Influence of Low-Level Environment on Severity of High-Plains Moist Convection

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ABSTRACT

Early afternoon environmental conditions preceding hail-producing thunderstorms are statistically compared with conditions for classes of less severe moist convection using only data from individual radiosonde releases collected during the National Hail Research Experiment in northeast Colorado. The ensuing analyses emphasize the thermodynamic characteristics of the mixed layer and immediate overlying free flow.

On days with hail-producing thunderstorms, the mixed layer tends to be particularly thin and moist. Energy required to initiate moist convection is found to be somewhat greater than normal, while energy required to further develop moist convection is substantially less than normal. Parcel energies are found to be quite sensitive to the level of parcel origin in the mixed layer.

1. Introduction

The present study seeks to relate the severity of moist convection to the preexisting structure of the low-level environment. Toward this goal, 1400 L (all times Mountain Daylight) radiosonde releases taken at Sterling, Colo., 1972–74, by the National Hail Research Experiment (NHRE) are analyzed (Fig. 1).

Considerable attention is devoted to the state of the mixed layer and differences between energy required to initiate moist convection and energy required for further development of convection once condensation is initiated. This natural partitioning of the lower troposphere is expected to allow more efficient identification of specific physical influences. For example, substantial observational evidence suggests that prior to the initiation of particularly severe moist convection, the tropospheric stratification (increase of potential temperature with height) is relatively weak except in the lower troposphere where the stratification is concentrated in an inversion capping the mixed layer (e.g., Humphreys, 1926; Fawbush and Miller, 1953; and Palmén and Newton, 1969). The role of the enhanced low-level inversion is to inhibit moist convection, allowing moisture in the low levels to build up (Fulks, 1951; Carlson and Ludlam, 1968). As a possible result, long-lived "super cells" over the high plains of the United States appear to prefer an environment where weaker moist convection is suppressed (Browning and Foote, 1976). Once initiated by a triggering or destabilizing mechanism, moist convection under such conditions may take advantage of any destabilization aloft and at the same time have exclusive rights to the low-level moisture without competition from widespread cumulus development.

Principally due to the low-level inversion, the updraft is generally observed to be negatively buoyant at cloud base (e.g., Marwitz 1972a; Foote and Fankauser, 1973) and is thought to be driven at these levels by a storm induced, nonhydrostatic vertical gradient of pressure. The negative buoyancy of the updraft at low levels might control the updraft speed of the storm, allowing it to be properly matched to the moist inflow, and enabling the updraft to be persistent. Moncrieff and Green (1972) formulated this matching requirement in terms of the convective Richardson number.

Motivated by the above observations and suggestions, the present study seeks to examine differences in the low-level structure between environments leading to hail-producing thunderstorms and environments leading to weaker moist convection. It is not our goal to construct forecasting tools. In fact, we do not explicitly estimate the dynamic forcing presumably required to initiate moist convection in the presence of the low-level inversion.

In Section 2, we reconsider the representation of the mixed layer. Statistical procedures are discussed in Section 3 and results in Sections 4–6.

2. Mixed-layer representation

Since the principal goal of this study is to identify physical differences in the low-level environment accompanying different classes of moist convection severity, we partition the lower troposphere in as natural a manner as possible. Instead of mixing a layer of fixed thickness and lifting a corresponding representative parcel to a predetermined level such as the 500 mb surface, we divide the atmosphere into the

