomials in the second column are P-, F-, V- and ∇- polynomial in this order. The last two columns list the linking numbers and the writhe.

We use the results from Chapter III.2 about the format of the polynomials for links. The P-polynomial can be written as $P(L)(v,z) = z^{1-c(L)}\,(p_0(v) + z^2p_1(v) + \ldots + z^{2r}p_r(v))$ where the p_i are odd polynomials in v, if 1-c(L) is odd, and even polynomials, if 1-c(L) is even. Therefore we write 1-c(L) in braces and then every other coefficient of $p_i(v)$ between the i-th pair of parenthesis. If p_i is an even polynomial, we enclose the constant term in brackets, if it is odd an asterisk is placed between the coefficients of powers -1 and 1. The other polynomials are written in similar manner. Since $F(L)(a,x) = x^rf_r(a) + x^{r+1}f_{r+1}(a) + \ldots + x^sf_s(a)$, where f_i is an odd or even polynomial in a, if i is odd or even, respectively, we write {r} and then the coefficients of f_{r+i} between the i-th pair of parenthesis with the same convention for the use of '[]' and '*'. The Jones- and Conway-polynomials are polynomials in one variable. In the case of the Jones-polynomial the number in braces is the lowest power of this variable, for the Conway-polynomial we write 1-c(L) in braces. The numbers in parenthesis are coefficients of consecutive terms. The next column lists all linking numbers between components of L. They are written in the form of the right upper triangle of the linking matrix $(lk(L_i, L_j))_{i,j=1,..,k}$ and the coefficients are in the order induced by the oriented standard sequence. Finally, the writhe w(L) is in the last column. The following example should help to read the table:

Consider the projection with the sequence 6 10 | 4 12 | 2 14 8. The standard sequence is 6 10 | 2 12 | 4 14 8. So the data of the corresponding link can be found in the group after this sequence in the second table. The listed invariants are:

6 10| 4 12| 2 14 8	{ -2}([0] 0 1 -2 1)([0] 0 3 -3)([0] 0 3 1 -1)([0] 0 1 1)	1 1	7
	{ -2}(-1 -2 -1 0[0])(2 2 0 0 *)(3 5 3 0[0])		
	(-3 -3 0 0 *)(-3 -5 -5 -3 0[0])(1 -3 -4 0 0 0 *)		
	(3 3 1 1 0[0])(3 4 1 0 0 *)(1 1 0 0[0])	1	
	{ 4/2} (1 -1 4 -3 4 -3 3 -1)		
	{ -2} (0 0 3 2)		

This means that

$P(L)(v,z) = z^{-2}[(v^4-2v^6+v^8)+z^2(3v^4-3v^6)+z^4(3v^4+v^6-v^8)+z^6(v^4+v^6)],$

$F(L)(a,x) = x^{-2}(-a^{-8}-2a^{-6}-a^{-4})+x^{-1}(2a^{-7}+2a^{-5})+(3a^{-8}+5a^{-6}+3a^{-4})$

$+x(-3a^{-7}-3a^{-5})+x^2(-3a^{-10}-5a^{-8}-5a^{-6}-3a^{-4})+x^3(a^{-11}-3a^{-9}-4a^{-7})$

$+x^4(3a^{-10}+3a^{-8}+a^{-6}+a^{-4})+x^5(3a^{-9}+4a^{-7}+a^{-5})+x^6(a^{-8}+a^{-6}),$

$V(L)(t) = t^{4/2} - t^{6/2} + 4t^{8/2} - 3t^{10/2} + 4t^{12/2} - 3t^{14/2} + 3t^{16/2} - t^{18/2}$ and

$\nabla(L)(z) = 3 + 2z.$

The linking numbers are $lk(L_1, L_2) = 1$, $lk(L_1, L_3) = 1$ and $lk(L_2, L_3) = 1$. The writhe of the link is 7.

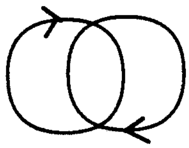
When reading these invariants one should keep in mind that everything was done for one choice of the quadrupel of possibly different links that originate from taking the mirror image or reversing all components. Therefore the numerical invariants of the other possibilities could differ by their sign and some of the polynomials could change according to 3.8 and 3.13.

Table of Projections

Projections of Oriented Links with 2 Crossings and 2 Components

Oriented Stand. Seq. | Other Projections

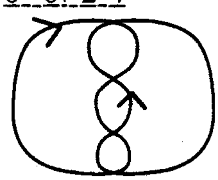
4| 2



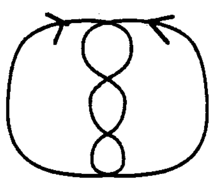
Projections of Oriented Links with 4 Crossings and 2 Components

Oriented Stand. Seq. | Other Projections

6 8| 2 4



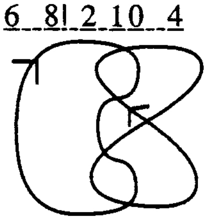
6 8| 4 2



Projections of Oriented Links with 5 Crossings and 2 Components

Oriented Stand. Seq. | Other Projections

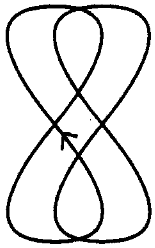
6 8| 2 10 4



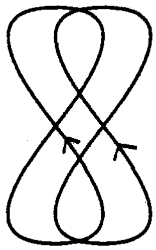
Projections of Oriented Links with 6 Crossings and 2 Components

Oriented Stand. Seq.

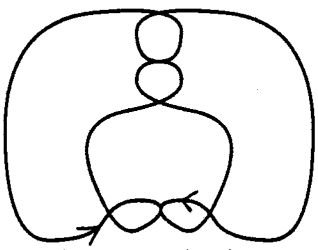
4 8 10| 2 12 6



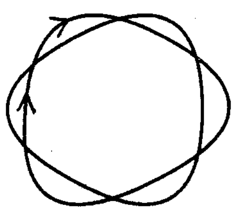
4 8 12|10 2 6



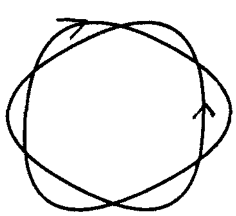
8 10 12| 2 6 4



8 10 12| 6 2 4

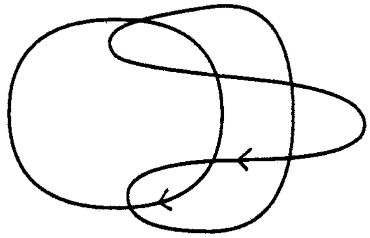


8 12 10| 2 6 4

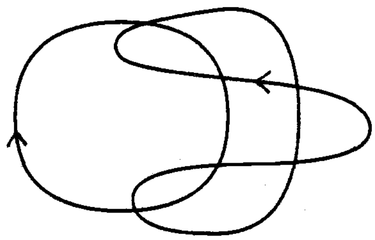


Other Projections

6 10| 4 12 2 8

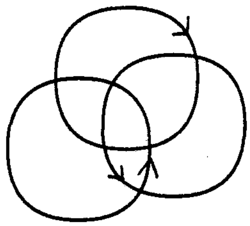


6 10| 2 12 4 8

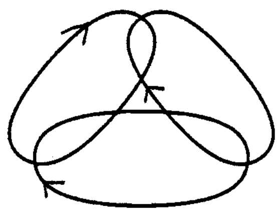


Projections of Oriented Links with 6 Crossings and 3 Components

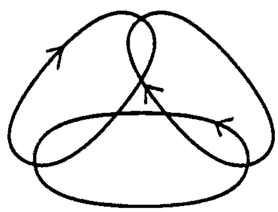
Oriented Stand. Seq.	Other Projections
6 8|10 12| 2 4	



6 10| 2 12| 4 8



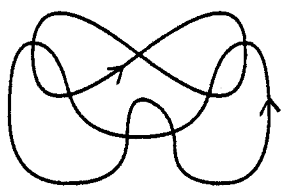
6 10| 2 12| 8 4



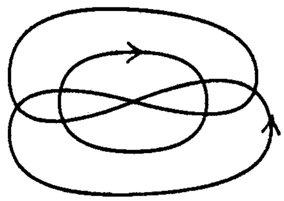
Projections of Oriented Links with 7 Crossings and 2 Components

Oriented Stand. Seq.

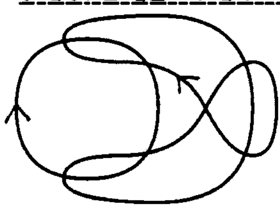
4 8 12| 2 14 6 10



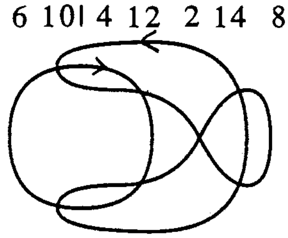
6 8|10 12 14 2 4



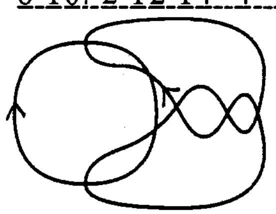
6 10| 2 12 4 14 8



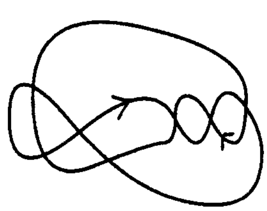
6 10| 4 12 2 14 8



6 10| 2 12 14 4 8

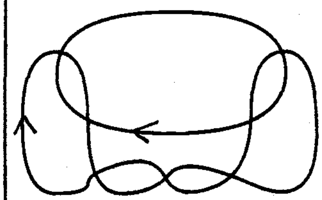


8 10 12| 2 6 14 4

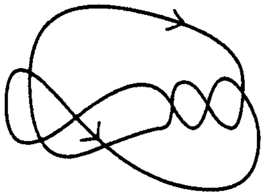


Other Projections

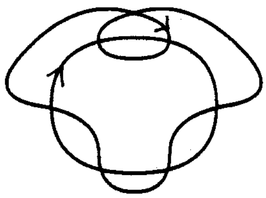
6 10| 2 14 12 4 8



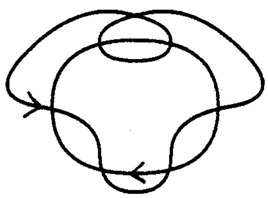
8 10 12| 4 2 14 6



8 10 12| 2 14 4 6



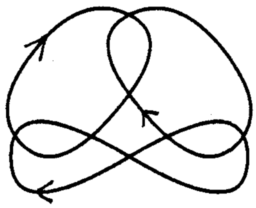
8 12 10| 2 6 14 4



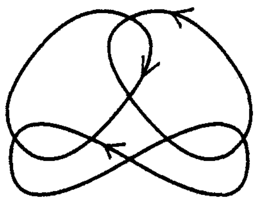
Projections of Oriented Links with 7 Crossings and 3 Components

Oriented Stand. Seq.

6 10| 2 12| 4 14 8

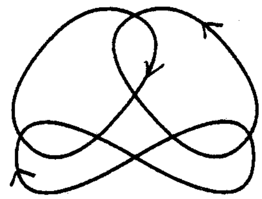


6 10| 4 12| 2 14 8



Other Projections

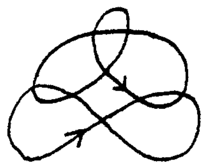
6 10| 4 14|12 2 8



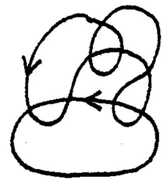
Projections of Oriented Links with 8 Crossings and 2 Components

Oriented Stand. Seq. | Other Projections

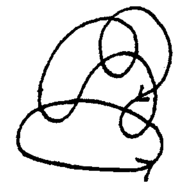
4 8 12| 2 14 6 16 10



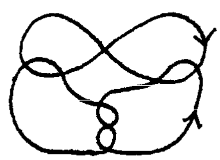
6 10| 2 14 12 4 16 8



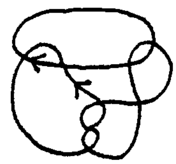
6 10| 2 16 14 4 8 12



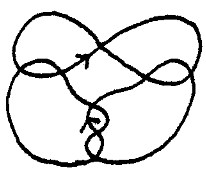
4 8 12| 2 14 16 6 10



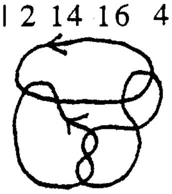
6 12| 4 14 16 2 10 8



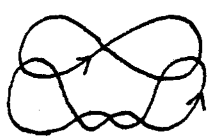
4 8 14|12 2 16 6 10



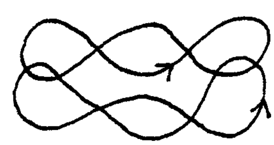
6 12| 2 14 16 4 10 8



4 8 12| 2 16 14 6 10



4 10 8 14| 2 16 6 12



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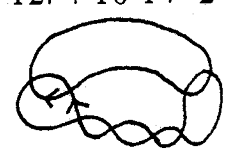
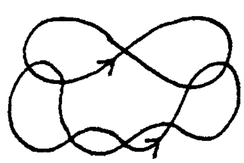


4 8 14|10 2 16 6 12



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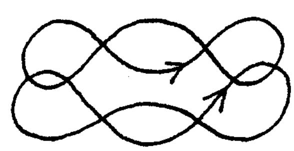

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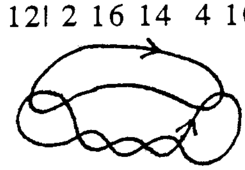

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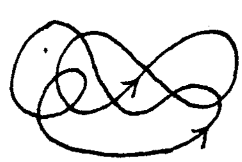

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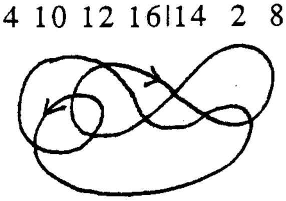
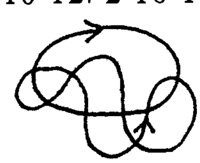


8 10 12| 2 16 14 6 4



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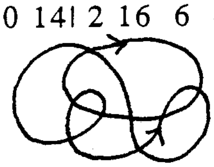
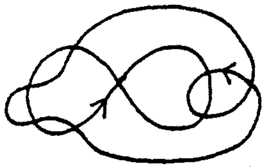


4 10 12 14| 2 16 6 8



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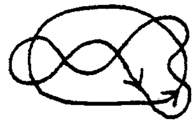

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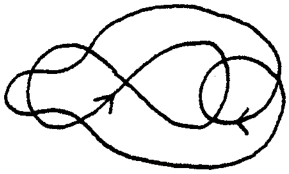

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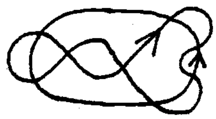

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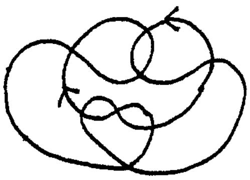

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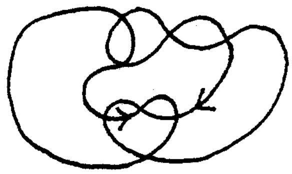

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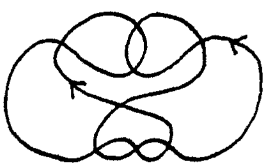

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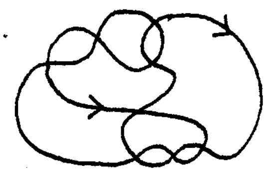

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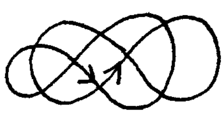

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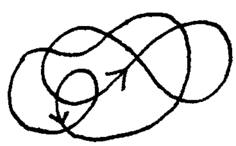

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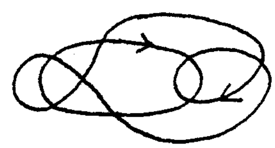

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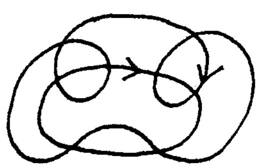

