AN ABSTRACT OF THE THESIS OF

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Title: Impact of Asset Usage Preferences on Parallel Replacement Decisions

Abstract approved:

David S. Kim

Data from a state department of transportation fleet shows that the usage of a typical asset decreases as it ages. One possible explanation for decreasing asset usage with age is operator preference for using the newest available asset. In this research the long-term cost and replacement implications of such a "newest first" usage practice are investigated for a fleet of similar assets with a fixed replacement age (age standard). Capital constraints on replacement are not considered. Comparisons are made to a "random" usage practice that results in constant expected usage as an asset ages.

For most major cost categories, the newest first usage practice results in greater or equal costs. It is also shown that asset usage and fixed replacement age are interdependent under the newest first usage practice. The implications of this are that simple approaches found in the literature to address decreasing usage with age give incorrect results.

Impact of Asset Usage Preferences on Parallel Replacement Decisions

by
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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Tristan John Wagner, Author

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INTRODUCTION

In 2005, U.S. manufacturing establishments spent over $109 billion on machinery and equipment (U.S. Census Bureau, 2006). Some of this was due to new growth, but much of it was replacement of equipment that was either too costly to retain or superseded by new technology. The area of replacement analysis is concerned with optimizing such equipment replacement decisions. Replacement decisions are not limited to the realm of manufacturing, but are often important to government and other organizations as well.

Fleets of motor vehicles provide a common opportunity for application of replacement analysis methodologies. The Oregon Department of Transportation (ODOT) Fleet Services group operates such a fleet for road and highway maintenance purposes within the state. In 2008, the fleet consisted of approximately 5,000 pieces of equipment representing an investment of over $340 million. An interesting characteristic of fleet operations is that usage and repair costs of assets in the ODOT fleet generally decrease with age. For example, mileage versus age data for one class of vehicles – heavy diesel trucks – is presented in Figure 1. Many replacement models traditionally assume that operating and maintenance (O&M) costs, such as repair costs, increase with age – the opposite of what occurs in the ODOT fleet.

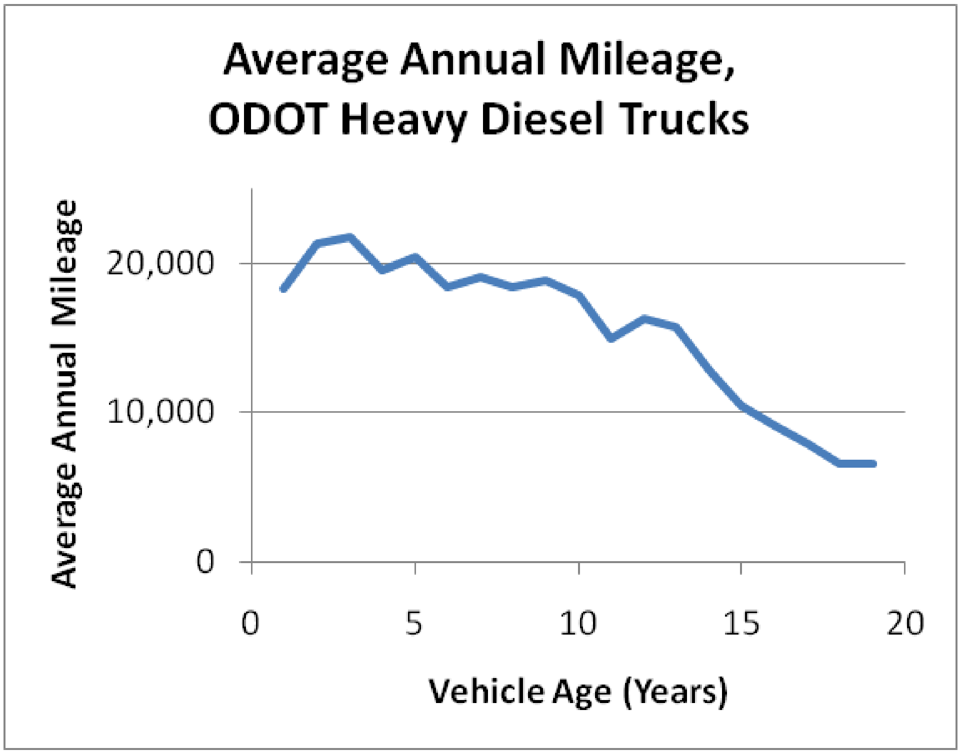


Figure 1: Average annual mileage vs. age for ODOT heavy diesel trucks

This pattern of decreasing usage with age is also reported independently in several contributions to the literature (e.g., Dietz & Katz 2001, Buddhakulsomsiri & Parthanadee 2006). However, such an observation raises serious issues for many other parallel replacement methodologies that do not explicitly address asset usage and the impact of asset usage on costs and economic life. Without increasing O&M costs, there may not be a finite economic life nor optimal replacement age.

Furthermore, much of the literature that does address this issue assumes that decreasing usage values are independent of other factors, such as replacement decisions. One possible explanation for the decreasing usage pattern could be a preference among fleet operators for using the newest available asset when given a choice. Conversations with ODOT work crews confirm that such usage decisions do occur frequently in practice.

This research investigates the logical implications of employing such a “newest first” usage practice. The following section discusses relevant elements of the replacement analysis literature. The general assumptions behind the fleet model used for analysis are then discussed, and details of the newest first usage practice and the baseline usage practice to which it is compared are outlined. The impact of the newest first practice on several categories of costs is analyzed (primarily annual-usage-dependent and cumulative-usage-dependent costs). Finally, several methods are considered for setting age standards to make replacement decisions in the context of a newest first usage practice, and conclusions are summarized.

LITERATURE REVIEW

Replacement problems can be split into two major categories: serial and parallel replacement problems. A serial replacement problem considers a scenario where assets are purchased, used to provide service, and are retired – one at a time, in series – over the course of a specified time period (the planning horizon). For the case of an infinite horizon with known economic parameters, a standard approach utilizing economic life calculations is commonly covered in textbooks on engineering economics such as Park (2006). Terborgh (1949) provides good explanations of many of the basic elements of equipment replacement analysis. The optimal replacement decision depends upon balancing O&M costs (that are typically assumed to increase over time) with one-time capital costs. Other approaches include geometric programming (Cheng, 1992), graphical sensitivity analysis (Walker, 1994), statistical simulation using Markov chain matrices (Kobbacy & Nicol, 1994), modeling with fuzzy arithmetic (Chang, 2005), and use of integral equations (Yatsenko & Hritonenko, 2005). Some of these use a finite horizon while others use an infinite horizon. Hartman & Murphy (2006) use a dynamic programming formulation to compare optimal decisions under finite and infinite horizons.

Serial replacement decisions may also be made on the basis of technological change. If newer assets are assumed to have lower O&M costs due to technological improvement, at some point the increased savings make purchasing a replacement worthwhile. However, technological change can be difficult to predict. One way around this is to find the optimal first-period decision and reevaluate in future periods. Sethi &

Chand (1979) use a forward algorithm based on a dynamic programming approach to find the optimal first-period decision, while Karsak & Tolga (1998) describe a stochastic dynamic programming approach to technological change with several additional features: overhaul capacity and differential interest rates. Yatsenko & Hritonenko (2008) analyze integral equations to reveal several qualitative properties related to technological change. Marsh & Nam (2003) show that technological change can also drive replacement decisions by increasing the levels of quality and precision demanded by customers.

Although serial replacement problems can provide much insight, in many practical situations multiple assets are used to meet demand. For example, the Oregon Department of Transportation (ODOT) uses a fleet of hundreds of vehicles to maintain state transportation infrastructure. Optimizing replacement decisions for this fleet is an example of a parallel replacement problem. If replacement decisions for multiple assets are independent, then the parallel replacement problem can simply be split into several serial replacement problems. In this case, the significant complexity introduced by the parallel replacement problem may be avoided by applying serial replacement analysis to an “average” asset (Waddell, 1983). Realistically, independence is commonly violated by concerns such as economies of scale in purchase of replacements, budget constraints, or asset utilization levels.

Several authors have noted similarities between parallel replacement problems and capacity expansion problems. Capacity expansion problems often consider economies of scale, but do not necessarily factor in the replacement of equipment. On the other hand, replacement problems do not necessarily consider changes in demand.

Rajagopalan (1998) unifies the two types of problems using an integer programming formulation over a finite horizon. A more efficient dual-based solution approach is also described, based on an alternate disaggregate formulation of the problem. Several ways in which the model can be easily extended to incorporate alternative technologies and suppliers, multiple demand types, and quantity discounts are also described. Chand et al. (2000) describe a similar model that incorporates the ability for assets to remain idle (a possible result of economy-of-scale purchases). A heuristic algorithm that can be used to solve the integer programming problem more efficiently is also defined. Another variation, introduced by de Matta & Hsu (2006), measures capacity continuously with solution procedures that use Lagrangian relaxation and tabu search. In replacement problems, capacity is typically measured discretely since it is assumed that assets cannot be split and are commonly identical.

