Uplink Performance Characterization and Analysis of Two-Tier Femtocell Networks

Nessrine Chakchouk and Bechir Hamdaoui
School of EECS, Oregon State University
chakchon@eecs.oregonstate.edu, hamdaoui@eecs.oregonstate.edu

Abstract

This paper provides a cross-layer analysis of uplink performance in femtocell networks. It characterizes the uplink physical interference in femtocell networks and studies its impact on the delay and data loss rate of constant-bit-rate (CBR) traffic, as well as on the maximum achievable femto-user throughput. Our work derives data-link layer QoS performances as a function of physical layer parameters, thereby establishing key cross-layer relationships that can be useful for designing efficient resource allocation techniques for FC networks.

Index Terms—Uplink Interference, Outage Probability, Link Delay and Capacity, Femtocell Networks.

I. INTRODUCTION

Femtocell (FC) is a new networking paradigm that has emerged as a response to the wireless operators needs of providing high capacity and high coverage for wireless users. A FC is a low power, small-area-covering wireless cellular network consisting of one femto access point (FAP) and multiple stationary or low-mobility femto users (FUs) deployed in an indoor environment such as a home or an office environment. Characterizing the performance of the uplink (UL) communication from a FU to its associated FAP is an essential step that helps understanding the various factors that impact FC system performance, and that can in turn help designing efficient strategies for optimizing such FC systems. It is well known that co-channel interference is the main cause of performance degradation in wireless systems. Therefore, in this paper, we focus on characterizing UL interference and studying its impact on FC network performance. Specifically, we first start by modeling and analyzing the behavior of UL interference, as well as the Signal-to-Interference Ratio (SIR) and the outage probability in power-controlled FC networks. Then, we characterize and study the impact of these physical parameters on data-link layer QoS performances for CBR delay-constrained type of traffic.

A. Related Work

Characterizing and analyzing interference is becoming more and more important in modern wireless communications mainly due to the emergence of new communication and networking paradigms, such as femtocell and cognitive radio networks, which necessitate and call for the sharing of the radio spectrum more than ever. Therefore, it has been the focus of many recent works, ranging from hardware-level design and optimization [1]–[5] to system-level analysis and characterization [6]–[15].
the following, we overview some of these works, highlight their limitations, and state how our work differs from them. Researchers at both academia and industry have been studying and analyzing interference since the emergence of cellular networks. Similar to our work, in [6]–[11], the authors provide a system level analysis of the FC interference power and outage probability while taking into account the users’ spatial distributions, the wireless propagation gain, etc. However, these works present some limitations. In fact, [6] only applies to single-tier networks. In addition, [10] and [11] analyze UL interference in two-tier networks while differentiating between two types of users: licensed primary users (PUs) and unlicensed secondary users (SUs) whose activity depends on the strength of the signal transmitted by the PUs. In our work, we consider a MC network overlaid with multiple FC networks where all considered active users (FUs and MUs) are licensed users sharing the same radio resource and their activity is independent of one another. On the other hand, [7]–[11] address two-tier wireless networks, but they did not consider the impact of using power control by the cellular users (CUs). In our work, however, we assume that both MUs and FUs use fractional power control. Moreover, we provide a statistical characterization of the SIR auto-correlation per FU for the case of mobile and stationary CUs, which represents a novel contribution that may be used in the design of more efficient retransmission schemes. Other prior works analyze the UL interference spectrum while taking into account physical layer issues that involve modulation and coding [2]–[5]. These works may have applications in hardware radio design and optimization, but do not provide enough statistics for the analysis of the QoS experienced by the CU. For instance, in [2], the symbol and packet error probability are derived with respect to two different spread spectrum techniques: Direct Sequence and Frequency Hopping, while taking into account the channel fading and the interferers’ spatial distribution. The packet error probability models the block/bit error probability in a given packet at the receiver. That is, it characterizes the outage probability from a packet viewpoint. While such characterization could be useful/helpful for the study and design of error correcting schemes/codes, it doesn’t allow us to assess the QoS experienced per user, namely the per-user transmission outage probability and delay. In fact, in our scheme, we are interested in the outage probability from a system level viewpoint rather than a link-level viewpoint. That is, we aim at characterizing the transmission outage probability (from a user viewpoint) in order to characterize the MAC performance metrics such as delay and data loss rate. In [3] and [4], the authors provide a system characterization that incorporates metrics such as error probability, channel capacity, power spectral density, and aggregate RF emission of the network for different linear modulation schemes (M-PSK and M-QAM). These characterizations could be helpful for hardware RF emission standardization to ensure proper functioning of different co-existing networks such as GPS, cellular networks, etc. In our analysis, however, we make abstraction of the modulation and coding part and analyze the interference power statistics rather than its temporal/spectral properties since we target the characterization of our FC system from a higher level, i.e. MAC layer level. Indeed, in our work, we propose a cross layer analysis, in which we study the impact of the PHY performance metrics on the MAC-related ones (delay, data loss rate, throughput), in power-controlled FC networks, thereby providing useful statistics/metrics and open new horizons for future applications design such as call admission control design [16], [17].

B. Our Contributions

In this paper, we derive UL physical interference in FC networks, and study its impact on the outage probability, packet delay, and the maximum achievable FU throughput for CBR, delay-constrained traffic. Our key contributions are:
First, we provide statistical characterizations of UL interference, SIR, and outage probability as functions of design parameters, such as the power control exponents and the cellular users’ densities. These characterizations can be used for optimizing the fractional power control exponents to mitigate the UL interference. In addition, we derive the temporal auto-correlation of the SIR, which can be used for predicting SIR changes during the coherence interval, thereby providing useful techniques for designing mechanisms, such as call admission control.

Second, we propose a novel cross-layer UL performance analysis of two-tier FC networks. Specifically, we provide a system modeling that links the data-link layer performance metrics to those of the physical (PHY)-layer, and characterizes their interactions. In this model, an active FC (i.e., the FAP and its active FUs) is assimilated to a D/G/1 queueing system, thus linking two different layers: (i) The queue belonging to the data-link layer, which represents the waiting time of the FUs to access the wireless channel; (ii) The server belonging to the PHY-layer, which presents in our case the wireless channel. The packet service time in our case is nothing but the time required for a successful reception of a packet at the FAP. Using this model along with some queueing theory and effective capacity theory results, we characterize key QoS metrics related to CBR traffic in FC networks, namely the average delay, the asymptotic delay, the data loss rate, and the effective network capacity/throughput.

The remainder of this paper is organized as follows. Section II describes the system model. Section III characterizes the UL interference of FC systems. Section IV studies the SIR, its temporal correlation and the outage probability. Section V evaluates the FC system delay and derives upper bounds on its achievable throughput. Section VI presents system evaluation via simulations. Section VII concludes the paper.

II. System Model

A. Network Model

We consider a single-carrier two-tier cellular system consisting of FCs (with average coverage radius $R$) overlaid on one MC (with coverage radius $R_M \gg R$), all operating over an identical carrier frequency $f$. In our model, we assume that the FAPs are spatially distributed according to a homogeneous PPP with mean $\lambda_{FAP}$. We model the spatial distribution of the FUs and the MUs using two independent homogeneous PPPs, $\phi_1$ and $\phi_2$, in the two-dimensional plane, with intensities $\lambda_1$ and $\lambda_2$ respectively. For a PPP with intensity $\lambda$, the probability of $n$ nodes being inside a region $Z$ depends only on the total area $A_Z$ of $Z$ and is given by [18]:

$$\mathbb{P}(n \in Z) = \frac{(\lambda A_Z)^n}{n!} e^{-\lambda A_Z}$$

Here $\lambda$ is the spatial density of interfering nodes (in our case $\lambda_1$ for FUs and $\lambda_2$ for MUs), in nodes per unit area. Once scattered over the geographic area, each FU is associated with the closest/nearest FAP in its neighborhood. This is just a graphical model that we use to mimic real deployments of FCs. In fact, in real deployment scenarios, it is not unlikely that FUs are not associated with their closest FAP; this might especially happen in areas with a high density of FCs. But we still assume that such minor variations/exceptions although not taken into account still do not hurt our system analysis, since we primarily aim to characterize cross-layer (physical and data link) performance parameters from a statistical viewpoint.
In this work, we consider the UL communication stream; i.e., communication from the MUs to the macrocell base station (MBS) and from the FUs to their corresponding FAPs. We assume that TDMA is used by the CUs (MUs and FUs) to access the wireless channel, and that the UL communications at the FCs are synchronized with those at the MC [19], and consequently are mutually synchronized. It is worth mentioning, that our statistical characterization is still valid under the assumption of asynchronous FU/MU operation. However, intra-FC synchronization needs to be maintained. We further assume that FUs residing in the same FC do not interfere with each other since they are scheduled in different time slots (TSs). Moreover, we assume that a MU that lies within the coverage area of a FC still communicates with the MBS, but it is scheduled on a TS that is orthogonal to the rest of the TSs used by the active FUs belonging to that FC. Hence, at any TS, there is at most one active user per FC. Although in our model, we consider TDMA as the MAC scheme, our system could be mapped into a TH-CDMA (Time-Hopping Code Division Multiple Access) system, where unlike [7], orthogonal codes are used by the users inside the same FC. In our model, each CU can only be in one of two states: On or Off; we use $\delta_i(t)$ to indicate CU $i$’s activity/state:

$$\delta_i(t) = \begin{cases} 
1 & \text{if user } i \text{ is active (On) at time } t \\
0 & \text{if user } i \text{ is inactive (Off) at time } t 
\end{cases}$$

