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CRITICAL BUCKLING STRENGTH OF STIFFENED FLAT PLYWOOD PLATES IN COMPRESSION AND SHEAR—CLOSELY SPACED STIFFENERS

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CRITICAL BUCKLING STRENGTH OF STIFFENED FLAT PLYWOOD PLATES IN COMPRESSION AND SHEAR -- CLOSELY SPACED STIFFENERS[1]

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Summary and Conclusions

The equations previously[2] derived for the critical stresses in flat plywood plates subjected to edgewise compression or shear were applied to plywood stiffened with a multiplicity of parallelly placed stiffeners. It was found that these equations apply with reasonable accuracy if the over-all elastic properties of the stiffened plates are substituted in them. In the verification of the equations, the elastic properties used were determined for the plates tested by auxiliary tests.

Methods were derived for the computation of the over-all elastic properties of multiply stiffened plywood plates from the elastic properties of the plywood and the stiffeners and the dimensions and spacing of the stiffeners. These methods were verified by test and found to yield good approximations of the over-all properties.

Introduction

In the construction of wood aircraft structures plywood is sometimes stiffened with a multiplicity of stiffeners. Identical stiffeners are arranged parallel to each other, uniformly spaced, and are cemented to the inner face of the plywood. The critical loads of such plates subjected to edgewise compression or shear are required for purposes of design.

If the spacings between the stiffeners are sufficiently small, the plate, as a whole, may be considered as a unit and the critical loads determined by means of the formulas developed for unstiffened plywood,[2] provided the

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[2]Forest Products Laboratory Report No. 1316, "Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear."

properties, such as stiffness in directions parallel and perpendicular to the direction of the stiffeners and flexural rigidity, are known. These properties can be determined by tests of the stiffened plates or computed by the methods described in this report.

The report is, therefore, divided into two major sections. The first is an experimental verification of the use of the formulas developed for plywood, for the determination of the critical loads of plywood stiffened with a multiplicity of stiffeners. For use in this section the properties of the stiffened plates were determined by test. The second section contains the derivations of the formulas required for computing the properties of multiply stiffened plates and the results of their experimental verifications.

Each of these sections is divided into parts. The first is divided into two parts, one dealing with edgewise compression and the other dealing with edgewise shear. The second is divided into four parts: (1) flexural stiffness in the direction of the stiffeners; (2) flexural stiffness perpendicular to the direction of the stiffeners; (3) shear stiffness; and (4) flexural Poisson's ratio effect.

Section I. Verification of Formulas for Critical Loads

Part 1. Edgewise Compression

Formula to be Verified

As shown in Forest Products Laboratory Report No. 1316,[2] the following equation is that for the critical stress of a plywood plate simply supported at its edges and subjected to a uniformly distributed compressive stress over two opposite edges. It is:[3]

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[3] In dealing with two dimensional problems concerning an orthotropic material such as wood, the (x) and (y) axes are chosen in the directions of the natural axes of the material. Some of the elastic properties are associated with one or the other of these axes; others are associated with both. The subscripts used denote this association.

In dealing with plywood, over-all elastic properties are employed. These properties are divided into two groups; those associated with direct stress and those associated with flexure. For those associated with direct stress the subscripts (a) and (b) are employed in place of (x) and (y), and for those associated with flexure, the subscripts (1) and (2) are employed. For example, let any elastic property that applies to wood be denoted by (H). It may have the subscripts (x), (y), or (xy), depending upon its direction and the particular elastic property it represents. It also may have different values in the different plies of the plywood. The directions (x) and (y) are the same for all plies, thus (x) may be with the grain in one ply and across it in another. The over-all elastic properties of the plywood can be computed by the following formulas:

(Continued on p.3)

$$p_{cr} = \frac{\pi^2}{12\lambda}\left[E_1 \frac{b^2}{n^2a^2} + 2A + E_2 \frac{n^2a^2}{b^2}\right]\frac{h^2}{a^2} \qquad (1)$$

in which:

b = length of plate in the direction stress, in inches.
a = width of plate in the direction perpendicular to the direction of stress, in inches.
h = thickness of plate, in inches.
n = number of half waves into which the plate buckles. The integer is chosen that results in the least value of the critical stress.

$$H_a = \frac{1}{h}\sum H_{xi}\ h_{xi}$$

and similarly for H_b, H_{ab}, and H_{ba}.

$$H_1 = \frac{12}{3}\sum\left[\frac{h_i^3}{12} + h_i\ y_i^2\right]H_{xi}$$

where

H_{xi} = the elastic property for the ith ply of the plywood in the (x) direction.
h = thickness of the plywood.
h_i = thickness of the ith ply of the plywood.
y_i = distance of the center of the ith ply from the neutral axis of the plywood

and similarly for H_2, H_{12}, and H_{21}. The position of the neutral axis of the plywood can be obtained by the following formula:

$$c = \frac{\sum h_i\ H_i\ c_i}{\sum h_i\ H_i}$$

where:

c = the distance of the neutral axis from one face of the plywood.
c_i = the distance of the center of the ith ply to the same face of the plywood.

The summations are taken over all of the plies of the plywood. If the plywood is symmetrically constructed about its central plane both as to ply thickness and species, the neutral plane will be identical with its central plane. If the plywood is not so constructed, the neutral plane will not be identical with the central plane and may be located differently for different over-all elastic properties.
This matter is discussed more fully in Forest Products Laboratory Report No. 1312, the No. 1316 series, and others.

E_1 = the flexural modulus of elasticity of the plywood in the direction perpendicular to that of the stress.

E_2 = the flexural modulus of elasticity of the plywood in the direction of the stress.

$A = (E\sigma)_{12} + \mu_{12}$

$\lambda = 1 - \sigma_{12}\,\sigma_{21}$, approximately 0.99 for wood and plywood.

μ_{12} = the flexural modulus of rigidity of the plywood.

$(E\sigma)_{12}$ = the flexural modulus, including the Poisson's ratio effect.

σ_{12} = Poisson's ratio, in flexure of the plywood, of the strain in the direction (1) of (b) to the strain in the direction (2) of (a) due to a stress in the direction (1).

σ_{21} = is similarly defined.

In the application of equation (1) to stiffened plates, it is not clear what value to use for the over-all thickness of the combination. The same difficulty is encountered in the computation of the over-all elastic properties.

This difficulty can be avoided by setting up equation (1) in terms of the critical load per inch of width rather than in terms of stress. Thus the equation becomes:

$$P_{cr} = \frac{\pi^2}{\lambda a^2}\left[E_1 \frac{h^3}{12}\frac{b^2}{n^2a^2} + 2A\frac{h^3}{12} + E_2\frac{h^3}{12}\frac{n^2a^2}{b^2}\right] \tag{2}$$

Let $D_1 = \dfrac{E_1h^3}{12\lambda}$

$D_2 = \dfrac{E_2h^3}{12\lambda}$

$D_{12} = \dfrac{Ah^3}{12\lambda}$

Then $$P_{cr} = \frac{\pi^2}{a^2}\left[D_1\frac{b^2}{n^2a^2} + 2D_{12} + D_2\frac{n^2a^2}{b^2}\right] \tag{3}$$

in which the elastic properties of the stiffened plate are contained in the values of D_1, D_{12}, and D_2 that are to be determined by auxiliary tests and $n = 1$ for a plate that breaks into a single buckle.

The values of D_1 and D_2 can be determined by bending tests of the plate in which two edges are supported and the load applied along a central line parallel to the supported edges. The usual formula for a centrally loaded wide beam yields

$$D_1 = \frac{L}{d}\frac{a^3}{48\lambda} \tag{4}$$

$$D_2 = \frac{L}{d}\frac{b^3}{48\lambda} \tag{5}$$

in which L and d are the loads applied and the deflections measured respectively, and λ, because wood is the material dealt with, can be considered to be unity.

The value of D_{12} is divided into two parts:

$$D_{12} = \frac{h^3 (E\sigma)_{12}}{12\lambda} + 2\,\frac{h^3 \mu_{12}}{12} = F + 2B \qquad (6)$$

The first part will be represented by F and the second by 2B. Values of B can be experimentally obtained by supporting the plate at two diagonally opposite corners, loading it on the other two corners, and measuring the amount of twist obtained by the method described in a previous report,[4] thus:

$$B = \frac{P}{W}\,\frac{u^2}{8} \qquad (7)$$

in which P and W are the load applied and the deflection measured by the method described in report No. 1301, and u is the diagonal distance from the center of the plate to the position at which W is measured.

A method of test is not available for the determination of F. The following approximate expression for its value, however, is derived in Section II, part 4:

$$F = \sqrt{\sigma_{TL}\,\sigma_{LT}\,D_1\,D_2} \qquad (8)$$

Description of Specimens

Five plates, approximately 24 inches square, were fabricated for the compression buckling tests. Three of these were made of five plies of 1/48-inch yellow-poplar veneer, with grain directions of adjacent plies perpendicular to each other, and two of four plies of 1/20-inch veneer, with grain directions of the two central plies parallel to each other and perpendicular to the remaining plies. One plate in each group had the length of the stiffeners oriented perpendicular to the grain of the outside plies of the plywood, while the others had the length of the stiffeners parallel to the face grain of the plywood. In all cases the plates were tested with the grain direction of outside plies of the plywood parallel to the direction of the load.

Each plate had 13 stiffeners of quarter-sawn Sitka spruce, 3/4 of an inch in width, and either 1/8 or 1/4 of an inch in thickness. The length of the stiffeners was equal to the length of the plywood plates, and the stiffeners were spaced 2 inches, center to center, and were glued to the plates with a resorcinol-resin adhesive.

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[4] Method for Measuring the Shearing Moduli of Wood. Forest Products Laboratory Report No. 1301. The nomenclature used here is that of report No. 1301.

After each plate was tested it was cut to a smaller size and retested. This procedure was repeated several times in some cases. Also, some of the tests were repeated. A total of 21 tests were made.

Methods of Test

The apparatus used in making the buckling tests is shown in figure 1. The tests were carried out in accordance with the method for unstiffened flat plywood panels described in Forest Products Laboratory Report No. 1554, "Methods of Conducting Buckling Tests of Plywood Panels in Compression." The panels were placed in the apparatus so that the neutral plane, computed by the usual method and assuming bending in the direction of the stiffeners, contained the axes of the cylindrical segments, described in report No.1554, through which the load was applied to the panel.

Figure 2 shows the test employed to determine values of D_1 and D_2. The span of the beam is not quite the full length of the plate and the values of a and b used in formulas (4) and (5) were reduced accordingly.

Figure 3 shows the test employed in the determination of the second part of D_{12}. It is described in report No. 1301.

Results of Tests

The results of the tests are given in table 1. The values of D_1, D_2, and B were obtained from tests according to formulas (4) to (7), inclusive. The computed critical loads were then obtained by use of formula (3). The observed critical load was obtained by means of the test previously described and illustrated in figure 1.

The computed critical loads can be conveniently compared with the observed critical loads by plotting one against the other. Figure 4 is such a plot in which both the computed and observed loads have been divided by the estimated ultimate load in compression. This estimate was made by adding the compressive strength of the stiffeners to that of the plywood, with the compressive strengths being computed from the results of compression tests on specimens cut from the plates. In figure 4, then, the points representing the various specimens are arranged in order according to the value of the stress associated with the critical load. It may be noted that the highest direct stress encountered was about 45 percent of the ultimate compressive stress and that the proportional limit of the material was not exceeded.

The straight line in figure 4 passes through points indicating absolute agreement between the computed and observed critical loads. It is evident from the grouping of the plotted points around this line that the formula yields a reasonable estimate of the critical loads obtained by test.

Part 2. Edgewise Shear

Formula to be Verified

Equation (20) in report No. 1316[2] is the formla used for determining the critical buckling stress in shear for a flat plywood plate. It is

$$q_{cr} = \frac{c_a h^2}{3\lambda a^2} (E_1{}^3 E_2)^{1/4} \tag{9}$$

With a knowledge of the elastic properties of the plywood, it is a simple matter to compute the critical buckling stress in shear. For a plate stiffened with closely spaced stiffeners, however, the question arises as to what value to use for the over-all thickness of the combination; also, what method to use to compute the over-all elastic properties. As in the preceding discussion of compression buckling, this difficulty can be avoided by setting up equation (9) in terms of the shearing load per inch of width and using the stiffness factors obtained from static tests.

In terms of shearing load per inch of width, equation (9) is:

$$Q_{cr} = 4 \frac{c_a}{a^2} \left[\left(\frac{E_1 h^3}{12\lambda}\right) \left(\frac{E_2 h^3}{12\lambda}\right) \right]^{1/4} \tag{10}$$

and making the substitutions indicated in Part 1:

$$Q_{cr} = 4 \frac{c_a}{a^2} (D_1{}^3 D_2)^{1/4} \tag{11}$$

in which the value of c_a is determined from figure 8 of report No. 1316 by use of the parameters α and β_a. It is evident from report No. 1316 that the values of these parameters are given by:

$$\alpha = \frac{D_{12}}{(D_1 D_2)^{1/2}} \tag{12}$$

$$\beta_a = \frac{a}{b} \left(\frac{D_2}{D_1}\right)^{1/4} \tag{13}$$

The values of D_1, D_2, and D_{12} contain the elastic properties and can be obtained by tests of the plate as described in part 1 of this section.

Description of Specimens

Six stiffened plywood plates, similar to those described in part 1 of this section, approximately 18 inches square, were fabricated for the buckling tests in shear. Three plates were made of three-ply and three plates of five-ply 1/48-inch yellow-poplar veneers. Each plate was provided with nine Sitka spruce stiffeners, 1/2 by 3/4 inch in cross section, running parallel to the grain of the face plies of the plywood and spaced 2 inches center to center.

Method of Test

The apparatus employed is shown in figure 5. It consists of eight steel channels, 3 inches deep and 18 inches in length; four sets of steel rollers and pins, and V-shaped, hardened-steel loading blocks. Each channel has a strip of fine sandpaper glued to its face to reduce the slippage between the plate and the channels when load is applied.

In preparation for test, the channels are grouped into four pairs. Each pair is fastened along one edge of the plate with the flat sides of the channel in contact with the faces of the plate. Gripping of the plate between each set of channels is secured by bolts inserted through holes in the channels and the plate.

Load is transmitted from the testing machine to the test plate through the V-shaped blocks to rollers attached to a steel plate welded to one end of each channel. When the apparatus is properly assembled, the rollers are located near the ends of one diagonal of the rectangle formed by the steel frame, and the ends of the other diagonal are restrained from moving under load except in the direction of the plane of the plate.

In preparation for test the channels were attached to the plate and the rollers placed in the proper location. Because of the weight of the apparatus, the plate had a tendency to bend along its horizontal diagonal when in an upright position and before it was subjected to the imposed loads. Consequently, it was necessary to attach metal straps in a vertical position across the ends of the channels on both sides of the frame as shown in figure 5. The plate was then placed in the testing machine and alined so that the plane of the plate was parallel to the applied load and a uniform bearing was obtained between the two "v" blocks and the four rollers.

A lateral deflection gage was positioned at the center of the plate, and load-lateral deflection readings were recorded at uniform increments of load. Load-deflection curves were plotted and the critical load estimated by the position of the "knee" of the curve.

