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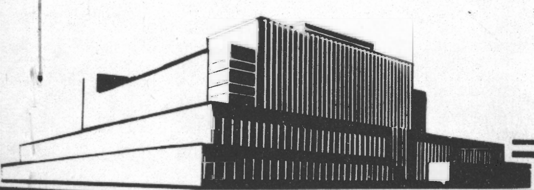
METHODS OF COMPUTING THE RATE OF TEMPERATURE CHANGE IN WOOD AND PLYWOOD PANELS WHEN THE TWO OPPOSITE SURFACES ARE MAINTAINED AT DIFFERENT TEMPERATURES

Information Reviewed and Reaffirmed

September 1956

No. 1406



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UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
In Cooperation with the University of Wisconsin

METHOD OF COMPUTING THE RATE OF TEMPERATURE CHANGE IN WOOD AND PLYWOOD PANELS WHEN THE TWO OPPOSITE SURFACES ARE MAINTAINED AT DIFFERENT TEMPERATURES

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It is sometimes useful, as in hot-press gluing, to know the rate of temperature change in wood and plywood panels of any given thickness when the two opposite faces are held at different temperatures. This information is applicable in estimating the approximate time required to obtain a given temperature at a specified distance from the surface, as, for example, in finding the time required to reach the maximum temperature at any point in a panel; in studying the effect of temperature differences between two hot plates while gluing; in estimating the amount of temperature change that will occur when there is a delay in closing the hot press after loading; and in sterilizing wood vats or churns with hot water or steam. Methods of making such temperature determinations by means of charts which eliminate much tedious calculation are here presented as developed by studies at the Forest Products Laboratory.

RELATION OF TEMPERATURE AND THICKNESS

If two panels, one $\underline{a}$ inches thick and the other $\underline{b}$ inches thick, are heated under the same conditions until a temperature $\overline{U}$ is obtained at a given distance $\underline{X_a}$ from the surface of the one $\underline{a}$ inches thick, and at the same

[1]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

proportional distance $\underline{X_b}$ from the surface of the one $\underline{b}$ inches thick, where

$$\frac{X_a}{a} = \frac{X_b}{b}$$

then the temperature distribution will be the same throughout each panel. That is, the temperatures will be equal at the same proportional distances from the surface of each panel. If, for convenience, $\underline{a}$ is taken as unity, $\underline{X_b} = \underline{X_a b}$ and $\underline{X_a} = \underline{X_b/b}$. The time required to reach the same temperature at the same proportional distances from the surface, such as (0.1 $\underline{a}$) and (0.1 $\underline{b}$); or (0.2 $\underline{a}$) and (0.2 $\underline{b}$), will be inversely proportional to the diffusivity and directly proportional to the square of the thickness. (See references 1-8 for a discussion of diffusivity.)

As an illustration, assume that two panels are composed of the same material, one being 1 inch thick and the other 3.5 inches thick, and that the initial wood temperature and temperatures maintained on opposite faces of each panel are the same. Let one surface of each panel be placed in contact with a hot plate at 300° F. and the opposite face be maintained at a temperature of 50° F. Neglecting heat losses from the edges, assume a temperature of 150° F. has been reached at a distance of $\underline{X_a} = 0.4$ inch from the hot surface of the panel 1 inch thick after it has been heated for $\underline{T}$ minutes. The corresponding distance $\underline{X_b}$ in the panel 3.5 inches thick will be (0.4)(3.5) or 1.4 inches. The time required to reach the temperature of 150° at 1.4 inches from the hot surface of the panel 3.5 inches thick will be

$$(3.5)^2\ T = 12.25\ (T)\ \text{minutes}$$

or over 12 times as long as is required to reach this temperature at 0.4 inch from the surface of the 1-inch panel. After the panel 3.5 inches thick has been heated 12.25 ($\underline{T}$) minutes, the computed temperature distribution will be the same in both panels, the only difference being that distances between points of equal temperature in the two panels will, of course, be 3.5 times greater in the thicker panel than in the 1-inch panel. For example, temperatures at the center will be the same, but the distance from the surface to the center of the panel 3.5 inches thick is

(0.5)(3.5), or 1.75 inches in comparison with 0.5 inch for the panel 1 inch thick.

When the diffusivity, thickness of panel, and heating conditions are known, a formula can be derived for computing the temperature that should be obtained at any distance from the surface of a panel after it is heated for a given period. Similarly, if a particular temperature is desired at some point, the required heating period can be computed. (See appendix for formula derived for this purpose.)

In order to eliminate computations, three charts of temperature curves have been prepared which provide a simplified method for computing either the temperature obtained after a given heating period or the heating period required to obtain a given temperature, for any assumed heating conditions. The use of these curve charts, figures 1, 2, and 3, will be illustrated in the examples that follow.

COMPUTATION OF DATA FOR CURVES

The formula used for the computation of the temperature curves is given in the appendix (equation 2). It may be noted that this formula contains two converging series that can be separated into two parts. If $\underline{U_1}$, which is outside the parenthesis in the first line of the formula, is placed before the plus sign in the third line, the latter part of the formula would be the same as the formula used for computing temperatures when panels are heated between hot plates held at the temperature $\underline{U_1}$ (8). The remaining part of the formula, which is the series in parenthesis multiplied by ($\underline{U_2}$ - $\underline{U_1}$), would be used to compute the temperature to be subtracted from that determined by means of the equation for finding the wood temperature when the two opposite faces are heated at the temperature $\underline{U_1}$.

In this discussion, $\underline{U_x}$ will designate the temperature determined from the first mentioned part of the equation and $\underline{U_z}$ will represent the temperature to be subtracted from $\underline{U_x}$. The temperature at any point $\underline{X_b}$ will be designated as $\underline{U_w}$, which equals $\underline{U_x}$ - $\underline{U_z}$. Temperatures from figure 1 will be designated as $\underline{U_v}$, those from figure 2 as $\underline{U_k}$, and those from figure 3 as $\underline{U}$.

In most cases for which computations are to be made, the initial wood temperature U_a and the lower temperature U_c maintained at one surface will probably be about the same and only figure 1 will be needed for computing the temperature at any particular point.

When the temperature of the hot surface is not much greater than 220° F. and heating is done by means of a hot plate, no allowance need be made for cooling because of moisture evaporation even when heating is continued until the maximum temperature conditions are obtained. In fact, it is evident that, with relatively high temperatures applied to one surface only, moisture evaporation would have much less effect on the rate of temperature change than when heat is applied from both sides (8). For example, if one surface is heated by a hot plate at 300° F. and the cold surface is maintained at 100° F., after constant temperature conditions are reached all wood temperatures beyond a distance from the hot surface of 0.44 of the total thickness will be below 212° F. In this case, less than half the panel could reach a temperature at or above the boiling point, whereas the entire panel could be heated to 300° F. if both surfaces were held at this temperature. If the cold surface is assumed to be at 70° instead of 100° F., only about three-eighths of the wood could be heated to a temperature greater than 212° F. That is, as the temperature of the cold surface is lowered, a progressively smaller proportion of the wood will be at or above 212° F.