Also, we assume that all FUs and MUs have the same average activity rate, which is denoted by $\bar{\delta}$. According to our model, there is at most only one active FU ($FU_i$) in each femtocell $FC_i$ at a given time slot $t$. Hence, we are interested in the interference caused by the neighboring active FUs and the neighboring active MUs at $FAP_i$. Given the users are located according to PPP, we model the interference’s spatial distribution as follows: We consider that $FAP_i$ is located at the center of a disk of radius $R$ representing the area of $FC_i$ covered by $FAP_i$. Since only $FU_i$ is active at time slot $t$ inside $FC_i$, then the interference at $FAP_i$ is only caused by out-of-cell interference and it comes from the active FUs located in the annulus $Z_1$ (delimited by the radii $R$ and $R_1$) and from the active MUs located in the annulus $Z_2$ (delimited by the radii $R$ and $R_2$) as shown in Fig. 1. $R_1$ and $R_2$ are chosen such that the interference due to FUs beyond $R_1$ (respectively MUs beyond $R_2$) is negligible. Although in real world settings wireless signals emitted by cellular users are subject to shadowing (slow fading), fast fading, and pathloss, we here assume that cellular users are slowly-moving or fixed so that their transmitted signal degradation is mainly dominated by shadowing (slow fading) and pathloss effects in compliance with ITU specifications. In this paper, we distinguish between three values of the pathloss exponent depending on the position of the cellular user (i.e. MU or FU). Let us denote $\alpha$ the pathloss exponent and $r_j$ the distance between cellular user $j$ and $FAP_i$. We have:

$$\alpha = \begin{cases} 
2 & \text{if } r_j < R \\
\alpha_1 & \text{if } R \leq r_j < R_1 \\
\alpha_2 & \text{if } r_j \geq R_1 
\end{cases}$$

with $\alpha_2 > \alpha_1 > 2$. This propagation model has been widely used to model the transmission in FC networks. We also adopt it in our work in order to gain some insights on the physical characteristics of FCs and their impact at the data link layer.

\footnote{Once turned on and before initiating any communication, FCs get synchronized to the cellular core network using an asymmetric communication link such as xDSL thanks to an enhanced version of IEEE 1588 [19].}
Unfortunately, if we consider the combined action of shadowing and fast fading, the problem becomes analytically intractable and difficult to come up with some insightful/useful results. Therefore, we assume that the physical channel gain is represented by a combination of path-loss and log-normal shadowing in compliance with the ITU specification [20]. Hence, the amplitude of the signal received by FAP$_i$ placed at a distance $r_j$ from FU$_j$ is $A_{ji} = S_j r_j^{-\alpha_1} P_j$, where $\alpha_1$ denotes the path loss exponent associated with the interfering FUs in the zone $Z_1$, $P_j$ the transmission power of FU$_j$, and $S_j$ the log-normal shadowing coefficient for the signal propagating from FU$_j$ to FAP$_i$ given as follows [21]:

$$S_j = 10^{-a(\xi_j/10)} 10^{-b(\xi_{ji}/10)}$$

(2)

where $a = b = \frac{1}{\sqrt{2}}$, $\xi_j$ and $\xi_{ji}$ are two independent realizations from a zero-mean normal random variable (RV) with standard deviation $\sigma_{\xi_j}$. $\xi_j$ represents the propagation environment local to FU$_j$ (the near field), while $\xi_{ji}$ deals with the propagation environment of the path between FU$_j$ and FAP$_i$ (the far field). It is also important to mention that for two different FUs $j$ and $m$, $\xi_{ji}$ and $\xi_{mi}$ are two independent identically distributed RVs. Likewise, the amplitude of the signal received by FAP$_i$ placed at a distance $r_k$ from MU$_k$ is $A_{ki} = S_k r_k^{-\alpha} P_k$, where $\alpha$ denotes the path loss exponent associated with the interfering MUs, $P_k$ the transmission power of MU$_k$, and $S_k$ the log-normal shadowing coefficient for the signal propagating from MU$_k$ to FAP$_i$ given as follows:

$$S_k = 10^{-a(\xi_k/10)} 10^{-b(\xi_{ki}/10)}$$

(3)

where $\xi_k$ and $\xi_{ki}$ are two independent realizations from a zero-mean normal RVs with standard deviation $\sigma_{\xi_m} > \sigma_{\xi_j}$. $\xi_k$ represents the propagation environment local to MU$_k$ (the near field), while $\xi_{ki}$ deals with the propagation environment of the path between MU$_k$ and FAP$_i$ (the far field). Also in this case, for two different MUs, $k$ and $m$, $\xi_{ki}$ and $\xi_{mi}$ are two independent identically distributed RVs.
B. Fractional Power Control

UL power control is considered as one of the fundamental approaches that helps mitigate the interference experienced by base stations in order to enhance the reliability and QoS of wireless networks. In this paper, we assume that both FUs and MUs implement and use the recently proposed fractional power control approach [22], which is being investigated by some wireless operators such as Motorola [23] and Siemens [24]. In our work, we use the fractional power control scheme proposed in [22], tailored to the case where the wireless propagation environment is rather dominated with log-normal shadowing. Recall that in our analysis we assume that the amplitude of the signal received at FAP$_i$ located at a distance $r_i$ from its associated FU$_i$ is:

$$A_{ii} = 10^{-a(\xi_i/10)} 10^{-b(\xi_{ii}/10)r_i^{-2}} P_i$$ (4)

Moreover, we assume that both $\xi_i$ and $\xi_{ii}$ are constant during the coherence interval (slow fading), and that its values could be obtained at FU$_i$ from its associated FAP$_i$. Based on this assumption, our fractional power control scheme is designed in order to get rid of the near-field shadowing $\xi_i$ and to reduce the impact of the far-field shadowing $\xi_{ii}$ as follows:

$$P_i = \frac{10^{a(\xi_i/10)} 10^{s_1(\xi_{ii}/10)} P_{fu}}{E \left[ 10^{a(\xi_i/10)} 10^{s_1(\xi_{ii}/10)} \right]}$$ (5)

In (5), $s_1$ is an exponent chosen from the interval $[0, 1]$ in order to compensate the effect of the far-field channel propagation loss $\xi_{ii}$. $P_{fu}$ is the average FU transmission power satisfying $0 < P_{fu} \leq P_{f}^{max}$, with $P_{f}^{max}$ being the maximum transmission power allowed per FU. Moreover, observe that in the power control rule (5), we used the normalizing factor $E \left[ 10^{a(\xi_i/10)} 10^{s_1(\xi_{ii}/10)} \right]$ so that on average we have $E[P_i] = P_{fu}$; that is the average transmission power per FU does not exceed the maximum power $P_{f}^{max}$. We also assume that the same power control policy is used by the MUs. Hence, the UL transmission power of MU$_k$ is:

$$P_k = \frac{10^{a(\xi_k/10)} 10^{s_2(\xi_{ko}/10)} P_{mu}}{E \left[ 10^{a(\xi_k/10)} 10^{s_2(\xi_{ko}/10)} \right]}$$ (6)

where $\xi_k$ represents the propagation environment local to MU$_k$, $\xi_{ko}$ deals with the propagation environment of the path between MU$_k$ and its MBS, and $P_{mu}$ is the average MU transmission power satisfying $0 < P_{mu} \leq P_{m}^{max}$, with $P_{m}^{max} > P_{f}^{max}$ being the maximum transmission power allowed per MU.