Values of D_1, D_2, and D_{12} were determined by the methods described in part 1 of this section.

Results of Tests

The results of the tests are given in table 2. The values of D_1, D_2, and B were obtained from tests according to formulas (4) to (7), inclusive. The computed critical loads were then obtained by use of formula (11) with figure 8 of report No. 1316, with the values of the parameters being obtained from formulas (12) and (13). The observed critical loads were obtained by the method described above and illustrated in figure 5.

The computed critical loads are plotted aginst the observed critical loads in figure 6. The straight line in this figure passes through points indicating absolute agreement between the two. It is evident from the positions of the plotted points that formula (11) yields values that are reasonable estimates of the observed values.

Section II. Methods of Computing Elastic Properties of Multiply Stiffened Plywood Plates

Part 1. Flexural Stiffness in the Direction of the Stiffeners

Derivation of Formula

It is assumed that the stiffeners are closely spaced and that, therefore, the usual elementary method can be employed to determine the position of the neutral axis and then the stiffness. The formula obtained is:

$$D_1 = D_p + \frac{W_s E_s h_s}{12W_p}\left[h_s^2 + \frac{3h_c^2}{\frac{W_s E_s h_s}{W_p E_a h_p} + 1}\right] \tag{14}$$

in which D_1 = the effective stiffness of the stiffened plate per unit width

D_p = the stiffness of the plywood alone, in the direction of the stiffeners $= \frac{E_1 h_p^3}{12}$

E_a = the direct-stress modulus of elasticity of the plywood alone, in the direction of the stiffeners

E_s = the modulus of elasticity of the stiffeners in their longitudinal direction.

The remaining symbols have to do with the dimensions of the stiffened plate and are defined by figure 7.

Description of Specimens

Twelve specimens were fabricated. Plywood made of nine plies of 1/16-inch yellow-poplar veneers was used in five of these specimens, and four plies of 1/16-inch yellow-poplar in the remaining seven. The grain directions of the two central plies of the four-ply plywood were parallel to each other, so that the construction was substantially three ply of 1:2:1 type. The material used for stiffeners was quarter-sawn Sitka spruce in thicknesses of 1/8, 1/4, 1/2, 3/4, and 1 inch. This material was edge-glued into slabs equal in size to the plywood plates, and one such slab was glued to one face of each plywood plate.

The specimens were then tested for stiffness in the manner illustrated in figure 2, with the grain of the spruce stiffeners running from support to support, and the values of loads limited so that the proportional-limit stress of the material was not exceeded. A series of grooves were then cut, by means of a rotary saw 1/4-inch thick, in the spruce slabs down to the face of the plywood. These grooves were spaced 2 inches on center. This resulted in plywood panels stiffened with stiffeners 1-3/4 inches wide spaced on 2-inch centers. The specimens were then tested for stiffness again. The width of the stiffeners was reduced to about 1-1/2 inches by means of the saw, and the specimens were retested. This process was repeated until the stiffener material was completely removed from the plates and the plates, alone, tested for stiffness.

Results of Tests

The results of the tests are tabulated in columns 10 and 11 of table 3. The values in column 10 were obtained from the tests by means of formula (4). Those in column 11 were computed by use of formula (14). The value of E_a substituted in this equation for each plate was chosen to be consistent with the flexural moduli of elasticity E_1 and E_2 obtained by the bending tests after the stiffeners were completely removed, and to be consistent with the constructions of the various plates. The value of E_s was then computed by means of the results of the first tests, in which the stiffeners completely covered the plates, by use of equation (14) by substituting the values of D_1 obtained from these tests, equating W_s to W_p, and solving for values of E_s. These values of E_a and E_s were then substituted in equation (14) and the values of D_1 obtained, which are tabulated in column 11 of table 3.

In figure 8 the computed values of D_1 are plotted against the values obtained from test. The straight line in the figure passes through points indicating absolute agreement between the computed and observed values. The grouping of the plotted points around this line indicates that formula (14) yields values of D_1 that are good estimates of the values obtained from tests.

Part 2. Flexural Stiffness Perpendicular to the Direction of the Stiffeners

Derivation of Formula

A stiffened plywood plate subjected to a static-bending test simply supported at the ends, loaded at the center of the span, and oriented such that the stiffeners are perpendicular to the span, does not have a uniform elastic curve across the length of the specimen when under load. The curvature for the nonstiffened portion of the plate is greater than for the stiffened portion.

Also, stress concentrations occur at the edges of the stiffeners that cause sharp kinks in the elastic curve at these locations. Figure 9 is a sketch of a portion of the elastic curve that is indicated by line HBCDF. The arc CDF of radius R_{sp} represents the elastic curve at one of the stiffeners. The arc HBC of radius R_p represents the elastic curve of the plate alone between the stiffeners. The dotted arc HB'CD'F of radius R_a represents the average elastic curve. The kink caused by stress concentrations is represented by the angle ϕ. Thus the two arcs HBC and CDF are not quite tangent at point C. The width of the stiffener is represented by W, and the spacing of the stiffeners center to center by S.

From the geometry of the figure

$$\text{Angle MCE} = \phi$$

$$\text{Angle CME} = \frac{W}{2R_{sp}}$$

Considering triangle CME

$$\text{Angle PEA} = \text{angle MCE} + \text{angle CME} = \phi + \frac{W}{2R_{sp}}$$

Further, angle EPA $= \frac{S-W}{2R_p}$

and considering triangle EPA

$$\text{Angle BAD} = \text{angle PEA} + \text{angle EPA} = \phi + \frac{W}{2R_{sp}} + \frac{S-W}{2R_p}$$

Now angle BAD is approximately equal to $\frac{S}{2R_a}$, therefore:

$$\frac{S}{2R_a} = \phi + \frac{W}{2R_{sp}} + \frac{S-W}{2R_p} \qquad \text{(approx)}$$

$$\text{or} \qquad 2\phi = \frac{S}{R_a} - \frac{W}{R_{sp}} - \frac{S-W}{R_p} \qquad \text{(approx)}$$

Using the general equation

$$\frac{1}{R} = \frac{M}{D}$$

in which R = radius of curvature
M = bending moment
D = stiffness of the section considered

$$2\phi = M\left[\frac{S}{D_2} - \frac{W}{D_{sp}} - \frac{S-W}{D_p}\right] \qquad \text{(approx)} \qquad (15)$$

It is now proposed to build up arbitrarily another expression for 2ϕ consistent with the assumption that this angle is due to stress concentrations. It is evident that ϕ must be small if D_{sp} and D_p are very large; also, that it must be finite if D_{sp} is very large and D_p is finite. These requirements are met if ϕ is proportional to $(\frac{1}{D_{sp}} + \frac{1}{D_p})$. It is evident also that ϕ must be zero when $D_{sp} = D_p$, and finite when D_{sp} is very large. These requirements are met if ϕ is proportional to $(1 - \frac{D_p}{D_{sp}})$. Further ϕ must be some function, z, of the stiffener width, W, and proportional to the bending moment M. The following expression satisfies all these conditions:

$$2\phi = M\left(\frac{1}{D_{sp}} + \frac{1}{D_p}\right)\left(1 - \frac{D_p}{D_{sp}}\right) z \qquad (16)$$

Eliminating 2ϕ from equations (15) and (16)

$$\frac{D_p}{D_a} = \frac{z}{S}\left[1 - \left(\frac{D_p}{D_{sp}}\right)^2\right] - \frac{W}{S}\left[1 - \frac{D_p}{D_{sp}}\right] + 1 \qquad (17)$$

The relation between $\frac{z}{S}$ and $\frac{W}{S}$ will be determined experimentally, and equation (17) be verified by test.

Method of Test

The method of test used is illustrated by figure 2, except that the stiffeners were placed parallel to the supports. The specimens described in part 1 of this section were used. Each specimen was tested in this way immediately after the corresponding test described in part 1 was performed. The values of D_a were obtained by use of formula (5) and are tabulated in column 12 of table 3.

Determination of the Relation Between $\frac{z}{S}$ and $\frac{W}{S}$

Equation (17) was used in this determination. For each plate a value of D_{sp} was obtained from the first test, and a value of D_p from the last test, by use of equation (5). Various values of D_a were determined from the intermediate tests. These values were substituted in equation (17), and associated values of $\frac{W}{S}$ and $\frac{z}{S}$ were obtained. These values are tabulated in columns 20 and 21 of table 3. Values for the first five plates of the table were not determined. These plates had a tencency to warp as the size of the stiffeners was reduced. The warpage of the plates was sufficient to mask the effect of the decrease in the size of the stiffeners.

Values of $\frac{z}{S}$ are plotted against $\frac{W}{S}$ in figure 10. The plotted points scatter considerably due to the fact that it is necessary to take differences of numbers of the same order of magnitude in solving equation (17) for $\frac{z}{S}$. It is evident, however, that the curve is roughly parabolic. The equation

$$\frac{z}{S} = 0.312 \left[1 - \left(1 - \frac{W}{S}\right)^2\right] \tag{18}$$

fits the data reasonably well. The proportionality factor 0.312 was obtained by the method of least squares.

Substituting the value of $\frac{z}{S}$ given by equation (18) in equation (17), the complete formula becomes:

$$\frac{D_p}{D_2} = 0.312 \left[1 - \left(1 - \frac{W}{S}\right)^2\right] \left[1 - \left(\frac{D_p}{D_{sp}}\right)^2\right] - \frac{W}{S}\left[1 - \frac{D_p}{D_{sp}}\right] + 1 \tag{19}$$

This formula was verified by test.

Verification of Formula

Values of D_{sp} and D_p obtained by means of formula (5), from the first and last test of each plate, were substituted in formula (19) with the appropriate values of $\frac{W}{S}$, and values of D_2 were obtained. These values are tabulated in column 13 of table 3. They are plotted against the values obtained by test in figure 11. The straight line in this figure passes through points indicating absolute agreement between computed and observed values. The grouping of the plotted points around this line indicates that the formula gives reasonable approximations of the observed values.

Part 3. Shear Stiffness

Derivation of Formula

Consider a rectangular prism twisted by two couples at its ends. The value of the applied torque is:[5]

$$T = \left(\frac{16}{3} - \lambda\frac{h}{w}\right)\frac{wh^3}{16}\mu\theta \qquad (20)$$

in which:

T = applied torque
h = thickness of prism
w = width of prism
θ = angle of twist per unit length of prism
μ = modulus of rigidity of the material
λ is a function of $\frac{w}{h}$ and is 3.36133 for $\frac{w}{h}$ greater than 4

Equation (20) may be written:

$$\frac{3T}{\theta} = \overset{\mu}{\cancel{wh^3}} - \frac{3}{16}\lambda h^4\mu \qquad (21)$$

The term $\frac{3T}{\theta}$ is proportional to the torsional stiffness of the prism. Equation (21) shows that the stiffness of a wide prism (w greater than 4h) is a linear function of its width.

It is convenient to think of torsion in terms of the soap-film method.[6] If a soap film is stretched across an opening equal in size and shape to that of the cross section of the prism and a slight difference in air pressure is maintained across the film, it will assume a shape such that the slope of the film at any point in the cross section will be proportional to the shear stress at the corresponding point in the prism. Also, the volume enclosed by the film and the plane of the opening will be proportional to the torsional stiffness of the prism.

Such a film stretched across a narrow rectangular opening will assume the shape of a parabolic cylinder except at the ends of the opening. Here the film will be pulled down to meet the ends of the opening. Equation (21) shows that the cross-sectional area of the parabolic cylinder formed by the film is μh^3, and that $\frac{3}{16}\lambda h^4\mu$ is the amount of volume lost from the parabolic cylinder due to the pulling down of the film at the ends of the

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[5] "The Torsion of Members Having Sections Common in Aircraft Construction" by G. W. Trayer and H. W. March. National Advisory Committee for Aeronautics Report No. 334.

[6] Ibid. Other references given there.

rectangular opening. If the rectangular prism is so wide that this loss of volume can be neglected, the second term in the right-hand member of equation (21) can be omitted. The equation then becomes identical to that derived for a flat plate supported at two diagonal corners and loaded at the other two corners.[3]

The torsional stiffness of a stiffened plywood plate may be analyzed in a similar manner. Figure 7 shows the cross section of such a plate and the nomenclature used. The stiffeners are all of equal widths and thicknesses. It is first necessary to determine the $\mu \frac{h^3}{12}$ values for the plywood plate alone and for the combination of the plate and the stiffener material. This value for the plate alone is determined in the usual way (see footnote 3). The value for the combination is obtained in the same manner except that the stiffener is assumed to be part of the plywood, i.e., an extra ply, or a stiffener having a width equal to the width of the plate. The value for the plywood alone is represented by B_p and the value for the combination of plywood and stiffener material by B_c.

Consider a soap film stretched over an opening the shape of the cross section shown in figure 7. The value of B_p will be proportional to the cross-sectional area of the parabolic cylinder formed by the film at section A-A if the stiffeners do not come too close to this section. The value B_c will be proportional to the similar area at section B-B provided that the stiffener is sufficiently wide. The film forms a continuous surface between the two sections. Figure 12 shows an estimate of the contours of such a surface and the cross-sectional areas involved.

Following equation (21), the over-all value of B of the stiffened plate is given by the formula:

$$B = \frac{1}{w_p} \left[(w_p - w_s) B_p + (w_s - n\epsilon h_c) B_c \right] \tag{22}$$

in which ϵ is a correction factor due to the "pulling down" of the soap film at the edges of the stiffeners. From equation (22), this correction factor is found to be:

$$\epsilon = \frac{1}{n_c n B_c} \left[(w_p - w_s) B_p + w_s B_c - w_p B \right] \tag{23}$$

It should be noted that the correction factor (equation 23) does not reduce to zero when the plate is completely covered by stiffeners. The reason for this is illustrated by figure 13, which is an estimate of the contours of the soap film surface at the juncture of two stiffeners placed side by side.

The error ϵ affects only one term of the four of equation (22) and, therefore, if a reasonable approximation can be found for this error, accurate values of B can be obtained by use of this equation.

Method of Test

The method of test used is illustrated in figure 3 and described in report No. 1301. The specimens described in part 1 of this section were used. Each specimen was tested in this way immediately after the corresponding test described in part 2 was performed. Values of B were obtained by use of formula (7) and are tabulated in column 8 of table 3.

Evaluation of the Error ϵ

Values of B_c and B_p were determined from the first and last tests of each plate and values of B from the intermediate tests. Values of ϵ were then obtained by use of equation (23) and are tabulated in column 14 of table 3. These values are plotted against values of $\frac{w_s}{nh_s}$ in figure 14. It will be noted that the curves obtained seem to approach horizontal asymptotes at various values of ϵ. The positions of these asymptotes were estimated by averaging the values of ϵ associated with a number of points at the right end of each curve and are tabulated in column 16 of table 3. The asymptotic values of ϵ are designated by ϵ_{max} and are plotted against the parameter $\left[\frac{B_c h_c}{B_p h_p}\right]^{1/4}$ in figure 15. The form of this parameter was suggested by the second term of the right-hand member of equation (21). It will be noted that a reasonably smooth curve can be drawn through the points.