When steam or water is used, as in sterilizing churns or tanks, it would, of course, be unnecessary to consider moisture evaporation, regardless of the temperature, since moisture could not evaporate from the hot surface.

It should be mentioned here that if an exact formula is derived to apply in problems of heating cylindrical shells, such as round churns, tanks, etc., the equation would be more complicated and more difficult to use than the one employed for these computations, which applies to flat surfaces. Because of the large radius in proportion to thickness, however, the data for flat surfaces will be entirely suitable to use for circular containers of this kind.

METHOD OF USING TEMPERATURE CURVES

Definitions of the symbols used in the present discussion, and simple formulas to be used in conjunction with figures 1, 2, and 3, when any heating conditions are employed, are given in the appendix.

In computing the temperature curves shown in figures 1, 2, and 3, the temperature of the hot surface was taken as 250° F., the initial wood temperature was taken as 60° F., and the diffusivity as 0.00020. The temperature maintained on the cold surface was assumed as 60° F., in computing figures 1 and 2. Experiments with various heating mediums show that the diffusivity tends to decrease as the specific gravity increases. Table 1, col. 3, shows average diffusivity factors that can be used for various ranges of specific gravity. These factors are based on results from experiments with hot plates. The diffusivity values shown can be increased about 15 percent if steam or hot water is used as the heating medium in direct contact with the wood, since wood heats faster in steam or hot water. When panels are glued with phenolic-resin glue, and the veneer thickness is 1/10 inch or less, the large proportion of glue per unit thickness has a retarding effect on the rate of heating. Experiments indicate that in order to compensate for this effect, the heating period for such conditions should be increased by about the percentage shown in table 2.

Although the computations for figures 1, 2, and 3 were based on assumed temperature values of 60° F. for the initial wood temperature and cold-surface temperature, 250° F. for the hot-surface temperature, and a diffusivity value of 0.00020, the curves can be used for any other temperature conditions and any given diffusivity $\underline{h_x^2}$ different from those used, as will be illustrated in the following examples.

Figure 1 shows the temperature distribution for a panel 1 inch thick for various heating periods up to the time when constant temperature conditions are reached. Distances from the hot surface are given in decimal fractions and, therefore, represent proportional distances. The time values in minutes given on the curves plotted in figures 1, 2, and 3 may be considered as time factors, while decimal values on the bottom scale may be considered as proportional distances based on total panel thickness.

The following examples will illustrate the use of figure 1:

Example 1

Method of computing heating period ($\underline{T_b}$) for any thickness when initial wood temperature $\underline{U_a}$, and cold surface temperature $\underline{U_c}$, are the same.

Data given:

Temperature of heating medium = 320° F. = U_b.

Initial temperature of wood and temperature of cold surface = 90° F. = U_a and U_c.

Diffusivity = 0.00025 = h_x^2.

Thickness = 2.5 inches = b.

Required: Find time T_b required to reach a temperature U_w of 230° at a distance X_b = 3/8 inch from the surface.

Solution: The corresponding proportional distance X_a from the surface of a panel 1 inch thick is 3/8 ÷ 2.5 = 0.15 inch.

First, by using equation (F_1), appendix, find U_v corresponding to a temperature U_w = 230. In this case

$$U_v \doteq 250 - \left[\frac{(320 - 230)(190)}{(320 - 90)}\right] = 176° \text{ F.}$$

The second step is to find from figure 1, the time T required to reach a temperature of 176° at a distance of 0.15 inch from the surface. This is found to be about 3.5 minutes. The third and last step is to find T_b. Since the diffusivity h^2 used in computing the curves for figure 1 is 0.00020 and the diffusivity assumed for the material 2.5 inches thick is 0.00025, T_b will be determined by means of equation (D_1), appendix, which gives,

$$T_b = (2.5)^2 (3.5) \left[\frac{0.00020}{0.00025}\right] = \underline{17.5} \text{ minutes}$$

Example 2

Method of computing temperature $\underline{U_w}$ at any distance from surface when heating period $\underline{T_b}$ is known and initial wood temperature and cold surface temperature are the same.

Data given:

Temperature of heating medium = 275° F. = $\underline{U_b}$.

Initial wood temperature and temperature of cold surface = 75°.
In this case $\underline{U_a} = \underline{U_c} = 75°$.

Diffusivity = 0.00018 = $\underline{h_x^2}$.

Thickness = 2 inches = $\underline{b}$.

Required: Find temperature $\underline{U_w}$ at 1/2 inch from the surface after heating 50 minutes, = $\underline{T_b}$.

Solution: The corresponding proportional distance $\underline{X_a}$ from the surface of a panel 1 inch thick is $(\frac{0.5}{2}) = 0.25$.

First, using equation (C_1), appendix, compute $\underline{T}$. From this equation

$$T = \frac{50}{(2)^2}\left[\frac{0.00018}{0.00020}\right] = 11.25 \text{ minutes.}$$

Second, from figure 1 find temperature $\underline{U_v}$ obtained in 11-1/4 minutes at a distance of 0.25 inch from the surface. This is found to be about 179° F.

Third, find $\underline{U_w}$, using equation (E_1), which gives,

$$U_w = 275 - \left[\frac{(275 - 75)(250 - 179)}{190}\right] = \underline{200°} \text{ approximately.}$$

The procedure illustrated in examples 1 and 2 would be used in computing the temperature at any given distance from the interior surface of tanks or churns, since in most cases the initial wood temperature and outer surface temperature could be assumed as approximately the same.

Computation of Temperature $\underline{U_w}$ When the Temperature $\underline{U_c}$ of Cold Face is Different from Initial Wood Temperature $\underline{U_a}$

If the temperature of the wood and temperature of the cold face are different, the temperature $\underline{U_w}$, obtained after a given heating period, can be found by using figures 2 and 3. The procedure will be illustrated by the following examples:

Example 3

Method of computing temperature $\underline{U_w}$ at any distance from surface when heating period $\underline{T_b}$ is known and initial wood temperature $\underline{U_a}$ is different from the cold surface temperature $\underline{U_c}$.

Data given:

Temperature of hot side, $\underline{U_b}$ = 300°.

Temperature of cold side, $\underline{U_c}$ = 80°.

Initial wood temperature, $\underline{U_a}$ = 40°.

Diffusivity = 0.00022 = $\underline{h_x^2}$.

Heating period = 60 minutes = $\underline{T_b}$.

Thickness = 2.25 inches = $\underline{b}$.

Required: Find temperature at 1 inch from the surface of the panel 2.25 inches thick.

Solution: The corresponding proportional distance X_a for a thickness of 1 inch will be $1 \div 2.25 = 0.445$ inch.

