AN ABSTRACT OF THE
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Title: Middle School Students' Mathematical Dispositions in a Problem-Based Classroom

Abstract appr

Signature redacted for privacy.

Diànne K. Erickson

The purpose of this study was to describe middle school students' mathematical dispositions in a problem-based learning [PBL] classroom. Eight volunteer students from one 6th grade mathematics classroom participated in this study. The curriculum used was the Connected Mathematics Project [CMP]. The main sources for data collection were classroom observations, the Attitudes and Beliefs questionnaire, teacher interviews, and student interviews. The CMP class routine consisted of four phases: Warm-up, Launch, Explore, and Summarize. The teacher in this study had her students investigate mathematics problems within cooperative small groups and share their ideas in large group discussions. The teacher acted as a facilitator and encouraged her students to try new ideas without fear of making mistakes.

The findings revealed that almost all of the students in this study demonstrated positive mathematical dispositions. They volunteered and shared their ideas, both in small cooperative group investigations and in large group discussions. They believed

mathematics was about "learning new ideas" and mathematics was "life" because it was everywhere in their lives. They also mentioned the usefulness of numbers, measurement, and geometry in their daily lives. All eight participants liked hands-on activities and working on a mathematics project. Most of them agreed that they liked mathematics because it was fun and interactive. Most also saw themselves as good at mathematics. All of them agreed that mathematics was useful, and that one's mathematics ability could be increased by effort. They also believed that there were no gender differences in mathematics, even though in their class, they realized that boys outperformed girls. Most of the students agreed that they could solve time-consuming mathematics problems and that it was important to understand mathematical concepts. None of them had negative feelings about group work; they learned from each other.

Finally, an analysis of the participants' mathematical dispositions was discussed. Based on the Taxonomy of Educational Objects: Affective Domains by Krathwohl, Bloom, and Masia (1964), the participants were categorized into three disposition levels: Level 1: "receiving;" Level 2: "responding;" and Level 3: "valuing." Half of the participants demonstrated dispositions at the high level (Level 3: "valuing") because of their willingness to pursue and/or seek to do mathematics outside the classroom. Three of them were in mathematics disposition Level 2.3: "satisfaction in response" because they usually participated in the class activities. They were satisfied and enjoyed doing mathematics. One of them demonstrated mathematical disposition Level 1.2: "willingness to receive" because she listened to the whole class and group discussions without sharing any ideas or asking for help when she needed it.

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Middle School Students' Mathematical Dispositions in a Problem-Based Classroom

by

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CHAPTER I

THE PROBLEM

Introduction

Middle school students are at a stage in their lives where they are rapidly developing both physically and emotionally. These adolescents are often shifting from concrete to abstract thinking, as well as acquiring new self-concepts and social skills (Krulik, Rudnick, & Milou, 2003). Also, they are developing lasting attitudes and beliefs about learning, work, and other adult values (Clewell, Anderson & Thorpe, 1992). Middle school students often make permanent decisions based on their mathematical dispositions that may affect the rest of their lives (National Research Council [NRC], 2000). Therefore, it is critically important for teachers to develop effective teaching approaches aimed at positively enhancing students' mathematical experiences.

> An understanding of the general developmental needs of middle school students, regardless of sex or race, can help establish guidelines for determining teaching strategies, including planning teacher-student activities, selecting materials, and organizing learning situations, and thus help make the teaching and learning processes profitable and rewarding experiences for both educators and students. (Clewell, Anderson & Thorpe, 1992, p. 18)

Statement of the Problem

In the past, many students believed that mathematics was about the process of memorizing teacher directions. Many students saw mathematics as a tedious subject because of the way they were taught, which was to listen to what their teachers said, memorize what their teachers did, and follow the teachers' procedures without necessarily trying to understand the concepts (Ridlon, 1999). Many youth seem to lose their inspiration to learn mathematics by the time they reach high school because of their attitudes and beliefs about mathematics (Clewell, Anderson, & Thorpe, 1992). Moreover, some middle school students had difficulty applying the mathematics they learned in schools to their daily lives or future careers (Saxe, 1988). As a result, recent research shows that middle school students still do not perform well on mathematical tasks that require mathematical understanding and problem-solving skills, even though they have significantly improved their mathematics on computational skills since 1973 (Reese, Miller, Mazzee, & Dossey, 1997).

At the age of early adolescence, affective domains often impact the teenagers' discussions and lives. Even though they still depend on their parents, they begin to look for independence and freedom (Krulik, Rudnick, & Milou, 2003). Many issues emerge during the middle school years. For instance, teenagers begin to shape their views of femininity and masculinity (Clewell, Anderson, & Thorpe, 1992). Teenage girls' confidence in their abilities to do mathematics decreases throughout the middle school grades (Fennema, 1996). In addition, middle school students have to consider academic options and make decisions about course selections that will affect their future career choices (Throndike-Christ, 1991). Therefore, the middle school years are an opportune time to explore affective and cognitive issues.

> Mathematics is a key to many opportunities. It opens doors to careers and enables informed decisions (NRC, 1989). However, a large segment of our society is willing to make statements like 'I hated math in school;' 'math, yuk;' and 'I did not do well in math.' These cannot continue to be acceptable statements… It is our commission to set the wheels in motion that will change this attitude. (Brumbaugh & Rock, 2001, p.4)

Mathematics educators have long been aware of the need to improve student cognition and affective factors. Since 1989, the National Council of Teachers Mathematics [NCTM] developed affective goals related to student understanding, including the value of mathematics and appreciation of the role of mathematics in society as ways of promoting students' positive dispositions toward mathematics.

The International Association for the Evaluation of Educational Achievement [IEA]'s Third International Mathematics and Science Study [TIMSS] revealed a positive relationship between middle school students' positive attitudes toward mathematics and higher mathematics achievement in several countries such as England, Denmark, Singapore, Thailand, and the United States. Additionally, most 8th grade students in several countries expressed that they liked mathematics to some degree. Nevertheless, in some countries such as Germany, Japan, Korea, and the Netherlands, almost half of their 8th grade students indicated that they disliked mathematics (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996).

In the United States, research on mathematical dispositions, including attitudes and beliefs about mathematics has made significant contributions over the past 30 years. This has provided insights about students' dispositions in learning mathematics. Prior to the 1970s, research about students' mathematical dispositions was limited. During the 1970s, research in this area began to appear in the literature. Fennema and Sherman (1976), for example, developed scales measuring attitudes and beliefs about mathematics. Research on attitudes toward mathematics was mostly focused on the relationships between gender differences and mathematics achievement. In early studies, Aiken (1976) reported that attitude toward mathematics was related to achievement in mathematics. More studies provided information for gaining a better understanding of mathematical dispositions, especially students' beliefs about mathematics and their reaction in mathematics classrooms (McLead, 1992; Kloosterman, 1991). Mathematical dispositions also had a significant impact on students' decisions about their willingness to take optional mathematics coursework and pursue careers related to mathematics in the future (Thorndike-Christ, 1991). These dispositions extend beyond a mere liking for mathematics to forming attitudes and beliefs about mathematics, which are related to mathematics performance (NCTM, 1989). Knowing students' dispositions toward mathematics may motivate teachers to provide appropriate classroom environments and activities.

According to NCTM (1989) and Krathwohl, Bloom, and Masia (1964), mathematical dispositions are defined as follows:

1. Willingness to receive, respond, and explore in solving mathematics
2. Attitudes toward mathematics
3. Beliefs about mathematics
4. Perseverance in doing mathematics
5. Tendency to reflect on one's own thinking
6. Values and interests in mathematics subject
7. Desire to become a life-long learner of mathematics

Mathematical dispositions have been studied in traditional classrooms for years. Attitudes and beliefs are important elements in dispositions toward a subject. According to Fishbein and Ajzen (1975), "Attitudes are feelings that generally have a moderate level of intensity, can be either unfavorable or favorable in direction, and are typically directed toward some subjects" (p. 222). Rajeki (1990) suggested that social learning is related to the process of attitude formation.

The author suggested that students' attitudes are influenced by their society such as parents, peers, teachers, and other informational sources such as commercial media. Based on social learning, Bandura (1997) noted that people form their dispositions by observing and modeling others' behaviors. In addition, Aiken (2002) noted, "As children mature, their attitudes, although typically remaining somewhat similar to those of their parents, become more like those of their age-mates and other people in their expanding social world"(p. 56).

Clearly related to attitudes are beliefs. Beliefs are defined as, "Confidence in the truth or exercise of something that is not immediately susceptible to rigorous proof. Beliefs are less certain than knowledge but more certain than attitudes..." (Aiken, 2002, p. 6). In the process of belief formation, students first think of how reliable the given information is. Then, students basically think of possible explanations for the information according to their expectations and mental models (Elster, 1999). Frijda and Manstead (2000) stated, "A belief is plausible when it agrees with mental mechanisms, and with expectations. These include the opinions current in the social environment, as well as the attribution rules that underlie the emotional instrumentality of beliefs" (p. 69).

As focus on mathematical dispositions, including attitudes and beliefs about mathematics, has increased; educators have been discussing the importance of promoting positive mathematical dispositions. Recently, in one of their publications, *The Principles and Standards for School Mathematics* (2000), the NCTM called for reform in teaching and learning mathematics. Two important goals of the reform are to improve students' mathematical dispositions and their mathematics achievement. The NCTM promotes students' realization of the value of mathematics and their desire to pursue becoming life-long learners. Both of these are part of a student's mathematical disposition, which in turn may foster achievement.

Additionally, mathematics educators have accomplished some gains in attempting to increase students' mathematics achievement. The National Assessment of Educational Progress [NAEP] (Reese, Miller, Mazzee, & Dossey, 1997) reported that 8th grade students had significantly higher performance on computational tasks than they had 30 years ago. However, the 8th grade students still did not perform well on mathematical tasks that required mathematical understanding, such as solving multi-step problems, or explaining reasons for solving problems. To enhance students' understanding, current research has suggested that mathematics teachers should build students' learning from the students' prior knowledge (e.g., Hiebert, Carpenter, Fennema, Fuson, Wearne, Murry, Oliver, & Human, 1997; Hiebert, 1998). In constructing knowledge, people have an active role in the knowing or coming to know process (Piaget, 1970). "Piaget proposed that one's concept development could not be done by observation, but in fact, it must be constructed personally. This is the foundation of the constructivism framework" (Miller, 1993, p. 2).

The effects of constructivism on learning and teaching mathematics have been discussed for years. Vygotsky (1978) proposed that students generate new concepts by reflecting on and visualizing meaningful concepts in their mind during communications and interactions. Fosnot (1996) added that Vygotsky defined the "zone of proximal development" [ZPD] as the zone of the difference between what learners can accomplish by themselves and what they can attain only with guidance or help from capable peers or adults. With this idea, scholars are beginning to look at the value of the group work process.

Moreover, Svincki (1999) suggested that group learning promotes an individual's interpretations of his/her circumstances and improves his/her understanding. Goodman (1984) suggested that to be successful in group work and problem solving, the students must be capable of constructing conceptualizations by using their previous knowledge or experiences in group discussions.

The concept of a student-centered approach, including peer communication, has led to continual calls for reforming the mathematics preparation of students (Loveless, 2001). As teachers search for new teaching approaches, problem-based learning [PBL] is one such approach that teachers believe potentially increases students' interpersonal, active learning, and problem-solving skills (Sage, 1996). Savery and Duffy (1999) noted that "PBL might be one of the best examples of constructivist learning" (p.21).

Kelly and Lesh (2000) noted that the PBL approach, which they called the "construct-eliciting" approach, leads students to solve problems that involve not only finding an answer but also involve a series of interpreting questions in the problems. They also added that the construct-eliciting activities are related to many processes such as integrating, modifying, refining, and revising in order to solve problems. This approach also includes developing self-directed learning skills, thinking skills, problem-solving skills, learners' knowledge acquisition and use, and cooperative learning.

> PBL is a method based on the principle ofusing problems as the starting point for the acquisition of new knowledge. Pivotal to its effectiveness is the use of problems that create learning through both new experience and the reinforcement of existing knowledge. Situations that are in the learner's real world are presented as problems and stimulate the need to seek out new information and synthesize it in the context of the problem scenario. (Lambros, 2002, p. 1)

Torp and Sage (1998) discussed the core elements in problem-based learning and concluded that

> PBL is a student-centered pedagogical strategy that creates an active small group learning environment around an investigation and resolution of real-world problems while teachers coach students thinking, guide students' inquiry, and provide opportunities for reflection to learn as they develop conceptual understanding and thinking skills. (p. 15)

More recently, researchers have found that the PBL approach seems to reinforce positive mathematical dispositions including attitudes and beliefs about mathematics among middle school students (e.g., Boaler, 1998, 2002; Brenner, Mayer, Moseley, Brar, Duran, Reed, & Webb, 1997; Gallaher, 1997, Yu-Chen Hsu, 1999). However, more comprehensive studies are needed on the PBL approach and how students react, think, and feel about it. Therefore, the purpose of this study is to describe the middle school students' mathematical dispositions in a PBL classroom environment that involves cooperative learning and real-world problems.

Significance of the Study

Current empirical research can identify and strengthen effective mathematics classroom practices. Gaining insights about students' mathematical dispositions will help educators, the teachers, and pre-service teachers adjust new teaching strategies that will benefit all students. These may include planning student-centered activities and organizing new learning situations as well as enhancing classroom environments, in order to promote students' positive perceptions of mathematics and their ability to do mathematics.

Moreover, findings about teachers' experiences in implementing the PBL instructional method can demonstrate the effectiveness and challenges of this approach. Knowledge about teachers' successes and obstacles can help to improve implementation of the PBL approach. Knowing how to implement the PBL approach more effectively may contribute to a classroom environment, which enhances students' mathematical dispositions. This knowledge may help mathematics educators find suitable direction and guidance for professional development of both pre-service teachers and mathematics teachers leading to the goal of promoting positive mathematical dispositions in PBL classrooms.

Furthermore, the effects of the PBL instructional approach on students' learning and dispositions will be important in developing mathematics curriculum more relevant to students' interests and future needs. Recently, new Standards-based curricula have been developed in order to enhance student learning in mathematics. At the middle grade levels, five new mathematics curricula funded by the National Science Foundation [NSF] are available: Connected Mathematics [CMP] (EDC, 1999); Mathematics in Context [MiC] (EDC, 1999); MathScape; Seeing and Thinking Mathematically [MathScape] (EDC, 1999); MATHThematics or Six Through Eight Mathematics [STEM] (EDC, 1999); and Middle-school Mathematics Through Applications [MMAP] (EDC, 1999). Other new curricula programs, including problem-based and cooperative learning, can be developed to meet the increasingly diverse needs of the mathematics classrooms. There is still a need for more empirical studies to show whether teachers' use of these curricula have a positive effect on students mathematics learning in ways that enhance students' mathematical dispositions as well as their achievement.

CHAPTER II

REVIEW OF LITERATURE

Introduction

The objective of this chapter is to provide a foundation for this study. The chapter consists of three literature review sections. First, literature related to the topics of a problem-based theoretical frame work is reviewed. Second, implementation of the problem-based approach by teachers is presented. Third, students' mathematical dispositions in problem-based contexts are provided.

Theoretical Framework

> Mathematics education is struggling mightily to escape the stranglehold of the outdated and inadequate behaviorist learning theory that has dictated the course of mathematics teaching for more than forty years. In its place, all current major scientific theories describing students' mathematics learning agree that mathematical ideas must be personally constructed by students as they intentionally try to make sense of situations. (Loveless, 2001, p.53)

Because the workplace of the 21st century requires life-long learners, educators are searching for ways to assist students in building their own knowledge and skills (Loveless, 2001). Constructivist pedagogical concept has been emphasized (Evensen & Hmelo, 2000). Constructivism, the process of coming to know, involves a knower and the known. It describes constructing knowledge by the knower investigating the environment. Consequently, knowledge is built from responses to the students' world (Grigoruk, 1997).

Over 60 years ago, Jean Piaget introduced an idea that serves as the foundation of constructivism (Fosnot, 1996). The complementary processes for cognitive change that Piaget described are assimilation and accommodation. Assimilation happens when an individual takes in information. There are many sources of the information, such as playing with Legos, reading books, or listening to conversations. The individual adds new information to his/her existing knowledge through such experiences. The process of accommodation takes place when this information causes individuals to transform their understanding of an idea or event. For example, a child might believe that 2x is always greater than x, when x is any amount. If then the teacher introduces the idea that when x = 0, 2x equals x, to accommodate his/her cognitive system to the new information, he/she might change his/her cognitive system to include the idea that 2x = x when x = 0. The transformation of an idea that results in a new cognitive structure is called equilibration. Equilibration has taken place after an individual's perceptions, which are in conflict with new information, are resolved in the process of learning. Assimilation and accommodation are consistently occurring in the process of equilibration (Schunk, 2000). Additionally, Piaget suggested that social interactions are desired in the accommodation process (Fosnot, 1996).

Since the mid-1980s, the learner-centered constructivist approach has been emphasized (Fosnot, 1996). In fact, constructivism was categorized from the simple to complicated contexts in making sense of new information while depending on prior knowledge (Berieter, 1990). It is called situated cognition when learners focus on the situation itself (Brown, Collins, Duguid, 1989). It is called social construction when learners focus on the situation in a cooperative group setting (Bruner, 1986). Here the foundation for understanding by learners implicates not only their knowledge and current experience, but also that of other individuals who engaged in the situation.

Knowledge construction within social interaction is the core concept of Vygotsky's work. Fosnot (1996) concluded that Vygotsky believed learners' previously learned concepts together with social interaction and communication with capable peers or adults are the factors that assist learners to establish new knowledge. Fosnot (1996) added that the interactions occurred when a knowledgeable adult or peer aided a child in completing tasks that the child could not accomplish alone. These tasks are supposed to develop in the Zone of Proximal Development [ZPD]. Steel and Reynolds (1999) noted that in the ZPD, the child understands previously learned concepts, which need to be organized systematically in order to learn new concepts. The competent adult or peer can offer a scaffold to cooperate with the child to help the child learn new concepts. Huethinck and Munshin (2000) stated, "scaffolding can be as simple as giving leading questions to enable the child to see the steps required to work a word problem. The constructivist approach to teaching requires establishing a community of mathematics learners" (p. 51).

Today, constructivist epistemology is well known among mathematics educators. "Current translations of a constructivist epistemology into mathematical pedagogy are based on the assumption that children can reconstruct mathematical ideas and processes for themselves if they can be engaged in a social enterprise driven by investigation, negotiation, and renegotiation of the meaning of concepts" (Lesh & Doerr, 2002, p.445).

Constructivism Application to the Classroom

As constructivism gained popularity among educators, teachers realized the need to be up to date in the teaching and learning process. Noddings (1990) noted that teachers benefit from constructivism epistemology in developing their teaching strategies and gaining better understanding in learning mathematics.

Knowledge about how constructivism applies in classrooms is varied in the literature. Contemporary research suggests that students are not passive receivers of information teachers provide; students should be involved in active learning and concrete experiences to establish new knowledge based on existing knowledge (Brophy, 1992).

Cobb, Wood, Yackel, and Wheatley (1993) suggested natures of mathematics-classroom principles. They believed that students' new mathematical concepts are built through active participation, involving subsequent reflections. Furthermore, the knowledge construction process is "continual revision." In this process, students themselves make sense of mathematical concepts as they engage socially and "resolve conflicting points of view."

Brooks and Brooks (1999) noted, "We construct our own understanding of the world in which we live" (p. 4). They recommended five principles for teaching strategies based on the constructivist perspective. First, teachers need to pose problems that are relevant to students' experiences. The authors suggested according to Joel Greenberg (1990) that good problem solving incorporates students' predictions, has enough complexity to stimulate students to create a variety of problem-solving strategies, and involves group work. Second, teachers should design learning environments around students' existing concepts or prior knowledge and using "big ideas" (broad concepts) to stimulate students to participate "irrespective of individual styles, temperament, and dispositions" (Brooks & Brooks, 1999). Third, teachers should search for and value students' perspectives. The teachers should encourage students to express their views in order to grasp students' conceptual understanding. Fourth, teachers need to adjust curriculum when needed to challenge students' current assumptions. Fifth, teachers need to monitor students' learning throughout teaching context. The authors suggested that teachers should view themselves as "cognitively linked with their students in order to evaluate students' cognitive learning and their dispositions" (Brooks & Brooks, 1999).

Fosnot (1996) recommended investigations of open-ended problems because this provided students opportunities to predict and make conclusions by themselves. Reflection and disequilibrium generate meaningful learning. Students' errors can be viewed for their misconceptions in progressing toward conceptual understanding. In addition, the authors noted that communication is also a key element in the process of learning, allowing students to articulate and crystallize their thoughts in discussion.

Szabo and Lambert (2002) detailed basic principles of constructivism in classrooms: learning is "an active rather than a passive process," which occurs socially as learners interact to solve problems. Learners must have chances to "make sense of new knowledge" individually and cooperatively. The learning process is enriched by "reflection and metacognition." Finally, "new learning is mediated by prior experience, values, and beliefs."

Schmidt and Moust (2000) presented a model of the PBL approach based on Schmidt and Gijselaers's study in 1990. Schmidt and Moust (2000) suggested that problem-based learning begins with providing student groups with "minimal information" based on the students' prior knowledge or concepts. Then, the students search for and decide what materials or knowledge is available and what information they need to know and learn to accomplish the task. During the searching process in group work, the students take particular roles to help their group succeed in solving the problem. The students need to negotiate and reflect their thoughts through the group process. The teacher as a facilitator acts as a person who provides metacognitive questions to stimulate reflective thinking such as, "asking students to explain why they consider a particular solution to be good, or why they need a particular piece of information about the problem" (Evensen & Hmelo, 2000, p.3).

Schmidt and Moust (2000) noted that the PBL approach focuses on having students understand the cause/concept of the problem more than getting the solution. In this model, the outcome of the learning process was not only academic achievement, but also interest in the subject matter. The authors propose the model of this PBL approach, according to Schmidt and Gijeselaers, 1990. It is shown in Figure1.

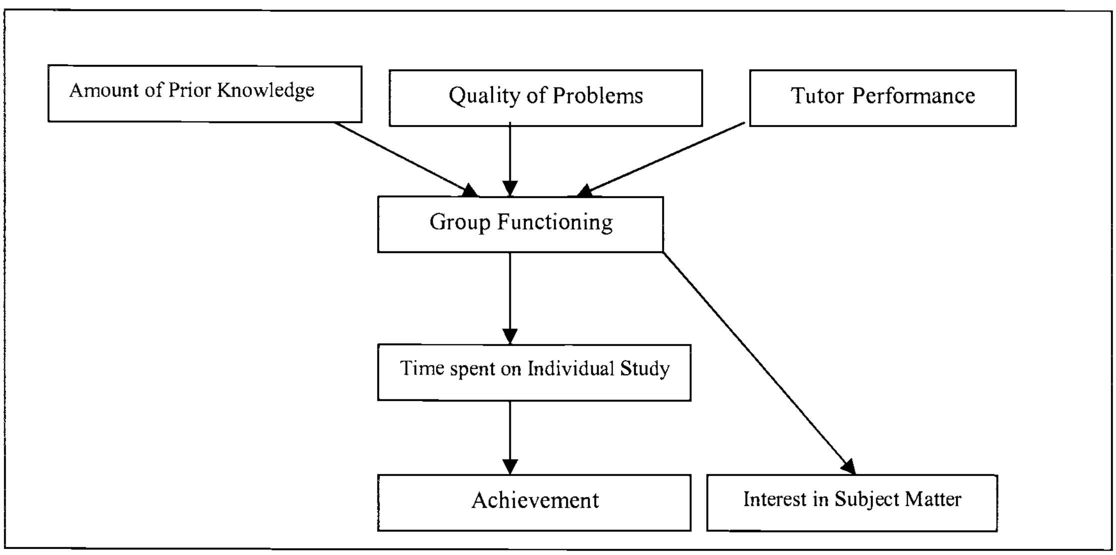


Figure 1. Theoretical Model of PBL as Schmidt and Moust (2000) adapted from Schmidt and Gijeselaers (1990).

Summary

The review includes literature related to the constructivist epistemology in order to provide the background of current research that highlights the construction of knowledge. Several researchers provided different views of ho w constructivism can be applied in mathematics classrooms. These applications help mathematics teachers gain practical help and understand the significance of a mathematics classroom that cultivates new understanding through learners' previous knowledge, social interaction, and communication.

Teachers' Implementation of the Problem-Based Learning [PBL] Approach

Several recent studies have focused on issues of the PBL curriculum implementation, providing details of successes and difficulties of the reform curriculum. The following section highlights findings from five studies, related to implementation of the PBL approach at middle school and high school levels: Rickard (1995) was interested in a case study of one teacher who implemented the Connected Mathematics Project [CMP]; Lloyd and Wilson (1995) explored an experienced teacher's concepts and their influence on his implementation of the PBL approach; Wilson and Lloyd (1995) investigated three mathematics teachers implementing the new approach; and Schoen, Finn, Griffin, and Fi (2001) provided a description of classroom practices and teachers' concerns in using the Core Plus Mathematics Project [CPMP]. Cain (2000) studied the use of the CMP curriculum in 6th, 7th, and 8th grade levels.

At the middle grade level, Rickard (1995) conducted a case study to document a 6th grade teacher's use of Connected Mathematics Project [CMP] in a measurement unit (Covering and Surrounding). During the seven-week study, the researcher collected data by taking field notes and audio taping classroom verbal interactions. The teacher in this study had 25 years experience, but she had not received any prior training in the CMP curriculum. The findings suggested that the teacher shaped the unit to meet her own problem-solving goals (to reinforce the correct application of formulas and procedures), rather than using the unit to help her develop her practice to more closely reflect NCTM-recommended reform, such as helping students to make mathematical connections. Nevertheless, some students who made mathematical connections between parallelograms and rectangles or between rectangles and triangles demonstrated their understanding of concepts, even though their conceptual understanding was not an explicit goal of the teacher.

Lloyd and Wilson (1995) conducted a case study to investigate concepts of a veteran mathematics teacher who implemented the Core-Plus materials for the first time in teaching a function unit. During six weeks, the authors collected data by using interviews, observations, and classroom artifacts. The findings indicated that teaching for conceptual understanding was influenced by the teacher's expertise. This study demonstrated that teachers who can incorporate various approaches can teach beyond traditional curriculum.

In a later study, Wilson and Lloyd (1995) reported a broad picture of first time teachers' experiences and conceptions in using the Core-Plus Mathematics Project [CPMP]. This study focused on how teachers shifted from a teacher-centered to a student-centered classroom. The authors also investigated teachers' and students' beliefs about mathematics. Three teachers (one female and two male) were selected from two high schools. All three had taught for 10 or more years, but none had previously taught using CPMP materials. The researcher collected data through interviews, observations, and students' and teachers' written work and plans.

The researchers found that the teachers' main concern was that their students would not make correct connections without direct instruction, which resulted in small group work often being disbanded. As they gained experience with the CPMP program, the teachers believed that their facilitator roles came more naturally.

The three teachers believed that whole-class discussions were important, but one felt uncomfortable moving between group work and teacher-centered instruction because of earlier difficulties in managing the transitions. One of the three teachers was adept at direct whole-class discussion, nevertheless viewed group work as the key to meeting students' needs. He also explained that many students found group explorations were more stimulating than teacher lectures. Overall, the teachers found that shifting to student-centered learning was more challenging to them than to their students, who described the new approach as being meaningful to them.

Schoen et al., (2001) explored teaching behaviors and concerns of CPMP teachers that correlated positively with growth in their students' mathematical achievement in Contemporary Mathematics in Context [CPMP] curriculum. The researchers obtained data from various sources such as observations, surveys, and tests. The researchers profiled the effective CPMP teachers, detailing several beneficial characteristic s.

A CPMP teacher whose behavior is positively associated with enhancement in students' achievement either had strong preparation in reform curriculum and teaching before his/her first CPMP class, or he/she had completed a workshop to specially prepare him/her to teach the curriculum. The effective CPMP teacher also used the various parts of the CPMP lessons in ways that aligned well with the developers' expectations. He/She used the CPMP recommendations for homework, keeping in mind that in each lesson the recommendations involved several choices for teacher and students. A variety of assessment techniques were used, including group observations, written and oral reports, and take-home exams. About 50% of his/her grading was based on in-class exams and quizzes, another 20% on homework, about 10% on group work and the reminder spread among written and oral reports, notebooks, and attendance/class participation.

The effective CPMP teacher was not likely to supplement or revise the assessment materials except possibly to combine similar questions or mix forms of a test or quiz. In particular, the teacher was not inclined to make either the materials or the assessment more structured or skill-oriented. Finally, by year-end these teachers had few concerns about the CPMP curriculum. The teachers felt well informed about the curriculum, its goals and resources. The teachers felt confident of their ability to manage his/her class in group and pair investigations and felt comfortable with the changes required, inc luding in their role as teachers.

Cain (2002) conducted a study to evaluate the use of the CMP curriculum in four middle schools (6th, 7th, 8th grades) in one year by using several data sources, such as achievement test scores, observations, interviews, and attitude and belief questionnaires. The researcher found that most mathematics teachers in this study (n=22) liked the CMP curriculum better for a variety of reasons than other programs they had taught. Some of them explained that the CMP curriculum had many side benefits such as improving students' problem-solving skills, developing students' higher-thinking skills, and providing an active environment. In addition, the teachers felt they had better understanding of mathematics through teaching the program. All of the teachers agreed that to be effective CMP teachers, they needed to receive training about using the program. The teachers also agreed that the CMP curriculum was outstanding in providing an interactive learning environment, and enhancing students' critical thinking and communication skills.

A similar questionnaire was administered to 300 students about their feelings toward the new curriculum. The researcher found that most students who participated in the study liked the CMP curriculum because it was fun and it was related to "real-life stories." They felt that their problem-solving skills were improved. They felt that the new mathematics was harder than the mathematics they had learned the previous year. Higher-grade level students (7th and 8th grades) had a stronger belief about the importance of conceptual understanding. Half of the students in this study liked group work activities.

Students' Mathematical Dispositions in a PBL Context

Learning is a combination of the cognitive and affective domains. It is often impossible to separate these domains (Maker, 1982). The affective domain, related to personality, values, attitudes, and beliefs, cannot be seen, but the effects can be observed (Vachon, 1984). Numerous research studies examined the relationship between mathematics performance and mathematical disposition factors. For instance, students' attitudes toward mathematics (e.g., confidence in doing mathematics, liking or disliking mathematics) and beliefs about mathematics (e.g., perception of the utility of mathematics and mathematics as a male domain) all affected students' performance (e.g., Fennema & Sherman, 1976; Kloosterman, 1991; Kloosterman & Stage 1992; Loebl, 1993; Ma & Kishor, 1997; Reynolds & Walberg, 1992). In addition, some researchers found that students' beliefs about mathematics affected their behaviors in learning mathematics. Kloosterman and Cougan (1994) reported that students believed that mathematics was rules and procedures. They tried to memorize those rules and procedures instead of attempting to understand or make sense of them. Also, beliefs about learning mathematics, such as the importance of understanding in mathematics, or whether everyone can learn mathematics if they try hard, appeared to be significant factors in motivating students to learn mathematics. Additionally, Kloosterman (1991) noted that students who believed that mistakes are acceptable were likely to be higher achievers than those who believed that mistakes were indicators of low ability. In brief, the more researchers studied dispositions in learning mathematics, the more they realized the significant role of mathematical dispositions and performance in mathematics.

Recently, as an alternative approach such as problem-based learning is promoted, mathematical dispositions in this new classroom environment should be studied. Several researchers have been interested in observing and studying students' mathematical dispositions in the PBL classrooms, especially attitudes and beliefs about mathematics. For instance, at the middle school level, Higgins (1997) investigated the impact of a PBL approach on students' attitudes and beliefs about mathematics development. Bay et al. (1999) explored students' reactions to the Standards-based approach by collecting over 1,000 students' feedback on their experiences in the Standards-based classes. Staffaroni (1996) used the Fennema-Sherman Mathematics Attitudes Scale to study students' dispositions toward mathematics, finding that students in CMP classes showed stronger positive attitudes and beliefs about mathematics than students in traditional classes. At the high school level, Schoen and Pritchett (1998) conducted a longitudinal study about the influence of the PBL approach on students' affective domain. Austing and Hirstein (1997) were not only interested in the effects of the PBL approach on students' affective domain, but also on students' problem solving skills. Madsen and Lanier (1992) looked at whether the new approach enhanced students' attitudes and beliefs about mathematics. Boaler (1998) conducted a longitudinal study to gain insight into the difference between PBL and traditional classrooms. Recently, Boaler (2002) compared the impact of the traditional approach and the PBL approach on students' attitudes and beliefs about mathematics. A description of each of these studies follows.

At the middle grade level, Higgins (1997) carried out a study better understand the consequences of a problem-based approach on middle school students' attitudes and beliefs about mathematics in three different schools in Oregon. The findings showed that Standards-based students believed that mathematics involved practical problem-solving, not just rote memorizing. To them it was more important than to their counterparts do well in mathematics. They said they needed to think harder versus being told the answers, compared to traditional students who relied more on their teachers or textbooks to get the right answers.

Most of the traditional students rarely mentioned working on problem-solving challenges, but discussed their teacher's instructional aspects, such as classroom management issues. Both groups of students recognized their teachers' concern for students, and both preferred specific mathematics activities. Both groups had common dislikes, such as tests or some specific mathematical topics.

Most of the Standards-based students were able to apply problem-solving skills that they had learned during the five-week instruction: guess and check, look for pattern, make a list, make a drawing or a model, and eliminate possibilities. Most traditional students considered solving a problem as computation, which involved addition, subtraction, multiplication, and division.

Regarding perceptions of school mathematics and classroom practices, all Standards-based students believed that mathematics was useful and they were able to provide various samples of its usefulness in their daily lives, such as in baking, building, and shopping. For the traditional students, belief about the usefulness of mathematics decreased according to their ability level. The high-ability students had stronger beliefs in mathematics' usefulness. Interestingly, alternative students critiqued the problems as being irrelevant to the real world, whereas the traditional students merely claimed the problems were boring. Moreover, two-thirds of the alternative students agreed that memorizing was important in some particular areas, such as rules or formulas. The other one-third indicated that memorization was unimportant. In contrast, most of the traditional students believed that memorization was extremely important in learning mathematics. Additionally, all alternative students interpreted their mathematics understanding as their ability to solve problems in different ways, make generalizations, and communicate their thinking. On the other hand, some traditional students regarded understanding as getting high scores; some related their understanding to their problem-solving speed.