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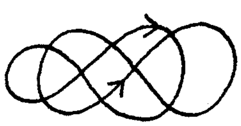

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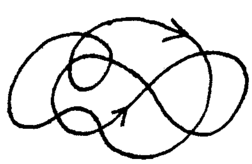

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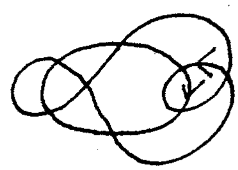
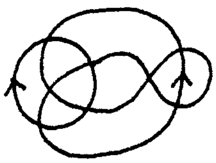


6 8|10 14 12 16 2 4



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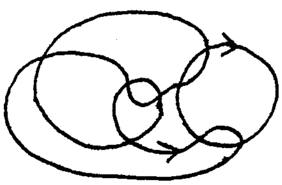

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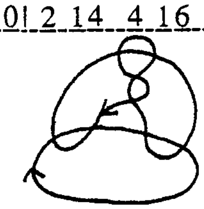
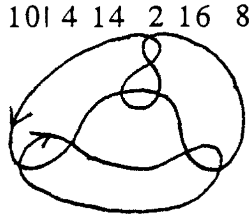


6 10| 4 14 2 16 8 12



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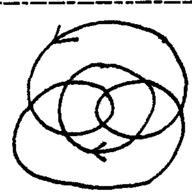

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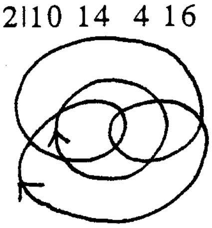
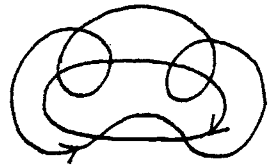


8 14 10| 2 12 6 16 4



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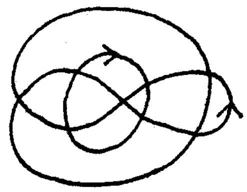

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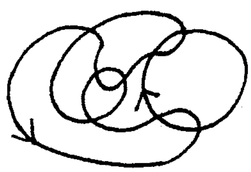
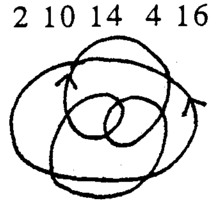
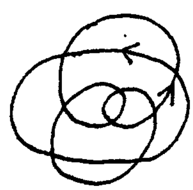


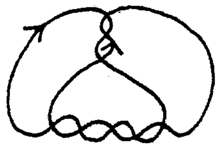
6 12| 2 10 14 4 16 8



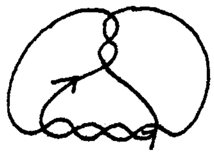
6 12| 4 10 14 2 16 8



10 12 14 16| 2 8 4 6



10 12 16 14| 4 2 8 6



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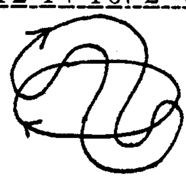

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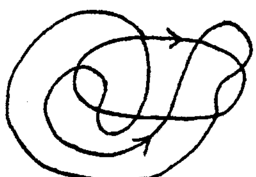
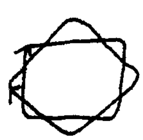
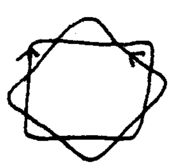


10 12 14 16| 8 2 4 6



10 16 14 12| 2 8 6 4



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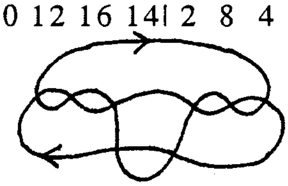
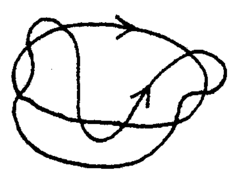


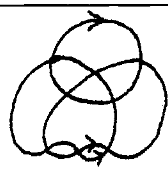
10 12 16 14| 4 2 6 8



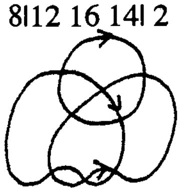
Projections of Oriented Links with 8 Crossings and 3 Components

Oriented Stand. Seq.   Other Projections

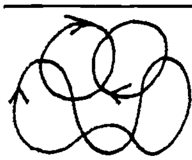
6 8|12 14 16|10 2 4



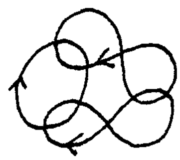
6 8|12 16 14|2 10 4



6 10|2 14|4 16 8 12



6 12|2 10 14|4 16 8



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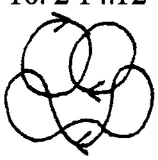

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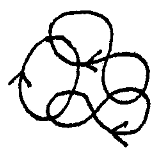

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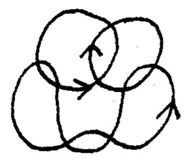

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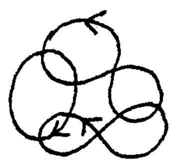
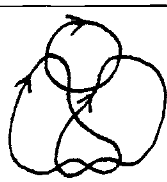
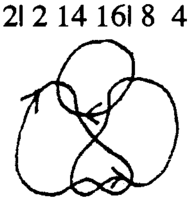


6 12|2 14 16|4 10 8



6 12|2 14 16|8 4 10



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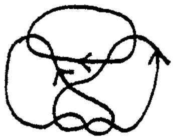

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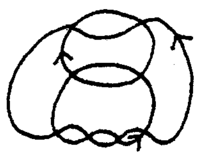

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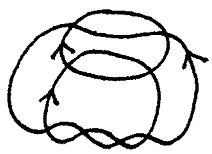
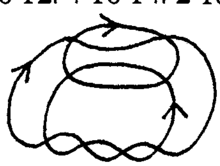
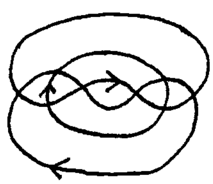


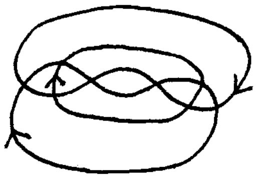
6 12| 4 16 14| 2 10 8



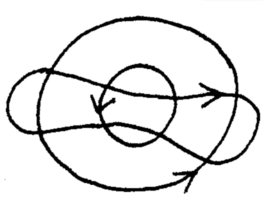
6 12|14 16 2| 8 10 4



6 12|16 14 2|10 8 4



10 14|12 16| 2 6 4 8



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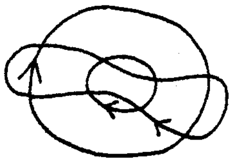

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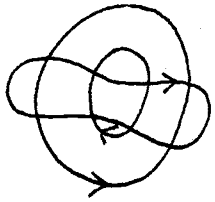

Projections of Oriented Links with 8 Crossings and 4 Components

Oriented Stand. Seq.	Other Projections

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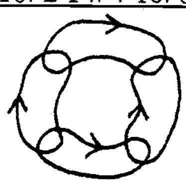
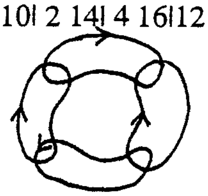
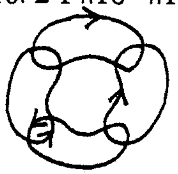


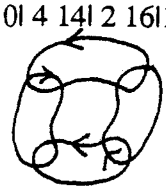
6 10| 2 14| 4 16|12 8



6 10| 2 14|16 4|12 8



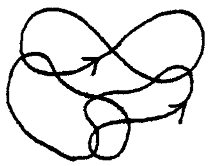
6 10| 4 14| 2 16|12 8



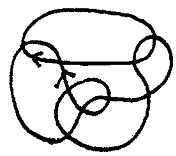
Projections of Oriented Links with 9 Crossings and 2 Components

Oriented Stand. Seq. | Other Projections

4 8 12| 2 14 16 6 18 10



6 12| 4 14 16 2 10 18 8



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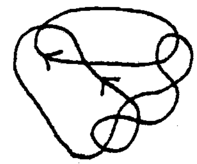
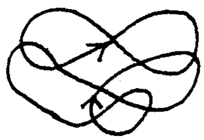
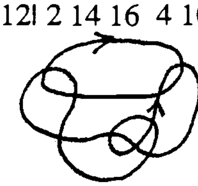


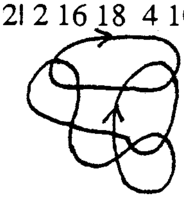
4 8 14|12 2 16 6 18 10



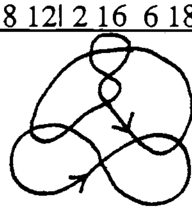
6 12| 2 14 16 4 10 18 8



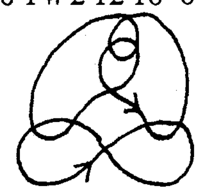
6 12| 2 16 18 4 10 8 14



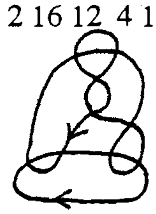
4 8 12| 2 16 6 18 10 14



4 8 14| 2 12 16 6 18 10



6 10| 2 16 12 4 18 8 14



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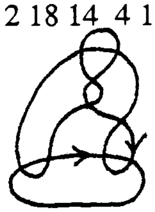

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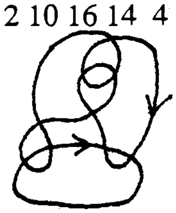

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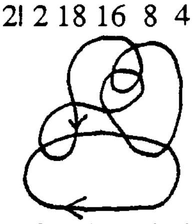

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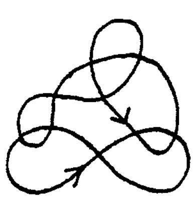

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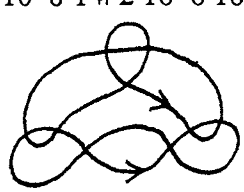

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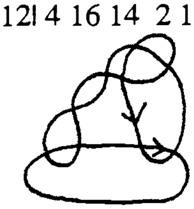

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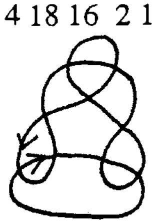

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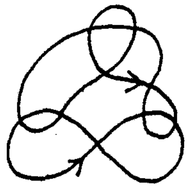

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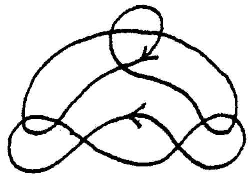

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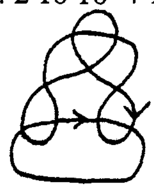

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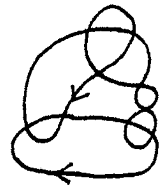

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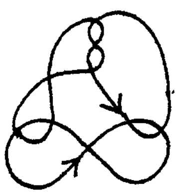

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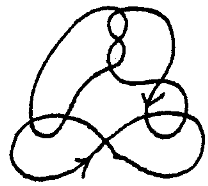
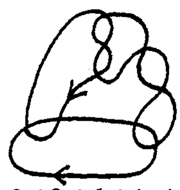


6 12| 2 16 18 14 4 10 8



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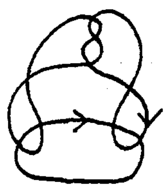

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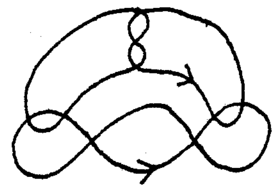

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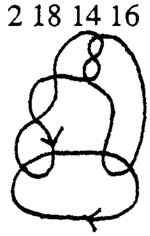

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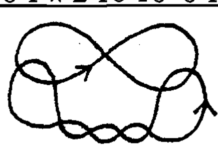

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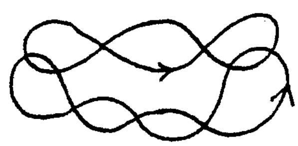

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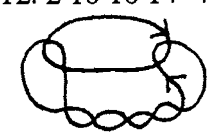

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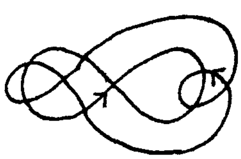

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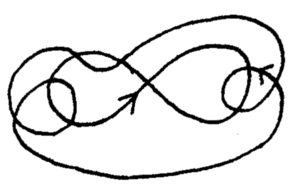

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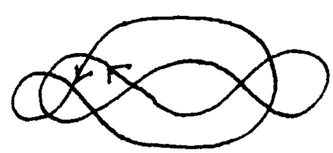

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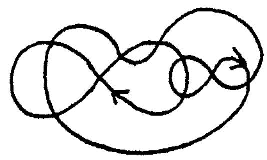

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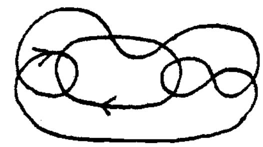

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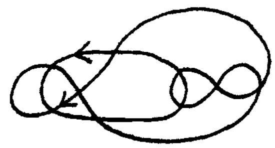

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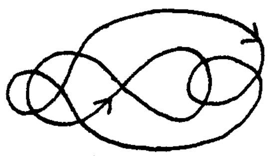

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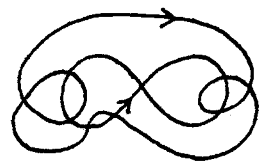

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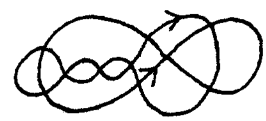

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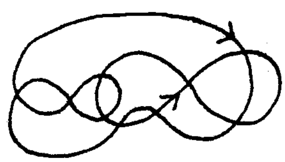

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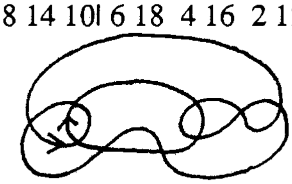

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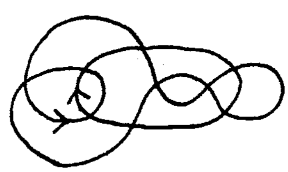

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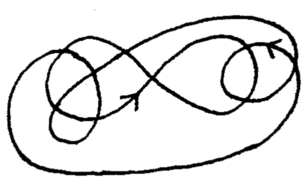

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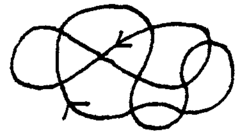

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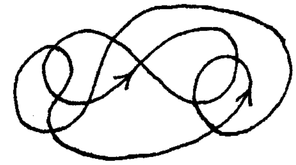

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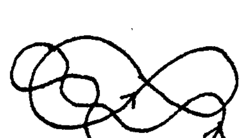

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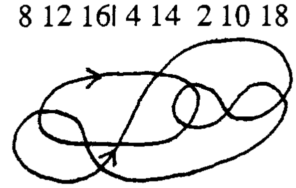

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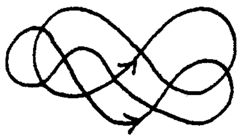

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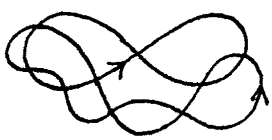

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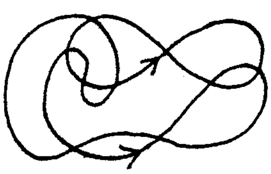

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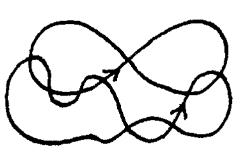

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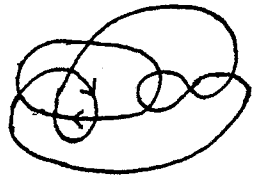

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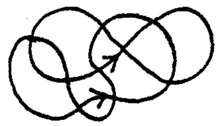

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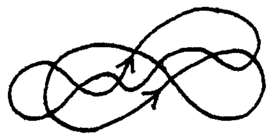

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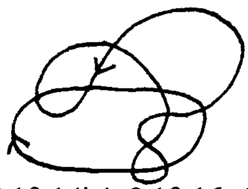

