AN ABSTRACT OF THE THESIS OF

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Title: THE DEMAND FOR AND SUPPLY OF THE CHARACTERISTICS
OF A NEW RESIDENCE AND THE RESIDENTIAL LOCATION
DECISION

Abstract approved: ___________________________________________
Herbert H. Stoevener

This study provides a theoretical and econometric framework
which may be used to explain residential land values and to explore,
in a micro-economic context, the technical and behavioral relation-
ships underlying locational choice in a multi-center, multi-faceted,
urban environment. An understanding of the factors that influence
the residential location decision may be helpful in the development of
land use controls implemented to mitigate the market failure problems
inherent to the residential land/housing market.

A residence is defined by a set of \( n \) characteristics (including
the accessibility of and publicly provided services at the residential
site) which are assumed to be arguments in the utility (production)
function of the household (supplier of the residence). Household bid
(Supplier offer) functions are defined to give the maximum (minimum)
price that the household (supplier) could pay (receive) for residences
embodying any given combination of characteristics and maintain a constant level of utility (profit). Marginal to a bid (offer) function there is an inverse compensated demand (supply) function for each of the $n$ characteristics; it gives the household's (supplier's) implicit demand (supply) price for each quantity of a characteristic. In market equilibrium and for all residences traded, a bid of each utility-maximizing household is equal to an offer of a profit-maximizing supplier, and a market price function relates the market implicit prices of the characteristics and the amount of each characteristic embodied in a residence to the market price of the residence. The solution for the system of compensated demand and supply functions marginal to the equilibrium offer and bid functions for each residence gives both the quantity of each characteristic embodied in the residence and its market implicit price. The household's location is defined by its utility maximizing position in characteristics space. Explicit consideration is given to the influence of commuting costs and property taxes on the location decision.

The econometric model consists of the market price function and a system of $2n$ simultaneous equations representing the compensated demand and supply functions of the theoretical model. Specification in the demand (supply) function of variables descriptive of household (supplier) attributes will identify statistically estimated supply (demand) functions if all households (suppliers) are not identical in
in their attributes. The market price function is estimated by regressing the market prices of the residences on variables descriptive of their characteristics so that the estimated market implicit prices, which are the dependent variables in the system of supply and demand functions, may be obtained.

An econometric model was estimated using data descriptive of 81 new, speculatively-built, single-family dwellings, of the households that purchased them and of the developers who supplied them. The estimated market price function, which included variables descriptive of the house **per se**, the neighborhood of the house, and the accessibility of the house to retail centers and the household's place(s) of employment, explained about 80 percent of the variation in market price. However, as there was relatively little variation in most of the characteristics of the residences and in the attributes of the households (which were measured primarily in terms of income and wealth), only the demand function for the dwelling area of a residence could be identified.

It is suggested that further research be directed toward relaxing certain of the model's simplifying assumptions and toward the identification of variables which are descriptive of those characteristics of the urban environment which land use controls are designed to regulate.
The Demand for and Supply of the Characteristics of a New Residence and the Residential Location Decision

by

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THE DEMAND FOR AND SUPPLY OF THE CHARACTERISTICS OF A NEW RESIDENCE AND THE RESIDENTIAL LOCATION DECISION

I. INTRODUCTION

Need for the Study

Residential settlement patterns, the macro result of locational decisions made by individual buyers and suppliers of housing, give rise to several contemporary problems. First, because of the pervasive technological externalities inherent to residential land use decisions, not all of the concomitant costs of a residential location decision are likely to be internal to the private benefit-cost decision calculus. Second, the residential settlement pattern per se has public good aspects—the benefits of which are likely to remain external to the individual location decision and which, consequently, will not be optimally provided in a decentralized market. Third, the level of provision, spatial distribution, and method of funding public services both influences and is influenced by the residential settlement patterns; thus decisions as to the former frequently become a focus of public controversy. Each of these land use problems, which may be

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1 The buyers are also referred to herein as either individuals or households. The suppliers are also referred to as either developers or developer-sellers.
placed in the general category of market failure problems, inhibits the attainment of a maximum of net social benefits from residential land use.

These problems may provide much of the impetus for land use controls. If public intervention in the residential land/housing market is to increase the net social benefits from residential land use, controls must be designed so as to ensure that each individual either considers or is directed to locate as if he had considered the technological economies and/or diseconomies that may result from his location decision in the benefit-cost calculus of that decision. In addition, the controls must be designed so as to guide location decisions toward a settlement pattern wherein, for each of its public good aspects, the sum of the marginal benefits accruing to all members of the relevant public at each location is equal to the marginal cost of provision of the public good at that location. If the net social benefits from residential land use are to be maximized, the land use controls must, in the limit, cause each urban resident to be located such that no individual could increase his welfare, without diminishing the welfare of another, by moving to a different position.  

\[\text{Though these economic efficiency conditions are necessary for a welfare maximum, they are not sufficient. The social welfare maximizing distribution of location related goods and services must also be determined. Nevertheless, for any given distribution, an increase in social efficiency will result in an increase in welfare. Stoevener (1975) has argued that among the societal goals for land use}\]
Clearly, detailed information is a prerequisite to the design of land use controls—be they either direct controls, which generally prohibit land uses which are at variance with a predetermined, socially optimum, land use plan or indirect controls, which attempt to influence locational behavior so as to achieve an optimum land use pattern. The design of such controls requires that the decision variables which may not be optimally adjusted in private benefit-cost calculations be identified. Knowledge is also required of the public good aspects of the settlement pattern which are not optimally provided by the unregulated market. In addition, indirect controls require knowledge of the behavioral and/or technical relationships underlying the location decisions of the buyer and suppliers of housing so that methods of influencing the locational decisions of the acting planning are the following:

1. the elimination of certain land related Pareto-relevant externalities,
2. the optimum level of provision of certain public goods, and
3. the minimum cost provision of certain public services.

But, the implementation of land use controls frequently alters the distribution of property rights in land and thus the distribution of location related goods and services. Therefore, though economic efficiency may be improved by the implementation of land use controls, the attendant redistribution of location related goods and services may cause a decline in welfare; as a result, the inefficient state, without the controls, may be socially preferred to the efficient state with the controls. However, given the technological and public good externalities inherent to residential land use, the attainment of maximum welfare on the utility possibility frontier would ultimately require that market regulatory devices be implemented to increase economic efficiency in land use.
and/or affected parties may be designed. Knowledge of these technical and behavioral relationships is also necessary to the design of direct controls if unexpected results from the imposition of the controls are to be avoided. That is, the design of the land use controls requires not only that the goals of the controls be identified, but also that the behavioral and technical relationships underlying the location decisions of those individuals who are the target of the controls be identified.

This study attempts to describe the technical and behavioral relationships underlying the residential location decision of the buyers and sellers of new, single-family, dwellings. These individuals are, undoubtedly, a principle target of residential land use controls.

Objective of the Study

This study focuses on the demand for and supply of a set of location related goods that are hypothesized to affect the location decision of households that have chosen a general area in which to locate and have resolved the "rent versus buy" decision in favor of the latter. Neither the "rent versus buy" decision nor the location decision of those who opt to rent is studied. Similarly, the locational behavior of those who choose to buy used rather than new housing is not studied. The decision as to place of employment or choice of area of retirement is not considered so as to further simplify the
problem. Given that a household has decided to purchase a new house in a given area, an attempt is made to identify those factors affecting the choice of the location of the residence within the area. Since this choice cannot, in general, be made independently of the location of the available residences, it is also necessary to identify those factors influencing the characteristics of the residences offered for sale by the developers.

The specific objectives of this study are to:

(1) identify the variables entering the location decision of both the buyers and the suppliers of newly constructed residences and describe the interrelationships of these variables through a set of demand and supply equations for location related goods;

(2) statistically estimate the parameters of the system of supply and demand equations developed under objective (1).

Fulfillment of objective (2) will serve to empirically test the validity of the residential location model developed under objective (1).

Organization of the Study

The remainder of this thesis is divided into five chapters and three appendices. A review of the economic theory of residential location and urban land values and of selected empirical studies of the determinants of urban land values is presented in Chapter II. A
theoretical model of the residential location decision is developed in Chapter III and is used as the framework for the corresponding econometric model developed in Chapter IV. The sources of the data used to estimate the parameters of the econometric model are also described in Chapter IV. The procedures used in and the results of the statistical estimation of the econometric model are described in Chapter V. The conclusions and recommendations for further research are presented in Chapter VI. Supplementary statistical results and the computer programs developed to estimate and/or transform the observations on certain variables of the econometrics model are presented in the Appendices.
II. REVIEW OF LITERATURE

Introduction

Economists have long recognized that a relationship exists between the characteristics of land, the use of land (or the location of the users of land) and the value of land. A basic tenet of land economics holds that economic rents allocate the land resource between alternative uses or users. That use or user which enjoys the highest economic rent is able to place the highest bid for the use of the land and will, in the absence of institutional constraints, gain the use of the land. Further, assuming perfectly competitive product and factor markets, competitive bidding for the use of the land will ensure that the payment for the use of the land equals the economic rent, which, in turn, is identically equal to the value of the marginal product of the land in the equilibrium use. It is the relative productivity of land as determined by its characteristics, one of which may be location, that determines, ceteris paribus, differential economic rents and thus differential values.  

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3 The value is obtained by the capitalization of all future expected economic rents. Factors influencing expectations and capitalization rates are generally ignored in this study. The concern herein is with the determinants of economic rent; they are also determinants of value.
The preceding brief statement of modern rent doctrine belies both its complexity and the intellectual struggle underlying its development. Ricardo is generally credited for the insights that provide the foundation for the modern theory of rent, though there were few salutary moves toward the modern formulation. Mills (1969, p. 231), in his very brief 'cooks tour' through the history of land rent doctrine, asserts that "to compare a careful modern statement of rent theory with Ricardo's is to affirm that economics is a cumulative science".

Neither a review of the history and development of rent theory nor a comprehensive review of the theories, and/or models, or urban land rents and use will be provided here. In particular, neither urban land use simulation models nor the so-called "new urban economics" will be reviewed. A review of the former prior to 1972 is provided by Brown et al. (1972). The latter, a brief review of which can be found in Mills and Mackinnon (1973), are primarily neoclassical adaptations of the von Thunen prototype, and rely on control theory or programming to analyze optimum or market equilibrium land use. A brief review will be provided of the economic theories of the process by which urban residential land values and settlement patterns are

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4 Brown et al. (1972, p. 79) state, in regard to urban simulation models: "In particular, insufficient effort has been made to model the underlying reasons for locational decisions by households and firms. Existing models would be better able to explain the changing patterns of urban growth if they paid more attention to these determinants."
determined, and the variables entering that process. Particular attention is paid to those theoretical models explicitly built upon the economic theory of consumer behavior. In addition, a review of several empirical studies of the determinants of urban residential property values will be provided. A comprehensive survey of the urban economics literature has been provided by Goldstein and Moses (1973).

**Early Theories of Land Rent and Location**

The early economists recognized that differential land rents stemmed, in part, from the differences in transportation costs at different locations. Adam Smith (1937, p. 147)\(^5\) noted that:

The rent of land not only varies with its fertility, whatever be its produce, but with its situation, whatever be its fertility. . . . it must always cost more to bring the produce of the (more) distant land to market.

Ricardo (1895, p. 46), though he based his theory of rent on the differential fertility levels of agricultural land, also recognized the

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\(^5\) The dates provided in the parenthesis refer to the publication dates of the works referenced in this study, and provide no historical perspective as the works of several of the earlier economists have been republished at later dates. Adam Smith's *Inquiry Into the Nature and Causes of the Wealth of Nations* was first published in 1776 (Heilbroner, 1969, p. 37). David Ricardo's *Principles of Political Economy and Taxation* was originally published in 1817 (Heilbroner, 1969, p. 86). Von Thunen's *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie* was published in 1826, and Alfred Marshall's *Principles of Economics* was first published in 1890 (Heilbroner, 1968, p. 189).
importance of transportation costs when he stated that:

It is only, then, because land is not unlimited in quantity and uniform in quality, and because in the progress of population, land of an inferior quality, or less advantageously situated, is called into cultivation, that rent is ever paid for the use of it.

As is to be expected, the early economists focused on the determinants of the rents of agricultural lands.

Smith (1937, pp. 791-792), however, provided a brief discussion on the determinants of urban land rent in his writings on housing taxes. After distinguishing between "building rent" and "ground rent" and defining ground rent as the difference between the total rent paid the landlord and the building rent sufficient for affording a "reasonable profit", he attributes this "surplus" or ground rent to:

. . . the price which the inhabitant pays for some real or supposed advantage of the situation. . . . whatever be the reason for that demand, whether for trade and business, for pleasure and society, or for mere vanity and fashion.

Marshall provided a discussion of urban land rents in Book V, Chapter XI and Appendix G of his Principles of Economics. He defines the "site value" of a parcel of land to be the sum of its "situation value" and agricultural value. Situation value is said to exist if a producer has advantages of situation at a location in that "costs of carriage" are lower or if he has better "access to a labor market specially adapted to his trade" relative to another location.

"The extra income which can be earned on the more favoured site
gives rise to what may be called situation rent" (Marshall, 1947, p. 441). Marshall (1947, p. 445) further notes that the various users bid for the land and that the land will be placed in that use "subject to local building bylaws . . . from which the most profitable results are anticipated. . . ." He also observed the relationship between intensity of use and rent when he stated that (Marshall, 1947, p. 448): "If land is cheap he (the developer) will take much of it; if it is dear he will take less and build high." Marshall, it can be seen, concentrated on the determinants of the value of land use for commercial and industrial purpose; he did not explicitly consider residential land.

Von Thunen (1966), writing shortly after Ricardo and before Marshall, provided the first theoretical link between location, transportation, the spatial utilization of land, and land values. As his model provides the basis for many modern urban land rent models, it shall be briefly reviewed. 6

Von Thunen (1966, pp. 7-8) posed the following problem:

Imagine a very large town, at the centre of a fertile plain which is crossed by no navigable river or canal. Throughout the plain the soil is capable of cultivation and of the same fertility. Far from town, the plain turns into an

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6Von Thunen's isolated state is an abstract model in the sense that modern economists use the term. Von Thunen was the first economist to use such abstraction as an analytical tool (Schumpeter, 1954, p. 466). This was only one of von Thunen's contributions. Among other notable firsts was his use of the calculus as a form of economic reasoning (see Schumpeter, 1954, pp. 466-467).
uncultivated wilderness which cuts off all communications between this State and the outside world.

The problem we want to solve is this: what pattern of cultivation will take shape in these conditions?; and how will the farming system of the different districts be affected by their distance from the town?

The von Thunen model emphasizes the role of transportation costs in determining land value and use, and can be conveniently summarized using Figures 1 and 2. Assume that there are four different crops that may be cultivated on the plain. Let the annual revenue net of all costs, save those of transportation and payments for the use of the land, be given by $R_i$ for each of the four crops $(i = 1, \ldots, 4)$. Let the annual transportation costs $(T_i)$ for the $i$th crop, including the opportunity cost of time spent on transport, be given by $0T_i$ (Figure 1). The economic rent $(r_i)$ of the land at each location (as given by the distance from the town) is then obtained by deducting transportation cost from net revenue $(r_i = R_i - T_i)$. As can be seen in Figure 2, the economic rent in each use will diminish with the distance from the town (as transportation costs increase) and will equal zero at that point where transportation costs equal net revenue; this distance is the extensive or no-rent margin for that use.

If the four uses bid against one-another for the use of the land, the maximum amount that they can bid at any given distance from the town is given by $r_i$; if they were to exceed this bid, net revenues, after deduction of transportation costs and the payment for the use of
land, would be negative. Use (1), the most expensive crop to transport, can out-bid all other uses up to distance $A$ from the town. The highest bidder for lands lying between distance $A$ and $B$ will be use (2), and so on--as depicted in Figure 2. As can be seen, the von Thunen model focuses on the role of transportation costs in determining land use and value.

![Diagram](image)

**Figure 1.** The relationship between transportation costs, net revenue, and location in the von Thunen model.
Though von Thunen did complicate his analysis by allowing for small villages and differences in soil fertility in the Isolated State, his primary conclusion was (von Thunen, p. 8):

... near the town will be grown those products which are heavy or bulky in relation to their value and hence so expensive to transport that remoter districts are unable to supply them. Here too we shall find the highly perishable products which must be used quickly. With increasing distance from town, the land will progressively be given up to products cheap to transport in relation to their value.

For this reason alone, fairly sharply differentiated concentric rings or belts will form around the town, each with its own particular staple product.
Thus von Thunen provided a theory of location differential land rents wherein each use utilizes that land where it can afford the highest bid and the value of the land is equal to the value of its product less transport and production costs.

The von Thunen theory of agricultural land utilization and value has been further refined by Dunn (1970) and Isard (1970). More recently, the von Thunen model of agricultural land value and land utilization has been given a neoclassical reformulation (Beckman, 1972; Katzman, 1974). However, "urban economics is the field where the (von Thunen) paradigm has had its greatest impact and enjoyed its theoretical elaboration" (Katzman, 1974, p. 683).

Modern Theories of Urban Residential Land Rent and Location

The modern formulation of urban land use and rent stems, primarily, from the work of Wingo (1961), Alonso (1964), and Muth (1969). Both Alonso's and Muth's models are firmly anchored in the neoclassical theory of consumer behavior. Though the ancestry of Wingo's model is particularly evident, all are derivatives of the von Thunen prototype.

Models Based on the von Thunen Prototype

Wingo: Transportation and Urban Land. Wingo (1961) assumes a spatially uniform urban area wherein all employment is concentrated
in the central business district (CBD) and the residences (land plus
structure) are homogeneous in all respects except lot size. The
demand for space is identical across all individuals and is of constant
elasticity. Parallel to the von Thunen model, land rents are assumed
to decrease sufficiently to just offset the increase in transportation
costs with increasing distance from the CBD (i.e., land rents and
transportation costs are assumed to be complementary).

The assumption of complementarity of land rents and
transportation costs arises from Wingo's explicit assumptions that the
marginal value of leisure is constant across all workers (or house-
holds) and that the transportation costs include the value of time spent
in the journey to work. Since equilibrium requires that the marginal
return to labor be equal to the marginal value of leisure, it follows
from his assumptions that the wage rate is equal for all workers.
However, this is true only in terms of "manifest" hourly wage rate.
If the net return to labor is defined to be the gross return, or manifest
hourly wage rate, less transportation costs and land rent, then
workers will locate farther from the CBD, and incur higher travel
costs, only if land rents decline sufficiently to offset the higher travel
costs. Thus, land rents and transportation costs must be

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7Wingo (1961) makes no explicit assumptions concerning housing. He implicitly assumes that the land is ready for occupancy and explicitly assumes that the individual's demand for land is a function only of land rent.
complementary if workers located at various distances from the CBD are to have equal net return to labor.

However, the assumptions of Wingo's model are not sufficient to ensure that land rents and transportation costs will be complementary. Due to lower transportation costs, those residences located near the CBD have an economic advantage for the workers, and bidding will ensue to capture the advantage. "The 'price' of any individual location will be bid up to the point where all workers but one are excluded, with the result that part or all of the value of the locational advantage is absorbed" (Wingo, 1961, p. 64). However, in the absence of further restrictive assumptions, the highest bidder may enjoy a consumer's surplus and, therefore, not all of the travel savings at a particular location are necessarily captured in land rent. Consequently, Wingo explicitly assumes that all advantages of location are captured in land rent and, therefore, rent differentials are assumed to fully compensate for differences in travel costs at different locations.

Given his assumption, and the additional requirement that the urban area be just sufficiently large to contain its exogenously given

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8 Wingo (1961, p. 64) states that it seems that all advantages of location would be captured in land rents "only under the restrictive assumption of perfect competition and where all bidders are of the same industry (uniform technology) and equally efficient."
population, Wingo is able to determine the equilibrium rent-distance and population density functions. Assuming an inelastic demand for land (or housing), he shows that population density and per unit rents will decline with distance from the CBD.

However, Mills (1972, p. 65) has shown that Wingo's model leads to an "implausible" conclusion if the demand for land (or housing) is not inelastic. If the demand is assumed to be elastic, it is a consequence of the model that rent and population density increase with distance from the CBD. If the demand for housing is of unit elasticity, the model does not have a solution unless commuting costs are zero. This is of interest in that Wingo (1961, p. 81) states that "the demand for space by households appears to be elastic over most of its range," and Muth (1969, p. 72; 1971, p. 248) estimates the elasticity of demand for housing to be about unitary.

The household plays only a token role in Wingo's model as its location decision is assumed to be entirely budgetary given the assumption that transport costs plus the amount spent on land must equal a constant sum regardless of location. Wingo considers neither the possibility of the substitution of capital (housing structure) for land by the household nor the possibility of the substitution of other consumption activities for housing and/or land. As all households are assumed uniform in tastes and income and land rent is assumed to be a fully compensating differential, the homogeneous households
are locationally indifferent. Given Wingo's assumptions, there are no individually unique households and thus no individually identifiable residential location decisions.

Alonso: Location and Land Use. Alonso (1964) provided a general theory of urban land use and value, though he primarily focused on residential land. He, like Wingo, assumed a featureless plain with a CBD where all employment is concentrated. The residences are assumed to be homogeneous in all respects except lot size. Alonso first developed a theory of the household in urban space then turned to conditions for a market equilibrium.

The individual household's utility is assumed to be a function of land (or housing), a composite good (which may be a vector of goods) and distance from the CBD which represents the (assumed) disutility of commuting. The household adjusts these three variables so as to maximize its utility subject to its budget constraint. The prices of the composite good and land are independent of the amounts purchased. The unit price of land, which is given by the "price structure" (Alonso's term for the rent-distance function), is assumed to vary

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9 Alonso (1964, p. 15) makes no explicit assumptions about housing. He assumes that the land is ready for occupancy.

10 Hartwick (1971, pp. 7-12) argues that distance is a proxy variable for leisure. Accordingly, he revises Alonso's model by assuming the household's utility is a function of leisure available, which, in turn is assumed to vary inversely with the distance from the CBD. All of Alonso's results follow.
inversely with distance from the CBD and to be taken by the households as beyond their individual influence. Commuting costs are assumed to vary directly with distance from the CBD. The budget constraint requires that the expenditure on land, commuting, and the composite good equal the household's income, which is exogenously determined.

Alonso (1964, p. 59) introduces the concept of a bid price function which gives:

... the set of prices for land the individual would pay at various distances while deriving a constant level of satisfaction; that is to say, if the price of land were to vary with distance in the manner prescribed by the bid price curve, the individual would be indifferent among locations.\footnote{Stull (1973, pp. 108-109) argues that a more precise definition of the bid price function is that it "is a locus showing the maximum price that the household can pay for land at various locations while deriving a constant level of satisfaction" (emphasis added).}

As commuting costs increase with distance from the CBD, and thus reduce the amount of money that the household has at its disposal to spend on land and the composite good, the amount that the household would be willing to pay for a unit of land would decrease with distance from the CBD. If the land were cheaper at greater distances from the CBD, the household could maintain a constant level of satisfaction with its reduced effective income. Thus commuting costs reduce the bid price as distance from the CBD increases. More precisely, the bid price for land "must drop just sufficiently for the resulting
income-effect to offset the increase in commuting costs" (Alonso, 1964, p. 64). Formally, the bid price function is obtained from the first-order conditions for a constrained utility maximum, the budget constraint, and the indifference surface. There is one bid price function for each level of household utility.

Household equilibrium is determined in a manner similar to that in the nonspatial theory of the consumer. Each bid price curve represents a set of combinations of land prices and distances among which the household is indifferent, with higher bid price curves representing lower levels of utility. The land price structure represents the locus of opportunities available to the household similar to the budget constraint in the usual problem. In maximizing utility, the household chooses that location where the lowest bid price function is tangent to the price structure given certain assumptions about the shape of both functions. The interpretation is similar to that of tangency between the indifference surface and the budget constraint in the nonspatial theory of the household.

In market equilibrium, at least one of the bid price functions of each user of land is tangent to the price structure, where the price structure is the envelope of the equilibrium bid price curves. Market equilibrium requires that no user of land be able to increase his

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12 The concept of a bid function is used in this study. See Chapter III for a mathematical derivation of the function.
profits or utility by changing location or by purchasing a different quantity of land, and that no landlord can find it possible to increase his revenue by changing the price of land. In addition, the quantity of land demanded must equal the quantity supplied given the price structure.

In Alonso's simplest formulation, each household's bid price curves constitute a family of parallel straight lines. The slope of the bid price function may vary between households. The equilibrium bid price function \( B_i \) of four households \( (i = 1, \ldots, 4) \) and the price structure \( P \) might appear as illustrated in Figure 3. In this formulation, Alonso concludes that the households (or other establishments) with the steeper bid price functions will locate closer to the CBD. Households will be ranked by distance from the CBD according to the rank by slope of their bid price curves, as depicted in Figure 3.  

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13 Mills (1972, pp. 69-71) shows that Alonso's assumptions are not sufficient to establish this conclusion. Mills argues that the additional assumptions necessary are those of perfect competition; in particular, an assumption that, for each family of bid function, there is a very large number of households with the same bid functions in order to ensure competitive bidding.

It seems that an assumption or set of assumptions sufficient to ensure competitive bidding is necessary to ensure that the bid function is a "locus showing the maximum price that the household can pay" (see footnote 11).
Though it is a result of Alonso's model that households that locate far from the CBD will, ceteris paribus, purchase a greater quantity of land than those closer to the CBD, it is not necessarily the case that higher income households will locate further from the CBD than those with lower incomes. The slope of the bid price function depends, in part, upon the specific form of the household's utility function or, more precisely, upon the marginal rate of substitution between distance from the CBD and the composite good (Alonso, 1964, p. 70). In general, nothing can be said about the effect of income on
the slope of the household's bid function and, therefore, on location (Alonso, 1964, pp. 106-109).

In Alonso's formulation, the theory of residential location is firmly grounded in the theory of consumer behavior as he starts with the households' utility functions in analyzing their behavior. His model explicitly considers the possibility of substitution between land and distance (commuting), and the possibility of substitutions between either of these and the composite good. However, he does not allow the possibility of substitution between either land and the housing structure or between housing structure and either distance or the composite good.

Muth: *Cities and Housing*. Muth (1969) developed a residential location model based on the von Thunen prototype, and that is similar to those of Alonso and Wingo. In the first part of his book, he provides a theory of the supply of housing services as well as a theory of residential site selection. He also discusses other determinants of residential land use, such as racial segregation and discrimination, preferences for different housing types, and various neighborhood characteristics. He devotes much of the remainder of his book to a description of his empirical tests and findings. Only his basic theoretical model will be reviewed here.

Though Muth does not begin his analysis by explicitly assuming an urban area located on a featureless plain, he does assume a CBD
wherein all employment is concentrated and to and from which transport costs are uniform in all directions. The identical housing supply firms provide homogeneous housing services, though they may do so using different ratios of land and non-land inputs. All firms and households are assumed to be competitive in both product and factor markets, and thus can influence the prices they pay or receive only through choice of location.

Muth assumes that the household's utility is a function of the housing services consumed and dollars of expenditure on all other commodities (or a composite good) except transportation but including the money value of time spent in leisure. The budget constraint equates income to expenditure on the composite good, housing, and transportation. In this formulation, income includes both money income and money value of travel and leisure time (Muth, 1968, p. 21). 14 Muth assumes that transportation costs are a function of

14 As Goldstein and Moses (1973, p. 476, footnote 20) point out, Muth's approach is somewhat confusing. As the value of leisure is included in the composite commodity, the implication is that leisure and hours of work are choice variables. Therefore, there should be a time constraint as well as a budget constraint on the household. Further, if the value of leisure changes with income, "then as income changes the value of the composite commodity changes. This will affect the housing decision. . . ."

An alternative approach would be to assume that the price of leisure is independent of money income; then the price of the composite commodity could be given to the household. However, this is a rather strong assumption, and one that Muth does not explicitly make.
both income and distance from the CBD, as the time cost of commuting is paid in part by foregone leisure and by foregone income—both of which are components of income. The household is assumed to make a fixed number of trips to the CBD per unit of time. Location influences the household's consumption only through its effect on the price of housing, which is assumed to vary with distance from the CBD, as Muth ignores the effects of tastes and preferences for different types of housing services. The household is assumed to maximize its utility subject to its budget constraint.

Using household equilibrium conditions, Muth demonstrates that the price of the homogeneous housing services must decline (at an increasing rate) with distance from the CBD if transport costs increase with distance. For (Muth, 1969, p. 23):

If the price of housing were the same everywhere but the marginal cost of transport increases with distance, any move toward the CBD would increase the real income of the household, and everybody would try to live as close to

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15 If the number of trips changed as the cost per trip changed due to a change in location (i.e., if the demand for trips were not perfectly inelastic), the change in transportation costs due solely to a change in distance would not be an accurate measure of the change in welfare and thus of the compensating rent differential.

16 The price of housing refers to the price of the flow of one unit of housing service per period of time. The service can be produced by different combinations of land and non-land. The housing variable in the utility function refers to housing services consumed, not to the flow of services from the land and non-land inputs used to produce it.
(the CBD) as possible. . . . the price of housing would be bid up while housing prices in more distant locations would be bid down.

The first-order conditions for a utility maximum require that, in equilibrium, the household savings in housing expenditure for a very short move away from the CBD be exactly offset by an increase in transport costs. Therefore, the change in housing consumption with a small move in the vicinity of the household's equilibrium location is given by the household's real income compensated housing demand curve, as real income is constant with a small change in location (Muth, 1969, p. 24).

Many of Muth's results pertaining to residential location follow only with additional assumptions. For example, the effect of income on the household's equilibrium location cannot be determined without knowledge of the income elasticity of the demand for housing and the elasticity of transport costs with respect to income. If the effect of a higher income on housing consumption outweighs the increased valuation of travel time, the higher income households will locate at a greater distance from the CBD than will the lower income households, ceteris paribus (Muth, 1969, pp. 31-32). Many of these assumptions reflect, directly or indirectly, on the nature of the households' utility functions; i.e., on their tastes and preferences.

In his theory of the supply of housing services, Muth starts with the assumption that the homogeneous housing services are produced
by profit-maximizing firms with identical production functions using homogeneous land and non-land inputs. The price of the non-land input is assumed to be constant throughout the urban area, but the price of the land input is assumed to vary with distance from the CBD. The housing producers are assumed to adjust the ratio of land to non-land inputs with any change in location, and thus the input price ratio, so as to maximize profits.

Locational equilibrium of the housing producers requires that they locate such that their incomes cannot be increased by any change in location. However, as Muth assumes identical production functions for all producers of housing services, the condition for locational equilibrium reduces to the requirement that their profits be the same everywhere and independent of location (Muth 1969, p. 49).

As the price received for housing varies inversely with distance from the CBD, equality of profits requires that land rent vary directly with the housing price received at the point of production. For, if the producers of housing were to pay the same price for both land and non-land inputs regardless of location (Muth, 1969, p. 52):

... firms located close to the city center would earn greater incomes than those located farther away. It would then be in the interest of firms located at greater distances to offer more for land located close to the CBD than centrally located firms were currently paying. Land rentals would thus rise in the central locations and fall in the more distant ones,...
Thus, land rentals, like housing prices, will decline with distance from the CBD.

First-order conditions for a profit maximum require that the producers of housing services use more land relative to the non-land factor as they move away from the CBD, as the ratio of land price to the price of the non-land factor decreases with distance from the CBD.

Additional assumptions concerning the production functions of the housing suppliers are necessary to obtain many of Muth's results. For example, by assuming, in effect, a linear homogeneous production function, Muth shows that the value of housing produced per unit of land will decline with distance from the CBD (Muth, 1969, pp. 54-56).

In Muth's formulation, the demand for residential land is derived from the demand for housing services. Though land and structure (or non-land) are substitutes in the production of housing services, they are not substitutes in consumption. Housing services, not land per se, is an argument in the household's utility function.

The residential location models of Wingo, Alonso, and Muth can be criticized on several common points. Each assumes that the urban area is a featureless plain; i.e., that the quality of the urban environment does not vary with location. The different characteristics of different locations, such as neighborhood quality, levels of air and noise pollution, etc., are ignored as is the role of these
characteristics in the household's location decision. Similarly, no consideration is given the role of variation in publicly provided services and/or taxes within the urban area in the residential location decision. None of the models explicitly considers the possibility that land and capital (structures) may be substitutes in the composition of housing; each assumes that housing is a homogeneous good. The assumption, common to each of the models, that the households travel to and from, and/or prefer to be accessible to, only one central location is an obvious oversimplification. All of these criticisms are aimed at the simplifying assumptions of the models. The authors, especially Muth, explicitly acknowledge these simplifications, but apparently introduce them so as to develop a model amenable to simple mathematical formulation and analytical (rather than numerical) solution for market equilibrium conditions. 17

Adaptations of the Alonso and Muth Models. Efforts have been made to relax some of the simplifying assumptions of the models derived from the von Thunen prototype. However, a mathematical derivation of the condition of a market equilibrium in such models often proves to be intractable.

17 A substantial portion of Muth's (1969) book, Cities and Housing, is devoted to relaxing the assumptions of and elaborating his basic model. However, in the main, his approach is verbal rather than mathematical.
Hartwick (1971) retains the concept of a CBD, but extends Alonso's model to consider alternative modes of transportation, variations in the quantity of public goods available, variations in taxes at alternative sites, and the role of other neighborhood, or environmental, characteristics in the residential site selection problem. In his formulation, the various dimensions of the urban environment enter the households' utility functions as variables which the consumer can affect by changing location and/or expending alternative amounts of income on travel. His is only a partial equilibrium model; he does not consider market equilibrium. Households are assumed to consider all feasible alternative sites, and choose that site at which the first-order conditions for equilibrium are satisfied and the highest level of utility is achieved. Hartwick (1971, p. 21) states that the usefulness of his approach depends upon "the possibility of conceptually separating a property of the environment from a conventional good or service."

Papageorgiou and Casetti (1971) introduce \( n \) business centers into Alonso's model. Each of the centers provides a composite good, which differ in the set of goods and services included in them. Each of the \( n \) composite goods has a fixed price, but transport costs and the price of land at each location are functions of the set of distances from the location to each of the \( n \) centers. The households are assumed to maximize utility, a function of the set of composite goods
consumed, land, and the set of distances, subject to the usual restriction that total expenditures equal income.  

A major criticism of their model, to this point, is that their mathematical formulation of the household's problem is analytically inoperative as it requires that the utility function be maximized with respect to each of the \( n \) distances to the \( n \) centers. Yet once any three distances are selected, the remaining distances are determined if the individual is to locate on the two-dimensional settlement plain. The problem is analogous to the generalized Weber (1929) problem which requires that a location be chosen so as to minimize transport costs to \( n \) given points. There appears to be no non-trivial analytical solution to the problem, though methods have been developed that enable a numerical solution (Kuhn and Kuenne, 1962).

Papageorgiou and Casetti (1971) approach the locational choice problem by assuming the household to be initially at a given location. The household then selects a utility maximizing set of composite goods and quantity of land given the set of distances and its income. They state that the households will have no incentive to relocate, and a spatial equilibrium will result, "if and only if the optimal utility level

\[18\] Papageorgiou and Casetti (1971) make no assumption about the marginal utility of distance, though, presumably, they consider it to be negative.

\[19\] An additional constraint is required to ensure that the individual locates on (not above or below) the plain.
attainable by the households is constant throughout the region" (Papageorgiou and Casetti, 1971, p. 386). This approach seems to be of little more than heuristic value.

Wright (1971) extends Muth's model by adding a vertical dimension to the model. He introduces vertical as well as horizontal locational preferences and a transportation cost function. One result is that the height of buildings decrease at an increasing rate with distance from the CBD; that is his "high-rise" gradient. He also derives a population density gradient from the "high-rise" gradient. The "high-rise" gradient is analogous to Muth's (1969) measure of the variation in the ratio of land to non-land costs with distance from the CBD; both provide a measure of the variation in the intensity of residential land use with location.

Nelson: The Theory of Residential Location

Nelson (1972, p. 249) points out three major deficiencies of the theories of residential location developed by Wingo, Alonso, and Muth.

First, typical measures of accessibility focus on the distance and travel time to one central location, neglecting distances and travel times to other important locations. Second, wide variations in the types of housing occupied by renters (and purchasers) at different locations are often ignored. The third deficiency is that the different site advantages of different locations, such as different views, different levels of air and noise pollution, different neighborhood attractiveness, and so forth are usually either neglected or ignored.
Nelson (1971) describes a residence by the set of attributes, categorized by accessibility, housing facilities, and site advantages, associated with each residential location so as to avoid the simplifying assumptions of a CBD, a featureless plain, and homogeneous housing.

Following Becker (1965), Nelson (1971) defined the accessibility at any given location by the set of "time prices" faced by the household in the consumption of those goods which require travel. The total price of each good is the sum of the monetary expenditure required to obtain one unit of it plus the monetary value of the time spent in consumption of it. Differences in travel times at different locations influence the set of time prices, and thus total prices, faced by the household. The journey (more specifically, the time) to work is treated as a consumption good. No consideration is given to the utility of travel per se. The monetary cost of travel is not considered, though his model could easily be expanded to include it.

The locational characteristics (i.e., housing facilities and site advantages) of a residence are assumed to be constrained to a fixed (and measurable) amount at each location. The housing facilities consist of the characteristics of the housing structure and lot. The site advantages consist of the various neighborhood characteristics (e.g., race of inhabitants, average income level, and average value of structures) and the publicly provided services (e.g., availability of public water and sewer and quality of schools) existing at a given
location. The locational characteristics are assumed to enter directly into the household's utility function along with "normal" market goods; therefore, variations in their level of availability at different locations directly influence the household's utility.

Given a location, the households, which in Nelson's simplest formulation are assumed to have identical utility functions and incomes, maximize their utility subject to an income constraint, a time constraint (which requires that the sum of the products of the quantity of each good consumed and the time spent consuming is not to be greater than the total amount of time available), and one constraint for each locational characteristic. The households are assumed to take the monetary prices of the "normal" market goods and the rent function, which gives the rent (price) that they must pay for a residence with any given combination of housing facilities, accessibility, and site advantages, as given and beyond their individual influence.

The households' time prices are assumed to be fixed at any given location and vary only between locations with different accessibilities; i.e., their magnitude is independent of the quantities of goods consumed. If a household locates at a given location, it must utilize the full (constrained) amount of the locational characteristics.

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20 It is assumed that the housing facilities and site advantages available at a location do not enter the household's time constraint.
and pay that "price" for them that is dictated by the rent function. The households, therefore, seek that location where, given their preferences, constraints, market prices, and the rent function, the constrained amounts of the locational characteristics permit them to satisfy the conditions for a utility maximum.

The housing suppliers, or "landlords", are assumed to own all land and construct, and lease to households, housing units with those combinations of housing facilities that "maximize the landlord's net return" (Nelson, 1971, p. 109)--where the net return is defined to be "the total rents collected from renters minus the total costs of facilities" (Nelson, 1971, p. 109). As each landlord is assumed to be associated with a given location and plot of land, he has no control over the accessibility (the set of time prices) or site advantages of the residences associated with his location. Landlords take the rent function, which gives the price that can be obtained for housing units with different combinations of facilities and has, as parameters, the site advantages and time prices at their location, as given and beyond their individual influence. It is assumed that there is no collusion between landlords. The cost of constructing the housing unit is assumed to depend upon the number of units constructed and the combination of housing facilities included in each unit. Given the rent and cost functions, the landlord selects both that number of housing units and combination of housing facilities that will maximize his net return.
subject to the constraint that the number of housing units require no
more land than the landlord possesses.

To determine the rents (or prices of the housing units with
different combinations of housing facilities, accessibilities and site
advantages) and housing facilities that will exist at different locations
in equilibrium, Nelson introduces a number of assumptions concern-
ing the urban area. The area is assumed to consist of a large number
of subareas, each of which has different locational advantages (i.e.,
different accessibilities and locational characteristics), and thus dif-
ferent rent functions. The points to which households consider
expending time in travel (e.g., places of employment and recreation)
are assumed to be fixed and unaffected by the equilibrium solution.
The site advantages are assumed to be independent of the equilibrium
solution so that, in the aggregate, locational choices will not alter site
advantages (e.g., neighborhood quality or prestige). Given these,
and another set of assumptions that ensure that all households are
housed and that there are no vacant housing units, Nelson discusses
the nature of the equilibrium solution. A necessary condition for
equilibrium is that the rents paid by the households located in different

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21 Presumably the subareas are homogeneous in locational advantages within their boundaries.

22 The model is too complex to permit a mathematical derivation of the equilibrium conditions.
subareas differ such that, between any two areas, the difference is "just such that it equals the value in terms of income" (Nelson, 1971, p. 147) of the differences in time prices between the two areas and the differences in site advantages between the two areas. As the households are homogeneous in all respects, this implies that, in equilibrium, they are all at the same level of indifference (i.e., the rent differentials are fully compensating), and that no household can increase its utility by moving. Similarly, no landlord can increase his net return by constructing different housing facilities. Nothing, in general, can be said about the housing facilities except that the housing units will be of higher density the greater is the rent that can be obtained. However, greater locational advantage does not necessarily imply that higher rents and/or density will result. All other things equal, greater accessibility, for example, implies higher rents. All other things do not remain equal, however, as higher rents imply higher density. But greater density may result in less desirable housing facilities (less space per housing unit, e.g.), thus causing rent to fall. The outcome is not unambiguous and depends upon the specific utility function assumed for the homogeneous households. Similarly, relaxing the assumption of equal household incomes, Nelson finds that there is no unambiguous relationship between either income and location or income and housing expenditure.
Nelson's (1971) *The Theory of Residential Location* is perhaps best considered, in its entirety, not as a specific model of the urban residential economy designed to yield testable hypotheses, but rather, as an attempt to provide a more general theory of the household and housing supply in urban space. His principal contributions lie in his exposition of the derivation of compensating rent differentials and in his integration of these differential rents with familiar concepts of conventional (non-spatial) price theory (see Nelson, 1972). Excepting his theory of housing supply, Nelson's (1971) theory would, with the introduction of the appropriate behavioral and simplifying assumptions, yield Alonso's, Wingo's and Muth's less general models.

Nelson's development of the theory of housing supply can be criticized on at least two points. First, as he started the theory of household behavior with the utility function, he might also have started the theory of supplier behavior with the production (rather than cost) function. Further, in his discussion, it is not clear whether all suppliers are assumed to face the same cost function or if it might vary with location and/or between suppliers. Second, the suppliers of housing are not conceptually divorced from the owners of land. Rather, the landowners, all potential housing suppliers, sit on their plots and respond, subject to their costs and plot size, to the stimuli provided by the households. It follows that the housing facilities provided at the various (and possibly adjacent) locations, and thus the
households choosing to locate there, are not independent of the exogenously determined distribution of land among housing suppliers. A more general theory of housing supply would conceptually separate housing suppliers and landowners, explicitly allow for variations in the production capability of housing suppliers, and allow housing suppliers to treat land as a variable (rather than constrained) input.

Summary of the Modern Theories of Residential Location

The principal hypothesis that emerges from the body of residential location theory reviewed here is that land prices (or the price of housing services) will decline with distance from the CBD, ceteris paribus. In the urban world assumed by the theory, the household's only locational choice is between either higher commuting costs and lower priced land (housing services) as it moves away from the CBD or lower commuting costs and higher priced land as it moves closer in. Without knowledge of (or subsidiary hypotheses concerning) the nature of the households' utility functions, the outcome of this trade-off cannot be predicted, and no other hypotheses emerge from the theory.

With the introduction of the auxiliary hypothesis that the households' income elasticity of demand for land (housing services) is greater than the households' percentage increase in travel costs (direct and indirect) given a one percent increase in income, it can be
hypothesized that higher income households will tend to locate farther from the CBD than will lower income households.

The residential location theory developed by Wingo, Alonso, and Muth focuses on accessibility to a central point. It ignores variations in both the housing available to consumer and to the urban environment.

Attempts have been made to expand the theory of residential site choice to consider consumer preferences for both different house types and differences in the urban environment at different locations. The complexity of these models is such, however, that they do not readily permit deduction of testable hypotheses concerning consumer behavior in residential location.

**Empirical Studies of the Determinants of Residential Property Values**

Though the economic theory of residential location has focused on the role of accessibility in residential site selection, empirical studies of residential location behavior have considered consumer preferences for many other attributes of the residence—including housing characteristics and neighborhood and other location related attributes. The literature is extensive, and includes a number of consumer attitudinal studies (e.g., Charde, date unknown; Blake et al., 1974; Highway Research Board, 1969). No attempt is made to present a comprehensive survey. Rather, this review will be limited
to selected economic studies of the determinants of residential
property values.

Empirical studies of residential land and/or housing values are
usually conducted under the assumption (either explicit or implicit)
that, within any given market, competitive bidding for the houses
and/or land with the more desirable attributes will result in a higher
market price for such houses and/or land. All of the studies
reviewed here have regressed a measure of the value of residential
property on measures of its attributes in an attempt to determine the
relative implicit value of the attributes.

These hedonic price studies,\textsuperscript{23} designed to provide a measure of
the relative strength of consumer preferences for housing character-
istics and/or locational attributes are, for review purposes, placed
in one of two categories: (1) studies of the determinants of residential
land values and (2) studies of the determinants of urban house values.
Each of the categories contains an extensive literature. However,
many of the studies have a common methodology, and, therefore, only
a select few of the more recent studies will be reviewed. In the fol-
lowing two sections, the studies will first be independently reviewed
and then collectively critiqued.

\textsuperscript{23} See Griliches (1971) for a description and various applications
of the technique in constructing price indexes.
Empirical Studies of the Determinants of Residential Land Values

Brigham (1965) studied variations in the property tax appraised value of land utilized for single-family residences in Los Angeles County, California. Two arbitrary rays were drawn out from the Los Angeles Civic Center (CBD), and the five-block, moving average, appraised land value per square foot was computed for the single-family residences on each block along each ray. In addition, observations were made of the level of seven attributes associated with each block. The attributes considered were:

Accessibility

(1) distance to the CBD,

(2) an accessibility potential index,\(^{24}\)

Neighborhood Amenity

(3) average appraised value of the dwelling unit structures,\(^{25}\)

(4) median family income,

(5) percent of dwelling units with more than 1.01 persons per room, and

(6) a dummy variable to allow for topography differences.

\(^{24}\) The accessibility potential index was based on employment opportunities available in the vicinity of each block. It varied directly with the number of employment positions surrounding the block and inversely with the distance between the block and these employment positions (Brigham, 1965, p. 328).

\(^{25}\) Used as a measure of crowding within dwelling units.
The topography dummy variable was included as an amenity variable, but it also reflects costs of development.

The value per square foot of the land was regressed on the set of attributes. It was found that distance to the CBD, the median family income, building value, and the topography dummy variable explained the most variation \( (R^2 = .87) \) in land values along one ray, and the accessibility index, percentage of dwelling units with more than 1.01 persons per room, building value, and the topography dummy explained the most variation \( (R^2 = .89) \) along the other ray. All of the independent variables, except median family income, had the expected sign.

Harris et al. (1968) studied variations in the amenity value of residential land in Raleigh, North Carolina. Twenty clusters of three to five contiguous blocks were randomly selected and the average land value per square foot was estimated for each cluster using tax appraisal records. The average actual travel costs (including monetary value of time spent in travel) of each household were estimated for the households in each cluster using household survey data. Average estimated travel costs from the travel margin (defined as the point beyond which household travel savings have no effect on urban land values—that point on the city periphery where land has only agricultural value) were also estimated for the households in each cluster utilizing the information about household travel habits obtained
by the survey. The travel savings value of land per square foot was then defined, for each cluster, as the capitalized difference between the estimated average travel costs from the travel margin and the average actual travel costs of the households in the cluster. The average amenity value per square foot was then defined to be, for each cluster, the average value of the land per square foot minus the average travel savings value of the land per square foot for the households in the cluster minus a constant (equal to the estimated agricultural value of the land per square foot plus the cost per square foot of the installed utilities).

Several factors thought to affect amenity levels in each cluster were also measured. The explanatory variables considered were:

1. average lot size (considered a measure of population density),
2. several dummy variables denoting the zoning in and around each cluster,
3. a dummy variable for the social class predominance in each cluster,
4. the percent the largest social class was of all residents,
5. the percent upper class residents were of all residents,
6. the percent of occupants who were tenants,
7. the percentage of housing units receiving enough maintenance to preserve their present condition, and
(8) the average value of land and buildings in each cluster.

Harris, Tolley, and Harrell (1968) then regressed the average amenity value per square foot on the measured levels of the neighborhood (cluster) amenity variables. The set of statistically significant variables explaining the most variation in amenity values was comprised of average lot size, the percent of the cluster occupants who were tenants, the percent which upper class residents were of total residents in the cluster, a dummy variable indicating that the cluster was zoned exclusively for residential use and was surrounded by areas similarly zoned, and a dummy variable indicating that the cluster was zoned for residential use but bordered on an area zoned to nonresidential use. They reported that all variables had the expected sign except average lot size\(^2\) and the dummy variable indicating that a cluster and the area bordering it were zoned for residential use only; both of these variables were of questionable statistical significance.

Brodsky (1970), using property tax assessed land value and the census tract as a unit of analysis, regressed the average residential

\(^{26}\)Lot size had a negative sign; Harris et al. (1965, p. 245) expected a positive sign. The author would argue that the expected sign is negative given that they are regressing amenity value per square foot on lot size. Each additional square foot of lot would be expected to add less amenity value than did the previous assuming a diminishing marginal utility of lot (or space). Had they regressed total value per lot on lot size, the expected sign would be positive.
land value per square foot in each Washington, D.C. census tract
(with the exception of CBD and institutional tracts) on the tract's dis-
tance from the CBD, its population density, and its median family
income (used as a measure of neighborhood amenity). All variables
were significant at the one percent level of significance in a double-
logarithmic equation explaining 86  \((R^2 = .86)\) percent of the valua-
tion in land values per square foot. The coefficients of income and
density had positive signs, and the coefficient of the distance variable
was negative.