First, find T, using equation (C_1). This gives

$$T = \frac{60}{(2.25)^2} \left[\frac{0.00022}{0.00020}\right] = 13.2° \text{ or approximately 13 minutes.}$$

Second, from figure 2, at a distance of 0.445 inch from the surface find the temperature U_k obtained in 13 minutes. This is found to be about 59° F.

Third, from equation (G) determine U_z, the temperature to be subtracted from U_x.

$$U_z = 59 \left[\frac{300 - 80}{190}\right] = 68° \quad \text{approximately.}$$

Fourth, from figure 3, at a distance of 0.445 inch from the surface, find the temperature U obtained in 13 minutes. This is found to be about 199° F.

Fifth, using equation (E_2), compute U_x. Substituting temperature values in this equation gives:

$$U_x = 300 - \left[\frac{(300 - 40)(250 - 199)}{190}\right] = 230° \text{ F.}$$

The temperature U_w obtained after heating for 60 minutes is then

$$U_x - U_z = 230 - 68 = \underline{162}° \text{ F.}$$

In the examples given, it has been shown that when the heating period T_b is known or assumed, figure 1 can be used directly for finding T_b if a given temperature is to be obtained at any point distant X_b from the surface if the initial wood temperature U_a and cold-surface temperature U_c

are assumed to be the same. Likewise, figures 2 and 3 can be used directly for computing the wood temperature $\underline{U_w}$ which depends on the temperature of the hot surface $\underline{U_b}$, the initial wood temperature $\underline{U_a}$, and the temperature of the cold surface $\underline{U_c}$.

If however, the initial wood temperature is different from the cold-surface temperature and the problem is to find the required heating period, the time $\underline{T_b}$ can be determined approximately by first making a tentative computation using figure 1, assuming for the purpose of computation that the initial wood temperature and the cold-surface temperature are the same. The time $\underline{T_b}$ thus determined can then be checked and adjusted by means of figures 2 and 3. The procedure will be illustrated by the following example, in which a fairly large difference in the initial wood temperature and cold-surface temperature is assumed. The problem is simpler when the difference between these two temperatures is small.

Example 4

Method of computing the heating period $\underline{T_b}$ required to obtain a given temperature $\underline{U_w}$ when the initial wood temperature $\underline{U_a}$ and the surface temperatures $\underline{U_b}$ and $\underline{U_c}$ are all different.

Data given:

Temperature of hot side = 325° = $\underline{U_b}$.

Temperature of cold side = 100° = $\underline{U_c}$.

Initial wood temperature = 45° = $\underline{U_a}$.

Diffusivity = 0.0024 = $\underline{h_x^2}$.

Thickness = 3 inches = $\underline{b}$.

Distance from surface $\underline{X_b}$, at which temperature is desired, = 2-1/8 inches.

Required: Find time $\underline{T_b}$ required to reach a temperature $\underline{U_w}$ of 130° F. at the distance given.

Solution: The distance of 2-1/8 divided by the total thickness of 3 inches = 0.708, which is the corresponding distance from the hot surface of a panel 1 inch thick.

The first step is to decide what temperature between the initial wood temperature of 45° and the cold-surface temperature of 100° should be assumed for substituting in place of $\underline{U_a}$ and $\underline{U_c}$ in order to make the tentative computation of $\underline{T_b}$ from figure 1. Since the point is 2-1/8 inches from the hot plate surface, it is 7/8 inch from the cold surface, which is at 100° F. The assumed temperature will then be taken a little higher than the average of 72.5° = $\left[\frac{100 + 45}{2}\right]$. For the trial computation, this will be taken as 80°.

The second step is to compute $\underline{U_v}$, using equation (F_1) in the appendix, assuming $\underline{U_a} = \underline{U_c} = 80°$. Substituting the temperature values indicated gives

$$U_v = 250 - \left[\frac{(325 - 130)(190)}{(325 - 80)}\right] = 250 - 151 = 99° \quad .$$

From figure 1 it is found that a temperature of 99° at a distance of 0.708 inch from the surface is obtained in approximately 15 minutes = $\underline{T}$. The next step is to check this period by means of figures 2 and 3. From figure 2 the temperature $\underline{U_k}$, for a distance of 0.708 and a heating period of 15 minutes, is found to be about 118. From formula (G),

$$U_z = 118 \left[\frac{325 - 100}{190}\right] = 140° \quad .$$

From figure 3 the temperature $\underline{U}$ is found to be about 217° at a distance of 7/8 ÷ 3 or 0.292, when the heating period is 15 minutes. (Since the curves in figure 3 are symmetrical about the center axis, because the same temperature is assumed to be applied at each face, they are plotted for one-half the thickness only. A temperature distant 0.708 inch from one side would then evidently be the same as the temperature at (1 - 0.708) or 0.292 inch from the opposite side).

Since the temperature $\underline{U}$ from figure 3 was found to be about 217° substituting in equation (E_2),

$$U_x = 325 - \left[\frac{(325 - 45)(250 - 217)}{190}\right] = 325 - 49 = 276^\circ .$$

The temperature obtained in 15 minutes would then be,

$$U_w = U_x - U_z = 276 - 140 = 136^\circ\ F.$$

This is slightly higher than the temperature of 130° which is required. A closer value will, of course, be obtained by assuming a heating period a little less than 15 minutes.

Taking the heating period $\underline{T}$ = 13 instead of 15 minutes, the temperature $\underline{U_k}$ from figure 2 is found to be about 114° F. after heating 13 minutes, and $\underline{U_z} = 114 \left[\frac{325 - 100}{190}\right] = 135^\circ$.

The temperature $\underline{U}$ from figure 3 is found to be about 209° F. after heating 13 minutes and, from equation (E_2),

$$U_x = 325 - \left[\frac{(325 - 45)(250 - 209)}{190}\right] = 265^\circ .$$

The wood temperature $\underline{U_w}$ for this period is then, $\underline{U_x} - \underline{U_z} = 265 - 135 = 130^\circ$. This temperature $\underline{U_w}$, the temperature desired, is obtained in 13 minutes when the thickness equals 1 inch and the diffusivity is 0.00020.

The last step is to find $\underline{T_b}$ for a thickness of 3 inches when the diffusivity is 0.0024. This is readily found by using equation (D_1),

$$T_b = (3)^2\ (13) \left[\frac{0.00020}{0.00024}\right] = 97.5$$

or approximately 98 minutes.

Example 5

Method of computing heating period required to obtain a specified temperature at a given distance from the heated surface, in bag-molding operations.

In certain bag-molding operations, the plywood may be heated by applying heat over one surface of the mold while the opposite mold surface is at approximately room temperature or at some temperature ($\underline{U_c}$) lower than that of the plywood face.

The following computation will illustrate the procedure for finding the time required to obtain a given temperature at a specified distance from the heated surface.

Data given:

Thickness of bag = 0.05 inch. Since the bags or coverings used are comparatively thin, the diffusivity of the bag may be assumed to be the same as that of the wood. The bag thickness is then merely added to that of the plywood for the purpose of computation.

Thickness of wood mold = 4 inches = $\underline{b}$.

