Computational Modeling of Photon Scattering In Gamma Ray Bursts Using Monte Carlo Techniques

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Abstract

The current model for gamma ray bursts (GRBs) assumes that internal shocks are responsible for the emission of γ -rays. Internal shocks occur when the fast expanding shell collides with slowly expanding shells. The light curve of a GRB depends on the initial state and the expanding speed of the photosphere. This research focuses on examine the photon scattering process when and after the shells collide. Using Monte Carlo Methods and computer simulations, we exam two different models of GRB with different parameters for the models. The results for the non-expanding centered photon model shows that changing the radius of the photosphere for cases with the same opacity and changing the opacity for cases with opacity > 100 do not change the observed light curve. For the non-expanding uniformly distributed photon model, cases with the same opacity but different radius produce the same observed light curve. However, cases with different opacity do not produce the same observed light curve. Lastly, we simulate the expanding centered photon model at different velocities and compare the results with the data from GRB 7475. The goodness tests, χ^2 , with $\alpha = 0.001$ show that null hypothesis can be rejected for the case with the expanding velocity of v= 0.2c.

1 Introduction

1.1 Origin of GRBs

Gamma ray bursts were first detected in the late 1960s by military satellites [1]. They are jets of high energy photons, usually above 0.1 MeV, that peak in the γ -spectrum and often outshine all other sources in the sky [2]. Figure 1 shows the energy levels observed from Earth for several GRBs and cosmological objects [3]. These objects are much closer to Earth compared to the GRBs.



Figure 1: The energy levels of several GRBs and cosmological objects observed from Earth [2]. The GRB sources are at greater distance from Earth compared to the cosmological objects.

GRBs were originally thought to come from within the Milky way galaxy, but it could not be proven because only the γ -spectra could be detected and their durations were too short to pinpoint their locations. In 1997, Italian-Dutch satellite Beppo-SAX successfully captured the first high resolution X-ray images of the afterglow of a burst, which had a longer duration than its counterpart, γ - spectrum, and made it possible to accurately measure its position [1]. With several detections of GRBs' afterglows later on, it was certain that GRBs occurred at cosmological distances.

GRBs are very diverse in terms of duration, temporal profiles, and spectral variations. Figure 2 shows some burst morphologies and durations. They can be categorized into two types: short/hard GRBs that last less than 2 seconds and long/soft GRBs that last longer than 2 seconds. The two have different progenitors and surrounding environments.



Figure 2: The time profiles, intensities, and durations of γ -ray bursts from BATSE Catalog [2].

1.2 Types of GRBs

1.2.1 Short GRBs

Short gamma ray bursts (SGRBs) are the results of merging of two neutron stars (NS-NS) or a neutron star and a black hole(NS-BH) [4]. Figure 3a provides the process of SGRBs. The two compact objects are bound by a strong gravitational force and slowly spiral inward. When the two finally merge, their matter, which can exceed 18 billion degrees Fahrenheit in temperature, forms an accretion disk around their axis of rotation. The magnetic field strength is amplified up to ten thousand times the individual's initial strength, while a black hole forms at the center [5]. Both the matter and the magnetic field of the system gradually become more organized with the magnetic field taking the shape of a jet. Figure 4 shows the evolution of the magnetic field in a GRB. SGRBs have relatively short durations compared to LGRBs' durations, hence the name [5]. The bursts themselves last for less than 2 seconds and are less energetic than long gamma ray bursts. They are likely to occur in early and late type galaxies with low star formation rate and are associated with an old stellar population [4].

1.2.2 Long GRBs

Long gamma ray bursts (LGRBs) are those that have duration longer than 2 seconds. The most widely accepted model for LGRB progenitors is the collapsar model which was proposed by Woosley [6]. The model suggests that massive stars, at the end of their lives, collapse under the force of gravity and undergo supernovae. Their cores collapse further into black holes. As in the case for SGRBs, the process causes an

accretion disk to form around their axes of rotation and emit γ -rays via the jet perpendicular to the disks [7]. Figure 3b shows the process of a collapsing star. Observation data suggest that LGRBs are located in high star-formation regions [5]. Theoretically, LGRBs' progenitors should have a low metal content in order to meet the angular momentum and the mass requirements for the progenitors, which range from primordial iron abundance, $Z^{0.86}$, to 1% solar's metallicities [6].