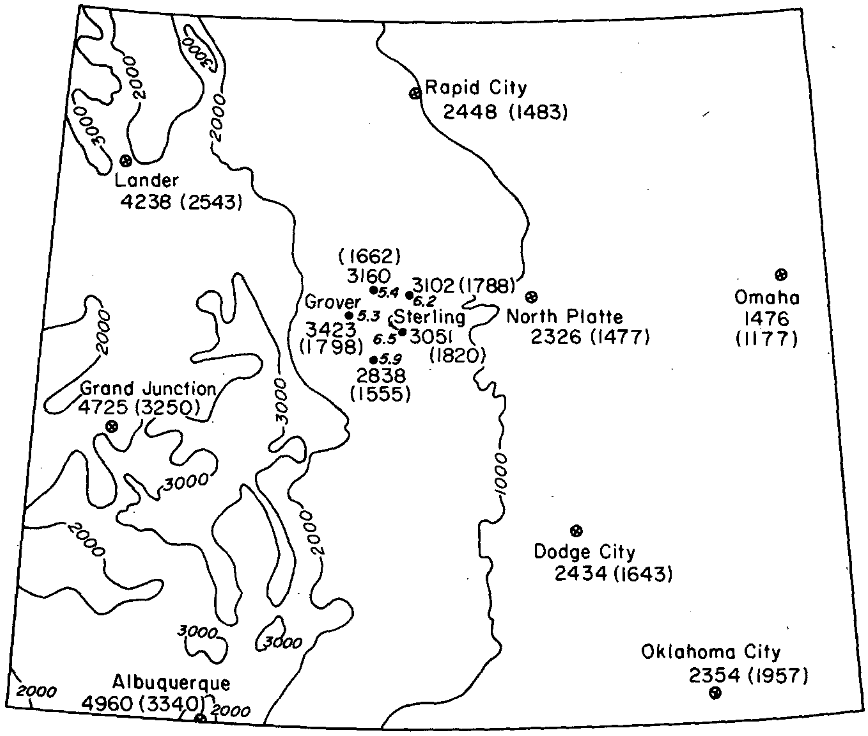

FIG. 1. Mean elevation of mixed layer top above sea level (m) and above ground level (parentheses). Values for five NHRE radiosonde stations (dots) were calculated at 1400 L, late May to early August 1973. For this discussion only, mixed layer depths are computed objectively as the depth where the potential temperature over a two-contact point interval first exceeds the potential temperature measured at the first radiosonde contact point (~10 mb above the surface) by 0.5°C. The actual days used in the calculation vary from station to station which may in part explain the low value at Ft. Morgan. Values at stations denoted by ⊗ are estimates of June-August average diurnal maximum of the mixed layer depth computed by Holzworth (1964). Also indicated at the five NHRE stations is the average 1400 L mixing ratio in the lowest 100 mb for the above data period. Contours indicate surface station elevation (m).

mixed layer, the layer between the top of the mixed layer and the lifted condensation level (LCL) and a layer of fixed thickness above the LCL.

We presently devote considerable attention to the selection of variables representing the state of the mixed layer, since the statistical inferences developed in Section 3 are limited by linearity and by themselves do not indicate cause-and-effect relationships. In particular, the determination of a representative parcel is complicated by vertical gradients of specific humidity which, under certain circumstances, survive vigorous turbulent mixing. In the lowest 50 m or so (surface layer), both specific humidity and potential temperature decrease with elevation above ground. Use of surface values of temperature and moisture normally underestimates the condensation level associated with mixed-layer initiation of clouds (Petterssen *et al.*, 1945; Malkus, 1958). Above the surface layer, gradients of potential temperature and specific humidity are generally small, although slight gradients in specific humidity can significantly complicate estimation of the condensation level (Betts, 1976). Over the high plains of the United States, the vertical decrease of specific humidity is often significant throughout the mixed layer (Danielsen *et al.*, 1972; Mahrt, 1976). The specific humidity often decreases particularly rapidly with height just below the top of the mixed layer (Melgarejo and Deardorff, 1974; Schaefer, 1976; Burk, 1977). The level of moisture origin appropriate to the initiation and development of moist convection depends on the type of moist convection. Convective clouds initiated by penetrating thermals probably depend on some yet-to-be-determined, weighted vertical average of potential temperature and specific humidity. That is, such plumes or thermals tend to organize 100 m or so above the surface, incorporating smaller scale surface layer plumes, but then suffering significant entrainment as they rise through the remainder of the turbulent mixed layer. The estimation of the entrainment rate is quite complex since self-similarity parameterization of the sub-thermal-scale turbulence is expected to be invalid (e.g., Telford, 1970). Entrainment into the thermal might be particularly complex near the mixed layer top if shearing instability is present. After development of severe well-organized moist convection, updrafts seem to originate close to the surface and appear to be sufficiently large and intense to minimize the influence of entrainment even in the subcloud layer (Marwitz, 1973; Browning and Foote, 1976; Fankhauser, 1976). For example, cloud base properties of such storms are similar to near-surface values of moisture, although inferences are complicated by horizontal inhomogeneity and updraft tilt. Fankhauser found the updraft to draw in air in the lowest half kilometer over an extensive horizontal region, perhaps 10 km wide.

The significant vertical structure of the mixed layer at Sterling, Colorado is indicated by 1400 L composite profiles of specific humidity (Fig. 2), potential temperature, relative humidity and energy (Fig. 3) required to lift a parcel initially in equilibrium with its environment to its condensation level (hereafter referred to as lifted energy). Composite profiles are shown for days on which hail was observed at two or more locations within 75 km of Sterling between 1400 and 2000 L and no hail reported between 0800 and 1400 L. These days are hereafter referred to as "hail days." The classes of weaker moist convection are defined in Section 3. The mixed layer depth on individual days is estimated to the nearest radiosonde contact level primarily from potential temperature profiles, although in ambiguous cases profiles of specific humidity and relative humidity as well as several objective schemes are also used. Approximately 5% of the days which contained no definable mixed layer had to be discarded. Such a procedure, in part, filters out substantial distortion of low-level flow due to downdrafts from moist convection prior to the radiosonde release. Several soundings containing elevated superadiabatic temperature drops in excess of 1 K or incomplete winds were also excluded. There remain 38 hail days which represent about 15% of the days from 15 May to early August 1972–74 and roughly half of the days on which hail was reported by Modahl (1975) over a somewhat larger geographical region with no restriction on time of day.