All but one alternative student favored the alternative teaching approach. They preferred mentally stimulating and creative problems. The majority of both groups disliked computation problems for different reasons. The traditional students claimed they were less successful at computation problems due to inadequate instruction, resulting in their dislike of these problems. Alternative students viewed computation problems as lacking and less reflective of real life, thus not worth learning.

Regarding the ability to solve non-routine mathematics problems, the alternative students had higher achievement on all dimensions. The Standards-based students outperformed the other groups in "(a) making generalizations of their solutions; (b) verifying their findings by using estimation; (c) communicating their solution strategies; and (d) approaching problems through reasoning and logic."

The author noted that alternative students and teachers could explain the rationale for their strategies in solving problems by using the vocabulary of the problem-solving skills they learned during the problem-solving instruction at the beginning of the school year, but the traditional groups had difficulties in providing problem-solving explanations.

Students were given freedom to choose problems-solving strategies. They were also encouraged to create ideas and work cooperatively. Working on these kinds of problems also tended to increase students' perseverance in problem solving. Additionally, the better performance of the alternative students in solving the given problems at the end of the interviews suggested that one year of alternative teaching approach (PBL), which the author called systematic problem-solving instruction, improved skills in problem solving.

The next study was conducted by Bay et al. (1999) in order to provide information about the responses of 6th and 7th grade students to Standards-based curriculum: the Connected Mathematics Project [CMP] or the Six Through Eight Mathematics [STEM] or [MATHThematics]. The present study involved a total of more than 1,000 students of 15 teachers from five different school districts varying from small towns to a large city. The teachers participated in a National Science Foundation [NSF] project, using cooperative curriculum investigation as a means for professional development.

During the one school year, the teachers were implementing Standards-based curriculum. The curriculum emphasized using mathematical problems or tasks as the center of teaching and learning. Then, at the end of the school year, the students were asked to write letters to reflect on their experiences in their mathematics classrooms, which used new curriculum materials over the year. The students were not directed by their teachers to comment in any particular manner.

A set of common themes with a range of student reflection was created from the letters. The common themes across classrooms included hands-on activities, application, affective factors, group work, mathematical content, student difficulty with real-world problems, and positive self-assessment.

The main finding was that as the students were more exposed to a Standards-based curriculum and became familiar with a variety of classroom activities, they came to see the new curriculum as better when compared to the previous year's materials, which were considered a traditional approach. Most of the students would rather be in the Standards-based class than the traditional environment.

Regarding common themes across classrooms, the students liked and valued hands-on activities in the Standards-based curriculum. They perceived the hands-on activities as a vehicle for helping them better learn mathematics. Moreover, the students noted that the activities made the mathematics lessons more interesting and more fun for them. Focusing on the application theme, one-fourth of the students mentioned the new materials were rich in real-world mathematical applications, which appeared to challenge them. The students realized the mathematics projects helped them to learn how to use and apply mathematics in real-life situations. However, not all students had positive attitudes toward the applications. Some had a difficult time with complicated and multi-step solutions, which caused them to think hard about the problems. Interestingly, most of the students realized that in the new curriculum environment, they had increased perseverance in solving problems.

In the affective factor theme, the new approach appeared to have an effect on positive attitudes toward mathematics. The students were interested and enjoyed learning mathematics, which in turn promoted their mathematics understanding. Even the students who considered themselves as low-ability agreed that mathematics could be fun and not boring. Most students indicated that teachers had an important role in creating enjoyable and interesting coursework. The students had a positive feeling while working with groups.

Regarding the mathematics content theme, their comments about content in the new curriculum were positively related to expected topics (fractions and percents) and advanced topics (probability, algebra, and geometry). The Standards-based students noted that mathematics content reflected everyday life and future jobs.

The themes were set according to teachers' choices. Some teachers had their students make many comments; some did not. None of the students from six of the 15 teachers provided negative comments about the curriculum. In the nine remaining teachers' classes, 2% to 25% of the students expressed negative comments on some aspects of the new approach. For example, most of the comments were directed toward the end-of-unit projects, which took several days to complete.

As reported on the self-assessment, 40% to 70% of each class felt positive about their progress or talent in mathematics. They were proud of themselves and believed their ability in doing mathematics had improved a great deal in the Standards-based environment.

The researchers concluded that teachers played a crucial role in students' attitudes and beliefs about mathematics, regardless of the curriculum used. Even though the teachers in this study had continued professional development in the use of the Standards-based curriculum, there was still difficulty in implementing the curriculum effectively. This highlighted the significance of supporting teachers to implement the new approach in middle school mathematics classes.

Staffaroni (1996) was also interested in comparing Standards-based students and traditional students' attitudes and beliefs about mathematics. The researcher investigated 8th grade students' confidence in learning mathematics and their perception of the usefulness of mathematics by using the Fennema-Sherman Mathematics Attitude Scale. The researcher found that students who attended the MATH Connections program developed more confidence in learning mathematics and a stronger belief in the usefulness of mathematics than traditional students. In addition, the researcher also found that trained-teachers in the Standards-based curriculum had more potential to improve students' confidence in mathematics, as well as their belief about its usefulness, than untrained-teachers.

At the high school level, Schoen and Pritchett (1998) conducted a longitudinal study of high school students' beliefs and attitudes toward the Standards-based mathematics classes using the Core-Plus Mathematics Project [CPMP]. The researchers classified six logical categories in comparative findings from the survey: course difficulty; problem solving, reasoning and sense making; learning in groups; communicating mathematics; realism and general interest.

In regard to course difficulty, at the beginning of Course 1, there were no significant differences between the CPMP and the traditional students either on their perceptions of their mathematics grades, or how well they understood mathematical concepts. The CPMP students viewed their course as challenging and difficult. However, at the end of Courses 2 and 3, the CPMP students expressed increasing satisfaction with the level of their understanding. Many CPMP students indicated that the courses were related to real life; they also noticed that working with the curriculum materials helped them to understand the mathematical ideas of the course, whereas the traditional students expressed opposite points of view.

Regarding problem solving, reasoning and sense making, the CPMP students strongly believed that the projects they participated in had benefited them in making sense of mathematical ideas. Their positive opinions about their ability in mathematical problem solving and reasoning increased compared to their counterparts. At the end of Course 2, there was a significant increase in confidence among the CPMP students in doing mathematics.

Both CPMP and traditional groups enjoyed working in groups and thought this helped them learn mathematics. At the end of Course 1, a significant difference appeared between these two groups. The CPMP students had stronger beliefs that they had learned more mathematics by working in groups than the traditional students did.

Regarding communication, the authors noted that it was not surprising that the CPMP students had a stronger belief that their mathematics course helped them to think and write about mathematical concepts than their counterparts, since the CPMP students had more opportunities to do so in their classroom activities.

When looking at realism and general interest, most of the CPMP students enjoyed and were interested in their course because of the realistic problem contexts. However, a few CPMP students said that the lessons were not interesting because of their difficulty or because of not having enough practice at mathematical skills.

Austing and Hirstein (1997) conducted a study about the effects of the Systemic Initiative for Montana Mathematics and Science [SIMMS] on students' attitudes and beliefs about mathematics as well as on problem-solving skills. During 1992-1993, the authors had 22 classrooms of mostly 9th graders implementing the SIMMS project (Level 1) for the whole academic year. Of these SIMMS classrooms, the authors had 11 classrooms that were considered focus classes. Teachers paid attention to detailed feedback and students' responses on an attitude and belief questionnaire.

In the results of the study, the SIMMS teachers and students expressed enjoyment in working with the curriculum materials. The authors noted that classroom observational data revealed that the students were occupied by the materials and were working together to accomplish the open-ended tasks. They also noted that the new teaching approach was more challenging. Teachers in the new approach classrooms had to carefully prepare the materials, especially the open-ended tasks, as well as evaluate students' work on these tasks.

The following study was conducted by Madsen and Lanier (1992). They were interested in whether the PBL approach endangered students' computational capabilities. They also investigated the effects of this teaching approach on students' attitudes toward mathematics in the high-school setting.

The researchers had two study groups. One group of two classes learned mathematical concepts rather than computational procedures; the other group of two classes focused on practicing only computational procedures without understanding the processes. Then, a test of arithmetic operations (the Shaw-Hiele test) was administered to compare their computational achievement.

Overall, Standards-based classes had lower mean scores of the correct items than traditional classes on the pretest and interim test. However, the result was opposite on the posttest. Standards-based students outperformed traditional students in their computational achievement.

The students responded to the Shaw-Hiele test in three different ways: (a) attempting with correct answers, (b) attempting an answer but getting it wrong, and (c) no attempt to get the answers at all. The mean of attempted items was averaged from all successful and failure efforts. At the beginning of the study, both Standards-based and traditional students had the same mean number of attempted items. In the posttest, Standards-based classes had a higher than the mean number of attempted items than traditional classes. The traditional students expressed that if they thought a problem was too hard, or they forgot the procedure, they would not even try. Moreover, Standards-based classes had developed their computational capabilities from the pretest to the posttest. The students' raw scores in the posttest were converted to grade equivalent. Standards-based students gained higher-grade equivalence in computational skills than traditional students did.

From the observations, the researcher found that Standards-based students had developed a positive attitude toward mathematics. They turned out to be more confident in their mathematics abilities. Standards-based classes expressed more confidence in working with whole number items than with fraction, decimal, and percent items. Standards-based classes were continually improving in whole-number performance across the year from pretest to posttest; their comparison classes went in the reverse direction.

In conclusion, this study showed that Standards-based classes were able to gain a level of computational competence. The conceptual instruction (Standards-based approach) did not ignore computational procedures, but the instruction enhanced students' making sense of computational procedures by using a variety of problem contexts, which in turn, increased their retention and computational competence. The students in this study improved their confidence in difficult mathematics topics.

In the United Kingdom, Boaler (1998) designed a study to compared process-based (problem-based) mathematics classrooms and content-based (traditional) mathematics environment. She conducted a three-year case study of two schools to observe the relationships between students' mathematics classroom experiences and their improvement of mathematics understanding.

The researcher reported that, at the traditional school, students (from year 9 to 11) identified their mathematics classrooms as "boring and tedious." They expressed strong reactions to their mathematics work as uninteresting and they avoided engagement. The students also believed that mathematics was all about memorizing rules, formulas, and equations. This belief affected the following mathematical behaviors:

1. "Rule-following behavior." The students rarely tried to think about what to do in solving mathematics problems; they believed they needed to rely on memorized rules and procedures only. As a result, they struggled with unfamiliar situations requiring similar mathematical concepts.

2. "Cue-based behavior." The students were more concerned with teachers' expectations than with the actual questions, leading them to choose faulty work strategies.

Gender differences were found in the traditional school setting. Boys were significantly more positive and confident than girls in doing mathematics; girls just quit when they met challenging problems. And boys had higher mathematics achievement than girls.

At the school that used a problem-based approach, students were encouraged to think and solve mathematics problems on their own. Classroom environments were more flexible and relaxed than in the traditional school. The students participated in solving real-world open-ended problems and working with heterogeneous groups. Each problem lasted a few weeks and required handing in a description of their work and mathematical activities. The students in this school setting believed that mathematics involved "active and flexible" thinking, which translated into their ability to choose various means to solve problems.

The students in the problem-based approach also indicated that their mathematics classrooms were interesting. Most of the students paid attention to their work even though the classroom climates were noisy. However, the author found that some students preferred the traditional approach and avoided working on assignments, since they liked neither the open-teaching style nor the accompanying independence. A small number of the students in this school believed that mathematics was memorizing. In contrast to the traditional approach group, gender differences were not found in the problem-based school setting. Boys and girls were equally confident and enjoyed learning mathematics as well as attaining similar mathematics achievement.

Boaler (2002) reported some of the main findings of the first year of a study comparing two different mathematics curricula in mathematics classes: the Interactive Mathematics Program (IMP) vs. a traditional algebra-geometry sequence. The study was conducted by the author and Stanford University researchers.

The author reported one-year findings from one high school, which offered 9th-grade students a choice to take either the IMP program or the traditional sequence. Students in each year of the IMP sequence were provided practice in applying problems in several content areas: algebra, geometry, statistics, and probability. Students in the traditional approach learned one content area at a time. The researcher team conducted observations, interviews, and administered a questionnaire about students' mathematical experiences and their attitudes and beliefs about mathematics for both groups.

At the beginning, no significant differences were found in mathematics attainment between the traditional and IMP students. Both groups of students had the same achievement in algebra. The researcher noted that this finding was interesting because the IMP group (who spent the year not only on algebra, but also on geometry and probability) gained similar achievement to the students in a year-long algebra class.

Additionally, the IMP students and the traditional students had developed different attitudes and beliefs about mathematics, even though their mathematics outcomes from the different approaches were the same. Students' questionnaire responses showed that the students built on different motivations in learning mathematics. The IMP students expressed higher levels of intrinsic interest; the algebra 1 students felt mathematics grades were most important. In addition, more of the IMP students expressed that they studied hard in mathematics because it interested them, but the algebra 1 students liked their mathematics class when it required little effort. The IMP students demonstrated stronger convictions that mathematics was useful for their daily lives.

In addition, traditional students perceived themselves in mathematics classes as receivers of knowledge from their teachers; then they worked to replicate what the teachers showed them in solving problems. In contrast, the IMP students believed that they were expected to think, explore mathematics with different problem-solving strategies, and apply mathematical concepts. Interestingly, both traditional and IMP students appreciated their teachers and the way they taught them mathematics. However, the traditional students did not see the use of mathematics in real lives. They also preferred to have a chance to perceive the ways mathematics is connected to their life. The IMP students had developed their interest and enjoyment in learning mathematics from seeing the connection and application of mathematics in the real world.

Summary

In the research of the effects of the problem-based approach in secondary school grade levels, several important findings are revealed. First, teachers' implementation of the PBL approach was explored among middle school and high school teachers. Rickard (1995) found that teachers still hold beliefs that procedures and roles are important in solving mathematics, to the extent that they tended to overlook facilitating students' mathematics connections. Lloyd and Wilson (1995) found that the more comfortable the teachers became with their new role in the PBL classroom, the more freedom and energy they were able to devote to developing even more meaningful opportunities for students learning. Also, Wilson and Lloyd (1995) noted that teachers struggled longer than students to change their traditional views about mathematics. Schoen et al. (2001) summarized some characteristics of effective PBL teachers, which either had strong preparation in the PBL curriculum before teaching their classes or completed a workshop to specially prepare them to teach the PBL curriculum. The characteristics included: having a wide range of mathematical interests and aptitudes; using the curriculum materials for homework for the most part; using a variety of assessment techniques; and having confidence in their ability to teach the curriculum, as well as manage classes in groups and pair investigations. Finally, Cain (2002) documented middle school teachers' and students' reactions to the CMP curriculum. The researcher found that mathematics teachers perceived that the CMP curriculum not only benefited their students, but also themselves in gaining better understanding concepts in mathematics. The teachers liked the curriculum because of its benefits to their students in various ways; it involved active learning, higher-thinking skills, and communication skills, as examples.

Second, there was evidence for the benefits of the open-ended approach (PBL) to students' attitudes and beliefs about mathematics. Madison and Lanier (1992) found that the PBL approach did not negatively affect students' attitudes toward mathematics or ignore their computational skills. Rather, the reform approach appeared to positively affect students' attitudes and beliefs about mathematics (Bay et al., 1999). In comparison studies, inconclusive findings were reported. Some studies showed that the PBL students had more positive attitudes and beliefs about mathematics than their counterparts did (Austing & Hirstein, 1997; Higgins, 1997; Madison & Lanier, 1992; Schoen & Prichett, 1998; Staffaroni, 1996). Cain (2002) found that mathematics teachers who were trained in using the Standards-based approach had the potential to have positive effects on students' confidence and perception of the usefulness of mathematics more than non-trained teachers. Boaler (2002) found different results. She found that both PBL and traditional classrooms had developed positive attitudes and beliefs about mathematics, if both groups had good mathematics teachers. However, the PBL students had a higher level of intrinsic interest and expressed stronger beliefs about the usefulness of mathematics; traditional students had more concern about higher grades in mathematics. In addition, there was evidence showing that the PBL approach equalized girls' and boys' attitudes and beliefs about mathematics (Boaler, 1998). There were inconclusive findings about whether the PBL approach reduced gender differences in mathematics achievement (Battista, 1999; Boaler, 1998).

CHAPTER III

METHODOLOGY

Introduction

This study was designed to provide descriptive information about students' mathematical dispositions in a problem-based mathematics environment. The primary research question for this study was: What are middle school students' mathematical dispositions in a problem-based classroom? This chapter presents the methodology used to conduct a ten-week study. A description about the teacher and the problem-based classroom is included providing the context for the study. Data collection methods including participant observations, individual interviews, and documents are described along with data analysis.

Design of the Study

The rationale for pursuing a case study as the design for this study was that little research has been presented about middle school students' thoughts, feelings, and behaviors in their problem-based classrooms. Most studies in the literature focused only on students' beliefs and attitudes toward their PBL classrooms. Gall, Borg, and Gall (1996) suggested that one important purpose for conducting a case study was to depict and conceptualize a phenomenon. By taking the case study approach, the research design would supply an in-depth picture of the problem-based classroom experience and students' mathematical dispositions within it.

The study was conducted during spring term, 2003, over a period of approximately 10 weeks. This study involved one classroom's unit on measurement during four weeks of instruction. The intensive observation was conducted while the teacher was teaching the Covering and Surrounding unit focused on perimeter and area. Only a single classroom was focused on because of the time-consuming process of observation and data collection.

Finding the Teacher

Finding a problem-based mathematics teacher who was willing to participate in this study began four months before the data collection process. Four criteria were designed to select a potential volunteer mathematics teacher, including: (a) at least three-years of experience in teaching mathematics, (b) at least two-years of teaching experience in using the PBL approach, (c) a classroom routine closer to the PBL approach, which has mathematics problems as the center of group learning, and (d) the teacher's willingness to participate in this study.

The researcher found a mathematics teacher called "Ms. Smith" for anonymity. She showed an interest in this study and met the minimum requirements of the criteria. A formal teacher interview was conducted before the classroom observation period, to obtain in-depth information about the teacher's experiences in teaching mathematics and using the PBL instructional approach, as well as her expectations for the students. She was in her second year of using the Connected Mathematics Project [CMP] along with her mathematics teacher colleagues. After the first year of teaching with CMP, she and her colleagues thought their students liked the curriculum because it was fun. Ms. Smith and her colleagues also thought the new curriculum (CMP) helped her students learn more because they had chances to investigate a variety of mathematics content. Therefore, she and her mathematics teacher colleagues decided to keep using the CMP as their curriculum.

Ms. Smith's Classroom and Her Students

Ms. Smith used the Connected Mathematics Project [CMP] in her 6th grade mathematics classroom, consisting of 20 students. Ten students, including seven boys and three girls, volunteered to participate in this study, were given pseudonyms to protect their anonymity. Eight participants (five boys and three girls), selected according to their varied mathematics achievement, were called Bill, Bob, Cindy, Jim, Mary, Mike, Nicole, and Tony. All volunteer students had been taught mathematics in the traditional way (a teacher-centered approach) at the elementary school level. The participants' mathematics achievements in the previous term were varied: four of the participants were advanced mathematics achievers; three of them were average mathematics achievers; and one was a low mathematics achiever, based on their previous term's mathematics grades provided by Ms. Smith.

Ms. Smith graduated with a Master's degree in mathematics education, curriculum, and instruction from a university in 1992, the same year that she received her teaching certificate. Right after she graduated, she began teaching at a middle school. She had worked at that same middle school for 11 years. She taught 6th grade for 10 years and 7th grade for one year. She piloted the CMP curriculum in the prior year along with her 6th grade mathematics teacher colleagues. This school year, 2002-2003, was the second year that she and her colleagues had used the curriculum. They had participated in one workshop about implementing the curriculum. Ms. Smith liked the curriculum because she thought it was active and fun for her students.

A Description of the School and Classroom

The School and Classroom

The school population consisted of approximately 900 middle school students (6th to 8th grades) and 65 faculty and staff. The school had two main buildings. The first building housed the administration offices and some classrooms. The second building was a newer building and many student activities took place there because the building had a meeting room, a stage, and a big hallway. Ms. Smith and her 6th grade teacher colleagues had classrooms in the first building. During mathematics, the students' desks were arranged in groups of four throughout the classroom as in Figure 2.

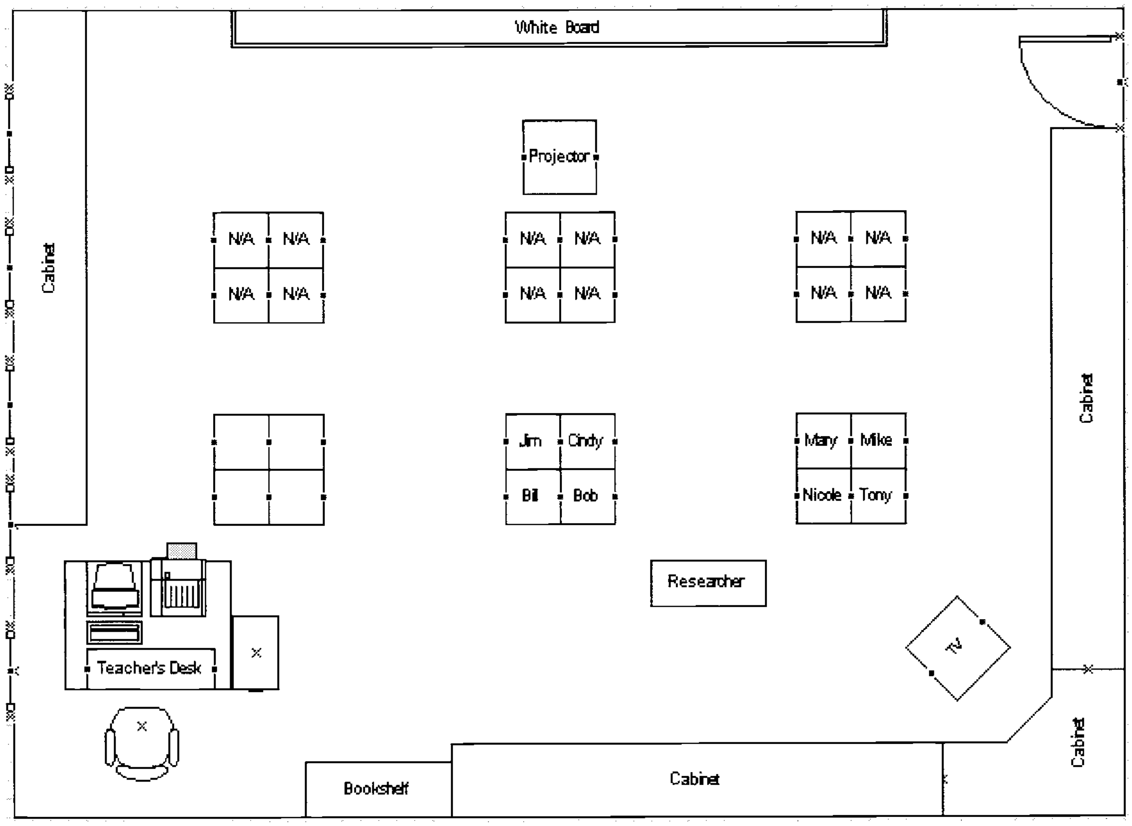


Figure 2. Physical arrangement of the classroom.

There was a white board and an overhead projector on a cart in the front of the classroom, which Ms. Smith usually used when she was teaching the class. Individual pictures of each student were posted in the front of the classroom to the right of the whiteboard. Ms. Smith posted students' work on the other side of the whiteboard, which she rotated periodically. Ms. Smith's desk was in the back of the classroom with a computer, a color printer, and a phone on it. Some dictionaries and the CMP books were stored on a bookshelf along the back wall, beside cabinets storing manipulative objects, construction paper, compasses, and rulers. The mathematics class met for 60 minutes every weekday morning.

Gaining Entry: The First Day of Fieldwork

Right after presenting the proposal to her department and doctoral committees, the researcher contacted several mathematics teachers in the surrounding areas (in order to observe their classrooms). It turned out that these mathematics classrooms did not use the PBL approach. Looking beyond the surrounding area to increase the chance of finding a potential volunteer teacher, the researcher contacted approximately 15 school principals in urban and other regions looking for a classroom that used the PBL approach. After searching for four months, the researcher found a potential volunteer teacher, located in a small town in the Pacific Northwest. After Ms. Smith agreed to participate in the study, the researcher then asked permission from the middle school principal to do the study. The researcher provided the principal all documentation about the research, including a brief description of the study, the script to ask for volunteer students, an information sheet describing the study, and the consent form. The school principal was very pleasant and supportive toward the researcher regarding the research in his school.

Next, the researcher recruited volunteer students from Ms. Smith's mathematics classroom. The researcher introduced the study in front of the classroom and asked the students to discuss the matter with their parents or guardian. The researcher asked the students to return consent forms to the teacher, Ms. Smith, within a week. While waiting for the consent forms, the researcher started to observe the classroom. Although the teacher reminded her students about the consent form deadline, several more days were needed before all consent forms were returned. Some students still had not talked to their parents about whether they could participate in the study, some of the students forgot to have their parents sign the form, and some forgot to take the form from their house to return. The researcher and Ms. Smith agreed to wait two more days to get consent forms back from the students. Ten students volunteered to participate in this study. Based on the variety of students' mathematical abilities, Ms. Smith chose eight students and grouped them into two groups of four varied mathematics achievers.

Data Collection

Seven data sources were administered in this study: (a) classroom observations; (b) the Attitude and Belief Questionnaire; (c) student interviews; (d) classroom artifacts; (e) students' mathematics achievement information; (f) teacher interviews; and (g) researcher journal. The data was collected for eight weeks, during spring term 2003. For triangulation purposes, all of the data resources used in this study were considered together in order to confirm the findings, with observations as the primary source.

Prior to the study period, the researcher conducted pilot students' interviews and observations using the classroom observation protocol with some non-participating students from different middle schools. The purpose of the pilot interviews and observations was to improve and verify questions in the interviews and observation protocol; the findings are not reported in this study.

Classroom observation (see Appendix I) was one of the most important data resources, providing the researcher with the classroom context and students' reactions in class and in cooperative PBL group assignments. The researcher used a classroom observation protocol in order to keep track of classroom elements such as daily routine, participants' reactions and roles in the PBL groups. The observation protocol included demographics, activities, and materials in the classroom; student/teacher interactions; and group activities. Additionally, videotapes and audiotapes were used during the observations and interviews in order to provide additional support for the data.

The Attitude and Belief Questionnaire (see appendix G), which was developed from several previous studies about students' mathematical dispositions, included attitudes and beliefs about mathematics, mathematics coursework plans, and career interests in middle school levels. The completed questionnaire provided corresponding data for students who had experienced the PBL approach. Moreover, a set of open-ended questions in the Attitude and Belief Questionnaire provided more details about each student's attitudes and beliefs about mathematics, mathematics coursework plans, and career interests.

Student interviews (see Appendix H) provided them opportunities to express verbally how they felt about mathematics, what they believed, and why. In student interviews the researcher also observed facial expression, tone of voice, and body language to gain further insight into students' attitudes and beliefs about mathematics and their experiences in small groups in the PBL context.

In addition, classroom documents or artifacts provided rich contexts of the PBL classroom. The artifacts could be anything the teacher employed in the PBL classroom such as textbooks, worksheets, homework assignments, quizzes or tests. The artifacts were collected in all observations.

Students' mathematics scores for the observed unit and from the previous term were collected in order to detect potential patterns or relationships among students' mathematics achievement, students' participation in PBL group work, and students' attitudes and beliefs about mathematics, mathematics coursework plans, as well as career interests. The students' mathematics scores were obtained directly from their teacher.

Teacher interviews (see Appendix H) included formal and informal formats. The pre-formal interviews were conducted with the volunteer teacher and provided the teacher's previous teaching experiences in both traditional and PBL approaches. In addition, the classroom textbook, the curriculum that the teacher used, and the teacher's expectations of her students were discussed in the pre-formal interview. The teacher's post-formal interview was conducted at the end of the study in order to gain in-depth details about the teacher's current teaching experience with PBL.

Next, informal teacher interviews were conducted when possible and appropriate before each lesson of the observed unit in order to gain broad ideas of the observed classroom's contents, goals, and routines. After each lesson, two debriefing interviews provided details about the teacher's opinions on each PBL lesson. Audiotapes (for all formal interviews and all observations) and videotapes (for all observations) were used to record the data as back-up data resources. Classroom artifacts were collected from teachers and/or students following each observation. During the last week of the study, the Attitude and Belief Questionnaire was administered in one mathematics classroom period to 10 volunteer students. The questionnaire results were based on data from all 10 volunteer students.

The researcher documented notes in a research journal to describe the classroom physical environment and to record additional circumstances of the interviews and observations. The researcher journal was kept daily during the eight-week observation period. The researcher recorded events before and after the observation sessions that might affect observed behaviors. The journal was reviewed during analysis of the data in order to reduce the researcher's bias.

Data Sources

Classroom Observation

The researcher observed the classroom as a non-participant observer. Each observation took place in the same mathematics class. The researcher was at the school site for approximately 10 weeks. The researcher observed a 6th grade class of the volunteer teacher for eight weeks including two weeks before and two weeks after a four-week period of one complete measurement unit (Covering and Surrounding). During the observed unit, the researcher observed the PBL classroom's context in general and focused on the two target groups, while they were working on group tasks. The classroom observations were guided by the classroom observation protocol (see Appendix J). The researcher used field notes, audiotapes, and videotapes to record the observed data. Throughout each observed lesson, the researcher used a classroom observation protocol to help the researcher focus on classroom activities, materials, students and student/teacher interactions, and group activities (student to student interactions). In addition, the researcher wrote comments, perspectives, interpretations, feelings, and frustrations in the field notes.

The researcher started to use the video and audio tape recorder as soon as parental consent was given. The recording was established about a week before the observed unit in order to familiarize the teachers and the students with the equipment.

The researcher visited Ms. Smith's classroom every weekday morning from 9:30 to 10:30 AM. The researcher normally arrived at the school 30 minutes earlier in order to prepare videotape and audiotape to record the volunteer students participating in the classroom activities. The researcher usually talked with the teacher approximately five to ten minutes before the class started in order to perceive a general idea about the classroom's activity for each day. The researcher normally took notes during the daily informal interview.

During the observations, the researcher remained as unobtrusive as possible in the back of the classroom between the two focus groups. The first two weeks before the observed unit, the researcher conducted a broad observation to minimize the appearance of the observer influence in classroom activities and to capture the classroom routine. To confirm the classroom routines, the researcher continued to observe the classroom daily for two more weeks after observing the four-week observed unit. This additional observation was to capture the consistency of the mathematics classroom routine.

Both target groups were audio taped from the beginning of the observed unit. However, one of the two focus groups was comfortable being videotaped; the other group felt uncomfortable. Therefore, one of the two focus groups (the Rhombus Square group) was not videotaped in the first two weeks of the observation unit. Halfway through the observation unit, that group told the researcher that they were now comfortable, so in the second half of the observation period, both groups were videotaped. The audiotapes and videotapes of both groups were transcribed and used to provide data sources for describing students' mathematical dispositions in the classroom.

In developing the Attitude and Belief Questionnaire, and the observation and interview protocols, the researcher looked for available suggestions and protocol frames within the literature. As suggested by Johnson and Christensen (2004), the researcher sought and constructed the questionnaire and the protocols along with the study's goals and design. As Patton's (1990) recommendation, the content of questions in the questionnaire and the protocols were divided into several categories, including background/demography, experience/behavior, opinion/value, sensory description, and feeling. Three mathematics educators from Oregon State University read through the questionnaire and the protocols in order to assure that the items would be related to the intended content for face validity.

The researcher made an effort to establish rapport with the teacher and the students by handing out worksheets or equipments. Also, the researcher volunteered to help with other school activities such as the school day fair, kindergarten day fair, and parent's tea.

Attitude and Belief Questionnaire

The attitude and belief questionnaire was presented to the 10 volunteer students in a mathematics class period toward the end of the observed unit (see Appendix G). The questionnaire consisted of a cover sheet asking for the student's name, and his/her identification number. Information about confidentiality and how to answer the items were described. The second part included demographic information questions.

Then, questions about attitudes toward mathematics and beliefs about mathematics (using the five-point Likert scale) and mathematics future coursework plans were added. An open-ended question was provided for additional data about students' future mathematics coursework plans and career interests. Next, a set of open-ended questions was followed-up by attitude and belief questions in order to determine the main reasons why students felt positive or negative about mathematics, and in order to determine students' mathematical dispositions. In addition, questions were asked about why (or why not) the students were likely to take future mathematics coursework and why the students were interested in particular careers.

Students' Demographic Information Sheet. To gather students' backgrounds, demographic questions were asked. It included questions that asked the student's name, identification number, gender, grade level, age, previous mathematics class description, the participation in related mathematics activities outside their mathematics classrooms, and the frequency of doing homework assignments.

Students' Attitude and Belief about Mathematics Questions. The scale statements in the Attitude and Belief Questionnaire included affective motivation in mathematics, confidence in mathematics, and mathematics as a male domain. These scales were adapted from the Fennema-Sherman Attitudes Scale (Fennema & Sherman, 1976). Additional scale statements, consisting of the usefulness of mathematics; the importance of understanding concepts in mathematics; the increase of mathematical ability by effort; and persistence with time-consuming mathematics problems, were adapted from the Indiana Mathematics Beliefs Scale (Kloosterman & Stage, 1992). Students were asked to respond to statements in the Attitude and Belief Questionnaire on a five-point Likert scale. Each response to a "positive statement" was given a score of five for "strongly agree," four for "agree," three for "undecided," two for "disagree," and one for "strongly disagree." The scores were reversed for a negative statement.

Reliability had been provided from the previous studies (Kloosterman & Stage, 1992; Muhern & Rae, 1998). For effective motivation in mathematics the scale was .86; the confidence in mathematic scale was .91; mathematics as a male domain scale was .85; the usefulness of mathematics scale was .86; the importance of understanding concepts in mathematics scale was .76; the increasing mathematical ability by effort scale was .84; and the time-consuming mathematics problems scale was .77. The researcher also provided students an opportunity to respond to open-ended questions about attitudes and beliefs about mathematics.

The students' mathematics coursework plans were gathered from the questionnaire that was adapted from Thorndike-Christ's (1991) study. For the future mathematics coursework plan, the students were asked to rate on a five-point Likert scale list from one (definitely will not continue to take mathematics as much as required for graduation at the college level) to five (definitely will continue to take mathematics as much as required for graduation at the college level) in order to indicate the likelihood of willingness to take future mathematics coursework. The collected information about career interests, open-ended questions were developed from Olsen (1998).