Plywood is 5-ply, 1/32-inch yellow-poplar veneer.

Diffusivity = 0.00023 = $\underline{h_x^2}$.

Temperature applied to surface of bag = 300° F. = $\underline{U_b}$.

Initial temperature of material (plywood, mold, and bag) = 70° F. = $\underline{U_a}$.

Room temperature = 80° F. = $\underline{U_c}$.

Assuming steam is applied on the surface of the bag, the faster rate of heating with this heating medium will compensate for the increased heating period indicated in table 2 for veneer of this thickness.

Required: Find time $\underline{T_b}$ required to obtain a temperature of 250° at the lowest glue line.

Solution: Since the thickness of the bag is assumed as 0.05 inch and the veneer thickness is 1/32 inch, the distance from the outer surface of the bag to the last glue line = 0.05 + 4 (1/32) = 0.175 inch.

The corresponding proportional distance for a thickness of one inch is X_a = (0.175) ÷ (4 + 5/32 + 0.05) = (0.175) ÷ (4.206) or 0.0416.

The distance from the hot surface is small, hence the wood temperature of 70° F. will be used for both U_a and U_c for the tentative computation of T_b from figure 1.

First, solve for U_v, using equation (F_1):

$$U_v = 250 - \left[\frac{(300 - 250)(190)}{(300 - 70)}\right] = 209° \text{ F.}$$

From figure 1, it is found that a temperature of 209° F. at a distance of 0.0416 inch from the surface is obtained in approximately 1 minute.

Second, check this time by means of figures 2 and 3. From figure 2 the temperature U_k for a distance of 0.0416 from the surface is found to be zero, hence the rate of heating to 209° for this short distance from the surface will be the same as when both surfaces, 4.206 inches apart, are heated at 300°. Only figure 3 will be needed for making the computation under these conditions.

Figure 3 indicates the temperature U is about 209° F. for a heating period of 1 minute and a distance of 0.0416 inch from the surface.

Solving for U_x from equation (E_2),

$$U_x = 300 - \left[\frac{(300 - 70)(250 - 209)}{190}\right] = 250$$

which is the desired temperature.

This computation applies for a thickness of 1 inch and a diffusivity of 0.00020.

Third, determine from equation (D_1) the time T_b for the thickness of 4.206 inches, the distance of 0.175 inch, and a diffusivity of 0.00023.

Substituting 1 minute for T and 4.206 inches for b,

$$T_b = (4.206)^2 \ (1) \left[\frac{0.00020}{0.00023} \right] = 15.4$$

or approximately 15 minutes.

In the foregoing example it may be noted that, because of the short distance from the surface and the short time required to obtain the specified temperature of 250°, the temperature U_k taken from figure 2 is zero and the rate of heating for the depth assumed is the same as when both surfaces are at 300° F. This, however, would not hold for long heating periods nor for points a considerable distance from the surface.

This can be shown by assuming, for example, that the desired temperature is 285° F. instead of 250. For this case,

$$U_v = 250 - \left[\frac{(300 - 285)(190)}{(300 - 70)} \right] = 238° \ .$$

Using figure 1, a temperature of 238° F. at a distance of 0.0416 inch from the surface is obtained in about 11.5 minutes = T.

From figure 2, U_k is found to be about 4 degrees, and from equation (G),

$$U_z = \frac{4 \ (300 - 80)}{190} = 4.6$$

or about 5° F.

From figure 3, U is found to be about 242° F. at 0.0416 inch from the surface, for a heating period of 11.5 minutes.

Using equation (E_2),

$$U_x = 300 - \left[\frac{(300 - 70)(250 - 242)}{190} \right] = 290° \ \text{approximately}$$

and

$$U_w = U_x - U_z = 290 - 5 = \underline{285°} \quad .$$

A temperature of about 285° would then be obtained in about 11.5 minutes at a distance of 0.0416 inch from the surface of a panel 1 inch thick with a diffusivity of 0.00020. The time $\underline{T_b}$ required to obtain a temperature of 285° at the same proportional distance (0.175 inch from the surface) when the thickness = 4.206 inches and the diffusivity = 0.00023 is found from equation (D_1) to be,

$$T_b = (4.206)^2 \ (11.5) \left[\frac{0.00020}{0.00023}\right] = 177 \text{ minutes approximately.}$$

If, in the foregoing example, both surfaces had been maintained at 300° F., the time required to reach 238° F. would, of course, be determined from figure 3. This is found to be about 8.2 minutes. Substituting 8.2 in equation (D_1) for $\underline{T}$,

$$T_b = (4.206)^2 \ (8.2) \left[\frac{0.00020}{0.00023}\right] = 126 \text{ minutes,}$$

or nearly 1 hour less than when one surface is assumed to be held at 80°, while the opposite surface is at 300° F.

A comparison of the time needed to obtain a temperature of 250° at the depth given, with the time necessary to reach the higher temperature of 285°, shows that when the required temperature becomes fairly close to that of the heating medium, the heating period becomes relatively long. In such cases, the heating period can be considerably shortened by using a somewhat higher surface temperature. If, in the example given, the heating temperature is assumed as 325° instead of 300°, it will be found that, with the opposite surface at 80° F., the time required to reach a temperature of 285° at the distance of 0.175 inch from the surface will be about 25 minutes. Calculations of this kind will be found convenient for comparing the heating periods required for different temperatures applied to the surface of the plywood.

Since, in bag molding, the plywood is usually fairly thin compared with panels commonly heated in the hot press, and because pressure on the bag helps retard moisture evaporation, the cooling effect from evaporation is small and can generally be neglected as a factor affecting the results.

Example 6

Method of computing heating period $\underline{T_b}$ required to obtain a given temperature $\underline{U_w}$ when difference between the two surface temperatures is relatively small.

Assume there is a difference in the temperatures of the two plates on a glue press and it is desired to find the time required to reach a given temperature in a panel $\underline{b}$ inches thick.

Data given:

Temperature of one plate = 320° F. = $\underline{U_b}$.

Temperature of opposite plate = 300° F. = $\underline{U_c}$.

Initial wood temperature = 75° F. = $\underline{U_a}$.

Thickness of panel = 3/4 inch = $\underline{b}$.

Diffusivity = 0.00023 = $\underline{h_x^2}$.

Required: Find heating period $\underline{T_b}$ required to obtain a temperature of 275° F. at the center of the panel.

Solution: When the two surface temperatures are fairly close, as in this example, the tentative computation of $\underline{T_b}$ can be made by using figure 3 and assuming both surfaces at a temperature which is about the average of the two or, in this instance, 310° F.

The first step is to find $\underline{U}$. Substituting in equation (F_2),

$$U = 250 - \left[\frac{(310 - 275)(190)}{(310 - 75)} \right] = 222^\circ \text{ F.}$$

Second, determine the time $\underline{T}$ from figure 3 for a temperature of 222° F. at the center. This is found to be slightly over 18 minutes.