Figure 3: a)The merging process of two neutron stars. The stars slowly spiraling towards each other and their orbital speeds increase. The result of the merger is a black hole with two γ -ray jets moving at a relativistic speed parallel to the black hole's axis of rotation. b)The force of gravity causes the star to collapse onto its core, resulting in supernova explosion while a black hole is forming at the stars center. The surrounding matter fall into the black hole and causes γ -burst.(Credit: http://ecuip.lib.uchicago.edu/)



Figure 4: After the merging of the two neutron stars, the magnetic field of the system gradually becomes more organized and slowly taking a jet-like shape. a)The magnetic field at 13.8 milliseconds after merging. b) The magnetic field 15.26 milliseconds after merging. c) The magnetic field 21.2 milliseconds after merging. d) The magnetic field 26.5 milliseconds after merging. [5]

1.3 Theory

1.3.1 Fireball Model

The most accepted model of GRBs is the "Fireball Model". The outflow from a GRB's engine has a wide range of velocities and finite durations, hence they can be modeled as multiple discrete relativistic

expanding shells [8]. In the Fireball model, a thermally- or magnetically- driven central engine releases energy after the merger or while the star is collapsing [9]. The engine causes intense pressure inside the fireball. The ball then, behaving as a shock wave, expands outward at a relativistic speed. Inside the ball, the temperature, pressure, and density vary, which cause the discrete shells to move at different velocities [5]. At this state, most of the thermal energy has already been converted to the outflow's kinetic energy, while some has been released as photons in the first shell known as the photophere [9]. The fast expanding shell continues to expand at a relativistic speed and collides with the slowly moving shells. This process is called "internal shock" and it is responsible for GRBs [10]. The shock wave further collides with the surrounding matter and produces X-ray emission and later on, radio waves[10]. This process is known as the external shock and it is responsible for the afterglow. The representation of the model is shown in Figure 5.



Figure 5: The engine releases its energy and causes intense pressure inside the fireball. The pressure and temperature inside the ball vary which causes the shells to expand at different speeds. GRBs occur when the faster moving shell collides with a more slower moving shell in the Fireball model. The shells further collide with the surrounding gases and produce X-ray. (Credit: NASA's Goddard Space Flight Center)

1.3.2 Internal Shocks

As mentioned previously, internal shocks occur when the faster expanding shell collides with the slower expanding shell. This occurs at:

$$R_{int} = c\delta t \Gamma^2 = 3 \times 10^{14} cm \Gamma_{100}^2 \tilde{\delta} t \tag{1}$$

where Γ_{100}^2 is the Lorentz factor with units of 10^2 and δt is the time difference between the emission of the two shells, which is roughly equal to δt , the observed fluctuation in the light curve of the burst [7]. The energy after the collision is:

$$E_{int} = \Gamma_m \epsilon_{int} \tag{2}$$

where ϵ_{int} is the internal energy in the local frame, and Γ_m is the resulting Lorentz factor in an collision collision of the shells, which turns out to be:

$$\Gamma_m \approx \sqrt{\frac{m_r \Gamma_r + m_s \Gamma_s}{m_r / \Gamma_r + m_s / \Gamma_s}}, \Gamma_r \ge \Gamma_s >> 1$$
(3)

where m_r , Γ_r, m_s , and Γ_s are the masses of the faster expanding shell and slower expanding shell, and their Lorentz factors respectively [7].