III. Interference Analysis

In this section, we derive a statistical characterization of the UL interference in FC networks. We first derive its average and variance, and then derive its probability density function (PDF). In our FC network, we assume a TDMA operation where only one FU is active per FC per time slot. However, when the femto user FU$_i$ is communicating with its associated FAP$_i$ at time slot $t$, its signal may be affected by the transmissions of the neighboring active FUs and MUs. Hence the interference at FAP$_i$ at time slot $t$ can be expressed as:

$$I(t) = \sum_{j \in \mathcal{A}_{F}} \delta_j(t) r_j^{-\alpha_1} S_j(t) P_j(t) + \sum_{k \in \mathcal{A}_{M}} \delta_k(t) r_k^{-\alpha} S_k(t) P_k(t)$$ (7)
The interference expression consists of two sums: the first one is over the set of neighboring active FUs, $Z_{F1}$, confined in the region $Z_1$, and the second one is over the set of neighboring active MUs, $Z_{M2}$, confined in the region $Z_2$. Let $X_j(t) = S_j(t)P_j(t), \forall j \in Z_{F1}$ and $X_k(t) = S_k(t)P_k(t), \forall k \in Z_{M2}$.

$$X_j(t) = 10^{-\alpha \xi_j(t) - \beta \xi_j(t)/10} \left( \frac{10^{\alpha \xi_j(t)/10}}{\mathbb{E} \left[ 10^{\alpha \xi_j(t)/10} \right]} \right)$$

$$= \frac{10^{((\alpha \xi_j(t) - \beta \xi_j(t))/10)} P_{fu}}{\mathbb{E} \left[ 10^{((\alpha \xi_j(t) + \beta \xi_j(t))/10)} \right]}$$

(8)

with $\xi_{jj}, \xi_{ji}$, are i.i.d (independent identically distributed) whose distribution is a Gaussian with zero mean and standard deviation $\sigma_{\xi_j}$ for all $j \in Z_{F1}$. Hence, $X_j(t)$s are i.i.d log-normal RVs with mean $\mu_1$ and variance $\sigma_1^2$ for all $j \in Z_{F1}$. Using some basic operations on independent normal variables as well as relationship between the statistics of a log-normal RV and its associated normal variable we can easily show that:

$$\mu_1 = \frac{\mathbb{E} \left[ 10^{((\alpha \xi_j(t) - \beta \xi_j(t))/10)} \right] P_{fu}}{\mathbb{E} \left[ 10^{((\alpha \xi_j(t) + \beta \xi_j(t))/10)} \right]} = P_{fu}$$

(9)

$$\sigma_1^2 = \left( \exp \left( \left( \frac{s_1^2 + b^2}{2} \right) \ln \left( \frac{10}{\sigma_{\xi_j}} \right) - 1 \right) \right) P_{fu}^2$$

(10)

Likewise, as far as the MUs are concerned, we have:

$$X_k(t) = 10^{-\delta \xi_k(t) - \mu \xi_k(t)/10} \left( \frac{10^{\delta \xi_k(t)/10}}{\mathbb{E} \left[ 10^{\delta \xi_k(t)/10} \right]} \right)$$

$$= \frac{10^{((\delta \xi_k(t) - \mu \xi_k(t))/10)} P_{mu}}{\mathbb{E} \left[ 10^{((\delta \xi_k(t) + \mu \xi_k(t))/10)} \right]}$$

(11)

with $\xi_{k0}, \xi_{ki}$, are i.i.d RVs distributed according to a zero-mean Gaussian with standard deviation $\sigma_{\xi_m}$ for all $k \in Z_{M2}$. Hence, $X_k(t)$s are i.i.d log-normal RVs with mean $\mu_2$ and variance $\sigma_2^2$ for all $k \in Z_{M2}$.

$$\mu_2 = \frac{\mathbb{E} \left[ 10^{((\delta \xi_k(t) - \mu \xi_k(t))/10)} \right] P_{mu}}{\mathbb{E} \left[ 10^{((\delta \xi_k(t) + \mu \xi_k(t))/10)} \right]} = P_{mu}$$

(12)

$$\sigma_2^2 = \left( \exp \left( \left( \frac{s_2^2 + b^2}{2} \right) \ln \left( \frac{10}{\sigma_{\xi_m}} \right) - 1 \right) \right) P_{mu}^2$$

(13)

Thus, we have shown that the interference $I(t)$ experienced at $FAP_1$ is the sum of the independent log-normal RVs related to the FU interferers and the MU interferers: $X_j(t), j \in Z_{F1}$ with mean $\mu_1$ and variance $\sigma_1^2$ and $X_k(t), k \in Z_{M2}$ with mean $\mu_2$ and variance $\sigma_2^2$ respectively. In the rest of the paper, we will use the interference expression given by (14) to carry out our statistical analysis.

$$I(t) = \sum_{j \in Z_{F1}} \delta_j(t) r_j^{-\alpha_1} X_j(t) + \sum_{k \in Z_{M2}} \delta_k(t) r_k^{-\alpha} X_k(t)$$

(14)

For ease of derivation, we use the following notation: $I(t) = I_1(t) + I_2(t)$, with $I_1(t) = \sum_{j \in Z_{F1}} \delta_j(t) r_j^{-\alpha_1} X_j(t)$ and $I_2(t) = \sum_{k \in Z_{M2}} \delta_k(t) r_k^{-\alpha} X_k(t)$. 
Theorem 1: The average $\mu_I$ and the variance $\sigma_I^2$ of the interference at $FAP_i$ can be expressed as

\[
\sigma_I^2 = \frac{\pi \delta (\lambda_1 (\sigma_1^2 + \mu_1^2) + \lambda_2 (\sigma_2^2 + \mu_2^2))}{\alpha_1 - 1} \left( \frac{1}{R_2^{2(\alpha_1 - 1)}} - \frac{1}{R_1^{2(\alpha_1 - 1)}} \right)
+ \frac{\pi \delta \lambda_2 (\sigma_2^2 + \mu_2^2)}{\alpha_2 - 1} \left( \frac{1}{R_2^{2(\alpha_2 - 1)}} - \frac{1}{R_1^{2(\alpha_2 - 1)}} \right)
\]

(15)

\[
\mu_I = \frac{2\pi \delta (\lambda_1 \mu_1 + \lambda_2 \mu_2)}{\alpha_1 - 2} \left( \frac{1}{R_2^{\alpha_1 - 2}} - \frac{1}{R_1^{\alpha_1 - 2}} \right)
+ \frac{2\pi \lambda_2}{\alpha_2 - 2} \delta \mu_2 \left( \frac{1}{R_2^{\alpha_2 - 2}} - \frac{1}{R_1^{\alpha_2 - 2}} \right)
\]

(16)

Proof: The proof of this theorem uses the law of total expectation, the law of total variance and Campbell’s theorem for PPP [25]. We have $\mu_I \triangleq \mathbb{E}[I(t)] = \mathbb{E}[I_1(t)] + \mathbb{E}[I_2(t)]$. Moreover the two sums $I_1(t)$ and $I_2(t)$ are independent since the two PPPs $\phi_1$ and $\phi_2$ are independent, the activity of MUs and FUs are independent, and the shadowing factors of the different interfering users are also mutually independent. Hence, $\sigma_I^2 \triangleq \mathbb{V}[I(t)] = \mathbb{V}[I_1(t)] + \mathbb{V}[I_2(t)]$. In the rest of this proof, we will only present the derivation of $\mathbb{E}[I_1(t)]$ and $\mathbb{V}[I_1(t)]$ (the derivation of $\mathbb{E}[I_2(t)]$ and $\mathbb{V}[I_2(t)]$ uses exactly the same techniques). Using the law of total expectation,

\[
\mathbb{E}[I_1(t)] = \mathbb{E}_r \left[ \mathbb{E}_X \left[ I_1(t) | r, \delta \right] \right] = \mathbb{E}_r \left[ \mu_1 \delta \sum_{j \in Z_{F_1}} r_j^{-\alpha_1} \right]
\]

By applying Campbell’s Theorem, we get:

\[
\mathbb{E}[I_1(t)] = \int_R^{R_1} \frac{\mu_1 \delta}{r^{\alpha_1}} 2\pi \lambda_1 r \, dr = \frac{2\pi \lambda_1 \mu_1 \delta}{\alpha_1 - 2} \left( \frac{1}{R_2^{\alpha_1 - 2}} - \frac{1}{R_1^{\alpha_1 - 2}} \right)
\]