Assuming 18 minutes as the time $\underline{T}$ for a panel 1 inch thick, the time can be checked by means of figures 2 and 3.

The third step is to find $\underline{U_k}$ from figure 2. This is found to be about 81° at the center (a point 1/2 inch from the surface) when the heating period is 18 minutes. For the heating conditions assumed, $\underline{U_z}$ is found from equation (G) to be,

$$\frac{81\,(320 - 300)}{190} = 8.5 \text{ or approximately } 9° .$$

Fourth, from equation (E_2),

$$U_x = 320 - \left[\frac{(320 - 75)\,(250 - 222)}{190}\right] = 284° .$$

Then,

$$U_w = 284 - 9 = 275° .$$

The required heating period is then about 18 minutes for a thickness of 1 inch and a diffusivity of 0.00020.

Since the panel in question is 3/4 inch thick and the diffusivity is assumed as 0.00023, the time required to reach a temperature of 275° at the center of this panel is found by means of equation (D_1) to be,

$$T_b = (3/4)^2 \;(18) \left[\frac{0.00020}{0.00023}\right] = 9 \text{ minutes}.$$

If, in this example, both plates had been assumed to be held at 320° F., the time required to reach a temperature of 275° would be determined from figure 3 as follows:

First, determine $\underline{U}$ from equation (F_2).

$$U = 250 - \left[\frac{(320 - 275)\,(190)}{(320 - 75)}\right] = 215 .$$

From figure 3, the time $\underline{T}$ required to reach a temperature of 215° F. at the center, for a thickness of 1 inch, is found to be about 16 minutes.

This is, therefore, the time required to reach a temperature of 275° when both hot plate temperatures are 320° and the initial wood temperature is 75°. For the panel 3/4 inch thick, with the diffusivity given, the time $\underline{T_b}$ is found by using equation (D_1),

$$T_b = (3/4)^2 \;(16) \left[\frac{0.00020}{0.00023}\right] = \text{approximately 8 minutes.}$$

Since 9 minutes are required when one plate is at 300° F., or 20° lower than the temperature applied to the opposite surface, the required heating period in the latter case is more than 12 percent greater than when both plates are at 320° F.

In selecting an average temperature to be substituted for $\underline{U_a}$ and $\underline{U_c}$ in the tentative solution when $\underline{U_a}$ and $\underline{U_c}$ are not greatly different, consideration should be given to the distance of the point under consideration from the hot surface. If the point is fairly close to the center, an average value of the initial wood temperature and cold-surface temperature might be taken. If the point is near the hot surface, a temperature somewhat closer to the initial wood temperature could be assumed, while for a point near the surface at the lower temperature a temperature nearer that of the cooler surface might be taken for the tentative determination of $\underline{T}$ as illustrated in example 4. In any case, after the approximate value of $\underline{T}$ is determined from figure 1, it is easy to estimate a value of $\underline{T}$, which can be checked by using figures 2 and 3, to obtain the specified temperature as closely as desired.

COMPUTATION OF MAXIMUM TEMPERATURE AT ANY DISTANCE FROM SURFACE

After the steady state is reached, the temperature distribution between the two faces is represented by a straight line connecting the higher temperature of one face with the lower temperature of the opposite face. This is shown in figure 1 when the maximum temperature is reached in about 70 minutes. Since there is less than a 1/2-degree temperature change between the heating periods of 50 and 70 minutes, for practical purposes one might consider the maximum temperature for figure 1 as reached in about 50 minutes.

If U_b is the temperature of the hot surface and U_c the temperature of the cold surface, the maximum temperature of any point distant X_b from the hot surface of a panel b inches thick will be,

$$U_x = U_b + \frac{X_b}{b}\left[U_c - U_b\right]$$

Similarly, for the plotted data where the temperature of the hot side was assumed as 250°, the temperature of the cold side as 60°, and the thickness as 1 inch, the maximum temperature of any point at a distance X_a from the surface would be,

$$U = 250 + X_a\,(60 - 250)$$

or,

$$U = 250 - 190\,X_a \quad .$$

The time required to reach the maximum temperature will be the same for all points between the two surfaces, and it is convenient to consider the midpoint in making computations of T_b.

At the midpoint,

$$U_w = \left[\frac{U_b + U_c}{2}\right]$$

and

$$U_v = \left[\frac{250 + 60}{2}\right] = 155$$

Example 7

Data given:

As an example, assume a panel is 2-1/2 inches thick, the temperature of the hot surface U_b = 315° F., the temperature of the cold surface U_c = 85°, and the diffusivity = 0.00023.

Required: Determine the time required to reach the maximum temperature.

Solution: From figure 1, the time required to reach the maximum temperature (155°) at the center is found to be 70 minutes = T.

Substituting in equation (D_1),

$$T_b = (2.5)^2 \; (70) \left[\frac{0.00020}{0.00023} \right] = 380 \text{ minutes}$$

or 6 hours and 20 minutes.

As previously mentioned, the change in temperature between the heating periods of 50 and 70 minutes, for the thickness of 1 inch, is less than 1/2 degree, hence, for most purposes it would be sufficiently close to substitute 50 instead of 70 minutes for the time T. In the foregoing example, T_b would then be computed as,

$$(2.5)^2 \; (50) \left[\frac{0.00020}{0.00023} \right] = 271 \text{ minutes}$$

or 4 hours and 31 minutes.

In computing the time required to reach the maximum temperature or steady state condition, the initial wood temperature does not enter into the computations because the portions of equation 2 which are in the parentheses become zero.

APPENDIX

Formula Used for Computing the Rate of Temperature Change in Wood Panels When the Two Opposite Surfaces are Maintained at Different Temperatures

Symbols:

U_1 = Temperature of the hot surface.

U_2 = Temperature of the opposite surface when at a lower temperature.

U_o = Initial temperature of the wood before heat is applied.

x = Distance from the hot surface.

U = Temperature at distance $\underline{x}$ from the surface.

t = Time (in seconds) heat is applied to panel.

a = Thickness of panel.

h^2 = Diffusivity factor. This is assumed to be constant over the range of heating temperatures that may be employed, and is expressed in square-inch-per-second units.