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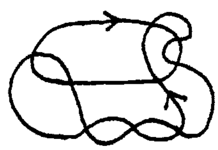

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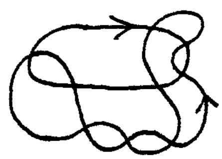

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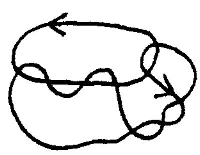

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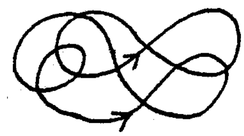

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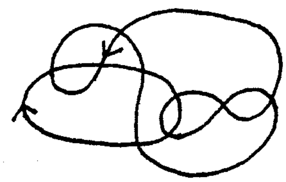

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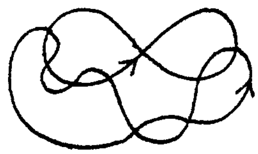

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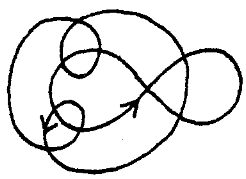

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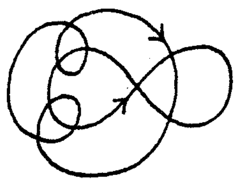

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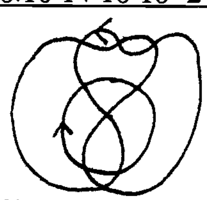

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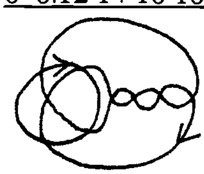

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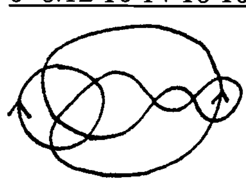

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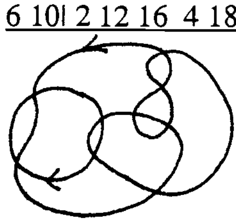

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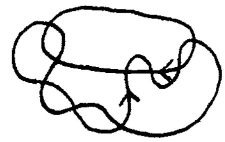

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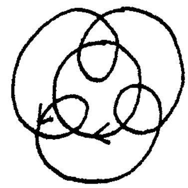

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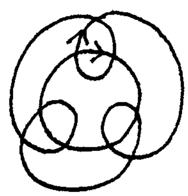

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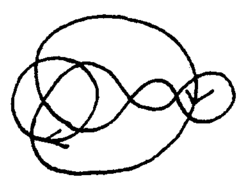

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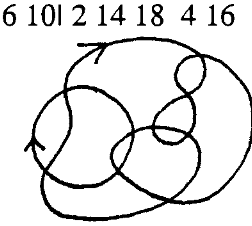

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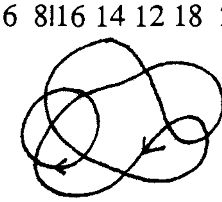

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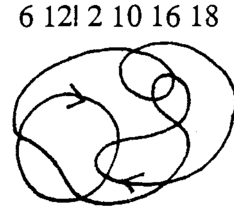

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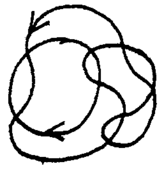

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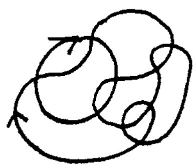

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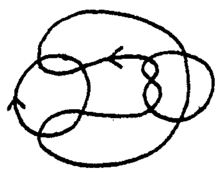

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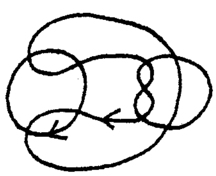

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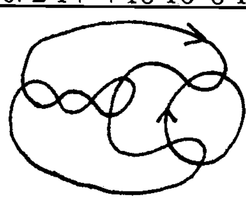

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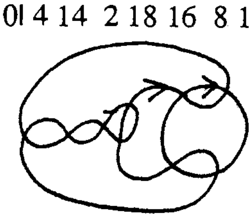

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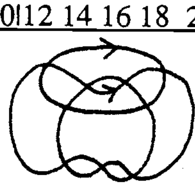

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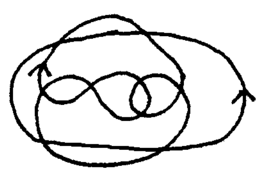

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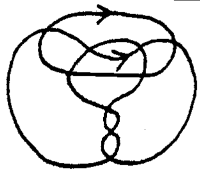

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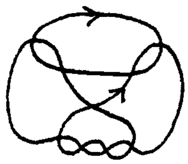

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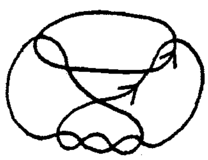

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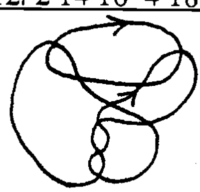

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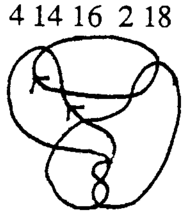

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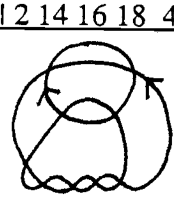

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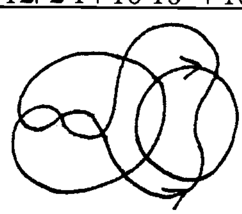

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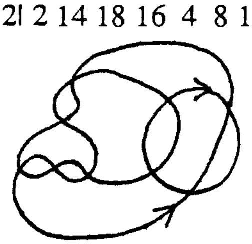

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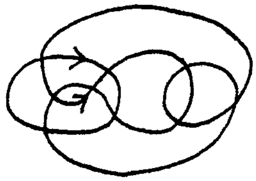

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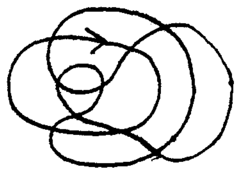

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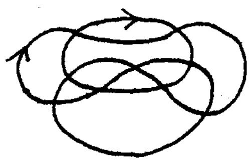

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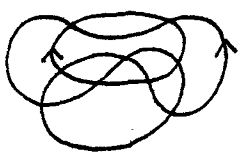

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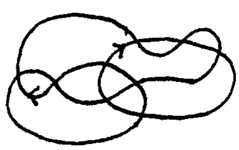

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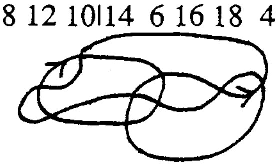

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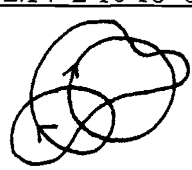

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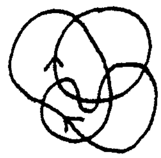

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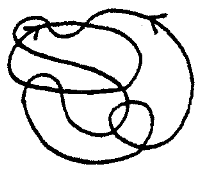

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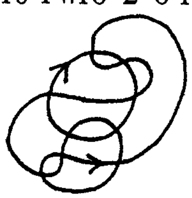

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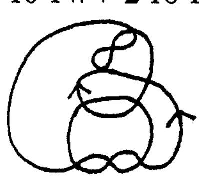

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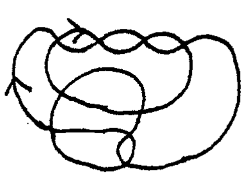

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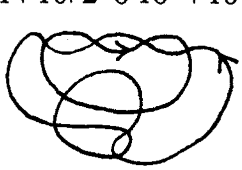

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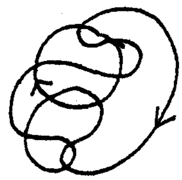

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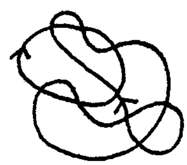

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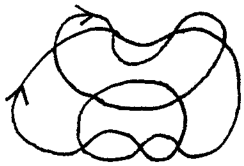

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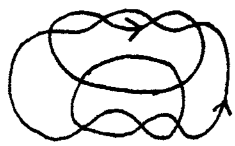

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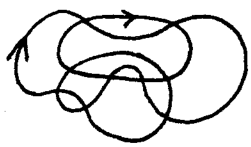

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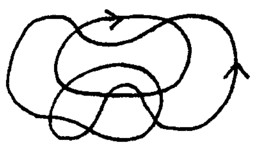

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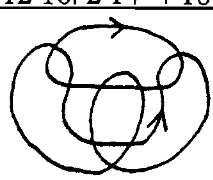

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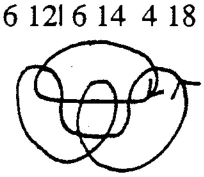

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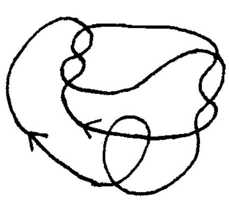

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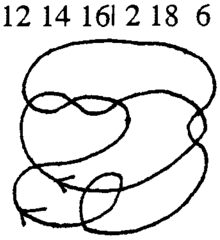

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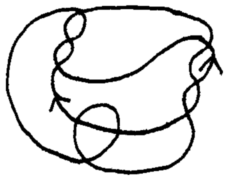

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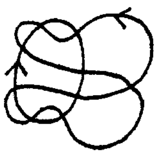

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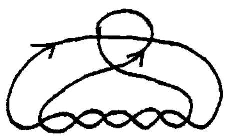

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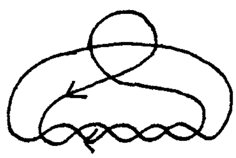

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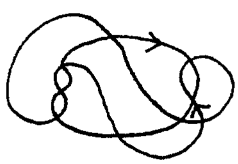

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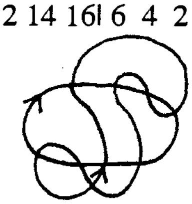

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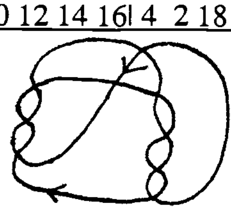

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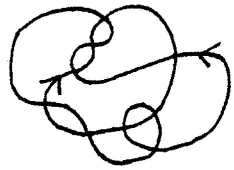

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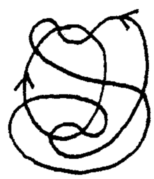

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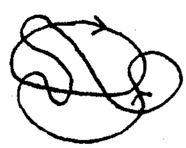

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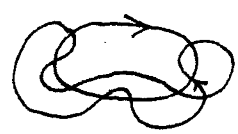

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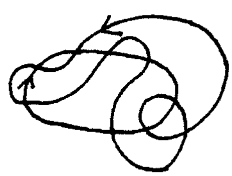

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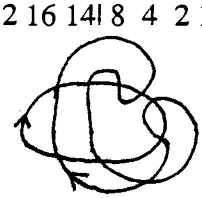

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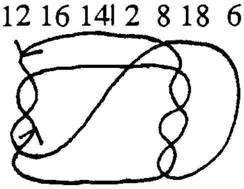

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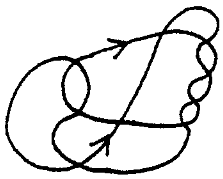

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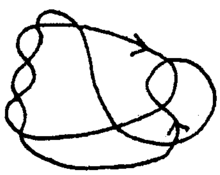

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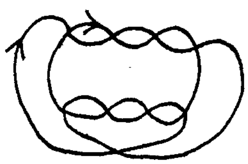

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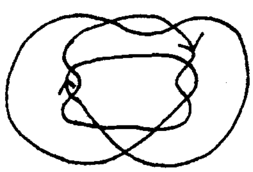

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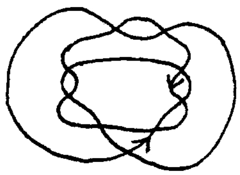

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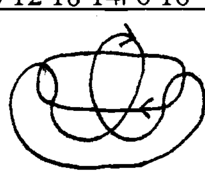

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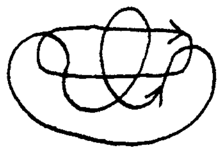

Projections of Oriented Links with 9 Crossings and 3 Components

Oriented Stand. Seq. | Other Projections

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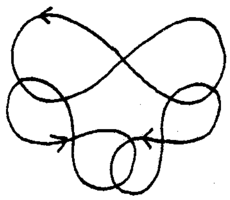

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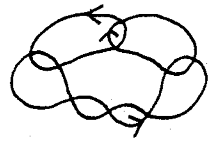

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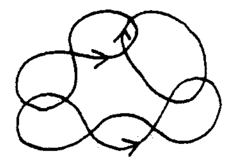

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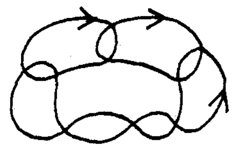

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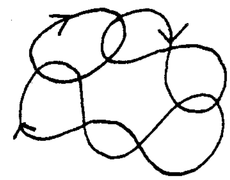

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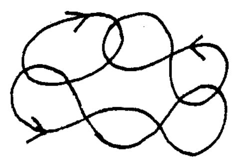

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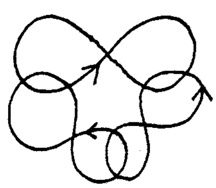

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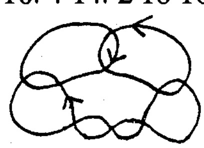

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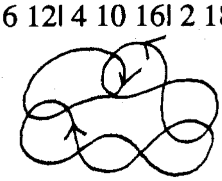

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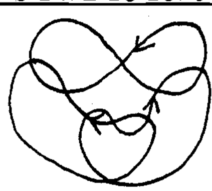

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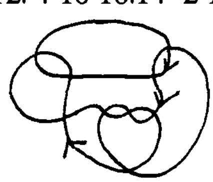

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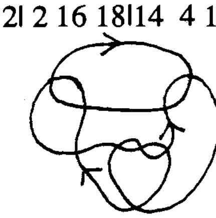

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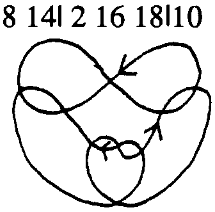

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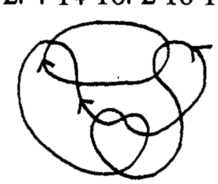

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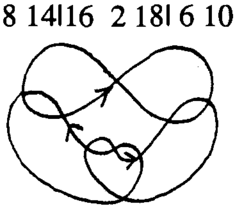

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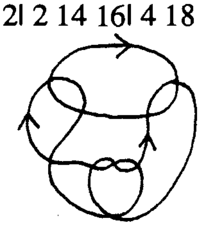

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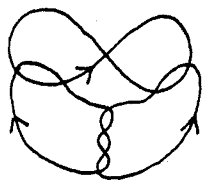

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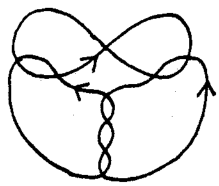

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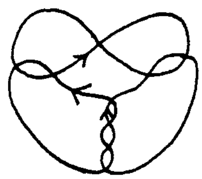

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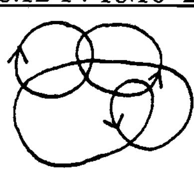

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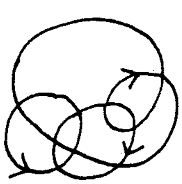

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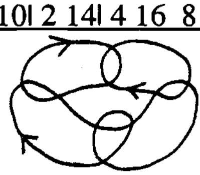

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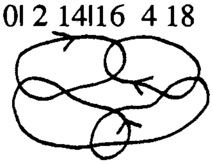

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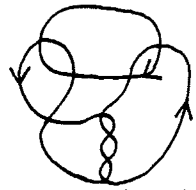

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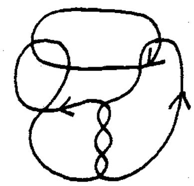