On the other hand, using the law of total variance we have $\mathbb{V}[I_1(t)] = \mathbb{V}[\mathbb{V}(I_1(t)|r, \delta)] + \mathbb{V}[\mathbb{E}[I_1(t)|r, \delta]]$ with:

\[
\mathbb{E}[\mathbb{V}[I_1(t)|r, \delta]] = \sigma_I^2 \delta \mathbb{E} \left[ \sum_{j \in Z_{F_1}} (r_j^{-\alpha_1})^2 \right]
\]

\[
\mathbb{V}[\mathbb{E}[I_1(t)|r, \delta]] = \mu_1^{-2} \delta^2 \mathbb{E} \left[ \left( \sum_{j \in Z_{F_1}} \frac{1}{r_j^{\alpha_1}} \right)^2 - \sum_{j \in Z_{F_1}} \frac{1}{r_j^{2\alpha_1}} \right]
+ \mu_1^{-2} \delta \left( \mathbb{E} \left[ \sum_{j \in Z_{F_1}} \frac{\delta}{r_j^{\alpha_1}} \right] - \mathbb{E} \left[ \sum_{j \in Z_{F_1}} \frac{\delta}{r_j^{\alpha_1}} \right]^2 \right)
\]

On the other hand, we have:

\[
\mathbb{E} \left[ \left( \sum_{j \in Z_{F_1}} \frac{1}{r_j^{\alpha_1}} \right)^2 \right] = \mathbb{E} \left[ \sum_{j \in Z_{F_1}} \frac{1}{r_j^{2\alpha_1}} + \sum_{i \neq j \in Z_{F_1}} \frac{1}{(r_i r_j)^{\alpha_1}} \right]
\]

\[
= \mathbb{E} \left[ \sum_{j \in Z_{F_1}} \frac{1}{r_j^{2\alpha_1}} \right] + \mathbb{E} \left[ \sum_{j \in Z_{F_1}} \frac{1}{r_j^{\alpha_1}} \right]^2
\]
Hence, $\mathbb{V}[\mathbb{E}[I(t)|r, \delta]] = \mu_1^2 \delta \mathbb{E} \left[ \sum_{j \in Z_{F1}} (r_j^{-\alpha_1})^2 \right]$. Thus:

$$\mathbb{V}[I(t)] = \delta (\sigma_1^2 + \mu_1^2) \frac{\pi \lambda_1}{\alpha_1 - 1} \left( \frac{1}{R^2(\alpha_1 - 1)} - \frac{1}{R_1^2(\alpha_1 - 1)} \right)$$

Knowing the statistics of UL interference is useful to the design of FC networks and to the improvement of its PHY layer performance. For instance, it could be used to optimize the fractional power control exponent $s_1$ used by the FUs so that the average UL interference experienced at the FAP is minimized. Deriving the PDF of the interference is useful as well, for e.g. in non-cooperative systems whose operations rely on the estimation of the interference [26]. Observe that the interference expression at $FAP_i$ is nothing but the sum of independent log-normal RVs. Hence, using the Fenton-Wilkinson approximation [27] about the distribution of the sum of log-normal RVs, it follows

**Corollary 1:** At any time slot $t$, $I(t)$ is a log-normal random variable whose PDF is

$$f_I(x) = \frac{1}{\sqrt{2\pi} x \sigma_{eq}} \exp \left( -\frac{(\ln x - \mu_{eq})^2}{2\sigma_{eq}^2} \right)$$

(17)

where $\mu_{eq} = \ln \left( \frac{\mu_1^2}{\sqrt{\sigma_1^2 + \mu_1^2}} \right)$ and $\sigma_{eq}^2 = \ln \left( \frac{\sigma_1^2 + \mu_1^2}{\mu_1^2} \right)$.

**Proof:** Providing an accurate PDF of the UL interference $I(t)$ is mathematically intractable since it is expressed as the sum of log-normally distributed RVs and the number of summands follows a Poisson distribution. Therefore, we approximated it using the following approach. We have divided the problem of finding the PDF of $I(t)$ into two sub-problems: (i) Determining the nature of the probability distribution that statistically characterizes the aggregate interference at $FAP_i$ (normal, lognormal, etc.) (ii) Characterizing the shape of this distribution via its associated mean and variance. In order to answer part (i), we have used the Fenton-Wilkinson approach which states that the sum of a finite number of independent log-normal distributions is a log-normal distribution. This approach is actually more accurate than the central limit theorem since it applies independently of the number of RVs in the sum (whether it is high or low), moreover it is more specific since it only applies to the log-normal distribution type of PDF. Part (ii) has already been computed in Theorem 1, in which we have taken into account that the interferer locations are described by homogeneous PPPs.

### IV. SIR AND OUTAGE PROBABILITY

In this section, we first derive some statistical characteristics of the UL signal to interference ratio (SIR) that allowed us characterize the link outage probability. Then, we study the temporal auto-correlation of the SIR for the case of stationary CUs, as well as for the case of slowly-moving CUs using the uniform mobility model.
A. Statistical Characterization

Taking into account the wireless propagation model and the fractional power control described in Section II, the signal transmitted by FU to its FAP placed at a distance $r_i$ is:

$$A_{ii}(t) = \frac{10^{((s_l-b)\xi_{ii}(t)/10)} P_{fu} r_i^{-2}}{\mathbb{E} \left[ 10^{\xi_{ii}(t)/10} 10^{s_l(\xi_{ii}/10)} \right]}$$  \hspace{1cm} (18)

Hence, the SIR of $FU_i$ transmitting at time slot $t$ to its associated $FAP_i$ can be written as:

$$\gamma(t) = \frac{10^{((s_l-b)\xi_{ii}(t)/10)} P_{fu} r_i^{-2}}{\mathbb{E} \left[ 10^{\xi_{ii}(t)/10} 10^{s_l(\xi_{ii}/10)} \right] I(t)}$$  \hspace{1cm} (19)

We notice from (19) that the SIR $\gamma(t)$ is equal to the ratio of two independent log-normal random variables. Hence, we conclude that $\gamma(t)$ is log-normally distributed as shown in Theorem 3 and its proof.

**Theorem 2:** At any time slot $t$, the SIR $\gamma(t)$ of the transmission from $FU_i$ to $FAP_i$ in $FC_i$ is a log-normal random variable whose PDF is

$$f_{\gamma}(u) = \frac{1}{\sqrt{2\pi u\sigma_{s-eq}}} \exp \left( -\frac{(\ln u - \mu_{s-eq})^2}{2\sigma_{s-eq}^2} \right)$$  \hspace{1cm} (20)

where $\mu_{s-eq} = \ln \left( \frac{\mu_s^2}{\sigma_s^2 + \mu_s^2} \right)$, $\sigma_{s-eq}^2 = \ln \left( \frac{\sigma_s^2 + \mu_s^2}{\mu_s^2} \right)$, and $\mu_s$ and $\sigma_s$ are the average and variance of the SIR, given by

$$\mu_s = P_{fu}(\bar{r})^{-2} e^{-\mu_{eq} + \frac{\sigma_{eq}^2}{2\pi\lambda FAP}}$$  \hspace{1cm} (21)

$$\sigma_s^2 = \frac{P_{fu}(\bar{r})^2}{\pi^2} \left( \frac{\sigma_{eq}^2}{2\pi\lambda FAP} \right) \left( \frac{\ln(10)}{10} \sigma_j \right)^2 + \sigma_{eq}^2 - 1 \right) e^{\sigma_{eq}^2 - 2\mu_{eq}}$$  \hspace{1cm} (22)

And $\bar{r}$ is the average distance between $FU_i$ and its $FAP_i$

$$\bar{r} = \frac{1}{2\sqrt{2\pi\lambda FAP}}$$  \hspace{1cm} (23)

