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Abstract Approved:
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The purpose of the thesis is to give a better understanding of the radial loading capacity of anti-friction bearings and to show why a formula cannot be prepared that can be universally applied to bearings of different types and of different manufacture.

The early investigations of Hertz and Stribeck are useful for the determination of the static loading capacity. Latter investigators formulated methods of calculating bearing capacity from the fatigue standpoint.

Very extensive laboratory tests have been conducted on ball and roller bearings to determine the relation between load and life. The relation is, the life is inversely proportional to the load raised to the ten-thirds power. The capacity of an anti-friction bearing is meaningless unless combined with a time factor, since it has been definitely established that ultimate failure is due to fatigue. The results of tests also show that a certain dispersion of life exists. Even in cases where the same life should be expected, that is, where a number of bearings of the same size and type, ident-

ical conditions of heat treatment, material, and manufacture were tested under identical conditions, there appeared a wide range of life for the groups of bearings.

Since there is such a large spread of life in bearings operating under identical conditions it is necessary to select some definite point on the curve as a basis for arriving at an allowable bearing capacity, and also to state the average life expectancy.

When it is desired to compare two catalog bearing ratings using different life bases the values of one must be corrected before a comparison is made with the other.

Methods of calculating bearing capacity found in engineering literature are presented and applied on a number of ball and roller bearings of approximately the same size. The results of these comparative calculations show a great variation from the manufacturers' ratings. The manufacturers' ratings are also compared and found to vary greatly. Due to the knowledge of the reaction of the material to stress, and to experience acquired by practical experiments, bearing manufacturers have developed formulae which represent a relation between load rating and life. However, a life formula can be applied only to a certain definite type manufactured from certain definite materials and with a definite degree of accuracy; therefore, no formula can be prepared which can be universally applied to bearings of different material and different manufacture.

THE RADIAL LOAD RATING OF ANTI-FRICTION BEARINGS

by

ALBERT WILLIAM ANDERSON

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During the construction of Bonneville Dam several tests were made on large roller bearings to be used for the gate wheels of the project. These tests were made by Professor S. H. Graf, Head of the Mechanical Engineering Department, at the request of R. R. Clark, Engineer.

The author, in assisting with these tests, became interested in the study of bearing ratings, and as a result this thesis is written.

Much credit is due Professor Graf for his help and criticism, and the author wishes to thank him and members of the Mechanical Engineering Department staff for their cooperation.

A.W.A.

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PART I

INTRODUCTION

During the last few years consumers of ball and roller bearings have been particularly interested in obtaining information which would permit them to compare the carrying capacity of the bearings of one manufacturer with those manufactured by another concern. A formula which could be universally applied to bearings of different types and of different manufacture would be welcomed by their users.

It is the purpose of this paper to give a better understanding of the radial loading capacity of anti-friction bearings and to show why such a universal formula cannot be prepared.

In order to get a better understanding of how the ratings of ball and roller bearings are prepared, methods that have been and are used are presented together with comparative calculations which show a wide divergence from the manufacturer's ratings. The influence of design and materials is also considered.

Requests were sent to several manufacturers of ball and roller bearings for information on methods used for obtaining the radial load ratings of their bearings. These requests were not granted, with the explanation that this type of information could not be divulged. Consequently, the information presented was gathered from manufacturer's catalogs, engineering literature, and handbooks as given in the bibliography.

PART II

EARLY INVESTIGATIONS BY HERTZ AND STRIBECK

In 1898 Professor R. Stribeck began an investigation of (3)* the principles of ball bearing design. The research undertaken by him was based on the mathematical treatment of the behavior of bodies in contact as studied by Heinrich Hertz.

The problem was to determine, for the ball bearing elements, what relation existed between ball diameter, shape of surfaces in contact, and load, in terms of the resulting temporary and permanent deformation. Hertz made the following assumptions to simplify mathematical treatment:

1. That the force between the bodies in contact is normal at the contact surface and that neither body exerts a tangential force on the other.
2. That the bodies are elastic.
3. That they are homogeneous.
4. That they are at rest relative to each other.
5. That the elastic limit is not exceeded.

The primary conclusions arrived at were the following formulae which deal with the case of two spheres in

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*Numbers in brackets refer to bibliography.

PART II

EARLY INVESTIGATIONS BY HERTZ AND STRIBECK

contact: (10)

$$1.\quad d/2 = 1.23 \sqrt[3]{P^2 \alpha^2 (r_1 + r_2)/r_1 r_2}$$

$$2.\quad a = 1.11 \sqrt[3]{P \alpha r_1 \; r_2/r_1 + r_2}$$

$$3.\quad P_o = 0.388 \sqrt[3]{\frac{P}{\alpha 2}(r_1 + r_2/r_1 r_2)^2}$$

where d/2 is the compression, that is, the distance by which the two spheres approach each other under load P, r_1 and r_2 are the radii of the two spheres, α is the elasticity coefficient and is equal to 1/2,120,000 (this is equivalent to a modulus of elasticity of 30,000,000 lb/sq in), a is the radius of the pressure area, and P_o is the maximum pressure which acts at the center of the pressure area.

Figure 1 shows the results of Stribeck's experiments and also shows the application of Hertz's equations using balls of 5/8 inch diameter. The total approach measured between centers of the outside balls is d, and half of this is the approach of the two spheres. This figure also shows the temporary and permanent deformations in a 5/8 inch diameter ball, and it is seen that the elastic property of the steel is evident under comparatively small loads. It was revealed that minute permanent sets were registered at the ball surfaces for quite small loads, and from an analysis of the circumstances it appeared that this deformation displayed its maximum effect for a mean com-

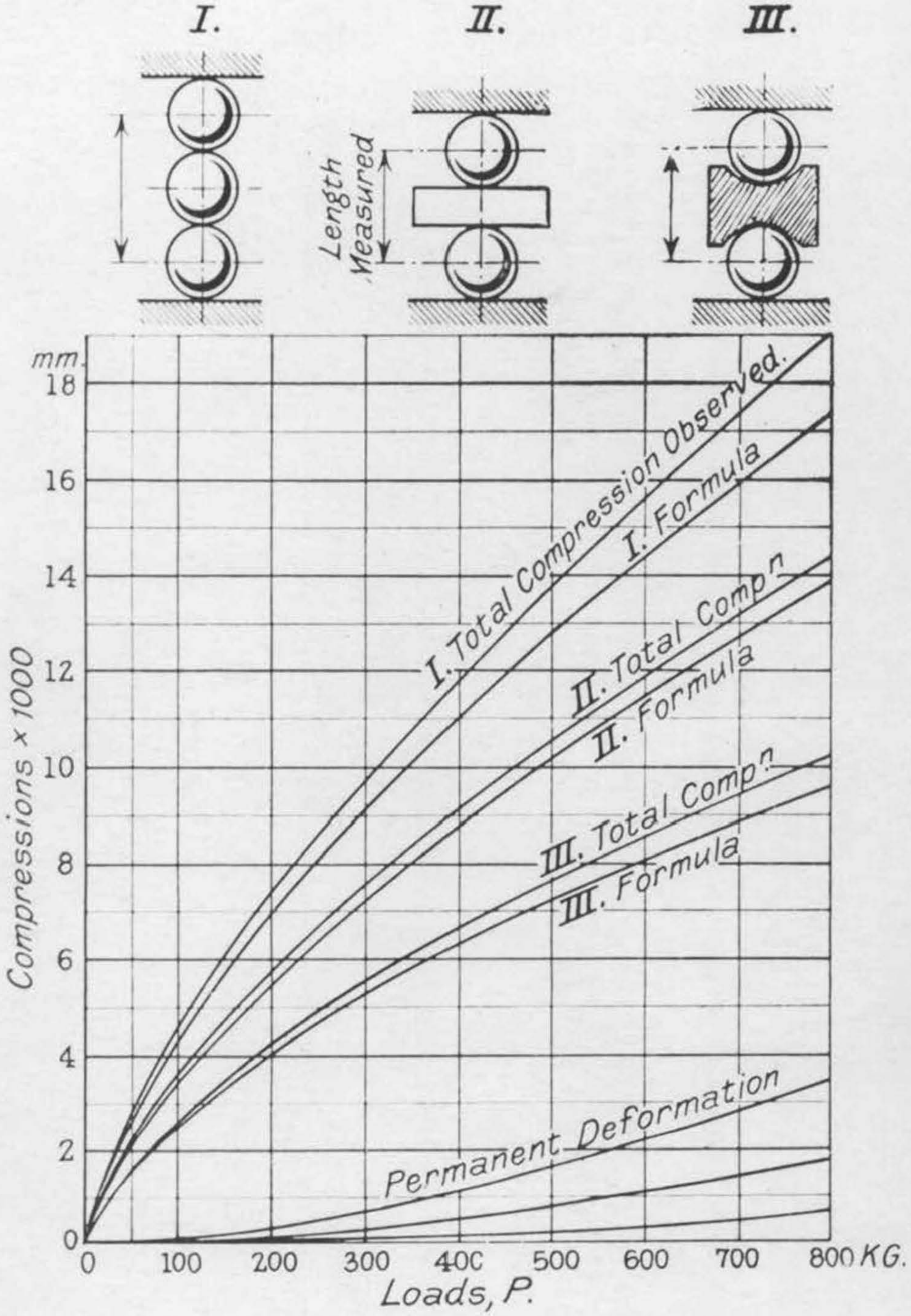

FIG. 1.

I. 3 balls $\frac{5}{8}''$ diameter. Compression according to Hertz: $\delta/2 = 0{\cdot}0001014 \sqrt[3]{P^2}$

II. 2 balls $\frac{5}{8}''$ diameter and plate. Compression according to Hertz: $\delta/2 = 0{\cdot}0000850 \sqrt[3]{P^2}$

III. 2 balls $\frac{5}{8}''$ diameter and grooved cylinder. Compression according to Hertz: $\delta/2 = 0{\cdot}000057 \sqrt[3]{P^2}$

Elastic Coefficient $= \dfrac{1}{2120000}$.

pressive stress between the surfaces of about 160,000 pounds per square inch.

Stribeck anticipated that there was a constant relation between ball diameter and load, and assumed $P = kD^2$. It also seemed feasible that there was a constant relation between permanent set and the ball diameter. The series of curves shown in Figures 2 and 3 were used in the investigation of these assumptions. Figure 2 is a series of curves showing the relation between load and permanent compression for balls of different diameters under Condition I in Figure 1. He accordingly took, say, a ball of 5/8 inch diameter and dividing the permanent set, d_b, under various loadings by this diameter, obtained a curve typified by the expression $d_b/d = 0.00025$. Since d_b is the permanent set for the system shown in I Figure 1, the permanent set for one ball is $d_b/4 = 0.0000625\ d$.

In order to get the loads which for various ball diameters cause the permanent sets $d_b = 0.00025\ d$, a diagonal $d_b/d = 0.00025$ was drawn through the curves in Figure 2. The abscissa of each intersection gives a diameter. The corresponding load is read at the curve intersected. The squares of these diameters and the curves corresponding to them are plotted in Figure 3 and a fairly straight line passing through the origin is obtained, for since $d_b/d = 0.00025$ it is true that $P = kD^2$. Taking