The values of ϵ for each specimen were divided by the appropriate value of ϵ_{max} and are tabulated in column 18 of table 3. These values are plotted against the parameter $\frac{w_s}{nh_s}\left[\frac{B_c h_c}{B_p h_p}\right]^{1/4}$ in figure 16. It will be noted that they fall, roughly, on the curve drawn in this figure.

A reasonable estimate of the error ϵ can be obtained for any particular stiffened plate by the use of the curves drawn in figures 15 and 16.

Verification of Formula (22)

Values of B_c and B_p were obtained by means of formula (7) applied to the results of the first and last tests of each plate. Values of $\frac{\epsilon}{\epsilon_{max}}$ were obtained from the curve of figure 11 and multiplied by values of ϵ_{max} obtained from the curve of figure 16 to determine estimates of values of ϵ. These values were substituted in equation (22) and estimates of B obtained. These estimates are tabulated in column 9 of table 3 and plotted against the corresponding observed values in figure 17. The straight line in this figure passes through points representing absolute agreement between computed and observed values. The grouping of the plotted points around this

line indicates that formula (22) yields reasonable estimates of the values obtained by test.

Part 4. Flexural Poisson's Ratio Effect

Values of the Poisson's ratio effect for plywood are given by the first term of the right-hand member of equation (6).

$$\frac{h^3 (E\sigma)_{12}}{12\lambda} = F = \sum \left[\frac{h_i^3}{12} + h_i y_i^2 \right] \frac{E_{xi}\, \sigma_{yxi}}{\lambda} \tag{24}$$

in which h_i = thickness of an individual ply

y_i = distance from the neutral axis of the plywood to the center of the individual ply

E_{xi} = the modulus of elasticity of the individual ply in the x direction (either parallel or perpendicular to the grain)

σ_{yxi} = Poisson's ratio of the individual ply (the ratio of the strain in the x direction to the strain in the y direction due to a stress in the y direction)

$\lambda = 1 - \sigma_{yxi}\, \sigma_{xyi}$ (taken as unity for wood),

with the summation being taken over all the plies of the plywood.

Making use of Maxwell's relation

$$E_{xi}\, \sigma_{yxi} = E_{yi}\, \sigma_{xyi}$$

equation (24) becomes

$$F = \sum \left[\frac{h_i^3}{12} + h_i y_i^2 \right] \frac{\sqrt{E_{xi}\, \sigma_{yxi}\, E_{yi}\, \sigma_{xyi}}}{\lambda} \tag{25}$$

making use of the fact that

$$\sigma_{yx}\, \sigma_{xy} = \sigma_{LT}\, \sigma_{TL}$$

in which L and T refer to the longitudinal and tangential directions in each ply, and that this product has about the same value for various species of wood; also remembering that:

$$D_1 = \sum \left[\frac{h_i^3}{12} + h_i y_i^2 \right] \frac{E_{xi}}{\lambda}$$

$$D_2 = \sum \left[\frac{h_i^3}{12} + h_i y_i^2 \right] \frac{E_{yi}}{\lambda}$$

equation (25) becomes

$$F = \sqrt{\sigma_{LT}\,\sigma_{TL}\,D_1\,D_2} \tag{26}$$

It is assumed that this expression can be used approximately in connection with multiply stiffened plywood as well as for plywood alone.

Table 1.--Results of compression buckling tests of plywood plates stiffened with stiffeners spaced 2 inches on centers

Plate No.	Number of stiffeners	Number of plies	Nominal ply thickness	Plate dimensions: Thickness	Plate dimensions: "a" width	Plate dimensions: "b" length	Stiffener dimensions: Thickness	Stiffener dimensions: Width	D_1	D_2	B	σ_{LT}	σ_{TL}	Computed critical load	Observed critical load	Obs. / Comp.	Computed ultimate load	Obs. crit. / Comp. ult.	Comp. crit. / Comp. ult.	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
			In.	In.	In.	In.	In.	In.	In.-lb.	In.-lb.	In.-lb.			Lb.	Lb.	Percent	Lb.			
A	4	5	0.020	0.104	8.50	22.75	0.251	0.750	74.7	3,074.3	103.75	0.406	0.020	1,962	1,800	91.7	8,803	0.204	0.223	Well-defined buckle
														1,962	1,620	82.6	8,803	.184	.223	Do.
														1,962	2,100	107.0	8,803	.239	.223	Do.
														1,962	2,000	101.9	8,803	.227	.223	Do.
														1,962	1,800	91.7	8,803	.204	.223	Do.
A_1	5	5	.020	.106	10.50	22.75	.248	.750	74.7	3,074.3	104.50	.406	.020	1,550	1,500	96.8	10,975	.137	.141	Do.
														1,550	1,500	96.8	10,975	.137	.141	Do.
														1,550	1,800	116.1	10,975	.164	.141	Do.
C	13	5	.020	.102	24.74	24.75	.126	.750	61.1	913.6	39.17	.406	.020	476	432	90.4	19,225	.022	.025	Apparent buckle
C_1	8	5	.020	.102	16.25	24.75	.126	.750	61.1	913.6	37.33	.406	.020	465	600	129.0	12,309	.049	.038	Well-defined buckle
C_2	4	5	.020	.100	8.75	20.77	.116	.750	61.1	913.6	30.92	.406	.020	893	1,000	112.0	6,164	.162	.144	Apparent buckle
D	13	4	.047	.195	24.75	24.75	.128	.750	218.5	2,119.5	133.17	.406	.020	1,222	.....	.....	21,349	.....	.057	Did not buckle
D_1	8	4	.047	.195	16.73	24.75	.128	.750	218.5	2,119.5	127.00	.406	.020	1,294	1,300	100.5	13,960	.093	.093	Apparent buckle
D_2	4	4	.047	.195	8.75	20.79	.128	.750	218.5	2,119.5	124.33	.406	.020	2,595	3,200	106.8	7,182	.446	.417	Well-defined buckle
E_1[1]	13	5	.020	.103	24.73	24.74	.124	.750	856.7	144.4	39.00	.406	.020	418	550	131.6	11,538	.048	.036	Apparent buckle
E_1[1]	13	5	.020	.101	16.78	24.74	.127	.750	856.7	144.4	39.00	.406	.020	607	.....	.....	7,678	.....	.079	No definite buckle
E_2[1]	10	5	.020	.101	16.73	20.75	.127	.750	856.7	144.4	36.50	.406	.020	575	700	121.7	7,678	.091	.075	Buckle in double wave
E_3[1]	8	5	.020	.101	16.78	16.77	.127	.750	856.7	144.4	36.25	.406	.020	618	700	113.3	7,678	.091	.080	Do.
E_4[1]	7	5	.020	.101	16.73	14.76	.127	.750	856.7	144.4	36.08	.406	.020	685	750	109.5	7,678	.098	.089	Apparent buckle
F_1[1]	13	4	.047	.191	24.73	24.72	.124	.750	1,715.3	715.6	124.75	.406	.020	1,325	1,500	113.2	23,129	.114	.101	Do.
F_1[1]	13	4	.047	.190	16.77	24.72	.128	.750	1,715.3	715.6	126.00	.406	.020	1,850	1,700	91.9	8,838	.192	.209	Do.

[1] Stiffeners perpendicular to face grain of plywood and direction of load.

Table 2.--Results of shear buckling tests of plywood plates stiffened with nine stiffeners spaced 2 inches on centers

Plate No.	Number of plies	Nominal ply thickness	Plate dimensions: Thickness	Plate dimensions: "a" width	Plate dimensions: "b" length	Stiffener dimensions: Thickness	Stiffener dimensions: Width	D_1	D_2	B	σ_{LT}	σ_{TL}	Computed critical load	Observed critical load	Obs. / Comp.	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
		In.	In.	In.	In.	In.	In.	In.-lb.	In.-lb.	In.-lb.			Lb.	Lb.	Percent	
4s	3	0.020	0.063	18.06	18.07	0.125	0.767	431.0	4.96	15.38	0.406	0.020	572	625	1.093	Apparent buckle
4z	3	.020	.062	18.12	18.09	.129	.763	450.6	4.60	18.42	.406	.020	610	600	.984	Do.
4aa	3	.020	.063	18.10	18.11	.127	.760	449.2	4.96	14.97	.406	.020	584	625	1.070	Do.
16z	5	.020	.107	18.03	18.12	.128	.759	881.0	44.6	36.70	.406	.020	1,563	1,560	.998	Do.
16cc	5	.020	.112	18.11	18.09	.124	.760	790.0	48.1	39.16	.406	.020	1,486	1,475	.893	Do.
16dd	5	.020	.104	18.06	18.09	.130	.767	849.0	52.0	30.18	.406	.020	1,523	1,650	1.083	Do.
Av.															1.037	

Table 3.—Results of tests in bending and shear of stiffened plywood plates and the computations related to them

Test No.	Plate dimensions: Width	Plate dimensions: Length	Plate dimensions: Thickness	Stiffener dimensions: Width	Stiffener dimensions: Thickness	Stiffener dimensions: Number	Shear stiffness (B) (Fig. 17): Test	Shear stiffness (B) (Fig. 17): Computed	Bending stiffness: Parallel to stiffeners (D_1) (Fig. 8): Test	Bending stiffness: Parallel to stiffeners (D_1) (Fig. 8): Computed	Bending stiffness: Perpendicular to stiffeners (D_2) (Fig. 11): Test	Bending stiffness: Perpendicular to stiffeners (D_2) (Fig. 11): Computed	ϵ (Fig. 14)	$\frac{W_s}{nhs}$ (Fig. 14)	ϵ_{max} (Fig. 15)	$\left[\frac{(Bh)_c}{(Bh)_s}\right]^{\frac{1}{4}}$ (Fig. 15)	$\frac{\epsilon}{\epsilon_{max}}$ (Fig. 16)	$\frac{W_s}{nhs}\left[\frac{(Bh)_c}{(Bh)_s}\right]^{\frac{1}{4}}$ (Fig. 16)	$\frac{w}{s}$ (Fig. 10)	$\frac{\epsilon}{s}$ (Fig. 10)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
	In.	In.	In.	In.	In.		In.-lb.	In.-lb.	In.-lb.	In.-lb.	In.-lb.	In.-lb.								
SPECIMEN "A" (NINE 1/16-INCH PLIES)																				
1	18.01	18.00	0.523	18.01	0.127	1	2,030		29,365		8,205									
2	17.76	18.00	.523	1.75	.127	9	1,843	1,832	28,470	28,160	8,010		0.144	13.76	0.129	1.197	1.117	16.49		
3	17.50	18.00	.523	1.50	.127	9	1,716	1,738	26,840	26,850	7,380		.105	11.81	.129	1.197	.811	14.14		
4	17.26	18.00	.523	1.26	.127	9	1,670	1,645	24,900	25,470	7,540		.123	9.92	.129	1.197	.956	11.88		
5	16.99	18.00	.523	.99	.127	9	1,562	1,537	23,470	23,770	6,760		.124	7.80	.129	1.197	.960	9.33		
6	16.74	18.00	.523	.74	.127	9	1,423	1,434	21,560	22,010	8,000		.176	5.83	.129	1.197	1.367	6.97		
7	16.52	18.00	.523	.52	.127	9	1,383	1,350	19,450	20,300	8,270		.102	4.09	.129	1.197	.790	4.90		
8	16.23	18.00	.523	.25	.127	9	1,280	1,274	16,940	17,970	8,220		.052	1.97	.129	1.197	.637	2.36		
9	16.13	18.00	.523	.13	.127	9	1,254	1,251	15,510	16,820	9,220		.044	1.02	.129	1.197	.344	1.22		
10	16.00	16.00	.523			0	1,229		15,510		5,304									
SPECIMEN "AA" (NINE 1/16-INCH PLIES)																				
1	18.02	18.00	.525	18.02	.129	1	2,075		30,609		7,366									
2	17.73	18.00	.525	1.75	.129	9	1,905	1,857	29,530	29,320	7,720		.104	13.57	.120	1.212	.864	16.45		
3	17.52	18.00	.525	1.49	.129	9	1,789	1,748	28,390	27,780	6,760		.115	11.55	.120	1.212	.954	14.00		
4	17.25	18.00	.525	1.26	.129	9	1,681	1,651	26,260	26,330	7,030		.131	9.77	.120	1.212	1.087	11.84		
5	16.99	18.00	.525	1.00	.129	9	1,570	1,537	25,680	24,470	6,370		.127	7.75	.120	1.212	1.063	9.40		
6	16.76	18.00	.525	.74	.129	9	1,448	1,419	22,840	22,350	7,890		.134	5.74	.120	1.212	1.111	6.95		
7	16.52	18.00	.525	.52	.129	9	1,320	1,326	19,850	20,340	7,620		.170	4.03	.120	1.212	1.412	4.89		
8	16.24	18.00	.525	.25	.129	9	1,221	1,244	16,930	17,510	7,820		.129	1.94	.120	1.212	1.072	2.35		
9	16.13	18.00	.525	.12	.129	9	1,215	1,218	16,190	15,970	6,470		.053	.93	.120	1.212	.438	1.13		
10	16.00	16.01	.525			0	1,197		14,480		5,124									
SPECIMEN "B" (NINE 1/16-INCH PLIES)																				
1	20.05	20.03	.539	20.05	.249	1	3,429		49,163		10,345									
2	19.77	20.03	.539	1.75	.249	10	2,674	2,739	47,820	46,800	8,330		.366	7.03	.356	1.425	1.031	10.02		
3	19.51	20.03	.539	1.50	.249	10	2,473	2,471	46,120	44,140	[illegible]		.321	6.02	.356	1.425	.901	8.59		
4	19.27	20.03	.539	1.27	.249	10	2,133	2,223	43,010	41,370	7,410		.388	5.10	.356	1.425	1.089	7.27		
5	19.00	20.03	.539	1.00	.249	10	1,873	1,929	39,570	37,560	7,060		.357	4.02	.356	1.425	1.004	5.72		
6	18.74	20.03	.539	.73	.249	10	1,576	1,682	34,290	33,010	7,130		.348	2.93	.356	1.425	.977	4.18		
7	18.48	20.03	.539	.51	.249	10	1,480	1,510	29,510	28,610	7,570		.237	2.05	.356	1.425	.666	2.92		
8	18.23	20.03	.539	.25	.249	10	1,235	1,345	23,670	22,860	7,320		.192	1.00	.356	1.425	.541	1.43		
9	18.13	20.03	.539	.12	.249	10	1,142	1,279	20,110	18,480	7,460		.147	.48	.356	1.425	.414	.69		
10	16.00	16.00	.539			0	1,215		14,568		5,425									
SPECIMEN "BB" (NINE 1/16-INCH PLIES)																				
1	20.04	20.03	.537	20.04	.243	1	3,377		48,506		10,784									
2	19.77	20.03	.537	1.75	.243	10	2,592	2,679	46,940	46,180	8,140		.397	7.20	.360	1.439	1.103	10.36		
3	19.53	20.03	.537	1.50	.243	10	2,419	2,412	42,910	43,560	7,270		.326	6.17	.360	1.439	.905	8.88		
4	19.27	20.03	.537	1.27	.243	10	2,170	2,162	40,820	40,830	7,310		.326	5.23	.360	1.439	.906	7.52		
5	19.01	20.03	.537	1.01	.243	10	1,768	1,874	37,320	37,230	6,850		.391	4.16	.360	1.439	1.086	5.98		
6	18.73	20.03	.537	.74	.243	10	1,613	1,613	32,580	32,780	7,750		.293	3.05	.360	1.439	.815	4.38		
7	18.49	20.03	.537	.52	.243	10	1,385	1,442	28,350	28,460	7,650		.271	2.14	.360	1.439	.753	3.08		
8	18.24	20.03	.537	.26	.243	10	1,185	1,275	21,750	22,220	7,620		.191	1.07	.360	1.439	.531	1.54		
9	18.14	20.03	.537	.12	.243	10	1,190	1,203	18,720	18,200	7,450		.070	.49	.360	1.439	.193	.71		
10	16.00	16.00	.537			0	1,143		14,308		5,593									
SPECIMEN "C" (NINE 1/16-INCH PLIES)																				
1	24.03	24.03	.536	24.03	.495	1	7,550		94,100		16,993									
2	23.79	24.03	.536	1.73	.495	12	4,865	4,626	104,970	88,620	8,230		.474	3.49	.504	1.910	.941	6.68		
3	23.51	24.03	.536	1.49	.495	12	3,962	3,909	100,920	83,220	8,040		.514	3.01	.504	1.910	[illegible]	5.75		
4	23.27	24.03	.536	1.27	.495	12	3,138	3,334	96,290	77,560	7,920		.544	2.57	.504	1.910	1.079	4.90		
5	23.00	24.03	.536	1.01	.495	12	2,528	2,719	83,660	69,700	7,370		.484	2.04	.504	1.910	.961	3.90		
6	22.74	24.03	.536	.74	.495	12	1,898	2,163	78,100	59,720	7,640		.418	1.49	.504	1.910	.829	2.86		
7	22.52	24.03	.536	.52	.495	12	1,444	1,777	60,930	49,700	7,070		.346	1.05	.504	1.910	.687	2.01		
8	22.26	24.03	.536	.25	.495	12	1,195	1,428	40,670	33,870	6,550		.182	.51	.504	1.910	.362	.96		
9	22.12	24.03	.536	.12	.495	12	1,152	1,258	26,720	24,260	6,080		.085	.24	.504	1.910	.169	.46		
10	16.00	16.00	.536			0	1,090		13,733		5,267									
SPECIMEN "D" (FOUR 1/16-INCH PLIES)																				
1	20.02	20.02	.247	20.02	.492	1	3,176		46,759		7,281									
2	19.76	20.02	.247	1.68	.492	10	1,949	1,981	42,950	43,080	800	735	.549	3.41	.721	2.854	.900	9.74	0.840	0.261091
3	19.52	20.02	.247	1.50	.492	10	1,943	1,726	39,570	40,980	620	642	.774	3.05	.721	2.854	1.074	8.70	.750	.317160
4	19.27	20.02	.247	1.27	.492	10	1,235	1,384	35,060	37,950	550	558	.745	2.58	.721	2.854	1.034	7.37	.635	.284189
5	18.99	20.02	.247	1.01	.492	10	873	992	31,620	34,040	520	492	.715	2.05	.721	2.854	.992	5.86	.505	.225590
6	18.73	20.02	.247	.74	.492	10	595	669	28,870	29,140	450	445	.596	1.50	.721	2.854	.827	4.29	.370	.179702
7	18.48	20.02	.247	.51	.492	10	366	444	22,930	23,900	420	415	.484	1.04	.721	2.854	.672	2.96	.255	.122460
8	18.22	20.02	.247	.26	.492	4	196	273	15,250	16,290	400	391	.296	.53	.721	2.854	.411	1.51	.130	.049232
9	18.12	20.02	.247	.13	.492	4	168	212	13,570	10,440	370	381	.150	.26	.721	2.854	.208	.75	.065	.063428
10	8.00	8.00	.247			0	143		2,301		372									

