AN ABSTRACT OF THE THESIS OF

Robert B. Avery for the degree of Master of Science in Forest Engineering presented on January 4, 1984

Title: Mathematical Model for Determining the Position and Line Tensions for a Tethered Logging Balloon

Abstract approved: Eldon D. Olsen

Eldon D. Olsen

A program was developed on a desktop computer to determine the cable tensions and position of a balloon tethered by two or more guylines. The program was specifically applied to the analysis of the static load lifting capability of the Pendulum Swing Balloon System. This system uses a tethered balloon and a gravity assisted swing to provide lift and movement to a load of logs.

Due to fundamental differences between conventional cable yarding systems and the pendulum ballon system, a catenary analysis is used to determine cable tensions, balloon position and available lift at a specified load location. Available lift is determined by the tautness of the pendulum, or load carrying line. The length of this line is altered to induce tension into it. The balloon position changes in response to the length adjustment. A gradient search procedure is used in combination with the appropriate catenary equations to conduct the analysis.

Comparison of calculated line tensions with selected measurements obtained in a separate field study revealed an average difference of ± 11.7 percent. Calculated balloon position coordinates were, on the average, within ± 0.01 percent of the measured positions.

A hypothetical setting consisting of a uniform, 60 percent slope extending 2000 feet horizontally was used to evaluate available lift of the system. Guyline placement and corridor orientation were selected so as to facilitate the analysis. The effect of the pendulum swing was not considered. Results obtained are as follows:

1. Total cable weight has a dramatic impact on available lift. If the balloon is to be placed at a high elevation above the ground, guylines should be made of a material having a higher strength to weight ratio than wire rope.

2. Shortening the pendulum line causes a transfer of tension from adjacent guylines to the pendulum line. The relation of the load position to the balloon and the original amount of vertical tension in the guylines determines the amount, and rate, of increase in available lift. In most circumstances, a shortening of 15 feet or less will provide a significant tension transfer without incurring significant balloon movement. Excessive balloon movement will alter the swing capability.

3. If lightweight guylines are used, the balloon should be placed between 1000 and 1500 feet above the 60 percent slope to obtain satisfactory lift at each end of the corridor.

The developed model can be used to study the effects of wind, live guylines and cable stretch on the system. Organization of the program is such that harvesting plans for a proposed setting can be developed quickly and easily.

Mathematical Model for Determining the Position and Line Tensions for a Tethered Logging Balloon

by

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Mathematical Model for Determining the Position and Line Tensions for a Tethered Logging Balloon

INTRODUCTION

Balloons fulfill an important role in providing economical access to difficult terrain. They are used in logging applications to supplement the lift of conventional cable yarding systems. These balloon-assisted systems have several advantages over more standard cable yarding methods (McIntosh, 1968):

1. The balloon lift alleviates problems of inadequate skyline deflection associated with long unbroken slopes, intervening obstacles and other types of unfavorable topography. The reach of conventional systems is extended as a result.

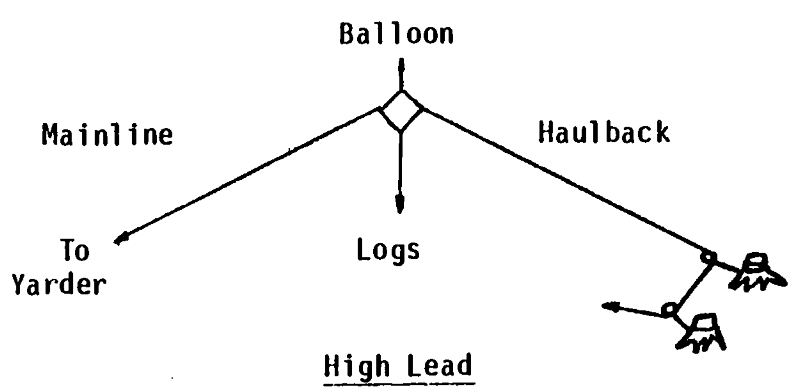
2. Less land area is lost to roads with a subsequent reduction in road construction costs. However, the reduced access may offset road construction savings by incurring higher costs for fire prevention and post-harvest activities.

3. Soil disturbance and subsequent erosion potential is minimized since the logs are flown, rather than dragged, over the ground.

4. Log breakage during the yarding phase is greatly reduced.

5. Greater flexibility is achieved in harvesting isolated patches of windthrown or insect infested timber.

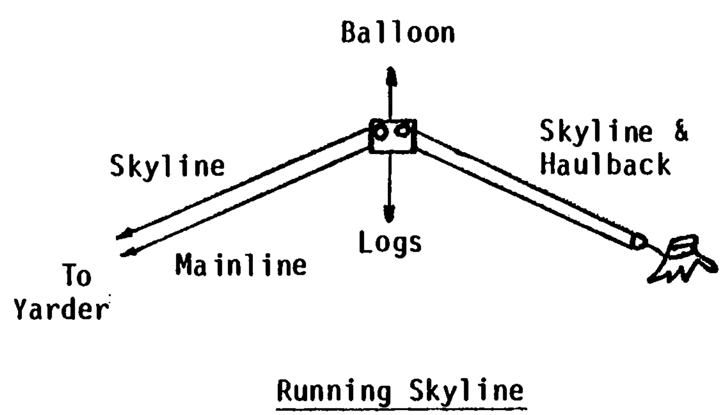
Figures 1 and 2 show the arrangement of the yarding lines and equipment for four balloon yarding systems which have seen operational use in the Western United States and Alaska (Peters, 1973).



Used for downhill logging in clearcuts. Maximum yarding capability is about 3000 feet.

Cycle description: Haulback line pulls balloon from landing to load position. Logs are attached to butt rigging beneath balloon. Tension on haulback released, balloon rises and supports load. Mainline pulls balloon and logs to landing.

Disadvantage: Haulback line can create a fire hazard.

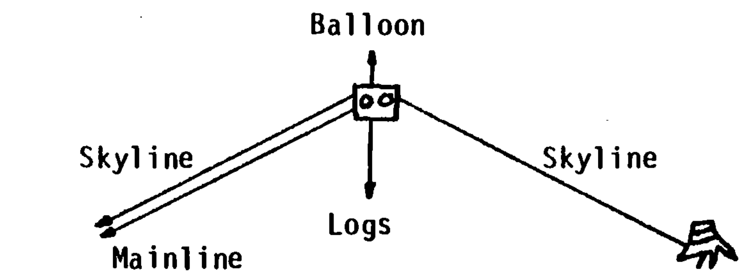


Same description as high lead system above but with following advantages:

- Greater pull down capability of balloon at load position.
- Increases productive yarding time by reducing the number of block changes on a corridor.

Disadvantage: Potential line breakage due to sawing of the haulback lines.

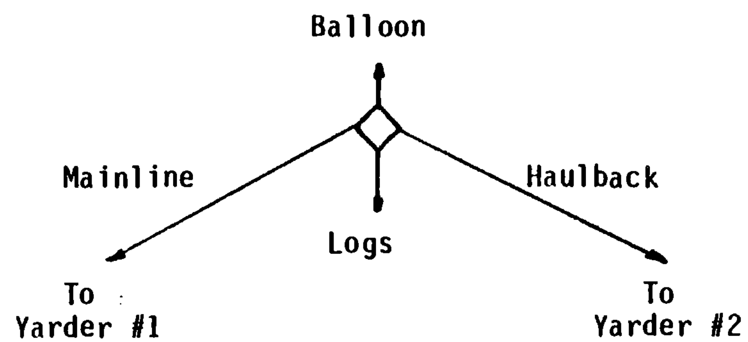
Figure 1. Arrangement of cables and description of yarding cycle for the high lead and running skyline balloon yarding systems.



Inverted Skyline

Used for downhill logging in clearcuts. Maximum yarding capability is about 5000 feet.

Cycle description: Release mainline tension and balloon lift pulls carriage up and along skyline to load position. Skyline is tensioned to lower chokers to ground. Releasing skyline tension raises balloon and load. Mainline brings balloon and logs to landing.



Yo-Yo

Used for uphill or downhill logging in clear-cutting operations. Can yard up to 7000 feet although 5000 is more practical limit.

Cycle description: Similar to high lead except 2 yarders are used. One yarder line is the mainline and the other acts as a haulback. Two yarders can also sit adjacent to each other and operate as highlead or inverted skyline systems.

Figure 2. Arrangement of cables and description of yarding cycle for inverted skyline and yo-yo balloon yarding systems.

A brief description of each system is shown adjacent to the diagram of the lines.

These four systems should be considered as those which have been used or are being used today. Research is continuing in an effort to improve existing systems and to identify and evaluate alternative methods of balloon yarding.

In December, 1981 the Forest Engineering Department at Oregon State University began research on the Pendulum Skyhook balloon concept. This concept was proposed by John Bell in 1973 as a viable alternative to conventional aerial logging methods.

The Pendulum Skyhook system consists of a 1,100,000 cubic foot natural shape balloon tethered by three guylines. Lift is provided by a pendulum line and a diesel powered winch fixed to the base of the balloon. The theoretical advantage over the balloon systems presented earlier is that the balloon is stationary and does not move from the load position to the landing with every load of logs. Since the lift and drag of the balloon do not have to be overcome, a reduction in the yarding cycle time may be possible. The method calls for fixing the balloon at a position on the slope with the guylines. Its vertical lift is employed at various load locations and a pendulum action assisted by gravity and the yarder brings the load of logs to the landing. Repositioning of the balloon is accomplished by adjusting the lengths of the appropriate guylines.

The proposed system is shown in Figure 3. The balloon is initially tethered to three guyline anchors at A, B and C. The haulback

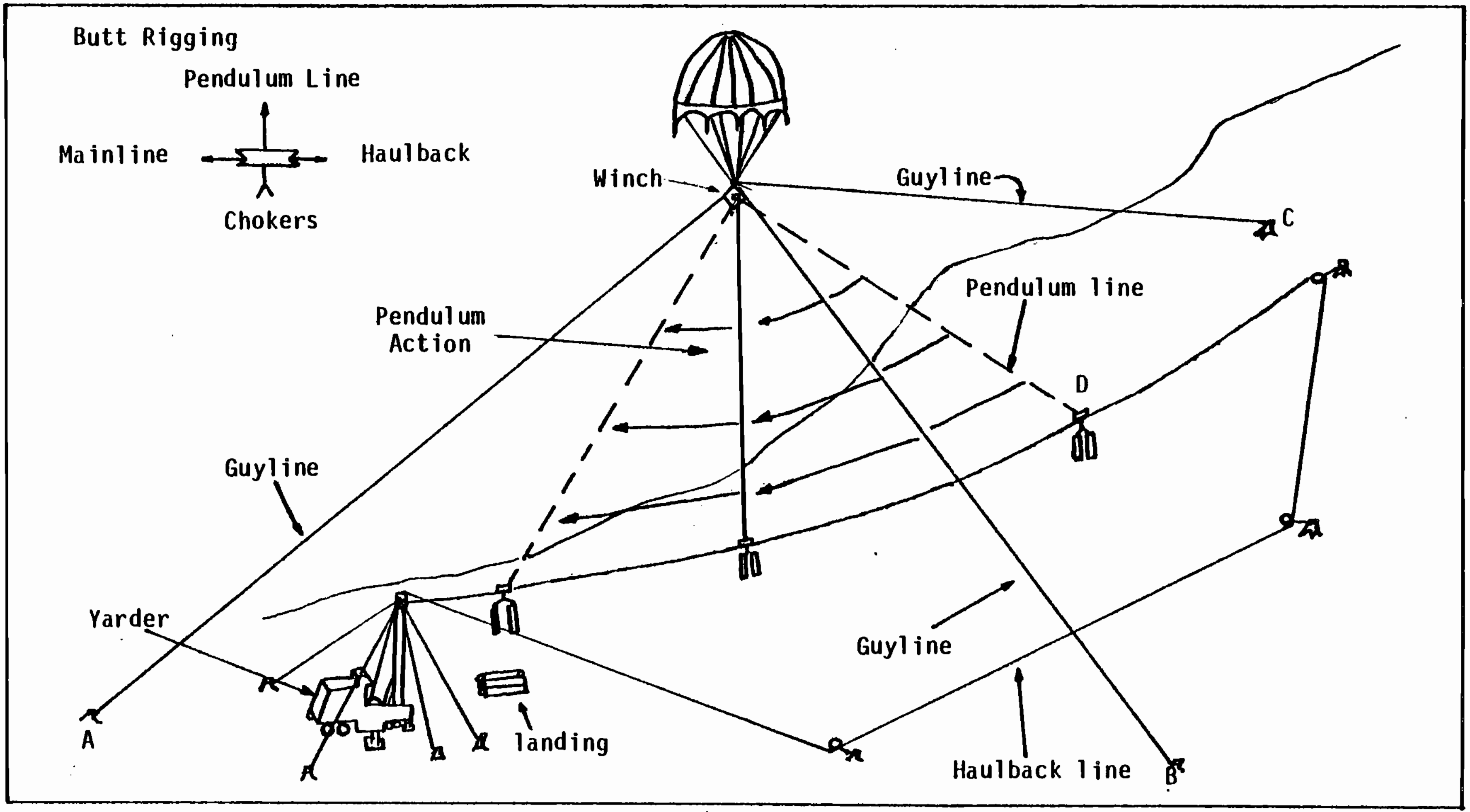


Figure 3. Arrangement of lines and equipment for the pendulum balloon concept.

line brings the butt rigging and chokers to the load location, D. The winch mounted underneath the balloon spools or unspools the pendulum line as required to give sufficient reach at the load location. Chokers are then attached to the logs which are to be yarded to the landing. After attachment, the pendulum line is reeled in while the haulback line is held taut. Vertical tension in both these lines lifts the load into the air. When the load height is sufficient to clear potential obstacles, the tension on the haulback line is released. The load of logs begins to move toward the landing underneath the influence of gravity. The haulback line serves as a brake to control the rate of swing. The mainline is taken up by the yarder and the pendulum line is paid out by the winch beneath the balloon as the load moves downhill toward the landing. This prevents a substantial elevation gain as the swing progresses. When the swing begins to slow down, the mainline may be used to pull the logs the remaining distance to the landing while the balloon suspends the load.