In addition, the reliability coefficients for this study of all seven scales in the questionnaire were computed. From this study, the ten volunteer students took the attitude and belief questionnaire. Cronbach Alpha coefficients were computed for each sub scale of each scale in the questionnaire. The Attitude and Belief Questionnaire, for the 10 students, had a reliability coefficient of .89. Most of the subscales in the Attitude and Belief Questionnaire also contained average to high reliability coefficients as follows: motivation in learning mathematics subscale had a reliability coefficient of .52; confidence in mathematics subscale has a reliability coefficient of .96; mathematics as a male domain subscale has reliability coefficient of .56; the time-consuming subscale had a reliability coefficient of .80; the importance of understanding concepts in mathematics subscale had reliability coefficient of .76; the increasing mathematical ability by effort subscale had a reliability coefficient of .86, and the usefulness in mathematics subscale had a reliability coefficient of .70. Normally, a reliability coefficient of .70 or higher is considered respectable; however, sometimes lower coefficients are acceptable (Henerson, Morris, & Fitz-Gibbon, 1978). In this study coefficients of .70 and higher were considered reliable, while caution should be used in interpreting the results with coefficients between .50 and .70. The low reliability for motivation scale may be a result of students' inconsistency in liking mathematics; the low reliability of the mathematics as a male domain may be a result of student confusion with the question. Some students came to the researcher for explanation of the questions in this scale.

Student Interviews

Students' informal interviews (see Appendix H) were conducted at least once a week for approximately five minutes with each student after they experienced the lesson's group work or project work. Then, students' formal interviews (see Appendix H) were conducted with the eight target students. The formal interviews took place for approximately 15 to 30 minutes for each target student and was audio taped. The interviews aimed to gather more insight about students' mathematics learning background, and to determine why students had particular attitudes and beliefs about mathematics, mathematics coursework plans, as well as career interests. The researcher would use student's responses to the open-ended questions in the Attitude and Belief

Questionnaire as a guideline to obtain in-depth information about their mathematical dispositions. In addition, the researcher asked the students questions, adapted from a previous study (Mingus, 1996). The questions focused on students' experiences in group activities and interactions with their peers.

Students' Mathematics Achievement Information

Students' previous mathematics grades were obtained and the mathematics scores for the observed unit were collected at the end of the study. The teacher was asked to provide each student's previous mathematics grade and the mathematics scores for the observed unit on the appropriate class list and to return only the portion of the sheet containing the student's scores and grades.

Artifacts

To complete a picture of the PBL classroom, all mathematics class artifacts were collected, such as individual student or group worksheets, the CMP textbook, curriculum materials, homework assignments, and classroom tests or quizzes. The volunteer teacher required that her students keep all of their mathematics work in a binder. At the beginning of the observed unit, the researcher asked all target students to keep their entire work sample for the unit in the binder to be collected at the end of the unit. Later, however, the researcher found that some of the students had lost some of their work, meaning some participants did not have a complete work sample in their mathematics binder.

Teacher Pre- and Post-Formal Interviews

Teacher interviews included pre- and post-formal interviews (see Appendix I) as well as informal interviews. The pre-formal interview was conducted with the volunteer teacher for approximately 40 minutes. The goal for the pre-formal interview was to obtain data about the volunteer teacher's experience with teaching mathematics and using the PBL instructional approach, and to obtain the teacher's expectations of students' learning. The informal interviews were conducted before and after the classroom observations (when possible and appropriate). If it was not possible to do so immediately after the lesson, other arrangements were scheduled. The informal interview focused on daily instructional objectives, the teacher's assessment of student learning, whether or not the class went as expected, and whether the teacher was going to use the same approach to teach this topic/lesson again. At the end of the study, a formal post-interview was conducted for approximately 45 minutes to gather the teacher's opinions about the PBL teaching experiences, and students' and classroom interactions during the eight-week period. For back-up information, tape recorders were used to record all the formal interviews, and notetaking was used for the informal interviews. In order to avoid the possibility that the volunteer teacher might change her normal classroom routine, the teacher was not told the specific research question for the study.

Researcher Journal

As recommended by researchers (Bogdan & Biken, 1992; Gall M., Borg & Gall J., 1996), there is a need to minimize possible bias and misinterpretations of collected data. Since the researcher was the main instrument for collecting the observational data, a researcher's journal was used to avoid or identify possible bias problems. It contained the descriptive and reflective aspects of the collected data, portraits of the classroom, the physical setting, accounts of particular events, and the observer's thoughts and behaviors. The researcher used the researcher journal to record daily activities and circumstances of the classroom during the observation period to reduce any possible bias from the researcher's perspective.

The Researcher

Since the researcher was the main instrument in collecting data, it was important to identify the researcher's background and educational area of interest. The researcher graduated with a Bachelor's degree in Mathematics Education in Thailand. She had mathematics teaching experiences at middle and high school levels for four years before pursuing higher education in the United States, obtaining a Master's degree in Mathematics Education. In her master's thesis, she studied Thai middle school girls' and boys' attitudes and beliefs about mathematics. In the pursuit of higher education, the researcher continued in a doctoral program in Mathematics Education in preparation for a position as a mathematics educator in Thailand. The researcher first encountered problem-based learning [PBL] in mathematics classrooms in her doctoral program. Since then, researchers were interested this new instructional approach because this approach appeared to have potential to increase students' mathematical dispositions and mathematics achievement.

Data Analysis

In order to gain an insight into middle school students' mathematical dispositions in a classroom using the PBL approach, a case study was employed in this research. Classroom observation provided a rich description of the PBL classroom including the whole- group and small-group discussions. In addition, teacher interviews, student responses to a set of open-ended questions in the Attitude and Belief Questionnaire, and student interviews provided additional descriptive data on the questions of interest. The students' questionnaire was the focus for the analysis of quantitative data. The reliability of each scale of the questionnaire was calculated.

To portray the teacher's implementation of the PBL approach and context of the PBL classroom, first the teacher pre- and post-formal interview data were transcribed. The transcript for the teacher's teaching background, the classroom's routine, and the reflections on her teaching were written. Continuing analysis proceeded during observation of the classroom to capture possible variables, relationships, or patterns that might impact each student's mathematical dispositions, mathematics coursework plans, and career interests.

The case study approach was applied to analyze data on each student participant. The first step of interpreting involved reading through the transcribed data in order to capture important words and phrases. From these words and phases, the researcher identified and classified key concepts into categories for a "Master list" of repeating ideas (Auerbach & Sinerstein, 2003). The researcher used this list to code segments considered meaningful and relevant to the researcher's concern.

In the coding system, the researcher labeled segments with abstract descriptors such as "classroom environment," which would include "furniture arrangement" and "teaching materials," for example (Auerbach & Sinerstein, 2003). If the researcher found that some segments of information in the developed coding system were incongruent, the researcher revised the "Master list" of repeating ideas until it was suitable. After coding all segments, the researcher put together all segments that were classified into the same category for a final check on the coding system and then drew conclusions.

Johnson and Christensen (2004) suggested that researcher bias is "one potential threat to validity." A researcher's background, opinions, and perspectives may "affect how data are interpreted and how the research is conducted." To establish validity, Johnson and Christensen recommended two main strategies: "reflexivity" and "negative-case sampling." They defined "reflexivity" as a researcher's self-reflection on possible biases and previous dispositions. The authors commented that the process of "reflexivity" helped researchers become cautious and attempted to limit their biases. Negative-case sampling, the authors noted, is a way to seek for examples that disconfirm researchers' expectations and to find tentative explanations. This method helped researchers to be aware of all details and information in the study. Regarding the validity of this study, during data collection and interpretation, the researcher attempted to use self-reflection and negative-case sampling to reduce the effect of the researcher's bias.

The researcher used the taxonomy of educational objectives (Krathwohl, Bloom, & Masia, 1964) as a guideline to determine each student's mathematical disposition based first on their observed behaviors and second on their interviews and questionnaire. Krathwohl, Bloom, and Maria searched for and described an affective domain continuum, its nature, and its structure in order to classify student dispositions. Krathwohl, Bloom, and Maria classified educational objectives in affective domain into five main categories: "receiving", "responding", "valuing", "organization", and "characterization" (see Appendix K).

At Level 1: the "receiving," the learner was considered to be aware of the existence of a certain mathematics learning situation. This level was divided into three sub-categories including awareness, willingness to consider, and controlled or selected attention to designate different levels of considering a situation. For example, at the level the students listened to the class discussion without avoidance. The second level was responding. This level considered the response after receiving the mathematical learning situation. The response was reacted to actively at this level. Also, there are three sub-categories in this level, including acquiescence in responding, willingness to respond, and satisfaction in response. At this level, the student became actually involved in the learning situation, such as volunteering to participate in the class discussion.

The third level was valuing with sub-categories of acceptance of a value, preference for a value, and commitment. This level was used to detect whether or not a situation or a subject has worth for a student. A student who showed behavior at this level perceived it as valuing of the mathematics topic or the activity. For instance, students who sought to do additional mathematics problems in their free time would be concluded as valuing.

The fourth level was organization. This category aspired to describe the beginning of the construction of the value system. It was categorized into two levels, "conceptualization of a value" and "organization of a value system," because a prerequisite for interrelating was the conceptualization of the value in a form that allows organization, such as attempting to identify the characteristic of mathematics subject that he/she admired.

The fifth category was characterization by a value or value complex with two sub divisions: "generalized set" and "characterization." At this level, the values had a place in a student's value hierarchy. The student acted consistently in accordance with the values he or she had "internalized" at this level.

In conclusion, classroom transcript and documents were placed by classroom activities and then by student. The data were read several times in order to develop initial picture of the classroom environment. Then, the data analysis involved searching through the data and writing down words and phrases for regularity patterns in the PBL classroom. The words and phrases that were generated from this search were used as coding categories to sort the data. Finally, the researcher considered all data sources. These sources included data from classroom observations, teacher formal interviews, the attitude and belief questionnaire, student formal interviews, and the researcher journals. Finally, the taxonomy of educational objectives (Krathwohl, Bloom, & Masia, 1964) helped the researcher in discriminating the students' mathematical dispositions based on observations, interviews, and questionnaire to determine their behaviors, thoughts, and feelings. The researcher used all collected information to describe the students' mathematical dispositions in the PBL context.

CHAPTER IV

RESULTS

Introduction

This chapter provides the findings of this study on the dispositions of middle school students toward mathematics in a problem-based classroom. The study describes students' mathematical dispositions relating to patterns of classroom interaction in a problem-based environment, using the PBL approach: the Connected Mathematics Project [CMP].

Chapter four begins by looking at the volunteer teacher's teaching experiences. Next, the chapter details an overview of the ten-week study during the spring term of 2003. The unit, "Covering and Surrounding," focused on measuring areas and perimeters. Then, two cooperative groups from the CMP class, with four participants in each group are described. Later, the dispositions of those eight participants are reported. This section reports data collected from observations in the class and the questionnaires and interviews after participants' one-year experience in the CMP 6th grade class. An analysis of participants' behaviors in class is presented. Finally, the major findings are summarized.

The Connected Mathematics Project [CMP] Teacher's Teaching Experiences

The CMP classroom teacher, Ms. Smith was a member of the 6th grade teacher team with four other teachers. Ms. Smith and her team taught in self-contained classrooms. Each of the teachers in her team had the same group of students all day for mathematics, language-arts, social studies, and science. Ms. Smith and her colleagues were the teachers for all of the subjects except physical education, art, and music. Ms. Smith noted that her teacher team preferred the self-contained model because they could get bored with teaching the same lessons to several groups of students each day and they enjoyed doing different things during the school day. This meant the teachers completed several teaching preparations in order to get ready for each day. Ms. Smith admitted that sometimes she tended to ignore some important tasks, like preparing a detailed plan at the beginning of each unit, since she had to plan several activities for each day. However, she found that she could not ignore planning with the CMP curriculum, because it required more preparation than traditional mathematics curriculum. Ms. Smith had taught the CMP curriculum for almost two years and she hoped that once she had taught it for three or four years, she would feel more comfortable. She said she still felt a little nervous at the beginning of each lesson.

Ms. Smith and her 6th grade teacher team had started to pilot the CMP curriculum during the past year. She mentioned in her interview that she and her teacher colleagues had a chance to test and try the CMP 6th grade curriculum mathematics even though they did not all review the whole curriculum. Some of them successfully completed five pilot units in the first year, all units from the 6th grade except the unit about probability. On the basis of this pilot, the teachers decided to use CMP for their new curriculum. She and her 6th grade teacher team had the most experience using this curriculum in her school. Ms. Smith mentioned that they also had a chance to learn more about the new curriculum from participating in a CMP workshop the previous year. The school paid for the workshop for the teacher team.

The workshop provided examples of how to teach the curriculum units and how to use and organize the new curriculum material more effectively. For example, since the teacher's guidebook seemed somewhat confusing, the workshop instructor suggested that they tear the teacher's guide out and put it in a separate binder, and then put the materials together in a more effective order. Based on another suggestion from the workshop, Ms. Smith divided her students into four categories in order to create student groups. She defined her students as being high, medium, low, or nice students. The nice students were defined as the ones who were willing to help or offer help to their peers. Ms. Smith said, "…it was brought to our attention again in that workshop, we refocus on building our groups with a high student, a medium student, a low student, and a nice student." She explained that high students could be nice students because they were helpful, but not always. Sometimes, medium or low students were actually more helpful than high students and would fit better into the nice student category. She created various groups of high, medium, low, and nice students and alternated them for each unit.

In so doing, Ms. Smith was committed to providing her students with experience real workplace situations by giving them a chance to work with different people having varied abilities. She showed that she valued the significance of cooperative groups by adding, "It's just a skill. The kids need to be taught how to work in groups and I am not going to say I have taught them well (laughing), but we keep telling them that, when they go out the real world, their employers are going to put them on a team and they need to know how to work with people."

The workshop also encouraged Ms. Smith to do her unit planning ahead of time. She said, "I got kind of inspired by that (the workshop). I have been writing unit plans for each unit. Before, I just did it a chapter at a time. I found planning units ahead of time is real valuable. It takes several hours to do that, but once you've got it done, you're all set for 5 to 6 weeks." At the end of the teacher initial interview, Ms. Smith again expressed her concern about being prepared to teach the CMP curriculum. She said that she was uncomfortable teaching it, "…without being really prepared…where (laughing) more traditional textbooks might be a little easier just to grab…five minutes before the kids come (laughing). That's made me nervous to do that (last minute preparations) with the (CMP) curriculum. I really feel like I need to take more time with it."

Ms. Smith said that the middle school provided 50 minutes of teacher preparation time for mathematics on Friday mornings. All mathematics teachers meet the first and third Fridays of every month. The 6th, 7th, and 8th grade mathematics teachers were all at these meetings and exchanged their experiences with their mathematics classes, either using the CMP curriculum or other mathematics curriculum such as MathScape curriculum (EDC, 1999) or Chicago Mathematics Project (EDC, 1999). Ms. Smith said, "It's been very valuable to have time on Friday mornings." She explained that in the first half of the year, the mathematics teachers exchanged suggestions during these meetings. The 7th and 8th grade mathematics teachers provided useful information to the 6th grade mathematics teachers by sharing their views on what was important for the students to learn before entering the upper grades. Ms. Smith added that in the last half of the year, the teachers spent time organizing curriculum materials during these meetings. Based on these teachers' experiences, the school had an important role in supporting the mathematics teachers by providing time for them to prepare and implement the CMP curriculum.

Regarding teacher expectations, Ms. Smith wanted her students to pay attention to her teaching, to try hard and do their best to understand the class activities and concepts. She said she felt that students met these expectations when "...they're here, paying attention, and being kind to each other and completing their work to their best ability, with their best effort. If they are not successful on a certain unit or certain task, they will make an effort to remedy it on their own or come in to get extra help."

Ms. Smith mentioned that she felt pressure to get all the CMP units taught, since the school had cut 10 days from this school year as budget reductions. Ms. Smith and her teacher team had sent the probability unit home and asked students and their parents to work on some investigations from the unit. She said the probability unit was a friendly one, in which students and parents could play games together like flipping coins and spinning dice. Most of the students had completed the activities at home with their parents. The teacher had her students return a letter from their parents to confirm that they had done the probability activities together at home.

Overall, after she finished teaching the CMP curriculum this term, Ms. Smith said, "I am pleased. I still like the curriculum a lot. However, there are some difficulties that I am working out." She added, "The assessments are hard for a lot of kids. Sometimes, I think the scores are not reflecting what they really learned." Regarding the difficulty of using the CMP, Ms. Smith was concerned about some of the language in the curriculum, and the complexity of the problems. She said, "It used a lot of idioms and the problems may be too complicated..." In her opinions, the CMP curriculum is "... just fine for both medium and higher kids, but I am still stuck with the lower kids." She felt she might need to develop an alternative assessment. She added, "I am still very pleased with it. It's life fully. It's interactive. I think that kids enjoy it more than the traditional lessons. I think they try harder." Ms. Smith added that the CMP curriculum approach is about presenting the concepts, getting the students "going", and letting "the group work out the ideas." She said, "The teacher is more like a facilitator than the traditional one, pouring knowledge into their head." She added, "That's my style. That's how I see myself as the teacher, as the facilitator of learning, not the dispenser of knowledge, I feel like I am the learner right along with them. I am happy to admit that (laughing)."

Ms. Smith admitted, in the interview after the observation, that she had not done a good job of teaching group work at the beginning of the school year. She said, "I made an assumption that they come to me knowing how to work in a group. My expectations are that they're helping each other, but not just giving each other the answers." She added "I still have kids that just kind of hurry all by themselves and say I'm done. Well, what about the person sitting next to you? They're lost, you know (laughing)." She concluded, "So, I need to think about that and do a better job at the beginning of the year of teaching them what I expect as far as group work will go. Kids don't naturally do cooperative learning. It's a skill, I think."

Reflecting further on her experiences teaching group work, Ms. Smith said, "When I first started teaching, I had just been recently trained all about that and I did a better job at the beginning of the year. Then I kind of forgot about things and just tried to jump in and start doing (laughing). So, there is something I have to keep in my mind."

Concerning the program's successes, Ms. Smith said, "...I think all kids, but particularly the ones on the lower end appreciate and learn better with all (the) nice manipulatives that come with the program." She explained that the use of manipulatives, such as "blocks" and "poly strips" that can be used to create different shapes, have been helpful "...especially to the kids that are still a little bit on the concrete side and haven't crossed over to abstract thinking yet." She also added that, "The higher kids, the ones who are thinking abstractly, I wonder about them. I wonder if it is challenging enough."

She added that she provided some "extension problems" to the more advanced student so that they had opportunities to "think a little harder" and "apply (concepts) in different ways." She wondered whether she should find other sources to challenge advanced students. She added, "I think they have a good time. I think they like working with their friends, but I was hoping that they wouldn't be bored."

In summary. Ms. Smith was an experienced mathematics teacher, having taught mathematics in elementary schools, especially the 6th grade level, for 10 years. She had some experience in teaching the CMP curriculum. She also attended a workshop on how to use the CMP curriculum and had teacher colleagues to consult in using the CMP curriculum. Ms. Smith was willing to learn more about the CMP curriculum and worked hard at getting more familiar with it. Overall, Ms. Smith liked and enjoyed using the CMP curriculum. She thought the curriculum was fun with hands-on activities for students. She sometimes learned with her students, when they explored something new or unexpected. However, she commented that, on group work teaching, she needed to be more concerned about teaching her students how to work with groups at the beginning of the school year, in order to have them work together effectively.

On the other hand, Ms. Smith found that the CMP assessment package sometimes did not make sense to her students, especially those who had lower mathematics achievement, because of the language in the test problems, which might be too complicated for the low mathematics achievers. Furthermore, due to budget cuts, the school had to reduce the school year by 10 days, which placed more stress on teaching time. However, Ms. Smith commented that the school had provided preparation hours for all mathematics teachers to meet and discuss teaching mathematics with the new curriculum, which helped her to implement the program effectively.

Overview of Ms. Smith's CMP Class Routine

The purpose of this section is to present an overall description of the "Covering and Surrounding" unit in the CMP class. The material used in this mathematics class was the Connected Mathematics Project [CMP], which was designed to develop middle school students' mathematics knowledge by using an active-group learning environment around investigations of real-world problems. The section on "Covering and Surrounding" included basic two-dimensional geometric measurements such as perimeter and area. The goals for the Covering and Surrounding unit are provided in Appendix L. Prior to this unit, the 6th grade participants had completed five units in the following order: "Data about Us" (data analysis and probability), "Prime Time" (number and operations), "Bits and Pieces I" (number and operations), "Bits and Pieces II" (number and operations), and "Shapes and Designs" (geometry). The general picture of Ms. Smith's CMP class is detailed in the following section.

The instruction for the CMP class took place for 60 minutes in the morning, five days a week. There were four segments in Ms. Smith's mathematics instruction following the recommendation of the curriculum: Warm-up, Launch, Explore, and Summarize. A description for each segment follows.

Warm-up

Typically, Ms. Smith started her CMP mathematics class with warm-up activities. The activities included extra individual practice on prior lessons, mathematical reflection problems, Applications-Connections-Extensions [ACE] problems, or homework. An example of Ms. Smith's warm-up activity follows.

After the teacher let her students check their homework with their peers for approximately 10 minutes, she discussed the homework with the whole class.

> T: Ok, describe an effective way to find the area of triangle. Be sure to mention the measurements you would need to make and how you would use them to find the area.

Bob raised his hand and shared his idea (he was the only volunteer to share his knowledge at this time).

> Bob: Half of the base times height.
>
> T: So, you need to measure the base and the height?

Bob: Yes. Or base times height divided by two.

From Bob's ideas, the teacher asked if "…we can cut the measurements in half to get it…?" Mike responded and added his alternative idea (he was also the only volunteer to share his idea).

Mike: Yes, I made a square out of the triangle then found the area of the triangle by dividing it by two, because the area of the triangle is half of the rectangle or square or parallelogram.

Mike and Bob demonstrated their conceptual understanding of why they needed to divide by two. They also demonstrated that they knew dividing by two was the same as taking half of the product.

T: Ok, next, describe an effective way to find the perimeter of a triangle. Be sure to mention the measurements you would need to make and how you would use them to find the perimeter.

Tony and Mike raised their hands.

T: Tony.

Tony: You measure all sides and combine them.

T: So, typically, you measure all three edges and then add them up together?

Tony: Yes.

T: Ok, summarize what you have discovered about finding areas and perimeters of rectangles, parallelogram, and triangles. Describe the measurements you need to make to find the area and perimeter of each figure.

Mike raised his hand again.

T: Mike.

Mike: For all perimeters, you get to measure around the edges outside. For the area of a rectangle, you do length times width equals area. For the area of a parallelogram, you do base times height equals area. For the area of a triangle, you do base times height divided by two.

T: Ok, good.

Jim: Nice.

T: Ok, today we're going to look at...

During Warm-up activities, Ms. Smith provided opportunities for her students to recall previously learned concepts by having the students work individually first and then share their ideas with the whole class. Some target students (Jim, Bob, Mike, and Tony) often volunteered to participate in the discussion by raising their hands. Therefore, most of the students that the teacher called on were higher achievement boys from the two cooperative groups. While Ms. Smith limited her choices of respondents to volunteer students, she did provide opportunities for them to provide explanations of their understanding. After refreshing the memory of her class with warm-up activities, Ms. Smith then introduced her class to a new concept in the Launch section.

Launch

Ms. Smith would launch a problem for the class investigation by using real-world situations, which are situations in context or previously learned concepts that are familiar to the students. For example, the teacher used a pizza problem to introduce connections between the diameter, radius, area and circumference of a circle.

The teacher introduced the Launch by talking about the different pizza sizes and the assumption that the bigger pizza is the better deal. She added that, "sometimes it doesn't work that way" and that the class was going to investigate "if [they] really are getting a better buy by getting a bigger one."

T: Please, look at page 69: "The Sole D' Italia Pizzeria sells small, medium, and large pizzas. A small is 9 inches in diameter, a medium is 12 inches in diameter, and a large is 15 inches in diameter. Prices for cheese pizzas are $6.00 for small, $9.00 for medium, and $12.00 for large.

Before class started, the teacher had already drawn three circles of 9, 12, and 15 inches on the white board.

Mike: Is it the real sizes?

T: Yes, I measured the diameters.

T: All right, you're going to use grid paper and pretend like a square of grid paper is one inch and draw your three circles and then, we're going to estimate the radius, circumference, and the area of the pizzas. At this point, we're just going to use the things that we already did for the parallelogram. We'll use similar strategies to measure the circumference and find the area.

Mike expressed his interest about the investigation by asking for more information about the assignment.

Mike: And then what?

T: So, which measurement --radius, circumference, or area-- seems most clearly related to the prices? It you look at your measurement, which one do you think they used to set the price? You're going to work through this yourself and then with your group "which is the best value?

Then, the teacher checked to make sure her students understood the assignment.

T: Questions before we start?

S2: So, how much for each one?

T: We've got 6 dollars, 9 dollars, and 15 dollars and we're going to make measurements. You may need to set up a table and to decide which one has the most value.

T: Ok, let's turn your desks around and start working.

In addition, Ms. Smith encouraged her students by giving them confidence to "try out new ideas" without fear of making mistakes as in the following example.

T: Today, we're going to look at seven parallelograms that are on page 47. I'll give you a lab sheet that you can start discovering the area of those and anything interesting about that. Don't be afraid to try out new ideas…it's ok to make mistakes…

In another example, when a student expressed frustration with the given assignment, she expressed her confidence in him.

Mike: It kind of hard to figure out about how many radius squares are in a circle…

T: You can do it. I am challenging you...

In the Launch activities, Ms. Smith provided her students directions for the investigation. She checked their understanding about the assignment before starting the investigation. In addition, she repeated her directions twice for her students when it seemed that some students still were not sure what they were expected to do. Also, she encouraged her students to not be afraid of trying new ideas. The teacher also tried to maintain the level of the tasks.

Explore

In the Explore section, Ms. Smith allowed the students to investigate the problem with a pair or with their cooperative group. This study focused on the exploration of the two cooperative groups of four participants each. According to Ms. Smith's expectations about group work, her students in each group were supposed to work individually at the beginning of the investigation, and then compare their work with their group members.

However, sometimes when the students worked or explored with their group, the class became loud and disorderly, as the students tried to talk across the classroom with their friends who were in other groups. However, the students were usually still working on the classroom task. Ms. Smith told her students to work with their groups, not to talk across the classroom. After Ms. Smith advised the students to stay focused, they followed her directions for a few days and then returned to the same actions.

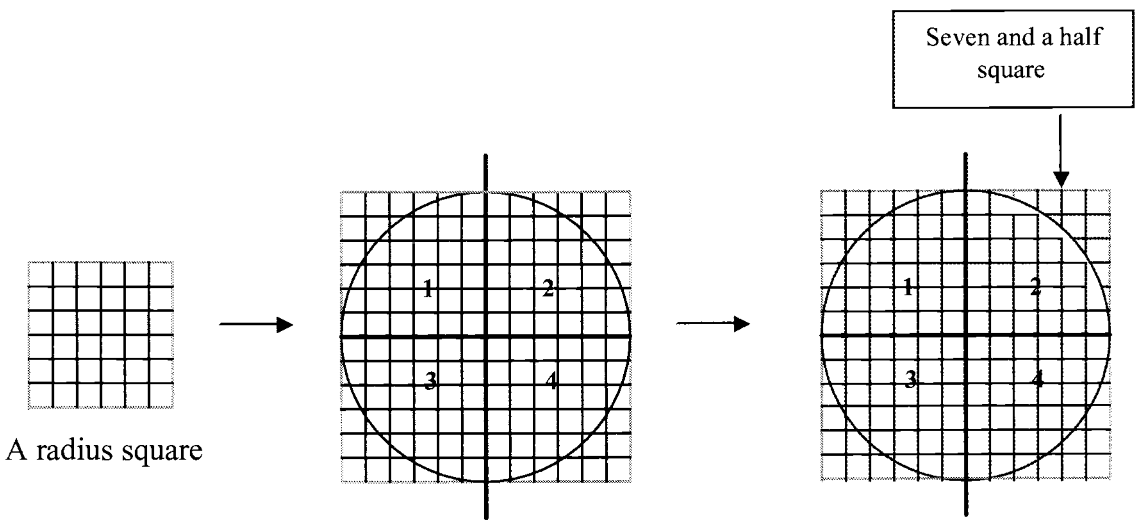
In Ms. Smith's post-formal interview, she admitted that she was tired of trying to quiet the class, and her students needed more discipline. However, in her opinion, it was not that serious. One reason for her class becoming disorganized, she proposed, might be because of the students' familiarity with each other. The teacher noted that the class consisted of students who graduated from three out of five different elementary schools in town. Therefore, the students who came from the same elementary school knew each other quite well and tended to collaborate on their work.

The exploration took approximately 15 to 20 minutes. (see the description of the two cooperative groups investigation in Group Work in the CMP Class in the discussion on p. 70). During the group work, Ms. Smith occasionally walked around the class, checked on the groups, and guided the students. Two examples are provided as follows.

The first example was collected from the Cubic Melon group while they were working on finding how many radius squares would fit in a circle (squaring a circle investigation).

Bob called for the teacher and asked for advice.

Bob: [Ms. Smith]...what if I put four of them in here (four radius squares). Then, I subtract seven and a half (squares)?



T: From where? Four radius squares?

Bob: Yes, would that work?

T: What are you going to do with that?

Bob: I'll divide it (seven and a half) by four.

Bob was confused.

Jim: No, no...

Bob: Wait.

T: Ok, we want to know how many of these (radius) squares would fit in the circle, right?

Jim and Bob: Yes.

Bill and Cindy listened but made no responses.

T: Now, you put four radius squares in the circle, then you subtract seven and a half squares from the four radius squares. Do you think it will fit now?

Bob: No, we need to minus (seven and a half squares) three more times.

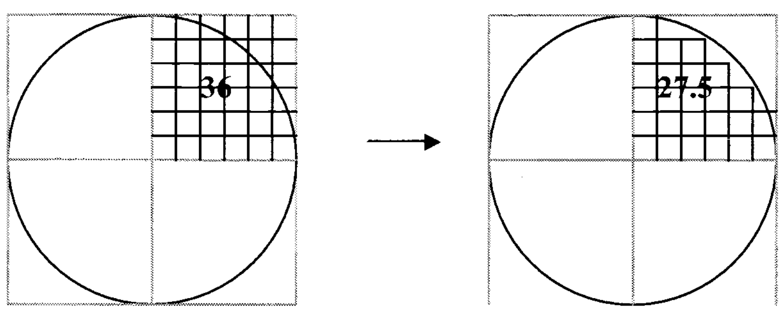
T: Ok, …then you need to find out how many radius squares actually fit in there…keep thinking about that...

The teacher moved to other groups after helping the students in Cubic Melon group clarify and expand their thinking.

The second example was gathered from the Rhombus Square group while working on the radius square problem stated in the prior example. The teacher stopped by and checked on the group work.

T: Where are you guys now?

Mike: We used about 27.5 squares out of 36, in each radius square, we did not use the whole (radius) square.



T: Ok, how could you find out how many radius squares you need for the circle?

Mike: We need to figure out how many percent of 36 of 27.5 or 28. So, it's 27.5 divided by 36 or about 28 divided by 36.

Tony agreed with Mike's idea.

Tony: Yes, yes.

Mike and Tony used a calculator to find out the answer.

Mike: It's about 80%...then, times four.

T: Why did you times four?

Mike: Because it's 80% of each radius square to fit in one part of the circle. We need 80 % of four radius square to fit in the circle...

Mike and Tony carried on calculating the answer using their calculators.

Mike: So, 0.8 times 4 ...it's 3.2.

Tony: Yes, it's about 3.2 radius squares.

T: Good, find out about the other circles, ok.

Again, the teacher provided opportunities for students to clarify and explain their thinking without telling what to do.

Later on, the teacher called her students to attention. She collected each group's findings for the data on the three circles including radius of the circles, area of radius squares, area of the circles, and number of radius squares needed. With the lead of the teacher's questions, the class came up with a conclusion that the area of a circle was a bit more than three times of the area of the radius square.

In Explore activities, Ms. Smith had her students explore the squaring a circle investigation with groups. Occasionally, the class was loud because the students talked across the classroom. Ms. Smith thought it was not serious, since the students were still on task. She thought the students wanted to discuss mathematics problems with their friends in other groups because of students' familiarity with each other. Additionally, Ms. Smith had each small group explain and share their thinking in the whole class discussion, which led to consensus about their conceptual understanding.

Summarize

In the Summarize classroom activity section, Ms. Smith had the class summarize and discuss what they had found from their investigations or explorations. The classroom environment was open and informal. Ms. Smith provided a friendly atmosphere by joking, smiling, and laughing while she was teaching. The students were welcome to ask questions and share their ideas in the class. Ms. Smith would ask volunteers, or specific students with questions that related to previous concepts, in order to guide the class progress in solving problems. Also, Ms. Smith encouraged her students to share their ideas within their cooperative group and with the whole class. Two examples of the CMP class in the summarizing process are provided. The first example (finding area of circles in the pizza problem) involved the class sharing the discovery of a mathematical relationship from group work. In the second example, the class discovered a mathematical pattern (finding the largest or smallest possible area for a rectangle of a fixed perimeter) during the whole class discussion. The first example follows.

T: Ok, Cubic Melon group, what do you think?

Jim answered almost immediately as if he knew that he was the group's representative.

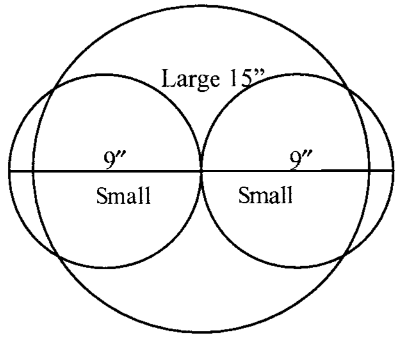
Jim: We think the larger pizza was the better deal because you got 14.75 square inches per dollar. For the small (one), you got 10.30 square inches per dollars, and the medium (one), you got 12.50 square inches per dollar.

T: Ok…

The teacher did not usually ask any follow-up question or ask for clarification from her students. The teacher moved on to the Rhombus Square group.

T: Ok, Rhombus Square, what do you think is the best value?

Mike: We just talked about one way to do that. We are not sure, but it worked. If we took the area of the two small pizzas, then we put it into the large and we say 65 (area of the 9 inches small pizza) times two is 130. The price you pay is going to be 12 dollars. So, we pay the same amount on the two small pizzas and the large, but you're not getting nearly the area of 176 squared inches (the area for the 15 inches large pizza). That's why we say the larger one is a better deal than the small one.



