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Mary D. Beisiegel

A teacher's mathematical knowledge for teaching has been shown to have a positive correlation with their students' success (Monk, 1994). So, when half of the students that start out in a STEM major switch out to a non-STEM major before graduation to in large part to instructors pedagogical methods and inadequate teaching we must ask what does the mathematical knowledge for teaching look like at this level (Lowery, 2010; Seymour & Hewitt, 1997). To address this, there has been a lot of research in developing professional training activities for graduate teaching assistants that would build their mathematical knowledge for teaching. This research is being conducted without ever having looked at the mathematical knowledge for teaching that these graduate teaching assistants possess. In this research over a series of interviews with four graduate teaching assistants asking them questions about their mathematical knowledge for teaching. The results of this study are present here in this thesis.

The Mathematical Knowledge for Teaching of First Year Mathematics Graduate Teaching Assistants: The Case of Exponential Functions

by
Matthew Keeling

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Major Professor, representing Mathematics

Chair of the Department of Mathematics

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Matthew Keeling, Author

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Table of Contents

LIST OF FIGURES

Chapter 1: Introduction

Since Lee Shulman (1986) identified the different forms of knowledge needed by teachers and then Ball, Thames, and Phelps (2008) adaptation of those forms to a framework for the knowledge needed to teach mathematics, researchers have been studying teachers' mathematical knowledge for teaching. Ball et al.'s framework consisted of six categories of knowledge: common content knowledge, specialized content knowledge, horizon content knowledge, knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum. The results of these studies have shown that there is a positive correlation between students' success and teachers' mathematical knowledge for teaching (Monk, 1994). However, the research of Monk (1994) and many others has been conducted at the K-12 level. This leads to a natural question of whether or not this relation holds at higher levels.

In order to study knowledge of mathematics for teaching at higher levels one needs to establish a framework at that level (Speer, King, Howell, 2014). In an attempt to do this there have been numerous frameworks created: the knowledge quartette (Rowland, Huckstep, Thwaites, 2003), knowledge of algebra for teaching (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012), and pedagogical content knowledge for secondary and post-secondary mathematics (Hauk, Toney, Jackson, Nair, & Tsay, 2014) to name a few. While these frameworks have been designed for higher level mathematics, one framework is still considered to be the best and that is Ball et al.'s (2008) mathematical knowledge for teaching. While this framework has some issues when being adapted outside of the K-8 grades, such as the differentiation between common content knowledge and specialized content knowledge (Speer et al., 2014), it is still one of the most widely used frameworks.

Why does this matter? Half of the students that start out in a STEM major at the university level will switch out to a non-STEM major before graduation (Lowery, 2010). In a study by Seymour and Hewitt (1997), they found that concerns about faculty's pedagogical methods and inadequate teaching were among the leading reasons for students to switch. These issues of pedagogy have been found to be most problematic in mathematics (Daempfle, 2002; Pemberton et al., 2004). So, where do most university faculty begin to develop their sense of mathematical knowledge for teaching? Many university faculty begin to develop their sense of mathematical knowledge for teaching as first year graduate teaching assistants in mathematics departments while earning their graduate degrees. What do these first year mathematics graduate teaching assistants know about mathematical knowledge for teaching?

Chapter 2: Literature Review

The research on mathematical knowledge for teaching at the college level is minimal in relation to the research on mathematical knowledge for teaching at the elementary levels, for which it was developed (Speer & King, 2009). Since Lee Shulman (1986) identified different forms of knowledge needed for teaching and the later adaptation of these forms to mathematics instruction by Ball, Thames, and Phelps (2008), there have been a multitude of research studies on mathematical knowledge for teaching at the elementary school setting. They have tested the effects of mathematical knowledge for teaching both empirically and qualitatively and showed a significant positive correlation between teachers' mathematical knowledge for teaching and students' mathematical achievements (Hill, Rowan, & Ball, 2005). However, within the college setting, the basic definitions of mathematical knowledge for teaching need to be reassessed to accommodate the fact that teachers' educational levels have changed from elementary educators to collegiate instructors and because the content, students and approaches to teaching are different (Speer & King, 2009; Speer, King, & Howell, 2014).

Within the area of mathematical knowledge for teaching research at the postsecondary level is the research investigating mathematics graduate teaching assistants' preparation, teaching, and mathematical knowledge for teaching. Graduate teaching assistants and pre-service teachers are not unlike one another, where pre-service teachers will go on to become teachers at the K-12 levels, many graduate teaching assistants will go on to become college instructors (Speer, Gutmann, & Murphy, 2005). The research articles on post-secondary teachers' mathematical knowledge for teaching evaluate different aspects of mathematical knowledge for teaching or look at it through different topics of post-secondary education (Holmes, 2012; Speer, Gutmann, & Murphy, 2005; Speer, Gutmann, & Murphy, 2009). However, they also state that more research is needed to better understand how the ideas of mathematical knowledge for teaching, as developed at the elementary level, adapt to post-secondary levels (Speer, King, & Howell, 2014). They point to the need for research on post-secondary instructors' mathematical knowledge for teaching using an appropriate adaptation of the concepts of mathematical knowledge for teaching (Speer, King, & Howell, 2014).

In this chapter I will provide descriptions of the theories of teacher knowledge, a detailed description of the mathematical knowledge for teaching framework, the effect of mathematical knowledge for teaching on students' achievement, and research on graduate teaching assistants. Finally, this will lead to my research question, what does that mathematical knowledge for teaching of first year graduate teaching assistants look like?

2.1 Theories of Teacher Knowledge

One question that is often asked is “what do teachers need to know to teach?” Many researchers have developed frameworks for looking at teachers’ knowledge, some look purely at the teacher's content knowledge (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956; Hiebert & Carpenter, 1992; Porter, 2002; Skemp, 1976; Skemp, 1993; Webb, 1997) while others focused on a more complete picture of knowledge needed for teaching (Ball, Hill, & Bass, 2005; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012; Rowland, Huckstep, & Thwaites, 2003; Shulman, 1986). Some of these frameworks may appear to be a framework for categorizing education goals, they can be connected to teacher knowledge through comparisons with the Diagnostic Teacher Assessment of Mathematics and Science. The Diagnostic Teacher Assessment of Mathematics and Science is a tool that assesses the depth of both a teachers’ conceptual knowledge and pedagogical content knowledge (Holmes, 2012). This assessment is comprised of items that reflect teachers’ depth of content knowledge with three levels of difficulty: memorization, understanding, and problem solving/ reasoning (Holmes, 2012). I will briefly go over all of these frameworks except for Ball et al.'s (2005) mathematical knowledge for teaching framework, which I will go into more detail in a later section of this review.

Bloom's Taxonomy

One of the first attempts to classify intellectual learning was conducted by behavioral psychologists and their resulting work yielded Bloom's Taxonomy of Educational Objectives, a framework that contained six levels of knowledge (Bloom & Krathwohl, 1956; Holmes, 2012). These six levels of knowledge were not just categories, but had a ranking structure that allowed the researchers to classify their levels from least complex to most complex as follows: knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom & Krathwohl, 1956; Krathwohl, 2002). These levels would later be revised by Anderson, Krathwohl, and Bloom (2001). This revision consisted of primarily renaming the levels and switching the last two, synthesis and evaluation (Krathwohl, 2002). For the purposes of this review I will focus on Bloom and Krathwohl's (1956) original levels and names. The first level consists of knowing and being able to recall previously learned information. The comprehension level is a basic understanding of the material. The third level, application, is using or carrying out of procedures in new situations. Analysis describes the ability to deconstruct material into its base parts to be able to realize new meanings. Synthesis is then the ability to put together these parts into something new. Lastly we have evaluation, this level refers to the ability to judge items and assesses value (Bloom & Krathwohl, 1956; Holmes, 2012; Krathwohl, 2002). These six levels make up Bloom's Taxonomy.

How does Bloom's Taxonomy relate with teacher knowledge? We can see similarities between Bloom's Taxonomy and the Diagnostic Teacher Assessment of Mathematics and Science as described by Holmes (2012) in figure 2.1. Here Holmes draws connections to Bloom's knowledge and the Diagnostic Teacher Assessment of Mathematics and Science's memorize, Bloom's Comprehension and Application levels with Diagnostic Teacher Assessment of Mathematics and Science's understanding, and lastly Bloom's analysis, synthesis, and evaluation with Diagnostic Teacher Assessment of Mathematics and Science's problem solving and reasoning.

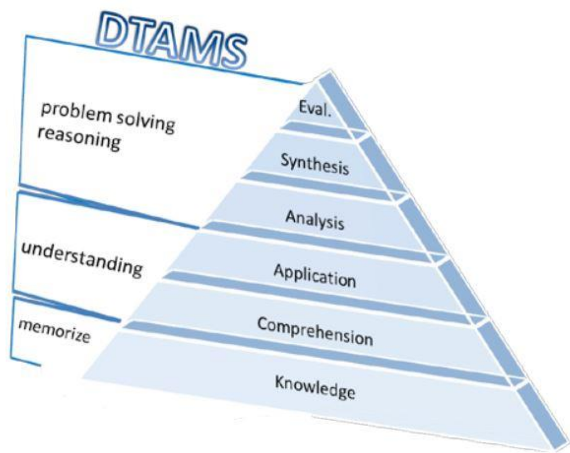

Figure 2.1: Comparison between Bloom's Taxonomies and the Diagnostic Teacher Assessment of Mathematics and Science.

Skemp's Instrumental and Relational Understanding Framework

Skemp's (1976) framework was different from Bloom and Krathwohl (1956) in that he only had two classes of teachers' mathematics knowledge- relational and instrumental understanding (Skemp, 1976). Much like Bloom et al., Skemp's two classes are hierarchical with relational understanding being the higher level of understanding than instrumental. To Skemp, relational understanding meant that the person had a "deep, conceptual understanding of the material" (Holmes, 2012, p. 58). While instrumental understanding is a basic understanding of the material, or "rules without reason" (Skemp, 2006, p.89). Skemp explained that simply memorizing the processes, algorithms, and facts of material leads to an instrumental understanding and that to achieve relational understanding you need to engage in deliberate conceptual comprehension (Holmes, 2012). Even though Skemp partitioned teachers' mathematics knowledge into two classes, one is able to identify Skemp's framework with Bloom's Taxonomy. In figure 2.2, we can see Bloom's Taxonomy depicted in the pyramid and its relationship to Skemp's framework as depicted by Holmes (2012).

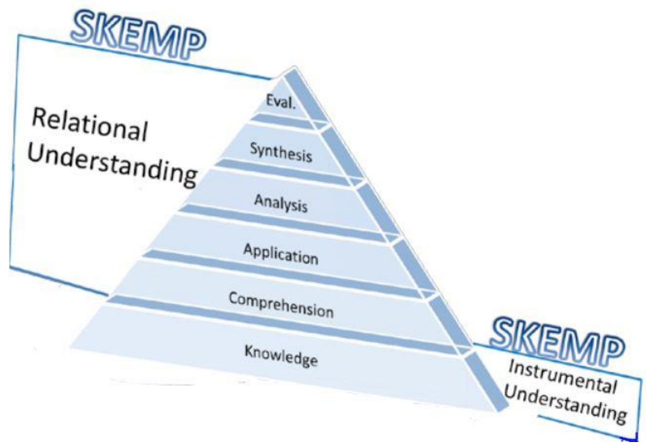

Figure 2.2: Bloom's Taxonomies compared with Skemp's Framework.

Hiebert and Carpenter's Procedural and Conceptual Understandings Framework

Similar to Skemp (1976), Hiebert and Carpenter (1992) used two categories to classify the mathematics knowledge used by teachers (Holmes, 2012). They classified knowledge into procedural knowledge, which can be thought of as the knowledge of formal language and symbols, rules and algorithms, and the fundamental how-to procedures of material. Their second category is conceptual knowledge, which is the knowledge of the connections and relationships between concepts and the processes (Hiebert & Carpenter, 1992; Holmes, 2012). What makes Hiebert and Carpenter's framework different from that of Skemp (1976) and Bloom and Krothwohl (1956) is the codependence of the two classifications (Holmes, 2012). Unlike the other two frameworks Hiebert and Carpenter (1992) contend that both knowledge types are a requirement for mathematical excellence.