Formula:

The following partial differential equation must be solved to meet the requirements of the boundary conditions given:

$$\frac{\delta U}{\delta t} = h^2 \left[\frac{\delta^2 U}{\delta x^2} \right] \quad . \tag{1}$$

Boundary conditions:

$$U = U_1 \quad \text{when } x = 0$$

$$U = U_2 \quad \text{when } x = a$$

$$U = f(x) \quad \text{when } t = 0 \quad .$$

When differential equation (1) is solved to meet these boundary conditions, the general equation for computing the temperature U at any point x after heating for a given time T becomes:

$$U = U_1 + (U_2 - U_1)\left[\frac{x}{a} - \frac{2}{\pi}\left(\sin\frac{\pi x}{a}\,\epsilon^{-\frac{\pi^2 h^2 t}{a^2}} - \frac{1}{2}\sin\frac{2\pi x}{a}\,\epsilon^{-\frac{4\pi^2 h^2 t}{a^2}}\right.\right.$$

$$\left.\left. + \frac{1}{3}\sin\frac{3\pi x}{a}\,\epsilon^{-\frac{9\pi^2 h^2 t}{a^2}} - \frac{1}{4}\sin\frac{4\pi x}{a}\,\epsilon^{-\frac{16\pi^2 h^2 t}{a^2}} + \text{------}\right)\right]$$

$$+ (U_o - U_1)\frac{4}{\pi}\left[\sin\frac{\pi x}{a}\,\epsilon^{-\frac{\pi^2 h^2 t}{a^2}} + \frac{1}{3}\sin\frac{3\pi x}{a}\,\epsilon^{-\frac{9\pi^2 h^2 t}{a^2}}\right.$$

$$\left. + \frac{1}{5}\sin\frac{5\pi x}{a}\,\epsilon^{-\frac{25\pi^2 h^2 t}{a^2}} + \text{------}\right] \quad . \qquad (2)$$

Method of Using the Figures When Both the Diffusivity and Heating Conditions are Different from Those Used in Computing Data for Time-Temperature Curves

The following equations are simple relations of time, temperature, diffusivity, and panel thickness that are convenient to use in connection with the data plotted in figures 1, 2, and 3.

These are derived from the relations shown by formula (2), which was used in computing the temperature data. In these computations, the heating temperature $\underline{U_1}$ was taken as 250° F., the initial wood temperature and temperature of the cold surface (temperatures $\underline{U_o}$ and $\underline{U_2}$ were assumed as 60° F., and the diffusivity $\underline{h^2}$ was taken as 0.00020.

By using equations (A) to (G) (which follow) as needed, the plotted data can be employed when the diffusivity, thickness, initial wood temperature, and temperatures of the two faces are different from those assumed in making the computations.

Symbols applying to computed data (figs. 1, 2, and 3):

U_o = Initial wood temperature assumed as 60° F.

U_1 = Temperature of hot surface assumed as 250° F.

U_2 = Temperature of cold surface assumed as 60° F.

h^2 = Diffusivity assumed as 0.00020.

U = Temperature taken from figure 3. This is the temperature obtained in a given time $\underline{T}$ at some particular distance from the hot surface. Values of $\underline{U}$ are calculated temperatures within the wood at the distance from the surface shown by the figure.

U_k = Temperatures taken from figure 2.

U_v = Temperatures taken from figure 1.

T = Time of heating (in minutes) necessary to obtain the temperature $\underline{U}$ or $\underline{U_v}$. This period of heating is found from the chart used.

X_a = Distance from hot surface when thickness = 1 inch.

Symbols applying to temperature computations when diffusivity, heating conditions, or both are different from those used in computing the curves:

U_a = Any initial wood temperature.

U_b = Any assumed temperature at hot surface.

U_c = Any assumed temperature at cold surface.

h_x^2 = Any diffusivity assumed for the wood.

U_w = Temperature obtained at some specified point in panel when surface temperatures, initial wood temperature, or all three temperatures, are different from values used in computing $\underline{U_v}$ in figure 1.

U_x = Temperature obtained at a particular distance from the hot surface when the initial wood temperature, hot plate temperature, or both differ from 60° and 250° F., the values assumed in making the computations for figure 3.

U_z = Temperature corresponding to the temperature $\underline{U_k}$ from figure 2, when different heating conditions are used.

T_a = Time of heating (in minutes) required to obtain the temperature $\underline{U_w}$ when panel is $\underline{a}$ inches thick.

T_b = Time required to obtain the temperature $\underline{U_w}$ when panel is $\underline{b}$ inches thick.

When the diffusivity is the same for panels having thicknesses of $\underline{a}$ and $\underline{b}$ inches respectively,

$$T_a = \left[\frac{a^2}{b^2}\right] T_b \tag{A}$$

$$T_b = \left[\frac{b^2}{a^2}\right] T_a \quad . \tag{B}$$

If the diffusivity of a panel with thickness $\underline{a}$ inches is $\underline{h^2}$, and the diffusivity of another panel with thickness $\underline{b}$ inches is $\underline{h_x^2}$, then

$$T_a = \left[\frac{a^2 h_x^2}{b^2 h^2}\right] T_b \tag{C}$$

$$T_b = \left[\frac{b^2 h^2}{a^2 h_x^2}\right] T_a \quad . \tag{D}$$

If, in equations (C) and (D) we let $\underline{T_a} = \underline{T}$, which designates the time found from the charts; $\underline{h^2} = 0.00020$, which is the diffusivity assumed in computing the data; and let $\underline{a} = 1$ inch;

$$T = \frac{T_b}{b^2}\left[\frac{h_x^2}{0.00020}\right] \tag{C_1}$$

$$T_b = b^2 T \left[\frac{0.00020}{h_x^2}\right] \quad . \tag{D_1}$$

In using figure 1, $\underline{U_w}$ can be determined from the relation,

$$U_w = U_b - \left[\frac{(U_b - U_a)(U_1 - U_v)}{(U_1 - U_o)}\right]$$

$$= U_b - \left[\frac{(U_b - U_a)(250 - U_v)}{190}\right] \qquad (E_1)$$

where $\underline{U_w}$ is the temperature obtained at any given distance from the surface when $\underline{U_b}$, $\underline{U_a}$, or both are different from 60° and 250°, the temperatures assumed in making the computations, and $\underline{U_v}$ is the temperature from figure 1. It may be noted by comparing equations (E_1) and (E_2) that $\underline{U_w}$ becomes the same as $\underline{U_x}$ when both surfaces are held at the same temperature. Similarly, by comparing equations (F_1) and (F_2) it may be noted that $\underline{U_v}$ becomes the same as $\underline{U}$ when the two surface temperatures are the same.

If $\underline{U_w}$ is assumed, solving for $\underline{U_v}$ gives,

$$U_v = U_1 - \left[\frac{(U_b - U_w)(U_1 - U_o)}{(U_b - U_a)}\right]$$

$$= 250 - \left[\frac{(U_b - U_w)(190)}{(U_b - U_a)}\right] . \qquad (F_1)$$

Similarly, when values of $\underline{U}$ are taken from figure 3,

$$U_x = U_b - \left[\frac{(U_b - U_a)(U_1 - U)}{(U_1 - U_o)}\right]$$

$$= U_b - \left[\frac{(U_b - U_a)(250 - U)}{190}\right] \qquad (E_2)$$

and

$$U = U_1 - \left[\frac{(U_b - U_x)(U_1 - U_o)}{(U_b - U_a)}\right]$$

$$= 250 - \left[\frac{(U_b - U_x)(190)}{(U_b - U_a)}\right] \qquad (F_2)$$

where $\underline{U_x}$ is the temperature at any given distance from the surface when either $\underline{U_a}$, $\underline{U_b}$, or both are different from 60° and 250°, respectively.

$$U_z = \frac{U_k (U_b - U_c)}{(U_1 - U_2)} = \frac{U_k (U_b - U_c)}{190} \qquad (G)$$

where $\underline{U_z}$ is the temperature corresponding to $\underline{U_k}$ obtained from figure 2, when the surface temperatures $\underline{U_c}$ and $\underline{U_b}$ are different from 60° and 250°, the temperatures assumed in computing the values of $\underline{U_k}$.