**Proof:** For analytical tractability, in this proof, we will replace $r_i$ the distance separating $FAP_i$ from $FU_i$ in the SIR expression (19) by $\bar{r}$ defined as the average distance between a FU and its associated FAP. It has been shown that the distance between a point $u$ and the nearest point from a point process $X$ with intensity $\lambda$ is Rayleigh-distributed with mean $m = \frac{1}{2\sqrt{2\pi}\lambda}$ [25]. By applying this to our network settings, we get the average distance between a FU and its associated FAP, which happens to be the nearest one among its neighboring FAPs, is $\bar{r} = \frac{1}{2\sqrt{2\pi}\lambda_{FAP}}$. Let $Y = 10^{((s_l-b)\xi_{ii}(t)/10)} = e^{(s_l-b)\xi_{ii}(t)(\ln(10)/10)} = e^Z$, with $Z \sim \mathcal{N} \left( 0, (s_l^2 + b^2) \left( \frac{\ln(10)}{10} \sigma_j \right)^2 \right)$. Moreover, from the analysis made in the previous section, the interference $I(t)$ experienced at $FAP_i$ is log-normally distributed. That is $I(t) = e^X$, with $X \sim \mathcal{N} \left( \mu_{eq}, \sigma_{eq}^2 \right)$. Hence, we can write:

$$\gamma(t) = \frac{P_{fu}(\bar{r})^{-2} e^{Z-X}}{\mathbb{E} \left[ 10^{\xi_{ii}(t)/10} 10^{s_l(\xi_{ii}/10)} \right]}$$

with $Z$ and $X$ being two independent normal variables. The independence property of these two variables can be easily deduced from the fact that the random variables $\xi_{ii}$, $\xi_{jj}$, $\xi_{ji}$, $\xi_{kk}$ and $\xi_{kj}$ are mutually independent $\forall j \in Z_{F1}, j \neq i$ and $\forall k \in Z_{M2}$.
Hence, \((Y - Z) \sim N \left( -\mu_{eq}, \left( (s_1^2 + b^2) \left( \frac{\ln(10)}{10} \sigma_f \right)^2 + \sigma_{eq}^2 \right) \right) \). Consequently, \(\gamma(t)\) is log-normally distributed, with mean

\[ \mu_s \triangleq \mathbb{E}[\gamma(t)] = P_{fu}(\bar{r})^{-2}e^{-\mu_{eq} + \frac{\sigma_{eq}^2}{2}} \]

and variance

\[ \sigma_s^2 \triangleq \mathbb{V}[\gamma(t)] = \frac{P_{fu}^2}{\bar{r}^4} \left( e^{(s_1^2 + b^2) \left( \frac{\ln(10)}{10} \sigma_f \right)^2 + \sigma_{eq}^2} - 1 \right) e^{\sigma_{eq}^2 - 2\mu_{eq}} \]

In addition, we assume that the transmission from \(FU_i\) to \(FAP_i\) fails if its SIR \((\gamma)\) is below a certain defined threshold \(\gamma^{th}\). This is the case if the interference at \(FAP_i\) is high enough compared to the amplitude of the signal transmitted by \(FU_i\), so that this FAP cannot detect it.

**Corollary 2:** The outage probability \(P_o \triangleq \mathbb{P}(\gamma < \gamma^{th})\) of \(FU_i\’s\) transmission to \(FAP_i\) is:

\[ P_o = \frac{1}{2} \text{erfc} \left( \frac{-\ln(\gamma^{th}) - \mu_{s-\text{eq}}}{\sqrt{2\sigma_{s-\text{eq}}^2}} \right) \quad (24) \]

**B. The Temporal Auto-Correlation of the SIR**

In previous works, SIR realizations are assumed independent across time. However, this is not always the case, especially when the interferers positions are correlated across time. In our analysis, we assume that the nodes are stationary or (at most) moving slowly. Therefore, in the following we derive the temporal autocorrelation of the UL SIR \(\gamma\) corresponding to the transmission of \(FU_i\) to \(FAP_i\) at two different TSs \(t_1\) and \(t_2\). These TSs are chosen from a given coherence interval during which the channel propagation gain as well as the transmission power used by \(FU_i\) remain constant, so that the signal received at \(FAP_i\) from \(FU_i\) could be approximated by a constant \(K_i\). Here, we distinguish between two cases:

**Case(1)—Mobile interferers:** We consider that the interferers, the MUs and the FUs, are moving with constant speeds \(v_1\) and \(v_2\) respectively, and their displacement direction is described by an angle \(\theta\) uniformly distributed in \([0, 2\pi]\).

**Case(2)—Stationary interferers:** We consider \(v_1 = v_2 = 0\).

**Theorem 3:** The temporal autocorrelation of SIR \((\gamma)\) of the transmission from \(FU_i\) to \(FAP_i\) at slots \(t_1\) and \(t_2\) \((t_1 < t_2)\) is:

**Under case(1)—Mobile interferers:**

\[ R_\gamma(\tau) = \frac{K_i^2 \mathbb{E} \left[ \frac{1}{\bar{r}^4 (\beta_1 X_j + \beta_2 X_k) (\beta_3 X_j + \beta_4 X_k)} \right] - \mu_s^2}{\sigma_s^2} \] \quad (25)

where \(\tau = t_2 - t_1\), \(X_j\) and \(X_k\) are log-normal shadowing coefficients related to FUs and MUs respectively (as defined in (14)), and

\[ \beta_1 = \frac{2\pi \lambda_1}{\alpha_1 - 2} \left( \frac{1}{R_{\alpha_1} - 2} - \frac{1}{R_{\alpha_1}^{\gamma_1} - 2} \right) \]

\[ \beta_2 = \frac{2\pi \lambda_2}{\alpha_1 - 2} \left( \frac{1}{R_{\alpha_2} - 2} - \frac{1}{R_{\alpha_2}^{\gamma_1} - 2} \right) + \frac{2\pi \lambda_2}{\alpha_2 - 2} \left( \frac{1}{R_{\alpha_2} - 2} - \frac{1}{R_{\alpha_2}^{\gamma_2} - 2} \right) \]
\[
\beta_3 = \int_{R} \int_{0}^{2\pi} \frac{\lambda_1 r}{(r^2 + (\bar{\tau}^2 r)^2 + 2\bar{\tau} r \cos(\theta))^\frac{\alpha_1}{2}} \, dr \, d\theta
\]
\[
\beta_4 = \int_{R} \int_{0}^{2\pi} \frac{\lambda_2 r}{(r^2 + (\bar{\tau}^2 r)^2 + 2\bar{\tau} r \cos(\theta))^\frac{\alpha_2}{2}} \, dr \, d\theta
\]

**Under case (2)—Stationary interferers:**

\( \beta_1 = \beta_3 \) and \( \beta_2 = \beta_4 \), thus:

\[
R_\gamma(\tau) = \frac{K_i^2 \mathbb{E}\left[ \frac{1}{\sigma^2 (\beta_1 X_j + \beta_2 X_k)^2} \right] - \mu_s^2}{\sigma_s^2}
\] (26)

**Proof:** Given that SIR realizations are identically distributed but correlated across time, the SIR temporal autocorrelation at the time slots \( t_1 \) and \( t_2 \) \( (t_1 < t_2) \) is:

\[
R_\gamma(\tau) = \frac{\mathbb{E}[\gamma(t_1)\gamma(t_2)] - \mu_s^2}{\sigma_s^2}
\]

where

\[
\mathbb{E}[\gamma(t_1)\gamma(t_2)] = K_i^2 \mathbb{E}\left[ \frac{1}{I(t_2)I(t_1)} \right]
\]

\[
= K_i^2 \int_0^{+\infty} \int_0^{+\infty} \mathbb{E}\left[ e^{-(xI(t_1)+yI(t_2))} \right] \, dx \, dy
\]

By further decomposing the interference into two interference terms induced by the neighboring FUs and MUs as in (14), it follows that

\[
\mathbb{E}[\gamma(t_1)\gamma(t_2)] = K_i^2 \left( \int_0^{+\infty} \int_0^{+\infty} \mathbb{E}\left[ e^{-(xI_1(t_1)+yI_2(t_2))} \right] \, dx \, dy \right) \mathbb{E}\left[ e^{-(xI_2(t_1)+yI_2(t_2))} \right]
\]

(27)

When considering mobile interferers, for all \( j \in Z_{F_1} \) and \( k \in Z_{M_2} \), we have

\[
r_j(t_2) = \sqrt{r_j(t_1)^2 + (\bar{\tau}^2 r)^2 + 2\bar{\tau} r \cos(\theta)}
\]

(28)

\[
r_k(t_2) = \sqrt{r_k(t_1)^2 + (\bar{\tau}^2 r)^2 + 2\bar{\tau} r \cos(\theta)}
\]

(29)

On the other hand for any point process \( \phi \), its Laplace functional is defined as

\[
L_\phi(f) \triangleq \mathbb{E}\left[ e^{-\int_{x \in \mathbb{Z}} f(x) \phi(dx)} \right] \mathbb{E}\left[ e^{-\sum_{x \in \mathbb{Z}} f(x)} \right]
\]

(30)