Webb's Depth of Knowledge Framework

Webb's (2002) depth of knowledge (DOK) framework was designed with standards and assessments alignment in mind. However it still gives a classification of mathematical knowledge and so I will address it briefly. The DOK framework has four levels that resemble somewhat Bloom's Taxonomy (Holmes, 2012). His levels are as follows: recall, skill/ concept, strategic thinking, and lastly extended thinking (Webb, 2002). His first level can be thought of much like the first levels of all the frameworks we have discussed, recalling facts, definitions, terms, processes, and algorithms. His second level requires "engagement of some mental processing beyond a habitual response" (Webb, 2002, p. 5). This level would be things such as graphing and function classification (Holmes, 2012). The third level of DOK involves planning reasoning at a higher level than the previous two (Webb, 2002). Webb's final level of DOK includes reasoning, planning and developing over an extended period of time (Webb, 2002).

Porter's Cognitive Complexity Framework

Porter (2002) went about a similar path as Webb, as both were interested in assessment and standards alignment (Holmes, 2012). Porter's framework was divided into five different categories: memorize, perform procedures, communicate understanding, solve non-routine problems, and conjecture/generalize/prove (Porter, 2002). Porter's categories are labeled in such a way as they are both a label and a description. Porter's framework and Webb's DOK cover similar range of cognitive behaviors with the exception that Porter splits Webb's skill/concept into two categories: perform procedures and communicate understanding (Holmes, 2012). In figure 2.3 one can see two pyramids representing Webb and Porter's frameworks with arrows between

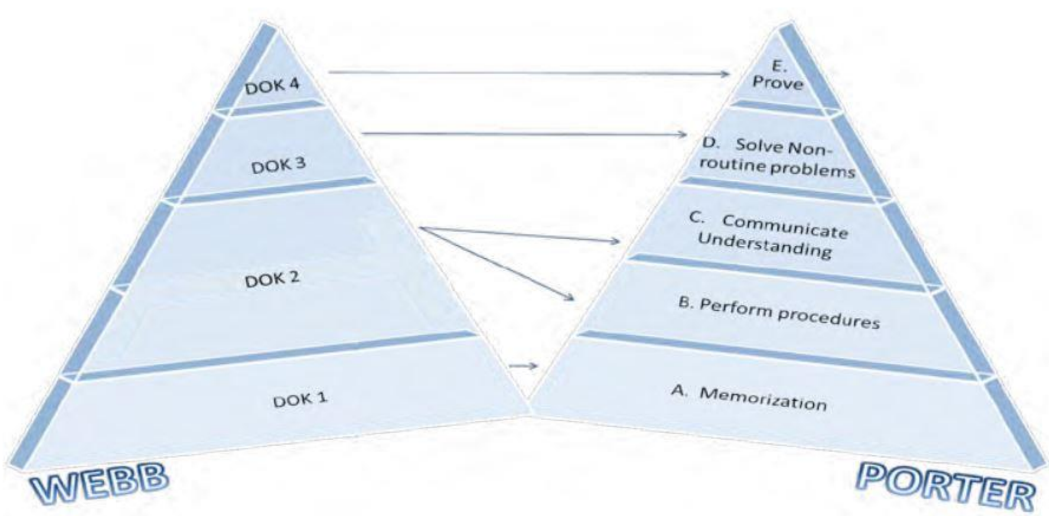

Figure 2.3: The frameworks of Webb (2002) and Porter (2002).

similar categories.

Now I will describe the frameworks that focused on a more complete picture of the knowledge needed for teaching.

Shulman and Pedagogical Content Knowledge

In the 1980s there was a gap in the education research field in regards to *teacher* content knowledge (Petrou & Goulding, 2011). In an attempt to fill in this gap, Lee Shulman and his team asked about what types of knowledge teachers use in their daily teaching tasks (Holmes, 2012; Shulman, 1986). In answering this question, Shulman (1986) developed a framework that involved three dimensions: content knowledge, pedagogical content knowledge, and curricular knowledge. He then broke these dimensions down further. For mathematics he broke down content knowledge into substantive knowledge, understanding and explaining key facts, concepts, and principles; and then syntactical knowledge, or the knowledge of the underlying mathematical concepts (Shulman 1986; Shulman & Grossman, 1988).

His second dimension, pedagogical content knowledge, is knowledge of the content that is most relevant to the act of teaching (Shulman, 1986). This type of knowledge contains items such as the most useful representation of different topics, powerful examples, and analogies of material. In addition to these items, this dimension of Shulman's framework includes the understanding of what makes a given topic easy or difficult for students as well as what types of preconceptions that students will bring with them to the lesson (Shulman, 1986).

In his final dimension, he considered a teacher's understanding of curricular knowledge, or the knowledge of the range of programs for a topic and their corresponding material for any given level (Shulman, 1986). This type of knowledge is important in allowing a teacher to have an understanding of what material was covered in previous courses their students have taken and what material will be covered in subsequent courses thus allowing to effectively plan their current course. This type of curricular knowledge is referred to as vertical curricular knowledge (Shulman, 1986). He then also discussed what he called lateral curricular knowledge. This is the knowledge of curriculum materials that the teachers are currently using for teaching. This knowledge allows the teacher to relate the content of their course with the content of other courses their students are currently taking (Shulman, 1986).

Knowledge Quartette

In 2003 Rowland, Huckstep, and Thwaites analyzed 24 videotaped lessons in an effort to create a framework depicting how a teacher's content knowledge played out in their teaching. This resulted in the creation of four superordinate dimensions comprised of a total of 18 subcategories (Huckstep, Rowland, & Thwaites, 2003; Rowland et al., 2003). These four dimensions are: foundation, transformation, connection, and contingency. The first of these dimensions is the knowledge of mathematics that we develop in the academic setting as learners (Rowland et al., 2003). This dimension also includes the beliefs that we develop as learners in the nature and purpose of mathematics as well as the "ability to articulate pedagogical issues" (Huckstep et al., 2003, p. 39).

The second dimension of the knowledge quartet is the dimension that refers to the teachers "knowledge-in-action" as they plan and carry out lessons in the classroom (Rowland et al., 2003, p. 98). This is the knowledge of how the teacher interacts with their pupils during a lesson or demonstration (Huckstep et al., 2003). This dimension contains the knowledge of different examples and representations that are more or less impactful within a given topic. Transformation and Shulman's pedagogical content knowledge have many similarities in that they both primarily consist of the knowledge or behavior that is directed towards learners.

In the connection dimension, Rowland et al. (2003) consider the knowledge of coherence between lessons, episodes, and courses. This is typically seen in either the planning to, or actual act of, teaching where teachers are making deliberate decisions on sequencing and connecting topics. They also consider the understanding of the cognitive demands being placed on individuals (Huckstep et al. 2003; Rowland et al., 2003).

The first three dimensions of the knowledge quartet have a rather clear relation to the three dimensions of Shulman (1986). However, in the knowledge quartet there is the fourth and final dimension of contingency. This final dimension is the ability to “think on one’s feet” (Huckstep et al., 2003 p. 41; Rowland et al., 2003, p. 98). Contingency is the understanding of when and how to go away from a planned lecture and how to respond to the unpredictable questions and answers that students inevitably provide. This dimension is distinct from mere possession of background knowledge and the deliberation and judgement involved in lesson planning (Huckstep et al., 2003, p. 41).

Knowledge of Algebra for Teaching

McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) developed a framework for the knowledge needed for the effective teaching of algebra. This framework differs from those previously mentioned in that the researchers focus on teaching a specific topic within mathematics. In their framework, McCrory et al. (2012) split the knowledge needed into two dimensions and then further divide these dimensions into three categories each. The two dimensions are mathematical content knowledge and mathematical uses of knowledge in teaching. The first dimension is further divided into school knowledge, advanced knowledge, and teaching knowledge. The mathematical uses of knowledge in teaching is divided into trimming, bridging, and decompressing.

McCrory et al. (2012) give a succinct simplification of each of their categories of mathematical content knowledge as follows:

> Knowing what they will teach (school knowledge of algebra); knowing more advanced mathematics that is relevant to what they will teach (advanced knowledge); and knowing mathematics that is particularly relevant for teaching and would not be typically taught in undergraduate mathematics courses (teacher knowledge). (p. 595)

For their categories of mathematical uses of knowledge in teaching, they relate them to categories of knowledge of previous researchers. Decompression is described as the ability of a teacher to take a finalized idea in mathematics that has been compressed down and work backwards towards its essential parts. Trimming is the act of scaling up or down, omitting details intentionally, or modifying the level of rigor to better deliver an idea or topic at the level of the students. This category also includes the ability to recognize when something has been trimmed too severely. The last category of bridging is similar to Shulman's (1986) category of curricular knowledge. Bridging is the work to "connect and link mathematics across topics, courses, content, and goals" (McCrory et al., 2012, p. 606).

Pedagogical Content Knowledge for Secondary and Postsecondary Mathematics

In a proposed model for pedagogical content knowledge in secondary and postsecondary mathematics, Hauk, Toney, Jackson, Nair, and Tsay, (2014) describe a model that includes four sectors. They use the three sectors that are laid out by Ball, Thames, and Phelps (2008) in their framework for pedagogical content knowledge, and then they add one more called knowledge of discourse (Hauk et al., 2013). Since I will discuss the three sectors of pedagogical content knowledge defined by Ball et al. (2008) later in more detail, I will focus on the fourth aspect of pedagogical content knowledge and how Hauk et al. (2013) describe the interactions between each sector. Before we can talk about knowledge of discourse (Hauk et al., 2013), we must first distinguish between what they call "little d" discourse and "big D" Discourse. They define "little d" discourse as language-in-use (Hauk et al., 2013). "Big D" discourse is comprised of "little d" discourse and "nuances of gesture, tone, hesitation or wait time, facial expression, hygiene, and other aspects that make for authenticity in an interaction" (Hauk et al., 2013, p. A22). Using these understandings of the ideas of discourse and Discourse Hauk et al. (2013) defines knowledge of Discourse as:

> Knowledge about the culturally embedded nature of (big D) discourse, including inquiry and forms of communication in mathematics (both in and out of educational settings). It includes what a teacher knows of normative and non-standard mathematical vocabularies, representations, and artifacts. (p. A26)

This framework can be displayed in a tetrahedron with a base consisting of Ball et al.'s (2008) pedagogical content knowledge and the other vertex being Hauk et al.'s (2013) knowledge of Discourse as shown in figure 2.4. The edges of the tetrahedron represent the interaction between the three categories of Ball et al. (2008) framework and the addition of the knowledge of Discourse. These categories of interaction are curricular thinking, anticipatory thinking, and implementation thinking (Hauk et al., 2013).

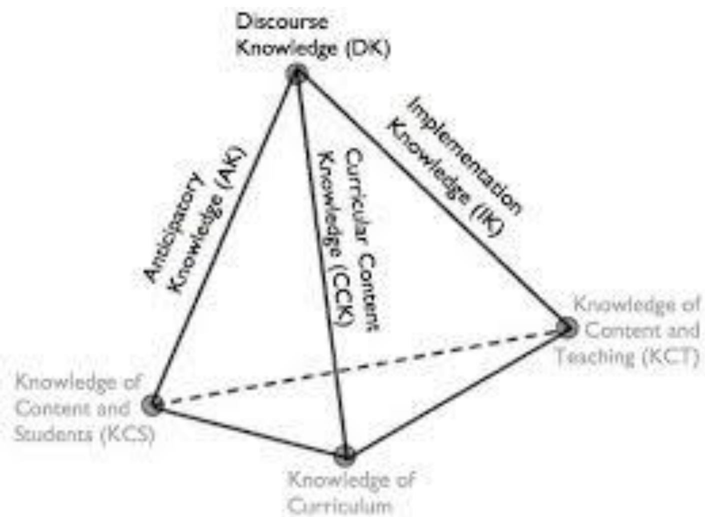

Figure 2.4: Tetrahedron model for visualizing pedagogical content knowledge components and their relationships to Discourse Knowledge.

2.2 Mathematical Knowledge for Teaching

Researchers have identified many domains of mathematical knowledge in general as well as for teaching, as we can see from above. While the specific boundaries and the names of the categories may vary, in this research I focus on one of the most agreed upon sets of categories (Firouzian, 2014): Ball, Thames, and Phelps's (2008) model (figure 2.5). To create these categories for teaching, Ball et al. (2008) observed elementary classrooms and tried to answer questions such as, "What are the recurrent tasks and problems of teaching mathematics? What do teachers do as they teach Mathematics?" (p. 395). While for years it was thought that all one needs to know to teach is the mathematics in the curriculum and some number of additional years of study, or the content in the curriculum only deeper in some way (Ball et al., 2008; Monk, 1994). The research of Ball and her team found that the content knowledge needed to teach was different from that needed by a research mathematician (Brodie, 2004; Hill & Ball, 2009). So what is it that mathematics teachers need to know that other mathematicians don't?