The studies just reviewed point out one problem faced by most
empirical land value (and housing value) studies. The actual market
price and associated attributes of urban property are frequently either
not available to or not readily obtainable by the researcher. If the
actual sales price is available, it is often the case that there are not
a sufficient number of transactions during any given time period and/or
sufficient variation in the attributes of the traded properties to pro-
vide a satisfactory sample. As a consequence, the researcher is often
required to resort to either tax assessment records or aggregate
data, such as the U.S. Census of Housing census tract data, to obtain
information on the value of properties and their associated attributes.

The use of the appraised value of the property in lieu of market
value presents two problems. First, usually not all property in an
area is appraised on a single date; as a consequence, the researcher
must adjust the value of properties appraised at different times for temporal changes in their value. Secondly, the tax appraisers may not assess the properties at either market value or at a consistent proportion of market value. In short, the use of appraised values, rather than market values, may introduce extra-market factors (usually unspecified) into the problem.

In regression studies, such as the three just reviewed here, the use of aggregate data deliberately grouped so as to minimize the within-group variation in the individual observations will lead to a higher $R^2$ than would have been obtained by using individual observations (Johnston, 1972, pp. 228-232). It is worthy of note that Brigham deliberately used a five-block moving average to reduce "spurious variations" in land values (Brigham, 1965, p. 331) and Harris et al. calculated averages for their clusters because "an average for a group of similar blocks tends to minimize effects of assessment errors due to qualitative differences of individual properties" (Harris et al., 1968, p. 244). Brodsky used census

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27 In a study by Berry and Bednarz (1975), wherein both market price and assessed value were regressed, in turn, on a set of attributes of a sample of Chicago residences, the attributes of the residences explained more of the variation in market price than in assessed value.

28 Brown and Nawas (1972) show that the use of aggregate data can also lead to problems of multicollinearity among the explanatory variables if the data are grouped non-randomly.
tract averages in his study while noting that (Brodsky, 1970, pp. 236-237)

The effect of shape and size of lot, . . . , distance from locally desirable or undesirable land uses . . . should, for the most part, cancel out in census tract averages. These localized effects are important to an understanding of the real estate market for individual houses but . . . may serve only to obscure the broader aspects of market behavior upon which existing land value theory is based.

Indeed, census tracts are defined so as to encompass areas "relatively homogeneous in population characteristics, economic status, and living conditions" (U.S. Bureau of the Census, 1969, p. 18). Thus, though the variables used in the studies just reviewed did, in each case, explain a large percentage of the variations in land values, the use of aggregate data makes it difficult to discern the extent to which their effectiveness as explanatory variables is more apparent than real.

Each of the three land value studies reviewed herein experienced problems of collinearity between their explanatory variables. In some cases, there were reasons to believe, a priori, that the explanatory variables would be correlated.

Harris, et al. (1968, p. 245) dropped, among others, the variable representing the average value of land and structures in each cluster because of multicollinearity problems. It might have been hypothesized, a priori, that the average value of land and structures is a function of average lot size and of the proportion of occupants of
the clusters who were members of the upper class (presumably the classes were stratified by income)—two explanatory variables that remained in the final regression equation.

Similar comments apply to Brigham's (1965) final equations, as he includes the average appraised value of the structures, the median family income, and a measure of dwelling unit crowding for each block as explanatory variables. It should not be unexpected that median family income and the value of the structures are correlated—the income elasticity of demand for housing has been estimated to be approximately unitary (Muth, 1960, p. 408). Similarly, it could be hypothesized that lower income families tend to crowd dwelling units more than do higher income families and that, therefore, the percentage of upper class occupants in a cluster would be negatively correlated with the percentage of the dwelling units that are crowded.

Brodsky (1970, p. 239) also reported a high correlation between two of his explanatory variables: density and distance. Again, there is reason to expect this relationship. Muth (1969, pp. 70-93) developed a theoretical link between land value, distance from the CBD and population density, and has estimated the density gradients (distance-density relationships) for 46 U.S. cities (Muth, 1969, pp. 139-145). In 40 of the cities, there was a statistically significant (.01 level) decline in population density with distance from the CBD. A linear regression of density on distance explained about one-half of the
variation in density among census tracts in the 46 cities. It is not surprising, therefore, that Brodsky found a high correlation between density and distance from the CBD. 29

The result of the multicollinearity problems in the three studies reviewed here is to cast doubt on the precision of the coefficients of the explanatory variables, and, particularly, on the significance of their relative magnitudes. Brigham (1965, p. 324) noted that the coefficients in his model "are not stable from ray to ray and (that) this suggests (that) factors not included in the model are at work." It is worthy of note that multicollinearity between explanatory variables is sufficient to cause the estimates of the coefficients to be "very sensitive" to different samples of data (Johnston, 1972, p. 160).

The primary reason for multicollinearity between the explanatory variables may be that these variables are, in many land value studies, endogenous variables in the complex system that is the residential land market. In reality, land values and many of the variables thought of as determinants of neighborhood quality, such as density, average value of structures and socio-economic

29 It should be noted that the results of Muth's (1969) work were probably not available to Brodsky (1970) at the time he was conducting his study. Further, the multicollinearity problem is not likely to be serious if the explanatory variables do not have a highly linear relationship. Neither casual observation nor the theory suggests a linear relationship--though it does postulate a theoretical link between distance from the CBD and population density.
agglomeration, may be simultaneously determined. However, if the explanatory variables, though endogenous, do not have a highly linear relationship, multicollinearity will not be a serious problem.

But serious errors in the estimation of a single regression equation may result if the dependent variable and one or more of the explanatory variables are, in fact, more accurately defined as endogenous variables in a more complex system. Brodsky’s (1970) study, reviewed above, provides an excellent example of a case where, at least, one explanatory variable, population density, would have been more appropriately considered as an endogenous variable in a larger system. He regressed land value per square foot on density, but the line of causation may run both directions. Increased land values may result in more intensive residential use of the land (smaller lot sizes and, therefore, more households) and, consequently, a higher population density. But the higher population density may decrease neighborhood quality and thus lower land values. Although higher land values are generally associated with more intensive use, this is no indication that both of the processes described do not exist. The single ordinary least squares regression of land value on density ignores the simultaneity of the process. Similar criticism applies to Brigham’s (1965) use of the average appraised value of the

\[30\] This is not to say that time is not a factor; the process determining residential land use and values are surely dynamic.
structures, the median family income, and the percent of the dwelling units that are crowded on each block as independent explanatory variables. Also, Harris, et al. (1968, p. 245) point out that the line of causation between amenity value per square foot and both percent tenancy and percent upper class may point both directions. The estimation of a single equation system, rather than a multiple equation system describing the relationships between all of the endogenous variables, may result in biased and inconsistent estimates of the coefficients of the explanatory variables (Johnston, 1972, p. 343).

Though only three residential land value studies have been reviewed, there exists a substantial number of such studies. For example, Mills (1969) studied the relationship between urban land values and, primarily, distance from the CBD and zoning in Chicago. Czamanski (1966) examined the relationship between public investments and urban land values in Baltimore. Adams et al. (1968), Clonts (1970), Husak (1975) and Weiss et al. (1966) have studied the relationship between various factors and land values on the urban periphery. Many of the comments made concerning the three studies reviewed apply also to these studies.

**Empirical Studies of the Determinants of Urban House Prices**

For varying reasons, several studies have been conducted that explore the relationships between house prices (including the value of
site), the characteristics of the residence (e.g., floor area, age, lot size), and its locational attributes (e.g., various measures of neighborhood quality, air pollution, and accessibility). However, as many of the studies have a common methodology and common problems, it will be sufficient to review only a select few here. A comprehensive review of ten studies of urban house prices that were conducted between 1968 and 1973 has been provided by Ball (1973); these studies, with one exception, will not be reviewed here.

Musgrave (1969) examined the relationship between the sales price of new homes built for sale (i.e., neither built to lease nor for the exclusive use of the owner) and the characteristics of the houses per se in an attempt to develop a housing price index that would separate pure price changes from changes in housing quality. Though he did not consider locational attributes, his study is of interest because of its large sample size (Federal Housing Administration data on 7,000-10,000 new homes) and because he used actual sales price rather than assessed value as the dependent variable. From an apparently large list of characteristics, Musgrave selected the following eight after "a large number of regression runs using various combinations of characteristics and various functional forms of the regression" (Musgrave, 1969, p. 775):

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31 Unfortunately, the FHA data did not include lot size.
(1) floor area,
(2) number of stories,
(3) number of bathrooms,
(4) presence or absence of central air conditioning,
(5) garage or no garage,
(6) basement or no basement,
(7) location within the U.S. (in terms of 12 areas), and
(8) inside or outside a standard Metropolitan Statistical Area (SMSA).

Each of the eight characteristics was treated as a dummy independent variable in a linear regression equation which explained 71 percent of the variation \((R^2 = .71)\) in house prices.

Berry and Bednarz (1975) studied the relationship between the characteristics of 275 single-family dwellings and both their sales price and assessed valuation in an attempt to determine the equity effects resulting from a change in property tax assessment techniques in Chicago.

From 27 characteristics of the structure and lot, they selected the following explanatory variables after preliminary tests for collinearity and explanatory power:

**Housing Characteristics**

(1) floor area,
(2) age,
(3) lot area,
(4) dummy variable for air conditioning,
(5) dummy variable for improved attic,
(6) dummy variable for improved basement,
(7) number of bathrooms,
(8) dummy variable for garage,

**Neighborhood (census tract) Characteristics**

(9) median family income,
(10) percent of dwellings that are multiple-family dwellings,
(11) percent of families in tract living in a different tract five years before,
(12) black residents as percent of total population,
(13) Cuban/Mexican residents as percent of total population,
(14) Irish residents as percent of total population,

**Pollution**

(15) an air pollution index, and

**Accessibility**

(16) distance from downtown Chicago.

The characteristics of the houses and their sales price and assessed valuation were individually obtained, but the neighborhood characteristics were obtained from census data and pertain to the census tract in which each house is located. The air pollution variable pertains to the one-kilometer square in which the house is located.
The logarithms of sale price and assessed value were regressed, in turn, on the logarithms of the 16 explanatory variables. Seventy-nine percent of the variation \( R^2 = .79 \) in sales price and 64 percent of the variation \( R^2 = .64 \) in assessed value were explained by the estimated, double-log equations. Two of the ethnicity variables (black and Irish) and the migration variable changed from negative in the sale price regression to positive in the assessed valuation. The coefficients of both the pollution and the distance variable had the unexpected (positive) sign in both regressions.

Richardson, et al. (1974) examined the variation in house sale prices in Edinburgh, England, using several regression models. The sample of about 1200 houses, accounting for about one-fifth of the total residential properties sold in Edinburgh in 1966, was stratified by small zones of approximately 71 acres. The sale price of each house and, apparently, some of the characteristics of the house were obtained from a public land and property register. Unfortunately, both the description of the source of the observations on the explanatory variables and the definitions of these variables are somewhat vague--and in some cases nonexistent. For example, the apparently important variable "change in use" is not defined and is only alluded to in the discussion. Apparently many of the neighborhood characteristics are obtained from census data and pertain to the census enumeration district (ED) encompassing the zone in which a house is
located. The explanatory variables included in the "best fit" equation, a composite of several models designed to indirectly test different theories of residential location, were:

(1) house type (house of flat),

(2) age of structure,

(3) change in use (not defined),

(4) paid outright (perhaps a dummy variable indicating a cash payment for the dwelling unit—rather than a mortgage),

(5) Building Society finance (apparently a dummy variable indicating that the mortgage was carried by a building finance society rather than an alternative type of lender),

(6) owner-occupation (the proportion of dwelling units that were owner-occupied in, apparently, the ED in which the house is located),

(7) distance from the zone in which the house is located to the CBD,

(8) car ownership (the proportion of the residents owning automobiles in, apparently, the ED in which the house is located),

(9) zone in which the house is located is on the urban periphery (dummy variable?),

(10) social class (measurement device not described, but apparently based on occupation of residents of ED),
(11) no industry (apparently a dummy variable indicating no industry in the zone in which the house is located),

(12) rooms per person (not defined; an ED-wide measure?),

(13) percent of population under 15 years of age (in, apparently, the ED in which the house is located),

(14) percent of population of age 65 years and older (similar to above),

(15) open space (not defined),

(16) housing amenities (not defined), and

(17) direction NE (a dummy variable indicating that a house lies in the quadrant NE of the CBD).

A linear regression of house sale price on the 17 explanatory variables explained about 64 percent of the sample variation in sales price. All of the explanatory variables were statistically significant at the 0.1 significance level, though some, notably percent owner-occupied, had an unexpected sign.

Ridker and Henning (1967) studied the variation in house values in those St. Louis census tracts that had a 1960 population density of at least one person per acre and wherein at least 60 percent of the housing units were single-family dwellings. The study was conducted to examine the effects of air pollution on residential property values.

The dependent variable was median property value (median, owner-estimated value of owner-occupied single-family housing units),
and the explanatory variables in their "preferred" regression equation were the following:

1. an air pollution index,
2. median number of rooms per housing unit,
3. percent of housing units built between 1950 and 1960,
4. housing units per square mile,
5. a discrete variable denoting average rush hour travel time to CBD by express bus (residualized),
6. a dummy variable denoting whether or not census tract touched a major highway,
7. a dummy variable denoting whether or not a census tract was in a school district of above average quality,
8. ratio of number craftsmen, foremen, operatives, and laborers to total number of employed persons (residualized),
9. average number of persons per dwelling unit,
10. nonwhite owner and renter occupied housing units as percentage of all occupied units (residualized),
11. a dummy variable indicating whether or not property located in Illinois (residualized), and
12. median family income (residualized).

Though the data were obtained from diverse sources, all of the explanatory variables representing neighborhood characteristics, including air pollution, pertain to census tracts.
An attempt was made to correct for high multicollinearity using a technique referred to as "residualization". That is, a new variable was obtained by subtracting from the observed values of a variable the computed values obtained from a regression of that variable on the variables with which it is highly correlated. For example, median family income, used as a surrogate for those neighborhood characteristics not explicitly considered, was "residualized" against median number of rooms per unit, houses per square mile and the occupation ratio. The new "residualized" variable was then utilized in lieu of the original variable.

The "preferred" regression equation explained about 97 percent ($R^2 = .94$) of the variation in median property values. However, the high $R^2$ may be partly due to the use of grouped data.

Unexpected results included a positive sign for the coefficient of percent nonwhite, a negative sign for school quality, and the result that the sign and coefficients of the travel-time-to-CBD variable indicated that median property values decreased up to a travel time of about 28 minutes and increased thereafter. Air pollution had the expected (negative) impact on property values.

As with the land value studies, it is apparent that many of the variables that could be (and are) used to explain house prices are highly correlated. Though none of the studies reviewed here presented the matrix of simple correlation coefficients for the explanatory
variables included in their regressions, each of them mentioned problems of multicollinearity. To the extent that there is, a priori, reason to believe that a given set of variables is necessary (even if inter-correlated) for a properly specified model, the analyst is left with little choice but to attempt to correct for the multicollinearity by some method other than dropping variables from the set. 32 Ridker and Henning (1967), however, provide the only house price study reviewed here, wherein an attempt was made to correct for multicollinearity. Consequently, the coefficients of the explanatory variables in the regression equations must be interpreted with caution.

Further, there is reason to believe that there may be simultaneous equation bias in the estimated coefficients of the regression equations. For example, both Berry and Bednarz (1975) and Ridker and Henning (1967) include median family income in their models as a proxy variable for neighborhood quality. If households prefer to live, for whatever reason, in neighborhoods where median family incomes are higher, it is reasonable to hypothesize that they will bid up the prices of the houses in those neighborhoods. But in so doing, families with lower incomes may be excluded from buying a house in the neighborhood. Therefore, it is also reasonable to

32 For example, Brown (1973) and Brown and Beattie (1975) have proposed ridge regression rather than the omission of relevant variables (thus introducing specification bias) as one method of mitigating the problems of multicollinearity.
hypothesize that the incomes of families who live in the neighborhood are a function of house prices in that neighborhood. That is, the line of causation between neighborhood median family income and house price (and between neighborhood median family income and land value) runs both ways. The least squares estimates of the coefficients of single equation systems, which ignore the simultaneity of house price and neighborhood median family income, will be biased and inconsistent.

Because they hypothesized that "rich households will spend more than poor on housing and transportation expenditures," Richardson et al. (1974) include a surrogate ("car ownership") for zonal or neighborhood income in their "best fit" equation. However, household income should not be included in a hedonic price regression designed to estimate the market implicit price of the determinants of total house (or land) prices. As will become more apparent in the next chapter, the hedonic price equation is solely a creature of market equilibrium relating market demand for and supply of each of the house's characteristics and locational attributes. In a hedonic price regression, the coefficients of the variables will reflect the market valuation of the determinants as determined, in part, by the distribution of income among the buyers. Inclusion of household income as a variable in the regression constitutes a mis-specification of the model
and will result in biased coefficients on the explanatory variables.  

The "best fit" equation in Richardson et al. (1974) might be regarded as an attempt to estimate an expenditure equation.  

Further, the inclusion of median family income in the equation estimated by both Ridker and Henning (1967) and Berry and Bednarz (1975), though a surrogate for neighborhood quality, makes it difficult to distinguish them from expenditure equations—especially the former as its dependent variable was median property value.  

As mentioned previously, many other studies attempting to determine the relative importance of house price determinants have

---

33 Ball (1973, p. 224) also makes this point in his review of Anderson and Crocker's (1971) study of the effects of air pollution on residential property values in St. Louis. Ball states that the "income coefficient (in the regression) equation cannot be regarded as reflecting all the impact on house prices of differences in income. Each characteristic will have a different income elasticity, and, consequently, the relative prices of characteristics will, in part, depend on the level and distribution of income. . . . It seems more plausible to treat the price a household is prepared to pay for a house as being the summation of the valuations of the characteristics embodied within it. . . . However, in no way can income be treated as entering this demand equation directly; instead its influence is indirect, via its effect on the valuation of characteristics."

Ball (1973, p. 231) later recognizes that the estimated regression equation is not a demand equation.

34 If so, the variables "paid outright" and/or "Building Society finance" might be interpreted as an indication of the asset position of the home buyer. But their interpretation of the negative sign on the coefficient of the "Building Society finance" variable as indicating "that the main alternative source of finance, insurance companies, favoured the more expensive house" (Richardson et al., 1974, p. 196) is inexplicable.
been conducted. A number of studies, Kain and Quigley (1970), Wilkinson (1973) and Davies (1974) among them, have used factor analysis to group related neighborhood characteristics and locational attributes into factors. The house prices were then regressed on the individual factor scores to obtain the implicit prices of these factors. A problem with this approach is that the factors, and therefore their estimated coefficients (implicit prices), must be subjectively interpreted. In addition, the results are not easily duplicated as the choice of factors (groupings) is arbitrary and relies on the judgement of the analyst.

**Summary of the Empirical Studies of the Determinants of Urban Property Values**

Though the specific explanatory variables used as determinants of urban property values vary between studies, there seems to be general agreement as to the generic description of the variables that should be included in a properly specified model---namely, the characteristics of the house *per se*, socio-economic characteristics of the neighborhood, measures of accessibility and, less generally, characteristics of the physical environment (topography, pollution, etc.). In many cases, the specific characteristics used as explanatory variables are, to some extent, predetermined by the data available to the analyst.
The high \( R^2 \) achieved in each of the studies attests to the effectiveness of the specific variables used as determinants in each case. However, those studies explaining the most variation in house prices did so with the use of grouped data, and thus the \( R^2 \) may be inflated beyond that which would have been attained with the use of individual observations.

The estimated implicit prices of the determinants of urban property values must, in most cases, be interpreted with caution because of the multicollinearity between the explanatory variables and, especially, because of the possibility of simultaneous equation bias in the estimated coefficients. In many of the studies reviewed, the line of causation between the dependent variable and one or more of the explanatory variables could reasonably be hypothesized to point in both directions.

Further, both the absolute and the relative magnitudes of the estimated implicit prices of the determinants apply only in the market, or city, for which they were estimated. Ball (1973, p. 231), Richardson (1974, p. 197), and others have emphasized that the hedonic price function which they estimated is not a demand function but is, rather, a reduced form equation for some unspecified structural model relating housing (or residential land) demand and supply in a particular city. Therefore, even if there were reason to believe that a particular housing (or residential land) demand function had
some validity beyond the specific city for which it was estimated, there is no reason to expect that the implicit prices resulting from the interaction of the supply and demand in a particular city have any validity beyond that market (or point in time) for which they were estimated.

Under certain assumptions (which will be made explicit in the next chapter), the price functions estimated in these urban property value studies are market equilibrium functions relating the implicit prices of the attributes of the property and the attributes per se to the market price of the property. The magnitude of the implicit prices, as with all prices, is determined jointly by the aggregate demand for and supply of the various attributes. Thus, their magnitudes, both absolute and relative, are only partially a reflection of consumer preferences; they also depend upon the distribution of income within the market and the costs of providing the attributes.

**Summary**

The economic theory of residential location as developed, primarily, by Wingo, Alonso and Muth is a derivative of the von Thunen paradigm. Like the von Thunen prototype, it focuses on accessibility (transportation costs) to a single central point where all employment is concentrated and ignores all variations in the urban environment. Housing is assumed to be a homogeneous good--whatever
its location. The household's only choice is to locate either farther from the CBD where land (housing) is less expensive but commuting costs are higher or closer to the CBD where land (housing) is more expensive but commuting costs are lower. Given a set of assumptions about the household's preferences concerning land (housing), the utility of time spent in travel and/or the time costs of commuting, the direct costs of travel, the price of all goods (including land or housing), and the household's income, it is possible to determine the outcome of the trade-off between land (housing) and travel (distance from the CBD). Given further assumptions concerning the distribution of income and preferences, it is possible to make conclusions concerning the spatial utilization of urban land.

The empirical studies of the determinants of urban residential property values recognize, or confirm, that many aspects of the urban environment, other than accessibility, enter the individual residential location decision calculus. These studies estimate a market equilibrium price function relating the equilibrium values (implicit prices) of the attributes of the property and the attributes of the property per se to the total market price of the property. Among the variables found to be important in explaining the variation in residential property values are those describing the residence itself, those describing the socio-economic characteristics of the neighborhood within which the residence is located, and, less generally, variables
describing the physical environment in the neighborhood of the
residence. A measure of accessibility is generally included in the
price function along with the other determinants of urban property
values.

The existing economic theory of residential location is, clearly,
a macro-spatial model. Its objective is to provide a theory explaining
existing patterns of urban land use--more precisely, a theory explain-
ing urban residential settlement patterns. Further, the implicit
prices of the determinants of residential property values (i.e., the
implicit value of factors which households' presumably consider of
some importance when choosing a residence) as estimated by hedonic
price regressions are market prices. Both the implicit prices and
the residential settlement patterns are the macro result of micro
decisions made by both the suppliers and the buyers of housing.

The implicit prices of the attributes of a residence are the
result of the interaction of the demand for and supply of these attrib-
utes. Very little has been done to explore these forces underlying
the individual residential location decision. 35 The next chapter

35 A number of empirical studies have estimated the demand for
housing (considered as a homogeneous good), where the quantity of
housing consumed was measured by the household expenditure on
housing (e.g., Muth, 1960, 1971; Malone, 1966; Reid, 1962; deLeeuw,
1971).

Straszheim (1973) estimated the demand for selected attributes
of housing including the number of rooms in the dwelling unit, the age
provides a theoretical framework of the housing market. Though too complex to permit a market equilibrium solution for residential settlement patterns, it provides a basis for empirical exploration of the economic forces underlying the individual residential location decision.

Granfield (1974) provides a three equation, recursive, simultaneous equation system wherein the household first chooses an annual expenditure on housing, then chooses a location (distance from the CBD) and, finally, chooses a type of residence (single-family, duplex, or multifamily). Annual expenditure on housing is a function of household income, median income of census tract in which located, and occupational class of head of household. Location is a function of annual housing expenditure, race, median income of census tract, the cost of commuting, and mode of travel. The type of residence is a function of annual expenditure, location, and family size.
III. THE DEMAND FOR AND SUPPLY OF THE CHARACTERISTICS OF A RESIDENCE AND THE RESIDENTIAL LOCATION DECISION: A THEORETICAL FRAMEWORK

The purpose of this chapter is to develop a theoretical framework that can be used to explain urban residential land values and the individual location decision. To facilitate understanding of the theory, a basic model is first introduced and explored. Then the model is expanded to include the effect of property taxes, publicly provided services, and commuting costs on the household's residential choice. The model is also expanded to include the effects of zoning and certain residential development regulations and taxes or fees on the suppliers of residences.

The Problem: Buyers and Sellers

The individual (head of household) who is choosing a residence is assumed to have in mind a set of characteristics that he would like the residence to have and an idea of what those characteristics are worth to him. Given all available choices, it is assumed that the individual can make a bid on each residence based on its characteristics. The bid is assumed to depend upon the type and level of the characteristics comprising the residence as well as on the bidder's preferences and income.
For each individual, it is assumed that there is some level of bid beyond which he would be worse off with the residence and less money to spend for other purposes than he would be without the residence and all of his money to spend for other purposes—including some other residence. It is assumed that the individual will not exceed the bid that would, if successful, make him worse off than he would be if he had not made that bid.

The suppliers of residences are assumed to have an array of residences with varying characteristics that they offer for sale at differing prices. The offer price is assumed to depend upon the type and level of the characteristics comprising the residence and the cost of producing those characteristics. The costs are assumed, in turn, to be influenced by the nature of the developer-seller firms (their production functions) and the price of the factors used to produce the residence. It is assumed that the seller will not offer the residences for sale at some price which, if the offer is accepted, will result in a lower level of profit than the seller could have otherwise realized—perhaps by a higher offer or by, initially, putting his resources into a different combination of characteristics to produce a different residence.

The market price of the residence is determined by the bid-offer process. The buyer must bid the amount at which the residence if offered for sale or exceed the amount bid by the highest bidder,
whichever is greater, if he is to take possession of the residence. The market price is determined by the offer and the highest bidder, and it will depend upon the costs of producing the residence and the preferences and income of the highest bidder.

Given this overview of the bid and offer process which results in a sale, or market, price for residences, a more technical exposition is developed and the concepts of bid price and offer price are related to the traditional concepts of demand and supply.  

The Residence and the Market Price Function

The Residence

Any given residence \((Q)\) is defined by the flow of services \((q_i)\) of the various \((n)\) housing and location related characteristics \((Q_i)\) comprising the residential package. Then any given residence with \(n\) characteristics is defined by the vector:

\[
Q = (Q_1, Q_2, \ldots, Q_n), \quad i = 1, \ldots, n.
\]

36 In reality, the bid-offer process may be iterative.

37 The theoretical framework of the residential location model presented herein is based on the general theory of hedonic (or implicit) prices developed by Rosen (1974). In this chapter, his general model is adapted to the market for urban residences.
Similarly, the flow of services provided by the residence is given by the vector:

\[(2) \quad q = (q_1, q_2, \ldots, q_n), \quad i = 1, \ldots, n.\]

The \( q_i \) are measured such that they, in general, have positive marginal valuations by consumers and thus may be treated as goods. It is assumed that there is a sufficiently large spectrum of residences available so that the elements of \( q \) may be considered to be independently and continuously variable.

The characteristics are categorized by the market and nonmarket, nonpublicly provided characteristics, the publicly provided characteristics of the residence, and the accessibility of the residence to various locations and services. The market provided characteristics consist of the measurable, observable, characteristics of the house and lot--such as floor area, lot size, number of bathrooms, and size of the garage. The nonmarket, nonpublicly provided characteristics consist of the socio-economic and physical attributes of the neighborhood in which the residence is located--examples of which are the racial and occupational composition of the neighborhood in which the residence is located, the average value of the structures within the neighborhood, the state of repair of the neighboring structures, the tenancy status (whether renters or owners) of the occupants of the neighborhood, and the levels of air and noise.
pollution in the neighborhood. The publicly provided characteristics consist of such items as the availability of public water and sewer, police and fire protection, quality of schools, etc. The final category of characteristics, accessibility, has both a time and a distance dimension. Most individuals desire to be within a "reasonable" commuting time and distance of their place of employment. In addition, individuals may desire to be close to (or distant from) relatives, open space, natural attractions, and other places and/or features on the settlement plain. Any, or all, of the characteristics of the residence might enter the utility function of the individuals attempting to choose a residence and thus, to the extent that these characteristics vary from location to location, the individual's utility will vary between alternative locations on the settlement plain.

38 At this point, the distinction between market and nonmarket, nonpublicly provided goods as one category of characteristics and publicly provided goods as another is not particularly useful, but it becomes a convenient dichotomization when the property tax is added to the model later in the chapter.

The term "nonmarket, nonpublicly provided" is somewhat confusing. It is intended to describe those characteristics which are not explicitly market goods in the sense that a developer intentionally produces them for sale but are, rather, the macro result of the individual buyer and supplier decisions (e.g., the socio-economic characteristics of the neighborhood). However, certain of these characteristics may be provided by a developer if he produces the entire neighborhood—for example, the average value of the structures within a new subdivision produced entirely by one developer.

Finally, certain of the characteristics do not always fall into the same category. For example, water and sewer facilities may be supplied by some developers at some locations and thus would be classified as market provided rather than publicly provided.
The Market Price Function

Each residence has both a final sale or market price and a fixed value of the vector $q$. Therefore, the market reveals a price function of the form:  

$$ (3) \quad P(q) = P(q_1, q_2, \ldots, q_n), \quad i = 1, \ldots, n. $$

For purposes of exposition, it is assumed that this function possesses continuous first and second-order derivatives. Further, as it has been assumed that the $q_i$ are measured such that they all have positive marginal valuations, it is assumed that the $\partial P / \partial q_i > 0$ for $i = 1, \ldots, n$. In the following section of this chapter it is assumed that both the homebuyers and the developers have knowledge of this price function and take it as given and beyond their individual influence.

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39 As used herein, "price" does not refer to the capitalized value of the residence; it refers to the annual payment for the use of the services of the residence. Market value is the capitalized value of the stream of all expected future annual payments. As stated in Chapter II, the factors influencing expectations and capitalization (or discount) rates are generally ignored in this study. Similarly, the complications of the theory of capital and investment introduced by the possibility of the resale of used residences are ignored so as to simplify the exposition.

40 Initially, the existence of a market price function is taken as a maintained hypothesis and its theoretical and technical underpinnings are hypothesized. The technical conditions for a market equilibrium, prerequisite to the existence of a stable price function, have been discussed by Rosen (1974), and will not be repeated here. The existence of a price function and its empirical form are the subject matter of the following chapters.
The Demand for Location Related Goods

The Household

The household is defined by its utility function, or by the utility function of the head of the household who is assumed to have the best interests of the household in mind, and by its level of income. Let the household's utility function be given by:

\[ U = U(q_1, q_2, \ldots, q_n, x); \]

where \( x \) is the level of "all other goods" \((X)\) consumed by the household. Let the price of \( X \) be unity. Let the household's disposable (after tax) income be \( y \), and assume that the household purchases only one residence defined by \( q^* = (q_1^*, q_2^*, \ldots, q_n^*) \), where the \( q_i^* \) are chosen to maximize the utility function \((4)\) subject to the budget constraint:

\[ y = P(q) + x. \]

The necessary first-order conditions for a maximum are given by:  

Equations (6) and (7) are formed from the partial derivatives (with respect to each of the \( q_i \) and \( x \)) of the Lagrangian \((L)\):

\[ L = U(q_1, q_2, \ldots, q_n, x) + \lambda(y - P(q) - x), \]

where \( \lambda \) is the Lagrangian multiplier. They, with the additional requirement that \((y - P(q) - x = 0)\), constitute the first-order conditions for an income-constrained utility maximum. The second-order conditions, not explicitly considered here, must also hold. See Henderson and Quandt (1971, pp. 405-406).
Equations (6) state that, for each of the \( q_i \), the marginal rate of substitution of \( Q_i \) for the numeraire \( X \) (i.e.,
\[
\frac{\partial U}{\partial q_i} / \frac{\partial U}{\partial x} = \frac{\partial P}{\partial q_i}, \quad i = 1, \ldots, n
\]
and
\[
\frac{\partial U}{\partial q_i} / \frac{\partial U}{\partial q_j} = \frac{\partial P}{\partial q_i} / \frac{\partial P}{\partial q_j}, \quad i, j = 1, \ldots, n, \quad i \neq j.
\]

Equations (6) state that, for each of the \( q_i \), the marginal rate of substitution of \( Q_i \) for the numeraire \( X \) (i.e.,
\[
(\frac{\partial U}{\partial q_i} / (\partial U / \partial x))
\]
must equal the market implicit price of the \( Q_i \) (i.e., \( \frac{\partial P}{\partial q_i} \)). For example, if one of the \( Q_i \) is accessibility to place of employment, the rate at which the individual is willing to substitute accessibility for other consumption activities (other than residential) must equal the market's assessment of the contribution of the accessibility of a particular residence to that residence's market value. In general, if \( X \) is considered as money, then the rate at which the individual is willing to substitute any given \( Q_i \) for money must equal the market's assessment of the rate at which that \( Q_i \) contributes to market price. This must be the case since if the individual's marginal bid were less than the implicit market price for any given level of \( Q_i \), the individual would not obtain that \( q_i \). If the individual's marginal bid were, \textit{ex ante}, greater than the implicit market price, it would, \textit{ex post}, become the market implicit price--assuming a perfectly competitive market for the \( Q_i \). The implications of Equations (6) are stressed by considering the household's bid
function and its derivative—the (inverse) compensated demand function.

**Bid Functions.** Following Alonso (1964; p. 59), a bid function for an individual is defined as the set of maximum prices for residences that he could pay given various combinations of the characteristics comprising the residential package while maintaining a constant level of utility. That is, the bid function gives the maximum amount the individual could bid for any given residence such that he would be neither worse off nor better off if the bid were successful than he would be without the residence and with the amount of the bid to be spent for other purposes.

More explicitly, define an indifference surface

\[ U(q_1, q_2, \ldots, q_n, x) = u \]

---

42 The concept of a bid (or value) function has been primarily used in urban economics. See, for example, Alonso (1964) and Nelson (1972).

The bid function is not necessarily observable from the bidding (and counter-offers) that occur in a market transaction. However, points on an individual's bid function for a particular object are observable at an auction if the bids are increased incrementally. The bid level beyond which a bidder drops out of the bidding defines a point on his bid function--up to that level he feels he would be better off without his money and with the object of the bid; beyond that level he apparently feels that he would be better off without the object and with his money to spend for other purposes. The highest bid may or may not be a point on the highest bidder's bid function. The second highest bidder may have dropped out before that level was reached, and the successful bidder may retain some consumer's surplus.
on which the individual is indifferent between various combinations of the characteristics \((Q_i)\) and \(X\). As in the usual optimization problem of the household, introduce the budget constraint:

\[
y = \beta(q) + x.
\]

Let \(\beta(q)\) be the amount of the bid and \(y\) and \(x\) be as previously defined. From (9), (8) may be written as:

\[
U(q_1, q_2, \ldots, q_n, y - \beta(q)) = u
\]

The bid function is obtained by solving (10) for \(\beta(q)\):

\[
y - \beta(q) = U^{-1}(q_1, q_2, \ldots, q_n, u)
\]

or

\[
\beta(q) = \beta(q_1, q_2, \ldots, q_n, u, y).
\]

Partial differentiation of (10) with respect to \(q_i\) gives:

\[
\frac{\partial U}{\partial q_i} + \frac{\partial U}{\partial x} \frac{\partial x}{\partial q_i} \frac{\partial \beta}{\partial q_i} = 0 \quad \text{for } i = 1, \ldots, n.
\]

Therefore,

\[
\frac{\partial \beta}{\partial q_i} = \frac{\partial U/\partial q_i}{\partial U/\partial x} > 0 \quad \text{for } i = 1, \ldots, n,
\]

since, from (9), \(\partial x/\partial \beta = -1\). Also, given the usual assumption that
\( \frac{\partial U}{\partial q_i} \) and \( \frac{\partial U}{\partial x} > 0 \), and that the utility function is strictly concave, then:

\[
(14) \quad \frac{\partial^2 \beta}{\partial q_i^2} = \frac{1}{(\partial U/\partial x)^3} \left[ (\frac{\partial U}{\partial x})^2 \frac{\partial^2 U}{\partial q_i^2} - 2 \frac{\partial U}{\partial q_i} \frac{\partial U}{\partial x} \frac{\partial^2 U}{\partial x^2} + (\frac{\partial U}{\partial q_i})^2 \frac{\partial^2 U}{\partial x^2} \right] < 0.
\]

Equations (13) and (14) state that the bid for any and each of the \( Q_i \) will increase at a diminishing rate as the \( q_i \) increases.

Note, especially, that the bid function refers specifically to one individual, one level of utility, and one level of income. If the individual's level of utility and/or his income changes, the bid function will change. Income held constant, a higher \( \beta(q) \) will result in a lower \( x \) and thus be associated with a lower level of utility \( u \).

Specifically, implicit differentiation of (10) with respect to \( u \) gives:

\[
\frac{\partial U}{\partial x} \frac{\partial x}{\partial \beta} \frac{\partial \beta}{\partial u} = 1.
\]

Therefore:

\[
(15) \quad \frac{\partial \beta}{\partial u} = -\frac{1}{\partial U/\partial x} < 0.
\]

Also, as income increases, all other things— including utility—held

\[\text{43} \quad \text{Strict concavity of the utility function requires that the Hessian matrix of second-order partial derivatives be negative definite (Intriligator, 1971, p. 147). See also Henderson and Quandt (1971, pp. 15-16).}\]
constant, all additional income will go toward higher bids.\footnote{Otherwise, it could only go toward expenditure on $x$ and consequently, $u$ would increase contrary to assumption.} Implicit differentiation of (10) with respect to $y$ gives:

$$\frac{\partial U}{\partial x} \left( 1 - \frac{\partial \beta}{\partial y} \right) = 0,$$

since, from (9),

$$\frac{\partial x}{\partial y} = 1 - \frac{\partial \beta}{\partial y}.$$

Therefore,

(16)$$\frac{\partial \beta}{\partial y} = 1.$$

From these results, it is clear that the bid function only states that if the individual's income level were such, his utility held constant at such, then he would be willing to bid a maximum amount, $\beta(q)$, for a given residential package. If the individual's tastes and preferences, as given by his utility function, change, it is clear that the bid function will also change.

To further clarify the concept of a bid function assume that only $q_1$ and $x$ are allowed to vary, thus allowing a graphic depiction of the bid function. Further assume, for ease of exposition, that the $q_2, \ldots, q_n$ are held constant at their utility maximizing quantities given $P(q) = P(q_1, q_2, \ldots, q_n)$ and $y$. Let the constrained
maximum utility level be \( u^* \). Then:

\[
(17) \quad U(q_1', x; q_2^*, \ldots, q_n^*) = u^*
\]
or

\[
(18) \quad x = x(q_1'; q_2^*, \ldots, q_n^*; u^*)
\]

Assume, initially, that \( q_1 = q_1^* \) and \( x = x^* \) (Figure 4) and that the individual's bid is \( \beta(q^*) = y - x^* \).

![Figure 4. An indifference curve between one characteristic and the numeraire.](image)

If the individual has the opportunity to purchase a residence

(with \( q_1 = q_1'' \)) differing only from the utility maximizing residence

(with \( q_1 = q_1^* \)) only in that \( q_1'' > q_1^* \), the maximum additional
amount of $X$ that he would be willing to sacrifice (i.e., the increase in his bid) is $(x^*-x^*)$. His bid on the residential package with $q_1'' > q_1^*$ is:

\begin{equation}
\beta(q_1'') = \beta(q_1^*) + (x^* - x^*) > \beta(q_1^*),
\end{equation}

or, from (18):

\begin{equation}
\beta(q_1'') = \beta(q_1^*) + (x^* - x(q_1'')),
\end{equation}

where

\begin{equation}
\beta(q_1^*) = y - x^*.
\end{equation}

Similarly, if the individual were offered the alternative $q_1' < q_1^*$, his bid would be:

\begin{equation}
\beta(q_1') = \beta(q_1^*) + (x^* - x') < \beta(q_1^*)
\end{equation}

or

\begin{equation}
\beta(q_1') = \beta(q_1^*) + (x^* - x(q_1')).
\end{equation}

Given the initial bid $(\beta(q_1^*))$, the utility level $(u^*)$, and income $(y)$, the bid function can be written as:

\begin{equation}
\beta(q_1) = \beta(q_1^*) + (x^* - x(q_1)).
\end{equation}

The bid function corresponding to the indifference curve depicted in Figure 4 is given in Figure 5.
Given the usual assumption of strict concavity of the utility function, the bid function will be convex, as shown in Figure 5. This can be seen by total differentiation of (17):

\[(24) \quad \frac{\partial U}{\partial q_1} dq_1 + \frac{\partial U}{\partial x} dx = 0\]

or

\[(25) \quad - \frac{dx}{dq_1} = \frac{\partial U / \partial q_1}{\partial U / \partial x} > 0\]

by assumption. Differentiating (23):
The positive slope of the bid function follows from the assumption, made above, of the positive marginal evaluation of the $Q_i$. Further, if strict concavity of the utility function is assumed, then from (25):

\[
\frac{d^2 x}{dq_1^2} = \left(\frac{\partial U}{\partial x}\right)^2 \frac{\partial^2 U}{\partial q_1^2} - 2 \frac{\partial U}{\partial q_1} \frac{\partial U}{\partial q_1 \partial x} + \left(\frac{\partial U}{\partial q_1}\right)^2 \frac{\partial^2 U}{\partial x^2} < 0.
\]

Therefore,

\[
\frac{d^2 \beta(q_1)}{dq_1^2} < 0.
\]

Inequalities (26) and (28) state simply that the individual's bid for characteristic $Q_1$, all other characteristics, income, and utility held constant, will increase at a diminishing rate as $q_1$ increases.

**Inverse Compensated Demand Functions.** The first partial derivatives of the bid function with respect to any one of the $q_i$ [Equation (13)] gives the inverse of the individual's income compensated demand function for that $q_i$. The compensated demand function for $Q_i$ gives the level of $Q_i$ that would be purchased (and consumed) if the market implicit price of $Q_i$, $\partial P/\partial q_i$, were some specified amount and if utility were held constant at a specific level. Conversely, the inverse compensated demand function gives the
implicit price, \( \partial \beta / \partial q_i \), necessary to induce the household to consume some specified level of \( Q_i \) if utility were held constant.

Equation (13) states that the inverse compensated demand function for the \( Q_i \) is the marginal rate of substitution between the \( i^{th} \) characteristic and the numeraire or money. The \( \partial \beta / \partial q_i \) gives the individual's marginal valuation of \( Q_i \) in terms of \( X \) at a given income and utility level and thus gives his demand price for an additional unit of \( Q_i \).

The effect of a change in income upon the household's implicit price for a characteristic, ceteris paribus, is obtained by differentiation of (13) with respect to income:

\[
\frac{\partial^2 \beta}{\partial q_i \partial y} = \frac{\partial^2 U}{\partial q_i \partial x} \frac{\partial x}{\partial y} - \frac{\partial^2 U}{\partial q_i \partial x^2} \frac{\partial x}{\partial y} = 0 \quad \text{for} \quad i = 1, \ldots, n,
\]

since \( \partial x / \partial y = 0 \) by the assumption that utility does not change and, thus, \( x \) and \( q \) are held constant. That is, a change in income, holding the level of utility \( (u) \) and \( q_i \) constant, will not change the household's implicit price for the characteristic \( Q_i \). 46 Thus,

\[45\] The same result is obtained by differentiation of (16) with respect to \( q_i \) since \( \partial^2 \beta / \partial y \partial q_i = \partial^2 \beta / \partial q_i \partial y \).

\[46\] This does not mean that the bid price \( (\beta(q)) \) will be invariant with respect to income. For any given level of a characteristic, the
the inverse compensated demand function for \( Q_i \), along which the level of utility is held constant, is invariant with respect to income.

However, an increase (decrease) in income will always increase (decrease) the level of attainable utility.\(^{47}\) Therefore, the direction (not the magnitude) of a change in the implicit price of a characteristic due to a change in the level of attainable utility resulting, in turn, from a change in income can be determined by differentiation of (13) with respect to the level of utility \( u \) while holding all elements of \( q \) and income \( y \) constant. Then:\(^{48}\)

\[
\frac{\partial^2 \beta}{\partial q_i \partial u} = \frac{\partial U}{\partial x} \frac{\partial^2 U}{\partial q_i \partial x} - \frac{\partial U}{\partial q_i} \frac{\partial^2 U}{\partial x \partial \beta} \frac{\partial \beta}{\partial u} - \frac{\partial U}{\partial q_i} \frac{\partial^2 U}{\partial x \partial \beta} \frac{\partial \beta}{\partial u}
\]

for \( i = 1, \ldots, n \).

The bid associated with the new level of income will be given by the bid associated with the previous level of income plus a constant equal to the magnitude of the change in income (since, from (16), \( \partial \beta / \partial y = 1 \)). But the implicit price is independent of income since the demand curve is real income compensated and thus does not contain income as an argument.

\(^{47}\) It can be shown, from the first-order conditions for an income-constrained utility maximum that, in household equilibrium, \( \partial U / \partial x = \lambda = \partial U / \partial y \), where \( \lambda \) is the Lagrange multiplier in the Lagrangian equation formed from the utility function and the income constraint (see footnote 41, p. 77). By assumption, \( \partial U / \partial x > 0 \). Therefore, \( \partial U / \partial y > 0 \).

\(^{48}\) The same result is obtained by differentiation of (15) with respect to \( q_i \) since \( \partial^2 \beta / \partial u \partial q_i = \frac{\partial^2 \beta}{\partial q_i \partial u} \).

\[\]
Substitution of \( \partial x / \partial \beta = -1 \) and, from (15), \( \partial \beta / \partial u = -1/(\partial U / \partial x) \)

into (30) gives:

\[
\frac{\partial^2 \beta}{\partial q_i \partial u} = \frac{\frac{\partial U}{\partial x} \frac{\partial^2 U}{\partial q_i \partial x} - \frac{\partial U}{\partial q_i} \frac{\partial^2 U}{\partial x^2}}{\left(\frac{\partial U}{\partial x}\right)^3} \quad \text{for } i = 1, \ldots, n.
\]

Thus, the direction of change in the household's implicit price for the characteristic \( Q_i \) as a result of a change in utility depends upon the numerator of (31) determines the sign of the income elasticity of demand for \( Q_i \) in the standard theory where the effect of a change in income upon the quantity of \( Q_i \) demanded (rather than the effect of a change in utility upon the implicit price of \( Q_i \)) is calculated. Differentiation of (12) with respect to income \( (y) \) and holding "price" \( (\partial \beta / \partial q_i) \) and all elements of \( q \) except \( q_i \) constant gives:

\[
\frac{\partial^2 U}{\partial q_i \partial y} + \frac{\partial^2 U}{\partial q_i \partial x} \frac{\partial x}{\partial y} - \left[ \frac{\partial^2 U}{\partial x \partial q_i} \frac{\partial x}{\partial y} + \frac{\partial^2 U}{\partial x^2} \frac{\partial x}{\partial y} \right] \frac{\partial \beta}{\partial q_i} = 0
\]

Substitution of \( \frac{\partial x}{\partial y} = 1 - \frac{\partial \beta}{\partial q_i} \frac{\partial q_i}{\partial y} \) and (12) into the above, and solving \( \partial q_i / \partial y \) gives:

\[
\frac{\partial q_i}{\partial y} = \frac{\frac{\partial U}{\partial x} \frac{\partial^2 U}{\partial q_i \partial x} - \frac{\partial^2 U}{\partial x^2} \frac{\partial U}{\partial q_i}}{\frac{\partial U}{\partial x} \left[2 \frac{\partial^2 U}{\partial q_i \partial x} \frac{\partial U}{\partial q_i} - \frac{\partial^2 U}{\partial q_i^2} (\frac{\partial U}{\partial x})^2\right]}.
\]

Since \( \partial U / \partial x \) and the bracketed term in the denominator of the preceding equation are greater than zero by the assumption that the second-order conditions for a utility maximum are satisfied, the sign of the numerator determines the sign of the income elasticity of demand \( (\partial q_i / \partial y)(y / q_i) \). (See Henderson and Quandt, 1971, pp. 31-32).
sign of the numerator of (31). The implicit price will be independent of the level of utility, and thus of income, only if \( \partial^2 \beta / \partial q_1 \partial u = 0 \).

The conditions under which the implicit price of a characteristic would be independent of utility, and therefore of income, can be determined by setting the numerator of (31) equal to zero. Then:

\[
\frac{\partial^2 U / \partial q_1 \partial x}{\partial U / \partial q_1} = \frac{\partial^2 U / \partial x^2}{\partial U / \partial x}.
\]

That is, there would be no change in the household's implicit price for the characteristic \( Q_i \), at the given level \( q_i \), if the percentage change in the marginal utility of the characteristic due to a change in the level of money \( (x) \) equaled the percentage change in the marginal utility of money due to a change in the level of money.

If the marginal utility of the characteristic and the marginal utility of the numeraire both change by the same proportion, the marginal rate of substitution of money for the characteristic \( ((\partial U / \partial q_1) / (\partial U / \partial x)) \)

\[50\] Interpretation of the numerator of (31) is facilitated by noting that the second-order partial derivatives are taken with respect to the level of the numeraire or money. An increase (decrease) in the level of utility, income and the level of the characteristic held constant, results in a decrease (increase) in the level of money to be spent on "other things" \( (x) \) since \( y = x + \beta(q) \). Reference to the numerator of (30) and to (13) clarifies this point.
would remain constant. That is, the household's compensated demand function for the characteristic $Q_i$ would be invariant with respect to changes in utility and, therefore, to changes in income.

The compensated demand function and its inverse are geometrically identical. Again assuming that only $q_i$ and $x$ are allowed to vary, the inverse compensated demand function given by Equation (25) and corresponding to the bid function in Figure 5 is depicted in Figure 6. Note that, from (26, 27, and 28), the slope of the inverse compensated demand function is given by the second derivative of the bid function and is, therefore, negative.

\[ \frac{2U}{x^2} = 0 \]

An extreme example would be provided by a utility function wherein (1) the marginal utility of money is constant \( \left( \frac{2U}{x^2} = 0 \right) \) and (2) \( q_i \) and \( x \) are separable \( \left( \frac{2U}{q_i x} = 0 \right) \).