Lifting capability of the system is influenced by the size of the guylines used, geometry of the guyline anchors, position of the load and the initial tethered balloon position. Required resources for field testing these variables are prohibitively expensive so it was necessary to develop other approaches to evaluate lifting capability. The project was organized into four major parts:

1. Evaluate static load lifting capability with a prototype version. A trial arrangement of guyline anchors and an initial balloon position were established. Static tensions were measured in

each guyline and the pendulum line for a number of load locations. The balloon used had a nominal 2000 lb. lifting capacity. This project began in the summer of 1982.

2. Examine the additional forces induced by the pendulum action. A small, functional version of the system was constructed using cables, weights and pulleys. Tension was monitored in each of the lines as a given weight was allowed to swing freely through a prearranged angle. This part was completed in the summer of 1983.

3. Engineering analysis of the productivity and economic potential of a full-scale system. Cycle times, required equipment and probable payloads were estimated for the pendulum balloon system. A time study was conducted on a balloon logging show near Coquille, Oregon in the summer of 1983. Results from this study and other published sources of balloon logging productivity will be analyzed to estimate operational characteristics of a full-scale system.

4. Develop a computer-based mathematical model capable of being used to determine how static load-lifting capability of the system is affected by the following variables:

a) Initial balloon position

b) Load position

c) Line weights and lengths

d) Guyline anchor geometry

e) Ground slope

f) Gross balloon lift.

Several additional objectives will be fulfilled by the model. These are:

1. Use the model to determine static payload capacity at various points along a 60 percent slope extending 2000 feet horizontally.

2. Provide sufficient flexibility within the model to allow for future modification.

3. Examine the effect of balloon elevation on lift capability.

4. Completely document the model and its results so that future users have minimum difficulty applying it to their specific needs.

DIFFERENCES BETWEEN MODELING THE PENDULUM BALLOON AND CABLE YARDING SYSTEMS

Theoretical static lift capability of any cable yarding system is typically determined by using the following assumptions:

1. Yarder, tailhold, carriage and load locations are fixed, known with certainty and can be represented as points.

2. Line sizes and weights per foot are known and remain constant.

3. For maximum payload calculations, one of the lines is assumed to be at the safe working limit. This limit is intended to prevent a permanent deformation in the cable with a subsequent reduction of its breaking strength. If the tension in wire rope exceeds one-half the breaking strength, a permanent deformation is imparted to the line. The safe working limit recommended for logging applications is one-third the breaking strength for a static situation. This provides a margin of safety for dynamic and sudden shock loads. The first cable to reach its safe working limit becomes the limiting line in the system. A maximum payload is calculated by knowing the tension in one of the lines and the geometry of the other lines at the carriage. Determination of this payload assumes the yarder remains securely fixed and is capable of delivering the specified tension.

4. Virtually all existing programs for computer analysis of cable yarding systems use the rigid link method to calculate cable

tensions. The critical assumptions of this method will be reviewed later.

These assumptions were examined for their applicability in determining the static load lifting capability of the pendulum balloon. Assumptions 1, 3 and 4 were considered inapplicable for the following reasons:

1. Tensions imposed upon the balloon are a function of the cable geometry, line weight and available balloon lift. If the balloon position is mathematically represented as a static point in space, all forces must balance in the X, Y and Z directions. The presence of a force imbalance causes the balloon to move to a new location according to the magnitude and direction of the imbalance.

2. Vertical lift available at the load location is determined by the horizontal distance from the base of the balloon to the load position and the degree of pendulum line tautness. The load position acts as a fixed anchor until the pendulum line supplies sufficient lift to support the load. This is accomplished by tightening the pendulum line with the winch attached to the base of the balloon. As the line is taken up, its changing tension creates a force imbalance. In turn, the balloon must move to restore a force balance. In most instances, the direction of balloon movement is toward the load position. This movement is resisted, but not prevented, by the opposing guylines.

3. As the pendulum line tension increases, one or more of the guylines must slacken due to the fixed amount of balloon lift. While

the proportion of lift in each line varies, the total must remain the same. As the guylines slacken, the error inherent in straight line analysis methods is magnified and a catenary analysis is required to accurately determine line tensions.

4. Initial tension or deflection in the guylines or pendulum line may not be known. Generally, the balloon is incapable of tensioning the lines to their safe working limits. Consequently, a range of cable tautness conditions exists. Some of these conditions may be physically impossible to attain using the assumed tension values and guyline lengths.

5. Determining a force balance at the load location is impractical due to the range of functions the haulback and mainline perform. Initially, the haulback supplies additional lift to raise the load, but when it is slackened to start the swing, this lift is lost. The loss of lift due to the magnitude and direction of the mainline tension cannot be established due to the indeterminate coincidence of the load position with the end of the swing phase. Termination of the swing at a point other than the landing may require use of the mainline to move the load the remaining distance.

6. Calculating available lift at any given load position requires a known static balloon position. This position is determined by the lengths of the guylines and pendulum line.

PROBLEM DEFINITION

The goal of the program development was to create a model capable of meeting the following requirements:

a) Applicable to a wide range of problems and conditions associated with cable logging.

b) Very high probability of success in establishing a balanced balloon position and associated cable tensions under all conditions of cable tautness.

c) High degree of accuracy in the position and tension calculations.

d) Uses only those variables which can reasonably be expected to be known when logging plans are being formulated.

e) Reasonably representative of the operation of the full scale system.

f) Capable of being fully automated in evaluating a number of balloon and load positions.

g) Relatively easy to use, modify and understand.

h) High execution speed yet economical to use.

No computer models of moored balloons or sea buoys meeting all the above requirements were found in the literature. Therefore, a suitable model had to be developed.

The static analysis of the pendulum balloon presented in this paper is based on Newton's first law of motion and a fundamental hypothesis. Newton's first law states that every particle continues in a state of rest or uniform linear motion unless compelled to change that state by forces imposed upon it.

The fundamental hypothesis is that a unique point can be found between the ground and balloon where all lines meet in such a way that forces are balanced in all directions.

This hypothesis simplifies and defines the problem. Given the appropriate information, the program must provide a method of systematically searching for an intersection point for a force balance.

The forces existing at the balloon are:

1. Gross lift of the balloon.
2. Upper end tensions in the guylines and pendulum line.
3. Weight of the balloon fabric, harness, winch and fuel.

The weight of the items in (3) can be subtracted from the gross lift of the balloon to establish net lift. This net lift must be offset by the vertical tensions in the guylines and the pendulum line to achieve a vertical force balance.

Three methods of analysis are available to determine the tension in a cable. In order of increasing complexity these are, weightless line, rigid link and catenary.

The weightless line approach employs the following assumptions:

1. The effect of cable weight is negligible.
2. The cable is a straight line between points of attachment.
3. The tension at any two points on the cable is the same.

This method is fast, easily applied and accurate if the assumptions are not violated. This approach is unsuitable, however, for slack cables or those which differ substantially from a straight line assumption.

Rigid link analysis uses a different set of assumptions:

1. The effect of cable weight is not negligible.
2. The cable is straight between points of attachment.
3. The tension at the upper of two points on the cable is equal to the tension at the lower point plus the product of the line weight in lbs/foot and the difference in elevation between the two points.

Results given by this method are very close to those obtained by a catenary analysis if the ratio of tension to line weight is greater than 15:1 (FE 560). The difference is negligible for lines tensioned to one-third their breaking strength. The rigid link method has two weaknesses, however. First, the formulas require a tension value or a horizontal component of tension. The second weakness is its inapplicability to slack cables or those which do not satisfy a straight line assumption.

A catenary analysis is the most accurate of the three methods but also the most time consuming. The method uses assumptions 1 and 3 from the rigid-link approach and may be used to evaluate any condition of cable tautness except a straight line. This latter condition implies the existence of an infinite amount of tension in the line.

Catenary is the name given to the shape adopted by a uniform, completely flexible, cable, rope, chain or string fastened at each end and hanging freely under the action of its own weight. The shape of the wires between two telephone poles is an example of a catenary. Equations used to analyze these shapes involve hyperbolic and logarithmic functions and identities. Frequently, an iterative solution procedure for the tension must be undertaken because a direct algebraic solution is not possible. Successive passes are made through one or more equations with a new estimate refined from the results of the previous guess until a satisfactory accuracy is obtained. The nomenclature and methods of solution used by logging engineers to analyze a catenary is that published by Carson (1977). Appendix A shows the general shape of a catenary and the equations used to analyze it.

INFORMATION AND ASSUMPTIONS REQUIRED FOR A CATENARY ANALYSIS

A catenary analysis was judged to be the most appropriate model for calculating cable tensions. The essential information which will meet the previously stated model requirements and allow the use of a catenary based approach is:

1. X, Y, Z coordinates of each guyline anchor and the load location.
2. Diameter, modulus of elasticity and weight per foot of line for all cables.
3. Lengths of all guylines.
4. Initial pendulum line length.
5. Net balloon lift available to hold the cables and a payload of logs aloft.

There must be sufficient line on the drum of the winch suspended beneath the balloon to reach the back of the corridor and the landing. Therefore, several more inputs are required.

6. X, Y, Z coordinates of landing and furthest point away from landing where a load is to be picked up.
7. Amount of additional line stored on the drum beyond that required to reach the farthest point.

The conditions and simplifying assumptions made to allow use of this information are:

1. The balloon location can be represented as a point where the intersection of the guylines and pendulum line exists.

2. Balloon lift is a constant force located directly at this intersection point and is acting entirely vertically upward.
3. The X, Y, Z coordinates of the landing, farthest point away from landing, load position and the guyline anchor locations are known exactly and remain fixed.
4. The location of the items in (3) can be represented as points.
5. All coordinates of all points must be greater than zero.
6. Line weight per foot, diameter and modulus of elasticity are constant and greater than zero.
7. Net balloon lift exceeds the weight of all lines.
8. The length of line not required to reach a given load location is stored on the winch underneath the balloon where its weight is subtracted from net balloon lift.
9. Neither the ground nor the other cables interfere with the shape of the lines as they become slack or taut.
10. Balloon elevation must be above all locations specified in (3).
11. Anchor locations are points just forward of any shackles or other types of connectors used to fasten the cable to a fixed object. For example, if each guyline is wrapped around a stump several times and clamped off, the clamped portion cannot be considered to act as a caten-

ary. Only the free portion of the lines can be considered.

12. The contribution or loss in payload capacity due to the haulback and mainline tensions at the load location is ignored. This means the lift calculated is what the ballon alone can contribute. There are other tensions present at the load location, but their magnitude cannot be determined without more assumptions and data.

