AN ABSTRACT OF THE THESIS OF

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Title: Brake-Based Wheel Speed Control Design of a Rear Wheel Open Defferential Vehicle.

Abstract approved:

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A brake-based wheel speed control system for a rear-wheel drive vehicle is developed and simulated in this thesis. The OSU mini-Baja vehicle team will use this study in the development and implementation of a similar system for upcoming competitions. Vehicle submittals must differ to a specified degree from previous year's designs, according to the Society of Automotive Engineers (SAE) competition guidelines. The OSU team intends to satisfy this requirement by implementing an electronic traction control system, hereafter referred to as the Smart Brake System (SBS). The SBS design will not only enable OSU to satisfy SAE guidelines, but will reduce undesired drive torque distribution to the wheels. The development of SBS is based on a rear-wheel drive, open-differential vehicle and turning dynamics data gathered by the 2004 OSU Baja team. The vehicle model and the control system are designed and simulated using MatLab.

Brake-Based Wheel Speed Control Design of a Rear Wheel Open Differential Vehicle

by

Eli Jeon

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APPROVED:

Major Professor, representing Mechanical Engineering

Head of the Department of Mechanical Engineering

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Eli Jeon, Author

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LIST OF SYMBOLS

English Symbols

F_v aerodynamic and viscous forces on vehicle, $[N]$

$F_{T,i}$ longitudinal traction force between ground and the i-th wheel (the wheel under consideration), $[N]$

$F_{z,i}$ a quarter of the normal force from the ground to the vehicle, $[N]$

$T_{d,i}$ driving torque applied to the i-th wheel, $\left[\frac{N}{m}\right]$

$T_{b,i}$ braking torque applied to the i-th wheel, $\left[\frac{N}{m}\right]$

$T_{b,o}$ braking torque applied to the o-th wheel (the wheel “opposite”, with respect to the differential, of the wheel in consideration, $\left[\frac{N}{m}\right]$

C_v is the aerodynamic drag coefficient

m mass of the vehicle, [kg]

t track width of the vehicle [m]

r effective rolling wheel radius, [m]

V longitudinal vehicle velocity, $\left[\frac{m}{s^2}\right]$

g acceleration of gravity, $\left[\frac{m}{s^2}\right]$

LIST OF SYMBOLS (Continued)

English Symbols

x vehicle position with respect to the inertial frame, [m]

Greek Symbols

μ coefficient of normalized traction friction force between the tire and terrain

λ longitudinal wheel slip parameter

* over-dots represent time derivatives, subscript "d" refers to "desired value"

Brake-Based Wheel Speed Control of an Open Differential, Rear-Wheel Drive Vehicle

Introduction

The automobile industry desires quality vehicle performance due to customer demand and competition between different automobile manufacturers. The industry measures vehicle performance in many categories, such as vehicle safety and handling. Performance in racing competitions is often measured by the time it takes for the vehicle to complete a given course and efficient vehicle dynamics with respect to the terrain in these competitions is highly desired. A variety of traction control systems, such as the limited slip differential and active differential, both of which produce desirable vehicle dynamics, currently exist in the automobile industry. In fact, many higher-end automobiles come equipped with these features for increased safety of both the driver and passengers.

The SBS vehicle platform developed in this work consists of a rear-wheel open differential with a steering wheel position input obtained via potentiometer, and wheel speeds obtained via Hall effect sensors. Unlike a locked differential, an open differential allows individual wheels to spinning at different rates, which leads to effective turning. The design of the SBS takes advantage of this kind of wheel motion by controlling the distribution of input drive torque via braking. A previous OSU mini-Baja electronic traction control team related steering wheel angle to vehicle turning radius during minimal slip conditions. This data is used to represent desired wheel speed dynamics as a function of steering wheel position and provides the basis for the SBS control algorithm. The SBS control is tested in an open differential model which is describe in the 'Background' section. It is shown that the SBS reduces undesirable drive torque distribution and as a result improves longitudinal vehicle acceleration and prevention of undesired turning dynamics.

Background

Available Traction Control Systems

A variety of automotive performance enhancing control systems have been modeled, developed, and implemented. The more widely used systems are the Yaw Stability Control (YSC), Anti-Lock Brake System (ABS), and Traction Control Systems (TCS). These control systems have different optimization purposes, which include vehicle handling, cornering performance, traction performance, and passenger safety.

The YSC forces the vehicle to track a model vehicle with desired yaw rate states. Vehicle handling and passenger safety during slip situations is improved with this concept. In the event a vehicle equipped with YSC would start to 'fishtail' or show undesirable yaw characteristics, the controller would apply the brakes accordingly to prevent 'fishtailing'. In the development of the YSC seen in [6], the angular rates of the driven wheels and the vehicle longitudinal speeds are assumed to be measured states. The controller structure of the YSC in [6] is based on brake pressure input to the wheels.

Prior to the advent of the ABS, when brakes were fully applied, the wheels would lock and bring the car into a skid. The traction force at the wheels is not at its optimum since the traction coefficient of the wheels on the surface actually decreases in a skid. ABS was developed to prevent the occurrence of skidding situations. The brakes of an ABS are applied in a manner that keeps the traction force at the most optimum value possible given the vehicle speed and the wheel speeds. Many different ABS control schemes have been developed. ABS keeps the traction force at its peak value by keeping a parameter known as the slip ratio at ideal values. The slip ratio is the difference between the angular speed of the wheel and the vehicle velocity. The relationship between the slip ratio parameter and traction will be explained in the *Model Derivation* section.

In [1] and [2], Anwar uses the Generalized Predictive Algorithm as the control method for developing both a YSC and ABS system. In [1], he develops a set of equations where the slip ratio is the state variable. In [3], Buckholtz develops a Sliding Mode Control algorithm to reduce the error between desired slip ratio and actual slip ratio. In any case, the method of developing the state equations with the slip ratio as the state variable requires the measurement of the wheel speed and vehicle velocity. Achieving accurate measurements of vehicle velocity may be troublesome with simply a radar sensor, especially in cases when all the vehicle wheels are not on the ground, and at lower vehicle speeds. There has been thorough work done in incorporating a combination of many other sensor readings to achieve more accurate vehicle velocity measurements. In [5], the author develops an observer to reduce vehicle velocity reading errors to be used in an effective ABS scheme.

Automobile differentials

The majority of currently manufactured vehicles are furnished with open differentials and have the option of being locked manually, electronically, or with some type of traction control applied to the open differential. Some examples of open differentials are the limited slip differential and active differential. An automobile differential will receive engine power via drive shaft and distribute engine torque to the driving wheels, which dictates the rotation rate of the wheel; whatever engine power is produced is distributed at the wheels. The type of differential that is distributing the drive torque determines whether the distribution is equal or varied. A locked differential will always evenly distribute drive torque and an open differential can distribute the drive torque unevenly which allows the wheels to turn at different rates which is necessary for effective turning.

Since a locked differential evenly distributes engine torque to the driving wheels they are forced to spin at the same rate. On loose terrain, driving wheels may slip and the locked differential is forced to provide engine torque evenly to both the slipping wheel and non-slipping wheel. Consider the case when one driving wheel is off the ground and the other remain on the ground. A locked differential will force the wheels to spin at the same rate and enables the vehicle to take advantage of the tractive force of the wheel on the ground. However, a locked differential provides poor performance during non-slipping turning maneuvers. In a turning situation where the vehicle follows a circular path, the outer wheel travels a longer path than the inner wheel as seen in Fig.1.

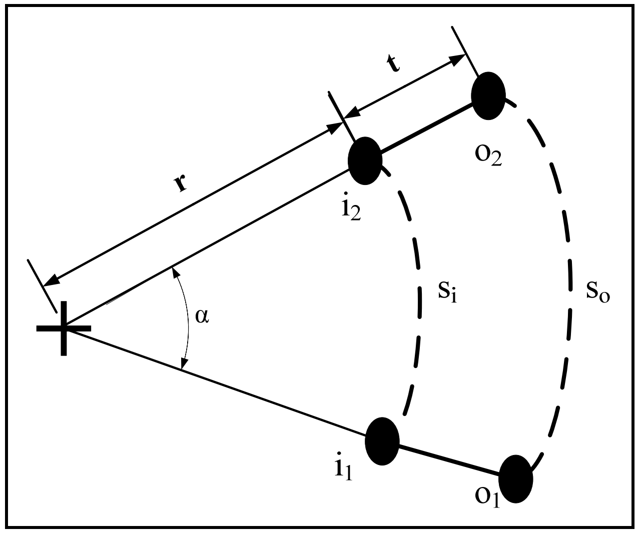

Figure 1: Vehicle turning geometry used in DWSR derivation.

In Fig.1, the letter "o" refers the outer wheel, the letter "i" refers to the inner wheel, the subscripts "1" and "2" denotes start and end positions, respectively, α is the displacement angle between the starting and ending positions, "r" is the turning radius followed by the inner wheel, "t" is the track width of the vehicle, and "s" represents the path lengths traveled by the wheels. The path lengths of the inner and outer wheel, s_i and s_o, respectively, are given by Eqs. [1a] and [1b]:

$$s_i = r\alpha \quad [1a]$$

$$s_o = (r+t)\alpha \quad [1b]$$

The wheels complete the turn in the same amount of time and are rotating at the same rate but the inner wheel has a shorter path. Turning maneuvers with a locked differential cause the inner wheel to spin at the same rate as the outer wheel and the excess spin causes unnecessary damage to the tire and road surface. Since open differentials allow wheels to turn at different rates, they are used in most track racing situations where there are many turning maneuvers performed.

The capability for a rear wheel drive open differential to allow for different turning rates can limit efficient power transfer during instances when one of the driving wheel's tractive force is much less than the tractive force on the other driving wheels. In the case where one driving wheel traverses on a slippery surface and the other driving wheel remains on a tractive surface, the open differential provides greater drive torque to the wheel on the slippery surface since there is less resistance from the ground. As a consequence, less drive torque will reach the wheel with greater traction thereby limiting its traction force and the vehicle wheel traction is limited to the traction provided by the slipping wheel. A solution to this torque distribution problem is approached using electronically controlled wheel brake commands to distribute the engine torque in order to achieve desired traction performance.

Development of SBS

The SBS control is developed using an open differential vehicle model. The SBS control tracks a desired wheel speed ratio (DWSR) which is derived from a combination of turning characteristics based on vehicle geometry and desirable turning characteristics at extremes of the steering wheel positions. SBS does not require vehicle velocity readings since the state variables used are the wheel rotation rates. The main purpose of the SBS is to reduce unnecessary torque input to the slipping wheel and in doing so, provide engine torque to the non-slipping wheel.

SBS platform

The SBS system uses braking torque of one wheel to distribute drive torque via the open differential to the other wheel. The model used in the design of the SBS is a 2 input / 2 output system: a braking input for each driving wheel and two output wheel speed states. An explicit model following control scheme is developed for the SBS since the wheel speeds were to track desired wheel speeds ratios empirically derived in 2004.

The primary brakes of this system are a conventional setup of master and slave cylinders with a brake pedal that is actuated by the driver's foot. The conventional brakes are hooked up to the primary brake cylinder and the computer controlled SBS is connected to the auxiliary brake cylinder. This setup allows the two systems to operate independently. Stepper motor linear actuators drive the master cylinder that provides the hydraulic force that will actuate the piston on the braking caliper. The system uses two separate motors and a master cylinder combination so that separate left and right side braking is achieved.

Two Hall effect sensors measure rear wheel speeds and a potentiometer measures steering wheel position. The Hall effect sensors are located near the rear wheel brake calipers and use evenly spaced, drilled holes in the discs brake plates to determine wheel speeds; the steering wheel potentiometer is connected to the steering wheel shaft and turns with the steering wheel.

Longitudinal vehicle motion

A basic dynamic system is developed and serves as a platform to develop the model following control scheme. The main forces present in the overall vehicle system are depicted in Fig.2.

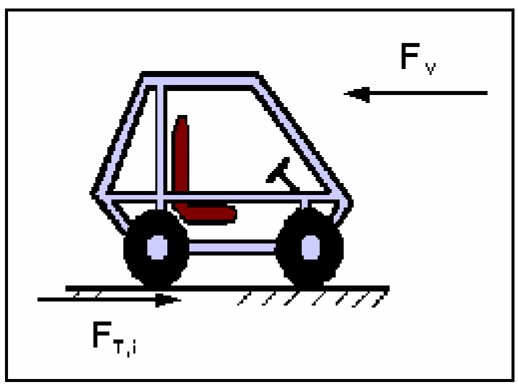

Figure 2: Free-body diagram of the vehicle.

$F_{T,i}$ represents the traction force at the i-th tire, and F_v the aerodynamic and other rolling resistant forces.

In order to capture the dynamics between the wheels and the ground, a quarter model of the vehicle is used. This quarter model is depicted in Fig.3 and is used to create equations of motion that incorporate the wheel speed state information.

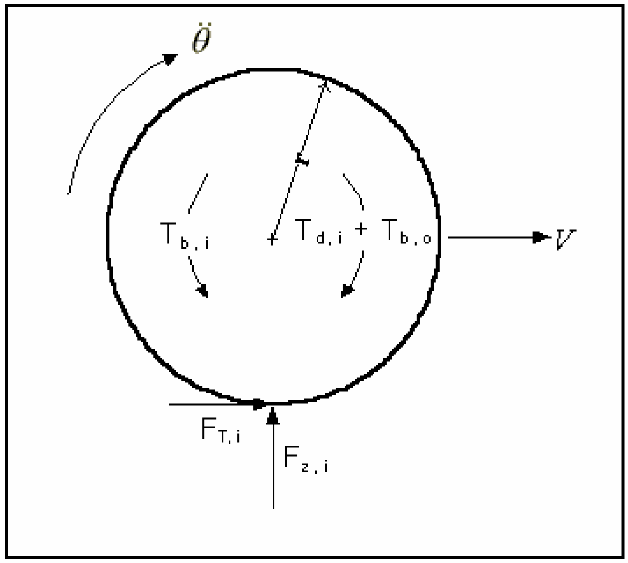

Figure 3: Free-body diagram of a quarter model of the vehicle wheels.

$T_{d,i}$ represents the driving torque applied to the i-th wheel, $T_{b,i}$ is the braking torque at the i-th wheel, $T_{b,o}$ is the braking torque at the o-th wheel or the opposite wheel, $F_{T,i}$ is the longitudinal friction force between the i-th wheel and the terrain, $F_{z,i}$ is the normal force from the ground, r is the effective wheel rolling radius, $\ddot{\theta}$ is the angular acceleration of the wheel, and V is the vehicle longitudinal velocity.

Equation [2] describes the overall vehicle longitudinal motion with respect to the ground:

$$m\dot{V} = F_{T,1} + F_{T,2} + F_{T,3} + F_{T,4} - C_v V^2 \quad [2]$$

where, C_V is the aerodynamic drag coefficient, V vehicle velocity, and m is the mass of the vehicle. The equation of motion for a rear-wheel drive vehicle can be written as Eq. [3] since there are two driving wheels that directly affect the acceleration.

$$m\dot{V} = F_{T,1} + F_{T,2} - C_v V^2 \quad [3]$$

The traction force $F_{T,i}$ is defined as:

$$F_{T,i} = \mu_i F_{z,i} \,. \qquad [4]$$

μ_i is called the coefficient of friction or coefficient of normalized traction force [6] and $F_{z,i}$ is the normal force on the wheel. μ_i is a function of longitudinal wheel slip, which, is defined as:

$$\lambda = \frac{r\dot{\theta} - V}{\max\{r\dot{\theta}, V\}} \,. \qquad [5]$$

Using the definition of λ in Eq. [5], a positive μ_i value is associated with vehicle acceleration and a negative μ_i value is associated with vehicle deceleration via braking. For the purposes of this paper, only the positive μ_i values will be used since the simulations assume there is a constant drive torque applied and the brakes simply distribute the drive torque to one wheel or the other. When μ_i has a value of +1, the wheel is spinning without causing vehicle acceleration and μ_i has a zero value. Figure 4 shows a simplified Pacejka magic curve traction model [6] which shows the correlation between μ_i and λ_i on various surfaces:

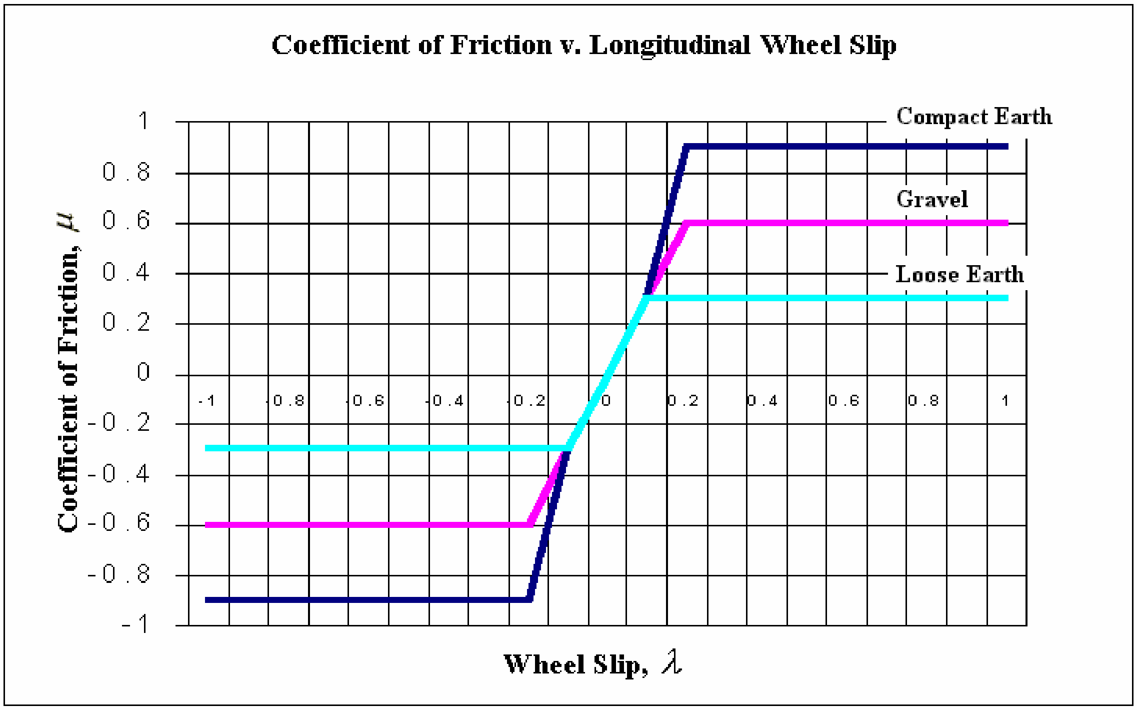

Figure 4: Simplified Pacejka model showing correlation between coefficient of friction and longitudinal wheel slip.