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Table 1. --Average diffusivity factors determined in heating with hot plates for wood of different specific gravities and moisture contents up to 15 percent

Specific gravity range	Common woods included	Approximate diffusivity factor h_x^2	$\frac{0.00020}{h_x^2}$
(1)	(2)	(3)	(4)
0.30 to 0.40	Basswood, eastern white pine, sugar pine, ponderosa pine, western white pine, redwood, spruces, yellow-poplar, northern white-cedar, western red-cedar, white fir	0.00023	0.87
0.40 to 0.50	Douglas-fir, hemlock, Port Orford white-cedar, eastern redcedar, mahogany	.00022	.91
0.50 to 0.60	Black tupelo, black walnut, southern yellow pines	.00021	.95
0.60 to 0.70	Yellow birch, sugar maple, oak	.00020	1.00

Table 2. --Correction factors to compensate for the retarding effect of phenolic-resin glue on the rate of heating veneers of various thicknesses

Veneer thickness	Approximate increase in heating period required over that determined from figures 1, 2, and 3
Inch	Percent
1/100	35
1/32 to 1/50 inclusive	20
1/16 to 1/28	15
1/12	10
1/10	5
1/8 and thicker	0

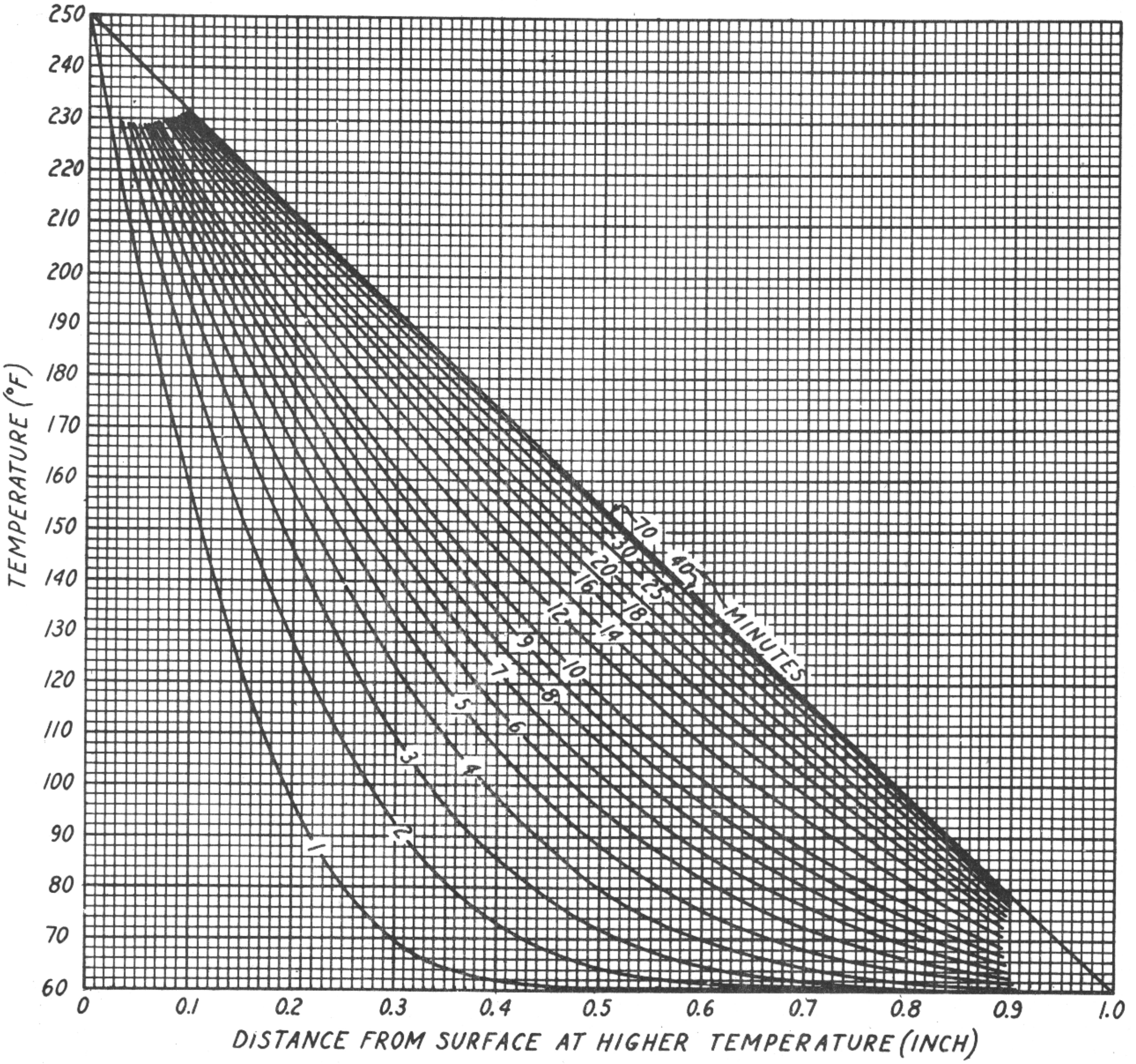

Figure 1.--Temperature distribution in a panel 1 inch thick (with opposite surfaces at different temperatures) for different periods up to the time that constant temperature conditions are reached.

Temperature of hot surface	=	250° F.
Temperature of cold surface	=	60° F.
Initial wood temperature	=	60° F.
Diffusivity	=	0.00020