(cont.)

Z M 77403 F

Table 3.--Results of tests in bending and shear of stiffened plywood plates and the computations related to them (cont.)

Test No.	Plate dimensions: Width	Length	Thickness	Stiffener dimensions: Width	Thickness	Number	Shear stiffness (S) (Fig. 17): Test	Computed	Bending stiffness: Parallel to stiffeners (D_1) (Fig. 8): Test	Computed	Perpendicular to stiffeners (D_2) (Fig. 11): Test	Computed	ϵ (Fig. 14)	$\frac{W_n}{nbs}$ (Fig. 14)	$\epsilon_{max.}$ (Fig. 15)	$\left(\frac{Sb}{D_1}\right)^{\frac{1}{2}}$ (Fig. 15)	$\frac{\epsilon}{\epsilon_{max}}$ (Fig. 16)	$\frac{W_n}{nbs}\left(\frac{Sb}{D_1}\right)^{\frac{1}{2}}$ (Fig. 16)	$\frac{W}{S}$ (Fig. 10)	$\frac{\epsilon}{S}$ (Fig. 10)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
	In.	In.	In.	In.	In.		In.-lb.	In.-lb.	In.-lb.	In.-lb.	In.-lb.	In.-lb.								
SPECIMEN "E" (FOUR 1/16-INCH PLIES)																				
1	24.03	24.02	0.245	24.03	0.749	1	6,610		98,705		16,522									
2	23.76	24.02	.245	1.74	.749	12	3,769	3,780	96,870	91,540	830	0.756	0.620	2.32	0.638	3.806	0.972	8.85	0.870	0.264257
3	23.50	24.02	.245	1.50	.749	12	2,871	3,006	90,920	84,620	620	.616	.662	2.00	.638	3.806	1.039	7.63	.750	.283408
4	23.27	24.02	.245	1.27	.749	12	2,119	2,266	85,140	77,490	540	.530	.665	1.70	.638	3.806	1.044	6.46	.635	.254538
5	22.97	24.02	.245	1.00	.749	12	1,445	1,506	73,360	68,340	500	.462	.603	1.34	.638	3.806	.946	5.08	.500	.172096
6	22.75	24.02	.245	.74	.749	12	859	935	64,650	58,120	450	.416	.519	.99	.638	3.806	.814	3.76	.370	.125891
7	22.47	24.02	.245	.51	.749	12	445	556	49,280	47,320	400	.386	.413	.68	.638	3.806	.648	2.59	.255	.114087
8	18.24	24.02	.245	.26	.749	10	250	302	33,470	31,710	360	.362	.222	.35	.638	3.806	.345	1.38	.130	.075292
9	14.12	24.02	.245	.12	.749	8	186	211	21,320	18,970	330	.351	.103	.16	.638	3.806	.161	.61	.060	.055920
10	8.00	8.00	.245			0	128		1,970		344									
SPECIMEN "F" (FOUR 1/16-INCH PLIES)																				
1	24.00	24.00	.244	24.00	.997	1	12,692		184,612		30,523									
2	23.75	24.00	.244	1.72	.997	12	6,185	6,097	161,890	169,020	850	.736	.811	1.73	.614	4.712	.994	8.13	.884	.246033
3	23.53	24.00	.244	1.49	.997	12	4,529	4,679	165,520	155,660	620	.604	.841	1.49	.614	4.712	1.043	7.04	.745	.276886
4	23.29	24.00	.244	1.25	.997	12	3,127	3,206	145,180	140,980	550	.515	.628	1.25	.614	4.712	1.022	5.91	.625	.224490
5	22.99	24.00	.244	1.00	.997	12	1,935	2,026	124,630	124,510	520	.453	.578	1.00	.614	4.712	.941	4.73	.500	.145737
6	22.73	24.00	.244	.74	.997	12	1,115	1,105	109,240	105,170	440	.407	.472	.74	.614	4.712	.768	3.50	.370	.124860
7	22.48	24.00	.244	.50	.997	12	533	515	86,760	84,040	420	.377	.351	.50	.614	4.712	.571	2.36	.250	.054434
8	18.23	24.00	.244	.25	.997	10	252	255	57,010	55,240	360	.353	.185	.25	.614	4.712	.302	1.18	.125	.057260
9	14.12	24.00	.244	.12	.997	8	199	208	35,310	33,610	330	.343	.088	.12	.614	4.712	.143	.57	.060	.064076
10	8.00	8.00	.244			0	131		2,030		335									
SPECIMEN "FF" (FOUR 1/16-INCH PLIES)																				
1	24.02	24.01	.244	24.02	1.000	1	12,845		185,868		28,585									
2	23.75	24.01	.244	1.74	1.000	12	6,246	6,288	163,520	171,170	1,220	.759	.627	1.74	.624	4.683	1.005	8.15	.870	.137416
3	23.50	24.01	.244	1.50	1.000	12	4,633	4,795	168,980	157,060	620	.615	.642	1.50	.624	4.683	1.028	7.02	.750	.287666
4	23.27	24.01	.244	1.24	1.000	12	3,290	3,195	145,850	140,720	540	.518	.604	1.24	.624	4.683	.967	5.81	.620	.236580
5	22.97	24.01	.244	1.00	1.000	12	2,048	2,076	126,050	124,790	480	.458	.566	1.00	.624	4.683	.907	4.68	.500	.201383
6	18.75	24.01	.244	.73	1.000	10	1,201	1,113	111,110	105,090	460	.411	.456	.73	.624	4.683	.730	3.42	.365	.106620
7	18.46	24.01	.244	.51	1.000	10	551	574	87,260	85,920	420	.382	.358	.51	.624	4.683	.573	2.39	.255	.060669
8	14.23	24.01	.244	.26	1.000	8	262	278	57,000	58,030	370	.358	.193	.26	.624	4.683	.309	1.22	.130	.037334
9	14.12	24.01	.244	.12	1.000	8	199	222	34,800	34,090	330	.347	.088	.12	.624	4.683	.142	.56	.060	.067987
10	8.00	8.00	.244			0	136		1,900		339									
SPECIMEN "SO-1" (FOUR 1/16-INCH PLIES)																				
1	20.01	20.02	.265	20.01	.493	1	2,827		42,833		7,168									
2	19.77	20.02	.265	1.76	.493	10	1,874	1,858	41,550	40,420	880	.657	.607	3.57	.621	2.782	.978	9.93	.880	.187611
3	19.52	20.02	.265	1.51	.493	10	1,492	1,536	39,500	37,720	640	.535	.660	3.06	.621	2.782	1.063	8.52	.755	.285874
4	19.24	20.02	.265	1.23	.493	10	1,159	1,164	35,120	34,360	460	.451	.625	2.49	.621	2.782	1.007	6.94	.615	.252522
5	19.02	20.02	.265	1.01	.493	10	899	877	28,900	31,370	420	.406	.591	2.05	.621	2.782	.952	5.70	.505	.216395
6	18.75	20.02	.265	.74	.493	10	567	594	26,090	27,030	370	.366	.551	1.50	.621	2.782	.888	4.18	.370	.185859
7	18.49	20.02	.265	.51	.493	10	345	399	22,020	22,400	350	.341	.460	1.03	.621	2.782	.740	2.88	.255	.116286
8	14.24	20.02	.265	.26	.493	8	180	250	14,670	15,370	320	.320	.283	.52	.621	2.782	.456	1.44	.128	.068596
9	14.12	20.02	.265	.13	.493	8	154	196	10,030	10,120	320	.312	.147	.26	.621	2.782	.237	.73	.065	.029508
10	7.50	7.50	.265			0	135		2,165		305									
SPECIMEN "SO-3" (FOUR 1/16-INCH PLIES)																				
1	12.02	12.01	.255	12.01	.253	1	840		12,727		1,985									
2	11.76	12.01	.255	1.75	.253	6	643	645	11,490	12,170	560	.532	.550	6.92	.561	1.940	.979	13.42	.875	.277740
3	11.52	12.01	.255	1.52	.253	6	550	569	10,820	11,580	450	.461	.628	6.01	.561	1.940	1.118	11.65	.760	.305314
4	11.24	12.01	.255	1.23	.253	6	469	469	10,420	10,700	400	.401	.540	4.86	.561	1.940	.961	9.43	.615	.267097
5	11.01	12.01	.255	1.00	.253	6	385	386	8,920	9,870	390	.368	.545	3.95	.561	1.940	.971	7.67	.500	.277168
6	10.74	12.01	.255	.74	.253	5	287	290	7,820	8,680	330	.340	.545	2.92	.561	1.940	.971	5.67	.370	.216571
7	10.50	12.01	.255	.49	.253	5	207	215	6,570	7,180	310	.320	.465	1.94	.561	1.940	.828	3.76	.245	.172185
8	10.26	12.01	.255	.26	.253	5	140	162	4,660	5,320	310	.307	.359	1.05	.561	1.940	.640	2.03	.132	.083159
9	10.13	12.01	.255	.13	.253	6	117	140	3,360	3,800	300	.300	.226	.51	.561	1.940	.403	1.00	.065	.050629
10	7.49	7.50	.255			0	118		1,878		295									
SPECIMEN "SO-4" (FOUR 1/16-INCH PLIES)																				
1	24.02	24.02	.261	24.02	.734	1	6,426		106,646		19,317									
2	23.73	24.02	.261	1.74	.734	12	3,738	3,659	89,070	98,730	750	.746	.597	2.37	.619	3.688	.966	8.74	.870	.303037
3	23.53	24.02	.261	1.50	.734	12	2,817	2,918	85,160	90,950	610	.607	.653	2.04	.619	3.688	1.056	7.54	.750	.290719
4	23.25	24.02	.261	1.24	.734	12	2,128	2,107	73,580	82,140	520	.513	.616	1.69	.619	3.688	.996	6.23	.620	.264245
5	22.99	24.02	.261	1.02	.734	12	1,455	1,517	67,430	74,140	460	.458	.608	1.39	.619	3.688	.983	5.12	.510	.241829
6	22.74	24.02	.261	.74	.734	12	847	927	57,490	62,580	390	.409	.517	1.01	.619	3.688	.835	3.72	.370	.230519
7	18.49	24.02	.261	.52	.734	10	456	574	47,600	52,280	370	.381	.418	.71	.619	3.688	.675	2.61	.260	.169550
8	14.23	24.02	.261	.26	.734	8	218	315	32,110	35,940	330	.356	.234	.36	.619	3.688	.378	1.32	.131	.138455
9	14.12	24.02	.261	.14	.734	8	174	236	20,870	24,100	330	.346	.126	.19	.619	3.688	.204	.70	.070	.079679
10	7.49	7.50	.261			0	132		2,115		337									