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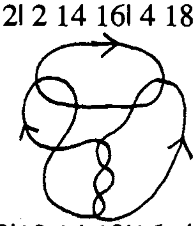

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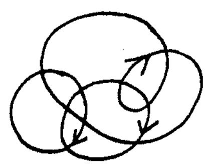

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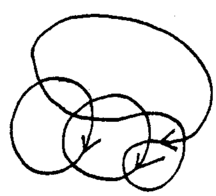

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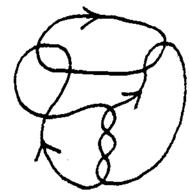

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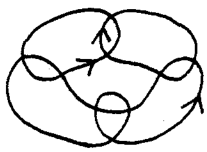

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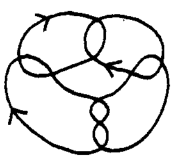

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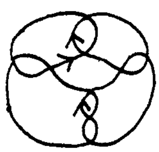

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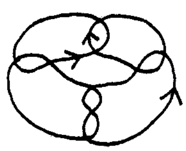

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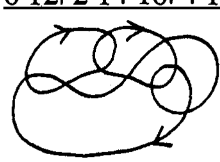

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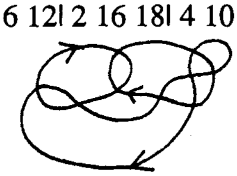

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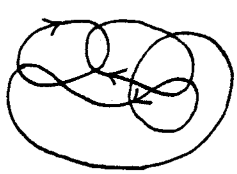

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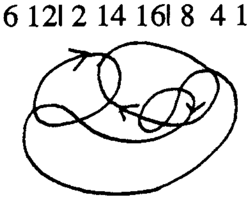

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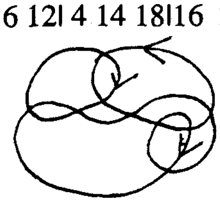

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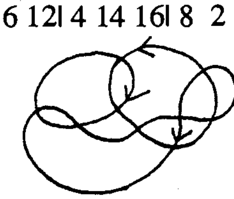

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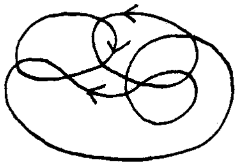

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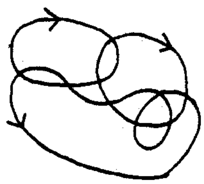

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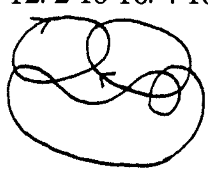

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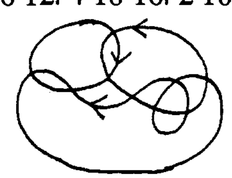

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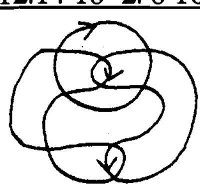

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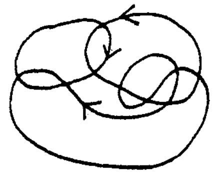

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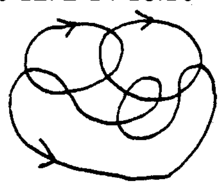
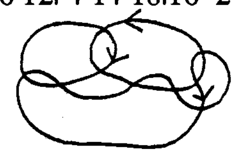
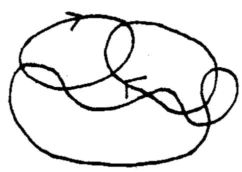


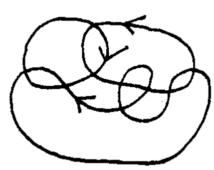
6 12| 4 14 18|16 2 8 10



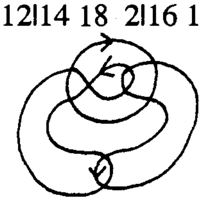
6 12| 2 16 14| 4 10 18 8



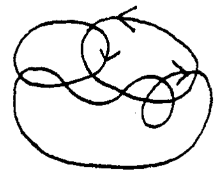
6 12| 4 16 14| 2 10 18 8



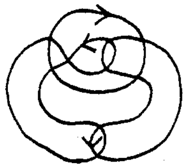
6 12|14 18 2|16 10 4 8



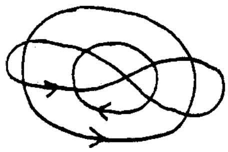
6 12| 4 14 16|10 2 18 8



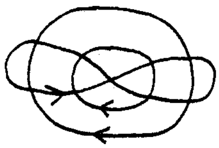
6 12|18 14 2|10 16 8 4



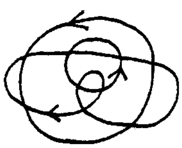
10 12|14 18| 6 16 8 2 4



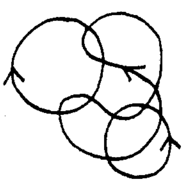
10 12|14 18| 6 16 8 4 2



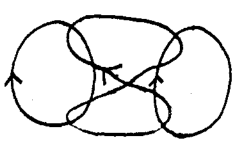
10 14|12 16| 2 6 4 18 8



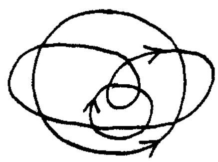
10 14|12 16| 2 18 6 4 8



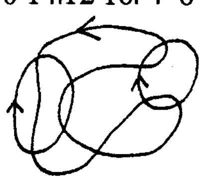
10 14|12 16| 2 6 18 4 8



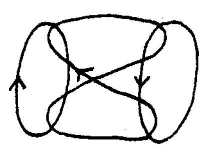
10 14|12 16| 2 8 4 18 6



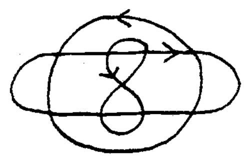
10 14|12 16| 4 6 2 18 8



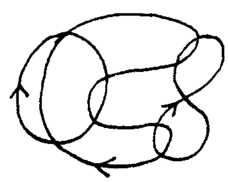
10 14|12 16| 2 8 18 4 6



12 16| 8 14 18| 2 6 4 10



12 16| 8 14 18| 4 6 2 10



<u>Projections of Oriented Links with 9 Crossings and 4 Components</u>

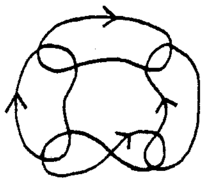
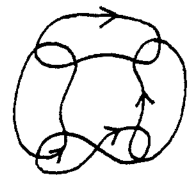
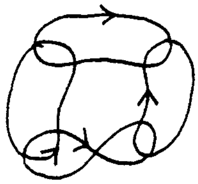
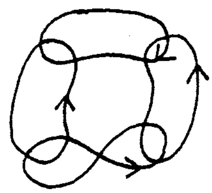
<table>
<tr><th><u>Oriented Stand. Seq.</u></th><th><u>Other Projections</u></th></tr>
<tr><td><u>6 10| 2 14| 4 16| 8 18 12</u>
</td><td>6 10| 2 14|16 4| 8 18 12
</td></tr>
<tr><td>6 10| 2 14|18 4|16 8 12
</td><td>6 10| 4 14| 2 16| 8 18 12
</td></tr>
</table>

Table of Invariants

Oriented Alternating Links with 2 Crossings and 2 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
4| 2	{ -1}(* 1 -1)(* 1)	1	2
	{ -1}(-1 -1 *)(1 [0])(1 1 *)		
	{ 1/2}(-1 0 -1)		
	{-1}(0 1)		

Oriented Alternating Links with 4 crossings and 2 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
6 8| 2 4	{ -1}(* 0 1 -1)(* 1 1)	2	4
	{ -1}(1 1 0 *)(-1 0 [0])(-3 -2 1 *)(1 1 [0])(1 1 0 *)		
	{ 1/2}(-1 1 -1 0 -1)		
	{ -1}(0 2)		
6 8| 4 2	{ -1}(1 -1 0 *)(1 -3 0 *)(-1 0 *)	2	4
	{ -1}(* 0 1 1)([0] 0 -1)(* 0 -3 -2 1)([0] 0 1 1)(* 0 1 1)		
	{ -11/2}(-1 1 -1 0 -1)		
	{ -1}(0 -2 -1)		

Oriented Alternating Links with 5 crossings and 2 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
6 8| 2 10 4	{ -1}(1 * -1)(-1 2 * -1)(1 *)	0	-1
	{ -1}(1 * 1)([-1])(-2 * -4 -2)([-1] 0 1)(1 * 3 2)([1] 1)		
	{ -7/2}(1 -2 1 -2 1 -1)		
	{ -1}(0 0 1)		

Oriented Alternating Links with 6 crossings and 2 Components

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td>4 8 10| 2 12 6</td><td>{ -1}(* 0 1 -1)(* 0 2 1 -1)(* 0 1 1)
{ -1}(1 1 0 *)(-1 0[0])(-1 0 -1 -2 0 *)(-3 -3 0 0[0])(1 0 0 1 0 *)
(2 3 1 0[0])(1 1 0 0 *)
{ 3/2} (-1 1 -3 2 -2 2 -1)
{ -1} (0 2 2)</td><td>2</td><td>6</td></tr>
<tr><td>4 8 12|10 2 6</td><td>{ -1}(1 -1 0 *)(-2 1 * -1)(1 *)
{ -1}(* 0 1 1)([0] 0 -1)(-1 * 0 -1 -2)([-3] -3)(1 * 0 0 1)([2] 3 1)(* 1 1)
{ -9/2} (-1 1 -3 2 -2 2 -1)
{ -1} (0 -2 1)</td><td>-2</td><td>-2</td></tr>
<tr><td>8 10 12| 2 6 4</td><td>{ -1}(* 0 0 1 -1)(* 0 2 2 -1)(* 0 1 1)
{ -1}(-1 -1 0 0 *)(1 0 0[0])(-2 3 3 -2 0 *)(-1 -2 -1 0[0])(1 -2 -2 1 0 *)(1 2 1 0[0])
(1 1 0 0 *)
{ 3/2} (-1 1 -2 2 -2 1 -1)
{ -1} (0 3 2)</td><td>3</td><td>6</td></tr>
<tr><td>8 10 12| 6 2 4</td><td>{ -1}(* 0 0 1 -1)(* 0 0 6 -3)(* 0 0 5 -1)(* 0 0 1)
{ -1}(-1 -1 0 0 *)(1 0 0[0])(1 -1 4 6 0 0 *)(1 -2 -3 0 0[0])
(1 -4 -5 0 0 *)(1 1 0 0[0])(1 1 0 0 *)
{ 5/2} (-1 0 -1 1 -1 1 -1)
{ -1} (0 3 4 1)</td><td>3</td><td>6</td></tr>
<tr><td>8 12 10| 2 6 4</td><td>{ -1}(* 0 0 1 -1)(* 1 1 1)
{ -1}(-1 -1 0 0 *)(1 0 0[0])(6 4 -1 1 *)(-3 -2 1[0])(-5 -4 1 0 *)(1 1 0[0])(1 1 0 0 *)
{ 1/2} (-1 1 -1 1 -1 0 -1)
{ -1} (0 3)</td><td>3</td><td>6</td></tr>
</table>

<u>Oriented Alternating Links with 6 crossings and 3 Components</u>

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td>6 8|10 12|2 4</td><td>{ -2}(1[-2] 1)([0])(-1[2] -1)([1])</td><td>0 0</td><td>0</td></tr>
<tr><td></td><td>{ -2}(1[2] 1)(-2 * -2)([1])()(-4[-8] -4)(1 -1 * -1 1)(3[6] 3)(2 * 2)</td><td>0</td><td></td></tr>
<tr><td></td><td>{ -6/2} (-1 3 -2 4 -2 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 0 1)</td><td></td><td></td></tr>
<tr><td>6 10| 2 12| 4 8</td><td>{ -2}([0] 0 1 -2 1)([0] 0 3 -3)([0] 1 2)</td><td>1 1</td><td>6</td></tr>
<tr><td></td><td>{ -2}(-1 -2 -1 0[0])(2 2 0 0 *)(3 5 3 0[0])(-3 -3 0 0 *)(-3 -9 -5 1[0])(-1 1 2 0 *)
(1 4 3 0[0])(1 1 0 0 *)</td><td>
1</td><td></td></tr>
<tr><td></td><td>{ 2/2} (1 -2 3 -1 3 -1 1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 3)</td><td></td><td></td></tr>
<tr><td>6 10| 2 12| 8 4</td><td>{ -2}(1 -2[1])(1 -3[2])(1 -3[1])(-1[0])</td><td>1 -1</td><td>-2</td></tr>
<tr><td></td><td>{ -2}([-1] -2 -1)(* 2 2)([3] 5 3)(* -3 -3)([-3] -9 -5 1)(* -1 1 2)([1] 4 3)(* 1 1)</td><td>-1</td><td></td></tr>
<tr><td></td><td>{ -10/2} (1 -2 3 -1 3 -1 1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 -1 -1)</td><td></td><td></td></tr>
</table>

Oriented Alternating Links with 7 crossings and 2 Components

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td>4 8 12| 2 14 6 10</td><td>{ -1}(1 * -1)(1 * -1 -1 1)(* -1 -1)
{ -1}(1 * 1)([-1])(2 2 -2 * -2)(-2 0 2[0])(-5 -5 1 * 1)(1 -1 -1[1])(2 3 1 *)(1 1[0])
{ -3/2} (-1 1 -3 3 -3 2 -2 1)
{ -1} (0 0 -2)</td><td>0</td><td>3</td></tr>
<tr><td>6 8|10 12 14 2 4</td><td>{ -1}(1 * -1)(1 -2 * 1)(1 -3 * 1)(-1 *)
{ -1}(1 * 1)([-1])(2 * 4 2)(-1[-2] -3 -2)(-6 * -12 -5 1 (1[-1] 1 3)(3 * 7 4)([2] 2)
{ -9/2} (-1 3 -4 4 -5 3 -3 1)
{ -1} (0 0 -1 -1)</td><td>0</td><td>-1</td></tr>
<tr><td>6 10| 2 12 4 14 8</td><td>{ -1}(* 0 0 2 -3 1)(* 0 1 4 -3)(* 0 1 2)
{ -1}(-1 -3 -2 0 0 *)(1 3 3 0 0[0])(2 8 5 -1 0 *)(-2 -7 -6 -1 0[0])
(-4 -10 -5 1 0 *)(1 2 3 2 0[0])(2 5 3 0 0 *)(1 1 0 0[0])
{ 3/2} (-1 2 -4 3 -4 3 -2 1)
{ -1} (0 2 3)</td><td>2</td><td>7</td></tr>
<tr><td>6 10| 4 12 2 14 8</td><td>{ -1}(-1 * 3 -2)(-2 * 6 -2)(-1 * 4 -1)(* 1)
{ -1}(-2 -3 * -1)(3[3] 1)(-1 5 8 * 2)(-1 -6[-7] -2)(1 -5 -10 * -4)(2 3[2] 1)(3 5 * 2)
(1[1])
{ -5/2} (1 -2 3 -4 3 -4 2 -1)
{ -1} (0 2 2 1)</td><td>2</td><td>1</td></tr>
<tr><td>6 10| 2 12 14 4 8</td><td>{ -1}(-1 3 -2 *)(-2 5 -3 *)(-1 4 -1 *)(1 0 *)
{ -1}(* -2 -3 -1)([0] 3 3 1)(* 5 9 4)([0] -2 -6 -3 1)(* -4 -12 -6 2)
([0] -2 1 3)(* 1 4 3)([0] 1 1)
{ -13/2} (1 -2 3 -3 2 -3 1 -1)
{ -1} (0 0 2 1)</td><td>0</td><td>-3</td></tr>
</table>