Notice that the peak values will be higher or lower depending on the dryness and stability of the road surfaces.

The normal force, F_Z, can be defined as:

$$F_z = \frac{1}{4} mg . \qquad [6]$$

Solving for $F_{T,i}$ results in Eq.[7]:

$$F_{T,i} = \frac{\mu_i mg}{4} . \qquad [7]$$

Using Fig.3, the wheel rotational dynamics are derived as shown in Eq. [8]:

$$T_{d,i} - T_{b,i} - F_{T,i} r_i - C_{w,i} \dot{\theta} = I_i \ddot{\theta}_i \qquad [8]$$

where $C_{w,i}$ represents the viscous friction acting on the i-th wheel.

SBS test environment

The following assumptions form the environment in which the control is tested. During a straight line travel and no slip, the input drive torque to each wheel is kept constant. In order to model the open differential, it is assumed that a change of net torque on one wheel will cause an equal and opposite change in torque on the other wheel. Equations [40a] and [40b] show the wheels are coupled through the open differential via the terms δ_L and δ_R, and by adding a brake torque on one wheel while subtracting that same brake torque on the wheel.

$$\dot{x}_2 = \frac{1}{I_L}\left[(T_{d,L} - \delta_R) - (\mu_L F_N r - \delta_L) - u_L + u_R - C_w x_1\right] \quad [9a]$$

$$\dot{x}_3 = \frac{1}{I_R}\left[(T_{d,R} - \delta_L) - (\mu_R F_N r - \delta_R) + u_L - u_R - C_w x_2\right] \quad [9b]$$

where, $u_L = T_{b,L}$, $u_R = T_{b,R}$ and the state variables are defined as:

$$x_2 = \dot{\theta}_L \quad [10a]$$

$$x_3 = \dot{\theta}_R . \quad [10b]$$

The variable x_1 is reserved for vehicle velocity, which is not analyzed quantitatively in this paper.

Equation [9a] and [9b] can be combined in matrix form as shown in Eq.[11]:

$$\begin{Bmatrix} \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} -\frac{C_w}{I_L} & 0 \\ 0 & -\frac{C_w}{I_R} \end{bmatrix} \begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} -\frac{1}{I_L} & \frac{1}{I_L} \\ \frac{1}{I_R} & -\frac{1}{I_R} \end{bmatrix} \begin{Bmatrix} u_L \\ u_R \end{Bmatrix} + \begin{bmatrix} \frac{1}{I_L}(T_{d,L} - \delta_R - \mu_L F_N + \delta_L) \\ \frac{1}{I_R}(T_{d,R} - \delta_L - \mu_R F_N + \delta_R) \end{bmatrix} \quad [11]$$

During non-slip situations, the input drive torque to the left and right wheels are kept constant ($\delta_L = 0$ and $\delta_R = 0$). During slip situations the traction coefficient μ of the slipping wheel will decrease, reducing the traction force of that wheel. The non-slipping wheel drive torque will decrease by the same amount slowing down the wheel and causing a loss in the tractive force it can provide according to the Pacekja model. In the simulations μ's are kept constant, however, the changes in applied wheel torque caused by slip of traction force are represented by δ_L and δ_R.

Consider the case where the left wheel slips without control and the traction force torque term drops by δ_L. Note that this change in the tractive force on the left wheel will cause the net torque on the right wheel to decrease by δ_L. Now consider the case where the left wheel slips with control; u_L is activated, slowing down the left wheel and speeding up the right. This simulation environment is suitable for testing the control method developed earlier.

Eq.[11] has the form:

$$\dot{x} = Ax + Bu + F(t) \qquad [12]$$

where, x represents the system states, A represents the system matrix, B represents the distribution of the input brake vector u, and F(t) is the state independent time-varying inputs into the model which represents net torque result of drive and traction torque on the wheel. During a non-slip situation the traction on each wheel is assumed to be the same and the components of F(t) are the same value.

Controllability

Before deriving a control techniques for this system, a quick check is made to ensure the system is indeed controllable. By calculating the controllability matrix, $\mathfrak{C}$., it can be shown that the system states, in this case rear wheel speeds, are controllable. According to control theory [4], the number of states that are controllable in a system is the rank of the controllability matrix.

$$\mathcal{C} = [\mathbf{B} \quad \mathbf{AB}]. \tag{13a}$$

From Eq. [10b] and [10c] , $\mathcal{C}$ is calculated to be:

$$\mathcal{C} = \begin{bmatrix} -\dfrac{1}{I_L} & \dfrac{1}{I_L} & \dfrac{C_w}{I_L} & 0 \\ \dfrac{1}{I_R} & -\dfrac{1}{I_R} & 0 & \dfrac{C_w}{I_R} \end{bmatrix}. \tag{13b}$$

Equation [13b] shows that the rank of the matrix is 2, which shows that the states of interest are controllable.

Desired wheel speeds

In 2004 the OSU mini-Baja team recorded the turning radius achieved with varying steering wheel positions. This set of turning radii represents the ideal path with respect to the steering wheel position since the tests were done under minimal slip conditions. To achieve the ideal turning radii given a steering wheel position during minimal slip conditions, the wheel speed of one wheel relative to another is varying. To form a starting point on which to define a set of desired wheel speed ratios as a function of steering wheel position, both vehicle geometry and collected data will be used.

In order to derive the DWSR as a function of steering wheel position, Eqs.[1a] and [1b] were rewritten with α isolated:

$$\alpha = \frac{s_i}{r} \tag{14a}$$

$$\alpha = \frac{s_o}{r+t}. \tag{14b}$$

Eq.[14a] and [14b] were then equated:

$$\alpha = \frac{s_i}{r} = \frac{s_o}{r+t}. \qquad [15]$$

Next, Eq.[15] is manipulated to form the following ratio:

$$\frac{s_i}{s_o} = \frac{r}{r+t}. \qquad [16a]$$

The path lengths s_i and s_o can also be written in terms of the number of times the wheels have turned:

$$\frac{s_i}{s_o} = \frac{2\pi R_i N_i}{2\pi R_o N_o} = \frac{r}{r+t} \qquad [16b]$$

where N_i and N_o are the number of revolutions undergone by the inner and outer wheel respectively. Since the wheels are identical, the Eq.[16b] can be simplified to:

$$\frac{N_i}{N_o} = \frac{r}{r+t}. \qquad [17]$$

By differentiating the left side of the equation with respect to time Eq.[17] is written in terms of a speed ratio as shown in Eq.[18]:

$$DWSR = \frac{\dot{\theta}_{inner\_wheel}}{\dot{\theta}_{outer\_wheel}} = \frac{r}{r+t}. \qquad [18]$$

Equation [18] is used as a basis of deriving desired wheel speeds with respect to steering wheel positions. Table I contains the steering wheel position versus turning radius test results collected by the 2004 OSU mini-Baja team.

TABLE I: Turning radius achieved at specified steering wheel positions.

Steering Wheel Angle, Φ [deg]	Turning Radius of inner wheel, r [ft]
0	0
90	22
120	18
180	11
212	9
245	4.5

At the 0° steering wheel position indicated in the table refers to the position when the wheels are headed straight. Positive angles indicate clockwise motion of the steering wheel from the 0° position and negative angles indicate counter-clockwise motion. The 245° steering wheel position is the greatest angle the wheel can turn.

Table I data provides some insight on how to define DWSR values. Using Eq. [19] and a 4.2 feet vehicle track width (t_w = 4.2 feet) the DWSR values are calculated. Table II shows DWSR values corresponding to the recorded steering wheel angles. The DWSR for the first entry at $\Phi = 0°$ does not use the formula since Eq. [19] was derived from turning geometry. Instead a unity value of DWSR at Φ=0° is entered because it is desired that the wheels turn at the same rate during a straight travel with minimal slip.

TABLE II: DWSR according to Eq.[19].

Steering Wheel Angle, Φ [deg]	DWSR for a left turn
0 (straight heading)	1
90	0.84
120	0.81
180	0.72
212	0.68
245 (fully turned steering wheel)	0.51

Table II contains the DWSR determined strictly according Eq.[19]. Using Table II as a starting point to define a set of DWSR's and desired performance characteristics, such as fully braking the inside wheel when the steering wheel position is at an extreme. Table III contains some user-defined DWSR values.

TABLE III: DWSR according to Eq. [19] and user-defined DWSR at 0° and 245°.

Steering Wheel Angle [deg], Φ	DWSR
0 (straight heading)	1
90	0.84
120	0.81
180	0.72
212	0.68
245 (fully turned steering wheel)	0

It is assumed that similar turning radius is obtained for the steering wheel angles in the counter-clockwise directions and that the DWSR for counter-clockwise turns will be the same. Using Microsoft Excel, Φ was plotted against DWSR and the points are fitted to a polynomial curve. Figure 5 shows the left and right turn maneuvers are combined into one graph.

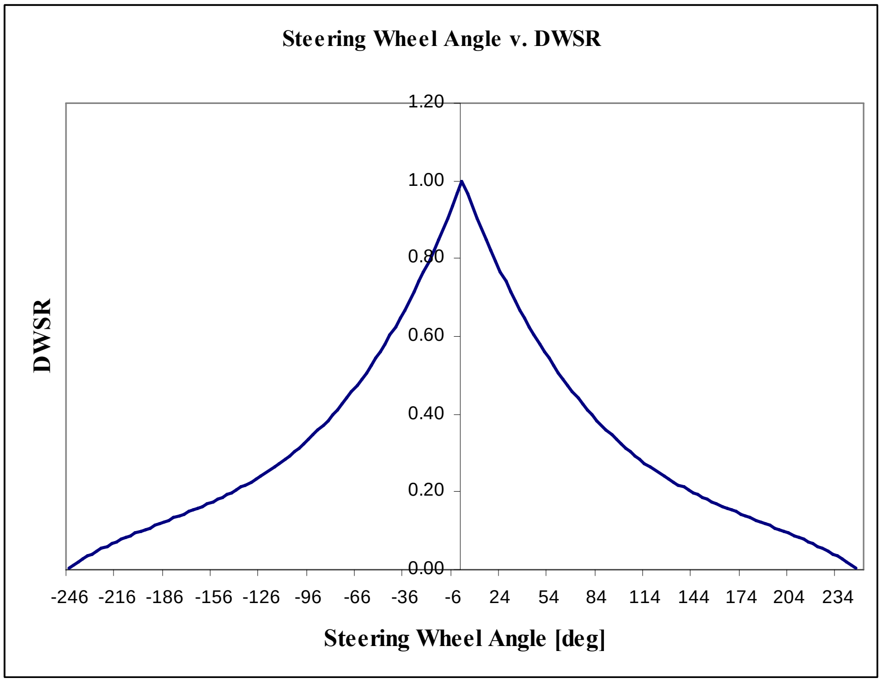

Figure 5: DWSR as a function of Φ.

For the curve left of the graph and right of the graph, Excel provides the following fitting equations, respectively:

$$DWSR_{left\_turn} = (9E-8)\Phi^3 + (5E-5)\Phi^2 + 0.0109\Phi + 1 \qquad [19a]$$

$$DWSR_{right\_turn} = (-9E-8)\Phi^3 + (5E-5)\Phi^2 - 0.0109\Phi + 1 \qquad [19b]$$

It will be convenient to avoid excess braking control effort in actual implementation. One solution to this problem would have an error buffer zone. This buffer-zone would reduce the amount of control effort that would occur for insignificant differences between the actual and desired wheel speeds. Another method of reducing excess control effort would be to associate a range of steering wheel positions to the same DWSR. This will eliminate the need for every steering wheel position to be associated with a slightly different DWSR. The depiction of these buffer-zones are shown in Fig.9.

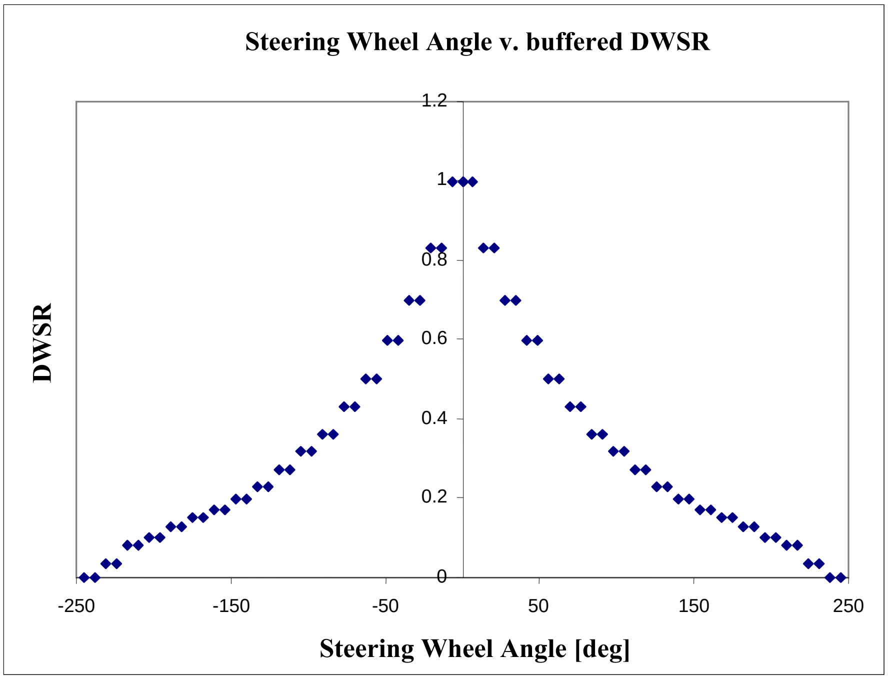

Figure 6: Buffered DWSR as a function of Φ.

It is desired that the actual wheel speed ratio (AWSR) achieve the DWSR shown in Fig.5. The DWSR provides a way to derive the desired wheel speeds (DWS) that will be compared against the actual wheel speeds (AWS) that are fed back in the control loop.

The WSR is defined as:

$$WSR = \frac{\dot{\theta}_{inner\_wheel}}{\dot{\theta}_{outer\_wheel}}. \quad [20a]$$

This definition will allow a zero value to be read during the event when the inner wheel is fully braked. The variable $\dot{\theta}$ is assumed to be the actual wheel speed state when it is written without the *d* subscript. Following this notation the DWSR may be written as follows:

$$DWSR = \frac{\dot{\theta}_{inner\_wheel} \pm X}{\dot{\theta}_{outer\_wheel} \mp X} \ , \qquad [20]$$

where, "X" is the speed change that must be made by the wheels in order to get the AWSR to the DWSR. In Eq.[20] it is assumed that braking a slipping wheel to slow it down by a certain amount speeds up the other wheel by the same amount via the open differential. Equation [20] leads to DWS calculations for four cases.

The first two cases are for a right turn maneuver :

1. AWSR < DWSR (left wheel slip), and
2. AWSR > DWSR. (right wheel slip)

The next two cases are for a left turn maneuver:

1. AWSR < DWSR, (right wheel slip), and
2. AWSR > DWSR. (left wheel slip)

Consider a turn maneuver when the AWSR is defined with the right wheel speed state in the numerator. The following inequality can be written for the case where AWSR < DWSR :

$$\frac{\dot{\theta}_R}{\dot{\theta}_L} < \frac{\dot{\theta}_{R,d}}{\dot{\theta}_{L,d}} = DWSR \qquad [21]$$

In this case, the left wheel is slipping causing the AWSR to be smaller than the desired.

Equations [22] – [25] show how the DWS is derived for a right turn case when AWSR < DWSR. First, in Eq.[21], the DWSR is defined for a right turn case when AWSR < DWSR (left wheel is slipping).

$$DWSR = \frac{\dot{\theta}_{R,d}}{\dot{\theta}_{L,d}} = \frac{\dot{\theta}_R + X}{\dot{\theta}_L - X} \qquad [22]$$

Note the signs on the "X" in the equation. In order to slow down the slipping left wheel to the desired state it is slowed by an amount "X", which in turn speeds up the right wheel by "X". Next, "X" is solved for in Eq.[23]:

$$X = \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR}. \qquad [23]$$

Substituting Eq.[23] into [22] yields Eq.[24a] and [24b] :

$$\dot{\theta}_{R,d} = \dot{\theta}_R + \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR} \qquad [24a]$$

$$\dot{\theta}_{L,d} = \dot{\theta}_L - \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR}. \qquad [24b]$$

The X value is defined to be a positive value in which a wheel is desired to speed up or down. The DWS for the other three cases are derived in the same fashion. Shown in Table IV are the DWS for the different cases.

TABLE IV: Desired wheel speed calculations.

Slip Case	DWSR	Change in speed, X	DWS Equations
<u>Right Turn</u> Left Slip	$\frac{\dot{\theta}_{R,d}}{\dot{\theta}_{L,d}} = \frac{\dot{\theta}_R + X}{\dot{\theta}_L - X}$	$X = \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR}$	$\dot{\theta}_{R,d} = \dot{\theta}_R + \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR}$ $\dot{\theta}_{L,d} = \dot{\theta}_L - \frac{-\dot{\theta}_R + DWSR * \dot{\theta}_L}{1 + DWSR}$
Right Slip	$\frac{\dot{\theta}_{R,d}}{\dot{\theta}_{L,d}} = \frac{\dot{\theta}_R - X}{\dot{\theta}_L + X}$	$X = \frac{\dot{\theta}_R - DWSR * \dot{\theta}_L}{1 + DWSR}$	$\dot{\theta}_{R,d} = \dot{\theta}_R - \frac{\dot{\theta}_R - DWSR * \dot{\theta}_L}{1 + DWSR}$ $\dot{\theta}_{L,d} = \dot{\theta}_L + \frac{\dot{\theta}_R - DWSR * \dot{\theta}_L}{1 + DWSR}$
<u>Left Turn</u> Right Slip	$\frac{\dot{\theta}_{L,d}}{\dot{\theta}_{R,d}} = \frac{\dot{\theta}_L + X}{\dot{\theta}_R - X}$	$X = \frac{-\dot{\theta}_L + DWSR * \dot{\theta}_R}{1 + DWSR}$	$\dot{\theta}_{R,d} = \dot{\theta}_R - \frac{-\dot{\theta}_L + DWSR * \dot{\theta}_R}{1 + DWSR}$ $\dot{\theta}_{L,d} = \dot{\theta}_L + \frac{-\dot{\theta}_L + DWSR * \dot{\theta}_R}{1 + DWSR}$
Left Slip	$\frac{\dot{\theta}_{L,d}}{\dot{\theta}_{R,d}} = \frac{\dot{\theta}_L - X}{\dot{\theta}_R + X}$	$X = \frac{\dot{\theta}_L - DWSR * \dot{\theta}_R}{1 + DWSR}$	$\dot{\theta}_{R,d} = \dot{\theta}_R + \frac{\dot{\theta}_L - DWSR * \dot{\theta}_R}{1 + DWSR}$ $\dot{\theta}_{L,d} = \dot{\theta}_L - \frac{\dot{\theta}_L - DWSR * \dot{\theta}_R}{1 + DWSR}$

Explicit model-following control method

A general diagram of a model following control is shown in Fig.8.