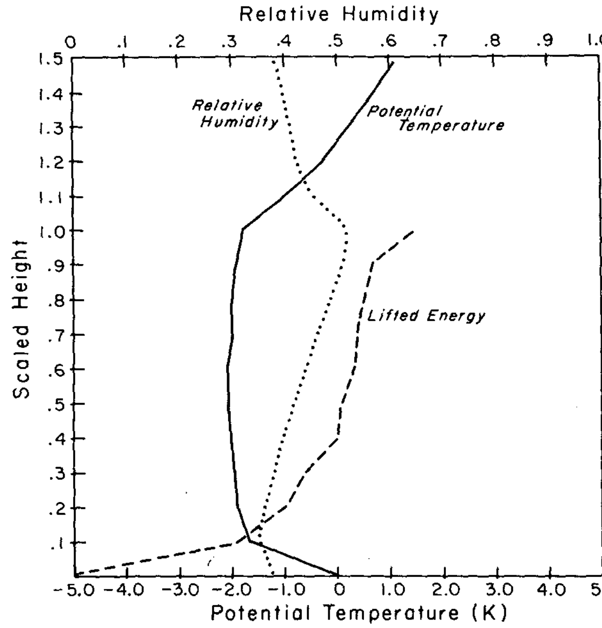

FIG. 3. Composite profiles of the potential temperature excess over the surface value (solid line), relative humidity (dotted line) and energy (dashed line) required to lift a parcel to its condensation level scaled with the mixed-layer average of this energy. The average and standard deviation of surface potential temperature is 313.1 and 3.2 K, respectively, on hail days and 310.5 and 5 K on cumulus congestus days. See Fig. 2 for further explanation.

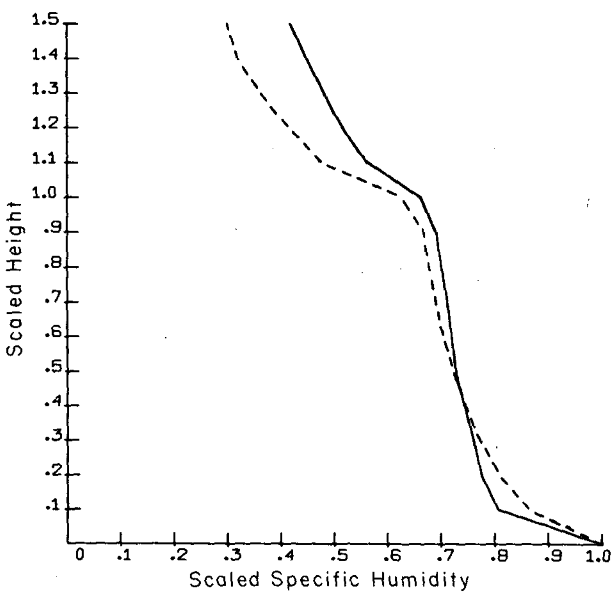

FIG. 2. Composite 1400 L profile of specific humidity for hail (solid) and cumulus congestus days (dashed). Profiles are constructed by scaling specific humidity with the surface value and elevation above ground with mixed layer depth for each day and then averaging. Vertical gradients of moisture cannot be compared since the average mixed layer depth is greater on cumulus congestus days (2064 m) than on hail days (1360 m). The average and standard deviation of surface specific humidity is 10.2 and 2.1 g kg^{-1}, respectively, for hail days and 7.4 and 2.7 g kg^{-1} for cumulus congestus days.

Fig. 3 indicates that the modest decrease of moisture with elevation above ground in the mixed layer causes the required lifted energy to increase substantially with increasing elevation of parcel origin. That is, parcels originating higher in the mixed layer, compared to parcels of lower origin, have essentially the same potential temperature but less moisture and, therefore, a higher condensation level. This gradient of energy offers a distinct advantage to organized moist convection with nearly undiluted updrafts originating from near the surface. Browning (1977) suggests that such a vertical decrease of moisture with height in the mixed layer over the high plains may be responsible for the