The teacher positively commented on the group work.

T: Ok, that is a good comparison.

Mike: So, …we think the large one is the better deal than the smaller.

T: Ok, and do you find any measurement more correlated with the price than the other measurements? Did the diameter or radius?

Jim raised his hand and shared his answer.

Jim: We found that the price is three less than the diameter. It's three dollars less than the diameter.

T: So, the relationship is?

Jim: The diameter and the price.

T: Ok. Any other groups have anything to add to the question?

There was no response from the class. Then, the teacher moved on without asking any further questions or clarification. The teacher missed the opportunity for students to find alternative ways to solve the problem.

T: All right, I will just put the estimate on the board that you can check real quick. Let me know if you have something that's way off of this. We've kind of run out of time.

Ms. Smith put the measurements (see Figure 3) on the three circles that she had made in advance on the overhead projector.

Size	Diameter (Inches)	Radius (Inches)	Circumference (Inches)	Area (Square inches)
Small	9	4.5	28.3	63.6
Medium	12	6	37.7	113.1
Large	15	7.5	47.1	176.7

*Figure 3.*The measurements of the three circles (small, medium, and large).

T: If you're off, let's hear why? Remember that it's an estimate, so you maybe quite a bit off.

Mike: We are off a lot on all circumferences.

T: All of them?

Mike: Yes.

T: Tell me what you did? How did you measure that?

Ms. Smith walked to Mike's group, but still talked to the whole class.

Mike: What I did, I used the string to measure the outside of the circle like this.

T: How did you measure the string?

Mike: I measured the outside by using centimeters.

Ms. Smith still talked out loud to the whole class.

T: Oh! That's why you're off. You used the wrong unit; actually you need to measure in inches not centimeters.

At this point, the teacher did not ask students to make conjectures; she solved her student's dilemma, instead. Since the class session was ending soon,

Ms. Smith rushed back to front of the classroom.

T: Ok, anybody else have a problem? What is going on here is… you have to use the right measurement unit.

T: Okay, eyes up here, please. Homework for tonight for the weekend is to do the follow up, write a report, and ACE problem #15 on page 78. (The ACE problem #15: Which measurement of a circular pizza--diameter, radius, circumference, or area--best indicates its size?)

The homework problem was about measurement of a circle, which was related to today's topic.

In the second example, the CMP class discovered a mathematical pattern during the summarizing process following students' cooperative group investigation. First, Ms. Smith had her class search for possible rectangles with whole-number side lengths and a perimeter of 24 square units. Next, Ms. Smith collected all data from her students and put the information on a table on the white board. Then she led class discussion about a pattern that the students found about the perimeter of figures (see Figure 4).

Length	Width	Perimeter	Area (Square meters)
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35
8	4	24	32
9	3	24	27
10	2	24	20
11	1	24	11

*Figure 4.*The list of possible rectangles with whole number side lengths and a perimeter of 24 square units.

The teacher started the discussion about the patterns that the students found from data collected in the small group reports. Some students noticed that the lengths went from 1 to 11 whereas the widths went from the other way (11 to 1). Bob found that the area went up to 36 then the area went down. Bill noted that the perimeter stayed the same. Mike added another pattern that he noticed. The description of the patterns in the figure was as follows:

Mike: The length and the width is half of the perimeter.

S2: Ok, because like each rectangle and square have 4 sides.

T: Ok.

S2: So, the length and the width, there are always 2 of them. So, the length and the width is half of it [the perimeter].

T: Ok, because you really have 2 lengths and 2 widths, right?

S2: Yes.

T: Ok, can you think of any mathematical way to talk about how to get the perimeter, not just with the numbers, just the length and the width?

Mike: Well, it's like the length times 2.

T: Wait, wait the length times 2, ok.

Mike: Um…plus the width times 2.

T: Equals?

Mike: Equals perimeter.

T: Excellent.

Then, Ms. Smith wrote on the overhead projector: $(l \times 2) + (w \times 2) = p$

T: Anybody has another way to write it up?

Jim: I have the length plus the length plus the width plus the width equals perimeter.

T: Ok, the length plus the length plus the width plus the width equals perimeter. That would work.

Ms. Smith wrote on the overhead projector: $l + l + w + w = p$.

T: Ok, anybody else?

Mike: Um… perimeter minus width plus width…

Mike tried to move the variables around.

T: (laughing) Usually, math people would think of the shorter way to write the formula. Can you think of the short way to write this?

Jim: Length plus width times 2.

T: Let's say l = length, w = width, and p = perimeter. So, length plus width and then times 2, in math we do this (write (…)), that means times. That works pretty good.

Ms. Smith wrote $2(l + w) = p$ on the overhead projector.

T: And you guys should know that when we can put letters next to numbers that means times. Can we do this: $2l + 2w = p$?

Tony: Yes.

T: So, you know that we can use the distributive rule $a (b + c)= a \times b + a \times c$, right! So, all of them are right. Choose your favorite short one. Good job you guys! Any questions?

There was no response from the class.

T: Ok, we have 5 minutes left so let's start your homework on page 38 ACE Problem #1- 4 and 14 (an example of the problems: If you have 72 centimeters of molding to make a frame for a painting, how should you cut the molding to give the largest possible area?).

In the Summarizing activities, Ms. Smith helped her students develop their understanding in mathematical concepts without direct instruction. She gathered information from the class and then put it together using students' ideas until they described the mathematical relationships. Her students were able to write the relationships between the perimeter, the length, and the width in various forms. However, the teacher did not ask follow-up questions or clarifications in summarizing. Most of the students who participated in the activities were boys. Fewer girls participated in the activities; they rarely raised their hands to share their ideas, indicating a need for extra encouragement from the teacher by calling on non-volunteer students.

Ms. Smith's summarizing provided opportunities for her students to share and discuss ideas concluding the new concepts that they had learned that day. Even though Ms. Smith attempted to focus on students' understanding and tried to draw conclusions from her students instead of telling what they had learned, there sometimes was not enough time to do this. For example, when some of her students were able to tell the relationship between perimeter (p), length (l) and width (w), which was $(l \times 2) + (w \times 2) = p$ or $l + l + w + w = p$, Ms. Smith asked for other alternative ways to write the relationship. One of her students tried to move the variables (p, l, and w) in the relationship around, which was a good opportunity to check students' understanding of the relationship. Ms. Smith just told him to find or use the less complicated form of the relationship instead of having him verify his understanding of the relationship. This might have been due to time constraints. In the teacher's interview, Ms. Smith revealed that she had a time line concern for her CMP class, since the school days were cut 10 days for budget reductions.

Additionally, Ms. Smith had encouraged her students to share their mistakes with the class. Ms. Smith never blamed her students for making mistakes. She tried to help her students figure out what went wrong, as demonstrated during the class discussion about the pizza problem (see Summarize section p. 62). When Mike reported that his group had different measurements than the rest of the class, she helped them to find out why, and she did not think that it was a serious mistake. Ms. Smith showed her students that they could learn from their mistakes and that making mistakes was acceptable. She would smile and then work with them to figure out what went wrong. This made the students feels comfortable with sharing their mistakes.

Summary

Ms. Smith's CMP class consisted of four main components. First, she warmed up the class by having her students work on additional practice, mathematics reflection problems, Applications Connections and Extensions [ACE] problems, or homework. Second, she launched the investigation to the whole class using real-world situations, or previously learned concepts, and using a group-learning format to motivate the students' learning. Third, after launching the investigation, the teacher normally had her students explore the situations with their partner or with a group. Sometimes, the class discussion was loud because the students discussed mathematics problems with their friends across the classroom. However, the teacher thought this was not a serious issue because the students were still on task. Fourth, in the Summarize section, Ms. Smith led the whole classroom discussion and encouraged a variety of ideas. She provided opportunities for students to share their ideas and conjectures from the large group exploring relationship between ideas and coming to class consensus. She showed her students that making mistakes was acceptable. Sometimes, she had the students share their ideas in front of the class, but more often she had the students share their ideas from their seats and she expanded on the ideas for the whole class. In addition, the teacher needed to be aware of calling only volunteer students, which might lead to having mostly boys talk and share their ideas than girls.

Overview of the Two Participant Groups

The study was introduced to the 20 students in Ms. Smith's class, who were asked to volunteer to participate in this study using information gathered from class observations, questionnaires, and interviews. With direction from Ms. Smith, eight volunteer students were divided into two groups of four students with varied mathematics achievement. The two groups named themselves "Cubic Melon" and "Rhombus Square," adopting the teacher's suggestion to use a name with mathematical words in it. The eight participants were called by pseudonyms in order to assure the anonymity of the participants.

The Cubic Melon group was comprised of three boys and one girl, going by the pseudonyms of Bill, Bob, Cindy, and Jim. According to the teacher's information, in this group, Jim had the highest mathematics achievement. Bob had the second highest mathematics achievement, whereas Bill and Cindy had average mathematics achievement in this group.

The Rhombus Square group had two boys and two girls. The Rhombus Square group members were named Mary, Mike, Nicole, and Tony. In this group Mike had the highest mathematics achievement. Tony had the second highest mathematics achievement. Mary had average mathematics achievement, and Nicole had low mathematics achievement.

The participants ranged in age from 12 to 13 years old. Jim, Mike, and Tony (all high mathematics achievers) came from elementary school A. Bill, Cindy, and Mary (all average mathematics achievers) came from elementary school B.

Bob (a high mathematics achiever) and Nicole (a low mathematics achiever) came from elementary school C. All of the eight participants experienced mathematics classes in elementary school in the same traditional way; teachers used direct instruction from the front of the class in their mathematics lessons. Students listened to the teachers as they presented examples of the procedures to be learned, and then individually worked on provided mathematics worksheets.

Group Work in the CMP Class

The two groups of participants, Cubic Melon and Rhombus Square, are the focus in this section. Group work was carried on during class exploration of new mathematics concepts. Ms. Smith would normally have her students work as a group for about 15 to 20 minutes in the exploration time. The participants worked in their group with a variety of mathematics achievers. The Cubic Melon group contained high and average mathematics achievers, whereas the Rhombus Square group had high, average, and low mathematics achievers. The details on how each group interacted are as follows.

The Cubic Melon Group

The Cubic Melon group was led either by Jim (the highest mathematics achiever) or Bob (the second highest mathematics achiever). Bill and Cindy usually followed the other boys' lead. Jim and Bob each led the group with different styles. Jim always preferred to finish an assignment individually, and afterward explain or discuss the outcomes to his group. In contrast, Bob usually tried to divide the assignment into smaller parts and then gave each group member a different part. Jim and Bob always led the discussion during the investigations. Cindy and Bill were sometimes lost and needed help from their group members to accomplish the investigation. However, Bill seemed to be more easily distracted than Cindy. The following describes the Cubic Melon group work on the pizza investigation[1] described in detail earlier in Ms. Smith Connected Mathematics Class Routine (see Launch section, p. 57). This activity was chosen as an example of the typical working of the two groups.

After the teacher had given direction for the investigation, Cindy went to get some strings from the front of the classroom. At the beginning of the group's interaction, Jim and Bob tried to initiate the problem-solving in different ways[2].

Jim: Everybody here!

Cindy came back to her seat.

Bill: What?

Bob: Ok, here's what we're supposed to do. We need to find the areas and perimeters. Maybe each of us draws a different circle and then finds the area of each circle.

Jim: The area of the small one is ...

Jim's voice was fading as he focused on a calculation and did not response to Bob's idea about dividing the task into small parts.

As the Cubic Melon group worked together, Jim seemed to ignore his group and work ahead on his own. Following suit, the rest of the group members worked individually. Bob liked to hum when he was on task, showing that he enjoyed doing mathematics. Bob was also the one who tried to help and encourage group members to work with confidence.

Cindy: I am not gonna draw the circles.

Bob: Ok, then you're gonna get a bad grade. You have to draw the circle #1, #2, and #3.

Cindy: I can't draw a circle.

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[1] The Pizza problem: "The Sole D' Italia Pizzeria sells small, medium, and large pizzas. A small is 9 inches in diameter, a medium is 12 inches in diameter, and a large is 15 inches in diameter. Prices for cheese pizzas are $6.00 for small, $9.00 for medium, and $12.00 for large.

A. Draw a 9-inch, a 12-inch, and a 15-inch "pizza" on centimeter grid paper. Let 1 centimeter of the grid paper represent 1 inch on the pizza. Estimate the radius, circumference, and area of each pizza.
B. Which measurement-radius, diameter, circumference, or area-seems most closely related to price? Explain your answer."

[2] T means the teacher (Ms. Smith), S means an anonymous student, Ss means the whole class. The students who are named are students from the two target groups.

Bob: No, Cindy you can do it. You can do it. You can do it.

Bob encouraged Cindy to keep trying.

Cindy: That's not good. No, I can't.

Cindy tried to draw a circle by hand.

Bob: See, yours is better than mine...

After working on the task individually, the Cubic Melon group members finally had a chance to share and exchange ideas about their outcomes. Bill and Cindy usually brought up questions. At this point, group members were helpful to each other. From the group's discussion, Jim even found out that he was headed in a wrong direction.

Cindy: Ok, how do you know what the diameter is?

Jim: First, you draw the diameter line.

Bob (thinking out loud): What's half of 15? (The diameter of the circle was 15 inches) It's six. Oh! It's 7.5.

Jim: Wait a minute, what am I doing?

Jim realized he had used the diameter as the radius. He had forgotten to divide the diameter by two in order to find the radius.

Bill: Ha! Jim's off. (laughing) So, Jim we all have days like that.

The group continued to work on the pizza problem. Bob kept humming while working on the task. After working for a while, the Cubic Melon group members came up with ideas for solutions to the problem.

Bill: What's the cheapest?

Bob: The cheapest...

Cindy: I think the smallest one.

None of the group members responded to Cindy's comment.

Bob: I think it's related to the diameter.

Bill: So what? It's either two smalls or two mediums have the best value because two nine inches are 18 inches and that's the same price of the large.

The teacher stopped by and checked on the Cubic Melon group.

T: What are you guys doing?

Jim: I'm gonna try to find the area.

The teacher looked at Jim's paper with no comment.

T: Ok.

Then, the teacher walked on to the next group. As some group members continued measuring, Bob deduced the two small pizzas would be the better buy based on the sum of their diameters, 18 inches versus the large size 15 inches. While Jim, using a calculator with 'π', to find the area of the small pizza.

Bob: Look! This is 18. Six times two is 12. Wow! It's a lot cheaper. You got 18 inches of the pizza for 12 dollars.

Bob focused on only one variable (diameter), which led him to a faulty conclusion.

Bob: Hey! Jim what are you doing?

Jim: Ok, I got the area for the small one, it is 63.585. Do you think so?

Bill was still measuring diameters individually, while Cindy was working on finding the areas along with Jim and Bob.

Cindy: I have 63.

Jim: Ok.

Jim agreed with Cindy's comment.

Bob: I got 225 square inches for the large.

Cindy: No way, that's way too much.

Cindy's comment led Bob to check back on his answer, while Bill showed no response.

Bob: We found the radius. (Um) I think I got something...

While Bob was still checking on the area measurements, Jim led the group discussion about the answers to the problem.

Jim: Well, I think the diameter's pretty closely related to the price because the price is the diameter minus three.

Bill: The price is related to the diameter?

Jim: Yes, ok, guys, do you know what the question for b is?

Cindy: Draw and measure the diameters and circumference.

Bob: Ok, I have the radius and circumference.

Cindy: For the medium one.

Jim: You have the circumference. How did you get the circumference?

Cindy: I drew it and then I measured it. I put the string around.

Cindy: It's 36…Oh! Oh! That's for the medium one, but the smallest one is 26.

Jim confirmed Cindy's answers

Jim: Oh! Yes, ok.

Bob: I got the circumference for the large one; it's 48.

Bob used the string to measure the circumference of the large circle that he drew.

The teacher stopped by and checked on their progress. Bill turned his attention back from looking outside the classroom. The teacher glanced at everybody's work in the Cubic Melon group, but did not comment because she noticed that Jim was the only one working on the task, whereas the others just looked at him. Ms. Smith gave the group advice that everybody in the group should be on task, not just waiting for someone in the group to finish the assignment. Without offering specific advice about how to work together as a group or how to rotate in their group roles, she moved on to other groups.

Acting on Ms. Smith's suggestion, everybody in the Cubic Melon group resumed measuring individually. Bill asked Jim how to use string to measure circumference, but figured it out himself. Jim agreed with his idea and added his advice.

Bill: Jim, …I don't know how to do it (measuring the circumferences).

Bill: Oh! Yes, you put it around and then you measure it.

Jim: Yes.

Bill: So, scissors!

Jim: You don't need scissors just mark it with your thumb.

Jim: Ok, and then b, it asks you what measurement --radius, diameter, circumference, or area-- seem most closely related to the price? Um, I said diameter because the price is three lower than the diameter.

Bill: How do you measure this because you can't measure it, eventually?

Bill's comment revealed his misconception that a circle's circumference cannot be measured. Jim replied, but instead of answering how to do it, he gave the answers. This showed a lack of understanding about working together as a group because he just gave the answers instead of suggesting a method, which would help his group members work toward the solutions together.

Jim: Ok, that one, the circumference is 26. That one is 36. And that one is 46.

Bob and Cindy also listened to Jim's explanation.

Bob: Jim, you always find the trick.

Bill: So, I did it all for nothing!

Jim and Bob encouraged Bill that he could learn from making mistakes.

Jim: You learned.

Bob: Yes, you learned.

Jim and Bob debated which size was the best deal.

Bob: If you buy two small, we'll get the most pizza for the money.

Jim: No.

Bob: Yes.

Jim: Its area is 64. You pay six dollars.

Bob: No, look!

Jim explained his reasoning by comparing the area per dollar.

Jim: Look! The area is 64 inches and you pay six dollars. So, that's about 10 inches a dollar. This one, its area is 178 inches and you're paying 12 dollars. So, you pay less, you pay less...you're getting more for your money.

The group was solving an area problem but they tended to use incorrect units, inches, instead of square inches. Bob was still confused on the diameters.

Bob: No, Jim. You get 18 inches for 12 dollars. It's nine inches across. So, you get two of nine inches, you get 18 inches...

Bill and Cindy listened to Jim and Bob's discussion. Cindy was quiet, but Bill was not able to follow the discussion and expressed his confusion after looking out through the classrooms window to the parking lot outside.

Bill: I'm lost!

Bill often could not follow the group discussion because he was easily distracted. Jim and Bob continued their exchange without any responses to Bill's comment.

Bob: Jim, you think buying the large one is the best deal?

Jim: I think so.

Bob: Jim, Jim. So, two of nine inches is 18 inches across for 12 dollars and then you buy the big one for 12 dollars, you got 15 inches.

Jim: Yes, but see, multiply its area by two.

Bob: It's only um…

Bob used a calculator to find out.

Jim: No, just do it in your head. 64 plus 64 is 128 and this one (the large one) is 176.

Bill and Cindy eventually understood the solution.

Bill: Oh! Yes. I got it. I'm smart. (Laughing)

Bill was confident in his mathematics ability, even though he came up with the solution by listening to his group members' discussion, not by his own thinking. He complimented himself.

Cindy: Uh huh.

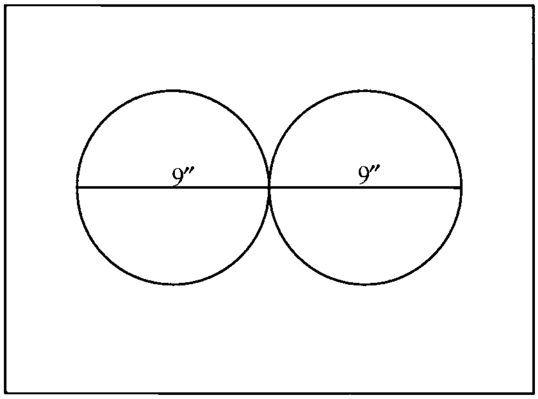
Cindy also found the solution.

Bob: So, we are buying the large one.

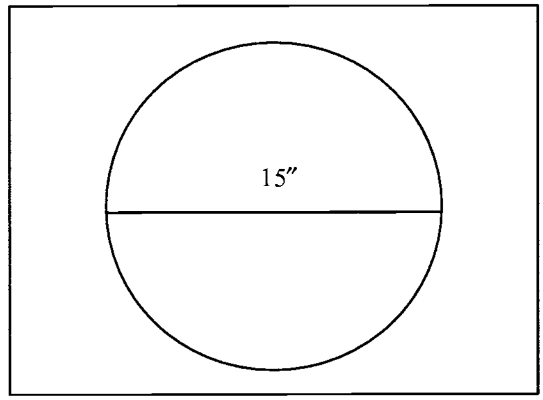
Jim: Yes.

At this point, Ms. Smith called for attention to start the class discussion.

From the group interaction, Jim demonstrated his strong mathematics knowledge. Bob sometimes focused on one aspect of the problem without looking at the whole picture. As shown from the previous example, he only focused on the diameters of the pizzas to determine which one was the best deal. Bob thought the sum of the diameters indicated a larger area. Thus, the diameter of the two small circles (18 inches) exceeded the biggest circle itself (15 inches), providing more pizza for the same price, in his reasoning.



The two small circles



The biggest circle

Bob only looked at the sum of the diameters of the two small circles, not the sum of their areas. However, Jim suggested that he find the areas before comparing, not just the diameter, which was the correct factor for this solution. The area of the small (9") circle was 63.6 square inches. The area of the two small circles was 127.2 square inches. In contrast, the area of the biggest (15") circle was 176.7 square inches. Therefore, at the same price, the biggest circle provided more pizza than the two small circles.

Bill, who expressed confusion at the beginning, and Cindy, who was quiet until the end of the dialog, both developed understanding by listening to the exchange. This example showed that group discussion helped students gain a better understanding of the mathematical concepts.

In summary. The Cubic Melon group was most likely to work individually at the beginning of group investigations. Then group members would share their findings. Jim was the one who usually worked ahead. However, when his group members needed help, he did not hesitate to help, especially after he had finished his work. Bob usually tried to cooperate in group work and encouraged his group members to keep trying. Cindy and Bill usually needed help, but sometimes they brought up interesting comments to the group, like when Cindy commented that Bob's area measurement was too much. This spurred him to recheck his answer. However, Bill usually got lost because he was easily distracted. The group discussion during the investigation benefited all Cubic Melon group members, allowing them to articulate their thinking and to catch and correct errors.

The Rhombus Square Group

In the Rhombus Square group, Mike and Tony were leaders. Both were high mathematics achievers who usually worked together in the group. Mary and Nicole who were average and low achievers, normally received help from Mike and Tony. This group worked together in a more interactive way than the Cubic Melon group. During group work, Mike and Tony always explained to their team what they were doing. They usually thought out loud, so their team could follow, and shared their ideas and suggestions. The Rhombus Square group worked on the pizza investigation as follows.

At the beginning of the investigation, group members worked individually. Mary was not sure how to start; she asked for some advice from Mike. Mike explained what he did to measure the diameter and showed her his work.

Mary: "How'd you measure?"

Mike: "Counting."

Mike pointed to small grids that were along the diameter line. He drew the circle on grid paper and pointed at one side of the circle's diameter. The group continued to work individually for awhile. Mary, who was usually less talkative, started the group discussion. Then, Mike and Tony dominated the discussion.

Mary: Somebody got anything yet?

Mike started to use the string to measure the circumference.

Mary: Wait, I'm gonna miss that one. That looks like it would work.

Tony: That would work. I was gonna do that.

Mike: The circumference is three hundred something.

Tony: The circumference is around; you use the string to figure it out.

Mike: Well, the small one is 7.5.

Tony confirmed Mike's answer as he often did in his group. Mike and Tony made progress toward goals of solving the problems together. They were able to find the diameters and circumferences. And later on, they moved on to find the areas.

Tony: Yes, 7.5.

Mike: 7.5 what? You used inches or centimeters?

Tony: Inches.

Mike: Ok, inches. Oh! Wait! I think we're supposed to measure it in centimeters. So, that's wrong (um) wait…

Mike: Inches…I got 7.5 and in centimeters…I got 19.

Tony: Ok, so that's it.

The atmosphere in this group was more cooperative than the Cubic Melon group. Mike and Tony kept sharing their ideas while exploring the problems, not waiting until the group work was completely done as happened in the Cubic Melon group.

Tony and Mike both were nice students in this group. They would offer help or share their work with their group members when they needed help. Mary was usually the one who asked for help or explanations (mostly from Mike), but Nicole appeared too shy to ask for help. So, Tony often offered to help her.

Tony: Do you know where we are now? Do you know how to find the diameter?

Nicole gave no response, but smiled.

Tony: The line across is the diameter and half of it is the radius.

Nicole: Ok.

The Rhombus Square group continued on task and discussed finding the circumference. Shortly, Tony changed their focus to the area.

Tony: Now, we need to estimate the area.

Mary: Hard!

Tony: How about we just count the whole squares in there and add a little to it.

Mike: Ok, I think we just count the whole squares and then later, I'll do something else. I'll try to figure out something.

The teacher stopped by and confirmed their idea. "Just estimate. You can count the squares." Then, all of them except Nicole shared their answers.

Mary: It's hard to find the areas.

Tony: Just count the squares. Ok, for the first one, I got 52. For the second one, I got 90.

Mary: 90.

Tony: Yes.

Mike: For the medium one I got 110...

Ms. Smith occasionally walked around her class and checked the progress of her students' group work. In helping students in their investigation, she sometimes would ask her students to explain why and how instead of telling them whether they were right or wrong. Also, when she found out that the group had accomplished the minimum requirement of the investigation, she would ask them to work on additional problems. The following example shows this group's responses to her guidance.

T: Well, where are you guys now?

Tony: We started to answer the problem with finding the areas.

T: Ok, and explain your answer. What do you think? and how do you find the answer?

Mike: Maybe we can find the fraction between what percent the price is of the area and compare it.

Tony: Oh! Yes, yes.

T: So, how could you...

Tony: Oh! Oh! The area is about twice as big.

The teacher did not give an answer but asked them to verify it.

T: I don't know, check it out.

Tony: Because the area of the small one is 62 and the large one, the area is 148.

T: So, if you got two smalls?

Mike: Two smalls would be like a hundred and...

Tony: A hundred and twenty-four.

T: Ok.

Tony: Oh! Two smalls is a better deal because you got bigger area.

The teacher accepted the student's answer without judging it right or wrong.

T: So, do two smalls have a bigger area than the large?

Tony appeared confused but Mike offered his idea.

Tony: Oh! No. No. Never mind. Never mind.

Mike: The large is the better deal.

Tony: Yes, yes.

The teacher accepted their answer a second time.

T: So, a large is the better deal than two smalls?

Mike and Tony: Yes.

At this point, the teacher did not ask why or check on Mike's and Tony's understanding. She just directed them to an additional problem.

T: So, we know the large is a better deal than two smalls. How about the medium one? How does that work here? Think about it, ok…

At that, the group went back to working individually for a time. But punctuating the silence, the Rhombus Square group members still shared ideas as they progressed.

Mike: Ok, I did the price by area.

Tony: What? What did you divide the price by?

Mike: Area.

Tony: Ok.

Tony: How do you find how much per square inch? Is that like the price by…

Mike: It's area by the price. Actually, I think it doesn't matter.

Mike showed his conceptual understanding about ratio. He noticed that the ratio between area and price or the ratio between price and area could be used for finding the better deal.

Tony: Oh! Ok. So, because of that I think the largest is the best deal.

Nicole: Why?

Tony: Because two smalls equal the price of the large, but they're smaller area than the large one. So, we get more square inches per dollar.

Mary: So, the large one is the better deal.

Mike: Yes, yes.

Nicole gave no response to Tony's explanation.

In summary. The Rhombus Square group was an interactive group. The group had two leaders (Mike and Tony) with high mathematics achievement. During group work, the two leaders usually discussed the problem allowing the other group members to follow the investigation. Also, the two leaders were willing to help and share their knowledge with their team. Mary, who was an average mathematics student, appeared to gain understanding from the Mike's and Tony's exchange. At the beginning, she commented, "hard", but by the end she appeared to be following the discussion and able to give an answer. In contrast, Nicole, who was a low mathematics student, remained silent during the discussion. Nicole still did not understand about the solution, even after group members explained it to her. She participated less and she understood less. For the discussion, three out of four of the group members showed their success in solving the problem.

Summary

In the Cubic Melon group, Jim, the highest mathematics achiever in the CMP class, normally led this group. Jim led his group to work individually at the beginning of the group work, and then let them share their work at the end of the investigation. Bob, a high mathematics achiever, sometimes tried to lead the group in a different way. He tried to break up group assignments into smaller parts, then gave each group member different parts to work with. Bill and Cindy, the average mathematics achievers, usually followed the group's leader. Bill was easily distracted; therefore, sometimes he got confused. However, with help from his group members, he was able to follow the group work. Bill was confident in his mathematics ability. He commented to himself that he was "smart" when he was able to accomplish the task (with help from his group). Cindy did not participate a lot in the group discussion, but she usually listened to the exchange and double checked on the group work.

In the Rhombus Square group, Mike and Tony were co-leaders in the group work. Both high mathematics achievers usually checked their answers and shared their ideas through the group work activities. They usually thought out loud while they were working on tasks, which helped the rest of the group members to follow them. This group had a more dynamic atmosphere than the Cubic Melon group, which did not share their work until the end of the group work. Mary, an average mathematics achiever, usually listened to the exchange and tried to follow the discussion. She asked for help and questioned when she did not understand. Nicole, a low mathematics achiever, was quiet most of time during the group work, not asking for help even if she needed it. Her comment on why the solution worked, at the end of the group discussion, showed that she was still confused. After a group member explained the solution to her, she still kept quiet and showed no sign whether she understood it.

Students' Mathematical Dispositions in the CMP Class

This section discusses mathematical dispositions of the eight participants in the two cooperative groups. The findings from the observations, the Attitude and Belief Questionnaires and students' interviews are discussed. Each participant in the two groups (Cubic Melon and Rhombus Square) receives in this section.

From the previous section, the dynamic of group work between the Cubic Melon and the Rhombus Square was shown to be quite different. The Cubic Melon group, which was usually led by Jim, was less interactive than the Rhombus Square group, which was usually led by Mike and Tony. Jim, one of the highest mathematics achievers in the CMP class, tended to work individually at the beginning of the group work and then shared his work at the end. In contrast, in the Rhombus Square group, Mike and Tony worked together and thought out loud so the other two group members could follow them. The details of the eight participants' mathematical dispositions were gathered from the observations, questionnaires, and interviews.

Even though most of the volunteer students rarely did extra mathematics related activities, two of the eight participants did engage in mathematics activities outside their mathematics classrooms. Cindy from the Cubic Melon group was, in student a council in her 5th grade year. She mentioned that she was a treasurer and she had to count money and analyze the council budget. In 5th grade, Mike from Rhombus Square group had participated in the Mathematics Olympic program, where he worked on complex mathematics problems individually and with groups. In this program, he had to complete several mathematics tests and he won first place. In addition, since elementary school level, Mike often worked on sets of mathematics problems during his free time. His father, a university professor, created mathematics problems for him to practice. However, he noted that in middle school, he did not have as much free time as in elementary school to work on extra mathematics problems outside the classroom.

The Cubic Melon Group's Mathematical Dispositions

The details of the Cubic Melon group members' mathematical dispositions are described according to their roles in the group. First, Jim was usually the leader. Second, Bob was sometimes the leader. Third, Cindy was usually the verifier. She usually rechecked the group work. Fourth, Bill was usually the follower. He usually needed help from his group members.

Jim. Jim was usually the leader. He was the most advanced student in the Cubic Melon group and also in this CMP class. In his opinion, Jim thought mathematics consisted of numbers. He said, "I think it is dealing with numbers to find the answers for problems. It is about using different numbers and you can add them, subtract them, whatever, to find the answer to your questions." His opinion about mathematics showed that he saw mathematics as it applied in the classroom, but not necessarily as it applied in everyday life. However, later on, he did mention that mathematics was involved with adding up the prices in shopping, finding fractions in cooking, and measuring money.

Jim did not realize how the usefulness of mathematics was important for daily life at his age, but he knew mathematics would be more important for adults. He said, "...not for me, but more helpful for, like, adults. Like when you go shopping, you wanna add the prices, and when you are cooking (using) fraction parts..." Since Jim still did not know what future career he aspired to do, he commented that the question about the usefulness of mathematics in his future career was hard to determine. However, he knew that he wanted to do anything that made a lot of money. He was able to give examples of the usefulness of mathematics in future life. He said, "…if you have a career, like a scientist or accountant or something, you really need to know math, especially when you are in a career to measure money." Jim viewed only the usefulness of mathematics as it could help people succeed in their careers, not necessarily how mathematics was useful in his immediate everyday life.

In addition, Jim believed that getting the right answer was more important than understanding why the answer worked. Jim said, "…if you get the right answer then it is good enough for me…" This suggested that Jim did not have intrinsic motivation for learning mathematics. He did not show curiosity beyond getting the answers, but was satisfied more by succeeding than through understanding. However, Jim did realize the importance of understanding as a connection to similar problems or situations. He said, " …if you come across problems that are a little different, if you understand how to do the types, it might help you to figure them out." He knew that in approaching related problems previous understanding would benefit him.

Jim also agreed that trying hard in doing mathematics helped to increase one's mathematics achievement. He said, "I know that with other things in life, trying hard makes you better. And I try hard in math, and I am good at it. So, I guess it works." His life experience helped him to see that effort could improve one's mathematics ability.

Jim considered himself a good mathematics student, since he was always one of the top mathematics students in his class. He added, "I am always the one who explained to the group what to do and I usually have a good grade in math."

He felt that this CMP class was tedious, since he already knew what to do. He commented that the class would be more fun for him if there were more challenging problems. He learned some of the content from his elementary school in a previous year. Other than being a bit simple for him, he said this CMP class was "pretty good." He usually studied the materials before each lesson. He liked hands-on activities because they were more fun than just writing answers. Jim said mathematics was his favorite subject and he enjoyed mathematics. However, he thought that mathematics was easy.

Jim usually liked the beginning of a new unit in his CMP class. He said, "I like the beginning of the unit usually because that is where I learn the most, when I am first introduced to new ideas." He liked to learn new concepts. He did not like this CMP class when the class repeated the same topic over and over again. He added that the teacher was helpful when she explained or guided her students through the lesson until they understood a concept. Jim did not think that he needed extra help from the teacher. "I don't need help. I usually get it," he said.