Other authors have more directly extended serial replacement analysis to the parallel case, while incorporating economies of scale in asset purchase. Through a dynamic programming formulation solved as a linear programming problem, Jones et al. (1991) propose a model that can be used with time-varying economics over a finite horizon or time-invariant economics over an infinite horizon. They also prove several useful rules based on the model that allow for a significant reduction in state space (and hence, improved solution efficiency). The first rule is the no-splitting rule, or NSR, which states that all assets of equal age in a given period will either be kept together or replaced together. The second rule is the older-cluster replacement rule, or OCRR, which is based upon some additional assumptions. The OCRR states that older clusters of assets with equal age in a given period are replaced before younger clusters. Using a weaker set of assumptions, Tang & Tang (1993) proved an even stronger rule than OCRR: the all-or-none rule, or AONR. Although weaker, the assumptions of Tang & Tang may not hold for assets where salvage cost decreases quickly. Hopp et al. (1993) give an additional proof of the OCRR, based on even weaker assumptions. Using a forward-time dynamic programming algorithm, McClurg & Chand (2002) take advantage of a dominance property to reduce the number of checked nodes and further improve solution efficiency. Chen (1998) takes a different approach by formulating the problem as a shortest path integer programming problem and applying Bender's decomposition to solve. Finally, Jones & Zydiak (1993) discuss optimal fleet designs that are stable over the long term for cases with purchasing economies of scale and maintenance dis-economies of scale.

Incorporating economies of scale in asset purchase is one way dependence is introduced to decisions in a parallel machine problem, but dependence may also result from capital rationing constraints. This is a common practical limitation, since many organizations must work within the context of a budget. A simple, though not necessarily optimal, approach is to assign a replacement score to each machine in service (Dietz & Katz, 2001). Karabakal et al. (1994) find optimal solutions by solving an integer programming model with capital rationing constraints. Solving such formulations can be extremely difficult due to the large number of integer decision variables, however, and much of the paper is spent discussing efficient solution methods based on Lagrangian relaxation. Hartman (2000) formulates an integer programming problem with fixed replacement costs in addition to capital constraints and shows that relaxation of the integer programming formulation has integer extreme points for a specific class of sub-problems. This reduces the state space and improves solution efficiency. Keles & Hartman (2004) conduct a case study on bus fleet replacement where they further develop the model to include multiple challengers (i.e., multiple vehicle options from several suppliers).

Another difficulty presented by many practical parallel replacement problems is interdependence of asset utilization – which can, in turn, affect replacement decisions. Bethuyne (1998) creates a calculus-based serial replacement model that includes asset utilization in each time period. Using this model, it is demonstrated that the economic life of an asset can be extended by reducing the level of utilization over its lifetime. Hartman (1999) uses an integer programming formulation of a parallel replacement problem with a three-dimensional network structure to create a model that includes discrete utilization by period and multiple types of challengers. Capital constraints may be imposed as well. However, finding solutions can quickly become very time-consuming – the number of nodes in the problem is the approximate product of the maximum utilization, maximum age, and decision horizon (each in integer units). Hartman (2001) introduces a modified model, where utilization in each period depends probabilistically on utilization in the previous period. Under certain assumptions, the state space in this model can be limited by using an "economic life frontier." In yet another model, Hartman (2004) introduces stochastic demand, and utilization returns to being treated as a decision variable. Several methods of reducing state space are discussed.

Redmer (2005) presents a simple engineering economics model for replacement decisions where utilization is specified as a parameter dependent upon age. Although this model does not incorporate the interdependence of assets in the parallel replacement problem, it does offer the ability to include a trend of decreasing utilization with age. Similarly, Buddhakulsomsiri & Parthanadee (2006) adapt the model of Hartman (1999) to include decreasing usage as an age-based parameter instead of as a decision variable.

FLEET AND USAGE PRACTICE DEFINITIONS

The analysis completed is based upon a generic sub-class of the fleet of vehicles managed by ODOT Fleet Services. Assets in the model are replaced using an age standard; that is, assets are replaced when they reach the fixed age of L and not before or after. Assets are acquired to provide a minimum level of service, such that the fleet can meet its obligations.

A survey of DOTs for states across the U.S. showed that the most common policy for vehicle replacement decisions was to use standards based on criteria such as age or cumulative usage (Kriett, 2009). None of the DOTs surveyed employed more complex approaches involving dynamic programming, integer programming, etc. One advantage of age/usage standards is that they make it easy for fleet managers to justify their decisions to non-specialists (i.e., equipment operators). Furthermore, finding an optimal solution using some of the approaches described in the Literature Review section may require excessive computing resources when applied to a very large fleet. Although an age-standard policy may not be the optimal policy for fleet management, the approach has been shown to have significant practical value (as evidenced by its common use).

The fleet model consists of N assets which all have the same cost function. This function may be dependent upon several variables, such as age or usage. Capital limitations and technological improvement are not considered within the scope of this research, so that all assets requiring replacement are replaced, and are replaced with a technologically identical asset

Requests for service (i.e., customers) arrive to the system with interarrival times according to some probability distribution. The need for a piece of equipment is often time-sensitive, so the system does not include a queue, nor are requests rescheduled. If all assets (also referred to as servers) in the system are busy, the request is rejected.

Two usage practices are evaluated: the random usage practice and the newest first usage practice. The names of these usage practices are based upon the mechanism of selecting among available assets upon customer arrival.

Under the newest first practice, the request for service is assigned to the newest available asset by age. Assets may also be thought of as having a priority ranking, with the newest assets having the highest priority. This differs from queuing systems that may prioritize jobs; it is assets which are being prioritized here. If multiple assets of equal age are available, the request is assigned to one of them randomly with equal probability.

Under the random practice, the prioritization portion of the procedure is skipped, and the request is randomly assigned among all available assets with equal probability.

Once a request is assigned to an asset, the asset remains busy with the request for a service time length generated from another general stochastic process. All service times are assumed to come from the same distribution regardless of which asset is performing the job. After the request is completed, the asset returns to idle status and is ready to accept a new request.

Due to the stochastic nature of this system, usage (and any costs dependent upon usage) will display some amount of variability even under identical circumstances. Expected values of these variables are used for purposes of analysis. Hence, the results of the analysis are applicable to long-run operation, and results may differ significantly from the analysis for a system that is only operated over a short period of time.

EXPECTED USAGE UNDER THE NEWEST FIRST PRACTICE

Assets in a fleet under the newest first practice experience different usage levels dependent upon their relative age within the fleet. Prior to analyzing the impact of this on fleet-wide costs, it is helpful to establish that expected long-run usage under the newest first practice is non-increasing with respect to age. A procedure for calculating expected usage values under certain distributional assumptions is also described.

Analysis Using Little's Law

The relationship between long-run asset utilizations for different asset priority classes can be established using Little's Law (Hillier & Lieberman, 2006). Priority classes are numbered from low to high (i.e., i=1 indicates the highest priority asset). The utilization of an asset in priority class i can be found using the application of Little's Law in equation (1) with the following variable definitions.

λ_i Long-run arrival rate to a single asset in the i^{th} priority class

W_i Long-run average time waiting and being served by a single asset in the i^{th} priority class

L_i Long-run average number of customers in the system

$$L_i = \lambda_i W_i \tag{1}$$

Since arrivals to the system are rejected if no assets are available (i.e., no queue forms), and each asset serves a single customer at a time, L_i equals the long-run utilization of a single asset in priority class i, and W_i is the average service time for a customer. The average service time for a customer is the same for all priority classes ($W_i = W$, for all i), so assets with greater long-run arrival rates will have greater utilization.

Figure 2 illustrates an example of a system with two priority classes over time. In the illustration, priority class i contains two assets and priority class i+1 contains one asset. Arrivals are marked by vertical lines indicating arrival time. If there is an idle asset in priority level i (white background in the "Priority level i assets busy" row) when a customer arrives, the asset will begin working on the arrival. However, if all assets in priority level i are busy (shaded background in the "Priority level i assets busy" row), the arrival will be sent to the next lowest priority level (i+1). This is illustrated by continuing the arrival line, as a dotted line, to the next row. If all assets in the last priority level are busy, the arrival will be rejected.

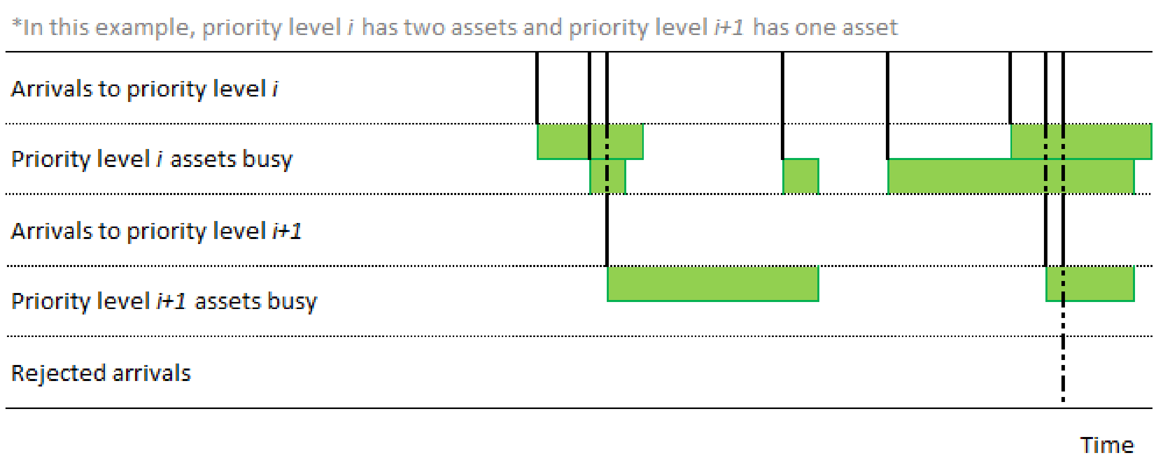


Figure 2: Illustration of a system with two priority levels

Requests arrive to a lower priority asset if and only if all higher priority vehicles are already in use. Therefore, the effective arrival rate to a lower-priority asset is less than or equal to the rate for a higher-priority asset, and the long-run utilization of a lower-priority asset is less than or equal to the utilization of a higher-priority asset according to Little's Law.

$$\lambda_i \geq \lambda_{i+1}$$

$$\lambda_i W \geq \lambda_{i+1} W$$

$$L_i \geq L_{i+1} \tag{2}$$

Since older assets have lower priority under the newest first usage practice, expected usage of older assets will always be less than or equal to expected usage of newer assets under the newest first usage practice.