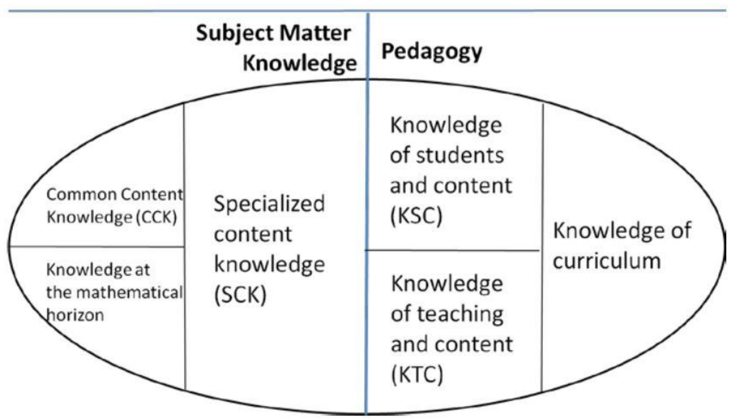

Figure 2.5: The Mathematical Knowledge for Teaching Framework (Ball, Thames, Phelps, 2008).

The basis out of which mathematical knowledge for teaching was developed comes from the 1986 paper by Lee Shulman. For many years it was asserted that someone either knows content and then pedagogical knowledge is secondary and less valuable, or that someone knows pedagogy and they are not accountable to content knowledge (Shulman, 1986). However, the current idea on knowledge for teaching is that it has an overlapping mixture of both content knowledge and pedagogical knowledge termed pedagogical content knowledge (Shulman, 1986). This theory was further developed in the area of teaching mathematics and became what is called mathematical knowledge for teaching (Ball, 2000; Ball, Lubienski, & Mewborn, 2001; Ball et al., 2008). The idea of mathematical knowledge for teaching is that a teacher's knowledge of mathematics for teaching can be divided into two main categories, each of which is divided into three distinct but overlapping categories (Ball et al., 2008). The two primary categories are subject matter knowledge and pedagogical content knowledge, with common content knowledge, horizon content knowledge, and specialized content knowledge making up the category of subject matter knowledge, and knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum making up pedagogical content knowledge. These categories were meant to allow scholars to be able to better identify and test a teacher's content knowledge for teaching (Ball et al., 2008).

In the overarching category of subject matter knowledge, two of its categories have a difficult to distinguish boundary. The category of common content knowledge is defined by Ball et al. (2008) to be the "mathematical knowledge and skill used in settings other than teaching" while they define specialized content knowledge as "the mathematical knowledge and skill unique to teaching" (p. 399-400). These two categories can be difficult to distinguish from each other since the general assumption is that "common content knowledge is knowledge held or used by an average mathematically literate citizen and that specialized content knowledge is different" (Speer, King, Howell, 2014, p. 114). The second of the two overarching categories, pedagogical content knowledge, is very broad and thus difficult to properly identify. Shulman (1986) defines pedagogical content knowledge as the knowledge of appropriate representations and formulations of the content that make it accessible and comprehensible to others (p.9). These categories were constructed from research into elementary teachers' practices, and as such must be looked at critically any time a researcher attempts to utilize this framework at a different level.

The adaptation of these categories from the elementary level to the post-secondary level will require a reworking not necessarily of the definitions themselves but of the relationships between the categories. Elementary teachers and postsecondary teachers typically differ in the extent of their content preparations. Most post-secondary mathematics instructors have at least a bachelor's degree in mathematics or a related field while elementary teachers typically don't hold such a degree. Therefore we must consider that what is specialized content knowledge for elementary teachers may very well be common content knowledge for post-secondary instructors (Speer et al., 2014). Speer and King (2009) call for more research to be conducted at the post-secondary level to identify parts of the existing theory that apply to this level and those parts that need to be refined in order to better fit the knowledge of post-secondary teachers and their peers. Knowledge such as that needed to look at student work and recognize its validity would be well within the category of specialized content knowledge in the elementary level, but that is something that research mathematicians do regularly and therefore it could fall into the common content knowledge when evaluating postsecondary teachers' mathematical knowledge for teaching (Speer et al., 2014). This is why we must refine our notions of the categories of mathematical knowledge for teaching from the elementary level when we apply them to post-secondary teachers.

2.3 Effects of Mathematical Knowledge for Teaching

With the mathematical knowledge for teaching framework at hand, many researchers have tested its relation to student achievement and have found significant relations between a teacher's mathematical knowledge for teaching and their students' achievements at the elementary levels (Hill et al., 2005). However, this does not mean that simply having teachers take more and higher level mathematics courses will improve students' achievements as teachers who have more content courses are not strongly correlated to higher student achievements (Monk, 1994). Both Monk (1994) and Hill et al. (2005) utilized quantitative methods to draw out their findings and they both found that a teacher's mathematical knowledge for teaching was positively correlated to students' success.

Other research has evaluated teachers' mathematical knowledge for teaching in qualitative ways. These studies typically involve the observation or interview of one or more teachers and then assessing their mathematical knowledge for teaching through thematic analysis or other qualitative methods of analysis. One such study was conducted by Hill and Ball (2009) where they observed teachers modeling subtraction involving negative integers. From this they noted that the mathematical knowledge needed for teaching was more than purely being able to solve problems and get to a correct answer (Hill & Ball, 2009). This result would imply that there is some kind of specialized knowledge needed to teach mathematics as was described by Ball et al. (2008). They go on to say that "conventional content knowledge seems to be insufficient for skillfully handling the mathematical tasks of teaching" (Hill & Ball, 2009, p.69).

To better understand the influence that pedagogical content knowledge has on a teacher's ability to effectively teach we look at the results of Johnson and Larsen (2012) and their observation of a college instructor's teaching practice. They found that the area of mathematical knowledge for teaching that restricted the ability of the instructor the most was their pedagogical content knowledge, primarily their knowledge of content and students. This reinforces the findings of Speer and Wagner (2009) that mathematicians, meaning teachers at the postsecondary level with doctorates in mathematics, may struggle when interpreting the mathematics posed by their students. Even for post-secondary mathematics teachers that have sufficient pedagogical content knowledge, the ability to bring the two together can prove to be very difficult as the two types of knowledge are typically taught as two disjoint concepts and rarely overlap (Ball, 2000). Ball (2009) identifies three things that we need to know to be able to bridge the gap between the two forms of knowledge: what types of content knowledge is needed for teachers at the postsecondary level, how that knowledge needs to be held, and what it takes for teachers to use that knowledge in practice.

2.4 Research on Graduate Teaching Assistants

While there is a wealth of studies that have developed and polished the definition of mathematical knowledge for teaching, developed assessment tools to evaluate a teacher's mathematical knowledge for teaching, and identified direct connections between a teacher's mathematical knowledge for teaching and their students' success, it was done predominantly from research on the practices of elementary teachers (Speer et al., 2014). With these developments, it was determined that there is a need for K-8 teachers to be better prepared for the practice of teaching. Recently, a similar acknowledgment has been made for post-secondary teachers and their preparation. Coinciding with this acknowledgement, researchers have also recognized that there is little research of mathematical knowledge for teaching at the post- secondary level (Hauk, Toney, Jackson, Nair, & Tsay, 2013; Speer et al, 2014). Further, Speer, Smith, and Horvath (2010) found that, while there are some research studies on post-secondary mathematics teaching, they were able to characterize the majority of them into two categories: reflections of past teaching experience and studies of student learning. This illustrates that the body of research on post-secondary mathematics teaching fails to ask the questions of what are teachers thinking, doing, saying, and asking in the classroom, and thus what their mathematical knowledge for teaching is (Speer et al., 2010). These are the types of questions that need to be answered in order to attempt to identify what theoretical pieces from the existing K-8 framework can be adapted and what pieces need to be refined. Just as the existence of the mathematical knowledge for teaching framework has allowed researchers to develop assessment tools to measure a K-8 teacher's mathematical knowledge for teaching, researchers could do the same at the post-secondary level and identify the relationships to student success in the post-secondary classroom.

The current body of research on graduate teaching assistants chiefly addresses professional development activities and programs, as well as their beliefs about the nature of teaching and learning mathematics (DeFranco & McGivney-Burelle, 2001). None of the research on mathematics graduate teaching assistants directly address the issues of the mathematical knowledge for teaching of a graduate teaching assistant.

Overall the research that has been conducted on mathematical knowledge for teaching at the postsecondary level is in its infancy when compared to the relatively new area of mathematical knowledge for teaching at the elementary level. Significant research is still to be conducted and is needed. As Kuhn says, effective research scarcely begins before a scientific community has answered questions like, what are the fundamental entities of which the universe is composed (2012, p. 5). In the case of post-secondary mathematical knowledge for teaching research, the universe would be knowledge needed to teach mathematics at the postsecondary level. Through the investigations of

Shulman (1986) and Ball, Thames, and Phelps (2008) as well as more recent investigations, there are numerous potential frameworks through which researchers could identify a framework for mathematical knowledge for teaching at the post-secondary level.

Once there is an established framework, researchers will be able to excel in this area and we will be able to gain an insight into the knowledge of post-secondary teachers. In doing this, our post-secondary students would, theoretically, have a better understanding of the content that is being taught to them. This would then cause those that eventually go on to become K-12 teachers to be better prepared to teach their students.

From the literature presented above we can see that there are many gaps in the research that remain to be filled. While a consistent framework of teacher knowledge that is generally agreed upon in the research on undergraduate mathematics education community is clearly needed, we can still proceed cautiously without one. After we look past the need for a framework we can see that the research being done at the post-secondary level is mostly reflections of past teaching experience and studies of student learning (Speer et al., 2010). This shows the hole in the body of knowledge with regard to the types of knowledge held by instructors and graduate teaching assistants at the post-secondary level.

2.5 Exponential Functions

In mathematics education, it is difficult to study each aspect of mathematics. Thus, some researchers will focus on a topic and make generalizations from their observations within that topic to other areas of mathematics. We can see Doerr (2006) and McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) taking a particular focus. What about exponential functions then? Ball (2003) identified algebra as one of three focal areas in her proposed research agenda for the RAND Mathematics Study Panel (2003). Exponential functions have also been cited as a critical research cite because of their importance in modeling populations growth, radioactive decay, compound interest, musical scales, complex analysis, calculus, and differential equations (Confrey, 1994; Confrey & Smith, 1995; Weber, 2002a; Weber, 2002b). The topic of exponential functions is a rich area for investigating mathematical knowledge for teaching as research has shown that students' understanding of exponentials is limited and they make fundamental errors when solving problems (Hewson, 2013; Weber, 2002a; Weber, 2002b).

This leads into my primary research question; what does the mathematical knowledge for teaching exponential functions of first year mathematics graduate teaching assistants look like?

Chapter 3: Methods

In this chapter I will describe the process used to recruit participants to enroll in this study and the population of graduate teaching assistants. I will also explain the data that was collected and the rationale for the data collection methods. Finally, I will explain the process of analyzing the data.

3.1 Recruitment and Graduate Teaching Assistant Population

The data for this research study was collected over the winter and spring terms in 2015 at a university in the Pacific Northwest. In an attempt to gain an understanding of the mathematical knowledge for teaching of first year mathematics graduate teaching assistants, I wanted to recruit three to five first mathematics graduate teaching assistants. The reason for choosing first year math graduate teaching assistants instead of any graduate teaching assistant is that I wanted to explore the mathematical knowledge for teaching that graduate teaching assistants possess at the beginning of their time as a graduate teaching assistant. In order to recruit first year mathematics graduate teaching assistants, I obtained a list of all the first year graduate teaching assistants from the mathematics department website and placed a recruitment letter in each of the first year graduate teaching assistants' department mailboxes. One week later, I sent out a recruitment email to all of the first year graduate teaching assistants. Both the original letter and the follow-up email can be found in appendix A. Out of all the first year graduate teaching assistants contacted, four self-selected to participate in the study. All four of these graduate teaching assistants completed the entire study.

Each of the four participants was in charge of teaching three to four recitation sections for math courses. In the table below there are both personal and academic attributes of the four participants that are relevant to this thesis.