Even though Jim preferred to work individually at the beginning of the assignment, he added that he liked group work. He noticed that he was the group leader and he helped his group members often. However, sometimes when he did something wrong, his group members had been helpful to him. He said, "…a lot of times I have to teach them (his group members) stuff and explain to them…but, sometimes, when I mess up…working with them helps me to find out what was wrong…" This showed that a high performance student also gained benefit from group work, which helped him find out what he did wrong in his methods.

Jim also enjoyed working on the mathematics project (see Appendix M) at the end of the unit. He liked to design the project either by himself or with a group. He said "I guess it would be fun with friends, but on my own is fine too, because I can see how much I can do by myself…" His comments showed that he gained satisfaction from accomplishing some work on his own even though working with the group would add fun to the project.

Jim noticed that in the CMP class, it seemed the boys did better than the girls, but he believed that boys and girls do equally well in mathematics. He said, "…in our class, it seems like boys do a little bit better, but I know…a lot of girls are really good at math, too." His comment on gender differences in the CMP classroom brings up a question of why boys outperformed girls in this class. Whatever reasons, extra encouragement for the girls to share their ideas was needed.

Bob. Bob, sometimes the leader in the Cubic Melon group, was an advanced student, sharing his experiences in the CMP class. Bob believed that mathematics was "solving problems, answering problems with addition, subtraction, multiplication, and stuff like that and learning new ideas, figuring out problems using numbers." When he was asked what he thought helped him to do well in mathematics, Bob explained, "I think, just trying hard and liking it, that really helps. Like, if you don't like it, it's hard for you to learn, and if I make a mistake, then I learn from it. Also, I always try as much as I can. If I don't try it at all, then, I am not gonna get the answers. If you're really bad at math, but you at least try, then you'll get better." He also expressed that when he encountered a time-consuming mathematics problem, he usually spent time on it and tried to figure it out as much as possible.

He was able to provide more examples of how mathematics was useful for daily and future life. He said:

> Math is pretty much everywhere. Like, if you're gonna buy a set of pens, you'll need 24. If it comes in a pack of six, you'll need four packs. If you're paying cash for something that $16.99 and then you have to know how many dollar bills and change you need…in the future, I wanna be a polar-bear researcher. So, if I need to know how far it is from one check point to the other, I need to measure by using yards and I need to divide if I need to get it in feet…

These comments clearly showed that he knew how mathematics was related to his life outside the mathematics classroom.

Additionally, Bob believed that understanding was more important than getting the right answers. He explained, "…if you go 2+2=4, but you don't know why it works…[it] isn't gonna help you in your life…it's gonna help you [if you know] 2 of these plus 2 of these equals 4 of these..." Bob revealed his concern about how understanding concepts could apply in daily life. Bob also agreed that effort could help one increase his/her mathematics ability. In addition, he believed that he could solve time-consuming mathematics problems.

Bob thought that he was good at mathematics in this CMP class. He said, "I think I am good at math, not the best but good enough, because I have a good grade. Usually, I am above an A- in math and I mean that's not hard for me." Here Bob expressed confidence in his mathematics ability.

He added, "The CMP class was really fun. Just sometimes, she (the teacher) tells us the same thing over and over again. I think we're learning the same thing over and over again…it's kind of annoying. I still have fun. I learn a lot." Bob mentioned that he liked mathematics because he liked to solve problems, get answers, and find out new ideas. He said, "I like math a lot, I like mathematics because it is just fun solving puzzles and finding answers." Bob liked hands-on activities because it was fun to play with manipulatives. However, he disliked it when he was not allowed to take a shortcut to figure out an answer. He mentioned that the teacher helped the class to learn by guiding them to find the answer on their own. Bob did not want any extra help from the teacher. He thought he understood it "pretty well." Seeing mathematics as a fun puzzle to solve showed Bob's intrinsic motivation in learning mathematics.

In addition, Bob liked working with groups. He said, "Working with groups is more fun than working by yourself. It's more interactive. I mean I'll get it wrong maybe when we work together, then we can check all of the problems together that might help me to realize about getting the wrong answer." He added, "Sometimes, it could be annoying if a few people in your group don't do their work, like they just sit there and then you got a bad grade because of them." When Bob was asked what would he do if this happened again, he said he would just ignore them. He would just do the work for them or he would not give them any work to do, since he did not want to get a bad grade because of them. However, Bob said, "It does not happen a lot."

Sometimes, Bob would lead, as well as Jim, in the Cubic Melon group. Bob said, "I don't wanna be bossy, but I just tell everybody that they need to do that, they need to do this, to get the job done." Bob usually tried to break the group work into smaller parts and then assigned a different part to each group member. He explained that it was easier to delegate tasks than to have people arguing over them. He said, "I think it's faster that way." Speed and efficiency motivated Bob to attempt to lead the group dividing the work. He remembered task division from 2nd or 3rd grade. He said, "We usually got different parts of work when we did the groups."

Bob also realized that group members were very helpful. He said:

> If you have some not so smart people and some medium people and some smart people, then, you can use them, the smart people and the medium people to check over where everybody is. So, you have a better chance of getting it all right because you have a group opinion rather than just one person's opinion. So, if it's to yourself, 'I think it's right', but it might not be right. And like that's what you could think and you could be thinking wrong, like just a common mistake and you would think it's right, but then someone else would tell you and then you realize that was wrong.

Bob realized that a group with various mathematics abilities benefited every member in helping them verify their methods. Bob added that he took everybody's advice in his group, not just the smart ones. They then sorted out the reasonable suggestions and the unreasonable ones and decided together.

Bob did the park project at the end of the Covering and Surrounding unit on his own. He needed no help from friends or parents. He said the project was "pretty easy." He liked doing the project because he liked to create things. The difficulty in doing the project for him was drawing accurately. He said, "I am not good at drawing." He added that the only thing he did not like was to design a big park on small paper. He added that sometimes he got aggravated during work on the project. He got bored easily and when he got bored with doing the project, he took a break. He said, "I took a five minute break or went to play outside, or watched television and then came back to work."

In addition, Bob noticed that in this CMP class most boys were better than girls in mathematics, but girls could be better than boys. "Sometimes, I think they (girls) don't really try that hard. I mean I think girls are better in spelling in our class, but some boys also are really good in spelling. It is kind of mixed. I think girls could be good. It's just sometimes they got off the subject and they can try harder. There are a lot of girls that are really good at math, but the average of them just don't try as hard as they could." Bob provided an assumption about this CMP class; girls might not try hard enough to do mathematics, so the boys outperformed them.

Bill. Bill, the follower, was an average mathematics student in the Cubic Melon group. In his opinion, mathematics was dealing with numbers. He said, "...you learn how to do cool stuff in life with math and then you'll see how much it involves." Bill thought that mathematics was pretty easy when he understood it. He said, "...you have to understand it to do it. If you don't understand it, you can't do the math." Bill believed that trying hard helped to increase mathematical ability. He said, "You have to push yourself harder to get better."

Bill believed that mathematics was useful. He said, "Everywhere is math because you have to measure the football field, the soccer field, basketball course, a house, a garden, or street. You even have to measure the street size for the car to fit. Everywhere you are, there is math. It's always math some ways." He explained his opinion more about the usefulness of mathematics in future life as follows, "I want to be a soccer player or a basketball player. Well, we know who won by the scores..."

Bill also believed that there was no difference between boys and girls doing mathematics. He said, "There's not any difference. They can be good in math as long as they try harder. They can both be as smart because they are both people." Moreover, Bill added he normally would not quit easily when he had to solve a mathematics problem that took time to figure out. He said, "I keep trying and trying until it's done and I get to rest."

In the 6th grade CMP class, Bill said, "It is my worst subject now, mathematics!" He explained, "I don't know, I don't really know why. I just did bad on a couple math tests and that's really dropped my grade down...I think I'm just not doing well on my grade, but I think I am smart in math." One thing he was sure about regarding his grade reduction was that he kept forgetting to turn in his mathematics assignment or homework.

Bill commented that he liked the CMP mathematics a lot, even though it was harder than elementary school mathematics. He said, "It's pretty fun. It's more fun than 5th grade, but there're still some days that are kind of not really fun." He explained, "It was not fun when it's not active or kind of sitting there and listening to the teacher. Well, it is fun with the teacher, but sometimes it's boring listening to the teacher. I like getting to the point of my working." He explained, "Well, you know what the basketball is? Well, when you play basketball, it wouldn't be fun if you have to listen to the coach all the time. You have to play. (It's the) same thing with math." Bill added that he received a lot of help from the teacher. He said, "She (the teacher) guided me." However, he did not need much help from the teacher.

Bill liked working with groups. He thought working with groups was fun. He said, "I like to work with groups on homework or investigations because you have someone to ask, to help, and work with and not going to the teacher all the time." He explained more about how his group worked. Bill received help from everybody in his group and had a lot of help from Jim, the most advanced student in his group. Bill said, "Jim usually figured out the problems that I didn't know and then he teaches me the strategy. If I'm stuck, he just helps." Bill usually asked for help when he was stuck.

Bill liked to work on the park project. He said, "It's really fun. I like to do a big fun project like that to create your own thing and stuff. It's related to a real life situation. If you want to go do a dream park, it's a perfect blueprint." Bill had a bit of help from his mother who gave him an idea for a tennis court. Bill preferred to work on the park project alone instead of working with a group. He explained, "I like to do things, like the park project, by myself because I like to use my own ideas on it, and not share other ideas with the other people because they might want to have something on it, that I didn't. So, it just works out better if I just do it myself." Finally, he believed that there was no gender difference in learning mathematics.

Cindy. The verifier, Cindy was an average mathematics student in this CMP class. Interestingly, Cindy thought, "Mathematics is life." She said:

> Mathematics is life because in life everybody uses math. I use math to calculate (um) how many clothes I'll need, how much time I'll need, how much sunscreen I'll need, how much water, how much food, how much money (um) how much money I have, and I need to use, and how much change that I can give to someone else to use that they're short on without running short myself. I am doing math right now in my head, figuring out all the different ways to say all the different things. All the different times that I do math in the day. And I do

> math on my fingers. I do math in my head. I do math on calculators. I do math on paper, on a chalkboard, on a whiteboard, you name it.

Cindy's expression about mathematics was so unique. Her definition about mathematics, which was not just numbers, showed that she definitely saw mathematics related to real life outside the classroom.

About the usefulness of mathematics, Cindy agreed that it was helpful for both daily and future life. She said:

> I use math in my daily life when I am going to buy something, like for a birthday party for a friend of mine and today I am going to use math because I am going to Dairy Queen and when I go to the Dairy Queen and I have 10 dollars, I need to figure it out, since one of my friends doesn't have any money for anything. So, I said I'll loan him some money, but he has to pay me back. So, I have to take out the money for that and then I need to take out the money for my own food and then I need to add it together and see what the total is to figure out how much of the 10 dollars I'll need, and how much could I get back to my dad in change.

Cindy added, "I think it's also really helpful in the future. If I'll be a scientist, I'm gonna need mathematics everyday to figure out, say a formula for an experiment and how many times I would need to use that formula ...and for shopping with my salary, I would need to use it wisely. I would need to have enough to pay the rent and enough to buy clothes per month." Cindy's explanation about the usefulness of mathematics showed that she did realize how to use mathematics in her daily and future life. She had a very broad perspective about the usefulness of mathematics.

Cindy also agreed that understanding is important in learning mathematics. She said, "I think understanding is important because if understanding was not important then we would not need to know anything (laughing)." Cindy also believed that trying hard would definitely help everybody to be good at mathematics. She said, "The more I study, the more I'll learn because without studying it's just like not doing anything. Studying for me is like doing everything in life that I can, but not studying is like sitting at home and saying I can't. And sitting at home and saying I can't is really, really super boring! I really don't like to be bored." Her expression showed that she was an active person and was willing to learn and was trying hard to be successful. She also believed that she would be able to solve time-consuming mathematics problems.

In this class Cindy did not see herself as good at mathematics as she was in the elementary school. She stated her reason; "I am not really good in math in 6th grade because it just does not click very fast. Sometimes, it'll take me an hour to do a simple problem when it'll take others 10 minutes." Cindy realized that the CMP class this year was different from the mathematics classes she was familiar with in elementary school. She said, "Last year, I got an A in math, but this year, I got a C because this math isn't what I am used to. I have only been doing the Connected Math this year. So, I did not really know what it was and I have never worked with it until now, and now, I understand it (smiling and laughing) I really want to go back and take all the tests I ever had (this year) and hopefully I would get an A on all of them." Cindy added, "I think it is pretty hard (the CMP mathematics), but once you got to understand it and how everything works and why it works, then it's really easy."

Cindy enjoyed the CMP this year. She said, "It's pretty hard, but it's not very bad. Since the stuff we are doing now with the Connected Math, it is a lot more fun and easier than what I was doing last year. Last year, we had tests and the tests for just mathematical equations and we had to figure it out in our head, no calculators, no talking, no breaks, until math was over." She also provided other reasons for why she liked mathematics. She said, "I do enjoy math because it's just really stimulating. I enjoy math as much as I do reading. When I am doing math problems, if it draws a picture in my mind, I can get lost in it when I try to figure it out. Same with reading, if the piece that I am reading is good enough to paint a picture of the scene, then I forget where I am and I am in the book and I am watching as everything is happening..." Cindy showed that she had imagination and creativity. Learning mathematics in a new way was hard for her at the beginning, but later on she liked it because it stimulated her thoughts.

In addition, Cindy liked hands-on activities and a chance to do investigations in the CMP class. She said, "I am a hands-on learner and I am also a pretty visual learner. If you explain something by a picture, I would get it like that. If you explain something just by saying that, it'll take me a while to draw a picture in my head." However, she disliked the CMP class, when the teacher gave the instructions or explanations because she already knew what she was supposed to do. Cindy usually read the lesson ahead of time. She felt that she just did nothing but listen.

Cindy liked the way that the teacher provided guidance for her students when they needed it. The teacher gave them clues instead of telling them the answers. Cindy said, "She guides us. She would leave it, she makes it vague, but it's still hard. And once she makes it vague, she gives us enough hints that I can find it out by myself." Generally, Cindy noted that she did not really need any other help from her teacher.

Additionally, Cindy enjoyed working with her group. She realized how useful the group work was to her. She said, "I like working with a group because if I don't have a group, I'd be totally lost." She explained more about her role in the Cubic Melon group as follows, "I was pretty much like a verifier. I would go back and I would double check and I'd verify everyone and everything, just to make sure that everyone is on the same track…and everyone understands it."

In addition, Cindy liked doing the mathematics project on a park project because she liked to design things. However, the only difficulty she found in doing the project was making decisions about how to place everything in the park. Cindy did not know how big the trees really were. Her father was helpful to her in giving some advice about how big the things in the park would be. She said, "…I asked him, what kinds of flowers would be in a garden and how much space would they take up. And about how big the path would be…my dad is very helpful." Cindy preferred to work with a group on the project. She said, "I wish I could work with my friends because they would give me feedback and if something was too big or too small or visually unrealistic. So, I really wish I could have worked with a group…I am not good at working alone." Cindy's description of the group members' feedback was that it was valuable for her.

Cindy also commented about having limited time to work on the park project. Ms. Smith assigned one week for her students to work on the project. Cindy said, "…I was pressured for time and I couldn't do that much in the time frame that she (the teacher) said. If we had more time or could have done it two weeks ago…it would be really good. And I'll put the most effort in it as I could..." This showed the mathematics project required time to complete it and indicated that more time might be needed for this particular project.

In summary. In the Cubic Melon group, Jim defined mathematics as "dealing with numbers." He explained that mathematics was about using numbers in counting, measurement, addition, and subtraction in order to find the answers to mathematics problems. Bob thought mathematics was "solving problems" by using addition, subtraction, and multiplication. He also noted that mathematics was about learning new ideas. Bill had the same belief about mathematics as Jim. Bill believed that mathematics was "dealing with numbers." However, Bill was able to see mathematics applications outside the mathematics classroom more than Jim. Bill explained that mathematics related to people's lives in some ways. In Cindy's opinion, mathematics was defined as "life." She expressed mathematics in a broader view. She realized that mathematics was everywhere in our daily lives was also able to provide examples how mathematics was used in her immediate and future life, such as loaning people money, shopping, and managing her income.

The group members believed that mathematics was useful. All of them were able to provide examples of how to use mathematics in their daily and future lives such as in shopping and cooking. Three out of four group members (Bob, Bill, and Cindy) did mention that mathematics was everywhere in their lives. Only Jim was more concerned about using mathematics in the classroom than his daily life. However, he agreed that eventually it could benefit his future career.

All but Jim thought that understanding concepts in mathematics was more important than getting a right answer. Bob, Bill, and Cindy agreed that understanding concepts in doing mathematics would help them to apply the concepts to their real life situations. However, Jim realized that understanding helped him make connections in learning mathematics, but seemed like he was convinced that being successful (getting the right answers) was more important than knowing why and how.

All of the group members were confident in their ability to solve time-consuming mathematics problems. Jim clarified that from his life experience he saw that people were successful when they tried hard. He believed that one of the reasons he was good at mathematics was because he tried hard.

Jim and Bob had strong confidence in their mathematics abilities, whereas Bill and Cindy had less confidence in their mathematics abilities. From their explanations, the group members had realistic levels of confidence based on their mathematics achievement in the CMP class.

Jim thought that this CMP class was a bit easy for him, since he had learned some topics from the previous year. Jim's explanation showed that he had less curiosity in learning mathematics in this class, whereas Bob liked mathematics because he liked to explore and investigate. Bill and Cindy enjoyed mathematics because it was interactive. Cindy had also done related mathematics outside the classroom. In this group, Bob showed his curiosity in learning mathematics and Cindy did extra mathematics activities outside the classroom. These actions revealed their intrinsic motivation.

In addition, this group enjoyed both working with a group and individually. All of the Cubic Melon group members liked working with a group because they could learn from each other. They also liked working on the park project because they had a chance to design things using their own ideas.

Finally, all of the Cubic Melon group members believed that there were no gender differences in mathematics in general, although in this CMP class boys outperformed the girls. However, all of the group members believed that they would see more girls being successful in mathematics outside their mathematics classroom.

The Rhombus Square Group's Mathematical Dispositions

The Rhombus Square group members' mathematical dispositions are presented according to the roles in this group. First, Mike was usually the group leader. Second, Tony was also the group leader along with Mike. Next, Mary was a follower. She usually was the one who asked for help. Next, Nicole was also a follower. However, she was the one who needed help most, but rarely asked for it.

Mike. The leader, Mike was the highest mathematics achiever in the Rhombus Square group. He was also the second highest mathematics achiever in this CMP class. In his opinion, mathematics was about understanding numbers. He said, "It's knowing how to use numbers in your life. I think it's just understanding numbers and understanding how to add, subtract, ...in real life, building something or tiling surfaces or something...which you needed to know relating sides to angles... so, you just need to understand how numbers work." So Mike believed that mathematics was related to arithmetic and geometry. Mike believed that it was, in fact, important to understand mathematics. He said, "If you can do one problem and just barely get through it and get it right, then, maybe a week later you come up with the same problem, you don't know how to do it. So, you really need to understand why it works and how it works." Mike's explanation disclosed that he saw mathematics as not only understanding numbers, but also how to apply and use them in real life. He realized how important it was to understand concepts in mathematics in order to make connections to similar mathematics problems.

Mike also believed that mathematics was useful. At first, Mike thought that mathematics was more important in future life than in daily life. He said, "... in daily life, it may not be too much in daily life, but I think eventually it'll get really helpful to know math, like when I get some jobs, like maybe an accountant or selling. Even if I work at a cash register, it's important to know math because if there're some mistakes, you should be able to get back and check it quickly and everything. So, like tiling, if I build a building, you need to know how long you need to cut materials and how to lay everything, like the angle." Mike was uncertain what he would be when he grew up. He said, "I would like to be an athlete or maybe even get a job that involves math." Later on, he realized that mathematics also could be helpful for his daily life. He said, "...Oh! Mathematics is helpful when we're buying things and stuff ... it's helpful to know how much change you're going to get so they can't cheat you."

In reference to a belief about how effort can help people increase in their mathematics ability, he explained that people who get good grades now were trying hard in the past. He added that people who "start trying hard now" would get better grades later on. He said, "You need to ask a lot of questions. I ask a lot of questions. When I don't understand, I ask a lot of questions until I understand. So, you need to work hard, yes, you have to study, but you have to have studied hard in the past, too. If you just start trying hard now, it'll take a while." Interestingly, Mike's comment on trying hard in doing mathematics was a bit different from other participants. He was the only one who mentioned that in order to gain success from trying hard, it would take time. As he explained, being successful now meant people had to try hard in the past and being successful in the future; they have to keep trying hard from now on. Mike also believed that he would be able to solve time-consuming mathematics problems.

In general, Mike saw himself as good at mathematics. Mike noticed patterns in the CMP class. He said, "I think math lessons that we do out of the book, it seems like we are repeating everything (Launch, Explore, and Summarize). We are doing it over and over again, but once we do the ACE problems (Applications, Connections, and Extensions) as the homework…it's challenging…I like that most…" He also mentioned that in the CMP class, he liked hands-on activities. Mike said, "It is fun actually, like doing something not just all in your head. So, it's fun to actually be doing that."

Mathematics was Mike's favorite subject. Mike expressed that he liked mathematics because it was fun to learn new concepts, even though sometimes it was boring when people asked a lot of the same questions over and over. Mike also mentioned that he liked the beginning of the classroom activities, since the introduction part was challenging for him. He did not like the follow-up because it repeated what he already knew. He added, "I think that they should have to ask everyone in their groups, if you don't understand it and then after that you can come to the teacher, because I think people ask for help from the teacher too much. I feel like I want to move on. I want to do some other new math." This statement showed that he had intrinsic motivation because he enjoyed mathematics and was curious to learn more new concepts in mathematics.

When Mike was stuck in mathematics, he asked for help from his group first, and then he would go to the teacher if his group could not help him. He said, "When I don't understand, I would usually ask my group, and then I could go up and ask her (the teacher)." When Mike encountered a hard mathematics problem, he would just "hang in there, trying to solve it even though it would take time for a while to complete it."

Mike liked working with groups. He said, "I really like working with a group because you really get a chance to discuss and talk about it. You don't just hear it." Mike saw himself as the group's leader and explainer with his partner Tony. He said, "I and the other guy (Tony), we like to work together and explain things to the others." Mike added that, "I think our group work is pretty good. It's good to have like two people that work well together in one group and then we can help out the other two people. So, it's fun because we sort of like that. We work together… " Mike found that his group members were helpful, especially his partner. He said, "I like to have one other person that you can work with a lot and then the other two people that's fun to teach. So, it's really helpful to have one person to share and teach someone else." Mike also thought the people that he helped in his group were very helpful to him, too. He explained, "They help me to understand better because when I explain something to them and then I kind of pick up something that I didn't know before. So, sometimes it helps."

Regarding the park project, Mike said:

> I like the project because I like art, too. I like to draw and design stuff and I like to think about how to figure out, that I like the most, and I probably dislike all the writing you have to do about explaining the areas and perimeters. That's because when I actually designed the project, I knew all the stuff already. It seems like repeating it. A little difficulty in doing the project was to find how many square yards of grass field, because you have all weird shapes and you have to add a bunch of them together.

Mike's expression revealed that the mathematics project was fun for him because he had a chance to draw and create things. However, since he already knew all the dimensions and sizes of his designs, he felt that it was repetitive to write all the details and explanations again. This comment indicates that in doing mathematics projects students may benefit from choosing their own way to present the information.

The difficulty he faced in doing the project was finding the areas of weird shapes. This challenge might help him to realize the usefulness of mathematics in the real world, where shapes are not normally perfect.

Mike added that he finished his mathematics project on his own without any help from his parents. He commented, "If it's a big project, it's fun to work with groups and if it's smaller, it's fun to work alone to see how I can make it by myself." His comments showed that the complication of the mathematics project affected his desire to work with a group or individually.

Based on his experiences, Mike was still unsure whether or not boys and girls do equally well in mathematics. He said, "I don't know, it seems like girls are a little better in writing and boys a little better in math. I don't know, I think when the classroom gets bigger and when I am in more classes, it might change." Again, this showed that in this CMP classroom boys had higher achievement in mathematics than girls. However, Mike did not conclude that gender differences exist in learning mathematics. He preferred to see a big picture of how girls did in mathematics classroom and he tended to believe that in general both boys and girls do equally well in mathematics.

Tony. A co-leader of the group, Tony was an above average mathematics student. Tony described mathematics as knowing how to use numbers and figuring out about mathematics problems. He said, "It's kind of how you use the numbers and figure out things how we' re using the numbers like prices or how many to add up stuff. That's mathematics." Tony was able to articulate how mathematics involved real-world situations.

In addition, Tony explained how mathematics was important for daily life and in the future "When you go to the grocery store, when you're buying stuff, you probably want to know what it costs ahead of time. So, you need to add it up…you need it (mathematics) in paying bills, taxes, organizing stuff and showing it in graphs and a lot more… And in most jobs, you need to know math, at least basic stuff like adding or basic arithmetic. Currently, I want to be an architect. I need to know how to measure and all that kind of stuff."

Tony also believed that understanding in mathematics was important. He said, "You can't really get anywhere in math if you don't know how." He also believed that trying hard would help everybody get better in mathematics. He said, "I kind of like to try to figure out as much as is possible." Tony did not agree that if he could not solve a mathematics problem in a few minutes, he probably could not do it at all.

In the 6th grade CMP mathematics, Tony believed that he was good in mathematics. He said, "…it's kind of steady. I usually get an A, when I get most of the problems right." However, he added that bad grades usually came from his confusion about the wording of the problems. Tony confirmed that the mathematics teacher helped him by guiding instead of telling what the answers were. He rarely needed any other extra help from his teacher in this 6th CMP class. In addition, Tony said that he enjoyed learning mathematics because "Knowing how important it is in life, it's good knowing. I know something that will help me later." Tony's expression showed that he realized how mathematics was related to his life outside the classroom. Also, he was concerned that knowing mathematics would help him in the future.

In the CMP class, Tony stated that he enjoyed figuring out mathematics problems. Also, he liked hands-on activities because they were fun. He enjoyed exploring mathematics problems. Moreover, he liked to learn new mathematics concepts.

He felt neutral about working with a group because he got tired with helping people in his group. He said, "Working with groups, you're supposed to get help, but it turns out that you are the one that can help. I am the one who is trying to explain things to them or everybody who doesn't get it. Well, I like to explain, but when it's too much I get tired." From the observations, Tony often offered help to his group members. He offered a lot of help to one of his group members who needed help most in the classroom. It might cause him to feel exhausted with helping one of the low achievers in his class. Even though he felt tired with helping and explaining to his group members, he continual offered them help. This action showed that he felt he had a responsibility to help his group members learn.

However, he liked sharing ideas with his group. He saw himself as the leader or the explainer with his partner (Mike). Tony explained how his group worked together "We sort of like trying to figure out the way how to do it and then when it works we just try another way together." Tony also mentioned that his group members were helpful for him. He said, "They point out something. I mean both my partner and the other two people." Tony explained more, "For the one (Nicole) that needed help from me, she usually pointed out things to me and when I explained to her, I found out something about the problems. And the other one that helped me (Mike), usually shared the ideas with me, he was checking and verifying things (solutions)..." Tony's explanation revealed that he benefited from working with group. He received suggestions and different ideas from his partner who had high achievement in mathematics. He also gained useful comments from his group members who had low mathematics achievement in pointing out something that he might not have realized before. His expression confirmed that group work benefits for group members who had various performance levels.

Tony worked on the park project on his own without help from anybody. He liked designing and figuring out things. Since the teacher only gave the students a week to complete the project, he felt he needed more time to work. He said, "We had only a few days. It's kind of hard because I just roughly sketched that…" The only difficulty in doing the project for Tony was the big size of grid paper. He said, "Everything was big. I mean you have to kind of really think more because I am used to the smaller grid paper. It's kind of hard to think that way." It did not matter to Tony whether he worked with a group or alone on the project. He enjoyed working on the project either alone or with a group.

He also believed that girls and boys do equally well in mathematics. He said, "Probably in our class boys might be smarter than girls in math, but in mathematics generally I don't think so." Tony did not form a conclusion from what he observed in his mathematics classroom, but he rather believed that there was no gender difference in learning mathematics over all.

Mary. Mary, a follower, was an average mathematics student in the Rhombus Square group. In Mary's opinion, mathematics was learning. She said, "Mathematics is about learning how to do work with numbers so you can get a good job. It's knowing about how to do multiplication, addition, subtraction, and some things like that." Mary defined mathematics in pretty narrow point of view. Even though she knew that knowing mathematics would help her to get a better job, she saw mathematics as dealing with numbers by different operations.

Mary was able to give some examples about the usefulness of mathematics in daily life and in the future. She explained, "In daily life, we use it (mathematics) just to do ordinary problems, like if you're going to build a corral for your horse or something, you'd have to know how many panels you'd need and how big you'd want it...I want to be a vet. So, I have to know the math, like how many shots I need to do for my animals." She believed that learning mathematics would help her to get a better job in the future. Although previously Mary defined mathematics as involving with only numbers and operations, here she expressed her broader perspective of mathematics by being able to provide examples of how to use mathematics in her daily and future life.

Mary believed that in doing mathematics, understanding was important. She said, "It (understanding) is important because if you don't understand it, you would not know how to do it." Also she believed that trying hard would lead to increased mathematics achievement. She said, "I think trying harder and harder, then you will learn more and get better." She also believed she would be able to solve time-consuming mathematics problems.

Mary thought the CMP class was different from her 5th grade mathematics. She thought the 6th grade CMP mathematics was "more interactive." She said, "It was harder. The class was more active and had more group work." She saw herself as an average mathematics student because she had average grades in mathematics. She said, "Sometimes, I don't get it or some of them I do. So, I am okay, not the best." When Mary received some help from the teacher, she said her teacher guided her to figure out the mathematics problems instead of telling her the answers. She usually asked for help from her group first before going to ask for help from the teacher. Mary was not quite sure about her ability to solve time-consuming mathematics problems; however, she believed she would try to figure it out first and then seek help after trying for a while.

She enjoyed learning mathematics when it was not hard. She liked hands-on activities and working with a group. However, she did not like doing homework because when she got stuck, she couldn't have help from her group. She added, "It's boring to work alone. I work better when I work with people."

Mary enjoyed working with groups because she received an opportunity to share her work. She agreed that her group members were helpful because if she had a wrong answer, they would show her how to fix it. Mary explained, "We sort of like to do the problems together, and sometimes we'll do them separately and then compare answers later."

Working on the park project, Mary said, "It is ok as long as you have time to do it." She said she got a lot of help from her mother figuring out how much material she needed. Mary preferred to work on the project with a group because if she had a hard time, the group could help her. Mary did not have any particular difficulty in doing the project. However, she wished she could have more time, so she could "create more and more." She expressed a comment frequently heard from others about the time restraint, indicating that this project needed more time than allowed.

Mary also believed girls and boys do equally well in mathematics, even though in the CMP class she noticed that boys seemed to be a little bit "smarter" than girls in mathematics as frequently mentioned by other participants. She still believed that in general they both were equal in mathematics ability.

Nicole. Nicole, a follower, was the low mathematics student in the Rhombus Square group and she was one of the low mathematics achievers in this CMP class. Nicole had difficulty defining mathematics. She said, "It's about (um) about (um) you should know math because it might help you in life. If you'd like to get a good job, you need to do a lot of mathematics." When asked to explain further she said, "Mathematics is…um…(pause)…I don't know…um …(pause)…um…it's figuring out about numbers…(um)…I don't know, I can't think of anything else…"

However, Nicole did realize that mathematics was important in daily life and in the future. She was able to explain how to use mathematics outside the classroom. She said, "…it helps when you buy something with your money. The teacher taught us that how to get the better price, the cheaper price of something. For a future career, if I want to be a designer, mathematics would help me to do my work, like measuring things and stuff." However, Nicole did not know for sure what she would like to be when she grew up, but she believed as Mary did that learning mathematics would help her to get a better job.

Nicole did say she believed that understanding mathematics concepts was important and trying hard in mathematics helped increase one's mathematics ability. "I'm sure that people can figure out always, like, if they try hard, they probably figure it out and if you are trying hard and listen to your teacher, then you'll get better." However, she was not certain of her ability to solve time-consuming mathematics problems.

Nicole thought this CMP class was harder than elementary school mathematics. She said, "I'm kind of glad that now we're doing easy stuff, because I am struggling with the harder book we did, I mean the Fraction book. I like the Covering and Surrounding book because they are kind of fun." Nicole expressed that she did not like mathematics that much. She said, "I hate it when I don't get some of the stuff. So, I can't really do my homework. I don't think it's very fun when I don't even know what are they talking about or I don't understand it." Nicole sought help neither from her friends nor from her teacher. She said, "Sometimes, a few times, I went to the teacher in the morning and I did tell her 'well, I don't get this,' and mostly I don't like to ask her because I just think I'd probably like to be able to figure it out at home." She added, "…my sister (a college student) like always gets tired of helping me. And my friends --I don't really ask them because I don't know any of their phone numbers." This information showed that she had limited resources to ask for help when she got stuck in doing mathematics.

Nicole also preferred her CMP mathematics teacher to give the directions on how to get the answers to problems, instead of guiding her. Nicole liked being able to redo the tests in this CMP class. She said, "I liked that we got a chance to redo the tests, so we could get extra scores." Nicole found that mathematics problems requiring a long time to solve sometimes bothered her because when she got stuck, she did not want to go to ask the teacher too often and she did not know a lot of friends to ask for help.

Nicole liked working with a group because she got help from them. She liked being in groups when they asked her whether she understood and offered to help. Nicole described herself as the one who got help. She said, "Sometimes, like we can ask each other and they explain it, sometimes, I said, like 'how did you get this answer' and then he said (Tony) 'I did this,' and then oh! I got it." Her expression showed that she felt comfortable working with group and her group members were helpful for her in explaining concepts or methods to her. In addition, Nicole thought girls and boys might do equally well in mathematics, even though she noticed that in the CMP class, boys seemed to do better in mathematics.

Nicole liked working on the park project because it was fun. She said, "I kind of like everything about the project. I liked to make my park more exciting." Nicole gained some help from her older sister and her mother explaining how big everything is. The difficulty of the project park for her was that she was confused about square yards. She said, "I was kind of confused like…how big the square yard was, but I got it now." Nicole got help from her family in explaining the size of things like trees, the playground, and including the size of the square yard so that she now knows how many things she can put in one square yard. Also, she added that working on the project with a group would be fun, too.