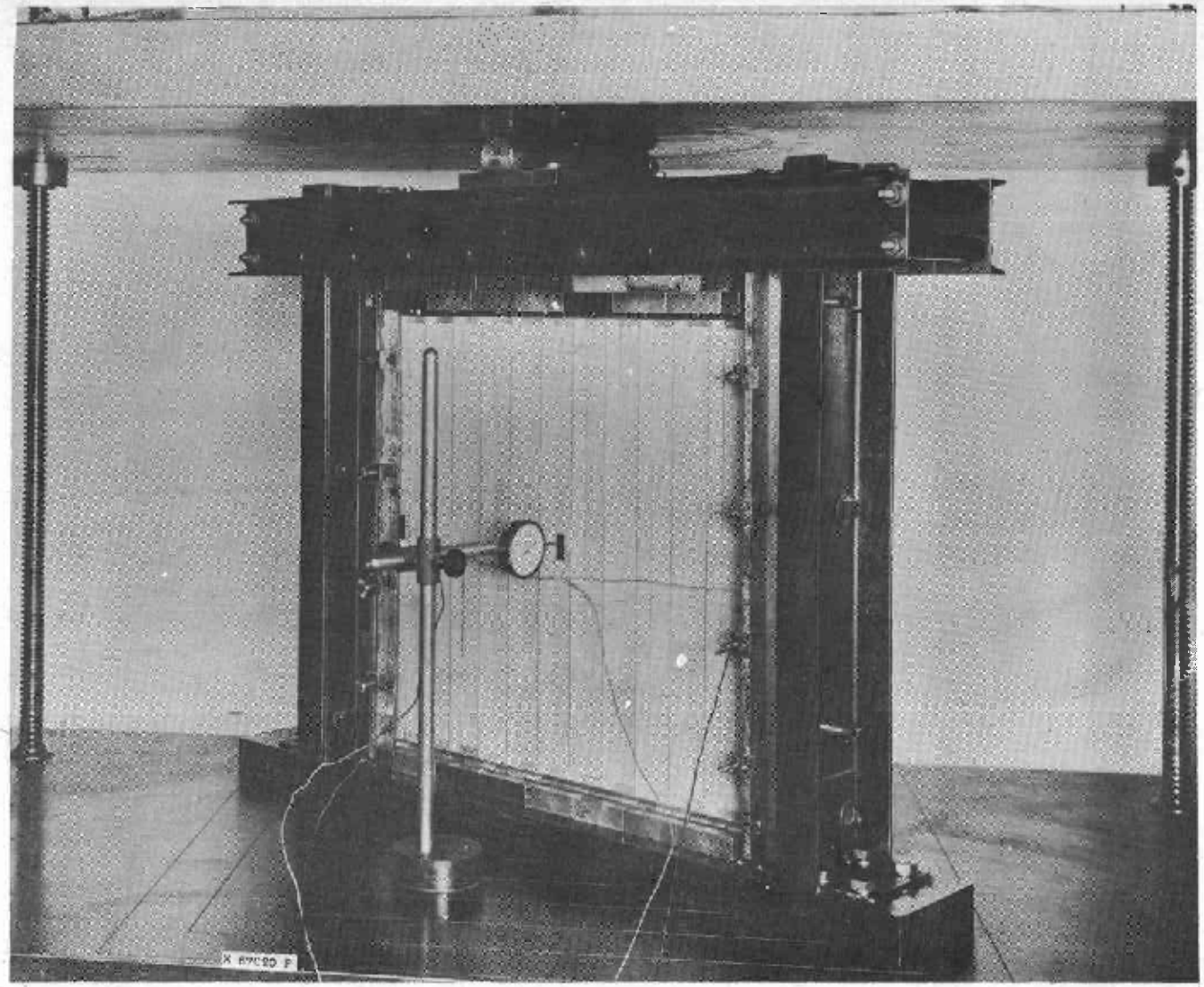

Figure 1.--Plywood panel stiffened with closely spaced stiffeners and supported in the compression-buckling apparatus in preparation for test.

Z M 77415 1

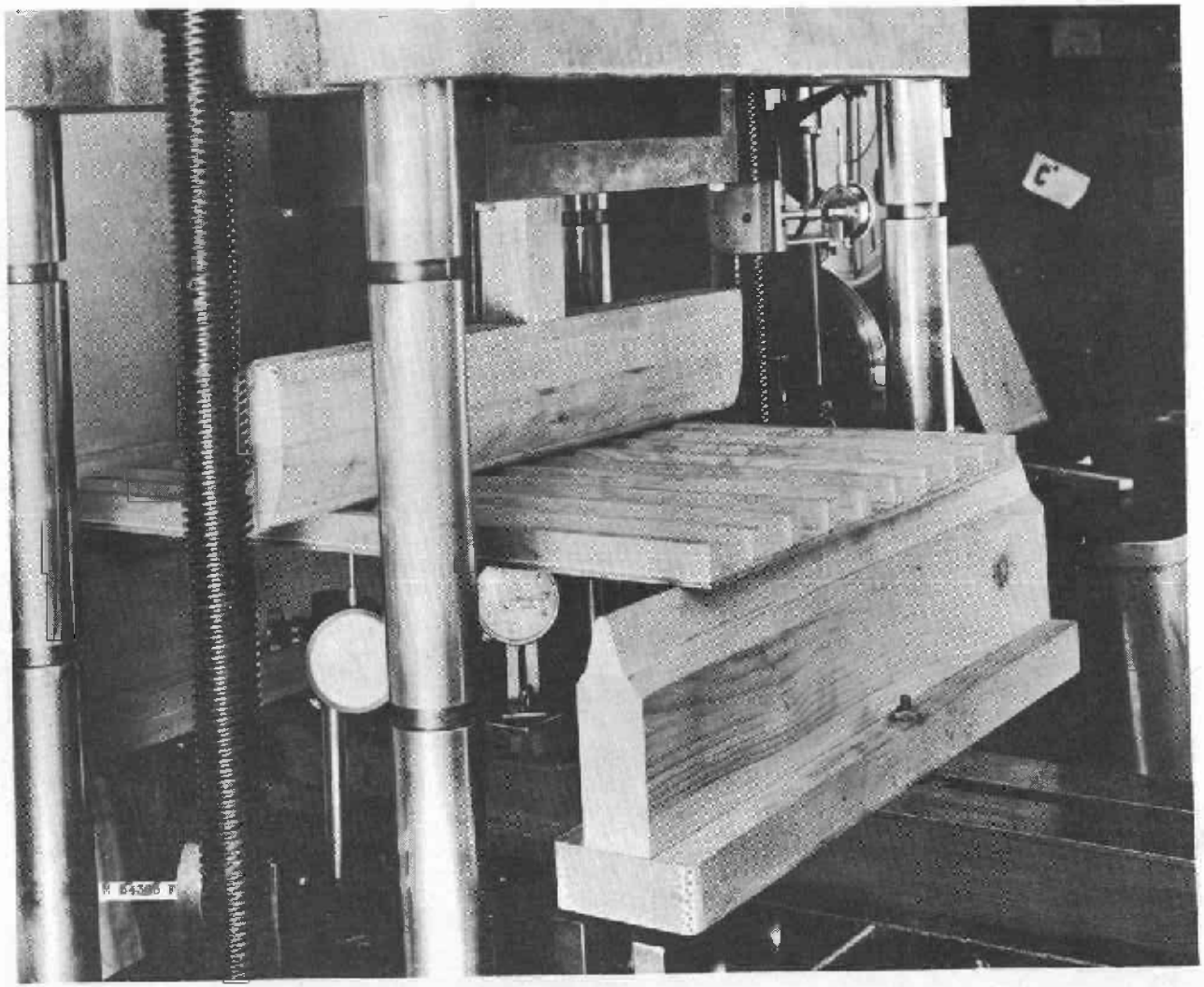

Figure 2.--Yellow-poplar plywood plate stiffened with closely spaced Sitka spruce stiffeners supported in the static-bending apparatus for the determination of the flexural stiffness in the direction of the stiffeners.

Z M 77416 F

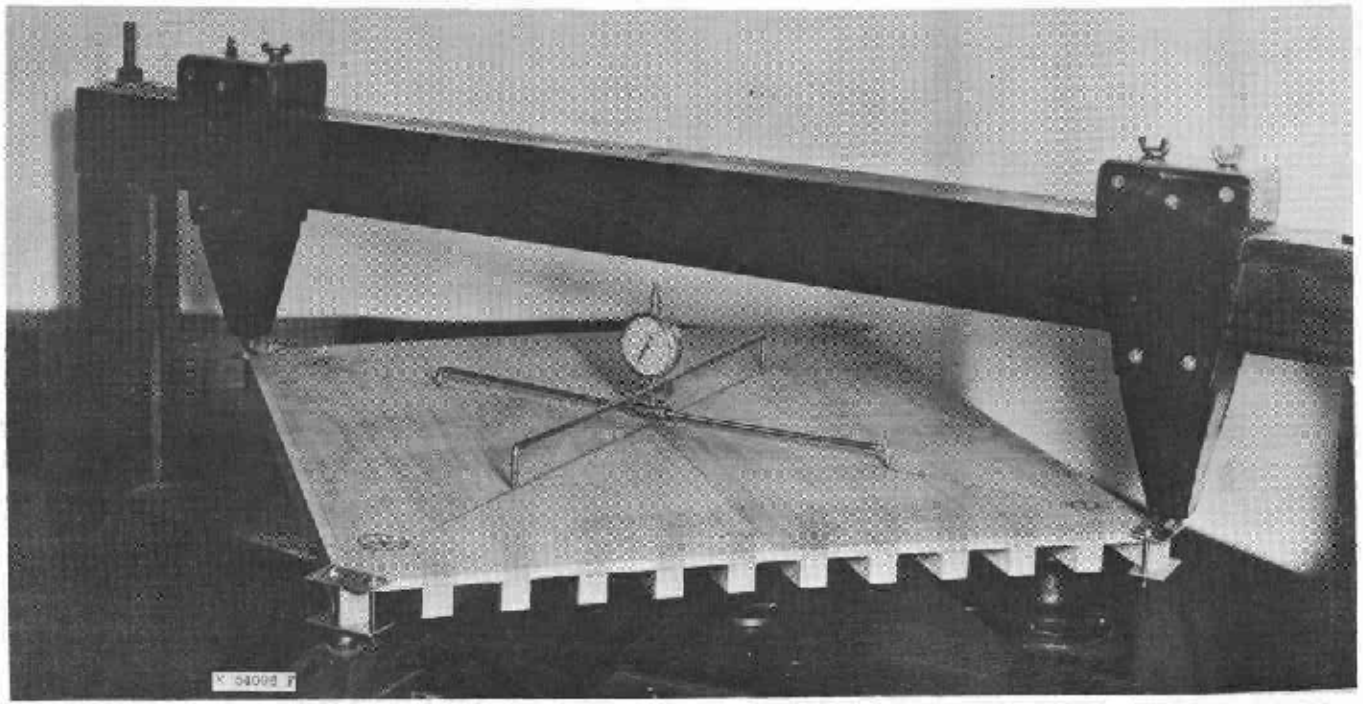

Figure 3.--Yellow-poplar plywood plate stiffened with closely spaced Sitka spruce stiffeners supported in the plate-shear apparatus for the determination of the flexural rigidity in the plane of the plate.

Z M 77417 F

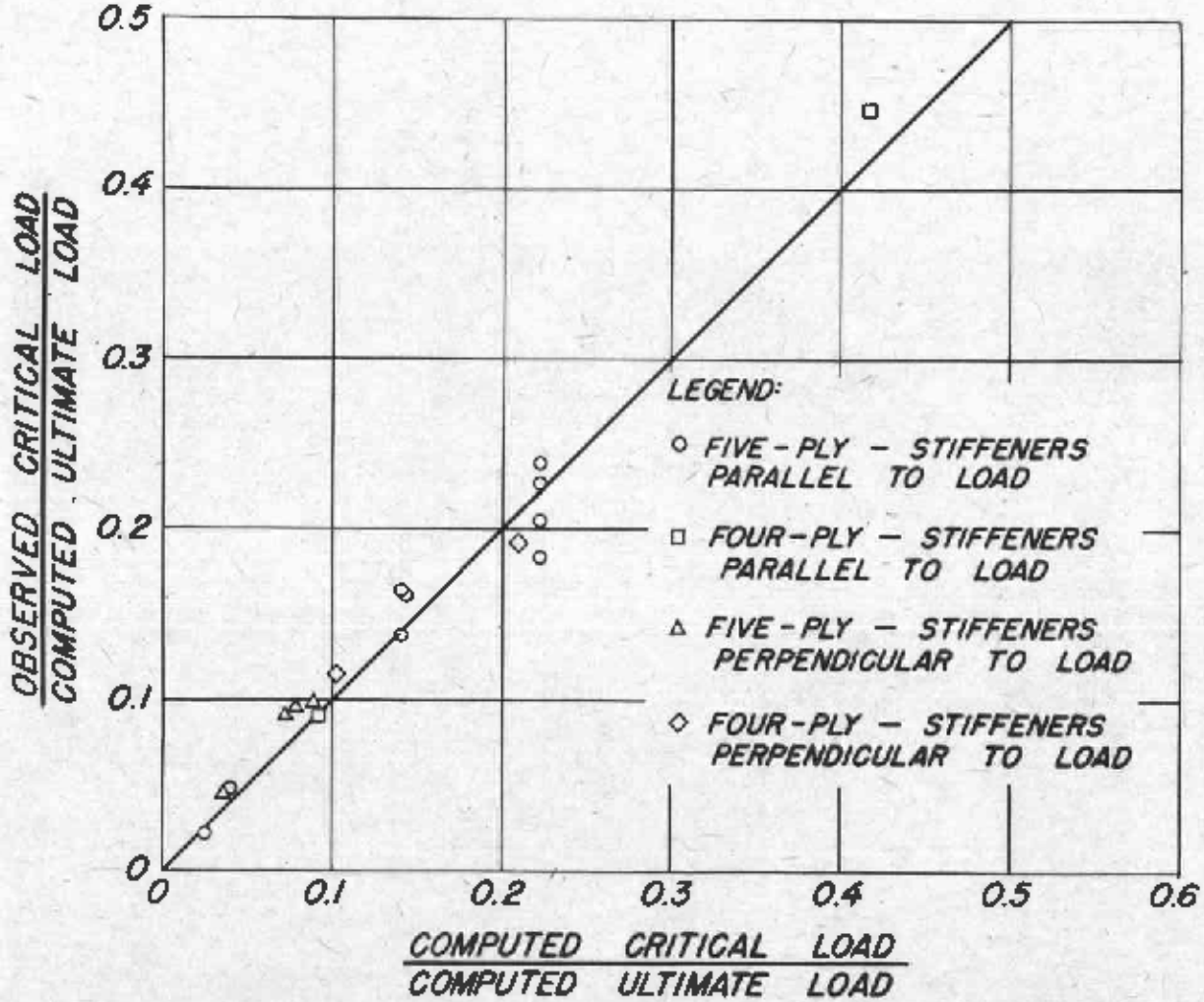

Figure 4.--Comparison of the observed critical loads in compression with the computed values.