	Pam	Jim	Kelly	Michael
Age	23	23	22	32
Educational background	BS Mathematics	BS Mathematics	BS Mathematics	BS Business
Planned final degree	Ph.D.	Ph.D.	MS	Undecided
Major	Mathematics	Mathematics	Mathematics	Statistics
Prior teaching experience	None	None	Job Shadow in community college classroom.[1]	Taught two non-math courses.[2]
Have you TA'd College Algebra	Yes	Yes	Yes	No

1: Participant job shadowed a community college instructor in two courses, Introductory Algebra and Trigonometry.
2: Participant taught a philosophy and a dream analysis course.

The study consisted of three interviews, each interview lasting approximately one hour, so it is possible that the time commitment required of the graduate teaching assistants could have influenced their decision to self-select. The graduate teaching assistants were not paid for their participation in the study. Prior to the first interview each participant was assigned a random research identification number and told of the potential risks associated with participating in the study. They were also told of the measures being taken to protect their identity.

3.2 Interview Protocols

Participants were interviewed three times over the course of 4 weeks. The interviews were originally scheduled to be done over three weeks, but due to a one week break in school some of the participants were unavailable during this time. This was done to allow for easier scheduling of the interviews, as well as to allow for some minor question alterations between interviews. The participants were scheduled on an individual basis for their first of the three individual interviews. For each interview, the four participants were scheduled as close together as possible by day and then each subsequent interview was scheduled at similar time intervals to avoid any one participant having significantly more or less time between interviews. Each interview was both audio and video recorded for later analysis.

As there is very little research on mathematical knowledge for teaching at the post-secondary level, the interview protocols for this study were designed by me to be similar to questions from studies done at the K-8 levels. Each item was designed to target specific areas of the mathematical knowledge for teaching framework from Ball et al. (2008). It was the goal that every item was mathematically valid and grounded in the work of teaching so as to allow for the best possible image of the participants' mathematical knowledge for teaching. Below I will discuss some of the questions from each interview, how they are mathematically valid and grounded in the work of teaching, and what category of the mathematical knowledge for teaching framework the responses were hoped to elicit. Please see appendix B for the complete interview protocols. While each interview had a heavier focus on one or two categories of the mathematical knowledge for teaching framework, they all had questions that were either specifically targeted to each category or would open the participant up to giving responses in each area. The focus of each interview was chosen in a way that would allow for the potential to build on previous responses.

3.2.1 Interview 1

Interview one had questions that focused on each area of the mathematical knowledge for teaching framework. I didn't focus on any singular category because I wanted to get a sense of the participants' knowledge in each of the categories before asking them any questions that may create new knowledge in a section.

First in interview one, the participants were asked "When you hear the term function what do you think of?" This question was posed in hopes to elicit a response in the common content knowledge category. I wanted to see what each participant thought of when thinking about functions. I then asked this same question two more times replacing function with exponential and then exponential function. These two subsequent questions were also aimed at getting responses in the common content knowledge category. These questions were grounded in the act of teaching since what a teacher thinks of for any singular topic can greatly affect their teaching of the topic.

The participants were then asked about when exponential functions would occur within a school setting and outside of a school setting. These questions were aimed at the participants' knowledge of content and curriculum as well as their horizon content knowledge. With these questions, the mathematical validity is not relevant but these questions are grounded in the work of teaching as our ideas of a topic can shape how we both understand and teach that topic.

Lastly, the participants were asked a questions focused on two functions, $f(x) = 2^x$ and $g(x) = x^2$. The participants were asked how the functions varied from one another and how they were similar. This line of questioning was targeting three different categories of the mathematical knowledge for teaching framework: common content knowledge, specialized content knowledge, and knowledge of content and teaching. The questions are mathematically valid as these two functions have properties in common and also have significant differences. These questions are grounded in the work of teaching since the ability to draw connections between old and new ideas as well as identify their differences is paramount in teaching.

3.2.2 Interview 2

The focus in the second interview was more on the common and specialized content knowledge categories. The reason for this is to allow the participants to continue from where the first interview left off. One of the first questions that I asked them was to define an exponential function mathematically. This question was not only to see if they knew the definition of an exponential function but also to see how rigorously they defined it. This allowed the participants to respond in ways that incorporated other categories such as knowledge of content and teaching. If the participants gave a full and correct definition of an exponential function then I would move on to the next question, but if they didn't mention a piece of the definition I would then ask a question that would lead them to including the missing piece. An example of a question like this could be for a participant that gave a symbolic definition such as $f(x) = a^x$, I would then ask them how they would model a situation where there were initially 100 rabbits and each month they would double in population. This question is simple enough for the participants to answer and their answers usually resulted in their realization of the missing part of their definition. Once participants had a mathematically accurate symbolic definition of an exponential function I asked them "What do each of the variables represent? Are there any restrictions on any of the terms of an exponential function?" These two questions continued to aim at the participants' common content knowledge while allowing for the participants to expand into multiple other categories of knowledge. These questions were about the ability to rigorously define and restrict mathematical functions, an important ability for a mathematics teacher.

After the participants had established a mathematically accurate definition of exponential functions I asked them to compare two functions, $f(x) = -(2)^x$ and $g(x) = -2^x$. After hearing their responses to this question I then included one more function, $h(x) = (-2)^x$. I asked each participant what the differences and similarities between the functions *f* and *h* are. Again this question is primarily focused on the participants' common content knowledge but it allowed for them to expand into specialized content knowledge as well as knowledge of content and teaching and even knowledge of content and students. These functions represent common mistakes made by students when attempting to write negative exponential functions. They are all mathematically valid functions and the ability to differentiate these functions is an important work of teaching.

Following this I asked each participant to solve an exponential equation without using logarithms. The equation they were trying to solve was $2^{3x} = 8^{-x+2}$. This equation can be solved in multiple ways and so I wanted to see what each participant's approach to the problem would be. After they solved this problem the participants were shown three different student solutions to this problem and were asked describe what each student was attempting to do, if the students had made any mathematical errors, and if each students solution process would be generalizable to any similar problems; see appendix C for all interview handouts. This line of questioning was aimed to get responses in the categories of common and specialized content knowledge as well as knowledge of content and teaching and knowledge of content and students. After the participants answered these questions they were asked to solve another similar problem, $2^{5x} = 8^{-x+2}$. They were then shown student work from the same three students and asked to do the same things as with the previous problem. This second problem was posed in order to demonstrate a deficiency in one of the student solutions. After having explored each of the three students' solutions to the problems as well as their own solutions, the participants were asked if either of the two problems would be better to ask students in order to gain an insight into the students' understanding of the process of solving exponential equations without using logarithms. This last question was firmly aimed at gaining insight into the participant's knowledge of content and teaching and specialized content knowledge. 3.2.3 Interview 3

In the third and final interview, the questions started out in much the same way as the last few questions of interview two. The participants were asked to solve a word problem and were then given a student solution to the same problem; see appendix B for both the problem and the student solution. They were asked to identify what the student was trying to do in their solution attempt and how they would help the student to correct their solution. These questions are targeted for responses demonstrating both common and specialized content knowledge as well as knowledge of content and both teaching and students. These questions were followed up with a second problem and an attempted student solution. They were again asked to identify what the student was attempting to do with their solution as well as what they would do as instructors to help the student. These questions were written as to be mathematically correct with the exception of the student's work that was designed to demonstrate common student mistakes. The act of solving problems, assessing and evaluating student work and helping to correct student misconceptions is essential in the work of teaching.

After this, the questions took a turn away from the participant's common content knowledge and focused away from the common content knowledge. Here they were asked questions like:

> What should a teacher know in order to teach about exponential functions?
> What kinds of examples would be good/bad to show students when they are first learning about exponential functions?

These questions were aimed at the participant's knowledge of content and teaching and even some specialized content knowledge.

Some interview questions were posed in each of the three interviews. For example, at the beginning of each interview the participants were asked in one way or another, "what is an exponential function?" This was asked each time to see if the participant's idea or definition changed at all from interview to interview. Also at the end of each interview, they were asked if there was anything they would like to add, and if they had learned anything about their own knowledge of exponential functions through the process of the interview.

3.3 Coding and Analysis

To code and analyze the data, I utilized the mathematical knowledge for teaching framework as put forth in the 2008 article by Ball et al. While Hauk et al. (2013), Hill, Schilling, & Ball (2004), Speer & King (2009), Speer et al. (2010) all state that the current framework, mathematical knowledge for teaching, is not necessarily suitable to an immediate adaptation to the post-secondary level and needs to be reassessed, Speer et al. (2014) states that the current framework can be used, provided that we acknowledge its potential shortcomings and very carefully define the categories that cause problems. The categories that Speer et al. (2014) specifically identified as being problematic are common content knowledge and specialized content knowledge.

3.3.1 Definitions of Mathematical Knowledge for Teaching

For the purposes of this Study each of the categories of the mathematical knowledge for teaching framework must be carefully defined to account to for the differences present at the post-secondary level. Therefore, the definitions to follow are the ones that were used in the coding and analysis of the data. The definitions of subject matter knowledge and pedagogical content knowledge will remain the same. First I will define the three categories of mathematical content knowledge. Common content knowledge is the knowledge of mathematical content that someone with an equivalent educational background who does not teach would know. Notice that the main difference in this definition and Ball et al.'s (2008) definition is the addition of an equivalent educational background. Specialized content knowledge is defined much as it was by Ball et al. (2008), the knowledge of mathematical content that is specific to teaching. The last category of mathematical content knowledge is horizon content knowledge. The definition of this category is the awareness of how mathematical topics are related over the span of mathematics. While the definitions of mathematical content knowledge needed to be redefined, the definitions of the categories of pedagogical content knowledge will remain the same as those stated by Ball et al. (2008).

3.3.2 Coding

The method of coding used consisted of categorizing each participant's response into one or more of the six categories of the mathematical knowledge for teaching framework and then giving it a rating of either: high level of knowledge, medium level of knowledge, low level of knowledge, or no knowledge. To ensure that there was consistency across participants, a rubric was created with which to code each interview. To create the rubric, I began by listening and familiarizing myself with the interviews from each participant for each interview. Then I identified an example of each level of knowledge for each question. As each question could be answered in a number of ways, and each way could contain elements from multiple categories of the mathematical knowledge for teaching framework, I did not want to specify a category for each question. While the questions were designed to elicit responses that demonstrated knowledge from a specific category or categories, I also wanted to analyze each response for what category the response represented without any predetermined category in mind. The point of this approach was to reduce the chances of getting tunnel vision on a single category and assigning a level in that category, and missing part of the response that applied to a separate category. Appendix D contains a table with responses of each level for each of the six categories. While many of the examples are from the interviews, there were some categories that did not have a response for each level of knowledge. For these situations, I created an example of a response that would be coded in the necessary way.

3.3.3 Analysis

Once all three of the interviews for each participant were coded, I was then able to begin analyzing the data for patterns. To do this I looked for patterns for each participant, each interview, each category, as well as each question. To look for these patterns I printed out the coded interview data and arranged them in columns by participant with interview one on top then down to interview three on the bottom. This allowed me to be able to aspect of knowledge I was looking at. To identify a pattern, I looked over the coding very carefully looking for high rates of similar levels for any given aspect. For the purposes of this research, a pattern could be different depending on what aspect of the data was being analyzed. A pattern when analyzing a single category of a participant could be simply seeing that the majority of their responses are coded as one level of knowledge. It could also be seeing that each time the participant's response was coded at a certain level the questions were all asking them to do the same action. When analyzing across all participants, patterns could be seeing a question where each participant's response was coded the same or all but one was coded the same. The results of this analysis are given in the following chapter.

Chapter 4: Data Analysis and Discussion

In this chapter, I will present the results from analyzing the coded interview data. After coding the interview data using the mathematical knowledge for teaching framework, I began to examine the coded matrix for patterns as described in chapter three. I will fist discuss the patterns and findings from analyzing each participant individually, and then I will discuss some other interesting results from all four of the participants.

4.1 Individual Results

Each participant completed a series of three interviews and with each participant many interesting patterns were found within their responses. Here I will use pseudonyms for the participants to further protect their identities. I will call the participants by the following names: Pam, Michael, Kelly, and Jim. If I consider my own personal anecdotal evidence, it is reasonable to think that mathematics graduate teaching assistants would have a good level of common and specialized content knowledge, but are likely to have low knowledge of content and teaching and students. We can think this from their extensive backgrounds in mathematics courses at the post-secondary level and from their limited background in teaching training. What of the other two categories of the mathematical knowledge for teaching framework?