<u>In summary</u>. In the Rhombus Square group, Mike defined mathematics as "understanding numbers." He explained that mathematics was about knowing how to use numbers in real life such as in building or tiling surfaces. Tony also thought mathematics was about "…knowing how to use numbers in…life." He added that using numbers in finding prices or adding something up all involved with mathematics. Mary expressed mathematics was "learning how to do work with numbers…" She mentioned that mathematics was involved with multiplication, addition, subtraction, and so on. At the beginning, Nicole had a hard time defining mathematics. However, later on, she

mentioned that mathematics was about numbers, which would help people in daily life such as finding the better prices and measuring "things and stuff." All of the opinions about mathematics revealed that all of the Rhombus Square group members, even though some of them hard a hard time defining it, realized that mathematics was not just numbers and operations but involved how to use them in their lives. All of them could see how mathematics would be involved with real-world situations outside the mathematics classroom.

The group members agreed that mathematics was useful. All of them were able to give examples of how mathematics was used in their daily and future life such as in building, selling and buying, and measuring. Even the low mathematics achiever, the follower, was able to articulate how mathematics was useful for her life such as in buying "things" and measuring "stuff."

In addition, all of the group members thought that understanding concepts in mathematics was important in addition to getting a right answer. Mike pointed out that making connections among similar mathematics problems would not occur without understanding the related concepts. Tony confirmed that without understanding concepts, in doing mathematics was useless because one could not apply the concepts to related situations.

All of the group members, except Nicole believed that they were able to solve time-consuming mathematics problems. Also they believed that effort could help to improve one's mathematics ability. Mike and Tony had stronger beliefs in their abilities to accomplish time-consuming mathematics problems than Mary and Nicole. Mike added that trying hard was also a time consuming process. He explained that to be good at mathematics, one had to try hard in the past. If he/she just started trying hard now, it would take a while to see the result.

In this group, Mike had strong confidence in his mathematics ability. Tony also saw himself as a good mathematics student because he thought he was able to understand mathematics concepts. However, he commented that sometimes he did not get good scores because of confusing wording in the problems. Mary had less confidence in her mathematics ability, whereas Nicole had the least confidence in her mathematics ability in this group.

Additionally, Mike liked mathematics because it was fun for him to learn new concepts. In addition, Mike related mathematics activities outside the classroom and participated in a mathematic contest. Tony liked mathematics because he liked to figure out mathematics problems and he thought it was good to know how important mathematics was. Mary liked mathematics when it was active. Nicole did not like mathematics when it did not make sense to her. The findings showed that Mike and Tony showed intrinsic motivation in learning mathematics because they were willing to learn new concepts and/or interesting in doing extra mathematics activities outside the class.

Also, almost every group member liked working with a group because they thought they could learn from each other. Only Tony felt neutral about that since he felt tired helping and explaining to his group members, especially the low achiever. However, he kept offering to help his group members, showing responsibility in helping his group members work together. In addition, all of the Rhombus Square group members liked working on the park project, since they enjoyed creating ideas and designing things. Most group members expressed that working on the project with a group or individually was fine. They liked to see how far they could design the project on their own. Also they thought it would be fun to work on the project with a group sharing their ideas. Only one of the group members, Nicole, preferred working with a group on the project. She thought that the group could help her when she got stuck.

Finally, almost all of the Rhombus Square group members believed that there were no gender differences in mathematics. Only Mike was not sure about gender differences in mathematics. He did not indicate whether there was a gender difference in learning mathematics, even though he saw that in this CMP class, boys outperformed girls. He would rather see more examples from other mathematics classrooms or gain a broader view before making a conclusion.

Conclusion

Overall, most of the students in this study had positive mathematical dispositions in the CMP class. They believed that mathematics was knowing and understanding how to deal with numbers. Some of them explained in a narrow view that mathematics was about using numbers in counting, measurement, addition, and subtraction in order to find the answers to mathematics problems. However, most of them described mathematics in a broader perspective. They believed mathematics was about "learning new ideas" and mathematics was "life" because it was everywhere in their lives. It was about figuring out real life situations by using numbers, operations, measurement, geometry, and data analysis. Although at the beginning, one low mathematics achiever (the follower) had a hard time defining mathematics, in the end she was able to explain that mathematics was dealing with numbers in daily life such as "finding the better prices" and "measuring."

All students in this study agreed that mathematics was useful for their lives. Some of them were concerned more about the usefulness of mathematics in their future careers, but most of them realized the benefits of mathematics in their daily lives, such as shopping and cooking and in their future lives, such as in building, buying and selling, applying their knowledge in their jobs, and managing their salaries. In addition, some of them believed that knowledge of mathematics would help them to get a better job in their future lives.

Most of the students agreed that it was important to understand concepts in mathematics in addition to getting a right answer because it would help them to apply the concepts to similar mathematics problems and even to situations in the real world. However, one of the students in this study (the leader) did not agree. He thought it was "good enough" if he could get a right answer in order to get a good grade. Although he did realize that understanding helped him in making conjectures while learning mathematics, it seemed he was convinced that being successful (getting the right answers) was the most important thing.

All students in this study agreed that effort could help increase their mathematics ability. High performance students (the leaders) had stronger beliefs than the intermediate and low students (the verifier and the followers) about their ability to solve time-consuming mathematics problems. Most of them, based on their experiences seeing people, gained accomplishment from their efforts. One high performance student (the leader) noted that trying hard in doing mathematics was not a short process, but a long-term process. People had to keep trying hard for a while to show improvement.

All high performance students (the group leaders) had strong confidence in doing mathematics. Average performance students (the followers and the verifier) had intermediate confidence in doing mathematics, whereas a low student had low confidence in doing mathematics. Seemingly, the students perceived the amounts of their confidence based on their mathematics performance in the CMP class.

Most of high performance students (the group leaders) had a willingness to learn and explore new ideas. They liked mathematics when the problems were challenging. They enjoyed figuring out mathematics puzzles. Average performance students (the verifier and the followers) liked mathematics because it was interactive. All of the students also liked hands-on activities because these were fun. However, a low performance student did not like mathematics when she did not know what to do. The students, who were willing to learn new ideas, have fun exploring mathematics problems, and doing additional activities related to mathematics outside the classroom had revealed intrinsic motivation.

None of the participants expressed negative feelings about group work because they believed that they learned from each other. They found that their group members either helped them to verify their thoughts or provide them suggestions. However, they also liked to work individually on the park project because they liked to have a chance to design and create things on their own. All of them agreed that it also would be fun to work on the project with the group.

Most of the students believed that there were no gender differences in mathematics in general, even though they admitted that boys did better in this class. They thought that boys and girls could do equally well in mathematics in general.

However, one of the leaders was not certain about this, since in his CMP class boys had higher achievement in mathematics than girls. However, he believed that in the bigger population, their achievement might even out.

An Analysis of Students' Mathematical Dispositions in the CMP Class

For each cooperative group, the analysis of each group member's mathematical dispositions is presented, according to the collected data from the observations during the whole class discussion and cooperative group work. Also, the analysis of the data gathered from interviews and questionnaires are included in order to determine the best descriptive level of each of the participants' mathematic dispositions in this CMP class. The analysis was based on the students' dispositions table found in Appendix K. Finally, the main outcomes are summarized according to the Krathwohl, Bloom, and Masia Dispositions Taxonomy (1964). In brief, the levels were as follows:

1. Level 1: receiving
   1.1 awareness
   1.2 willingness to receive
   1.3 controlled or selected attention
2. Level 2: responding
   2.1 acquiescence in responding
   2.2 willingness to respond
   2.3 satisfaction in response
3. Level 3: valuing
   3.1 acceptance of a value
   3.2 preference for a value
   3.3 commitment
4. Level 4: organization
   4.1 conceptualization of a value
   4.2 organization of a value system

5. Level 5: characterization by a value or value complex
    5.1 generalized set
    5.2 characterization

The Cubic Melon Group

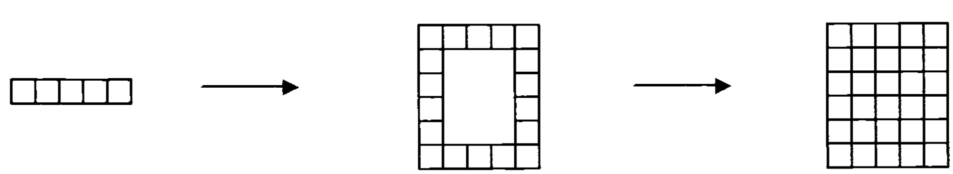
The group members were Jim (the leader), Bob (sometimes the leader), Bill (the follower), and Cindy (the verifier). This group normally would start to work on the assignment individually and then share their work at the end of the investigation. Jim played an important role in leading the group to work in a less interactive format.

Jim. Jim, the leader, participated in all required mathematical tasks in the classroom. In the whole class activities he usually listened to the teacher and followed the directions. He usually stayed on task during the class activities. Moreover, he participated in the whole class discussions almost every day. He often raised his hand and shared his ideas and explanations of his idea with the class. The following is an additional example of Jack's behaviors in the whole class activities. When the teacher taught the relationship between areas and perimeters by looking at pentominos, she asked for ideas on how to add more tiles to the pentominos in order to make the biggest area with a perimeter of 18.

T: Anybody want to share an idea with the class?

Jim raised his hand and the teacher called on him.

Jim: I made it (pentominos and additional tiles) to a rectangle and then I added from the outside to inside.



T: Good.

Jim smiled and went back to work.

As demonstrated in the example from the Pizza problem (see Summarize p. 62), Jim was usually the one who presented his group conclusions or ideas to the class during the class discussions. He often took the role as his group representative. Most of the time he shared the group findings or facts, but not creative ideas or alternative ways to solve the problems.

Jim usually led the group work in his cooperative group. He tended to work by himself at the beginning of the group work and then share his work later. He was a helpful group member. He sometimes helped his group members to able follow the group work during the group activities as shown in the example of the Cubic Melon group work (see p. 71).

However, Jim did not show interest in doing additional problems either in the whole class discussion or in the cooperative group work, if the teacher did not assign them or if he could not get extra credit points for them. When Jim was able to solve the assignment problems, he rarely showed interest in finding alternative ways to solve the problems. Instead, he would help his group members solve the problems and he just waited for the teacher to assign the next task.

From the interview, he explained that he would do extra mathematics as long as he got extra credit for doing so. He did not participate in any mathematics activities outside the classroom, except sometimes when his friends called and asked him to help or advise them on mathematics homework on the phone. However, Jim expressed that he liked mathematics when it challenged him. He also liked the beginning of each unit because he was first introduced to new ideas. In addition, Jim believed in the usefulness of mathematics.

The Attitude and Belief Questionnaire provided some additional data about Jim. He expressed that he liked mathematics. Also, he strongly agreed that he was good at mathematics. He agreed that boys and girls could do equally well in mathematics. He strongly agreed that trying hard could increase one's ability to do mathematics and also strongly agreed that he could solve time-consuming mathematics problems. He agreed that mathematics is useful. However, he disagreed that understanding why the answer works was more important than getting a right answer in mathematics (see Table 1). In addition, he indicated he intended to take as many mathematics classes as possible at the high school level, because he thought mathematics was the easiest subject for him. Jim was not sure what he would like to be when he grew up.

Table 1
The Results from Jim's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales[3] (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	3.80
2. Confidence in doing mathematics	5.00
3. Gender difference in mathematics	3.65
4. Effort can increase mathematical ability	5.00
5. Ability to solve time-consuming mathematics problems	4.80
6. The usefulness of mathematics	3.50
7. The importance of understanding concepts in mathematics	1.80

Overall, Jim participated in the CMP class activities according to the teacher's requirement. He listened to the teacher's instruction; he participated in both class discussion and group discussion; he helped and gave suggestions to his group members. Jim's behaviors showed a mathematical disposition higher than Level 1: "receiving," which required that he paid attention in learning mathematics. However, his scores did not reach Level 3: "valuing," because he was not intrinsically motivated. He showed no interest or curiosity in doing more mathematics than required. Jim always completed his homework and classroom assignments, evidence of his acquiescence in responding. In addition, he volunteered and shared his ideas in the class activities, which demonstrated willingness to respond. Furthermore, Jim expressed that he enjoyed doing mathematics. He smiled when he got the teacher's compliments from volunteering and/or sharing his

[3] The scores on the table are the average scores of the agreement on the five-point Likert scales from six questions for each variable.

ideas. Therefore, Jim's mathematical disposition scored at Level 2.3: "satisfaction in response."

Bob. As a leader, Bob participated in the mathematics class more than the minimum requirement. He was usually on task, listened, and followed the teacher's instructions. He volunteered or shared his ideas almost everyday. For example, when the teacher asked for a volunteer to describe an effective way to find the area of a triangle, Bob raised his hand to answer the question.

> T: Ok, describe an effective way to find the area of a triangle. Be sure to mention the measurements you would need to make and how you would use them to find the area.

Bob raised his hand.

> Bob: Half of the base times height.
>
> T: So, you need to measure the base and the height?
>
> Bob: Yes, or base times the height divided by two.
>
> T: So, we kind of discover that we can cut that measurement in half to get it?
>
> Bob: Yes.

Moreover, the observational data revealed an additional fact about Bob. He was the only one who hummed while working on tasks, indicating that he enjoyed working one assignments. At least once a week, he would hum a song, whether or not he was working with his group or individually. Sometimes, Bob asked questions about the specific topics, which illustrated his interest in the topics. For example, when the teacher taught the class about how to measure the height of a parallelogram by imagining dropping a rock from the corner of the parallelogram, Bob seriously wondered what would happen if he threw the rock instead of dropping it.

> T: "Let's pretend we are standing on top corner of the parallelogram and you drop a rock. It will go straight down to the base, right?"
>
> Bob: "What if we threw the rock?"

Bob gestured his hand like a projectile curve with his hands showing how the rock would arc and fall.

> T: "Well, the rock will go in a different direction…"

In addition, Bob's interview information revealed that he liked mathematics because it was fun for him to solve problems and to get solutions. Bob realized how important it was for his daily life and future career. He was able to explain the usefulness of mathematics when he did grocery shopping and how mathematics would be useful in his future career.

The questionnaire data revealed that Bob definitely would take mathematics in the future. He strongly agreed that he liked mathematics and that he was good at mathematics. Moreover, he strongly agreed that girls could do just as well as boys in mathematics, and that trying hard could improve one's ability in mathematics. He agreed that in addition to getting a right answer in mathematics, it was important to understand why the answer worked (see Table 2). However, he preferred to take an average number of mathematics courses in high school because he thought other subjects like science were also important. Bob said he would like to be a polar bear researcher. Bob believed mathematics was the second most important subject, next to science, for his future career.

Table 2
The Results from Bob's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	4.00
2. Confidence in doing mathematics	4.65
3. Gender difference in mathematics	5.00
4. Effort can increase mathematical ability	5.00
5. Ability to solve time-consuming mathematics problems	4.65
6. The usefulness of mathematics	5.00
7. The importance of understanding concepts in mathematics	3.50

In general, Bob was an active mathematics student with a willingness to learn new things each day in his mathematics class. He carried out all class assignments and got involved in discussions with his group and with the whole class. He stayed on task during the classroom activities. He expressed his enjoyment of mathematics through humming, a sign of mathematical dispositions Level 2.3: "satisfaction in response." Moreover, he was curious to learn new ideas and liked to explore mathematics puzzles. These statements revealed his mathematics disposition at Level 3.1: "acceptance of a value."

Bill. A follower, Bill complied with what the teacher required him to do in both the whole class activities and the cooperative group work. In the whole class discussion, he sometimes shared his ideas with the whole class, as in the following example.

When the teacher let the students draw a circle and provided data about its radius, the area of the radius square, the area of the circle itself, and the number of radius squared that was needed to fill the circle, Bill raised his hand to share his data.

Bill: I got a circle with the radius of 5 and then the radius square is 25.

T: The radius square is 25...ok, how about the area of the circle?

Bill: It's about 78.

Bill counted all of squares in the circle.

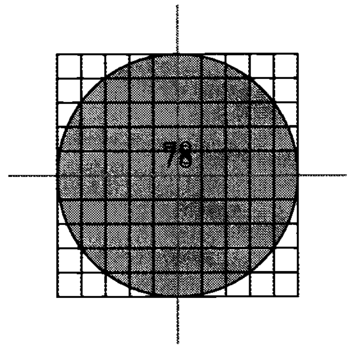
T: Ok, and how many radius squares do we need?

Bill: I got about 3.12.

T: You did it with counting the whole little squares (in grid paper), right?

Bill: Yes, I counted how many radius squares in there and then I got three radius squares in there and I got three little squares left over and then I did three divided by 25 and I got .12.

Bill divided 78 squares into 75 squares plus three squares. Next, he knew that 75 squares were equal to three radius squares because one radius square had 25 squares. (75 divided by 25). Then, he divided the left over three squares by 25. He got twelve hundredths. Thus, he concluded that 78 squares equaled 3.12 radius squares.



T: Good…

The teacher moved on with the lesson.

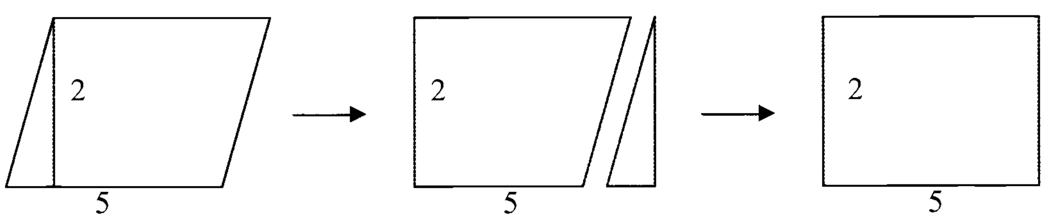
In the cooperative group activities, Bill usually followed other group members' leads. Bill was easily distracted. When he was not sure what to do or did not understand the concepts, he usually asked for help from his group members.

When the teacher gave an assignment to the group to find the area of the parallelograms, Bill got stuck for a while, and then asked for help from Jim.

Bill: Jim! How to find the area? Give me a clue? (pause) I really don't know how to find the area.

Jim: Ok, what do you see?

Jim showed Bill the picture of moving a small piece of a triangle on the side of the parallelogram to the other side.



Bill: Ah ha! I got it. This should be 5 and this should be two, so the area would be 10…I'm smart.

Bill smiled after he solved the problem.

Even though many times he found out that he was wrong, Bill sometimes tried to find a variety of ways to do the assignment. However, he did not consistently participate in class activities, but sometimes lost interest and was easily distracted by a ringing phone or outdoor movement.

In the student interview, Bill was able to describe how mathematics was useful for daily life in shopping, and for his future career. Bill wanted to be a soccer player or a basketball player. He explained that mathematics helps in soccer games or basketball games in scoring, in order to know who won the game. Additionally, he believed that understanding concepts in doing mathematics was important and effort could help one improve his/her mathematics ability. Bill expressed that mathematics was fun for him. He liked an active classroom. He liked doing mathematics by himself instead of observing someone else doing it. It appeared that Bill's motivation in learning was having fun in doing mathematics and realizing the usefulness of mathematics. Finally, the student interview revealed that he did not believe there were gender differences in learning mathematics.

In the questionnaire, the data confirmed that he believed that mathematics was useful for his life. However, he was not sure whether getting a right answer in mathematics was more important than understanding why the answer worked.
He agreed that trying hard would help one's mathematics ability. He believed that he could get a good grade in mathematics. In addition, mathematics was fun for him when it was interactive. He liked working with a group. He agreed that boys and girls could do equally well in mathematics (see Table 3). In addition, Bill was willing to take some optional mathematics classes.

Table 3
The Results from Bill's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	3.00
2. Confidence in doing mathematics	4.30
3. Gender difference in mathematics	4.30
4. Effort can increase mathematical ability	4.30
5. Ability to solve time-consuming	4.15

mathematics problems	
6. The usefulness of mathematics	4.15
7. The importance of understanding concepts in mathematics	3.30

Bill was involved with his CMP class activities to the minimum requirement. Even though he was sometimes distracted, most of the time he listened to the teacher's direction. He participated in both class discussion and group discussion, but he did not provide help. He received and asked for help from his group members or the teacher. Bob's behaviors showed at least an initial mathematical disposition Level 1: "receiving," which required the students' attention to the mathematics classroom activities. Bill claimed that he had completed his homework or classroom assignments, but he often forgot to turn them in. He sometimes volunteered and shared his ideas in the class activities, which meant he had willingness to respond. Bill expressed that he enjoyed doing mathematics and smiled and complimented himself when he was able to solve problems. Taking into account these indicators, his mathematical disposition extends to Level 2.3: "satisfaction in response," which required a feeling of satisfaction such as pleasure and enjoyment.

Cindy. The observational data revealed that Cindy, the verifier, was usually on task, listened and followed the teacher's directions. Her class participation was generally average. When she did participate in whole class discussion, she usually shared her answers to low level questions such as questions about facts or yes or no questions. An example of her behaviors follows:

T: What is the name for the measurement all around the circle, it is kind of the perimeter in a polygon?

Cindy raised her hand.

Cindy: Circumference.

T: And then, we have one more: the measurement across the circle from one side to the circle, all the way through the other side?

Cindy: Diameter.

T: Ok.

When the teacher asked how to find the area of a rectangle, she again raised her hand to answer the question. Cindy replied "Length times height.

T: And how to find the area of a parallelogram?

Cindy: It's the same, which is length times height.

T: Ok.

Cindy rarely showed or created alternative ways to solve mathematics problems. She would just share her answers about facts.

In the cooperative group discussion, Cindy usually listened to the group discussion and stayed on task during the small group work. She received some help and encouragement from her group members when she was stuck. In turn, she helped her group members by rechecking their answers.

The data from Cindy's interview showed that she had not gotten used to the new mathematics approach this year, but after a year of the new mathematics classroom experience she expressed her willingness to go back and redo all her mathematics work in this class, believing she would do better. However, she did not actually do it. Moreover, Cindy mentioned the value of the group work. She said, "If I didn't have a group, I'd be totally lost." She also realized how important mathematics was in daily life and for a future career.

From the questionnaire, Cindy expressed that she would continue to take mathematics. She was interested in becoming a scientist. She agreed that she liked mathematics. She enjoyed learning mathematics because it was fun and she would like to be a smart person. She also agreed that girls and boys could equally do well in mathematics. However, she did not have confidence in her mathematical ability. She believed that mathematics was useful for her life (see Table 4). However, she would take an average number of mathematics classes because she thought other subjects, such as science and reading, were important, too.

Table 4
The Results from Cindy's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	4.00
2. Confidence in doing mathematics	3.65
3. Gender difference in mathematics	4.00
4. Effort can increase mathematical ability	4.00
5. Ability to solve time-consuming mathematics problems	3.80
6. The usefulness of mathematics	4.00
7. The importance of understanding concepts in mathematics	4.00

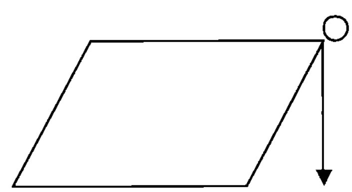
Generally, Cindy participated in the CMP class activities at least at a minimum level. In the whole class discussion, she paid attention to the teacher instruction. She participated in both class discussion and group discussion. In the cooperative group discussion, she followed the group's leader. These behaviors meet the Level 1 mathematics disposition: "receiving." But Cindy additionally exhibited a higher level of disposition, completing the mathematics assignments, which showed "acquiescence in responding." She sometimes volunteered and shared her ideas in the whole class discussion. She usually helped group members by double checking their work, meaning that she had "willingness to respond." Additionally, Cindy expressed that she enjoyed doing mathematics and it was fun. Such behaviors demonstrated a mathematics disposition of level 2.3: "satisfaction in response," which required an emotional response involving pleasure or enjoyment. Beyond that, Cindy expressed her willingness to go back and redo all mathematics work in this year CMP class. She believed that she would do better. Even though she did not go back and redo all mathematics work in this CMP class, her willingness showed her intrinsic motivation to develop her mathematics abilities. This revealed her mathematics disposition reached Level 3.1: "acceptance of a value." Furthermore, she participated in activities related mathematics outside the classroom. Therefore, her mathematics disposition was considered as Level 3.2: "preference of a value," which required a desire to do and seeks for ways to develop mathematics ability.

Rhombus Square group

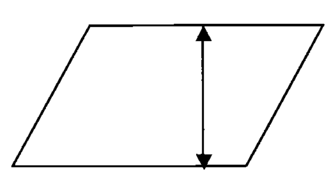
The Rhombus Square group members were Mike (the leader), Tony (the co-leader), Mary and Nicole (the followers). This group was an active group. Mike usually started the group discussion. Mike and Tony usually dominated the discussion; while Mary would ask for advice when she could not follow them. However, Nicole rarely participated in any activities. Most of the time, she just listened and did not respond.

Mike. Almost everyday, Mike was involved in the whole class discussions and the group discussion. Moreover, he often asked questions about the topics and suggested alternative ideas within the whole class discussions. When the teacher reminded the students about how to find the height of a parallelogram by imagining a rock dropped from the top to the bottom of the parallelogram, Mike suggested an alternative way.

T: The height of the parallelogram, do you remember that? Drop the rock from the top down to the bottom, that's the height.



Mike: Teacher! Can we go like measure two flat parts of the parallelogram? (The distance between the top and the bottom)



T: Yes, it is another way to do this…

The teacher did not ask for an explanation.

In another example, when the teacher taught about the height of any triangle, Mike asked questions again as follows:

T: Where is the height of the triangle? It depends on where the base is.

Mike: It (the height) has to be from the base to the vertex, right?

T: Yes, it has to be from the vertex to the opposite side (the base) in a perpendicular line, which makes a right angle. Thank you for bringing that up.

Mike smiled after the teacher commented on his work.

Mike also actively participated in the group activities. He usually led the group discussion, and exchanged his ideas with the other group leader, Tony. Mike was always helpful to his group. Helping them did not bother him at all. He liked to learn from them, too.

The observed data revealed that Mike actively participated in the classroom activities and group work. Also, from time to time, he shared and suggested ideas or asked questions about mathematical topics, as in the following. When the teacher introduced her students to finding a parallelogram's area, Mike asked, "If we measure one side length and measure the base, could the length times the base be the area?" The teacher responded, "You could try that, try it out, we can play this around today..." Mike's question revealed his interest about the topic, although his conjecture was inconclusive and incorrect.

During the student interview, Mike stated that mathematics was his favorite subject. He also liked hands-on activities. Mike liked mathematics because it was fun for him to learn new concepts, to explore, and do challenging or complicated problems. In addition, Mike revealed his willingness to learn in different ways. He mentioned that he liked to learn from and to teach his peers. When he had free time, he sometimes did some additional mathematics problems outside the classroom. Moreover, he had participated in a mathematics contest. He noted that he liked challenging problems in the contest and he won first place.

Responding to the questionnaire, Mike was not sure what would he like to be when he grew up, but he knew that mathematics was involved in many careers, like being a cashier, an accountant, or a salesman. He agreed that mathematics would eventually become very useful for his future life. He would definitely take available mathematics courses when he was in high school. However, he would take an average number of classes because he thought other subjects were important as well. Additionally, Mike agreed that he liked mathematics. He thought it was fun to "think deep inside" and not just merely remember. He strongly agreed that he was good at mathematics. Boys and girls could do equally well in mathematics in his opinion. He agreed that trying hard would help one get smarter in mathematics and he believed that understanding is more important than getting right answer (see Table 5).

Table 5

The Results from Mike's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	4.15
2. Confidence in doing mathematics	4.00
3. Gender difference in mathematics	3.50
4. Effort can increase mathematical ability	4.15
5. Ability to solve time-consuming mathematics problems	4.30
6. The usefulness of mathematics	4.80
7. The importance of understanding concepts in mathematics	4.30

Overall, Mike actively participated in the CMP class and his group activities. Mike paid attention to the teacher instruction. He participated in both class discussion and group discussion. He helped and exchanged ideas with his group members. Mike's behaviors demonstrated that his mathematical disposition was beyond Level 1: "receiving." Mike always finished his homework and classroom assignments, showing that he had "acquiescence in responding." In addition, he volunteered and shared his ideas in the class activities, which meant he had "willingness to respond." Mike expressed that he enjoyed doing mathematics, showing "satisfaction of response" by smiling when the teacher gave him feedback. These actions showed that he had a mathematical disposition higher than Level 2: "responding." He liked to learn new ideas and to solve challenging problems. These actions showed his intrinsic motivation arising out of his curiosity and interest, indicating a mathematical disposition Level 3.1: "acceptance of a value." In addition, when Mike had free time, he worked on additional mathematics problems. Also, he had participated in a mathematics contest, which he enjoyed doing. Therefore, Mike's mathematical disposition was considered at Level 3.2: "preference for a value," which required a commitment to pursue doing mathematics.

Tony. The observations revealed that Tony, the co-leader in the Rhombus Square group, did not participate in the whole class discussion as actively as Mike. Tony sometimes tried alternative ways of doing the class activities. For instance, when the class was assigned to measure the diameter of a circle in grid paper square units, most of the class cut a strip from the grid paper and then used the square strip to measure the diameter, as the teacher suggested. Tony found an effective way to measure. He used a compass to measure the diameter and then compared the compass width with the grid paper.

T: Wow! Cool! Awesome! Tony! Do you want to share with the class?

Tony: Yes.

Tony smiled and prepared to present his idea to the class.

T: Hey! Guys could I get your attention, please…we got a new trick going on for measuring the side length.

Tony: We can find the diameter in a unit of the square grid paper without cutting the square strip, by using a compass. Um…what I do, I measure it…sort of like straighten it (the compass) out and compare it to the grid paper…

T: If you don't have a compass, you can use the string with the same idea to measure from one point to the other point. That's clever…

In the cooperative group work, Tony was usually on task, discussing the group assignment with his group members. He usually tried to help other group members, whether or not they asked for it.

Tony stated in his interview that he liked hands-on activities. He enjoyed doing mathematics because it was fun to learn new concepts. Tony also realized how useful mathematics was, especially for his future career. He felt neutral about working with his group because he got of tired helping and explaining to other group members.

In the questionnaire responses, Tony expressed that he liked mathematics because "It was good to know something that will help me out later." Also, he liked to learn and discover new ideas. He believed he was good at mathematics. He commented that "even though I might not get really good grades on tests, I think I know the concepts pretty well. And the bad grades are usually from confusion about the wording in the problems." Tony also strongly agreed that trying hard would definitely help anyone increase his/her mathematical ability. He thought it was important to understand why the answer was correct, in addition to getting a right answer in mathematics. Furthermore, Tony strongly agreed that there was no gender difference in doing mathematics. He also strongly agreed that he would definitely take optional mathematics courses because he strongly agreed that mathematics was very useful and relevant to his life (see Table 6). However, he would take an average amount of mathematics classes in high school because he would like to learn other things, too.

Table 6

The Results from Tony's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	4.00
2. Confidence in doing mathematics	4.30
3. Gender difference in mathematics	4.00
4. Effort can increase mathematical ability	4.15
5. Ability to solve time-consuming	4.30

mathematics problems	
6. The usefulness of mathematics	4.80
7. The importance of understanding concepts in mathematics	4.65

Overall, Tony met the requirement for participation in the CMP class activities. He listened to teacher instruction. He participated in both class discussion and group discussion, more actively in the latter. This observation provides evidence for a mathematical disposition of Level 1: "receiving." Offering help to his group members and finishing his homework assignments showed that he had "acquiescence in responding," which required willingness to participate with others and/or to complete his homework/assignments. In addition, he sometimes volunteered and shared his ideas in the class activities, which verified his "willingness to respond." Additionally, Tony expressed that he enjoyed learning mathematics. This observation brings Tony's mathematical disposition to Level 2.3: "satisfaction in response," which required delightful response to the classroom activities. Moreover, he valued mathematics, commenting that it was good to know mathematics and that it would help him in the future. Also, he was willing to learn and explore new ideas. These actions showed his intrinsic motivation. In conclusion, Tony's mathematics disposition was considered at Level 3.1 "acceptance of a value." However, he did not show any signs of seeking to do mathematics-related activities outside the classroom. Therefore, he was not considered to have a higher mathematical disposition.

Mary. Mary, the follower, paid attention to the teacher and followed the teacher's direction, but participated only sometimes in the whole class discussion. However, she often participated in the cooperative group.

In the small group discussion, Mary usually followed the group leaders. When she got stuck, she did not hesitate to ask for help from her group members, as in the following example.

Mary: How do you find the area of a circle?

Mike: How to find the area of a circle, ok…do you know what the radius is?

Mary: Yes.

Mike: You get the radius from here to here, right? (from the center to the side)

Mary: Uh huh.

Tony: And then you do the square.

Mike: Yes, you do the radius times itself and then times 'π', you'll get the area...and for the circumference, you do the diameter times 'π'.

Tony: Yes.

Mike: The diameter is the whole way across.

Mary: Wait…wait…again.

Mike: So, the circumference, you get from diameter times 'π' and the area you get from doing radius times itself times 'π'…

Mary: Ok.

In her interview Mary expressed that she liked mathematics because it was interactive. She also liked the hands-on activities. It was fun for her to play with manipulatives, like using tiles to design a figure with the same areas, but different perimeters. Mary also realized how useful mathematics was for her future career. It appeared that Mary's inspiration for learning mathematics was from the benefit it would provide for her future life. She liked working with her group as well, since she got a lot of help from her group members. She found it boring to work alone.

In addition, Mary expressed in the questionnaire that she liked mathematics because it was fun. However, she disagreed that she was good at mathematics, because she did not get good grades. Mary strongly agreed that boys and girls could do equally well in mathematics. In addition, she believed that understanding was more important in learning mathematics than just getting a right answer. She believed that trying hard could help anyone increase his/her mathematical abilities, as well. Mary strongly believed that mathematics was a useful subject (see Table 7). She would take available mathematics courses in high school. However, she also would take an average number of high school mathematics classes because she would like to study other subjects.

Table 7

The Results from Mary's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	3.50
2. Confidence in doing mathematics	2.30
3. Gender difference in mathematics	4.30
4. Effort can increase mathematical ability	3.80
5. Ability to solve time -consuming mathematics problems	3.50
6. The usefulness of mathematics	4.65
8. The importance of understanding concepts in mathematics	4.50

Overall, Mary showed no signs of avoidance during the whole classroom discussion. Even though she did not participate in the class discussion a lot, she often participated in the group discussions by asking for help from the group's members. She paid attention to the help or explanations she got from her group members. Mary usually did her homework, showing "acquiescence in responding," which required compliance to complete mathematics assignments. In addition, she asked for help and questioned when she was not able to follow the assignment. Furthermore, Mary expressed that she enjoyed doing mathematics. Therefore, Mary's mathematical disposition was at Level 2.3: "satisfaction in response," which required feeling of satisfaction.