The bid price, however, is not necessarily invariant to changes in utility. If the marginal rate of substitution of money for the characteristic does not change with a change in utility, the indifference curve associated with the new level of utility is vertically parallel, at the given \( q_i \), to the indifference curve associated with the previous level of utility. Since the bid function (11) is the inverse of the monotonic decreasing indifference function (10), it is monotonic increasing. Therefore, indifference curves that are vertically parallel, at the given \( q_i \), in the \( x - q_i \) plane will cause the bid functions to be similarly vertically parallel in the \( \beta(q) - q_i \) plane. Thus, the bid associated with the new level of utility would be equal to the bid associated with the previous level of utility plus a constant equal to the magnitude of \( \frac{\partial \beta}{\partial u} = -1/(\partial U/\partial x) \) at the given level of the characteristic; but the slope of both bid functions at this point would be equal. The bid price is invariant to changes in utility only in the event that the marginal utility of money approaches infinity.

If the marginal utility of money is constant and if the utility function is separable in \( q_i \) and \( x \), the indifference curves and thus the bid functions will be vertically parallel for all values of \( q_i \).
Figure 6. The inverse compensated demand function and the market implicit price function for $Q_1$ in household equilibrium.

**Household Equilibrium.** The maximum amount that the individual is willing to pay for a package of residential characteristics (a residence) is given by $\beta(q)$ whereas the minimum amount that he must pay in the market is $P(q)$. Utility maximization requires, then, that $\beta(q^*) = P(q^*)$ and that, from (6) and (13):

$$\frac{\partial \beta}{\partial q_i} = \frac{\partial U/\partial q_i}{\partial U/\partial x} = \frac{\partial P}{\partial q_i} \quad \text{for} \quad i = 1, \ldots, n;$$
where \( q^* \) represents the utility maximizing vector of residential characteristics. In other words, the two surfaces \( \beta(q^*) \) and \( P(q^*) \) must be tangent. The solution for \( q^* \) only is depicted in Figures 5 and 6.  

The Supply of Location Related Goods

The Developer-Seller

The residence developer-seller is faced with the problem of simultaneously choosing the residence design \( q = (q_1, q_2, \ldots, q_n) \) and the number of residences \( (N) \) to be constructed so as to maximize profits \( (\pi) \). The cost of the factors used in the production of the residential package, the market price of the residence, \( P(q) \), and the technical relationships between the various factors \( (z_i) \), the characteristics \( (q_i) \), and the number of residences constructed \( (N) \), will influence the maximum level of profits that the developer-seller can obtain.

The developer-seller firm is defined by its multiple-output production function:

\[
F = (N, q_1, \ldots, q_n, z_1, \ldots, z_m) = 0. \tag{34}
\]

The curve \( \mathcal{P}(q_1) = P(q_1; q^*_2, \ldots, q^*_n) \) has been arbitrarily drawn; there is no \textit{a priori} reason to put any constraints on the form of the curve other than that \( \frac{\partial P}{\partial q_1} > 0 \) (from (13)).
The production function gives the technical relationships and rules by which the various inputs \((z_i)\) that go into the production of a residence (construction materials, labor, land, accessibility, neighborhood characteristics, etc.) are transformed into a residence. The form of the production function \((F)\) depends upon the firm's production processes and its level of managerial ability.

It is especially worthy of note that the form of the production function may not be independent of some of the site characteristics (Weiss et al., 1966, p. 67). For example, preparation of the land (lot) that is to support the structure may require considerably more capital and labor if the site is situated on a steep hillside rather than on flat land, or if the land is heavily wooded rather than barren of trees. Therefore, the choice of site characteristics may not, under some circumstances, be made independently of the choice of the form of the production function.

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54 Recall that a residence is defined solely by the vector of characteristics \(q = (q_1, q_2, \ldots, q_n)\) describing it. The developer-seller is assumed to choose the level of each of the characteristics \((q_i)\) and thus assemble the residential package. As is discussed later, the level of some of the characteristics may be determined simply by the choice of location of the housing structure, and may emerge from the production process untransformed.

55 The firm would, therefore, be required to choose an optimum production function in conjunction with the characteristics in solving the formal mathematical problem of maximizing profits. The problem will not be considered here.
In some instances many of the inputs may be relatively unchanged in the process of being transformed into characteristics. Accessibility may be one example, unless, as Weiss et al. (1966, p. 69) point out, the development of a subdivision by a firm includes production of a shopping center or employment center. Further, the production of accessibility requires more than location; it requires streets, and, thus, all of the factors that go into the production of streets. Another example of a broad category of factors of production that may pass through the production process unchanged is neighborhood characteristics. But, then again, the neighborhood characteristics would be a direct result of the production process and their production subject to the technical rules of the production function if the developer-seller firm produces the entire neighborhood. Those instances where the inputs emerge untransformed as outputs may, therefore, not be so prevalent upon reflection of the nature of the production process.

Assume either that each firm produces only one design of residences or that each production unit within a firm producing a

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56 If the developer-seller chooses a location where several of the inputs will emerge from the production process untransformed (e.g., an established neighborhood with existing streets, water and sewer facilities, etc.) rather than a location where these characteristics must be individually produced, he must purchase these characteristics as a "package" along with the land. That is, the cost to the developer-seller of the land includes the value of these characteristics and will be higher where these characteristics exist.
number of designs specializes in only one design, that there are no cost spillovers between production units, and that each production unit acts independently of all other units in maximizing profit. The assumption that each firm, or production unit, produces only one design is required to avoid the analytical difficulties arising from cost interdependencies in production of more than one design or model of residence.  

Let the costs of the factors required to produce \( N \) residences be given by:

\[
(35) \quad C = f(z_1, z_2, \ldots, z_m).
\]

Assume that the firm minimizes the factor costs (35) subject to the technical constraints of its production function (34) to obtain its cost function:

\[
(36) \quad C = C(q_1, q_2, \ldots, q_n, N) = C(q, N).
\]

The cost function relates total factor costs to the characteristics of the residence and the number of residences produced. Assume that \( \partial C/\partial q_i \) and \( \partial^2 C/\partial q_i^2 > 0 \) for all \( i \) and that \( \partial C/\partial N > 0 \); that is, the marginal cost of increasing each of the characteristics in a residence and the marginal cost of producing one more unit of a given

design is positive and increasing at an increasing rate. The specific form of the cost function \( C \) will depend upon the form of the production function \( F \) and the factor prices.

The firm's profits are given by:

\[
\pi = N \cdot P(q) - C(q, N),
\]

and the first-order conditions for a profit maximum are given by:

\[
\frac{\partial P}{\partial q_i} = \frac{\partial C/\partial q_i}{N} \quad \text{for } i = 1, \ldots, n,
\]

and

\[
P(q) = \frac{\partial C}{\partial N}.
\]

Equations (38) state that the market implicit price of each characteristic must equal the marginal cost per residence of producing the characteristic. Equation (39) states that the market price of the residence must equal the marginal cost of producing the residence for a profit maximum to exist.

Equations (38), the developer's counterpart to the household's requirement that \( \frac{\partial P}{\partial q_i} = \frac{(\partial U/\partial q_i)/(\partial U/\partial x)} \), are of direct interest.

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58 It is assumed that the second-order conditions for a profit maximum are satisfied, and that, thus, marginal cost is increasing at an increasing rate. For a statement of these conditions see Intriligator (1971, pp. 22-27, 96).
to this study. Their implications can be stressed by considering the concept of the offer function.

**Offer Functions.** An offer function specifies the price, \( \phi(q) \), at which the firm is willing to offer various designs of a residence for sale given a constant level of profit \( \Pi \), and given that the optimum (profit maximizing) number \( N^* \) of residences to be produced has been chosen. Then, for any given offer, the profit is given by:

\[
\pi = N^* \phi(q) - C(q, N^*).
\]

The offer function is obtained by solving (40) for \( \phi(q) \):

\[
\phi(q) = \frac{\pi}{N^*} + \frac{C(q, N^*)}{N^*}.
\]

The offer function is seen to be simply the total cost per residence plus a constant mark-up in the amount of the profit per residence. Therefore, the developer-seller will deviate from his optimum design if he can receive the total cost per residence of producing the alternative design plus the profit he would have achieved on the profit-maximizing design.

Note that, as \( \frac{\partial \phi}{\partial \pi} = 1/N^* > 0 \), higher offers are associated with higher profits if the offer is accepted and lower offers will be associated with a lower level of profits.
To further clarify the concept of an offer function, assume that only \( q_1 \) is allowed to vary. Further assume that all other characteristics are held constant at their profit maximizing levels 
\( (q_i = q_i^*, \ i = 2, \ldots, n) \) and that profit is held constant at the maximum level \( (\pi^*) \). Then the offer function, depicted in Figure 7, is given by:

\[
\phi(q_1) = \pi^* + \frac{C(q_1; q_2^*, \ldots, q_n^*, N^*)}{N^*}.
\]

\( P(q_1), \phi(q_1) \)

\( \phi(q_1^*) \)

\( P(q_1^*) \)

\( q_1^* \)

\( q_1 \)

Figure 7. One dimension of the offer function and the market price function in market equilibrium.
Profit Compensated Inverse Supply Functions. The first derivative of the offer function (41) with respect to any one of the \( q_i \) gives the profit compensated inverse supply function for that characteristic. This function is given by:

\[
\frac{\partial \phi}{\partial q_i} = \frac{3C/\partial q_i}{N^*} \quad \text{for} \quad i = 1, \ldots, n.
\]

The inverse compensated supply function is given by the marginal cost of producing an additional unit of \( Q_i \). It gives the implicit price per unit of \( Q_i \) necessary to induce the developer to vary the level of any characteristic from the optimum level for each residence. The inverse compensated supply function is geometrically identical to the profit compensated supply function and is also identical to the marginal cost per residence of the characteristic.

Again assuming that only \( q_1 \) is allowed to vary, the inverse profit compensated supply function corresponding to Equation (43) and to the offer function depicted in Figure 7 is depicted in Figure 8. Note that, given the assumption that \( \partial^2 C/\partial q_1^2 > 0 \), the offer function will increase at an increasing rate as depicted.

Developer Equilibrium. The price the developer-seller is willing to accept on each residence, given a constant profit, is given by \( \phi(q) \) whereas the market price of the residence is given by \( P(q) \). Consumption of the sale requires that \( \phi(q) = P(q) \) and that,
from (38) and (43):

\[
\frac{\partial P}{\partial q_i} = \frac{\partial C/\partial q_i}{N^*} = \frac{\partial \phi}{\partial q_i} \quad \text{for} \quad i = 1, \ldots, n.
\]

In other words, the offer and market price surfaces must be tangent.

The solution for \( q_1 \), ceteris paribus, is depicted in Figures 7 and 8.

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**Figure 8.** The inverse compensated supply function and the market implicit price function for \( Q_1 \) in developer equilibrium.
Market Equilibrium

The price structure has been assumed to be taken as given by both the household and the developer-seller. Yet the individual utility maximizing actions of all households and the individual profit maximizing actions of all developers determines the market price function, which is considered to be given by each acting alone.

The conditions necessary for equilibrium in the market are similar to those in the market for any other commodity. The quantity of residences of each design demanded must equal the quantity of residences of each design supplied and a single market price function must exist. In equilibrium no developer-seller is able to increase his profits by producing a different design and/or number of residences and no household is able to increase its utility by purchasing a different residence.

To determine the equilibrium market price, it is necessary to find a function $P(q)$ such that the quantity demanded of all types or designs of residences equals the quantity supplied. However, in the market for residences, there is a large number of goods (the $n$

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59. If each design is considered to be in a market of its own, then the usual condition of single price, rather than price function, applies. However, this study focuses on the problem of choice between designs and it is more fruitful to consider the market for residences as a single market comprised of smaller markets for each design that are interrelated through the price function.
characteristics) but only one transaction and one price \( (P(q)) \) for each residence. Therefore, the quantity demanded of each of the \( n \) characteristics and the quantity of those characteristics supplied depends on the entire market price function.

In the market for residences, therefore, the simple requirement of the equality of demand and supply prices is much more complicated than in the market for a single homogeneous good. For example, suppose that the quantities supplied and demanded of a particular design of residence do not match at a particular price. The effect of a change in the prices for that design of residence is not restricted to the residences of that design but induces substitution and design changes in the production and purchase of all designs of residences (Rosen, 1974, p. 44).

Some of the conditions describing the relationship between buyer and seller in equilibrium follow directly from the preceding behavioral theory of the household and the developer-seller. The

\[60\]

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A rigorous development of the process by which equilibrium is attained, and of the necessary and sufficient conditions for the existence of a unique and stable equilibrium requires further work and greater understanding and is beyond the necessary scope of this study. For an example of a theoretic game solution for equilibrium in a two characteristic (distance from city center and lot size) model, see Alonso (1964, pp. 76-100). Rosen (1974, pp. 44-48) uses a one characteristic model and illustrates the nature of the equilibrium by use of a specific example. For a description of a bid-offer process (auction) lending to market equilibrium for one good, see Henderson and Quandt (1971, pp. 114-115).
relationship between the bid function, offer function, and market price function in equilibrium is developed from the necessary conditions for utility maximization by the households and for profit maximization by the developers.

Among the conditions that must exist between each buyer and seller of a particular design is the trivial condition that bid price equal offer price and hence, if the sale is consummated, they both equal market price. This

\[ \beta(q) = P(q) = \phi(q) \]  

assumes, of course, that the buyer has full knowledge of all other bids on the residence and that the seller has knowledge of the amount at which all other residences of the same design are being offered for sale.

Equations (6), (13), (38), and (43) require that the following conditions hold between each buyer and seller:

\[ \frac{\partial \beta}{\partial q_i} = \frac{\partial U}{\partial q_i} = \frac{\partial P}{\partial q_i} = \frac{\partial C}{\partial q_i} = \frac{\partial \phi}{\partial q_i} \quad \text{for } i = 1, \ldots, n. \]

In other words, for each characteristic and between each buyer and seller, the household's marginal rate of substitution between the characteristic and the numeraire (its implicit valuation of the \( q_i \)) must equal the marginal cost per residence of producing the
characteristic and both, in turn, will equal the market implicit price of the characteristic.

The equilibrium condition between one buyer and one seller assuming that only \( q_1 \) can vary and represented by Equation (45) is depicted in Figure 9. Note that, by Equations (46), the offer and bid functions must be tangent for each characteristic. Further note that, if all households had identical utility functions and incomes and if all developers had identical production functions and faced the same factor prices, all households would purchase the same level of characteristic \( q_1 \) at the constant market price \( P(q_1) \) since all bid functions would be identical and all offer functions would be identical.

The equilibrium condition between one buyer and one seller assuming that only \( q_1 \) can vary and represented by Equation (46) is depicted in Figure 10. Note, again, that the existence of identical buyers and sellers implies identical inverse compensated demand and supply functions and, therefore, a constant market implicit price \( \frac{\partial P}{\partial q_1} \) for \( q_1 \).  

\[ \text{61} \] Though the market price is constant it is not independent of the level of \( q_1 \). If \( q_1 \) is lot size and all individuals purchase a lot of 1/4 acre for $5000 \( (q_1^* = 1/4, \; P(q_1) = \$5000) \) that does not imply that a 1/2 acre lot does not have a higher market price. In fact, both bid price and offer price may be greater than $5000—not equal.

\[ \text{62} \] A constant market implicit price for \( q_1 \) would require that \( P(q) \) be linear in \( q_1 \) and that the market implicit price not vary with the quantity of \( q_1 \) purchased; for example, a ten acre lot would sell for exactly forty times as much as a 1/4 acre lot, ceteris paribus.
Figure 9. Equilibrium between one buyer and one seller: one dimension of the bid and offer functions.

Figure 10. Equilibrium between one buyer and one seller: the inverse compensated demand and supply functions.
It is not, of course, the case that all individuals have the same utility function and/or incomes. Differences in the sociological attributes of households, such as the age of the head of the household, educational level of the head and/or spouse of the head of the household, and the number of children at home may result in different preferences for residences. The differences in social attributes, as well as differences in incomes, may result in different sets of bid functions between households.

Similarly differences in developer-seller firms' production functions or in the factor prices that they face may provide comparative advantages in the production of different designs and/or number of residences. The business of residential development is an extremely complex one requiring dealings with contractors, public officials, financial intermediaries, etc. (see Weiss et al., 1966, p. 30). Developer-seller firms with a higher level of managerial ability may be able to more effectively coordinate the development of a large number of homes, entire subdivisions or neighborhoods for example, and thus produce a different set of characteristics comprising their residences than would smaller developer-seller firms. Further, large-scale developers may be able to obtain quantity discounts or more lenient credit terms in the purchase of some of the factors of production and thus have a comparative advantage over firms of a smaller scale. Anything that might affect the developer's
production function and/or his factor prices will be reflected in his cost function and, therefore, in his set of offer functions.

The profit maximizing offer function of four different (nonidentical) developers and the utility maximizing bid functions of four nonidentical households are depicted in Figure 11. The conditions for market equilibrium, to the extent discussed in this study, are met by the firms and households depicted. It is, again, assumed that only characteristic \( Q_1 \) varies so as to allow graphical presentation of the conditions. The developer-seller represented by offer function \( \phi^1(q_1) \) has a comparative advantage in the production of \( Q_1 \) up to that level of \( Q_1 \) given by \( q'_1 \) --perhaps because of differences in factor prices or in the production functions. As depicted in Figure 11, the household represented by bid function \( \beta^3(q_1) \) is willing and able to outbid the household with bid function \( \beta^2(q_1) \) for that level of \( Q_1 \) greater than \( q''_1 \). The ability and the desire to outbid household number (2) may arise from a greater income and/or differences in taste. Below that level of the characteristic represented by \( q''_1 \), household (3) may not be willing, though perhaps able, to outbid household (2) for that level of the characteristic represented by \( q''_1 \). If \( Q_1 \) represents lot size for example, household (3) may not find a lot of size less than \( q''_1 \) to be as desirable as does household (2). If there is a large number of households and sellers, say \( M \), the locus of points
(q_1^*, q_2^*, \ldots, q_M^*) forms a continuous curve, P(q_1), that is the joint envelope of the M offer and bid functions.

There need not be an equal number of buyers and developer-sellers. There will be a bid function for each residence as well as an offer function for each residence; in equilibrium, they must be tangent.

Figure 11. One dimension of the bid and offer functions of four different developers and households and one dimension of the market price function in market equilibrium.
Market equilibrium and the conditions necessary for identification of a market price function can also be illustrated using the curves marginal to the bid and offer functions—as in Figure 12. Again, it is assumed that only characteristic \( Q_1 \) can vary and that all other characteristics are held constant at their optimum levels.

The necessary conditions represented by Equation (46) for \( i = 1 \) are satisfied at the points of intersection of the (inverse) compensated supply and demand functions. The points of intersection of the functions for the four hypothetical households and developers trace out the market implicit price function \( (\frac{\partial P}{\partial q_1})_{q_1^*, \ldots, q_n^*} \). Note that, if all developers were identical, only one supply function would exist and observed market implicit prices and quantities would trace out that supply function; the implicit price function would be coincident with the inverse compensated supply function and the inverse compensated demand function would not be identifiable using market observations. If all households were identical, only one observable demand function would exist and the implicit price-quantity observation would trace out that demand function; the implicit price function would be coincident with the inverse compensated demand function and the inverse compensated supply function could not be identified from market observations of implicit price and quantity.
Figure 12. The inverse compensated demand and supply functions of four different developers and households and the market price function.

It is important to note that the bid and offer functions depicted in Figure 11 are equilibrium functions. Each household has a set of bid functions for each residence and each developer-seller has a set of offer functions for each residence he offers for sale. If, and only if, each household can not increase its level of satisfaction by moving
to a different residence and if, and only if, each developer-seller
has no opportunity to increase his level of profits by producing a dif-
ferent design or number of residences, then and only then, can the
equilibrium bid and offer functions be determined. Consequently, the
demand and supply functions can be derived only in equilibrium. It
follows that the market price function and therefore the implicit
price function exists only in equilibrium.

The market price function, given by the joint envelope of the bid
and offer functions, is increasing in each of the characteristics
\( \frac{\partial P}{\partial q_i} > 0 \) so long as the marginal cost of producing each of the
characteristics is positive. This result follows directly from the
assumption that the characteristics are measured in such a manner
that they have positive marginal valuations and from the requirement
that these marginal valuations equal marginal cost per residence
(see (46)).

**Location**

The household's (and developer's) location in n-dimensional
space is given by the solution vector \( (q_1^*, q_2^*, \ldots, q_n^*) \) which satisfies
the equilibrium conditions given by (46) as well as those for utility
maximization by the household (Equations (5), (6), and (7)) and those
for profit maximization by the firm. As the solution vector is deter-
mined by the point of tangency of the bid and offer surfaces, location
is a function of both the household's and the developer's decision process—neither is independent of the other.

The household's location can be uniquely mapped from the n-dimensional characteristics space to the two-dimensional settlement plain only to the extent that the $q_i^*$ are unique levels of the characteristics uniquely located on the settlement plain. But, as there is likely to be duplication as to both levels and location of the characteristics in geographic space, such a unique mapping will not generally be possible. The household would, however, be indifferent between the different locations on the settlement plain represented by the n-dimensional optimum solution vector.

It is a clear consequence of this model that households with similar preferences and incomes will tend to locate at the same points in the n-dimensional characteristics space; that is, they will tend to choose the same residential packages. To the extent that these packages map from n-dimensional space to neighboring points in geographic space, like-minded individuals with similar incomes will tend to group together on the settlement plain. This pattern will be reinforced to the extent that such community homogeneity per se is a good to be considered in the individual residential location decision. 64

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64 The tendency for households to segregate by taste and income groups is also a result of the Tiebout (1956) model wherein the public
Extensions of the Model

The residential choice model proposed in this chapter is quite abstract, but there is no need to belabor its obvious oversimplifications. However, the model is easily extended to consider the role of commuting (transport) costs in the location decision of the household, and the effects of zoning, certain subdivision regulations, and certain local government residential development charges on the developer-seller.

To simplify exposition of the basic model, it has been implicitly assumed that the household’s disposable income (income after taxes—including property and income taxes) is independent of the household’s residential choice. However, the residential choice may affect disposable income in several ways—two of which are: (1) the property tax, and therefore disposable income, may not be unrelated to the public services provided with a residence (i.e., may not be unrelated to the "q" vector); (2) the property tax is usually based on either the market capital value of the residence or on a constant proportion thereof, and therefore will vary directly with the capital goods provided by local governments are the characteristics of choice. Elickson (1971) provides a rationale for stratification of communities by wealth. Sociologists have, in particular, commented on the tendency of people with similar social attributes to select the same type of residential area (see Charde).
expenditure on the residence, ceteris paribus. Therefore, the model should be expanded to include the interdependence of disposable income, the property tax, and residential choice.

Local Public Services, the Property Tax, and the Location Decision

The provision of publicly provided services generally varies between locations—as do property tax rates. Assume (1) that households have full knowledge of the differences in the rate of provision of

However, the property tax is deductible from taxable income in computing the U.S. federal income tax. Thus disposable income is not diminished by the full amount of the property tax—the extent of the decrease depends upon the level of the household's taxable income as the tax rate on taxable income is progressive.

In addition, homeowners receive two income tax benefits under U.S. federal income tax laws: (1) the total annual interest payment under the mortgage purchase contract is deductible from taxable income. Thus, the amount of the mortgage (generally directly related to the capital expenditure on the residence) affects disposable income—though the effect diminishes as the mortgage approaches the end of its term and the annual interest payment diminishes; (2) the imputed rent (the value of the annual flow of services \( P(q) \) from the residence) is not included in taxable income, and is exempt from the federal income tax—though it is income-in-kind to the homeowner. Aaron (1970) has shown that these tax benefits are equivalent to a reduction in the price of housing to homeowners. Thus, the effect of the tax benefits is to induce a substitution of housing for other goods in household consumption. To the extent that the increased expenditures on housing as a result of the tax benefits are for location related characteristics, these tax benefits will affect the location decisions of households. Further, the effects of the tax benefits, and thus the effective reduction in housing prices, do not apply equally to all income levels as the U.S. federal income tax is progressive. The interrelationships between rent \( P(q) \), capitalized value, the income
publicly provided services and differences in property tax rates between alternative residential locations, and (2) that there exists a continuous spectrum of choices of public services.\textsuperscript{67} Given these assumptions, households can be expected to consider, in their location decision calculus, the differences in the rate of provision of public services and in property taxes between alternative locations.\textsuperscript{68}

To explicitly consider the role of public services in the constrained utility maximizing problem of the household, rewrite the utility function (4) as:

\begin{equation}
U = U(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n, x),
\end{equation}

where the $q_i$ ($i = 1, \ldots, p-1$) denote the flow of services stemming from the market goods and nonmarket goods not publicly provided at a given residence. The $q_j$ ($j = p, \ldots, n$) denote the flow of services stemming from the publicly provided services at a given residence.

These assumptions are similar to Tiebout's (1956) assumptions (2) and (3) in his model designed to explain the aggregate level of local government expenditures on public goods.

It is an assumption of the Tiebout (1956) public expenditures model that individuals will move to that local government justification (i.e., location with that level of publicly provided services and tax rates) where their preference functions are maximized subject to the relevant constraints, \textit{ceteris paribus}.
Similarly, the market price function may be written as:

\[ P(q) = P(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n) \]

To simplify the residential choice model, two extreme relationships between the property tax rate and the level of the publicly provided services will be considered.\(^{69}\) Case I: An explicit relationship, invariant with location, is assumed to exist between the type and level of publicly provided services available at any given location and the property tax rate at that location.\(^ {70}\) Case II: the property tax rate is assumed to be invariant with location and/or the public services provided at any given location. Case I is the more extreme of the two, and is not likely to exist in reality. Case II, a special case of Case I, is more likely to exist within a local government's jurisdiction (within a city, for example) wherein the level of public services is not uniformly distributed but the property tax rate

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\(^{69}\)The relationship between the property tax rate and the type and/or level of public services provided at a given location is a complex subject in itself, and an exploration of this relationship is beyond the scope of this study. For a discussion of the interrelationships between local government expenditures, transfer payments, and local government tax rates, see Godwin and Fitch ("Two Stage Least...", 1975) and Goodwin and Fitch ("The Impacts of...", 1975).

\(^{70}\)This is not to say that property taxes are assumed to provide the sole source of revenue for funding publicly provided services; they may be funded through another tax--though that tax is assumed to be invariant with location and/or the level of provision of publicly provided services.
is, nevertheless, constant regardless of location within the jurisdictional area. These two cases (assumptions) will be considered in turn.  

**Case I: Property Tax Rate a Function of Publicly Provided Services.** Let the property tax rate, \( t(q) \), be given by:

\[(49) \quad t(q) = t(q_p, q_{p+1}, \ldots, q_n).\]

Then assuming that the residence is to be assessed and taxed at market price, the property tax \( T \) is given by:

\[(50) \quad T = t(q) \cdot P(q).\]

Again letting \( y \) be the household's disposable (after income tax) income, the household's budget constraint is then given by:

\[(51) \quad y = x + (1+t(q))P(q).\]

The necessary first-order conditions for an income-constrained utility maximum are obtained by maximizing (47) subject to (51).

---

\[\text{71} \] Note that in Case I the functional relationship between the property tax rate and the level of publicly provided services is assumed invariant with location whereas in Case II the property tax rate is assumed invariant with the level of public services provided \( (\partial t/\partial q_j = 0, \text{ for } j = p, \ldots, n) \) and/or location.

\[\text{72} \] Recall that \( P(q) \) is more like a contract rent. Similarly, the property tax referred to here is a tax on the amount of this rent rather than a tax on the capitalized value of the residence.
Equation (52) states that, for a utility maximum, the household's marginal evaluation of each of the market goods and of the nonmarket, nonpublicly provided goods must equal the market implicit price \( (\partial P / \partial q_i) \) of the goods plus the per unit tax on the goods \( (t(q)(\partial P / \partial q_i)) \). The property tax, in effect, increases the implicit price of the characteristics by a factor of \( (1 + t(q)) \). As a result, the household will reduce the amount of each of the market goods and nonmarket, non-publicly provided goods that comprise the optimum residence relative to the amount that would comprise the optimum residence in the absence of the tax.

Equation (53) states that, for a utility maximum, the household's net marginal evaluation of each of the publicly provided goods must equal the change in the market price of the residence as a result of the change in the availability of the publicly provided good at the

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73 Assume that the second-order conditions for a utility maximum are satisfied.
residence \((\partial P / \partial q_j)\) plus the change in the property tax on the residence as a result of the capitalization of the value of the services of the publicly provided good into the market price of the residence \((t(q)(\partial P / \partial q_j))\). \(^{74}\) The net marginal evaluation of the publicly provided good, given by the left side of Equation (53), is the marginal evaluation of the public service at the level provided at the residence \(((\partial U / \partial q_j) / (\partial U / \partial x))\) minus the change in the total property tax due to the change in the property tax rate \(((\partial t / \partial q_j)P(q))\) resulting from the change in the level of the publicly provided service.

The bid function can also be adapted to incorporate the role of publicly provided services and the property tax in the residential location decision calculus. As before, define an indifference surface along which the household is indifferent between various combinations of the characteristics and \(x\) as:

\[(54) \quad U(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n, x) = u.\]

Substitution of the constraint that

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\(^{74}\) The word "capitalize" is not used in its usual economic sense here. That is, it does not mean that the value of the (future) net benefits of the publicly provided service are discounted in calculating the present value of the residence. As used here, the term simply means the change in the market price of the residence as a result of the change in the availability of publicly provided services.
\[(55) \quad x = y - (1+t(q))\beta(q)\]

into (54) gives:

\[(56) \quad U(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n, y-(1+t(q))\beta(q)) = u,\]

where \(\beta(q)\) is, as before, the amount of the bid. Solving (56) for \(\beta(q)\) gives:

\[(57) \quad (1+t(q))\beta(q) = \beta(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n; y, u)\]

or

\[(58) \quad \beta(q) = \frac{1}{1+t(q)} \beta(q_1, q_2, \ldots, q_{p-1}, q_p, \ldots, q_n; y, u).\]

Comparison of the bid function for the household after incorporating the property tax (58) with the bid function in the absence of a property tax (11) reveals that the only difference is the presence of the term \((1+t(q))^{-1}\) in the former; the household discounts its bid by the amount of the property tax rate.

The effect of a change in any of the characteristics of a residence (or between residences) on the individual's bid for the residence is determined by partial differentiation of (56) with respect to the characteristic. For the market provided characteristics and the nonmarket, nonpublicly provided characteristic, partial differentiation of (56) with respect to \(q_i\) gives:
Similarly, for the publicly provided goods:

\[(60) \quad (1+t(q)) \frac{\partial \beta}{\partial q_j} = \frac{\partial U/\partial q_i}{\partial U/\partial x} - \frac{\partial t}{\partial q_j} \beta(q) \quad \text{for} \quad j = p, \ldots, n.\]

Recall that the partial derivative of the bid function with respect to any one of the characteristics gives the (inverse) compensated demand function for that characteristic. Thus, it can be seen, by dividing both sides of Equation (59) by \((1+t(q))\), that the effect of the property tax is to discount the households implicit demand price for the nonpublicly provided characteristics by the amount of the property tax rate. Similarly, it can be seen, by dividing Equation (60) through by \((1+t(q))\), that the effect of the property tax is to discount the household's net (net of potential property tax) marginal evaluation of the publicly provided service, and thus its marginal bid for that service, by the amount of the property tax rate.

Among the conditions for household equilibrium is the requirement that demand price (or marginal evaluation) equal the market implicit price for each characteristic. Combining Equations (52) and (59) gives, for the nonpublicly provided characteristics:
Similarly, Equations (53) and (60) give, for the publicly provided characteristics, the requirement that:

\[
\frac{\partial \beta}{\partial q_i} = \frac{1}{(1+t(q))} \frac{\partial U/\partial q_i}{\partial U/\partial x} = \frac{\partial P}{\partial q_i} \quad \text{for } i = 1, \ldots, p-1.
\]

There is nothing new in Equations (61); their interpretation is immediate.

Equations (62) are of particular interest for they give the conditions under which a residence with a publicly provided service (e.g., a public school) would have the net marginal benefits of that service capitalized into the market price of a residence. Consider the bracketed term, the net marginal evaluation \(NME\), in Equation (62):

\[
NME = \frac{\partial U/\partial q_j}{\partial U/\partial x} - \frac{\partial t}{\partial q_j}(q).
\]

The first term on the right side of Equation (63) gives the individual's demand price for the publicly provided service \(Q_j\). The second term gives, in effect, the supply price of the service. Equation (63),
for \( j = p \), is illustrated in Figure 13. 75 If circumstances allow the household to reach an optimum level of the use of the publicly provided service, it will "purchase" that level of service represented by \( q^*_p \) (Figure 13) where \( (\partial U / \partial q_p) / (\partial U / \partial x) = (\partial t / \partial q_p) \beta(q) \). That is, the household will include the publicly provided characteristic as a component of the residence up to that level where its net marginal evaluation of the characteristic is zero \((\text{NME} = 0)\). If the net marginal evaluation is zero, then, from (62), both the demand price \((\partial \beta / \partial q_p)\) and the market implicit price \((\partial P / \partial q_p)\) of the characteristic will be zero. Since both the benefits of the publicly provided service and the cost of the service to the household (the property tax) are equated at the margin, net marginal benefits will be zero and the market price of the residence will be unaffected by the availability of the publicly provided service at its location. That is, if all households can reach an optimum level of the publicly provided service, no household will make an after-tax bid greater than zero for the benefits of the publicly provided service and the market implicit price of the

Note that the bid price, \( \beta(q) \), is held constant along the curve given by \((\partial t / \partial q_p) \beta(q) \). This curve gives the (potential) change in property taxes due to a change in the property tax rate as a result of a change in the level of the publicly provided services. The total (potential) change in property taxes includes this effect plus the change in property taxes due to a change in the bid price of the residence as a result of the availability of the publicly provided service \((t(q)(\partial \beta / \partial q_p))\). The concern here is with the conditions under which the bid price would be influenced by the publicly provided service.
service will be zero—there would be no net benefits from the publicly provided services to be capitalized into land values.

\[
\frac{\partial t}{\partial q_p} \beta(q)
\]

\[
\frac{\partial U}{\partial q_p} \frac{\partial U}{\partial x}
\]

Figure 13. The household's marginal evaluation of a publicly provided service and the change in property taxes due to a change in the property tax rate.

If, however, all households could not incorporate the optimum level (i.e., that level where \( \text{NME} = 0 \)) of the publicly provided service into their residence, the net benefits of the publicly provided service will be capitalized into the market price of the residence. For example, if the supply of the publicly provided service is limited such that there is only \( q'_p < q^*_p \) per residence or if access to the public service is limited to \( q'_p < q^*_p \) by a nonprice mechanism such
as density zoning (see Figure 13), the net marginal evaluation of the
publicly provided service at that restricted level is given by:

\[ \text{NME} = \frac{\partial U/\partial q_p}{\partial U/\partial x} \bigg|_{q_p=q'_p} - \frac{\partial t}{\partial q_p} \beta(q) \bigg|_{q_p=q'_p} > 0. \]

The net marginal evaluation at the level \( q'_p \) is represented in
Figure 13 by the amount \( \theta - c = (b-0) \). Each household's demand
price is given by:

\[ \frac{1}{\partial q_p} = \frac{1}{1+t(q)} \left[ \frac{\partial U/\partial q_p}{\partial U/\partial x} \bigg|_{q_p=q'_p} - \frac{\partial t}{\partial q_p} \beta(q) \bigg|_{q_p=q'_p} \right] > 0. \]

That is, each household's marginal bid will be given by its net
marginal evaluation of the benefits of the publicly provided character-
istics \( Q_p \) discounted by the rate of the property tax. In equilibrium,
the highest marginal bid determines the market implicit price and
Equation (62) holds. The excess demand for the publicly provided
characteristic \( Q_p \) at the "supply price" \( (\partial t/\partial q_p)P(q) \bigg|_{q_p=q'_p} \)
results in a market implicit price of \( (\partial P/\partial q_p) \bigg|_{q_p=q'_p} > 0. \)

The increased market price of the residence will be realized
in the form of higher land values. Suppose for example, that location
within a particular school district yields a positive net marginal
evaluation in that the marginal benefits from the household's utiliza-
tion of the school exceed the marginal cost (increase in property
taxes) incurred as a result of locating within the school district. Thus
a residence located within the school district will have a positive
market implicit price for the location and a higher market price.
Therefore, developers building within that school district will realize
a larger profit than those building outside of the school district,
ceteris paribus. But, competitive bidding among developers for sites
within the school district can be expected to drive land prices up and,
as a consequence, the marginal cost of providing a residence with the
characteristic that it is in this particular school district will be posi-
tive and reflected in the marginal cost of lot size.

Further, assuming perfectly competitive bidding by the
developers, land prices will be bid up to that point where there is no
comparative advantage, for either the developers or the household,
to locating within the school district. That is, in equilibrium:

\[
\frac{\partial \beta}{\partial q_p} = \frac{1}{(1+t(q))} \left[ \frac{\partial U/\partial q_p}{\partial U/\partial x} - \frac{\partial t}{\partial q_p} \beta(q) \right] = \frac{\partial P}{\partial q_p}
\]

\[
= \frac{\partial C/\partial q_p}{N^*} = \frac{\partial \phi}{\partial q_p},
\]

where \( (\partial C/\partial q_p)/N^* \) is the marginal cost per residence of including
the publicly provided service as a characteristic of the residence.

A household may not be willing to remain at a location where its
net marginal evaluation of the level of the publicly provided service,
as given by Equation (63), is equal to zero. Such a location need not be an equilibrium one if the "supply curve", given by \( \frac{\partial t}{\partial q_j} \beta(q) \), is negatively sloped. However, this may be the case if the publicly provided service is produced under conditions of increasing returns to scale (declining average cost of provision) and if \( \frac{\partial t}{\partial q_j} \beta(q) \) is set, by public authorities, equal to either the marginal or average cost of production. Stability of equilibrium requires that the net marginal evaluation continue to diminish for those levels of provision of the public service greater than that level where the net marginal evaluation is equal to zero. That is, for a stable equilibrium, it is required that:

\[
\frac{\partial NME}{\partial q} = \frac{\partial (\frac{\partial U}{\partial x})}{\partial q_j} - \frac{\partial^2 t}{\partial q_j^2} \beta(q) < 0, \quad \text{for } j = p, \ldots, n.
\]

---

76 As was previously noted, an exploration of the relationship between the property tax rate and the level of provision of public services, is beyond the scope of this study. However, this relationship determines the form of the property tax rate function, as given by Equation (49) and thus that of the function \( \frac{\partial t}{\partial q_j} \beta(q) \) which is depicted in Figure 13. Therefore, unless the "supply price" given by \( \frac{\partial t}{\partial q_j} \) is the marginal social cost of providing the public service, satisfaction of the equilibrium conditions given by Equation (62) does not imply that social efficiency in the allocation of location related public services between households has been achieved.

See Edel and Sclar (1974) for a discussion of the interaction between taxes, public spending, and property values couched in terms of producers' and consumers' surplus, return to scale, and efficiency considerations.
This requirement states that the slope of the compensated demand function (given by the first term on the right side of Equation (66)) must be less than the slope of the "supply function" (given by the second term on the right side of Equation (67)). If the supply price were less than the demand price and if it were to decline at a faster rate than the demand price as the level of the publicly provided service increased, the household would continue to seek a location so long as the marginal benefits of the publicly provided service were greater than the marginal cost. If it could not find a location, over the range of provision of the service, where its net marginal evaluation of the publicly provided service reached zero and declined thereafter, the positive net marginal evaluation of the service at that location where its maximum level of provision occurred would, as previously explained, be capitalized into land values.

**Case II: Property Tax Rate is Invariant.** The benefits of a publicly provided service are not always uniformly distributed within a local government's (or tax authority's) jurisdictional area. The benefits of a city park, for example, may accrue more to those

77 That is, the "supply function" must intersect the implicit demand function from below. This is equivalent to the Marshallian stability condition wherein an equilibrium is stable if an increase in quantity reduces the excess demand price. As it is the effect of quantity supplied on demand price that is of concern here, it is the Marshallian stability condition and not the Walrasian, wherein the concern is with the effect of price on quantity supplied, that is relevant. See Henderson and Quandt (1971, pp. 133-134).
residences located immediately adjacent to the park than to those located across town from the park. However, property tax rates are usually constant within a city's boundaries; the city government somehow chooses a program of public services and sets a property tax rate sufficient to fund the program. To determine the effect of variation in public services within the city on the market price of residences within the city, assume that property tax rates are indeed constant throughout the city.

If \( \partial t/\partial q_j = 0 \) within a local government's jurisdiction, then Equation (62), one of the household equilibrium conditions, reduces to:

\[
\frac{\partial \beta}{\partial q_j} = \frac{1}{(1 + t(q_j))} \frac{\partial U/\partial q_j}{\partial U/\partial x} = \frac{\partial P}{\partial q_j} \quad \text{for } j = p, \ldots, n.
\]

As the marginal bid price for the publicly provided characteristic is positive, the market implicit price of the characteristic will be positive. This implies that those residences with a locational advantage to publicly provided services will have the benefits of that service capitalized into the market price of the residence.

This section can be briefly summarized by noting that, if all households can locate at a site where their net marginal evaluations of a publicly provided service that they desire to include in their residential packages are each equal to zero, the marginal bid price and
thus the market implicit price of the publicly provided characteristic will equal zero. However, it is not necessarily the case that all households can locate at a point where their net marginal evaluations of the publicly provided characteristics, and thus their marginal bid prices or demand prices are all equal to zero. The market implicit prices of the publicly provided characteristics may or may not be greater than zero; there is no way, in general, to predict.

**Commuting Costs and the Household**

The role of commuting, or access, costs in the household's residential location decision can be readily included in the model. This can be accomplished by specifying the accessibility variables in the utility function and the household's commuting costs as a function of accessibility.

Accessibility to various locations, natural attractions, and services is of concern to the household primarily, or perhaps only, because of the time and/or monetary cost of overcoming distance. Alonso (1964, p. 34) states that "... a near location is preferred to a distant one (from the central business district) since commuting is generally regarded as a nuisance." Hartwick (1971, p. 7) states that: "one has disutility for distance traversed because it represents

78 Accessibility will be explicitly defined in the following chapter.
leisure time foregone." Nelson (1971, p. 73) argues that the disutility of commuting stems primarily from the often high time price of commuting and from uncertainty as to the time price of any given trip. Solow (1973, p. 162) is not sure that accessibility, per se, is either good or bad and chooses not to include accessibility (distance from the central business district) in the individual's utility function. Nevertheless, if travel were both instantaneous and free, it seems that the utility or disutility of travel would be of little concern as there would be neither time nor money costs involved.

Accessibility is included as an argument in the utility function of households in this model. It is assumed that accessibility to those attractions or services that enter the household's utility function is a good or, at least, never a bad. Specifically, it is assumed that the marginal utility of accessibility is nonnegative; it may be zero.

Denote the variables specifying the level of accessibility to, for example, place of employment, retail establishments, school, etc., for any residence by $q_j$ ($j = a, \ldots, n$). Then, the market price function may be written:

$$ (69) \quad P(q) = P(q_1, q_2, \ldots, q_{a-1}, q_a, q_{a+1}, \ldots, q_n) . $$

All other characteristics of the residence are denoted by $q_i$ ($i = 1, \ldots, a-1$).
Similarly, the household's utility function may be given as:

\[
U = U(q_1, q_2, \ldots, q_{a-1}, q_a, q_{a+1}, \ldots, q_n, x).
\]

Note that whereas in a particular household's utility function a \( q_j \) gives, for example, accessibility to a specific place of employment at a given residence that same \( q_j \) refers to accessibility to place of employment in general in the market price function.

It may be the case that the accessibility preferences of some households are highly individualistic. All other things equal (particularly income), the household with a highly individualistic accessibility preference or a very strong preference relative to other households for a particular location will be the highest bidder for that location.

However, the highest bidder need only bid marginally higher than the second highest bidder to be successful--provided he meets the offer. As an extreme example, suppose that all employment, save that of one individual, is concentrated in the central business district of a city. Assume that the excepted individual is the sole employee of a photography studio at accessibility (to employment in the central city) level \( q^a_1 \). The individual that is employed at the studio may be willing to bid relatively high to locate adjacent to the studio (at his accessibility to employment level \( q^a_1 > q^a_2 \) but at general accessibility to employment level \( q^a_1 \)). However, he need pay
no more for his accessibility to employment level $q_a''$ than any other individual pays for accessibility level $q_a'$.\textsuperscript{79} It is the marginal bidder who determines the market price for accessibility, given the supply of accessibility.\textsuperscript{80} The individual who would be willing to pay a higher price, but need not, reaps a consumer's surplus.

Let total commuting costs at a residence be given by:

\begin{equation}
M(q, y) = M(q_a, q_{a+1}, \ldots, q_n, y)
\end{equation}

As greater accessibility implies a lower commuting cost, assume that the $\partial M/\partial q_j < 0$ for all $j$. Also assume that the time costs of commuting will increase with income; then, $\partial M/\partial y > 0$. The specific form of the commuting cost function will depend upon the direct monetary costs of overcoming distance, the time costs of overcoming distance (see Becker (1965), Nelson (1971, pp. 53-80; 1973), and the number of persons in the household.

In addition, the commuting cost function may, itself, be a function of the characteristics of the residence. If, for example, the household has access to mass transport at a particular location and not at another or if the streets at a particular location are in such

\textsuperscript{79}Assuming all others are indifferent to accessibility to the studio.

\textsuperscript{80}A similar hypothesis is central to Harris, Tolley, and Harrell's (1968) study of the role of amenity in residence site choice.
poor condition as to be more damaging to motor vehicles than at another location, the commuting cost function is likely to vary in consideration of these characteristics. Analytical difficulties arising from variation in the commuting cost function with the characteristics of the residence are noted in passing. 

Incorporation of the commuting cost function (71) into the budget constraint (5) gives:

\[(72) \quad y = x + P(q) + M(q, y)\]

where \( y, x, \) and \( P(q) \) are as previously defined.

Explicitly considering commuting costs, the household's formal utility maximizing problem requires maximization of (70) subject to (72). Among the first order conditions are the following:

\[(73) \quad \frac{\partial P}{\partial q_j} = \frac{\partial U/\partial q_j}{\partial U/\partial x} \quad \text{for} \quad i = 1, \ldots, a-1\]

\[(74) \quad \frac{\partial P}{\partial q_j} = \frac{\partial U/\partial q_j}{\partial U/\partial x} - \frac{\partial M}{\partial q_j} \quad \text{for} \quad j = a, \ldots, n\]

---

\[81\] The household would be required, in maximizing utility subject to the constraints, to choose both a cost function and a set of characteristics. The problem will not be considered here.

\[82\] Assume all second-order conditions are satisfied.
Equation (73) has the usual interpretation.

Equation (74) states that, for a constrained utility maximum, the change in the market price of a residence as a result of a change in accessibility at a residence must equal the individual's marginal evaluation of the change in accessibility plus the change in travel savings as a result of the greater accessibility, *ceteris paribus*.\(^{83}\)

The role of commuting costs in the individual residential location decision can be stressed by considering the effect of commuting costs on the household's bid function. As before, define an indifference surface along which the individual is indifferent, at a constant level of utility \(u\), between all possible combinations of the characteristics and of the characteristics and \(x\):

\[
U(q_1, q_2, \ldots, q_{a-1}, q_a, q_{a+1}, \ldots, q_n, x) = u
\]

Introduce the constraint that (from (72)):

\[
x = y - \beta(q) - M(q, y)
\]

where \(\beta(q)\) is, once again, the amount of the bid. Then, substituting (76) into (75) gives:

---

\(^{83}\) Recall that \(\partial M/\partial q_j\) gives the change in travel costs as a result of a change in accessibility. The negative of \(\partial M/\partial q_j\) gives the change in travel *savings* as a result of a change in accessibility.
and solving for $\beta(q)$ gives the household's bid function:

(78) $\beta(q) = \beta(q_1, q_2, \ldots, q_{a-1}, q_a, q_{a+1}, \ldots, q_n, y-M(q, y), u)$.

Differentiating (77) with respect to any one of the accessibility variables gives:

(79) $\frac{\partial \beta}{\partial q_j} = \frac{\partial U}{\partial q_j} - \frac{\partial M}{\partial q_j}$ for $j = a, \ldots, n$.

Equation (79) states that the individual can, when considering an incremental change in accessibility, give a maximum bid equal to the amount of the marginal evaluation of the increased accessibility per se plus the marginal travel savings.

The accessibility bid function will have a positive slope even if the household's marginal evaluation of increased accessibility is zero. If the $\frac{\partial U}{\partial q_j} = 0$, the slope of the bid function in any one given accessibility dimension is $\frac{\partial \beta}{\partial q_j} = -\frac{\partial M}{\partial q_j}$ — which is positive by assumption.

Whether the bid for accessibility increases at an increasing rate or a decreasing rate depends upon the sign and relative magnitude of the second derivatives, with respect to accessibility, of the utility function and the travel cost function. By assumption, the
second derivatives of the utility function with respect to $q_j$ (the first derivative of $(\partial U/\partial q_j)/(\partial U/\partial x)$) is negative. That is, the increment added to the bid for accessibility, *per se*, will diminish with each incremental increase in accessibility. Therefore, the bid function for accessibility will increase at a diminishing rate if the second derivative of the commuting cost function is nonnegative $(\partial^2 M/\partial q_j^2 \geq 0)$, as the second derivative of the bid function is given by the sum of the second derivatives of the utility and cost functions. That is, the bid function for accessibility will be increasing at a diminishing rate if the marginal cost of travel decreases at either a constant or a diminishing rate with increasing accessibility.