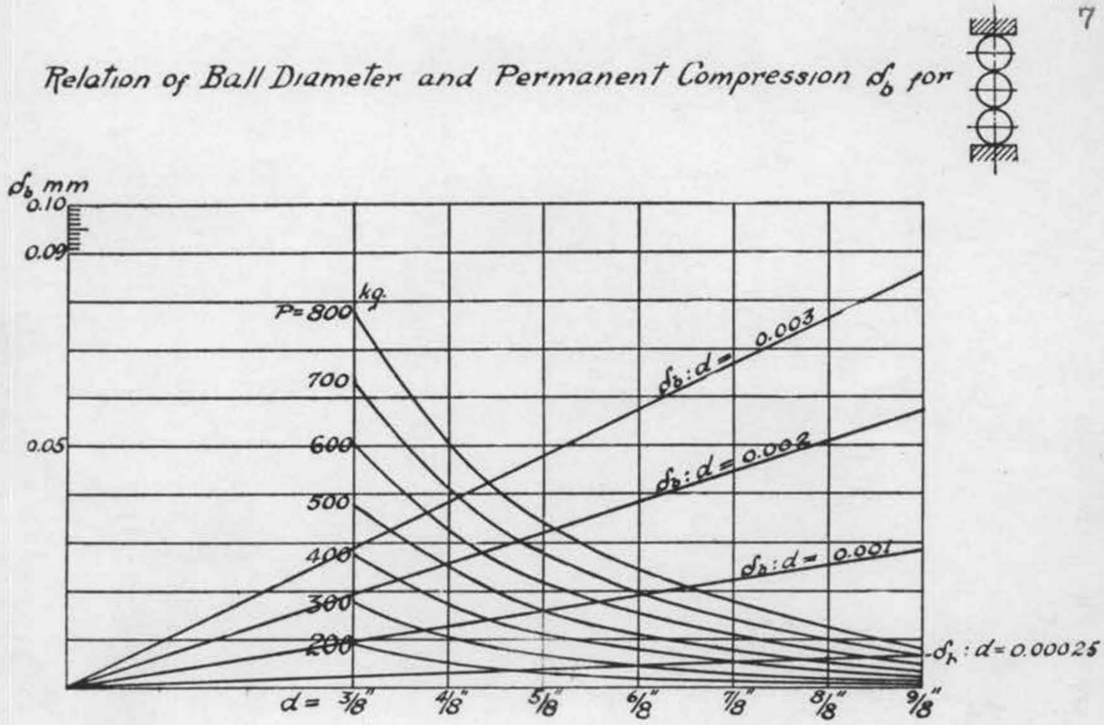

FIG. 2 PERMANENT COMPRESSION FOR DIFFERENT DIAMETERS

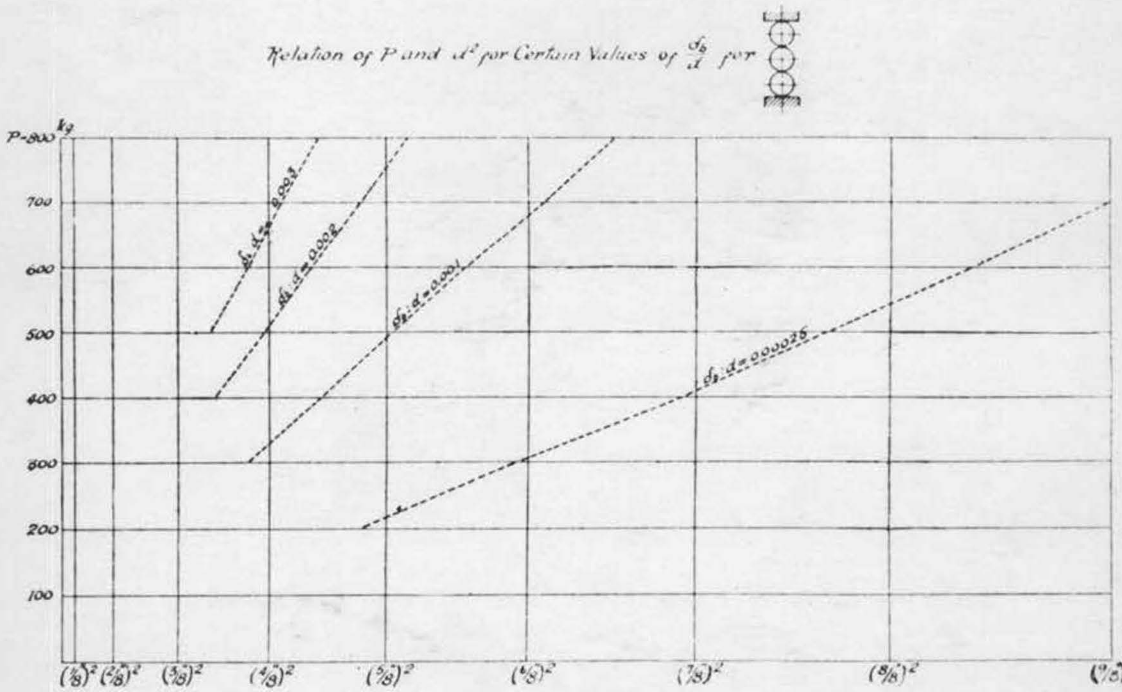

FIG. 3
DIAGRAM DEVELOPED FROM FIG. 2

numerical values found we get from Figure 2:

P in Kg.	200	300	400	500	600	700
d in inches	0.60	0.74	0.869	0.966	1.05	1.125
Therefore P/d^2	556	547	530	536	545	553

Figure 3 also shows curves for higher values of d_b/d. It can be seen that the smaller d_b/d, the less the curves deviate from straight lines starting from the origin, that is to say, the nearer we get to the elastic limit. The conclusions reached were:

1. That d_b/d is a constant within the limits of proportionality.
2. That when the elastic limit is approached, the loads are proportional to the square of the diameters.
3. That the resultant compressions are the same for any ball diameter.

A certain similarity to the usual stress-strain laws in strength of materials are suggested in these conclusions, but the elastic limit is not pronounced as a characteristic point in the curve. There is nothing to indicate when the elastic limit has been reached when two spheres are compressed, since as the limit is reached for one particle, the surface under pressure is enlarged, and more particles come under stress.

It is seen in Figure 1 that the form of the ball support is of great importance (12). Hertz showed that for the same average stress in the contact area, a ball resting upon a flat plate will support four times the load that it will when resting between two other balls of the same diameter. If, in the place of the two balls there are substituted plates with grooves having radii of curvature equal to twice the ball diameter, the carrying capacity will be increased sixteen fold.

Therefore, in order to lessen the stresses in the contact areas the curvatures of the raceways should be accurately controlled to closely confine the ball.

The relation between the curvature of the raceway and the ball is called "conformity", and from what has been said, it is evident that this factor has a very great influence upon the performance of the bearing and upon the formulae used to compute the capacity.

In the early days of the ball bearing it was feared that race grooves made with a radii less than two-thirds of the diameter of the ball would produce contact ellipses with too great a major axis, causing slip and loss of efficiency. Better materials, metallurgy, and methods of manufacture have permitted bearing makers to use much closer conformity without loss of efficiency and materially reducing the deformation, and thereby, the molecular work. In cases where a shaft is required to flex under load, more open curvature is needed to prevent the bearing from becoming too rigid.

On a basis of the rated capacity at a certain speed, the load (5) operating on the most heavily loaded ball during rotation is about 1000 lbs. in a normal single row bearing of 45 mm bore with 5/8 inch diameter balls. In applying the above formula to a design of normal race track gives 0.00192 in. as the approach of the two races, and the actual permanent set per ball surface is approximately 0.00003 in. This shows the difficulty in fixing a permissable temporary or permanent deformation with reference to capacity.

To show the actual stresses that occur in ball bearings they have been calculated (5) for two cases of single row and self-aligning standard ball bearings 50 x 110 x 27 mm. The single row bearing had thirteen 11/16 in. balls, and the self-aligning type had 26 9/16 in. balls. The applied load was 500 kg., and the pressure areas and normal stresses as calculated from the above formulae gave the following:

Outer ring of self-aligning bearing	Ball pressure 96 kg. Pressure area $0.732\,\pi \times .372$ mm^2 Max. normal stress 330 kg./mm^2 (210 tons per sq. in.)
Inner ring of self-aligning bearing	Ball pressure 96 kg. Pressure area $1.225\,\pi \times .188$ mm^2 Max. normal stress 200 kg./mm^2 (127 tons per sq. in.)

Outer ring of single row bearing	Ball pressure 192 kg. Pressure area 1.55 π x .318 mm^2 Max. normal stress 186 kg./mm^2 (118 tons per sq. in.)
Inner ring of single row bearing	Ball pressure 192 kg. Pressure area 2.05 π x .216 mm^2 Max. normal stress 207 kg./mm^2 (131 tons per sq. in.)

It can be seen from the above figures that the stresses in the ordinary ball bearing in service are considerably above the elastic limit, and Stribeck found it necessary to determine capacity by some more or less arbitrary method. In this connection he was assisted by a consideration of the behavior of the material with regard to deformation, as illustrated in Figure 4, which indicated that for K = 10 Kg. per 1/8 in. ball diameter, the slope of the curve representing d_b/d had attained its maximum value. Therefore this value for k was taken as the maximum allowable load for each 1/8 in. ball diameter and was called the specific load.

Passing to the application of these static tests to the completed bearing, it is necessary to add geometrically the various forces acting to determine the number of balls which might be assumed to operate during rotation of the race. Pure radial load, applied to a ball bearing is not evenly distributed on all the balls (12). Let us assume that such a load is applied vertically to the outer ring and that the position of the balls is as

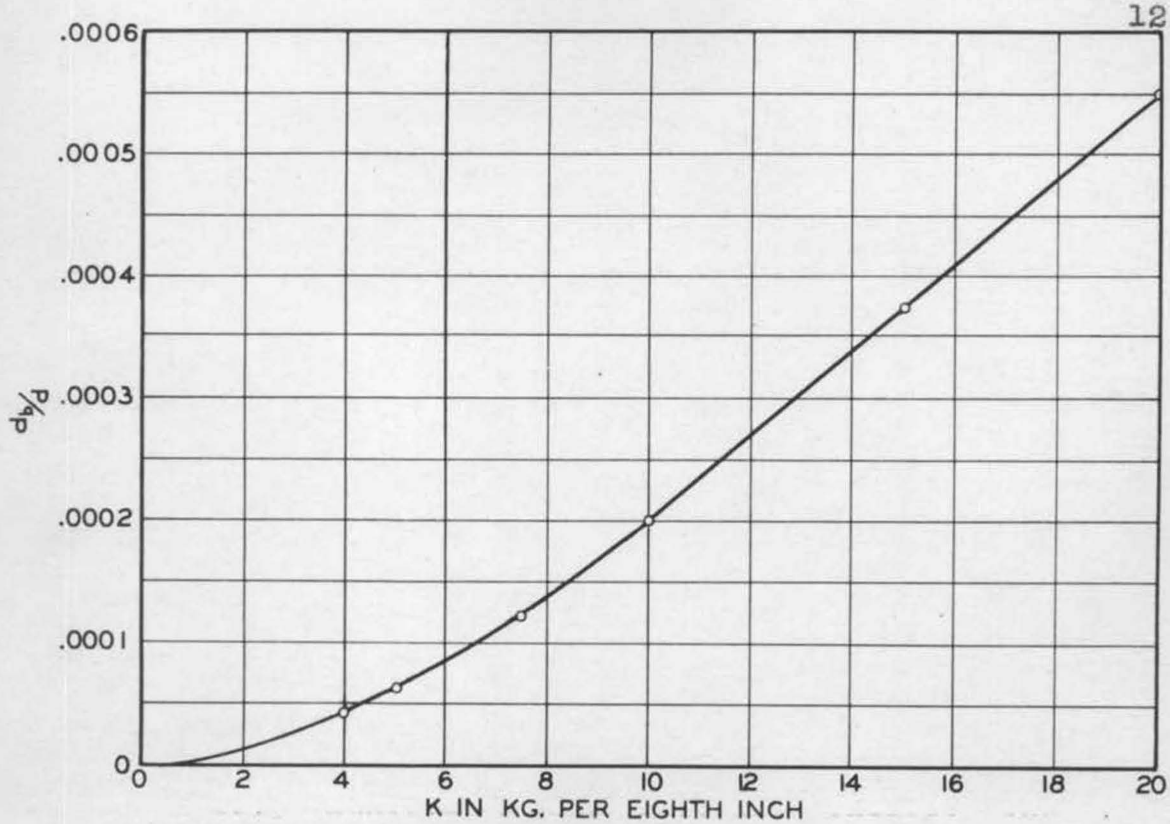

Fig.4 Relation Between d_b/d and Specific Load K.

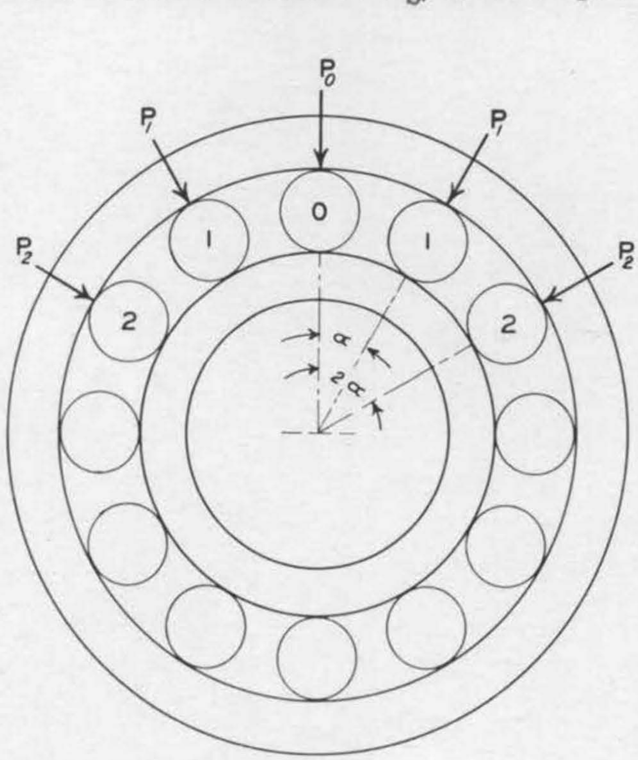

Fig.5 Load Distribution on Balls and Rollers.

shown in Figure 5. The topmost ball "O" will then be under the heaviest load, and the two adjoining it, one on each side, "1", will carry somewhat less but equal amounts. The next pair "2" will be equally loaded, but to a still lesser amount. Under this load, therefore, because of the deformation in the balls and races at the contact points, the outer race will move downward a small distance so that all the balls in the lower half of the bearing are relieved of load.

If we call P the total load applied to the row of balls, d the vertical approach between inner and outer races, then

$$P = P_o + 2\,P_1 \cos\alpha + 2\,P_2 \cos 2\alpha + \ldots\ldots\ldots + 2\,P_n \cos n\alpha$$

where n is less than one-fourth the total number of balls, and α the angle between the balls.

Therefore $n\,\alpha$ is less than 90 degrees. The approach d_o corresponds to the ball load P_o, d_1 to the load P_1, etc. Then, $d_1 = d_o \cos\alpha$, $d_2 = d_o \cos 2\alpha$, etc.

As was shown in Figure 1, Hertz and Stribeck agreed that the total compression d_o varies as $P^{\frac{2}{3}}$. Conversely, P_o varies as $d_o^{\frac{3}{2}}$. Thus

$$\frac{P_1}{P_o} = \left(\frac{d_1}{d_o}\right)^{\frac{3}{2}} , \quad \frac{P_2}{P_o} = \left(\frac{d_2}{d_o}\right)^{\frac{3}{2}} \quad , \text{etc.}$$

But $d_1 = d_o \cos\alpha$, and $d_2 = \cos 2\alpha$, etc.

Then by substitution and simplification,

$P_1 = P_0 \cos^{3/2}\alpha$ and $P_2 = P_0 \cos^{3/2} 2\alpha$, etc.

and

$$P = P_0 \,(1 + 2 \cos^{5/2}\alpha + 2 \cos^{5/2} 2\alpha + \ldots\ldots\ldots 2 \cos^{5/2} n\alpha)$$

If N is the number of balls in a complete bearing we have

$$\begin{array}{llll} N = & 10 & 15 & 20 \\ \alpha = & 36 \text{ deg.} & 24 \text{ deg.} & 18 \text{ deg.} \end{array}$$

$$\frac{P}{P_o}\begin{cases} = & 2.28 \quad 3.44 \quad 4.58 \\ = & \dfrac{N}{4.38} \quad \dfrac{N}{4.36} \quad \dfrac{N}{4.37} \end{cases}$$

$$P_o + 2\,P_1 + \ldots\, 2\,P_n = \quad 1.23\,P \quad 1.22\,P \quad 1.21\,P$$

The sum of the individual loads is almost invariable and the values of P/P_o are almost exactly equal to $N/4.37$. Therefore, for N = from 10 to 20 balls the largest load per ball is $P_o = 4.37\ P/N$ for bearings without any play. Usually there is slight play in a bearing and the value taken is $P_o = 5\ P/N$ or $P = N\ P_o/5$.