Nicole. The observed data showed that Nicole did not participate in the whole classroom discussion or in the cooperative group work. She never shared her ideas with the whole class or with her cooperative group. However, she followed and listened to the teacher's instruction. She did not show any sign of avoidance. However, she rarely asked for help from her group members, even when she needed help. Most of the time, she would just quietly listen to the whole class discussion or to her group discussion.

Once in awhile, she would ask for clarification for what her group members had just said.

She expressed in her interview that she did not like mathematics very much. She hated it when she did not understand, and did not know what to do. However, she liked the hands-on activities. Cutting and attaching paper or any manipulative activities was fun for her. Unfortunately, Nicole did not usually ask for any help from her group members or the teacher. She kept telling herself that next time she was going to ask for help.

She realized how useful mathematics was for her daily life, but she was not sure about the usefulness of mathematics in her future life. Nicole did not know for sure what she would like to be when she grew up. It appeared that Nicole did not have much motivation for learning mathematics.

From the questionnaire, Nicole expressed a different opinion about mathematics from the other participants. She revealed that she did not like mathematics that much. It was not fun for her when she got stuck. She agreed that mathematics was her worst subject because this year she got two 'Ds' in mathematics. However, she believed that understanding was more important than getting a right answer in mathematics, as well as that trying hard could help one's ability to do mathematics. She did not realize how mathematics would be important for her future life (see Table 8). She would not take optional mathematics classes. Furthermore, she would take as little high school mathematics as possible. She would do mathematics only as much as she was required to do.

Table 8

The Results from Nicole's Responses to the Attitude and Belief Questionnaire

Attitudes and Beliefs about Mathematics	Means of the Agreement on the five-point Likert scales (1=strongly disagree and 5=strongly agree)
1. Motivation in learning mathematics	3.15
2. Confidence in doing mathematics	2.15
3. Gender difference in mathematics	4.00

4 Effort can increase mathematical ability	3.50
5. Ability to solve time-consuming mathematics problems	2.65
6. The usefulness of mathematics	3.15
7. The importance of understanding concepts in mathematics	4.00

In general, Nicole rarely participated in either class discussion or group discussion. She needed help, but rarely asked for it. She passively received information with very few responses. As in the previous example in the Rhombus Square group work, when Tony offered her help and explained why the solution worked, she still kept quiet and gave him no response as to whether she understood it (see p. 82-83). However, she listened to the others with respect. This action revealed that Nicole's mathematics disposition was at Level 1.2: "willingness to receive," which required willingness to attend to necessary stimulus, but not to avoid it.

Summary

Based on the collected data and the Taxonomy of Affective Domain (Krathwohl, Bloom, & Masia, 1964), four (Bob, Cindy, Mike, and Tony) out of eight volunteer students reached the mathematical dispositions of Level 3: "valuing." Bob and Tony demonstrated their mathematical dispositions at Level 3.1: "acceptance of a value," since both of them showed curiosity in learning mathematics. They expressed that they liked to explore and learn new ideas, revealing their intrinsic motivation in learning mathematics. Cindy and Mike demonstrated a higher level of mathematical dispositions than Bob and Tony because they not only liked to learn and explore new ideas but also sought to do mathematics. They showed their intention to learn more about mathematics even outside the mathematics classroom. Cindy was willing to redo all of her mathematics assignments and she believed that she would do better next time. Mike would do additional mathematics problems in his free time. Therefore, both of them demonstrated mathematical dispositions of Level 3.2: "preference of a value," which requires seeking to do mathematics.

Three volunteer students (Bill, Jim, and Mary) had mathematical dispositions at Level 2.3: "satisfaction of response." Bill and Jim showed no interest in doing any additional mathematics assignments. Jim would just do the minimum required on the mathematics assignments to get a good grade. Mary also liked mathematics, but they did not provide any evidence about interest in doing mathematics more than what the teacher had assigned.

Finally, only one volunteer student (Nicole) had a mathematical disposition at Level 1.2: "willingness to receive." Nicole was a patient receiver. She listened to the whole class discussion and the small group discussion quietly, even though she appeared not able to follow the discussions. Nicole's mathematics disposition was not considered at Level 1.3: "controlled or selected attention" because she rarely showed signs of being aware of what was she listening to or where to direct her attention. She did not show that she had listened and tried to capture the main topics of the class discussions. She simply sat there in a passive manner.

Conclusion of the Findings

This study sought to describe middle school students' mathematical dispositions in a problem-based classroom. The findings indicate four main results as follows. First, the teacher in this CMP classroom was an experienced mathematics teacher for 11 years; however, she had experience in using the CMP curriculum for approximately two years. She attended a workshop in the first year of using the curriculum.

Second, the CMP class routine is revealed from an eight-week observation period. The CMP class consisted of four sections: Warm-up, Launch, Explore, and Summarize. The teacher acted as facilitator during the classroom activities. Students had a chance to work with a cooperative group during Explore section, and then in Summarize section, the teacher would lead the whole class discussion. The teacher encouraged the students to share their ideas and to not be afraid to make mistakes.

Nevertheless she rarely provided the students opportunities to clarify and explain their thoughts. Additionally, the teacher seldom called on non-volunteer students in order to promote girls participation in the classroom.

Third, the overview of the two participant groups is reported. Eight students from a 6th grade CMP class volunteered to participate in this study. The teacher helped to divide the volunteer students into two groups of four with mixed mathematics ability. The Cubic Melon group was less interactive than the Rhombus Square group. The Cubic Melon group usually was led by a high mathematics achiever, who usually preferred to work through the group assignment alone and then helped his group members. The Rhombus Square group usually worked together led by two leaders, who usually exchanged their thinking throughout the group work and helped their group members at the same time.

Fourth, the eight participants' mathematical dispositions are detailed. All eight participants liked hands-on activities and working on the mathematics project (the park project). The advanced students liked challenging mathematics problems. Most of the volunteer students agreed that they liked mathematics because it was fun and interactive. Most of the participants believed that mathematics involved figuring out new ideas that related to numbers and real-life situations. More than half of the participants mentioned that mathematics was everywhere in their lives. All of the participants agreed that mathematics was useful for their lives. Most of the participants were able to provide examples of how numbers, measurement, geometry, and/or data analysis benefit their daily and future lives. All advanced mathematics students saw themselves as good at mathematics, whereas low mathematics achievers did not feel sure about their mathematics abilities. This evidence showed that they were accurate in seeing their mathematics abilities in relation to their achievement. All of the participants agreed that mathematics was useful, one's mathematics ability was increased by effort, and no gender differences in mathematics occurred generally, even though in this class the students noticed that boys outperformed girls in mathematics. High and average mathematics achievers agreed that they can solve time-consuming mathematics problems and it was important to understand mathematics concepts.

Based on the Taxonomy of Affective Domain by Krathwohl, Bloom, and Masia (1964), the participants were in three different mathematical disposition levels. One of advanced mathematics participants, the leader, and one of average mathematics participants, the verifier, was categorized in mathematics disposition Level 3.2: "preference of value" because of their willingness to pursue and seek to do mathematics outside the classroom. Two of the advanced mathematics participants (the co-leader and sometimes the leader) were categorized in mathematics disposition Level 3.1: "acceptance of a value" because of their willingness to learn and explore new mathematical ideas. One advanced and two average mathematics achievers were in mathematics disposition Level 2.3: "satisfaction in response" because they enjoyed responding to the classroom activities. They liked doing mathematics because it was fun. However, they did not exhibit curiosity in learning mathematics. One participant in this study was categorized in mathematics disposition Level 1.2: "willingness to receive." She was a low mathematics achiever, the follower. She listened to the whole class discussion and her group discussion without sharing any ideas or asking for help. She was uncomfortable in doing mathematics. She hated mathematics when it did not make sense to her. She participated less and she understood less.

CHAPTER V

DISCUSSION AND CONCLUSION

Introduction

The purpose of this study was to investigate middle school students' mathematical dispositions in a problem-based classroom (using the Connected Mathematics Project [CMP]). The first section presents a discussion of the main findings. The second section describes limitations of the study. The third and final section provides implications for middle school mathematics education and includes recommendations for further research relating to understanding students' mathematical dispositions.

Conclusion and Discussion of the Main Findings

The discussion in this section is based on the main findings of the study with comparison to previous studies. First, Ms. Smith's experience in the CMP classroom is provided. Second, group work in the CMP class, including two cooperative work groups and whole class discussions, are considered. Third, students' mathematical dispositions are discussed.

Ms. Smith's Experience in the CMP Class

This section discusses Ms. Smith's classroom routine, which involved Warm-up, Launch, Explore, and Summarize. She demonstrated many positive teaching skills as well as a few areas where teaching strategies could be improved. The discussion includes the teacher's successes and difficulties in implementing the CMP curriculum.

After nearly two years experience teaching CMP, Ms. Smith taught each class in a sequence of four sections: Warm-up, Launch, Explore, and Summarize. She warmed-up her class by having them work on additional practice, mathematics reflection problems, Applications, Connections, and Extensions [ACE] problems, or homework. Next, she launched investigations to the whole class using real-world problems, or previously learned concepts. Then, she had her students explore the investigations in small groups of three or four. Finally, she led a whole class discussion of group results.

Mathematics teachers' weekly meetings allowed Ms. Smith to share and discuss ideas about using the curriculum with others. In addition, she learned how to use the CMP curriculum in more effective ways from a workshop she attended the preceding year in which she piloted the curriculum. Ms. Smith's experience reflected the same finding of Schoen, Finn, Griffin, and Fi' s study (2001), which noted that an effective Standards-based mathematics teacher needed to be well-prepared before teaching his/her Standards-based classes, including completing workshops to help them implement the curriculum.

"...Being well prepared in teaching mathematics classes, teachers need to be concerned with not only choosing appropriate tasks and structuring the lesson, but also forming good questions" (NCTM, 2003, p. 134). In preparation for teaching the teacher should be aware of providing stimulating higher-order thinking questions, which are difficult to conceive in the middle of the class (NCTM, 2003). Ms. Smith demonstrated her focus on preparation of tasks and lessons beforehand and less focus on higher-level questions. Although Ms. Smith sometimes asked her students to reason and make connections, she infrequently probed her students about their thoughts during classes. In the teacher interview, she did not mention preparing questions for her class ahead of time. To provide successful mathematics lessons, using questions is one of the means to spark intellectual involvement. Good questioning is valuable in directing students to develop their mathematical ideas and understandings (NCTM, 2003).

NCTM (2003) also noted that fully understanding a task is important in moving forward to complete the task. In her CMP class, Ms. Smith introduced the classroom tasks carefully. She helped her students understand the assignments by asking direct questions about what was given and what was to be found. She also repeated the assignments to the whole class when necessary. Finally, before starting activities, she usually asked again whether students understood the assignments or had questions, although she infrequently asked specific questions to check for conceptual understanding during classroom tasks.

Moust and Schmidt (1994) found that there were two important elements for facilitating. Facilitators must know how to establish a personal relationship and know what subject matter that students need to acquire. Ms. Smith's role in facilitating exemplified Moust and Schmidt's suggestions (1994). During the CMP class activities, she acted as a facilitator. She usually guided her students instead of telling them the answers. In a warm and approachable manner she asked her students about previously learned concepts in order to help them establish newly learned concepts. Ms. Smith usually smiled and was in a good mood when she was teaching her class.

Evenson and Hmelo (2000) noted that facilitators also performed a significant role in scaffolding students' thinking and self-assessment skills. Schmidt and Moust (2000) explained that facilitators should stimulate students' thinking skills by asking higher-level cognitive questions. In this CMP classroom, Ms. Smith asked few questions that stimulated students thinking skills.

In addition, facilitators need to help students become self-directed learners by asking particular questions such as "'Why did you request that information?' 'What do you especially hope to learn?' or 'What more do you need to know?'" (Hmelo & Lin, 2000, p. 230). Students' skills in self-directed learning will help them become life- long learners (Zimmerman & Lebeau, 2000).

During the CMP class sessions, Ms. Smith sometimes let her students talk across the class as long as they were on task. Ms. Smith did not regard the noise level as a classroom-management problem. Good and Brophy (1997) suggested that effective classroom management could be formed by carefully establishing classroom rules and procedures at the beginning of the school year and to constantly and reasonably implement the classroom rules and procedures in order to maintain effective classroom management.

Ms. Smith liked the CMP curriculum because it was meaningful and active, as well as fun, with hands-on activities for her students. However, she encountered some difficulties in implementing the curriculum. She found that the CMP assessment packages sometimes did not make sense to her students, especially to low achievers. She explained that the language in the test problems might be too complicated for them.

Also, due to budget cuts and reduction of school days, Ms. Smith felt stressed about teaching time. Nevertheless, the school provided valuable preparation time for the mathematics teachers in order to meet and discuss implementing problem-based mathematics curricula. Wilson and Lloyd (1995) reported that Standards-based mathematics teachers enjoyed implementing the curriculum since it had meaning and application. They also found that mathematics teachers who were new to a Standards-based curriculum had some difficulties directing students in group work. Ms. Smith sometimes had to remind her students that they all needed to do work and then share their work together.

Bay et al. (1999) noted that it was difficult to implement a Standards-based curriculum effectively. However, Lloyd and Wilson (1995) suggested that teaching a Standards-based curriculum would become easier as teachers gained experience in implementing the Standards-based curriculum. Ms. Smith felt she would learn more over time about teaching the curriculum. She also mentioned that she would feel more comfortable the longer she taught the program.

In summary, in the CMP classroom, Ms. Smith demonstrated having many good teaching skills, as well as a few skills she could develop to improve her teaching in the problem-based classroom. She was concerned about being well prepared before teaching the CMP lessons. However, good preparation also required forming good questions ahead of time in order to promote students' thinking skills. Ms. Smith presented her classroom assignments carefully and was aware of students' understanding of the assignments. During the classroom activities, she guided her students while they were solving problems. She used previously learned concepts to scaffold new concepts. She interacted with her students with a warm and approachable manner. For more effective facilitation, the facilitator also needs to stimulate students' thinking skills and self-assessment. These behaviors were not observed during this limited observation. Ms. Smith had success in using hands-on activities in her classroom. Her students enjoyed doing the activities. She also enjoyed teaching the CMP curriculum, although there were some difficulties for her in implementing it. She found that some of the language in the test problems was too complicated for some of her students. She expressed, having difficulty in directing student group work, as well as with classroom management. However, Ms. Smith felt that she would learn more about implementing the curriculum effectively over time. Bay et al. (1999) and Lloyd and Wilson (1995) had suggested that it was not easy to implement a Standards-based curriculum and it would become simpler with experience. Also, there was evidence that further professional development might assist teachers in implementing Standards-based curricula effectively (Cain, 2002; Schoen et al., 2001).

Group Work in the CMP Class

In Ms. Smith's class, the CMP class format included having students work in small groups as has been increasingly applied and recommended. Moreover, at the end of each lesson, the teacher had her students come together as a whole class. The details about the two cooperative work groups and the whole class discussion follow.

Two Cooperative Work Groups. In this CMP class the two cooperative work groups, the Cubic Melon and the Rhombus Square, showed three different leadership styles: independent, delegation of work, and co-leadership. The independent and delegation of work styles were found in the Cubic Melon group. Most often, the leader guided the group to work independently and then they shared their work later, which reflected the teacher's advice to their group. In contrast, the Rhombus Square group usually worked together led by the co-leaders. The group leaders usually shared their work and thought out-loud through the investigations. This group worked more cooperatively than the Cubic Melon group. The followers were able to get help and ask for help during the group work. High and low mathematics achievers played different roles in group work. The lower achievers usually followed the leads of the higher achievers.

The findings about the two cooperative groups revealed that the students were not formally taught how to work in a group. The two groups' interactions were different because they operated according to their own experiences and ideas of group work. Davidson (1990) noted that cooperative skills such as leadership communication must be taught as clearly and accurately as academic skills. In this CMP class, the teacher admitted in her interview that she should have taught her students how to work in groups at the beginning of the year. The teacher's role dramatically impacts students' engagement in group work (Sharan, 1990). Encouraging and stimulating student interaction during group work was one of the most important skills to help students learn (Sharan, 1990). Davidson (1990) added that students have to engage in face-to-face interaction while they are completing mathematics tasks. In this study, especially in the Cubic Melon group, the students were more often observed face-to-desk than face-to-face.

There is also a need for developing cooperative norms in order to prepare students to behave appropriately during group work (Sharan, 1990). In this CMP class, the teacher observed and occasionally gave feedback to her students when they worked in groups. She pointed out her main concern about group work, which was about individual responsibility for completing the tasks. She urged her students not to wait for someone in their group to finish the work for them, and emphasized that everybody had to be doing the tasks. However, positive interdependence among the students in each group was not observed during the study. Students need to gain "…the perception that one is linked with others in a way that one cannot succeed unless the others do…therefore, their work benefits one and one's work benefits them" (Davidson, 1990, p. 105). This view may be established through various methods such as a division of work or assigning specific roles to each group member (Davidson, 1990). NCTM confirmed that teachers could minimize the possibility of having "…the highest achieving students solve the problem and explain it to the other students" (NCTM, 2003, p. 135) by giving unique roles for each student to work on and insisting that each group member must be able to explain solutions.

The Whole Class Discussions. The teacher had her CMP class share and discuss their group work with the whole class following their investigation. The teacher called on students by names as well as calling for volunteers. In this CMP class, many boys raised their hands and volunteered to share their ideas; a few girls did this. More boys participated in the whole class discussion than girls, which Evensen and Hmelo (2000) have previously noted. They said the literature showed that grade K-12 teachers tend to call on boys more often and give them more positive feedback than girls. In Ms. Smith's class, girls received less feedback because of their lack of participation. A strategy to call on and provide feedback for both boys and girls equally in order to encourage both genders to participate in class activities might diminish gender differences in this CMP class.

NCTM (2003) suggested that teachers' reactions to students' responses and questions significantly affect the climate of the classroom, which impacts students' willingness to participate in class discussions. The CMP teacher in this study demonstrated her concerns about classroom atmosphere. She recognized students' efforts and mistakes as a way to learn. She also challenged her students to try to solve complex problems by expressing to them her confidence in their abilities. The teacher sometimes emphasized students' reasoning, but sometimes she moved to the new concepts or questions right after getting the answers from her students, without asking them to explain and/or verify their thoughts, or to check with other students for their thinking. This might have been caused by time constraints, which she had mentioned. The teacher stated in her post interview that she was pressured to complete the lesson in limited time because of the school day reduction.

Students' Mathematical Dispositions

A qualitative and quantitative analysis of all the findings revealed that the middle school students in this problem-based/Standards-based classroom had, for the most part, positive mathematical dispositions. In this study, the participants, who usually encountered real-world situations in the classroom investigations, believed that mathematics was about knowing how to apply mathematics concepts in real-world situations. They believed that mathematics was figuring out mathematics problems in their lives and they mentioned that mathematics was found everywhere in their lives. One of the participants even mentioned, "Mathematics is life." These beliefs correspond with the result from Higgins' study (1997), which noted that Standards-based students believed that mathematics was more than facts and procedures. Cobb, Wood, Yackel, and McNeal (1992) suggested that consistent systems of beliefs about mathematics are acquired as students participate in classroom activities. Accordingly, the CMP classroom context with rich real-world problems might allow students to perceive connections between mathematics in classrooms and in their daily and future lives.

Most of the students who participated in the CMP activities expressed positive attitudes and beliefs about mathematics. The students in the PBL-classroom viewed mathematics as a useful subject, both for their daily and their future lives such as in baking and shopping. Higgins (1997) and Bay et al. (1999) also found that Standards-based students were able to explain and provide various examples about the usefulness of mathematics in their lives, such as in shopping and cooking. Perception of the usefulness of mathematics is a significant element in students' willingness to take future mathematics coursework and in their career interests (Thorndike-Christ, 1991). In this study, all of the participants could articulate how mathematics was necessary for their future lives even though some of them had not yet settled their career goals. Moreover, most of the participants would definitely elect to take mathematics in high school as well as other subjects they thought would also benefit their lives. For example, when some participants were interested in being scientists; then they would take the same amount of mathematics and sciences.

The CMP study also revealed that most participants believed in their abilities to solve time-consuming mathematics problems. Additionally, they agreed that understanding concepts in mathematics was important. The participants believed that effort in doing mathematics could help to increase their mathematical abilities. Higgins (1997) also reported that students in Standards-based classrooms agreed that they had to try hard in order to succeed in mathematics. Bandura (1997) noted that when students perceived that they could accomplish an assignment or subject, they were more likely to carry it out and they increased the value of the subject. The beliefs about learning mathematics and beliefs about self among the CMP students might be formed during participation in the CMP activities. Kloosterman (1994) noted that students were shaping beliefs about how one learns mathematics while they received mathematics instruction. Bandura (1997) mentioned that beliefs about self, as a student, were developed through (mathematics) classroom experiences.

Most of the students in this study showed confidence in doing mathematics. They also enjoyed learning mathematics and working in groups and agreed that group work was useful. Most liked hands-on activities and mathematics projects. Previous research revealed similar findings. Bay et al. (1999) reported that Standards-based students realized that mathematics projects helped them to know how to use and apply mathematics in real-life situations. Additionally, Bay et al.(1999) and Schoen and Pritchett (1998) found that Standards-based students enjoyed learning mathematics, doing hands-on activities, and working in groups in a Standards-based classroom. Sharan (1990) noted that cooperative group learning promoted positive social skills among peers in small group work; it motivated group members to work together toward their goals.

Most of the participants displayed their mathematical dispositions at the responding level according to the Taxonomy of Affective Domain by Krathwohl, Bloom, and Masia (1964), which involved acquiescence, willingness, and satisfaction in response. These behaviors were observed during the classroom activities such as volunteering to answer questions. Other participants in this study at higher levels exhibited intrinsic motivation. They were interested and enjoyed exploring and figuring out mathematics problems rather than working only to attain a good grade. Two of the participants revealed that they valued learning mathematics. They participated in mathematics related activities outside their classrooms, such as doing additional mathematics problems during their free time. Boaler (2002) also found that Standards-based students demonstrated a higher level of intrinsic motivation than students in traditional mathematics classrooms. Pintrich, Marx, and Boyle (1993) suggested that students' intrinsic motivation in learning involved valuing learning something, such as realizing the importance of it for their future lives. Then they were aware of a need and/or interest to proceed to work toward thinking through instead of just finishing it.

All participants but one, who was not sure, agreed that there were no gender differences in mathematics. They believed that boys and girls could do well in mathematics. Although, in this CMP class, the students realized that girls did not achieve as high as boys in mathematics. Boaler (1998) reported that gender differences were not found in the problem-based approach setting. Perhaps, in this CMP setting, the teacher needed to focus on making sure that both boys and girls were equally called on during class activities as recommended by Gurian and Henley (2001). The authors suggested that teachers should be aware of bias against girls in the mathematics classroom. They added that "showcasing girls", by asking girls to take leadership roles, might be another solution. This action could help girls gain confidence in their achievement in mathematics.

In this study, students who actively participated in classroom discussions and cooperative group work demonstrated high-level mathematical dispositions. They usually volunteered or shared their ideas in the whole-class discussion or small group work. Students with high-level mathematical dispositions usually showed their curiosity in learning new mathematics by asking questions related to the topic or finding interesting and alternative ways to solve problems. Moreover, they pursued mathematics activities as much as possible, even outside the classroom or without assignments from the teacher.

In contrast, students at lower mathematical dispositions levels either participated in the classroom activities at the minimum requirement or did not show interest or curiosity in learning new concepts. The students who carried lower mathematical dispositions put less effort into solving difficult mathematics problems or were just satisfied with getting a good grade even if understanding was not achieved.

Two high and two average mathematics achievers demonstrated a mathematical disposition at Level 3: "valuing." They were curious about learning mathematics concepts and sought to do additional mathematics problems or related mathematics activities. The third high mathematics achiever and the other two average mathematics achievers showed their mathematical dispositions at Level 2: "responding." They volunteered in mathematics classroom activities and sometimes shared their ideas in the group and in class discussions. The low mathematics achiever performed at a low level of mathematical disposition at Level 1: "receiving." The low achiever rarely participated in any classroom activities; however, she listened to the whole class and small group discussions with respect. There could be a relationship between students' mathematical dispositions and mathematics achievement, but the findings in this study could not conclusively portray this relationship.

The findings about students' mathematical dispositions were revealed in small and large group work in the problem-based classroom using Krathwohl et al.'s taxonomy (1964) and were verified using interviews and questionnaires. Sometimes, observational data differed slightly from interview data regarding students' dispositions. The variety of sources of data helped to verify the students' mathematical dispositions. If such cases, observation data was considered first because it revealed the students' reactions during participating the CMP classroom activities. The taxonomy significantly helped in classifying each student's mathematical disposition level. The application of the Taxonomy of Affective Domain by Krathwohl, Bloom, and Masia (1964) is considered new information for describing students' dispositions for the literature in this area.

Limitations of the Study

This study aimed to provide descriptive information about one problem-based classroom. According to its design, it had several limitations. This study employed a qualitative method and design to describe 6th-grade students' mathematical dispositions in this classroom. Even though this study provided a detailed picture of middle school students' mathematical dispositions in one CMP classroom, generalizing the results was not a purpose of this study.

The participating school was a public middle school where the problem-based curriculum was implemented. The school was located in a mostly middle to upper level socio-economic community, which influenced both the school's culture and activities. The explanation of the school and classroom activities should be considered according to this context.

Also it should be noted that there was difficulty finding a volunteer teacher experienced in teaching mathematics and in using the problem-based approach. In addition, several schools using the problem-based curricula faced an economic crisis; their budgets and several school days were eliminated at the time of the research. Therefore, some of the more experienced teachers could not afford to spend time participating in extra research activities. Due to these facts, the teacher in this study may not necessarily represent all problem-based teachers in middle schools in the Northwest.

The data collection process limited the results of this study. First, this study was conducted during two months over one unit (measurement). The short time of the study may have limited the outcomes. The observations of an additional unit, such as geometry, numbers, or probability might provide more comprehensive conclusions about the CMP classroom.

Second, due to a reduction of school days, the classroom activities were rushed at the end of the study. Some students' interviews, which were conducted during the last week of the study, were also rushed. Some students who were interviewed had other assignments to finish that day and worried about getting the assignments done. Moreover, noises outside the interview room occurred as boxes of teaching materials were being moved from one classroom to another classroom, and as students were cleaning out their desks and lockers. This distracted both the researcher and the participants during the final student interviews.

Next, the students' responses to the questionnaire and the interview questions were another limitation of the study. The participants were supposed to respond to the questionnaire and interview questions honestly. However, it was possibly did not. They might not have realized that their authentic responses were more important than conceivable "right" answers in learning about students' mathematical dispositions in a problem-based/Standards-based classroom. It's also possible that their beliefs and attitudes were not yet well formed.

Finally, one of the major instruments in any qualitative study is the researcher, the researcher is the one who collects the data and interprets the findings. Therefore, some bias from the researcher was unavoidable. The researcher's perception may have generated bias during data collection. The classroom observation protocol and the interview protocol were established to reduce possible bias. However, sometimes during in the interviews, the researcher did not ask follow-up or probing questions. Also, there were some inconsistencies between students to the questionnaire and interview questions and their behaviors in the classroom. These discrepancies might have been due to the inexperience of the researcher, students' emotions during the observations or interviews, and/or the environment or atmosphere of the classroom. The researcher's journal, which was recorded daily during the study, was reviewed for any possible bias during the observations and interviews. To reduce bias, leading questions and matching responses were eliminated and not included in the analysis.

Implications and Recommendations for Further Research

The description of middle school students' mathematical dispositions in a problem-based classroom revealed several implications about teaching and learning mathematics. Additionally, the findings about middle school students' mathematical dispositions also provided information for making recommendations for future research for middle school education. The implications and recommendations are as follows.

First, the findings indicate that most participants had positive dispositions including attitudes and beliefs about mathematics. They liked mathematics and enjoyed hands-on activities. They believed that there were no gender differences in mathematics. All of them agreed that effort could help increase mathematical ability. Most of them believed that understanding concepts in mathematics was more important than getting a right answer. All of them expressed that mathematics was useful. They were able to provide or explain examples about the usefulness of mathematics. These comments show that the new problem-based approach might help students to see that mathematics is relevant in their lives. However, to confirm the findings, research about how the problem-based approach affects students' mathematical dispositions, including attitudes and beliefs about mathematics, needs to be conducted, not only in the United States, but also in other countries in order to gain a broader perspective.

Second, this study suggests that the teacher had an effect on cooperative group work. The Cubic Melon group tended to first work independently and then share their work later as the teacher suggested. In contrast, the Rhombus Square group worked together actively though the investigations. The two groups had different experiences with group work due to the different group approaches. Ms. Smith tried to avoid having only one person work through the assignment alone. She suggested that her students work individually and then share their work later. However, Johnson and Johnson (1994) suggested that cooperative group work involves more than sitting next to each other, discussing, helping, or sharing materials with each other; it requires positive interdependence, promoting each other's learning, fair sharing of the group work, and interpersonal and small group skills. Moreover, Johnson and Johnson (1994) noted that the teacher's role in facilitating cooperative group work was crucial. Since cooperative group learning is one of the important components of the problem-based/Standards-based approach, focusing on this aspect would be beneficial to Standards-based teachers. Further research about effective teachers' roles in cooperative group work in Standards-based classrooms is suggested.

Third, in order to effectively implement the problem-based curriculum, this study shows that mathematics teachers need more time for preparation. Schools have an important responsibility to provide time for teachers' preparation. Also, teachers could learn from their meetings and discussions about their experiences in using problem-based curricula. In addition, attending workshops about how to use problem-based curricula may be helpful for mathematics teachers. Bay et al.(1999) suggested that there were 10 essentials in implementing Standards-based mathematics curricula, including administrative support; opportunities to study; sampling the curricula; daily planning; interaction with experts; collaborating with colleagues; incorporating new assessments;

communicating with parents; helping students adjust; and planning for transition. Research on how to prepare mathematics teachers to effectively implement the problem-based approach in effective ways needs to be pursued.

Fourth, this study reveals possible gender differences in learning mathematics. In this CMP class, the students realized that boys outperformed girls. Some previous studies found no gender differences in Standards-based classes and suggested that Standards-based curriculum might have a potential to decrease gender differences in mathematics classrooms (Boaler, 1998; Boaler, 2002). Since, finding an answer to what factors increase/decrease gender differences in the PBL setting was not the purpose of this study, research on Standards-based curricula and gender differences needs to be expanded.

Fifth, the findings show that none of the participants were classified higher than mathematical disposition Level 3: "valuing," although the taxonomy had five levels. There were no students in this study who demonstrated mathematical dispositions at Level 4: "organization" and Level 5: "characterization by a value or value complex." Some factors, such as student age, may account for this gap. The students may develop higher levels of mathematical dispositions as they mature. Moreover, a variety of causes relating to promoting or prohibiting the development of students' mathematical dispositions probably exist. Future research in this area is recommended.

Finally, even though in this study, high mathematics achievers in this study seemed to have positive mathematical dispositions, this study did not aim to investigate about relationships between students' mathematical dispositions and mathematics achievement. Therefore, additional research may be conducted that focuses on the relationships between students' mathematical dispositions and mathematics achievement in problem-based classrooms.

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APPENDICES

APPENDIX A

Letter to a School Principal

Date___________

Dear __(School Principal's Name),

My name is Ms. Duanghathai Katwibun. I am a former middle school and high school mathematics teacher in Thailand and am currently pursuing my doctoral degree at Oregon State University.

I am interested in researching middle school students' mathematics dispositions, including attitudes and beliefs about mathematics, in a Standards-based classroom. I believe this study will benefit mathematics students, mathematics teachers, and the mathematics educators by providing in-depth information about how students think and feel about mathematics in a Standards-based classroom.

A teacher from your school, (teacher's name), is interested in participating in this study. I am therefore writing to request your permission to conduct the research at your school. The study description is enclosed (please see general information sheet). The study will be conducted for approximately three months (until the end of the school year, 2003) and will not interrupt the regular classroom activities. I would like to observe a mathematics classroom, administer an Attitude and Belief Questionnaire, and interview some students in order to collect the data for my dissertation. I assure you that the teacher, all individual students, and school identities will be kept confidential to the extent permitted by law.

If I get your permission to proceed, I will seek permission from parents and students to participate in the study.

This study is under the supervision of Dr. Dianne K. Erickson. For further information or to address any questions you may have, please do not hesitate to contact me or my advisor at:

Duanghathai Katwibun
Oregon State University
Dept. Science and
Mathematics Education
239 Weniger Hall
Corvallis, OR 97331
Phone: 541-753-9085
E-mail: katwibud@onid.orst.edu

Or

Dr. Dianne K Erickson
Oregon State University
Dept. Science and
Mathematics Education
239 Weniger Hall
Corvallis, OR 97331
Phone: 541-737-1821
E-mail: ericksod@onid.orst.edu

Thank you for your time and consideration, and I look forward to hearing from you by (MM/DD/YY).

Sincerely,

Duanghathai Katwibun

APPENDIX B

Script for Volunteers

March 2003

My name is Duanghathai Katwibun and I am a graduate student at Oregon State University. In the past I was a math teacher in Thailand. I would like to conduct a study about students' attitudes and beliefs about mathematics and I need some volunteers from your class.

I will be present in your classroom until the end of this school year (spring term, 2003), observing your work activities. For my study, I need some volunteers who will take a 20-30 minute questionnaire. I will also ask for your math grades, for the observed units only, from your teacher. In addition, 6-8 volunteer students will be selected for observation with audio and videotaping. During the study, the 6-8 students will be interviewed briefly for approximately 5 minutes once a week about their experience in the mathematics classroom. At the end, these 6-8 students will be interviewed individually one last time for 15-20 minutes to express their feelings and thoughts about mathematics. Your mathematics teacher will not participate in any student interviews.

Your identity will be protected. For example, you will be called by a fake name during the interviews. Assigned identification numbers will be given to all volunteer students to use on the Attitude and Belief Questionnaire. Also, the assigned numbers will be used in obtaining the volunteer students' mathematics grades for the units covered during the study. After the study is completed, all collected data will be destroyed.

Here is an information sheet for you and your parent/guardian to read and sign if you are willing to participate. If you or your parent/guardian have any questions about the study, please don't hesitate to contact me or my advisor. You do not have to volunteer and will not be punished if you choose not to participate. Also, you can withdraw from the study at any time. Whether or not you choose to participate in this study will not affect your mathematics grade or your relationship with your mathematics teacher. Students who do not want to participate in the observation phase will be kept off camera and their comments will be stricken from the audiotape.