To facilitate the analysis, an option is provided within the program to have the lengths of the guylines and pendulum line calculated by entering the coordinates of an initial weightless line balloon position. The coordinates are used to calculate Pythagorean distances between the balloon and each cable anchor location, including the load position. These distances are then used as actual catenary line lengths. The actual balloon position will always be below the position initially entered, due to the weight of the lines. If this is a problem, the balloon may be placed at a higher initial position to compensate for line weight. The discrepancy between a weightless and an actual balloon position becomes larger as cable weight increases. A small reduction in the Z coordinate of the weightless balloon position is necessary to perform a catenary analysis because the initial line lengths form straight lines between points of attachment.

If the above option is not selected, then a non-iterative method developed by Tuor (personal communication) for determining a balanced intersection point is used. This method assumes the lines are weightless and that the fourth line is slack and can be ignored in the force calculations. Tension in the lines is determined by a procedure similar to methods used to find the forces in the legs of a tripod. Inherent weaknesses in this method are:

1. The load position and pendulum line tautness may cause more than one line to become slack. Using the weightless line assumptions can result in overestimating the tension in the slack lines and underestimating lift available at the load position.
2. The difference between balance point coordinates and cable tensions determined by a catenary analysis of all four lines and those calculated by Tuors' method increases as the pendulum line is progressively shortened.
3. Ignoring the slack guyline may be suitable for analysis purposes, but if a more accurate balloon position must be known, this method will not provide one under all conditions of cable tautness.

For the above reasons it is recommended that the first option be used in developing harvesting plans.

COMPUTER REQUIREMENTS

Initial program development was done with a Hewlett Packard[1] 41 C/V programmable calculator. A program was written to perform both a weightless line determination of a balloon position and a catenary analysis of a given balloon position. No attempt was made to program a systematic search for a balanced balloon position. Catenary analysis of a given balloon position with four different lines takes about five minutes to run. Since 25 to 40 different positions may need to be evaluated for a satisfactory force balance, the analysis can require several hours for a single load position. The only recommended use of the calculator program is for an in-the-field check of a balloon position obtained by surveying instruments.

The entire program was first implemented on a Hewlett-Packard 86 desktop computer. Depending on the particular application, the time required to obtain a balanced balloon position varies from 5 minutes to several hours. The average time required is approximately 15 minutes. This time could be reduced 40 to 50 percent by specifying a lower level of desired accuracy in the results. This machine is capable of determining a force balance within ± 0.01 lbs. in each direction. Accuracy of the balloon position is within 0.0001 feet.

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[1] Use of trade names does not imply endorsement by Oregon State University.

The second machine used was an HP-9816 desktop computer. Its calculation speed for this application is approximately 15 times faster than the 86. A static balloon position can typically be found in several minutes or less. Furthermore, the accuracy of the force balance improved to ± 0.001 lbs. and the balloon position is accurate to within 0.00001 feet. All the results presented were obtained using this computer.

When formulating logging plans the level of accuracy recommended for obtaining a force balance is ± 10 lbs. in all directions.

MODEL VALIDATION

Two separate field studies were conducted to gain operational insight into a full scale system. The first study was designed to monitor static tensions in the guylines and pendulum line for a variety of load positions and was selected for model validation purposes. A small balloon having a maximum lift of 2000 lbs. was used. Surveying instruments were used to establish the cable anchor and balloon positions. Tensions at the lower end of each cable were measured electronically. Strip chart recorders were used to continuously record the tension measurements graphically.

These charts were subsequently analyzed to determine the mean line tensions existing for a given anchor and balloon configuration. The mean tensions, measured balloon position and cable anchor coordinates were used to calculate line lengths. These line lengths were input into the developed computer model to calculate line tensions and a balanced balloon position for comparison to the field information. The results of four individual trials are shown in Table 1. Measured field data is shown as such and calculated data was derived from the computer model. For the trials indicated, the range in tension differences is from ± 0.09 to 149.1 lbs. The average difference is approximately 58 lbs. or 11.7 percent. The average difference between measured and the calculated balloon positions is ± 0.01 percent. The last column in Table 1 is a determination of the force imbalances existing within the field data. Measured line tensions

Table 1. Comparison of measured and calculated cable tensions and balloon positions from prototype static balloon experiment

Trial #	Measured Cable Tensions (lbs.)		Calculated Cable Tensions (lbs.)	Measured Balloon Position (ft.)	Calculated Balloon Position (ft.)	Force Imbalance from Field Measurements (lbs.)
1.	Guy 1	15	15.57	X = 998.78	X = 998.7822	X = 14.73
	Guy 2	575	568.37	Y = 990.25	Y = 990.2769	Y = 44.74
	Guy 3	606	574.09	Z = 829.82	Z = 829.8315	Z = 11.66
	P.L.	579	628.21			
2.	Guy 1	737	789.46	X = 999.26	X = 999.2677	X = 137.44
	Guy 2	624	752.95	Y = 995.72	Y = 995.7198	Y = 26.04
	Guy 3	865	763.86	Z = 832.51	Z = 832.5117	Z = 55.67
	P.L	50	50.09			
3.	Guy 1	596	606.57	X = 999.61	X = 1000.0755	X = 99.56
	Guy 2	72	85.81	Y = 996.70	Y = 996.7007	Y = 15.93
	Guy 3	243	93.90	Z = 831.64	Z = 832.5409	Z = 28.95
	P.L.	1024	1150.24			
4.	Guy 1	702	782.85	X = 999.25	X = 999.2564	X = 100.98
	Guy 2	630	663.39	Y = 996.39	Y = 996.3936	Y = 92.88
	Guy 3	810	672.82	Z = 833.13	Z = 833.1290	Z = -18.94
	P.L.	145	144.73			

and calculated line lengths were resolved into X, Y and Z components at the measured balloon position. Forces were summed in each direction and results are indicated in the table. Potential sources of error within the field data include surveying measurements, tension monitoring equipment, wind and other external factors.

Results from this field study indicates the model can adequately predict line tensions and a balloon position provided the required input information is accurate. External factors such as wind, cable condition and measurement errors can be expected to make actual results significantly different from theoretical results.

BALLOON RIGGING AND LAYOUT CONSIDERATIONS

The size and type of line to be used in tethering the balloon and swinging the load is important. Conventional wire rope is relatively heavy and long lengths significantly detract from the lift performance of the system. For this reason, guylines made of Kevlar® were specified for this analysis. This material is light in weight, very strong and used in the manufacture of hawsers for tethering large, ocean-going freighters. Kevlar® lines were used in the static lift experiment on the prototype balloon. The size selected for tethering the full-scale balloon is 1 5/8 inches in diameter with a breaking strength of approximately 180,000 lbs. Its weight is 0.67 lbs. per foot. This size provides a significant margin of safety in the event of a guyline failure.

Kevlar® is not suitable for use as a pendulum line, however. It is much larger in diameter than a similar strength steel cable and does not possess comparable abrasion and shock resistance. A one inch extra improved plow steel (EIPS) wire rope was specified for use as the pendulum line. This cable weighs 1.85 lbs. per foot and has a safe working limit of 34,500 lbs. The pendulum line cannot be used exclusively to maintain the tethered position of the balloon during corridor changes on the setting, due to the net balloon lift of 47,000 lbs.

Selecting the initial elevation and horizontal position of the balloon requires careful consideration. If the swing portion of the yarding cycle is intended to carry the load to the landing, the balloon cannot be placed near the back of the setting. Given a fixed anchor geometry, the following advantages accrue as balloon elevation increases:

1. The guylines form a steeper angle with respect to the ground and acquire a greater proportion of vertical tension. Transfer of this tension to the pendulum line supplements load lifting capability.
2. There is a reduction in the angle the load must travel through from any given load position to the landing.
3. Less horizontal tension is necessary in the mainline and haulback lines to alter the direction of the load. The main component of tension in these lines is horizontal. Raising balloon elevation alleviates the problem of violating their safe working limits.

Disadvantages of positioning the balloon at a high elevation above the slope include:

1. The longer lengths of cable required to tether the balloon detract from its gross lift capability.
2. The balloon is more susceptible to wind forces because the guylines have less horizontal tension.
3. The pendulum line must be shortened more to lift a given load since the balloon provides less net lift. Balloon movement increases, which in turn affects the swing.

If a relatively low elevation is chosen for the balloon, the lines will have more horizontal tension in them. This limits the load lifting capabilities for positions which are further away horizontally. It also increases the angle of travel from any given load position to the landing. However, the balloon can resist wind induced movement more and has more net lift available due to the reduction in line lengths.

Initial guyline anchor coordinates and the orientation of the corridor were arbitrarily assigned to facilitate the analysis. Anchor locations for the guylines are given below. Guyline #1 was assigned initial coordinates of 10 for the X, Y, and Z directions because all coordinates must be greater than zero.

	Guyline #1	Guyline #2	Guyline #3
X coordinate	10	1010	510
Y coordinate	10	10	2010
Z coordinate	10	10	1210

A plan view of the setting is shown in Figure 4. Anchor positions of guylines 1 and 2 are on the same truck road and are 1000 feet apart horizontally. The landing is midway between them on the same truck road. The anchor location of guyline 3 is 2000 feet away horizontally from the landing and perpendicular to the truck road.

The proposed corridor has beginning and ending coordinates of X = 510, Y = 10 and Z = 10 and X = 510, Y = 1910 and Z = 1150, respectively. Its termination point is 100 feet short horizontally from the third guyline anchor. This guyline placement and corridor

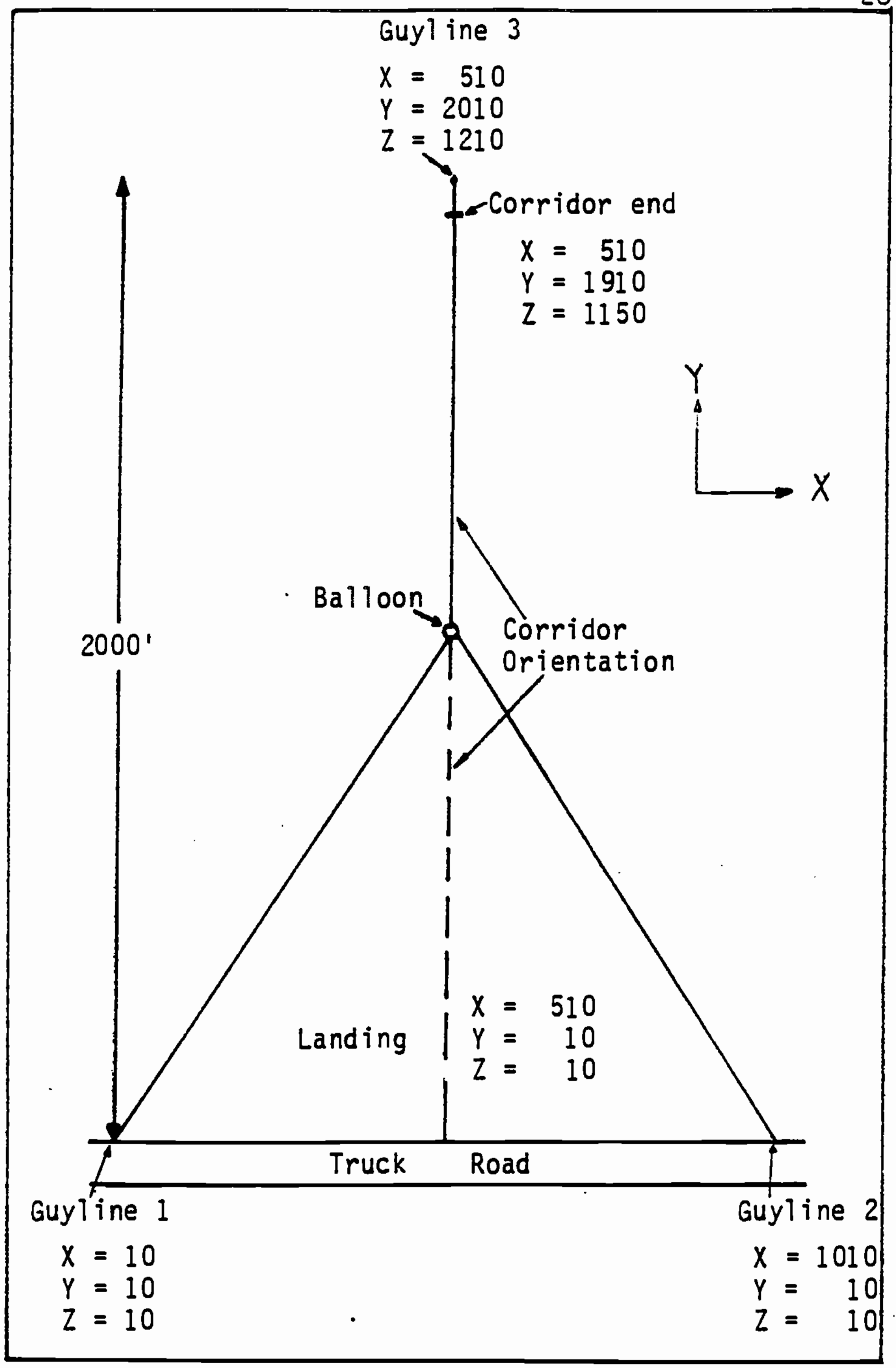

Figure 4. Plan view of guyline anchors and corridor orientation used to analyze full scale pendulum balloon system.

orientation cancel the forces in the X direction for all load positions on the corridor. The load positions are located at intervals of 100 horizontal feet from the landing and lie along a continuous 60 percent slope. The program is capable of evaluating other arrangements of guyline anchors and corridor orientations.