<table>
<tr><td>8 10 12| 2 6 14 4</td><td>{ -1}(* 1 -1)(1 * 0 -1 1)(* -1 -1)
{ -1}(-1 -1 *)(1[0])(3 6 2 * -1)(-2 -2 -2[-2])(-5 -8 -2 * 1)(1 0 1[2])(2 4 2 *)
(1 1[0])
{ -3/2} (-1 2 -3 3 -4 2 -2 1)
{ -1} (0 1 -2)</td><td>1</td><td>3</td></tr>
<tr><td>8 10 12| 4 2 14 6</td><td>{ -1}(* 1 -1)(-2 * 5 -2)(-1 * 4 -1)(* 1)
{ -1}(-1 -1 *)(1[0])(-1 2 6 * 3)(-2 -2[-2] -2)(1 -2 -8 * -5)(2 1[0] 1)(2 4 * 2)(1[1])
{ -5/2} (1 -2 2 -4 3 -3 2 -1)
{ -1} (0 1 2 1)</td><td>1</td><td>1</td></tr>
<tr><td>8 10 12| 2 14 4 6</td><td>{ -1}(1 -1 *)(-2 4 -3 *)(-1 4 -1 *)(1 0 *)
{ -1}(* -1 -1)([0] 1)(* 4 6 1 -1)([0] 1 -1 -1 1)(* -4 -9 -3 2)([0] -3 -1 2)(* 1 3 2)
([0] 1 1)
{ -13/2} (1 -2 2 -3 2 -2 1 -1)
{ -1} (0 -1 2 1)</td><td>-1</td><td>-3</td></tr>
<tr><td>8 12 10| 2 6 14 4</td><td>{ -1}(* 1 -1)(-1 1 * 2 -1)(1 * 1)
{ -1}(-1 -1 *)(1[0])(4 6 * 1 -1)(1[-1] -1 1)(-4 -9 * -3 2)(-3[-1] 2)(1 3 * 2)(1[1])
{ -7/2} (1 -2 2 -3 2 -2 1 -1)
{ -1} (0 1 2)</td><td>1</td><td>1</td></tr>
</table>

Oriented Alternating Links with 7 crossings and 3 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
6 10| 2 12| 4 14 8	{ -2}([1] -2 1)([2] -3 1)(-1[2] -2)([1])	1 1	1
	{ -2}(-1 -2[-1])(2 2 *)(3 5[3])(-3 -3 *)(-3 -5[-5] -3)(-4 * -3 1)		
	(1 1[3] 3)(1 4 * 3)(1[1])	-1	
	{ -6/2} (-1 3 -3 4 -3 4 -1 1)		
	{ -2} (0 0 -1 1)		
6 10| 4 12| 2 14 8	{ -2}([0] 0 1 -2 1)([0] 0 3 -3)([0] 0 3 1 -1)([0] 0 1 1)	1 1	7
	{ -2}(-1 -2 -1 0[0])(2 2 0 0 *)(3 5 3 0[0])(-3 -3 0 0 *)(-3 -5 -5 -3 0[0])		
	(1 -3 -4 0 0 0 *)(3 3 1 1 0[0])(3 4 1 0 0 *)(1 1 0 0[0])	1	
	{ 4/2} (1 -1 4 -3 4 -3 3 -1)		
	{ -2} (0 0 3 2)		

Oriented Alternating Links with 8 crossings and 2 Components

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td>4 8 12| 2 14 6 16 10</td><td>{ -1}(-1 3 -2 *)(-1 4 -4 * 1)(2 -3 * 1)(-1 *)
{ -1}(* -2 -3 -1)([0] 3 3 1)(2 * 7 7 2)(-1[-1] -2 -4 -2)
(-5 * -11 -9 -3)(1[-3] -5 0 1)(3 * 5 4 2)([3] 5 2)(* 1 1)
{ -11/2} (1 -2 4 -6 5 -6 4 -3 1)
{ -1} (0 0 0 -1)</td><td>0</td><td>-2</td></tr>
<tr><td>4 8 12| 2 14 16 6 10</td><td>{ -1}(* 0 0 2 -3 1)(* 0 0 5 -2 -1)(* 0 0 4 2 -1)(* 0 0 1 1)
{ -1}(-1 -3 -2 0 0 *)(1 3 3 0 0[0])(1 2 6 5 0 0 *)(-2 -1 -2 -3 0 0[0])
(1 -4 -6 -5 -4 0 0 *)(3 -2 -6 -1 0 0[0])(4 3 0 1 0 0 *)(3 4 1 0 0[0])(1 1 0 0 0 *)
{ 5/2} (-1 1 -4 4 -5 5 -4 3 -1)
{ -1} (0 2 5 2)</td><td>2</td><td>8</td></tr>
<tr><td>4 8 14|12 2 16 6 10</td><td>{ -1}(2 -3 * 1)(1 -5 * 3 -1)(-2 * 3 -1)(* 1)
{ -1}(-1 * -3 -2)(1[3] 3)(1 2 * 6 5)(-2 -1[-2] -3)
(1 -4 -6 * -5 -4)(3 -2[-6] -1)(4 3 * 0 1)(3[4] 1)(1 * 1)
{ -7/2} (-1 1 -4 4 -5 5 -4 3 -1)
{ -1} (0 -2 0 1)</td><td>-2</td><td>0</td></tr>
<tr><td>4 8 12| 2 16 14 6 10</td><td>{ -1}(* 0 1 -1)(* 0 2 0 1 -1)(* 0 1 1 1)
{ -1}(1 1 0 *)(-1 0[0])(1 0 -1 -2 -2 0 *)(5 5 0 0 0[0])(-3 0 3 1 1 0 *)
(-7 -8 0 1 0[0])(1 -2 -2 1 0 0 *)(2 3 1 0 0[0])(1 1 0 0 0 *)
{ 3/2} (-1 1 -3 3 -4 3 -2 2 -1)
{ -1} (0 2 3)</td><td>2</td><td>8</td></tr>
<tr><td>4 8 14|10 2 16 6 12</td><td>{ -1}(1 -1 0 *)(-2 0 * 1 -1)(1 * 1)
{ -1}(* 0 1 1)([0] 0 -1)(1 0 * -1 -2 -2)(5[5])(-3 0 * 3 1 1)
(-7[-8] 0 1)(1 -2 * -2 1)(2[3] 1)(1 * 1)
{ -9/2} (-1 1 -3 3 -4 3 -2 2 -1)
{ -1} (0 -2 2)</td><td>-2</td><td>0</td></tr>
</table>

<table>
<tr><td>4 10 12 14|2 8 16 6</td><td>{ -1}(* 1 -1)(-1 * 4 -3 1)(-1 * 3 -2)(* 1)
{ -1}(-1 -1 *)(1[0])(2 7 7 * 2)(-1 0 2[0] -1)(-5 -13 -13 * -5)
(1 -3 -8[-3] 1)(3 6 6 * 3)(3 6[3])(1 1 *)
{ -5/2} (1 -3 4 -6 6 -6 4 -3 1)
{ -1} (0 1 0 1)</td><td>1</td><td>2</td></tr>
<tr><td>4 10 12 14|2 16 6 8</td><td>{ -1}(* 0 0 1 -1)(* 0 0 4 1 -2)(* 0 0 4 3 -1)(* 0 0 1 1)
{ -1}(-1 -1 0 0 *)(1 0 0[0])(-1 0 -2 1 4 0 0 *)(-2 3 4 -1 0 0[0])(1 -1 2 0 -4 0 0 *)
(2 -3 -7 -2 0 0[0])(2 -1 -2 1 0 0 *)(2 3 1 0 0[0])(1 1 0 0 0 *)
{ 5/2} (-1 1 -3 3 -4 4 -3 2 -1)
{ -1} (0 3 6 2)</td><td>3</td><td>8</td></tr>
<tr><td>4 10 16 14|12 2 8 6</td><td>{ -1}(1 -1 0 0 *)(-2 0 0 * -1)(1 1 *)
{ -1}(* 0 0 -1 -1)([0] 0 0 1)(-1 * 0 -2 1 4)([-2] 3 4 -1)
(1 * -1 2 0 -4)([2] -3 -7 -2)(* 2 -1 -2 1)([0] 2 3 1)(* 0 1 1)
{ -13/2} (-1 1 -3 3 -4 4 -3 2 -1)
{ -1} (0 -3 2)</td><td>-3</td><td>-4</td></tr>
<tr><td>4 10 12 14|2 16 8 6</td><td>{ -1}(* 0 0 1 -1)(* 0 1 3 0 -1)(* 0 1 2 1)
{ -1}(-1 -1 0 0 *)(1 0 0[0])(2 1 4 4 -1 0 *)(4 3 -2 -1 0[0])(-3 -4 -6 -4 1 0 *)
(-6 -8 0 2 0[0])(1 0 2 3 0 0 *)(2 4 2 0 0[0])(1 1 0 0 0 *)
{ 3/2} (-1 2 -4 4 -5 4 -3 2 -1)
{ -1} (0 3 4)</td><td>3</td><td>8</td></tr>
<tr><td>4 10 14 16|12 2 8 6</td><td>{ -1}(1 -1 0 0 *)(1 -4 2 -2 *)(-2 3 -1 *)(1 0 *)
{ -1}(* 0 0 -1 -1)([0] 0 0 1)(* 2 1 4 4 -1)([0] 4 3 -2 -1)
(* -3 -4 -6 -4 1)([0] -6 -8 0 2)(* 1 0 2 3)([0] 2 4 2)(* 0 1 1)
{ -15/2} (-1 2 -4 4 -5 4 -3 2 -1)
{ -1} (0 -3 0 1)</td><td>-3</td><td>-4</td></tr>
<tr><td>4 10 12 14|8 2 16 6</td><td>{ -1}(1 -1 *)(1 -4 * 3 -1)(-2 * 3 -1)(* 1)
{ -1}(* -1 -1)([0] 1)(2 6 * 7 3)(-2 -2[2] 2)(1 -4 -12 * -10 -3)
(3 -1[-9] -5)(4 5 * 2 1)(3[5] 2)(1 * 1)
{ -7/2} (-1 2 -4 4 -6 5 -4 3 -1)
{ -1} (0 -1 0 1)</td><td>-1</td><td>0</td></tr>
</table>

<table>
<tr><td>4 10 16 14| 2 8 6 12</td><td>{ -1}(* 1 -1)(* 2 -3 3 -1)(* 1 -3 2)(* 0 -1)
{ -1}(-1 -1 *)(1[0])(2 6 7 3 *)(-2 -2 2 2[0])(1 -4 -12 -10 -3 *)
(3 -1 -9 -5[0])(4 5 2 1 *)(3 5 2[0])(1 1 0 *)
{ -1/2} (-1 2 -4 4 -6 5 -4 3 -1)
{ -1} (0 1 0 -1)</td><td>1</td><td>4</td></tr>
<tr><td>6 8|10 14 12 16 2 4</td><td>{ -1}(1 * -1)(* 1 -2 1)(-1 * 2 -2)(* 1)
{ -1}(1 * 1)([-1])(2 4 2 *)(-1 2 5[2])(-4 -10 -11 * -5)
(1 -5 -14[-7] 1)(3 4 5 * 4)(4 9[5])(2 2 *)
{ -5/2} (1 -4 5 -7 7 -7 5 -3 1)
{ -1} (0 0 -1 1)</td><td>0</td><td>2</td></tr>
<tr><td>6 10| 2 12 14 4 16 8</td><td>{ -1}(1 -2 * 2 -1)(1 -4 * 4 -1)(1 -3 * 2)(-1 *)
{ -1}(-1 -2 * -2 -1)([-1])(3 10 * 10 3)(3[5] 1 -1)
(-3 -13 * -17 -6 1)(-5[-12] -4 3)(1 3 * 7 5)(2[6] 4)(1 * 1)
{ -9/2} (-1 3 -5 5 -6 5 -4 2 -1)
{ -1} (0 0 0 -1)</td><td>0</td><td>0</td></tr>
<tr><td>6 10| 2 14 4 16 8 12</td><td>{ -1}(* 1 -2 2 -1)(1 * 1 -3 3)(* -1 -2)
{ -1}(-1 -2 -2 -1 *)(-1 0[0])(3 8 9 3 * -1)(2 6 3[-1])(-3 -11 -12 -3 * 1)
(-5 -11 -4[2])(1 2 4 3 *)(2 5 3[0])(1 1 0 *)
{ -3/2} (-1 2 -4 5 -5 4 -4 2 -1)
{ -1} (0 2 -3)</td><td>2</td><td>4</td></tr>
<tr><td>6 10| 4 14 2 16 8 12</td><td>{ -1}(* 1 -2 2 -1)(* 2 -3 4 -1)(* 1 -3 2)(* 0 -1)
{ -1}(-1 -2 -2 -1 *)(-1 0[0])(-1 3 9 8 3 *)(-1 3 6 2[0])(1 -3 -12 -11 -3 *)
(2 -4 -11 -5[0])(3 4 2 1 *)(3 5 2[0])(1 1 0 *)
{ -1/2} (-1 2 -4 4 -5 5 -4 2 -1)
{ -1} (0 2 0 -1)</td><td>2</td><td>4</td></tr>
</table>

<table>
<tr><td>6 12|10 14 2 16 8 4</td><td>{ -1}(* 0 0 2 -3 1)(* 0 0 5 -2 -1)(* 0 1 3 1)
{ -1}(-1 -3 -2 0 0 *)(1 3 3 0 0[0])(1 2 6 5 0 0 *)(5 4 -1 0 0[0])(-2 -5 -12 -8 1 0 *)
(-8 -16 -5 3 0[0])(1 -1 4 6 0 0 *)(3 8 5 0 0[0])(2 2 0 0 0 *)
{ 3/2} (-1 3 -6 5 -7 6 -4 3 -1)
{ -1} (0 2 5)</td><td>2</td><td>8</td></tr>
<tr><td>6 12|10 14 4 16 8 2</td><td>{ -1}(-1 * 3 -2)(1 -3 * 5 -1)(-2 * 3 -1)(* 1)
{ -1}(-2 -3 * -1)(3[3] 1)(5 6 * 2 1)(-1[4] 5)(1 -8 -12 * -5 -2)
(3 -5[-16] -8)(6 4 * -1 1)(5[8] 3)(2 * 2)
{ -7/2} (-1 3 -4 6 -7 5 -6 3 -1)
{ -1} (0 2 0 1)</td><td>2</td><td>0</td></tr>
<tr><td>10 12 14 16| 2 8 4 6</td><td>{ -1}(* 0 0 0 1 -1)(* 0 0 3 4 -3)(* 0 0 4 4 -1)(* 0 0 1 1)
{ -1}(1 1 0 0 0 *)(-1 0 0 0[0])(-2 1 -5 -5 3 0 0 *)(-1 3 5 1 0 0[0])(1 -1 8 6 -4 0 0 *)
(1 -2 -6 -3 0 0[0])(1 -4 -4 1 0 0 *)(1 2 1 0 0[0])(1 1 0 0 0 *)
{ 5/2} (-1 1 -2 2 -3 3 -2 1 -1)
{ -1} (0 4 7 2)</td><td>4</td><td>8</td></tr>
<tr><td>10 12 16 14| 4 2 8 6</td><td>{ -1}(* 0 0 0 1 -1)(* 0 2 1 2 -1)(* 0 1 1 1)
{ -1}(1 1 0 0 0 *)(-1 0 0 0[0])(3 -5 -5 1 -2 0 *)(1 5 3 -1 0[0]) (-4 6 8 -1 1 0 *)
(-3 -6 -2 1 0[0])(1 -4 -4 1 0 0 *)(1 2 1 0 0[0])(1 1 0 0 0 *)
{ 3/2} (-1 1 -2 3 -3 2 -2 1 -1)
{ -1} (0 4 3)</td><td>4</td><td>8</td></tr>
<tr><td>10 12 14 16| 2 8 6 4</td><td>{ -1}(* 0 0 0 1 -1)(* 0 1 3 1 -1)(* 0 1 2 1)
{ -1}(1 1 0 0 0 *)(-1 0 0 0[0])(4 -3 -3 3 -1 0 *)(1 0 -1 0[0])(-4 2 1 -4 1 0 *)
(-2 -5 -1 2 0[0])(1 -2 0 3 0 0 *)(1 3 2 0 0[0])(1 1 0 0 0 *)
{ 3/2} (-1 2 -4 4 -4 4 -3 1 -1)
{ -1} (0 4 4)</td><td>4</td><td>8</td></tr>
<tr><td>10 12 14 16| 4 2 8 6</td><td>{ -1}(* 0 0 0 1 -1)(* 0 0 4 2 -2)(* 0 0 4 3 -1)(* 0 0 1 1)
{ -1}(1 1 0 0 0 *)(-1 0 0 0[0])(-1 3 -3 -3 4 0 0 *)(-1 0 1 0 0 0[0])
(1 -4 1 2 -4 0 0 *)(2 -1 -5 -2 0 0[0])(3 0 -2 1 0 0 *)(2 3 1 0 0[0])(1 1 0 0 0 *)
{ 5/2} (-1 1 -3 4 -4 4 -4 2 -1)
{ -1} (0 4 6 2)</td><td>4</td><td>8</td></tr>
</table>