Using (28) and (29), and applying (30) yield

\[
\mathbb{E}\left[ e^{-(xI_1(s)+yI_1(t))} \right] = e^{-\bar{\tau} X_j(\beta_1 x + \beta_2 y)}
\]

where

\[
\beta_1 = \int_{R}^{R_1} r^{-\alpha_1} 2\pi \lambda_1 r \, dr = \frac{2\pi \lambda_1}{\alpha_1 - 2} \left( \frac{1}{R_1^{\alpha_1-2}} - \frac{1}{R_1^{\alpha_1-2}} \right)
\]
\[ \beta_3 = \int_R^{R_1} \int_0^{2\pi} \frac{\lambda_1 r}{r^2 + (v_1 r)^2 + 2v_1 r \cos(\theta)} \frac{dr}{r} d\theta \]

Likewise,
\[ \mathbb{E} \left[ e^{-(x I_2(t_1) + y I_2(t_2))} \right] = e^{-\mathcal{F}_k(\beta_2 x + \beta_4 y)} \]

where
\[ \beta_2 = \int_R^{R_2} \int_0^{2\pi} \frac{2\pi \lambda_2}{\alpha_1 - 2} \left( \frac{1}{R_1^{\alpha_1 - 2}} - \frac{1}{R_1^{\alpha_1 - 2}} \right) \]
\[ + \frac{2\pi \lambda_2}{\alpha_2 - 2} \left( \frac{1}{R_1^{\alpha_2 - 2}} - \frac{1}{R_2^{\alpha_2 - 2}} \right) \]
\[ \beta_4 = \int_R^{R_2} \int_0^{2\pi} \frac{\lambda_2 r}{r^2 + (v_2 r)^2 + 2v_2 r \cos(\theta)} \frac{dr}{r} d\theta \]

Hence, it follows that
\[ \mathbb{E} [\gamma(t_1) \gamma(t_2)] = K^2 \mathbb{E} \left[ \left( \int_0^{+\infty} \int_0^{+\infty} e^{-\mathcal{F}_j(\beta_1 x + \beta_3 y)} \right) \right] \]
\[ \mathbb{E} [\gamma(t_1) \gamma(t_2)] = K^2 \mathbb{E} \left[ \frac{1}{\mathcal{F}^2 (\beta_1 X_j + \beta_2 X_k)(\beta_3 X_j + \beta_4 X_k)} \right] \]

The characterization of the temporal auto-correlation of the SIR in FCs is important. In fact, it helps characterize the correlation of transmission failures over time. Thus, it provides useful information for the design of retransmission strategies, or power control schemes for efficient reliable FC networks.

V. SYSTEM CAPACITY AND DELAY PERFORMANCE

In this section, we characterize the asymptotic capacity (i.e. steady state capacity) of a FC network. We determine the delay characteristics for CBR (constant bit rate) traffic in FC networks, and derive an upper bound on the achievable asymptotic FC service/throughput while taking into account the interference analysis done in the previous sections. First, by assimilating a FC to a D/G/1 queuing system, we characterize the average delay per FU. Then, we derive the probability that it exceeds a certain delay threshold. We further explain the derived delay result through an example of CBR, delay constrained type of traffic: Voice over IP (VoIP). Finally, we study the asymptotic achievable throughput in FC networks.

A. Delay Characterization:

Since time is fairly shared among the FUs in the same FC, and the interferers are assumed to be spatially distributed according to a homogeneous PPP, then we can safely assume that the average packet delay experienced by any active FU in a given FC is the same. Therefore, to characterize the delay performance of a FC, it suffices to characterize it for one of its active FUs.
Moreover, from any active FAP’s viewpoint, the spatial and temporal distribution of the interferers have the same statistical characterization. Therefore, the statistical delay characterization that we provide hereafter for $FC_i$ applies for any active FC deployed inside our MC.

In this section, we characterize the per packet average delay at $FU_i$. Recall that in our system we assume that the FCs use TDMA as a channel access technique. That is, we assume that time is slotted and at every time slot only one FU is active per FC. Moreover, we assume that each active FU generates $\nu$ packets of voice traffic in its assigned time slot. Hence, given that $FC_i$ contains multiple active FUs ($n_f$ active FUs), $FAP_i$ experiences an arrival of traffic with a constant data rate equal to $\nu$ packets per time slot. Hence, $FC_i$ could be assimilated to a $D/G/1$ queuing system with a constant packet arrival rate equal to $\nu$ packets per time slot, served by a wireless channel with a per-packet average service time $\chi = \mathbb{E}[\chi]$, where $\chi$ is a random variable representing the packet service time. Our aim is to derive the packet’s average waiting time ($W$) in the queue of $FU_i$ as well as its average service time $\chi$, in order to deduce the per-packet average total delay ($D = W + \chi$) at $FU_i$. In our analysis, we further assume that FCs are heavily loaded. That is, the active FUs always have traffic to send in their assigned time slots. Hence, using Kingman’s heavy traffic approximation, the steady state average queuing delay in our system is:

$$ W = \frac{\nu(\chi^2 - (\mathbb{E}[\chi])^2)}{2(1 - \nu \mathbb{E}[\chi])} \quad (31) $$

In fact, Kingman’s heavy traffic approximation [28] states that for a heavy loaded $G/G/1$ system with an average packet arrival rate $\nu$ and average service time $\chi$, the average waiting time is $W = \frac{\nu(\sigma^2_a + \sigma^2_s)}{2(1 - \nu \chi)}$, with $\sigma^2_a$ being the variance of the packet inter-arrival times, and $\sigma^2_s$ the variance of their service times. Note that in our case the $D/G/1$ system is a particular case of the $G/G/1$ system with the difference that the packet inter-arrival times are deterministic in our case, that is $\sigma^2_a = 0$, leading then to Eq. (31). Now, all what remains to approximate $D$ is to derive the first and second order moments of the service time $\chi$ and $\chi^2$.

In our system, we define the average packet service time ($\chi$) as the average delay between the instant it is initially transmitted by $FU_i$ and the instant of its successful reception at $FAP_i$. Moreover, we assume that a transmission attempt failure is solely due to excessive interference; i.e., due to high transmission powers of neighboring interferers causing $\gamma < \gamma^{th}$. Hence:

$$ \chi \triangleq \sum_{k=0}^{+\infty} T(k)\mathbb{P}(success|k) $$

$$ = \sum_{k=0}^{+\infty} T(k)(1 - P_o(t_{k+1})) \prod_{j=1}^{k} \mathbb{P}(\gamma(t_j) < \gamma^{th}) $$

$$ = \sum_{k=0}^{+\infty} T(k)(1 - P_o(t_{k+1}))\mathbb{P}(\max_{j=1,k} \gamma(t_j) < \gamma^{th}) $$

$$ = \sum_{k=0}^{+\infty} T(k)(1 - P_o)P_o^k $$

where $T(k)$ denotes the delay corresponding to $k$ retransmissions, $t_j$ corresponds to the time slot of the $j^{th}$ transmission attempt of the packet, and the expression of the outage probability $P_o$ is given by Eq. (24). In the above derivation, we assumed that $\gamma(t)$ (for any time slot $t$) are i.i.d. Hence, basic order statistics (with some algebraic manipulation) yield the last line of the above derivation. Given our system settings, it is easy to show that the average delay of $k$ retransmissions is $T(k) = (1 + n_f k)$.
Plugging this value in the last line of the above derivation, we get two sums that, using some known results in geometric series, lead to the following expression

\[
\bar{\chi} = 1 + \frac{n_f P_o}{1 - P_o}
\]  
(32)

On the other hand, the second moment of the service time is

\[
\bar{\chi}^2 \triangleq \sum_{k=0}^{+\infty} (T(k))^2 (1 - P_o) P_o^k
\]

Using the same calculus techniques for the derivation of Eq. (32), we can write

\[
\bar{\chi}^2 = 1 + 2\nu P_o + \nu^2 P_o^2 \left(\frac{1 - P_o}{1 - \nu P_o}\right)^2
\]  
(33)

Thus, we conclude the following result:

**Theorem 4:** The average packet delay in a TDMA heavy-loaded FC system in which FUs are scheduled in a round-robin fashion is:

\[
D = 1 + \frac{n_f P_o}{1 - P_o} + \nu \left(\frac{\bar{\chi}^2 - (\bar{\chi})^2}{2(1 - \nu \bar{\chi})}\right)
\]  
(34)

where \(\bar{\chi}\) is given by Eq. (32) and \(\bar{\chi}^2\) is given by Eq. (33).