RESULTS

Application of the following results to an actual setting should be done with caution for several reasons. First, as the load moves, the tension in the pendulum line changes, which in turn causes the balloon to move. Because of this, the position of either the balloon or the load must be known to calculate the position of the other. In this study, a load position is assumed and the load that can be supported by the pendulum line is calculated.

A second consideration is that the balloon cannot support as great a load above the ground as it can at ground level. This is because as the load elevation increases the vertical angle measured from the base of the balloon to the load position increases. For an actual setting, be sure that specified load locations account for log length and potential obstacle clearance downslope. This is very important as balloon elevation decreases.

Third, one limitation of the catenary analysis used by the model is that a calculation of lift cannot be made for a load position directly beneath the balloon. This is because the horizontal distance between end points of the pendulum line is zero. A calculation can be made for positions within several feet of this point but considerable time may be required. This should be remembered when shortening the pendulum line because the final balloon position may be directly over a planned load location. The recommended minimum vertical angle to achieve after shortening is 0 degrees 45 minutes.

The angle is measured in a vertical plane from the balloon base to the desired load location. A rule of thumb is 5 feet from vertical for every 500 feet of balloon elevation above the slope. All results shown use an estimated lift value for load positions which are underneath the initial balloon location.

Effect of Line Weight on Available Lift

Figure 5 illustrates load lifting capability at various load positions for two different sizes and types of line. The lower curve represents lift capability if all guylines and the pendulum line were 1 1/4 inch, EIPS wire rope. Each line is capable of counteracting the entire net balloon lift of 47,000 lbs. without exceeding one-third breaking strength. The landing and guyline anchor positions are as previously specified in Figure 4. The pendulum line was shortened 30 feet from its initial Pythagorean length to tension it. Initial balloon position corresponds to X = 510, Y = 980 and Z = 2092. This is 1500 feet above the 60 percent slope. Also, payloads are shown on the figure for Kevlar® guylines and a one inch EIPS wire rope pendulum line, as discussed earlier. The pendulum line was shortened 13 feet from its initial length for each load position. The criterion used to determine the amount of shortening was the tautness of the pendulum line.

Load lifting capability is much greater with the Kevlar® guylines because of their lighter weight. At the 1010 foot load posi-

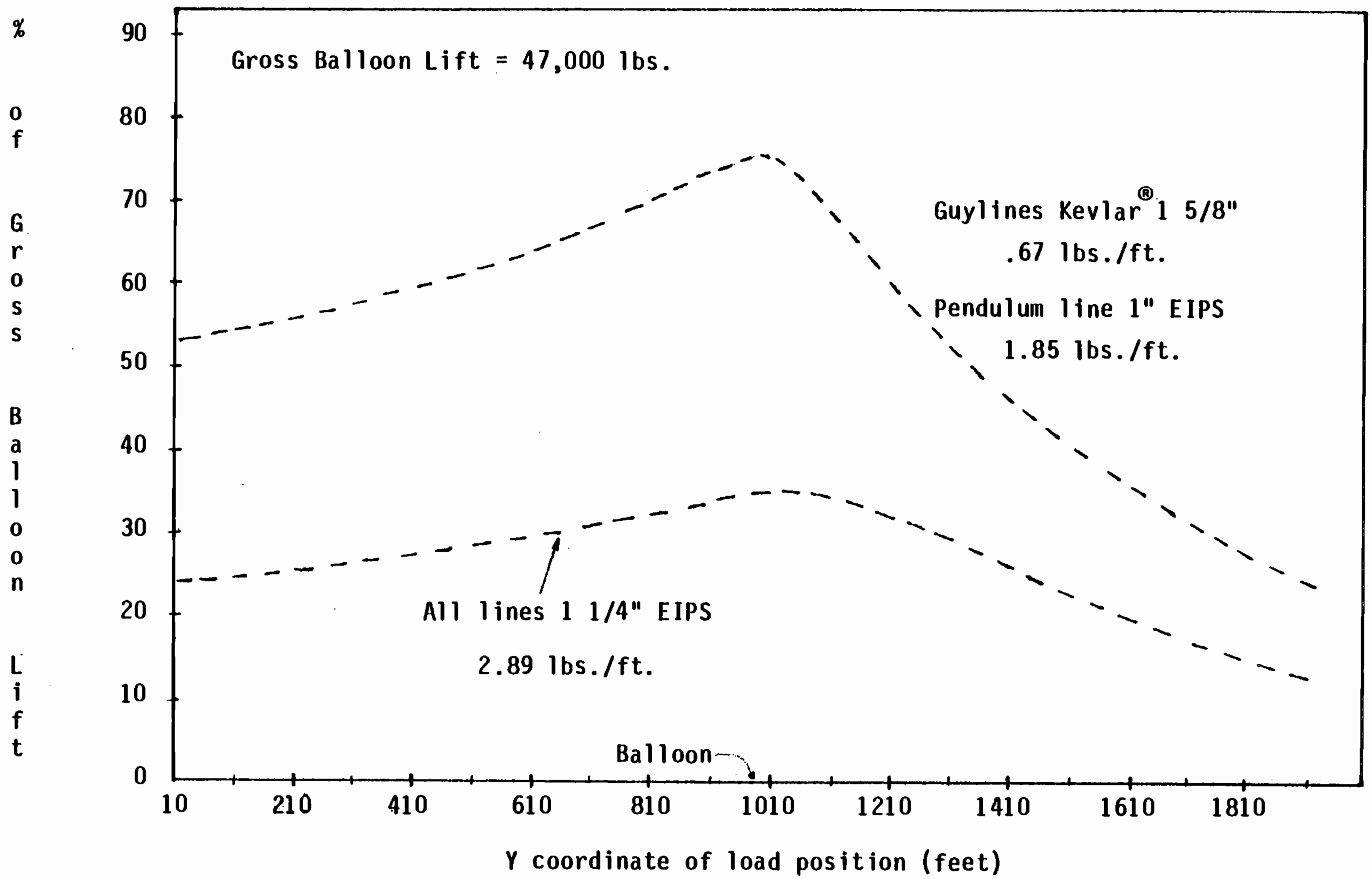

Figure 5. Comparison of payload capability at various load locations for Kevlar® and wire rope lines.

tion, the available lift is nearly 40 percent greater for the Kevlar® guyline configuration.

Effect of Pendulum Line Length on Balloon Movement

Figure 6 illustrates the movement of the balanced balloon position as the pendulum line is tightened using the winch underneath the balloon. The original balloon position is X = 510, Y = 980 and Z = 2092. This is 1500 feet above the 60 percent slope. Guyline anchor positions and the landing are at their previously specified locations. The load position is located at X = 510, Y = 1910 and Z = 1150. Initially, all line lengths were equal to the Pythagorean distances between the balloon and each anchor position. The pendulum line was shortened in 1 foot increments up to 20 feet. The lift available at the load location with the initial Pythagorean length of the pendulum line is approximately 8300 lbs. Figure 6 shows that balloon movement is in a straight line. Lifting capability after 20 feet of shortening increased to 11,735 lbs.

Further shortening of the pendulum line should cause the movement of the balanced balloon position to describe a curved path due to the position of the front two guylines. This is exactly what is shown in Figure 7. The balloon maintains a straight line for approximately 200 feet and then descends along an arc defined by the front two guylines as the rear guyline slackens. The pendulum line was shortened 1080 feet from its initial length. The final balloon position is X = 510, Y = 1844.45 and Z = 1384.75 feet. Available

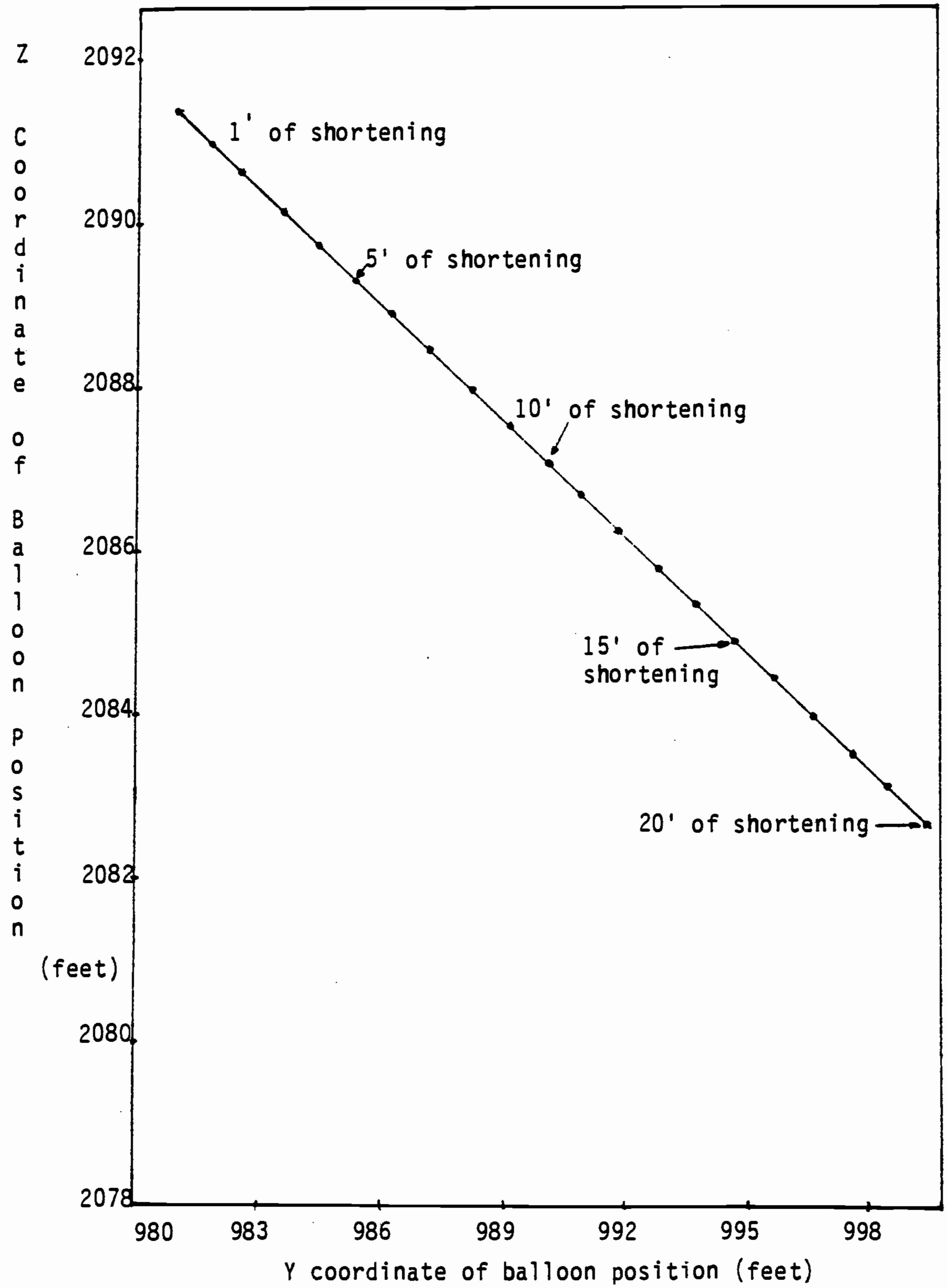

Figure 6. Y and Z coordinates of static balanced balloon position as pendulum line is shortened from 0 to 20 feet.