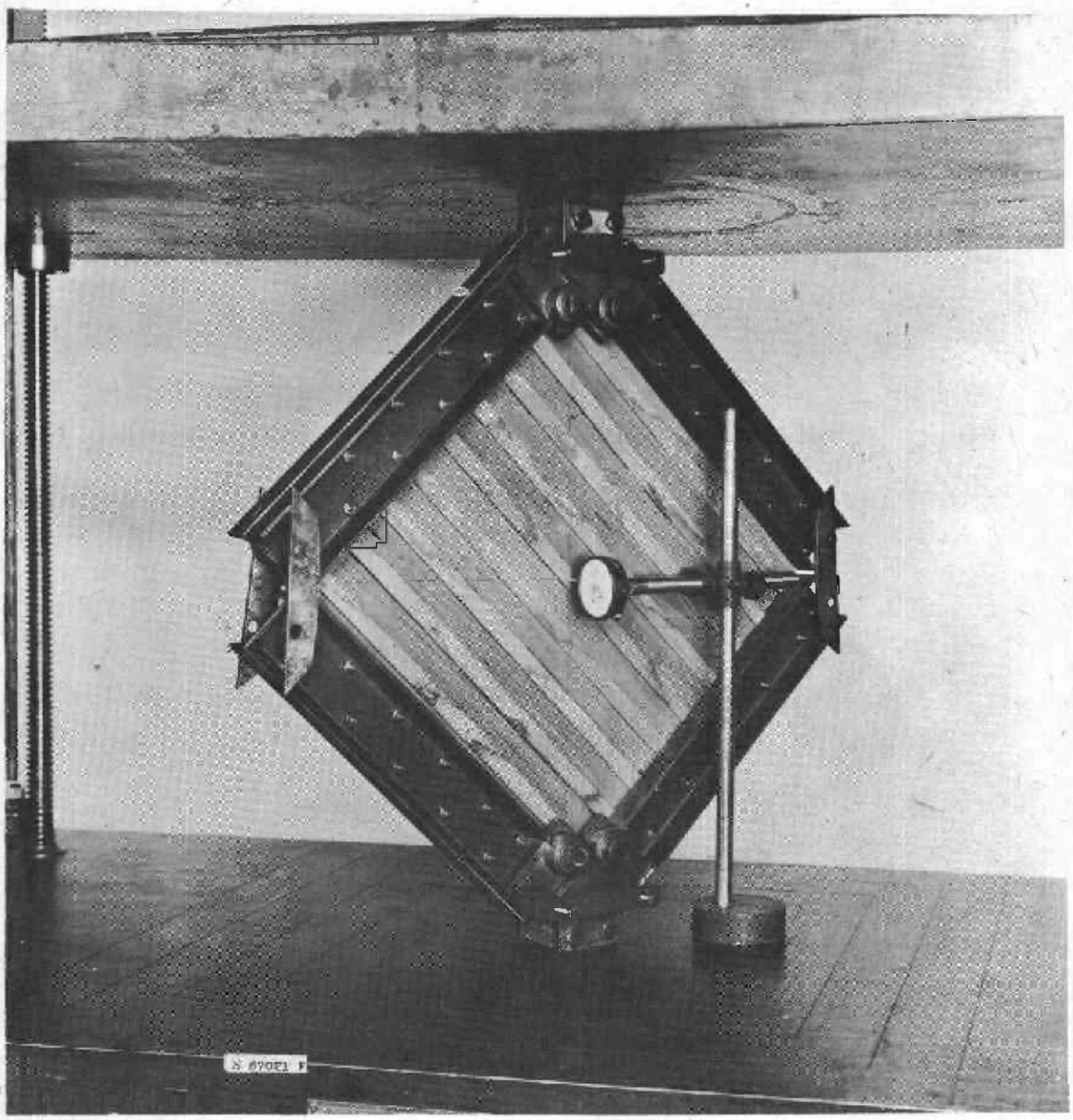

Figure 5.--Plywood panel stiffened with closely spaced stiffeners and supported in the shear-buckling apparatus in preparation for test.

Z M 77418 F

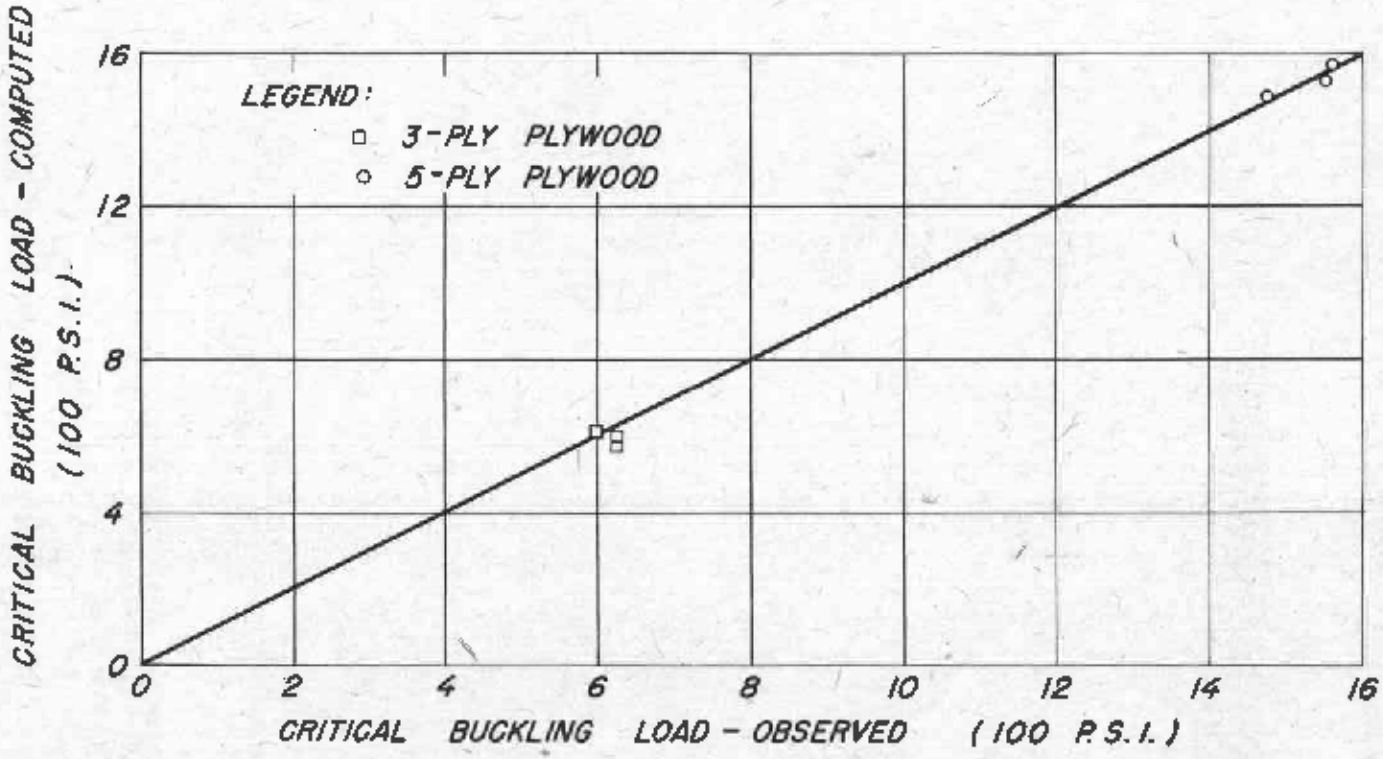

Figure 6.--Comparison of the observed critical loads in shear with the computed values.

ZN 77389 F

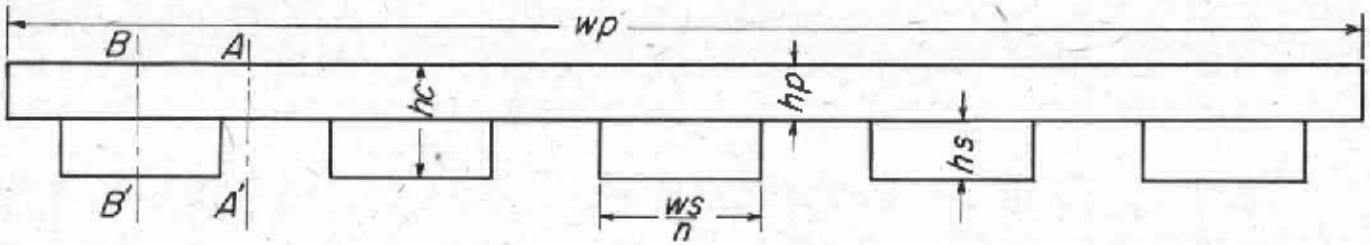

Figure 7.--Cross section of stiffened plywood plate and dimensional notation.

z N 77390 F

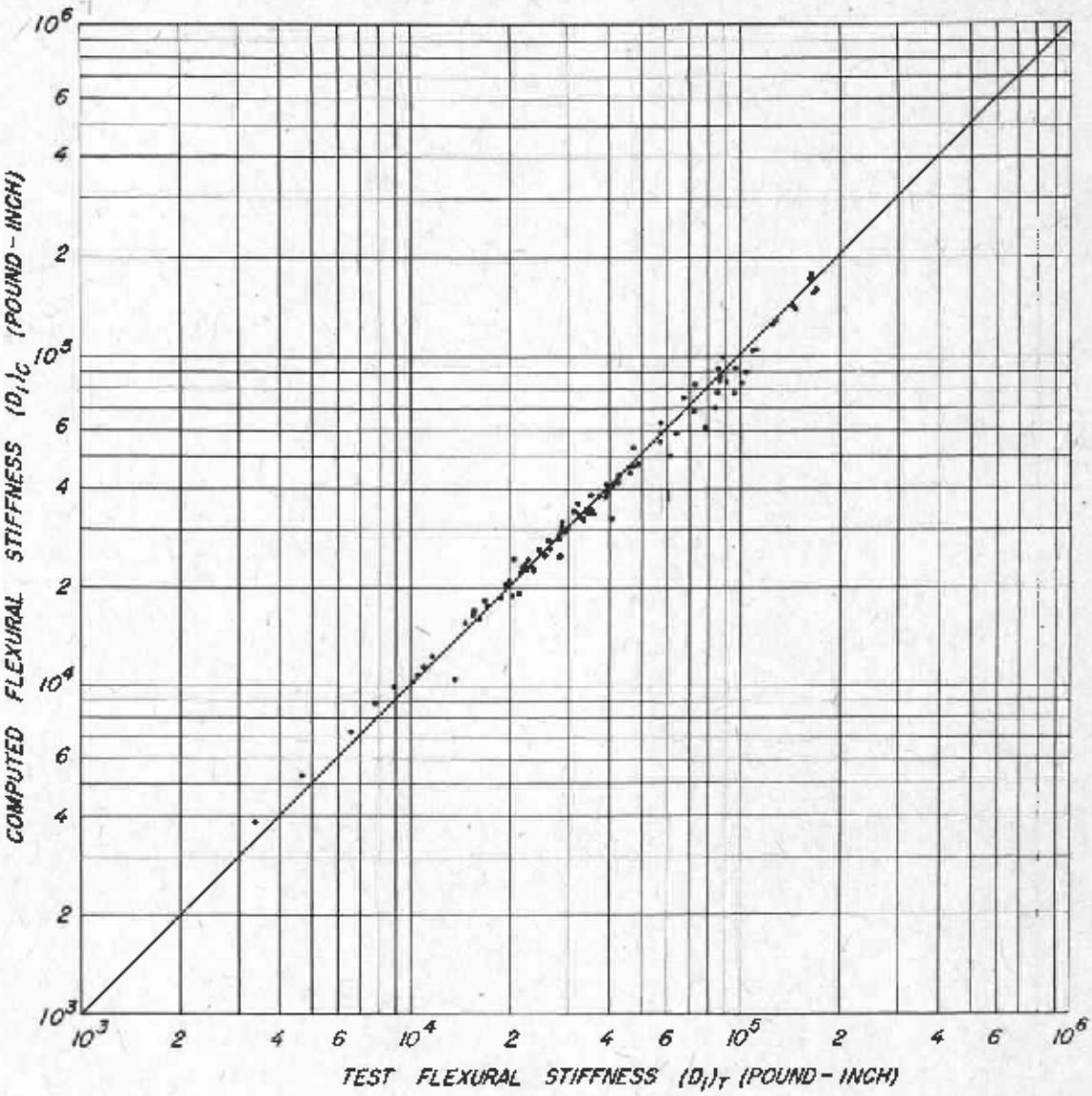

Figure 8.--Comparison of computed values of flexural stiffness with observed values; length of stiffeners parallel to the span of the plate.

2 N 77391 F

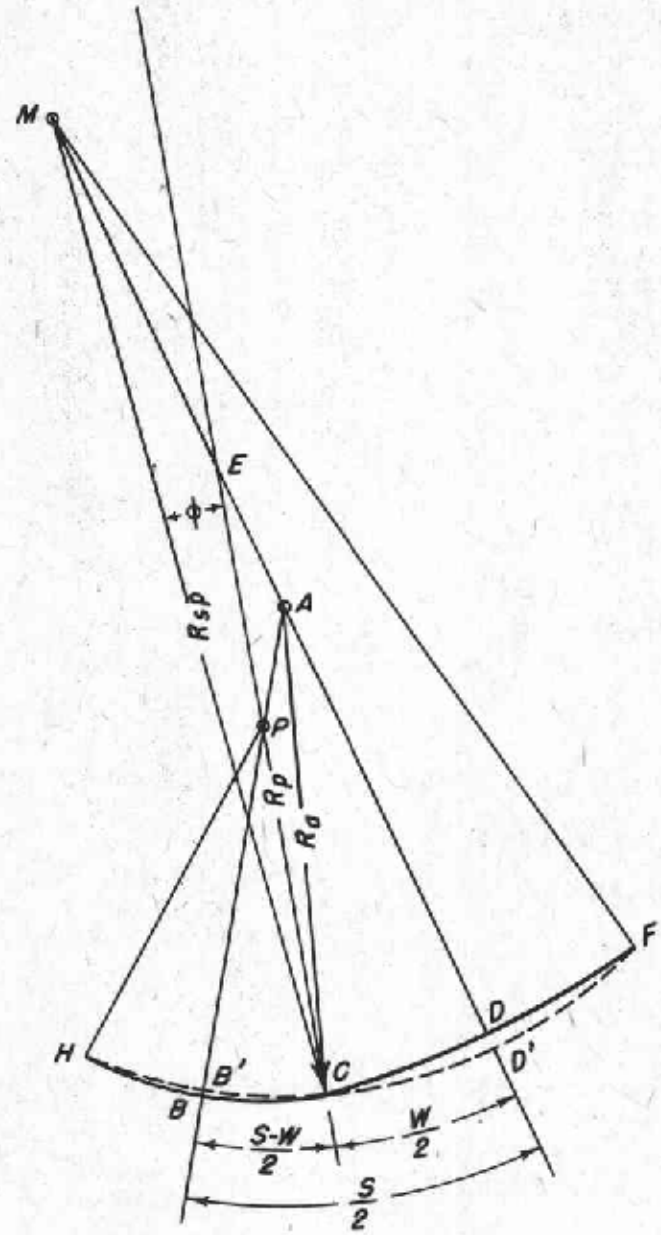

Figure 9.--Sketch of a portion of the elastic curve of a stiffened plate subjected to simple bending perpendicular to the length of the stiffeners.