Calculating Utilizations with Exponential Interarrival and Service Times

It is useful to have a method for calculating numerical usage values under the newest first practice to provide realistic examples and illustrations, etc. Numerical usage values can be found using Little's Law and by assuming exponentially distributed interarrival and service times.

Since the fleet has no queue, the arrival rate must be corrected to exclude rejected arrivals – that is, arrivals when all servers are busy. If the interarrival times are exponential (i.e., a Poisson arrival process), the probability that all assets in an *i*-asset system are busy ($P_{b,i}$) is equal to the fraction of arrivals that are rejected – so the effective arrival rate to an *i*-asset system is $\lambda(1-P_{b,i})$. This follows from the "PASTA" property (Poisson arrivals see time averages). If service times are also exponentially distributed with rate μ, the system can be represented as an *M/M/s/s* queue where *s* is the number of assets in the system. For example, a one-asset system is an *M/M/1/1* queue and a two-asset system is an *M/M/2/2* queue. A formula exists (Hillier & Lieberman, 2006) for finding $P_{b,i}$ for a *M/M/s/s* queue:

$$P_{b,s} = \left(\frac{\lambda}{\mu}\right)^s \frac{P_{0,s}}{s!}, \text{ where } P_{0,s} = \frac{1}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad (3)$$

Consider a system with one asset in each of two priority levels. The system as a whole can be analyzed using Little's Law with an adjusted arrival rate, as can the sub-system consisting of only the higher-priority asset (with a different adjusted arrival rate). Since the average number of assets busy in both the one-asset and two-asset systems can be determined, it is easy to find the utilization of the second asset by taking the difference in the average number of assets busy. This can be expressed generally for systems of any size where there is one asset per priority level, and can be adapted for systems with multiple assets of equal priority by averaging over all assets in a priority level. Let the long-run utilization of the i^{th} asset be denoted as u_i. This results in the following equation.

$$u_i = L_i - L_{i-1} = \frac{\lambda\left(P_{b,i-1} - P_{b,i}\right)}{\mu} \text{ for } i = 1,2, \ldots \quad (4)$$

MANAGING USAGE TO MINIMIZE COSTS

This section discusses the impact of the two usage practices (random and newest first) on several categories of O&M costs. O&M costs are assumed to be evaluated annually – i.e., an O&M cost value is calculated at the end of each year in an asset's life.

Costs that depend only upon age, or that are constant from year to year, do not depend upon usage. Under an age standard, these types of costs will be equal under either usage practice; no usage practice is preferred over the other. Costs that depend upon usage are split into two subcategories and discussed below in detail.

Usage-Dependent O&M Costs

Many types of assets can be used at varying levels over time. For example, a vehicle might be driven 10,000 miles in one year and 7,500 miles in the following year. Machine utilizations – often expressed on a scale from zero to one, representing the fraction of time in use – may likewise vary from period to period.

Two subcategories of usage-dependent costs are considered: annual-usage-dependent costs and cumulative-usage-dependent costs. Annual-usage-dependent costs depend upon the usage level in only the year for which costs are being calculated. For example, a vehicle driven 5,000 miles in one year might have a different cost in that year than a similar vehicle driven 10,000 miles in that same year.

On the other hand, cumulative-usage-dependent costs depend upon the total usage of the asset over its entire life up to the end of the year for which costs are being calculated. For example, two vehicles might be driven 5,000 miles in the same year, but their cumulative-usage-dependent costs could differ based on their previous usage history. Table 1 shows such an example. Two vehicles are each three years old, but vehicle A has been driven 5,000 miles per year for each of its first two years of life and vehicle B has been driven 10,000 miles per year for each of its first two years of life. Their cumulative usages at the end of the third year of their lives are 15,000 miles and 25,000 miles respectively.

Table 1: Example of cumulative usage for two similar vehicles

	Vehicle A			Vehicle B		
Year	Age	Annual Usage	Cumulative Usage	Age	Annual Usage	Cumulative Usage
2005	1	5,000	5,000	1	10,000	10,000
2006	2	5,000	10,000	2	10,000	20,000
2007	3	5,000	15,000	3	5,000	25,000

Annual-Usage-Dependent O&M Costs

Hartman (2004) considered several types of non-decreasing annual-usage-dependent cost curves (convex, concave, and linear, as shown in Figure 3) and discussed optimal asset utilization levels for a two-asset fleet in general terms.

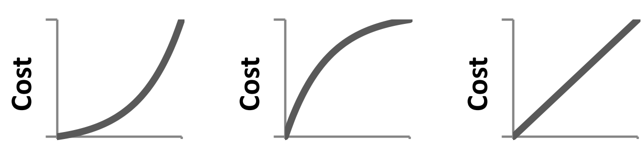


Figure 3: Cost as a function of annual usage (based on Hartman 2004); from left to right: convex, concave, and linear cost functions

Maximum per-asset utilization in a period was defined to be $\bar{u}$ (not necessarily on a scale from zero to one) and total demand for the period was defined as d_t. A convex cost curve demanded that each asset be used at the minimum level possible – both assets were used at the level $d_t/2$. In the concave case, costs were minimized by using one asset at full capacity ($\bar{u}$) and the second asset only enough to meet demand ($d_t - \bar{u}$). For the linear case, costs could be minimized through any linear combination of asset utilization levels summing to d_t.

These observations are extended to the comparison of random and newest first usage practices, and specifically proven for a fleet of N assets, using the following notation:

$C_t(u)$ Annual-usage-dependent O&M cost in period t as a function of usage u

u_{it} Expected usage of asset i in period t under the newest first practice for $i = 1, 2, \ldots, N$

q_t Expected usage of any asset in period t under the random practice

In order to compare usage practices fairly, the total fleet-wide usage in any period t should be equal for both practices. That is,

$$N \cdot q_t = \sum_{i=1}^{N} u_{it} \Leftrightarrow q_t = \frac{1}{N} \sum_{i=1}^{N} u_{it} \tag{5}$$

Convex $C_t(u)$ Case

Let $C_t(u)$ be convex over the interval containing all feasible usage levels, as is the case of (c) in Figure 3. The summation form of Jensen's inequality (Wolff, 1989),

$$f\left(\frac{1}{N} \sum x_i\right) \leq \frac{1}{N} \sum f(x_i) \tag{6}$$

may be used to show that, the random usage practice results in a total fleet-wide O&M cost that is less than or equal to that for the newest first practice:

$$N \cdot C_t(q_t) = N \cdot C_t\left(\frac{1}{N}\sum_{i=1}^{N} u_{it}\right) \leq \sum_{i=1}^{N} C_t(u_{it}) \tag{7}$$

Graphically, this may be interpreted as utilizing all assets equally at the lowest level possible (while meeting fleet-wide demand) to avoid higher marginal costs, as in Figure 4.

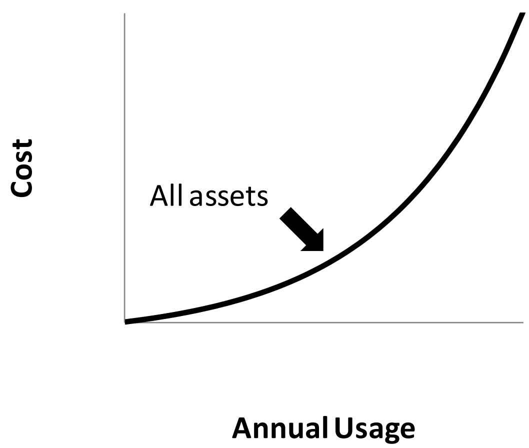


Figure 4: Using all assets equally to meet demand (as under the random practice) minimizes annual-usage-dependent costs for a convex cost function by avoiding higher marginal costs

Concave $C_t(u)$ Case

If $C_t(u)$ is concave, then Jensen's inequality is again applicable with the opposite result as for a convex $C_t(u)$ in equation (7):

$$N \cdot C_t(q_t) \geq \sum_{i=1}^{N} C_t(u_{it}) \tag{8}$$

The newest first utilization practice results in a fleet-wide total O&M cost less than or equal to that for the random utilization practice. Graphically, some assets are utilized at a high level to take advantage of marginally decreasing costs, while the remaining assets are utilized at lower levels (Figure 5).