If the household's commuting costs are influenced by the time costs of travel and, thus, by the household's income (i.e., if $\partial M/\partial y > 0$), an increase in income, utility held constant, will not increase the household's bid for a residence by the amount of the increase in income. Implicit differentiation of Equation (77) with respect to income, utility held constant, gives:

$$\frac{\partial U}{\partial x} (1 - \frac{\partial \beta}{\partial y} - \frac{\partial M}{\partial y}) = 0,$$

or

$$\frac{\partial \beta}{\partial y} = 1 - \frac{\partial M}{\partial y}.$$  

That is, as an increase in income will increase the time cost of
commuting at any given level of accessibility, the household will subtract this increase in commuting costs from the addition to the bid that is induced by the increase in income. Nevertheless, an increase in income can be expected to increase the household's bid for a given residence.\textsuperscript{84}

Similarly, an increase in income can be expected to increase the household's demand price for accessibility, as it can be expected to increase the household's marginal cost of commuting. Differentiation of (80) with respect to accessibility to the $j$th point gives:

\begin{equation}
\frac{\partial^2 \beta}{\partial y \partial q_j} = \frac{\partial^2 \beta}{\partial q_j \partial y} = -\frac{\partial^2 M}{\partial q_j \partial y}.
\end{equation}

$\partial^2 M/\partial q_j \partial y$ gives the change in the marginal cost of travel at a given level of accessibility due to an increase in income. As an increase in income can be expected to increase the marginal cost of travel due to the increased time cost of travel, $\partial^2 M/\partial q_j \partial y$ can be expected to be positive. Its negative gives, for a given level of accessibility, the decrease in marginal travel costs (i.e., the increase in marginal travel savings) that would be achieved by an incremental increase in accessibility given the increase in income. This (potential) increase

\textsuperscript{84} Note that if $\partial M/\partial y > 1$, an incremental increase in income will decrease the household's bid on a residence. It is not likely, however, that a one dollar increase in income will result in more than a one dollar rise in the household's time cost of commuting.
in marginal travel savings is the maximum amount by which the household could increase its bid for accessibility to the $j$th point, given its increased income, and remain at the original level of utility.

Among the first-order conditions necessary for household equilibrium is the condition that (from (74) and (79)):

$$\frac{\partial \beta}{\partial q_j} = \frac{\partial U / \partial q_j}{\partial U / \partial x} - \frac{\partial M}{\partial q_j} = \frac{\partial P}{\partial q_j} \quad \text{for} \quad j = a, \ldots, n.$$  

That is, the household will locate, with respect to each point that it desires to be accessible to, at that level of accessibility where its marginal bid price, or compensated demand price, equals the market implicit price for accessibility to those points.

**Zoning, Development Regulations and Charges, and the Developer-Seller**

The developer-seller is usually faced with a set of institutional constraints in the residential development process. Among these constraints are zoning and subdivision regulations and building and fire codes.

Certain zoning and subdivision regulations place constraints on the developer's production function that prohibit certain inputs and/or outputs (characteristics). For example, subdivision regulations may prohibit development on flood plains or on land with unsuitable soil
conditions. Another example can be found in zoning regulations that prohibit construction of single-family dwellings in an area zoned exclusively for business and/or industrial use. The intent of most of the prohibitionary constraints seems to be to limit residential development to certain areas of the settlement plain.

Some regulations take the form of conditional constraints on the developer's production function. For example, zoning regulations may not permit row houses in an area zoned for single-family and two-family dwellings (R2) or two-family dwellings in an area zoned for single-family dwellings (R1). If, for example, a developer's profit maximizing design requires that he construct row houses in an existing neighborhood of single-family and two-family dwellings, he may be required to construct in an area zoned R3. The regulation states, in effect: if row houses, then locate in an area zoned R3. The effect of many zoning regulations is to impose a conditional, "if-then", constraint on the developer-seller.

Another common form of regulation places a conditional constraint based on an inequality on the developer's production function. For example, areas zoned R1 may have subcategories R1-A, R1-B, and R1-C requiring progressively smaller minimum lot sizes or frontages. If the developer's optimum design requires a minimum lot size of say \( q^*_1 < q^*_1 \), too small for area R1-A where the minimum is \( q^*_1 \), he will be required to locate in an area zoned for a
smaller minimum lot size—say R1-B. The constraint then takes the form of: if \( q'_1 < q''_1 \), then locate in an area zoned R1-B. Other examples of inequality based conditional constraints may be found in regulations on height of structures, floor area of structures, setback requirements etc. These constraints constitute minimum standards applicable in certain areas on the settlement plain.

Still other minimum standards may apply uniformly across those portions of the settlement plain lying within one local government's jurisdiction. There may be minimum width and surface standards and maximum grade restrictions on streets. There may be a requirement that the developer either provide a complete water and sewer system or connect to the municipal system. These constraints on the developer's production function may, of course, vary across and sometimes within local government jurisdictions.

The building and fire codes place another set of inequality constraints on the developer's production function. For example, there may be minimum standards on plumbing and electrical systems. There are usually minimum standards on structural construction and design. These constraints, also, are likely to vary across local government jurisdictions.

To the extent that the developer's optimum design would not be in compliance with the various zoning and subdivision regulations and building and fire codes, the developer's regulation constrained
optimum location in n-dimensional space will differ from the location he would have chosen in the absence of the constraints. In some instances, for example prohibitionary constraints, the developer-seller will not be allowed to locate in certain areas on the settlement plain. In other instances the regulation imposed constraints on the developer's production function may affect the form of his cost function and thus the form of his offer function.\textsuperscript{85} Further, the effect of the constraints may not be limited only to those characteristics to which the regulations specifically apply but may induce substitutions in inputs and/or outputs (characteristics) and thus result in further design changes. As a result, the effects of the constraints are likely to be pervasive and not confined solely to the characteristics at which they are directed.\textsuperscript{86}

The developer-seller is usually required to pay a number of fees or charges in the process of residential development. The list of payments includes such items as hook-up charges for connecting a

\textsuperscript{85} If the regulations, and thus the constraints, vary across local government jurisdictions, the form of the cost function may also vary across local government jurisdictions. The developer-seller would then be required to choose a profit maximizing cost function and set of characteristics. The problem is not considered here.

\textsuperscript{86} Note that the imposition of inequality constraints will move the developer's formal optimization problem from the realm of classical programming to that of nonlinear programming (see Intriligator, 1971, pp. 44-64).
residence (or subdivision) to the municipal water and sewer system, building permit fees and inspection fees. These fees enter directly into the cost function and, as they often vary across local government jurisdictions, are likely to influence the location of the developers' activity on the settlement plane.

Those fees which are fixed in amount and do not vary with the design of the residence are, in effect, fixed costs. These fixed fees will shift the cost function and thus the offer function upward but will not affect the optimum design within any given local government jurisdiction.

Those fees which vary with the design of the residence, such as the Corvallis Systems Development Charge which varies with the floor area, lot size, and elevation, impose an additional variable cost. Their effect is to shift marginal cost, and therefore the compensated supply curve, upward and thus influence the optimum design of the residence.

**Summary of Extensions to the Model**

The basic theoretic framework provided to explain the residential location of households was expanded to include the effects of local public services and their concomitant property taxes and the effect of commuting or travel costs on the location decision. The effects of zoning and subdivision regulation and charges on the
Consideration of the effects of the property tax revealed that the household's bid on any given residence is discounted by the rate of the property tax; i.e., the bid function is shifted downward, \textit{ceteris paribus}. The effect of the property tax on the compensated demand functions for the characteristics depends upon whether or not the characteristic is publicly provided. The compensated demand functions for those characteristics which are market and nonmarket, nonpublicly provided goods are discounted by the amount of the property tax rate; i.e., the compensated demand function shifts downward. The cost of the publicly provided goods (the property tax) is subtracted from the demand price for those goods to obtain the net marginal evaluation of the publicly provided goods. The net marginal evaluation is discounted by the property tax rate to obtain the effective compensated demand function for the publicly provided characteristics. The result is to lower the compensated demand function for the publicly provided characteristic relative to what it would be if the good were provided at no cost to the household.

It was also shown that, if the supply of publicly provided goods were in long run equilibrium such that each household could consume the publicly provided goods at that level where their net marginal evaluation of each of the goods is zero, the implicit market price of the publicly provided goods would be zero. Under such conditions,
the publicly provided characteristics of a residence would enter the
market price function but their effect would be infra-marginal
\[(\partial P/\partial q_j = 0 \text{ for } j = p, \ldots, n).\]

Consideration of travel costs in the demand for accessibility
revealed that the bid function is lowered relative to what its position
would be if travel costs were not a factor to be considered in resi-
dential location.

The effect of development regulations on the developer's offer
function and compensated supply function is difficult to assess.
Many of the development regulations set minimum standards for the
residence and therefore impose an inequality constraint on the
developer's production function. To the extent that the developer-
seller would have met or exceeded these standards in the absence of
regulation, the offer function would not be affected by the regulations.
In those instances where the profit maximizing design in the absence
of regulation would fail to meet the minimum standards, the offer
function is likely to be shifted upwards as a result of the standards
since they impose a design that the developer-seller would not have
voluntarily chosen in his search for a profit maximizing design.

The effect of development charges and fees will, in all cases,
be to shift the offer function up since they so shift the cost function.
However, fixed fees will not alter the optimum design whereas vari-
able fees will.
In summary, the extensions to the model considered herein do not affect the basic structure of the model but simply act on the bid and offer functions and on their derivatives—the compensated demand and supply functions.

Summary

This chapter outlines a theoretical economic framework designed to aid in the explanation of the individual residential location decision. In choosing a residence, the individual is considered to select a vector of $n$ characteristics, including such characteristics as accessibility and neighborhood quality, that comprise a residence. There is assumed to be a sufficiently wide range of residences to consider so that the elements of the vector defining the residence may be considered to be independently and continuously variable. The problem is conceived, from the viewpoint of the household, as a process of simultaneously, and optimally, satisfying a set of demand equations for the various characteristics of a residence subject to the supply of the characteristics.

A set of bid functions having the $n$ characteristics of a residence as variables, the household's income as a parameter, considering variations in property tax with changes in the level of public services provided, and variations in travel costs with changes in accessibility is defined for the household. The bid function is
constructed to represent the hypothetical conditions (levels of the characteristics and price of the residence) under which the household would be indifferent between residences. Its form is, therefore, partially determined by the form of the utility function. There is a bid function for each level of utility.

The first derivative of any bid function with respect to any one of the $n$ characteristics gives the maximum amount the household could pay for an incremental increase in the level of the characteristic and remain indifferent, at a given level of utility, between the increased price with the increased level of the characteristic and the initial level of the characteristic with the lower price. The $n$ functions marginal to a bid function are, therefore, inverse compensated (real income held constant) demand functions for the characteristics. There is, in general, a set of compensated demand functions for each of the $n$ characteristics—one for each level of utility considered.

Symmetrical to the household, a set of offer functions having the $n$ characteristics of the residence as variables is defined for each developer-seller. The form of the offer functions depends upon the developer's cost function which in turn depends upon the production function, the factor costs, and the constraints placed on the developer-sellers and the fees charged them by the local governments in whose jurisdiction they choose to build. An offer function
represents the hypothetical conditions under which the developer-seller would be indifferent (at a constant level of profit) to producing residences of various designs. There is an offer function for each level of profit considered.

The first derivative of any offer function with respect to any one of the \( n \) characteristics comprising a residence gives the minimum amount the developer-seller would accept for providing an incremental change in the level of the characteristic of a residence and remain indifferent between providing the change in design with a new offer price and not changing the design and staying with the original offer price. The \( n \) functions marginal to an offer function are, therefore, inverse compensated (profit held constant) supply functions for the characteristics. There is a set of compensated supply functions for each of the \( n \) characteristics—one for each level of profit considered by the developer-seller.

To effect the purchase (and sale) of a residence, the bid price of a household must equal the offer price placed on the residence by the developer-seller. In market equilibrium, no developer-seller can increase his profits by increasing his offer price or by producing a different design and/or number of residences and no household can increase its utility by making a (successful) lower bid on a residence of the design it actually chooses or by purchasing a different residence. In market equilibrium, the lowest possible bid function of every
homebuyer is tangent to the highest possible offer function of a developer-seller such that no one can better his position, in terms of utility or profits, without detriment to the position of another. The points of tangency of all equilibrium offer and bid functions trace out the market price function.

Corresponding to the equilibrium offer and bid function for each sale, there are \( n \) compensated demand and \( n \) compensated supply functions for the characteristics of the residence—one pair for each characteristic. That is, there are \( 2n \) equations (the compensated demand and supply function) in \( 2n \) unknowns (the levels of the characteristics and their implicit prices). The points of intersection of each pair of these demand and supply functions gives the equilibrium level of the characteristics provided by the household and the market implicit price of the characteristics.

The market implicit price of each characteristic, given by the first derivative of the market price function with respect to the characteristic, is considered as given and beyond their influence by each household and each developer-seller.

The residential location of individuals is defined by their utility maximizing position in \( n \)-dimensional characteristics space as determined by their demand for and the developers' supply of each of the characteristics comprising a residence. The optimum position in
characteristics space may or may not correspond to a unique position in geographic space.

The mathematical aspects of the residential location model developed in this chapter are summarized below. Unless otherwise noted, the mathematical notation is the same as that used throughout this chapter.

I. The residence is defined by the following vector

\[ q = (q^H, q^P, q^A) \]

where:

\[ q^H = (q_1, q_2, \ldots, q_{p-1}) \]
\[ q^P = (q_p, q_{p+1}, \ldots, q_{a-1}) \]
\[ q^A = (q_a, q_{a+1}, \ldots, q_n) \]

and where:

\( q^H \) = rate of flow of services from the market and nonmarket publicly provided characteristics of the residence,

\( q^P \) = rate of flow of services from the publicly provided characteristics of the residence, and

\( q^A \) = accessibility of the residence to various locations and services.
II. The household's utility function is given by:

\[ U(q, x) = u \]  

where:

- \( x \) = the quantity of all other goods consumed, (it is treated as a numeraire and assigned a unit price), and
- \( u \) = the level of utility attained.

The household's budget constraint is given by:

\[ y = (1 + t(q^P)) \beta(q) + M(q^A, y) + x \]  

where:

\( \beta(q) \) = household's bid on residence defined by vector \( q \),

(this is more like a proposed contract rent),

\( y \) = the household's disposable (after income tax)

income,

\( M(q^A, y) \) = household's commuting costs at the location defined by \( q^A \) and given its income \( y \), and

\( t(q^P) \) = the property tax rate (actually more like a tax on the contract rent).

Substitution of (88) for \( x \) in (87) gives:

\[ U(q, y-(1 + t(q^P)) \beta(q) - M(q^A, y)) = u. \]
III. The bid function is obtained by solving (89) for $\beta(q)$:

$$\beta(q) = \frac{1}{1 + t(q)} \beta(q, y - M(q^A, y, u))$$

The real income compensated inverse demand functions are obtained by differentiating (89) with respect to $q_i$ and then solving for $\frac{\partial \beta}{\partial q_i}$. These functions are given by:

$$\frac{\partial \beta}{\partial q_i} = \frac{1}{1 + t(q)} \frac{\partial U}{\partial q_i} \frac{\partial U}{\partial x} \text{ for } i = 1, \ldots, p - 1,$$

$$\frac{\partial \beta}{\partial q_i} = \frac{1}{1 + t(q)} \left[ \frac{\partial U}{\partial q_i} \frac{\partial t}{\partial q_i} \beta(q) \right] \text{ for } i = p, \ldots, a - 1,$$

$$\frac{\partial \beta}{\partial q_i} = \frac{1}{1 + t(q)} \left[ \frac{\partial U}{\partial q_i} \frac{\partial M}{\partial q_i} - \frac{\partial M}{\partial q_i} \right] \text{ for } i = a, \ldots, n.$$ 

IV. The developer is defined by its multiple-output production function:

$$F(N, q^H, q^P, q^A, Z) = 0,$$

where:

$N =$ number of residences constructed per period of time, and

$Z =$ a vector of inputs used in the production of the residence.
The developer is assumed to minimize factor costs subject to the technical constraints of its production function and to the institutional constraints at the construction site to obtain its cost function:

\[ C = C(q, N). \]

The developer's potential profits are, then, given by:

\[ \pi = \phi(q)N - C(q, N), \]

where:

\[ \pi = \text{level of profits}, \quad \text{and} \]

\[ \phi(q) = \text{offer price of residence defined by } q. \]

V. Given that the profit maximizing number of units to be built \( (N^*) \) is selected, the offer function is obtained by solving (96) for \( \phi(q) \):

\[ \phi(q) = \frac{\pi}{N^*} - \frac{C(q, N^*)}{N^*} \]

The profit compensated inverse supply functions are obtained by differentiating (97) with respect to each of the \( q_i \):

\[ \frac{\partial \phi}{\partial q_i} = \frac{\partial C / \partial q_i}{N^*} \quad \text{for } i = 1, \ldots, n. \]
VI. Market equilibrium is attained when:

\[ \beta(q) = P(q) = \phi(q), \]

where \( P(q) \) is the market price function which relates the market price of the residence to its characteristics, and when:

\[ \frac{\partial \beta}{\partial q_i} = \frac{\partial P}{\partial q_i} = \frac{\partial \phi}{\partial q_i} \quad \text{for} \quad i = 1, \ldots, n, \]

where \( \frac{\partial P}{\partial q_i} \) is the market implicit price function which relates the market implicit price of each of the characteristics of the residence to the characteristics comprising the residence.

In the next chapter, an econometric model, based on this theoretical framework, is developed to provide a method of estimating the compensated supply and demand functions for some of the major characteristics of a residence.
IV. AN ECONOMETRIC MODEL OF THE DEMAND FOR AND SUPPLY OF THE CHARACTERISTICS OF A RESIDENCE

As was noted in Chapter III, it is the differences in household attributes and in developer attributes that, when related to market prices and characteristics of the residences, permit identification of demand and supply functions for the various characteristics. Given the market prices of the residences along with data descriptive of the characteristics of the residences, of the attributes of the developers that produce them, and of the attributes of the households that purchase them, it is theoretically possible to statistically estimate the demand and supply functions for each of the characteristics of a residence. The purpose of this chapter is to provide an econometric model, amenable to statistical estimation, of these demand and supply functions.

First, a general econometric model, which follows directly from the theoretical model developed in Chapter III, is provided. Then a reduced model, compatible with existing data, is specified.
Following Equations (91) to (93), (98), and (100), the system of supply and demand equations, one dimension of which is depicted in Figure 13, may be written as:

\begin{align*}
\beta_i &= f_i(q^H, q^P, q^A, y, t, \epsilon^d_i) \quad \text{for } i = 1, \ldots, a-1, \\
\beta_i &= f_i(q^H, q^P, q^A, y, t, M, \epsilon^d_i) \quad \text{for } i = a, \ldots, n, \\
\phi_i &= g_i(q^H, q^P, q^A, N, W_1, W_2, \epsilon^s_i) \quad \text{for } i = 1, \ldots, n, \\
\beta_i &= P_i = \phi_i \quad \text{for } i = 1, \ldots, n, \\
P(q) &= P(q^H, q^P, q^A, \epsilon_i) \quad \text{for } i = 1, \ldots, n;
\end{align*}

where \( q^H, q^P, q^A, y, t, \) and \( N \) are as previously defined and:

\begin{align*}
\beta_i &= \frac{\partial \beta}{\partial q_i} = \text{implicit demand price for the } i^{th} \text{ characteristic;} \\
\phi_i &= \frac{\partial \phi}{\partial q_i} = \text{implicit supply price for the } i^{th} \text{ characteristic;} \\
P_i &= \frac{\partial P}{\partial q_i} = \text{market implicit price for the } i^{th} \text{ characteristic;} \\
M &= \text{a vector of variables, other than those describing the}
\end{align*}

\textsuperscript{88} The econometric model presented here is based on the more general econometric model developed by Rosen (1974, p. 50).
accessibility of the residence, that enter the commuting cost function;

\[ V = \text{a vector of variables describing those attributes of the household that influence its preferences;} \]

\[ W_1 = \text{a vector of variables describing the attributes of the developer-seller that influence its cost function;} \]

\[ W_2 = \text{a vector of variables describing the institutional constraints placed on the developer at specific locations and any of the characteristics of a residence that may not enter the household's utility function but do influence the cost function;} \]

\[ \varepsilon_i = \text{a random disturbance term.} \]

Equations (101) through (104) provide a system of \( 3n \) equations (the \( n \) demand equations given by (101) and (102), the \( n \) supply equations given by (103), and the \( n \) equilibrium identities given by (104)) in \( 3n \) unknowns.

With the exception of household disposable income \( (y) \), each of the variables included in the preceding system follows directly from the theoretical model developed in Chapter III. Disposable income is not a variable in a compensated demand function—the level of utility is. However, as was discussed in Chapter III, the level of income influences the (unmeasurable) level of attainable utility, and therefore
is included as a variable in the demand functions.

The system of demand and supply equations is estimated in two steps:

1. The market prices of the residences are regressed on a set of measurements of the \( q_i \) for all residences in the sample to obtain an estimate \( \hat{P}(q) \) of the market price function (105). Then the partial derivative of \( \hat{P}(q) \) with respect to each of the \( q_i \) is obtained and the value of these derivatives calculated for each characteristic of each residence using the observed values of the \( q_i \). These calculated values provide the "observed" market implicit prices \( \hat{P}_i \) for each characteristic of each residence.

2. The \( \hat{P}_i \) are used as the dependent variables in the statistical estimation of the simultaneous equation system given by Equations (101) through (104).

The second step provides estimates of the system of supply and demand equations underlying the market price function estimated in the first step.

If the market price function estimated in step one is linear, the partial derivatives of the function with respect to the \( q_i \) will be constants, and the market implicit prices of the characteristics will be independent of the design of the residence (i.e., will be independent of the \( q_i \)). In that event, the market implicit prices would
not vary with the level of the characteristics, and the system of supply and demand equations could not be estimated.\footnote{That is not to say that the supply and demand functions do not exist, but, rather, that the supply and demand functions could not be statistically estimated as there would be no variation in the market implicit prices.}

However, the market price function would be linear only if the market implicit price of each characteristic were independent of both the level of the characteristic and of the other characteristics of the residence. Though there is no apparent reason why the market price function could not be linear, it seems more likely that the market implicit price of a characteristic will depend not only upon the level of the characteristic but also upon the other characteristics of the residence. That is, the relationship between the \( q_i \) in the market price function seems more likely to be multiplicative than additive. Nevertheless, the market price function is simply a locus of the points of tangency of the offer and bid functions—thus, it need not exhibit the properties of either of these functions.

A Specific Econometric Model

A specific econometric model of the residential location decision is presented in this section. Because certain aspects of the study area are relevant to the specification of the model, it is first described. Then, the model is specified as to the determinants of
market price and of the demand for and supply of the characteristics
of a residence. The variables included in the model are then
explicitly defined and the data sources are described. Then, after
discussion of an estimation problem, hypotheses are formulated as
to the specific variables included in the demand and supply equations
and as to their effects on demand and supply.

The Study Area

The study area is located in the northeastern corner of Benton
County in the center of western Oregon's Willamette Valley. There
are four more-or-less distinct areas within the study area: the
incorporated cities of Corvallis and Philomath, the rural areas
immediately to the north and west of Corvallis, and the North Albany
area which is an unincorporated suburb of Albany, Oregon. In 1970,
the study area had a total population of about 47,000 with about
35,000 in Corvallis and about 1600 in Philomath (U.S. Bureau of,
pp. 48-55). Employment is primarily centered in Corvallis, where
the largest single employer is Oregon State University, and in the
Albany area, where the largest employers are of an industrial nature.
Both Albany and Corvallis serve as retail centers and provide much
the same retail, commercial, and professional services whereas
Philomath provides the basic services generally associated with a city
of its size. The study area is depicted in Figure 14 (map pocket).
Specification of the Model

A Measurement Problem. In Chapter III, the residence was defined by the rate of flow of the services provided by its various characteristics per period of time. Consequently, the market price of the residence was defined as the periodic payment, similar to a contract rent, required to secure the services of the residence for the period. Though, in principle, it is possible to state the capitalized market value (i.e., purchase rather than rental price) of a residence in terms of a periodic contract rent (and vice-versa), there are, in practice, obvious problems associated with defining and measuring—both in a qualitative and in a quantitative sense—the flow of services provided by the characteristics of a residence.

Therefore, the econometric model will be specified in terms of the stock of characteristics associated with the residence rather than in terms of the flow of services provided by the characteristics of the residence. Similarly, the market price of the residence, \( P(q) \), is defined as the capitalized value, or purchase price, of the residence. As a consequence, the implicit prices referred to in the econometric model are interpreted as the present value, at the time of purchase, of the future flow of services stemming from the associated characteristics of the residence.
The Determinants of Market Price. A review, presented in Chapter II, of the empirical studies of the determinants of residential property values indicated that the characteristics of the house *per se*, the characteristics of the neighborhood in which the house is located, the accessibility of the house to retail and employment centers, and the public services available at the site of the house may be important determinants of residential property values. Therefore, these characteristics of a residence are, apparently, of some importance in the residential location decision, and thus measures of them have been incorporated in the model to the extent possible given the available data.

Those characteristics of the house *per se* incorporated in the model are measures of its lot size, floor area, number of bathrooms, and a variable denoting whether or not the house has a fireplace. Admittedly, measurements on these variables will provide only a minimum description of the house.  

90 Consideration was also given to including a measure of the number of bedrooms in the house and a variable denoting whether or not the house had a garage. However, the number of bedrooms was dropped from consideration because it was expected to be highly correlated with the floor area of the house as Musgrave (1969, p. 775) found, in a study of 7,000-10,000 new, single-family houses, that the number of rooms and number of bedrooms were highly correlated with both the floor area and the number of bathrooms. The variable denoting whether or not the house had a garage was dropped when it was found that all residences included in the sample had a garage.
Two measures of accessibility, accessibility to retail centers and accessibility to the places of employment of the members of the household, are included in the model. A distinction between these two dimensions of accessibility is made for several reasons. First, in the absence of information as to the travel habits of the household, particularly in regard to the frequency of travel to retail points as compared to points of employment, it seems desirable to distinguish between the two as the trips to the places of employment may not be made with the same frequency as trips to retail centers. If such is the case, the implicit demand price of accessibility to the retail centers and of accessibility to places of employment will differ. Secondly, the trips to the places of employment must, generally, be made with a minimum frequency at a given time regardless of the location of the residence whereas the frequency and timing of trips to retail centers are more discretionary and may not be independent of the choice of location of the residence. Third, though the monetary travel costs per unit distance may be the same for trips to the place of employment as for trips to retail centers, there is no reason to believe, a priori, that the utility (or disutility) of travel, and thus the implicit demand price for accessibility, to both is the same—as would

There may be, of course, some discretion in the frequency of trips to the places of employment. For example, an employed member of the household may return home for a noon lunch more frequently if he is highly accessible to his place of employment.
be implied by using one measure of accessibility for both places of employment and retail centers. However, because retail centers are also employment centers and because retail and employment centers tend to agglomerate, the two measures of accessibility may be highly correlated; if such is the case, they will be combined in one measure of accessibility.

The only variable distinguishing between the publicly provided services available at different locations within the study area is one indicating whether or not the residence is located within School District 509J. During 1974-75, the property tax rate in the 11 tax-code areas (i.e., geographic areas with different property tax levying authorities and/or rates) within School District 509J averaged $6.60 higher per 1000 dollars assessed valuation ("market value") than it did in the remaining 15 tax-code areas within the study area.

Further, the location of a tax-code area within School District 509J explained about 68 percent of the total variation in property tax rates between the tax-code areas within the study area. Though there are differences in the level of provision and/or quality of other public services (e.g., differences in roads or streets and in fire protection)

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92 School District 509J serves all of the study area except the North Albany area and the city of and area immediately surrounding Philomath, which are served by separate districts.

93 The information concerning property tax rates was obtained from the Benton County Assessor's office.
at different locations in the study area, none had neither the differential impact on property tax rates nor seemed as readily measurable as did the availability of the services of School District 509J. Therefore, location within School District 509J was the only measure of the publicly provided characteristics of a residence included in the model.

The inclusion, in the model, of variables descriptive of the neighborhood of the residence was severely limited by the availability of data. The only available data which was sufficiently disaggregated to allow a reasonably meaningful definition of "neighborhood" was a listing of all houses sold in Benton County during the study period (to be defined later). Using this data, it was possible to obtain observations on the following variables: (1) the standard deviation of the prices of the houses sold within the neighborhood, (2) the average age of the houses sold within the neighborhood, and (3) the number of multiple-family housing structures sold within the neighborhood.  

The standard deviation of the prices of the houses was included as a measure of neighborhood heterogeneity, and the average age of the houses and the number of multiple-family housing structures were included as measures of neighborhood quality—though the latter may

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94 Several other ad hoc variables descriptive of the neighborhood were developed from this data and tested in the model. Certain of these will be described in the next chapter.
also serve as a measure of neighborhood heterogeneity. Admittedly, these variables provide a rather "artificial" description of the socio-economic characteristics of the neighborhoods. \(^\text{95}\)

A variable describing the elevation of each residence above the flood plain of the Willamette River (estimated to be 200 feet above sea level) is included in the model as a determinant of the market price of the residence. In the study area, houses at the higher elevations tend to be situated on irregular terrain which is, generally, wooded and/or tends to have a view of either the Cascade mountain range to the east or the coastal range to the west. Houses at the lower elevations tend to be situated on flat terrain (the Willamette Valley floor) with few or no trees and with a more restricted view. In addition, casual observation suggests that the houses "on the hills" tend to be more expensive and thus form more aesthetically appealing neighborhoods. Thus, this variable provides a measure of a characteristic of the house per se as well as a characteristic of the neighborhood.

\(^{\text{95}}\) Census block data (U.S. Census, 1970) were available for approximately 600 city and census-defined blocks within the study area. The study area was selected partially on the basis of the availability of these data—which were to be utilized to describe the following characteristics of the neighborhood: (1) the percent of the dwelling units which were owner-occupied, (2) the percent of the dwelling units which were one unit structures, and (3) the average number of rooms per owner-occupied unit. However, the census data were found to be incomplete and, for some blocks, the data conflicted with that which was known about the blocks. Therefore, the census data was not utilized.
Finally, a variable denoting the month of the study year during which the deed to the house was transferred and, thus, the sale of the house consummated is included among the determinants of house price to control for the rate of inflation.

**The Determinants of Household Demand.** Those attributes of the household that may influence its demand for the characteristics of a residence are any that might influence the family's preferences and/or level of attainable utility. Four variables descriptive of the household are included on the "demand side" of the model: (1) the disposable (after income tax) income of the household, (2) the amount of equity that the family had in the house they owned prior to purchasing their new residence, (3) the number of children under 19 years of age in the household, and (4) the age of the head of the household. The variable describing the amount of equity in the previously owned residence is included in the model as one indication of the level of household wealth. The age of the head-of-household, in conjunction with income, also influences the level of wealth accumulated; though it may also serve to measure changes in preferences with age. The number of children under 19 years of age in the household is also considered to influence a household's locational preferences. In addition, the number of children in the household influences household disposable income as the household was allowed, in 1974, to deduct $750 from taxable income for each child under 19 years of age. Though
many other household attributes (e.g., occupation, educational level, male or female head of household) might influence the choice of residence, they are thought to be highly correlated with those included in the system—which are, primarily, measures or indicators of the level of household income and wealth.

Also included under the determinants of demand is a fifth variable which denotes the amount of credit to be applied against the family's federal income tax under the provisions of a federal law designed to stimulate the housing industry. The law allows the family to deduct, under certain conditions, up to the lesser of either five percent of the purchase price of the house or $2000 from their federal income tax. This tax credit can be viewed either as a reduction in the effective price of the residence or as a gift (i.e., an increase in household wealth) the receipt of which is contingent upon the purchase of a residence. As the model developed herein requires that the price of the residence be the same to both the buyer and the seller (i.e., the bid and offer prices must be equal), the latter alternative was selected and this variable is included, along with other variables designed to measure wealth, under the determinants of demand.

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96Not all residences qualify for the tax credit. Generally, the house must have been under construction prior to March 26, 1975. In addition, it must have been purchased between March 12, 1975 and January 1, 1976 (see U.S. Internal Revenue Service, 1975). The method of calculating this credit will be explained when the tax credit variable is explicitly defined in a following section.
The Determinants of Developer Supply. The attributes of the developers that may influence their supply of the various characteristics of a residence are any that might influence their cost functions (36); that is, any that might influence their factor costs and/or production functions (i.e., the efficiency of their production processes). In addition, any constraints placed on the developers and development fees charged them in the construction of a residence may influence their supply of various characteristics. The attributes of each developer are herein described by two sets of variables: (1) those descriptive of the developer per se and (2) those descriptive of the constraints placed on, and fees charged, the developer at a particular site.

Variables descriptive of four attributes of the developer are included in the "supply side" of the system; these are: (1) the developer's scale of operation as measured by the number of single-family residences on which construction was completed during the study period, (2) the diversity of the residences constructed as measured by the standard deviation of the construction cost per single-family unit completed during the study period, (3) the average quality of construction as measured by the average construction cost per square foot of the single-family dwelling units completed during the study period, and (4) the average cost of construction per single-family dwelling unit completed during the study period.
The reasons for including a measure of the builder's scale of operation follow directly from the development of the theoretical model and were discussed in Chapter III.

The variable denoting the standard deviation of the construction cost per residence is included in the model to control for any economies of specialization that might accrue to developers that specialize in a residence of a given design (more precisely, cost). Presumably such economies would influence the marginal cost per residence of provision of certain characteristics and thus would, by definition, affect the developer's implicit supply prices of the characteristics, ceteris paribus.

A variable measuring the average construction cost per square foot of dwelling unit area is included in the system as there may be considerable differences between developers in, for example, the quality of siding, millwork, flooring, and plumbing used in the construction of a house. These differences in the quality of construction are, it is hypothesized, reflected in the implicit supply price of certain characteristics of the residence.

The variable denoting the average construction cost of the residences constructed by a developer is included in the model for much the same reason as the variable denoting the average cost of
construction per square foot. All other things held constant, particularly the level of the characteristics of the house per se, a higher average cost of construction may be indicative of a higher quality of construction—which may be reflected in the implicit supply price of certain characteristics.

The variables descriptive of the constraints placed on, and fees charged, the developers at a particular construction site consist of two proxy variables which indicate whether or not the residence is located in either Corvallis or Philomath. Both cities have imposed zoning and subdivision regulations and require certain permits and/or permit fees which differ from Benton County—within which the cities are located. In addition, both cities have imposed a "systems

97 However, as will be explained when the variables are explicitly defined, measurements on these two variables are made over different sets of houses.

98 The building and mechanical permit fee and the plan check fee schedules are uniform throughout the study area and are based upon the value of the construction as determined by the city or county building officials. The electrical permit fee schedule is uniform throughout the State and is based on the total building area. The Benton County and City of Philomath plumbing permit fee varies with the number of bathrooms, and the City of Corvallis plumbing permit fee varies with the number of fixtures (sinks, bathtubs, etc.).

The building code is uniform throughout Oregon. However, in a telephone conversation of the author with the Executive Secretary of the Albany-Corvallis Homebuilders Association, the latter expressed the opinion that the City of Corvallis, is much more rigid in its interpretation of the code than is either the City of Philomath or Benton County; thus, construction costs may be higher in Corvallis, ceteris paribus.
development charge" which varies with certain characteristics of the residence. Thus, the two proxy variables are included in the system to measure the impact of the additional constraints and fees imposed by Corvallis and Philomath on the implicit supply price of the characteristics of the residence.

Finally, a variable denoting whether or not a house has "custom features" is included with the determinants of supply to measure the effect of buyer-imposed constraints on the developer. Buyers occasionally request that the developer change the design of the residence in some way—particularly if they agree to buy the house before construction is completed. For example, the buyer may request double-pane windows instead of single-pane (or vice-versa). The buyer may request that the builder leave certain details of construction (e.g., painting) unfinished or that the developer do more (e.g., finish the basement) than he had originally intended. These customer

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99 The City of Corvallis Systems Development Charge (SDC) varies with the value of the construction, the lot size, the total floor area of the building, and the elevation of the lot above some baseline. For example, the SDC for a 1500 square foot house, the construction cost of which is estimated at 25 dollars per square foot and which is situated on a 10,000 square foot lot, would range from about $600 to $775 depending upon the elevation of the lot (Corvallis, City of, Ordinance . . .).

The City of Philomath SDC varies with the lot size and the number of waste fixtures (sinks, toilets, dishwasher, etc.). For a house similar to that described above, the SDC would be a minimum of about $700 (information obtained from City of Philomath building inspector in telephone conversation with author).
requests may be reflected in the implicit supply price of certain characteristics of the residence.

Data Sources and Definitions of the Variables

A Note on Model-Imposed Data Constraints. The model developed herein limits the study of residential choice to the buyers and developers of new, speculatively-built (i.e., built-for-sale rather than built-for-owner) homes. The model requires that each residence have a market determined price. This requirement eliminates from consideration built-for-owner homes since, though they have a potential market value, their value has not been established by a market transaction. 100 Also, there is not a clear distinction between buyers and suppliers in the case of built-for-owner homes as these homes are, generally and to varying degrees, built to the buyer's specifications. Further the theory does not provide a rationale for the emergence, on the market of a used residence (for

100 For such homes, it would be possible to use, as an approximation of market price, the appraised value of the residence as determined by the county assessor's office. The disadvantages of this approach were briefly discussed in Chapter II. Further, an analysis of unpublished data obtained from the Benton County Assessor indicated that the appraised value of new and used residences sold in Benton County during the study period ranged from about 23 to 166 percent of the sale price, with the appraised value within 20 percent of market price for about 65 percent of the sales.
either sale or rent) with a given set of characteristics; this limits consideration herein to new residences. In addition, a methodological problem is encountered in consideration of rental units in that the contract rent of the units and the purchase price of the residences would have to be converted to a common basis so as to allow comparisons. The implications of the restriction of this study to the buyers and developers of new, speculatively-built, residences will be discussed in Chapter VI.

The Source and Nature of the Data. Statistical estimation of the model required that data descriptive of the characteristics of new, speculatively-built, residences, of the attributes of the households that purchased them, and of the developers that supplied them be acquired.

A list of all residences sold and for which, in addition, a deed was recorded in Benton County between September 1, 1974 and August

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101 The inclusion of used residences in the model would require development of a theory of supply of such residences. It is difficult to envision such a theory which would not require consideration of physical depreciation and obsolescence (physical and fashionable) and of other changes in, especially neighborhood, characteristics with time. Used residences would, presumably, be described by a set of characteristics, some of which may be either depreciated or obsolete. But the theory would have to provide a rationale for placing such residences on the market. Though the seller may be profit-motivated once the decision to sell has been made, the decision to sell the residence, and thus the location of the residence in characteristics space, may not always be profit-motivated.
31, 1975 was obtained from the Benton County Assessor. 102 This list included all new, speculatively-built, houses sold in the county during that period except those for which a record of deed was not filed with the Benton County Office of Records and Elections. 103 Among the information provided for each residence (new and used) was the following:

1. the sale price of the residence, 104
2. the property tax account number of the residence,
3. the tax code number for the area in which the residence is located,
4. the instrument number under which the deed was recorded in the Benton County Office of Records and Elections,
5. the year in which the residence was built, and
6. the building class of the residence (whether a single or a multiple-family dwelling).

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102 The period of time from September 1, 1974 through August 31, 1975 will hereafter be referred to either as the "study period" or "study year".

103 The law does not require that a record of deed be filed. However, it is in the buyer's interest to record the deed, and, if a mortgage loan is involved, the lender generally requires that the deed be recorded. Further, it is required that, if a deed is recorded, it be recorded in the county wherein the property is located.

104 Oregon law requires that the "true and actual consideration" for which ownership of property is transferred be stated on the deed.
All residences built in 1974 and 1975 and sold for the first time during the study period were considered to be new, speculatively-built houses. Of the 950 residences sold in Benton County during the study period, 108 were identified as being new and speculatively-built. However, to include a residence in this study, it was necessary to have measurements of the relevant characteristics of the house per se--namely, the lot size, floor area, number of bathrooms, and the presence or absence of a fireplace. As these data were obtained from the files of the Benton County Assessor, it was also required that those residences included in the study be appraised. As seven of these residences had not yet been appraised, measurements of the characteristics of the residence per se were obtained for 101 houses.

To obtain the attributes of the household, an attempt was made to interview an adult member of each of the 101 households. Using

105 It is possible that a house could have been built and purchased by its first owner between Jan. 1, 1974 and August 31, 1975 and subsequently sold during the study period—thus being counted as a new, speculatively-built, house. However, data the same as that described for the study period were available for all houses sold between January 1, 1974 and August 31, 1974. None of the houses purchased during this period were resold prior to or during the study period.

106 More precisely, 33 of the 108 residences had not been appraised, but appraisers had obtained measurements of the characteristics of 26 of these 33 houses at the time the data were obtained. They had not yet been assigned an appraised value.
the questionnaire presented in Appendix A, measurements were obtained on the attributes of 81 households. Thus, only 81 residences and households are included in the study; their approximate location is depicted in Figure 14.

For each of the 81 residences, the developer was identified using building permit information obtained from the building officials of Corvallis, Philomath, and Benton County. The date on which the building permit was granted and the estimated value of construction, as determined by the appropriate city or county building authority, was obtained for each residence. Thirty-one developers supplied the 81 residences included in this study.

A commercial survey firm was contracted to conduct the interviews during the month of January, 1976. After at least three attempts, they were unable to contact three households. Nine households refused to answer the entire questionnaire, and six refused to reveal their income and/or the questions pertaining to the purchase and resale price of their previous residence.

Two respondents stated, in response to the second question on the questionnaire, that they had their house built for them, and thus they were dropped from the study. A possible reason for a deed transfer in the case of a built-for-owner house may be that the developer owned the lot, that the household entered into a contract with the developer to build the house, and the deed was transferred and the sale consummated when the terms of the contract were fulfilled.

The building permit application requires that both the owner and the builder of the residence be identified. If a distinction between the owner and builder was made on the building permit report, the owner was considered to be the developer.
The number of residences and the estimated cost of construction of each residence constructed by the developers represented in the study were obtained by review of all building permits granted during 1974 by issuing authorities in Linn and Benton Counties. Measurements of the developer attributes were obtained for 1974 because building permits for all but one of the residences included in the study were issued during that year. Further, it was found that, for the residences included in the study, an average of 8.3 months elapsed between the date the permit was granted and the date the residence was sold. Thus, on the average, residences sold in September 1974 would have been in the planning and/or construction phase since, at least, January of 1974 and residences sold in August, 1975 would have been in this phase since, at least, December of 1974.

Given the description of the data sources, this study of residentail choice includes all of those families who purchased a new, appraised (by March, 1976), speculatively-built, house in northeastern Benton County, Oregon, between September 1, 1974 and August 31, 1975 and who, in addition, could and did cooperate in the study. That is, an attempt (limited by data availability) was made to study the location decision of all buyers and developers of new,

109 The issuing authorities were the building officials of Albany, Benton County, Corvallis, Lebanon, Linn County, Philomath, and Sweet Home.
speculatively-built, houses who chose to locate in the study area during the study period. After explicitly defining the variables included in the system, the limitations of this data set with respect to statistical inference will be briefly discussed.

The Characteristics of the Residence. The variables descriptive of the house per se are defined as follows:

1. Lot size (LOTSIZE): The area (square feet) of the lot on which the residence is situated,
2. Floor area (FLRAREA): The heated dwelling area (square feet) of the residence as measured using the exterior dimensions,
3. Number of bathrooms (NBATH): The number (in increments of one-half) of bathrooms in the residence,
4. Fireplace (FIREPL): A binary variable set equal to one if the residence had at least one fireplace and zero if it had none.

Measurements on each of these variables were obtained from the files of the Benton County Assessor.

A binary variable was used to indicate whether or not the residence was located within School District 509J (SD509J). The variable was set equal to one if the residence was located in either tax code area 901 or 905 and set equal to zero if it was located in any other tax code area.
The accessibility of the residence to the household's place(s) of employment (ACCESSM) was defined as the sum of the distances from the most accessible point to the place(s) of employment less the sum of the distances from the actual location of the residence to the place(s) of employment. The most accessible point was defined as that location where the sum of the distances to the place(s) of employment is a minimum. The location of the places of employment of only the male and female heads of the household was obtained by the household survey. Thus, assuming for purposes of illustration that both were employed, accessibility to employment was calculated as:

110 The calculation of distances required that grid coordinates be assigned to the residence and the places of employment. The first eight characters of the property tax account number of the residence provided a description of the location of the residence according to the government (or rectangular) survey system (see Barlowe, 1958, pp. 37-38). A computer program was developed to convert the property tax account number to grid coordinates that identify the location of the residence to within about 1000 feet (i.e., it was assumed that the residence was located in the center of the 40-acre square identified by its property tax account number; if it were in a corner of this square, it would be 933 feet from its assumed location). The addresses of the places of employment were obtained by household survey and were converted to a number similar to the tax account number by superimposing the rectangular survey grid on the appropriate map. The computer program used to calculate the grid coordinates for the residences is presented in Appendix Table C1, and an illustration of the government survey grid is provided in Appendix Figure C1.

111 With the exception of those households located in North Albany, it was assumed that travel is possible in all directions, and thus straight-line (linear) distances were calculated between all points. The distance between a residence in North Albany and a point in Albany was calculated as the sum of the linear distance from the
ACCESSM = \( d_{12} - (d_1 + d_2) \leq 0 \)

where

\( d_{12} = \) distance (in miles) between the places of employment (the sum of the distances from the most accessible point),

\( d_1 = \) distance from the residence to the place of employment of the male head of the household, and

\( d_2 = \) distance from the residence to the place of employment of the female head of the household.

Therefore, ACCESSM may be interpreted as the change in the total (one-way) journey-to-work distance that the household could achieve by moving to its most accessible (with respect to employment) location.\(^{112}\)

Seven of the 81 households included in the study did not have a place of employment at the time they purchased the residence (six were retired and one was unemployed). These households were

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\(^{112}\) This assumes, of course, that the members of the household do not travel to work together.
assigned the mean value of ACCESSM as calculated for the remaining 74 households.  

The variable describing the accessibility of a residence to retail centers was defined analogously to that which was used to describe accessibility to employment. That is, it was defined as the sum of the distances from the most accessible point to a set of retail centers less the sum of the distances from the actual location of the residence to the set of retail centers, where the most accessible point was defined as that location where the sum of the distances to the set of retail centers is a minimum. Therefore, each observation on the variable measuring accessibility to retail establishments may be interpreted as the change in the total (one-way) journey-to-shop distance that the household could achieve by moving to its most accessible (with respect to retail centers) location.

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113 The unemployed household may have considered the probable location of future employment in its location decision. Though accessibility to employment would not be a consideration in the location choice of a retired household, the resale value of the residence, and thus its accessibility to employment in general, may be a consideration. Thus, assignment of the mean value of ACCESSM to the residence may not be totally inappropriate.

114 As with accessibility to employment, linear distances were used in the calculation of accessibility to retail centers except for those households located in North Albany.

115 This assumes that the household does not take "short-cuts" by traveling between retail centers (i.e., it assumes that the household travels separately from the residence to each retail center).
The following locations were selected as retail centers on the basis that each is thought to be an area-wide, rather than neighborhood, retail center:

1. the center of the Albany CBD (assumed to be at the intersection of Ellsworth and Main Streets),
2. the intersection of Waverly Drive and S.E. Santiam Highway in Albany,
3. the center of the Corvallis CBD (assumed to be at the intersection of Fourth Street and Madison Avenue),
4. the intersection of Ninth Street and Circle Boulevard in Corvallis,
5. the intersection of Fillmore Avenue and Kings Boulevard in Corvallis, and
6. the center of the Philomath CBD (assumed to be at the intersection of Twelfth and Main Streets).

Those retail centers located away from the central business districts are shopping centers. The locations of the retail centers are depicted in Figure 14.

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Neighborhood retail establishments, such as grocery stores, automobile service stations, etc., are assumed to have a sufficiently uniform spatial distribution so that accessibility to them is not a major consideration in the location decision. Further, accessibility to retail centers will, generally, ensure accessibility to such establishments.
Due to the absence of information as to individual household shopping habits, the variable describing accessibility to retail centers at a given residence was measured under two different assumptions. These assumptions, and the name assigned to the corresponding accessibility variable, are that:

1. each household visits, with equal frequency, each of the retail centers in the city in, or closest to, which it is located (ACCESSR);

2. each household visits only, and with equal frequency, the CBD of the city in, or closest to, which it is located and the shopping center closest to the location of its residence (ACCESSR2).

The second assumption was introduced as there may be considerable substitutability between shopping centers, and, therefore, households may visit only that one which is most convenient.

Two additional accessibility variables were defined under the assumption that households make no distinction, in their location decision, between accessibility to employment and accessibility to retail centers. The assumptions under which these two variables are measured are:

1. each household visits, with equal frequency, its place(s) of employment and each of the retail centers in the city in, or closest to, which it is located (ACCESST); and
2. each household visits, with equal frequency, its place(s) of employment, the CBD of the city in, or closest to, which it is located, and the shopping center closest to the location of its residence (ACCESRM2).

Four sets of accessibility variables will be considered in the estimation of the market price function; they are: (1) ACCESST, (2) ACCESRM2, (3) ACCESSM and ACCESSR, and (4) ACCESSM and ACCESSR2. 117 That set which is most effective in explaining the variation in the market price of the residences will be selected.

The variables descriptive of the neighborhood in which the house is located are self-explanatory; they are:

1. the standard deviation of the prices of all houses sold within the neighborhood (STDEVPRN) during the study period,
2. the average age (in years) of all houses sold within the neighborhood (AVEAGEN) during the study period, and
3. the number of multiple-family dwelling units sold within the neighborhood (NMULTFN) during the study period.

Measurements of these variables were obtained from data, provided by the Benton County Assessor, which were descriptive of 950

117 The computer program used to calculate the various measures of accessibility is presented in Appendix Table C4. The program presented is that used to calculate ACCESST; the observations for all other accessibility variables were calculated by making only minor alterations to this program.
residences sold in Benton County during the study period.