<u>10 12 14 16| 8 2 4 6</u>	{ -1}(* 0 0 0 1 -1)(* 0 0 0 10 -6)(* 0 0 0 15 -5)(* 0 0 0 7 -1)(* 0 0 0 1) { -1}(1 1 0 0 0 *)(-1 0 0 0[0])(1 -1 1 -7 -10 0 0 0 *)(1 -2 3 6 0 0 0[0]) (1 -3 11 15 0 0 0 *)(1 -4 -5 0 0 0[0])(1 -6 -7 0 0 0 *)(1 1 0 0 0[0])(1 1 0 0 0 *) { 7/2} (-1 0 -1 1 -1 1 -1 1 -1) { -1} (0 4 10 6 1)	4	8
10 16 14 12| 2 8 6 4	{ -1}(* 0 0 0 1 -1)(* 1 1 1 1) { -1}(1 1 0 0 0 *)(-1 0 0 0[0])(-10 -7 1 -1 1 *)(6 3 -2 1[0])(15 11 -3 1 0 *) (-5 -4 1 0[0])(-7 -6 1 0 0 *)(1 1 0 0[0])(1 1 0 0 0 *) { 1/2} (-1 1 -1 1 -1 1 -1 0 -1) { -1} (0 4)	4	8

Oriented Alternating Links with 8 crossings and 3 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
6 8|12 14 16|10 2 4	{ -2}(1 -2[1])(-1[1])(-2 4[-2])(-1 4[-1])(1[0])	0 0	-2
	{ -2}([-1] -2 -1)(* 2 2)([1] 1 1)(* -1 -1)([6] 10 2 -2)		
	(-2 * -1 -3 -3 1)([-9] -17 -5 3)(1 * -3 0 4)([3] 7 4)(* 2 2)	-1	
	{ -12/2} (-1 3 -4 6 -5 6 -3 3 -1)		
	{ -2} (0 0 0 2 1)		
6 8|12 16 14| 2 10 4	{ -2}([1] -2 1)([1] -1)(-1[1] 1 -1)([1] 1)	0 0	2
	{ -2}(-1 -2[-1])(2 2 *)(1 1[1])(-1 -1 *)(6 10[2] -2)		
	(-2 -1 -3 * -3 1)(-9 -17[-5] 3)(1 -3 0 * 4)(3 7[4])(2 2 *)	1	
	{ -6/2} (-1 3 -4 6 -5 6 -3 3 -1)		
	{ -2} (0 0 0 2)		
6 10| 2 14| 4 16 8 12	{ -2}([0] 0 1 -2 1)([0] 0 3 -3)([1] -1 3)([0] -1)	1 1	4
	{ -2}(-1 -2 -1 0[0])(2 2 0 0 *)(3 5 3 0[0])(-3 -3 0 0 *)(-3 -6 0 2[-1])(2 -3 -5 *)		
	(1 2 -5 -5[1])(1 0 2 3 *)(1 4 3[0])(1 1 0 *)	1	
	{ -2/2} (1 -3 4 -4 6 -4 4 -1 1)		
	{ -2} (0 0 3 -1)		
6 10| 2 14|12 4 16 8	{ -2}(1 -2[1])(1 -3[2])(1 -1 -2[1])(-1 -1[0])	1 -1	-4
	{ -2}([-1] -2 -1)(* 2 2)([3] 5 3)(* -3 -3)([-3] -6 0 2 -1)		
	(* 0 2 -3 -5)([1] 2 -5 -5 1)(* 1 0 2 3)([0] 1 4 3)(* 0 1 1)	-1	
	{ -14/2} (1 -3 4 -4 6 -4 4 -1 1)		
	{ -2} (0 0 -1 -2)		
6 12| 2 14 16| 4 10 8	{ -2}([0] 0 0 1 -2 1)([0] 0 0 5 -6 1)([0] 0 2 6 -3)([0] 0 1 2)	1 1	8
	{ -2}(1 2 1 0 0[0])(-2 -2 0 0 0 *)(1 -3 -8 -5 0 0[0])(6 6 0 0 0 *)(-2 1 15 10 -2 0[0])		
	(-3 -6 -6 -3 0 0 *)(1 -2 -12 -8 1 0[0])(2 2 2 2 0 0 *)(2 5 3 0 0[0])(1 1 0 0 0 *)	2	
	{ 4/2} (1 -2 4 -4 6 -4 4 -2 1)		
	{ -2} (0 0 5 3)		

<table>
<tr><td><u>6 12| 2 14 16| 8 4 10</u></td><td>{ -2}(1 -2 1[0])(1 -3 1[1])(1 -2 -1[1])(-1 -1[0])
{ -2}([0] 1 2 1)(* 0 -2 -2)([1] -3 -8 -5)(* 0 6 6)([-2] 1 15 10 -2)
(* -3 -6 -6 -3)([1] -2 -12 -8 1)(* 2 2 2 2)([0] 2 5 3)(* 0 1 1)
{ -14/2} (1 -2 4 -4 6 -4 4 -2 1)
{ -2} (0 0 -1 -2)</td><td>1 -1

-2</td><td>-4</td></tr>
<tr><td>6 12| 4 14 16| 2 10 8</td><td>{ -2}(1[-2] 1)(3[-6] 3)(3[-9] 3)(1[-5] 1)([-1])
{ -2}(1[2] 1)(-2 * -2)(-5[-8] -3 1)(6 * 6)(-2 10[15] 1 -2)
(-3 -6 * -6 -3)(1 -8[-12] -2 1)(2 2 * 2 2)(3[5] 2)(1 * 1)
{ -8/2} (1 -2 4 -4 6 -4 4 -2 1)
{ -2} (0 0 -3 -3 -1)</td><td>1 1

-2</td><td>0</td></tr>
<tr><td><u>6 12| 2 14 16|10 4 8</u></td><td>{ -2}(1 -2 1[0])(2 -6 4[0])(3 -8 4[0])(1 -5 1[0])(-1 0[0])
{ -2}([0] 1 2 1)(* 0 -2 -2)([0] -5 -8 -3 1)(* 0 6 6)([0] 8 18 6 -3 1)(* 0 -1 -7 -4 2)
([0] -5 -16 -8 3)(* 0 -3 0 3)([0] 1 4 3)(* 0 1 1)
{ -16/2} (1 -2 3 -3 4 -2 3 -1 1)
{ -2} (0 0 -1 -3 -1)</td><td>1 -1

-2</td><td>-4</td></tr>
<tr><td>6 12| 2 16 14| 4 10 8</td><td>{ -2}([0] 0 0 1 -2 1)([0] 0 1 2 -3)([0] 1 2 2)
{ -2}(1 2 1 0 0[0])(-2 -2 0 0 0 *)(-5 -8 -3 1 0[0])(6 6 0 0 0 *)(8 18 6 -3 1[0])
(-1 -7 -4 2 0 *)(-5 -16 -8 3 0[0])(-3 0 3 0 0 *)(1 4 3 0 0[0])(1 1 0 0 0 *)
{ 2/2} (1 -2 3 -3 4 -2 3 -1 1)
{ -2} (0 0 5)</td><td>1 1

2</td><td>8</td></tr>
<tr><td>6 12| 4 16 14| 2 10 8</td><td>{ -2}(1[-2] 1)(2[-3] 0 1)(1[-3] -2 1)([-1] -1)
{ -2}(1[2] 1)(-2 * -2)(1 -3[-8] -5)(6 * 6)(1 -3 6[18] 8)
(2 -4 -7 * -1)(3 -8[-16] -5)(3 0 * -3)(3[4] 1)(1 * 1)
{ -6/2} (1 -1 3 -2 4 -3 3 -2 1)
{ -2} (0 0 -3 -2)</td><td>1 1

-2</td><td>0</td></tr>
<tr><td><u>6 12|14 16 2| 8 10 4</u></td><td>{ -2}(1[-2] 1)(1[-2] 1)(2[-5] 2)(1[-4] 1)([-1])
{ -2}(1[2] 1)(-2 * -2)(-2[-3] -2)(2 * 2)(-1 5[12] 5 -1)
(-4 -4 * -4 -4)(1 -7[-16] -7 1)(3 1 * 1 3)(4[8] 4)(2 * 2)
{ -8/2} (1 -3 5 -5 8 -5 5 -3 1)
{ -2} (0 0 -1 -2 -1)</td><td>1 -1

0</td><td>0</td></tr>
</table>

<table>
<tr><td>6 12|16 14 2|10 8 4</td><td>{ -2}([0] 1 -2 1)([0] 2 -2)([1] 0 -1 1)([0] -1 -1)
{ -2}(1 2 1[0])(-2 -2 0 *)(-2 -3 -2[0])(2 2 0 *)(-1 5 12 5[-1])
(-4 -4 -4 -4 *)(1 -7 -16 -7[1])(3 1 1 3 *)(4 8 4[0])(2 2 0 *)
{ -2/2} (1 -3 5 -5 8 -5 5 -3 1)
{ -2} (0 0 1 -2)</td><td>1 1

0</td><td>4</td></tr>
<tr><td><u>10 14|12 16| 2 6 4 8</u></td><td>{ -2}(1[-2] 1)(1 0[-2] 0 1)(-2[0] -2)([1])
{ -2}(1[2] 1)(-2 * -2)(1 -4[-9] -4 1)(8 * 8)(-2 5[14] 5 -2)
(-2 -12 * -12 -2)(1 -5[-12] -5 1)(2 5 * 5 2)(3[6] 3)(1 * 1)
{ -8/2} (1 -2 5 -5 6 -5 5 -2 1)
{ -2} (0 0 -4 1)</td><td>0 2

-2</td><td>0</td></tr>
<tr><td>10 14|12 16| 4 8 2 6</td><td>{ -2}(1[-2] 1)(4[-8] 4)(3[-10] 3)(1[-5] 1)([-1])
{ -2}(1[2] 1)(-2 * -2)(1 -4[-9] -4 1)(8 * 8)(-2 5[14] 5 -2)
(-2 -12 * -12 -2)(1 -5[-12] -5 1)(2 5 * 5 2)(3[6] 3)(1 * 1)
{ -8/2} (1 -2 5 -5 6 -5 5 -2 1)
{ -2} (0 0 -4 -3 -1)</td><td>0 2

-2</td><td>0</td></tr>
<tr><td>10 14|12 16| 2 8 4 6</td><td>{ -2}([0] 0 0 1 -2 1)([0] 0 1 3 -5 1)([0] 0 2 5 -3)([0] 0 1 2)
{ -2}(1 2 1 0 0[0])(-2 -2 0 0 0 *)(1 -4 -9 -4 1 0[0])(8 8 0 0 0 *)(-2 5 14 5 -2 0[0])
(-2 -12 -12 -2 0 0 *)(1 -5 -12 -5 1 0[0])(2 5 5 2 0 0 *)(3 6 3 0 0[0])
(1 1 0 0 0 *)
{ 4/2} (1 -2 5 -5 6 -5 5 -2 1)
{ -2} (0 0 4 3)</td><td>0 2

2</td><td>8</td></tr>
</table>

Oriented Alternating Links with 8 crossings and 4 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.#	W.
6 10| 2 14| 4 16| 8 12	{ -3}(* 0 0 1 -3 3 -1)(* 0 0 4 -8 4)(* 0 0 6 -6)(* 0 1 3)	1 1 0	8
	{ -3}(1 3 3 1 0 0 *)(-3 -6 -3 0 0[0])(-4 -9 -9 -4 0 0 *)(8 15 8 0 0[0])		
	(6 14 14 6 0 0 *)(-6 -12 -6 0 0[0])(-4 -11 -17 -9 1 0 *)(-5 -2 3 0[0])		
	(1 2 7 6 0 0 *)(1 5 4 0 0[0])(1 1 0 0 0 *)	0 1	
	{ 3/2} (-1 3 -6 4 -7 4 -5 1 -1)	0	
	{ -3} (0 0 0 4)		
6 10| 2 14| 4 16|12 8	{ -3}(1 -3 * 3 -1)(2 -6 * 6 -2)(1 -5 * 5 -1)(1 -3 * 2)(-1 *)	1 1 0	0
	{ -3}(1 3 * 3 1)(-3[-6] -3)(-4 -9 * -9 -4)(8[15] 8)(6 14 * 14 6)(-6[-12] -6)		
	(-4 -11 * -17 -9 1)([-5] -2 3)(1 2 * 7 6)(1[5] 4)(1 * 1)	0 -1	
	{ -9/2} (-1 3 -6 4 -7 4 -5 1 -1)	0	
	{ -3} (0 0 0 0 -1)		
6 10| 2 14| 16 4|12 8	{ -3}(1 -3 * 3 -1)(2 -6 * 6 -2)(1 -5 * 5 -1)(1 -3 * 2)(-1 *)	1 -1 0	0
	{ -3}(1 3 * 3 1)(-3[-6] -3)(-4 -9 * -9 -4)(8[15] 8)(6 14 * 14 6)(-6[-12] -6)		
	(-4 -11 * -17 -9 1)([-5] -2 3)(1 2 * 7 6)(1[5] 4)(1 * 1)	0 -1	
	{ -9/2} (-1 3 -6 4 -7 4 -5 1 -1)	0	
	{ -3} (0 0 0 0 -1)		
6 10| 4 14| 2 16|12 8	{ -3}(* 0 0 1 -3 3 -1)(* 0 0 4 -8 4)(* 0 0 6 -6)(* 0 0 4 1 -1)(* 0 0 1 1)	1 1 0	8
	{ -3}(1 3 3 1 0 0 *)(-3 -6 -3 0 0[0])(-4 -9 -9 -4 0 0 *)(8 15 8 0 0[0])		
	(6 14 14 6 0 0 *)(-6 -12 -6 0 0[0])(1 -9 -17 -11 -4 0 0 *)(3 -2 -5 0 0 0[0])		
	(6 7 2 1 0 0 *)(4 5 1 0 0[0])(1 1 0 0 0 *)	0 1	
	{ 5/2} (-1 1 -5 4 -7 4 -6 3 -1)	0	
	{ -3} (0 0 0 4 2)		