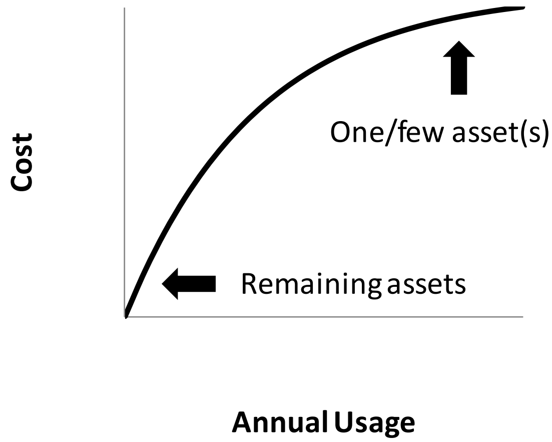


Figure 5: Using some assets a lot and others less (as under the newest first practice) results in lower annual-usage-dependent costs than using all assets equally for a concave cost function

Linear $C_t(u)$ Case

If $C_t(u)$ is linear, then $C_t(u)$ is both convex and concave and the total fleet-wide O&M cost is equal under either of the two usage practices. Neither practice is preferred over the other. Combining equations (7) and (8) gives the following result:

$$N \cdot C_t(q_t) = \sum_{i=1}^{N} C_t(u_{it}) \tag{9}$$

Cumulative-Usage-Dependent O&M Costs

For analysis of cumulative-usage-dependent costs, several further assumptions are made:

1. Costs are time-invariant

2. Usage practices are compared on the basis of a cycle *L* years in length
3. Total fleet-wide usage is the same in all periods

The first assumption (time-invariant costs) means that two assets with equal cumulative usage will have the same cost, even if that cumulative usage is reached in different years. For example, in Table 2 vehicle A is purchased in 2005 and accumulates 15,000 total miles by the end of 2007. Vehicle B is purchased in 2007 and accumulates 15,000 total miles by the end of 2009. The O&M cost for vehicle A in 2007 is equal to the O&M cost for vehicle B in 2009 (prior to discounting). This assumption also means that effects such as inflation or technological improvement are not included in the model.

Table 2: The assumption of time-invariant costs means that the 2007 cost for Vehicle A and the 2009 cost for Vehicle B are equal (before discounting)

	Vehicle A			Vehicle B		
Year	Age	Annual Usage	Cumulative Usage	Age	Annual Usage	Cumulative Usage
2005	1	5,000	5,000			
2006	2	5,000	10,000			
2007	3	5,000	15,000	1	5,000	5,000
2008	4	5,000	20,000	2	5,000	10,000
2009	5	5,000	25,000	3	5,000	15,000

The second assumption is required to eliminate the effects of “starting” cumulative usage in an initial fleet. Assets in an initial fleet may start out with cumulative usage values that are consistent with neither the newest first nor the random usage practice. Since the goal is to compare usage practices, this effect must be controlled. By comparing the two practices under long-term operation, cumulative usage will always be consistent with the appropriate practice. Assets are replaced when they reach the age standard of L years, so usage patterns will repeat over a cycle L years in length. By analyzing cycles L years in length, a fair comparison can be made.

The third assumption can be rewritten as an extension of equation (5):

$$L \cdot N \cdot q = \sum_{i=1}^{N} \sum_{j=1}^{L} u_{ij} \tag{10}$$

Total fleet-wide usage is the same in all periods, so there is no need to distinguish q from one period to another – q is equal in all periods and the subscript t has been dropped. Since total fleet-wide usage is still equal for both practices in any period, and total fleet-wide usage is the same in all periods, the total fleet-wide usage over an entire L-year-long cycle must also be equal for both practices. The right-hand side of equation (10) includes the usage of each asset at each age j (from 1 to L) under the newest first practice. This captures all of the usage that occurs over one L-year-long cycle. It does not make a difference whether an asset is a particular age at the beginning or end of a particular L-year-long cycle – its usage at that age will always be the same since the usage pattern in each cycle is the same. One of the subscripts on u has also been changed, from t in the previous section to j (representing age) in this section. Since analysis is over an L-year-long cycle, it is important to keep track of usage with respect to age to ensure a full cycle is covered.

Notation and AEC/V Criterion

Since the usage practices are compared over an L-year-long cycle, discounting must also be taken into account. First, some notation is defined.

$D(U)$ Cumulative-usage-dependent O&M cost for an asset as a function of cumulative usage U. $D(U)$ is assumed to be a non-decreasing function.

U_{ij} Expected cumulative usage of asset i of age j under the newest first practice, i.e.:

$$U_{ij} = \sum_{j=1}^{j} u_{ij} \text{ for } j = 0,1,\dots,L \quad (11)$$

Q_j Expected cumulative usage of any asset of age j under the random practice, i.e.:

$$Q_j = j \cdot q \text{ for } j = 0,1,\dots,L \quad (12)$$

m_{kj} O&M cost of a j-period old asset in asset age group k in the notation of Jones & Zydiak (1993)

n_j Number of j-period old assets at the end of a specific period. Using the notation of Jones & Zydiak (1993), the asset age group distribution in a period may be written in the form $\{n_1, n_2, \dots, n_L\}$

p Purchase cost of a new asset

s_j Salvage value of a j-period old asset

r Effective interest rate per period

In each period both usage practices result in equal fleet-wide output, so minimizing annual equivalent cost (*AEC*) may appear to be a reasonable criterion for comparison. However, different *AEC* values may be found for L-year-long cycles depending upon which starting period is used to evaluate a cycle, as shown by Jones & Zydiak (1993). Their criterion of annual equivalent cost per vehicle (*AEC/V*) does not depend upon the initial ages of assets, although it does depend upon the ages of assets relative to each other (relative age distribution). Assets are considered by age group k (up to the maximum age L), with n_k assets in each age group. Several factors outside of the scope of this paper (economies of scale in replacement and dis-economies of scale in maintenance) have been removed from the *AEC/V* formula, resulting in the following:

$$
\begin{aligned}
&AEC/V(n_1, n_1, \dots, n_L, L) \\
&= \frac{1}{N}\sum_{k=1}^{L}\left[pn_k - \left(\tfrac{1}{1+r}\right)^L s_L n_k + \sum_{j=1}^{L}\left(\tfrac{1}{1+r}\right)^j n_k m_{kj}\right]\left(\frac{r}{1-\left(\tfrac{1}{1+r}\right)^L}\right) \\
&= \left\{p - \left(\tfrac{1}{1+r}\right)^L s_L + \frac{1}{N}\sum_{k=1}^{L}\left[n_k \sum_{j=1}^{L}\left(\tfrac{1}{1+r}\right)^j m_{kj}\right]\right\}\left(\frac{r}{1-\left(\tfrac{1}{1+r}\right)^L}\right)
\end{aligned}
\tag{13}
$$

Several further modifications are made to make this more consistent with the notation described above. There is more than one way to ensure all assets are considered. Instead of considering each asset age group k and the n_k assets within it, each asset i from 1 to N is considered individually. In addition, $D(U)$ replaces m_{kj} as the O&M cost for the cumulative-usage-dependent case. This results in the following *AEC/V* formula:

$$
AEC/V = \left\{p - \left(\tfrac{1}{1+r}\right)^L s_L + \frac{1}{N}\sum_{j=1}^{L}\left(\tfrac{1}{1+r}\right)^j \sum_{i=1}^{N} D(U)\right\}\left(\frac{r}{1-\left(\tfrac{1}{1+r}\right)^L}\right) \tag{14}
$$

Note that the order of the summations has also been reversed so that the discounting term could be pulled out from the inner summation. From equation (14) it is easy to see that if the total fleet-wide cost for one practice is greater than the other at each age (from 1 to *L*), then the *AEC/V* for that practice is greater than the *AEC/V* for the other.

Asset Age Distributions under the Newest First Practice

While the relative distribution of asset ages does not impact *AEC/V* under the random practice, it can have a significant impact on *AEC/V* under the newest first practice since an asset's usage depends upon the number of newer assets in the fleet. For this reason, both a special "even-distribution" case and the overall general case are analyzed separately. The even-distribution case occurs when all non-empty asset age groups contain an equal number of assets, and each pair of adjacent non-empty asset age groups have an equal number of empty groups in between them. The "general case" covers all age group distributions, including the even-distribution case.

In the special even-distribution case, all assets have a single usage level at a given age under the newest first practice. In the general case, one asset may have a different usage level from another asset at a given age under the newest first practice. Simple two-asset examples of an even-distribution case (Table 3) and a general, non-even-distribution case (Table 4) are included below. Age distribution notation differs slightly from that of Jones & Zydiak (1993) in that specific assets are identified using capital letters instead of using counts. Cumulative usage versus age is also graphed for each asset in each example in Figure 6, clearly illustrating that assets in the general case may have different usages at the same age.