The neighborhood was spatially defined to include only those residences located on the 360 acres centered about the residence in question. This definition is arbitrary in that it considers neither barriers that may divide the area defined to be a neighborhood nor the spatial agglomeration of households according to socio-economic characteristics. However, it was chosen so as to allow, considering the data source, a "reasonable" number of observations (i.e., number of houses sold) for each neighborhood.

\[118\] More explicitly, the neighborhood was spatially defined to consist of all residences located on the 40-acre plot on which the residence in question is located plus all of those located on the eight 40-acre plots contiguous to it.

\[119\] One approach that would avoid the necessity of an explicit definition of neighborhood (an elusive concept) would be the use of "potential" variables similar to those often used by geographers wherein an observation on a particular variable is weighted by its distance from some point of interest (see, for example, Gould and White, 1974, pp. 21-24 and/or Ingram, 1971). For example, if the age of the residences in the vicinity (not explicitly defined) of a residence were one characteristic of a residence, then the variable descriptive of this characteristic might be defined as: \[\Sigma_i (A_i / d_i^a)\], where \(A_i\) is the age of the \(i^{th}\) residence in the sample, \(d_i\) is its distance from the residence in question, and \(a\) is a constant chosen, perhaps, so as to maximize the explanatory power of the variable in the estimation of the market price function. This approach has some intuitive appeal in that the external effects, on market price, of the other residences in the vicinity of a particular residence may be hypothesized to diminish with their distance from that residence. However, this method of measuring neighborhood characteristics was not used because it would be computationally expensive.
Using this spatial definition of neighborhood, there was an average of 23.51 houses sold in the neighborhood of each residence. The range in the number sold was from zero to 49, the median number sold was 20, and the standard deviation in the number sold was 14.87. There were two residences with no houses sold in their neighborhood during the study period; they were assigned the mean values of the neighborhood characteristics as estimated for the remaining 79 residences.

Measurements on the variable describing the elevation of the residence above the flood plain of the Willamette River were obtained from topographic maps with a scale of 2000 feet per inch and either a 10 or a 20 foot contour interval (U.S. Geological Survey).

The Attributes of the Household. Two of the attributes of the household were obtained directly from the questionnaire (Appendix A) used in the household survey; they are: (1) the number of children under 19 years of age who lived in the household either all or part of the year (NOCHILD), and (2) the age of the head of the household (AGEHEAD). The age of the male head of the household was used if there was a male head—otherwise the age of the female head of the household was used.

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120 The computer program used to calculate the observations on the variables descriptive of the neighborhood is presented in Appendix Table C2.
The disposable (after income tax) income of the household was estimated using the annual gross family income from all sources. It was assumed that the gross income of the household was equal to that income level at the midpoint of the income interval which the household checked on the questionnaire, and that the family used the standard deduction in calculating the state and federal income taxes.  

One household reported an income within the open income interval provided on the questionnaire, and thus it was necessary to estimate its income. It was assumed that, above the modal income, the income distribution of the buyers of new, speculatively-built houses in the study area followed a Pareto distribution such that:

\[ P = D Y_k^{-\alpha}, \]

where

- \( P \) = the percentage of the households included in the study that have an income greater than \( Y_k \),
- \( Y_k \) = the midpoint of the closed income intervals above the modal income level, and
- \( D \) and \( \alpha \) are positive constants.

It was further assumed that the income of the household reporting an income in the open interval was equal to the mean of the open interval, which is given, for a Pareto distribution, by:

\[ \bar{Y}_0 = (\alpha/\alpha-1)Y_0, \]

where \( \bar{Y}_0 \) is the mean of the open interval and \( Y_0 \) is the lower bound of the open interval (see Allen, 1938, pp. 407-408).

The Pareto distribution was estimated in double-log form, for incomes over $21,500, as:

\[ P^* = 55.5375 - 5.6578Y_k^*, \]

\[ R^2 = 0.839588, \quad F = 15.3585 \]

where the numbers in parentheses are t values and the asterisk indicates the natural logarithm of the variable. Therefore, the mean of the open interval, and the income assigned the household reporting
Other family attributes considered in the calculation of the tax were the marital status of the head of the household and the estimated number of exemptions that the household was entitled to claim.\(^{122}\)

The amount of equity that the household had in the house it owned prior to the residence considered in this study (EQUITY) was estimated using household survey data on the purchase price (PURPRI), resale price (RESALEPR), and the number of years it owned (YRSOWNED), the previous residence.\(^{123}\) In addition, it was necessary to make assumptions as to the amount of the down-payment

\[
\bar{Y} = \left(\frac{5.6578}{5.6578 - 1.0}\right) \times 40,000 \approx \$48,500.
\]

\(^{122}\) The income taxes and disposable income were calculated using the computer program presented in Appendix Table C3.

\(^{123}\) Six of the 54 households that previously owned a residence had either not yet sold their previous residence or refused to divulge the resale price, and thus it was necessary to estimate the resale value of the previous house. This was done, using the data which was available from 48 households, by regression of the resale price on the number of years the household owned the previous house and on the purchase price of the residence. After estimating a number of functional forms, the following equation was selected:

\[
\text{RESALEPR} = 1.0818(\text{PURPRI}) + 1.806.6(\text{YRSOWNED})
\]

\[
(19.257) \quad (4.102)
\]

\[- 44.004(\text{YRSOWNED})^2 , \]

\[
(-2.170)
\]

\[R^2 = .969651, \quad F = 479.25\]

where the numbers in parentheses are t values. This equation was then used to estimate the resale price of those residences for which such was not available.
on the previous residence and as to the term of, and interest rate
specified in, the mortgage contract for the residence. An annual con-
tact interest rate of 7.5 percent, a mortgage term of 25 years, and
a 20 percent (of purchase price) down-payment were assumed in the
calculation of the observations on the EQUITY variable. These
data and assumptions concerning the previously owned residence,
along with the additional assumptions of continuous payment on the
mortgage and continuous compounding of the interest due on the
balance of the loan, permitted the estimation of the amount of equity
as:

124 Those households that owned their previous dwelling had
owned it for an average of 5.28 years, and 72.2 percent of these
households had owned the dwelling for five years or less. First-
mortgage interest rates on home loans granted by conventional
lenders averaged 7.12 percent in 1968, 8.52 percent in 1970, and
7.75 percent in 1971, with the rate on FHA-insured loans averaging
slightly higher in those years. However, the first-mortgage interest
rates on home loans averaged only 5.83 percent on conventional and
5.47 percent on FHA-insured loans in 1965 (U.S. Bureau of Census,
1972, p. 455). Thus a contract interest rate of 7.5 percent, more
representative, but slightly lower than that, of recent years was
assumed.

During 1970-1971, the average "loan to price ratio" on
conventional first-mortgages ranged from about 71 percent to 75.5
percent whereas the same ratio for FHA-insured mortgages ranged
from about 92.5 to 93.1 percent during that period (U.S. Bureau of
Census, 1972, pp. 692-693). Therefore an average down-payment of
20 percent was assumed.

The average term to maturity on first-mortgage, conventional,
loans, ranged from about 24.8 to 26.4 years in 1970-1971. Over 99
percent of the FHA-insured mortgages issued in 1970-1971 had a
term of 30 years or more (U.S. Bureau of Census, 1972, pp. 692-
693). An average term of 25 years was assumed.
\[
EQUITY = \left[ e^{in + D(e^{-it} - e^{-in}) - 1} \right] P_1 + P_2,
\]

where

\( i = \) the mortgage contract rate of interest,
\( t = \) the term of the mortgage (in years),
\( n = \) the number of years the previous house was owned,
\( P_1 = \) the purchase price of the previous house,
\( P_2 = \) the resale price of the previous house, and
\( D = \) the percent of purchase price that was down-payment.

If the household did not own its previous dwelling unit, \( EQUITY \) was set equal to zero.

The estimated federal income tax credit (TXCREDES) which a household was allowed to claim if it purchased, after March 12, 1975, a house which had been under construction prior to March 26, 1975, was derived under three assumptions:

1. if the building permit for the residence was issued prior to March, 1975, and if the house was purchased either during or after March, 1975, it was assumed that the household was entitled to claim the credit;

\[125\] The major provisions of the Internal Revenue Code pertinent to this credit are given on Internal Revenue Service Form 5405.
2. the "adjusted basis" of the new residence, on which the computation of the credit is based, was assumed to be equal to the purchase price of the new residence (MKTPR) less any capital gain on the previous residence.

3. the capital gain on the sale of the previous residence was assumed to be equal to the resale price of the previously owned residence less its purchase price.

Given these assumptions, TXCREDES was set equal to the minimum of either:

1. \[0.05(MKTPR - (RESALEPR - PURPRI)) \geq 0,\]
2. $2000, or
3. the household's federal income tax as estimated using the standard deduction.

TXCREDES was set equal to zero for the 35 households that purchased a residence which did not entitle them to the credit.

**The Attributes of the Developers.** The observations on three of developers were estimated using data obtained from the building permits issued within Linn and Benton Counties; they are:

1. the sale of operation of the developer (BLDRSCAL) as estimated by the number of building permits granted to the developer for the construction of single-family residences,
2. the average cost of construction per residence (AVCONCST), and
3. the standard deviation of the cost of construction per residence (SDCONCST).

Each of these variables pertains to those residences for which the developer obtained building permits during 1974.

The observations on the variable describing the average cost of construction per square foot of floor area per residence constructed by each developer (AVCSTFT2) were estimated only for the residences included in the study. The cost of construction per square foot of heated dwelling area was estimated using the cost of construction as listed on the building permit and the heated dwelling area as estimated by the Benton County Assessor. The mean value of the cost of construction per square foot was then obtained for the residences constructed by each developer included in the study.

The construction cost is the estimated construction cost of the housing structure per se. Unfortunately, not all building officials use the same methods to estimate this cost, and in some cases these methods may be somewhat subjective. With the exception of Corvallis, all building officials in Linn and Benton Counties estimate the construction cost using guide lines provided by the International Conference of Building Officials (ICBO). The Corvallis building officials use a cost factor appraisal manual obtained from the Oregon State Department of Revenue; this manual provides parameters to
estimate construction costs for residential dwellings of eight
different quality levels.  

The three variables intended to be descriptive of the constraints
placed on the developer in the construction of a residence and of the
fees charged him by a local government are:

1. a binary variable set equal to one if the residence was located
   in Corvallis (CORVALLI) and set equal to zero if it was not,

2. a binary variable set equal to one if the residence was located
   in Philomath (PHILOMTH) and set equal to zero if it was not,
   and

3. a binary variable set equal to one if the household that
   purchased the residence placed any constraints on the
   developer or requested that any custom features (CUSTFET)
   be incorporated in the residence.

Observations on CUSTFET were obtained directly from the
questionnaire used in the household survey (Appendix A).

Summary statistics for each of the variables included or to be
considered in the model are presented in Table 1.

Reliability of the Data. It is not known if the data set is
representative of the population of buyers and developers of new,

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Information on methods of estimation were obtained by
conversations, either telephone or personal, with the building
officials of Albany, Benton County, Corvallis, Lebanon, Linn County,
and Sweet Home.
Table 1. Summary statistics for variables included in the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKTPR</td>
<td>Market price of residence (dollars)</td>
<td>37,241.42</td>
<td>8,304.67</td>
<td>39,500.00</td>
</tr>
</tbody>
</table>

**Characteristics of the Residence**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOTSIZE</td>
<td>Lot size (square feet)</td>
<td>17,267.95</td>
<td>15,938.82</td>
<td>99,806.00</td>
</tr>
<tr>
<td>FLRAREA</td>
<td>Dwelling floor area (square feet)</td>
<td>1,536.64</td>
<td>383.18</td>
<td>1,939.00</td>
</tr>
<tr>
<td>NBATHS</td>
<td>Number of bathrooms</td>
<td>1.90</td>
<td>.48</td>
<td>2.00</td>
</tr>
<tr>
<td>FIREPL</td>
<td>Residence has a fireplace (0, 1)</td>
<td>.95</td>
<td>.22</td>
<td>1.0</td>
</tr>
<tr>
<td>ACCESSSM</td>
<td>Accessibility to employment (miles)</td>
<td>-3.53</td>
<td>2.59</td>
<td>14.24</td>
</tr>
<tr>
<td>ACCESSR</td>
<td>Accessibility to retail centers (miles)</td>
<td>-4.13</td>
<td>2.60</td>
<td>11.49</td>
</tr>
<tr>
<td>ACCESSR2</td>
<td>Accessibility to CBD and closest shopping center (miles)</td>
<td>-2.96</td>
<td>1.98</td>
<td>7.10</td>
</tr>
<tr>
<td>ACCESSST</td>
<td>Accessibility to retail centers and employment (miles)</td>
<td>-6.07</td>
<td>3.62</td>
<td>16.24</td>
</tr>
<tr>
<td>ACCESRM2</td>
<td>Accessibility to employment, CBD, and closest shopping center (miles)</td>
<td>-5.08</td>
<td>3.14</td>
<td>13.98</td>
</tr>
<tr>
<td>SDS09j</td>
<td>Residence within School District 509j (0, 1)</td>
<td>.73</td>
<td>.45</td>
<td>1.0</td>
</tr>
<tr>
<td>STDEVPRN</td>
<td>Standard deviation of price of single-family dwellings sold in neighborhood (dollars)</td>
<td>7,071.51</td>
<td>3,125.32</td>
<td>16,707.00</td>
</tr>
<tr>
<td>AVEAGEN</td>
<td>Average age of single-family dwellings sold in neighborhood (years)</td>
<td>8.38</td>
<td>7.00</td>
<td>46.10</td>
</tr>
<tr>
<td>NMULTFN</td>
<td>Number of multiple-family structures sold in neighborhood</td>
<td>1.91</td>
<td>3.05</td>
<td>8.00</td>
</tr>
<tr>
<td>ELEVATN</td>
<td>Elevation of residence above Willamette River flood plain (feet)</td>
<td>120.68</td>
<td>117.90</td>
<td>680.00</td>
</tr>
</tbody>
</table>

**Attributes of the Household**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISPOSIN</td>
<td>Gross income less state and federal income taxes (dollars)</td>
<td>15,839.57</td>
<td>5,095.64</td>
<td>23,947.00</td>
</tr>
<tr>
<td>NOCHILD</td>
<td>Number of children under nineteen years of age</td>
<td>1.38</td>
<td>1.17</td>
<td>5.00</td>
</tr>
<tr>
<td>AGEHEAD</td>
<td>Age of head of household (years)</td>
<td>36.38</td>
<td>9.86</td>
<td>48.00</td>
</tr>
<tr>
<td>EQUITY</td>
<td>Equity in previously owned dwelling (dollars)</td>
<td>10,721.15</td>
<td>11,243.36</td>
<td>59,575.26</td>
</tr>
<tr>
<td>TXCREDRES</td>
<td>Tax credit allowed for purchase of certain residences (dollars)</td>
<td>898.54</td>
<td>838.85</td>
<td>2,000.00</td>
</tr>
</tbody>
</table>
Table 1. Continued.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes of the Developer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BLDRSCAL</td>
<td>Number of single-family residences constructed</td>
<td>29.84</td>
<td>28.31</td>
<td>81.00</td>
</tr>
<tr>
<td>AVCONCST</td>
<td>Average cost of construction per residence (dollars)</td>
<td>25,257.10</td>
<td>4,961.58</td>
<td>22,767.00</td>
</tr>
<tr>
<td>SDCONCST</td>
<td>Standard deviation of cost of construction per residence (dollars)</td>
<td>4,343.36</td>
<td>3,281.56</td>
<td>17,075.00</td>
</tr>
<tr>
<td>AVCSTFT2</td>
<td>Average cost of construction per square foot per residence (dollars)</td>
<td>17.03</td>
<td>1.83</td>
<td>10.15</td>
</tr>
<tr>
<td>CUSTFET</td>
<td>Residence has custom features (0, 1)</td>
<td>.14</td>
<td>.34</td>
<td>1.0</td>
</tr>
<tr>
<td>CORVALLI</td>
<td>Residence within Corvallis (0, 1)</td>
<td>.59</td>
<td>.49</td>
<td>1.0</td>
</tr>
<tr>
<td>PHILOMTH</td>
<td>Residence within Philomath (0, 1)</td>
<td>.06</td>
<td>.24</td>
<td>1.0</td>
</tr>
<tr>
<td>MOSOLD</td>
<td>Month of study period in which residence sold</td>
<td>6.70</td>
<td>3.10</td>
<td>11.0</td>
</tr>
</tbody>
</table>

*The range is the difference between the minimum and the maximum value.*
speculatively-built houses, and of the houses per se, within the study area during the study period. It is difficult to identify the population using public records; not all deeds need be recorded and "spec" houses cannot be identified with certainty using building permit records. Further, not all households which were identified as buyers of new "spec" houses agreed to participate in the study. As there is no apparent reason why those houses and households not identified and those for which data were not available might, as a group, have a different set of characteristics or attributes than those included in the study, it is thought that the data used herein are representative of the population of new, speculatively-built, single-family, dwellings and the buyers and developers thereof which existed within the study area during the study period.

Though the buyers, developers, and residences included in the study are thought to be representative of the population, the reliability of the observations on several of the variables is less certain. The data used to estimate the observations on the neighborhood variables and on the attributes of the developers are particularly suspect.

In the absence of any knowledge concerning the residential property in the neighborhoods of the residences, it is difficult to refute the null hypothesis that the residential property sold in the neighborhoods (on which the variables descriptive of the neighborhoods are based) is not representative of all residential property in the
neighborhood. If the newer, moderately priced, houses have a higher "turnover" rate (and thus a higher probability of being sold during the study period) than do the older houses and/or those toward the extremes of the price distribution, then the newer, more moderately priced, houses situated in neighborhoods of houses unlike them in age and/or value may be disproportionately represented in the neighborhood sales data, and have resulted in an underestimate of the average age and standard deviation of the prices of the houses sold in the neighborhood. However, to the extent that households with similar preferences and incomes group together and thus create homogeneous neighborhoods, as hypothesized in Chapter III, this effect will be lessened.  

The observations on those variables descriptive of the developer which are based on construction costs as estimated by the building officials are particularly subject to measurement error. In estimating the costs, the building officials use guide lines (generally furnished

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127 It has been estimated (State of Oregon, Housing . . . , 1974, p. 9) that there were 12,714 single-family dwellings and 5280 dwelling units in multiple-family structures in Benton County on July 1, 1974. The data on the 950 residential properties sold in Benton County during the study period indicated that 75 multiple-family structures containing 155 dwelling units and 875 single-family dwellings were sold. Thus about 6.9 percent of the single-family residences in the county and 2.9 percent of the multiple-family dwelling units are represented in the data used to estimate the observations on the neighborhood variables. But, this gives no indication of how representative these data are of individual neighborhoods.
by the ICBO) which are representative of average costs of construction in the Pacific Northwest. These guide lines, in the form of estimated costs per square foot of dwelling area and per square foot of garage area are updated periodically to reflect changes in costs, but are likely to lag behind actual costs. Therefore, estimates based on the ICBO guide lines are likely to be biased downward—but this bias should be the same for all estimates. However, conversations with building officials suggested that those of them in the smaller cities, particularly in Linn County, do not always attempt to assess cost of construction but, instead, apply the same, ICBO-provided, estimate of cost per square foot to dwellings within a wide range of quality. Thus, the variation in actual construction costs would be underestimated by that of the building-official-estimated costs. Finally, the appraisal of the quality, and thus cost, of construction is likely to be somewhat subjective and thus subject to measurement error.

An Estimation Problem

As the proposed system includes 11 variables descriptive of the characteristics of a residence, there are potentially 22 equations to be estimated (11 demand and 11 supply equations). Further, none of the variables representing the characteristics of the residence should be excluded, a priori, from any of the demand functions to aid
in their identification. As there are, potentially, 12 endogenous variables in each of the demand functions (those representing the 11 characteristics plus that representing the implicit demand price), the order condition for identification of a demand function that is to be statistically estimated requires that there be at least 11 exogenous variables excluded from that function. Generally, those excluded would be the variables descriptive of the attributes of the developer unless there is some reason to believe, a priori, that specific attributes of a household do not influence its demand for a particular characteristic. Yet, the proposed model includes only seven variables descriptive of the developer. Clearly, estimation of the entire system of demand and supply equations requires more information and resources than are available for this study.

Short of estimating the entire system, there appears to be only two alternatives: either (1) reduce the number of characteristics considered (i.e., the number of endogenous variables or (2) retain all of the characteristics and estimate only as many of the equations

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128 The interdependence or independence of the demand for two characteristics is a question of complementarity or competitiveness in consumption. Generally, there is not sufficient reason to believe, a priori, that the demand for one characteristic may be independent of the demand for another. However, the interdependence or independence of the supply of any two characteristics is, generally, a question of technical competitiveness or complementarity. On the supply side, there may be sufficient reason to believe, a priori, that two characteristics are independent in production.
as can be identified. The first alternative may result in specification error and the second may lead to simultaneous equation bias.

There appears to be no solution to the dilemma. However, "reasonably" accurate estimates of the "observed" market implicit prices require that the market price function be as completely specified as possible. Further, not all of the characteristics will be equally effective in explaining the variation in the market price of a residence. Therefore, the second alternative described above will be followed, and only those variables describing the characteristics which are most effective (i.e., those with the highest statistical significance) in explaining the variation in market price, as determined in the statistical estimation of the market price function, will be included as endogenous variables in the system of supply and demand equations.

Hypothesis Formulation

The econometric model presented in this chapter provides a framework for the statistical estimation of the market implicit prices of the characteristics of a residence. Given the definitions of the variables descriptive of these characteristics, the theoretical framework developed in Chapter III provides a basis for the formulation of hypotheses as to the effect of these variables on the market price of a residence. These hypotheses are presented in this section.
Similarly, the econometric model provides a framework for the statistical estimation of the demand and supply relationships underlying the implicit prices of the characteristics of a residence. Given the definitions of the variables descriptive of the attributes of the households and of the developers, the theoretical framework provides a basis for formulation of hypotheses as to the effect of these variables on the implicit demand and supply prices of the characteristics of a residence. However, it is not possible to specify, a priori, those characteristics of a residence which will most effectively explain the variation in market prices, and thus for which demand and supply functions will be estimated; that is an empirical problem, and one objective of this study is to identify those characteristics. Therefore, to illustrate the model, it is assumed that functions representing the demand for and supply of floor area, lot size, and accessibility to retail centers will be estimated, and hypotheses will be formulated as to the effect of the attributes of the household (developer) on the implicit demand (supply) prices of these characteristics of a residence.

The Market Price Function. Each of the variables descriptive of the characteristics of the house per se (LOTSIZE, FLRAREA, NBATHS, and FIREPL) is defined such that households can, generally, be expected to have a positive marginal evaluation of the characteristic. Further, each is expected to have a positive marginal (or
incremental) cost of provision. Therefore, each of these variables is expected to have a positive market implicit price.

Those variables which measure the accessibility of the residence are defined such that (whichever of the several measures or combinations of measures discussed is used) an increase in accessibility measures the decrease in travel distances associated with the increase in accessibility. As it was assumed, in Chapter III, that households do not have a negative marginal evaluation of accessibility per se, the positive travel savings associated with increased accessibility are sufficient reason to expect, \textit{a priori}, that accessibility will have a positive market implicit price.

The property tax rate for those residences (included in this study) which were located within School District 509J averaged \$7.87 higher than that for those located outside of the school district.\footnote{129 Though location within School District 509J explained 68 percent of the variation in property tax rates between tax-code areas within the study area, it explained over 84 percent of the variation in property tax rates between the residences included in the study.} If the marginal, school-oriented, benefits are greater (less) than the marginal cost (the increase in property taxes as a result) of locating in School District 509J, it is hypothesized that the household will submit an after-property-tax bid greater (less) than that attributable to the nonschool-oriented characteristics of the residence, and a
residence located within the school district will have a positive (negative) implicit demand price associated with that characteristic. If a residence located within School District 509J has a higher (lower) bid price than an identical one located outside of the district, it is hypothesized that the profit-maximizing developers will, among themselves, bid up (down) land prices within the district. As a consequence, the implicit supply price of lot size, and the offer price of a residence, is expected to be higher (lower) inside the district than it is outside. As a result of both higher (lower) bid and offer prices, the market price of a residence located within the district can be expected to be higher (lower) than that of those located outside of School District 509J, ceteris paribus. The outcome of this private benefit-cost calculus is not, in general, predictable, and therefore the null hypothesis is arbitrarily posited: the market implicit price of location within School District 509J is equal to zero.

The variables descriptive of the characteristics of the neighborhoods in which the residences are located have not been defined such that it may be assumed that the households will, in general, consider these characteristics to be "goods". Therefore, these characteristics may have negative market implicit prices.

It is thought that, in general, those neighborhoods containing houses of a wide and varied price range will tend to be less aesthetically pleasing and have a more heterogeneous population, in
terms of socio-economic characteristics, than will those with a smaller variance in the value of the houses. It is hypothesized that households prefer the less heterogeneous neighborhoods and, thus, it is expected that a negative market implicit price is associated with those neighborhoods which have a higher standard deviation in the prices of the houses within them, ceteris paribus.

The average age of the houses in the neighborhood of a residence (AVEAGEN) is thought to serve as a measure of neighborhood quality in that older houses may tend to be in a poorer state of repair and/or be less fashionable than newer houses. In addition, older neighborhoods may tend to be more densely populated and thus have less open space than do newer neighborhoods. However, some older neighborhoods have a certain "polish" and solitude not found in the newer neighborhoods which are still in the process of being developed. Nevertheless, it is thought that the negative aspects of age generally outweigh the positive, and it is expected that a negative market implicit price is associated with AVEAGEN.

The number of multiple-family structures within the neighborhood of a residence (NMULTFN) is thought to serve as a measure of neighborhood quality. Homeowners may be thought, by other homeowners, to make more desirable neighbors than do tenants. The owner of a residence is, to his neighbors, a "known quantity". The occupants of multiple-family dwellings, however, may move more
frequently than do homeowners, and thus neighboring homeowners have less certainty as to the identity and/or character of their neighbors. Further, the larger the number of multiple-family structures in the neighborhood of a given size, the higher may be the density of the population of the neighborhood and the greater may be the traffic flow through it. It is expected, therefore, that there is a negative market implicit price associated with NMULTFN.

It was previously hypothesized that, within the study area, houses at the higher elevations tend to have a better view of the surrounding landscape and tend to be more expensive—thus forming more aesthetically pleasing neighborhoods. Therefore, it is expected that, for the residences within the study area, elevation above the flood plain of the Willamette River (ELEVATN) will have a positive market implicit price.

It is expected that, due to inflation, the nominal market prices of the houses for which the deed was transferred during the latter months of the study period were higher than the market prices of those for which the sale was consummated at an earlier date, ceteris paribus. However, the month during which the deed was transferred (MOSOLD) need not correspond to the month during which the sale price of the residence was set by the parties to the transaction. The terms of the sale are sometimes set prior to the time that the construction of the residence is completed, and the deed is transferred
when the house is ready for occupancy. Therefore, the rate of inflation, as measured by the coefficient of MOSOLD in the market price function, may be biased downward.

**The Demand Functions.** In the demand functions, the direction of the relationship between the implicit demand price of a characteristic and each of the explanatory variables describing the characteristics of a residence depends upon the direction of the direct effects

\[
((\partial \beta_i / \partial q_i) = (\partial^2 \beta / \partial q_i^2)) \text{ and of the cross effects} \\
((\partial \beta_i / \partial q_j) = (\partial^2 \beta / \partial q_i \partial q_j)).
\]

As each of the demand functions is hypothesized to be real income compensated, the direction of the direct effect is expected to be negative for each characteristic. The direction of the cross effects (i.e., the effect of a change in the level of one characteristic on the implicit demand price of another characteristic) depends upon the complementarity \((\partial \beta_i / \partial q_j > 0)\), substitutability \((\partial \beta_i / \partial q_j < 0)\), or independence \((\partial \beta_i / \partial q_j = 0)\) of the services of the characteristics in consumption. \(^{130}\)

\[^{130}\text{The services of any two characteristics (}q_i\text{ and }q_j\text{) of a residence are defined to be substitutes (complements) in consumption if:}\]

\[
\frac{\partial^2 \beta}{\partial q_i \partial q_j} = \frac{\partial (\partial U/\partial q_i)}{\partial q_j} < 0 \quad (> 0).
\]

That is, the services of any two characteristics \((q_i\text{ and }q_j)\) are substitutes (complements) in consumption if the implicit demand price of one characteristic \((q_i)\) decreases (increases) when the
Neither the theoretical model presented in Chapter III nor the theory of residential location in general assists in deduction of the direction of the cross effects. Further, their direction may vary over the range of the variables describing the characteristics of a residence. The direction of the cross effects is an empirical question, and one objective of this study is to determine their direction. Therefore, hypotheses concerning the direction of the cross effects will not be specified.

The effects of a household's attributes on its demand for the three characteristics of a residence considered here (FLAREA, LOTSIZE, and ACCESSR) are measured, primarily, in terms of the effects of income and wealth. It is expected that each of the three characteristics is a "normal" good and that the marginal utility of money is not, in general, constant. Therefore, a higher quantity of the other characteristic \((q_j)\) consumed is increased.

For example if lot size and floor area are complements, an increase in floor area would not result in a desire for a smaller lot, but, rather, for a larger one, and thus increase the household's implicit demand price for land. In terms of the more conventional theory, a decrease in the price of dwelling floor area would not result in a substitution of floor area for lot size, but would lead to an increase in both floor area and lot size. Note, though, that the conventional theory considers the effect of a change in the price of floor area on lot size whereas the concern herein is with the effect of a change in floor area on the "price" per unit area of the lot; the second effect is not the inverse of the first.

The phrase "constant marginal utility of money", as used here, encompasses both the case of a constant marginal utility of money per se and the case where the indifference curves between a
household disposable income (DISPOSIN) and/or a greater equity in a previously owned house (EQUITY) is expected to increase the implicit demand price for each of the three characteristics, ceteris paribus.

However, disposable income is herein defined as gross income less income taxes as calculated using the standard deduction. But, near the beginning of the term of the mortgage, the mortgage interest deduction alone is likely to exceed the standard deduction. Therefore, most homebuyers are likely to itemize deductions to take advantage of the mortgage interest deduction in computing the federal income tax, and disposable income is likely to be underestimated herein. Further as a result of the mortgage interest deduction, disposable income is not independent of the amount of the mortgage and thus, generally, of the purchase price of the residence—as is implicitly assumed herein. The effect of the mortgage interest deduction is to reduce the effective price of the residence to the homebuyer. Moreover, this effect is greater for higher income households

characteristic and money (or "all other goods") are vertically parallel. Recall that, as was discussed in Chapter III, the compensated demand for a particular characteristic would be invariant with respect to income if the marginal utility of money were constant.

132 For example, the interest payments on a $35,000, 30 year, nine percent, mortgage total about $3100 during the first year. In 1974, the maximum standard deduction was $2000.
because the federal income tax is progressive. Given these considerations, the coefficient on DISPOSIN is likely to provide an underestimate of the effect of an increase in household disposable income on the implicit demand price for the characteristic in question.

The federal income tax credit (TXCREDES) which a household was allowed to claim is considered as an increase in wealth contingent upon the purchase of an eligible residence. Therefore, eligibility for, and an increase in, the tax credit is expected to increase the implicit demand price for each of the three characteristics of a residence considered herein, ceteris paribus.

However, as the tax credit has an upper limit equal to the federal income tax owed and because, as was previously explained, the federal income tax may be overestimated herein, the magnitude of the credit to be claimed by those eligible may be overestimated by TXCREDES. Further, the tax credit applied to, and reduced only, the federal income tax. As the federal income tax is deductible from taxable income in computing the state income tax, the reduction in federal income tax would increase the state tax, and thus TXCREDES overestimates the total reduction in income tax due to the tax credit.

\[\text{For example, a $3000 interest deduction provides a $1500 reduction in income taxes for a household with a taxable income in the 50 percent income tax bracket whereas it provides only a $450 reduction for a household in the 15 percent bracket.}\]
For these reasons, the coefficient on TXCREDES in a demand function may slightly overestimate the effect of the tax credit on the implicit demand price for the characteristic in question.

A variable denoting the age of the head of the household is included in the demand function as a proxy for the level of accumulated wealth; as such, it is expected that the implicit demand prices for the characteristics will increase with age. However, AGEHEAD may also serve to indicate changes in preferences with age. For example, the implicit demand price for lot size and for floor area may be expected to decrease and that for accessibility to retail establishments to increase as the head of the household nears or enters the retirement years. Therefore, the direction of the effect of AGEHEAD on the implicit demand prices for floor area and lot size is not certain. However, it is expected that the wealth dimension of AGEHEAD will prevail and that the implicit demand price of each of the three characteristics will increase with the age of the head of the household.

If the rearing of children may be viewed as a consumption activity, then the effect of an increase in the number of children under 19 years of age in the household (NOCHILD) on the household's demand for a characteristic of a residence will depend upon whether that characteristic is a complement or a substitute to child rearing. While it might be hypothesized that both floor area and lot size are
complements to NOCHILD, it seems probable that, over the range of at least one to two children, the effect of an additional child on the household's demand for both lot size and floor area will be infra-marginal as most speculatively-built houses are designed to accommodate a family of at least four or five persons. Therefore, it is expected that, if an increase in NOCHILD has any effect on the implicit demand prices of floor area and lot size, it will be positive.

The expected influence of the number of children in the household on the demand for accessibility to retail establishments is even less certain. It is possible that many parents prefer to raise their children in a more rural, or at least less commercial, environment, and therefore the presence of children in the household may reduce the implicit demand price for accessibility. However, the number of trips to, for example, music lessons, the swimming pool, and after-school activities may increase with the number and/or presence of children in the household. Therefore, the effect of an additional child on the implicit demand price for accessibility is not certain, but is expected to be either infra-marginal or positive.

The number of children under 19 years of age also has an impact on disposable income. In 1974, federal income tax regulations allowed a $750 deduction for taxable income for each child under 19 years of age. Therefore, the more children under 19 years of age, the higher the annual disposable income, ceteris paribus. However,
the income tax effect is sure to be more than offset by the necessary expenses of rearing the children, and thus is not likely to have a significant impact on the household's demand for the characteristics of a residence.

The Supply Functions. In the supply functions, the direction of the relationship between the implicit supply price of a characteristic and each of the explanatory variables describing the characteristics of a residence depends upon the direction of the direct effects

\[ \left( \frac{\partial \phi_i}{\partial q_i} \right) = \left( \frac{\partial^2 \phi}{\partial q_i^2} \right) \]  

and of the cross effects

\[ \left( \frac{\partial \phi_i}{\partial q_j} \right) = \left( \frac{\partial^2 \phi}{\partial q_i \partial q_j} \right) \].

As each of the supply functions is, in theory, a marginal cost function, the direction of the direct effect is expected to be positive for each characteristic. The direction of the cross effects (i.e., the effect of a change in the level of one characteristic on the implicit supply price, or marginal cost, of another characteristic) depends upon the technical complementarity

\[ \left( \frac{\partial^2 \phi}{\partial q_i \partial q_j} < 0 \right), \text{ competitiveness} \left( \frac{\partial^2 \phi}{\partial q_i \partial q_j} > 0 \right), \text{ or independence} \left( \frac{\partial^2 \phi}{\partial q_i \partial q_j} = 0 \right) \]  

of the characteristics in production. 134

134 Any two characteristics \((q_i\text{ and } q_j)\) of a residence are defined to be technically competitive (complementary) in production if:

\[ \frac{\partial^2 \phi}{\partial q_i \partial q_j} \cdot \frac{1}{N^*} \frac{\partial^2 \mathcal{C}}{\partial q_i \partial q_j} > (0). \]

That is, any two characteristics are technically competitive (complementary) in production if the implicit supply price (i.e., the marginal cost per residence) of one characteristic increases (decreases) when the quantity produced of the other characteristic is increased.
As with the demand functions, neither the theoretical model presented in Chapter III nor the theory of residential location in general assists in deduction of the cross effects. However, unlike the demand functions, technical (rather than behavioral) linkages through the cost function determine the effect of a change in the level of production of one characteristic on the implicit supply price of another characteristic. Thus, the direction of the cross effects in the supply functions can, generally, be specified with greater certainty than those in the demand functions. Therefore, in this section, the variables descriptive of certain of the characteristics of a residence are, a priori, excluded from the FLRAREA, LOTSIZE, and ACCESSR supply functions, and the expected directions of the cross effects are specified for each characteristic in each function.

It is thought that the implicit supply price of lot size is independent of all of the other characteristics (including floor area) of the house per se except, of course, lot size, and they are therefore excluded from the lot size supply function. In addition to lot size, those variables descriptive of the characteristics of a residence which are hypothesized to influence the implicit supply price of lot size include SD509J, ELEVATN, the ACCESS variables, and those descriptive of the neighborhood.

The implicit supply price of lot size is hypothesized to be independent of floor area (and vice-versa) even though zoning
regulations generally place an upper limit on the total area of a lot which may be covered by the housing structure. Usually, these regulations do not require a specific floor area-lot size ratio for single-family dwellings. The zoning regulation is usually in the form of an inequality constraint which requires, for a given minimum lot size, the area of the lot covered by the building to be less than some specific upper limit. For example, the most common minimum lot size in Corvallis, 8000 square feet (areas zoned R-1), affords a buildable area of approximately 3500 square feet on an interior lot and 2700 square feet on a corner lot after deducting minimum setback, side-yard, and rear-yard requirements (Corvallis, City of, Zoning. . .). As the height restriction is 30 feet and there is no restriction on basements, it is technically possible to construct a dwelling of considerable floor area on such a lot without increasing lot size. Although, if the buildable area of a lot were to be exceeded by a single story structure of a given floor area, the builder would have to add a story and/or a basement, and thus the marginal cost of floor area may not be independent of lot size. Alternatively, he might increase the lot size and retain the single story design, in which case the marginal cost of lot size would not be independent of floor area. However, considering that the houses included in this study are "spec" houses of relatively small floor area and considering minimum lot size requirements, it is thought that the marginal cost of lot size
is independent of floor area (and vice-versa).

Also, there is no apparent reason why the number of bathrooms and/or the presence of a fireplace in a residence should influence the implicit supply price of lot size. Therefore, variables representing these characteristics are excluded, a priori, from the lot size supply function.

The effect of location within SD509J on the implicit supply price of lot size is the result of the direct influence that this characteristic has on the price that developers must pay for undeveloped lots. If, for reasons previously discussed, homebuyers are willing to place a higher bid for residences located within the school district, profit-motivated developers will, it is hypothesized, bid up the price of land within the school district. Thus, if a location within the school district is, generally, preferred to one outside of it, the implicit supply price of lot size will be greater inside of the district, ceteris paribus. However, the direction of this relationship cannot, in general, be predicted as it depends upon the homebuyers' net marginal evaluation of location within the school district.

Similar logic applies to the expected influence of a change in the level of neighborhood quality, as measured by STDEVPRN and AVEAGEN, on the implicit supply price of lot size. If, as previously hypothesized, households prefer a lower to a higher level of STDEVPRN and AVEAGEN, it is expected that profit-motivated
developers will bid up the price of land in the neighborhoods where the observations on these variables are relatively low and, thus, it is expected that the implicit supply price of lot size will be negatively related to these variables, _ceteris paribus_. However, in those neighborhoods were there are multiple-family structures a different set of market forces, not considered herein, may influence the profit-motivated developers of multiple-family structures to bid up land prices so that the implicit supply price of lot size is greater than that where multiple-family structures do not exist. Thus, the implicit supply price of lot size is expected to vary inversely with STDEVPRN and AVEAGEN and to vary directly with NMULTFN.

Further, neighborhoods which are perceived by the households to be of a higher quality with a corresponding higher implicit supply price for lot size, may tend to exclude lower income households and thus result in a higher average income level in such neighborhoods. To the extent that this phenomenon, in itself, is thought by households to be conducive to a higher level of neighborhood quality, it may induce the developer to bid the price of undeveloped lots in such areas even higher and further increase the implicit supply price of lot size.

The effect of a change in the level of accessibility on the implicit supply price of lot size is, it is hypothesized, the result of competitive bidding by developers for the more accessible lots--for which, in turn, households are hypothesized to be willing to pay a
a higher price, *ceteris paribus*. Conversely, at a given level of accessibility, the developer who desires to provide a larger lot must bid against other developers for the land, and it is expected that the situation value of the land will be considered in the bidding. Thus an increase in the level of accessibility is expected to increase the implicit supply price of lot size, and an increase in lot size is expected to increase the implicit supply price of accessibility.

Residences situated at the higher elevations within the study area are hypothesized to have a better view of the surrounding landscape and/or to be located in neighborhoods of a higher quality. If, as hypothesized, households prefer these characteristics of a residence, profit-motivated developers can be expected to bid up the price of land at the higher elevations. Further, because the terrain at the higher elevations tends to be timbered and more irregular than that at the lower elevations, those costs of preparing the land for residential use which may vary with lot size (e.g., the installation of utilities) may be greater at the higher elevations. Thus, it is expected that the implicit supply price of lot size varies directly with ELEVATN, *ceteris paribus*.

For reasons previously discussed, it is hypothesized that the implicit supply price of floor area is independent of lot size. Also, there is no apparent reason why the characteristics of the neighborhood and/or the accessibility of a residence might influence the
implicit supply price of floor area. Therefore, variables descriptive of these characteristics are excluded, a priori, from the floor area supply function. In addition to floor area, those variables descriptive of a residence which are hypothesized to influence the implicit supply price of floor area include NBATHS, FIREPL, and ELEVATN.

The relationship between either the number of bathrooms or the presence of a fireplace in a house and the implicit supply price of floor area may not be so much a technical as it is an apparent relationship. That is, NBATHS and/or FIREPL may serve as measures of the quality of the dwelling area in that the presence of a fireplace and/or the number of bathrooms may be associated with other, unmeasured, characteristics of the house. For example, two-bathroom houses may tend to have fully carpeted floors and/or more expensive kitchen appliances than do those with one bathroom, or houses with a fireplace may tend to have more expensive trim on doors, windows, cabinets, etc. That is, these variables may have a qualitative as well as quantitative significance. Thus, the implicit supply price of floor area is expected to vary directly with NBATHS and FIREPL, but, due to their qualitative significance, the magnitude of the coefficient on each of these variables is likely to provide an overestimate of the incremental cost of providing these characteristics.

The implicit supply price of floor area is also expected to vary directly with ELEVATN. The houses built at the higher elevations
tend to be built on more rugged terrain which requires cut and fill procedures to provide a buildable area and/or a more elaborate foundation structure for the house. As the terrain tends to become more irregular with increased elevation, the cost of these procedures is likely to increase with increased elevation for a dwelling of a given floor area. However, this effect is likely to be mitigated by the fact that those houses located at the highest elevation may be located on the tops of the hills where the gradient of the lot is likely to be relatively small.\(^{135}\)

As there is no apparent reason why the characteristics of the house per se and of the neighborhood might influence the implicit supply price of accessibility to retail centers (ACCESSR), the variables descriptive of these characteristics are excluded, a priori, from the ACCESSR supply function. Those characteristics which are hypothesized to influence the implicit supply price of accessibility to retail centers include lot size (for reasons previously discussed) and, of course, ACCESSR.

If, ceteris paribus, the households prefer more accessibility to less, competitive bidding will ensure that the more accessible residences sell for a higher market price. Thus, the developers will bid up the price of the more accessible undeveloped lots; for, if they did

\(^{135}\) ELEVATN is used, here, as a proxy for the gradient of the lot--measures of which were not readily available.
not, those developers building at the more accessible locations would earn a higher profit, *ceteris paribus*, than would those building at less accessible locations. Thus, profit-motivated developer-bidding for the more accessible locations is expected to result in a higher implicit supply price of accessibility to retail centers at such locations.

The effect of a developer's attributes on its supply of the three characteristics of a residence considered herein (FLRAREA, LOTSIZE, and ACCESSR) are discussed under two categories: (1) the attributes of the developer *per se* (BLDRSCAL, AVCONCST, SDCONCST, and AVCSTFT2) and (2) the variables representative of the constraints placed on the developer at specific locations (CORVALLI, PHILOMTH, and CUSTFET). Where any of these variables are considered to be irrelevant to a specific supply function, they are excluded, *a priori*, from that function.

The developer's scale of operation (BLDRSCAL) is included in each of the three supply functions to measure any economies of scale that might accrue to large-scale developers. Such economies might result from the division and specialization of labor in the planning, management, and/or construction process and from technological factors such as greater use of prefabricated materials by larger developers. Further, large-scale developers may have, for example, their own electrical, heating, and plumbing systems installation crews
and thus rely less, or not at all, on sub-contractors for such work—
with possible savings in construction costs. Also, large-scale
developers may obtain quantity discounts when purchasing construc-
tion materials, appliances, heating and plumbing equipment, etc.
They may also receive more favorable credit terms than do smaller
developers and thus realize some savings in carrying costs. Because
of these possible economies of scale, it is expected that the larger the
scale of operation of the developer, the lower will be the implicit
supply price of both lot size and floor area.

The effect of the developer's scale of operation on the implicit
supply price of accessibility is less certain because it may be, gen-
erally, uninfluenced, by the production process. However, due to,
possibly, more favorable credit terms, the large-scale developers
may experience some savings in the carrying costs of the more
accessible, and thus more expensive, parcels of land. If these sav-
ings are realized and if they are passed on to the homebuyer, the
implicit supply price of accessibility will be lower for the large scale
developers. Thus, it is expected that, if BLDRSCAL has any effect
on the implicit supply price of accessibility, the supply price will
vary inversely with it.

The standard deviation of the value of construction per
single-family unit (SDCONCST) is included in the supply functions as
a measure of the degree of developer diversification. It is expected
that there are economies of construction cost to be achieved by specialization in the production of any particular type, or design, of a house. As a less perfect measure, it is expected that developers that specialize in housing structures of any given cost can produce each at less cost, *ceteris paribus*, than they could if they produced a variety of structures of different value. This effect is, however, expected to be limited primarily to the cost of producing the housing structure. Lot development costs (costs of grading, installation of utilities, etc.) are thought to be less variable with respect to developer specialization and, thus, the implicit supply price of lot size is not expected to increase with SDCONCST. Further, the implicit supply price of accessibility is thought to be relatively unaffected by the production process and is expected to be invariant with the degree of developer specialization. Only the implicit supply price of floor area is expected to increase with SDCONCST, though, if there is any effect on the implicit supply price of lot size and/or accessibility, it is expected that they too will increase with SDCONCST.

The average value of construction, as measured by both AVCONCST and AVCSTFT2, is included in the supply functions as a measure of the developer's average quality of construction, *ceteris paribus*. There may be considerable differences between developers in, for example, the quality of siding, millwork, flooring, and
plumbing used in the construction of a house. The implicit supply price of floor area is expected to increase with the average cost of construction, ceteris paribus. The implicit supply price of both lot size and accessibility is expected to be invariant with respect to both AVCONCST and AVCSTFT2.

CORVALLI and PHILOMTH are included in the model to measure the impact of the regulations and fees imposed on the developers by the Cities of Corvallis and Philomath. With the exception of the systems development charge (SDC), all major fees are either variable with the characteristics of the house, but not unique to Corvallis and/or Philomath, or they are fixed in magnitude. The fixed fees do not influence marginal cost and, therefore, implicit supply price—though they do increase the offer price. The SDC, which varies with both floor area and lot size, is expected to increase the implicit supply price of both of these characteristics within the boundaries of both cities, ceteris paribus.  

However, "all other things" do not remain equal if the developer builds within either of the cities rather than outside of them. Generally, if the developer builds within either Philomath or Corvallis, he must connect the house to city sewer and water systems and incur

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136 Only the Corvallis SDC is specifically based on floor area. However, the Philomath SDC is based on the number of waste fixtures in the house—which can be expected in increase with floor area.
the cost of so doing. These installation costs will vary with the length of the sewer and water mains and laterals which, in turn, will generally vary with lot size (especially lot frontage). This variable cost will be reflected in both the offer price and the implicit supply price of lot size. If the developer builds outside of the cities and installs a common water and sewer system for an entire subdivision, the costs of the sewer and water system can also be expected to vary with lot size. However, more commonly, the developer will provide a separate water and sewer system (usually, a well, pump, and septic tank) for each house if he builds anywhere in the study area other than in the cities. The cost of the water and sewer systems, in this event, are fixed and independent of lot size—they do not affect the implicit supply price of lot size, but their magnitude does affect the offer price. Thus, the implicit supply price of lot size is expected to be greater, due to the variable costs of the systems development charge and of municipal water and sewer provision, if the developer

137 If the utilities are already installed in the lot, their cost will be reflected in the cost, to the developer, of the lot.

138 It should be noted, however, that lot size may not be independent of the choice of sewer system. The lot size required to gain sufficient separation between the well and the septic tank drainage field is not independent of soil conditions at the site. In addition, the lot size required to prevent surface seepage may not be independent of the gradient of the lot if a septic tank is installed. Thus, it is possible that the lot size, and therefore the implicit supply price of lot size, may not be independent of the decision to install a septic tank sewage disposal system.
builds within either Corvallis or Philomath. 139

The implicit supply price of accessibility to retail centers, as defined in this study, is thought to be independent of the constraints and fees imposed by the cities. Nevertheless, the implicit supply price of accessibility may be higher if a residence is located within either city simply because the accessibility of residences located within either city may tend to be greater than that of residences located outside of the cities (i.e., ACCESSR may be correlated with CORVALLI and PHILOMTH).

The variable denoting whether or not a house has "custom features" (CUSTFET) when purchased is included in the floor area supply function to measure the effect of buyer-imposed constraints on the developer's construction costs. These constraints may require that certain types of work be left undone as frequently as they require additional work of the developer. In addition, households may request that one feature be added and another feature be deleted from the design of the house. Therefore, the influence of CUSTFET on the implicit supply price of floor area is uncertain, and the null hypothesis that it has no influence is adopted.