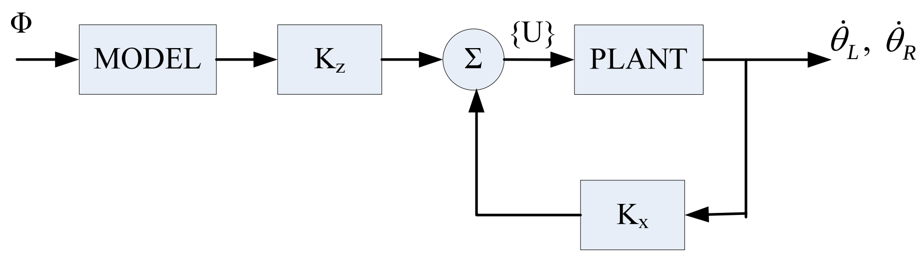

Figure 7 : Control Loop of SBS

The system input is the steering wheel position, Φ, the control inputs are the left and right wheel brakes represented by the vector, U, and the outputs are the wheel speeds which are fed back to compute any necessary control. The MODEL block takes in the reference steering wheel and outputs a desired wheel speed states. The K_Z and K_L are the gains for the input calculated according to the model following control method.

The state x_2 and x_3 track the desired wheel speeds derived earlier using an explicit model follower control scheme. A cost function that considers the error between a desired model and the actual system is formed in order to develop the control algorithm for this method. The desired model is represented by Eq. [25]:

$$\dot{z}_m = A_m z_m \tag{25}$$

where, z_m is a 2 x 1 vector of the desired wheel speed states and A_m is the desired system matrix. The cost function is shown in Eq. [26]:

$$J = \frac{1}{2}\int_0^{\infty}((y - z_m)^T Q(y - z_m) + u^T Ru)dt \tag{26}$$

where, Q is a 2 x 2 state weighting matrix, R is a 2 x 2 control weighting matrix, y is a 2 x 1 output vector, u is a 2 x 1 control input vector. Eq. [27] through Eq. [40] describe the process of obtaining the explicit model following control scheme for the cost function.

Consider the composite state vector, β, shown in Eq.[27]:

$$\beta = \begin{bmatrix} x_2 \\ x_3 \\ z_1 \\ z_2 \end{bmatrix}. \qquad [27]$$

The state equations can be written in terms of β as shown in Eq. [28]:

$$\dot{\beta} = \begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix} \beta + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad [28]$$

From Eq. [28], the following quantities are defined:

$$\widetilde{A} = \begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix} \quad \text{and} \quad \widetilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad [29]$$

Now, the difference between the actual output and the desired output is derived as follows:

$$y - z_m = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ z_m \end{bmatrix} - \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} x \\ z_m \end{bmatrix} \qquad [30]$$

where, I is the 2 x 2 identity matrix, C is a 2 x 2 output distribution matrix. Equation [30] simplifies to Eq.[31]:

$$y - z_m = \begin{bmatrix} C & -I \end{bmatrix} \beta . \qquad [31]$$

So the first term on the right of the cost function, Eq.[26] can be written as follows:

$$(y - z_m)^T Q(y - z_m) = \beta^T \begin{bmatrix} C^T \\ -\bar{I} \end{bmatrix} Q\begin{bmatrix} C & -I \end{bmatrix}\beta \qquad [32a]$$

$$= \beta^T \begin{bmatrix} C^T \\ -I \end{bmatrix} \begin{bmatrix} QC & -QI \end{bmatrix}\beta \qquad [32b]$$

$$= \beta^T \begin{bmatrix} C^T QC & -C^T Q \\ -QC & Q \end{bmatrix}\beta \qquad [32c]$$

and from Eq.[32c], the following term is defined:

$$\widetilde{Q} = \begin{bmatrix} C^T QC & -C^T Q \\ -QC & Q \end{bmatrix}. \qquad [33]$$

With β and $\widetilde{Q}$, the cost function can be rewritten as:

$$J = \frac{1}{2}\int_0^\infty (\beta^T \widetilde{Q}\beta + u^T Ru)dt\,. \qquad [34]$$

This form of cost function is recognized as the optimal regulator problem to which the solution is readily available in [8].

$$u = -R^{-1}\widetilde{B}^T P\beta\,, \qquad [35]$$

where, P solves the algebraic Ricatti equation:

$$P\widetilde{A} + \widetilde{A}^T P + \widetilde{Q} - P\widetilde{B}R^{-1}\widetilde{B}^T P = 0\,. \qquad [36]$$

P can be partitioned as shown:

$$P = \begin{bmatrix} P_{xx} & P_{xz} \\ P_{zx} & P_{zz} \end{bmatrix}. \qquad [37]$$

With P partitioned, the control input term can be expanded as:

$$u = -R^{-1}B^T P_{xx} x - R^{-1}B^T P_{xz} z_m . \qquad [38]$$

The gains, K_z and K_x, both 2 x 2 matrices, seen in Fig.8 are defined as:

$$K_x = -R^{-1}B^T P_{xx} \quad \text{and} \quad K_z = -R^{-1}B^T P_{xz} \qquad [39]$$

and can be substituted into Eq. [38] to get Eq. [40]:

$$u = K_x x + K_z z_m . \qquad [40]$$

MatLab functions are used to solve the required Ricatti equation and compute the gains of the control algorithm. MatLab is also used to integrate the equations of motion and show the results graphically. In the simulation the braking limits and actions are specified. For example, when there is a driving torque input into the system, the simulation will not allow both brakes to activate at the same time which in the real world would cause damage to the transmission. The limits of the possible brake torque in the simulation are approximated by the limits of the actual vehicle.

Simulation of SBS in a linear vehicle model

The effectiveness of the SBS is observed through simulations of uncontrolled slipping and controlled slipping situations. The slipping occurrences that will be simulated are:

1) Straight-line travel ($\Phi = 0°$) with one wheel slipping on a patch loose earth after both wheel reach a constant speed.
2) Straight-line travel with slip occurring at initial acceleration.
3) Constant turn (Φ = constant) with one wheel slipping (i.e. a wheel coming off the ground, or a wheel hitting loose earth.)
4) The occurrence of two slip events occurring: once at the start of acceleration and once during a right turn at $\Phi = 245$.

Shown in Table V are the specifications of the vehicle used in these simulations.

TABLE V: Vehicle specifications used in simulations

Vehicle Characteristic	Value	Units
Mass, m	176.9	Kg
Wheel Radius, r	0.2032	M
Wheel Inertia, I	0.65	kg-m^2
Coefficient of drag on vehicle, C_v	0.2	
Viscous coefficient on wheels, C_w	0.295	N-m

The Explicit model-follower control technique was used to calculate gains that helped the control input bring the states to desired quantities. The gains were then adjusted from the initial calculation to achieve desired performance of the brakes. Also, the SBS is commanded only to apply one brake at a time according to which wheel is slipping. This property was programmed into the explicit model- follower based control. Limits on the amount of braking torque were also set to 600 Nm.

Simulation Results

Each of the simulations run for t= 25 seconds and there is a constant input drive torque applied to the vehicle wheels which represents a throttle input. The wheel speeds and the error between desired and actual wheel speed ratios will be shown for both the uncontrolled and controlled situations. The braking control effort for the control case will be shown for the controlled situations.

Simulation Set #1

For the first set of simulations, Fig.10a-10l, the vehicle is traveling straight with the steering wheel at position Φ =0, making the DWSR =1. Between t=10 seconds and t = 15 seconds the tractive of the left wheel will decrease, which represents the wheel slipping. For completeness, the effect of increasing the state weighting value from unity to 100 will be compared in the first set of simulations. The state weighting value that achieves better tracking will be used for the rest of the simulations.

Simulation #1.1: System responses during a non-slipping situation.

Figures 10a-10c are the system responses during a non-slipping situation with the steering wheel at Φ = 0°.

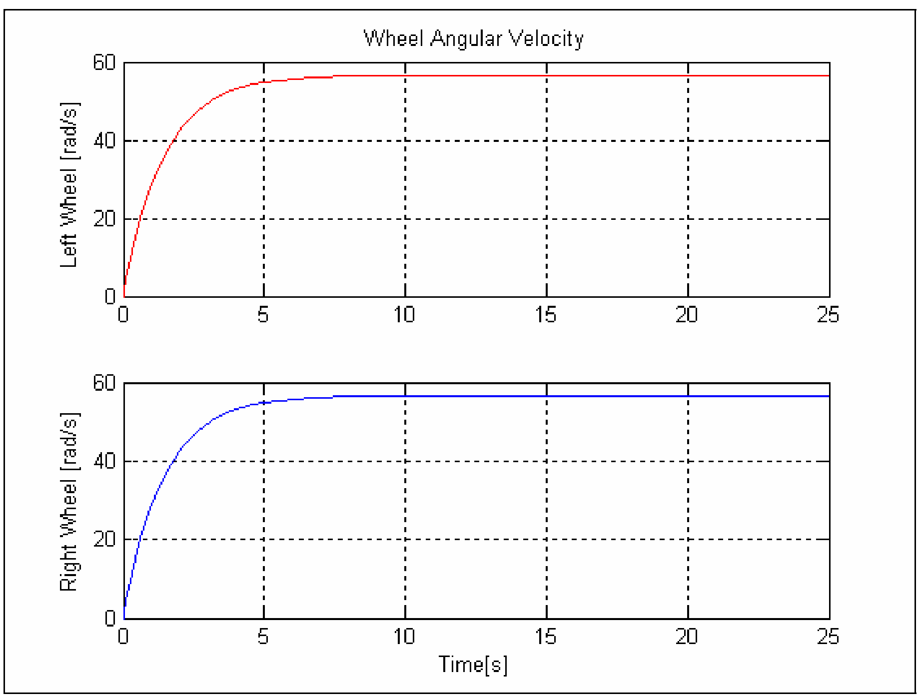

Figure 10a: Wheel speeds under no slipping.

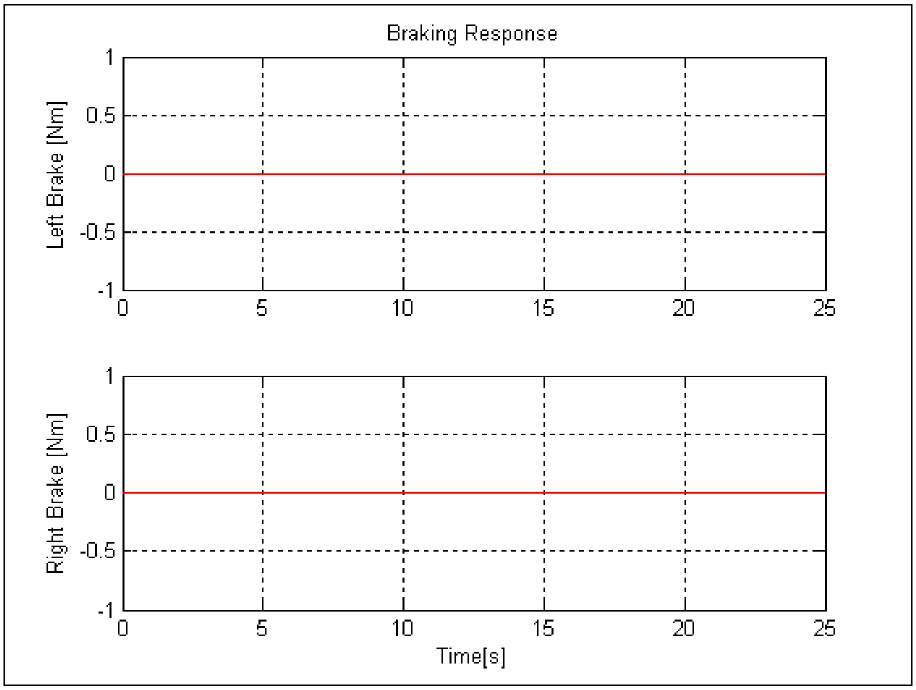

Figure 10b: Brake response for straight travel with no slipping.

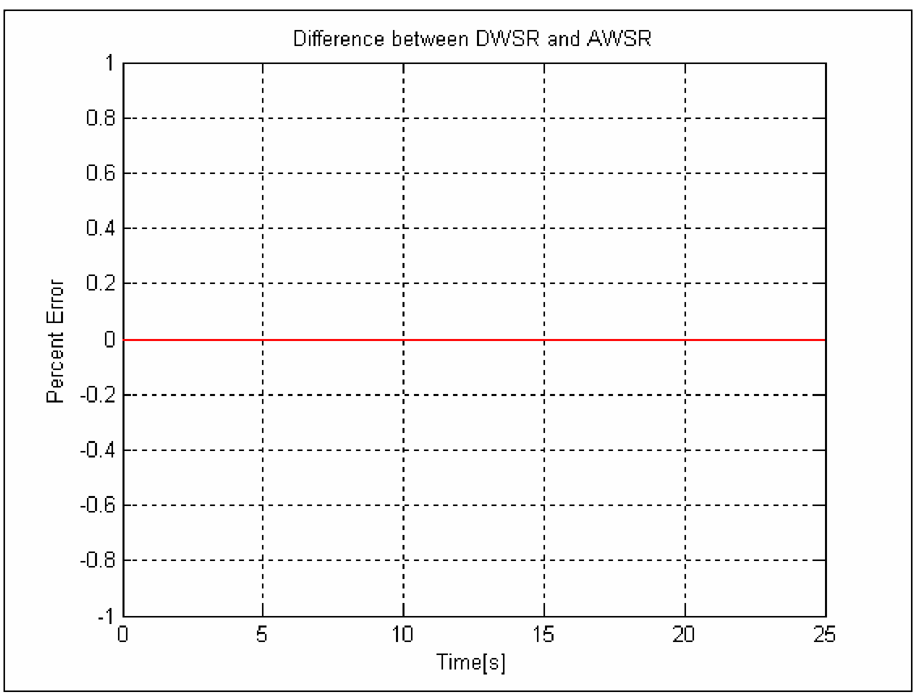

Figure 10c: Difference between DWSR and AWSR during straight line travel with no slipping.

<u>Simulation #1.2:</u> System responses during an uncontrolled slipping situation.

Figures 10d – 10f are the simulation results during an uncontrolled slip event, which happens at t = 10 seconds and ends at t = 15 seconds into the simulation.

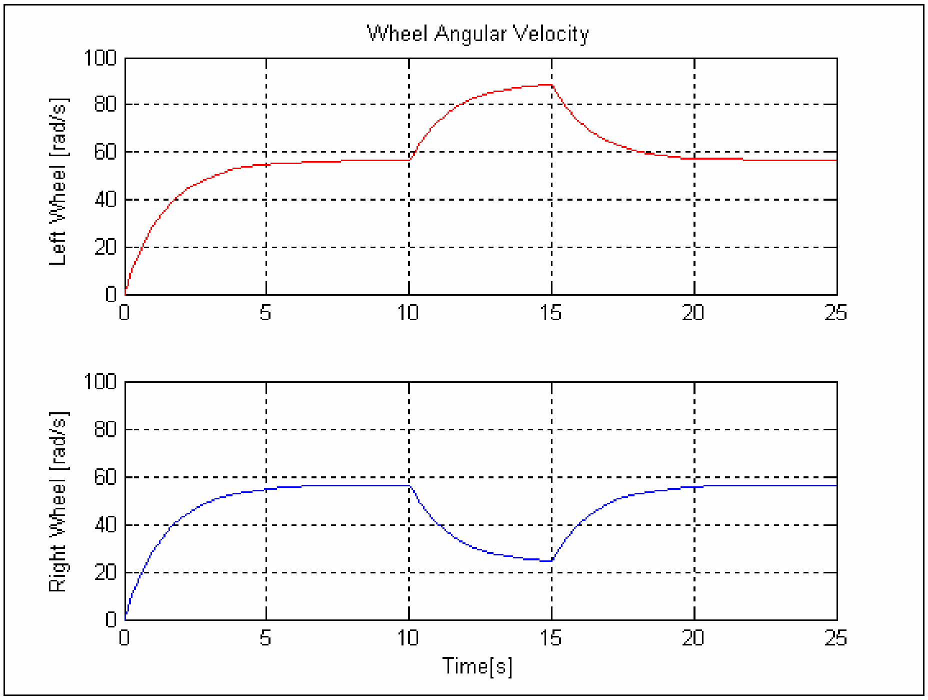

Figure 10d: Straight travel with slipping of the left wheel.

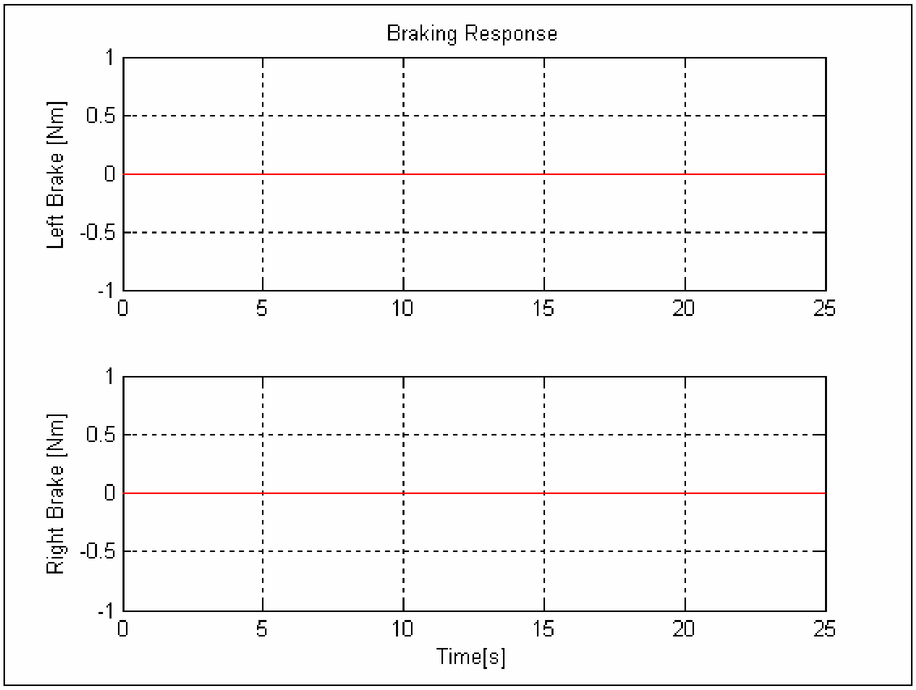

Figure 10e: Braking response during an uncontrolled straight travel with slip.

