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Diurnal/semidiurnal oceanic tidal angular momentum: Topex/Poseidon models in comparison with Earth's rotation rate

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Abstract. The oceanic tidal angular momentum (OTAM) has been demonstrated to be the primary cause for the diurnal and semidiurnal variations in the Earth's rotational rate, or ΔUT1. Three ocean tide models derived from the Topex/Poseidon altimetry mission are employed to yield predictions of ΔUT1 for eight major diurnal/semidiurnal tides. The predictions are compared with geodetic determinations of ΔUT1 from the very-long-baseline interferometry data both for long-term observations and during intensive campaigns. The agreement is good, with discrepancies typically within 2-3 μs. Systematic discrepancies observed for the well-determined M_2 tide clearly reveal the contribution of the Earth's spin libration.

Introduction

The luni-solar tidal force influences the Earth's rotation in a variety of ways [e.g., *Munk and MacDonald*, 1960]. It exerts direct torques on the Earth, resulting in the astronomical precession/nutation, libration [*Chao et al.*, 1991], and tidal braking. It also "excites" geophysical variations in Earth rotation under the conservation of angular momentum. The geophysical variations can be separated into two types: the (1-D) variation in the rotational rate, and the (2-D) variation in the rotation axis orientation known as the polar motion. They are continually excited by geophysical processes occurring on or within the Earth that involve motions and mass redistribution. The present paper treats the excitation of rotational rate variations by diurnal and semidiurnal oceanic tides (the corresponding problem for polar motion will be treated in a separate paper).

The rotational rate variation is usually expressed in terms of ΔUT1, or the deviation of the Universal Time determined by the Earth's rotation from the uniform atomic time. With the assumption of an axially symmetric Earth (and under the symmetry and orthogonality properties of spherical harmonics), only the zonal, long-period tidal forcing can excite ΔUT1 while only the tesseral, diurnal tidal forcing can excite polar motion. These are the only "first-order" tidal effects on Earth rotation. Second-order tidal effects on Earth rotation, such as the ones considered here, arise because of asymmetries in the Earth -- primarily the oceans and the lateral heterogeneities in the mantle and core.

Subject to its irregular geography and non-equilibrium behavior, the world's ocean couples spherical harmonics and generates zonal disturbances in response to diurnal (tesseral) and semidiurnal (sectorial) tidal forcings, leading to ΔUT1 at these short periods. The past few years have witnessed a dramatic confirmation of this effect [e.g., *Brosche et al.*, 1989; *Herring and Dong*, 1994; *Ray et al.*, 1994a], mainly due to the great advances in Earth rotation measurements in both precision and temporal resolution: the space geodetic technique of very-long-baseline interferometry (or VLBI, and to a lesser extent the satellite laser ranging and Global Positioning System) can now achieve μs-level precision in UT1 determination even at sub-daily time intervals. *Ray et al.* [1994a] reported rms deviation in ΔUT1 as small as 2 μs between VLBI observation and theoretical prediction computed based on the Schwiderski tidal height model [*Schwiderski*, 1980] for eight major tide constituents. That study unequivocally establishes the primary role of ocean tides in exciting diurnal and semidiurnal ΔUT1. The present paper builds on *Ray et al.* [1994a], but focusing on predictions from three new tide models derived from the Topex/Poseidon (henceforth T/P) satellite altimetry data, and their comparison with VLBI ΔUT1 observations.

Theory and Formulation

In general the Earth rotation variations can be excited by two different terms: the "mass term" associated with mass redistribution on or within the Earth that changes the Earth's inertia tensor, and the "motion term" associated with movement of mass that carries net relative angular momentum. In the case of the ocean tide, the mass term corresponds to the tidal height (designated "th" below), and the motion term the tidal current ("tc" below). As is now customary [e.g., *Barnes et al.*, 1983], we shall express the excitation in terms of the dimensionless oceanic tidal angular momentum (OTAM) function χ of time t; and we are here only concerned with its axial component:

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$$\chi_3(t) = \chi_3^{\mathrm{th}}(t) + \chi_3^{\mathrm{tc}}(t)\,; \qquad \text{where} \tag{1}$$

$$\chi_3^{\mathrm{th}}(t) = \left[0.753\, a^4\, \rho \,/\, C_{\mathrm{m}}\right] \int\!\!\int_{\mathrm{ocean}} \zeta(\Omega,t)\, \sin^2\theta\, d\Omega$$

$$\chi_3^{\mathrm{tc}}(t) = \left[0.998\, a^3\, \rho \,/\, C_{\mathrm{m}}\right] \int\!\!\int_{\mathrm{ocean}} u(\Omega,t)\, h(\Omega)\, \sin\theta\, d\Omega$$

In these equations, a is the Earth's mean radius; C_{m} is the axial moment of inertia of the mantle (assuming the fluid core does not participates in the ΔUT1 excitation process); $\rho = 1035$ kg m^{-3} is the mean density of seawater; Ω is an abbreviation for the co-latitude θ and longitude λ; $d\Omega = \sin\theta d\theta d\lambda$ is the surface element for the integral over the oceans. The tidal height relative to the seabed is ζ, and u is the eastward speed of the tidal current, which is assumed to be barotropic and hence uniform over the water column depth h. The numerical factors account for the elastic behavior and loading response of the solid Earth according to *Eubanks* [1994].