Since $P_o = k\ D^2$, where k is the specific load per 1/8 inch ball diameter, and D is the ball diameter in eighths of an inch, the total carrying capacity of the bearing may be expressed as $P = k\ N\ D^2/5$. Tests carried out with complete bearings under load to failure seemed to confirm the value of 10 Kg. for the specific load as already selected for k.

The above (12) equation furnished a simple basis of comparison for bearings of different sizes but of the same transverse sectional characteristics and material. However, in extending this formula from one size of bearing to another, no rational factor was introduced which compensated for changes in the curvatures in the central plane normal to the axis of the bearing. These variations greatly affect the shapes of the contact areas and therefore, influence the intensity of the contact stress, although the computed maximum allowable ball load is not exceeded.

The development of a rational bearing capacity rating must include a more comprehensive analysis of all the factors entering into the shape of the contact areas so that the average unit stress

$$S = \frac{P_o}{a} = \frac{\text{ball load}}{\text{contact area}}$$

is held within a safe value. In doing that, the rational expression for static capacity should read:

$$P = kCND^2/5,$$

where C is the conformity factor, a function of the curvature of the ball relative to both the transverse curvature and the curvature in the plane normal to the axis of the bearing.

For a safe load during running conditions the formula must include some factor based upon the endurance of the bearing under high stress slowly repeated or under low stress rapidly repeated. Regardless of the speed at which the bearing is running the rated load should be such that the life of the bearing will be the same.

PART III

INVESTIGATIONS FOR RUNNING CAPACITY.FATIGUE.

PART III

INVESTIGATIONS FOR RUNNING CAPACITY. FATIGUE.

1. GOODMAN'S METHOD. Professor John Goodman (5) carried on research to develop specific loads K to cover ball diameter and also speed. The results of his investigations are given in the formula:

$$P = \frac{k\ m\ d^3}{nD + cd}$$

Where D = inside diameter of the outer ball race in inches

m = number of balls in the bearing

d = ball diameter in inches

n = revolutions per minute of the shaft

k and c have the following values for radial bearings:

	k	c
Flat (i.e. not grooved) races	1,000,000	2,000
Hollow races of ordinary quality	2,000,000	2,000
Hollow races of best quality	2,500,000	2,000

If this formula is rewritten in the form

$$P = \frac{k\ m\ d^2}{\frac{nD}{d} + c}$$

it can be seen that the numerator is the same as Stribeck's formula, that D/d takes care of the relative speed of the balls and the ball race, and that c takes care of the static capacity.

The ball bearing manufacturer developed tables of constants for the factor k, and the early rated capacities were strictly based on these figures. Increasing experience modified the coefficients, however, and today a greater degree of arbitrariness is present. The load capacities arrived at had not any reference to the fatigue phenomenon in the material, and from what is to follow, it will be understood that the capacity depends upon the material and workmanship, as well as upon design.

2. PALMGREN'S INVESTIGATION. When a ball bearing at rest has a load applied to it deformation will take place in the balls and races and the points within the contact areas, as has been shown, will be principally under compressive stress. Beyond the confines of the contact areas the stress will be largely tensile since the surface metal surrounding these areas will be in pure tension.

Therefore when a bearing is put in motion and the balls roll in the races, each point which they pass is first under tension, then compression, followed by tension again.

The magnitude of the compressive stresses would vary depending upon the position of the balls in the loaded zone and the maximum values would be reached only once per revolution.

The useful life of a bearing is normally limited only by the length of time the races and balls will resist fatigue due to this cycle of stresses. If the load, which determines the intensity of the stress, is either reduced or increased, the fatigue life of a bearing will also be reduced or increased according to the change in stress.

It might be anticipated that the ball would be the weakest element, but this is not the case. During rotation, the balls in any design experience a certain amount of spin and the stress is consequently not imposed continuously at the same point, as occurs in the ball races. Experience indicates that in the single row type the inner race is usually the first member to fail. In the case of the self-aligning type it is the stationary race which fails first.

In 1923 A. Palmgren conducted tests on bearings in order to rationalize their capacity on the basis of the fatigue phenomenon in running.

As was stated previously, the fundamental values for the specific load k were developed by the bearing manufacturers and these took into account in a primary manner the variation in ball diameter and speed. In order to connect the specific load with the number of stresses it was necessary to introduce another factor which is called the "ideal specific load" K_i. The specific load k determined an arbitrary capacity for the bearing from a consideration of material, deformation, etc., while K_i modifies this figure when a certain determined life is to be obtained.

Palmgren made tests on about 300 bearings varying between light types of 15 mm bore to heavy types up to 175 mm bore, with a speed range from 300 up to 3000 rpm. The inner ring was the rotating element for all the journal bearing tests, which combined all types of load from pure radial to pure thrust.

Palmgren's analysis as related by A. W. Macaulay (5) follows:

"Double row self-aligning ball bearing.

Number of stresses on stationary race. ---- If a is the number of stresses to which a point on the stationary race is subjected during one revolution of the rotating race, and

r_y = the radius to the point of contact between ball and outer ring

r_i = the radius to the point of contact between ball and inner ring

N = the number of balls in the complete bearing.

Then Number of stresses per revolution

$$a = \frac{N}{1 + r_y/r_i} \quad \text{for stationary outer race} \quad (1)$$

$$a = \frac{N}{1 + r_i/r_y} \quad \text{for stationary inner race} \quad (2)$$

the stress reaching its maximum twice as each ball passes the point. The stress under consideration is not the simple compression stress at the centre of the contact surface, but the severe flexure stress at the edges. If the shaft has made n millions of revolutions, then the number of stresses to which the point on the ring has been subjected will be a.n millions.

Radial load. The connection between the various factors represented by these fatigue tests may be expressed in the following formulae -----

(millions of stresses) $a.n = \left(\frac{154}{K_i - 4.4}\right)^3 - 5$ (3)

while conversely the "ideal specific load" is

$$K_i = \frac{154}{\sqrt[3]{a.n + 5}} + 4.4/(1/8 \text{ in.})^2$$

This implies that for $K_i = 4.4$ per $(1/8 \text{ in.})^2$ the life of the bearing is infinite, thereby indicating the fatigue limit. It is impossible to determine this limit accurately as the curve within the entire range covering these tests is a declining one. The greatest number of stresses covered by the tests was actually 6,700 million.

The relation between the specific load and the "ideal specific load" is

$K_i = K(1 + 0.0001\ v)\ (1 + 0.007\ D^2)$ lb per $(1/8 \text{ in.})^2$ (5)

where v is the number of revolutions per minute,

D is the ball diameter in 1/8 in. units.

In Figure 6 is illustrated the experimental values for the number of stresses in millions obtained during the tests on spherical ball bearings. A line drawn through these has been so fixed that only 13.6 per cent came beneath it, and a conservative attitude has been adopted.

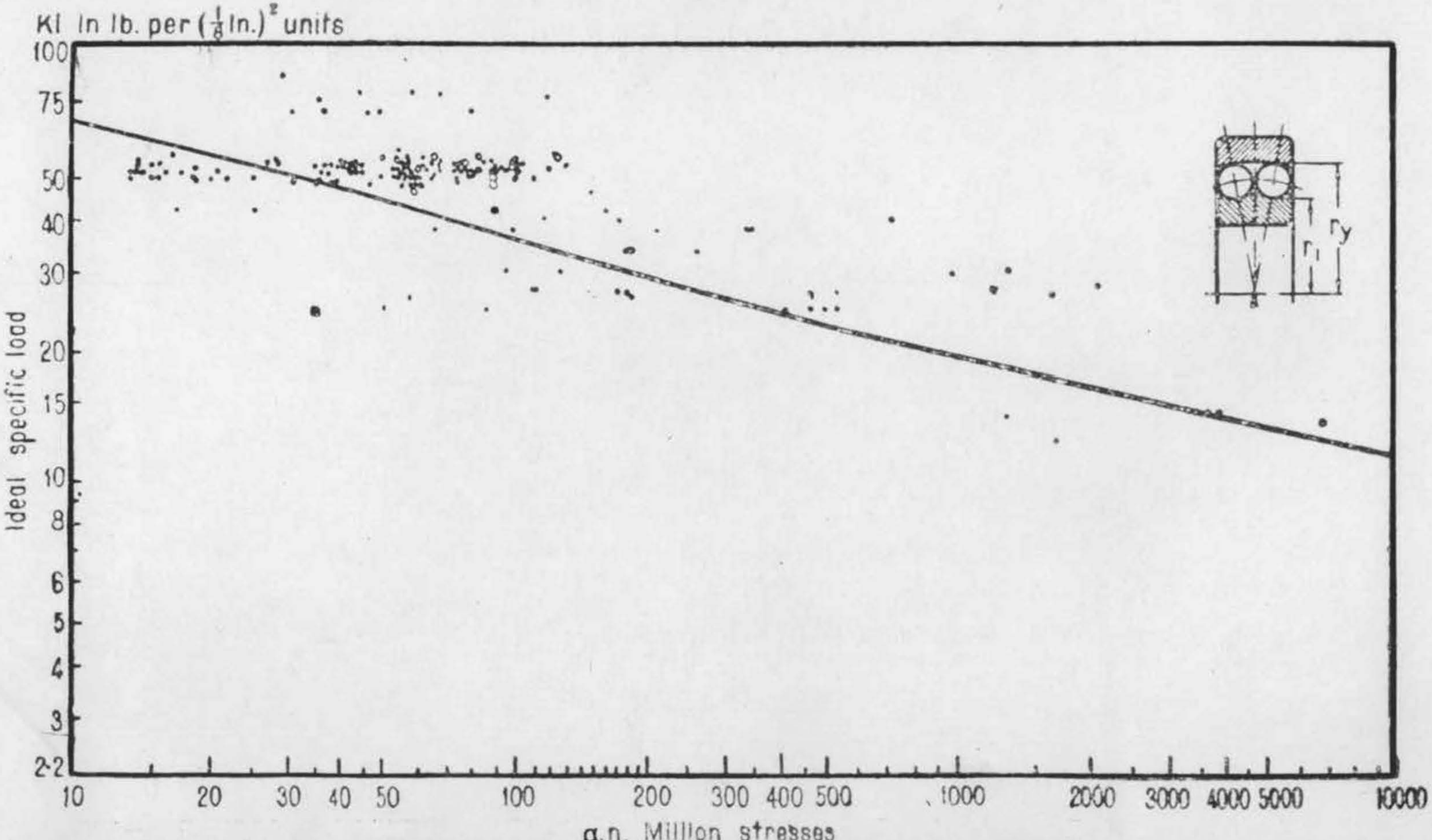

Fig.6 Relation Between Pure Radial Load and Life for Double-row Ball Bearings.

Life of Bearings. It will be appreciated that the bearings in many cases will give considerably longer life than indicated by the formulae, but it is necessary to cover not only the slight inequalities in manufacture which are constantly the subject of scrutiny, but also to cover the irregularities in loading present in many applications, these irregularities not being always amenable to calculation.

Considering the most common case of a rotating inner ring, the application of formula (1) and (3) give approximately a life of 1000 hours continuous running with the radial load as given in the rated capacity table, steadily applied. The corresponding life for an overload of 100 per cent is approximately 100 hours, and for the case where the load is half the value given as the rated capacity, the resultant life is about 10,000 hours.

The proportionate life for full load and half load suggests a cubic law, the resulting life varying inversely as the cube of the load variation, but this law does not hold with rigour where the running is intermittent.

A consideration of formula (2) shows that the number of stresses operating on the inner race, where the outer race is rotating, as in a front hub, is considerably greater, and the life correspondingly reduced. This reduction is approximately 30 per cent, which indicates that in order to obtain the same basic life of about 1,000 hours, a factor of 1.125 over the rated capacity should be adopted. The other relations as to proportionality follow on as before.

Application of Formulae. It is seen that for the general case a factor of about 2.5 indicates a life of, say, 24,000 hours, corresponding to 10 years operation under usual factory conditions of 8 hours per day for 300 days per year. As a matter of fact, the bearing will probably not be under load all the time, and there is little doubt that the factor of 2 will be sufficient to give 10 years service. The following figures provide the comparison between the loads at different speeds obtained in this way, and the loads according to the formula for the life for a standard double row self-aligning ball bearing, 30 mm bore x 72 mm outside diameter x 19 mm width.

rpm	10	25	50	100	250	550	1000	2500	5000
according to rated capacity with F.S.=2, lb	880	847	803	741	660	594	517	407	286
according to life formula, lb	1760	1364	1122	924	726	594	495	363	264

This reveals the fact that a decidedly higher load can be permitted at low speeds, while in the higher speed range the two values are in fair agreement.

Single row radial bearings. The corresponding data for single row journal bearings would appear to be covered in the following manner ----

$$a = \frac{N_1}{1 + r_i/r_y} \qquad \text{For stationary outer race} \quad (6)$$

$$a = \frac{2\,N_1}{1 + r_i/r_y} \qquad \text{For stationary inner race} \quad (7)$$

a = number of stresses at a point on inner race in both cases, where N_1 = the number of balls.