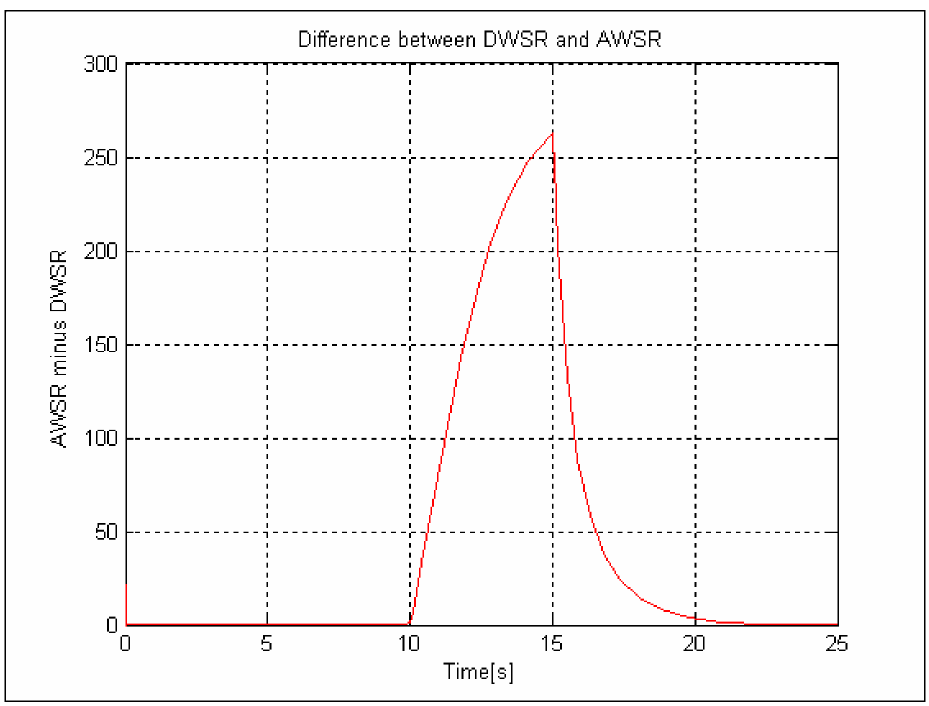

Figure 10f: Difference between DWSR and AWSR for an uncontrolled straight travel with slip.

<u>Simulation #1. 3:</u> System responses during a controlled slip situation with a unity state weighting value.

Figures 10g-10i show the results of a controlled slip situation. The conditions for slip are the same as simulation # 1-2, however, for this simulation, the control is activated and the DWSR is better tracked. The calculation of the control gains uses a state weighting value of one.

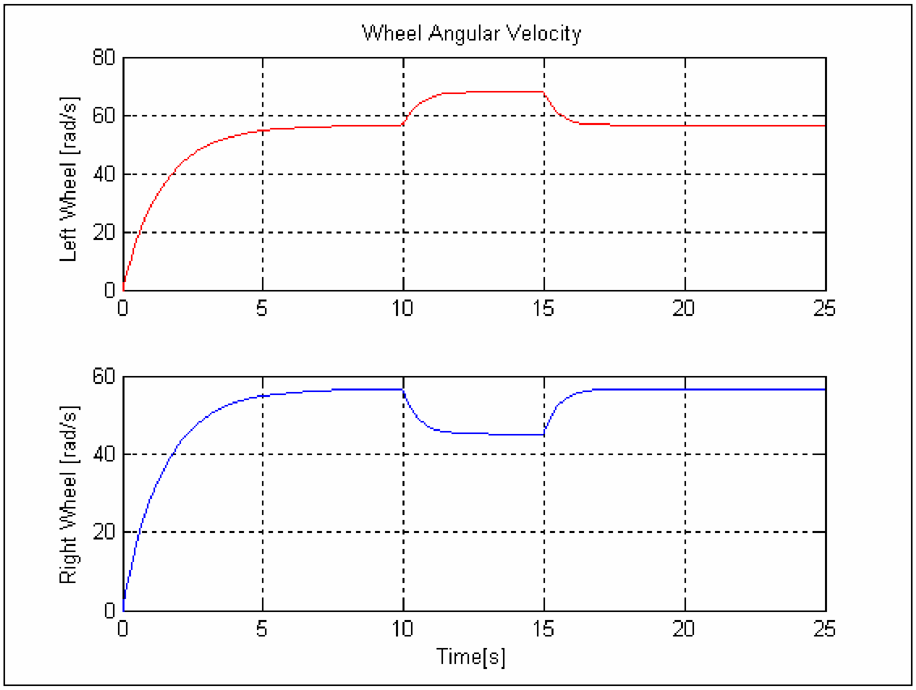

Figure 10g: Wheel speeds for a controlled straight line travel with slip (State weighting of unity).

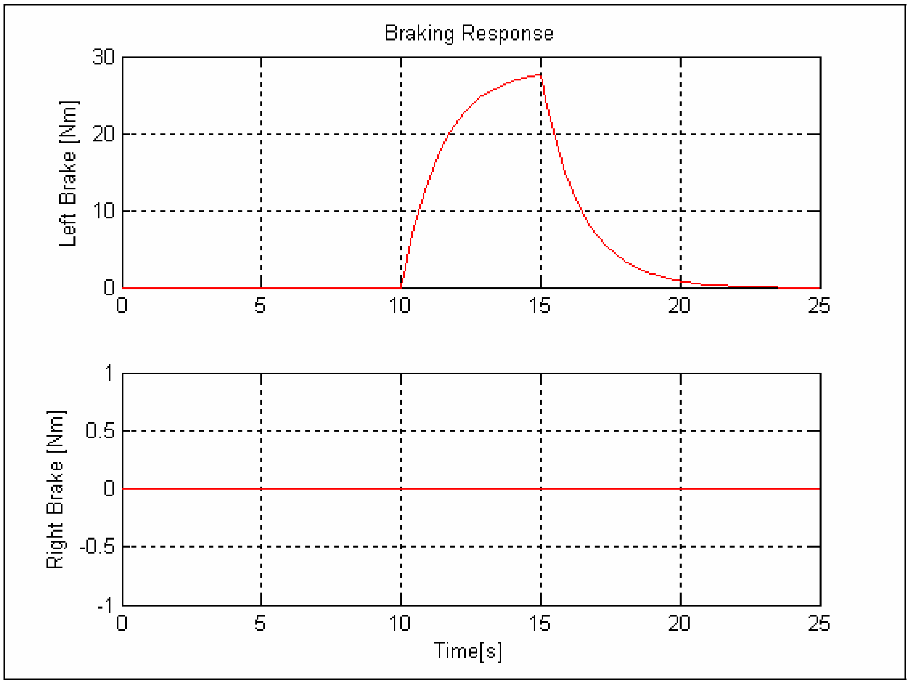

Figure 10h: Braking response for a controlled straight travel with slip (State weighting of unity).

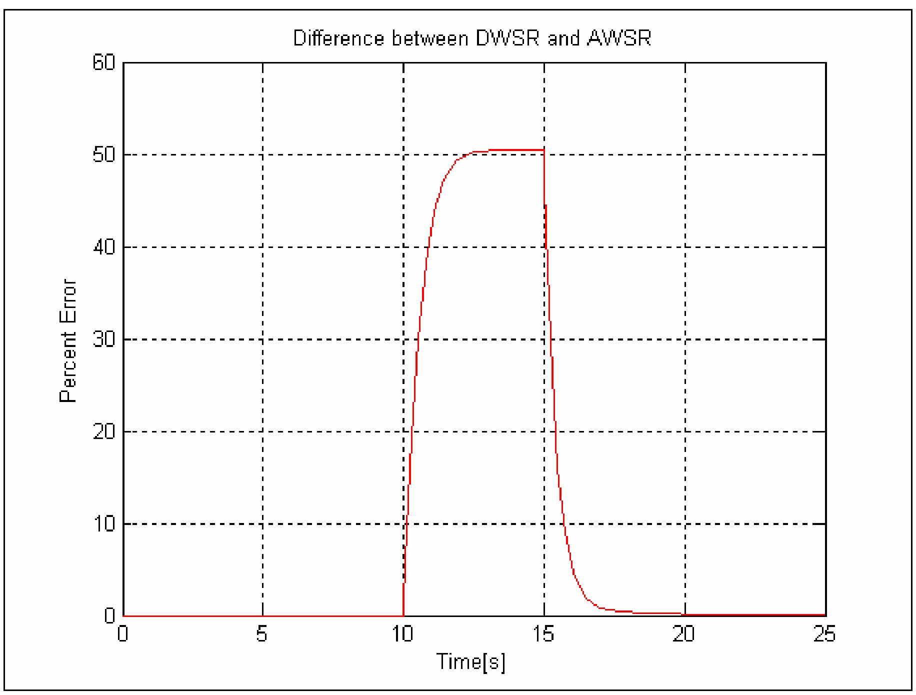

Figure 10i: DWSR v. AWSR for a controlled straight travel with slip (State weighting of unity).

<u>Simulation #1. 4:</u> System responses during a controlled slip situation with state weighting given a value of 100.

Figures 10g-10i show the results of a controlled slip situation. The conditions for slip are the same as simulation # 1-2, with the calculation of the control gains done with state weighting value of 100.

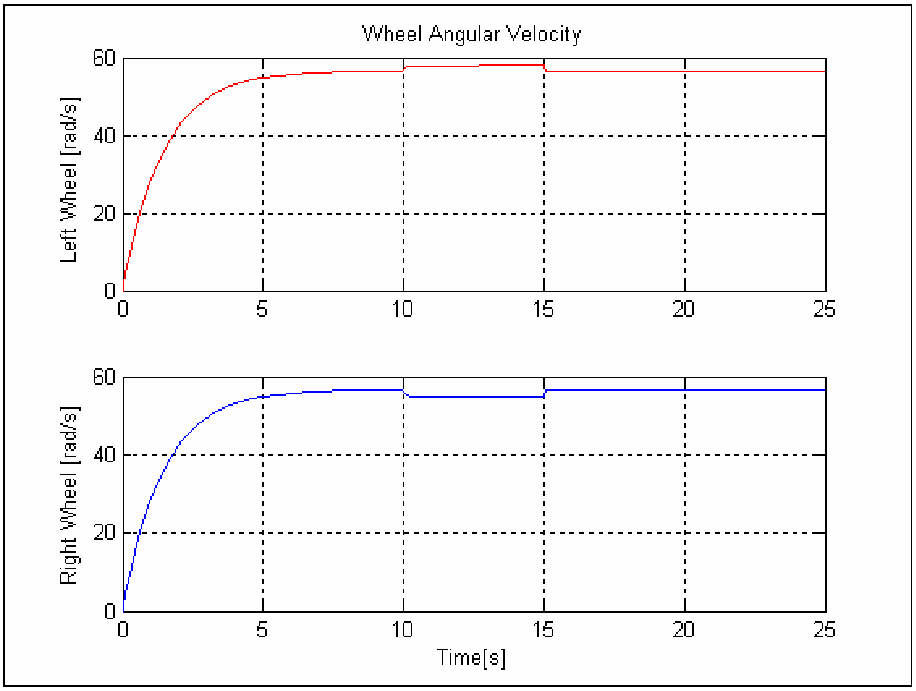

Figure 10j: Wheel speeds for a controlled straight line travel with slip (State weighting of 100).

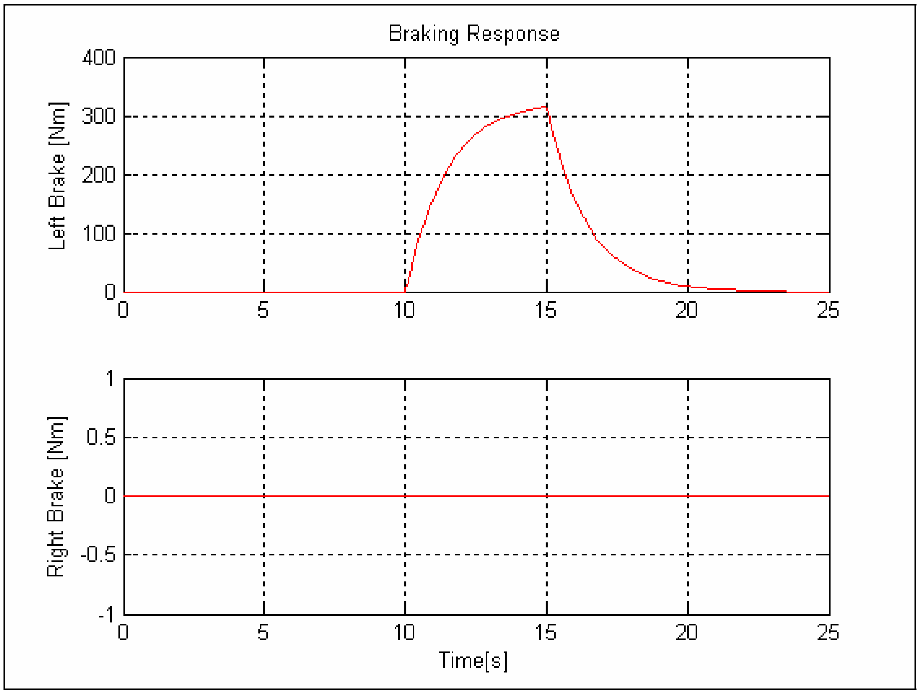

Figure 10k: Braking response for a controlled straight line travel with slip (State weighting of 100).

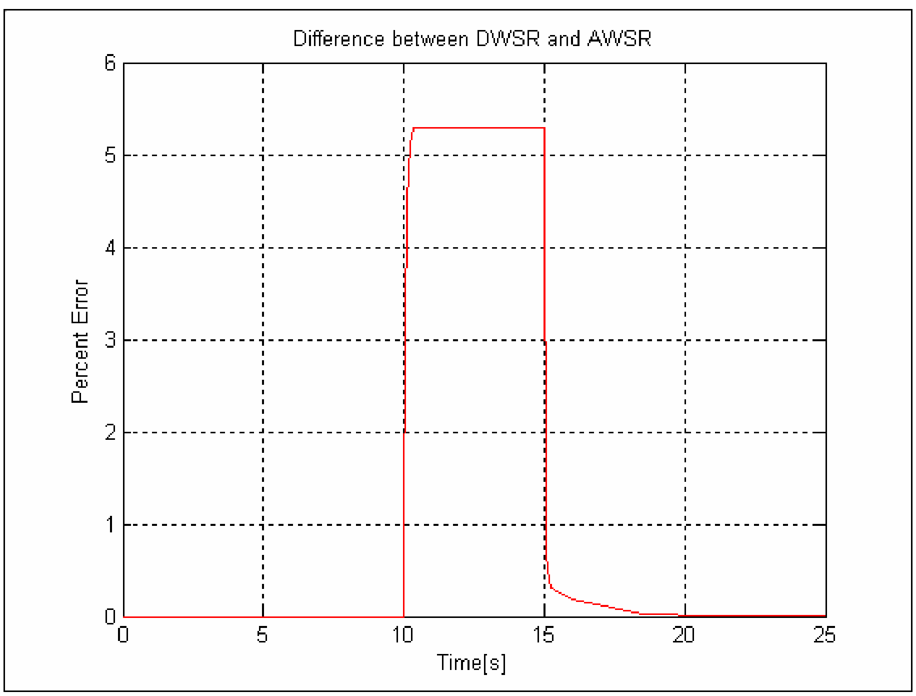

Figure 10l: DWSR v. AWSR for a controlled straight line travel with slip (State weighting of 100).

Figures 10a shows the wheel velocity profile for wheels undergoing no slip while the steering wheel is at $\Phi = 0°$, which makes DWSR =1. The left and right wheels maintain the same speed profile and the AWSR matches the DWSR. Figures 10b and 10c show the corresponding SBS braking response and the percent difference between the desired and actual wheel speed ratios, respectively.

Figures 10d -10f show the system response with the same conditions as simulation set #1.1 but with an uncontrolled slipping event at t = 10 seconds and ends at t = 15 seconds. Figure 10d shows the wheel velocity changes when the left wheel loses tractive force and causes an increase of drive torque to the left wheel and a loss of the same drive torque magnitude applied to the right wheel. The left wheel velocity increase is nearly equal to the decrease in right wheel velocity. Figures 10d and 10f are the corresponding braking response and error between the desired and actual wheel speed ratios.

Figures 10g -10i show the results of the controlled slip response with the control gains calculated with a unity state weighting value. The actual wheel speed ratio more closely follows the desired value of one, however, there still is a little difference. As the brake is applied, Fig.10h, the velocity of the left wheel due to slip is lowered. Braking the excess drive torque to the left wheel due to slip forces drive torque to be transferred to the right wheel speeding it up as a result.

Figures 10j - 10l show the results of the controlled slip response with the control gains calculated with a state weighting value of 100. The actual wheel speed ratio follows the desired wheel speed ratio more closely than the previous case. Increasing the state weighting value causes a greater brake torque response to slip.

Simulation Set #2

The second set of simulations, Fig. 11a – 11i, simulates an event where the right wheel slips at time t = 0 seconds during initial acceleration and stops slipping when it reaches more solid ground at t = 5 seconds.

Simulation #2.1: *System response during acceleration without slip.*

Figure 11a shows the results of a non-slip situation during initial throttle acceleration with a straight heading ($\Phi = 0°$). The braking control effort and the difference between desired and actual wheel speeds are not plotted since these are unaffected during a non-slipping situation.

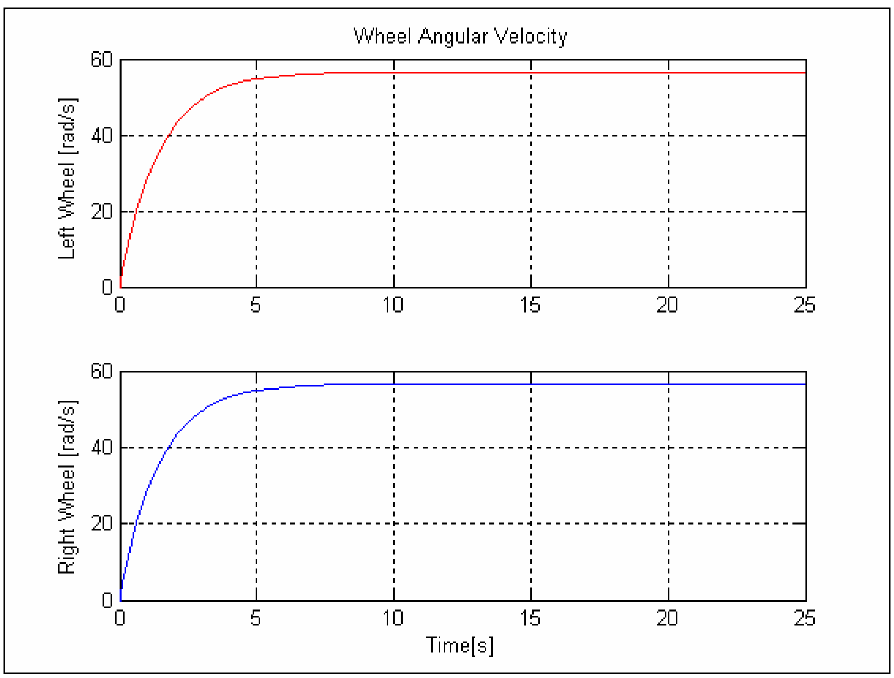

Figure 11a: Straight line acceleration with no-slip at initial take off.