In evaluating Equation (1), one should be aware of the fact that in general tidal height models do not enforce conservation of water mass for practical and numerical reasons. This inevitably introduces a small error in our calculation, especially in χ_3^{th}. Without knowledge of where this artifact originates, there is certain arbitrariness in the ways to deal with it. We choose to first decomposed χ_3^{th} into two terms [e.g., *Chao and O'Connor*, 1988]: one major term proportional to the tidal height-induced change in the Earth's oblateness parameter J_2, plus a minor term proportional to the (artificial) change in the total water mass. We then simply set the second term to zero. This practice is equivalent to compensating and then uniformly "spreading" the mass change over the entire globe, a distribution that is orthogonal to J_2. This minimizes the leakage of the non-conservation effect into our calculation. We further account for this effect due to the Arctic Sea (as the J_2 function has its anti-node at the poles) by supplementing the T/P models (which themselves do not include the polar regions) with independent Arctic data in a consistent manner (see below). In any event, the overall effect is small, as found in *Ray et al.* [in preparation, 1995].

The OTAM excites variations in the (solid) Earth's rotation (at rate ω) under the conservation of angular momentum: $\chi_3(t) = -\Delta\omega/\omega$. The corresponding ΔUT1 is therefore given by $-86400\int\chi_3(t)dt$ in seconds.

Model and Data

Tide Models. The T/P mission was launched in 1992 into a circular orbit of 66° inclination and 1336 km altitude. Its precise orbit determination has allowed ocean tide estimates unprecedented in accuracy [*Le Provost et al.*, 1995]. Three independent tide models for the major diurnal and semidiurnal tidal constituents will be used in this study, namely those by *Schrama and Ray* [1994, henceforth Model A], *Ray et al.* [1994b, henceforth Model B], and *Egbert et al.* [1994, henceforth Model C].

Both Models A and B consist of tidal heights derived through strictly empirical analyses of the altimeter data. Model A applied harmonic analysis to data captured in small bins covering the ocean surface, the bin size being dictated by considerations of tidal aliasing. The four major constituents were recovered. Model B employed a generalized global response method [*Cartwright and Ray*, 1990], with each response weight expressed as a series of special precomputed oceanic normal modes. A response analysis yields essentially all tides in the diurnal and semidiurnal bands, and we show here the UT1 results at eight frequencies.

Deep-ocean tidal currents for both Models A and B were computed from the gradients of tidal heights by solving the momentum equations of Laplace's tidal equations, with proper allowance for ocean loading and self-attraction [see *Cartwright et al.*, 1992]. Frictional dissipation, of little importance in the deep ocean, is ignored (although the global consequences of dissipation are apparent in the adopted height fields).

Model C was derived by assimilating T/P data into a hydrodynamic model. With this approach, both tidal heights and currents are estimated in a global inverse problem, and the solution represents a compromise between fitting the T/P data and satisfying the constraints of the Laplace tidal equations.

Compared to the other models, Model A exhibits slightly better agreement with a test set of tide-gauge measurements. However, we are certain its derived currents are the least accurate because its heights are somewhat noisy (particularly in the diurnal band); fortunately only the mean zonal transport is required here, so this noise by and large cancels. The Model B heights, which are smooth owing to the normal-mode basis functions, yield currents that appear satisfactory excepting near shallow seas. We presume that the currents of Model C are most accurate, because of its tighter constraints to the hydrodynamics through the continuity equation.

All three models are here supplemented in the polar regions with tidal heights adopted from either Schwiderski's model [*Schwiderski*, 1980] (for Model A), the hydrodynamic model of *Le Provost* [1994] (for Model B), or the Kowalik-Proshutinsky model [*Kowalik and Proshutinsky*, 1994] (for Model C). The detailed treatment can be found in *Ray et al.* [in preparation, 1995].

Observations. The ΔUT1 observations that will be used in this study are of two types, respectively called "long-term" and "intensive", both by VLBI. The long-term observations are based on multi-year series from various VLBI networks, typically with 24-hour measurement sessions separated by gaps of a few days, supplemented with (almost) daily 1-hour sessions. Independent algorithms have been applied to these raw data by various research groups [*Sovers et al.*, 1993; *Herring and Dong*, 1994; *Gipson et al.*, 1993], resulting in spectral estimates of the diurnal/semidiurnal tidal parameters (see Table 1 below).