4.1.1 Pam

Pam was a 23 year old graduate student with a bachelor's of science in mathematics who intends to get her Ph.D. in mathematics. Pam had no prior teaching experience before she entered graduate school. In her first term as a graduate teaching assistant she taught college algebra and during the interview process she was teaching differential calculus.

While Pam had no category of mathematical knowledge for teaching that she was exceptional in, she showed strength in both her common and specialized content knowledge. We can see this in an excerpt from her response when asked about the meaning of each of the constant terms in a general symbolic exponential function:

> Pam: The "C" is representing the initial value. So when x equals zero, that's the value that the function takes on. For this first case here, the "a" is representing the common ratio. So as you go up by one in x, that's what you're multiplying by in order to change your output. In the alternate expression the "k" is a rate of some sort. Really the way that I think about "k" is, "k" could be some number and we could write that "k" is equal to the natural log of some number, say "b". In which case this can go back to looking like the first expression. (Interview 2)

In her response the first case Pam is referring to is the form of an exponential function that looks like $f(x) = Ca^x$ and the alternate is $f(x) = Ce^{kx}$. Then when she says "this can go back to looking like the first expression" she is writing out on a piece of paper that $a = e^k$.

From this quote we can see that Pam has a level of understanding that allows her to easily identify what each of the constants of both forms of an exponential function represent, but also to identify the relations that they have with each other and with previous outputs. While this demonstrates a high level of common content knowledge it is important to make note of the language she is using. While teaching mathematics, it is important to not just convey algebraic processes, but also to convey the appropriate mathematical language attached with mathematics. We can see Pam doing this when she refers to “a” as a common ratio instead of calling it a slope or some other improper term. Doing this will allow for Pam’s students to make a connection with ratios and multiplication. Had she called “a” the slope, her students could have connected it with the idea of linear change and addition. Her response to this question demonstrates that Pam has a high level of common and specialized content knowledge for exponential functions.

However, Pam did not always demonstrate high levels of common and specialized content knowledge. In interview two, Pam was asked about two functions, $f(x) = -2^x$ and $g(x) = (-2)^x$. When she was asked what ways, other than symbolically, a teacher could use to represent these functions to show that they are different, she did not know. Since she was unable to even think of any alternate form to represent these functions, this shows a low level of knowledge in common and specialized content knowledge as well as in knowledge of content and teaching.

Pam showed that she had some holes in her common and specialized content knowledge, but overall she was strong in these two categories. However, when it came to the categories of knowledge of content and teaching and knowledge of content and students, she had the opposite results. Pam was shown a student's attempt to find an exponential function given two points (see appendix C, where the student had chosen to solve in a manner that left them with rational exponents and the student had gotten stuck. In response to how she would help this student, Pam said that she would teach them about rational exponents and how to use them in this situation. She did not mention providing the student with a method of solving this problem that would not involve rational exponents. Instead, she could have showed them that solving for the other unknown variable would lead to a simpler solution that would not involve rational exponents. While rational exponents are an important topic for students to know, they also need to be able to find solutions in an optimal way. Pam's decision to teach this student a potentially new topic instead of helping them find the optimal process shows a low level of knowledge of content and teaching and knowledge of content and students.

Even though Pam appears to have lower levels of knowledge in these two categories, she had some responses that demonstrated some higher levels of knowledge. After seeing student work on two problems, both where students had solved a given equation, Pam was asked which of the two problems would be better to test the conceptual understanding of the students. In her response she said that the second problem, $2^{5x} = 8^{-x+2}$, is better since it does not allow for students to accidentally get the same base as the first problem, $2^{3x} = 8^{-x+2}$. Being able to recognize that one of the common mistakes a student makes in this type of problem is to try and isolate the variable first shows some high level of knowledge of content and students.

The last two categories of the mathematical knowledge for teaching framework are horizon content knowledge and knowledge of content and curriculum. For Pam, and the other participants, these were troubling areas. Unlike the other four areas where Pam had some high level responses, Pam showed little to no evidence of high levels of knowledge in these two categories. Pam was asked when someone would see exponential functions both inside and outside of the classroom. In her response she said that "they are brought up in math courses mostly" (Pam, Interview 1) and she went on to say that they are also brought up in physics and biology, but she was not sure how. While Pam knows that exponential functions are used in other areas, she is not sure how they are used. This response along with her other responses in these categories illustrate that her knowledge might be sufficient in common and specialized content knowledge, but in the other four they are lacking sufficient knowledge.

4.1.2 Jim

Jim, like Pam, is a 23 year old graduate student with a bachelor's of science in mathematics who intends to get his Ph.D. in mathematics. Jim also had no teaching experience prior entering graduate school and in his first term as a graduate teaching assistant he taught Trigonometry, and he was teaching college algebra during the course of the interviews. Also similar to Pam is Jim's level of knowledge in each of the categories of mathematical knowledge for teaching. While they differ in minor ways from question to question, their overall patterns are similar. Thus, I will continue to describe the other participants and come back to Jim and Pam in section 4.2.

4.1.3 Kelly

Unlike Pam and Jim, Kelly is a 22 year old graduate student who also has a bachelor's of science in mathematics, but Kelly intends to finish graduate school with a master's of science in mathematics. Another place that Kelly differs from Pam and Jim is in her prior teaching experience. Kelly job shadowed in two different community college summer math courses during her time as an undergraduate; an introductory algebra course, and Trigonometry. The job shadowing consisted primarily of working in the classroom with the students and she had minimal lecture responsibilities. In her first term as a graduate teaching assistant she taught college algebra, and during the interview process she was teaching Trigonometry.

Both Pam and Jim showed high levels of knowledge in both the common and specialized content knowledge categories, as did Kelly. One example of this is her response when asked if these two functions $f(x) = -(2)^x$ and $g(x) = -2^x$ are the same and why. Her response was as follows:

> Kelly: These are the same.
> Interviewer: OK, how so?
> Kelly: Well I mean, well like, in this case if you have parenthesis around a positive number it's gonna kinda keep it positive. Like if the negative where on the inside, they would be different functions. But this is one quantity here and it's the same as that one. (Interview 2)

Here we can see that she is able to reason through why these two functions are equivalent and even how you would make them into different functions. This response differs from those given by the other participants in that she provided a reason why the functions were the same. While the other participants where not wrong in their answers, they showed a lack of the conceptual depth that Kelly demonstrated.

Even though Kelly appears to have a higher level of common and specialized content knowledge than Pam and Jim, the interesting result was in her knowledge of content and teaching and knowledge of content and students. While she had some prior experience in the classroom other than as a student, she showed similar results to that of Pam and Jim. When we look at Kelly's answer to the question of what problem is better suited to test students' conceptual understandings, $2^{3x} = 8^{-x+2}$ or $2^{5x} = 8^{-x+2}$, her response was unlike Pam and Jim. Even after coming across the issue where the $2^{3x} = 8^{x}$ in the first problem and then trying it again on the second problem, getting $2^{5x} = 32^{x}$, and then getting stuck herself. She felt that neither problem held any benefit over the other. She also stated that "it's hard to say with the first problem" if the student who tried to convert to the higher base got lucky or not. This shows a low level of knowledge of content and teaching.

4.1.4 Michael

Michael is a much different participant in terms of his background and demographics. Michael is a 32 year old graduate student with a bachelor's of arts in business in the statistics department and he was undecided on his end degree goal. Michael is also the only one of the participants to teach the same course for both of their first terms as a graduate teaching assistant integral calculus. Michael also differs in that he had teaching experience as a primary instructor prior to becoming a graduate teaching assistant. He developed and taught a course in philosophy and dream analysis at an undergraduate university. While it is interesting to note that Michael has taught two courses before, they were not math courses. I will revisit this when later when I discuss Michael's pedagogical content knowledge.

Unlike the other three participants, Michael's level of knowledge in the common and specialized content knowledge is not high. While he did correctly solve each of the problems posed to him during the interviews, he struggled with deciding how to solve the problems and was not sure of his answers once he attained them. I can see this happening while he is solving $2^{3x} = 8^{-x+2}$ for x.

> Michael: Here you have 2^{3x}, this I noticed is two to a power. So this is $2^3 = 8$. This may actually help us in this case. If you're typically going to clear this thing out you're going to want log base two. Since $8 = 2^3$, we can substitute this in here. So you have $2^{3x} = 2^{-3x+6}$. This is starting to look good because you no longer have two different bases of this thing. So if you're going to do log you can do log base 2. So then you have $\log_2 2^{3x} = \log_2 2^{-3x+6}$. I haven't worked with logs that are not base e in a while so I'm not 100% sure this is the proper way to do this. If I had base e. I see a lot more of like e^{3x}, in which case you're using ln to clear it out typically. So what this does, since we are using the same function in both cases, we may run into an issue since log is a function. I don't think we have any divide by zero problems. Typically you don't have any problems with ln I don't think with log base two you would have anything. Well let's forge on ahead. I'm going to put an asterisk because of my uncertainty. This is my uncertainty that there may be a divide by zero when you are using a log that's not base e. (Interview 2)

After this Michael went on to solve $3x = -3x + 6$ and get a solution for the problem. However, even after he obtained this solution he is not confident in his answer. Yet instead of checking his solution Michael decides that he is done and needs to do nothing further.

We can see that from the beginning Michael recognized that he can get a common base of two in the equation, $2^{3x} = 8^{-x+2}$, and he did so. After this Michael used a tool that he is uncertain of because it is what he does in other problems involving exponentials. Even though what Michael does is in no way wrong, it is his uncertainty that is interesting. Not being sure if the logarithmic function will equal zero for a base other than e illustrates that Michael's understanding of logarithms and how they relate to exponentials is procedural. Michael understands that if he takes $\ln e^x$ he will get x in return; and when you change the base of the log, Michael still knows that $\log_2 2^x = x$. However, Michael is confident that $\ln(x)$ will never equal zero, but is unsure that $\log_2(x)$ will also not equal zero. This uncertainty in this tool leads to Michael being unsure of what he is doing and even creating doubt in the final solution. Even more interesting is the fact that Michael is concerned with dividing by zero in a problem that involves no division. These issues from this response and others lead to a pattern of low levels of knowledge in both common and specialized content knowledge.

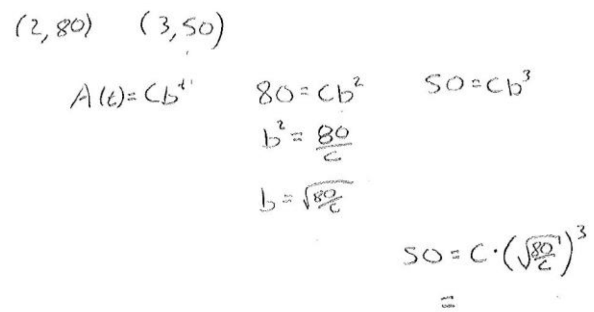

Figure 4.1: Student work from interview 3.

Similar to Michael's level of knowledge of common and specialized content knowledge, his level of knowledge for the categories of knowledge of content and teaching and knowledge of content and students is lower than the other three participants. When asked to assess a student's work on a problem and then describe how they would help this student, Michael's response showed a lower level of knowledge of content and teaching and knowledge of content and students. The problem was to give an exponential formula to model a situation. In the statement of the problem you were told that the function passed through the points (2, 80) and (3, 50). The student work Michael looked at is shown in figure 4.1:

> Michael: So you have the square root of this thing and you have this cubed, so I guess I would probably. Since they are obviously not thinking about it in like a formulaic way. They're not thinking about it as three divided by two in the exponent. I would probably go back to an earlier example. And I would say what happens if you have square root of four cubed. So in this situation I would ask them what is this? I think I would go through it this way first. I would hope they would get to two thirds equals eight. Then I would try to. Let's see, how would I do this? I guess I would try to also show that you could also do four cubed, square root. So you would have square root of 64. So you get eight. I guess I would try and show that it doesn't matter what the order of this is.
>
> Interviewer: Would you possibly want to give them an alternate way of doing this or would you want to just stick with what they have started?