<u>Simulation #2.2:</u> System responses during an uncontrolled slip of the right wheel at the start of acceleration.

Figures 11b -11c show the results of the controlled slip situation during initial throttle acceleration. The braking control effort is again unaffected since it is an uncontrolled situation and will not be plotted.

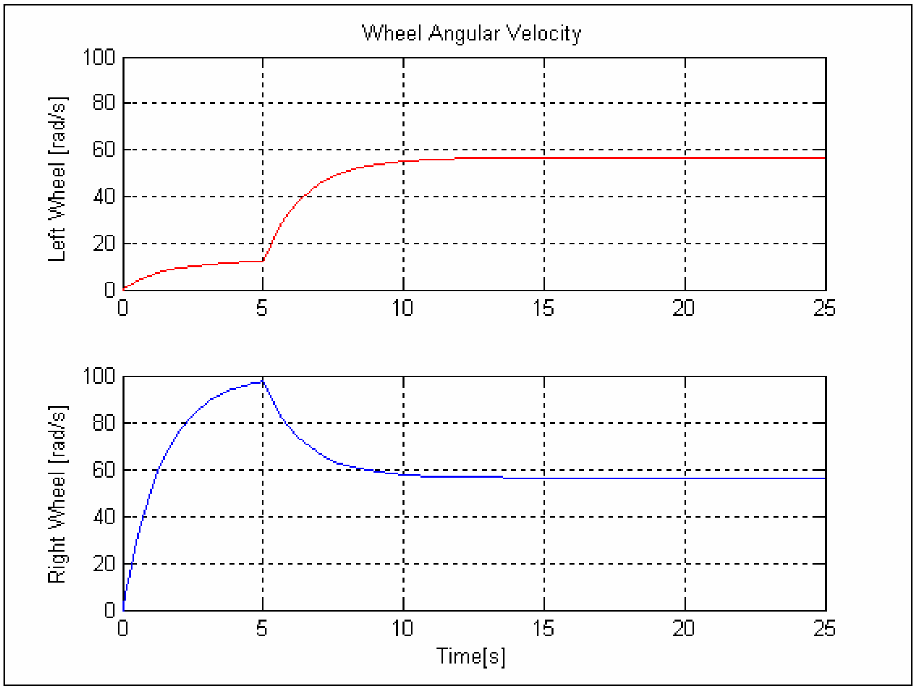

Figure 11b: Straight line acceleration with the right wheel slipping at start.

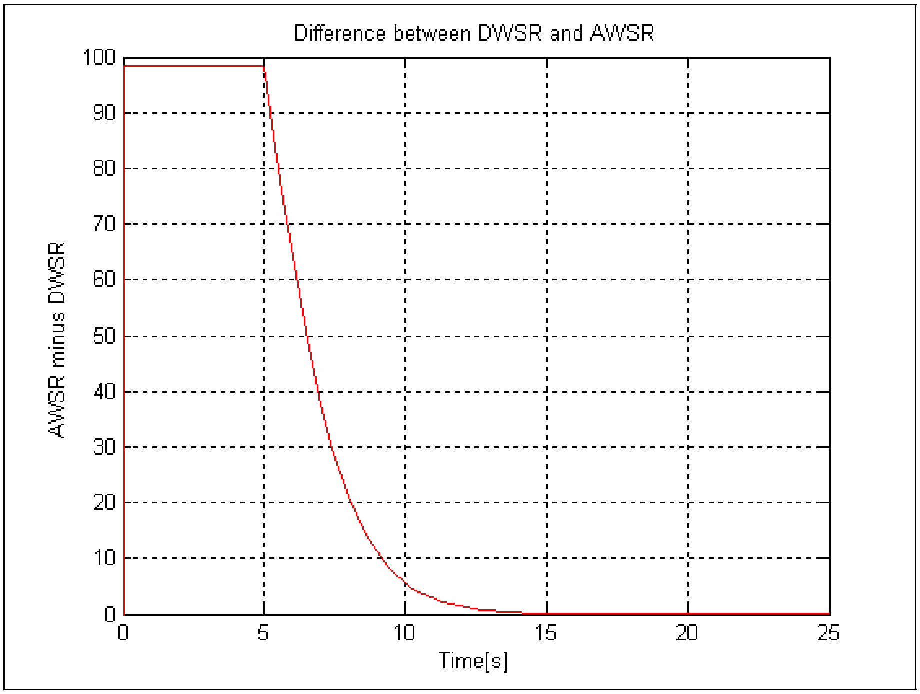

Figure 11c: DWSR v. AWSR with uncontrolled right wheel slipping at start.

<u>Simulation #2.3:</u> System responses during a controlled acceleration with slip.

Figures 11d - 11f show the results of a controlled slip situation occurring during initial acceleration. The conditions for slip are identical to simulation #1.2, however, SBS control is activated.

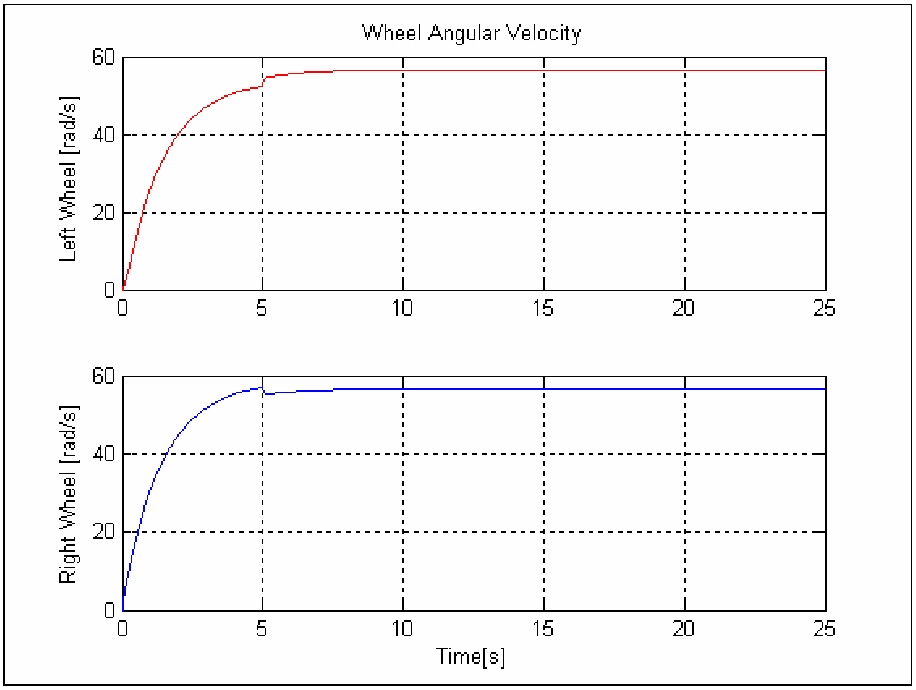

Figure 11d: Wheel speeds for a controlled straight line acceleration with slip at start.

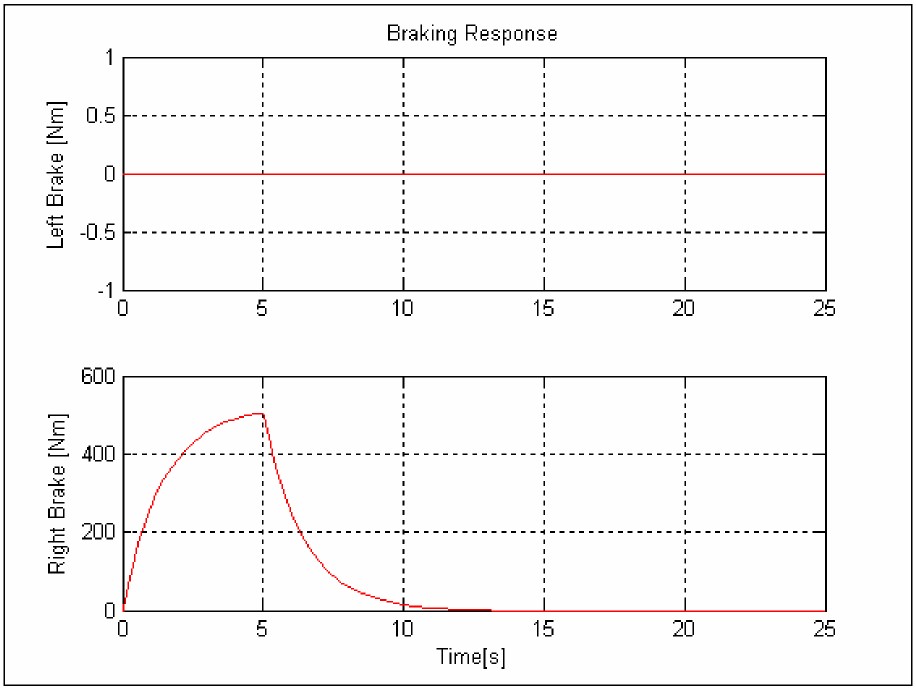

Figure 11e: Braking response for a controlled straight line acceleration with slip occurring at the start.

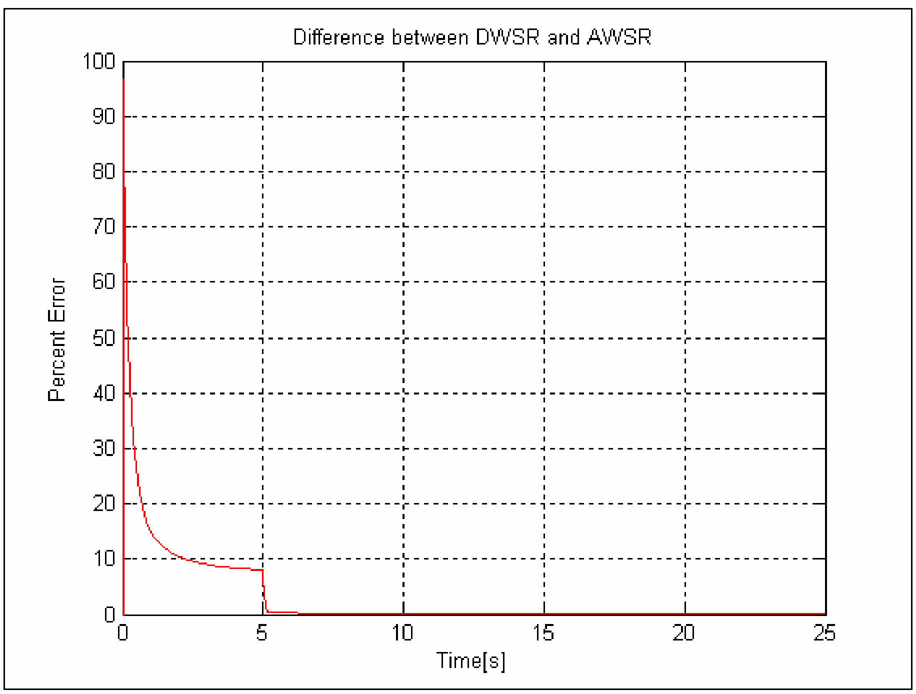

Figure 11f: DWSR v. AWSR for a straight line acceleration with slip occurring at the start.

Figure 11a shows the wheel velocities during acceleration without slip. Figures 11b -11c show the wheel velocities for an uncontrolled right wheel slip occurring at t=0 and ending at t= 5 seconds. The difference between the actual and desired wheel speed ratios eventually reach zero after about t = 13 seconds while the slipping wheel gradually slows down after regaining traction.

Figure 11d shows the controlled slip wheel speeds resulting from the SBS braking commands shown in Fig. 11e. The brake torque to the right wheel is increased gradually decreasing the error between DWSR and AWSR The AWSR is near the DWSR when the slipping wheel reaches a surface with good traction and the time for AWSR to match the DWSR is reduced from the uncontrolled case.

Simulation Set #3

The third set of simulations, Fig. 12a - 12f simulate a turning event where slipping occurs during a turn. At = 10 seconds the steering wheel has turned from $\Phi = 0°$ to $\Phi = 100°$ to the left and remains there for 3 seconds.

Simulation #3.1: System response to an uncontrolled slip during a turning maneuver with $\Phi = 100°$ to the left.

The plot of the steering wheel position is provided and can be used to observe how it affects when the brakes are applied, or more specifically, how it changes the DWSR value.

Figure 12a: Steering wheel position for simulation set #3.

The rotation rates of the wheels should differ during an effective turn. Keeping this in mind, Fig. 12b shows the case of a slipping situation with respect to the steering wheel position.

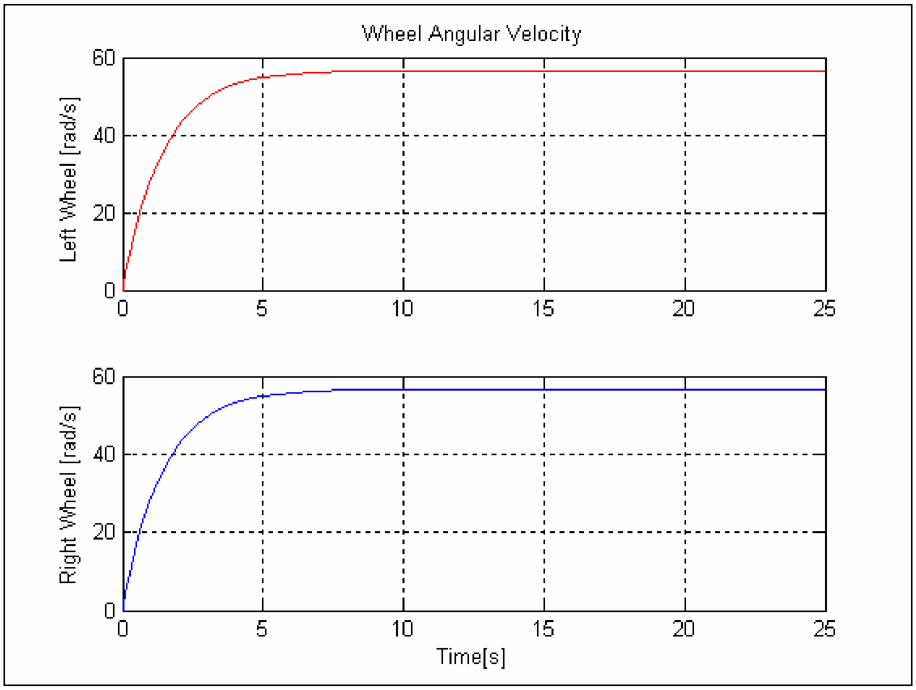

Figure 12b: Wheel speeds for an uncontrolled slip during a turning maneuver (Φ =100 to left)

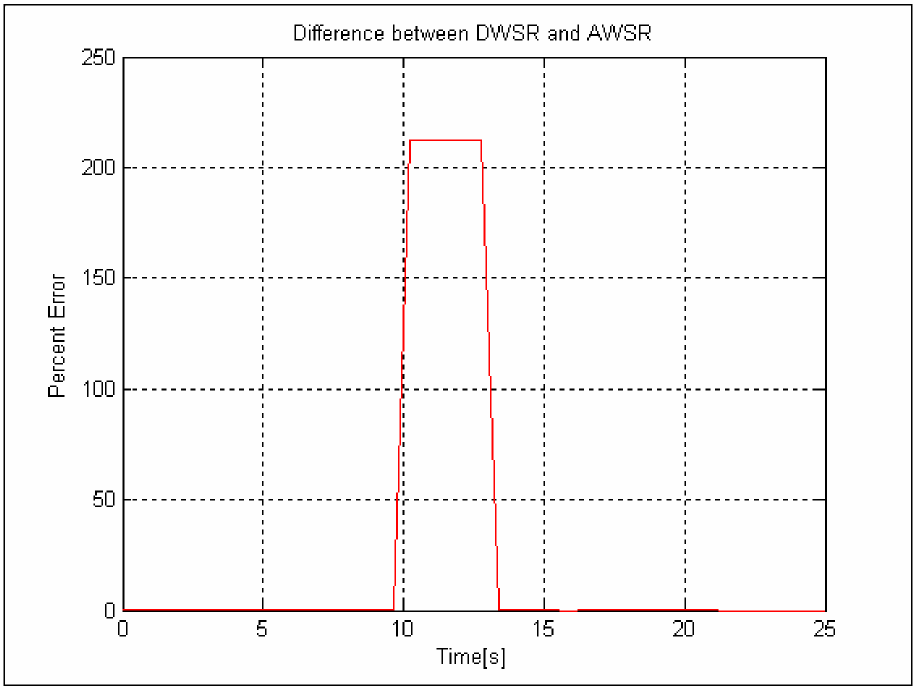

Figure 12c: DWSR v. AWSR for an uncontrolled slip during situation (Φ = 100°).

Simulation #3.2: *System response during an uncontrolled slip during a turning maneuver with* $\Phi = 100°$ to the left.