Oriented Alternating Links with 9 crossings and 2 Components

Sequence	P-, F-, V-, C- Polynomial	Lk.	W.
4 8 12| 2 14 16 6 18 10	{ -1}(* 1 -2 2 -1)(* 2 -2 2)(* 1 -2 -1 1)(* 0 -1 -1) { -1}(-1 -2 -2 -1 *)(-1 0[0])(4 8 7 3 *)(2 7 7 2[0])(-4 -7 -7 -7 -3 *) (1 -8 -17 -12 -4[0])(4 0 -5 0 1 *)(6 9 5 2[0])(4 6 2 0 *)(1 1 0[0]) { -1/2} (-1 2 -5 7 -9 9 -8 6 -4 1) { -1} (0 2 -1 -2)	2	5
4 8 14|12 2 16 6 18 10	{ -1}(1 -2 2 -1 *)(-3 4 -3 *)(3 -2 * 1)(-1 *) { -1}(* -1 -2 -2 -1)([0] 0 -1)(* 4 8 7 3)([2] 7 7 2)(-4 * -7 -7 -7 -3) (1[-8] -17 -12 -4)(4 * 0 -5 0 1)([6] 9 5 2)(* 4 6 2)([0] 1 1) { -13/2} (-1 2 -5 7 -9 9 -8 6 -4 1) { -1} (0 -2 2 -1)	-2	-3
4 8 12| 2 16 6 18 10 14	{ -1}(1 -2 * 2 -1)(1 -3 * 3 -2 1)(-2 * 2 -2)(* 1) { -1}(-1 -2 * -2 -1)([-1])(2 5 8 * 8 3)(-1 1 3[3] 2)(-4 -7 -8 * -8 -3)(1 -4 -10[-9] -4) (3 1 -3 * 0 1)(4 6[4] 2)(3 5 * 2)(1[1]) { -7/2} (-1 2 -5 7 -8 8 -8 5 -3 1) { -1} (0 0 -2 1)	0	1
4 8 12| 2 16 14 6 18 10	{ -1}(-1 * 3 -2)(-2 * 5 0 -1)(-1 * 3 2 -1)(* 1 1) { -1}(-2 -3 * -1)(3[3] 1)(1 0 4 7 * 2)(3 5 -1[-5] -2)(-2 0 0 -5 * -3)(-7 -11 -4[1] 1) (1 -5 -7 1 * 2)(3 4 3[2])(3 5 2 *)(1 1[0]) { -5/2} (1 -2 4 -7 7 -8 6 -5 3 -1) { -1} (0 2 3 2)	2	3
4 8 14|10 2 16 6 18 12	{ -1}(-1 3 -2 0 0 *)(3 -5 1 -1 *)(-3 2 -1 *)(1 0 *) { -1}(* 0 0 -2 -3 -1)([0] 0 0 3 3 1)(* 1 0 4 7 2)([0] 3 5 -1 -5 -2)(* -2 0 0 -5 -3) ([0] -7 -11 -4 1 1)(* 1 -5 -7 1 2)([0] 3 4 3 2)(* 0 3 5 2)([0] 0 1 1) { -17/2} (1 -2 4 -7 7 -8 6 -5 3 -1) { -1} (0 -2 -2 1)	-2	-5

<table>
<tr><td>4 8 14| 2 16 18 6 12 10</td><td>{ -1}(* 2 -3 1)(* 3 -4 0 1)(* 1 -3 -2 1)(* 0 -1 -1)
{ -1}(-1 -3 -2 *)(1 3 3[0])(-2 0 7 5 *)(-1 2 -1 -7 -3[0])(-4 4 5 -7 -4 *)
(1 -6 -2 4 -1[0])(3 -4 -7 1 1 *)(4 2 -1 1[0])(3 4 1 0 *)(1 1 0[0])
{ -1/2} (-1 1 -4 5 -7 7 -6 5 -3 1)
{ -1} (0 0 -3 -2)</td><td>0</td><td>5</td></tr>
<tr><td>4 8 14| 2 18 16 6 12 10</td><td>{ -1}(1 * -1)(1 * -1 0 -1 1)(* -1 -1 -1)
{ -1}(1 * 1)([-1])(-2 -2 0 -2 * -2)(3 0 -3 0[0])(10 10 0 1 * 1)
(-4 1 4 0[1])(-9 -11 -1 1 *)(1 -3 -3 1[0])(2 3 1 0 *)(1 1 0[0])
{ -3/2} (-1 1 -3 3 -4 4 -3 2 -2 1)
{ -1} (0 0 -3)</td><td>0</td><td>5</td></tr>
<tr><td>4 10 12 14| 2 18 16 6 8</td><td>{ -1}(* 1 -1)(* 2 -1 -1 1)(* 1 -2 -2 1)(* 0 -1 -1)
{ -1}(-1 -1 *)(1[0])(1 1 1 4 3 *)(-1 2 4 2 1[0])(-4 -2 0 -5 -3 *)(1 -5 -9 -7 -4[0])
(3 -1 -6 -1 1 *)(4 5 3 2[0])(3 5 2 0 *)(1 1 0[0])
{ -1/2} (-1 2 -5 6 -8 8 -7 5 -3 1)
{ -1} (0 1 -2 -2)</td><td>1</td><td>5</td></tr>
<tr><td>4 10 18 16|12 2 8 6 14</td><td>{ -1}(1 -1 *)(1 -3 * 2 -2 1)(-2 * 2 -2)(* 1)
{ -1}(* -1 -1)([0] 1)(1 1 1 * 4 3)(-1 2 4[2] 1)(-4 -2 0 * -5 -3)
(1 -5 -9[-7] -4)(3 -1 -6 * -1 1)(4 5[3] 2)(3 5 * 2)(1[1])
{ -7/2} (-1 2 -5 6 -8 8 -7 5 -3 1)
{ -1} (0 -1 -2 1)</td><td>-1</td><td>1</td></tr>
<tr><td>4 10 12 14| 2 18 16 8 6</td><td>{ -1}(* 1 -1)(-1 * 2 1 -1)(-1 * 2 2 -1)(* 1 1)
{ -1}(-1 -1 *)(1[0])(1 1 2 4 * 2)(4 7 5[1] -1)(-2 -1 -1 -6 * -4)
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(9 22 6 -6 1 0 *)(-3 -12 -21 -9 3 0[0])(-10 -26 -10 6 0 0 *)(1 -1 5 7 0 0[0])
(3 9 6 0 0 0 *)(2 2 0 0 0[0])
{ 3/2} (-1 3 -6 7 -9 8 -7 5 -3 1)
{ -1} (0 3 7)</td><td>3</td><td>9</td></tr>
<tr><td>8 16 12| 6 14 4 18 2 10</td><td>{ -1}(* -1 3 -2)(* -2 8 -3)(* -3 9 -3)(* -1 5 -1)(* 0 1)
{ -1}(2 3 1 *)(-3 -3[-1])(3 -6 -11 -2 *)(3 14 14[3])(1 -6 6 22 9 *)
(3 -9 -21 -12[-3])(6 -10 -26 -10 *)(7 5 -1[1])(6 9 3 *)(2 2[0])
{ -3/2} (1 -3 5 -7 8 -9 7 -6 3 -1)
{ -1} (0 3 3 3 1)</td><td>3</td><td>3</td></tr>
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{ -1}(1 1 0 *)(-1 0[0])(2 1 -6 -3 2 *)(-2 0 1 1 2[0])(-4 3 12 2 -3 *)(1 -2 0 -2 -5[0])
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(-5 -2 0[-2] 1)(1 -4 -10 -3 * 2)(2 0 0[2])(2 4 2 *)(1 1[0])
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10 12 14 16l 2 18 4 6 8	{ -1}(1 -1 0 *)(-3 7 -6 0 *)(-4 11 -5 0 *)(-1 6 -1 0 *)(1 0 0 *) { -1}(* 0 1 1)([0] 0 -1)(* 0 -7 -9 -2 -1 -1)([0] 0 0 2 0 -1 1)(* 0 11 21 6 -2 2) ([0] 0 6 0 -4 2)(* 0 -6 -15 -7 2)([0] 0 -5 -3 2)(* 0 1 3 2)([0] 0 1 1) { -19/2} (1 -2 2 -3 3 -3 2 -2 1 -1) { -1} (0 -2 2 4 1)	-2	-5
10 16 14 12l 2 8 6 18 4	{ -1}(* 0 1 -1)(-1 1 * 1 2 -1)(1 * 1 1) { -1}(1 1 0 *)(-1 0[0])(-7 -9 -2 * -1 -1)(2[0] -1 1)(11 21 6 * -2 2)(6 0[-4] 2) (-6 -15 -7 * 2)(-5 -3[2])(1 3 2 *)(1 1[0]) { -7/2} (1 -2 2 -3 3 -3 2 -2 1 -1) { -1} (0 2 3)	2	3
10 12 14 16l 2 18 8 6 4	{ -1}(1 -1 0 *)(-1 1 0 -2 *)(-1 2 2 -1 *)(1 1 0 *) { -1}(* 0 1 1)([0] 0 -1)(* 2 -2 -5 0 1)([0] 3 4 3 1 -1)(* -3 1 8 0 -4)([0] -5 -6 -7 -5 1) (* 1 -3 -9 -2 3)([0] 2 2 4 4)(* 0 2 5 3)([0] 0 1 1) { -17/2} (1 -3 5 -7 7 -8 6 -4 2 -1) { -1} (0 -2 2 2)	-2	-5
10 12 14 16l 6 4 2 18 8	{ -1}(* 0 1 -1)(-1 * 1 4 -2)(-1 * 2 3 -1)(* 1 1) { -1}(1 1 0 *)(-1 0[0])(2 -2 -5 0 * 1)(3 4 3[1] -1)(-3 1 8 0 * -4) (-5 -6 -7[-5] 1)(1 -3 -9 -2 * 3)(2 2 4[4])(2 5 3 *)(1 1[0]) { -5/2} (1 -3 5 -7 7 -8 6 -4 2 -1) { -1} (0 2 3 2)	2	3
10 12 14 16l 4 2 18 6 8	{ -1}(1 * -1)(-4 8 * -4)(-4 12 * -4)(-1 6 * -1)(1 *) { -1}(1 * 1)([-1])(1 -4 * -10 -4 1)(3[3] 3 1 -2)(-3 6 * 21 8 -4) (-6[-2] 0 -3 1)(1 -6 * -14 -5 2)(2[-1] -1 2)(2 * 4 2)([1] 1) { -11/2} (1 -2 3 -5 5 -6 4 -3 2 -1) { -1} (0 0 4 4 1)	0	-1

<table>
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{ -1} (0 2 4 4 1)</td><td>2</td><td>3</td></tr>
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{ -1}(* 0 1 1)([0] 0 -1)(-1 * 0 -5 -10 -4)([-2] 1 4 4 3)(1 * -1 7 20 11)([2] -2 -3 -3 -4)
(* 2 -5 -16 -9)([0] 2 -1 -2 1)(* 0 2 4 2)([0] 0 1 1)
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{ -1}(1 * 1)([-1])(4 2 -8 * -6)(1 -2 1 2[-2])(2 -8 -2 19 * 11)
(3 -6 -2[7])(4 -4 -14 * -6)(3 -2[-5])(2 3 * 1)(1[1])
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{ -1}(1 1 0 *)(-1 0[0])(-1 1 -4 -11 -5 *)(-1 6 10 7[4])(1 -2 4 17 10 *)
(2 -7 -13 -8[-4])(3 -5 -16 -8 *)(4 3 0[1])(3 5 2 *)(1 1[0])
{ -3/2} (1 -2 3 -6 5 -6 6 -4 2 -1)
{ -1} (0 2 5 4 1)</td><td>2</td><td>3</td></tr>
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10 14 12 16| 2 8 4 18 6	{ -1}(* 0 1 -1)(1 * 1 -1 0 1)(* -1 -2 -1) { -1}(1 1 0 *)(-1 0[0])(-5 -11 -4 1 * -1)(4 7 10 6[-1])(10 17 4 -2 * 1) (-4 -8 -13 -7[2])(-8 -16 -5 3 *)(1 0 3 4[0])(2 5 3 0 *)(1 1 0[0]) { -3/2} (-1 2 -4 6 -6 5 -6 3 -2 1) { -1} (0 2 -4)	2	5
10 12 18 14| 6 16 4 8 2	{ -1}(1 * -1)(-3 6 * -3)(-3 9 * -3)(-1 5 * -1)(1 *) { -1}(1 * 1)([-1])(1 -2 * -6 -2 1)(3[5] 6 3 -1)(-2 3 * 12 4 -3) (-6[-11] -12 -6 1)(1 -7 * -17 -6 3)(3[3] 5 5)(4 * 9 5)([2] 2) { -11/2} (1 -3 6 -9 9 -10 8 -6 3 -1) { -1} (0 0 3 3 1)	0	-1
10 14 12 16| 2 6 18 8 4	{ -1}(1 * -1)(-1 1 0 * 1 -1)(2 -1 * 2)(-1 *) { -1}(1 * 1)([-1])(1 -2 * -6 -2 1)(3[5] 6 3 -1)(-2 3 * 12 4 -3) (-6[-11] -12 -6 1)(1 -7 * -17 -6 3)(3[3] 5 5)(4 * 9 5)([2] 2) { -11/2} (1 -3 6 -9 9 -10 8 -6 3 -1) { -1} (0 0 3 -1)	0	-1

Oriented Alternating Links with 9 crossings and 3 Components

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td>4 8 14| 2 12 16| 6 18 10</td><td>{ -2}(1 -2[1])(1 -3[2])(-2[1] 1 -1)([1] 1)</td><td>1 -1</td><td>1</td></tr>
<tr><td></td><td>{ -2}([-1] -2 -1)(* 2 2)([3] 5 3)(* -3 -3)(3 2[-4] -6 -3)(-2 5 7 *)(-8 -5[4] 2 1)</td><td></td><td></td></tr>
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<tr><td></td><td>{ -8/2} (1 -1 4 -4 7 -6 5 -4 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 -1 2)</td><td></td><td></td></tr>
<tr><td>4 8 14| 2 12 18|16 6 10</td><td>{ -2}([0] 0 1 -2 1)([0] 0 3 -3)([0] 0 3 0 1 -1)([0] 0 1 1 1)</td><td>1 1</td><td>9</td></tr>
<tr><td></td><td>{ -2}(-1 -2 -1 0[0])(2 2 0 0 *)(3 5 3 0[0])(-3 -3 0 0 *)(3 2 -4 -6 -3 0[0])</td><td></td><td></td></tr>
<tr><td></td><td>(-2 5 7 0 0 0 0 *)(-8 -5 4 2 1 0[0])(1 -8 -9 1 1 0 0 *)(3 1 -1 1 0 0[0])</td><td></td><td></td></tr>
<tr><td></td><td>(3 4 1 0 0 0 *)(1 1 0 0 0[0])</td><td>1</td><td></td></tr>
<tr><td></td><td>{ 4/2} (1 -1 4 -4 7 -6 5 -4 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 3 3)</td><td></td><td></td></tr>
<tr><td>4 8 14| 2 16 18| 6 12 10</td><td>{ -2}(1 -2 1[0])(2 -6 4[0])(1 -6 6[-2])(-2 4[-1])(1[0])</td><td>1 -1</td><td>-3</td></tr>
<tr><td></td><td>{ -2}([0] 1 2 1)(* 0 -2 -2)([0] -5 -8 -3 1)(* 0 6 6)([5] 11 12 4 -2)(-2 * 2 -1 -7 -2)</td><td></td><td></td></tr>
<tr><td></td><td>([-8] -13 -10 -4 1)(1 * -6 -7 2 2)([3] 4 4 3)(* 3 5 2)([0] 1 1)</td><td>-2</td><td></td></tr>
<tr><td></td><td>{ -14/2} (1 -2 5 -6 8 -7 7 -4 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 -1 1 1)</td><td></td><td></td></tr>
<tr><td>4 8 14| 2 16 18|10 6 12</td><td>{ -2}([0] 0 0 1 -2 1)([0] 0 1 2 -3)([0] 0 2 4 0 -1)([0] 0 1 2 1)</td><td>1 1</td><td>9</td></tr>
<tr><td></td><td>{ -2}(1 2 1 0 0[0])(-2 -2 0 0 0 *)(-5 -8 -3 1 0[0])(6 6 0 0 0 *)(5 11 12 4 -2 0[0])</td><td></td><td></td></tr>
<tr><td></td><td>(-2 2 -1 -7 -2 0 0 *)(-8 -13 -10 -4 1 0[0])(1 -6 -7 2 2 0 0 *)(3 4 4 3 0 0[0])</td><td></td><td></td></tr>
<tr><td></td><td>(3 5 2 0 0 0 *)(1 1 0 0 0[0])</td><td>2</td><td></td></tr>
<tr><td></td><td>{ 4/2} (1 -2 5 -6 8 -7 7 -4 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 5 4)</td><td></td><td></td></tr>
<tr><td>4 8 14|16 2 18| 6 10 12</td><td>{ -2}(1[-2] 1)(1 0[-3] 2)(-2[-1] 1 -1)([1] 1)</td><td>-1 -1</td><td>1</td></tr>
<tr><td></td><td>{ -2}(1[2] 1)(-2 * -2)(-5[-8] -3 1)(6 * 6)(5 11[12] 4 -2)(-2 2 -1 * -7 -2)</td><td></td><td></td></tr>
<tr><td></td><td>(-8 -13[-10] -4 1)(1 -6 -7 * 2 2)(3 4[4] 3)(3 5 * 2)(1[1])</td><td>2</td><td></td></tr>
<tr><td></td><td>{ -8/2} (1 -2 5 -6 8 -7 7 -4 3 -1)</td><td></td><td></td></tr>
<tr><td></td><td>{ -2} (0 0 -3 2)</td><td></td><td></td></tr>
</table>