139 It does not necessarily follow that the offer price of the residence will be greater within the cities. The offer price is a function of total costs--fixed and variable. The variable costs may be higher within the cities due to the SDC and sewer and water systems installation costs, but the fixed costs of sewer and water provision may be higher outside of the cities.
Summary of the Econometric Model

Both the econometric model and the hypotheses formulated in this chapter are summarized in Table 2. The model depicted is presented under the assumption that FLRAREA, LOTSIZE, and ACCESSR will explain the most variation in the market price of residences; they are, therefore, treated as the endogenous variables in the system depicted in Table 2. In the next chapter, the results of the statistical estimation of the model developed in this chapter are presented.
Table 2. Summary of the econometric model of residential choice.*

<table>
<thead>
<tr>
<th>Market Price of Residence</th>
<th>Market Implicit Price of Characteristics</th>
<th>Dependent Variables</th>
<th>Explanatory Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demand Price of</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>LOTSIZE</td>
<td>FLRAREA</td>
<td>ACCESSR</td>
</tr>
<tr>
<td></td>
<td>Supply Price of</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>LOTSIZE</td>
<td>FLRAREA</td>
<td>ACCESSR</td>
</tr>
</tbody>
</table>

```
+   -   #   #   +   +   LOTSIZE
+   #   -   #   +   FLRAREA
+   #   #   #   +   NBATHS
+   #   #   #   +   FIREPL
+   #   #   #   +   ACCESSM
+   #   #   -   +   +   ACCESSR
#   #   #   #   #   SD509J
-   #   #   #   -   STDEVPRN
-   #   #   #   -   AVEAGEN
-   #   #   #   +   NMULTFN
+   #   #   #   +   +   ELEVATN
  +   +   +   DISPOSIN
  +   +   +   AGEHEAD
  +   +   +   NOCHILD
  +   +   +   EQUITY
  +   +   +   TXCREDES

-   -   -   -   BLDRCAL
  0   +   0   SDCONCST
  0   +   0   AVCONCST
  0   +   0   AVCSFT2
    +   +   +   CUSTFET
    +   +   +   CORVALLI
    +   +   +   PHILOMTH
    +   MOSOLD
```

*The table is read vertically. Each column represents an equation. A character entered in a column signifies that the corresponding variable is included in the equation and the expected direction of the relationship between the explanatory and the dependent variable; a pound sign (#) indicates that the expected direction has not been specified.
V. ESTIMATION OF THE ECONOMETRIC MODEL

As was explained in Chapter IV, estimation of the econometric model requires that the market price function first be estimated. Then the market implicit prices of the characteristics may be derived and used in the estimation of the system of supply and demand functions underlying the market price function.

The Market Price Function

The "Best Fit" Mathematical Form

Using the ordinary least squares (OLS) regression technique, the market price function was estimated in several forms (all intrinsically linear) and with different combinations of the variables descriptive of the accessibility of the residence to retail centers and to the household's place(s) of employment. Of the forms estimated, that form was selected which explained the most variation in the market prices of the residences and which appeared, upon examination of the residuals, to most nearly satisfy the assumptions

---

140 The "adjusted $R^2$" $= \bar{R}^2 = ((K-1)/(n-K))(1-R^2)$ was used as the choice criterion, where:

- $R^2$ = the coefficient of determination
- $K$ = the number of explanatory variables (including the constant term)
- $n$ = the sample size.
underlying the OLS technique. 141

In the selected form, which explained about 81 percent of the variation in market price, the natural logarithm of market price is expressed as a function of the untransformed binary variables (SD509J and FIREPL) and of the natural logarithms of all other variables--including a random error term. 142 This form implies that a one percent increase in the level of a continuously measured variable will result in a constant percentage change (given by the coefficient on the variable representing the characteristic) in the market price of the residence and that the presence of a characteristic measured by a binary variable will change the market price by a constant percentage. The parameters of the "best-fit" estimate of the market price function are presented in Table 3.

However, the "best-fit", nonlinear, market price function explained only slightly more of the variation in market price than did a linear function containing the same variables (see Table 4). A linear

141 The OLS technique requires that the residuals be serially independent (nonautoregressive), normally distributed about zero, and that they have a constant (homoskedastic) variance if the estimated parameters are to be best linear unbiased (see, e.g., Johnston, 1972, p. 121-122).

142 Because the observations on the accessibility variables are all negative, transformation of the observations to natural logarithms was made possible by addition of a constant equal to one plus the absolute value of the minimum observation on each variable to the observations on the variable. Thus, after the transformation to natural logarithms, these variables had a minimum value of zero.
Table 3. Estimate of the market price function in double-log form.+

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>5.5603</td>
<td>9.619**</td>
</tr>
<tr>
<td>LOTSIZE</td>
<td>-0.0230</td>
<td>-0.741</td>
</tr>
<tr>
<td>FLRAREA</td>
<td>0.6118</td>
<td>8.400**</td>
</tr>
<tr>
<td>NBATHS</td>
<td>0.2002</td>
<td>3.275**</td>
</tr>
<tr>
<td>FIREPL</td>
<td>0.0292</td>
<td>0.540</td>
</tr>
<tr>
<td>SD509J</td>
<td>0.0688</td>
<td>2.309**</td>
</tr>
<tr>
<td>ELEVATN</td>
<td>0.0078</td>
<td>0.687</td>
</tr>
<tr>
<td>STDEVPRN</td>
<td>0.0451</td>
<td>1.407*</td>
</tr>
<tr>
<td>AVEAGEN</td>
<td>-0.0123</td>
<td>-0.622</td>
</tr>
<tr>
<td>NMULTFN</td>
<td>0.0036</td>
<td>0.198</td>
</tr>
<tr>
<td>ACCESSR2</td>
<td>-0.0212</td>
<td>-0.687</td>
</tr>
<tr>
<td>ACCESSM</td>
<td>0.0148</td>
<td>0.568</td>
</tr>
<tr>
<td>MOSOLD</td>
<td>0.0449</td>
<td>2.293**</td>
</tr>
</tbody>
</table>

$R^2 = 0.813929$

$F_{12, 68} = 30.1619 > F_{.05, 12, 68} = 1.875$

* Significant at the .10 level ($t_{.10, 68} = 1.294$).

** Significant at the .05 level ($t_{.05, 68} = 1.668$).

+ Dependent variable and all explanatory variables except SD509J and FIREPL in natural logarithms.
Table 4. Estimate of the market price function in linear form.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>-212.73</td>
<td>-0.060</td>
</tr>
<tr>
<td>LOTSIZE</td>
<td>-0.0066</td>
<td>-0.192</td>
</tr>
<tr>
<td>FLRAREA</td>
<td>12.998</td>
<td>7.333**</td>
</tr>
<tr>
<td>NBATHS</td>
<td>5372.2</td>
<td>3.622**</td>
</tr>
<tr>
<td>FIREPL</td>
<td>1445.6</td>
<td>0.703</td>
</tr>
<tr>
<td>SD509J</td>
<td>2270.3</td>
<td>1.850**</td>
</tr>
<tr>
<td>ELEVATN</td>
<td>1.8824</td>
<td>0.399</td>
</tr>
<tr>
<td>STDEVPRN</td>
<td>0.2410</td>
<td>1.541*</td>
</tr>
<tr>
<td>AVEAGEN</td>
<td>-2.8425</td>
<td>-0.042</td>
</tr>
<tr>
<td>NMULTFN</td>
<td>92.31</td>
<td>0.477</td>
</tr>
<tr>
<td>ACCESSR2</td>
<td>-96.600</td>
<td>-0.281</td>
</tr>
<tr>
<td>ACCESSM</td>
<td>-32.780</td>
<td>-0.179</td>
</tr>
<tr>
<td>MOSOLD</td>
<td>278.57</td>
<td>1.857**</td>
</tr>
</tbody>
</table>

\[
\bar{R}^2 = 0.807753
\]

\[
F_{12, 68} = 29.0110 > F_{.05, 12, 68} = 1.875
\]

* Significant at the .10 level \(t_{.10, 68} = 1.294\).

**Significant at the .05 level \(t_{.05, 68} = 1.668\).
function implies that a unit increase in any one of the continuously measured variables changes the market price of a residence by a constant—\the magnitude and direction of which is given by the coefficient of the variable and which is independent of all other variables in the function; this constant is the estimated market implicit price of the characteristic. Therefore, the market implicit prices of the characteristics, as estimated using a double-log function which is closely approximated by a linear function, are likely to exhibit little variation with the level of the characteristics.

Inspection of the matrix of simple correlation coefficients, and of its inverse, indicated that there was not a serious problem of multicollinearity between the explanatory variables in the double-log function. Though there are no definite rules as to what constitutes an unacceptable degree of multicollinearity, one rule of thumb holds that a simple, pair-wise correlation \((r)\) as high as 0.8 or 0.9 is not unacceptable (Farrar and Glauber, 1967, p. 98). The highest pair-wise correlation, that between the natural logarithms of LOTSIZE and ACCESSR2, was -0.780. Another guideline states that the coefficient of multiple correlation \((R)\) between any given explanatory variable and the remaining members of the explanatory variable set should not exceed that between the dependent variable and the explanatory variable set (Farrar and Glauber, 1967, p. 98). The highest multiple correlation, that between FLRAREA and the
remainder of the explanatory variable set was 0.909 whereas the (unadjusted) $R$ for the double-log market price function was 0.918. Therefore, the degree of multicollinearity, a high level of which would render meaningless the ceteris paribus assumption underlying the calculation of the market implicit prices, was judged to be within acceptable bounds.  

**Examination of the Residuals in Characteristics Space**

A test for heteroskedasticity of the variance of the error term using Spearman's coefficient of rank correlation ($r_s$) between the

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The correlation matrix and its inverse, for the double-log market price function, are presented in Appendix Table B1. The coefficient of multiple correlation ($R_i$) between any one variable ($q_i$) and the remaining explanatory variables can be calculated, using the corresponding diagonal element ($r_{ii}$) of the inverse correlation matrix, as:

$$ R_i = (1 - \frac{1}{r_{ii}})^{1/2} $$

The coefficient of partial correlation ($r_{ij}$) between two explanatory variables ($q_i$ and $q_j$) can be calculated using the corresponding diagonal elements ($r_{ii}$ and $r_{jj}$) and the appropriate off-diagonal element ($r_{ij}$) of the inverse correlation matrix, as (Farrar and Glauber, 1967, p. 103):

$$ r_{ij} = \frac{-r_{ij}}{\sqrt{r_{ii}}\sqrt{r_{jj}}} $$

The correlation matrix and its inverse, for the linear function, are presented in Appendix Table B2. A comparison of Tables B1 and B2 reveals that the transformation of the variables to natural logarithms increased the multiple correlation between several of the variables and the remainder of the explanatory variables.
absolute value of the error term and, in turn, the dependent variable and each of the explanatory variables in the double-log market price function revealed that there was significant (at the 99 percent confidence level) positive correlation between the rank of the absolute value of the residuals and the rank of the observations on both FLRAREA and LOTSIZE. However, after visual inspection of the plots of the residuals against the explanatory variables, it was judged that the variance of the error term was not sufficiently heteroskedastic to warrant the use of generalized least squares techniques to reduce the heteroskedasticity and increase the efficiency of the estimated coefficients on LOTSIZE and FLRAREA.

The residuals from the double-log function were plotted against the explanatory variables and against the model-predicted market prices of the residences. In addition, the simple, pair-wise correlation between the residual and each of these variables was obtained. In each case, there was a complete lack of correlation ($r = 0$), and a visual examination of the plots did not reveal any nonlinear relationships between the variables and the residuals.

\[144\text{The Spearman coefficient of rank correlation between LOTSIZE and the absolute value of the error term was equal to 0.324 in the double-log model and 0.467 in the linear model. The coefficient between FLRAREA and the absolute value of the error term was 0.312 in the double-log model and 0.485 in the linear model. The critical value of } r_s \text{ at the 99 percent confidence level is 0.283. See Snedecor and Cochran (1967, p. 194) for a definition of the test statistic.}\]
However, a plot of the residuals from the double-log function against the market prices of the residences revealed that they were positively correlated \( r = 0.398 \). As negative residuals tended to be associated with the lower priced residences and positive residuals with the higher priced residences, the model overestimated the price of the lower priced residences and underestimated the price of the higher priced residences.

Correlation between the residuals and market price was observed with each of the several mathematical forms in which the market price function was estimated (including quadratic and third-order functions as well as a semi-logarithmic form in which the natural logarithm of market price was regressed on the untransformed variables). Further, tests for the presence of an autoregressive disturbance, a possible result of incorrect specification of the mathematical form of the regression equation (Kmenta, 1971, pp. 470-471), failed to reject the null hypothesis of nonautoregression.

---

145 \( r = 0.398 \) is significant at the 99 percent confidence level.

146 With the linear form, the simple correlation between actual market price and the residuals was equal to 0.404.

147 The Durbin-Watson statistic \( (d) \) was equal to 2.115 for the double-log function and 2.145 for the linear function. Though the available tables of critical values for this statistic did not contain entries for equations with more than five explanatory variables, the value of \( d \) for a random (rather than autocorrelated) series of residuals ranges around 2.0 (Johnston, 1972, p. 251). Further, the von
inspection of both a histogram and a normal plot of the residuals revealed that, especially for the double-log function, they were approximately normally distributed with zero mean, though the distribution is slightly skewed to the left as the median, equal to -0.02147, is less than the mean. Ramsey (1969) has shown that if either the mathematical form of the regression equation is misspecified or if a relevant variable is omitted, the distribution of the residuals will remain normal, but will have a non-zero mean. However, if more than one specification error is committed, the net effect may be to leave the residual distribution with a zero mean (Ramsey, 1969, p. 369). Thus, though it cannot be definitely concluded that the mathematical form of the market price function is not mis-specified, there is no strong evidence that the correlation between Neumann ratio \( \frac{\delta^2}{S^2} \), which is, for large \( n \), approximately normally distributed with \( \mathbb{E}(\frac{\delta^2}{S^2}) = \frac{2n}{(n-1)} \) and \( \text{var}(\frac{\delta^2}{S^2}) = \frac{(4n^2(n-2))/((n+1)(n-1)^3)}{n-1} \) is related to \( d \) as \( \frac{\delta^2}{S^2} = \frac{dn}{(n-1)} \) (Johnston, 1972, pp. 250-251). Therefore,

\[
Z = \frac{\left( \frac{\delta^2}{S^2} - \mathbb{E}(\frac{\delta^2}{S^2}) \right)}{\sqrt{\text{var}(\frac{\delta^2}{S^2})}} = \frac{\left( \frac{dn}{(n-1)} - \frac{2n}{(n-1)} \right)}{\sqrt{\frac{(4n^2(n-2))/((n+1)(n-1)^3)}{n-1}}} = \frac{d-2}{2\sqrt{\frac{(n-2)}{(n-1)}}}
\]

The value of \( Z \) for the double-log function was 0.524 and for the linear function was 0.661. As \( Z_{0.05} = 1.96 \), the null hypothesis of no autocorrelation is not rejected at the 95 percent confidence level.
the market prices of the residences and the residuals stems from such mis-specification.

A possible explanation for the correlation between the market prices and the residuals may be that certain relevant aspects of the residence may not have been explicitly considered in the specification of the market price function. This mis-specification would take the form of the omission of an explanatory variable descriptive of some characteristic of the residence which is relevant in the determination of its market price. Also, a variable included in the function may have a qualitative aspect which varies with the observations on the variable; that is, that which is being measured may not be of homogeneous quality. If the omitted variable is correlated with an included variable and/or if the qualitative aspect of a characteristic varies nonrandomly with either the variable measuring its quantitative aspect or with the level of another explanatory variable, the estimated coefficients on the variables included in the function will be biased (Kmenta, 1971, pp. 392-396).

Among other variables descriptive of the characteristics of a residence, FLRAREA, in particular, may have a qualitative aspect in that the dwelling area of larger residences may tend to be of a higher quality than that of smaller ones. Though, as was discussed in Chapter IV, NBATHS and FIREPL may also serve as indicators of the quality of dwelling area, they are likely to be imperfect measures
of it. Thus, if FLRAREA is assumed to be a homogeneous variable, there is an implicit assumption that the quality level of that which is being measured by the variable is equal to that at its mean value. Therefore, if the quality of the dwelling area varies directly with FLRAREA, the level of quality may tend to be overestimated for the lower observed values of FLRAREA and underestimated for the higher values. As a result, the regression coefficient on FLRAREA may be biased downward. Further, as FLRAREA is a highly significant determinant of market price \((t = 8.400)\), this would provide an explanation for the correlation between the residuals and market price.  

However, if the quality of the dwelling area varies directly with FLRAREA and if, therefore, the regression coefficient on FLRAREA is biased downward, the observed values of the residuals could be expected to be positively correlated with FLRAREA. That is, the residuals associated with those values of FLRAREA below its mean could be expected to be negative while those associated with values above the mean FLRAREA could be expected to be positive. As was previously noted, FLRAREA was not correlated with the residuals. Thus, there is no evidence to suggest that FLRAREA has a qualitative

\[148\] A regression of market price on FLRAREA alone explained about 73 percent of the variation in market price \((R^2 = 0.726078)\).
aspect which is not considered, perhaps implicitly by NBATHS and FIREPL, in the model.

Similarly, if a relevant explanatory variable has been omitted from the market price function, the estimated coefficient on those explanatory variables which are correlated with the omitted variable will be biased (Johnston, 1972, pp. 168-169). Thus, if a relevant variable which is correlated with an included explanatory variable has been omitted from the function, the residuals could be expected to be correlated with the included variable and/or exhibit a nonrandom pattern when plotted against the correlated variable. As none of the explanatory variables were correlated with the residuals and because an inspection of the plots of the residuals against each of the explanatory variables revealed no systematic relationship, there is no evidence to suggest that a variable which is correlated with the explanatory variables in the market price function has been omitted.

Examination of the Residuals in Geographic Space

If a relevant variable which is descriptive of a location related characteristic of the residence has been omitted, and if that characteristic is not uniformly distributed throughout geographic space, the market price function could not be expected to predict with equal precision throughout the study area. To determine if the double-log form of the market price function tended to predict with less precision
in certain areas (i.e., if the direction and magnitude of the residuals form some nonrandom pattern), the geographic location of each residence was plotted with a symbol to indicate the direction of the associated residual and whether or not the estimated price of the residence was within (an arbitrarily selected) five percent of the actual market price (see Figure 14—map pocket). \(^{149}\)

Though a visual examination of Figure 14 does not reveal any readily apparent patterns as to the magnitude and/or direction of the residuals, several patterns appear to emerge upon close examination. First, the model underestimated the market price of each of the six residences located in north and northeast rural Corvallis whereas it overestimated the market price of four of the five residences located in west rural Corvallis (off Oak Creek Drive). Also, it appears that the model more frequently underestimated the price of the residences located in north Corvallis (north of Grant St.) whereas it may have more frequently overestimated the price of the residences located in south Corvallis. Finally, the model estimated the price of 68 percent of the residences in Corvallis within five percent of the actual market

\(^{149}\) The direction of the arrow used, in Figure 14, to signify the location of a residence indicates the direction of the associated residual; it points up (down) to signify a positive (negative) residual. Thus the model overestimated the price of those residences represented by a "down" arrow and underestimated the price of those represented by an "up" arrow.

A large arrow indicates that the model did not estimate the price of the residence within five percent of its actual market price.
price whereas it achieved the same accuracy for only 39 percent of the residences in the remainder of the study area. However, it is not at all apparent that these patterns are statistically significant (i.e., could not be attributed to a random disturbance).

Using the information provided in Figure 14 and a contingency table, a chi-square statistic was used to test several *ad hoc* hypotheses concerning the independence of location and either the direction or the magnitude of the residual. The test statistic used was (Brunk, 1965, pp. 278-279):

\[ D^2 = \frac{(f_{ij} - F_{ij})^2}{F_{ij}} \]

where:

- \( f_{ij} \) = the observed frequency in the cell at the \( i^{th} \) row and \( j^{th} \) column of the contingency table,
- \( F_{ij} \) = the expected frequency in the cell at the \( i^{th} \) row and \( j^{th} \) column,
- \( F_{ij} = \frac{(n_i \cdot n_j)}{n} \) where \( n_i \) = sum of the observed frequencies in the \( i^{th} \) row, \( n_j \) = sum of the observed frequencies in the \( j^{th} \) column, and \( n = n_i + n_j \).

The hypotheses tested here are *ad hoc* in that they were not developed from some prior reasoning which resulted in expectations as to the pattern of the residuals. Rather, the tests were suggested by the observed patterns of the residuals. Nevertheless, the observed patterns and the results of the tests may suggest hypotheses that would aid in the specification of the market price function in further studies.
For each of these tests, the null hypothesis is that the probability of obtaining a residual of either a given direction or absolute magnitude (large or small) within a given sub-area is equal to that which was observed for the study area as a whole. If the observed frequency of residuals of a given direction or magnitude is significantly different from the expected frequency within the sub-area, the null hypothesis is rejected.

Given the sample size and spatial distribution of the residuals, the selection of sub-areas is limited by the requirement that the expected frequency in each cell of the contingency table be equal to at least five (Brunk, 1965, p. 276). For example, it was not possible to test a hypothesis concerning the independence of the residual and a location in north and northeast rural Corvallis as there were only six residences located in this area included in the sample.

However, a contingency table was constructed to test the hypothesis that the direction of the residual was independent of a location in north Corvallis and north and northeast rural Corvallis versus a location elsewhere within the study area (see Table 5). The null hypothesis was rejected at the 95 percent confidence level—which indicates that the double-log market price function more frequently underestimated the price of residences located in the north Corvallis area and more frequently overestimated the price of residences located elsewhere.
Table 5. A test for independence of direction of the residual and location. *

<table>
<thead>
<tr>
<th>Direction of Residual</th>
<th>North Corvallis and North and Northeast Rural Corvallis</th>
<th>Remainder of Study Area</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive residual</td>
<td>24</td>
<td>13</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>(18.27)</td>
<td>(18.73)</td>
<td></td>
</tr>
<tr>
<td>Negative residual</td>
<td>16</td>
<td>28</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>(21.73)</td>
<td>(22.27)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>41</td>
<td>81</td>
</tr>
</tbody>
</table>

\[ D^2 = 6.54 > D^2_{0.05, 1} = 3.84 \]

* The numbers in parentheses are the expected cell frequencies.

The tendency of the model to underestimate the market price of residences located in the north Corvallis area is particularly interesting in that the primary growth direction of Corvallis is to the north. One explanation for this phenomenon may be that a portion of the households' bids for residences located in this area may be a speculative bid greater than that for a location elsewhere in the study area, ceteris paribus. Moreover, a new high school (Crescent Valley School--see Figure 14) and community hospital were recently constructed in the north Corvallis area. To the extent that the services of the new school are generally preferred to those of the older school and that a location nearer the hospital is preferred to a less accessible location, the estimated market price function, which does
not contain variables representing these characteristics, could be expected to underestimate the prices of residences located in the north Corvallis area.

Contingency tables (not presented here) were constructed to test, at the 95 percent confidence level, several other hypotheses. Among the null hypotheses tested, and the results, were the following:

1. The direction of the residual is independent of location in Corvallis versus elsewhere in the study area. Not rejected \( D^2 = 0.001 < D^2_{0.05,1} = 3.84 \).

2. The absolute magnitude (large or small) of the residual is independent of location in Corvallis versus elsewhere in the study area--where a large residual was defined as one which resulted in a greater than five percent error in the estimated market price. Rejected \( D^2 = 6.87 > D^2_{0.05,1} = 3.84 \).

The latter test revealed that the model tended to estimate the price of residences outside of Corvallis with less accuracy than it did that of those located within the city. However, the model neither underestimated nor overestimated the price of the residences outside of Corvallis with a significantly different frequency than it did that of those residences within the city.

It is not clear that the tendency of the model to predict with less precision for those residences located outside of Corvallis is indicative of a specification error in the model. The residuals will not all
be of equal magnitude if they are normally distributed about zero, and
the model will predict most accurately for those residences which are
priced nearest the mean price of all residences and predict with less
accuracy for those residences with prices nearer the extremes of the
observed price range. Both the lowest priced residences and the
most expensive residences included in the sample tended to be located
outside of Corvallis. Thus it may be expected that the larger resi-
duals will tend to be associated with residences located outside of
Corvallis.

To summarize, an examination of the distribution of the
residuals in both geographic and characteristics space revealed two
inadequacies of the model:

(1) it tended to overestimate the price of the lower priced
residences and underestimate the price of the higher priced
residences—thus, the residuals are correlated with market
price; and

(2) it tended to underestimate the price of residences located in
the north Corvallis area and overestimate the price of the
residences located elsewhere in the study area.

Recall that the confidence interval for the dependent variable
in a regression equation will be narrowest when the value of each
explanatory variable is equal to its sample mean and will get progres-
sively wider as the values of the explanatory variables move away
from their sample means (see Kmenta, 1971, pp. 363-364).
Therefore, if the higher-priced residences were located in the north Corvallis area, a consistent explanation would be provided for these two estimation errors.

However, an examination of Table 6—in which is presented, for selected areas, the mean residual, geometric mean of the ratio of actual market price to estimated market price, and the mean value of the residences included in the sample—does not provide a consistent explanation. Using the information in the table, the mean price of the residences located in the north Corvallis area (i.e., in northeast and northwest Corvallis and north and east rural Corvallis)

\[ e^*_i = \ln e_i = \ln P_i - \ln \hat{P}_i, \]
\[ \sum_{i=1}^{n} e^*_i = \sum_{i=1}^{n} \ln e_i = \sum_{i=1}^{n} (\ln P_i - \ln \hat{P}_i), \]
\[ \frac{1}{n} \sum_{i=1}^{n} e_i = \frac{1}{n} \sum_{i=1}^{n} \ln e_i = \frac{1}{n} \sum_{i=1}^{n} (\ln P_i - \ln \hat{P}_i), \]
\[ \exp\left(\frac{1}{n} \sum_{i=1}^{n} e_i^*\right) = \left(\prod_{i=1}^{n} e_i\right)^{1/n} = \left(\prod_{i=1}^{n} (P_i / \hat{P}_i)\right)^{1/n}, \]

where

- \( e_i \) = the residual associated with the \( i^{th} \) residence,
- \( P_i \) = the actual market price of the \( i^{th} \) residence,
- \( \hat{P}_i \) = the estimated market price of the \( i^{th} \) residence,
- \( n \) = the number of residences in the area.

The last term in (4) above is, by definition, the geometric mean of the ratio of the actual market price to the estimated market price (Li, 1964, p. 547). The geometric mean was obtained for each area by exponentiation of the mean residual for the area.
Table 6. The mean residual, mean market price, and the geometric mean of the ratio of market price to estimated market price for selected areas. *

<table>
<thead>
<tr>
<th>Area</th>
<th>Number of Residences</th>
<th>Mean Residual</th>
<th>Geometric Mean of Ratio of Market Price to Estimated Price</th>
<th>Mean Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Study Area</td>
<td>81</td>
<td>0.000000</td>
<td>1.0000</td>
<td>$37,241</td>
</tr>
<tr>
<td>Urban Areas</td>
<td>53</td>
<td>-0.000355</td>
<td>0.9996</td>
<td>35,160</td>
</tr>
<tr>
<td>Philomath</td>
<td>5</td>
<td>-0.013970</td>
<td>0.9861</td>
<td>28,460</td>
</tr>
<tr>
<td>Corvallis</td>
<td>48</td>
<td>0.001063</td>
<td>1.0011</td>
<td>35,859</td>
</tr>
<tr>
<td>Northeast&lt;sup&gt;a&lt;/sup&gt;</td>
<td>19</td>
<td>0.013086</td>
<td>1.0132</td>
<td>32,705</td>
</tr>
<tr>
<td>Northwest&lt;sup&gt;b&lt;/sup&gt;</td>
<td>15</td>
<td>0.002091</td>
<td>1.0021</td>
<td>39,262</td>
</tr>
<tr>
<td>South&lt;sup&gt;c&lt;/sup&gt;</td>
<td>14</td>
<td>-0.016357</td>
<td>0.9838</td>
<td>36,513</td>
</tr>
<tr>
<td>Rural Areas</td>
<td>28</td>
<td>0.000673</td>
<td>1.0007</td>
<td>41,168</td>
</tr>
<tr>
<td>No. Albany</td>
<td>17</td>
<td>0.004109</td>
<td>1.0041</td>
<td>40,782</td>
</tr>
<tr>
<td>Rural Corvallis</td>
<td>11</td>
<td>-0.004638</td>
<td>0.9954</td>
<td>41,766</td>
</tr>
<tr>
<td>North and East</td>
<td>6</td>
<td>0.052817</td>
<td>1.0542</td>
<td>39,467</td>
</tr>
<tr>
<td>West</td>
<td>5</td>
<td>-0.073582</td>
<td>0.9291</td>
<td>44,526</td>
</tr>
</tbody>
</table>

* The mean residual and geometric mean refer to residuals from the double-log function.

<sup>a</sup> That area of Corvallis which is east of Kings and north of Circle Blvds.

<sup>b</sup> That area of Corvallis which is west of Kings Blvd. and north of Grant St.

<sup>c</sup> That area of Corvallis which is south of Highway 34.
was found to be $36,178 whereas the mean price of the residences located elsewhere in the study area was $38,279, as compared to a mean price of $37,241 for all residences. Thus, the prices of the residences located in the north Corvallis area, which tended to be lower than average, are underestimated by the model whereas the price of the residences located elsewhere in the study area, which are higher than average, are overestimated by the model. This pattern is the reverse of that which is expected given the positive correlation between market price and the residual. Thus, an examination of the residuals in both geographic space and characteristics space reveals no apparent statistical or theoretical reformulation of the market price function which would remove the correlation between market price and the residuals.

However, an examination of Table 6 reveals that the geometric mean of the ratio of market price to estimated price is reasonably close to unity for all areas except the two in rural Corvallis. This suggests that the correlation between market price and the residuals is independent of location. Though it seems unlikely that the relatively large prediction errors in rural Corvallis are random errors, it cannot be definitely concluded that they are not. Perhaps, contrary to assumption, the market is not in equilibrium, and the houses in north and east rural Corvallis are overpriced and those in west rural Corvallis underpriced relative to those elsewhere in the study area.
Explaining the Variation in Market Price

As was previously discussed, the double-log form of the market price function was found to explain the most variation in the market prices of the residences. Though the linear form explained nearly as much of the variation in prices, it will not be considered in this section. Note, however, that a comparison of Tables 3 and 4 reveals that the set of significant determinants is the same for both mathematical forms of the function.

The Significant Determinants. As is indicated in Table 3, the statistically significant determinants of the variation in the market prices of the residences were found to be FLRAREA, NBATHS, SD509J, STDEVPRN, and MOSOLD. The significant determinants which had the expected signs were FLRAREA, NBATHS, and MOSOLD.

The direction of the sign (and magnitude of the coefficient) on SD509J was hypothesized to depend upon the households' net marginal evaluation of a location within the school district. The results of the estimation indicate that a location within the school district will increase the market price of the residence by about 6.88 percent, 

\textit{ceteris paribus}.

However, location within School District 509J explains neither all of the variation in property tax rates nor all of the variation in publicly provided services. Therefore, neither the sign nor the
magnitude of the coefficient on SD509J can be solely attributed to the
homebuyers' net marginal evaluation of the school oriented benefits to
be obtained by a location within the school district. Because the City
of Corvallis is located entirely within School District 509J and because,
of those residences in the sample, 48 of the 59 residences located
within the school district are also located in Corvallis, it is possible
that the coefficient on SD509J is influenced by the homebuyers' net
marginal evaluation of a location within the city.\footnote{The simple correlation (r) between SD509J and a variable indicating whether or not a residence was located in Corvallis (CORVALLI) was equal to 0.736.} However, CORVALLI was not statistically significant ($t = 0.344$) when intro-
duced into the market price function, and its introduction reduced the
coefficient on SD509J by only 0.0125 (from 0.0688 to 0.0563). There-
fore, it appears that SD509J measures, primarily, what it was
intended to measure—the homebuyers' net marginal evaluation of the
school oriented benefits to be obtained by locating within School
District 509J.

Perhaps one explanation for the unexpected sign on STDEVPRN
can be provided by noting that the lower priced, often nearly identical,
"tract" houses tend to be build in large developments. Because of the
relatively large number of lower priced, nearly identical, houses in
these developments, STDEVPRN could be expected to be relatively
low for neighborhoods encompassing such "tract" developments. Conversely, the higher quality, speculatively-built, houses may tend to be built among custom-built houses and/or "spec" houses of a higher, but less uniform, price than the "tract" houses. Thus STDEVPRN may serve less as a measure of neighborhood heterogeneity than it does as an indicator of the level of the prices of the houses in the neighborhood. Therefore, because neighborhoods with higher priced houses may tend to be more aesthetically pleasing, and because it is thought that higher priced homes tend to be built among other houses of similar prices, it could be hypothesized that STDEVPRN has a positive market implicit price.

In an attempt to determine if STDEVPRN was serving both as a measure of neighborhood heterogeneity and as a measure of the level of the prices of the houses in the neighborhood, it was replaced with two variables: (1) the coefficient of variation of the prices of the houses sold within the neighborhood (COVARPRN), and (2) the ratio of the average price of the houses sold within the neighborhood to the average price of all houses sold in Benton County during the study period (NAPRATIO). COVARPRN was obtained by dividing STDEVPRN by the average price of the houses sold within the neighborhood, and thus, is a measure of the variation of the prices of the houses sold in the neighborhood relative to their average price. NAPRATIO served as a measure of the relative price level of the
houses sold in the neighborhood. Thus, the two characteristics of the neighborhood thought to be measured by STDEVPRN, the heterogeneity and relative price level of the houses, were described by separate variables.

If the households prefer the less heterogeneous neighborhoods, as was hypothesized in Chapter IV, and if they prefer neighborhoods where the houses have a higher relative price ratio, then COVARPRN would be expected to have a negative and NAPRATIO a positive market implicit price. However, when these variables were substituted for STDEVPRN in the market price function, the coefficient on COVARPRN had a positive sign and that on NAPRATIO a negative sign. But, both variables were insignificant at the 90 percent confidence level, though COVARPRN had a larger t-value ($t = 1.272 < t_{10,67} = 1.294$) than did NAPRATIO ($t = -0.229$).

Given the positive coefficient on both STDEVPRN and COVARPRN and the nonsignificance of NAPRATIO, it appears that households prefer more neighborhood heterogeneity to less. However, it must be emphasized that, as defined herein, heterogeneity is measured by the variation in the prices of the houses in the neighborhood—not by the variation in the socio-economic attributes.

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154 The adjusted $R^2$ for this model was equal to 0.810349; which is slightly less than that for the "best-fit" model ($\bar{R}^2 = 0.813929$).
of the households in the neighborhood.

The Nonsignificant Variables. The variables which were not significant as determinants of the variation in market price were FIREPL, ELEVATN, AVEAGEN, NMULTFN, the variables descriptive of the accessibility of the residence (those variables prefaced with ACCESS), and LOTSIZE. None of these variables were found to be statistically significant at the 90 percent confidence level.

It is not surprising that FIREPL was found to be not significant as a determinant of the variation in market price. There was relatively little variation in FIREPL as 95 percent of the residences included in the study had a fireplace.

ELEVATN was included in the market price function as a proxy for the view from the residential site. It was also hypothesized to serve as a measure of neighborhood attractiveness since the higher terrain tends to be wooded and, also, it was thought that, within the study area, the higher-priced homes tend to be located at elevations which are high relative to the immediately surrounding terrain. Obviously, ELEVATN is a poor measure of these characteristics. First, ELEVATN was measured relative to the Willamette River flood plain--a rather distant datum for many locations. Secondly, the view from a site depends not only upon the elevation of the site relative to the surrounding terrain but also upon the extent and the aesthetics of the panorama. Thus, ELEVATN may not have been significant
because the characteristics of which it was to be descriptive are not relevant in the location decision and/or because it was not descriptive of these characteristics; the latter explanation seems more likely.

It also seems probable that NMULTFN, the number of multiple-family dwelling structures sold within the neighborhood during the study period, served as an inaccurate measure of the number of multiple-family dwelling structures within the neighborhood. As was noted in Chapter IV, about 6.9 percent of the single-family and 2.9 percent of the multiple-family dwelling units in Benton County were sold during the study period. Given that such a small percentage of the total number of multiple-family dwellings was traded during the study period, it does not seem probable that the number of such dwelling units traded within any given neighborhood would be, across all neighborhoods, a consistent proportion of the total number within the neighborhood.

In Chapter IV, it was hypothesized that the average age of the single-family residences within the neighborhood, as measured by the average age of those sold in the neighborhood during the study period (AVEAGEN), has both positive and negative aspects as an indicator of neighborhood quality. It was hypothesized that the negative aspects would outweigh the positive and that, thus, the coefficient on AVEAGEN would be negative—-as it is in the estimated market price function. However, it is possible that the negative and positive
aspects offset one-another so that the net effect of AVEAGEN on market price was not significant.

Finally, the effectiveness of the variables descriptive of the neighborhood (AVEAGEN, NMULTFN, and STDEVPRN) in explaining the variation in market price is not independent of the definition of neighborhood on which the measurements on these variables are based. If that which was defined to be the neighborhood bore no relationship to that which the household considered to be the neighborhood in their location decision, there would be little reason to hope that the variables descriptive of the neighborhood would have any significance in explaining the variation in market price and/or that the estimated coefficients on these variables would have any significance other than statistical.

Variables which provided four alternative measures of accessibility, as defined in Chapter IV, were introduced into the market price function. The different combinations of ACCESS variables introduced were: (1) ACCESST, (2) ACCESSRM2, (3) ACCESSR and ACCESSM, and (4) ACCESSR2 and ACCESSM. The estimated market price function which included ACCESSRM2 explained about the same amount of variation in market price as did that which included ACCESSR2 and ACCESSM ($R^2 = 0.814$ for both). The latter

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155 Definitions of these variables were provided in Table 1.
combination was chosen on the basis that, unlike all of the other ACCESS variables, ACCESSM had the expected sign. However, none of the ACCESS variables were statistically significant.

One explanation for the nonsignificance of the ACCESS variables may be that, given the ease of travel and relatively short distances to be traversed regardless of location within the study area, the households' accessibility decisions are of an infra-marginal rather than marginal nature. That is, there may be some minimum threshold level of accessibility which the households desire. But, after reaching that threshold, they may be indifferent to any further increase in accessibility.

The ACCESS variables may also be nonsignificant if, contrary to assumption, there does not exist an independent choice of the levels of accessibility and neighborhood amenity. If, in reality, the level of neighborhood amenity tends to vary inversely with the level of accessibility, the household's marginal evaluation of an increase in the level of accessibility may consider, and be net of, any change in

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156 An examination of the distribution of ACCESSRM2 revealed that 75 percent of the households included in the study could have reduced their total travel distance (i.e., the sum of the one-way distances) from their residence to the nearest CBD, shopping center, and their places of employment by less than 6.7 miles if they were to move from their actual location to the location where their total travel distance would be a minimum. Using ACCESSRM2 as a measure of accessibility, the least accessible household could have reduced its total travel distance by 14.5 miles.
the marginal evaluation of neighborhood amenity as a result of a decrease in the level of neighborhood amenity. If this net marginal evaluation either approached zero or became negative, accessibility may have either a zero or a negative market implicit price. Thus, an inverse relationship between neighborhood amenity and

\[ U(q_1, q_2, x) = u, \]

where:
- \( q_1 \) = the level of accessibility, and
- \( q_2 \) = the level of neighborhood amenity.

Assume that \( q_2 = q_2(q_1) \) and that \( dq_2/dq_1 < 0 \). Also, introduce a budget constraint similar to that given by (71):

\[ y = \beta(q) + M(q_1, y) + x. \]

Substitution of the budget constraint into the function which defines the indifference surface gives:

\[ U(q_1, q_2(q_1), y - \beta(q) - M(q_1, y)) = u. \]

Differentiating the latter with respect to \( q_1 \) and solving for \( \partial \beta / \partial q_1 \) gives:

\[ \frac{\partial \beta}{\partial q_1} = \frac{\partial U/\partial q_1 + (\partial U/\partial q_2)(dq_2/dq_1)}{\partial U/\partial x} - \frac{\partial M}{\partial q_1}, \]

which defines the implicit demand price for accessibility and would be identical to Equation (79) where \( dq_2/dq_1 = 0 \). The right hand side of the last equation above gives the household's marginal evaluation of an increase in the level of accessibility ((\( \partial U/\partial q_1 \))/((\( \partial U/\partial x \))-(\( \partial M/\partial q_1 \))) net of any change in the marginal evaluation of neighborhood amenity as a result of a decrease in the level of neighborhood amenity, resulting, in turn, from an increase in the level of accessibility ((\( \partial U/\partial q_2 \))/((\( \partial U/\partial x \))(dq_2/dq_1))).
accessibility could explain the nonsignificance of and/or a negative
sign on the ACCESS variables.

Though the coefficients on those variables either descriptive of
accessibility to retail centers or descriptive of accessibility to both
retail centers and employment were negative, the coefficient on
ACCESSM was positive. Given the inability, or failure, to control
for neighborhood amenity with changes in accessibility, the opposite
signs on those variables measuring accessibility to retail centers
and that measuring accessibility to employment may be explained by
the neighborhood amenity levels about the places of employment.
Over 38 percent of the households included in the study had at least
one member employed at either Oregon State University (OSU) or at a
location immediately adjacent to the OSU campus. Those neighbor-
hoods to the north and south of the OSU campus which contain houses
included in this study have few, if any, commercial establishments,
and a move to these neighborhoods from less accessible locations
would not necessarily decrease the level of neighborhood amenity.
Thus, the positive market implicit price of accessibility to employ-
ment may be due to the concentration of employment at and about the
OSU campus.

Finally, it was thought that one reason for the nonsignificance of
the ACCESS variables (except ACCESSM) may have been their rela-
tively high correlation with LOTSIZE. The simple, pair-wise
correlation between LOTSIZ and ACCESSR2, for example, was equal to -0.780 and the partial correlation between these two variables was equal to -0.660.\textsuperscript{158} As this collinearity increases the estimated standard error of the regression coefficient on both LOTSIZ and ACCESSR2 (Johnston, 1972, pp. 160-162), it was thought that the nonsignificance of ACCESSR2 may have been due to its correlation with LOTSIZ. However, deletion of LOTSIZ from the estimated market price function did not cause ACCESSR2 to become significant (and vice-versa).

It is rather difficult to rationalize the lack of significance of LOTSIZ as a determinant of the variation in market price. There was a strong expectation that an increase in lot size would increase the market price of a residence, \textit{ceteris paribus}.

It was thought likely that there may have been differences in the land market and in lot development costs between various areas (particularly urban and rural areas) such that a single LOTSIZ variable could not explain, \textit{ceteris paribus}, the variation in market price without explicit consideration of these differences. Therefore, binary variables which denoted whether or not a residence was located in one of two or more mutually exclusive sub-areas of the study area were introduced into the market price function to consider the

\textsuperscript{158} These correlations are between the natural logarithms of the variables.
differences in the land market and lot development costs between various areas. These binary variables allowed for possible differences in the equation intercept term and in the LOTSIZE regression coefficient between the various areas (see Kmenta, 1971, pp. 409-425). Though several sets of these variables were tried, none were significant as a set when introduced into the double-log form of the market price function which contained all of the variables originally hypothesized to be determinants of market price.

Among the sets of binary variables introduced into the market price function were the following, which indicated whether or not a residence was located in either:

1. a rural area or an urban area,
2. Corvallis, Philomath, or elsewhere in the study area,
3. Corvallis, rural Corvallis, or elsewhere in the study area,
4. Corvallis, Philomath, rural Corvallis, or North Albany,
5. North Albany or elsewhere in the study area.

A residence was considered to be in an urban area if it was in either Corvallis or Philomath. With each set, one of the binary variables was excluded from the estimated function to avoid a singular data matrix.

In each case, the null hypothesis was that LOTSIZE did not influence market price. Thus, the appropriate test of the hypothesis required the F statistic (Kmenta, 1971, p. 456):

\[ F = \frac{(SSR_Q - SSR_K) / (Q-K)}{SSE_Q / (n-Q)} \sim F(Q-K, n-Q), \]

where:

- SSR = regression sum of squares,
- SSE = error sum of squares,
- n = number of observations,

and where the subscript K denotes the value of SSR or SSE pertaining to the original set of explanatory variables, and the subscript Q denotes the values pertaining to the extended set of explanatory variables containing the binary variables.
However, the market price function was estimated in a number of forms in which LOTSIZE was significant if certain explanatory variables were dropped from the function. The best of these functions, in that it had the highest $R^2$ of these reduced models and had a positive market implicit price for LOTSIZE, is presented in Table 7. This reduced model, in which the natural logarithm of market price was regressed on the untransformed explanatory variables, did not have as high an $R^2$ as did the double-log form of the market price function which was presented in Table 3.

Table 7. Estimate of the market price function wherein LOTSIZE is significant.*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>9.8579</td>
<td>9.778**</td>
</tr>
<tr>
<td>(LOT SIZE)$^{-1}$</td>
<td>-698.75</td>
<td>-1.598*</td>
</tr>
<tr>
<td>FLRAREA</td>
<td>0.0003986</td>
<td>11.055**</td>
</tr>
<tr>
<td>FIREPL</td>
<td>0.09862</td>
<td>1.754**</td>
</tr>
<tr>
<td>AVEAGEN</td>
<td>-0.002317</td>
<td>-1.322*</td>
</tr>
<tr>
<td>ACCESSM</td>
<td>0.007406</td>
<td>1.582*</td>
</tr>
<tr>
<td>MOSOLD</td>
<td>0.006362</td>
<td>1.614*</td>
</tr>
</tbody>
</table>

$R^2 = 0.750638$

$F_{6, 75} = 41.365 > F_{0.05, 6, 74} = 2.15$

* Significant at the .10 level ($t_{0.10, 74} = 1.293$).

** Significant at the .05 level ($t_{0.05, 74} = 1.665$).

The dependent variable is the natural logarithm of market price.

The correlation matrix for this function is presented in Appendix Table B3.
Though this semi-log, reduced, form of the market price function is not a "best-fit" model, it is of interest because of the variables which it does not contain. Each of the variables listed in Table 8 was statistically significant at, at least, the 90 percent confidence level when separately introduced into the market price function.\(^{162}\) However, as is shown in Table 8, their introduction caused LOTSIZE to become insignificant. Note that, with the exception of NBATHS and SD509J, the variables which caused LOTSIZE to become insignificant were intended to be descriptive of the neighborhood in which the residence was located.

Table 8. The individual effect of the separate introduction of selected variables on the significance of LOTSIZE in the reduced, semi-log, form of the market price function.

<table>
<thead>
<tr>
<th>Variable Entering</th>
<th>t-Value of Entering Variable</th>
<th>t-Value for (LOTSIZE)(^{-1})</th>
<th>Correlation (r) Between Entering Variable and (LOTSIZE)(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBATHS</td>
<td>4.485</td>
<td>-0.622</td>
<td>-0.581</td>
</tr>
<tr>
<td>SD509J</td>
<td>2.568</td>
<td>-1.258</td>
<td>0.148</td>
</tr>
<tr>
<td>STEVPRN</td>
<td>1.756</td>
<td>-1.151</td>
<td>-0.299</td>
</tr>
<tr>
<td>ELEVATN</td>
<td>1.557</td>
<td>-0.951</td>
<td>-0.521</td>
</tr>
<tr>
<td>NAPRATIO</td>
<td>3.095</td>
<td>-0.885</td>
<td>-0.413</td>
</tr>
</tbody>
</table>

\(^{162}\)Recall that NAPRATIO was defined as the ratio of the average price of the houses sold within the neighborhood to the average price of all houses sold within Benton County during the study period.
The individual effect of the three variables descriptive of the neighborhood on the significance of LOTSIZE as a determinant of market price may provide one explanation for the nonsignificance of LOTSIZE in the "best-fit" model presented in Table 3. Note that each of these neighborhood variables has a positive market implicit price when introduced into the market price function, and that this price would be reflected in the price of the land in the neighborhood. Furthermore, each of the neighborhood variables is negatively correlated with the inverse of LOTSIZE, and therefore is positively correlated with LOTSIZE. Thus, if these variables are omitted from the market price function, their effect may be measured by LOTSIZE. However, when these variables are introduced into the function, an increase in LOTSIZE, *ceteris paribus*, does not have a significant effect on market price. This implies that lot area *per se* may not have a positive market implicit price. However, those neighborhood characteristics which are associated with lots of a larger size may have a positive market implicit price. Thus, it may be that it is not lot area, in itself, which is important as a determinant of market price, but, rather, those neighborhood characteristics which tend to be associated with a larger lot area that are the significant determinants of market price. If these neighborhood characteristics are not explicitly considered in the specification of the market price function, lot area may have an apparent influence on market price.
This rationale for the nonsignificance of LOTSIZE may be weak, however, since both NBATHS and SD509J had much the same effect on the significance of LOTSIZE as did the neighborhood variables. But, a location within School District 509J was found to have a positive market implicit price, which would be reflected in land values within the school district, and thus the presence of SD509J in the market price function would lessen the significance of LOTSIZE. Note, however, that the correlation between (LOTSIZE)^{-1} and SD509J was very low, and that its effect on the t-value for (LOTSIZE)^{-1} was not as large as that of the other variables listed in Table 8. Also, the effect of NBATHS on the significance of (LOTSIZE)^{-1} may be due to its correlation with that variable. Finally, one neighborhood variable (AVEAGEN) remained in the market price function and did not cause (LOTSIZE)^{-1} to become insignificant—though it did change the t-value on (LOTSIZE)^{-1} from -1.92 to -1.60. Nevertheless, it cannot be definitely concluded that lot area per se is not a significant determinant of market price.

Selection of the "Best" Estimate of the Market Price Function and the Choice of Endogenous Variables

As each of the various forms in which the market price function was estimated had explanatory variables with associated t-statistics that were less than 1.0 in absolute value, it was possible to increase
the $R^2$ of the double-log form of the function presented in Table 3 from 0.813929 to 0.824070 by dropping all of the variables for which $t < 1.0$ (except FIREPL, which obtained a t-value of 1.152) from the function. However, the $R^2$ was increased by very little (never by more than 0.03), and this procedure always increased the correlation between market price and the residuals (it increased the correlation from 0.398 to 0.403 for the function presented in Table 3). Therefore, it was decided to select the fully-specified, double-log, form of the market price function, as presented in Table 3, as the best of the estimated functions.