Now that we have derived the per-packet average delay, we further assume that our system is delay sensitive and has a delay constraint expressed as

\[
P(D > D_{max}) < \epsilon
\]  
(35)

where \(D_{max}\) is the maximum allowed per packet delay, and \(\epsilon\) is a design parameter that will be explained later in this section through an example. It has been shown in [29] that given a system with a constant data arrival rate \(\nu\) and a variable channel capacity \(C(t)\), the probability of \(D(t)\) exceeding a delay bound \(D_{max}\) satisfies:

\[
\sup_t P(D(t) > D_{max}) \approx f(\nu)e^{-g(\nu)D_{max}}
\]

where \(D(t)\) denotes the delay experienced by the packet generated at time \(t\), and \(f(\nu), g(\nu)\) are two functions of the source data rate \(\nu\). Note that this implicitly assumes that the \(t^{th}\) packet delay \(D(t)\) is exponentially distributed. Hence, we can easily show that \(\frac{f(\nu)}{g(\nu)} = \mathbb{E}[D(t)]\) and \(f(\nu) = P(D(t) > 0)\). In our system, we are making discrete time analysis (time is slotted) where the delay is expressed in terms of number of time slots. Therefore, we will use the geometric distribution (with parameter \(p = 1 - P_o\)) as a discrete approximation of the exponential distribution to derive \(f(\nu)\). This approximation is legitimate when the time slot duration is small enough, which is the case in cellular networks in general where a time slot is approximately equal to 1 to 2 milliseconds. Thus, knowing that by definition \(D(t) \geq 0\), we have:

\[
f(\nu) \triangleq P(D(t) > 0) = 1 - p = P_o
\]
and consequently
\[ g(\nu) \triangleq \frac{f(\nu)}{\mathbb{E}[D(t)]} = \frac{p_o}{\overline{D}}. \]

**Theorem 5:** For CBR traffic, the probability of the packet delay exceeding a threshold \( D_{\text{max}} \) in FC networks satisfies
\[
\sup_t \mathbb{P}(D(t) > D_{\text{max}}) \approx p_o e^{-p_o \frac{D_{\text{max}}}{\overline{D}}}
\]  
(36)

This result can be useful for many applications, such as call admission control, cross-layer QoS-aware network design, etc.

**Illustrative example:** Consider a FC network whose FUs are scheduled in a TDMA fashion, and where each FU sends \( \nu \) voice packets at its assigned slot. We know that in order to have an acceptable QoS for voice, the per-packet delay should not exceed \( D_{\text{max}} = 400 \text{ ms} \), and the packet loss rate should not exceed about 3% [30]. Therefore, if a packet delay exceeds \( D_{\text{max}} \), it is considered lost. The maximum allowed packet loss rate is nothing but the parameter \( \epsilon \) introduced in Eq. (35). From Eq. (36), it then follows that the delay constraint is
\[
p_o e^{-p_o \frac{D_{\text{max}}}{\overline{D}}} \leq \epsilon
\]  
(37)

Using the expression of \( \overline{D} \) given in (Eq. 34) and solving (Eq. 37) for \( p_o \) yield the maximum allowed physical outage probability tolerated per FU in order to satisfy (37).

**B. Asymptotic Capacity:**

The instantaneous channel capacity (at time slot \( j \)) is defined via the Shannon formula as \( C(j) = b \log(1 + \gamma(j)) \), where \( b \) denotes the channel bandwidth. Hence, for FC networks, the service provided to FU \( i \) by the wireless channel is defined as 
\[
S(t) \triangleq \sum_{j=1}^{t} C(j).
\]

Inspired by the effective bandwidth, Wu and Negi proposed the effective capacity theory [29], which is the dual of the original effective bandwidth theory [31]. The effective capacity is defined as the maximum constant arrival rate that a given service process can support in order to guarantee a QoS requirement specified by the QoS exponent \( g(\nu) \). In our case, the effective capacity is nothing but the maximal achievable throughput under the maximum delay constraint, specified by the QoS exponent \( g(\nu) = \frac{p_o}{\overline{D}} \).

Analytically, the effective capacity can be formally defined as follows. Let the sequence \( C(j); j = 1, 2, \ldots \) (\( C(j) \) is the channel capacity at time slot \( j \)) denote a discrete time stationary and ergodic stochastic service process and \( S(t) \triangleq \sum_{j=1}^{t} C(j) \) be the partial sum of the service process. Assume that the Gärtner – Ellis limit of \( S(t) \), expressed as
\[
\Lambda_c(u) = \lim_{t \to +\infty} \frac{1}{t} \log \left( \mathbb{E} \left[ e^{uS(t)} \right] \right).
\]
exists and is a convex function differentiable for all real \( u \). Then the effective capacity of the service process, denoted by \( E_c(u) \), where \( u > 0 \), is defined as [29]:
\[
E_c(u) \triangleq -\Lambda_c(-u) u = -\lim_{t \to +\infty} \frac{1}{ut} \log \left( \mathbb{E} \left[ e^{-uS(t)} \right] \right)
\]  
(38)

Based on our physical-layer framework developed in Section IV, we now derive an upper bound on the network effective capacity, which is stated in the following theorem.
Theorem 6: In the high-SIR regime, the effective capacity of a FC network is upper bounded as follows:

$$E_c(u) < \mathbb{E} [\log(\gamma)]$$ (39)

Proof:

Case 1: Assuming i.i.d. SIR realizations across time: At the high-SIR regime, for any time slot $j$, we have $\log(1 + \gamma(j)) \sim \log(\gamma(j))$, with $\log(\gamma(j))$ is normally distributed with parameters: $\mu \triangleq \mathbb{E} [\log(\gamma(j))] = \mu_{s-eq}$ and $\sigma^2 \triangleq \mathbb{V} [\log(\gamma(j))] = \sigma_{s-eq}^2$. Let $Y_j = \log(\gamma(j))$ where the index $j$ refers to the $j^{th}$ time slot. Hence, $Y_j \sim \mathcal{N}(\mu, \sigma)$. Since the random variables $Y_j; j = 1, 2, \ldots$ are assumed i.i.d., the sum $Y = \sum_{j=1}^{N} Y_j$ is normally distributed with mean $\mu_Y = N\mu$ and variance $\sigma_Y^2 = N\sigma^2$. It follows that

$$\mathbb{E} [e^{-uS(t)}] = \lim_{N \to +\infty} \mathbb{E} \left[ e^{-u \sum_{j=1}^{N} (\frac{\gamma}{1}) \log(1+\gamma(j))} \right],$$

with:

$$\mathbb{E} \left[ e^{-u \sum_{j=1}^{N} (\frac{\gamma}{1}) \log(1+\gamma(j))} \right] \approx \mathbb{E} \left[ e^{-u \sum_{j=1}^{N} (\frac{\gamma}{1}) Y_j} \right] \leq \mathbb{E} \left[ e^{-u(\frac{\gamma}{1}) Y} \right] \leq M_Y \left( -\frac{u}{N} \right) = e^{-\left(\frac{t}{mu}\right) + \frac{1}{2} \left( \frac{\mu_y}{u^2} \right)^2}$$

Here, $M_Y$ denotes the moment generating function of the random variable $Y = \sum_{j=1}^{N} \log(\gamma(j))$. Calculating the limit of the obtained result as $N$ the number of time slots/samples goes to infinity yields

$$\mathbb{E} [e^{-uS(t)}] \leq e^{-\left(\frac{t}{mu}\right)}$$ (40)

Thus, the asymptotic network capacity, assuming time independent SIR realizations, is upper bounded as follows:

$$E_c(u) \triangleq - \lim_{t \to +\infty} \frac{1}{ut} \log \left( \mathbb{E} \left[ e^{-uS(t)} \right] \right) \leq \mu$$ (41)