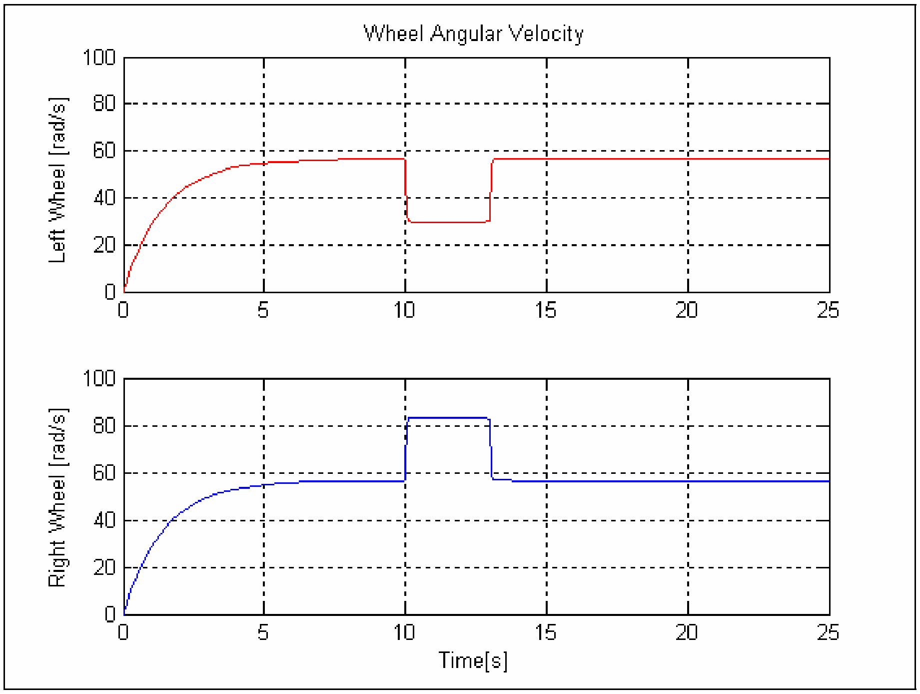

Figure 12d: Wheel speeds during a controlled slip during a turning maneuver ($\Phi = 100°$).

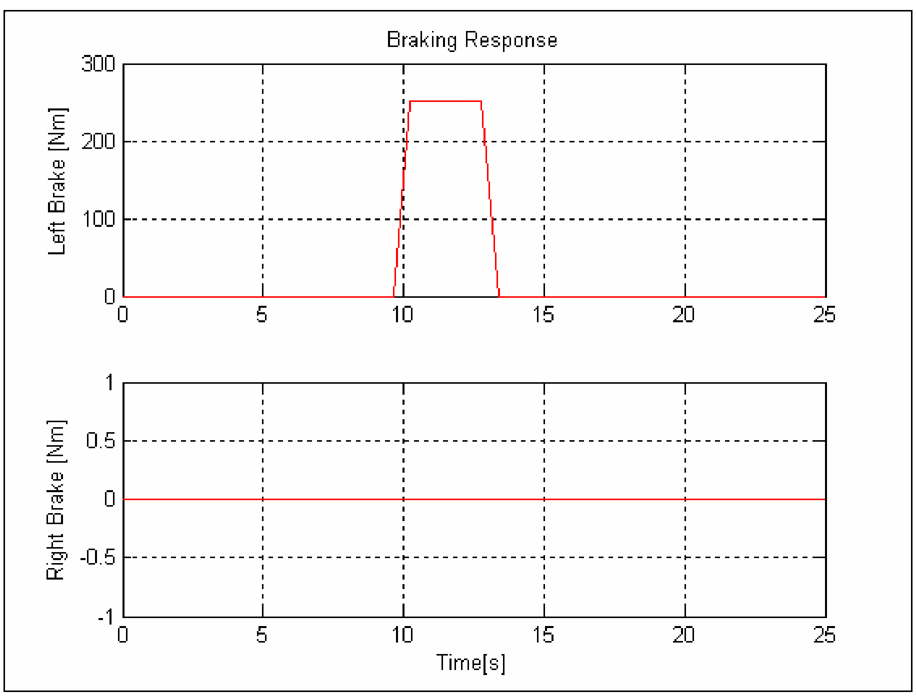

Figure 12e: Braking response during a controlled slip during a turning maneuver ($\Phi = 100°$) .

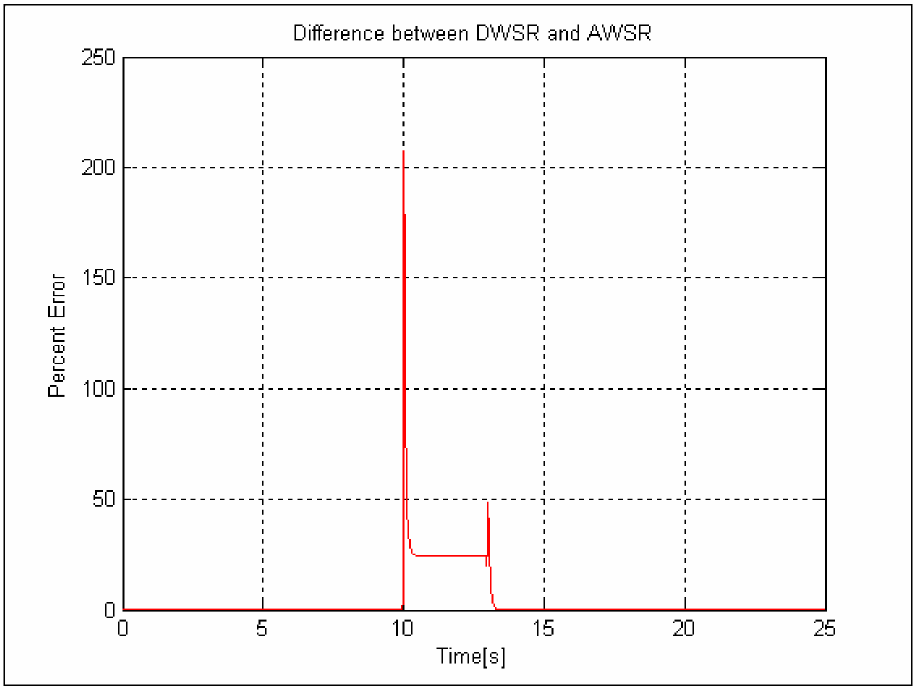

Figure 12f: DWSR v. AWSR during a controlled slip during a turning maneuver (Φ = 100°)

Figure 12a shows the steering wheel positions that are under consideration for this set of simulations. At about t = 10 seconds, the steering wheel is turned from 0° to 100° left or counter-clockwise and back to 0° at about t=13 seconds.

Figure 12b - 12c show the wheel speeds and the difference between the DWSR and AWSR, respectively, during an uncontrolled slip occurring during a turning maneuver. Fig.12b shows that at t=10 seconds, the wheel speeds remains the same or the wheel speed ratio is one. When the vehicle is undergoing a turn with the steering wheel at $\Phi = -100°$, it is desired that the outer wheel turns faster than the inner wheel. Figure 12c shows the difference between the actual and desired wheel speed ratios during the uncontrolled slip while turning.

Figures 12d - 12f show the wheel speeds, braking response, and differences between AWSR and DWSR, respectively, during a controlled slip while turning. When the

DWSR is issued by the steering wheel at about t =10 seconds, and the AWSR remains at one, the SBS reacts as shown in Fig. 12e. The vehicle is turning left and the DWSR is such that the left wheel is to turn slower than the right. The left wheel brake is applied such that the DWSR is tracked through the slipping occurrence. The peak error between the uncontrolled case and controlled case is about 200% improvement as seen by Fig.12c and 12f. In the error plot for the controlled case, Fig.12f, there is a second peak. This reflects the DWSR changing from the value associated with $\Phi = -100°$ to the DWSR value associated with $\Phi = 0°$. The peaks occur because the DWSR values are changed instantaneously as the steering wheel positions are changed instantaneously. The controlled wheels are forced to track the quickly changing DWSR which is represented by the peaks.

.

Simulation Set #4

The fourth set of simulations, Fig. 13a - 13f simulate an event with two slipping situations. The first slip situation occurs when the left wheel slips at initial acceleration from t = 0 seconds to t = 2 seconds. The second slip situation occurs at t = 10 seconds, when the steering wheel is turned $\Phi = 245°$ to the right. At $\Phi = 245°$, the control is commanded to lock the inside wheel (i.e. the right wheel) and transfer the drive torque to the left wheel. The turn will last from t = 10 seconds to t = 12 seconds

<u>Simulation #4.1:</u> System response during an uncontrolled slip situation at t=0 and during a turning maneuver with $\Phi = 245°$.

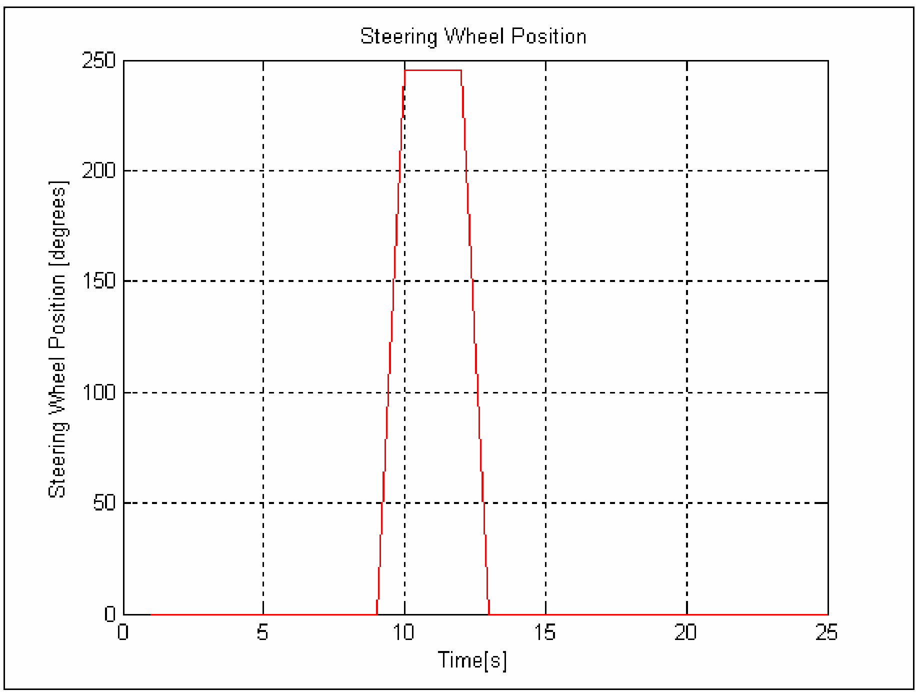

Figure 13a: SW for simulation set #4.

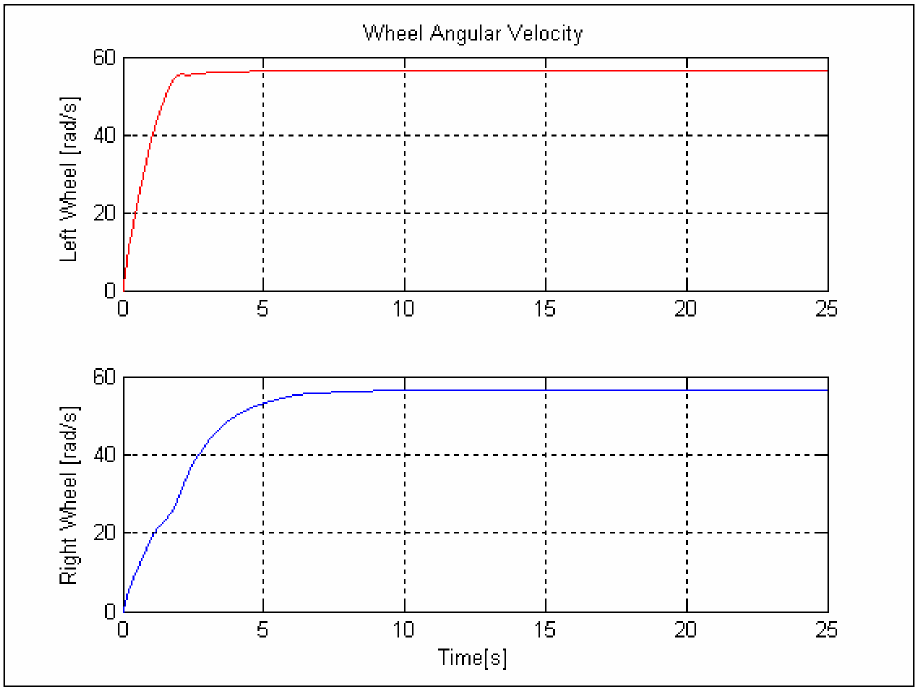

Figure 13b: Wheel speeds for uncontrolled slip during initial acceleration and during a turning maneuver of Φ = 245° issued at t=10 seconds and ending at t =15 seconds.

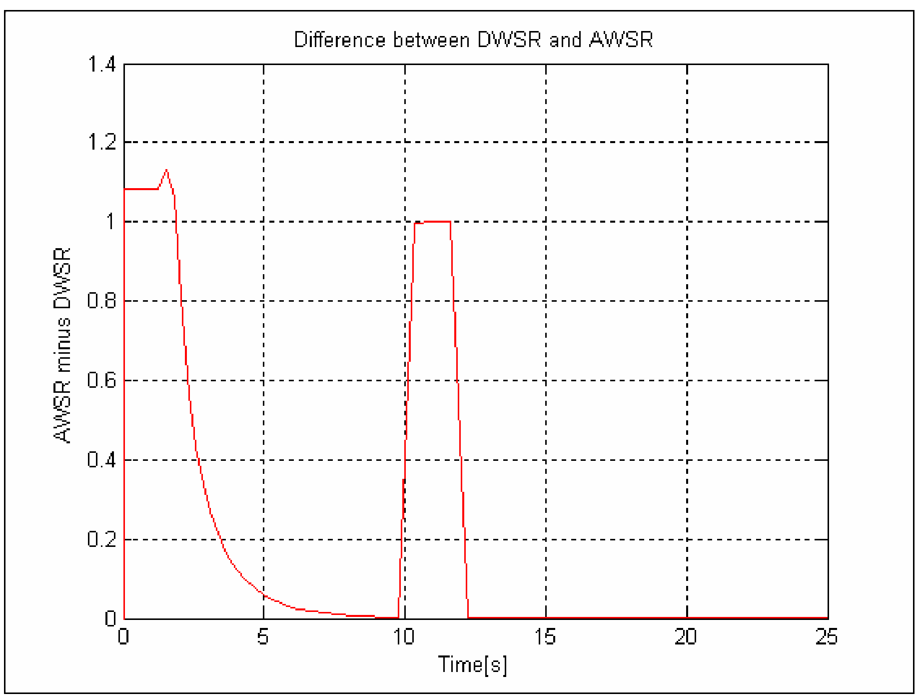

Figure 13c: DWSR v. AWSR for an uncontrolled slip during initial acceleration and during a turning maneuver of Φ = 245° issued at t=10 seconds and ending at t =15 seconds.

<u>*Simulation #4.2:*</u> *System response during a controlled slip situation at initial acceleration and during a turning maneuver with* $\Phi = 245°$.

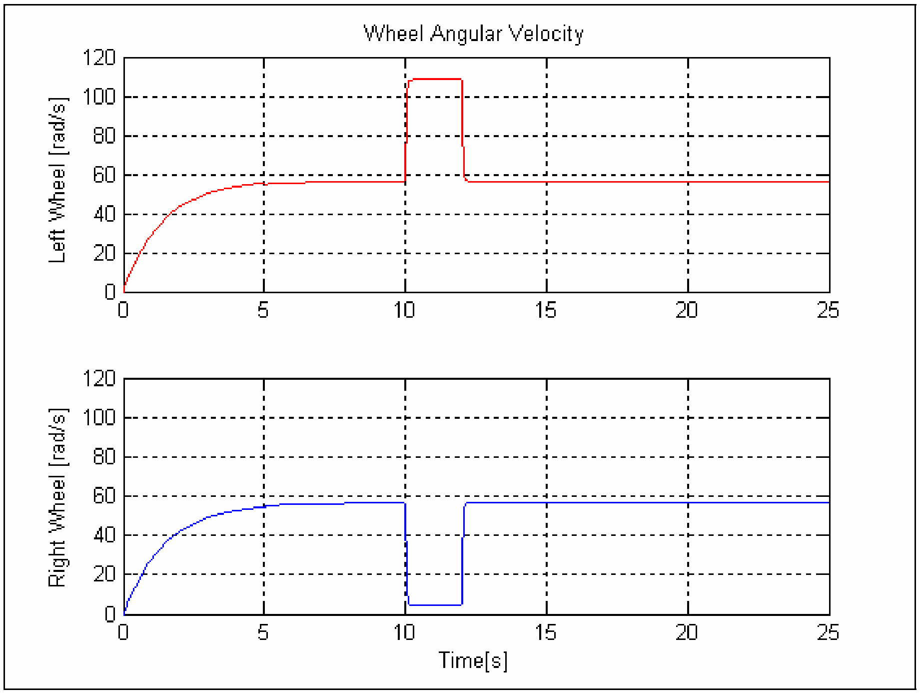

Figure 13d: Wheel speeds during the controlled slip situations during initial acceleration and during a turning maneuver of $\Phi = 245°$ issued at t=10 seconds and ending at t =15 seconds.

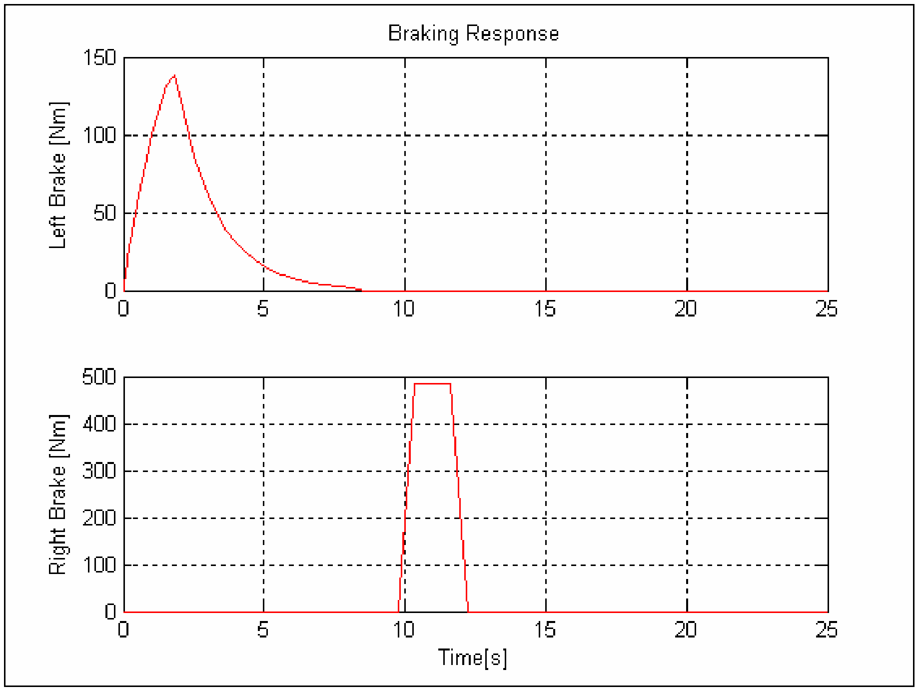

Figure 13e: Braking response during a controlled slip situation during initial acceleration and during a turning maneuver of Φ = 245° issued at t=10 seconds and ending at t =15 seconds.