<table>
<tr><td><u>4 8 14| 2 16 18|12 6 10</u></td><td>{ -2}([0] 0 0 1 -2 1)([0] 0 0 5 -6 1)([0] 0 0 8 -1 -2)([0] 0 0 5 3 -1)([0] 0 0 1 1)
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(-2 0 5 11 8 0 0[0])(1 -3 1 2 -3 0 0 0 *)(3 -4 -8 -6 -5 0 0[0])
(4 -4 -10 -2 0 0 0 *)(4 2 -1 1 0 0[0])(3 4 1 0 0 0 *)(1 1 0 0 0[0])
{ 6/2} (1 -1 4 -4 6 -6 6 -4 3 -1)
{ -2} (0 0 5 7 2)</td><td>1 1

2</td><td>9</td></tr>
<tr><td>4 8 14| 2 18 16| 6 12 10</td><td>{ -2}(1 -2 1[0])(1 -3 1[1])(-2 1[1] -1)(1[1])
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(3[-4] -8 -6 -5)(4 * -4 -10 -2)([4] 2 -1 1)(* 3 4 1)([0] 1 1)
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{ -2} (0 0 -1 2)</td><td>1 -1

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(3 -4 -8[-6] -5)(4 -4 -10 * -2)(4 2[-1] 1)(3 4 * 1)(1[1])
{ -6/2} (1 -1 4 -4 6 -6 6 -4 3 -1)
{ -2} (0 0 -3 1 1)</td><td>-1 -1

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<tr><td><u>6 8|12 14 18|16 2 4 10</u></td><td>{ -2}([1] -2 1)([1] -1)(1[-3] 3 -1)(1[-3] 2)([-1])
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<tr><td>6 8|12 16 14|10 18 2 4</td><td>{ -2}([1] -2 1)([1] -1)([-1] 3 -3 1)([-1] 3 -2)([0] 1)
{ -2}(-1 -2[-1])(2 2 *)(1 1[1])(-1 -1 *)(4 12 12[4])(-3 -2 4 2 * -1)
(1 -9 -25 -23[-8])(4 -4 -17 -8 * 1)(7 11 8[4])(6 11 5 *)(2 2[0])
{ -4/2} (-1 4 -6 10 -10 12 -9 7 -4 1)
{ -2} (0 0 0 0 1)</td><td>0 0

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<table>
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(3 14 23 11 -1 0[0])(-3 -11 -16 -10 -2 0 0 *)(-4 -16 -24 -11 1 0[0])
(1 1 -3 0 3 0 0 *)(2 6 10 6 0 0[0])(2 6 4 0 0 0 *)(1 1 0 0 0[0])
{ 4/2} (1 -3 7 -7 10 -8 8 -5 2 -1)
{ -2} (0 0 3 4)</td><td>1 1

1</td><td>9</td></tr>
<tr><td>6 10| 2 14|16 4 18 8 12</td><td>{ -2}(2[-5] 4 -1)(3[-9] 8 -2)(2[-8] 6 -1)(1[-4] 2)([-1])
{ -2}(1 4[5] 2)(-1 -5 -9 * -5)(-2 -10[-14] -7)(3 13 21 * 11)(3 14[23] 11 -1)
(-3 -11 -16 * -10 -2)(-4 -16[-24] -11 1)(1 1 -3 * 0 3)(2 6[10] 6)(2 6 * 4)
(1[1])
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-1</td><td>1</td></tr>
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([1] 1)
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{ -2}(2[5] 4 1)(-5 * -9 -5 -1)(-7[-14] -10 -2)(11 * 21 13 3)(9[19] 16 6)
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-1</td><td>1</td></tr>
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6 12l 2 14 16l 4 10 18 8	{ -2}([0] 1 -2 1)([0] 3 -5 3 -1)([1] 1 -4 3)([0] -1 -2) { -2}(1 2 1[0])(-2 -2 0 *)(-2 -8 -9 -4[0])(1 3 5 3 0 *)(5 21 24 7[-1]) (-2 -4 0 -1 -3 *)(-7 -24 -26 -8[1])(1 -3 -10 -3 3 *)(3 8 10 5[0])(3 7 4 0 *) (1 1 0[0]) { -2/2} (1 -3 6 -7 10 -8 8 -5 3 -1) { -2} (0 0 1 -3)	1 1 0	5
6 12l 2 14 18l16 4 10 8	{ -2}(1[-2] 1)(2[-5] 4 -1)(2[-7] 5 -1)(1[-4] 2)([-1]) { -2}(1[2] 1)(-2 * -2)(-2 -8[-9] -4)(1 3 5 * 3)(5 21[24] 7 -1)(-2 -4 0 * -1 -3) (-7 -24[-26] -8 1)(1 -3 -10 * -3 3)(3 8[10] 5)(3 7 * 4)(1[1]) { -8/2} (1 -3 6 -7 10 -8 8 -5 3 -1) { -2} (0 0 -1 -1 -1)	1 -1 0	1
6 12l 4 14 16l 2 10 18 8	{ -2}([0] 1 -2 1)([-1] 5 -5 1)([-2] 7 -5 1)([-1] 4 -2)([0] 1) { -2}(1 2 1[0])(-2 -2 0 *)(-4 -9 -8[-2])(3 5 3 * 1)(-1 7 24 21[5]) (-3 -1 0 -4 * -2)(1 -8 -26 -24[-7])(3 -3 -10 -3 * 1)(5 10 8[3])(4 7 3 *) (1 1[0]) { -4/2} (-1 3 -5 8 -8 10 -7 6 -3 1) { -2} (0 0 1 1 1)	1 1 0	3
6 12l 2 14 16l10 4 18 8	{ -2}(1[-2] 1)(-1 4[-5] 2)(-2 6[-6] 1)(-1 4[-2])(1[0]) { -2}(1[2] 1)(-2 * -2)(-4[-9] -8 -2)(3 * 5 3 1)(6[22] 23 6 -1)(3 * 3 -5 -4 1) (-4[-19] -27 -9 3)(-6 * -14 -3 5)(1[3] 8 6)(2 * 6 4)([1] 1) { -12/2} (-1 3 -5 7 -7 8 -5 5 -2 1) { -2} (0 0 -1 1 1)	1 -1 0	-1

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{ -6/2} (-1 3 -5 7 -7 8 -5 5 -2 1)
{ -2} (0 0 1 3)</td><td>1 1

0</td><td>3</td></tr>
<tr><td>6 12| 4 18 16| 2 10 8 14</td><td>{ -2}([0] 1 -2 1)([0] 3 -5 3 -1)([0] 3 -6 5 -1)([0] 1 -4 2)([0] 0 -1)
{ -2}(1 2 1[0])(-2 -2 0 *)(-2 -8 -9 -4[0])(1 3 5 3 0 *)(-1 6 23 22 6[0])
(1 -4 -5 3 3 0 *)(3 -9 -27 -19 -4[0])(5 -3 -14 -6 0 *)(6 8 3 1[0])(4 6 2 0 *)
(1 1 0[0])
{ 0/2} (1 -2 5 -5 8 -7 7 -5 3 -1)
{ -2} (0 0 1 -1 -1)</td><td>1 1

0</td><td>5</td></tr>
<tr><td>6 12|14 18 2| 8 16 10 4</td><td>{ -2}(1 -2 1[0])(1 -3 1[1])(1 -4 3[-1])(-2 3[-1])(1[0])
{ -2}([0] 1 2 1)(* 0 -2 -2)([1] -3 -8 -5)(* 0 6 6)([3] 9 17 10 -1)(-1 * 3 2 -4 -2)
([-8] -18 -21 -10 1)(1 * -9 -16 -3 3)([4] 6 8 6)(* 5 10 5)([0] 2 2)
{ -14/2} (1 -3 7 -8 11 -10 9 -6 4 -1)
{ -2} (0 0 -1 0 1)</td><td>1 -1

-2</td><td>-3</td></tr>
<tr><td>6 12|18 14 2|16 8 4 10</td><td>{ -2}([0] 0 0 1 -2 1)([0] 0 0 5 -6 1)([0] 0 1 7 -2 -1)([0] 0 1 3 1)
{ -2}(1 2 1 0 0[0])(-2 -2 0 0 0 *)(1 -3 -8 -5 0 0[0])(6 6 0 0 0 *)
(3 9 17 10 -1 0[0])(-1 3 2 -4 -2 0 0 *)(-8 -18 -21 -10 1 0[0])
(1 -9 -16 -3 3 0 0 *)(4 6 8 6 0 0[0])(5 10 5 0 0 0 *)(2 2 0 0 0[0])
{ 4/2} (1 -3 7 -8 11 -10 9 -6 4 -1)
{ -2} (0 0 5 5)</td><td>1 1

2</td><td>9</td></tr>
<tr><td>6 12|18 14 2|10 16 8 4</td><td>{ -2}(1[-2] 1)(3[-6] 3)(-1 4[-8] 2)(2[-4] 1)([-1])
{ -2}(1[2] 1)(-2 * -2)(-5[-8] -3 1)(6 * 6)(-1 10[17] 9 3)(-2 -4 * 2 3 -1)
(1 -10[-21] -18 -8)(3 -3 * -16 -9 1)(6[8] 6 4)(5 * 10 5)([2] 2)
{ -10/2} (-1 4 -6 9 -10 11 -8 7 -3 1)
{ -2} (0 0 -3 -1 -1)</td><td>1 1

-2</td><td>-1</td></tr>
</table>

10 12|14 18| 6 16 8 2 4	{ -2}(1[-2] 1)(1[-3] 3 -1)(1[-4] 4 -1)(1[-3] 2)([-1]) { -2}(1[2] 1)(-2 * -2)(-2 -6[-5] -2)(1 3 3 * 1)(5 17[17] 5)(-2 -1 3 * 0 -2) (-6 -22[-28] -11 1)(1 -5 -17 * -7 4)(3 7[12] 8)(4 11 * 7)(2[2]) { -8/2} (1 -4 8 -9 12 -10 10 -6 3 -1) { -2} (0 0 0 0 -1)	0 0 0	1
10 12|14 18| 6 16 8 4 2	{ -2}(1[-2] 1)(-1 3[-3] 1)(-1 4[-4] 1)(2[-3] 1)([-1]) { -2}(1[2] 1)(-2 * -2)(-2[-5] -6 -2)(1 * 3 3 1)(5[17] 17 5)(-2 0 * 3 -1 -2) (1 -11[-28] -22 -6)(4 -7 * -17 -5 1)(8[12] 7 3)(7 * 11 4)([2] 2) { -10/2} (-1 3 -6 10 -10 12 -9 8 -4 1) { -2} (0 0 0 0 -1)	0 0 0	-1
10 14|12 16| 2 6 4 18 8	{ -2}([0] 1 -2 1)([1] 1 -4 3 -1)([1] 0 -4 3)([0] -1 -2) { -2}(1 2 1[0])(-2 -2 0 *)(-2 -9 -10 -3[1])(1 3 7 5 0 *)(5 20 20 3[-2]) (-2 -5 -2 -1 -2 *)(-7 -22 -19 -3[1])(1 -3 -8 -2 2 *)(3 7 7 3[0])(3 6 3 0 *) (1 1 0[0]) { -2/2} (1 -2 5 -7 9 -7 8 -5 3 -1) { -2} (0 0 0 -3)	0 2 0	5
10 14|12 16| 2 18 6 4 8	{ -2}([0] 1 -2 1)([-1] 6 -7 2)([-2] 7 -6 1)([-1] 4 -2)([0] 1) { -2}(1 2 1[0])(-2 -2 0 *)(1 -3 -10 -9[-2])(5 7 3 * 1)(-2 3 20 20[5]) (-2 -1 -2 -5 * -2)(1 -3 -19 -22[-7])(2 -2 -8 -3 * 1)(3 7 7[3])(3 6 3 *) (1 1[0]) { -4/2} (-1 3 -5 8 -7 9 -7 5 -2 1) { -2} (0 0 0 1 1)	0 0 2	3
10 14|12 16| 2 6 18 4 8	{ -2}(1[-2] 1)(-2 6[-6] 2)(-2 7[-6] 1)(-1 4[-2])(1[0]) { -2}(1[2] 1)(-2 * -2)(-4[-11] -12 -4)(2 * 6 6 2)(6[26] 30 10)(4 * 3 -7 -5 1) (-4[-22] -35 -14 3)(-6 * -16 -4 6)(1[4] 11 8)(2 * 7 5)([1] 1) { -12/2} (-1 3 -6 8 -7 9 -6 5 -2 1) { -2} (0 0 0 1 1)	0 0 0	-1

<u>Oriented Alternating Links with 9 crossings and 4 Components</u>

<table>
<tr><th>Sequence</th><th>P-, F-, V-, C- Polynomial</th><th>Lk.#</th><th>W.</th></tr>
<tr><td><u>6 10| 2 14| 4 16| 8 18 12</u></td><td>{ -3}(* 1 -3 3 -1)(* 3 -7 5 -1)(* 3 -6 3)(-1 * 2 -3)(* 1)
{ -3}(1 3 3 1 *)(-3 -6 -3[0])(-4 -9 -9 -4 *)(8 15 8[0])(6 14 14 6 *)
(-6 -12 -6[0])(-4 -12 -8 -4 * -4)(2 -7[-8] 1)(1 3 -4 -2 * 4)(1 1 6[6])
(1 5 4 *)(1 1[0])
{ -5/2} (1 -4 6 -8 7 -10 5 -5 1 -1)
{ -3} (0 0 0 -2 1)</td><td>1 1 0

0 1
-1
</td><td>3</td></tr>
<tr><td>6 10| 2 14|18 4|16 8 12</td><td>{ -3}(1 -3 3 -1 *)(1 -5 7 -3 *)(-3 6 -3 *)(-1 1 3 -1 *)(1 1 0 *)
{ -3}(* 1 3 3 1)([0] -3 -6 -3)(* -4 -9 -9 -4)([0] 8 15 8)(* 6 14 14 6)
([0] -6 -12 -6)(* -4 -12 -8 -4 -4)([0] 0 2 -7 -8 1)(* 1 3 -4 -2 4)([0] 1 1 6 6)
(* 0 1 5 4)([0] 0 1 1)
{ -17/2} (1 -4 6 -8 7 -10 5 -5 1 -1)
{ -3} (0 0 0 2 2)</td><td>1 -1 0

0 1
-1
</td><td>-5</td></tr>
</table>