> Michael: I guess that's a good point. Yeah, um. I mean you can do it this way. If they are only stuck on this one idea then they can probably solve it from there. Yeah this would be. That's a good point. This could take a really long time. In that case you can solve it a lot more simply. Yeah I don't know. That wouldn't be my natural tendency. (Interview 3)

From this I can see a couple issues about Michael's knowledge of content and teaching and his knowledge of content and students. First of all, when Michael says that the student is not thinking of it in a formulaic way, he is referring to the student not converting between radical notation and rational exponents. While this may show some level of knowledge of content and students, what he is saying does not make sense. In mathematics, when converting between radical notation and rational exponents, there is no formula to follow only the properties of exponents. Then he says that he will go back to a previous example. This is very good and has the potential to show a good level of knowledge of content and students. However, he chooses a random problem and has the student think about what would happen if they had $\sqrt{4}^3$. This example provides no intuitive access point for a student to gain understanding with the idea of converting from radical notation to rational exponents. An example one could use is to show a student a series of equations and have them solve them. One possible series of equations is

$$\sqrt{x^2} = \qquad (x^2)^? = x \qquad \sqrt{x} = x^?.$$

This series can help students to see the connection between the properties of exponents and radical notations. Michael's example is simply doing a basic calculation based on order of operations.

After Michael finished describing his way of trying to help this student, I asked if he would want to give them an alternate way. Michael's response was essentially-not unless the student was not able to figure out the problem with rational exponents. This shows a low level of knowledge of content and students in the thinking that a student is going to attempt the most appropriate solution processes. It also shows a low level of knowledge of content and teaching as you must know if it is important to continue down a potential solution path or if you need to abandon that approach for one that is better suited to the problem. A pattern of responses like this one demonstrates that Michael has a low level of knowledge of content and teaching and knowledge of content and students.

4.1.5 Horizon Content Knowledge and Knowledge of Content and Curriculum

I have discussed each of the four participants' levels of knowledge in four of the six categories of the mathematical knowledge for teaching framework. The two that I have yet to discuss are horizon content knowledge and knowledge of content and curriculum. The reason for this is that each of the four participants showed a low level of knowledge in both of these categories.

In the first and third interviews, the participants were asked questions such as when they would see exponential functions inside and outside of a classroom setting and what is important to cover in a course that has exponential functions. The participants' responses to these questions were overall superficial and lacked any further explanation. Most of the participants were able to say that exponential functions are observed in growth and decay and finance, but only two of the four participants mentioned seeing them in subsequent math courses. The more interesting thing is how the participants were able to say that exponential functions occur in areas outside of math but with the exception of growth, decay, and finance, they were not able to identify any other areas that we would see exponential functions. Then in the third interview the participants were asked what is important for students to understand from a course covering exponential functions. While one of the participants were able to put together a response that showed a high level of knowledge, the rest of them did not. One of the participants said that the important thing for students to understand about exponential functions is retirement and credit card debt. Then Jim said that students need to understand:

> That they're (exponential functions) everywhere. Definitely everywhere. It's in loans, buying a house, buying a car, population type models. (Interview 3)

None of the participants mentioned anything about the properties of exponential functions or the rules that guide the algebraic manipulation of these functions. The inclusion of something like this would have shown a medium level of knowledge based upon the rubric for coding. These responses pointed to a pattern for each participant of low levels of knowledge of content and curriculum and horizon content knowledge.

These low levels of knowledge in these two categories are significant because without knowledge of how the content relates throughout the course of mathematics and how it relates to other fields of study, it is difficult for teachers to plan what to teach to students and how to teach it to students to prepare them for other courses. It is also important since these types of knowledge help to inform teachers of what students should know and by what point they should know it.

4.2 Group Results

While I began to discuss results and patterns found among all four participants in the last section of 4.1, I will further expand on some of these results here.

4.2.1 Totals

When I first began looking for patterns in the data as a whole I noticed some overall trends. The participants showed higher levels of knowledge in both common content knowledge and specialized content knowledge and medium to low levels of knowledge in; horizon content knowledge, knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum. These patterns were supported further when I compiled a count of responses into a table.

	High Level of Knowledge				Medium Level of Knowledge				Low Level of Knowledge				Total
	P	J	K	M	P	J	K	M	P	J	K	M	
CCK	13	8	12	7	4	5	6	5	8	10	6	12	40/20 /36
SCK	6	10	7	8	6	6	4	2	4	2	4	5	31/18 /15
HCK	3	0	4	0	0	3	5	3	5	3	1	5	7/11 /14
KCT	6	7	9	1	7	3	1	7	7	8	4	9	23/18 /28
KCS	5	3	1	3	3	3	4	3	2	6	5	2	12/13 /15
KCC	1	0	1	0	2	2	6	4	3	2	1	4	2/14 /10

This table shows the number of responses for each participant in each category for each level of knowledge. The final column is the total high/ medium/ low responses for that category of the mathematical knowledge for teaching framework. I can see from this totals column that common and specialized content knowledge both tend towards more high level responses. One thing of interest here is that in the common content knowledge category we can see a large number of low level responses. Many of these low level responses were due to quick and possibly careless responses. In future it would be worthwhile to investigate this further.

Another thing this table shows us is that the categories of knowledge of content and teaching and knowledge of content and students show a trend in the direction of a medium level of knowledge. Lastly the categories of horizon content knowledge and knowledge of content and curriculum shows a low level of knowledge. Now I will show a few more examples of questions and responses from the participants to demonstrate these levels.

4.2.2 How are they Similar?

During the first interview, each participant was asked about two functions, $f(x) = x^2$ and $g(x) = 2^x$. The participants were asked how these two functions were different and if those differences were significant. The participants mentioned how one is an exponential and the other is a quadratic function. They discussed end behavior of the two functions, and there was even mention of the curvature of the two functions. While each participant's response to this question was rooted in common content knowledge, they did not show a depth of mathematical knowledge for teaching.

The next question the participants were asked was, "how are those two functions similar?" Each of the participants struggled to begin answering this question. In contrast, the participants took no time when asked the previous question. After some thought, three of the participants were able to describe how the two functions were similar. The other participant started to give possible answers a couple of times but in the end said simply "I've never thought of that before. I don't know" (Pam, Interview 1). Jim answered that the domain and range are the same for the two functions. Then he lightheartedly added that they have the same characters. Kelly was only able to say that they are both non-negative. Each of these three responses shows a low level of knowledge in the specialized content knowledge category.

The fourth participant had a much more detailed answer for this question. As we see in this excerpt from Michael's interview, he saw multiple similarities between the two functions:

> Michael: In order to solve for x, you're going to need a different method. They're both dealing with exponents. You could look at it as, in this case if x=2 you have 2^2 which happens here also if x=2 you would have 2^2. So there are cases where these things would be exactly the same. So there are places where they will intersect. (Interview 1)

In his response, Michael described how these two functions require methods outside of the basic four arithmetic operations in order to solve for the variable. He also stated that both of these functions involve exponents and that they are even equal to each other at multiple points. After carefully playing with the functions and their graphs, Michael was able to say that these two functions intersect at three points. We can see from a graph similar to the one drawn by Michael in figure 4.2 that this is in fact true.

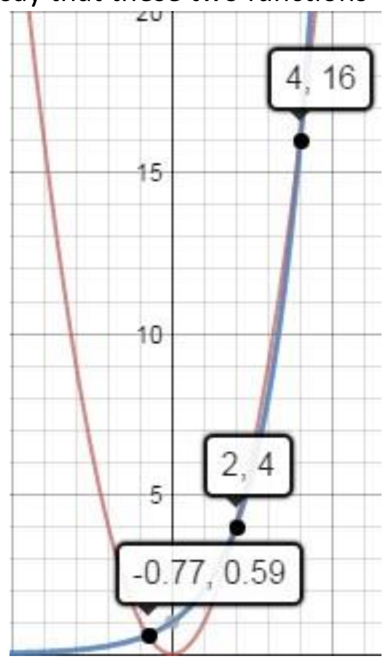

Figure 4.2: A graph of $f(x) = x^2$ and $g(x) = 2^x$

While it is interesting that only one of the four participants was able to give an answer to this question that included more than one similarity, it is more interesting to think about how the one participant who saw multiple similarities was the participant with a degree in business and who is not a mathematics graduate student. Even though this one instance with these four participants does not create a rule, it does prompt us to ask if this difference in response is due to the participants' different graduate programs.

4.2.3 Other representations

During interview two, the participants were asked a variety of questions regarding two functions, $f(x) = -(2)^x$ and $g(x) = (-2)^x$. One of the last questions involving these two functions was "what are some other ways that you could choose to represent these functions that would emphasize their differences and similarities?" Two of the participants gave an alternate method to show the difference between the two functions and the other two did not come up with another representation. A standard textbook for college algebra gives four different representations for a function: symbolically, graphically, verbally, and through a table (Rockswald G., Krieger T., and Rockswald J., 2012).

Pam thought about the question for a moment before saying that she did not know of any other ways to represent these functions beyond the way they currently were. Jim said that there was no need to use any other representations because students just need to get used to the mathematical conventions and know the difference. These two participants demonstrate a low level of specialized content knowledge in this response, as they either do not know any other representations or they do not think they are useful. In contrast, the other two participants said that the best way to represent these functions is to demonstrate their similarities and differences by using a table. They both spoke of the importance of being able to give students a visual model and that by choosing the independent values carefully, a teacher can accurately demonstrate the similarities and differences of these two functions.

The knowledge that I am addressing here would fall into the knowledge of content and teaching category. So it appears to be quite telling that the two participants who gave responses showing low levels of knowledge were the two participants that had no prior teaching experience and the ones that showed high levels of knowledge were the two who had prior teaching experience. This further demonstrates one of the primary deficiencies of first year graduate teaching assistants' mathematical knowledge for teaching, their low levels of knowledge of content and students and their knowledge of content and teaching. However, even with their prior teaching experience, neither Kelly nor Michael showed higher levels of knowledge in any of the pedagogical content knowledge categories when compared to their peers without prior teaching experience.

4.2.4 What a teacher needs to know

Towards the end of the third interview, after the participants had been asked questions about the types of things a teacher would need to know in order to teach exponential functions, they were asked "what a teacher needs to know in order to teach exponential functions?" This question allowed the participants to condense many of the different topics and concepts that were discussed throughout the course of the interviews. The responses that were given were more like the response of teaching philosophy before Shulman (1986). Michael's response was "I don't know, more than me. They would probably need to know all of the tricks for every one of the homework problems." This shows that

Michael thinks that the main thing a teacher needs to know is what we have defined as common content knowledge.

Similar to Michael's response is Jim's response. He also said that teachers need to know all of the tricks to solve the problems. However, Jim went on to say that teachers also need to know some of the basic and natural examples. He then added that teachers need to know the different properties and rules for exponential functions and how they relate to logarithms. Jim's response shows a deeper understanding of what a teacher needs to know to be able to teach exponential functions. With his response he touched on five of the six categories of the mathematical knowledge for teaching framework, only missing knowledge of content and students.

Kelly's answer was simple enough, "everything in the textbook." This response can be interpreted in a couple different ways. The first of which is that she meant that a teacher needed to know: all of the properties and rules, the solution methods, the terminology, the examples, and the applications. However, the other, and more likely, interpretation is that Kelly believes that a teacher simply needs to know the basic solution methods and computational tools. This illustrated that Kelly's understanding of what a teacher needs to know to teach exponential functions is primarily focused on the common and specialized content knowledge areas and leaves out the entirety of the pedagogical content knowledge half of the mathematical knowledge for teaching framework.

Lastly we have Pam's response to the question. She said that teachers should know the reasoning behind why the constants are restricted the way they are and they should be able to find an exponential function given two points. She went on to sum up what she thinks a teacher needs to know by saying that the teacher needs to understand everything that they are talking about. This shows a very limited understanding of what a teacher needs to know to teach exponential functions. She fails to mention anything that represents knowledge of content and teaching, knowledge of content and students, horizon content knowledge, or knowledge of content and curriculum.

We can condense these responses down to the response that all four participants had- teachers need to know how to solve the problems they are assigning. Jim added to that by saying that teachers also needed to know how exponential functions related to logarithms and what the basic examples are. The idea that teachers need to know how to do the problems they are assigning is important, but on its own it does not give anything close to a complete picture of the teacher knowledge that is required every day.

In the analysis above, I explored the differences and similarities of mathematics graduate teaching assistants' mathematical knowledge for teaching in the context of exponential functions. The four participants demonstrated varying depths of mathematical knowledge for teaching over the course of the three interviews. In the final chapter, I will explore the findings and implications of the study.