Table 3: Example of an even-distribution case

Period	Age at End of Period			Annual Usage		
t		$i=1$	$i=2$	u_{1t}	u_{2t}	q_t
1	{A,0,B,0}	1	3	7	3	5
2	{0,A,0,B}	2	4	7	3	5
3	{B,0,A,0}	3	1	3	7	5
4	{0,B,0,A}	4	2	3	7	5

Age	Cumulative Usage		
j	U_{1j}	U_{2j}	Q_j
0	0	0	0
1	7	7	5
2	14	14	10
3	17	17	15
4	20	20	20

Table 4: Example of a non-even distribution (general) case

Period	Age at End of Period			Annual Usage		
t		$i=1$	$i=2$	u_{1t}	u_{2t}	q_t
1	{A,B,0,0}	1	2	7	3	5
2	{0,A,B,0}	2	3	7	3	5
3	{0,0,A,B}	3	4	7	3	5
4	{B,0,0,A}	4	1	3	7	5

Age	Cumulative Usage		
j	U_{1j}	U_{2j}	Q_j
0	0	0	0
1	7	7	5
2	14	10	10
3	21	13	15
4	24	16	20

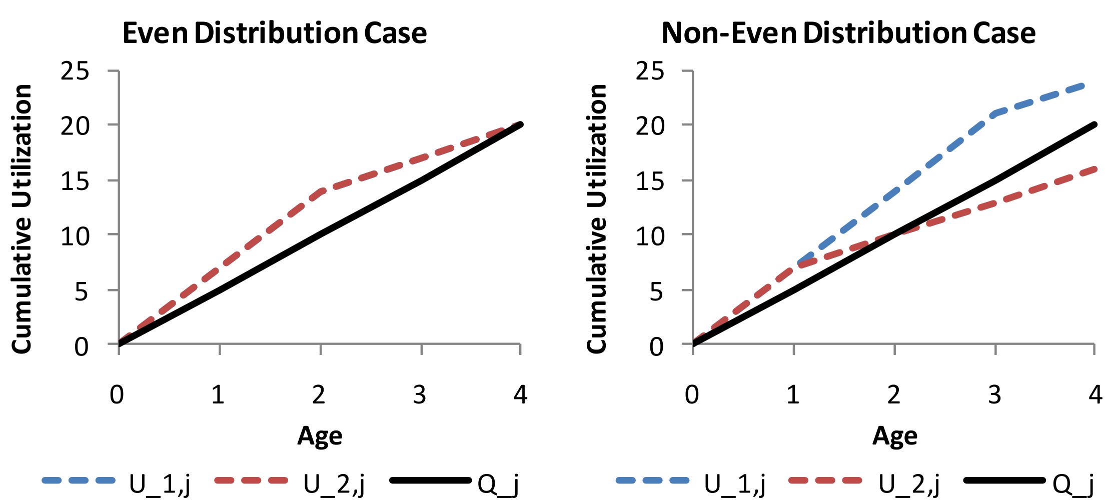


Figure 6: Graphs of cumulative usage versus age for examples of an even distribution case and a non-even distribution (general) case

Even-Distribution Case

In the even-distribution case, cumulative-usage-dependent O&M costs will have a greater or equal *AEC/V* under the newest first practice than under the random practice if costs are non-decreasing with respect to cumulative usage. The reasoning used to come to this conclusion follows.

Equation (10), along with equations (11) and (12), can be used to prove that Q_L will be the average of all U_{iL} in the general case (including the even-distribution case):

$$Q_L = \frac{1}{N}\sum_{i=1}^{N} U_{iL} \tag{15}$$

Since all assets have a single usage level at a given age for the even-distribution case, equation (15) implies that $U_{iL} = Q_L$ for all i=1, 2, …, N. The graph of Q_j over j is a straight line connecting two points on the graph of U_{ij}, at j=0 and j=L.

According to equation (2), usage is non-increasing with respect to age under the newest first practice, and therefore cumulative utilization U_{ij} is a concave function of age j over the interval from 0 to L. The graph of U_{ij} will remain above or on the graph of Q_j for this interval, so

$$U_{ij} \geq Q_j \Leftrightarrow D(U_{ij}) \geq D\left(Q_j\right) \text{ for } j = 0, 1, \dots, L \tag{16}$$

since *D(U)* is non-decreasing. By summing both sides of the inequality over all assets, it becomes clear that a fleet in the even-distribution case with cumulative-utilization-dependent O&M costs will have a greater or equal *AEC/V* under the newest first practice than under the random practice.

$$D(U_{ij}) \geq D(Q_j)$$

$$\left\{p - \left(\tfrac{1}{1+r}\right)^L s_L + \frac{1}{N}\sum_{j=1}^{L}\left(\tfrac{1}{1+r}\right)^j \sum_{i=1}^{N} D(U_{ij})\right\}\left(\frac{r}{1-\left(\frac{1}{1+r}\right)^L}\right)$$

$$\geq \left\{p - \left(\tfrac{1}{1+r}\right)^L s_L + \frac{1}{N}\sum_{j=1}^{L}\left(\tfrac{1}{1+r}\right)^j \sum_{i=1}^{N} D(Q_j)\right\}\left(\frac{r}{1-\left(\frac{1}{1+r}\right)^L}\right)$$

$$AEC/V_{newest\ \ first} \geq AEC/V_{random} \tag{17}$$

Under such conditions, the random practice is preferred regardless of the shape of *D(U)*, so long as *D(U)* is non-decreasing.

General Case

Observe that the graph of U_{1j} remains above the graph of Q_j for all $j=0, 1, \ldots, L$ in Figure 6. Hence, the asset-specific conclusions from analysis of the even-distribution case analysis apply to this asset also. However, the graph of U_{2j} does not remain above the graph of Q_j for all $j=0, 1, \ldots, L$. Therefore, the conclusions from the even-distribution case apply to neither the second asset ($i=2$) nor the fleet as a whole. Nevertheless, it is possible to reach similar conclusions by using an intermediate scenario for comparison if *D(U)* is convex.

Consider a third, intermediate usage practice where each asset has the same end-of-life cumulative usage as under the newest first practice, but is used at a constant level from year to year instead of having a decreasing usage level. The expected cumulative usage of asset *i* at age *j* is denoted V_{ij} in this intermediate scenario. V_{ij} for a given asset *i* and age *j* can be calculated according to equation (18).

$$V_{ij} = \frac{j}{L} \cdot U_{iL} \text{ for } j = 0,1,\dots,L \tag{18}$$

Table 5 extends the non-even-distribution example from Table 4 to include cumulative usages for the intermediate scenario, and Figure 7 illustrates these values graphically.

Table 5: Example of cumulative usage for a non-even distribution (general) case with addition of intermediate scenario

Age	Cumulative Utilization				
j	U_{1j}	U_{2j}	V_{1j}	V_{2j}	Q_j
0	0	0	0	0	0
1	7	7	6	4	5
2	14	10	12	8	10
3	21	13	18	12	15
4	24	16	24	16	20

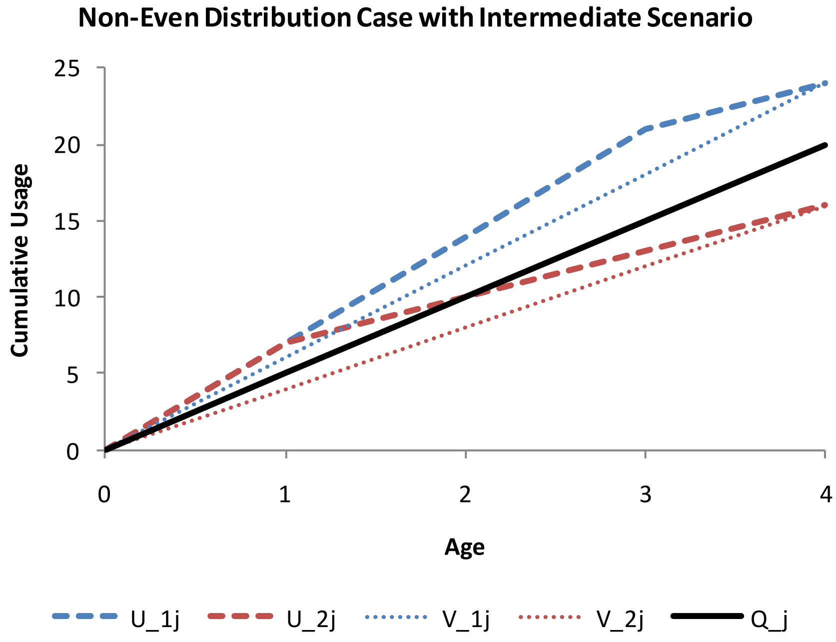


Figure 7: Graph of cumulative usage versus age for non-even distribution (general) case example with addition of intermediate scenario

The first part of the analysis of the even distribution case may now be applied individually to each asset to show that for each asset *i=1, 2, …, N* at each age *j=0, 1, …, L*, the cost under the newest first practice is greater than or equal to the cost for the same asset at the same age in the intermediate scenario. When the fleet-wide cost is summed for a particular age *j*, this results in equation (19).

$$\sum_{i=1}^{N} D(U_{ij}) \geq \sum_{i=1}^{N} D(V_{ij}) \text{ for } j = 0,1,\ldots,L \tag{19}$$

It remains to compare the intermediate scenario to the random practice. First, it is helpful to show that for any age *j* the cumulative usage under the random practice is equal to the average of the individual cumulative usages in the intermediate scenario as in (20).

Equation (20) can be shown to be true using the definitions of Q_j and V_{ij} – equations (12) and (18) – and equation (15), which states that Q_L is the average of all U_{iL}.

$$Q_j = j \cdot q = \frac{j \cdot Q_L}{L} = \frac{j}{L \cdot N} \sum_{i=1}^{N} U_{ij} = \frac{1}{N} \sum_{i=1}^{N} V_{ij} \tag{20}$$

Then, since *D(U)* is assumed to be convex, Jensen's inequality and equation (20) can be used to show the following:

$$\frac{1}{N} \sum_{i=1}^{N} V_{ij} = Q_j$$

$$N \cdot D\left(\frac{1}{N} \sum_{i=1}^{N} V_{ij}\right) = N \cdot D(Q_j)$$

$$\sum_{i=1}^{N} D(V_{ij}) \geq N \cdot D\left(\frac{1}{N} \sum_{i=1}^{N} V_{ij}\right) \text{ using Jensen's inequality}$$

$$\sum_{i=1}^{N} D(V_{ij}) \geq N \cdot D(Q_j) \tag{21}$$

Combining equations (19) and (21) gives the result,

$$\sum_{i=1}^{N} D(U_{ij}) \geq \sum_{i=1}^{N} D(Q_j) \text{ for } j = 0, 1, \dots, L \tag{22}$$

Therefore, if *D(U)* is convex (which includes the case where *D(U)* is linear), then a greater or equal *AEC/V* results for a fleet with cumulative-usage-dependent O&M costs under the newest first practice than under the random practice in the general case.