As FLRAREA, NBATHS, SD509J, and STDEVPRN were statistically significant variables in the estimated market price function, it would be possible, in principle, to estimate supply and demand functions for each of these characteristics. However, as such functions for NBATHS and SD509J were not thought to be especially interesting and because the meaning of such functions for STDEVPRN would be difficult to interpret, it was decided to attempt to estimate only the FLRAREA demand and supply functions. Though the estimation of only the FLRAREA demand and supply functions represents a rather extensive diminution of an original objective of this study, recall that a regression of market price on FLRAREA alone (both transformed to their natural logarithms) explained about 73 percent of the variation in the dependent variable.
The Calculation of the Market Implicit Prices of FLRAREA

Prior to estimating the demand and supply functions it was necessary to obtain the "observed" market implicit prices of FLRAREA. Using the estimated market price function presented in Table 3, the predicted natural logarithm of the market price of the $i^{th}$ residence is given by:

\[ \hat{P}_{i}^{*} = \ln \hat{P}_{i} = 5.5603 - 0.0230 \ln(\text{LOTSIZE}) \]
\[ + 0.6118 \ln(\text{FLRAREA}) + 0.2002 \ln(\text{NBATHS}) \]
\[ + 0.0292 \text{FIREPL} + 0.0688 \text{SD509J} \]
\[ + 0.0078 \ln(\text{ELEVATN}) + 0.0451 \ln(\text{STDEVPRN}) \]
\[ - 0.0123 \ln(\text{AVEAGEN}) + 0.0036 \ln(\text{NMULTFN}) \]
\[ - 0.0212 \ln(\text{ACCESSR2}) + 0.0148 \ln(\text{ACCESSM}) \]
\[ + 0.0449 \ln(\text{MOSOLD}). \]

The market implicit price of FLRAREA, in general, is given by the partial derivative of (108) with respect to FLRAREA:

\[ \frac{\partial \hat{P}_{i}^{*}}{\partial (\text{FLRAREA})} = \frac{1}{\hat{P}_{i}} \frac{\partial \hat{P}_{i}}{\partial (\text{FLRAREA})} = 0.6118 \frac{1}{\text{FLRAREA}}, \]

\[\text{The dependent variable in (108) is the estimated natural logarithms of the market price, not the natural logarithm of the estimated market price. However, it was necessary to assume the latter so as to obtain the market implicit price.}\]
or

\[ \frac{\partial \hat{P}_i}{\partial (\text{FLRAREA})} = 0.6118 \frac{\hat{P}_i}{\text{FLRAREA}}. \]

Then, the market implicit price of FLRAREA was obtained for each of the 81 residences included in the study by calculating the $\ln \hat{P}_i$ for each residence using Equation (108), obtaining the $\hat{P}_i = \exp(\ln \hat{P}_i)$, and substituting this estimated market price and the observation on FLRAREA into (109). These estimated market implicit prices of FLRAREA were used as the observations on the dependent variable in estimating the FLRAREA demand and supply functions.

The "observed" market implicit prices of FLRAREA ranged from about $12.15 to $17.70, with a mean value of about $14.94 and a median of about $14.92. As the standard deviation of the "observations" was equal to only about $1.21, there was relatively little variation in the estimated market implicit prices of FLRAREA; the coefficient of variation was equal to only 0.0811.

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If the "observed" market implicit prices of FLRAREA seem low relative to actual housing costs, recall that the market price function contains other variables descriptive of the house per se—namely, NBATHS and FIREPL.
The Floor Area Demand and Supply Functions

The Method of Estimation and Selection of the "Best" Functions

As the FLRAREA demand and supply functions are, in theory, a simultaneous system of supply and demand equations, they were estimated using the two-stage least squares (TSLS) technique (see Johnston, 1972, pp. 380-384, or Kmenta, 1971, pp. 559-564). However, in theory, not only are the magnitudes of the market implicit price of FLRAREA and of FLRAREA per se simultaneously determined, but so are the magnitudes of all other variables descriptive of the characteristics of the residence and their market implicit prices. Thus, those variables descriptive of the characteristics of a residence which were statistically significant in the estimated market price function were considered to be endogenous variables in the larger system of supply and demand functions—of which the FLRAREA demand and supply equations are a part.

Because each of the endogenous explanatory variables (FLRAREA, NBATHS, SD509J, and STDEVPRN) in the demand and supply functions is, in general, correlated with the error term associated with each function, the first stage of the TSLS procedure requires that each of these variables be regressed, using the OLS technique, on all exogenous variables which appear in the system. In these first stage regressions, all of those variables descriptive of the
characteristics of the residences which were not statistically significant in the estimated market price function were considered to be exogenous, as were the variables descriptive of the attributes of the households and of the developers. Using these estimated reduced form equations, the predicted values of the endogenous explanatory variables were obtained for use, in the second stage of the estimation, as the observations on these variables. Then these observations, along with observations on the appropriate exogenous explanatory variables were used in the separate estimation, by the OLS technique, of the FLRAREA demand and supply functions. Because the first stage regressions purge the endogenous explanatory variables of their correlation with the error term, the second stage estimates of the coefficients on these variables are consistent—which they would not be if OLS were used to directly estimate the demand and supply functions (Kmenta, 1971, pp. 533-534).

The FLRAREA demand and supply functions, as specified in Table 2 of Chapter IV, were each estimated in several different mathematical forms. As there are no measures of goodness-of-fit for equations estimated by the TSLS procedure, the "best" estimate of both the demand and supply equations was determined on the basis of an examination of the residuals for the nature of their distribution and for evidence of heteroskedasticity, autocorrelation, and specification error.
Of those forms in which the demand and supply functions were estimated, the double-logarithmic forms were judged to be the "best" estimates. These "best" estimates of the FLRAREA demand and supply equations are presented in Table 9.

Tests for heteroskedasticity using the Spearman coefficient of rank correlation \( r_s \) between the absolute value of the residuals and, in turn, each of the explanatory variables revealed that there was significant correlation (at the 99 percent confidence level) between the rank of the absolute values of the residuals and the rank of the observations on NOCHILD, EQUITY, and TXCREDES.\(^{165}\) However, after a visual examination of the plots of the residuals against these variables, it was judged that the heteroskedasticity was not of a sufficient degree to warrant the use of corrective techniques. There was no evidence of heteroskedasticity in the variance of the error term associated with the FLRAREA supply equation.

Tests for the presence of an autoregressive error term in the demand equation resulted in the failure to reject the null hypothesis of nonautoregression. However, the result of a test for an autoregressive error term in the supply equation was inconclusive, with the null hypothesis being rejected at the 95 percent confidence level but not

\(^{165}\) The \( r_s \) between NOCHILD and the absolute value of the residuals was equal to -0.319. The same statistics for EQUITY and TXCREDES were equal to -0.426 and -0.523, respectively. The critical value of \( r_s \) at the 99 percent confidence interval is 0.283.
Table 9. Estimates of the floor area demand and supply functions.\textsuperscript{a}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Demand Regression Coefficient</th>
<th>Demand t-Value\textsuperscript{b}</th>
<th>Supply Regression Coefficient</th>
<th>Supply t-Value\textsuperscript{b}</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>5.0422</td>
<td>5.473*</td>
<td>7.2447</td>
<td>6.090##</td>
</tr>
<tr>
<td>LOTSIZE</td>
<td>-0.0250</td>
<td>-1.021</td>
<td>-0.5354\textsuperscript{c}</td>
<td>-3.929##</td>
</tr>
<tr>
<td>FLRAREA\textsuperscript{c}</td>
<td>-0.4282</td>
<td>-3.600*</td>
<td>0.4138\textsuperscript{c}</td>
<td>4.023##</td>
</tr>
<tr>
<td>NBATHS\textsuperscript{c}</td>
<td>0.1532</td>
<td>0.905</td>
<td>0.0451</td>
<td>1.142</td>
</tr>
<tr>
<td>FIREPL</td>
<td>0.0446</td>
<td>1.052</td>
<td>0.0451</td>
<td>1.142</td>
</tr>
<tr>
<td>SD509J\textsuperscript{c}</td>
<td>0.0400</td>
<td>0.953</td>
<td>0.0078</td>
<td>0.825</td>
</tr>
<tr>
<td>ELEVATN</td>
<td>0.0099</td>
<td>1.137</td>
<td>0.0078</td>
<td>0.825</td>
</tr>
<tr>
<td>STDEVP2RN\textsuperscript{c}</td>
<td>0.0771</td>
<td>1.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVEAGEN</td>
<td>-0.0105</td>
<td>-0.495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMULTFN</td>
<td>0.0015</td>
<td>0.086</td>
<td>0.0078</td>
<td>0.825</td>
</tr>
<tr>
<td>ACCESSR2</td>
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<td>-1.196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACCESSM</td>
<td>0.0237</td>
<td>1.196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DISPOSIN</td>
<td>0.0213</td>
<td>0.660</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOCHILD</td>
<td>0.0021</td>
<td>0.121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGEHEAD</td>
<td>-0.0195</td>
<td>-0.442</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EQUITY</td>
<td>0.0005</td>
<td>0.210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TXGREDES</td>
<td>0.0049</td>
<td>2.237*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVCSTFT2</td>
<td>-0.0202</td>
<td></td>
<td>-0.212</td>
<td></td>
</tr>
<tr>
<td>BLDRSCAL</td>
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<td></td>
<td>-1.602#</td>
<td></td>
</tr>
<tr>
<td>SDGONCST</td>
<td>0.0032</td>
<td>0.643</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVGONCST</td>
<td>-0.0901</td>
<td>0.987</td>
<td></td>
<td></td>
</tr>
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<td>CUSTFET</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CORVALLI</td>
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<tr>
<td>PHILOMTH</td>
<td>-0.0652</td>
<td>-1.485#</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{*}Significant at the .05 level (t\textsubscript{64,.05} = 1.669).

\textsuperscript{#}Significant at the .10 level (t\textsubscript{69,.10} = 1.294).

\textsuperscript{##}Significant at the .05 level (t\textsubscript{69,.05} = 1.667).

\textsuperscript{a}The dependent variable and all explanatory variables except FIREPL, SD509J, CUSTFET, CORVALLI, and PHILOMTH, are in natural logarithms.

\textsuperscript{b}Reference to the t-distribution for tests of hypotheses may not always be a valid procedure (see Kmenta, 1972, p. 584).

\textsuperscript{c}These variables were treated as endogenous variables in the TSLS estimation of the equations.
rejected at the 99 percent level.\textsuperscript{166}

The residuals from both the supply and demand equations were plotted against their respective explanatory variables and equation-predicted implicit prices. In addition, the simple, pair-wise correlation between the residual and the respective explanatory variables was obtained. In each case, there was a lack of significant correlation, and a visual examination of the plots did not reveal any nonlinear relationships, symptomatic of specification error, between the variables and the residuals.

However, separate plots of the residuals from the demand and supply equations against the "observed" market implicit prices of FLRAREA revealed that there was a high, positive, correlation in both cases. The simple, pair-wise correlation was equal to 0.660 for the demand function and 0.713 for the supply function.\textsuperscript{167} Thus, the equations tended to overestimate the implicit demand and supply prices in those instances where FLRAREA was below the mean FLRAREA and underestimate the implicit price where FLRAREA was

\textsuperscript{166}For the demand equation, the Durbin-Watson statistic was equal to 1.741 and the von Neumann ratio was equal to -1.182. For the supply equation, the Durbin-Watson statistic was equal to 1.542 and the von Neumann ratio was equal to -2.089. The critical values for the von Neumann ratio are 1.960 and 2.326 at the 95 percent and 99 percent confidence levels, respectively.

\textsuperscript{167}Correlations of a similar magnitude were observed between the residuals and market implicit price with each of the forms in which these equations were estimated.
above the mean. Though disappointing, the existence of this phenomena could have been expected, as the "observed" market implicit prices of FLRAREA were estimated using an estimate of the market price function which tended to overestimate the market price of the more expensive homes. Remediation of this problem would require that the error, whatever its nature, in the specification of the system be identified.

Inspection of the correlation and inverse correlation matrices for both the demand and supply functions revealed that, in both functions, the highest pair-wise collinearity and multicollinearity existed between the endogenous explanatory variables. The high collinearity and multicollinearity between the endogenous variables is a result of the TSLS estimation procedure. The observations on these variables are the predicted values from the first stage regression equations which, in each case, contained all of the exogenous variables in the system. That is, observations on each of the endogenous explanatory variables were estimated as a function of the same set of variables, and thus each of these variables can be expected to exhibit multicollinearity with the remainder of the explanatory variable set.

The pair-wise correlations and multiple correlations between the exogenous variables were not as high as those between the endogenous variables. In the demand function, the highest pair-wise correlation between the exogenous variables was that between
LOTSIZE and ACCESSR2 \( (r = -0.780) \), and the highest multicollinearity was between ACCESSR2 and the remainder of the explanatory variables \( (R^2 = 0.819) \). The highest pair-wise correlation between the exogenous variables in the supply function was that between BLDRSCAL and CORVALLI \( (r = 0.615) \), and the highest multicollinearity was between BLDRSCAL and the remainder of the explanatory variables \( (R^2 = 0.860) \). Though the multicollinearity was quite high for ACCESSR2 and BLDRSCAL, it was judged to be of an insufficient extent to warrant special estimation procedures.

**Explaining the Variation in Implicit Demand Price**

**The Significant Determinants.** As was expected, the results of the estimation indicate that an increase in FLRAREA will decrease the households' implicit demand price for FLRAREA, *ceteris paribus*. More explicitly, if the FLRAREA demand function is, in fact, real income compensated, the rate at which the households' are willing to substitute money for dwelling area will decrease by about four-tenths of one percent \( (0.428\% = 0.00428) \) given a one percent increase in dwelling area, *ceteris paribus*.

---

168 The correlation and inverse correlation matrices for the FLRAREA demand function are presented in Appendix Table B4, and those for the FLRAREA supply function are given in Appendix Table B5.
The only other significant determinant of the variation in the implicit demand price for FLRAREA was the variable which indicated the magnitude of the federal income tax credit (TXCREDES) which a household received if it purchased a house which satisfied the prerequisites for claiming the credit. As was expected, an increase in the magnitude of the tax credit increased the demand for FLRAREA.

The Nonsignificant Variables. None of the variables descriptive of the characteristics of a residence were statistically significant as determinants of the demand for FLRAREA. If the coefficients on these variables do not differ significantly from zero, the implication is that the services of the characteristics which the variables represent are independent of those of FLRAREA in consumption. No hypotheses were formulated as to the relationships between the characteristics in consumption as the nature of these relationships was thought to be an empirical question; specifically, the services of the characteristics may be independent in consumption. If independence seems unlikely, the services of each of the characteristics may be interpreted as either a complement to or substitute for the services of FLRAREA in consumption according to the direction of the sign (positive or negative, respectively) on the regression coefficient associated with the variable descriptive of the characteristic. However, the probability that the sign on the coefficient is indicative of a relationship opposite that of the true relationship increases as the
t-value associated with the coefficient approaches zero.

None of the variables descriptive of the attributes of the household per se were statistically significant as determinants of the demand for FLRAREA. Though reasons for the possible nonsignificance of AGEHEAD and NOCHILD were discussed in Chapter IV, there was a strong expectation that the measures of household income and wealth (DISPOSIN and EQUITY) would be significant determinants of the demand for FLRAREA.

However, the theory presented in Chapter III does provide a rationale for the nonsignificance of income as a determinant of the demand for FLRAREA. If the households' marginal rate of substitution of money for FLRAREA were invariant with respect to an increase in utility and thus in income, the implicit demand price of FLRAREA would, by definition, be invariant with respect to income. The results of the estimation point toward the conclusion that the households' implicit demand price for FLRAREA is invariant with respect to income (and, thus, that the bid functions are vertically parallel in the FLRAREA dimension). However, such a conclusion is

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169 TXCREDES is not an attribute to the household per se. The magnitude of TXCREDES was a function of certain attributes of the household and was contingent upon the purchase of a residence which met the requirements outlined in Chapter IV.

170 Gross household income was not statistically significant when substituted for DISPOSIN in the estimated demand function.
rather strong and would be premature without consideration of other reasons why DISPOSIN might not be statistically significant as a determinant of the demand for FLRAREA; these reasons are discussed below under the topic of identification.

The nonsignificance of the variables descriptive of the attributes of the household has strong implications as to the nature of the functions which have been estimated—including the market price function. From a statistical viewpoint, the most important consequence of the lack of significant exogenous variables in the estimated demand function is that the supply function is underidentified. The order condition, for identification requires that, of all exogenous variables included in the system, the number excluded from the supply function must be at least as great as the number of endogenous variables included in the supply function less one (see Kmenta, 1971, p. 543). Prior to estimation, the two equation system presented in Table 9 satisfied the order condition. However, after estimation, when only the statistically significant variables count toward identification of the system, the supply function is underidentified. The implications of the failure to identify the supply equation will be discussed after a discussion of the results of the attempted estimation of the supply function.
Explaining the Variation in Implicit Supply Price

The Significant Determinants. As the supply function is underidentified, it is difficult to impute any meaning to the parameters of that function which has been estimated. To the extent that the variables in the function are clearly, from a theoretical standpoint, not descriptive of the determinants of demand and appear to be significant in explaining the variation in the market implicit price of FLRAREA, there is prima facie evidence that they are determinants of the supply of FLRAREA. However, the estimated parameters on these exogenous variables, and particularly, those on the endogenous variables must be interpreted with caution. There simply was not a sufficient number of statistically significant exogenous variables in the demand function to "trace out" all dimensions of the supply surface.

171 Since the parameters of an underidentified equation cannot be consistently estimated, the tests of hypotheses concerning their values are not valid (Kmenta, 1972, pp. 548-550). Therefore, the statistical significance of the estimated parameters of the supply function cannot, in a strict sense, be determined. As a consequence, it does not appear to be possible to determine if the demand function is identified by direct reference to the order condition for identification (i.e., by counting the "significant" variables in the supply function). Econometric textbooks do not, generally, discuss the problem of "ex post" identification of a system of estimated equations. Kmenta (1972, p. 546) notes the problem in passing and references a method of treating the identification of a system as a test of hypothesis. The problem is not pursued herein. It is assumed that those variables which appear to be significant in the supply function are, in fact, statistically significant, and the order condition is used to determine the identification of the estimated demand function.
As the supply function was not statistically identified, the estimated equation, whatever it may be, cannot be ascertained to be a supply function.

FLRAREA appears to be a significant determinant of the implicit supply price of FLRAREA, but the regression coefficient has the unexpected sign. The results of the estimation indicate that, contrary to expectation, the marginal cost of provision of FLRAREA declines with an increase in FLRAREA. The unexpected sign on the FLRAREA regression coefficient could be due to the underidentification of the supply function.

However, it may also be that the "wrong" sign was expected, and that the marginal cost of provision of FLRAREA is, in fact, declining. A "Cost Factor" manual prepared by the Oregon Department of Revenue (1971, p. 36.01 and pp. 26-29) to assist appraisers in calculating the replacement cost of residences indicates that the average "base" cost per square foot (ft$^2$) of ground floor area$^{172}$ for an "average quality home built for speculation" ranged, in April 1975, from $19.38/ft^2$ for a residence with 1000 ft$^2$ of ground floor area to $15.47/ft^2$ for a residence with 2000 ft$^2$ of area.$^{173}$ Further, for

172 Ground floor area is determined by the gross exterior dimensions of the house excluding the garage.

173 These costs are for a one-story residence without a basement, and include the costs of 1.5 bathrooms. The costs do not
residences of those quality levels considered, the "Cost Factor"
manual indicates that the average replacement cost per square foot
decreases over the entire range of floor area considered (up to
3000 ft$^2$). A declining average cost requires that marginal cost be
less than average cost, but marginal cost could be increasing.

The results indicate that NBATHS is a determinant of the supply
of FLRAREA, and that an increase in NBATHS increases the marginal
cost of FLRAREA. However, as NBATHS is an endogenous variable
which is also hypothesized to influence the demand for FLRAREA, it is
difficult, due to the underidentification of the supply function, to
determine if the indicated relationship between FLRAREA and NBATHS
is a supply rather than a demand relationship.

Of the exogenous variables descriptive of the attributes of the
developer, only BLDRSCAL was significant as a determinant of the
supply of FLRAREA. As expected, an increase in the number of
single-family residences constructed by the developer resulted in a
lower marginal cost (implicit supply price) of FLRAREA, ceteris
paribus.

The two variables intended to be descriptive of the regulations
and fees imposed on the developers by the Cities of Corvallis and
include the cost of either the heating system or a fireplace. However,
the cost of the heating system is estimated to decline with increases
in the heated dwelling area. All costs are for the Portland, Oregon,
area.
Philomath (CORVALLI and PHILOMTH) were both statistically significant. The results indicate that the marginal cost of FLRAREA is higher in Corvalis (as expected) and lower in Philomath (contrary to expectation) than it is in the remainder of the study area. However, CORVALLI and PHILOMTH may also have served to measure differences in the housing market (including rental and used housing) between Corvalis, Philomath, and the remainder of the study area, which were due to factors that were not explicitly considered in the specification of the model, and which may have influenced both the demand for and supply of FLRAREA. Therefore the direction and magnitude of the estimated coefficients on CORVALLI and PHILOMTH cannot necessarily be attributed solely to the effect of city-imposed development regulations and fees on the supply of FLRAREA.

The Nonsignificant Variables. Though each had the expected sign on their estimated coefficients, neither FIREPL nor ELEVATN were statistically significant as determinants of the supply of FLRAREA. Those possible explanations which were previously advanced for the nonsignificance of FIREPL and ELEVATN as determinants of the variation in market price apply here also and will not be repeated.

The variable denoting whether or not a household imposed a request for "custom features" (CUSTFET) on the developer was not statistically significant. As was hypothesized in Chapter IV, the
households' requests for such features may often be more in the nature of a trade, and thus neither add to nor subtract from the market price of the residence. Therefore, CUSTFET may, in fact, have no impact on the marginal cost of FLRAREA.

Finally, none of those variables which were based on the estimated construction cost of the residences (AVCSTFT2, AVCONCST, and SDCONCST) and which were intended to be descriptive of the developers were statistically significant as determinants of the supply of FLRAREA. As was discussed in Chapter IV, the reliability of the data which were used to obtain the observations on these variables is highly suspect and provides a probable explanation for the insignificance of AVCSTFT2, AVCONCST, and SDCONCST.

Identification

It was previously determined that, if only the statistically significant variables are counted, the estimated FLRAREA supply equation does not satisfy the necessary (or order) condition for identification. However, using the same criterion, the estimated FLRAREA demand function is identified. That is, the number of significant variables descriptive of the attributes of the developer was sufficient to "trace out" the FLRAREA demand equation, but the number of significant variables descriptive of the attributes of the household was not sufficient to "trace out" the FLRAREA supply equation.
The supply function is underidentified because the variables descriptive of the attributes of the households were not statistically significant in explaining the variation in the market implicit price of FLRAREA. There are at least three possible reasons for the nonsignificance of the set of household attribute variables:

1. the market price function may, in fact, be linear (at least in the FLRAREA dimension), and thus the market implicit price of FLRAREA is constant;

2. there may have been relatively little variance in the observations on the variables descriptive of the attributes of the household (i.e., the households may be nearly identical in all of those attributes which were measured), or

3. the households may not be identical, but the attributes of the household may not, in fact, be determinants of the implicit price of FLRAREA.

If the household attribute variables were not significant for either of the latter two reasons, the estimated market price function can be identified as (i.e., shown to be) the bid function--at least in its FLRAREA dimension.

As was previously discussed, an estimate of the market price function in linear form explained nearly as much of the variation in the market price of residences as did the double-log, "best", estimate of the market price function. In the linear form of the market price
function, the market implicit price of FLRAREA was estimated to be equal to about $13.00/ft^2$, and the bounds on the 99 percent confidence interval for this estimate were $8.30/ft^2$ and $17.70/ft^2$. The double-log form of the market price function resulted in estimates of the market implicit price of FLRAREA which ranged from $12.15/ft^2$ to $17.70/ft^2$—all within the 99 percent confidence interval for the market implicit price of FLRAREA as estimated using the linear function. However, reference to Table 4 reveals that the t-statistic for the regression coefficient on FLRAREA is higher in the double-log form of the market price function than it is in the linear model. Nevertheless, the estimates of the market implicit price of FLRAREA do have little variance.

The market implicit price function would be linear in the FLRAREA dimension if any one of the following conditions exist:

1. The developers differ in their attributes, but the buyers are identical and have a linear bid function;
2. The households differ in their attributes, but the developers are identical and have a linear offer function;
3. The developers and households are both nonhomogeneous in their attributes, but the points of tangency of their offer and bid functions happen to lie along a straight line.

The first condition implies that the market implicit price of FLRAREA (i.e., the households' marginal rate of substitution of money for
FLRAREA) does not, contrary to a basic assumption of the model developed in Chapter III, diminish with an increase in FLRAREA (income held constant). The second alternative implies that the marginal cost of provision of FLRAREA is constant, *ceteris paribus*. Neither of the first two conditions seems likely to exist, and the conditions under which the third alternative might exist must await an analytical solution for the conditions leading to market equilibrium for the model described in Chapter III.

If the market price function is not linear, then the nonsignificance of the variables descriptive of the household might be attributed to either the lack of variance in the observations on those variables or to the possibility that these variables are, in fact, not determinants of the households' implicit demand price for FLRAREA. A review of the standard deviation (see Table 1) and distribution of the observations on the household attribute variables reveals that the households are quite homogeneous in all of their (measured) attributes except EQUITY. In particular, about 63 percent of the households had gross incomes between $11,000 and $23,000, and about 37 percent had gross incomes between $17,000 and $23,000.\(^{174}\) The observations

\(^{174}\) Unlike the variance, the index of income concentration (or Gini coefficient) is a bounded measure of income equality; it ranges from zero to one, with zero indicative of a uniform distribution. For the households included in this study, the index of concentration for gross incomes was equal to 0.217. For comparison, the index of
on DISPOSIN have even less variance than do those on gross income, as the income taxes are progressive and thus the range of DISPOSIN ($5,851 to $29,800) is less than that of gross income ($6,500 to $48,500). Though the variables descriptive of the household attributes may be insignificant as determinants of demand because the households are highly homogeneous, it is difficult to discern whether they are insignificant because the households are nearly identical or because their attributes are, in fact, not determinants of demand. Whatever the reason, the effect is to make the households appear to be identical in their demand for FLRAREA.

If so, it immediately follows (from the discussion in Chapter IV) that the market price function and the bid functions are identical in their FLRAREA dimensions. Since the demand function is, in theory, the first-order derivative of the bid function with respect to FLRAREA, the integral of the demand function with respect to FLRAREA should be identical to the FLRAREA dimension of the estimated market price function. That is, using the notation of Chapter III:

---

(gross family) income concentration was equal to about 0.397 for Benton County, 0.367 for Oregon, and about 0.379 for the U.S. as a whole in 1969 (Fitch and Schefter, 1974, p. 10). Thus, as could be expected, there was much less variation in the gross income of the homebuyers included in this study than there was in the incomes of all families.
where:

\[ q_1 = \text{FLRAREA} \]
\[ \bar{q} = \text{a vector of the variables descriptive of all characteristics of the residence except FLRAREA, and} \]
\[ V = \text{a vector of the household attribute variables.} \]

The estimated FLRAREA demand function may be written as:

\[
\frac{\partial \hat{\beta}(q_1; \bar{q}, V)}{\partial q_1} dq_1 = -0.4282, \\
\]

where \( A \) is a constant obtained by setting each of the variables in \( \bar{q} \) and \( V \) equal to some value within their observed domain and evaluating the demand function at that point. Then:

\[
\hat{\beta}(q_1; \bar{q}, V) = \int \frac{\partial \hat{\beta}(q_1; \bar{q}, V)}{\partial q_1} dq_1 \\
= A \int q_1^{-0.4282} dq_1 = (A/0.5718)q_1^{0.5718} + C,
\]

where \( C \) is the constant of integration. Thus it is possible to compare the FLRAREA dimension of the bid function, as obtained by integration of the FLRAREA demand function, with the FLRAREA
dimension of the market price function only up to the level of some constant \( C \).

However, since the interest here is in a comparison of the slopes of the functions, the constant can be obtained by noting that, since the bid function and market price function are hypothesized to be equal in their FLRAREA dimension:

\[
\hat{\beta}(q_1; \bar{q}, V) = (A/0.5718)q^{0.5718} + C = \hat{P}(q_1; q).
\]

The value of \( C \) was obtained by setting each of the variables in the bid function equal to its geometric mean, setting \( \hat{P}(q_1; \bar{q}) \) equal to the geometric mean of the observed market prices, and then solving for \( C \).

With each of the variables except \( q_1 \) set equal to its geometric mean, the bid function which was obtained by integration of the estimated FLRAREA demand equation is given by:

\[
\hat{\beta}(q_1; \bar{q}, V) = 595.9649q^{0.57182} - 2549.5185.
\]

Evaluation of the market price function (108) at the same points gives:

\[
\hat{P}(q_1; \bar{q}) = 415.8003q^{0.61181}.
\]

Over the range of FLRAREA observed in this study, the bid price and market price estimates obtained from these functions are, for any
given level of FLRAREA, within one-half of one percent of each other.

Further, the 90 percent confidence interval for the coefficient on FLRAREA in the market price function ($0.61181$) ranges from $0.51756$ to $0.70606$—which includes the coefficient on FLRAREA in the bid function.

Note, also, that if the bid and market price functions are identical, the first derivative of the estimated market price function with respect to FLRAREA ($\frac{\partial \hat{P}(q_1; q)}{\partial q_1}$) should be equal to the estimated FLRAREA demand function ($\frac{\partial \hat{p}(q_1; q, V)}{\partial q_1}$) when evaluated at any given level of FLRAREA. These functions, with each of their variables except FLRAREA set equal to its geometric mean, are given by:

\begin{equation}
\frac{\partial \hat{P}(q_1; q)}{\partial q_1} = 0.61181 \frac{\hat{P}(q_1; q)}{q_1},
\end{equation}

which gives, by substitution of (110) into (111),

\begin{equation}
\frac{\partial \hat{P}(q_1; q)}{\partial q_1} = 254.3910q_1^{-0.38819}
\end{equation}

and

\begin{equation}
\frac{\partial \hat{\beta}(q_1; q, V)}{\partial q_1} = 340.7845q_1^{-0.42818}
\end{equation}

Both of these functions, evaluated over the range of FLRAREA encountered in this study, are depicted in Figure 15. The 90 percent
Figure 15. The estimated FLRAREA demand function and the first-order partial derivative of the market price function with respect to FLRAREA.
confidence interval for the coefficient on FLRAREA in the demand function (0.42818) ranges from 0.27425 to 0.58211—which includes the coefficient on FLRAREA in the derivative of the market price function.

Comparison of the estimated market price function with the integral of the estimated FLRAREA demand function as well as a comparison of the first-order derivative of the market price function with the estimated demand function indicates that the FLRAREA dimension of the market price function could be construed to be the FLRAREA dimension of the bid function (and vice versa). The household attribute variables provide no significant information in the estimation of the FLRAREA demand function, either because there are no substantial differences in the attributes of the households or because their attributes (even if they do differ) do not significantly influence the demand for FLRAREA. Therefore, if no relevant household attribute variables have been omitted from the model and if the market price function is not linear, that FLRAREA demand function which was estimated is apparently the FLRAREA demand function of the households included in this study.

If all of the households are identical in their demand for FLRAREA, it follows (from the theoretical model developed in Chapter III) that the households obtained different quantities of FLRAREA only because the developers produced residences of
different sizes (i.e., because the developers are not all identical).

Further, the estimated demand function is real income compensated and the households are indifferent between the residences with different floor areas if, as was assumed in the theoretical model, the market for new "spec" houses is perfectly competitive and in equilibrium. The assumptions underlying the theoretical model are satisfied only to varying degrees in reality. However, the results suggest that the differences in developers and/or the constraints placed on them at various locations may have played more of a role in determining the differences in the size of the dwellings that the households purchased than did the differences in the households.

**Summary of the Results**

An attempt was made to estimate the parameters of the econometric model which was developed in Chapter IV using data descriptive of 81 new, speculatively-built, single-family dwellings, of the households that purchased them, and of the developers that supplied them. The residences were all located in northeastern Benton County, Oregon and were traded between September 1, 1974 and August 31, 1975. As might be expected given such a narrowly defined sample, the households were quite homogeneous in their (measured) attributes, and thus the observations on the variables
descriptive of the attributes of the households had relatively little variance.

The first step in the estimation of the model consisted of the estimation of the market price function, which relates the characteristics of a residence and their market implicit prices to the market price of the residence. Of the various functional forms in which the function was estimated, a double-log form explained the most variation in the market prices of the residences ($R^2 = 0.814$). However, a linear form explained nearly as much variation in market price ($R^2 = 0.808$) as did the double-log form of the function. The same variables were statistically significant in both the linear and double-log forms of the function; they were: FLRAREA, NBATHS, SD509J, and STDEVPRN. In addition, a fourth variable denoting the month of the study period during which the residence was sold was statistically significant in both functions and indicated that the annual rate of inflation in the market price of the residences was equal to about ten percent (8.98 percent using the linear function and 11.16 percent using the double-log function). On the basis of its slightly better performance in explaining the variation in market price, the double-log form of the market price function was selected as the "best estimate" of the function.

However, the residuals were correlated with the market price of the residences in the double-log form of the market price function--
as they were in each of the functional forms in which the function was estimated. But, an examination of the distribution of the residuals in both characteristics space and geographic space revealed no apparent reformulation of the model that would remove the correlation between market price and the residuals.

Most notable among the variables which were not significant in explaining market price were lot size (LOTSIZE) and several measures of the accessibility of the residence to retail centers and to the households' places of employment (ACCESS). Though the evidence is not conclusive, the results indicated that lot size per se may not, in fact, be a significant determinant of market price. However, those characteristics which may generally be associated with neighborhoods wherein residential lots tend to be larger may have a positive market implicit price, and their effect may be reflected in the market implicit price of LOTSIZE if variables descriptive of these neighborhood characteristics are not included in the model. The ACCESS variables were thought to be insignificant because of either failure or inability to control for neighborhood amenity levels, which may tend to vary with, and thus diminish the net market implicit price of, accessibility. However, LOTSIZE and the ACCESS variables were retained in the "best estimate" of the market price function (as were all other insignificant variables) because the correlation between the
residuals and market price increased if they were dropped from the model.

After estimation of the market price function, the decision was made to calculate the market implicit price of and estimate only the FLRAREA demand and supply functions. These functions were not estimated for the other significant variables because they were thought not to be very interesting and/or meaningful.

The estimated market implicit price of FLRAREA had a mean of $14.94 and a standard deviation of only $1.21. The relatively small variation in the market implicit price of FLRAREA is a result of the near linearity of the market price function (at least in its FLRAREA dimension), which, in turn, may be due to the relatively small variation in the FLRAREA of the houses included in the study (the mean FLRAREA was equal to 1537 ft\(^2\) and the standard deviation was only 383 ft\(^2\)).

The estimated FLRAREA demand function contained only two significant variables: FLRAREA and the estimated magnitude of the federal income tax credit which the household received as a result of the purchase of the new house (TXCREDES). None of the variables descriptive of the attributes of the households were statistically significant in explaining the variation in the market implicit price of FLRAREA.
Other than FLRAREA and NBATHS, the only significant variables in the estimated FLRAREA supply function were (1) the scale of operation of the developer (BLDRSCAL) and (2) binary variables which denoted whether or not a residence was located within either Corvallis (CORVALLI) or Philomath (PHILOMTH). The results indicated that a location within Corvallis added about 7.5 percent to the marginal cost of FLRAREA, and a location within Philomath reduced the marginal cost of FLRAREA by about 6.5 percent relative to the remainder of the study area. However, it could not be ascertained that these cost differences were due solely to differences in local government building fees and regulations and not, in part, to differences in the housing market between the areas.

The results also indicated that, contrary to expectation, the marginal cost of FLRAREA declined with an increase in FLRAREA. Although there is strong evidence that the average cost of FLRAREA declines with an increase in FLRAREA, it could not be determined whether marginal cost was, in fact, declining or if the negative coefficient in the estimated supply function was due to the underidentification of that function.

The FLRAREA demand function satisfied the order condition for identification. However, the FLRAREA supply function was underidentified due to the nonsignificance of the household attribute variables.
The nonsignificance of the household attribute variables was attributed to either the lack of variation in the household attributes or to the fact that these variables may not be significant determinants of the household demand for FLRAREA. The effect is the same—the demand for FLRAREA appears to be invariant with the attributes of the household because either there are no significant differences in the attributes or any differences that do exist do not make a significant difference in the households' implicit demand price for FLRAREA.

As a further indication that the attributes of the households provided no information in the estimation of the demand for FLRAREA, the first derivative of the estimated market price function with respect to FLRAREA was shown to be virtually identical to the estimated FLRAREA demand equation. Similarly, the integral of the estimated demand function with respect to FLRAREA (i.e., the bid function) was shown to be almost identical to the FLRAREA dimension of the market price function. These results are to be expected, given the theoretical development in Chapter III, if the households are identical in their demand for FLRAREA.

The results of the estimation indicate that all of the households included in the study were identical in their demand for FLRAREA. Therefore, any differences in the size of the dwellings purchased by the households may be attributable more to the differences in the
developers who supplied the residences and/or the constraints placed on the developers and fees charged them by local governments than to differences in the households.
VI. SUMMARY, LIMITATIONS, AND RECOMMENDATIONS FOR FURTHER RESEARCH

In this chapter, the study is summarized, the limitations of both the theoretical and econometric model are discussed, and suggestions for further research are advanced. The results of the estimation of the statistical model are of interest in this chapter only to the extent that they assist in judging the validity of the theoretical model of locational choice, are indicative of improvements that could be made in the model, and are suggestive of problems for further research.

Summary

The Need for and Objectives of the Study

Neither the public good aspects of the residential settlement pattern nor the technological externalities resulting from individual land use decisions are necessarily considered in the private benefit-cost calculus of unregulated location decisions. These market failure problems may provide much of the impetus for land use controls to guide and/or direct individual location decisions so as to increase the net social benefits from residential land use. If public intervention in the residential land/housing market is to achieve the intended result and/or avoid unexpected results, knowledge of the technical and behavioral relationships underlying the location decisions


of the buyers and developers of housing is required.

The objective of this study was to describe the technical and behavioral relationships underlying the location decisions of the buyers and developers of new, single-family residences by:

1. identifying the variables entering their location decisions and describing the interrelationships of these variables through a system of demand and supply equations for location related goods, and

2. estimating the parameters of the system of demand and supply equations.

The technical and behavioral relationships are described in the terminology, and within the framework, of the economic theory of the household and of the firm.

Review of Literature

A review of the literature revealed that the economic theory of residential location developed by Wingo (1961), Alonso (1964), and Muth (1969) from the seminal work of von Thunen (1826/1969) is a macro-spatial model which focuses on accessibility to a single point (CBD). The model abstracts from the other aspects of the residential environment which may, on the margin, be no less important in the residential location decision than accessibility. Though there have been attempts to expand and/or reformulate the theory of residential
location to consider consumer preferences for several aspects of the residential environment, the resulting models are generally of such complexity that they do not readily yield testable hypotheses concerning individual locational behavior.

The empirical studies of urban residential property values have considered many aspects of the residential environment in addition to accessibility. In most of these studies, the value of the property was regressed on a set of variables descriptive of the characteristics of the property in an attempt to ascertain the determinants of residential property values and the implicit (or "hedonic") prices of the determinants. However, the specification and interpretation of these hedonic regression equations has often suffered from the lack of a theoretical basis for the technique.

The Theoretical Framework

This study provides an initial step toward a theoretical framework which can be used to explain residential land values and to explore, in a micro-economic context, the technical and behavioral relationships underlying locational choice in a multi-center, multi-faceted urban environment. The theoretical framework was developed around a general theory of hedonic prices provided by Rosen (1974).
In the theoretical model, a residence is defined by a set of \( n \) characteristics--including the accessibility of the residence to various points and the publicly provided services available at the site of the residence. The characteristics of the residence are assumed to be arguments in both the utility function of the household and the cost function of the supplier of the residence. Household bid functions, which are derived from the utility function and budget constraint, are defined to give the maximum price that the household could pay for residences embodying any given combination of the \( n \) characteristics and maintain a constant level of utility. Similarly, supplier offer functions, which are derived from the cost function, are defined to give the minimum price that the supplier could accept for residences embodying any given combination of the \( n \) characteristics and maintain a constant level of profits. Each household is assumed to have an array of bid functions wherein higher such functions correspond to lower levels of utility, and each developer is assumed to have an array of offer functions wherein the higher functions correspond to higher levels of profit. The \( n \) functions marginal to a bid function are shown to be inverse (real income) compensated demand functions for the characteristics, and the \( n \) functions marginal to an offer function are shown to be inverse (profit) compensated supply (identical to marginal cost) functions for the characteristics. The households are assumed to seek an income-constrained utility
maximum and the suppliers are assumed to maximize profits within the context of a perfectly competitive market.

In market equilibrium, the lowest possible bid function of every homebuyer is tangent to the highest possible offer function of a supplier such that no one can better his position, in terms of utility or profit, without detriment to the position of another. The points of tangency of the equilibrium offer and bid functions for all residences trace out the market price function which relates the market implicit prices of the characteristics and the characteristics of a residence to its market price.

Corresponding to the equilibrium offer and bid functions for each residence, there is a pair of demand and supply functions for each characteristic. That is, there are $2n$ equations (the compensated demand and supply functions) in $2n$ unknowns (the equilibrium levels of the characteristics and their market implicit prices), and the intersection of each pair of demand and supply functions determines the market implicit price and equilibrium level of each characteristic purchased by the household.

The residential location of a household is defined by its utility maximizing position in n-dimensional characteristics space as determined by its demand for and the supply of each of the characteristics comprising a residence. It is a consequence of the model that households with the same preferences and incomes will, subject to the
supply of the characteristics, locate at the same points in characteristics space; that is, they will choose the same residential packages. To the extent that these packages map from n-dimensional space to neighboring points in geographic space, like-minded individuals with similar incomes will tend to group together on the settlement plain.

In considering the effect of publicly provided services and property taxes on the household's location decision and on the market price of a residence, it was shown that the market implicit price of a publicly provided characteristic would be equal to zero if all households could locate at a site where their net (of property tax) marginal evaluation of the publicly provided service is equal to zero. That is, if the supply of a publicly provided service were such that no household would make an after-tax bid greater than zero for the benefits of the publicly provided service, there would be no net benefits from the publicly provided service to be capitalized into land values, and its market implicit price (given by the first-order partial derivative of the market price function with respect to the publicly provided characteristic) would be equal to zero.

It was also shown that, if the household's marginal evaluation of accessibility to any given point is nonnegative, the household's bid for accessibility to that point will increase with an increase in accessibility to the point, ceteris paribus (i.e., the bid function will
have a positive slope in that accessibility dimension). However, the
bid for accessibility was shown to increase at a diminishing rate, and
thus the accessibility demand function has a negative slope, if the
marginal cost of travel decreases at either a constant or a diminishing
rate with increasing accessibility.

The Econometric Model

The system of compensated demand and supply equations which
was developed in the theoretical model provided the basis for the
econometric model of locational choice. In general, all of the char-
acteristics of a residence are arguments in both the equation repre-
senting the demand for and that representing the supply of each
characteristic. In addition, the attributes of the household are argu-
ments in the demand functions and the attributes of the developers are
arguments in the supply functions. It is the differences in household
attributes and in developer attributes that, when related to the market
prices and characteristics of the residences, permits identification
of the demand and supply functions for the various characteristics.

A system of supply and demand equations is estimated in two
steps:

(1) The market price function is estimated by regressing the
market price of the residence on a set of variables descrip-
tive of the characteristics. Then the first-order partial
derivative of the estimated market price function with respect to each of the characteristics is obtained and evaluated for each residence using the observations on the variables descriptive of the characteristics of the residence. These calculated values provide the "observed" market implicit price for each characteristic of each residence given the quantity of each characteristic embodied in the residence.

(2) The simultaneous equation system of supply and demand functions is statistically estimated. The estimated market implicit prices serve as endogenous dependent variables and the variables descriptive of the characteristics of the residence are treated as endogenous explanatory variables. The variables descriptive of the attributes of the household and those descriptive of the attributes of the developer are treated as exogenous variables in the demand and supply functions, respectively.

An attempt was made to estimate the parameters of the econometric model using data descriptive of 81, new, speculatively-built, single-family, dwellings, of the households that purchased them, and of the developers that supplied them. The characteristics of the residences were represented by variables descriptive of the house *per se*, of the neighborhood in which the house was located.
and of the accessibility of the residence to retail centers and the household's place(s) of employment. The attributes of the household were, primarily, represented by variables which were intended to measure the income and wealth of the household. The attributes of the developers were represented by variables intended to be descriptive of the number, quality, and variety of residences produced by the developer and by binary variables indicating that the residence was constructed in one of two cities wherein the development regulations and fees differed from those elsewhere in the study area. All of the residences were located in Benton County, Oregon.

The Results of an Estimation of the Econometric Model

The market price function explained about 81 percent of the variation in the market price of the residences when estimated in either a double-log or a linear form—though the double-log form explained slightly more variation than did the linear form. None of the variables descriptive of the accessibility of the residence were statistically significant and only one variable descriptive of the neighborhood was significant. A regression of the market price of the residences on their floor area alone explained about 73 percent of the variation in market price.

There was an indication that the market price function was mis-specified as the residuals associated with the estimated function
were correlated with the market prices. However, an examination of the distribution of the residuals in both characteristics and geographic space revealed no apparent reformulation of the model that would remove the correlation between the residuals and the market prices.

Most notable among the variables which were not statistically significant in the market price function was lot size. Though the evidence is not conclusive, the results of the estimation indicated that lot size *per se* may not, in fact, be a significant determinant of market price. Rather, those characteristics which may generally be associated with neighborhoods wherein residential lots are larger may have a positive market implicit price, and their effect may be reflected in the market implicit price of lot size if variables descriptive of these neighborhood characteristics are not included in the model.

Because the variable representing the floor area of the residences was the only statistically significant variable for which supply and demand functions were thought to be of interest, such functions were estimated only for that characteristic. The only significant variables in the floor area demand function were floor area and the estimated magnitude of the federal income tax credit which the household received as a result of the purchase of its new house; both of these variables had the expected sign. None of the variables
descriptive of the attributes of the household were statistically significant.

Other than the floor area of and the number of bathrooms in the residence, the only significant variables in the estimated floor area supply function were the scale of operation of the developer and binary variables which denoted whether or not a residence was located within one of the two cities within the study area. The sign on the variable representing floor area indicated that, contrary to expectation, the marginal cost of floor area declined with an increase in floor area. However, as was discussed, there is evidence that the marginal cost of floor area may, in fact, be declining and that, therefore, the "wrong" sign was expected.

The unexpected sign on floor area in its supply function may also have been the result of the underidentification of the estimated supply equation. Though the floor area demand function was identified, the supply equation was underidentified because none of the household attribute variables were significant.

The nonsignificance of the household attribute variables may have been due to either the lack of variation in the household attributes or to the fact that these variables, which included income, may not be significant determinants of the household demand for floor area. In either case, the demand for floor area is apparently invariant with the attributes of the household. As a consequence, any differences in
the size of the dwellings purchased by the households may be less attributable to differences in the households than to differences in the developers who supplied the residences and/or the constraints placed on the developers and fee charged them by local governments.

Limitations of the Study

Limitations of the Theoretical Model

The theoretical model developed herein narrows the study of locational choice to the buyers of those dwellings which have a market determined price and the characteristics of which were selected by a profit-motivated supplier. Thus, the model does not consider the locational decision of the buyers of:

1. new custom-built houses as they have neither an established market price nor, necessarily, a set of characteristics which was chosen by a profit-motivated supplier; and
2. used dwelling units as the decision to place such units with a given set of characteristics on the market is not necessarily profit-motivated.

The inclusion of used dwelling units in the model would require that a theory of supply of such units be developed; such a theory would require consideration of obsolescence, physical depreciation, and of other temporal changes in the characteristics of the residence.
Given that the location decision of only the buyers of new, "spec" houses was to be considered, a number of aspects of their residential location decisions were ignored so as to further simplify the development of the model. The effective price which a household pays for a residence is not given by the original capital expenditure on (i.e., the market price of) the residence. The effective price will depend, in part, upon any capital gains to be achieved when and if the residence is resold, upon the mortgage interest rate, and upon the interrelationships that exist between the capital expenditure, the annual interest payment on the mortgage, the property tax on the residence, the gross income of the household, and the disposable (after income tax) income. The factors which influence the effective price which a household pays for a residence are complex, dynamic, subject to considerable uncertainty, and cannot all be incorporated in a static (or any other) model.

The factors influencing the household's decision as to whether it rents or purchases a residence were not considered in the theoretical model. It is expected that the "rent versus buy" aspect of the location decision is, in part, influenced by the effective price which a household must pay for a residence, which, in turn, would require consideration of those factors previously discussed.

The model of locational choice explicitly assumes that all of the characteristics of a residence are continuously and independently
variable so that any package of residential characteristics is feasible. Though this assumption is not satisfied in reality, it is necessary to distinguish between those cases where buyers do not have a continuous spectrum of choice because certain combinations of characteristics are not technically feasible and those cases where a continuous spectrum of choice does not exist because developers do not find it profitable to produce residences with certain characteristics.

Finally, the theoretical model does not provide a rationale for the household's decision as to place of employment and/or retirement. This is, obviously, an important aspect of the residential location decision which narrows the household's choice of options to those existing within some smaller area.

**Limitations of the Econometric Model**

All of the limitations of the theoretical model are reflected in the econometric model of locational choice. However, there are additional limitations of the econometric model which are due, primarily, to the specification of the model and/or the data used to estimate the model.

In the specification of the econometric model it was implicitly assumed that the household's utility function is separable in the characteristics of the household and "all other goods". This is not a realistic assumption as certain of the characteristics of a residence
may be substitutes or complements to other consumption activities of
the household. The econometric model would be improved by inclu-
sion of variables descriptive of certain household consumption
activities (e.g., annual expenditure on or investment in recreational
activities).

Estimation of the econometric model estimated herein was
hampered by the lack of data descriptive of the attributes of the
developers and by the very restricted sample of households. A larger
sample drawn from a more diverse population and more accurate
data on the attributes of the developers are required before conclu-
sions can be made concerning the validity of the model.