1.4 Focus of The Research

This research uses the Monte Carlo method(MCM) to study the photon scattering process in GRBs. This method is commonly used to model photons' propagation in a medium [11]. MCM uses large quantity of randomly generated numbers to obtain numerical results. It offers a powerful tool to study stochastic processes, such as particle transport and kinetic theory of gases[12]. These processes depend highly on random interaction between particles. For the first model, MCM is used to simulate random distribution of photons inside a spherical medium of uniformly distributed electrons. For the second model, the photons are positioned at the center of the sphere. The photons are also assigned with initial velocities using MCM, which are random in all directions. Next, the photons are scattered via Thompson scattering inside the medium until they are emitted. The photons are assigned with new velocities after each scattering until they leave the sphere. The time between each scattering is determined using Beer's law which relates the photons' motions with their medium. In the simulations, the photons are allowed to leave in all directions, but realistically, photons only leave the sphere in the directions of the jet. Once all the photons are emitted, the emission curve can be obtained by creating a histogram of the number of photons and the normalized emission time. The observation light curve is constructed after obtaining the emission light curve. Lastly, the medium is allowed to expand at non-relativistic/relativistic speeds for a new model. The same procedures will be used to generate light curves for the new cases.

1.4.1 Beer's Law and Poisson Distribution

The frequencies of collisions are different for the photons. This is determined using Poisson probability distribution function

$$P = \frac{N!}{n!(N-n)!} (\frac{v}{N})^n (1 - \frac{v}{N})^{N-n},$$
(4)

where N is the number of trials, n is the successes in N trials, and v is the expectation value for of n. As the number of trials get larger, Equation 4 becomes:

$$P = \frac{v^n e^{-v}}{n!},\tag{5}$$

Poisson distribution is a binomial distribution and is widely used for stochastic processes [13]. Combining Poisson distribution and Beer's law gives an expression for the stepsize between each scattering and it is :

$$\epsilon = e^{-\mu_t \Delta x},\tag{6}$$

where ϵ is a random variable between zero and one, μ_t is the property of the material, and Δx is the stepsize[11]. From Equation 6, the stepsize for each photon is:

$$\Delta x = -\frac{1}{\mu_t} \ln(\epsilon) \tag{7}$$

The stepsize can be converted into the time between each scattering by multiplying Δx by the speed of light c.

2 Methods

2.1 Summary

To mimic the photon scattering process in a GRB, a sphere of a known radius was created. The sphere had an initial opacity (τ) which was:

$$\tau = \sigma \times \rho_{electron} \times c \tag{8}$$

 σ , $\rho_{electron}$, and c were the effective cross section of scattering, the electron density, and the speed of light, respectively. The sphere also had a photon density, and a relativistic expanding velocity (v_{expand}). The

expanding velocity was zero for the non-expanding models. The photons were given initial positions within the sphere and initial velocities equal to unity. The photons were scattered, via Thompson scattering, until they were emitted. The emission times of the photons were recorded as well as the observation times. The simulation was done for various opacities (τ), to study the variations of GRBs' light curves.

2.2 Photon Scattering Process

Initially, the photons were positioned at the center of the sphere or randomly distributed inside the sphere, and they were each assigned a randomly normalized velocity equal to the speed of light in vacuum, c. Next, the photons were scattered and the times between the scattering were determined by:

$$\xi = e^{-\tau \Delta t} \tag{9}$$

where ξ is a random variable between 0 and 1. After each scattering, the photons were each assigned with a new randomly normalized velocity and different value for ξ . The radius of the sphere, R_{sphere} were increased by:

$$R_{new,sphere} = R_{old,sphere} + v_{expand}\Delta t \tag{10}$$

2.3 Generating the Light Curves

2.3.1 The Emission Light Curves

The times were incremented by Δt of the photons. When the photons escaped the sphere, the emission times were recorded. These were the times the photons took to escape the sphere and traveled some distances outside the sphere. The true times were found by:

$$T_{emission} = T_{recorded} - \frac{|P_{final}| - R_i}{c} \tag{11}$$

where $|P_{final}|$ were the final positions of the photons when the timer stopped.

2.3.2 The Observed Light Curves



Figure 6: When the photons escape the sphere, they have to travel some distance D_i to reach the detector. The distance the photons travel varies for the expanding models.