For single row bearings without filling slot

$$a.n = \left(\frac{264}{K_i - 8.8}\right)^3 - 5 \tag{8}$$

$$K_i = \frac{264}{\sqrt[3]{a.n + 5}} + 8.8 \text{ lb per } (1/8 \text{ in.})^2 \tag{9}$$

$$K_i = K\,(1 + 0.0001\,v)\,(1 + 0.007\,D^2) \tag{10}$$

For single row bearings with filling slot.

$$a.n = \left(\frac{198}{K_i - 6.6}\right)^3 - 5 \tag{11}$$

$$K_i = \frac{198}{\sqrt[3]{a.n + 5}} + 6.6 \text{ per } (1/8 \text{ in.})^2 \tag{12}$$

$$K_i = K\,(1 + 0.0001\,v)\,(1 + 0.007\,D^2) \tag{13}$$

The application of these formulae shows the resulting life on the basis of the rated capacities is approximately the same as for the self-aligning type, although in certain cases the rated capacity for the single row bearing could be slightly increased."

3. S. R. TREVES METHOD. In 1926 S. R. Treves (11) presented an article the object of which was to investigate the problem of the speed of the balls in a ball bearing and to obtain expressions for the relation between

this speed and the capacity factor K of Stribeck's equation for bearings of various types, including roller bearings.

Of particular importance are his equations which express very simply the circumferential speed of the balls or rollers around their centers, on which depends the coefficient of resistance k. These equations are as follows:

$$V = 0.0525\ n\ r \text{ for rotating inner ring}$$

$$V = 0.0525\ n\ R \text{ for rotating outer ring}$$

where V = the circumferential speed in meters per second, n = revolutions per minute, and r and R the radii in mm of the rolling circle of the inner and outer ring respectively.

The values of the coefficient k as a function of the circumferential speed V (expressed in meters per second) of the balls or rollers around their own axis are given below. These values are based on practical and experimental data of several foreign manufacturers of antifriction bearings.

When the diameter of the rolling elements exceeds a certain value resulting in lack of uniformity in the structure and hardness, the increase of the stress due to the centrifugal force of the balls or rollers cannot be neglected. Therefore, the values apply to the limiting diameters given (6).

V, meters per sec.	.25	1	2	3	4	5	6	7	8	10	12	14	16
k	18.2	13.3	10.4	8.3	7	9	5	4.6	3.6	2.6	1.8	1.3	1.0
Limiting diameter, mm		50	43	37	33	29	26	24	20	16	13	11	10

For the value thus obtained for the coeffieient k by use of the following table covering different types and shapes of bearings, we can determine the coefficient K of Stribeck's equation:

$$p = 0.02\ K\ z\ d^2 \quad \text{for ball bearings, and}$$

$$p = 0.02\ K\ z\ d\ l \quad \text{for roller bearings}$$

where p is the permissible load in kilograms, z is the number of the balls or rollers, d their diameter, and l the length of the rollers in mm.

It will be noticed that the value of the constant in the above formulae is not the same as was originally developed due to the fact that the diameters are to be given in mm instead of in 1/8 inch units.

Single row ball bearing, concave surface	K = k
Single row ball bearings, cylindrical surface	K = 0.4k
Single row ball bearing, convex surface	K = 0.1k
Double row ball bearing, concave surface	K = 0.66k
Double row ball bearing, cylindrical surface	K = 0.27k
Double row ball bearing, convex surface	K = 0.10k
Roller bearing	K = 1.5k

4. MUNDT ROLLER BEARING METHOD. The following method (8) of calculating the "specific wear" of a roller bearing is given by Von Dr.-Ing. Robert Mundt, Berlin in his paper Über die Tragfähigkeit von Zylinderrollenlagern which appeared in the May 21, 1931 issue of Maschinenbau. It will be seen that the "specific wear" is directly proportional to the bearing material and inversely proportional to the life in work hours. The translation is as follows:

The formula for fatigue. Specific loading.

When cylindrical bodies contact each other pressure strains will be found in the middle of the contact area, and tension strains at the edges of the contact area. It is unknown which strain or what combined stresses cause the greatest wear for the material. But in any case the experiments of Stribeck have shown that within the range of stresses which are used in roller bearings the specific loading can be considered as a measure of these pressures and tensions.

Q = the loading of the bearing in Kg.

d = diameter of the rollers in mm

L = length of the roller in mm

Z_W = number of rollers.

Specific loading $k = \frac{5\,Q}{Z_W\, d\, L}$ Kg/mm^2.

The most worn part of a bearing is the race, as the stresses there are the greatest. When moving one spot of the race on a loaded side a change between pressure and stress takes place. The destruction of a running bearing is, therefore, due to the fatigue of the material.

General fatigue experiments with bar-iron and also bearings showed accordantly that there is a relation between the stress, that is, the specific loadings and the number of cycles of loading. Drawn in the logarithmic coordinate system this relation shows a straight line so that the fatigue is

$$\log k = \log C' - 1/m \log Z$$

$$k = C'/Z^{\frac{1}{m}}$$

where

C' = bearing and material constant

Z = number of allowable cycles in millions

m = material constant determined by the fatigue experiment = 10/3

By substituting k as given in formula 1 and with m = 10/3 we find the allowable loading of roller bearing by

$$Q = \frac{C'}{5} \frac{Z_w \, dL}{Z^{.3}} \text{ Kg.}$$

Let $C'/5 = C$

$$Q = \frac{C \, Z_w \, d \, L}{Z^{.3}} \text{ Kg.}$$

When the constant C is known the allowable loading for a certain number of cycles can be found.

Number of cycles. In a single case it is complicated to calculate the number of cycles. Usually the life is determined in number of revolutions.

If c = number of cycles per revolution

N = life in a million revolutions

Then $Z = cN$ and

$$Q = \frac{C\, Z_W\, d\, L}{(cN)^{.3}} \text{ Kg.}$$

The determination of c depends on what race has the greatest wear. In most cases of practical machines the inner ring is rotating and gets the greatest wear. The number of cycles c can be determined by the formula:

(for rotating inner ring)

$$c = .17\, Z_W \frac{d_a}{d_a + d_i}$$

d_a = diameter of race of outer ring in mm

d_i = diameter of race of inner ring in mm.

If n is the number of rpm of the shaft the life B in work hours is

$$B = \frac{N\, 10^6}{60\, n} \text{ hours}$$

and $$Q = C\, 10^{1.8}\, Z_W\, d\, L/(60\, c\, n)^{.3}\, B^{.3} \text{ Kg.}$$

Herewith, the loading capacity of a roller bearing is given as a function of the life in work hours. For the calculation of capacity it is, therefore, necessary to state these capacities in dependance on the speed and hours of life.

As a measure of the wear that practically takes place we may call <u>k the specific wear</u>.

$k = Q\,(60c\, n)^{.3}/10^{1.8} Z_W\, d\, L$ where $k = C/B^{.3}$

PART IV

MODERN INVESTIGATIONS

PART IV

MODERN INVESTIGATIONS

1. GENERAL. In the last few years very extensive laboratory tests (12) have been conducted on ball and roller bearings to determine the relation between load and life. It can readily be seen that the capacity of an anti-friction bearing is meaningless unless combined with a time factor, since it has been definitely established that ultimate failure is due to fatigue. The point when flaking begins in the rolling elements or raceways as determined by a noise test is usually considered as the end of the useful life of a bearing. Increasing the load will make flaking appear sooner since it is caused by the intensity and number of stress cycles to which the elements of the bearing have been subjected. The number of stress cycles is constant for a given bearing during one revolution. Therefore, the life of a bearing under a certain load may be expressed in number of revolutions. The accumulated test results show that, within certain limits, the life of a bearing is independent of the speed when measured in number of revolutions.

The test results show that there is a definite relationship between the magnitude of the load and the number of revolutions that the bearing will run until fatigue failure occurs. This relation is, the number of revolutions is inversely proportional to the bearing load raised to the ten-thirds power. This equation may be used to graphically express the life curve of a bearing.

If we draw this curve of the relation between the number of revolutions and the load or specific load on a bearing in the logarithmic coordinate system a straight line will (8) result, the equation of which is $\log k = \log C' - 1/m \log N$ where k is the specific load, C' is a bearing and material constant, N is the number of revolutions in millions, and m is a constant which has been determined from fatigue experiments to be 10/3. This equation may be rewritten in the form $k = C'/N^{1/m}$ or $k = C' N^{-0.3}$. This is now in the form given in the preceeding paragraph.

The results of tests (9) on ball bearings to determine the variation of the <u>average</u> life with varying load are plotted on a logarithmic graph in Figure 7. For a decrease in load to 50 per cent the life is increased 10 times.

The slope of this line for average failures has been found (9) to be practically the same for other types of anti-friction bearings. Parallel lines can be plotted which represent various percentages of failure. The slope of the curve is constant for any bearing, but its position will depend on the type of bearing, its material, conformity, and looseness. For similar bearings the quality of the

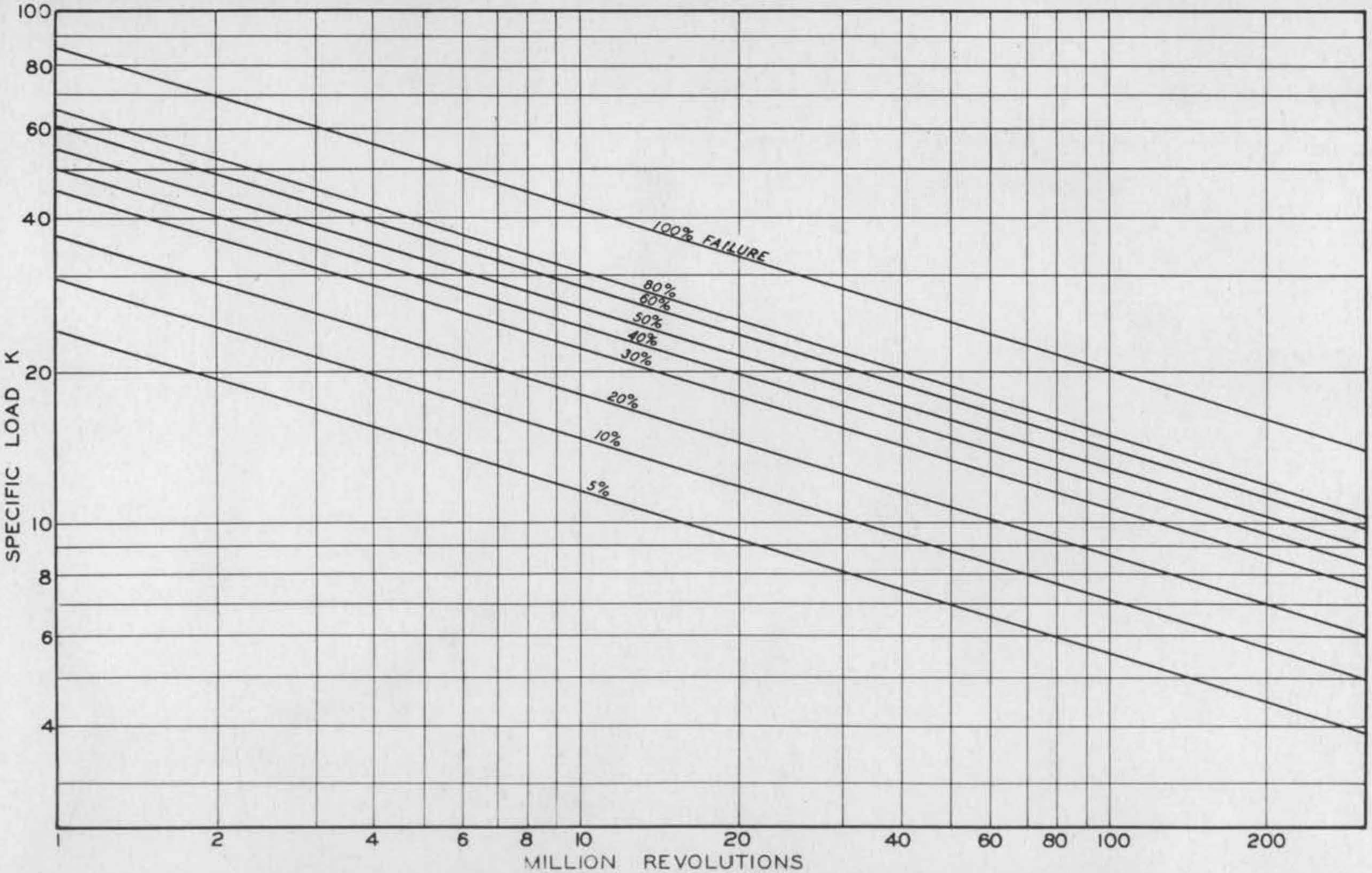

Fig.7 Load Life Relation for Ball and Roller Bearings.

material is of greatest importance.

2. THE DISPERSION OF LIFE. The results of a great number of laboratory tests (12) on bearings show that a certain dispersion or spread of life exists. Even in cases where the same life should be expected, that is, where a number of bearings of the same type and size, of identical conditions of heat treatment, material, and manufacture were tested under identical conditions, there appeared a wide range of life for the groups of bearings. The reason for this is hard to explain, but it is possible to ascertain that where a group of 30 or more bearings are subjected to identical load and speed, the average life of these bearings can be determined with a consistant degree of accuracy. Such a dispersion curve is shown in Figure 8. It has also been found that there is a definite relationship between the lives of the first bearings that show fatigue and the average life of the whole group of tested bearings.

3. DEFINITION OF BEARING CAPACITY. Due to the fact that there is such a large spread of life or dispersion in bearings operating under the same conditions, the necessity for selecting some point on the curve which will serve as a basis for arriving at an allowable load or bearing capacity can be seen.