Case 2: Assuming dependent but identically distributed SIR realizations across time: Note that in the case of identically-distributed but time-correlated realizations of the SIR, under the high-SIR regime, we can proceed the same way as in case 1 discussed above to characterize the asymptotic capacity of FC networks, with the only difference is the fact that $Y = \sum_{j=1}^{N} Y_j$ is no longer normally distributed (but we still have $Y_j \sim \mathcal{N}(\mu, \sigma)$). Hence,

$$\mathbb{E} \left[ e^{-u \sum_{j=1}^{N} (\frac{\gamma}{1}) Y_j} \right] \leq \int e^{-u \sum_{j=1}^{N} (\frac{\gamma}{1}) y_j} f_Y(y) dy$$ (42)

where $f_Y(y)$ is the multivariate normal joint distribution function of the RVs $(Y_j; j = 1, 2, \ldots, N)$ defined as:

$$f_Y(y) = (2\Pi)^{-\frac{n}{2}} |R_y|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} (y^T \mu^T) R_y^{-1} (y - \mu) \right)$$

where $R_y$ denotes their covariance matrix. Thus, all we need is to find an "adequately" chosen upper bound for the integral in (42), in order to obtain a finite upper bound of the asymptotic network capacity. Below, we present our approach to bound this quantity. Let us define the random vector $z = y - \mu$. Substituting this random variable in the right hand side (RHS) of the
The integral of (42) yields
\[
\int e^{-u \sum_{j=1}^{N} (\frac{1}{u})y_j} f_Y(y) dy = (2\Pi)^{-\frac{N}{2}} |R_y|^{-\frac{1}{2}} e^{-N\sum_{j=1}^{N} (\frac{1}{u})z_j} \times \left( \int e^{-u \sum_{j=1}^{N} (\frac{1}{u})z_j} e^{-\frac{1}{2}z^T R_y^{-1} z} |J| dz \right)
\]  
(43)

where \(|J|\) is the determinant of the Jacobian matrix (defined by: \(J_{mn} = \frac{\partial y_m}{\partial z_n}\)). Note that in this case, \(|J| = 1\). Moreover, we assume that at the high SIR-regime, \(Y_j > E[Y_j]\), for any time slot \(j\), and hence, \(Z_j > 0; \forall j\), implying that \(\forall u > 0, e^{-u \sum_{j=1}^{N} (\frac{1}{u})z_j} < 1\). Injecting this inequality in (25) yields
\[
E\left[e^{-u \sum_{j=1}^{N} (\frac{1}{u})Y_j}\right] \leq e^{-Nu \sum_{j=1}^{N} (\frac{1}{u})Z_j} (2\Pi)^{-\frac{N}{2}} |R_y|^{-\frac{1}{2}} \times \int e^{-\frac{1}{2}z^T R_y^{-1} z} dz
\]  
(44)

Moreover, we know that given \(K\) a symmetric positive definite matrix, the multidimensional Gaussian integral satisfies:
\[
\int \exp\left(-\frac{1}{2}x^T K^{-1} x\right) dx = (2\Pi)^{\frac{N}{2}} |K|^\frac{1}{2}
\]  
(45)

Since \(Y_j, j = 1, 2, ..., N\) are identically distributed, the covariance matrix \(R_y\) is symmetric positive-definite. Hence, the integral obtained in the RHS of (44) is nothing but the multidimensional Gaussian integral. Thus, by using the result in (45) and applying it to the RHS of (44), we get the same upper bound as in (40): \(E\left[e^{-u \sum_{j=1}^{N} (\frac{1}{u})Y_j}\right] \leq e^{-Nu \sum_{j=1}^{N} (\frac{1}{u})Z_j}\). Then, by taking the limit as \(N \rightarrow +\infty\), we get the same upper bound as in the time uncorrelated case: \(E\left[e^{-uS(t)}\right] \leq e^{-tu\mu}\). Thus, for this case, we also have:
\[
E_c(u) \triangleq - \lim_{t \rightarrow +\infty} \frac{1}{ut} \log \left( E\left[e^{-uS(t)}\right] \right) \leq \mu
\]

\[\square\]

As far as the low-SIR regime is concerned, for any time slot \(j\), \(\log(1 + \gamma(j)) \sim \gamma(j)\), with \(\gamma(j)\) log-normally distributed. Due to some computational complexity related to log-normal distribution, and the non-existence of a moment generating function for this type of distribution, we were not able to find an upper bound on the FC network asymptotic capacity at the low-SIR regime.

VI. NUMERICAL RESULTS

Using the physical model discussed in Section II, we apply Monte Carlo numerical techniques to simulate the co-channel interference observed at the FAP for \(10^6\) samples. At each sample instant, the locations of the active MU and FU interferers are generated as a realization of their corresponding PPPs, and their shadowing coefficients as realizations of their related log-normal distributions. In our simulation, we use the same PHY propagation parameters as in [32] and [20] and fix the PPP intensities, unless otherwise stated. The main simulation parameters are summarized in Table I.

In Fig. 2, we plot the theoretic outage probability derived in Eq. (24) and compare it with that obtained via Monte Carlo simulations. Note that there is a slight mismatch between the analytical curve and the simulated one since we derived the interference PDF using an approximation rather than an exact derivation (due to analytical intractability, as mentioned in Section III). The log-plot of this outage probability (see Fig. 3) shows that the analytical and the simulated outage probability
Fig. 2. The Physical Outage Probability

Fig. 3. The Physical Outage Probability (Log-Scale)
Fig. 4. The Outage Probability as a function of the FU and MU density

Fig. 5. The Outage Probability as a function of the FU density
### Table I
Summary of Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum FU Power $P_{f_{\text{max}}}$</td>
<td>0.125 Watt</td>
</tr>
<tr>
<td>Maximum MU Power $P_{m_{\text{max}}}$</td>
<td>1 Watt</td>
</tr>
<tr>
<td>Femto SINR Threshold $\gamma_{\text{th}}$</td>
<td>3.2 dB</td>
</tr>
<tr>
<td>FC Coverage Radius (R)</td>
<td>7 m</td>
</tr>
<tr>
<td>Interference Zone $Z_1$ radius ($R_1$)</td>
<td>50 m</td>
</tr>
<tr>
<td>Interference Zone $Z_2$ radius ($R_2$)</td>
<td>100 m</td>
</tr>
<tr>
<td>Indoor Pathloss Exponent</td>
<td>2</td>
</tr>
<tr>
<td>Pathloss Exponent $\alpha_1$</td>
<td>3</td>
</tr>
<tr>
<td>Pathloss Exponent $\alpha_2$</td>
<td>5</td>
</tr>
<tr>
<td>FU Shadowing Standard deviation $\sigma_{\xi_f}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>MU Shadowing Standard deviation $\sigma_{\xi_m}$</td>
<td>6 dB</td>
</tr>
<tr>
<td>Average activity rate $\delta$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>FU Spatial density $\lambda_1$</td>
<td>0.15</td>
</tr>
<tr>
<td>MU Spatial density $\lambda_2$</td>
<td>0.02</td>
</tr>
<tr>
<td>Power Control exponents $s_1$ and $s_2$</td>
<td>$\frac{1}{\sqrt{2}}$</td>
</tr>
</tbody>
</table>

![Figure 6](image.png)

**Fig. 6.** Delay Characterization for CBR traffic in FC network

coincide at high SIR regime, but they do not at low SIR regime (under -5 dB) and the gap between these two curves increases under -10 dB.

On the other hand, in Fig. 4 and Fig. 5, we illustrate the evolution of the outage probability as a function of the FU density for different MC loads (i.e. load in MUs). This curve is of a paramount importance since it constrains the density and consequently the number of active FUs that could be accepted in the underlying MC to meet a desired value of the outage probability. Hence, it would be useful for the design of admission control mechanisms. For instance, in order to maintain the outage probability at the FAP $P_o \leq 0.1$, the density of active FUs in the MC should not exceed 0.03 for a MU density $\lambda_2 \approx 0.001$. Finally, using the
theoretical delay derivation in Eq. (36), we plot the delay constraint \( P(D > D_{max}) \) for a CBR traffic with constant rate equal to 64kbps whose tolerated packet loss rate is \( \epsilon \leq 0.1 \). Fig. 6 shows that the packet loss probability \( P(D > D_{max}) \) increases slightly as the number of active FU's per FC \( (n_f) \) increases. However, it increases considerably as the physical outage probability \( P_o \) increases. Thus, a two tier FC/MC network with high density in FU's would have a low loss rate for CBR traffic only if its outage probability is maintained at a low level. One way to achieve this goal would be the design of interference-aware power control scheme.

VII. CONCLUSION

In this paper, we derived statistical characterizations of UL interference, SIR, and outage probability. Our analysis showed that UL interference and SIR in two-tier Poisson-distributed FC networks can be represented by a log-normal distribution. Moreover, we modeled and characterized link delay, data loss rate, and effective throughput of CBR delay-constrained traffic in two-tier FC networks. These derived results can be used to characterize many multimedia applications, such as interactive real-time gaming, voice, and video applications. This paper opens up several issues for future research on resource management in FC networks, including interference-aware fractional power control design, call admission control design, and the extension of the current results to multiple-tier heterogeneous networks.

VIII. ACKNOWLEDGMENT

This work was supported by NSF under NSF CAREER award CNS-0846044. The authors would like to thank Prof. Thinh Nguyen for his valuable feedback and comments.

REFERENCES