The intensive observations are those made during three special campaigns, each about 2 weeks long: the ERDE (October 1989), Search92 (July 31-Aug. 9, 1992), and Cont94 (Jan. 12-26, 1994). *Ray et al.* [1994a] examined 4 days data during Search92. The present paper will focus on Cont94 [*Gipson et al.*, 1994, see Figure 1 below].

Table 1. The amplitude (in μs) and phase angle (in degrees and is relative to the equilibrium tide on the Greenwich meridian as that used by *Gross* [1993]) of ΔUT1 variation for eight diurnal and semidiurnal tides. The observations are derived from long-term VLBI measurements. The predictions are computed according to ocean tide models: Models A [*Schrama and Ray*], B [*Ray et al.*], and C [*Egbert et al.*] all based on T/P, and Models S based on *Schwiderski* and *Cartwright et al.*

	Diurnal Tides				Semidiurnal Tides			
	Q_1	O_1	P_1	K_1	N_2	M_2	S_2	K_2
Observations								
Sovers et al.	6.6, 37°	21.4, 39°	7.2, 27°	15.5, 13°	3.0, 221°	18.2, 235°	5.2, 266°	2.8, 251°
Herring&Dong	5.3, 36°	23.6, 47°	7.1, 34°	18.9, 20°	3.2, 240°	17.9, 233°	8.6, 269°	3.8, 282°
Gipson et al.	5.3, 41°	21.9, 45°	5.9, 27°	16.8, 22°	4.1, 232°	18.2, 236°	8.7, 263°	3.8, 269°
Predictions								
Model A	—	20.5, 29°	—	22.3, 25°	—	19.4, 244°	7.7, 262°	—
Model B	4.8, 32°	23.2, 39°	8.3, 39°	24.2, 38°	3.8, 250°	17.6, 251°	7.7, 261°	2.1, 260°
Model C	5.6, 26°	20.1, 37°	5.9, 29°	19.7, 26°	4.1, 248°	17.7, 246°	7.6, 267°	2.0, 259°
Model S	4.3, 18°	20.4, 36°	5.9, 19°	21.2, 27°	3.6, 240°	18.7, 244°	7.7, 256°	1.9, 260°

Results and Discussion

We now compare the VLBI-observed ΔUT1 with the OTAM predicted according to Equation (1) by the T/P tide models.

"Long-Term" Comparison: Table 1 compares the tidal parameters for the eight major diurnal and semidiurnal tides. The upper panel shows the VLBI-observed long-term values determined by different authors mentioned above. *Sovers et al.* [1993] give a thorough discussion of the VLBI uncertainties. The lower panel gives the predicted values by the three T/P models, as well as Model S computed for the empirically constrained height model by Schwiderski [1980], with tidal currents computed by *Cartwright et al.* [1992].

The general agreement between observations and predictions is quite good, typically within 2-3 μs with the exception of K_1 where model predictions gives consistently larger amplitudes than observations. This level of agreement also provides a gauge for the uncertainties of the T/P model predictions. A more detailed study of T/P model uncertainty is underway [also C. K. Shum, personal communication, 1995].

In Table 1 only the total OTAM are given, but one should bear in mind that the tidal current contribution is generally larger than that of tidal height typically by a factor of 2-5 at the diurnal/semidiurnal periods [*Brosche et al.*, 1989; *Ray et al.*, 1994a]. For some tidal constituents there exist considerable variabilities among the height and current estimates from model to model, and sometimes fortuitous cancellations contribute to the general agreement of the net effect among models.

"Intensive" Comparison: Figure 1 plots the hourly VLBI observation of ΔUT1 (with standard errors) against predictions according to T/P tide Models B and C during Cont94. The observations are primarily those made by the NASA R&D network, with extension on both ends by one day and a one-day gap (Jan. 19) filled in with those from the NEOS network [*Gipson et al.*, 1994]. Longer trends in the time series are removed by fitting a 5th degree polynomials to the series.

The model predictions are simply computed using the parameters given in Table 1. Model A is not plotted because fewer constituents are represented. Model S is also omitted, as it is hardly discernible from Model C on the scale of Figure 1.