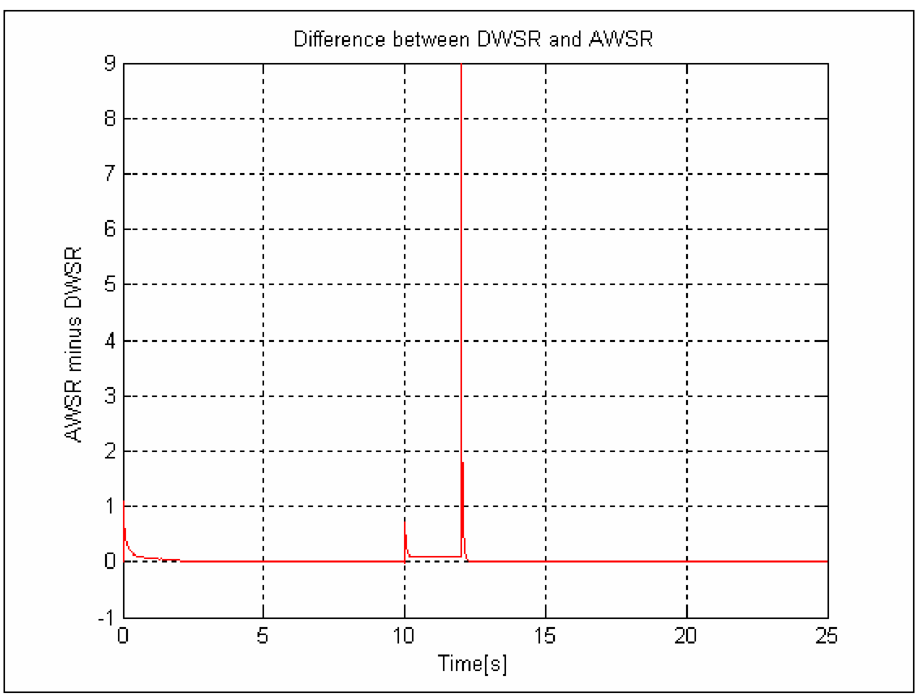

Figure 13f: DWSR v. AWSR during a controlled slip situation during initial acceleration and during a turning maneuver of Φ = 245° issued at t=10 seconds and ending at t =15 seconds.

Figure 13a shows the steering wheel input used during simulation set #3. Figure 13b shows the uncontrolled slip of the right wheel occurring at initial acceleration and at t=10 seconds when the DWSR is issued to zero that is associated with $\Phi = 245°$ to the right or clockwise. The error between DWSR and AWSR are shown in Fig.13c. The differences occur at initial take off and at t=10 seconds.

Figure 13d - 13f show the system results of the slip situations but with SBS control. Figure 13e shows that the braking is applied during the slipping occurring at the initial acceleration and when the turning maneuver is issued at t = 10 seconds. Figure 13f shows how the wheels respond and observe the decrease in error between the uncontrolled result in Fig.13c and controlled result in Fig.13f. Observe that the WSR does not exactly match the DWSR of zero value that corresponds to $\Phi = 245°$. The reason is due to rounding the AWSR to the nearest integer. In this case, the ratio is calculated as the speed of the right wheel (≈ 10 rad/s) divided by the speed of the left wheel (≈ 100 rad/s), which gives a rounded value of zero. The spike at t=12 seconds occurs in Fig.13f because the steering wheel position goes from $\Phi = 245°$ to $\Phi = 0°$ instantaneously which changes the DWSR from zero to one instantaneously.

Discussion

During a straight travel, non-slipping situation, the open differential will ensure that both wheels receive equal amounts of input from the drive shaft as shown in Fig.14a.

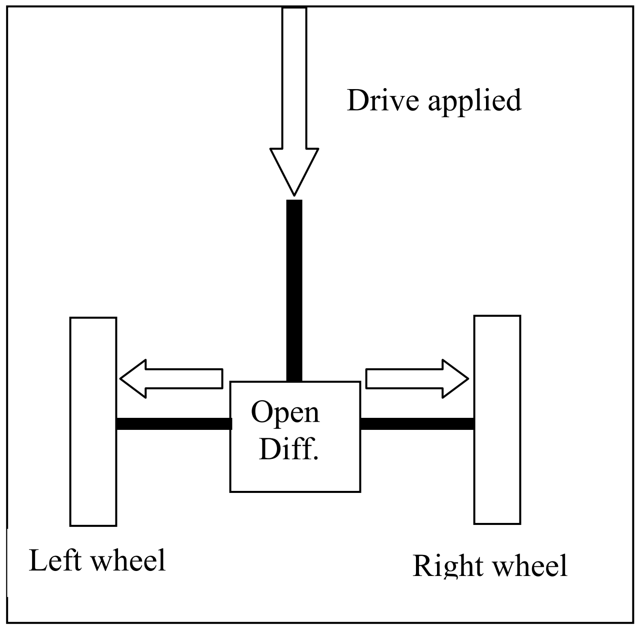

Figure 14a: Drive torque distribution to the wheels with no slip.

The size of the arrows pointing to the wheels represents the magnitude of drive torque applied to the wheels. During a straight line travel with no slipping, the wheels receive equal drive torque and the traction coefficient, μ, on both of the wheels are the same. The left and right wheel traction coefficients for the non-slipping case are located by a red dot on the simplified Pacejka model.

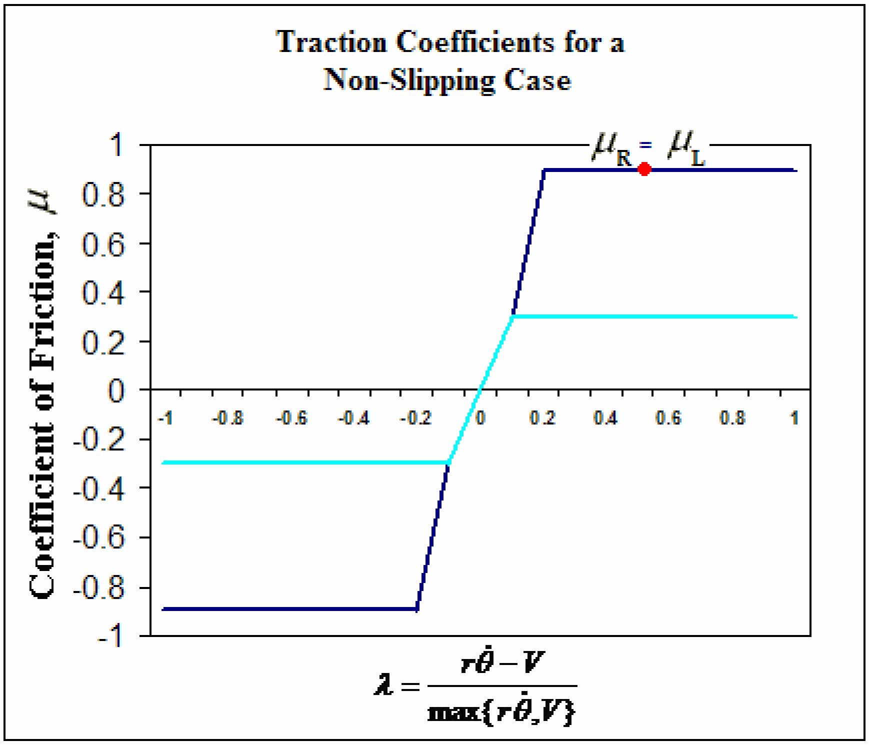

Figure 14b : Traction coefficient for the left and right wheel with no slip.

An uncontrolled open-differential operates such that equal traction force is always achieved at the wheels. When the tractive force of one wheel decreases, the tractive force on the other wheel will decrease to the same value. Consider the case when the left wheel is traveling along on a dry road and runs into a patch of ice. The uncontrolled open differential will distribute the drive torque to the slipping left wheel speeding it up and the right wheel will slow down because it is receiving less drive torque.

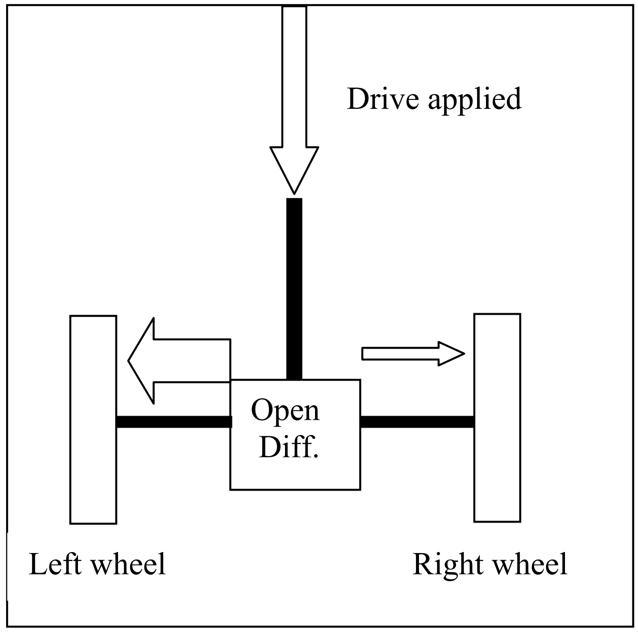

Figure 15a: Drive torque distribution to the wheels during slip.

Figure 15b shows how the traction coefficients of each wheel change during the slip event. μ_L has decreased because it is on another surface with a lower peak tractive coefficient. μ_R will stay on the Pacejka curve for dry ground but the value of μ_R will decrease since it receives less drive torque. As the right wheel receives less drive torque, the wheel slows down and the corresponding λ decreases.

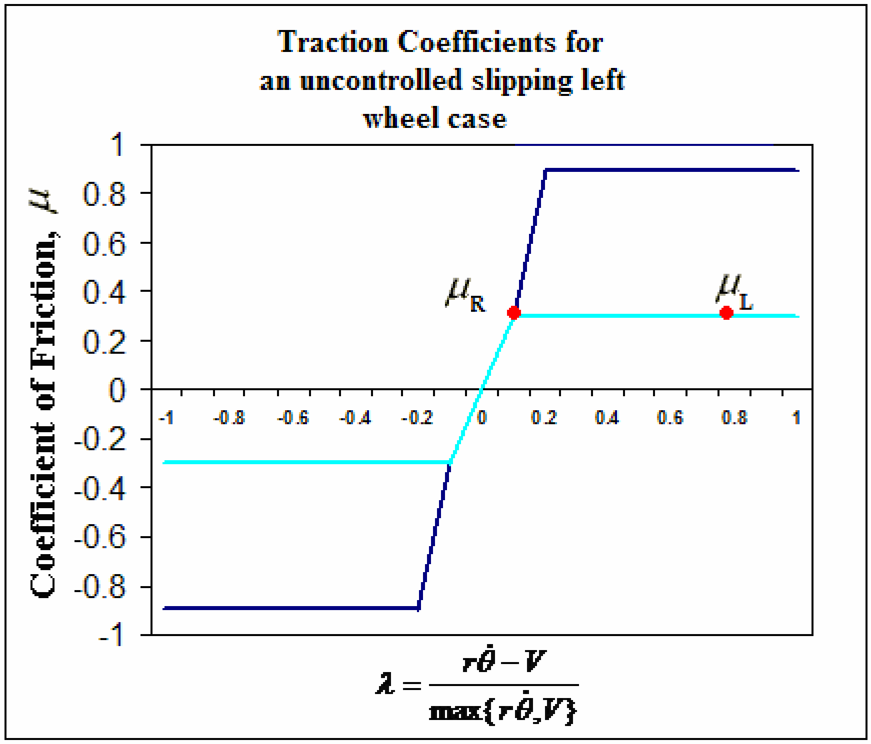

Figure 15b : Traction coefficient for the left and right wheel during an uncontrolled slipping left wheel.

By applying SBS during a slipping situation, the drive torque is transferred from the slipping wheel to the non-slipping wheel. In the case where the left wheel is slipping, the SBS will brake the left wheel and transfer drive torque to the right wheel. Figure 16a shows how the drive torque is transferred via the left brake which is represented by the shaded arrows.

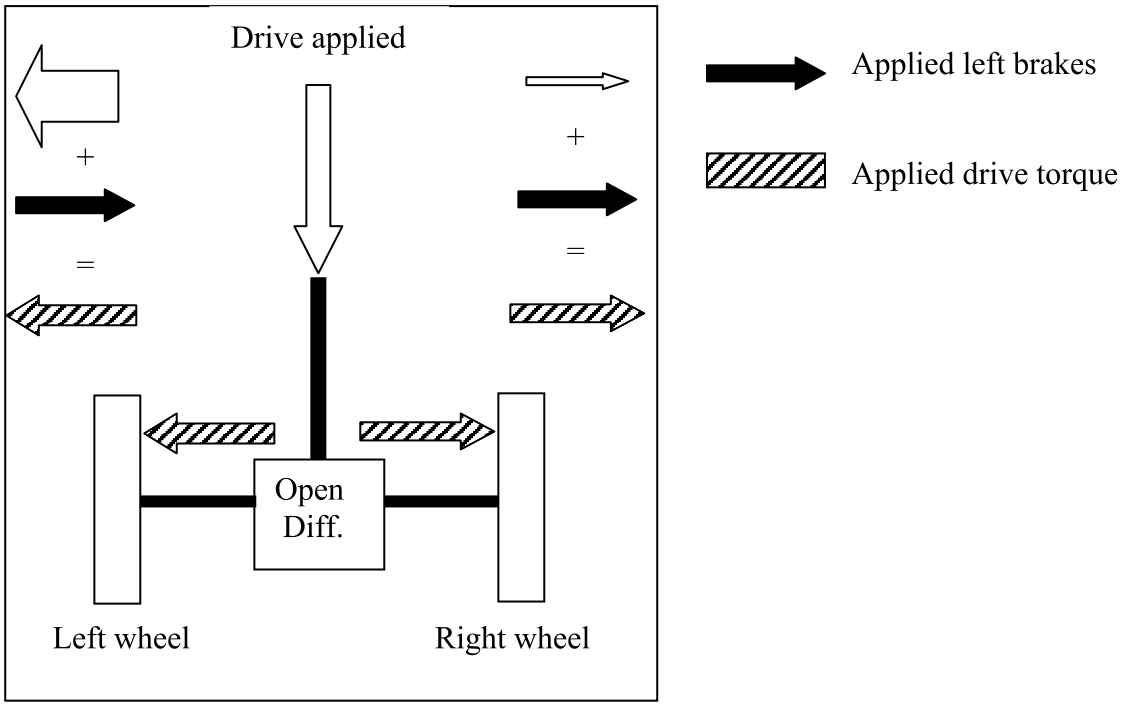

Figure 16a : Traction coefficient for the left and right wheel during a controlled slipping left wheel.

When the left wheel brake is applied, drive torque is transferred to the right wheel speeding it up. According to the Pacejka model, when the wheel velocity is increased, the longitudinal slip, λ, of that wheel increases. Thus with SBS, the wheel on dry ground receives drive torque and produces a greater tractive force to the ground, since μ has increased. Comparison of Fig.15b and Fig.16b depicts how the traction coefficients improve with SBS.

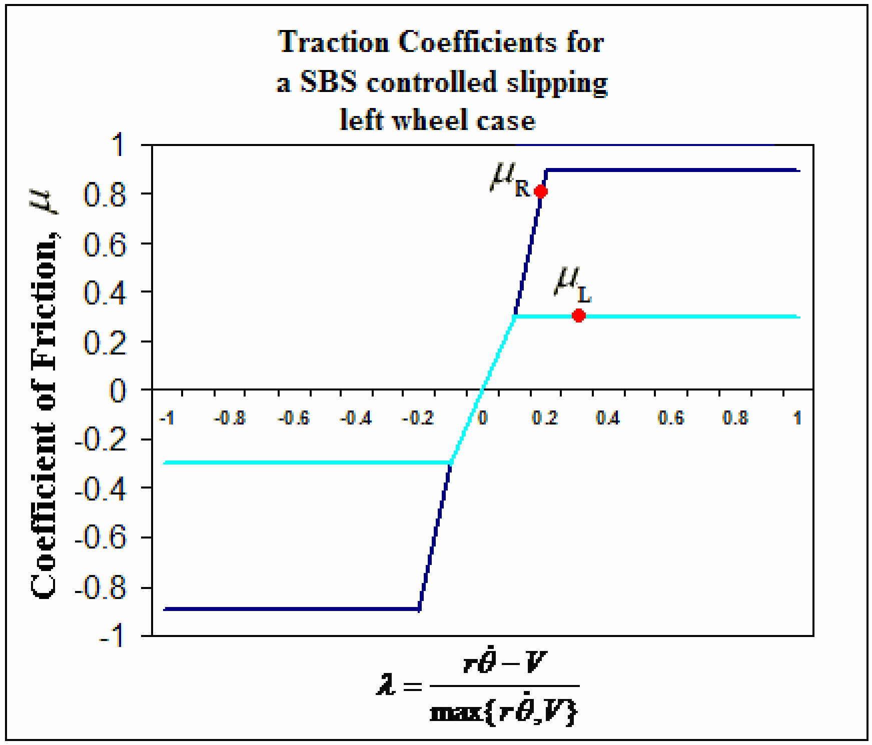

Figure 16b : Traction coefficient for the left and right wheel during a SBS controlled slipping left wheel.

Recall Eq. 3, which describes the vehicle longitudinal acceleration:

$$m\dot{V} = (\mu F_N)_1 + (\mu F_N)_2 - C_v \dot{V}^2$$

It is observed that by increasing the traction coefficient of the non-slipping wheel with SBS the longitudinal acceleration of the vehicle is improved.

In order to precisely measure the acceleration improvements that the SBS has during turns, the lateral dynamics of the vehicle must be modeled. However, the improvements that SBS has on turning dynamics may be measured by comparing the controlled wheel speed ratios during slip to the of the pre-defined desired wheel speed ratios. The pre-defined DWSR was derived under minimal wheel slip conditions and closely approximates ideal turning. Therefore, when the SBS is activated during turning situations and the AWSR tracks the DWSR it is preventing undesired turning dynamics.

Conclusion and future work

The open differential allows for effective turning because the wheels are allowed to turn at different rates which is useful during cornering because the outer wheel must travel a further distance than the inner wheel in the same amount of time. However, the uncontrolled open differential distributes drive torque such that the traction force applied to the ground by the wheels is always equal. This is detrimental during a slip situation, since the vehicle traction force is limited to the least tractive wheel because drive torque to the non-slipping wheel is diverted to the slipping wheel.

In this paper, the SBS was developed to reduce such unwanted dynamics. A set of desired wheel speed ratios were derived using vehicle geometry and data relating steering wheel position to turning radii. The DWSR is assumed to represent ideal vehicle dynamics. The SBS control logic is such that brake torque is applied such that the actual wheel speed ratios track the desired wheel speed ratios. The performance was evaluated according to how well the states track the pre-defined desired ratios.