Chapter 5: Conclusions

From the patterns discussed in chapter four, I will draw some preliminary conclusions regarding the mathematical knowledge for teaching of first year mathematics graduate teaching assistants. In this chapter I will present some conclusions that can be formed from the interview data, as well as some of the limitations of the research and the potential impact on future research and graduate teaching assistant training.

5.1 Discussion of findings

Based on the patterns in the coded interview data, I concluded that first year mathematics graduate teaching assistants have a high level of common and specialized content knowledge, but they have low to moderate levels of knowledge of content and teaching, content and students, content and curriculum, and horizon content knowledge. This is similar to the findings of Johnson and Larsen (2012), who observed that the area of the mathematical knowledge for teaching framework that was limited in college instructors was their pedagogical content knowledge. As we might expect from personal anecdotal evidence, mathematics graduate students are entering graduate school with a background of mathematics courses that provide them with a knowledge of the fundamentals of doing mathematics. However, with the findings of Monk (1994), that teachers who have more content courses are not strongly correlated to higher student achievements, and Hill and Ball (2009) who noted that “conventional content knowledge seems to be insufficient for skillfully handling the mathematical tasks of teaching” (p.69), we look to the pedagogical knowledge of these first year mathematics graduate teaching assistants. As we saw with all four of the participants, even Kelly and Michael with their previous teaching experience, their knowledge of mathematics and its relation to teaching is minimal. As the findings of Monk (1994) and Hill et al. (2005) found that a teacher’s mathematical knowledge for teaching was positively correlated to students’ success, we must now ask how can we improve the overall mathematical knowledge for teaching of first year mathematics graduate teaching assistants?

In chapter four, we saw the graduate teaching assistants struggle with many of the tasks from each of the six categories of the mathematical knowledge for teaching framework, we did see them excel with some tasks from each category. This shows that there is knowledge there to build off of, and that we need to continue to design professional development activities and programs, as we currently are (DeFranco & McGivney-Burelle, 2001), but now with the targeted area of pedagogical content knowledge.

From this study we now have a better understanding of a first year mathematics graduate teaching assistants’ mathematical knowledge for teaching as it relates to exponential functions. We also have modified definitions of the mathematical knowledge to teaching framework that can apply up to post-secondary education.

5.2 Limitations

The primary limitation of this study is the use of the mathematical knowledge for teaching framework. This framework has not been evaluated at the post-secondary level and so when using it the researcher must carefully define the categories of mathematical knowledge for teaching as they will be used throughout the study and be aware of the potential for incorrect results based on this framework (Speer, King, & Howell, 2014).

Although the study described in this thesis was designed to provide an initial image of first year mathematics graduate teaching assistants' mathematical knowledge for teaching exponential functions, the reality of this type of knowledge is that you can not necessarily gain an accurate image from any single form of inquiry. In order to create a more complete image of first year mathematics graduate teaching assistants' mathematical knowledge for teaching, researchers would need to combine interviews such as those conducted in this study with classroom observations. As the environment in a research interview can vary greatly from the environment inside an actual mathematics classroom, participants may not be demonstrating their full mathematical knowledge for teaching in interviews alone.

Another limitation of this study is simply the small size of the participant pool and where the participants attend school. In order to give a more generalized conclusion on first year graduate teaching assistants' mathematical knowledge for teaching, researchers would need to enroll more participants from universities in more regions than just the Pacific Northwest.

Also, the interviews conducted in this study were done so after the participants had finished a full term and during the interview process they finished their second term. Thus, by the time of the last interview the participants had approximately six months of teaching experience at the college level. To obtain a more accurate image of first year mathematics graduate teaching assistants' mathematical knowledge for teaching, future researchers should conduct the interviews prior to the beginning of the participants first year so that they would have baseline knowledge of graduate teaching assistants. The impact of this limitation depends upon the use of the image being created. If the goal is to identify areas of low levels of knowledge in order to provide targeted professional development and training for first year mathematics graduate teaching assistants, then one would want to conduct the research at a similar point in the year as when the training will take place.

The last limitation of this study is its focus on exponential functions as they arise in a college algebra course. While exponential functions are cited as being a critical research site (Confrey, 1991; Confrey & Smith, 1995; Ball, 2003) as well as a topic that is seen throughout mathematics and life (Castillo-Garsow, 2013; Webber, 2002a; Webber, 2002b), there are many different aspects of mathematics that were not covered.

5.3 Implications for future research

It is my hope that based on the results from this study researchers can begin to develop professional development activities and trainings for graduate teaching assistants that will be directly targeted at improving areas of their knowledge that are lower. It would be interesting to replicate this study with a larger group of first year mathematics graduate teaching assistants from a variety of schools to see if the levels of knowledge of this larger group differ from that of the participants in this study. Another future study that would be of value is one that covers either a larger area of mathematics or covers exponential functions as they arise throughout mathematics. It would also be of value to conduct a longitudinal study of first year graduate teaching assistants to see how their individual mathematical knowledge for teaching grows over the course of their first year, or their entire time in graduate school.

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Appendices

Appendix A: Recruitment Letter

Initial Letter for Recruitment (to be left in their mail boxes)

Dear [Mathematics Graduate Student],

My name is Matthew Keeling and I am a fellow graduate student in the Department of Mathematics. I am conducting a research study "Graduate Teaching Assistants' (GTA) Knowledge for Teaching Exponential Functions in a College Algebra Course" that investigates mathematics graduate students' knowledge of exponential functions. My research question is: what do GTAs know about exponential functions as they arise in the context of a college algebra course? This research study will contribute to the field in that it will give those in the field an insight into the knowledge that first year graduate teaching assistants have. From this insight people could design professional development to improve the abilities of graduate teaching assistants.

Participation in this study will include three interviews to be conducted between February and June 2015. Each interview will not last more than 1.5 hours. The total time that you will spend on this project is not more than 6 hours. The interviews will be video and audio recorded for analysis purposes.

Your confidentiality will be protected by de-identifying any information that you provide during the interviews. For example, you will be given a research ID number and all of the recordings of the interviews and transcripts thereof will be labeled solely with that research ID number. Your name will not, at any point, be directly associated with the data you provide. Additionally, interviews will be conducted outside of the department in a private room in the library as an additional way to protect confidentiality. Finally, I would also ask that you protect your own identification by not sharing with others in the department that you have or have not agreed to participate in this study.

Your participation in this study will not impact your standing in the department or people's perceptions of you.

If you would like to participate, please contact me via email at keelinma@onid.orst.edu or by phone at 541-218-3885. If you have any questions or concerns about this study, please contact Mary Beisiegel at

mary.beisiegel@oregonstate.edu.
Best Regards,

Matthew Keeling

Follow up Letter for Recruitment (to be emailed to potential participants)

Dear [Mathematics Graduate Student],

A week ago, I left a letter in your mailbox in the Faculty Lounge in the Department of Mathematics describing my study “Graduate Teaching Assistants' Knowledge for Teaching Exponential Functions in a College Algebra Course.” I am following up with you about your possible participation in my research study that will investigate mathematics graduate students’ knowledge of exponential functions. As the letter described, your participation includes three interviews to be conducted between February and June 2015. Each interview will not last more than 1.5 hours. The total time that you will spend on this project is not more than 6 hours. The interviews will be video and audio recorded for analysis purposes.

Your confidentiality will be protected by de-identifying any information that you provide during the interviews. For example, you will be given a research ID number and all of the recordings of the interviews and transcripts thereof will be labeled solely with that research ID number. Your name will not, at any point, be directly associated with the data you provide. Additionally, interviews will be conducted outside of the department in a private room in the library as an additional way to protect confidentiality. Finally, I would also ask that you protect your own identification by not sharing with others in the department that you have or have not agreed to participate in this study.

If you would like to participate, please contact me via email at keelinma@onid.orst.edu or by phone at 541-218-3885. If you have any questions or concerns about this study, please contact Mary Beisiegel mary.beisiegel@oregonstate.edu.

Best Regards,

Matthew Keeling

Appendix B: Interview Protocols

Interview I –

1) What's your name?
2) How old are you?
3) What is your educational background?
4) Why did you decide to pursue a graduate degree in mathematics?
5) What type of a degree do you currently plan on finishing with? (M.S. , PhD)
6) How long have you been a student at OSU?
7) How long have you been a graduate student?
8) How long have you been a GTA?
9) What courses have you taught, including this term?
10) Have you taught before becoming a GTA at OSU? If so, in what context – as a graduate student, instructor, etc.?
11) Have you tutored before becoming a GTA at OSU? If so, in what context – as a private one-on-one tutor, a tutor for a specific course, in a tutoring center, etc.?
12) When you hear the term function what do you think of?
13) When you hear the term exponential what do you think of?
14) What about when you hear the term exponential function, what does this make you think of?
15) In general what is an exponential function?
16) How do exponential functions differ from the other types of functions? (linear, quadratic, general polynomial, square root, absolute value)
17) How are they similar?
18) Why do you think we teach exponential functions?
19) When would an exponential function be seen…
    i) Inside of the classroom?
    ii) Outside of the classroom?
20) When should we try and teach students about exponential functions?
    i) Why not sooner?
    ii) Why not later?
21) Do you think that exponential functions are important to teach to students in college algebra (MTH 111)?
    i) If so, why?
    ii) If not, when should we teach students about exponential functions?
    iii) Or should they be taught at all?
22) How do $f(x)=2^x$ and $g(x)=x^2$ differ from each other?
    i) Is their difference significant?
    ii) What makes these functions similar?

23) Do you have any other comments or thoughts about exponential functions in general that you would like to add?

Interview II -

1) Have you looked up anything about exponential functions since our last meeting?
    i) If so what did you look up?
2) What is an exponential function?
3) How would you define an exponential function mathematically?
    i) What does that look like? (If the GTA gives a response that does not explicitly use a function form such as f(x) =C a^x or f(x) =Ce^{kx}).
4) What do each of the variables represent? (Referring to the C, a, e, etc.)
5) Are there any restrictions on any of the terms of an exponential function?
6) What happens if you use a value other than what is allowed?
    i) Will you still have an exponential function? A function?
    ii) What if "C" changes from positive to negative?
7) Is there any difference between f(x) =$-(2)^x$ and g(x) =-2^x?
    i) What can you tell me about these two functions?
8) What if I include the function $h(x) = (-2)^x$?
    i) What do these functions, *f* and *h*, have in common?
    ii) How are these two functions different?
    iii) Are these both exponential functions?
    iv) Are they functions?
    v) What other ways could you represent these functions to emphasize these similarities/ differences?
9) Solve the equation $2^{3x} = 8^{-x+2}$ for x.
10) Consider the following solutions to the problem above. (See Interview II handout a)
11) What where the students trying to do in each of the solutions?
12) In each of the solutions is the students thinking mathematically correct?
13) Will these methods for solving this problem work all the time, or will it only work in this specific situation?
    i) How do you know it will always work?
    ii) Why does it only work for this situation?
14) Now solve the equation $2^{5x} = 8^{-x+2}$ for x.
15) Again, consider the solutions from the same students to the problem. (See Interview II handout b)
16) What where the students trying to do in each of the solutions?
17) In each of the solutions is the students thinking mathematically correct?

18) Now do you think that these methods for solving this problem will work all the time, or only in this specific situation?
19) How could you help student 3?
20) Which problem do you think better allows a students to demonstrate conceptual understanding?
21) Have you learned anything from these interviews?
22) Do you have anything further you would like to add?

Interview III –

1) So have you looked up anything about exponential functions since our last meeting?
2) What is an exponential function?
3) How is it defined symbolically?
4) Talk me through how you would attempt to solve the following problem.
    - Chlorine is frequently used to disinfect swimming pools. The chlorine concentration should remain between 1.5 and 2.5 parts per million. On warm sunny days with swimmers agitating the water, 30% of the chlorine will dissipate into the air or combine with other chemicals.
        (a) Define a function $f(x)= Ca^x$ that models the amount of chlorine in the pool after x days. Assume that the initial amount is 2.5 parts per million and that no chlorine is added.
5) What would you think if you saw a student writing down this solution? (See interview III handout 1)
6) What do you think the student is trying doing here?
7) How would you explain this idea to a student?
8) Do you think this is a good question to ask students to get at their conceptual understanding?
9) Talk me through how you would attempt to solve the following problem.
    - If a car tire gets punctured and when then you take its pressure two minutes later and it has a tire pressure of 80 psi and a minute later the tire has a pressure of 50 psi. Find an exponential equation to model the tires pressure at time t.
10) As you look at this student work tell me what you think they are trying to do. (See interview III handout 2)
11) How would you help them solve this problem?
12) If you were teaching a class that covered exponential functions,
    i) What would you cover?
    ii) What is important for students to understand?

iii) What is something that would be bad to leave out?
iv) What things could you relate it to that the students already know?