Figure 8 shows no concentration of values around the

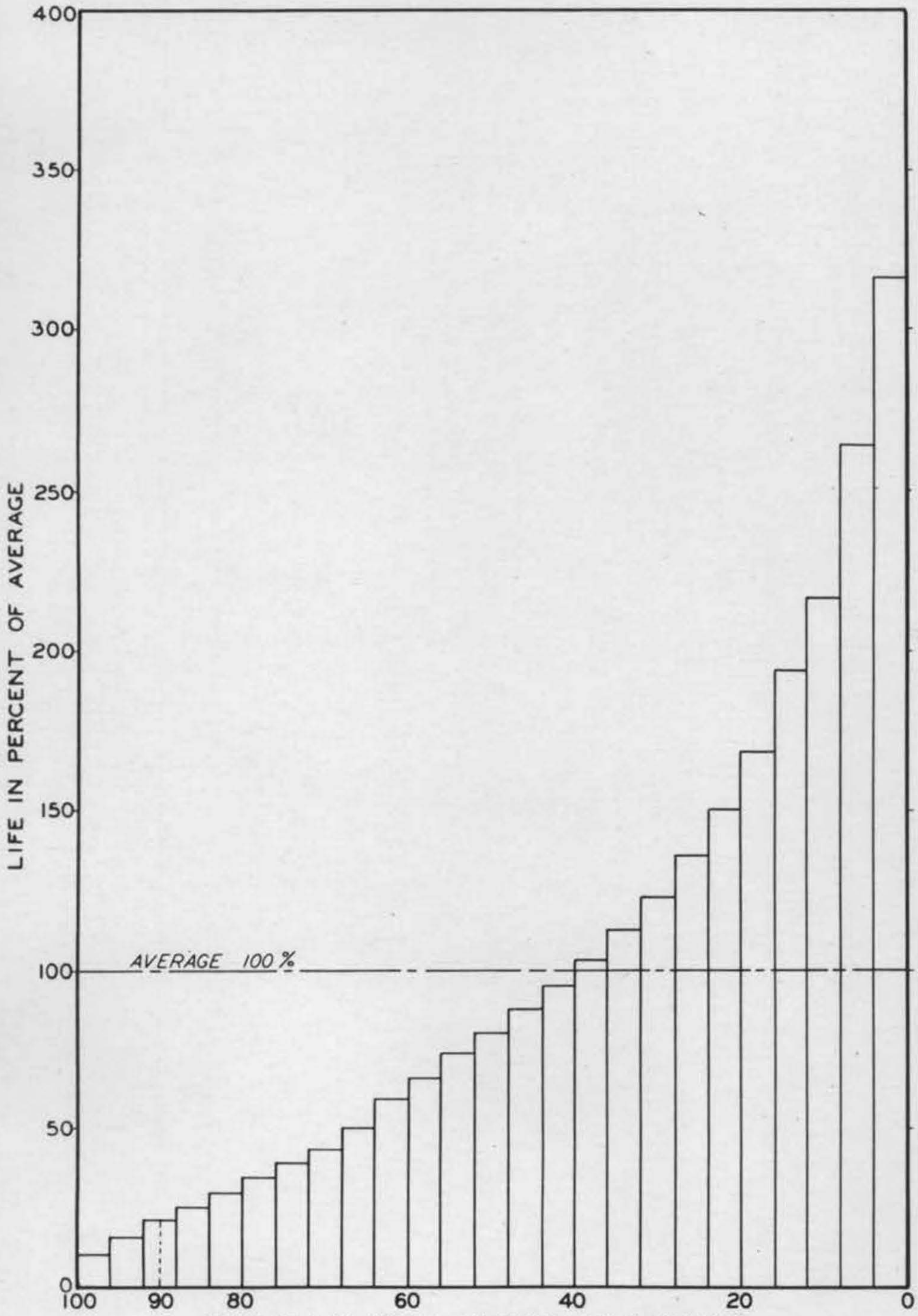

Fig.8 Typical Life Dispersion Curve.

average value. Therefore, average life should not be used for bearing selection. It is better to take into account the probability of early failures. From a technical and economical standpoint it has been found that, as a basis for the selection of a bearing of adequate capacity, they should be selected so that 90 per cent of the bearings will exceed a certain life, or in other words, that there will not be more than 10 per cent failures. That life is located on the curve at about 1/5 of the value of the average life or at the 20 per cent point on the curve.

No definition of carrying capacity can be established unless some agreement has been reached as to which point on the dispersion curve has been chosen as the basis.

So far bearing life has been expressed in number of revolutions, but load carrying capacity may also be referred to number of hours at a given speed. Then, if the life is fixed at a certain number of hours the life curve may be plotted against different speeds. This is the usual method of expressing bearing ratings. Thus, if a manufacturer rates his bearings on the basis of 500 hours minimum life, with not over 10 per cent failures, the average life corresponding to the catalog rating will be 5 x 500 or 2500 hours. The basic rating will then be given for a certain rpm, say 500, and the load which 90 per cent of all the bearings will carry during 500 x 60 x 500 = 15,000,000 revolutions. If the speed is higher than 500 rpm the total number of revolutions during 500 hours will be greater and the rating lower, since the corresponding point on the life curve has moved farther down. The opposite would be true for speeds lower than 500 rpm.

Thus it can be seen that if a fair comparison of bearing ratings is to be made consideration must be made of the point chosen on the life curve, and what average life expectancy is used as a basis. This information is usually given by the bearing manufacturers.

4. CORRECTING TO A STANDARD BASE. (1) When it is desired to compare two catalog bearing ratings using different life bases the figures of one must be corrected before a comparison is made with the other. The equation has been given, that: hours of life is inversely proportional to the bearing load raised to the ten-thirds power. If a curve is constructed having the equation $YX^{10/3}$, and when X = unity loading assign such a value to Y as will correspond to the standard number of hours used as a basis of bearing ratings by a particular manufacturer, a curve will be had which may be used to correct to any base and also to determine the probable life of any bearing of this make for which the load has been calculated in a given machine.

In Figure 9 is shown a series of curves that may be

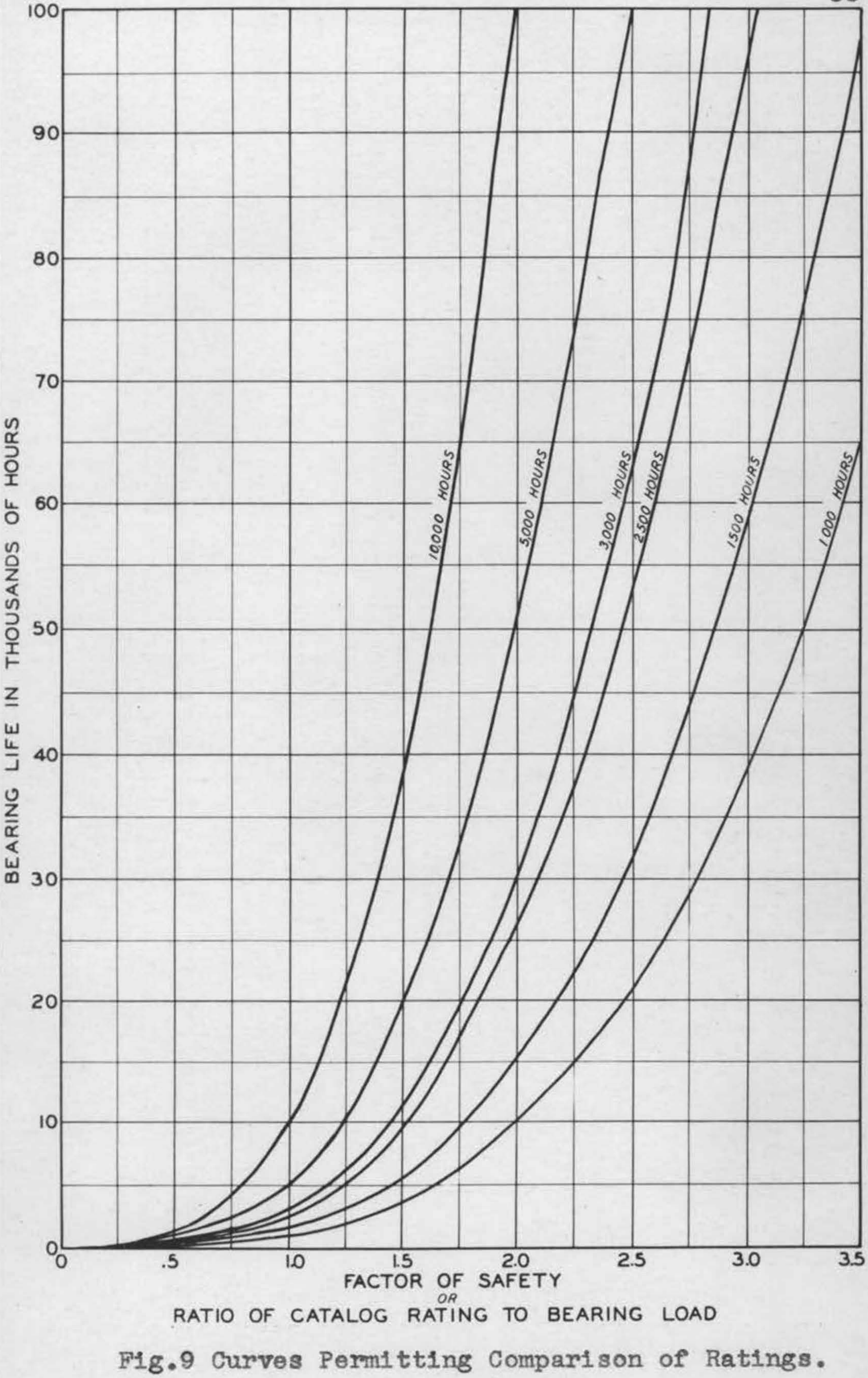

Fig.9 Curves Permitting Comparison of Ratings.

used in correcting to another base or calculating the probable bearing life according to bases in common use by manufacturers, namely 10,000, 5,000, 3500, 3000, 2500, and 1000 hours of average life. The ordinate of these curves represent the bearing life in hours and the abscissa represents factor of safety, in other words, if the catalog rating is based on 5000 hours average life the curve at 5000 hours of life will cross the abscissa at a factor of safety of one. This axis might also be called the ratio of catalog rating to bearing load.

From these curves it is seen by reading horizontally that the rating at 2500 hours is 1.25 times the rating at 5000 hours, or reading in the opposite direction, that the 5000 hour rating is equal to .80 times the 2500 hour rating.

The curves of Figure 9 may also be used in designing a machine that requires bearings made by different manufacturers using different basic load-life ratings. Let us assume, for an example, that a machine requires three types of bearings, #1, #2, and #3 which will fulfill the service requirements for three different parts of the machine. These bearings, being manufactured by different concerns, have different basic ratings, namely, 5000, 3000, and 1500 hours.

The design life of the machine is, say, 20,000 hours. A horizontal line through 20,000 hours intercepts the curves under consideration so that bearing #1 will require a factor of safety of 1.5, #2 a factor of safety of 1.75, and #3 a factor of safety of 2.2.

Therefore, to select the correct size of bearing from each of the three catalogs for #1, #2, and #3 so as to give them all an average service life of 20,000 hours, the load that has been calculated for each bearing must be multiplied by the corresponding factor of safety and then a selection is made of the bearing of the nearest size to correspond with this product.

By using the foregoing method the results will be as nearly correct as it is possible to calculate, and on a comparative basis the designer is at least dealing equitably with the competitive phases of the problem.

5. MODIFICATION FACTORS (12). The operating conditions for establishing the basic ratings must be more or less ideal. Loads are uniformly and constantly applied, the speed is constant, and the lubrication is controlled to standard test conditions. These operating conditions are necessary if the results are to show any degree of uniformity in the laboratory on which the life expectancy and load rating is based. These conditions are not met in actual practice, therefore, certain modification factors are applied to the basic catalog ratings to take care of the type of application, shock or steady load, continuous

or interrupted running, speeds different from basic speed, etc. These factors will not be discussed here, since bearing catalogs take this matter up in a very clear and understandable manner, and they do not enter into the basic ratings in any manner.

6. MODERN METHODS USED IN RATING DETERMINATION. (12) After the bearing manufacturer has decided on the life value which will be used in arriving at carrying capacity, it is necessary to determine the magnitude of the carrying capacity. As was said before, to determine the life of a single bearing under one given load. It is necessary to run, until fatigue failure occurs, several groups of bearings, using at least 30 bearings in each group so as to permit the law of averages to become adequately effective. Before a life formula can be written it is necessary to run groups under various loads and use bearings of different sizes. If this procedure were carried out to the fullest extent, a very large amount of equipment and time would be necessary. It is possible to use experimental data from one group of tests and apply it to other bearings of similar design, construction, and material, and arrive at a fairly close estimate of their life-load characteristics. However, a life formula can be applied only to a certain definite type manufactured from certain definite materials and with a certain definite degree of accuracy. This formula will

be applicable only when the finite conditions exist.

PART V

METHODS OF CALCULATING BEARING CAPACITY

PART V

METHODS OF CALCULATING BEARING CAPACITY

A. ROLLER BEARINGS

1. THE ROLLWAY BEARING COMPANY METHOD (12). From results of overload tests in the laboratory and in actual service, the Rollway Bearing Company has found the following formula to be very accurate in determining the capacity of a roller bearing that can be safely anticipated:

$$\text{Capacity at required rpm} = \frac{30{,}000 \times d \times L \times (n-3)}{\sqrt[3]{\text{rpm} \times (n+3)}}$$

where d is the diameter of the rollers in inches

L is the length of the rollers in inches

n is the number of rollers in the bearing.

30,000 is a constant derived from tests for rollers that are about the same length as the diameter. 22,500 is used as the constant when the length is twice the diameter or more.

From the results of laboratory tests, and records of bearings in service, it has been found that bearings loaded to the capacity, as found from the above formula, have an average life expectancy of 10,000 hours.

2. THE NAVY DEPARTMENT, which uses a large number of anti-friction bearings uses the following methods:
All roller bearings shall be figured for working capacity

by the use of the formula---

$$P = K D L N$$

For ball bearings the formula shall be

$$P = \frac{\pi D^2 KN}{16}$$

where

P = capacity in pounds

K = factor

D = diameter of roller or ball

L = Length of roller

N = number of rollers or balls

The factor "K" for flat thrust bearings is 7000 for a life of 3000 hours, diameters of rollers or balls 3 inches or more, and speeds of 5 rpm or less.