Z N 77392

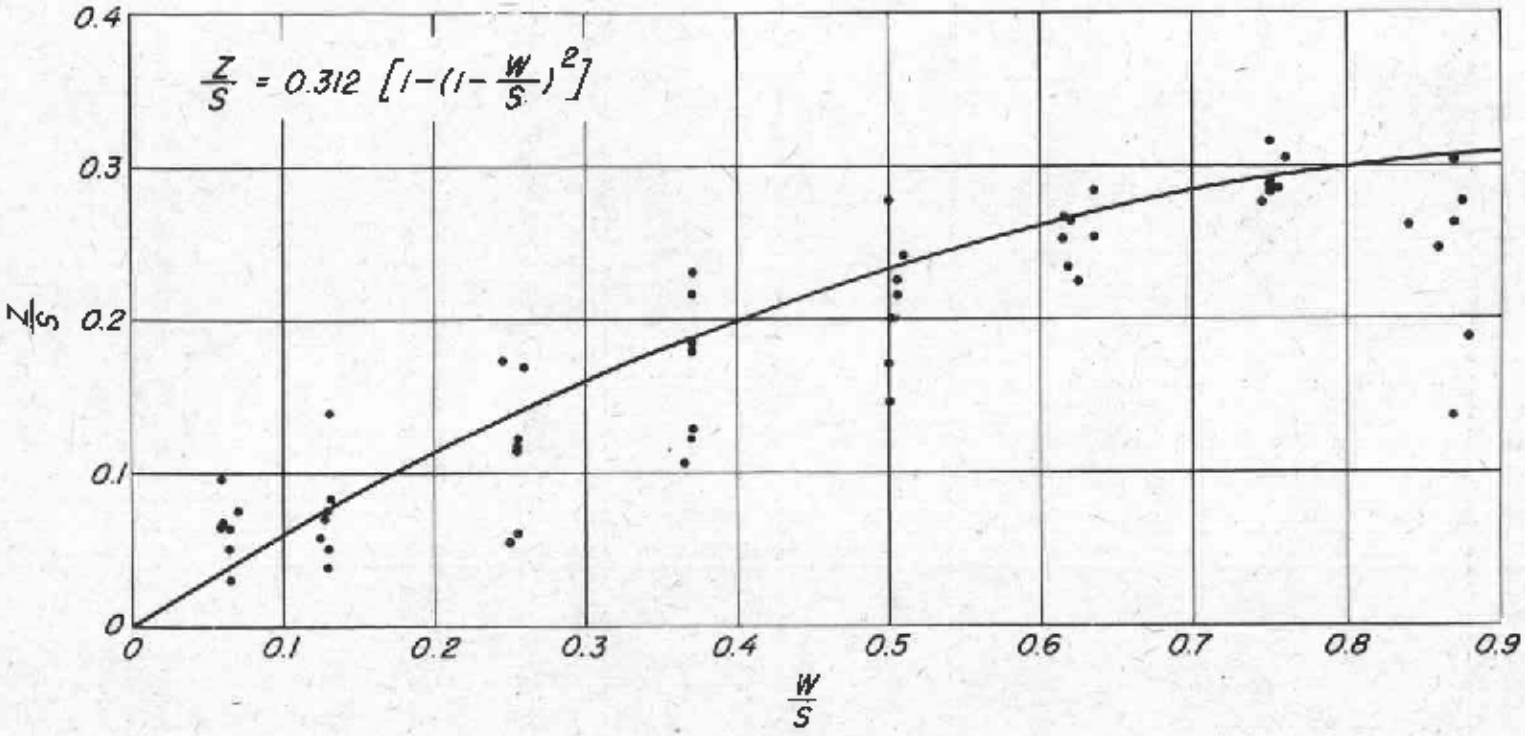

Figure 10.--Values of ratios of Z to S obtained from test plotted against values of the ratio W to S and the parabolic curve fitted to these ratios by the least-squares method.

Z K 77393 F

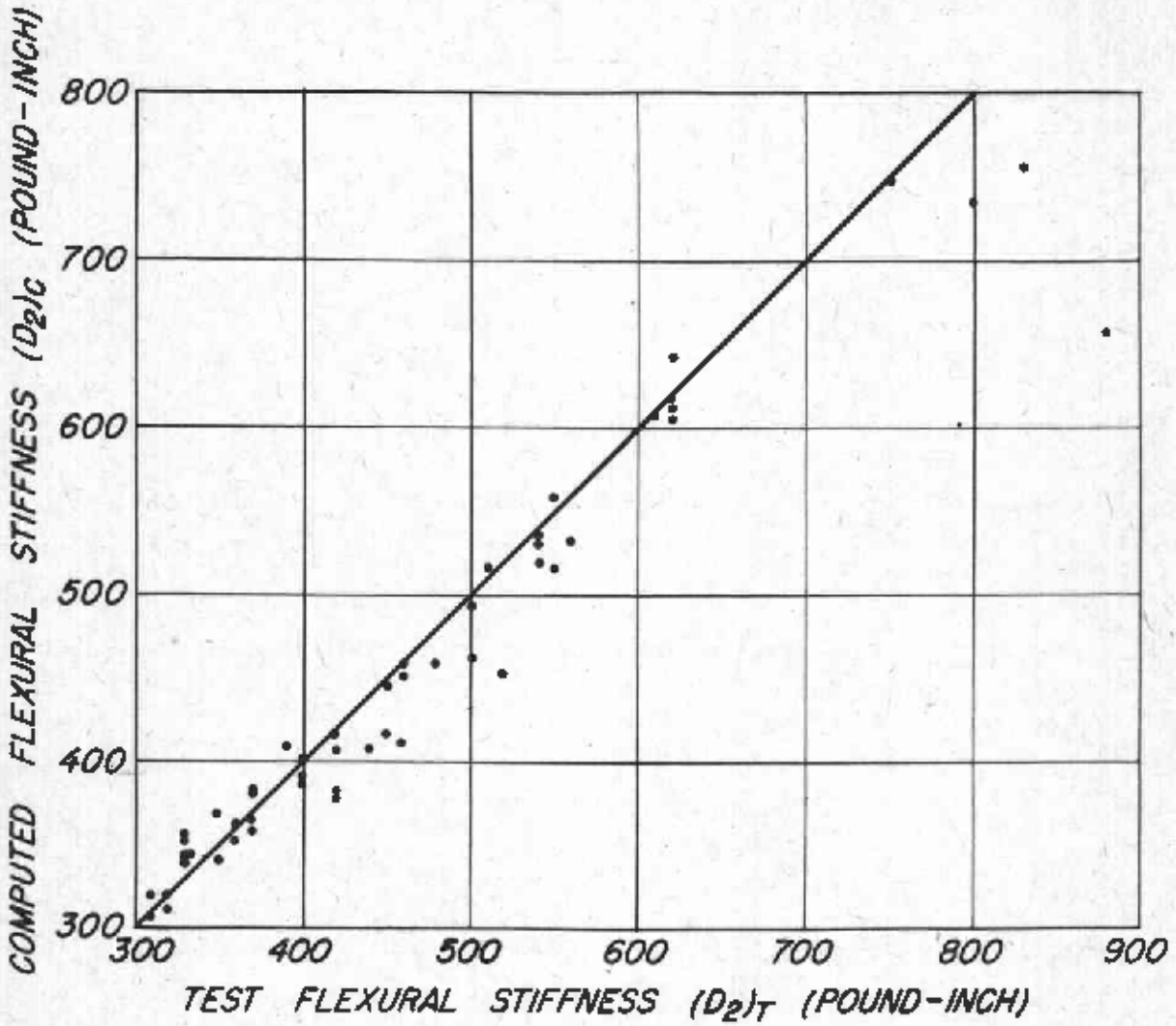

Figure 11.--Comparison of computed values of flexural stiffness to observed values; length of stiffeners perpendicular to the span of the plate.

Z M 7739⁴ F

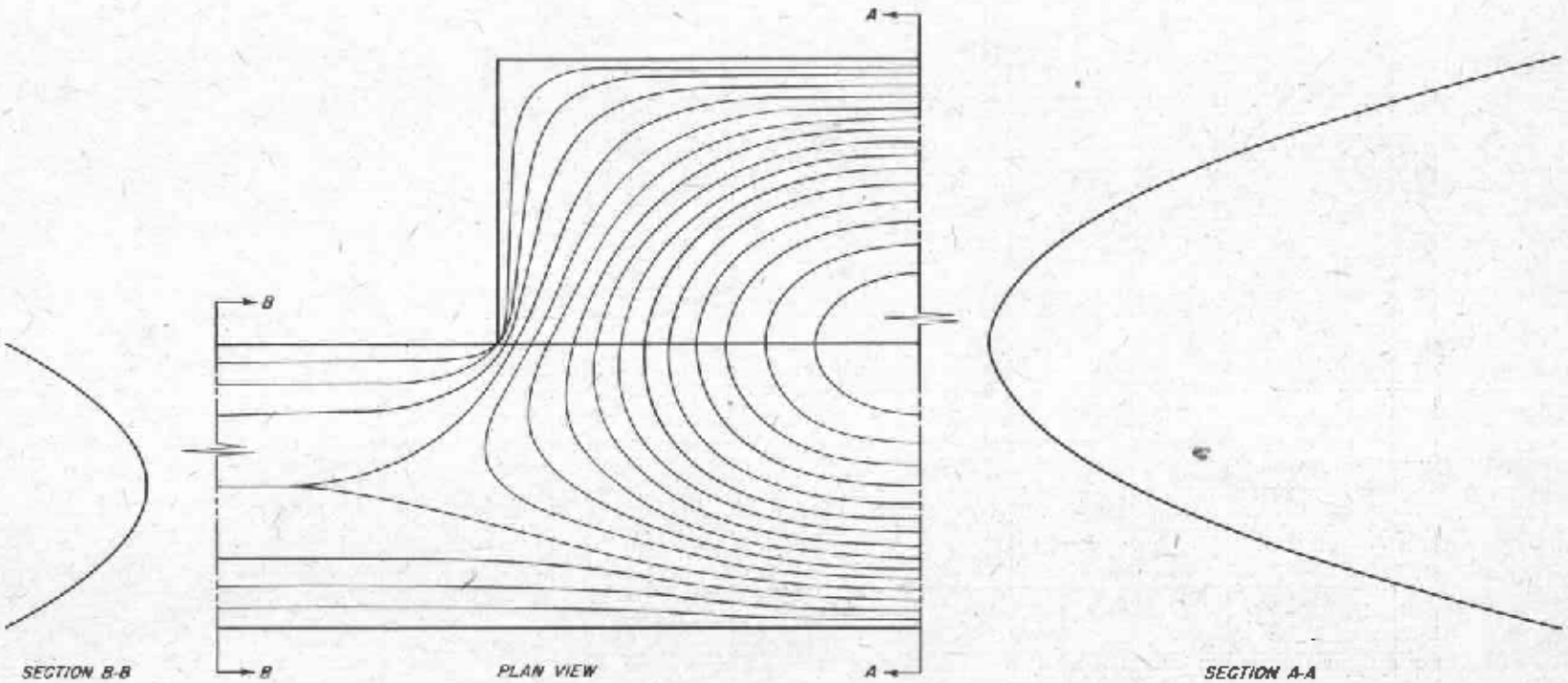

Figure 12.--Estimate of the contours of the surface of a soap film over a cross-sectional opening similar to the cross section of a stiffened plate; stiffeners separated.

Z M 77395 F

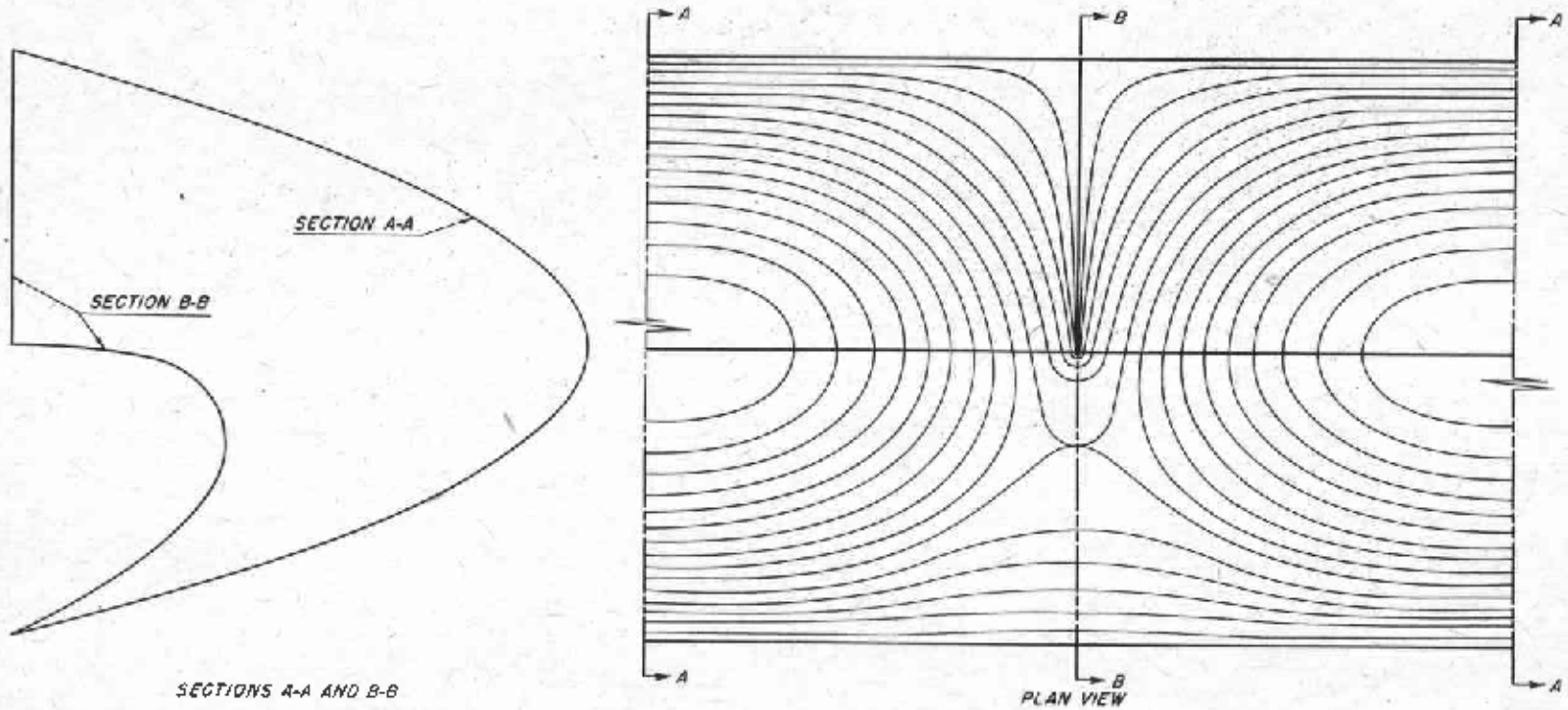

Figure 13.--Estimate of the contours of the surface of a soap film over a cross-sectional opening similar to the cross section of a stiffened plate; adjacent stiffeners touching but not solid.