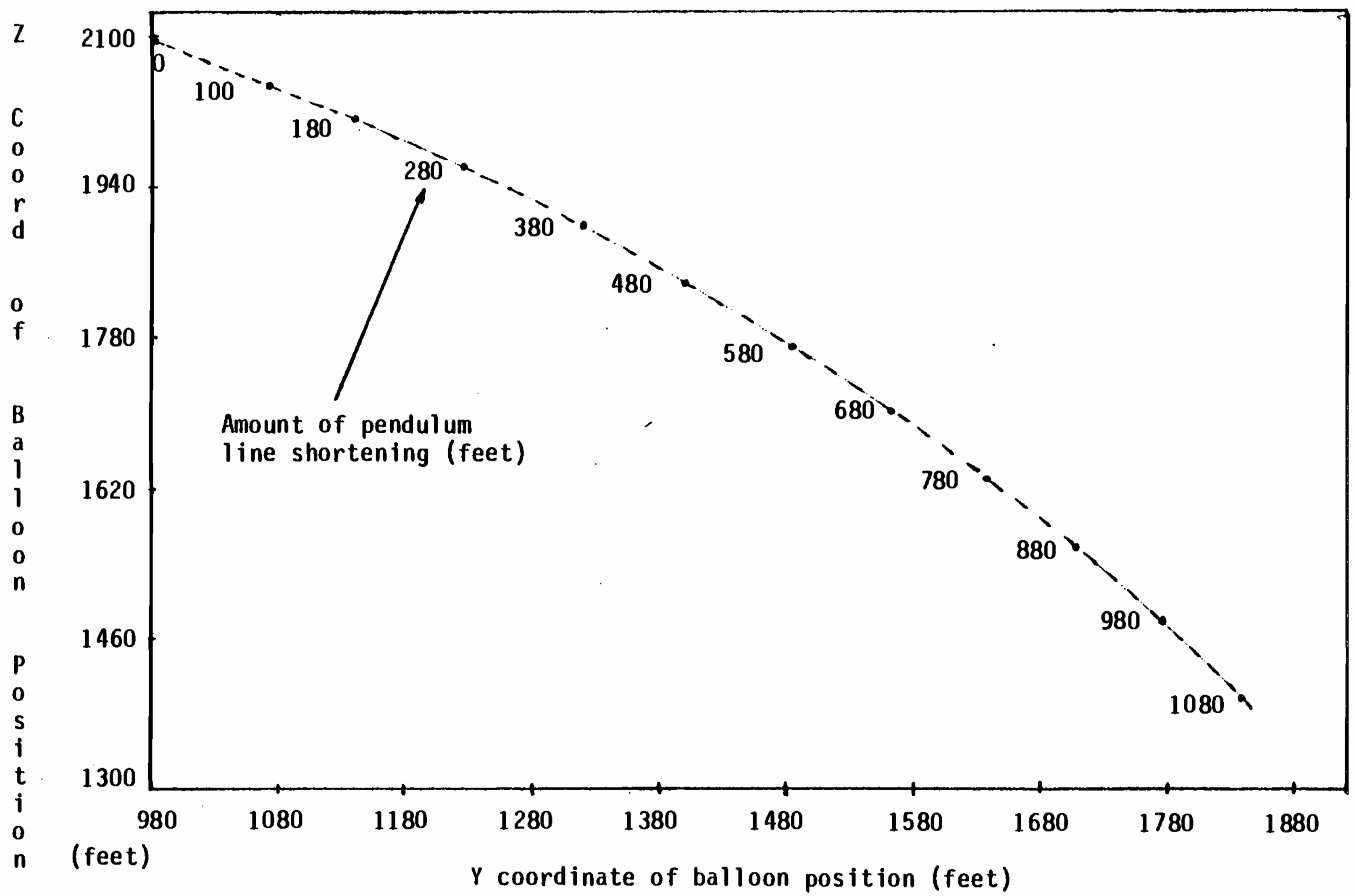

Figure 7. Y and Z coordinate of static balanced balloon position as pendulum line is shortened from 0 to 1080 feet.

lift at the load location is now 33,500 lbs. This figure is large because the balloon is 65 feet away horizontally from being directly over the load. The balloon moved 863.7 feet in the +Y and 706.7 feet in the -Z directions from its initial location.

The converse of this situation is important to understand. If a 33,500 lb. load had been positioned as indicated, the balloon must move over 1100 feet before the load could be lifted and the pendulum line must be shortened by 1080 feet. In an operational situation such balloon movement is not tolerable. The load of logs would have to be lightened considerably.

Effect of Pendulum Line Tautness and Load Location on Lift

If the pendulum line is allowed to remain at a Pythagorean length, it will not provide maximum lift at the load position. The reason is its capability of opposing the movement of the balloon will not be significantly different from the guylines. While at this Pythagorean length, its main function will be to balance any remaining horizontal forces imposed on the balloon by the other lines. If the load position is directly underneath the balloon and the pendulum line is at a Pythagorean length, the horizontal tension and the vertical lift available at the load location are equal to zero. However, if the line is shortened, it begins to oppose the upward lift of the ballooon more than any other line. Its tension increases very rapidly and the vertical lift available at the load position increases due to the transfer of vertical tension from the guylines to the pendulum line. The greatest amount of vertical lift the pendulum line can provide is equal to the difference between the net lift of the balloon and the weight of all the lines. This lift can only be realized when the load position is directly beneath the balloon. In this situation, the pendulum line essentially replaces all of the guylines.

As the horizontal distance between the balloon and the load position increases, the pendulum line begins to replace adjacent guylines. Shortening the pendulum line causes a transfer of tension from the guyline being replaced to the pendulum line. The magnitude of the change in vertical lift available at the load location is dependent upon the amount of vertical force initially present in the guyline being replaced. A point of diminishing return is eventually reached if the pendulum line is continually shortened.

Figure 8 shows how lift is affected by pendulum line tautness and the horizontal distance from the base of the balloon to the load location. The lower curve is for a pendulum line length equal to the Pythagorean distance between the initial balloon position of X = 510, Y = 860 and Z = 1520 and the various load locations. This balloon position is 1000 feet above the 60 percent slope. The guyline arrangement, landing location and line types are as previously specified.

The upper curve shows potential lift when the pendulum line is shortened 15 feet from its initial Pythagorean length. This amount

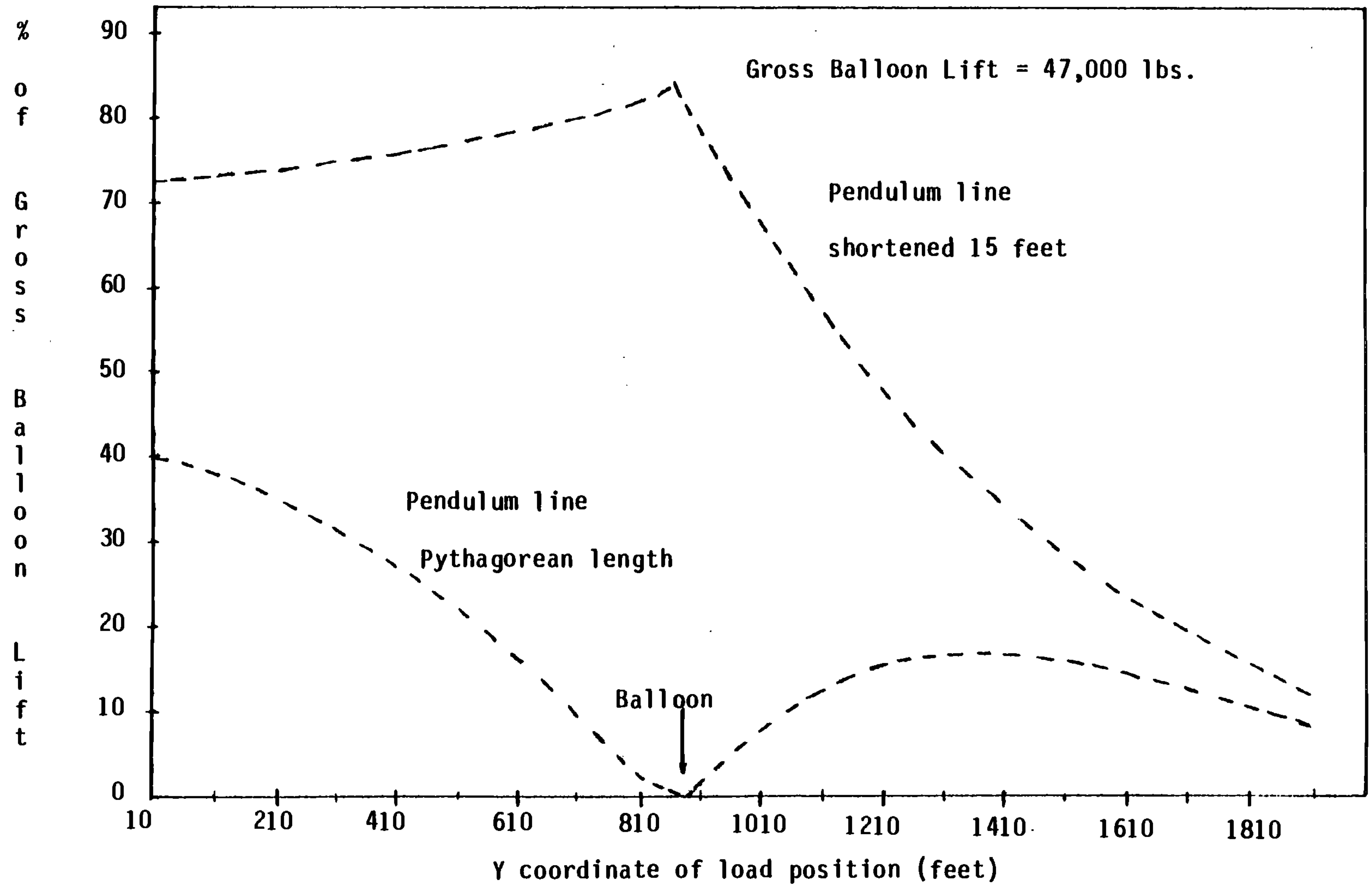

Figure 8. Percent of gross balloonlift available at various load locations as the pendulum line is shortened. Balloon at 1500' above ground.

is recommended as suitable for harvest planning purposes. Lift slowly decreases when the load position lies between the landing and the balloon. This is due to the steep angle of the front two guylines. Each guyline makes an angle of approximately 60 degrees above horizontal. Because the pendulum line is exactly between these lines, it replaces both of them. The loss in vertical lift is therefore gradual. The remaining guyline has an angle of approximately 15 degrees above horizontal and lies between the balloon and the back of the corridor. Thus, most of its tension is horizontal. As the pendulum line is shortened, the gain in lift is gradual until the load position is close to the balloon. Figure 8, therefore, is characteristic of the lift provided using the specified guyline anchor geometry and initial balloon position. Changing these variables would change the curves' shape considerably.

Figure 9 provides an example of how quickly tension changes in the pendulum line as it is shortened. The load position selected is 30 feet away horizontally from the base of the balloon. Balloon height is 1500 feet above the slope. The starting length for the pendulum line is the Pythagorean distance from the weightless balloon to the load position. For convenience, the upper tensions in the three guylines are also plotted.