As in the theoretical model, the econometric model was not
specified to consider those factors which may influence the "rent
versus buy" aspect of the location decision. Yet, differences in the
general housing market between various areas may not only influence
the "rent versus buy" decision but may also result in differences in
the price of housing between different points in space. The strength
of the interrelationship between the "rent versus buy" decision and the
price of owner-occupied residences depends, in general, upon the
cross price elasticity of demand between rental and owner occupied
housing and the responsiveness of both types of housing to either
excess demand or excess supply in their respective markets. Over
the small area and short time period considered in this study, it is
doubtful that failure to consider the impact of variations in the rental market and/or owner-occupied housing market in the location decisions of the households had an impact on the estimates of the parameters of the econometric model.

Finally, a more general limitation of this study is that, though an attempt was made to determine certain of the technical and behavioral relationships underlying the location decisions of the buyers and sellers of housing, the model was estimated using observations on the results of location decisions that had been made under a system of land use controls. Thus, to the extent that conclusions concerning locational behavior could, potentially, be derived by the use of the model proposed herein, it should be recognized that, in most cases, the parameters of the model will be estimated using data from an area in which land use controls already exist.

Recommendations for Further Research

Most of the limitations of the study provide potential areas for further research and all need not be repeated here. In addition, a number of problems that may be worthy of further investigation were encountered during the course of the study.

As a more accurate test of the validity of the model, it should be estimated using a larger sample drawn from a more diverse population than that which was used herein. However, if the model is
to be estimated over a larger area, it may be necessary to specify the model to consider differences in the general housing market between points within the area.

A major problem encountered in this study was the lack of data descriptive of the attributes of the developers. The definition of the developer attribute variables and the observations thereon were restricted by and to those data which could be obtained from building permits. For the reasons discussed in Chapter IV, it is recommended that very little confidence be placed in the accuracy of estimates of construction costs which are derived from building permit data. Research is also required to identify those attributes of the developers that may be relevant in their location decisions.

If the model developed herein were to be of use in either predicting the impacts of land use controls or providing information for the development of land use controls, it would be helpful to identify and define variables which are descriptive of those characteristics of the urban environment which such controls are designed to influence. For example, variables descriptive of the intensity and nature of the land use within the neighborhood of a residence would be helpful. In addition, it is necessary to either develop a meaningful operational definition of "neighborhood" or, as was suggested in Chapter IV, develop a method of obtaining measures of the impact of the neighborhood characteristics on the residential location decision.
without explicitly defining the neighborhood.

One result of the statistical estimation of the econometric model indicated that lot size *per se* may not be so much of a consideration in the residential location decision as are those neighborhood characteristics which may generally be associated with neighborhoods in which the size of the residential lots is relatively large. Zoning may often be used to regulate lot size so as to influence neighborhood characteristics. However, if similar neighborhood characteristics could be achieved by some means other than minimum lot size zoning, such zoning may result in an inefficient use of land. Thus, further research to determine the relationship between lot size and the characteristics of the neighborhood and to determine the demand for lot size *per se* may be of interest.

There is strong evidence that the average cost of an important characteristic of the residence, the floor area, may decline with an increase in floor area. It is suspected that similar evidence with respect to lot area exists. If average cost is declining, marginal cost may also be declining. Yet a basic assumption of the theoretical model is that the marginal cost of providing each of those characteristics which is a "good" is positive and increasing. Some conceptual work is required to determine the consequences of declining marginal cost of provision of a characteristic to the integrity of the theoretical model.
Finally, a methodological note. In a system of supply and demand equations such as that developed herein, the implicit price \( \frac{\partial P}{\partial q_i} \) of each of the characteristics is, in general, a function of all of the other characteristics of the residence and, in theory, the \( \frac{\partial^2 P}{\partial q_i \partial q_j} = \frac{\partial^2 P}{\partial q_j \partial q_i} \). That is, in the demand and supply equations, the effect of the \( j^{th} \) characteristic on the implicit price of the \( i^{th} \) characteristic is, in theory, equal to the effect of the \( i^{th} \) characteristic on the implicit price of the \( j^{th} \) characteristic.

If it does not exist, research to develop a means of using this information concerning the equality of the cross partial derivatives in the estimation of the supply and demand equations may help improve the estimates of the parameters of such equations.

**Concluding Remarks**

The purpose of this study was to develop and test a theoretic-economic model of the individual residential location decision. The theoretical framework which was outlined is quite abstract, and considerable development would be required to provide a general theory of residential location. However, the theoretical model does provide a conceptual framework which may be helpful in understanding the technical and behavioral forces which influence the residential location decision. Also, the conceptual leap required to move from the theoretical model to the econometric model of locational choice
does not appear to be as great as that currently required to move from the theory of residential location and land values to the empirical studies of residential property values.

Though the results of an attempt to statistically estimate the parameters of an econometric model which was based on the theoretical model were not particularly encouraging, it would be premature to reject the theoretical model on the basis of the statistical results obtained herein. The model should be estimated using a larger sample drawn from a more diverse population than that which was used in this study.

It would be difficult to draw strong policy recommendations as to either the design or impact of land use controls from this study; it was intended to be of a conceptual/methodological nature. However, if the theoretical model of locational choice developed herein has any validity whatsoever, it provides an indication of the complex set of interrelated behavioral and technical forces underlying the individual location decision. The model also provides an indication of the vast amount of information that may be required to obtain a working knowledge of the nature of those behavioral and/or technical forces which may be influenced by land use controls.
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APPENDICES
APPENDIX A

The Questionnaire Used in the

Household Survey
Hello, I'm __________. I'm working on a survey of new home buyers for Oregon State University. We understand that you recently bought this house, and I'd like to ask you a few interesting questions if you don't mind. PROMISE I'M NOT SELLING A THING!

1 - 1 Yes (CONTINUE) First, may I ask if you own or are buying this house?
   2 No (Terminate)

2 - 1 Yes - built for us (Ask #3) Did you have this house built for you? That is, did you buy it before construction was started?
   2 No (Skip to #4)

3 - ________________ (Month and Year) When did you buy this particular lot? (Just approximately when?)
   (INT: If you asked #3, skip now to #5)

4 - 1 After started (Ask #4a) At which stage of construction did you agree to buy this house -- after construction was started but not completed, or after construction was completed?
   2 After completion (Skip to #5)

4a - 1 Yes (Ask 4b) After construction was started, did you ask the builder to change the design or features of the house in any way?
   2 No (Skip to #5)

4b - 1 Yes (Ask 4c) Did the alterations or changes you made affect the purchase price of the house?
   2 No (Skip to #5)
   3 D.K. (Skip to #5)

4c - 1 Yes, raised price $______ Did the changes you made raise or lower the price of the house? About how much did they (raise) (lower) the price?
   2 Yes, lowered price $______
   3 D.K. (ASK OF EVERYONE)

5 - 1 Own (or buying) (Skip to #6) Before you moved here, did you own or rent the place where you were living?
   2 Rent (Ask 5a)

5a - 1 Temporary (Ask 5b) Was this rental unit just a temporary one while you were waiting for this house, or was it your permanent residence for awhile?
   2 Permanent (Skip to #7)

5b - 1 Owned (Ask #6) Did you own or rent the place where you lived just before this temporary one?
   2 Rented (Skip to #7)

6 - $______________ What was the purchase price of the house you bought before this one?
   2 D.K.

6a - 1 Sold (Ask 6b) Have you sold your previous house, or are you attempting to sell it, or are you renting it out or using it for some other purpose?
   2 Attempting sell (Ask 6b)
   3 Rent or other (Skip to 6c)

6b - $______________ About how much did you sell this previous house for? (OR) What is your best estimate of the amount you'll receive for the house when it's sold? (Just approximately)
   2 D.K.

6c - ________________ (Years) About how many years did you live in your previous house?
(ASK OF EVERYONE)

7 - ____________________________ Years/Mos.  
2 D.K.  About how long have you lived in the Albany-Corvallis-Philomath area?

8 - ____________________________ No.  
0 None  How many children 18 years or younger live with you in this household, either all or part of the year?

9 - ____________________________ Years  
2 D.K.  About how many years do you expect to live in this house? (INT: Record years or other answer such as "indefinitely", "as long as I live", etc.

10 - 1 Yes - married  
2 No  May I ask if you are now married?

11 -  
(1. Type and Industry)  
What type of work does the chief breadwinner in the family do?

11a-  
(2. Type and Industry)  
Is the (male head) (female head) of the household also employed?  
(If YES) What type of work does (he) (she) do?

12 - Now, to get some idea of the importance of distance between home and work, may I ask the address where the chief breadwinner in the family works? (INT: if unemployed get former work address)

<table>
<thead>
<tr>
<th>(Full Address or Description)</th>
<th>(City or Town)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12a- 1 Yes (Skip to #13)</td>
<td>Was the chief breadwinner employed at this address or expected to be employed at this address at the time you bought this house?</td>
</tr>
<tr>
<td>2 No (Ask 12b)</td>
<td></td>
</tr>
</tbody>
</table>

12b- At what address was the chief breadwinner employed or expected to be employed at the time you bought this house?

<table>
<thead>
<tr>
<th>(Full Address or Description)</th>
<th>(City or Town)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(INTERVIEWER: Refer to question 11a. If the (male head) (female head) is employed, ask question #13. If not employed, skip immediately to #14)</td>
<td></td>
</tr>
</tbody>
</table>

13 - What is the address of the place where the (male head) (female head) of the household works?

<table>
<thead>
<tr>
<th>(Full Address or Description)</th>
<th>(City or Town)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13a- 1 Yes (Skip to #14)</td>
<td>Was (he) (she) employed at this address or expected to be employed there at the time you bought this house?</td>
</tr>
<tr>
<td>2 No (Ask 13b)</td>
<td></td>
</tr>
</tbody>
</table>

13b- At what address was (he) (she) employed or expected to be employed at the time you bought this house?

| (Full Address or Description) | (City or Town) |
We're just about through now, but I'd like to ask you a couple of questions about yourself and your family, to allow the University to tabulate the results by different ages and types of families.

14 - ____________  First, would you mind telling me your approximate age?

15 - ____________  What about your (husband or man of the house) (wife or lady of the house -- what is (her) (his) approximate age?

16 - 1 (a) Less than $5,000  Last of all, here is a card containing some broad income brackets. Will you please look at it and tell me which one comes closest to your total family income from all sources for the (last) year, before taxes? Just call your answer by letter, please.
2 (b) $5,000 - $7,999
3 (c) $8,000 - $10,999
4 (d) $11,000 - $13,999
5 (e) $14,000 - $16,999
6 (f) $17,000 - $19,999
7 (g) $20,000 - $22,999
8 (h) $23,000 - $25,999
9 (i) $26,000 - $28,999
0 (j) $29,000 - $31,999
1 (k) $32,000 - $39,999
2 (l) $40,000 or over

17 - 1 Male  Time interview taken: ____________ AM ____________ PM
2 Female

I hereby certify this interview was actually taken at the following address, and represents a true and accurate account of the contact.

_________________________ (Address)  ______________________ (Date)  ___________________ (Interviewer's Signature)
_________________________ (Telephone Number)  ______________________ (Account Number)

FOR OFFICE USE ONLY

Interview verified by ______________________: ___ in person or ___ by phone.
APPENDIX B

Supplementary Statistical Results
Table B1. Correlation and inverse correlation matrices for explanatory variables in market price function (double-log model).*

<table>
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<th>NBAV1</th>
<th>FIREPL</th>
<th>SD509J</th>
<th>ELEVATN</th>
<th>STDEVPRN</th>
<th>AVEAGEN</th>
<th>NMULTFN</th>
<th>ACCESSR2</th>
<th>ACCESSM</th>
<th>MOSOLD</th>
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*The correlation matrix is presented above the diagonal and its inverse is presented on and below the diagonal. The elements on the diagonal are the variance inflation factors.
Table B2. Correlation and inverse correlation matrices for explanatory variables in market price function (linear model).*

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<th>SDS09J</th>
<th>ELEVATN</th>
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<th>AVEAGEN</th>
<th>NMULTFN</th>
<th>ACCESSR2</th>
<th>ACCESSM</th>
<th>MOSOLD</th>
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*The correlation matrix is presented above the diagonal and its inverse on and below the diagonal. The elements on the diagonal are the variance inflation factors.
Table B3. Correlation matrix for explanatory variables in the reduced, semi-log form of the market price function.

<table>
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<th>FLRAREA</th>
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<th>AVEAGEN</th>
<th>ACCESSM</th>
<th>MOSOLD</th>
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<td>FIREPL</td>
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* The correlation matrix is presented above the diagonal and its inverse is presented on and below the diagonal. The elements on the diagonal are the variance inflation factors.

The observations on this variable are those estimated in the first stage of the two stage least squares estimation procedure. This variable was considered to be endogenous to the system of supply and demand equations.
Table B5. Correlation and inverse correlation matrices for explanatory variables in floor area supply function.*

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<td>CORVALLI</td>
<td>-.083</td>
<td>1.223</td>
<td>0.355</td>
<td>-0.024</td>
<td>2.659</td>
<td>-.309</td>
<td>.367</td>
<td>.615</td>
<td>.079</td>
<td>-.280</td>
<td>.035</td>
</tr>
<tr>
<td>PHILOMTH</td>
<td>1.295</td>
<td>-0.128</td>
<td>0.448</td>
<td>-0.656</td>
<td>-.013</td>
<td>2.283</td>
<td>-.192</td>
<td>-.301</td>
<td>-.108</td>
<td>-.319</td>
<td>.048</td>
</tr>
<tr>
<td>AVCSTFT2</td>
<td>3.004</td>
<td>-0.734</td>
<td>0.248</td>
<td>-.324</td>
<td>-.324</td>
<td>0.747</td>
<td>2.304</td>
<td>.283</td>
<td>.271</td>
<td>-.094</td>
<td>.047</td>
</tr>
<tr>
<td>BLDRSCAL</td>
<td>2.435</td>
<td>-2.536</td>
<td>-.563</td>
<td>-.618</td>
<td>-.266</td>
<td>1.834</td>
<td>0.732</td>
<td>7.193</td>
<td>.547</td>
<td>-.448</td>
<td>.071</td>
</tr>
<tr>
<td>SDCONCST</td>
<td>-.048</td>
<td>-.0465</td>
<td>0.380</td>
<td>0.670</td>
<td>1.220</td>
<td>-.0972</td>
<td>-.0625</td>
<td>-3.561</td>
<td>3.239</td>
<td>.074</td>
<td>.134</td>
</tr>
<tr>
<td>AVCONCST</td>
<td>-2.550</td>
<td>-1.612</td>
<td>-0.529</td>
<td>0.595</td>
<td>-1.404</td>
<td>0.778</td>
<td>-1.061</td>
<td>3.551</td>
<td>-1.677</td>
<td>5.658</td>
<td>-.102</td>
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<tr>
<td>CUSTFET</td>
<td>.0527</td>
<td>.559</td>
<td>-.001</td>
<td>0.084</td>
<td>-.162</td>
<td>.061</td>
<td>0.010</td>
<td>.399</td>
<td>-.026</td>
<td>.273</td>
<td>1.079</td>
</tr>
</tbody>
</table>

*The correlation matrix is presented above the diagonal and its inverse is presented on and below the diagonal. The elements on the diagonal are the variance inflation factors.

The observations on this variable are those estimated in the first stage of the two stage least squares estimation procedure. This variable was considered to be endogenous to the system of supply and demand functions.
APPENDIX C

Computer Programs
### Table C1: Computer Program Used to Calculate the Grid Coordinates of the Residences

**Program Grid**

Given the Township (T), Range (R), Section (S), Quarter-Section (IQ), and Sixteenth-Section (IV) designation of a residential lot (I), this program assigns grid coordinates \((x(I), y(I))\) to the lot. Each lot is assumed to be in the center of the one-sixteenth section (40 acre) plot within which it is located. The origin of the grid is assumed to be to the NE direction at the NE corner of Township 10S, Range 3W.

**Dim**: \(T(81), R(81), S(81), IQ(81), IV(81), X(81), Y(81)\)

**Format**: (45X,3F2.0,2A1,27X)

**Variables**: \(T(I), R(I), S(I), IQ(I), IV(I), X(I), Y(I)\)

**Subroutine**: READ (40,100) \(T(I), R(I), S(I), IQ(I), IV(I)\)

**Variables**: \(F=1.0/24.0\), \(G=4.0/24.0\), \(H=1.0/48.0\)

**DO 600 1=1,81**

**Variables**: \(IS=0\), \(S1=0.0\), \(S2=0.0\), \(Q1=0.0\), \(Q2=0.0\), \(V1=0.0\), \(V2=0.0\), \(W1=0.0\), \(W2=0.0\)

**Subroutine**

- \(Q1=1.0\) if \(IQ(I) \neq 1\) or \(IQ(I) \neq 1\)
- \(Q1=2.0\) if \(IQ(I) = 1\)

- \(V1=1.0\) if \(IV(I) \neq 1\)
- \(V1=2.0\) if \(IV(I) = 1\)

**Subroutine**

- \(W1=Q1*V1*F\) if \(Q1 \neq 1\)
- \(W1=(Q1-V1)*F\) if \(Q1 = 1\)

**Subroutine**

- \(W1=(Q1+V1)*F\)

- \(W1=Q1*V1*F\) if \(Q1 \neq 1\)
- \(W1=(Q1-V1)*F\) if \(Q1 = 1\)

**Subroutine**: READ (40,100) \(T(I), R(I), S(I), IQ(I), IV(I), X(I), Y(I)\)

**Variables**: \(F=1.0/24.0\), \(G=4.0/24.0\), \(H=1.0/48.0\)

**DO 600 1=1,81**

**Variables**: \(IS=0\), \(S1=0.0\), \(S2=0.0\), \(Q1=0.0\), \(Q2=0.0\), \(V1=0.0\), \(V2=0.0\), \(W1=0.0\), \(W2=0.0\)

**Subroutine**

- \(Q2=1.0\) if \(IQ(I) \neq 1\) or \(IQ(I) \neq 1\)
- \(Q2=2.0\) if \(IQ(I) = 1\)

- \(V2=1.0\) if \(IV(I) \neq 1\)
- \(V2=2.0\) if \(IV(I) = 1\)

**Subroutine**

- \(W2=Q2*V2*F\) if \(Q2 \neq 1\)
- \(W2=(Q2-V2)*F\) if \(Q2 = 1\)

**Subroutine**: READ (40,100) \(T(I), R(I), S(I), IQ(I), IV(I), X(I), Y(I)\)

**Variables**: \(F=1.0/24.0\), \(G=4.0/24.0\), \(H=1.0/48.0\)

**DO 600 1=1,81**

**Variables**: \(IS=0\), \(S1=0.0\), \(S2=0.0\), \(Q1=0.0\), \(Q2=0.0\), \(V1=0.0\), \(V2=0.0\), \(W1=0.0\), \(W2=0.0\)

**Subroutine**

- \(Q3=1.0\) if \(IQ(I) \neq 1\) or \(IQ(I) \neq 1\)
- \(Q3=2.0\) if \(IQ(I) = 1\)

- \(V3=1.0\) if \(IV(I) \neq 1\)
- \(V3=2.0\) if \(IV(I) = 1\)

**Subroutine**

- \(W3=Q3*V3*F\) if \(Q3 \neq 1\)
- \(W3=(Q3-V3)*F\) if \(Q3 = 1\)
TABLE C1. CONTINUED

380 \[ W_2 = (Q_2 + V_2) \times F \]
C THE FOLLOWING ASSIGNS THE NUMBERS ONE THROUGH SIX PROCEEDING FROM EAST TO WEST \((S_1(I))\) AND, ALSO, FROM NORTH TO SOUTH \((S_2(I))\) TO ALL 36 SECTIONS WITHIN EACH TOWNSHIP.

400 I = \left( \left( S_1(I) - 1.0 \right) / 6.0 \right) + 1.0
GO TO 410 (410, 420, 430, 440, 450, 460), IS

410 \[ S_1 = S(I) \]
\[ S_2 = 1.0 \]
GO TO 500

420 \[ S_1 = 13.0 - S(I) \]
\[ S_2 = 2.0 \]
GO TO 500

430 \[ S_1 = S(I) - 12.0 \]
\[ S_2 = 3.0 \]
GO TO 500

440 \[ S_1 = 25.0 - S(I) \]
\[ S_2 = 4.0 \]
GO TO 500

450 \[ S_1 = S(I) - 24.0 \]
\[ S_2 = 5.0 \]
GO TO 500

460 \[ S_1 = 37.0 - S(I) \]
\[ S_2 = 6.0 \]
GO TO 500

C THE FOLLOWING CALCULATES \(X(I)\) AND \(Y(I)\), TO DETERMINE THE NE CORNER OF TOWNSHIP 10S, RANGE 3W, AS THE ORIGIN, TEN IS SUBTRACTED FROM EACH \(T(I)\) AND THREE IS SUBTRACTED FROM EACH \(R(I)\), TO MEASURE DISTANCE FROM THE CENTER OF EACH ACRE PLOT, 1/48 IS SUBTRACTED FROM EACH \(X(I)\) AND \(Y(I)\). TO CORRECT FOR THE "CORRECTIONAL JOGS" IN THE RANGE LINES AT THE TOWNSHIP LINE SEPARATING TOWNSHIPS WITH A 10S AND AN 11S DESIGNATION, 1/48 IS SUBTRACTED FROM EACH \(X(I)\) CALCULATED FOR LOCATIONS WITH \(T(I)\) GREATER THAN TEN. EACH \(X(I)\) AND \(Y(I)\) IS THEN MULTIPLIED BY SIX TO CONVERT THE \(X(I)\) AND \(Y(I)\) TO MILES WEST AND SOUTH, RESPECTIVELY, OF THE ORIGIN OF THE GRID.

500 IF \(T(I) \leq 10.0) \) 510, 520

510 \[ X(I) = 6.0 \times (R(I) - 3.0 + (S_1 - 1.0) \times G + W_1 - H) \]
GO TO 530

520 \[ X(I) = 6.0 \times (R(I) - 3.0 + (S_1 - 1.0) \times G + W_1 - F) \]
GO TO 530

530 \[ Y(I) = 6.0 \times (T(I) - 10.0 + (S_2 - 1.0) \times G + W_2 - H) \]
GO TO 530

600 CONTINUE

700 FORMAT(2(2X,F11.6))
DO 800 T = 1, 81

800 WRITE(45, 700) X(I), Y(I)
END
<table>
<thead>
<tr>
<th>R4W</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>R3W</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 sq. mile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>640 acres</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>29</td>
<td>28</td>
<td>27</td>
<td>26</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td></td>
</tr>
</tbody>
</table>

**Property tax account number begins with 100317AC**

**Figure C1.** An illustration of the rectangular survey system and the method of assigning property tax account numbers.
### TABLE C2. COMPUTER PROGRAM USED TO CALCULATE NEIGHBORHOOD DATA FOR RESIDENCES

| Program NA8RHO00 | Given the grid coordinates \((X_B,Y_B)\), year in which constructed \((A GE)\), sale price \((P R_3)\), and building classification \((I B L D C L S)\) of all residential property sold within Benton County during the period from August 31, 1974 to September 30, 1975 and the grid coordinates \((X_H,Y_H)\) and sale prices \((P R_H)\) of the houses included in the sample, this program calculates several measures of the characteristics of the neighborhood of the houses included in the sample. It calculates the standard deviation of the prices of all houses sold within the neighborhood \((S T O E V P R_B)\), the percentage of the houses which sold for a greater price than did the house in question \((P E R G T P R_H)\), the algebraic difference between the average price of the residences sold within the neighborhood and the sale price of the house in question \((D I F A V P R_B)\), the number of single-family \((S Z)\) and multiple-family \((N M F)\) residences sold within the neighborhood, the average age of the single-family residences sold \((A V E A G E_B)\), and the ratio of the average price of the single-family residences sold within the neighborhood to the average price of all single-family residences sold within Benton County \((Q U A L R A T_B)\). The measurements of the characteristics of the residences in question are not included in the calculation of its neighborhood characteristics. The neighborhood is spatially defined to be the 360 acres centered about the residence in question. |

**Program**

```fortran
PROGRAM NA8RHO00
C GIVEN THE GRID COORDINATES {XB,YB), YEAR IN
C WHICH CONSTRUCTED (AGE), SALE PRICE (PR3),
C AND BUILDING CLASSIFICATION (IBLDCLS) OF
C ALL RESIDENTIAL PROPERTY SOLD WITHIN BENTON
C COUNTY DURING THE PERIOD FROM AUGUST 31, 1974
C TO SEPTEMBER 31, 1975 AND THE GRID COORDINATES
C (XH,YH) AND SALE PRICES (PRH) OF THE HOUSES
C INCLUDED IN THE SAMPLE, THIS PROGRAM
C CALCULATES SEVERAL MEASURES OF THE CHARACTER-
C ISTICS OF THE NEIGHBORHOOD OF THE HOUSES IN-
C CLuded IN THE SAMPLE. IT CALCULATES THE
C STANDARD DEVIATION OF THE PRICES OF ALL
C HOUSES SOLD WITHIN THE NEIGHBORHOOD (STDEVPRB),
C THE PERCENTAGE OF THE HOUSES WHICH SOLD FOR A
C GREATER PRICE THAN DID THE HOUSE IN QUESTION
C (PERGTPRH), THE ALGEBRAIC DIFFERENCE BETWEEN
C THE AVERAGE PRICE OF THE RESIDENCES SOLD WITHIN
C THE NEIGHBORHOOD AND THE SALE PRICE OF THE HOUSE
C IN QUESTION (DIFAVPRB), THE NUMBER OF SINGLE-
C FAMILY (SZ) AND MULTIPLE-FAMILY (NMF) RESIDENCES
C SOLD WITHIN THE NEIGHBORHOOD, THE AVERAGE AGE
C OF THE SINGLE-FAMILY RESIDENCES SOLD
C (AVEAGEB), AND THE RATIO OF THE AVERAGE PRICE OF
C THE SINGLE-FAMILY RESIDENCES SOLD WITHIN THE
C NEIGHBORHOOD TO THE AVERAGE PRICE OF ALL SINGLE-
C FAMILY RESIDENCES SOLD WITHIN BENTON COUNTY
C (QUALRATB). THE MEASUREMENTS OF THE CHARACTER-
C ISTICS OF THE RESIDENCES IN QUESTION ARE NOT
C INCLUDED IN THE CALCULATION OF ITS NEIGHBORHOOD
C CHARACTERISTICS. THE NEIGHBORHOOD IS SPATIALLY
C DEFINED TO BE THE 360 ACRES CENTERED ABOUT THE
C RESIDENCE IN QUESTION.

DIMENSION X8(999),Y8(999),XH(81),YH(81),
IBLDCLS(999),
AVEAGEB(81),PR8(999),PRH(81),STDEVPRB(81),
PERGTPRH(81),
DIFAVPRB(81),AGE(999),AAEG(999),SZ(81),NMF(81),
QUALRATB(81)

10 FORMAT (2(2X,F11.6))
DO 20 I=1,81
20 READ (20,10) XH(I),YH(I)
30 FORMAT (F5.0,75X)
DO 40 I=1,81
40 READ (21,30) PRH(I)
DO 50 N=1,999
50 READ (22,10) XB(N),YB(N)
60 FORMAT (40X,I1,2X,F2.0,27X,=5.0,3X)
DO 70 N=1,999
70 READ(23,60) IBLDCLS(N),AGE(N),PR8(N)
SPR=0.0
ST=0.0
AVEPR=0.0
DO 75 N=1,999
IF (IBLDCLS(N).NE. 1) GO TO 75
SPR=SPR+(PR8(N))*10.0
ST=ST+1.0
75 CONTINUE
AVEPR=SPR/ST
DO 200 I=1,81
```

---
TABLE C2. CONTINUED

| S=0.0   |
| SPRB=0.0 |
| SSQPRB=0.0 |
| SAGE=0.0 |
| T=0.0   |

DO 100 N=1,999

IF (ABS(XB(N)-XH(I)) .GT. 0.30 .OR. ABS(YB(N)-YH(I)) .GT. 0.30)
1 GO TO 100

IF (IBDCLS(N) .NE. 1) GO TO 95

S=S*1.0

SPR3=SPRB*PRB(N)*10.0

SSQPR3=SSQPRB+(PR8(N)*10.0)**2

IF (AGE(N) .GT. 75.0) GO TO 30

AAGE(N)=75.0-AGE(N)

GO TO 90

80 AAGE(N)=175.0-AGE(N)

90 SAGE=SAGE-AAGE(N)

IF (PRB(N)*10.0 .LE. PRH(I)) GO TO 100

T=T+1.0

GO TO 100

100 CONTINUE

SPR9=SPRB-PRH(I)

SSQPR8=SSQPRB-PRH(I)**2

S=S-1.0

SZ(I)=S

IF (SZ(I).EQ. 0.0) GO TO 180

PERGTPRH(I)=T/S

AVEAGEB(I)=(SAGE)/S

DIFAVPRB(I)=SPR9-PRH(I)

QUALRATB(I)=(SPRB/S)/AVEPR

IF (SZ(I).EQ. 1.0) GO TO 190

STDEVPRB(I)=SQRT((SSQPR8+(SPRB**2)*S)/(S-1.0))

GO TO 200

180 PERGTPRH(I)=0.0

AVEAGEB(I)=0.0

DIFAVPRB(I)=0.0

QUALRATB(I)=0.0

185 FORMAT (F7.2,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0)

NO NAYDAT FOR I=*,I2

WRITE (61,185) I

190 STDEVPRB(I)=0.0

200 CONTINUE

300 FORMAT (F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0,1X,F7.0)

DO 400 I=1,81

400 WRITE (24,300) STDEVPRB(I),PERGTPRH(I),AVEAGEB(I),DIFAVPRB(I),SZ(I),NMF(I),QUALRATB(I)

ENDFILE 24

END
TABLE C3. COMPUTER PROGRAM USED TO ESTIMATE THE OREGON
AND FEDERAL INCOME TAXES AND THE FAMILY
DISPOSABLE INCOME

<table>
<thead>
<tr>
<th>PROGRAM OSPINC</th>
<th>THIS PROGRAM ESTIMATES DISPOSABLE INCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(OSPINC) BY ESTIMATING AND DEDUCTING 1974 FED-</td>
</tr>
<tr>
<td></td>
<td>ERAL AND OREGON INCOME TAXES (INCTXF AND INCTXO)</td>
</tr>
<tr>
<td></td>
<td>FROM TOTAL FAMILY INCOME (INC). IT IS ASSUMED</td>
</tr>
<tr>
<td></td>
<td>THAT THERE ARE NO ADJUSTMENTS TO INCOME SO THAT</td>
</tr>
<tr>
<td></td>
<td>TAXES AND &quot;ADJUSTED GROSS INCOME&quot; ARE EQUAL, THAT</td>
</tr>
<tr>
<td></td>
<td>THE STANDARD DEDUCTIONS (STDEDF AND STDEDO) ARE</td>
</tr>
<tr>
<td></td>
<td>USED IN COMPUTING THE TAXES, THAT MARRIED</td>
</tr>
<tr>
<td></td>
<td>COUPLES (MARIST .EQ. 1) FILE JOINT RETURNS, AND</td>
</tr>
<tr>
<td></td>
<td>THAT ALL SINGLE PERSONS (MARIST .EQ. 0) WITH</td>
</tr>
<tr>
<td></td>
<td>CHILDREN QUALIFY FOR THE &quot;HEAD OF HOUSEHOLD&quot; TAX</td>
</tr>
<tr>
<td></td>
<td>RATE. THE NUMBER OF EXEMPTIONS (EXEMP) IS BASED</td>
</tr>
<tr>
<td></td>
<td>ON THE NUMBER OF CHILDREN (NOCHLD) 18 YEARS OF</td>
</tr>
<tr>
<td></td>
<td>AGE OR YOUNGER THAT RESIDE WITH THE FAMILY</td>
</tr>
<tr>
<td></td>
<td>AT LEAST PART OF THE YEAR AND ON THE NUMBER OF</td>
</tr>
<tr>
<td></td>
<td>ADULT HEADS OF HOUSEHOLD AND THEIR AGES--IF</td>
</tr>
<tr>
<td></td>
<td>EITHER ONE OR BOTH ARE 65 YEARS OF AGE OR</td>
</tr>
<tr>
<td></td>
<td>OLDER. THE EQUATIONS USED TO ESTIMATE THE TAXES</td>
</tr>
<tr>
<td></td>
<td>WERE DERIVED BY REGRESSING FEDERAL TAX RATES, AS</td>
</tr>
<tr>
<td></td>
<td>GIVEN IN FEDERAL &quot;TAX RATE SCHEDULES X, Y, AND</td>
</tr>
<tr>
<td></td>
<td>Z&quot;, ON THE CORRESPONDING FEDERAL TAXABLE INCOME</td>
</tr>
<tr>
<td></td>
<td>(TXINCF) AND BY REGRESSING THE OREGON</td>
</tr>
<tr>
<td></td>
<td>INCOME TAX RATES, AS GIVEN BY &quot;GRADUATED RATE</td>
</tr>
<tr>
<td></td>
<td>CHARTS A AND B&quot; ON OREGON INDIVIDUAL INCOME</td>
</tr>
<tr>
<td></td>
<td>TAX RETURN FORM 40, ON THE CORRESPONDING OREGON</td>
</tr>
<tr>
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<td>TAXABLE INCOME (TXINCO) LEVELS. FOLLOWING THE</td>
</tr>
<tr>
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<td>1974 OREGON AND FEDERAL INCOME TAX REGULATIONS,</td>
</tr>
<tr>
<td></td>
<td>AND GIVEN THE PRECEDING ASSUMPTIONS, THE</td>
</tr>
<tr>
<td></td>
<td>EQUATIONS ESTIMATE THE TAX RATES, AND THUS THE</td>
</tr>
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<td>INCOME TAXES, GIVEN A TAXABLE INCOME BETWEEN</td>
</tr>
<tr>
<td></td>
<td>$1000 AND $44000, WITH THE MAXIMUM PERCENTAGE</td>
</tr>
<tr>
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<td>ERROR INDICATED BELOW THE EQUATION IN THE</td>
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<td>PROGRAM. THE COEFFICIENT OF DETERMINATION</td>
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<td>(R**2) OF THE EQUATIONS USED TO ESTIMATE THE</td>
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<td>CORRESPONDING TAX RATES ARE ALSO GIVEN BELOW</td>
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<td>THE EQUATION.</td>
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</tr>
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<td>IIAGEFEM(81),</td>
</tr>
<tr>
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<td>IINC(81),OSPINC(81)</td>
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<td></td>
<td>100 FORMAT (2F11.1X),2(I2,1X),F5.0,65X)</td>
</tr>
<tr>
<td></td>
<td>DO 200 I=1,81</td>
</tr>
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<td></td>
<td>200 READ(30,100) MARIST(I),NOCHLD(I),IAGEMAL(I),</td>
</tr>
<tr>
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<td>IIAGEFEM(I),</td>
</tr>
<tr>
<td></td>
<td>IINC(I)</td>
</tr>
<tr>
<td></td>
<td>DO 300 I=1,81</td>
</tr>
<tr>
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<td>EXEMP=0.0</td>
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<tr>
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<td>STDEDF=0.0</td>
</tr>
<tr>
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<td>STDEDO=0.0</td>
</tr>
<tr>
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<td>TXINCF=0.0</td>
</tr>
<tr>
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<td>DEDUCF=0.0</td>
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<tr>
<td></td>
<td>TXINCO=0.0</td>
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<tr>
<td></td>
<td>INCTXO=0.0</td>
</tr>
<tr>
<td></td>
<td>EXEMP=0.15*MARIST(I)+IAGEMAL(I)/65+IIAGEFEM(I)/65+</td>
</tr>
<tr>
<td></td>
<td>NOCHLD(I)</td>
</tr>
<tr>
<td></td>
<td>IF(0.15*IINC(I) .LT. 2000.0) GO TO 205</td>
</tr>
<tr>
<td></td>
<td>STDEDF=2000.0</td>
</tr>
<tr>
<td></td>
<td>GO TO 210</td>
</tr>
<tr>
<td></td>
<td>205 STDEDF=0.15*IINC(I)</td>
</tr>
<tr>
<td></td>
<td>210 IF(0.15*IINC(I) .GE. 1500.0) GO TO 215</td>
</tr>
<tr>
<td></td>
<td>IF(0.15*IINC(I) .LE. 1050.0) GO TO 220</td>
</tr>
<tr>
<td></td>
<td>STDEDO=0.13*IINC(I)</td>
</tr>
<tr>
<td></td>
<td>GO TO 225</td>
</tr>
</tbody>
</table>
TABLE C3. CONTINUED

215 STOEDO=1500.0
   GO TO 225
220 STOEDO=1050.0
225 TXINCF=INC(I)-STOEDF-EXEMP*750.0
   IF (TXINCF .LE. 0.0) GO TO 270
   IF (MARISt(I) .EQ. 1) GO TO 240
   IF (NOCHLD(I) .EQ. 0) 230, 235
   C IF MARRIED, 240 COMPUTES INCTXF, IF SINGLE AND
   C ONE OR MORE CHILDREN, 235 COMPUTES INCTXF. IF
   C SINGLE AND NO CHILDREN, 230 COMPUTES INCTXF.
   230 INCTXF=(0.1538442733+(5.22750756E-6)*TXINCF)*
      1TXINCF
   C MAXIMUM ERROR=9.7%, R**2=.996203
   GO TO 245
   235 INCTXF=(0.146188203+(4.66849401E-6)*TXINCF)*
      1TXINCF
   C MAXIMUM ERROR=7.8%, R**2=.998594
   GO TO 245
   240 INCTXF=(0.1391850286+(4.09313513E-6)*TXINCF)*
      1TXINCF
   C MAXIMUM ERROR=2.3%, R**2=.997934
245 IF (INCTXF .GE. 3000.0) GO TO 250
246 DEDUCF=INCTXF
   GO TO 255
250 DEDUCF=3000.0
255 TXINCO=INC(I)-STOEDF-EXEMP*675.0-DEDUCF
   IF (TXINCO .LE. 0.0) GO TO 275
   IF (MARISt(I) .EQ. 1) OR. MARISt(I) .EQ. 0 AND.
      1NOCHLD(I) .GE. 0) 265,260
   C IF MARRIED OR SINGLE WITH CHILDREN, 265
   C COMPUTES INCTXO. IF SINGLE WITH NO CHILDREN, 250
   C COMPUTES INCTXO.
   260 INCTX0=(1.43929585E-2)*(TXINCO**1.18411961)
   GO TO 280
   265 INCTX0=(7.530397912E-3)*(TXINCO**1.240025807)
   C MAXIMUM ERROR=5.5%, R**2=.976444
   GO TO 280
270 INCTXF=0.0
   GO TO 246
275 INCTX0=0.0
280 OSPINC(I)=INC(I)-INCTXF-INCTX0
300 CONTINUE
400 FORMAT (2X,F5.0)
   DO 500 I=1,81
500 WRITE (35,400) OSPINC(I)
   ENDFILE 35
END
TABLE C4. COMPUTER PROGRAM USED TO CALCULATE ACCESSIBILITY

<table>
<thead>
<tr>
<th>PROGRAM LOCATION</th>
</tr>
</thead>
</table>
| GIVEN THE LOCATION OF THE NTH HOUSEHOLD (XH(N), YH(N)), THE LOCATION OF ITS PLACE(S) OF EMPLOYMENT (XE(N), YE(N)), THE LOCATION OF THE RETAIL CENTERS TO WHICH IT TRAVELS (X(J), Y(J)), THE FREQUENCY WITH WHICH IT VISITS THE JTH CENTER OR PLACE OF EMPLOYMENT (F(N,J)), AND THE HOUSEHOLD'S COST PER UNIT DISTANCE OF TRAVEL TO THE JTH POINT (C(N,J)), THIS PROGRAM CALCULATES THE HOUSEHOLD'S MINIMUM TRAVEL COST (TCM(N)) LOCATION (XC(N),YC(N)). THE HOUSEHOLD'S TRAVEL COST AT ITS ACTUAL LOCATION (TCAC(N)) IS ALSO CALCULATED, AND ITS ACCESSIBILITY AT THAT LOCATION IS DEFINED TO BE ACCESS(N) = TCM(N) - TCAC(N).

IN THIS PROGRAM, EACH HOUSEHOLD IS ASSUMED TO VISIT, WITH EQUAL FREQUENCY, ONLY THOSE RETAIL CENTERS IN THE CITY CLOSEST TO IT, AND TRAVEL COSTS ARE ASSUMED TO BE EQUAL IN ALL DIRECTIONS. F(N,J) IS SET EQUAL TO ONE FOR THOSE RETAIL CENTERS CLOSEST TO THE HOUSEHOLD AND FOR THE HOUSEHOLD'S PLACE(S) OF EMPLOYMENT; IT IS SET EQUAL TO ZERO FOR ALL OTHER CENTERS. C(N,J) IS SET EQUAL TO ONE FOR ALL HOUSEHOLDS AND ALL POINTS.

WITH THE EXCEPTION OF THOSE HOUSEHOLDS LOCATED IN NORTH ALBANY, IT IS ASSUMED THAT TRAVEL IS POSSIBLE IN ALL DIRECTIONS AND, THUS, STRAIGHT-LINE DISTANCES ARE CALCULATED BETWEEN ALL POINTS. THE DISTANCE BETWEEN A RESIDENCE IN NORTH ALBANY AND A POINT IN ALBANY IS CALCULATED AS THE SUM OF THE LINEAR DISTANCE FROM THE RESIDENCE TO THE BRIDGE ON THE WILLAMETTE RIVER AT ALBANY AND THE LINEAR DISTANCE FROM THE BRIDGE TO THE POINT IN QUESTION.

THE ALGORITHM USED TO CALCULATE THE MOST ACCESSIBLE POINT WAS DEVELOPED BY KUHN AND KUENNE (1962).

THE EQUATION NUMBERS REFERENCED IN THE PROGRAM REFER TO THOSE IN THEIR ARTICLE.

DIMENSION X(8),Y(8),XH(81),YH(81),XE1(81),YE1(81),XE2(81),YE2(81),DIF(20),F(81,8),1G(81,8),W(81,8),XC(81),YC(81),ACCESS(81),
10 FORM(2(2X,F11.6))
20 FORMAT(2(2X,F11.6))

DO 20 N=1,81
20 READ (20,10) XH(N),YH(N)
DO 30 N=1,81
30 READ (30,10) XE1(N),YE1(N)
DO 40 N=1,81
40 READ (40,10) XE2(N),YE2(N)
TABLE C4. CONTINUED

THE FOLLOWING ASSIGNS EACH JOSEPH TO A SET OF RETAIL CENTERS ON THE BASIS OF ITS LOCATION RELATIVE TO PHILOMATH AND ALBANY.

DO 90 N=1,81
IF (XH(N) GE. 17.0 AND YH(N) GE. 12.0) GO TO 160
IF (XH(N) LE. 10.0 AND YH(N) LE. 11.0) GO TO 170
DO 50 J=3,5
F(N,J)=1.0
50 CONTINUE
GO TO 85
60 F(N,6)=1.0
GO TO 85
70 DO 80 J=1,2
F(N,J)=1.0
80 CONTINUE
85 IF(XE1(N) .EQ. 0.0 AND YE1(N) .EQ. 0.0) GO TO 190
F(N,7)=1.0
IF(XE2(N) .EQ. 0.0 AND YE2(N) .EQ. 0.0) GO TO 190
F(N,8)=1.0
90 CONTINUE
DO 150 N=1,81
X(7)=0.0
Y(7)=0.0
X(8)=0.0
Y(8)=0.0
X(7)=XE1(N)
Y(7)=YE1(N)
X(8)=XE2(N)
Y(8)=YE2(N)
DO 175 K=1,8
IF (X(K) .EQ. 0.0 AND Y(K) .EQ. 0.0) GO TO 175
DO 150 J=1,8
IF J .EQ. K OR X(J) .EQ. 0.0 AND Y(J) .EQ. 0.0 GO TO 15D
10.0) GO TO 150
DISKJ=0.0
DISKJ=SQRT((X(J)-X(K))**2+(Y(J)-Y(K))**2)
IF (DISKJ .EQ. 0.0) GO TO 150
COSJ(K,J)=(X(J)-X(K))/DISKJ
SINJ(K,J)=(Y(J)-Y(K))/DISKJ
150 CONTINUE
175 CONTINUE
DO 200 J=1,8
W(N,J)=F(N,J)*C(N,J)
200 CONTINUE
DO 500 K=1,8
IF(W(N,K) .EQ. 0.0) GO TO 500
SWCOSJ=0.0
SWSinJ=0.0
SWC2S2J=0.0
DO 400 J=1,8
IF (W(N,J) .EQ. 0.0) GO TO 410
SWCOSJ=SWCOSJ+W(N,J)*COSJ(K,J)
SWSinJ=SWSinJ+W(N,J)*SINJ(K,J)
400 CONTINUE
SWC2S2J=SQRT(SWCOSJ**2+SWSinJ**2)
IF (SWC2S2J .LE. 1.0000000001 AND SWC2S2J .GE. 10.999999999) SWC2S2J=1.0
THE FOLLOWING CHECKS FOR A CORNER SOLUTION,
EON(7).
IF(W(N,K) .GE. SWC2S2J) GO TO 1200
500 CONTINUE
SWX=0.0
TABLE C4. CONTINUED

350

| SWY = 0.0 |
| SW = 0.0 |
| DO 500 J = 1, 8 |
| SWX = SWX + W(N, J) * X(J) |
| SWY = SWY + W(N, J) * Y(J) |
| SW = SW + M(N, J) |
| 600 CONTINUE |

THE FOLLOWING CALCULATES THE WEIGHTED LEAST SQUARES POSITION AS A FIRST ESTIMATE OF THE MINIMUM TRAVEL COST POSITION (EQN(13)).

IF (SW .EQ. 0.0) GO TO 1500
XC(N) = SWX/SW
YC(N) = SWY/SW
M = 1
DIFF(M) = 0.0

THE FOLLOWING, UP TO 2 STATEMENTS BEFORE 1000, IS AN ITERATIVE PROCEDURE FOR DETERMINING THE MINIMUM TRAVEL COST POSITION.

700 SW1X = 0.0
SW1Y = 0.0
SW1 = 0.0
DO 600 J = 1, 8
IF (M(N, J) .EQ. 0.0) GO TO 600
W1 = 0.0
IF (X(J) .EQ. XC(N) .AND. Y(J) .EQ. YC(N)) GO TO 1800

1800 W1 = M(N, J) / SQRT((X(J) - XC(N)) ** 2 + (Y(J) - YC(N)) ** 2)
SW1X = SW1X + W1 * X(J)
SW1Y = SW1Y + W1 * Y(J)
SW1 = SW1 + W1

800 CONTINUE
XC0 = 0.0
YC0 = 0.0
YCD = YC(N)

THE FOLLOWING TWO ARE EQUATIONS (14)

XC(N) = SW1X / SW1
YC(N) = SW1Y / SW1
IF (M .EQ. 20) GO TO 1100
M = M + 1
DIFF(M) = SQRT((XC(N) - XC0) ** 2 + (YC(N) - YC0) ** 2)
IF (M .LE. 3) GO TO 700

THE FOLLOWING CHECKS FOR CONVERGENCE OF SOLUTION. IF SOLUTION DIVERGING OR IF LAST ITERATION PROVES LITTLE CHANGE IN X(N) AND Y(N), ITERATIVE PROCEDURE IS STOPPED.

IF (DIFF(M) .GE. DIFF(M-1)) GO TO 1100
IF (DIFF(M) .GT. 0.01) GO TO 700
GO TO 1300

1000 FORMAT (# DIFF NOT CONVERGING FOR N=*, I2, *)

1100 WRITE (61, 1000) N, M
GO TO 1300

1200 XC(N) = X(K)
YC(N) = Y(K)

1300 DO 900 J = 1, 8
IF (M(N, J) .EQ. 0.0) GO TO 900
DCJM(N, J) = SQRT((X(J) - XC(N)) ** 2 + (Y(J) - YC(N)) ** 2)
W1 = 0.0
IF (DCJM(N, J) .EQ. 0.0) GO TO 300
W1 = M(N, J) / DCJM(N, J)
IF (M(N, J) .GE. SWC2S2J) GO TO 900

THE FOLLOWING TWO EQUATIONS CALCULATE FIRST-ORDER CONDITIONS FOR A MINIMUM TRAVEL COST POINT (EQUATIONS 10). SUM OF EACH SHOULD BE EQUAL ZERO FOR A MINIMUM.

FORDX(N) = FORDX(N) + W1 * (X(J) - XC(N))
Figure 14. The study area and the location of the houses included in the study.

Legend:
- Price of house estimated within ±5% of market price
- Price of house not estimated within ±5% of market price
- Retail center
TABLE C4. CONTINUED

FOROY(N) = FOROY(N) + W1*(Y(J) - YC(N))

900 CONTINUE
C FOLLOWING CALCULATES ACCESSIBILITY.
TCA = 0.0
TCM = 0.0
DO 1400 J = 1, 8
IF(W(N, J) .EQ. 0.0) GO TO 1400
IF(X(J) .LE. 8.0, AND, Y(J) .LE. 8.0) GO TO 1350
TCA = TCA + W(N, J) * SQRT((XH(N) - X(J))**2 + (YH(N) - Y(J))**2)
1400 CONTINUE
C FOLLOWING CALCULATES ACCESSIBILITY CONSIDERING
C THE BRIDGE AT ALBANY.
1350 TCA = TCA + W(N, J) * SQRT((XH(N) - 5.75)**2 + (YH(N) - 6.975)**2) + SQRT((5.75 - X(J))**2 + (5.375 - Y(J))**2)
1375 TCM = TCM + W(N, J) * DCJM(N, J)
1400 CONTINUE
ACCESS(N) = TCM - TCA
1500 CONTINUE
1600 FORMAT(2X, F7.2)
DO 1700 N = 1, 81
1700 WRITE(25, 1600) ACCESS(N)
ENDFILE 25
1800 FORMAT(2(2X, F11.8))
DO 1900 N = 1, 81
1900 WRITE(35, 1800) FOROX(N), FOROY(N)
ENDFILE 35
DO 2000 N = 1, 81
2000 WRITE(45, 10) XC(N), YC(N)
ENDFILE 45
END