Figure 6 shows the process of the photons after they are emitted. D in the figure represents the distance from the center of the sphere to the detector on Earth, D_i is the distance the photons travel to reach the detector, R_i is the radius of the sphere at the instance the photon i is emitted, and v_{expand} is the velocity of the sphere. Using Figure 6, the observation time is:

$$t_{observation} = \frac{R_i}{v_{sphere}} + \frac{D_i - D}{c} \tag{12}$$

From the figure, $D_i - D = -R_i$. Substitute this into Equation 7:

$$t_{observation} = \frac{R_i}{v_{sphere}} - \frac{R_i}{c} = T_i - T_i \frac{v_{sphere}}{c},\tag{13}$$

where $T_i = t_{emission}$.

2.4 Program Verification

To check the validity of the written program, ten cases with different R_{sphere} , τ , and $P_{initial}$ were simulated to verify with the theoretical prediction for the average emission time which is:

$$< T_{emission} > = \frac{\tau^2}{\rho_e \sigma c \sqrt{2}},$$
(14)

and the average scattering which is:

$$\langle N_{scattering} \rangle = \frac{\tau^2}{\sqrt{2}}.$$
 (15)

Table 1 shows the parameters for the cases that were used to test the validity of the code. The results were plotted in Log-Log scale as well as the theoretical predictions.

Non-Expanding Centered Photons Model											
Case	1	2	3	4	5	6	7	8	9		
$R_{sphere}(meter)$	1.0	10.0	1.0	10.0	1.0	10.0	100.0	100.0	100.0		
Opacity (τ)	10.0	10.0	30.0	30.0	100.0	100.0	10.0	30.0	100.0		
N_e	$6.30\mathrm{E}25$	$6.30\mathrm{E}27$	1.90E26	1.90E28	$6.30\mathrm{E}26$	$6.30\mathrm{E}28$	6.30E29	1.90E30	6.30E30		

Table 1: The cases used to verify the written program. In each case, two parameters: the radius of the sphere, R_{sphere} , and the opacity, τ , were varied, which in turn changed the electron density, ρ_e .

2.5 The Non-Expanding Models

Once the program was verified, the non-expanding uniformly distributed photon models and the non-expanding centered photon models were simulated for different parameters. The simulated used the same sets of parameters for both models. Table 2 shows the parameters for the cases.

Non-Expanding parameters for both uniformly distributed photons cases and centered photons cases										
Case	1	2	3	4	5	6				
R_{sphere}	1.0	10.0	1.0	10.0	1.0	10.0				
Opacity (τ)	10.0	10.0	100.0	100.0	1000.0	1000.0				
N_e	6.30E25	6.30E27	6.30E26	6.30E28	6.30E27	6.30E29				

Table 2: The cases for the non-expanding models. These parameters were used in both the uniformly distributed model and the centered model.

The histograms of the cases were plotted with the count of photons against normalized observation time. The last step was to use the KolmogorovSmirnov test (KS) to find the correlation between the cases. This was determined by calculating the KS statistic, *D-value*, and the probability, *P-value*.

2.6 The Expanding Models

The sphere was allowed to expand at ten different speeds for the ten cases. The parameters are shown in Table 3. The sphere expanded by changing the radius as shown in Equation 5. The results were used to create the histograms as in the previous section.

Expanding Centered Photon Model										
Case	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$									
R _{sphere}	1.0									
Opacity (τ)	1000.0									
Expanding speed (v_{sphere}) 0.1c 0.2c 0.3c 0.4c 0.5c 0.6c 0.7c 0.8c 0.9c 0.									0.99c	

Table 3: The sphere were expanded at the relativistic speeds showing in the table with the same initial conditions. The radius rate of change depended on Δt

3 Results

3.1 Program Verification

The nine cases in Table 1 were simulated and the histograms of the results are shown in Figure 7. The vertical axes in both plots represent the count of emitted photons at a certain time and the horizontal axes represent the emission time in second. The legends in the plots show the colors corresponding to each of the cases.