If *D(U)* is concave, equation (21) is not necessarily true and the relationship between the random practice and the intermediate scenario is ambiguous. Cases where

D(U) was concave were investigated empirically, and in every case examined the random usage practice was at least as economical as the newest first practice (no counterexample was found).

HOW SHOULD AGE STANDARDS BE CALCULATED WHEN USAGE DECREASES WITH AGE?

In the previous section, an age standard was assumed. Assets were replaced upon reaching this age and not before or after reaching it. Determining this age standard is an important step in keeping total fleet-wide costs low. The impact of different usage practices on setting age standards is explored through several examples below.

Random Usage Practice Example Problem

Finding the replacement age that minimizes total fleet-wide cost under the random usage practice is fairly straightforward. Expected long-run usage is constant – effectively independent of other assets in the fleet and any replacement decisions – so replacement decisions can be broken down into individual serial replacement problems for each asset. Because asset cost functions are identical, the optimal serial replacement approach for one asset also applies to all of the other assets in the fleet. A single replacement age is selected based on the average experience of a large number of similar assets to minimize future costs that include variability. This is a standard approach, as described by Terborgh (1949).

Consider a fleet of 20 vehicles operating in a stable long-term pattern under the random usage practice. The maximum replacement age is 20 years and total fleet-wide usage (measured in miles) is the same in each period. The following additional parameter values have been selected for this example:

r = 10%	Effective interest rate per period
p = \$10,000	Purchase cost of a new asset
s = 0	Salvage value of a j-period old asset for all j = 1, 2, …, L
$m(u, U) = 0.005 \cdot u^{0.5} \cdot U^{0.7}$	O&M cost in a period for an asset with usage u during that period and cumulative usage U at the end of that period. This is an approximation of repair costs for ODOT sedans (based on historical data).
$q = 14487$	Expected usage of one asset in a period, measured in miles

Under the random usage practice, the expected usage in a period is the same at any asset age. The O&M costs for the example problem are shown in Table 6.

Table 6: Annual O&M cost by age for the example problem under the random usage practice

Asset Age	Annual Usage	Cumulative Usage	Annual O&M Cost ($)
1	14,487	14,487	492
2	14,487	28,974	800
3	14,487	43,461	1,062
4	14,487	57,948	1,299
5	14,487	72,435	1,519
6	14,487	86,922	1,725
7	14,487	101,409	1,922
8	14,487	115,896	2,110
9	14,487	130,383	2,291
10	14,487	144,870	2,467
11	14,487	159,357	2,637
12	14,487	173,844	2,803
13	14,487	188,331	2,964
14	14,487	202,818	3,122
15	14,487	217,305	3,276
16	14,487	231,792	3,428
17	14,487	246,279	3,577
18	14,487	260,766	3,723
19	14,487	275,253	3,866
20	14,487	289,740	4,007

The cost impact of varying replacement age from 1 to 20 years can be compared using the *AEC/V* criterion in equation (14), with *m(u,U)* substituted for *D(U)*:

$$AEC/V = \left\{ p - \left(\frac{1}{1+r}\right)^{L} s_L + \frac{1}{N}\sum_{j=1}^{L}\left(\frac{1}{1+r}\right)\delta^{j}\sum_{i=1}^{N} m(u,U) \right\} \left(\frac{r}{1-\left(\frac{1}{1+r}\right)^{L}}\right) \tag{23}$$

Table 7 includes the *AEC/V* value calculated for each replacement age from 1 to 20 years. Within this range, a replacement age of 13 years minimizes *AEC/V*, although the cost of a slightly shorter or longer cycle is not that much greater.

Table 7: AEC/V by replacement age for the example problem under the random usage practice

Asset Age	*AEC/V* ($)
1	11,492
2	6,400
3	4,788
4	4,036
5	3,624
6	3,378
7	3,224
8	3,127
9	3,065
10	3,028
11	3,007
12	2,997
13	**2,996**
14	3,000
15	3,009
16	3,021
17	3,034
18	3,049
19	3,065
20	3,082

Newest First Usage Practice Example Problem

Consider the same example problem, but under the newest first usage practice instead of the random practice. To keep things simple, only two replacement ages are initially compared – 20 years and 10 years. Assume that asset age groups are distributed as evenly as possible, so that any asset of a given age has the same expected usage and

O&M cost. For the 20-year replacement age, there is always one asset of each age in the range from 1 to 20. In the 10-year case, there are two assets of each age in the range from 1 to 10.

Example expected annual usage values for each age under a 20-year replacement age, along with cumulative usages and annual O&M costs, are presented in Table 8. These values were generated using equation (4) under the assumption of exponentially distributed interarrival and service times. Utilizations were multiplied by a factor of approximately 20,000 miles per year such that the total fleet-wide usage in any period was equal to that for the random usage practice problem described above, resulting in an average annual usage of 14,487 miles. Due to the even age distribution assumption, the 20-year cumulative usage for all assets is equal to 289,740 miles – the same as the random usage practice example.

Table 8: Annual O&M cost by age for the example problem under the newest first usage practice over a 20-year replacement cycle

Asset Age	Annual Usage	Cumulative Usage	Annual O&M Cost ($)
1	18,769	18,769	672
2	18,627	37,396	1,084
3	18,463	55,859	1,429
4	18,272	74,131	1,733
5	18,049	92,180	2,007
6	17,788	109,968	2,254
7	17,481	127,449	2,477
8	17,120	144,569	2,678
9	16,695	161,264	2,855
10	16,196	177,460	3,006
11	15,609	193,070	3,131
12	14,924	207,994	3,225
13	14,129	222,123	3,286
14	13,217	235,340	3,309
15	12,183	247,523	3,291
16	11,034	258,557	3,229
17	9,786	268,343	3,121
18	8,469	276,812	2,968
19	7,124	283,936	2,771
20	5,804	289,740	2,537

AEC/V was calculated according to equation (23) for each replacement age from 1 to 20 years using the data in Table 8 (results are shown in Table 9). This results in a correct value for a 20-year replacement age, but potentially incorrect values for replacement ages of length 1 through 19 because the total fleet-wide usage is not equal. For example, the usages in Table 8 result in a total fleet-wide usage of 354,922 in each period for a 10-year replacement age – nearly 22.5% greater than for the 20-year replacement age.

Table 9: AEC/V by replacement age for the example problem under the newest first usage practice, calculated using annual usages for a 20-year replacement age (resulting in incorrect AEC/V values for replacement ages of length 1 through 19)

Asset Age	*AEC/V* ($)
1	11,672
2	6,630
3	5,059
4	4,342
5	3,960
6	3,738
7	3,606
8	3,524
9	3,475
10	3,446
11	3,429
12	3,419
13	3,414
14	3,410
15	3,406
16	3,401
17	3,394
18	3,385
19	3,373
20	3,358

This is not surprising, since usage of an asset under the newest first practice is greater towards the beginning of its life. Only the early usage of assets with the 20-year replacement age is used to calculate the incorrect 10-year *AEC/V* value. This results in a greater total fleet-wide usage than the random practice or an asset with a 20-year replacement age under the newest first practice, as illustrated by Figure 8.