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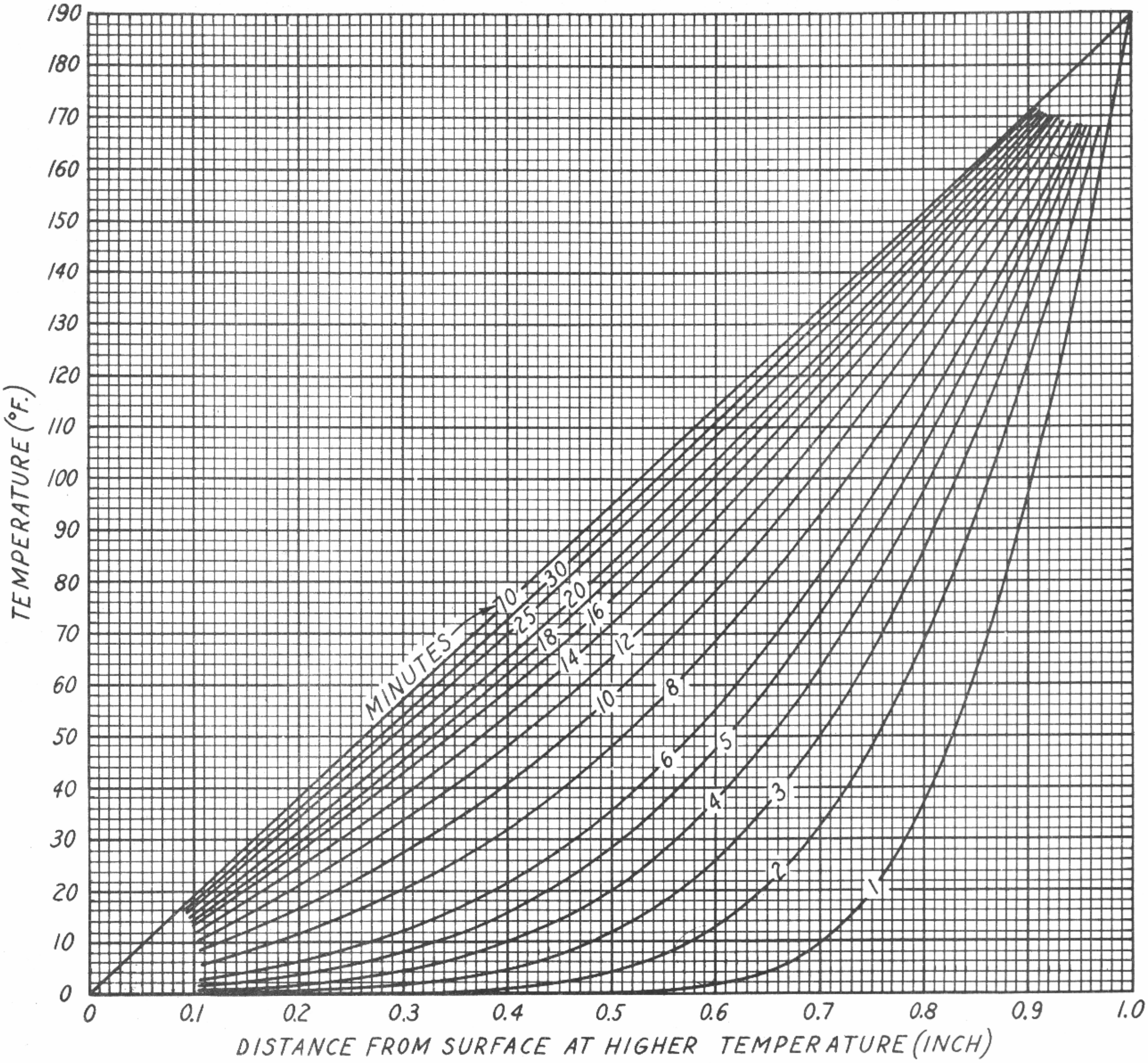

Figure 2.--Chart for finding temperature U_k.

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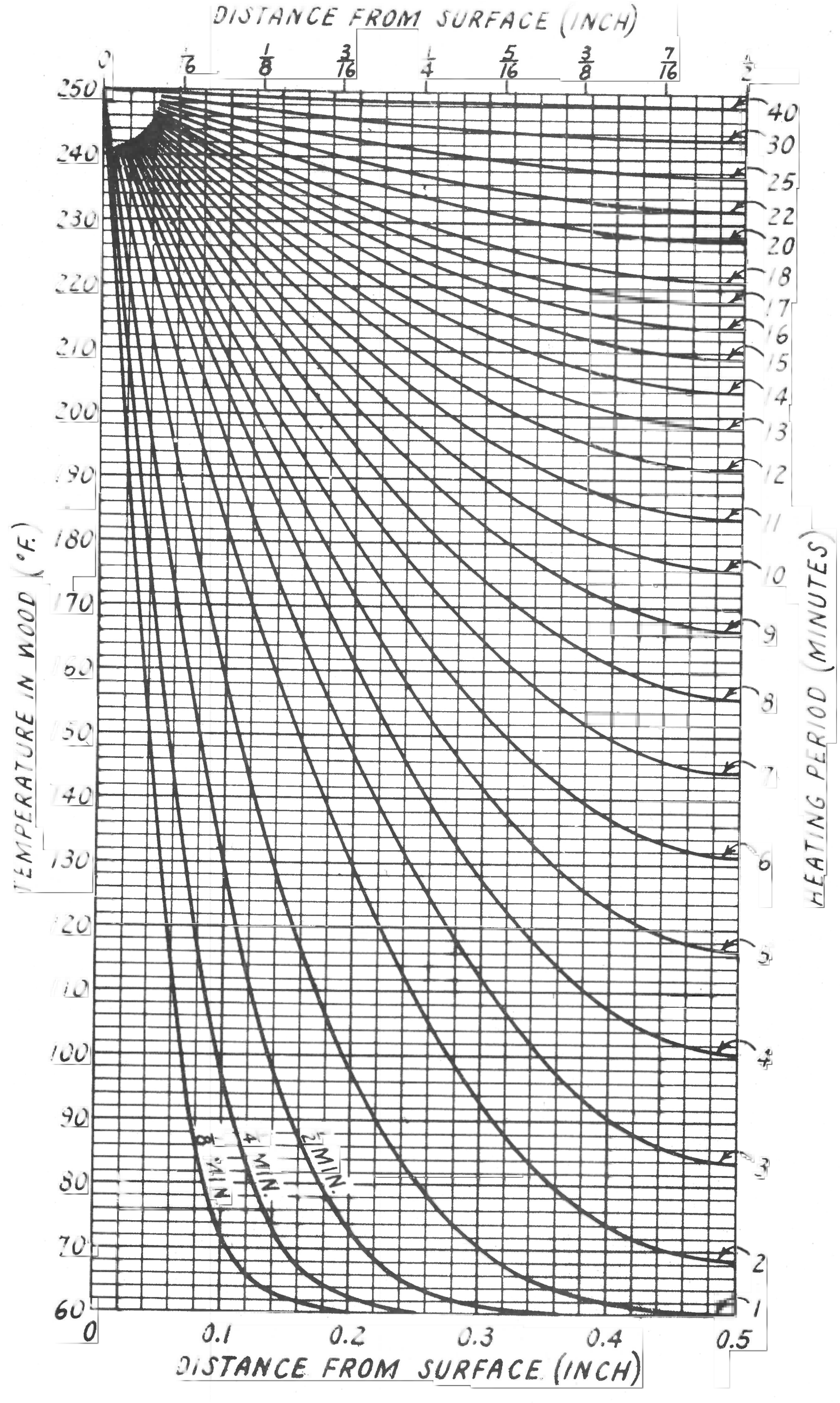

Figure 3.--Temperature distribution from surface to center of panel 1 inch thick after heating for various periods.

Hot plate temperature (each plate)	=	250° F.
Initial wood temperature	=	60° F.
Diffusivity	=	0.00020

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