Figure 7: a) Histograms of cases 1 to 5 from Table 1. The cases are labeled with colors in the legend. b) Histograms of cases 6 to 9 from Table 1. The legend shows the colors corresponding to the cases.

The average emission time was calculated for each of the cases as well as the average number of scattering. The average emission times were plotted against the corresponding $\frac{N_e}{R_{sphere}}$ values and the average numbers of scattering were plotted against $\frac{N_e^2}{R_{sphere}^4}$ values in Log-Log scale. Figure 7 shows the plots for the average emission time and the number of scattering in Log-Log scale. The theoretical results, Equation 9 & 10, are also included in the plots.



Figure 8: a) The Log-Log scatter plot for the average times and the corresponding $\frac{N_e}{R_{sphere}}$ for the six cases in Table 1. The solid line is the theoretical plot for the average time and $\frac{N_e}{R_{sphere}}$. b) The Log-Log scatter plot for the average number of scattering and the corresponding $\frac{N_e^2}{R_{sphere}^4}$. The solid line is the theoretical plot for the average number of collision.

3.2 The Non-Expanding Model and Kolmogorov-Smirnov Test

3.2.1 The Centered Photon Cases

The cases in Table 2 were simulated for the centered photon model and the histogram of the results was plotted. Figure 8 shows the histograms of the six cases with the vertical axis representing the photon count and the horizontal axis representing the normalized emission time. The parameters for the cases are also shown in the figure with each color coded.



Figure 9: The histograms for the non-expanding centered photon cases.

The KS tests were used to seek the relationship among the cases. The results are shown in Table 4. The values in each cell represent the KS statistic and the probability, respectively.

	KS tests for the centered photon model										
Case		2	3	4	5	6					
1	(D-value, P-value)	5.30E-03, 7.92E-01	1.01E-02, 9.32E-02	1.11E-02, 4.93E-02	9.47E-03, 1.35E-01	1.12E-03, 4.71E-02					
2	(D-value, P-value)		1.32E-02,1.06E-02	1.43E-02,4.40E-03	1.34E-2,8.82E-3	1.21E-02, 2.51E-02					
3	(D-value, P-value)			4.57E-03,9.13E-01	9.43E-03,1.38E-01	6.60E-03,5.29E-01					
4	(D-value, P-value)				7.67E-03,3.40E-01	4.07E-03,9.65E-01					
5	(D-value, P-value)					6.4E-03,5.69E-01					

Table 4: The results of KS tests of the cases. The first value in each cell corresponds to the D-value and the second value corresponds to the P-value.

3.2.2 The Uniformly Distributed Photon Cases

The cases in Table 2 were simulated for the uniformly distributed photon model. Figure 9 shows the histogram of the six cases, plotting the photon emission versus the normalized emission time. The KS tests were also used to show the relationship between the cases. The results for the tests are shown in Table 5 below.



Figure 10: The histograms for the non-expanding uniformly distributed photon cases.

	KS tests for the uniformly distributed model											
Case	1	2	3	4	5	6						
1	(D-value, P-value)	6.67E-03, 5.16E-01	9.86E-02, 1.99E-127	9.86E-02,2.43E-127	1.12E-01,4.39E-164	1.11E-01,4.81E-162						
2	(D-value, P-value)		9.77E-02,5.98E-125	9.86E-02,2.43E-127	1.12E-01,8.59E-164	1.10E-01,3.01E-159						
3	(D-value, P-value)			4.97E-03,8.52E-01	1.89E-01,4.34E-05	1.87E-02,5.24E-05						
4	(D-value, P-value)				1.71E-02,3.05E-04	1.69E-02,3.61E-04						
5	(D-value, P-value)					6.00E-03,6.51E-01						

Table 5: The results of KS tests of the cases. The first value in each cell corresponds to the D-value and the second value corresponds to the P-value.

The numbers in each cell represent the KS statistic, D-value, and the probability, P-value, respectively.

3.3 The Expanding Centered Photon Cases

The sphere was allowed to expand at difference relativistic speeds for the cases and the results are shown in Figure 11.