I hope you will be interested in participating in this study. If you think you would like to participate, take an information sheet and a consent form. The consent form must be signed by you and your parent/guardian. If you decide to participate, please hand in the consent form to your mathematics teacher by (MM/DD/YY). Remember, you can change your mind about participating at any time. Thank you for your participation.

APPENDIX C

Teacher Consent Form

Project Title:	**Middle School Students' Mathematics Dispositions in a Problem-Based Classroom**
Principal Investigator:	**Dr. Dianne K. Erickson**
Research Staff:	**Ms. Duanghathai Katwibun**

PURPOSE

This is a research study. The purpose of this research study is **to explore middle school students' mathematics dispositions, including attitudes and beliefs about mathematics, in a Standards-based classroom.** The purpose of this consent form is to give you the information you will need to help you decide whether to be in the study or not. Please read the form carefully. You may ask any questions about the research, what you will be asked to do, the possible risks and benefits, your rights as a volunteer, and anything else about the research or this form that is not clear. When all of your questions have been answered, you can decide if you want to be in this study or not. This process is called "informed consent." You will be given a copy of this form for your records.

We are inviting you to participate in this research study because you **have years of experience in teaching mathematics and implementing the Standards-based approach. One of your mathematics classes will be involved in this study.**

PROCEDURES

If you agree to participate, your involvement will last for approximately **three months (until the end of the school year, 2003).** The following procedures are involved in this three-phase study:

Phase I is the observation phase. With permission from the school and parents, one class of a volunteer mathematics teacher, using the Standards-based approach to teaching, will be observed daily for approximately three months. At the beginning of the observation phase, I will need your help in selecting 6-8 volunteer students with varying mathematics performance levels and to group these 6-8 students into two groups of 3-4 students to focus on during the observations. Audiotape and videotape recordings will be used during the observation phase for back-up data. Students who do not want to participate in the observation phase will be kept off camera and their comments will be stricken from the audiotape. Then, the 6-8 students will be observed as they participate in classroom activities and group work. Once a week, these 6-8 students will be informally interviewed, for approximately 5-10 minutes during their free time at school, about their classroom participation and mathematics dispositions. These interviews will not be audio or video taped. Additionally, classroom artifacts (including worksheets, tests, and quizzes) will be obtained from the teacher.

Phase II of this study involves the completion of the Attitude and Belief questionnaire. After the three-month observation, all volunteer students will be asked to respond to the Attitude and Belief Questionnaire during one math class period. The questionnaire is designed to take approximately 20-30 minutes. Students who choose not to participate in the questionnaire phase of this study will be asked to work independently in the school library. In addition, all volunteer students' mathematics grades (for the observed units only) will be obtained from the teacher. Since research has revealed inconclusive findings about the relationship between students' mathematics dispositions and mathematics performance, the researcher would like to collect students' mathematics grades (for the observed units only) to gain insight about students' mathematics dispositions and mathematics performance in this classroom.

Phase III incorporates student interviews. The 6-8 students will be interviewed individually during their free time at school for approximately 15-30 minutes in order to gain in-depth information about their mathematics dispositions. In the interview phase, audiotaping will be used for accuracy. The classroom teacher will not participate in student interviews. To protect the participants' confidentiality, each student will be assigned an identification (ID) number that will be used in place of his/her name. Students participating in the interview phase of this project will be given the opportunity to pick a pseudonym. In addition, audio and videotapes will be kept in a secure location. Finally, the selected teacher will be interviewed for approximately 45-60 minutes in order to obtain the teacher's opinions of his/her teaching with the Standards-based approach during the study.

RISKS

There are no foreseeable risks to participating in this study.

BENEFITS

The potential personal benefit that may occur as a result of your participation in this study is having an opportunity to share your thoughts and experiences in teaching mathematics using the Standards-based approach.

The researcher anticipates that the mathematics education society may benefit from this study by obtaining more in-depth information about students' mathematics dispositions, including attitudes and beliefs about mathematics, in a Standards-based setting.

CONFIDENTIALITY

Records of participation in this research project will be kept confidential to the extent permitted by law. All audiotapes, videotapes, and documents from the participants will be kept in a secure location.

Steps will be taken to preserve participants' identities, such as using assigned student ID numbers in obtaining students' responses to the Attitude and Belief Questionnaire and students' mathematics grades (for the observed units only), using pseudonyms during the audio taped interviews, allowing only the researcher access to the audio and videotapes, and destroying the tapes as soon as they have been transcribed. In the event of any report or publication from this study, participants' identities will not be disclosed. Results will be reported in a summarized manner in such a way that participants cannot be identified.

VOLUNTARY PARTICIPATION

Taking part in this research study is voluntary. You may choose not to take part at all. If you agree to participate in this study, you may stop participating at any time. If you decide not to take part, or if you stop participating at any time, your decision will not result in any penalty or loss of benefits to which you may otherwise be entitled. **Any data collected from the participant prior to withdrawal will be destroyed. Also, the participant is free to skip any questions in the Attitude and Belief Questionnaire and in the interviews that s/he would prefer not to answer.**

QUESTIONS

Questions are encouraged. If you have any questions about this research project, please contact: Dr. Dianne K. Erickson at (541) 737-1821 or at ericksod@onid.orst.edu or Duanghathai Katwibun at (541) 753-9085 or at katwibud@onid.orst.edu. If you have questions about your rights as a participant, please contact the Oregon State University Institutional Review Board (IRB) Human Protections Administrator, at (541) 737-3437 or by e-mail at IRB@oregonstate.edu or by mail at 312 Kerr Administration Building, Corvallis, OR 97331-2140.

Your signature indicates that this research study has been explained to you, that your questions have been answered, and that you agree to take part in this study. You will receive a copy of this form.

Participant's Name (printed)____

_________________________________ ______________________
(Signature of Participant) (Date)

RESEARCHER STATEMENT

I have discussed the above points with the participant or, where appropriate, with the participant's legally authorized representative, using a translator when necessary. It is my opinion that the participant understands the risks, benefits, and procedures involved with participation in this research study.

_______________________________________ __________________
(Signature of Researcher) (Date)

APPENDIX D

Letter to Parent/Guardian

To parent/guardian:

Hello! My name is Ms. Duanghathai Katwibun. As an Oregon State University doctoral student and former mathematics teacher in Thailand, I am interested in what middle school students believe and think about math. As part of my doctoral study, I will be conducting a study in your child's classroom. A description of the study is provided along with this letter (see general information sheet).

My research study has been designed to focus on students' dispositions, especially attitudes and beliefs about mathematics. This study is under the supervision of Dr. Dianne K. Erickson. The study will be conducted for approximately two to three months (until the end of spring term, 2003) and will not interrupt regular classroom activities.

Participation in the study is not required, but students who volunteer will help us learn more about how students feel and think in mathematics classrooms. At the end of the study, the volunteer students will be asked to respond to an Attitude and Belief Questionnaire and the researcher will ask for their math grades (for the observed units only) from their math teacher. If your son or daughter chooses to participate in this study, s/he may be selected as a focus student (6-8 students) in the observations. The selection of the 6-8 students will be based on their variety of mathematics performance levels. Once a week during the study, these 6-8 students will be informally interviewed during their free time at school for approximately 5-10 minutes about their classroom participation and mathematics dispositions. These interviews will not be audio or video taped. At the end of the study, the 6-8 students will be interviewed individually about their attitudes and beliefs about mathematics for approximately 15-20 minutes during their free time at school.

During the three-month observation, video and audio recordings will be used for back-up data. However, if you, as parent/guardian, are not comfortable with using videotape recordings during classroom observations, please feel free to contact me (the researcher) or my dissertation advisor in order to discuss your concerns. Whether or not a student participates in this study will have no effect on his/her math grade or his/her relationship with the teacher.

Also, the participant is free to skip any questions in the Attitude and Belief Questionnaire and in the interviews that s/he would prefer not to answer. Audiotape and videotape recordings will be used for back-up data. Students who do not want to participate in the observation phase will be kept off camera and their comments will be stricken from the audiotape. The classroom teacher will not participate in the student interviews. The 15-30 minute interview sessions will be audiotaped for back-up information as well.

The videotapes and audiotapes will be kept in a secure place with access only given to me, the researcher. Steps will be taken to preserve participants' identities, such as using assigned student ID numbers in obtaining students' responses to the Attitude and Belief Questionnaire and students' mathematics grades (for the observed units only), using pseudonyms (fake names) during the audiotaped interviews, allowing only the researcher access to the tapes, and destroying the tapes as soon as they have been transcribed. In the event of any report or publication from this study, your child's identity will not be disclosed. Results will be reported in a summarized manner in such a way that the participants cannot be identified.

I hope you will agree to allow your son or daughter to participate in this study. If you decide to allow participation, please sign the enclosed consent form.

The consent form must be signed by both student and his/her parent/guardian.

Note: Students can change their mind or drop out of the study at any time, even if they have signed the consent form, without any penalty.

Please return the consent form to (teacher's name) by (date). If you have any questions about the study, please contact me or my advisor at:

Duanghathai Katwibun
Oregon State University
Dept. Science and
Mathematics Education
239 Weniger Hall
Corvallis, OR 97331
Phone: 541-753-9085
E-mail: katwibud@onid.orst.edu

Or

Dr. Dianne K Erickson
Oregon State University
Dept. Science and
Mathematics Education
239 Weniger Hall
Corvallis, OR 97331
Phone: 541-737-1821
E-mail: ericksod@onid.orst.edu

Sincerely,

Duanghathai Katwibun

APPENDIX E

General Information Sheet

-What is the purpose of this study?

The purpose of this study is to investigate students' mathematical dispositions, which include attitudes and beliefs about mathematics, in a Standards-based classroom. This study is my dissertation for the doctoral program at Oregon State University.

-What will happen if students decide to participate in this study?

As part of the participation in this study, all students will be invited to respond to an Attitude and Belief Questionnaire, but only 6-8 students will be selected for the observations and to participate in the interview phase of this study. The selection of the 6-8 students will be based on their variety of math performance levels. Once a week during the study, these 6-8 students will be informally interviewed, during their free time at school for approximately 5-10 minutes, about their classroom participation and mathematics dispositions. These interviews will not be audio or videotaped. The math class will be observed until the end of the school year, 2003. During the observations, videotapes and audiotapes will be used for back-up data. After the observations, the 6-8 students will be interviewed on their attitudes and beliefs about mathematics for 15-30 minutes during their free time at school. I will audiotape students' interviews for accuracy. The teacher will not participate in interviewing the students. The 6-8 students will be called by pseudonyms (fake names) to use for the student interviews. At the end of the study, all volunteer students will be asked to respond to the Attitude and Belief Questionnaire for approximately 20-30 minutes during one math class. Students who choose not to participate in the questionnaire phase of this study will be asked to work independently in their school library. In addition, all volunteer student's mathematics grades (for the observed units only) will be collected from their teacher by using the students' assigned identification (ID) numbers. All collected data will be kept in a secure location.

-What will happen with the findings of this study?

The study's findings will be shared with my dissertation committee and the students and faculty at Oregon State University. Only the researcher can access the collected data, including observation and interview notes, videotapes and audiotapes, the students' questionnaire responses, classroom documents, and student mathematics grades (for the observed units only). The collected data will be kept confidential to the extent permitted by law.

-How will students' identities be kept confidential?

Each student will be assigned an identification (ID) number, which will be used in place of the student's name to record any information obtained from the student. Students participating in the interview phase of this project will be able to pick a pseudonym to be used during the interviews to help maintain each individual's confidentiality.

-Do students have to participate in this study?

Participation is voluntary and will not affect a student's grade or relationship with the teacher. Anyone can withdraw from the study at any time with no penalty.

If you would like more information about this study or specific procedures, please contact: Dr. Dianne K. Erickson at (541) 737-1821 or at ericksod@onid.orst.edu or Duanghathai Katwibun at (541) 753-9085 or at katwibud@onid.orst.edu. If you have questions about your child's rights as a research participant, please contact the Oregon State University Institutional Review Board (IRB) Human Protections Administrator, at (541) 737-3437 or by e-mail at IRB@oregonstate.edu or by mail at 312 Kerr Administration Building, Corvallis, OR 97331-2140.

APPENDIX F

Student/Parent Consent Form

Project Title: **Middle School Students' Mathematics Dispositions in a Problem-Based Classroom**
Principal Investigator: **Dr. Dianne K. Erickson**
Research Staff: **Ms. Duanghathai Katwibun**

PURPOSE

This is a research study. The purpose of this research study is **to explore middle school students' attitudes and beliefs about mathematics in a Standards-based classroom.** The purpose of this consent form is to give you the information you will need to help you decide whether to be in the study or not. Please read the form carefully. You may ask any questions about the research, what you will be asked to do, the possible risks and benefits, your rights as a volunteer, and anything else about the research or this form that is not clear. When all of your questions have been answered, you can decide if you want to be in this study or not. This process is called "informed consent." You will be given a copy of this form for your records.

We are inviting you to participate in this research study because you **are in a mathematics classroom using the Standards-based curriculum. Moreover, your mathematics teacher has years of experience in teaching mathematics and implementing the curriculum.**

PROCEDURES

If you agree to participate, your involvement will last for approximately **three months (until the end of the school year, 2003).** This study is divided into three phases as following:

Phase I is the observation phase. If you choose to participate in this study, you may be selected as a focus student (6-8 students) in the observations. The selection of the 6-8 students will be based on their variety of mathematics performance levels. Audiotape and videotape recordings will be used for back-up data. Students who do not want to participate in the observation phase will be kept off camera and their comments will be stricken from the audiotape. Then, the 6-8 students will be observed as they participate in classroom activities and group work. Once a week during the observations, these 6-8 students will be informally interviewed, for approximately 5-10 minutes during their school's free time, about their classroom participation and their attitudes and beliefs about mathematics. These interviews will not be audio or video taped. Additionally, classroom artifacts (including worksheets, tests, and quizzes) will be collected from the teacher.

Phase II of this study involves the completion of the Attitude and Belief Questionnaire. After observing the class for approximately three months, all volunteer students will be asked to respond to the Attitude and Belief Questionnaire during one math class period. The questionnaire is designed to take approximately 20-30 minutes. Students who choose not to participate in the questionnaire phase of this study will be asked to work independently in the school library. In addition, all volunteer students' mathematics grades (for the observed units only) will be obtained from the teacher. Since research has revealed inconclusive findings about the relationship between students' attitudes and beliefs about mathematics and mathematics performance, the researcher would like to collect students' mathematics grades (for the observed units only) for additional data in order to gain insight into students' attitudes and beliefs about mathematics and mathematics performance in this classroom.

Phase III incorporates student interviews. The 6-8 students will be scheduled for a final interview during their school's free time for 15-30 minutes individually in order to gain in-depth information about their attitudes and beliefs about mathematics. In the interview phase, audiotaping will be used for accuracy. The classroom teacher will not participate in student interviews. To protect the student participants' confidentiality, all students will be assigned identification (ID) numbers that will be used in place of student names in responding to the Attitude and Belief Questionnaire and obtaining volunteer students' mathematics grades (for the observed units only). Students participating in the interview phase of this project will be called by a pseudonym (fake name). In addition, audio and videotapes will be kept in a secure location.

RISKS

There are no foreseeable risks to participating in this study.

BENEFITS

The potential personal benefit that may occur as a result of students' participation in this study is having an opportunity to share his/her feelings and thoughts in more depth than may be possible without participating in this research project.

The researcher anticipates that the mathematics education society may benefit from this study by obtaining more in-depth information about students' attitudes and beliefs about mathematics in a Standards-based setting.

CONFIDENTIALITY

Records of participation in this research project will be kept confidential to the extent permitted by law. All audiotapes, videotapes, and documents from the participants will be kept in a secure location.

Steps will be taken to preserve participants' identities, such as using assigned student ID numbers in obtaining students' responses to the Attitude and Belief Questionnaire and students' mathematics grades (for the observed units only), using pseudonyms during the audiotaped interviews, allowing only the researcher access to the tapes, and destroying the tapes as soon as they have been transcribed. In the event of any report or publication from this study, participants' identities will not be disclosed. Results will be reported in a summarized manner in such a way that participants cannot be identified.

VOLUNTARY PARTICIPATION

Taking part in this research study is voluntary. You may choose not to take part at all. If you agree to participate in this study, you may stop participating at any time. If you decide not to take part, or if you stop participating at any time, your decision will not result in any penalty or loss of benefits to which you may otherwise be entitled. **Any data collected from the participant prior to withdrawal will be destroyed. A student's participation or lack thereof in this study will have no effect on his/her math grade or his/her relationship with the teacher. Also, the participant is free to skip any questions in the Attitude and Belief Questionnaire and in the interviews that s/he would prefer not to answer.**

QUESTIONS

Questions are encouraged. If you have any questions about this research project, please contact: Dr. Dianne K. Erickson at (541) 737-1821 or at ericksod@onid.orst.edu or Duanghathai Katwibun at (541) 753-9085 or at katwibud@onid.orst.edu. If you have questions about your child's rights as a participant, please contact the Oregon State University Institutional Review Board (IRB) Human Protections Administrator, at (541) 737-3437 or by e-mail at IRB@oregonstate.edu or by mail at 312 Kerr Administration Building, Corvallis, OR 97331-2140.

Your signature indicates that this research study has been explained to you, that your questions have been answered, and that you agree to take part in this study. You will receive a copy of this form.

Parent/Guardian:

Phase I: classroom observation (6 – 8 students only), which includes video and audio taping,

(Please initial) _______I give permission for my son or daughter to participate in the observation phase of this study and
☐ to be audio taped and/or ☐ video taped

(Please initial) _______I **DO NOT** give permission for my son or daughter to participate in the observation phase of this study and expect that he/she will not appear in the videotape and/or audiotape recordings.

<u>Phase II:</u> questionnaire, which involves administrating the Attitude and Belief Questionnaire and accessing your child's math grades (for the observed units only)

(Please initial) _______I give permission for my son or daughter to participate in the Attitude and Belief Questionnaire phase of this study and

☐ give permission for the research team to have access to my child's math grades (for the observed units only)

(Please initial) _______I **DO NOT** give permission for my son or daughter to participate in the Attitude and Belief Questionnaire phase of this study, or to have his/her mathematics grade (for the observed units only) disclosed.

<u>Phase III:</u> individual interviews (6 – 8 students only), which involve audio taping

(Please initial) _______I give permission for my son or daughter to participate in the individual interview phase of this study and

☐ to be audio taped

(Please initial) _______I **DO NOT** give permission for my son or daughter to participate in the individual interview phase of this study.

Students:

<u>Phase I</u>: classroom observation (6 – 8 students only), which includes video and audio taping

(Please initial) _______I would like to participate in the observation phase of this study and agree

☐ to be audio taped and/or ☐ video taped

(Please initial) _______I **DO NOT** want to participate in the observation phase of this study and expect that I will not appear in the videotape and/or audiotape recordings.

<u>Phase II:</u> questionnaire, which involves administrating the Attitude and Belief Questionnaire and accessing your mathematics grades (for the observed units only)

(Please initial) _______I would like to participate in the Attitude and Belief Questionnaire phase of this study and

☐ to give permission for the research team to have access to my math grades (for the observed units only)

(Please initial) _______I **DO NOT** want to participate in the Attitude and Belief Questionnaire phase of this study. I do not want my math grade (for the observed units only) to be disclosed.

Phase III: individual interviews (6 – 8 students only), which involve audio taping

(Please initial) _______I would like to participate in the individual interview phase of this study and agree

☐ to be audio taped

(Please initial) _______I **DO NOT** want to participate in the individual interview phase of this study.

Student's Name: ____

____ ____________________
(Parent or Guardian's signature) (Date)

____ ____________________
(Student's signature) (Date)

RESEARCHER STATEMENT

I have discussed the above points with the participant or, where appropriate, with the participant's legally authorized representative, using a translator when necessary. It is my opinion that the participant understands the risks, benefits, and procedures involved with participation in this research study.

____ ____________________
(Signature of Researcher) (Date)

APPENDIX G

Attitude and Belief Questionnaire

Part I: Demographic Information

Student's Identification Number: ____________________

Gender: __________

Grade Level: __________

Age: __________

1. Please, tell me about your 5th grade math:
    a. How did you learn math in 5th grade?

____

____

    b. Did you have an experience in working in a group in elementary grade level? Explain?

____

____

2. Please, tell me what your math classes were like in 1st to 4th grades:

    a. Did you just sit and listen to the teacher most of the time? Explain?

____

____

    b. Did you work in groups and discuss with each other most of the time? Explain?

____

____

c. Or other? Explain?

3. How did you do in math in 1st to 5th grades? Can you explain why?

4. How often (many days a week) did you do your math homework that was assigned by the teacher?

5. How many times each week did you finish your math homework?

6. Were there any important things in any of your elementary math classes that would help me to understand your math background?

Part II: Attitudes and Beliefs about Mathematics, Mathematics Coursework Plans, and Career Interest:

Please choose your choice of response to each statement.

If you strongly disagree with the statement given circle 1
If you disagree with the statement given circle 2
If you are undecided with the statement given circle 3
If you agree with the statement given circle 4
If you strongly agree with the statement given circle 5

	Statement	Strongly disagree	Disagree	Undecided	Agree	Strongly agree
1.	I like math puzzles.	1	2	3	4	5
2.	Males are not naturally better than females in mathematics.	1	2	3	4	5
3.	When a math problem arises that I can't immediately solve, I stick with it until I have a solution.	1	2	3	4	5
4.	Figuring out mathematical problems does not appeal to me.	1	2	3	4	5
5.	It's not important to understand why a mathematical procedure works as long as it gives a correct answer.	1	2	3	4	5
6.	I do as little work in math as possible.	1	2	3	4	5
7.	Girls can do just as well as boys in mathematics.	1	2	3	4	5
8.	I think I could handle more difficult mathematics.	1	2	3	4	5
9.	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.	1	2	3	4	5
10.	Girls who enjoy studying math are a bit peculiar.	1	2	3	4	5
11.	I'm not the type to do well in math.	1	2	3	4	5

12.	Ability in math increases when one studies hard.	1	2	3	4	5
13.	Studying mathematics is just as appropriate for women as for men.	1	2	3	4	5
14.	Hard work can increase one's ability to do math.	1	2	3	4	5
15.	I can get good grades in mathematics.	1	2	3	4	5
16.	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	1	2	3	4	5
17.	Males are not naturally better than females in mathematics.	1	2	3	4	5
18.	Knowing mathematics will help me earn a living.	1	2	3	4	5
19.	Time used to investigate why a solution to a math problem works is time well spent.	1	2	3	4	5
20.	I'm no good at math.	1	2	3	4	5
21.	It's hard to believe a female could be a genius in mathematics.	1	2	3	4	5
22.	I am sure I could do advanced work in mathematics.	1	2	3	4	5
23.	Mathematics is of no relevance to my life.	1	2	3	4	5
24.	It doesn't really matter if you understand a math problem if you can get the right answer.	1	2	3	4	5
25.	By trying hard, one can become smarter in math.	1	2	3	4	5
26.	Mathematics will not be important in my life's work.	1	2	3	4	5
27.	I can get smarter in math by trying hard.	1	2	3	4	5
28.	Math has been my worse subject.	1	2	3	4	5
29.	Math puzzles are boring.	1	2	3	4	5
30.	I can get smarter in math if I try hard.	1	2	3	4	5
31.	I study mathematics because I know how useful it is.	1	2	3	4	5
32.	Mathematics is for men; arithmetic is for women.	1	2	3	4	5

33.	Mathematics is a worthwhile and necessary subject.	1	2	3	4	5
34.	Working can improve one's ability in mathematics.	1	2	3	4	5
35.	Getting a right answer in math is more important than understanding why the answer works.	1	2	3	4	5
36.	Studying mathematics is a waste of time.	1	2	3	4	5
37.	I definitely will continue to take mathematics once participation becomes available	1	2	3	4	5

Part III: Open-ended Questions:

1. Imaging that you are in high school, how many math classes would you like to take? Why?

   ______as many as possible

   ______as an average amount

   ______as few as possible

Because____

____

____

2. Do you enjoy math? Why or why not?

____

____

____

3. Do you think you are good at math? Why or why not?

4. In your opinion, who is better at doing mathematics, boys or girls? Why or why not?

5. In your opinion, what is mathematics about?

6. Do you believe that understanding mathematics is important? Why or why not?

7. Do you believe that trying hard in mathematics class can increase your mathematical ability? Why or why not?

8. Do you think you can solve time-consuming mathematics problems? Why or why not?

9. Do you believe that mathematics is useful in daily life? Why or why not?

10. Do you believe that mathematics will be useful for your future career? Why or why not?

11. Do you like or dislike working in a group? Why?

12. What did you like or dislike about the mathematics project (The park project)?

13. What would you like to be when you grow up? Explain?

____

____

____

APPENDIX H

Student's Interview

Informal Interviews:

1. What do you think about today's classroom activities?
2. What do like or not like about today's classroom activities?
3. What do you feel about the group work today?
4. How did your group work together?

Post-Formal Interviews (possible questions as circumstances dictate):

1. Describe your previous experiences in mathematics

   -Talk about your experiences in this mathematics classroom.

2. Do you enjoy math? Why or why not?
3. Do you think you are good at math? Why or why not?
4. Do you think boys can do just as well as girls in math? Why or why not?
5. What is mathematics in your opinion?
6. What do you like and dislike most in your mathematics classroom?
7. Do you believe that understanding concepts is important in mathematics? Why or why not?
8. Do you believe that trying hard in doing mathematics can increase mathematical ability? Why or why not?
9. Do you believe that you can solve time-consuming mathematics problems? Why or why not?
10. Do you believe that mathematics is useful in daily life? Why or why not?
11. Do you believe that mathematics is useful for your future life? Why or why not?
12. What kind of help has your teacher provided for you?
13. What kind of help do you seek from your teacher?

14. What type of math related activities did you do outside math classrooms? (For example, read other math books, do non-assigned math problems, join a math/science club, tutor other friends, etc,) Please explain.
15. Tell me about working in groups

    -How did your group work together?

    -Do you find group members helpful?

    -Do you enjoy working in groups?

    -What kind of interactions, if any, did you have with your group members?

APPENDIX I

Teacher's Interview

Teacher's Pre-Formal Interview (prior to the observation):

1. Tell me about your mathematics teaching experience.
2. Tell me about your background in learning the Standards-based approach to teaching.
3. Tell me about your experiences in a Standards-based approach to teaching.
4. What textbooks and supplementary material do you use in your mathematics class?
5. What are your expectations for your students in your class?
6. Is there anything else that you think would be helpful for me to know?

Teacher's Informal Interview:

(Before class)

1. Tell me what concepts you plan on covering in class today.
2. What are today's classroom goals?
3. What do you expect your students to do in classroom today?

(After class)

1. Did the class go as you expected?
2. How would you assess your students' learning?
3. If you were to teach this topic/lesson again, what would you like to change?

Teacher's Post-Formal Interview:

1. Over all, what do you think about your mathematics class?
2. What would you like to change or not change if you were to teach your mathematics class again?
3. What are some of the successes and difficulties you have encountered in your mathematics class?
4. What are some of the successes and difficulties you think your students have encountered in your mathematics class?
5. Do you think that the textbooks and supplemental materials from the Standards-based curriculum are adequate and useful for your students?

APPENDIX J

Classroom Observation Protocol

Date ________________

Grade Level ___________

Note: Guidelines for Summary and Description of the Observed Lessons.

- I will take notes general notes about what is happening in the classroom. I will use the questions in the following list as a part of field notes protocol for focusing on classroom interactions before the teacher and her students and students with their peers.

1. Classroom demographics:

    -What is the total number of students in the class at the observation session?

    -What is the classroom space and classroom arrangement?

    -What mathematics content is being taught?

2. Classroom activities:

    -What is the classroom arrangement?

    -What are the objectives of today's classroom?

    -What is the main classroom activity for today?

    -How long was each classroom activity session?

3. Classroom materials:

    -What classroom materials are used in this class?

    -How are classroom materials being used in this class?

4. Target students/teacher interactions:

    -How do target students perform in classroom tasks?

    -How do target students participate in the large group (classroom) discussion?

    -How long do target students persist in doing math?

    -Do the target students try to solve problems in alternative ways?

    -Do the target students appreciate mathematics roles in real life?

    -Do the target students reflect on their own thinking in solving math problems?

    -What are target students' roles in groups?

    -How are the target students involved with the classroom activities?

    -How does the teacher facilitate target students learning in the classroom?

-How does the teacher control or manage the classroom when there are disruptions?

5. Group activities:

- How are the small groups organized?
- How did target students react in the cooperative small group work?
- How does the teacher support group-work activities?

APPENDIX K

The Affective Domain of the Taxonomy of Educational Objectives

(Focused on Level 1 to Level 3)
(Adapted from Krathwohl, Bloom, & Masia, 1964)

1.0 "RECEIVING"	At this level the learner realizes the expectation s for certain learning expectations or class activities; that is, that he/she be willing to receive or to attend to them. Three sub-categories indicate three different levels of attending to learning expectations and class activities. 1.1 "AWARENESS": Simple awareness without specific discrimination or recognition of characteristics of the mathematics learning expectations. The individual may not be able to verbalize aspects of the learning expectations. 1.2 "WILLINGNESS TO RECEIVE": At the minimum level, willingness to tolerate a given mathematics-learning situation, not avoid it. At best, the student is willing to take notice of the class activity or learning situation and give it his/her attention, such as attending carefully when others speak. 1.3 "CONTROLLED OR SELECTED ATTENTION": at a somewhat higher level, the differentiation of a given learning expectations or class activities at a conscious or perhaps semiconscious level. The student may not know the technical mathematical terms or symbols with which to describe it correctly or precisely to others.
2. "RESPONDING"	At this level we are concerned with responses, which go beyond merely attending to the learning expectation or class activity. Most commonly this indicates the desire of a student to engage with a mathematics learning expectation or class activity, seeking it out and gaining satisfaction from working with it. 2.1 "ACQUIESCENCE IN RESPONDING": The word "obedience" or "compliance" describes this behavior. The behavior is not initiated, and the classroom activity calling for this behavior is not subtle. The student makes the responses, but he/she has not fully accepted the necessity for doing it.

	2.2 "WILLINGNESS TO RESPOND": The key to this level is in the term "willingness," with its implication of capacity for voluntary activity. There is the implication that the learner is sufficiently committed to exhibited the behavior that he/she does so not just because of a fear of punishment, but "on his/her own" or voluntary, such as acceptance of responsibility for his/her own mathematical work and the group work. 2.3 "SATISFACTION IN RESPONSE": The element in the step beyond the willingness is that the behavior is accompanied by a feeling or satisfaction, an emotional response, generally of pleasure, zest, or enjoyment, such as finds pleasure in solving a difficult mathematics problem or puzzle.
3. "VALUING"	Behavior categories at this level are sufficiently consistent to have taken on the characteristics of a belief or an attitude. The learner displays this behavior with sufficient consistency in appropriate situations so that he/she comes to be perceived as holding a value. This activity is motivated, not by the desire to comply or obey, but by the individual's commitment to the value underlying the behavior, such as a beliefs that mathematics will be useful in their future. 3.1 "ACCEPTANCE OF A VALUE": At this level the learner can identify the value, and is sufficiently committed that he/she is willing to be so identified with it, such as accepting being known as a good mathematics student. 3.2 "PREFERENCE FOR A VALUE": The individual is more than willing to be identified with the mathematics expectation and activity; he/she pursues it, seeks it out, and wants it, such as doing mathematics related activities outside the classroom. 3.3 "COMMITMENT": The person displaying behavior at this level is clearly perceived as holding the value. He/She acts to further develop it or to deepen his/her involvement with it and with the thing representing it. He/She tries to convince others and seeks converts to his cause, such as telling group members to try hard and they will be able to be successful at solving a difficult mathematics problem.

APPENDIX L

Goals for the Covering and Surrounding Unit

(from Lappan, Fey, Fitzgerald, Friel, and Philips, 2002, p.1)

Covering and Surrounding was created to help students:

- Develop strategies for finding areas and perimeters of rectangular shapes and then nonrectangular shapes
- Discover relationships between perimeter and area
- Understand how the area of a rectangular is related to the area of a triangle and of a parallelogram
- Develop formulas or procedures--stated in words or symbols--for finding areas and perimeters of rectangles, parallelograms, triangles, and circles
- Use area and perimeter to solve applied problems
- Recognize situations in which measuring perimeter or area will answer practical problems
- Find perimeters and areas of rectangular and nonrectangular figures by using transparent grids, tiles, or other objects to cover the figures and string, straight-line segments, rulers, or other objects to surround the figures
- Cut and rearrange figures--in particular, parallelograms, triangles, and rectangles --to see relationships between them and then devise strategies for finding areas by using the observed relationships
- Observe and reason from patterns in data by organizing tables to represent the data
- Use reasoning to find, confirm, and use relationships involving area and perimeter
- Use multiple representations--in particular, physical, pictorial, tabular, and symbolic models-and verbally describe of data

APPENDIX M

The Unit Project: Plan a Park

(from Lappan, Fey, Fitzgerald, Friel, and Philips, 2002, p.82)

At the beginning of this unit, you read about Dr. Doolittle's donation of land to the city, which she designed as a new park. It is now time to design your plan for the piece of land. Use the information you have collected about parks, plus what you learned from your study of this unit, to prepare your final design.

Yours design should satisfy the following constraints:

- The park should be rectangular with dimensions 120 yards by 100 yards.
- About half of the park should consist of a picnic area and a playground, but these two sections do not need to be located together.
- The picnic area should contain a circular flower garden. There should also be a garden in at least one other place in the park.
- There should be trees in several places in the park. Young trees will be planted, so your design should show room for the trees to grow.
- The park must appeal to families, so there should be more than just a picnic area and a playground.

Your design package should be neat, clear, and easy to follow. Your design should be drawn and labeled in black and white. In addition to the scale of your design for the park, your project should include a report that gives:

1. The size (dimensions) of each item. These items should include gardens, trees, picnic tables, playground equipment, and anything else you included in your design.
2. The amount of land needed for each item and the calculations you used to determine the amount of land needed.
3. The materials needed. Include the amount of each item needed and the calculations you did you did to determine the amounts. Include the numbers and type of each piece of playground equipment, the amount of fencing, the numbers of picnic tables and trash containers, the amount of land covered by concrete or blacktop (so the developers can determine how much cement or blacktop will be needed), and the quantities of other items you included in your park.
4. A letter to Dr. Doolittle explaining why she should choose your design for the park. Include a justification for the choices you made about the size and quantity of items in your park.