13) What should a teacher know in order to teach about exponential functions?
14) What kinds of examples would be good/bad to show students when they are learning about exponential functions?
    i) Can you give me some examples of those examples?
15) What makes an example a good example?
16) What makes an example a bad example?
17) Over the course of these interviews have you learned anything about your own knowledge of exponential functions?
18) Is there anything that you want to add?

Appendix C: Interview Handouts and Student Work

Interview II Handouts-

Interview II handout a

Student 1:

$$2^{3x} = 8^{-x+2}$$
$$2^{3x} = (2^3)^{-x+2}$$
$$2^{3x} = 2^{-3x+6}$$

so

$$3x = \cancel{-3x} + 6$$
$$+3x \quad \cancel{+3x}$$
$$\frac{6x}{6} = \frac{6}{6}$$
$$x = 1$$

Student 2:

$$2^{3x} = 8^{-x+2}$$
$$(8^x)\, 2^{3x} = 8^2 \cdot \cancel{8^{-x}}\, \cancel{(8^x)}$$
$$(2^3)^x \cdot 2^{3x} = 8^2$$
$$2^{3x} \cdot 2^{3x} = (2^3)^2$$
$$2^{6x} = 2^6$$

so

$$\frac{6x}{6} = \frac{6}{6}$$
$$x = 1$$

Student 3:

$$2^{3x} = 8^{-x+2}$$
$$(2^3)^x = 8^{-x+2}$$
$$8^x = 8^{-x+2}$$

so

$$x = \cancel{-x} + 2$$
$$+x \quad \cancel{+x}$$
$$\frac{2x}{2} = \frac{2}{2}$$
$$x = 1$$

Interview II handout b

student 1: $2^{5x} = 8^{-x+2}$

$$2^{5x} = (2^3)^{-x+2}$$

$$2^{5x} = 2^{-3x+6}$$

so

$$5x = -3\cancel{x}+6$$

$$+3x \quad +\cancel{3}x$$

$$8x = 6$$

$$x = \frac{3}{4}$$

student 2: $2^{5x} = 8^{-x+2}$

$$(8^x)\ 2^{5x} = 8^2 \cdot 8^{-x}\ \cancel{(8^x)}$$

$$(2^3)^x \cdot 2^{5x} = 8^2$$

$$2^{3x} \cdot 2^{5x} = (2^3)^2$$

$$2^{8x} = 2^6$$

so

$$8x = 6$$

$$x = \frac{3}{4}$$

student 3: $2^{5x} = 8^{-x+2}$

$$(2^5)^x = 8^{-x+2}$$

$$32^x = 8^{-x+2}$$

Interview III Problems and Handouts-

- Chlorine is frequently used to disinfect swimming pools. The chlorine concentration should remain between 1.5 and 2.5 parts per million. On warm sunny days with swimmers agitating the water, 30% of the chlorine will dissipate into the air or combine with other chemicals.
  (a) Define a function $f(x) = Ca^x$ that models the amount of chlorine in the pool after x days. Assume that the initial amount is 2.5 parts per million and that no chlorine is added.

Interview III handout 1

part a) Since $f(0)=2.5$, $C=2.5$. The chlorine dissipates at 30% a day,
So $f(x)=2.5(.3)^x$.

- If a car tire gets punctured and you take its pressure two minutes later, it has a tire pressure of 80 psi. Another minute after that, the tire has a pressure of 50 psi. Find an exponential equation to model the tires pressure at time t.

Interview III handout 2

$(2,80)$ $(3,50)$

$A(t)=Cb^t$ $80=Cb^2$ $50=Cb^3$

$b^2=\frac{80}{C}$

$b=\sqrt{80/C}$

$50=C\cdot\left(\sqrt{\frac{80}{C}}\right)^3$

$=$

Appendix D: Table of Exemplar Responses

Common Content Knowledge

High Level of Knowledge This was a high level of knowledge because the response demonstrates a strong understanding of what the constant terms in an exponential function represent. The response also shows an understanding of the relationship between the "a" and "e^k".	Pam: The "C" is representing the initial value. So when x equals zero, that's the value that the function takes on. For this first case here, the "a" is representing the common ratio. So as you go up by one in x, that's what you're multiplying by in order to change your output. In the alternate expression the "k" is a rate of some sort. Really the way that I think about "k" is, "k" could be some number and we could write that "k" is equal to the natural log of some number, say "b". In which case this can go back to looking like the first expression. (Interview 2)
Medium Level of Knowledge This was a medium level of knowledge because the participant begins to solve the problem correctly but runs into difficulties when using logarithmic functions other than the natural log. They are unsure of the potential values it could take on.	Michael: Here you have 2^{3x}, this I noticed is two to a power. So this is $2^3 = 8$. This may actually help us in this case. If you're typically going to clear this thing out you're going to want log base two. Since $8 = 2^3$, we can substitute this in here. So you have $2^{3x} = 2^{-3x+6}$. This is starting to look good because you no longer have two different bases of this thing. So if you're going to do log you can do log base 2. So then you have $\log_2 2^{3x} = \log_2 2^{-3x+6}$. I haven't worked with logs that are not base e in a while so I'm not 100% sure this is the proper way to do this. If I had base e. I see a lot more of like e^{3x}, in which case you're using ln to clear it out typically. So what this does, since we are using the same function in both cases, we may run into an issue since log is a function. I don't think we have any divide by zero problems. Typically you don't have any problems with ln I don't think with log base two you would have anything. Well let's forge

	on ahead. I'm going to put an asterisk because of my uncertainty. This is my uncertainty that there may be a divide by zero when you are using a log that's not base e. (Interview 2)
Low Level of Knowledge This was a low level of knowledge because the participant does not know what the terms of an exponential function represent.	Interviewer: What do the constant terms of an exponential function represent? Kelly: That would translate it vertically. Umm. So "b" would scale it horizontally. I don't know, I'm trying to think of it in terms of transformations. Yeah that's as much as I would say.

Specialized Content Knowledge

High Level of Knowledge This was a high level of knowledge because the response shows a deep understanding of similarities between two functions that are typically not considered similar.	Interviewer: What do these functions ($f(x) = x^2$ and $g(x) = 2^x$) have in common? Michael: In order to solve for x, you're going to need a different method. They're both dealing with exponents. You could look at it as, in this case if x=2 you have 2^2 which happens here also if x=2 you would have 2^2. So there are cases where these things would be exactly the same. So there are places where they will intersect. (Interview 1)
Medium Level of Knowledge This was a medium level of knowledge because the participant thinks it will generalize (as it would) but is not sure and does not take the time to check if it does.	Interviewer: Will student two's solution generalize? (Interview 2 handout b) Kelly: I just never would have thought of student two's method. I think they were trying to get x to the left side. I never would have thought of it. It seemed to work for them both times. I think it will generalize but I'm not sure if I can make a definite claim.
Low Level of Knowledge This was a low level of knowledge because the response is actually incorrect.	Interviewer: What do these functions ($f(x) = x^2$ and $g(x) = 2^x$) have in common? The domain and range are the same for the two functions.

Horizon Content Knowledge

High Level of Knowledge This was a high level of knowledge because the response shows a deep understanding of multiple areas where exponential functions arise within future mathematics courses.	Question: When would an exponential function be seen inside of the classroom? **Created response**: Exponential functions are seen in a variety of different mathematics courses as well as non-mathematics courses, such as calculus, differential equations, biology, chemistry and finance.
Medium Level of Knowledge This was a medium level of knowledge because the response he does know that exponential functions are used in future math course but they are not sure if they already know them by the time they get to college algebra.	Interviewer: Do you think that exponential functions are important to teach to students in college algebra? Jim: Yes. If they want to go further in math then yes, they will need exponential functions. They should have them before college algebra. If not then yes in college algebra.
Low Level of Knowledge This was a low level of knowledge because the response fails to show any understanding of how exponential functions are used in future math courses.	Interviewer: What is important for students to understand about exponential functions? Jim: That they're (exponential functions) everywhere. Definitely everywhere. It's in loans, buying a house, buying a car, population type models. (Interview 3)

Knowledge of Content and Teaching

High Level of Knowledge This was a high level of knowledge because this response demonstrates a deep understanding of multiple things that students	Interviewer: What are some things that the students already know that you can relate to exponential functions? Kelly: The whole function thing for one. That's really the common denominator there. It's the same kind of thing. You're plugging in numbers and you're getting an output. You

should already know and how to relate these things to the concept of exponential function.	graph it the same way. It still has a domain, it still has a range. Start with just numbers together. Uhh. The applications should be stuff they're more familiar with. Yeah. You want to be able to solve everyday stuff with math.
Medium Level of Knowledge This was a medium level of knowledge because this response, while demonstrating an understanding of some good examples, does not discuss any sort of exponential equation examples or examples of finding an exponential model algebraically. They simple go from a basic table to straight to modeling.	Interviewer: What kinds of examples would be good to show students when they are learning about exponential functions? Pam: probably starting off with a basic example outside of a word problem. Like here's a table of values and then constructing an exponential function from that. Then going into some kind of modeling problem so students can see how exponential functions can be applied. Give an exponential function and ask what happens when we plug in this for x?
Low Level of Knowledge This was a low level of knowledge because the participant does not consider showing an alternate, and simpler, solution methods to this student.	Interviewer: How would you help this student (Interview 3 handout 2)? Michael: So you have the square root of this thing and you have this cubed, so I guess I would probably. Since they are obviously not thinking about it in like a formulaic way. They're not thinking about it as three divided by two in the exponent. I would probably go back to an earlier example. And I would say what happens if you have square root of four cubed. So in this situation I would ask them what is this? I think I would go through it this way first. I would hope they would get to two thirds equals eight. Then I would try to. Let's see, how would I do this? I guess I would try to also show that you could also do four cubed, square root. So you would have square root of 64. So you get eight. I guess I would try and show that it doesn't matter what the

	order of this is. Interviewer: Would you possibly want to give them an alternate way of doing this or would you want to just stick with what they have started? Michael: I guess that's a good point. Yeah, um. I mean you can do it this way. If they are only stuck on this one idea then they can probably solve it from there. Yeah this would be. That's a good point. This could take a really long time. In that case you can solve it a lot more simply. Yeah I don't know. That wouldn't be my natural tendency. (Interview 3)

Knowledge of Content and Students

High Level of Knowledge This was a high level of knowledge because this response demonstrated an understanding that students often attempt to isolate any terms involving the variable they are solving for.	Interviewer: What were the students trying to do in each of their solutions? (Interview 2 handout a) Pam: Students one and three took very similar approaches. Their working goal was to have a similar base for each one. Student one did the smaller and student three did the larger of the two bases. Student 2 put any term involving the x-variable on one side and made everything else constant on the other side.
Medium Level of Knowledge This was a medium level of knowledge because while some students might have this idea it is certainly not a fact of all students.	Interviewer: What are some things that the students already know that you can relate to exponential functions? Jim: They already have an intuitive sense of doubling or halving so start there.
Low Level of Knowledge This was a low level of knowledge because the response assumes that students in college algebra	Interviewer: Do you think that exponential functions are important to teach to students in college algebra? Why? Kelly: Students in college algebra aren't going on to grander things. It's a lower level class

are not going to do further mathematics.	and there's a large variety of majors.

Knowledge of Content and Curriculum

High Level of Knowledge This was a high level of knowledge since it covers a large variety of other areas that exponential functions are used.	Question: When would an exponential function be seen inside of the classroom? **Created response**: Exponential functions are seen in a variety of different mathematics courses as well as non-mathematics courses, such as calculus, differential equations, biology, chemistry and finance.
Medium Level of Knowledge This was a medium level of knowledge because the response only discusses other times exponential functions are seen.	Interviewer: What is important for students to understand about exponential functions? Jim: That they're (exponential functions) everywhere. Definitely everywhere. It's in loans, buying a house, buying a car, population type models. (Interview 3)
Low Level of Knowledge This was a low level of knowledge because the response failed to demonstrate any depth of knowledge of what it was that way required for future courses.	Interviewer: What would be something that is bad to leave out of a course that covers exponential functions? Michael: Anything that is a prerequisite for another course.