Figure 11: The histograms of the ten cases for the expanding centered photon model.

The results were normalized and modified so that all the light curves peaked at one second. Next, the light curve of GRB 7475 from BATSE was used for the fitting. Figure 12 below shows the fitting for the ten cases. All cases were stretched or shrunk in both vertical (a) and horizontal directions (b) and shifted in the horizontal direction (t_0) .



Figure 12: The ten expanding cases were modified to best fit GRB 7475 data.

4 Discussion

4.1 **Program Verification**

The average emission time and the average collision were determined based on the assumption that all photons were originated at the center of the fireball. The results indeed followed the theoretical prediction that the averages were expected to be $\frac{N_e}{R_{sphere}}$ and $\frac{N_e^2}{R_{sphere}^4}$. The simulated results were lower than the theoretical predictions which suggested that the predictions were over approximated.

4.2 The Non-Expanding Models

4.2.1 The Centered Photon Model

The light curves produced by the cases peaked at about the same interval of normalized time. This behavior is what we should expected for $\tau >> 10$. When the opacity is large, the shape of the curve is invariant [14]. The KS2 showed that the cases with the same opacity but different radii had high a P-value. The cases with the same $\frac{R_{sphere}}{\tau}$ also had high a P-value. Two observations were observed from the results for the non-expanding centered photon model: 1) Cases with the same opacity but different radii produce the same light curve. 2) Increasing the radius and the opacity by the same magnitude will not alter the observed light curve.

4.2.2 The Uniformly Distributed Photon Model

The light curves produced by the non-expanding uniformly distributed photon model showed that cases with lower opacity peaked at latter times compared to those with higher opacity. This model had the same property as the centered photon model that cases with the same opacity but different radii produced the same light curves and the KS2 test for those cases yielded a high P-value. The KS2 test showed that cases with the same τ : R_{sphere} ratio had a low P-values, suggesting that there was no correlation between the cases.

4.3 The Expanding Model

The fitting was done by changing the three degrees of freedom until the theoretical results closely resembled GRB 7475 data. Next, we calculated the χ^2 value for for the fitting and Table 6 below shows the changed values for the fitting and the χ^2 value for each of the results.

	GRB 7475 Fitting											
Case	1	2	3	4	5	6	7	8	9	10		
a	21	18.4	14.4	11.4	9.2	7.3	6	4.9	4.5	1.7		
t_0	-10	-7.2	-4.3	-2.1	-0.2	1.35	2.4	3.4	3.4	5.18		
χ^2	2543	1873	2096	3024	4484	6305	8075	10738	14105	15874		

Table 6: The values of a, t_0 , and χ^2 for GRB 7475 fitting. The values of a and t_0 were chosen so that the theoretical results closely resembled GRB 7475 data.

We chose the level of significance (α) to be 0.001 and calculated the χ^2 value for which the null hypothesis would be accepted or rejected. The value of χ^2 for $\alpha = 0.001$ and 1713 degrees of freedom, which is the difference of 1718 data points and the 5 fitting parameters, is 1900. The null hypothesis can be rejected if χ^2 from the fitting is less than 1900. From Table 6, only case 2 have $\chi^2 < 1900$. This suggested that the cases met the observed data distribution. All other cases have $\chi^2 > 1900$ which we have to take into account the null hypothesis.

5 Conclusion

For the non-expanding centered photon model, the light curves were invariant. Cases with the same value of opacity but different radii produced the same light curve and had a high P-value which rejected the null hypothesis. Also the light curve remained the same if the radius and the opacity increased by the same magnitude. For the non-expanding uniformly distributed photon model, the light curves peak at earlier times for cases with higher opacities. The KS2 tests showed that there were correlations between cases with the same opacity and no correlations between cases with the same $\tau : R_{sphere}$ ratio. For the expanding centered photon model, the null hypothesis was rejected for case 2 with $\chi^2_{case2} = 1873$. The null hypothesis were accounted for all other expanding cases with $\chi^2 > 1900$.

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