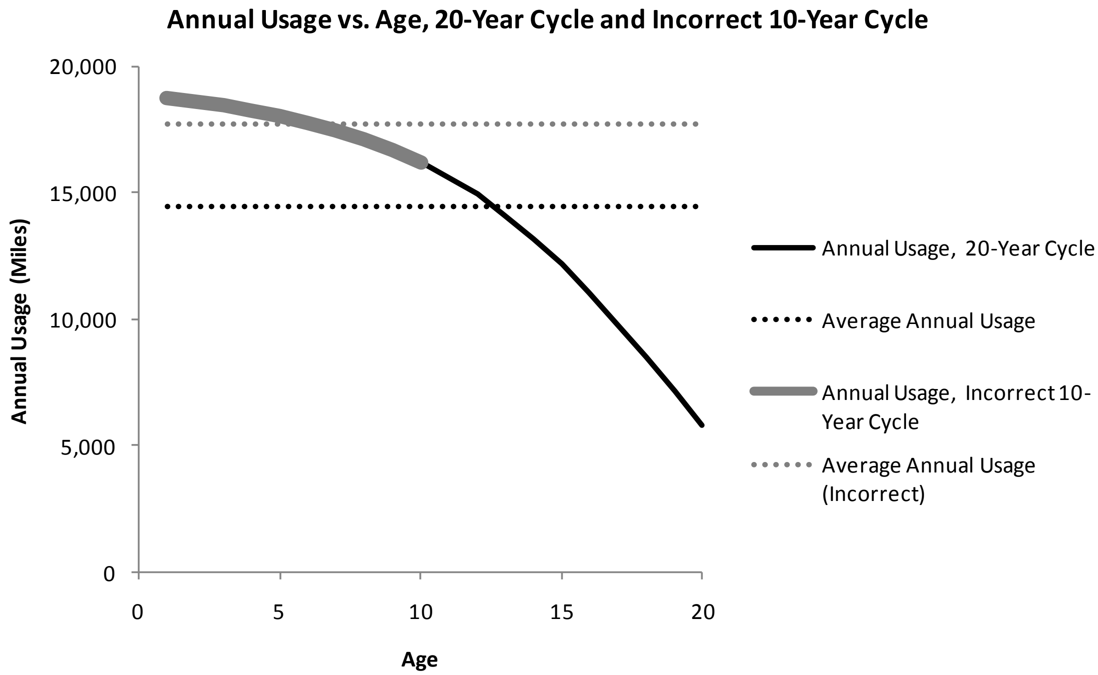


Figure 8: Annual usage vs. age, 20-year cycle and incorrect 10-year cycle

Since annual O&M costs in this example problem are usage-dependent, the difference in total fleet-wide usage cannot be ignored. Table 10 presents different usage values that correct this problem for a 10-year replacement age, resulting in an *AEC/V* of $3,160. These usage values were generated by calculating utilizations using equation (4) and multiplying the utilizations by 20,000 miles per year to get annual usage in miles. The corrected *AEC/V* value indicates that the 10-year replacement age is an improvement over the 20-year replacement age under the newest first practice, which is contrary to the conclusion one would draw from the incorrect value for the 10-year replacement age in Table 9.

Table 10: Annual O&M cost by age for the example problem under the newest first usage practice over a 10-year replacement cycle

Asset Age	Annual Usage	Cumulative Usage	Annual O&M Cost ($)
1	18,698	18,698	669
2	18,368	37,066	1,070
3	17,918	54,984	1,392
4	17,301	72,284	1,657
5	16,446	88,730	1,865
6	15,267	103,997	2,008
7	13,673	117,670	2,072
8	11,609	129,278	2,039
9	9,127	138,406	1,896
10	6,464	144,870	1,648

A graphical representation of the corrected usage values for the 10-year replacement age is included in Figure 9. The corrected average annual usage for the 10-year replacement age is equal to the average annual usage under the 20-year replacement age and the average annual usage under the random practice.

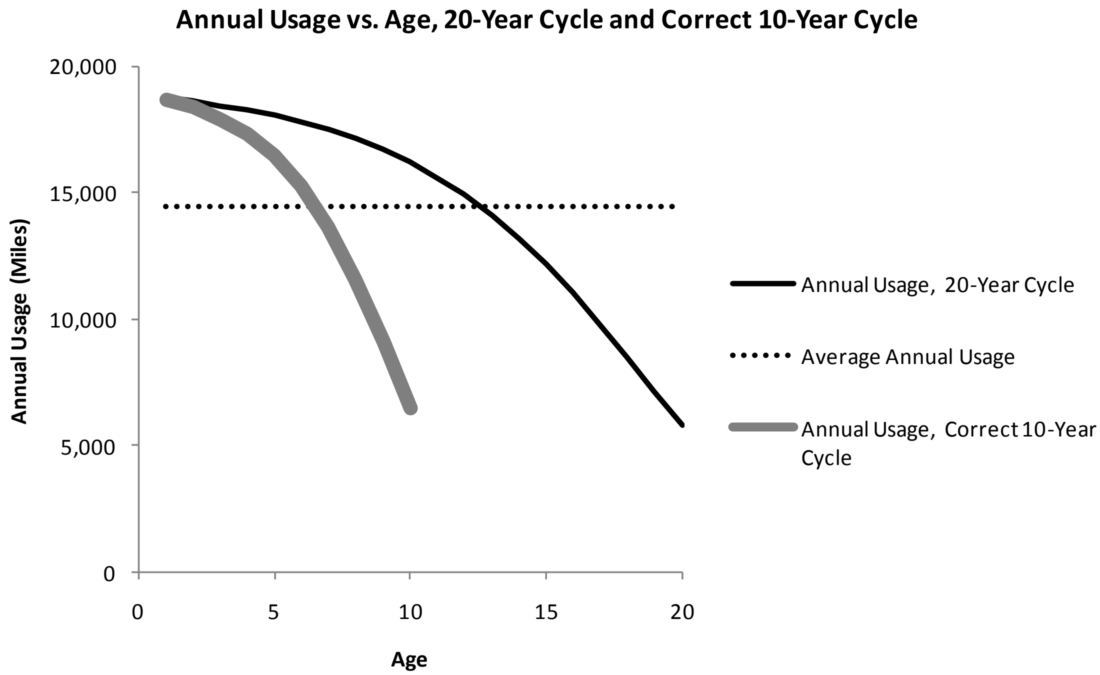


Figure 9: Annual usage vs. age for 20-year replacement age and correct 10-year replacement age

The annual usage vs. age curves in Figure 9 for the 10-year replacement age and for the 20-year replacement age are substantially different. Replacement problems with this feature will be referred to as having a “dynamic” usage vs. age curve – that is, the usage vs. age curve changes as the replacement age changes. ***If this change in usage impacts cost, then a new version of the dynamic usage vs. age curve must be calculated for each replacement age in order to correctly evaluate AEC/V.*** If the usage vs. age curve does not change based upon the replacement age (e.g., as under the random usage practice), then this will be referred to as a “static” usage vs. age curve.

Corrected annual usage schedules for each replacement age from 1 to 20 years were used to calculate the *AEC/V* values presented in Table 11. A replacement age of 11 years minimized *AEC/V* under the newest first practice for this example problem. Again, the difference in cost for a slightly shorter or longer replacement age is fairly small, but the more significant difference is between the two different usage practices. The *AEC/V* for the 11-year replacement age under the newest first practice was about 5% greater than the minimum *AEC/V* under the random practice (with a 13-year replacement age).

Table 11: AEC/V by replacement age for the example problem under the newest first usage practice, calculated using correct annual usages for each replacement age

Asset Age	*AEC/V* ($)
1	11,492
2	6,426
3	4,828
4	4,089
5	3,689
6	3,456
7	3,315
8	3,231
9	3,183
10	3,160
11	**3,153**
12	3,157
13	3,170
14	3,189
15	3,213
16	3,239
17	3,267
18	3,297
19	3,328
20	3,358

Critique of Several Other Approaches to Decreasing Usage with Age

Several other papers in the literature comment on the trend of decreasing asset usage with age, most commonly in vehicle fleets. Dietz & Katz (2001) and Redmer (2005) address this issue by comparing costs on a per-mile or per-kilometer basis. Buddhakulsomsiri & Parthanadee (2006) specifically hypothesize that a preference for newer assets may explain the trend of decreasing usage with age and adapt the model described by Hartman (1999). However, none of these approaches suitably address a case where assets are acquired for the purpose of meeting a minimum level of service and where the newest first usage practice results in a dynamic usage vs. age curve.

Dietz & Katz (2001) recognize a trend of decreasing asset usage with age in US West service vehicles, but follow a different and flawed approach in setting a replacement policy. In their approach an optimal per-mile cost is calculated for each vehicle type as part of a replacement scoring system. Per-mile cost is calculated for each possible replacement age by dividing the net present value (NPV) of all costs incurred by an asset for a given replacement age, by the cumulative mileage (according to a fitted quadratic curve) over the life of the asset. The minimum cost-per-mile value as a function of replacement age is used as the optimal per-mile cost. The NPV includes operating costs, purchase price, salvage value, and depreciation and tax effects.

The first problem is the use of the NPV to compare replacement cycles of different lengths. For example, the NPV of operating an asset for 5 years will likely be less than the NPV of operating an asset for 20 years just due to the shorter operation time (before considering any other ways in which costs may differ). If a minimum service level must be met over a 20-year period, a fairer comparison would be to repeat the 5-year replacement cycle four times so that an asset is available for the full 20 years in both cases. Calculating the equivalent annuitized payment for each project (based on 5 years and 20 years, respectively) would give the same result.

However, Dietz & Katz (2001) do not make either of these corrections. Instead, they compare NPVs of different length after dividing by cumulative mileage. For example, this approach can easily give two different per-mile costs for a 10-year replacement cycle based on whether costs are calculated over one cycle (10 years) or two cycles (20 years). An annuitized payment for a 10-year replacement cycle would not change regardless of how many cycles were used to calculate it, which makes for fairer comparisons between replacement cycles of different length.

Redmer (2005) also observes decreasing utilization intensity in vehicles as a function of age and calculates a per-km cost for each replacement age by dividing the NPV by cumulative usage. However, the author compares NPV over time periods of equal length, involving multiple consecutive replacements, and so avoids the first problem of Dietz & Katz (2001).

Both approaches are similar in that they essentially decompose a parallel replacement problem into independent serial replacement problems, as was solved above for the random usage practice. An effort is made to "correct" for unequal total usage at different replacement ages by dividing the NPV by cumulative usage to get a per-mile or per-km cost. In the case of the newest first usage practice, this is an attempt to deal with the increased total usage for shorter replacement ages (the problem illustrated by Figure

8). However, if the extra usage is not useful, using per-mile or per-km costs only obscures the problem – extra costs are being spread across capacity that may not actually be used.