The total capacity of the bearing for other hours of life shall be approximately--

$$P_1 = \frac{P}{\sqrt[3.3]{\frac{\text{proposed life in hours}}{3000}}}$$

where

P = capacity rating on basis of 3000 hours of work

and

P_1 = new capacity rating

K shall be taken as 10,000 for rollers or balls of 1/2 inch diameter, or less, and shall vary uniformly, according to the diameter of the roller or ball, to the value of 7000 as given above for rollers or balls 3 inches in diameter. For balls or rollers more than 3 inches in diameter the value of K shall be 7,000.

For speeds in excess of 5 rpm reductions shall be made in capacities of bearings, as determined above, by the use of the following coefficients:

rpm	per cent
50	70
100	60
250	44
500	35
1000	28
2000	23

For radial bearings the distribution shall be taken as 36 degrees, or on 1/5 of the rollers or balls in the lower half of the bearing.

No corrections need be made to the above formulae for tapered roller bearings on account of the taper, as the usual coefficient is approximately 0.97.

The above information is dated April 10, 1935.

3. MARK'S MECHANICAL ENGINEERS' HANDBOOK, Third Edition (6) gives the formula:

$P = kld^2/(ND + 2000d)$, in which N = rpm of the shaft; D = diameter of the sleeve or roller path, in.; and k = 1,200,000 to 2,000,000 for first-class workmanship, hardened steel rollers with $l = d$, running on hardened ground surfaces; k = 400,000 for ordinary workmanship and soft steel rollers running on a soft steel shaft.

4. THE STANDARD ROLLER BEARING CO. (6) determines load capacities of roller bearings by the formula $P = 130{,}000\ d^2nl/3s$, in which P = load on bearing, lb; d = diameter of rollers, in; n = number of rollers; l = length of each roller, in; and s = circumferential speed of each roller, ft per min. In bearings with conical rollers d is the diameter of the roller at its mid-length. The safe load per inch of length of a solid roller is taken at 2,000 lb, with the assumption that one-third the number of rollers take the whole load on the journal.

5. TREVE'S METHOD (11) is another means of determining the load which may be placed on a roller bearing. This method was described in detail and coefficients given in Part III.

6. Part III also gives MUNDT'S METHOD (8) of calculating specific wear in a roller bearing. If the material and bearing constant C is determined experimentally the load rating of a bearing may be determined by this method also.

B. BALL BEARINGS.

1. STRIBECK'S FORMULA (3) for determining the load capacity of a ball bearing was given in detail in Part II. This is recalled to be: $P = k\ N\ D^2/5$; where k was determined to be 10 Kg.; D was the ball diameter in 1/8 in. units;

and N the number of balls in the bearing.

2. PROFESSOR GOODMAN'S FORMULA (5) was given in Part III as: $P = k\,m\,d^3/(nD + cd)$. The constants k and c are given in Part III also and are seen to vary depending on the type of ball race and material used.

3. PALMGREN'S ANALYSIS (5) is used frequently in determining load capacities of ball bearings. The methods of calculation and application are given in Part III and an example for a radial single row bearing is given in Part VI.

PART VI

COMPARATIVE CALCULATIONS

PART VI

COMPARATIVE CALCULATIONS

In the preceding section a number of methods of calculating the loading capacity of roller and ball bearings were given. In order to see how the results of these various methods compare, calculations were made on a number of bearings. The results of these calculations are given in Table I for roller bearings and Table II for ball bearings.

These bearings were selected so as to be as nearly alike as possible and, therefore, bearings of approximately the same bore and outside diameter were chosen. These bearings were also selected, with the purpose in mind, of making a comparison of their catalog ratings.

Since the ratings of various manufacturers are based on different hours of average life all the calculations on the roller bearings were changed to an average life basis of 10,000 hours at 100 rpm.

Although the bearing bore and outside diameter are approximately the same in all the bearings listed it is noted that their widths, in the case of the roller bearings, differ considerably. Therefore, to compare the ratings on an equitable basis the ratings have been reduced to pounds per inch length of roller.

The methods of making the calculations and examples

Table 1

Comparative Calculations of Roller Bearings.

Bearing Make		S.K.F.	Norma Hoffman	Norma Hoffman	Norma Hoffman	Norma Hoffman	Hyatt	Hyatt	Hyatt	Bantam	Bantam	Bantam	Bantam	Bantam	Bantam	Bantam	Bantam	Bantam	Timken
Bearing Type		Two Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row	Single Row Taper	Single Row Taper
Bearing Number		22340	RLS-26	RMS-26	RLS26LL	RMS26LL	S-240	SW-240	AS240TS	ALF140	BLF140	ALF240	BLF240	ALF340	BLF340	HBCS	EBCS		
Bore	Inches	7.874	8.00	8.00	8.00	8.00	7.874	7.874	7,874	7.874	7.874	7,874	7.874	7.874	7.874	8.00	8.00	7.873	7.875
	mm	200	203	203	203	203	200	200	200	200	200	200	200	200	200	203	203	200	200
Outside Diameter	Inches	16.535	13.00	15.00	13.00	15.00	13.386	13.386	14.173	12.598	12.598	14.173	14.173	16.535	16.535	12.0	13.5	12.5	14.0
	mm	420	330	381	330	381	340	340	360	320	320	360	360	420	420	304	343	317	356
Number of Rollers		28	24	19	17	13	24	24	16	32	32	27	27	22	22	22	16	25	24
Contact Length of Rollers	Inches	1.90	0.875	1.375	0.875	1.375	3.60	5.67	3.53	0.875	1.875	1.094	2.344	1.5	3.25	3.937	3.937	1.25	1.75
	mm	48.5	22.2	34.9	22.2	34.9	91.5	144	89.6	22.2	47.6	27.8	59.5	38.0	82.5	100.0	100.0	31.8	44.5
Diameter of Rollers	Inches	2.362	1.0	1.5	1.0	1.5	1.375	1.375	1.50	1.00	1.00	1.25	1.25	1.75	1.75	1.00	1.375	1.00	1.125
	mm	60.0	25.4	38.0	25.4	38.0	34.9	34.9	38.0	25.4	25.4	31.8	31.8	44.5	44.5	25.4	34.9	25.4	28.6
Inner Race Diameter	Inches	10.197	9.578	10.04	9.578	10.04	9.245	9.245	9.535	9.222	9.222	9.773	9.773	10.45	10.45	9.0	9.5	9.187	10.25
	mm	259	246	255	246	255	235	235	242	234	234	248	248	265	265	229	241	233	260
Outer Race Diameter	Inches	14.921	11.578	13.04	11.578	13.04	12.00	12.00	12.54	11.222	11.222	12.273	12.273	13.954	13.954	11.0	12.25	11.187	12.5
	mm	379	294	332	294	332	304	304	320	285	285	312	312	354	354	279	311	284	318
Catalog Rating at 100 R.P.M., Lbs.		239,000	35,000	75,180	29,890	63,300	45,100	69,000	137,000	54,000	108,000	83,500	167,000	113,000	226,000	84,100	128,400	58,900	32,800
Basic average Life, hours		2500	10,000	10,000	10,000	10,000	5,000	5,000	5,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	15,000
Catalog Rating on 10,000 hour basis, Lbs.		160,000	35,000	75,180	29,890	63,300	36,600	56,000	110,000	37,000	75,000	58,000	116,000	78,600	157,000	58,600	87,400	41,000	37,000
Rating per inch length of roller, 10,000 hour basis, Lbs.		84,200	40,000	54,600	34,200	46,000	15,700	15,300	48,500	44,400	41,000	54,500	50,800	54,000	49,500	15,250	22,800	33,700	21,150
Rollway Formula. 10,000 hour basis, Lbs.		230,000	39,600	76,200	29,000	52,800	168,000	265,000	123,800	50,000	107,000	68,500	111,000	110,000	179,000	125,000	128,500	58,500	89,200
Navy Formula		605,000	82,700	143,800	58,200	98,400	545,000	853,000	384,000	158,000	339,000	201,000	430,000	294,000	637,000	488,000	456,000	176,000	160,000
$P = \frac{Klnd}{ND + 2000\ d}$			13,300	27,400	9,430	18,700				17,900		24,700		41,500				20,000	32,000
K of Mundt Formula		432,000	652,000	705,000	705,000	775,000	189,000	182,000	705,000	590,000	545,000	660,000	615,000	540,000	495,000	264,000	381,000	532,000	308,000
Treves Method		81,000	13,450	25,200	9,500	17,200	76,200	120,000	54,300	18,000	38,600	24,000	51,500	37,600	81,500	55,500	56,000	19,500	30,200
$P = \frac{130{,}000\ nld}{3\ 3}$		256,000	18,200	51,000	12,900	34,900	141,500	222,000	110,000	24,200	52,000	40,000	85,500	87,600	190,000	75,000	103,200	27,100	46,000

TABLE II

COMPARATIVE CALCULATIONS ON BALL BEARINGS

Bearing Make		M.R.C.	N.D.	Fafnir	Federal
Bearing Number		322M	7322	322-W	1322M
Bearing Type		Radial Fill. slot	Radial Fill. slot	Radial No slot	Radial No slot
Bore	Inches	4.33	4.33	4.33	4.33
	mm	110	110	110	110
Outside Diameter	Inches	9.448	9.448	9.448	9.448
	mm	240	240	240	240
Inner Race Radius	Inches	2.663	2.72		
	mm	67.75	69.10		
Outer Race Radius	Inches	4.226	4.220		
	mm	107.2	107.0		
Ball Diameter	Inches	1-9/16	1-$\frac{1}{2}$	1-5/8	1-$\frac{1}{2}$
	mm	39.8	38.1	41.25	38.1
Number of Balls		12	12	12	12
Catalog Rating (radial) at 100 rpm		22,100	19,630	32,470	19,520
Life Basis, avg.hours		3,000	Not stated	3,500	Not stated
Stribeck formula		8,250	7,600	8,950	7,600
Goodman Method (100 rpm)		32,300	29,600	35,000	29,600
Palmgren		8,040	7,700	10,200	10,230
for avg. life, hours		3,000	3,500	3,000	3,000
Navy Method at 100 rpm		30,200	28,000	32,300	28,000
Treve's Method (k = 17)		14,100	13,100	15,300	13,100

of the application of each method of load capacity determination are given in detail under Part VII Discussion of Calculations.

PART VII

DISCUSSION OF CALCULATIONS

PART VII

DISCUSSION OF CALCULATIONS

Comparative calculations have been made on a number of roller bearings of approximately the same bore and outside diameter. The physical dimensions of these bearings were obtained directly from the manufacturers.

The catalog ratings of these bearings are given in Table I for 100 rpm and are taken directly from the manufacturer's catalog or, in cases where the rating was not given for 100 rpm, the rating for this speed was calculated from the rating at the speed given. The bearing catalogs usually give the rating at some basic speed and if the rating at some other speed is desired the rating at the specified speed must be multiplied or divided by a speed factor, provided the same life is desired. For example: The S.K.F. bearing No. 22340 is rated at 140,000 pounds radial load at 500 rpm. At 100 rpm the rating would be: $140{,}000 \times (500/100)^{.3} = 239{,}000$ pounds. In other words, the lower the speed the higher the rating, or the ratio of the load rating will be as the speed ratio raised to the ten-thirds power.

The basis for the catalog ratings in terms of average life in hours is also given. In order to make these ratings comparable they have all been placed on a 10,000 hour average life basis. This was done with the use of

Figure 9. For example: The Bantam bearing No. ALF140 is rated at 54,000 pounds at 100 rpm on a basis of 3000 hours average life. From Figure 9 the factor of safety for the 10,000 hour life curve is, of course, 1, and for the 3,000 hour curve the factor of safety is 1.4. Then 54,000/1.4 = 37,800 pounds which is the radial capacity of this bearing on a 10,000 hour life basis.

It will be seen that a wide divergence exists in the ratings even on this basis. Perhaps this is because the bearings are not of the same length. To eliminate this factor the ratings on a 10,000 hour life basis have been reduced to the rating per inch length of roller. From an inspection of the figures given it will be seen that there is a great difference even on this basis. Especially is this true of the Hyatt S-240 and SW-240 bearings which are of the wound helix type, and consequently, would differ materially from the other bearings which are of the solid roller type.

Thus, it is seen from the above calculations that there is a wide divergence in the manufacturers' ratings of roller bearings of approximately the same bore and outside diameter. In the manufacture of these bearings the same material (2) is not used by all manufacturers and, consequently, accounts for some of the difference in the ratings. However, even if the same was used by all manufacturers the divergence in the ratings would still be great because of the quality of machining and grinding operations performed. A highly finished bearing can be rated considerably higher for the same life than a poorly finished one. Therefore, in considering the rating of a bearing account must be taken of the accuracy and quality of workmanship that goes into that bearing.

A number of formulae were given under Part V, "Methods of calculating bearing capacity". These have been applied to the bearings given in Table I in order to see how the results compare with the load ratings of the manufacturers.

The Rollway Bearing Co. formula is based on overload laboratory tests and on bearings in actual service. The Rollway Co. uses this formula for calculating the ratings on their bearings and state that it has been found very accurate in determining the capacity that can be safely anticipated. It is based on a 10,000 hour life basis. The results of using this formula on various bearings are shown in the table. In some cases it compares very favorably with the manufacturers' rating while in others it varies 300 or 400 per cent. As an example let us take the Norma-Hoffmann bearing No. RLS-26 where n is 24, d is 1.00 inch, and L is 0.875 inches.

$$\text{Capacity at required rpm} = \frac{30{,}000 \times d \times L \times (n-3)}{\sqrt[3]{\text{rpm} \times (n+3)}} \text{ lb}$$

$$\text{Capacity at 100 rpm} = \frac{30{,}000 \times 1.00 \times 0.875 \times (24-3)}{\sqrt[3]{100 \times (24+3)}}$$

$$= \frac{30{,}000 \times 1.00 \times 0.875 \times 21}{\sqrt[3]{2700}}$$

$$= 39{,}600 \text{ lb}$$

The Navy uses the formula P = K D L N for calculating the capacity of roller bearings for a life of 3,000 hours. The constant K seems to be entirely empirical. The value of K varies uniformly from 10,000 for rollers of ½ inch diameter to 7,000 for rollers of 3 inch diameter. If we take, for example, the Norma-Hoffmann bearing No. RLS-26 and calculate the capacity by this method it will be: D is 1.00 inch, L is 0.875 inch, N is 24, and K is 9400. For 100 rpm the capacity by the above formula must be multiplied by 0.60.