The Cont94 hourly Earth orientation measurements have formal errors of typically within 5-10 μs. The range of difference among T/P model predictions is somewhat smaller. Again, one indeed sees a striking general agreement between

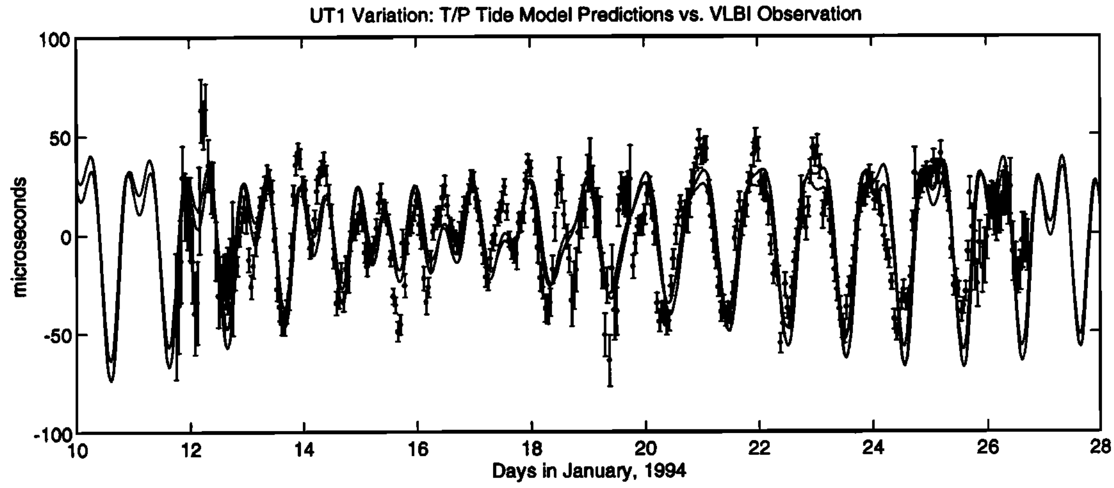

Figure 1. Comparison of ΔUT1 variations during Cont94 Campaign. The hourly observations with standard errors are those made by VLBI (primaily the NASA R&D network); the solid lines are predictions according to T/P tide Models B and C (see text) (Model B has the slightly larger amplitude).

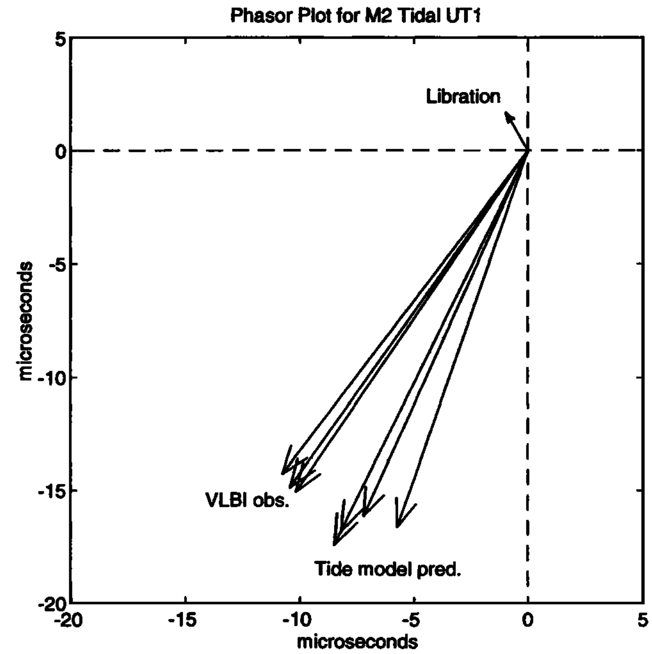

Figure 2. The phasor diagram for ΔUT1 variation at the M_2 period: Three VLBI-observed values and four tide-model predictions are given according to Table 1. The corresponding contribution of the Earth spin libration is also plotted.

observation and predictions. Yet the models tend to underpredict, especially for the semidiurnal variation which is particularly evident during Jan.16-20. The reason is not clear at present, but probably related to the fact that only selected tide constituents are represented here and to the presence of other geophysical contributions to ΔUT1, such as atmospheric and oceanic angular momentum variations and Earth librations (see below). A more detailed analysis is currently underway.

Semidiurnal Spin Libration: The largest tide M_2 is the best determined both in the observation and the model prediction. But as the phasor diagram of Figure 2 clearly shows, there exist systematic discrepancies that are too large and definite to be ascribed to random errors in VLBI or the tide models.

This points to the contribution of the Earth's spin librations to semidiurnal ΔUT1, the largest of which is for M_2 and is about 2 μs [*Chao et al.*, 1991]. Here we should point out that a minus sign was inadvertently dropped from Equation (8) of *Chao et al.* [1991] (this does not affect the conclusions of *Chao et al.* which did not discuss the phase). Comparing it with the expression of tidal potential for M_2 (that of a fictitious Moon circulating the Earth uniformly above the Equator at 24.83 hours), one finds the M_2 spin libration to be (1.9 μs, 120°). This libration indeed explains a large portion of the discrepancy of the model predictions from the observations (by "closing the gap" between the phasor vectors when added to the model predictions, see Figure 2). A more accurate determination of this parameter would bear on the equatorial ellipticity (hence triaxiality) of the core-mantle boundary [*Herring and Dong*, 1994].

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