Buddhakulsomsiri & Parthanadee (2006) do not use per-mile or per-km costs, but instead adapt the integer programming formulation of Hartman (1999). This is done by transforming the utilization-related decision variables into parameters, specifying utilization as a function of age. By doing so, the authors fall into the same larger trap as Dietz & Katz (2001) and Redmer (2005). That is, none of these approaches factor in the impact of replacement decisions on utilization (i.e., a dynamic usage vs. age curve). Taking a dynamic usage vs. age curve into account requires more complex calculations than any of these three approaches. An example of such an approach was used to solve the newest first usage practice example problem described above.

Approaches to Decreasing Usage with Age That Use Different Assumptions

This section describes two approaches to parallel replacement problems where asset usage varies, but these approaches are based on different assumptions than those outlined above. The approach taken by Hartman (1999) does not have the problem of excess usage occurring for shorter replacement ages. However, the author's formulation cannot easily be used to evaluate a specific usage practice (such as the newest first usage practice). Situations where additional usage may have economic value are also considered – i.e., situations where assets are not only acquired for the purpose of meeting a minimum level of service.

Hartman (1999) uses an integer programming formulation of a finite-horizon parallel replacement problem with individual asset usage levels treated as decision variables. There are several positive aspects to this approach. It allows more flexibility for the incorporation of capital constraints and multiple challengers. Usage levels are measured at evenly-spaced discrete intervals, which allows for complex formulas in determining costs at those specific points (taking other factors, such as inflation, into account). Although computational issues may exist, they are not related to fleet size. Since the structure is a three-dimensional network, the number of nodes – approximately equal to the product of the periods in the planning horizon, the maximum asset age, and the number of discrete values that cumulative usage can take – is proportional to the number of variables and constraints.

On the other hand, this approach is unable to easily answer questions about specific usage practices that may already be in place. One advantage to simple usage practices is that they are potentially easier to implement and manage. Not all fleets have the capability to control usage at the detailed level of this model (e.g., at some level assets may be under local control). Furthermore, it is possible that enforcing such a policy may incur additional administrative costs that are not factored into the model.

The number of discrete usage increments also has a significant impact on solution time. It may be difficult to find solutions in a reasonable amount of time for assets with relatively long lives, since this increases all three parameters that determine the number of nodes. Maximum age and cumulative usage must be increased for an asset with a longer life, and the planning horizon must also be increased to minimize end-of-study effects. Alternately, it may be desirable to increase the resolution of utilization decisions by adding more discrete increments. Solutions to the vehicle-based example problems solved by Hartman (1999) were found relatively quickly, but the largest problem only included four usage levels per year (0; 10,000; 20,000; and 30,000 miles per year) with a maximum cumulative usage of 100,000 miles, a maximum asset age of 6 years, and a planning horizon of 20 years. A vehicle with a maximum life in the range of 30 years would result in a much larger mixed-integer problem with many more variables and constraints. Fleet records indicate that some vehicles, such as heavy diesel trucks, may have physical lives of this length.

In some equipment fleets, such as a department of transportation fleet, assets are acquired to meet a minimum level of service. For the example problem outlined above, a greater total fleet-wide usage provides no additional value but does incur additional costs. However, additional output may be valuable in other situations (e.g., perhaps for a fleet of rental vehicles). In this case, it is possible to add a revenue term, *R(u)*, to the *AEC/V* formula to reflect the value of any additional usage:

$$AEC/V = \left\{ p - \left(\frac{1}{1+r}\right)^{L} s_L + \frac{1}{N} \sum_{j=1}^{L} \left(\frac{1}{1+r}\right)^{j} \sum_{i=1}^{N} [m(u,U) - R(u)] \right\} \left(\frac{r}{1 - \left(\frac{1}{1+r}\right)^{L}} \right) \tag{24}$$

This is similar to the common practice in engineering economics of evaluating net present value to compare projects with both income and expenses (Park, 2006). Allowing such variability in usage may also drastically increase the difficulty of finding an optimal replacement policy, however, since the solution space includes both replacement ages and different usage schedules.

CONCLUSIONS

Billions of dollars of equipment are replaced by manufacturing establishments every year in the U.S. – and that excludes equipment that is managed by government or other organizations. Optimizing replacement of such equipment can have a large financial impact. Replacement problems are commonly classified into serial and parallel replacement categories based on how many assets are operated at a time. Parallel problems may be broken into several serial problems if assets are independent of each other. However, economies of scale in asset purchase, budget constraints, and flexible usage policies are several factors that can introduce interdependence.

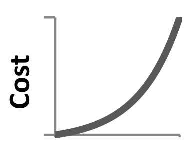
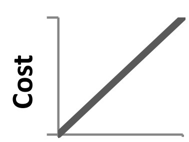
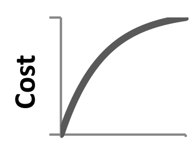
Observations of vehicle fleets, such as ODOT's, reveal that asset usage tends to decrease with age. Operator preference for newer assets is one possible explanation for this, and this paper investigated the implications of such a "newest first" usage practice. Long-run behavior of a fleet modeled on assets with identical cost functions under an age standard replacement mechanism was used for analysis.

Analysis of the fleet model using Little's Law resulted in a general procedure for calculating expected asset usage. It was shown that the newest first practice results in decreasing expected usage with age. A procedure for numerically calculating expected usage was also described under the assumption of exponentially-distributed interarrival and service times using an *M/M/n/n* queuing model.

The impact of the newest first practice on several types of O&M costs was evaluated as compared to the "random" usage practice (where expected usage is constant with respect to age). Impact on two usage-dependent cost categories was evaluated:

annual-usage-dependent costs and cumulative-usage-dependent costs. Different usage practices have no impact on non-usage-dependent costs. The preferred practice for annual-usage-dependent costs depended upon cost curve shape. A convex curve resulted in a preference for the random practice, while a concave cost curve resulted in a preference for the newest first practice. A linear cost curve resulted in no preference between practices. If asset ages are distributed evenly, the preference in the cumulative-usage-dependent cost case is for the random practice. This remains true even if asset ages are not evenly distributed if the cost curve is convex or linear, but was not proven for non-even age distributions where the cost curve is concave. A summary of the results is provided in Table 12.

Table 12: Summary of preferred usage practices for analyzed cost categories

		Economically Preferred Usage Practice		
		Cost Curve Shape **(with respect to cost category)**		
		 (Convex)	 (Linear)	 (Concave)
Cost Category	Constant, age-dependent, other non-usage dependent	Either		
	Annual-usage dependent	Random	Either	Newest first
	Cumulative-usage dependent	Random	Random	Random (if even age distribution)

In general, it appears that operating a fleet using the newest first usage practice results in greater or equal costs than the random practice (with the exception of annual-usage-dependent costs with a concave cost curve). This suggests that fleet operators may want to consider changing practices if the newest first usage practice is currently used. However, there is also room for further analysis by incorporating such concerns as technological advancement.

The impact of the newest first usage practice on replacement decisions was also considered, specifically when using an age standard (which is a common practice among DOTs). *AEC/V* was calculated for each replacement age up to 20 years for an example fleet under each usage practice, using data generated under a 20-year replacement age. The age standard was selected as the replacement age that minimized *AEC/V*. This was equivalent to decomposing the parallel replacement problem into individual serial replacement problems for each asset. While this did not raise any issues for the random practice, it led to incorrect results for the newest first practice. Using the first several years of usage for a longer replacement age to calculate *AEC/V* for a shorter replacement age resulted in excess fleet-wide usage (it was assumed that the fleet was operated for the purpose of meeting a minimum level of service). Excess usage resulted in excess costs.

Generating different newest first usage versus age curves for each replacement age – with total fleet-wide usage equal to that under the random practice – resulted in different *AEC/V* values and a different age standard for the fleet. The phenomenon was described as a “dynamic” usage versus age curve, since the usage versus age curve varied based on replacement decisions. A shorter age standard was selected under the newest first practice than under the random practice for the example problem (11 years versus 13 years), with the random practice resulting in a lower minimum *AEC/V*.

This approach was then compared to several alternate approaches taken in the parallel replacement literature, specifically in cases where usage was observed to decrease with age. While Dietz & Katz (2001) and Redmer (2005) based their approaches on per-mile and per-kilometer costs to adjust for extra total usage under shorter replacement ages, their approach did not address whether the additional usage was useful. Buddhakulsomsiri & Parthanadee (2006) adapted the model of Hartman (1999) by replacing asset utilization as a decision variable by an age-based parameter. However, none of these three approaches considered the effects of a dynamic usage versus age curve, where usage patterns depend upon replacement decisions.

Hartman (1999) described an approach that included individual asset utilization levels as discrete decision variables. While a useful approach allowing for fleet-wide optimization, general usage practices such as the newest first practice cannot be addressed. Such an approach is likely to only be useful in fleets where there is a very high degree of centralized control, and where maximum asset ages are relatively short to avoid computational issues.

For fleets that do operate under something similar to a newest first usage practice, it appears that the most practical approach to replacement decisions is that taken by this paper. The fleet is simulated under each replacement option (i.e., various replacement ages) and resulting *AEC/V* values are compared. Further opportunities for study might include investigating other types of replacement standards (such as usage standards, or standards that are based on a combination of age and usage) as applied to the newest first usage practice, or incorporating some of the other features that have been included in replacement models elsewhere, such as technological advancement, multiple challengers, or budget constraints.

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