P = K D L N x .60 = 9400 x 1.00 x 0.875 x 24 x .60 = 118,500 lb. For 10,000 hours life

$$P_1 = \frac{118{,}500}{\sqrt[3.3]{10{,}000/3{,}000}} = 82{,}700 \text{ lb}$$

This value is over 100 per cent more than the manufacturers' rating at 10,000 hours life.

The results in Table I were obtained by using the manufacturers' rating in the formula. By noting the results obtained by this method of calculating capacity, it can be seen that they are not reasonable.

The formula $P = (klnd^2)/(ND + 2000d)$, which is obtained from Mark's Mechanical Engineers' Handbook, is to be used where the roller length l is approximately equal to the diameter d, in which case K = 2,000,000. D is the outer race diameter. Only a few of the bearings listed have a roller length approximately equal to the diameter. Therefore, this formula is applicable on only a few of the bearings listed. The results in the few cases where it was used are very low compared to the manufacturers' ratings.

In the Mundt formula $Q = \frac{C \times 10^{1.8} Z_w d L}{(60 c n)^{.3} B^{.3}}$ Kg. where C is a bearing and material constant, B is the life in work hours, Z_w is the number of rollers, d is the diameter in mm, L the length in mm, c the number of cycles per revolution, and n the rpm of the shaft. Mundt calls $C/B^{.3}$ the specific wear, K. Then $\frac{C}{B^{.3}} = K = \frac{Q (60 c n)^{.3}}{10^{1.8} Z_w d L}$

c is given as $.17 Z_w d_a/(d_a + d_i)$ for rotating inner ring, as explained previously, then

$$K = \frac{Q \left(60 \times .17 Z_w \frac{d_a}{d_a + d_i} \times 100\right)^{.3}}{10^{1.8} Z_w d L}$$

In this formula Q is in Kg. and d and L are in mm. Changing Q to pounds and d and L to inches and simplifying we get:

$$K = \frac{180 Q (d_a/d_a + d_i)^{.3}}{Z_w^{.7} d L}$$

Since the ratings of the bearings listed in Table I are all given for 10,000 hours life and the dimensions are also given we can calculate K for all the bearings and make a comparison. It is seen from the results that there is a great difference in the values obtained between the bearings on this basis also.

Treves' method, which was also given previously, is used in the calculations. Since the calculations are for 100 rpm and the bearing dimensions are nearly the same, the value of k is practically the same for all bearings listed and is taken as 15. The constants used in the formula are based on the actual experience of some foreign manufacturers, but it is seen that the results do not agree at all with the ratings of the manufacturers of the bearings listed.

The Standard Roller Bearing formula given in Mark's Handbook, $P = (130,000\ n\ l\ d^2)/3\ S$ is also used in the calculations. According to "Elements of Machine Design" by Hoffman and Scipio (4), in their discussion of this formula take S, which is the circumferential speed of the rollers about their centers in feet per minute, as 50 when it figures 50 or less. Since this is the case with all the bearings listed in the table the value of 50 is used. The results are seen to vary greatly from the catalog ratings when using this formula.

In the comparative calculations of ball bearings of the radial type given in Table II it is rather difficult to make a comprehensive comparison since the life basis is not given in many of the catalogs. The bearings listed are given in catalogs as interchangeable bearings; however, it is seen that there is quite a wide range of capacity although the physical sizes are practically the same.

The application of the Stribeck formula $P = KND^2/5$ is seen to give very low results for the bearings listed. It does not take into account the rpm of the shaft and the same constant is used for all types of bearings. The application of this formula for the MRC No. 322M bearing is as follows: Diameter of the balls = 1-9/16 inch, or 12.5 one-eighth inch units; the number of balls N is 12; and the constant, K, is 10 Kg.

$$P = KND^2/5 = 10 \times 12 \times 12.5^2/5 = 3750 \text{ Kg.}$$

or $$3750 \times 2.2 = 8,250 \text{ lb}$$

The Goodman formula $P = k\ m\ d^3/(nD + cd)$, where $k = 2,500,000$ for hollow races of best quality, and $c = 2,000$ is also used to calculate the load which may be applied on these bearings. This formula takes into account the speed of the shaft and also the material used. The results are higher than the manufacturers' ratings. As an example, this formula is used on the MRC No. 322M bearing. The number of balls $m = 12$, $d = 1\text{-}9/16$ inches, $n = 100$ rpm, $D = 4.22$ inches and k and c as given above.

$$P = \frac{2,500,000 \times 12 \times 1.5625^3}{100 \times 4.22 + 2,000 \times 1.5626} = 32,300 \text{ lb}$$

The Palmgren formula for the average hours of life stated in Table II gives results about 70 per cent lower than the manufacturers' ratings. Of course, this would be conservative for calculating permissable loads on a bearing but very uneconomical. The method was explained in detail in Part III and its application to the New Departure bearing No. 7322 for 3500 hours life is given as an example. The number of stresses, a, per revolution = $N/(1 + r_i/r_y)$ where N is the number of balls and r_i and r_y the radius of the inner and outer race respectively.

$$a = \frac{12}{1 + 2.66/4.22} = 7.36 \text{ stresses per revolution.}$$

100 rpm = 6,000 rph and with 3500 hours life = 21 million stresses. a.n. = 7.36 x 21 = 154.56 million stresses. Since the bearing has a filling slot we will use

$$K_i = \frac{198}{\sqrt[3]{a.n. + 5}} + 6.6 \text{ lb}$$

$$K_i = \frac{198}{\sqrt[3]{154.56 + 5}} + 6.6 = 43.3$$

$$K_i = K\,(1 + .0001\ V)(1 + .007D^2)$$

$$= K\,(1 + .0001 \times 100)(1 + .007 \times 12^2) = K\ 2.028$$

$$K = K_i/2.028 = 43.3/2.028 = 21.35$$

$$W = 21.35 \times 12 \times 12^2/5 = 7.700 \text{ lb}$$

It is seen that the Navy method gives values much too high for three bearings while in the fourth it is equal to the manufacturer's rating. The bearing material is taken into account in the constant and the formula also considers the speed of the shaft. This formula applied to the New Departure bearing No. 7322 is as follows:

$P = \frac{\pi}{16} \times d^2 N K \times .6$ (for 100 rpm)

$d = 1.5$ in, $N = 12$, $K = 8800$

$P = 3.1416/16 \times 1.5^2 \times 12 \times 8800 \times .6 = 28{,}000$ lb

Treve's Method gives values that are quite low for all the bearings considered. Its application for the New Departure bearing No. 7322 is given as an example:

$P = .02\ K\ N\ D^2$ Kg.

$P = .02\ K\ N\ D^2 \times 2.2$ lb

K is a constant depending on the circumferential speed, V, of the balls about their centers.

$V = 0.0525 \times 100 \times 69{,}10 = 366$ mm or .366 meters

For $V = .366$ $K = 17$ approximately.

$P = .02 \times 17 \times 12 \times 38.1^2 \times 2.2 = 13{,}100$ lb

It is seen from the above calculations for roller and ball bearings that none of the general formulae available at present seem to give satisfactory results. Most of these methods take into account bearing material and also speed, which, of course, must be done. However, none of them take into account the type of bearing and bearing finish or workmanship in any satisfactory manner.

Ball bearings are mostly all made of the same material

(2)(12), namely, S.A.E. No. 52100 steel, which has a carbon range of .95 - 1.10, manganese .20 - .50, and chromium 1.20 - 1.50, and with a definite heat treatment to give it an average Rockwell "C" hardness of 60. However, the bearings are of many types, for instance light, medium, and heavy; and their finishes vary from pressed rings to those of a very finely ground finish.

Roller bearings, on the other hand, are not made of the same material (2) by all manufacturers and consequently, from that standpoint alone will not have the same rating. Also, their types and finish vary greatly, the same as in ball bearings.

In view of the above calculations, and also the diagrams and discussion of the influence of bearing design and materials which follows in Part III, it is seen that it is impossible to prepare a formula that may be used for calculating bearing capacity for bearings of different material and manufacture.

In support of the above statement the following is offered: A series of tests were run recently by the Navy to establish an acceptable list of ball bearing manufacturers complying with their specifications, in respect to design and construction and to establish comparative standards of performance from their relative endurance in special testing machines.

Due to the magnitude of an investigation including several sizes of various types it was necessary to limit the tests to one size bearing in each of the types submitted by the several exhibitors. For this purpose, medium duty bearings having bores of 65 mm and outside diameters of 140 mm were arbitrarily selected. The bearings submitted for test were submitted with the understanding that they were representative of the various exhibitor's products and not special or selected bearings.

Eight manufacturers submitted 25 bearings in one or more types.

The tests were divided into two parts or series; a series of tests which consisted of a determination of dimensions and physical properties of balls and races in conformance with Navy specifications and a series of tests to determine the relative endurance of the bearings in special testing machines. Five bearings of each type were used for determining dimensions and physical properties of the several parts, and the remaining 20 bearings were used for endurance testing.

Measurements were made on the five bearings with Johansson gage blocks and a Zeiss Optimeter.

The remaining 20 were used in the endurance tests. Breakdown time was decided upon as the test criterion.

The following test conditions were used for the different bearings:

Single row No. 313 bearings and	:	2000 rpm
Double row No. 1313 bearings	:	5000 lb radial load
Double row No. 5313 bearings	:	2000 rpm 4000 lb thrust load; 2000 lb radial load.

Four bearings of the same make and type were mounted at a time and run continuously on an average of about 130 hours per week with idle periods over week-ends until failure occurred or until bearings had run 1000 hours. Attendants held the speed at 2000 rpm and ingoing oil temperature at 90°F.

The results of the test showed: that in general the data taken in the measurement tests showed close conformity to the specification requirements. The endurance tests showed that, under the carefully controlled identical test conditions, a rather wide range of average hours of running for the various makes of bearings resulted in variations from 272 to 846 in the case of the single row No. 313 bearings and from 247 to 810 in the case of double row No. 5313 bearings. The only double row No. 1313 bearing showed an average running of 742 hours. Weighted figures based on the actual number of bearings of all makes on which tests were completed, gave an average running time of 574 hours for the No. 313 single row bearings and 595 hours for the No. 5313 double row bearings.

A general consideration of the results of all tests indicated that variations from the specifications, "either in physical characteristics or dimensions of the bearings are insufficient to account for the variations in relative endurance of the several makes of bearings. The variations in the performance of the several makes of bearings is undoubtedly due to variations in materials and methods of manufacture."

"Further analysis of the general results indicates that there appears to be no formula applicable for predicting the probable life of bearings produced by a number of manufacturers. On the other hand, since only one size of a given type of bearing manufactured by each exhibitor was tested, there is insufficient information available to determine the relation between the probable life of different sizes of bearings of the same type and make."

PART VIII

INFLUENCE OF BEARING DESIGN AND MATERIALS ON LOAD CAPACITY

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To give some idea of the progress made in the improvement in the design and manufacture of bearings manufactured by one concern (12) and the resulting increase in capacity, Figure 10 is shown. This curve shows the relative capacity of ball bearings based on 1927 ratings. Concerns making improvements in carrying capacity must likewise change their formulae. Consequently, the preparation of a formula must take into account the factors previously mentioned, but it must be adjusted as improvements are made in materials and workmanship.

The manner in which materials affect the load rating of (12) bearings is shown in Figures 11 and 12. These diagrams show dispersion curves with the number of bearings plotted as abscissa, and bearing life plotted as ordinate. The load and speed to which each group of bearings was subjected was the same, that is, Figure 11a, 11b, and 11c were all subjected to the same operation, while the bearings in Figure 12a, 12b, and 12c were also subjected to the same operation. Figure 11a and Figure 12a show the dispersion curve which resulted from the use of ore and production customarily used in making one certain make of ball and roller bearings. Figure 11b and 11c and Figure