A substantial portion of the tension transfer occurred in the first 0.8 feet of shortening. Less than 4 feet of shortening was required to change the upper end tension in the pendulum line from

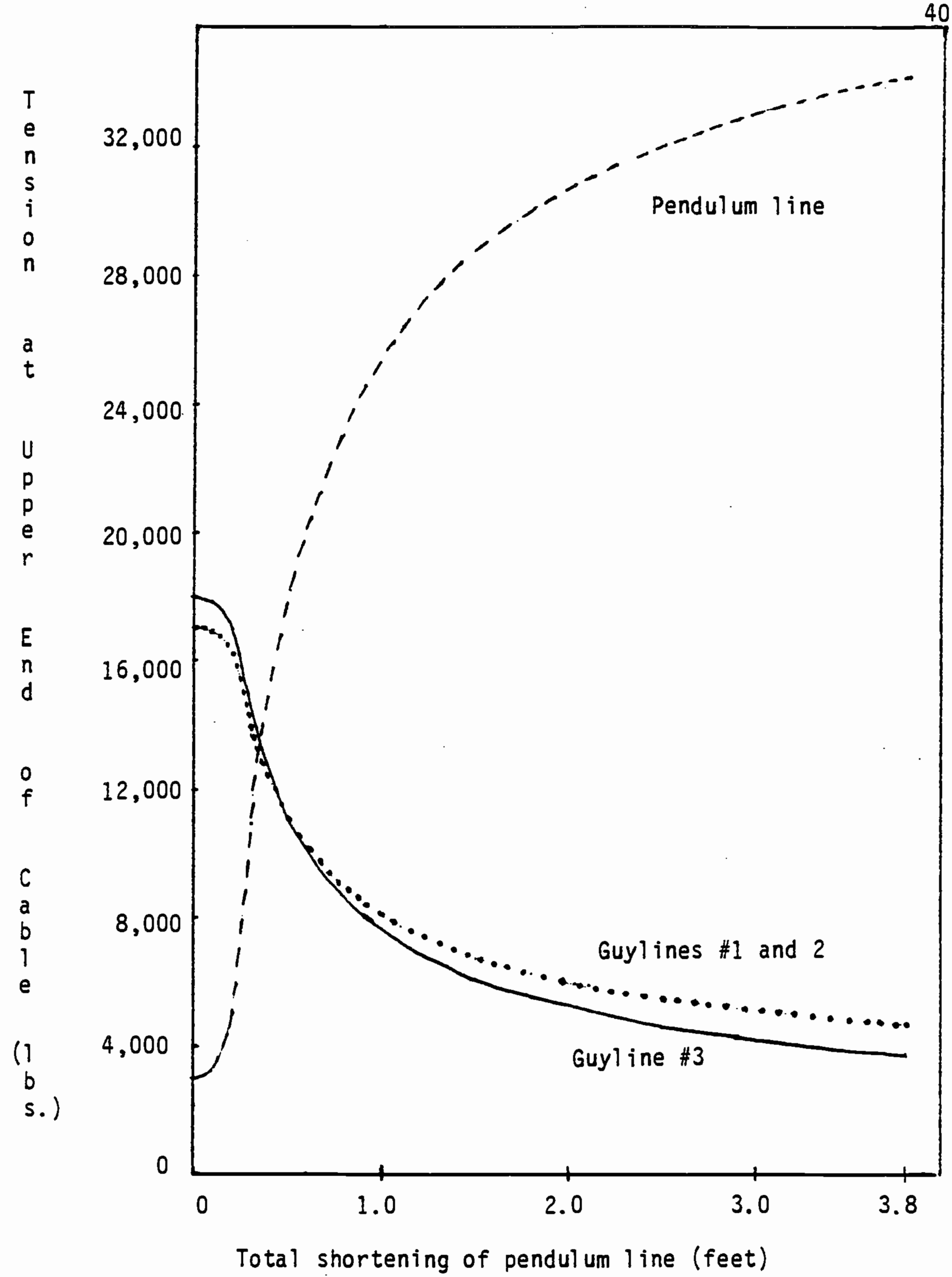

Figure 9. Cable tension at upper end of each line as the pendulum line is shortened.

approximately 3000 lbs. to very near its safe working limit of 34,500 lbs. Lift in the pendulum line changed at the same rate as tension.

Conversely, as the ground begins to support the load, lengthening the pendulum line a small amount will slacken it considerably and cause, in turn, a rapid change in the balloon position. The balloon's inertia will delay the position change however.

Effect of Balloon Elevation on Lift

Figure 10 illustrates the effect of balloon elevation on lift available at a load location. The horizontal balloon position is X = 510 and Y = 980 feet. Payload capabilities at various points on the 60% slope are shown as balloon elevation above the slope changes from 1000 to 2000 feet. For all three curves, the landing is the farthest point from the balloon. Further than about 900 feet from the landing, a larger load can be supported as the elevation of the balloon increases. However, as the load position moves toward the landing, lift diminishes with height due to the increasing length of the pendulum line.

Curves for each height were drawn using different amounts of pendulum line shortening. For the 1000 foot level, the amount is 15 feet. For the 1500 feet and 2000 feet elevations above the slope, the amount of shortening is 19 and 23 feet, respectively.

Lift available at the back end of the corridor is well below that at the landing. The balloon's horizontal distance from the landing is 970 feet in Figure 10, and the most distant load position

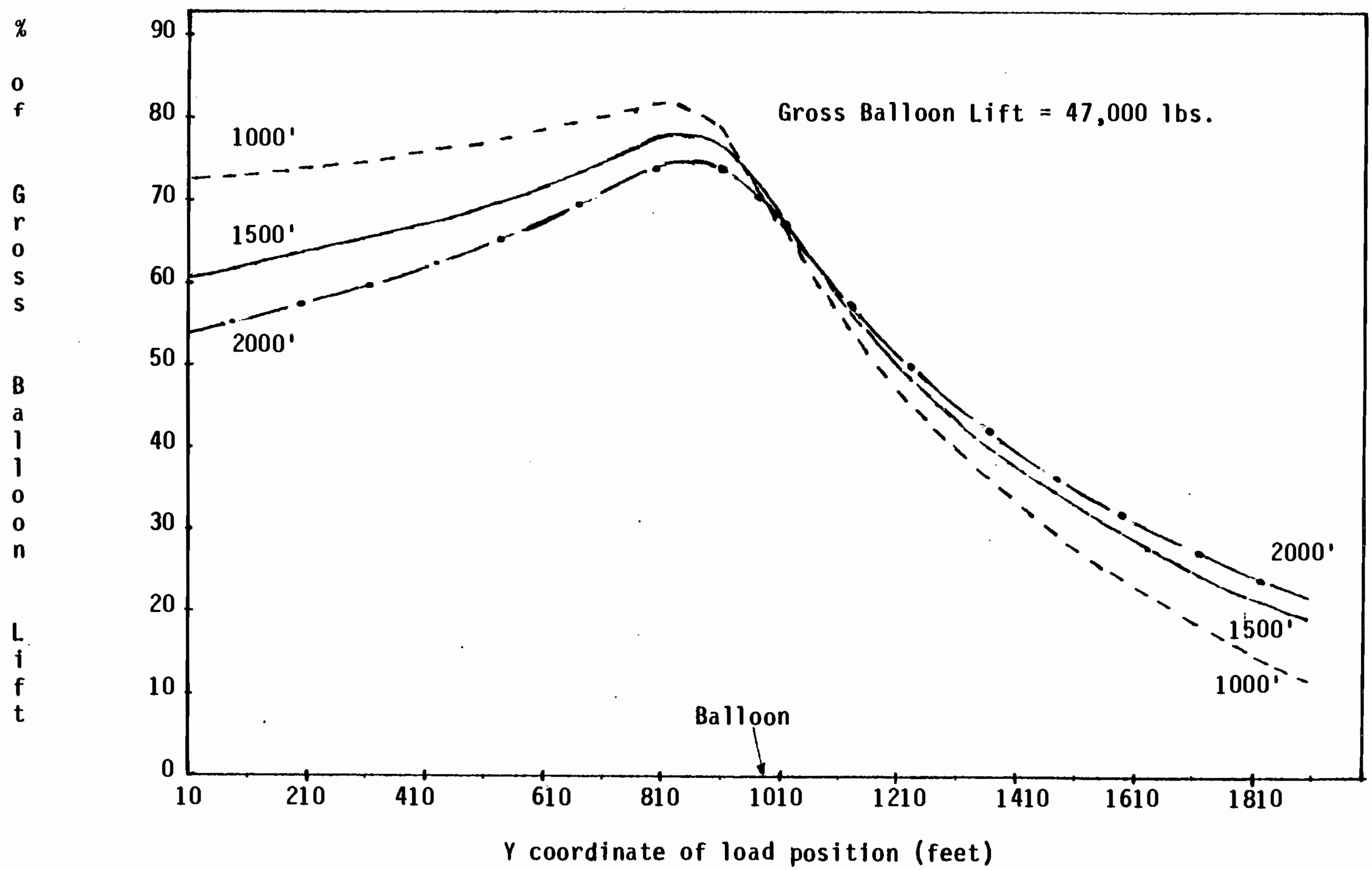

Figure 10. Payload capability at various locations as balloon height is varied.

is 1900 feet from the landing, horizontally. If more lift is needed at the furthest point on the corridor, the balloon must be moved in that direction. However, its swing capabilities will be reduced.

Balloon Placement and Minimum Economic Payload

One of the fundamental problems of the pendulum balloon system is placement over the harvest unit. Given a configuration of guyline anchors, the balloon should be placed to provide sufficient vertical lift at least equal to a minimum economic payload at each end of a given corridor. Safe working tension in any of the lines may not be exceeded, nor is it desirable to induce a negative vertical force in the lower end of any guyline which slackens as the pendulum line tightens.

An example illustrates the concept.

Given:

1. 10,000 lb. minimum payload at each end of corridor.
2. Safe working tension in pendulum line is 34,500 lbs.
3. Horizontal yarding distance equals 2000 feet.
4. Slope yarding distance equals 2330 feet.
5. Guyline anchor and landing coordinates are as previously specified in Figure 4.
6. Assume furthest point from landing to be yarded is at the same coordinates as guyline #3.

Find: The Y and Z coordinates of the balloon such that for a given balloon height above the slope the payload at the furthest point and at the landing is at least 10,000 lbs. and minimize the horizontal distance from the landing to the balloon.

The following points meet the specified criteria.

	Balloon Height Above Slope (feet)			
	500	1000	1500	2000
X coordinate	510	510	510	510
Y coordinate	1490	1160	980	920
Z coordinate	1398	1700	2092	2556
Payload-upper end (lbs.)	10,890	10,600	9998	10,005
Payload-lower end (lbs.)	28,185	28,265	28,370	27,555

The Y coordinate represents the horizontal distance from the landing to the initial balloon position.

Note that only by flying the balloon at 1500 feet above the slope or higher will the balloon be less than halfway to the furthest point along the corridor.

Figure 11 illustrates the relationship between yarding distance, balloon height and horizontal distance to the initial balloon position which satisfies all the constraints listed above. Yarding distance varies from 1000 to 4000 feet horizontally, and balloon height varies from 500 to 2000 feet above the ground.

This figure indicates that for the constraints specified there are significant advantages in flying the balloon at least 1000 feet above the ground, and perhaps up to 1500 feet, if one wishes to minimize the horizontal distance to the balloon. Table 2 contains the coordinates of the balloon locations which satisfy the constraints.

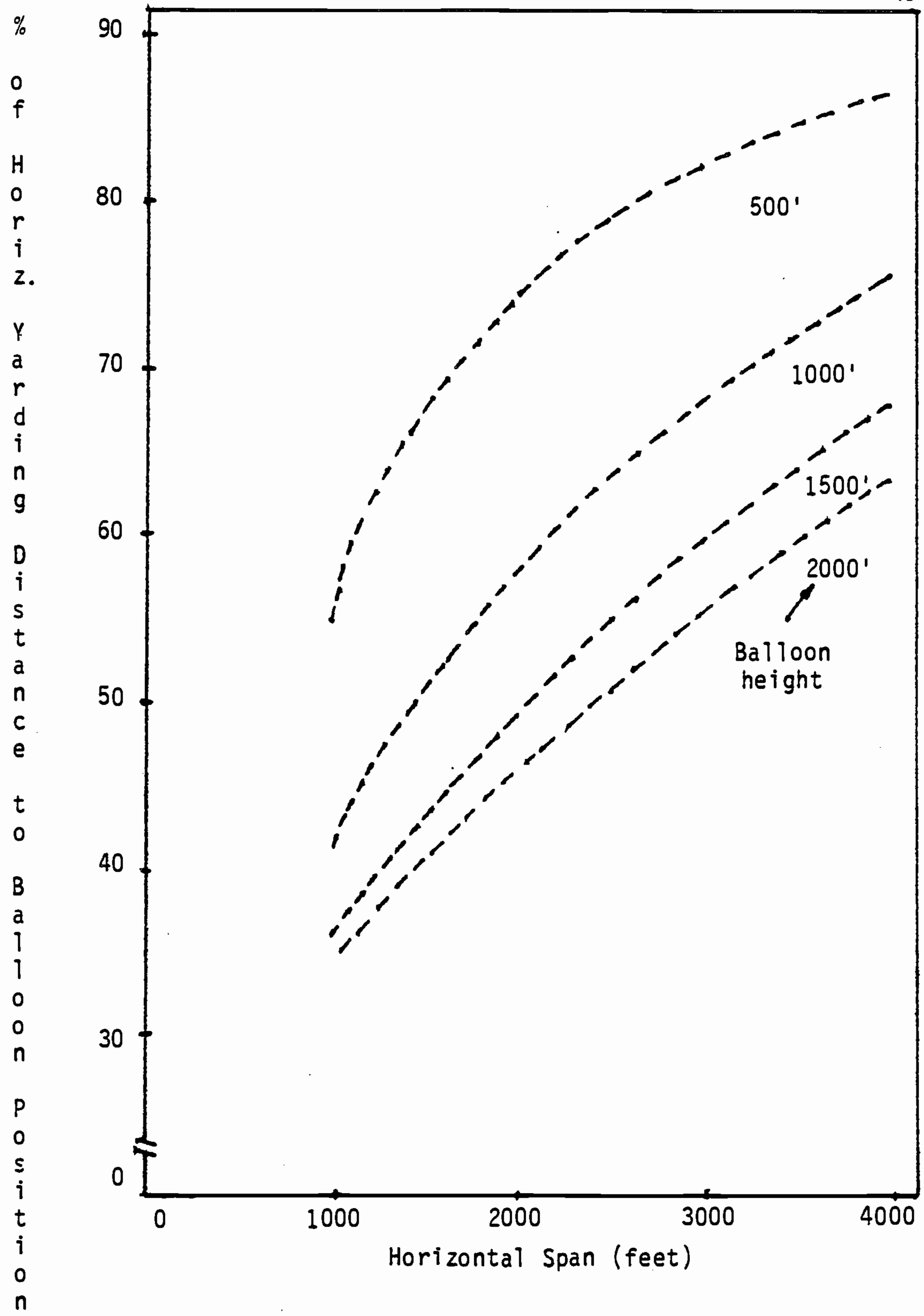

Figure 11. Percent of distance to initial balloon position meeting minimum economic payload requirements to total horizontal yarding distance as horizontal yarding distance and balloon height above slope vary.