Further modeling of overall vehicle dynamics, such as lateral motion, would provide insight on what effect the SBS has on overall vehicle performance. Also, physical aspects of the brake system, such as actuator dynamics, and brake line pressure dynamics should be modeled to provide more insight on SBS effectiveness during slip situations.

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Appendix A – SBS simulation code

The simulation program is separated in three programs. The function 'sbs' runs the actual simulation. The function 'diffsurf' is where slip events are defined and where the equations of motion reside. The function 'init' initializes the system matrices and calculates the gains used in the SBS.

Part I: This following is the MatLab program that runs SBS.

```
function sbs;
% Description: incorporating straight travel slipping with SBS

clear all; format long;
% calling car specs, calc gains, system matrices, damping coeffs,
% d = damping coeff on wheels, cv = drag coeff on vehic
% b = transfered torque amount

[constval_contrl, Kx, Kz, A, B, d, cv, b] = init;

m = constval_contrl(1); r = constval_contrl(2); IR = constval_contrl(3); IL =
constval_contrl(4);
I  = constval_contrl(5); b = constval_contrl(6); g = constval_contrl(7);


% Initial Vehicle Wheel speeds and end time declaration
% x1 = vehic vel, x2 = left ws, x3 = right ws
xo = [0; 0; 0] ;  ti = 0;   tf = 25;

% Event time declaration
% t_constant = time at which engine torque input becomes constant,
% t_slip = slipping occurence , t_regrip = when slipping wheel meets

t_const = 5; t_slip = 10; t_regrip = 15;
```

```
% Storage arrays and ode options to be used for graphical purposes
z = [];   SW_emp = [];  u1val = [];  u2val = [];
simoptions=[];

% some params: traction effectiveness, input torque rate
eff_s = 0.65; eff_ns = 0.90; T = 140;

% RK4 integrations and integration specifications: see help odeset and ode45
[t, x] = ode23s(@diffsurf, [ti, tf], xo, simoptions, T, eff_s, eff_ns, t_const, t_slip , t_regrip,
tf);
v_val = x(:,1);, x2val = x(:,2);,   x3val = x(:,3);
lent = length(t);

% Running through logic to have u's, z's, SW's stored into a storage matrix
% for plotting use

for i = 1:1:lent,
   if t(i) < 2
      SW=0;
   elseif t(i) < 10, % before slipping, set input torque at constant level and even dist to
wheels
      SW = 0;
   elseif t(i) < 12, % one wheel slips, the effectiveness of force transfer of slipping wheels
is changed
      SW = 0;
   elseif t(i) <= tf, % back to same surface for both wheels
      SW = 0;
   end

   x(2) = x2val(i);
```

```
    x(3) = x3val(i);
    if(x(2)==0 & x(3)==0)
        u(1)=0;
        u(2)=0;
        z1 = 0;
        z2 = 0;
    end
% % Left hand turn or straight path cases
            if  ( SW <= 0 & -245 <= SW ),                    % Left hand turn cases
        DWSR = (9*10^-8)*SW^3 + (5*10^-5)*SW^2 + 0.0109*SW + 1;
        if (SW == -245)
             DWSR = 0;
        end
        if(x(3)==0)
            x(3)=0.001;   %zero in the denominator will cause error
        end
        if(x(2)==0 & x(3)>0)
            AWSR = 0;
        end
        if(round(x(2))==0 & round(x(3))==0)
            AWSR =1;
        end
        if(x(2)>0 & x(3)>0)
            AWSR = x(2)/x(3);
        end
        if(round(AWSR*100) == round(DWSR*100)),
            u(1)=0; u(2)=0;
        end
        if(round(AWSR*100) > round(DWSR*100)),              %Left wheel is slipping
            DELTA = (x(2) - DWSR*x(3) ) / (1 + DWSR);
            z1 = x(2) - abs(DELTA);
```

```
        z2 = x(3) + abs(DELTA);
        if(round(z1*100) < round(x(2)*100)),
          u(1) = Kx(1,1)*x(2) + Kx(1,2)*x(3) + Kz(1,1)*z1 + Kz(1,2)*z2;
%           u(2)= Kx(2,1)*x(2) + Kx(2,2)*x(3) + Kz(2,1)*z1 + Kz(2,2)*z2;
          u(2) = 0;
           if(u(1)>=600),
              u(1)=600;
           end
           if(u(1)<=0),
              u(1)=0;
           end
        end
        if(round(z1*100) == round(x(2)*100)),
           u(1) = 0; u(2) = 0;
        end
     end
     if(round(AWSR*100) < round(DWSR*100)),              % Right wheel is slipping
        DELTA = (-x(2) + DWSR*x(3) ) / (1 + DWSR);
        z1 = x(2) + abs(DELTA);
        z2 = x(3) - abs(DELTA);
        if(round(z2*100) < round(x(3)*100)),
           u(1) =0;
%           u(1) = Kx(1,1)*x(2) + Kx(1,2)*x(3) + Kz(1,1)*z1 + Kz(1,2)*z2;
           u(2)= Kx(2,1)*x(2) + Kx(2,2)*x(3) + Kz(2,1)*z1 + Kz(2,2)*z2;
           if(u(2)>=600),
              u(2)=600;
           end
           if(u(2)<=0),
              u(2)=0;
           end
        end
```

```
        if(round(z1*100) == round(x(2)*100)),
            u(1) = 0; u(2) = 0;
        end
    end
        end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if(0 < SW & SW <= 245),             %Right hand turn cases
    DWSR = (-9*10^-8)*SW^3 + (5*10^-5)*SW^2 - 0.0109*SW + 1;
    if(SW == 245),
        DWSR = 0;
    end
    if(x(2)==0)
        x(2) = 0.001; % zero in the denominator will cause error
    end
    if(x(2)>0 & x(3)==0)
        DWSR = 0;
    end
    if(round(x(2))==0 & round(x(3))==0)
        AWSR =1;
    end
    if(x(2)>0 & x(3)>0)
        AWSR = x(3)/x(2);
    end
    if(round(AWSR*100)==round(DWSR*100)),
        u(1)=0; u(2)=0;
    end
    if(round(AWSR*100) < round(DWSR*100)),     % Left is slipping
        DELTA = (-x(3) + DWSR*x(2) ) / (1 + DWSR);
        z1 = x(2) - abs(DELTA);
        z2 = x(3) + abs(DELTA);
```

```
    if(round(z1*100) < round(x(2)*100)),
        u(1) = Kx(1,1)*x(2) + Kx(1,2)*x(3) + Kz(1,1)*z1 + Kz(1,2)*z2;
        u(2)=0;
        if(u(1)>=600),
            u(1)=600;
        end
        if(u(1)<=0)
            u(1)=0;
        end
    end
    if(round(z1*100) == round(x(2)*100))
        u(1) = 0; u(2) = 0;
    end
end
if(round(AWSR*100) > round(DWSR*100)),  % Right is slipping
    DELTA = (x(3) - DWSR*x(2) ) / (1 + DWSR);
    z1 = x(2) + abs(DELTA);
    z2 = x(3) - abs(DELTA);
    if(round(z2*100) < round(x(3)*100)),
        u(1)=0;
        u(2)= Kx(2,1)*x(2)+ Kx(2,2)*x(3) + Kz(2,1)*z1 + Kz(2,2)*z2;
        if(u(2)>=600),
            u(2)=600;
        end
        if(u(1)<=0),
            u(1)=0;
        end
    end
    if(round(z2*100) == round(x(3)*100)),
        u(1)=0; u(2)=0;
    end
```

```
        end
          end
    u1val = [u1val; u(1)];
    u2val = [u2val; u(2)];
%     u1=max(u1val);
%     u2=max(u2val);
    z(i,1) = z1;
    z(i,2) = z2;
    SW(i) = [SW_emp; SW];
    err1 = abs(x(i,2)-z(i,1));
    err2 = abs(x(i,3)-z(i,2));
    err(i,1) = err1;
    err(i,2) = err2;
    ERR_RATIO(i,1) = (abs(DWSR-AWSR)/ DWSR)*100;
%     ERR_RATIO(i,1) = (AWSR-DWSR);
end


%error array
err = [];
for i =1:1:lent,
    err1 = x(i,2)-z(i,1);
    err2 = x(i,3)-z(i,2);
    err(i,1) = err1;
    err(i,2) = err2;
end

for i=1:9,
    SW(i,1) = 0;
end
for i = 10:12,
```

```
   SW(i,1)=0;
end
for i=13:25,
   SW(i,1)=0;
end



%Plot Steering Wheel position
figure(1)
plot( SW(:,1),'r'), title('Steering Wheel Position'), xlabel('Time[s]'), ylabel('Steering
Wheel Position [degrees]'), grid on;

%Control effort
figure(2)
subplot(2,1,1), plot(t, u1val,'r')
title('Braking Response'), ylabel('Left Brake [Nm]'), grid on;
subplot(2,1,2), plot(t,u2val,'r')
ylabel('Right Brake [Nm]'), xlabel('Time[s]'), grid on;

%Wheel Speeds
figure(3)
subplot(2,1,1),  plot(t,x(:,2), 'r'), title('Wheel Angular Velocity'),ylabel('Left Wheel [rad/s]
'), grid on;
axis([0 25 0 150])
subplot(2,1,2), plot(t,x(:,3), 'b'),ylabel('Right Wheel [rad/s]'), xlabel('Time[s]'), grid on;
axis([0 25 0 150])

%Difference between AWSR and DWSR
figure(4)
plot(t, ERR_RATIO(:,1),'r'), title('Difference between DWSR and AWSR')
```

```
xlabel('Time[s]'), ylabel('AWSR minus DWSR'), grid on;
```

Part II: The following is the program where slip events are defined.

```
%EOM and logic
function dx = diffsurf(t,x,T, eff_s, eff_ns, t_const, t_slip , t_regrip, tf);

format short;

[constval_contrl, Kx, Kz, A, B, b, cv] = init;

m = constval_contrl(1); r = constval_contrl(2); IR = constval_contrl(3); IL =
constval_contrl(4);
I  = constval_contrl(5); b = constval_contrl(6); g = constval_contrl(7);

if t < 2,
  SW = 0;
elseif t < 10, % before slipping, set input torque at constant level and even dist to wheels
  SW = 0;
elseif t < 12, % one wheel slips, the effectiveness of force transfer of slipping wheels is
changed
  SW = 0;
elseif t <= tf, % back to same surface for both wheels
  SW = 0;
end

if(x(2)<=0)
    x(2)=0;
end
if(x(3)<=0)
    x(3)=0;
end
```

```
%Controller Logic
if  ( SW <= 0 & -245 <= SW ),                        % Left hand turn cases
   DWSR = (9*10^-8)*SW^3 + (5*10^-5)*SW^2 + 0.0109*SW + 1;
   if (SW == -245)
       DWSR = 0;
   end
   if(x(3)==0)
      x(3)=0.001;   %zero in the denominator will cause error
   end
   if(x(2)==0 & x(3)>0)
      AWSR = 0;
   end
   if(round(x(2))==0 & round(x(3))==0)
      AWSR =1;
   end
   if(x(2)>0 & x(3)>0)
      AWSR = x(2)/x(3);
   end
   if(round(AWSR*100) == round(DWSR*100)),
      u(1)=0; u(2)=0;
   end
   if(round(AWSR*100) > round(DWSR*100)),              %Left wheel is slipping
      DELTA = (x(2) - DWSR*x(3) ) / (1 + DWSR);
      z1 = x(2) - abs(DELTA);
      z2 = x(3) + abs(DELTA);
      if(round(z1*100) < round(x(2)*100)),
         u(1) = Kx(1,1)*x(2) + Kx(1,2)*x(3) + Kz(1,1)*z1 + Kz(1,2)*z2;
         u(2) = 0;
         if(u(1)>=600),
            u(1)=600;
```

```
      end
      if(u(1)<=0),
        u(1)=0;
      end
    end
    if(round(z1*100) == round(x(2)*100)),
      u(1) = 0; u(2) = 0;
    end
  end
  if(round(AWSR*100) < round(DWSR*100)),          % Right wheel is slipping
    DELTA = (-x(2) + DWSR*x(3) ) / (1 + DWSR);
    z1 = x(2) + abs(DELTA);
    z2 = x(3) - abs(DELTA);
    if(round(z2*100) < round(x(3)*100)),
      u(1)=0;
      u(2)= Kx(2,1)*x(2) + Kx(2,2)*x(3) + Kz(2,1)*z1 + Kz(2,2)*z2;
      if(u(2)>=600),
        u(2)=600;
      end
      if(u(2)<=0),
        u(2)=0;
      end
    end
    if(round(z1*100) == round(x(2)*100)),
      u(1) = 0; u(2) = 0;
    end
  end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if(0 < SW & SW <= 245),         %Right hand turn cases
```

```
  DWSR = (-9*10^-8)*SW^3 + (5*10^-5)*SW^2 - 0.0109*SW + 1;
  if(SW == 245),
     DWSR = 0;
  end
%    if(x(2)==0)
%        x(2) = 0.001; % zero in the denominator will cause error
%    end
  if(x(2)>0 & x(3)==0)
     AWSR = 0;
  end
  if(round(x(2))==0 & round(x(3))==0)
     AWSR =1;
  end
  if(x(2)>0 & x(3)>0)
     AWSR = x(3)/x(2);
  end
  if(round(AWSR*100)==round(DWSR*100)),
     u(1)=0; u(2)=0;
  end
  if(round(AWSR*100) < round(DWSR*100)),     % Left is slipping
     DELTA = (-x(3) + DWSR*x(2) ) / (1 + DWSR);
     z1 = x(2) - abs(DELTA);
     z2 = x(3) + abs(DELTA);
     if(round(z1*100) < round(x(2)*100)),
        u(1) = Kx(1,1)*x(2) + Kx(1,2)*x(3) + Kz(1,1)*z1 + Kz(1,2)*z2;
        u(2)=0;
        if(u(1)>=600),
           u(1)=600;
        end
        if(u(1)<=0)
           u(1)=0;
```

```
            end
        end
        if(round(z1*100) == round(x(2)*100))
            u(1) = 0; u(2) = 0;
        end
    end
    if(round(AWSR*100) > round(DWSR*100)),  % Right is slipping
        DELTA = (x(3) - DWSR*x(2) ) / (1 + DWSR);
        z1 = x(2) + abs(DELTA);
        z2 = x(3) - abs(DELTA);
        if(round(z2*100) < round(x(3)*100)),
            u(1)=0;
            u(2)= Kx(2,1)*x(2)+ Kx(2,2)*x(3) + Kz(2,1)*z1 + Kz(2,2)*z2;
            if(u(2)>=600),
                u(2)=600;
            end
            if(u(1)<=0),
                u(1)=0;
            end
        end
        if(round(z2*100) == round(x(3)*100)),
            u(1)=0; u(2)=0;
        end
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %Brake inputs
 u(1)=0;
 u(2)=0;
```

```
% %  EOM and logic

if t < 5,
        Y = 0.2;
        fn1 = Y*m*g;
            fn2 = Y*m*g;
        Td1 = 0.5*T;
        Td2 = 0.5*T;
        mu1 = 0.7;
        mu2 = 0.7;
        del_L = 0;
        del_R = 0;
elseif t < 10, % before slipping, set input torque at constant level and even dist to wheels
        Y = 0.2;
        fn1 = Y*m*g;
            fn2 = Y*m*g;
        Td1 = 0.5*T;
        Td2 = 0.5*T;
        mu1 = 0.7;
        mu2 = 0.7;
        del_L = 0;
        del_R = 0;

elseif t < 15, % one wheel slips, the effectiveness of force transfer of slipping wheels is
changed
        Y = 0.2;
        fn1 = Y*m*g;
            fn2 = Y*m*g;
        Td1 = 0.5*T;
        Td2 = 0.5*T;
        mu1 = 0.7;
```

```
        mu2 = 0.7;
        del_L = 0;
        del_R = 20;
elseif t <= tf, % back to same surface for both wheels
        Y = 0.2;
        fn1 = Y*m*g;
            fn2 = Y*m*g;
        Td1 = 0.5*T;
        Td2 = 0.5*T;
        mu1 = 0.7;
        mu2 = 0.7;
        del_L = 0;
        del_R = 0;
end

dx(2,1) =  (1/IL)*(Td1 - del_R - u(1) + u(2) - (mu1*fn1*r - del_L) + A(1,1)*x(2));
dx(3,1) =  (1/IR)*(Td2 - del_L + u(1) - u(2) - (mu2*fn2*r - del_R) + A(2,2)*x(3));
t
disp('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%')
```

Part III: This program initializes the system matrices and calculates the control input gains.

```
function [constval_contrl, Kx, Kz, A, B, d, cv, b] = sys_init_contrl;

% Vehic specs and properties
m = 176.9;      % kg
r = 0.2032;     % m
IR = 0.65;      % kg-m^2
IL = 0.65;      % kg-m^2
I = 0.65;       % kg-m^2
```

```
b = 1;          % amount transferred torque to specific wheel via opposite brake
g = 9.81;       % kg-m/s^2
Lam1 = -5;      %closed loop poles
Lam2 = -5;

% viscous damping - gears, etc..
d = -0.295;

% damping coeff on vehic
cv = 0.2;

% storing constants into array
constval_contrl = [m r IR IL I b g]';

m = constval_contrl(1); r = constval_contrl(2); IR = constval_contrl(3); IL =
constval_contrl(4);
I  = constval_contrl(5); b = constval_contrl(6); g = constval_contrl(7);

% w/o control%
A = [d/IL 0; 0 d/IR];
B = [-1/IL, b/IL; b/IR, -1/IR];
C = [1 0; 0 1];
D = [0 0; 0 0];

% w/ control or desired system
Ad = [Lam1 0 ; 0 Lam2];
Bd = [-1  1 ; 1  -1];
Cd = [1 0; 0 1];
D =  [0 0; 0 0];

%system used in LQR development
```

```
Ahat = [A zeros(2,2) ; zeros(2,2) Ad];
Bhat = [B ; zeros(2,2)];
W_Q = 100;   %state weighting
Q = W_Q*[1 0; 0 1];
W_R = 1;     %control weighting
R = W_R*[1 0; 0 1];
Qhat = [C'*Q*C  -C*Q ; -Q*C  Q];

% Solving Ricatti
[P, l, g] = care(Ahat, Bhat, Qhat, R);
Pxx = [ P(1,1)  P(1,2) ; P(2,1)  P(2,2) ];
Pxz = [ P(1,3)  P(1,4) ; P(2,3)  P(2,4) ];

%calculating gains
Kx = -inv(R)*B*Pxx;
Kz = -inv(R)*B*Pxz;
```