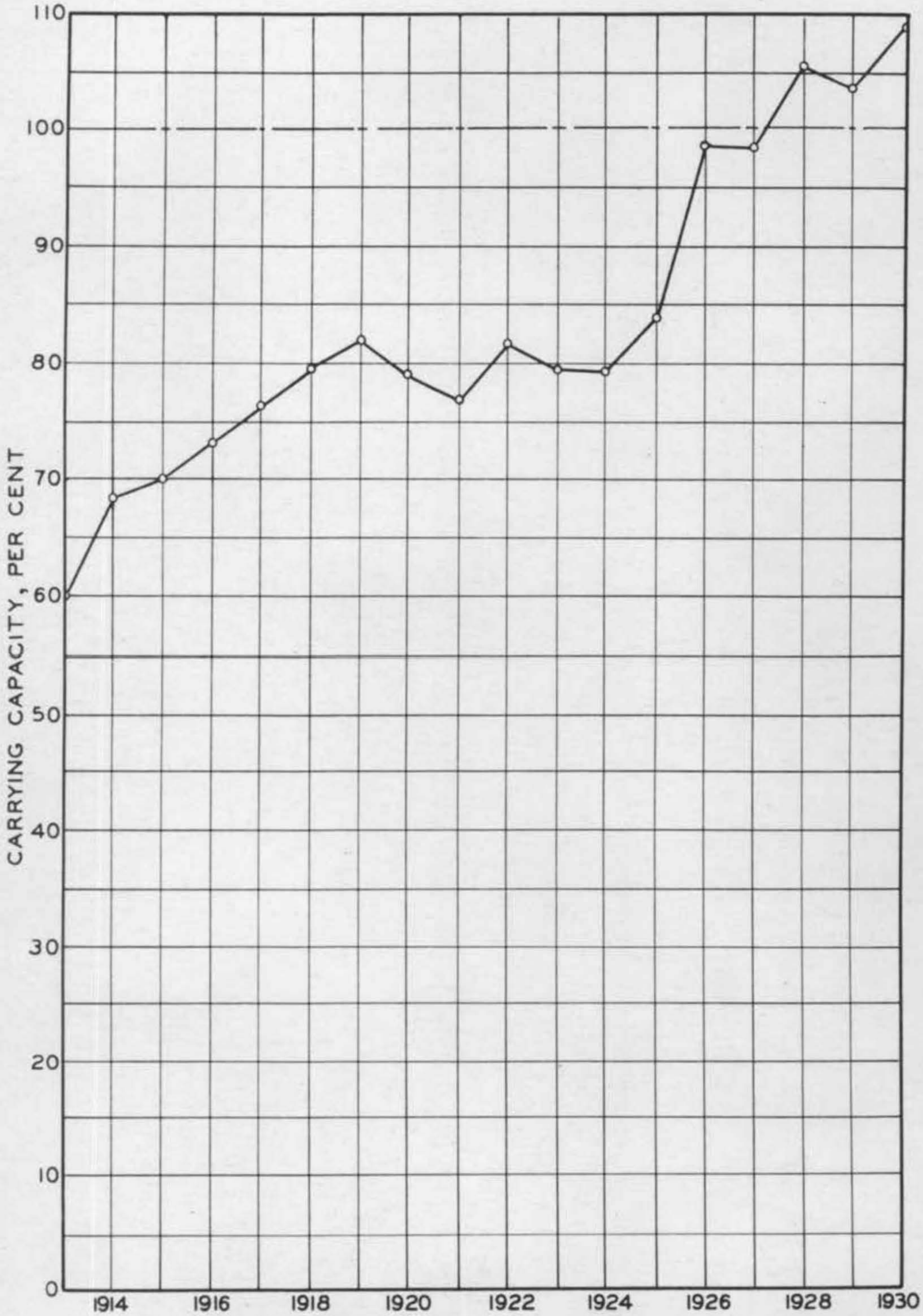

Fig.10 Increase in Bearing Capacity Due to Improvements in Manufacturing and Quality.

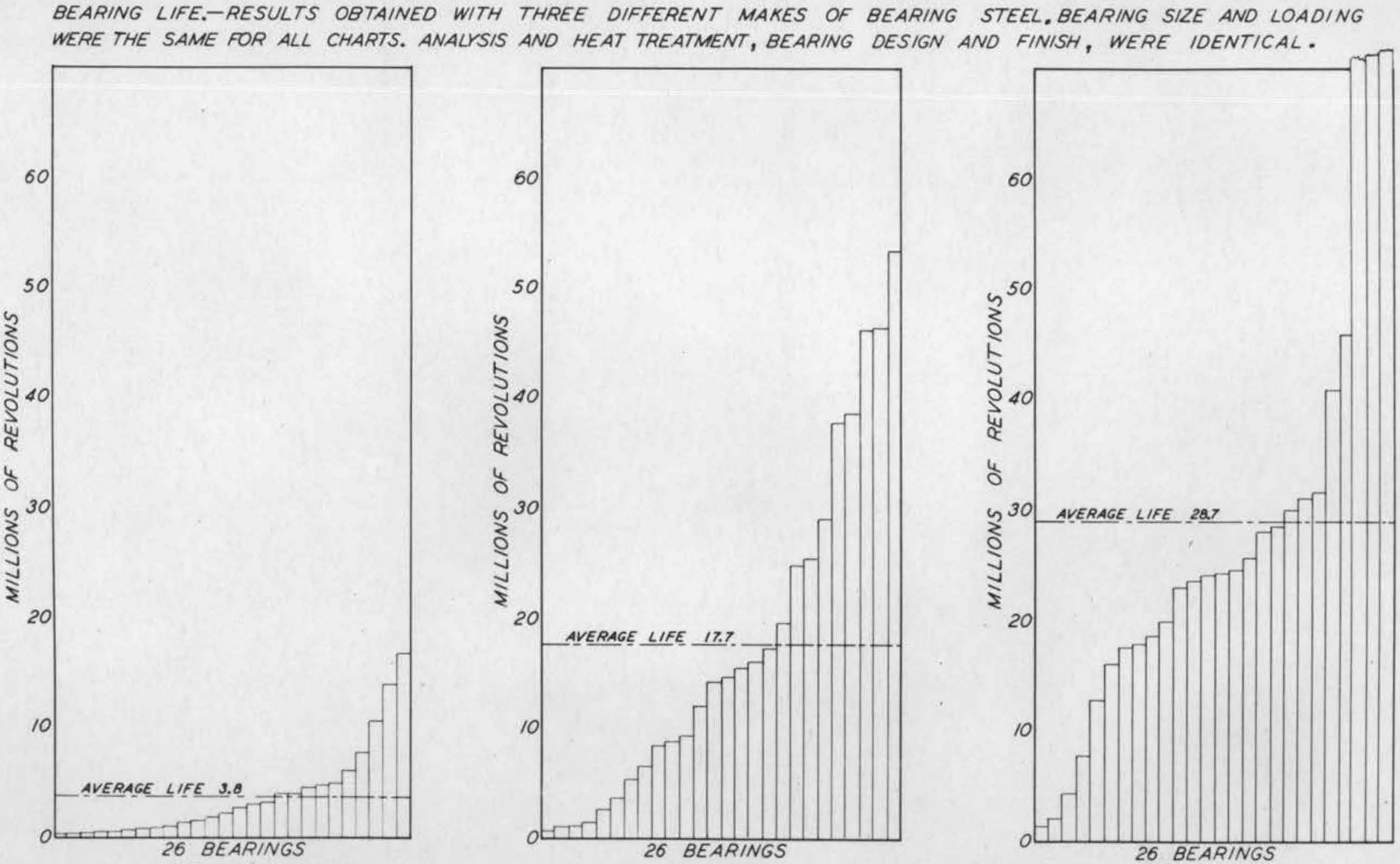

Fig.11a Fig.11b Fig.11c

Fig.11 Life Dispersion Curves Obtained Using Three Different Makes of Bearing Steel.

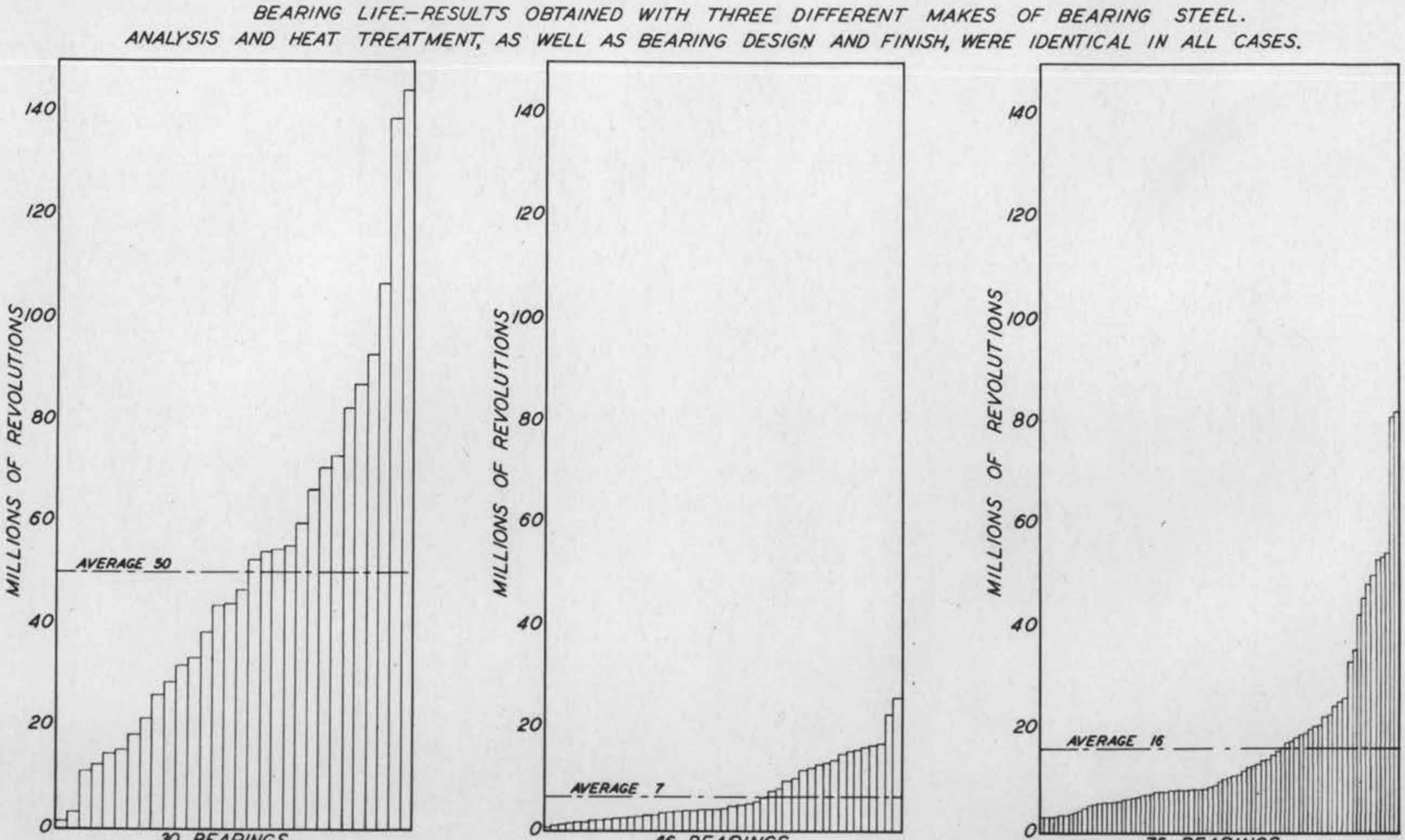

Fig.12a Fig.12b Fig.12c

Fig.12 Life Dispersion Curves Obtained Using Three Different Makes of Bearing Steel.

12b and 12c show the results of making bearings from different ores, although the chemical analysis and method of treating and finishing were identical in all cases.

It is rather difficult to explain the reason for such a wide variation in these dispersion curves. The reason may be partly explained as follows:[1] If we study Figure 9 it will be seen that with a variation of bearing load from 0.5 "factor of safety" to 2.45, which is 490 per cent, the life will vary from about 500 to 100,000 hours, which is 20,000 per cent. Therefore, in making overload tests in the laboratory, it does not take a very great variation in the imposed load to make a very great difference in life and it is quite likely that there would be a considerable variation in performance among a group of practically identical test bearings that are run as nearly as possible under the same set of conditions.

The great variation in the curves of Figure 8, Figure 11, and Figure 12 is most likely due to chemically indetectable particles of slag or oxidized iron, and of internal or microscopically invisible external cracks in the material used.[7] These irregularities cause high localized stresses due to the repeated loading. High localized stresses cause cracks to start. These cracks extend the discontinuity of the material, and at the roots the localized stresses are still further intensified. Not every localized stress develops a crack to failure of the part, but every high localized stress is a potential source of progressive failure. Therefore, the greatest homogeneity of internal structure and smoothness of surface are very important in bearing materials.

Nevertheless, it is seen in these diagrams, the great influence material has upon bearing life or capacity.

A clear understanding of how the improvements in bearing material and workmanship, and of how bearing material affects life and carrying capacity will show that it is impossible to apply a given formula to bearings made of different materials and by different manufacturers.

PART IX

CONCLUSION AND SUMMARY

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The wear in an anti-friction bearing is due, in general, to the fatigue of the material; for the calculation of the radial load rating, therefore, the process of fatigue must be the basis. Due to the knowledge of the reaction of the material to stress, and to the experience acquired by practical experiments, bearing manufacturers have developed formulae which represent a relation between load rating and life. However, a life formula can be applied only to a certain definite type manufactured from certain definite materials and with a definite degree of accuracy; therefore, no formula can be prepared which can be universally applied to bearings of different material and of different manufacture. If a bearing load rating is stated by a manufacturer it is meaningless, unless the life on which that rating is based is also stated.

The life of anti-friction bearings shows great variation; the experiments on a great number of bearings show that the dispersion or spread of life takes place according to a certain law. Therefore, if a load rating with a certain life is stated, it is furthermore necessary to state what is understood by that life.

PART X

APPENDIX

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BIBLIOGRAPHY

1. Haynes, J. L. "How to Compare Load Ratings of Antifriction Bearings." Machine Design, Vol. 8, No. 8, July 1936 p. 36-38.

2. Haynes, J. L. Private Communication to Author, December 4, 1936.

3. Hess, Henry "Ball Bearings." Trans. A.S.M.E., Vol. 29, 1907, p. 420-467.

4. Hoffman and Scipio. Elements of Machine Design, Ginn and Company, Boston, Mass.

5. Macaulay, A. W. Ball and Roller Bearings, Sir Isaac Pitman & Sons, Ltd., New York, N. Y.

6. Marks, Lionel S. Mechanical Engineers' Handbook, Third Edition, McGraw-Hill Book Company, New York, N. Y.

7. Moore, H. F. Materials of Engineering, McGraw-Hill Book Company, New York, N. Y.

8. Mundt, Robert. "Über die Tragfähigkeit Zylinderrollenlagern" Maschinenbau, Band 10, Heft 10, May 21, 1931, p. 354-357.

9. Styri, Haakon. "General Properties of Ball Bearings." Mechanical Engineering, Vol. 47, No. 6, June 1925, p. 490-492.

10. Timoshenko, S. Theory of Elasticity, McGraw-Hill Book Company, New York, N. Y.

11. Treves, S. R. "New Constants Determined for Ball Bearing Capacity Formula." Automotive Industries, Vol. 55, December 16, 1926, p. 1014-1016.

12. Various Manufacturers' Catalogs and Literature, Noteably those of S.K.F. Industries, Inc., and New Departure Manufacturing Co.