Table 2. Coordinates of points satisfying minimum economic payload requirements as balloon height and horizontal yarding distance varies.

Horizontal Yarding Distance (feet)		Balloon Elevation Above Slope (feet) 500	1000	1500	2000
1000	X coordinate	510	510	510	510
	Y coordinate	550	410	360	350
	Z coordinate	834	1250	1720	2214
2000	X coordinate	510	510	510	510
	Y coordinate	1490	1160	980	920
	Z coordinate	1398	1700	2092	2556
3000	X coordinate	510	510	510	510
	Y coordinate	2470	2050	1795	1660
	Z coordinate	1986	2234	2581	3000
4000	X coordinate	510	510	510	510
	Y coordinate	3475	3030	2725	2535
	Z coordinate	2589	2822	3139	3525

FUTURE ADAPTATIONS

The present form of the computer model is flexible enough to handle a variety of applications. Once the user is familiar with its operation small changes will make the following possible:

1. Calculating lift with more than three guylines. The model currently includes three but is capable of dealing with any number of guylines greater than one. Lower anchor position, size and weight of each line must be known.

2. Calculating lift capability at points selected automatically. An equation of a circle, ellipse, square or other pattern can be used to describe the corridor and calculate the coordinates of the next load position. Elevation of the load position also must be calculated. The program can be set to analyze different load positions, different balloon positions, or both.

   In conjunction with this, an array containing load location coordinates can be evaluated or a digitizer can be used to enter coordinates of guylines, landing and load locations.

3. Analyzing any load which has to be suspended by means of cables. The accuracy available from a catenary analysis, as well as the consideration of stretched or original cable lengths can be obtained.

4. The length of any guyline may be changed to reposition the balloon.
5. Evaluating the effects of external forces, such as wind provided the following simplifying assumptions are made:
   a.) External forces are of constant magnitude and direction.
   b.) All forces are applied solely at the intersection point of all the lines.
   c.) These forces do not affect the catenary shape of the cables nor do they change the direction or the location of the point where balloon lift is concentrated.

CONCLUSIONS

A model for evaluating the load lifting capability of the pendulum balloon system has been developed and discussed. The focus of the analysis has been to determine the contribution of the balloon lift at the load location. A catenary analysis is the most suitable procedure to use in determining the balloon position and associated cable tensions. In order to retain flexibility in the application of the analysis procedure, care has been taken to incorporate only that information which can be reasonably expected to be known when formulating logging plans. Information related to the size and position of the mainline, haulback and yarder is purposefully not considered.

Application of the model to the hypothetical setting revealed the following:

1. Available lift at a load location is significantly affected by the weights and lengths of the guylines used to tether the balloon. If the balloon is to be placed at a substantial height above the ground, the guylines should be made of a material having a higher strength to weight ratio than wire rope.
2. The majority of the vertical tension transfer between the guylines and the pendulum line is accomplished within the first 15 feet of pendulum line shortening. Additional shortening will bring about excessive balloon movement with a subsequent reduction in the swing capability of the system.

3. Reliance on the gravity assisted swing to move a load of logs from the back end of the corridor to the landing will require positioning the balloon between 1000 and 1500 feet above the ground.

BIBLIOGRAPHY

Carson, Ward W. 1977. Analysis of the Single Cable Segment, Forest Science, Vol. 23, (2):238-252.

FE 560, Aerial Mechanics, class notes and references.

Inglis, Sir Charles E. 1963. Applied Mechanics for Engineers, illus. New York, Dover Publications, Inc.

McIntosh, J. A. 1968. Production Analysis of Balloon Logging, Forest Products Journal, Vol. 18 (10): 35-41

Peters, Penn A. 1978. Balloon Logging: A Look at Current Operating Systems, Journal of Forestry, Sept., 577-579.

Tuor, Brian, personal communication.

APPENDICES

APPENDIX A. CATENARY EQUATIONS AND ANALYSIS

Analysis of the Pendulum Swing balloon utilizes a procedure developed to analyze each line and use the results to locate a position where all forces are balanced. The procedure assumes lines function as catenaries. Figures 12 illustrates the general case for a given catenary cable segment.

Important assumptions in the analysis are:

1. The tension is derived from the geometry of the cable.
2. The applicable equation for determining the cable tension is:

$$s^2 = h^2 + (2m \sinh (d/2m))^2$$

where

s = the stretched catenary length of the cable segment

h = vertical distance between cable supports

d = horizontal distance between supports

m = the ratio of horizontal tension to line weight per foot

$\sinh (x)$ = hyperbolic sine = $(e^x - e^{-x})/2$

e = 2.7182818289...

The hyperbolic sine can also be expressed as follows:

$$\sinh (x) = x + x^3/3! + x^5/5! + \cdots$$

where the (!) symbol is for the factorial.

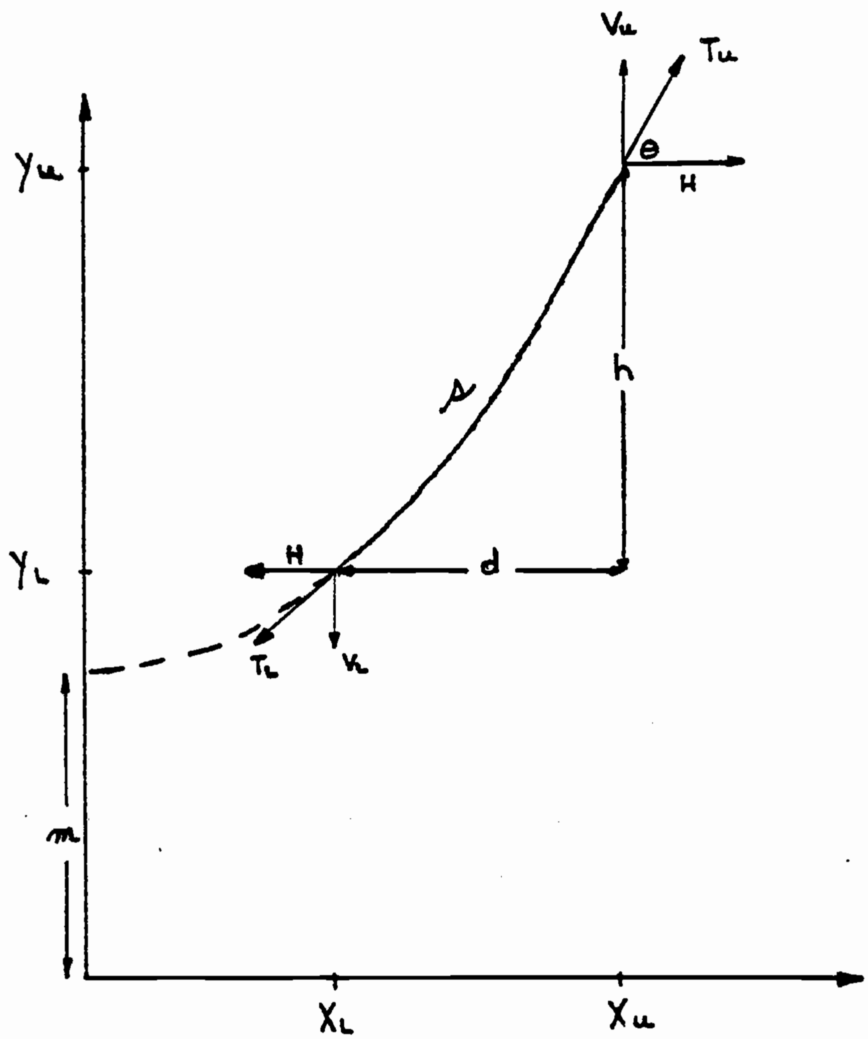

Figure 12. Catenary cable segment - general case

For the general case cable segment shown above, the following equations apply:

a. $Tu = (w/2)\ (s \coth (d/2m) + h)$

b. $T_L = (w/2)\ (h \coth (d/2m) + s)$

c. $Vu = (w/2)\ (h \coth (d/2m) + s)$

d. $V_L = (w/2)\ (h \coth (d/2m) - s)$

e. $V_L = Vu - ws$

f. $s^2 = h^2 + (2m \sinh (d/2m))^2$

g. $m = H/w$

h. $Y_L = m \cosh (X_L/m)$

i. $X_L = m \cosh^{-1} (Y_L/m)$

j. $Y_u = m \cosh^{-1} (X_u/m)$

k. $X_u = m \cosh^{-1} (Y_u/m)$

where:

s = the length of the cable segment between supports

T_u = the upper tension of the cable segment

T_L = the lower tension of the cable segment

V_u = the upper vertical component of the tension

V_L = the lower vertical component of the tension

d = the horizontal span between supports

h = the elevation difference between supports

m = the ratio of horizontal tension to line weight

H = the horizontal component of cable tension

w = line weight in pounds/foot

X_u, Y_u = catenary coordinates of upper end of cable segment

X_L, Y_L = catenary coordinates of lower end of cable segment

$\text{Cosh}(x) = 1 + x^2/2! + x^4/4! + x^6/6! + \ldots$

$\text{Sinh}(x) = x + x^3/3! + x^5/5! + x^7/7! + \cdots$

3. If the assumption is made that s, d, w and h are known, then the only parameter to be solved for is m.

4. The solution of the equation can be carried out by a binary, Newton or Secant search procedure for the value of

m which will satisfy the equation. Of these three, the Newton is by far the most preferable due to its speed of solution. The method employs the following iteration formula:

$$m_{i+1} = m_i - f(m_i)/f'(m_i)$$

where

m_{i+1} = next guess for m

m_i = current value of m

$f(m_i)$= value of the equation using the current value of m

$f'(m_i)$=the value of the first derivative using the current value of m

$$f(m_i) = s^2-h^2 - (2m \sinh (d/2m))^2$$

$$f'(m_i)= -8m [\sinh (d/2m)] [\sinh (d/2m) -d/2m \cosh (d/2m)]$$

A good initial guess for m is $m = d^2/(12(s^2-h^2-d^2))^{.5}$

The solution procedure uses this initial approximation for m and completes successive passes through the equations until f(m) is less than or equal to some acceptable tolerance on the accuracy of the solution desired.

The binary solution method is slightly more accurate than the Newton, but far slower in execution.

5. Expanding the calculation of the hyperbolic sine and cosine on a term by term basis was more preferable than using the e^x and e^{-x} terms. This was because small

rounding errors occurred which seriously impaired the convergence success of the f(m) equation to a satisfactory tolerance.

6. After finding a value for m, the tensions in the cable can be calculated and resolved into X, Y and Z directions.
7. When all cables have been analyzed, forces are summed in all directions. If a satisfactory balance has not been achieved, then the balloon position must be moved in the direction of the force imbalance.