Z M 7096 F

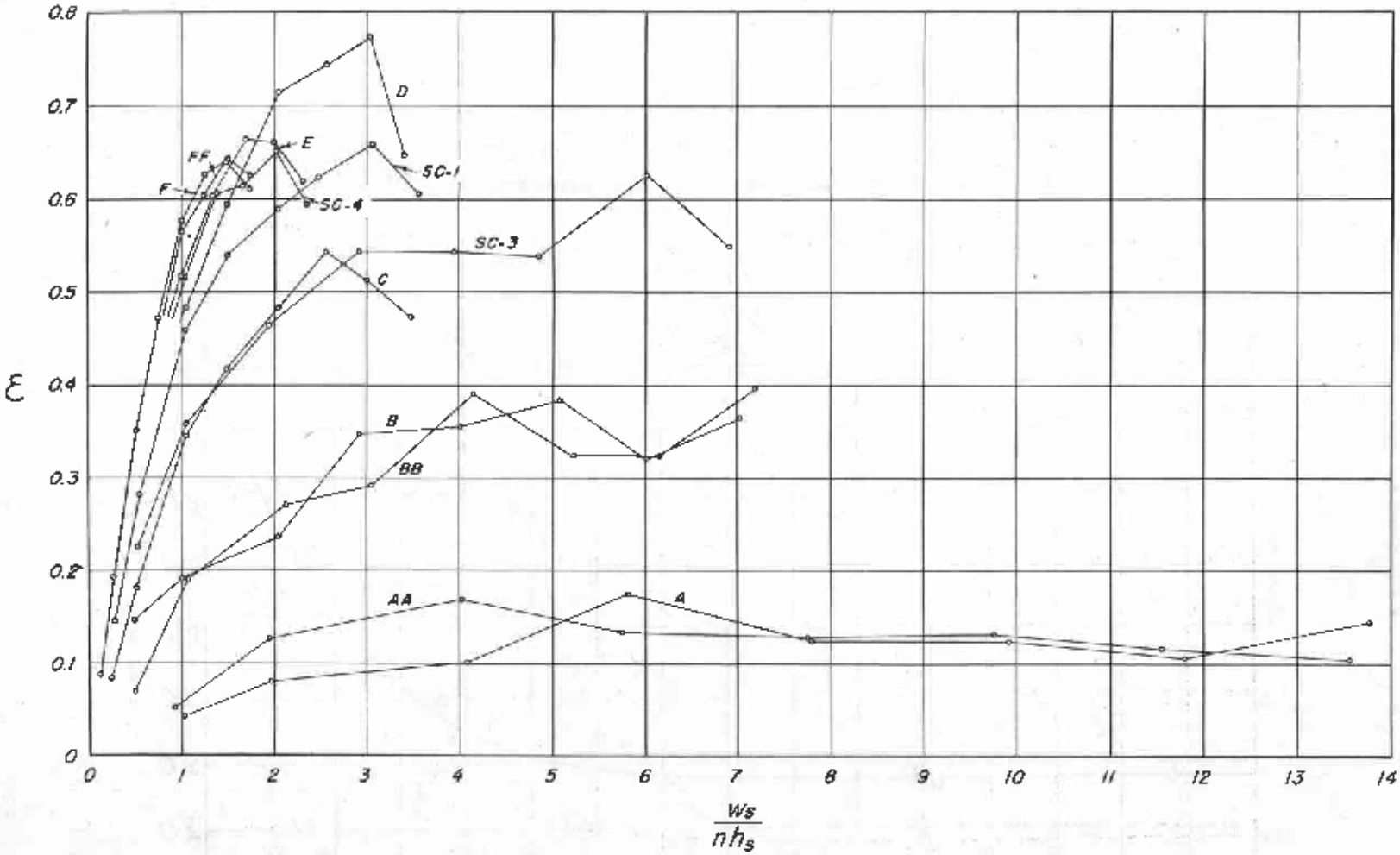

Figure 14.--Correction factor ε (equation 23) vs. the ratio of stiffener width to depth. Stiffened plates tested in shear. (Letters on individual curves designate the individual specimens (table 3)).

Z M 77397 F

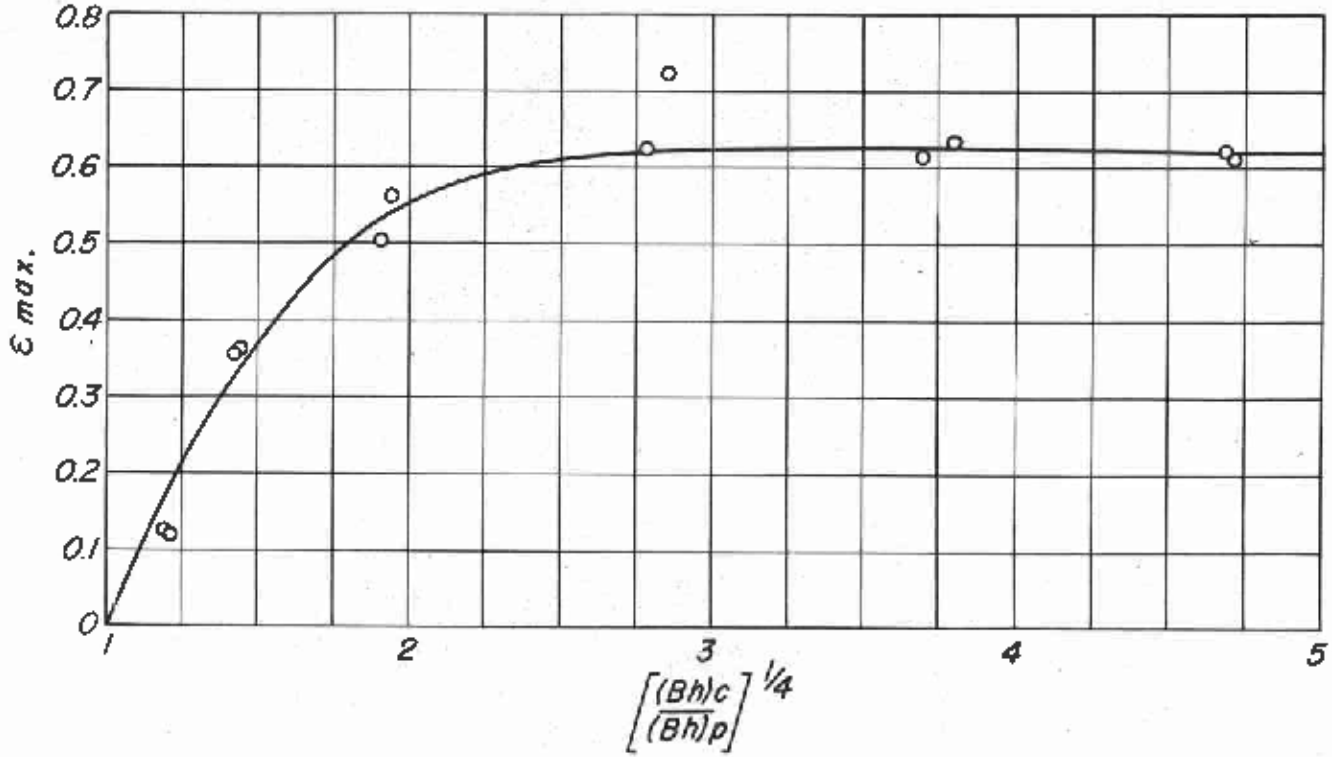

Figure 15.-- $\epsilon_{max.}$ vs. parameter as determined from shear tests of stiffened plates.

Z X 77398 F

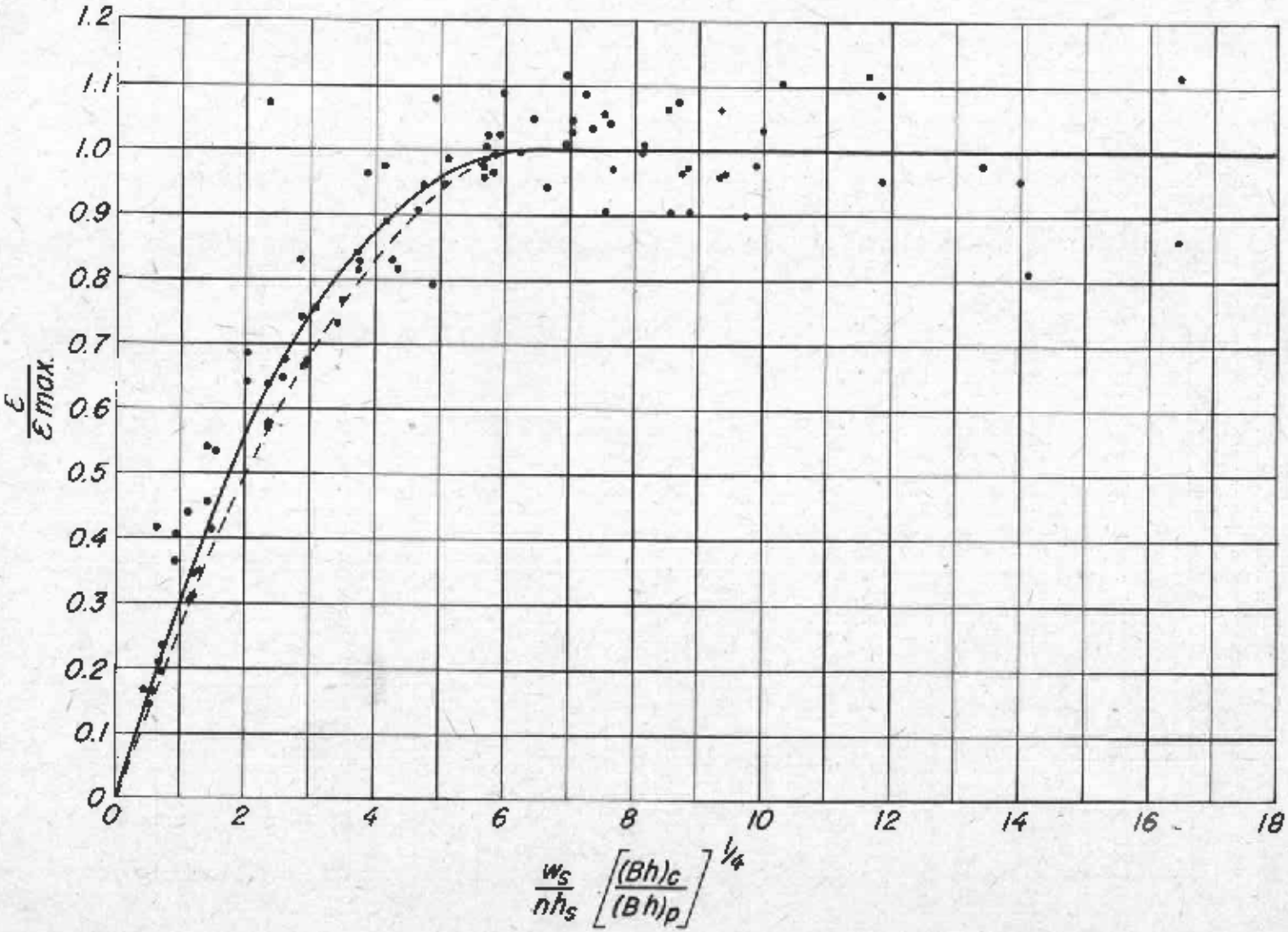

Figure 16.--The ratio of ϵ to $\epsilon_{max.}$ plotted against the parameter chosen.

2 M 77399 F

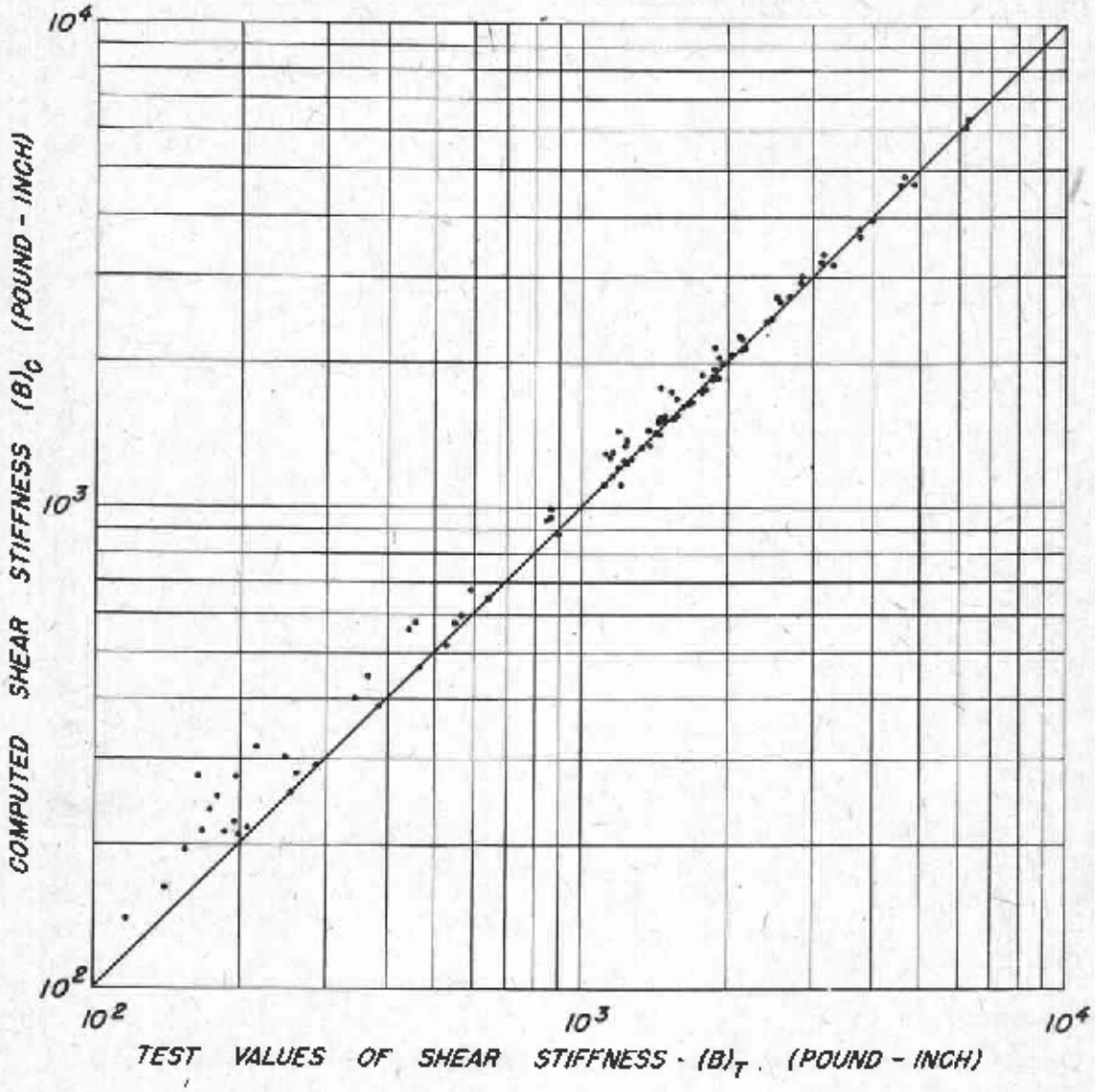

Figure 17.--Comparison of observed values of shear stiffness with computed values.

2 ✱ 77400 F