APPENDIX B. FINDING A BALANCED BALLOON POSITION

Locating a position where a balance of forces occurs is the goal of the program. This section provides a brief discussion of the search procedure used.

1. After the required information has been entered, the program checks to insure the weight of the cables does not equal or exceed the net lift of the balloon. All line lengths are multiplied by their weight per foot and added. The length of pendulum line beyond that required to reach the load position is also accounted for. This total weight is divided by the net balloon lift to yield a dimensionless ratio. If the ratio equals or exceeds unity a warning message is printed and the program stops. If less than unity, the ratio is subtracted from the Z coordinate of the balloon position. This is necessary because the lines are straight initially and cannot be analyzed as catenaries. The downward movement slackens all the cables.
2. The horizontal and vertical distance between the balloon and each cable anchor location is calculated using the new coordinates of the balloon position. The load location is assumed to function as an anchor.
3. A right triangle analysis is performed with each cable to determine the maximum Z coordinate attainable assum-

ing it's horizontal orientation and length is to remain fixed. The line length is assumed to be the hypotenuse of a right triangle. The horizontal distance between end points is assumed to be correct as calculated in the previous step. The length of the other leg of the triangle, which is the vertical distance between the ends of the cable is calculated by the Pythagorean Theorem. This result is added to the Z coordinate of the cable anchor position to obtain a maximum possible Z coordinate for the cable.

4. Horizontal and vertical distances obtained in step 2 are used to calculate the hypotenuse of a right triangle. This calculation serves as a check of guyline length. If the result obtained is equal to or greater than the initial Pythagorean length of the cable calculated earlier, the balloon position being evaluated cannot be analyzed by catenary methods. For the first pass through, since the balloon was lowered vertically downward no problem will exist. However, as a horizontal force balance is sought, the lines may not be capable of reaching the new point selected by the program to evaluate. If there is an error, the line causing the problem is identified and the maximum Z coordinate it could attain is assigned to the balloon position coordinates. This makes this particular cable a straight line, so a

downward adjustment on the Z coordinate is necessary. How this is done will be covered in a later step. Once the error is corrected, control returns to step 2 since new vertical distances exist between points of attachment for each cable.

5. The minimum of the maximum attainable coordinates calculated in the previous step is the limiting one. The balloon position cannot exceed this because the line length and horizontal distance between cable endpoints are fixed.

6. Each cable has a parameter representing the ratio of horizontal tension to line weight. Insufficient information prevents a direct algebraic solution for the parameter. Therefore, an iterative process is used to determine its value. Once established, vertical and horizontal components of cable tension are calculated.

7. Tension at the upper end of each cable is resolved into X, Y and Z components. When all cables have been evaluated, forces are summed in each direction at the balloon position. Some tolerance must be allowed in the maximum amount of imbalance acceptable.

8. If the vertical tension of all cables does not approximately balance out net balloon lift, the Z coordinate of the balloon position must be changed. The sign of the force imbalance indicates the direction the balloon

should be moved. If positive, the balloon must be moved upward since it is providing more lift than the cables exert downward. The problem is to determine how much to change the position. For the first guess the assumption used is that a one foot movement up or down will change the cable forces acting in the Z direction by 100,000 lbs. Thus, the force imbalance is divided by 100,000 and the result is added or subtracted from the Z coordinate of the balloon position. Should the new Z coordinate of the balloon be higher than the maximum allowable calculated in step 5, the new Z coordinate is halfway between the previous position and the maximum allowable one. The program returns to step 2 to repeat the analysis. For subsequent guesses at how much to change the position, a gradient procedure is used. The previous Z coordinate and the previous Z force imbalance are stored. The difference between the previous Z force imbalance and current Z imbalance is divided by the difference between the current Z coordinate and the previous one. The absolute value of this ratio is used to compute the next Z coordinate of the balloon position. The current Z coordinate replaces the previous Z coordinate and the current Z force imbalance replaces the previous imbalance. A check is made to ensure the new position does not exceed the maximum allowable one.

Control is sent back to step 2. After the Z forces have been balanced to a reasonable degree, examination of the force imbalances in the X and Y directions is made. The larger of the two is selected first for modification. Since the amount of adjustment required is unknown, an assumption must be used. As an initial estimate, an assumption of 50,000 pounds per foot of movement in the X or Y directions is used. The X or Y coordinate of the balloon position is changed accordingly. The horizontal distance between cable ends is assumed to be correct. The program returns to step 2. A gradient approach similar to that used in finding the Z coordinate of the balloon is employed. However, a limit is imposed on the change in the X and Y coordinates. This is necessary to avoid convergence problems.

Changing any balloon position coordinate changes the magnitude of the forces acting in all directions. For example, changing the Z coordinate of the balloon changes the forces acting in the X, Y and Z directions. Furthermore, the direction of the change in the forces acting in the X and Y directions is not always predictable. For this reason, only one coordinate is changed at a time. The Z forces must usually be rebalanced after X or Y has been changed. This force is balanced by the methods described above. The difference between the maximum allowable Z and the Z coordinate giving a vertical force balance is calculated. This is used as a correction factor to apply to the maximum allowable Z coordinate determined in step 4.

9. When all forces balance within 10 pounds in each direction, a change in accuracy is specified. Initially each cable parameter calculated by the iteration procedure was done to a level of .001 accuracy. After the change occurs the new level is .000001. Then Z forces must balance within .001 lbs. before anything is done to correct X or Y force imbalances. When all forces balance to at least .001 lbs., the analysis stops and results are printed.

Interested individuals may contact the Forest Engineering Department at Oregon State University for a copy of the program listing.

APPENDIX C. EFFECT OF CABLE STRETCH ON TENSION CALCULATIONS

A cable subjected to tension elongates. The amount of stretch which occurs is related to the cable's modulus of elasticity, cross sectional area, length and magnitude of the applied tension. A critical assumption made in the catenary analysis is that the line lengths entered are stretched cable lengths. Inglis (1963) developed the following equation to calculate the amount of stetch in a stretched cable:

$$dL = \frac{wm}{2AE}\left[(X_u - X_1) + \frac{m}{2}\left[\left(\sinh\frac{2X_u}{m} - \sinh\frac{2X_1}{m}\right)\right]\right] - \left(\frac{wm}{AE}\right)^2 \int_{X_1}^{X_u} \cosh^3\left(\frac{X}{m}\right)\, dx$$

where:

dL = the amount of stretch (feet)

A = metallic cross sectional area of cable (square inches)

E = modulus of elasticity (lbs/square inch) other variables are defined in Appendix A.

The formula assumes the cross sectional metallic area of the cable remains constant as the tension is applied.

The terms following the brackets comprise a correction factor which, by earlier analysis, proved to be negligible. It is not incorporated into the stretch calculations performed by the model. An evaluation of the above integral was also done but is not presented here due to space limitations.

The assumption that the initial line lengths satisfy the assumption of being stretched catenary lengths has an error in it which is associated with the line weights per foot. Since the amount of existing stretch in each cable is unknown, the nominal weight per foot given by the manufacturer for each size of cable is used to calculate catenary tensions. This weight is slightly high because each line has changed its length while retaining the same total weight. The procedure used to estimate the error in the use of the nominal weights was as follows.

The initial Pythagorean line lengths and nominal weights per foot were used to find a balanced balloon position. The appropriate information for each line was then used in the Inglis equation to calculate the change in length. This change in length was added to the original length of the cable. The weight per foot of the cable was changed by dividing the new cable length by the total weight of the cable. The assumption used here is that the change in the cable weight per foot is uniform over the entire length of the cable. A more complex analysis would involve an integration over the length of the cable for the difference between the actual weight per foot of the cable and the average weight per foot calculated above.

The new weights per foot and the new cable lengths were used to find a new balanced balloon position. Cable tensions change each time a new set of lengths and weights is used. Therefore, the entire procedure must be repeated until all line converge to a specified tolerance of length. The tolerance used was 0.0001 feet.

The final line weights determined by this procedure were then used in conjunction with the original cable lengths determined by the Pythagorean Theorem. The difference in the payload capacity between using the nominal and estimated stretched weights per foot was less than 0.1%. This indicates that the error in assuming the Pythagorean lengths are stretched lengths and using a nominal weight per foot is very small. Furthermore, since the nominal weights are heavier than the stretched weights, the payload estimate is slightly low. This means that slightly more can be lifted at the load location than presently calculated by the program.

The program does not undertake a determination of stretched line weights in the tension calculations. However, the entire procedure described above is present as an optional subroutine within the program for interested parties.

APPENDIX D. PROGRAM OUTPUT

Figure 13 is a sample of the results provided by the program for each load position evaluated. Numbered items will be taken up in order:

1. Coordinates of weightless balloon position in feet. If Pythagorean line lengths were calculated by the program then the coordinates used to calculate line lengths appear here. If line lengths were entered then the coordinates established by Tuor's method appear here.
2. Catenary balloon position in feet. This is the calculated position where a satisfactory force balance has been achieved.
3. Unbalanced forces in the system. Shows the unbalanced forces in lbs. existing at the given catenary balloon position.
4. Specifies which of the four cables contains the pendulum line information.
5. X, Y and Z coordinates of each cable anchor position in feet including the load position.
6. Weight per foot of each cable in lbs/foot.
7. Modulus of elasticity for each cable in lbs/sq. inch.
8. Ratio of horizontal tension to line weight. This parameter is used to calculate cable tensions. An

```
                    BALLOON POSITION

               WEIGHTLESS          CATENARY

X COOD     510                     510
Y COOD     980        1            983.744760295       2
Z COOD     2092                    2077.06858973

THE UNBALANCED FORCES IN THE SYSTEM ARE SHOWN BELOW
     UNBALANCED X FORCE =  0 LBS.
     UNBALANCED Y FORCE =  2.25966059377E-6 LBS.
     UNBALANCED Z FORCE = -3.96668037865E-5 LBS.          3
CABLE # 4 CORRESPONDS TO THE PENDULUM LINE                4

           1             2                  3                    4

X COORD= 10           1010               510                  510
Y COORD= 10           10                 2010           5     1010
Z COORD= 10           10                 1210                 610

WT/FT= .67            .67                .67      6           1.85
E VALUE= 8.E+6        8.E+6              8.E+6          7     1.3E+7
M VALUE= 1492.593563  1492.593563        1663.843297  8      359.161711
H TENS=  1000.04      1000.04            1114.78          9   664.43
VU=      2759.83      2759.83            1425.80   10        38501.32
VL=      1184.89      1184.89            517.26         11   35786.32
TU=      2935.43      2935.43            1809.87  12         38507.06
TL=      1550.50      1550.50            1228.93        13   35792.99

     THE PENDULUM LINE WILL SUPPORT  35786.8177369 LBS.        14
  THE NET AVAILABLE BALLOON LIFT IS  45446.7972689 LBS.   15
THE LENGTH OF GUYLINE # 1 IS 2350.66458688 FEET
THE LENGTH OF GUYLINE # 2 IS 2350.66458688 FEET              16
THE LENGTH OF GUYLINE # 3 IS 1356.03244799 FEET
 THE LENGTH OF THE PENDULUM LINE IS 1467.30061262 FEET        17

XU=     2596.511850    2596.511850       1772.964629         1706.991926
XL=     1501.898284    1501.898284        746.709390   18    1680.776686
YU=     4381.240556    4381.240556       2701.299429        20814.628360
YL=     2314.171967    2314.171967       1874.230840        19347.559771
RES= 0                 0            19   4.65661287308E-10  -7.30869942345E-9
```

Figure 13. Sample of program output.

iterative procedure is employed to determine its value because an algebraic solution is not possible.

9. Horizontal tension existing in each cable in lbs.
10. Vertical tension in lbs. at upper end of each cable.
11. Vertical tension in lbs. at lower end of each cable.
12. Total tension in lbs. at upper end of cable.
13. Total tension in lbs. at lower end of cable.
14. Vertical tension at lower end of pendulum line in lbs.
15. Gross lift of balloon minus weight of fabric, harness, winch, fuel and amount of cable on drum of winch.
16. Length of each guyline in feet.
17. Length of pendulum line in feet.
18. Catenary X and Y coordinates in feet of the upper and lower end of each cable. These are used in calculation of stretched line lengths.
19. Residual value resulting from the following equation

$$\Delta = s^2 - h^2 - (2m \sinh (d/2m))^2$$

where Δ is the residual value. This indicates the accuracy of the solution for m in the above equation.