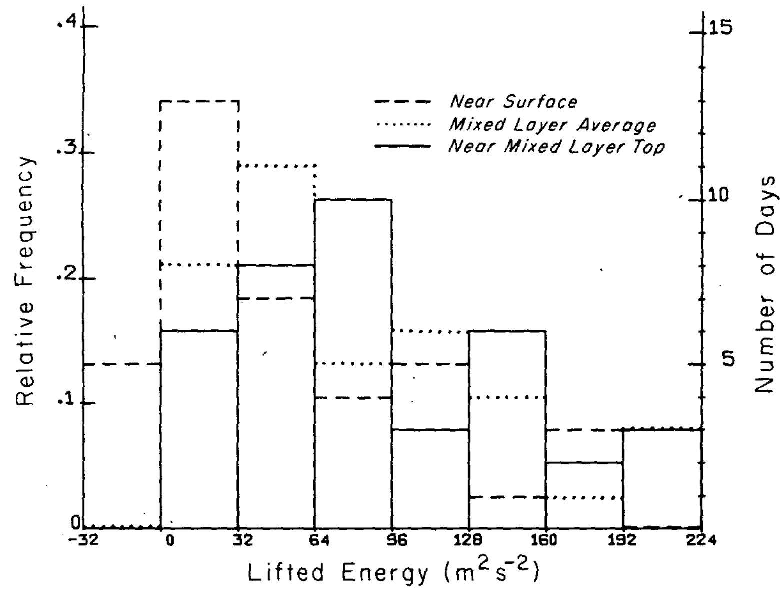

FIG. 4. Frequency distributions for energy required to lift a parcel to its condensation level for parcels with environmental properties (a) at one contact level above the surface (dashed line), (b) for the vertically averaged mixed layer (dotted line) and (c) at one contact level below the mixed layer top (solid line).

relatively high frequency of long-lived supercells in this region. The vertical gradient of lifted parcel energy is also evident in Fig. 4 which shows hail-day frequency distributions of lifted energies for parcels in equilibrium with the environment at the first radiosonde contact level (~100 m above the ground), one radiosonde contact level below the mixed-layer top, and the mixed-layer average environment (not including surface values).[1]

As a result of the complex initiation stage, vertical structure of moisture and limited ability of radiosondes to interrogate the mixed layer, the choice of a representative parcel to estimate parcel energies in the initiation stage is not obvious. Here we simply choose a parcel in equilibrium with the vertically averaged environment of the mixed layer, hereafter referred to as "mixed-layer averaged parcel." We interpret small lifted energy to indicate likelihood of widespread initiation of cumulus while large required lifted energies are interpreted here as indication of significant forcing or triggering required to start cloud development. We will therefore define required parcel energies to be positive when the parcel requires an external source of energy to overcome negative buoyancy.

[1] Measured surface values are not used since they are thought to be, in part, unrepresentative of synoptic-scale surface conditions due to a concentration of irrigation in the South Platte River Valley in which the radiosonde station is located. Analyses not reported here indicate that with cross valley winds of at least a few meters per second, the internal boundary layer induced by such surface inhomogeneity is not expected to significantly influence the first pressure contact level above ground although such estimates are quite tenuous.

Based on the above considerations, we choose the following parameters to represent the state of the mixed layer: mixed-layer depth, specific humidity and potential temperature at the first radiosonde level above ground (intended to be representative of storm updraft air), energy required to lift a mixed-layer-averaged parcel to its condensation level (intended to represent the cloud initiation stage), the gradient of such energy computed using the first radiosonde contact level above ground and one contact level below the mixed-layer top and, finally, the vertically averaged horizontal wind components in the lowest kilometer. Winds are averaged over the lowest kilometer and not the mixed-layer depth since momentum tends to increase with height so that winds averaged over the mixed layer depth would be correlated with the substantial day-to-day variations in mixed-layer depth. Low-level winds are important through advections and are probably highly correlated with synoptic-scale convergence and upslope vertical motion.

To represent the likelihood of extensive vertical cloud development after initiation, we also compute the energy required to lift the mixed layer averaged parcel from its LCL to 300 mb above this level, hereafter referred to as "required above LCL energy." We also compute the tropospheric shear as estimated from averaged wind components in the lowest kilometer and averaged wind components in the layer from 4 to 5 km above ground level. This shear is intended to be a crude indicator of surface inflow potential relative to the storm motion. The motivation for use of these latter variables is discussed further in Section 4.

3. Statistical and representativeness considerations

To exemplify the character of the low-level environment preceding the occurrence of hail-producing thunderstorms, we statistically compare 1400 L conditions on 38 hail days (defined in Section 2) with 1400 L conditions on 1) the 20 days during the observational periods on which cumulus congestus is the most developed form of convection, 2) the 33 days on which precipitating convection develops (but no thunderstorms), and 3) the 30 days with only nonhail-producing thunderstorms (Modahl, 1975). The 1400 L Sterling sounding is not always the optimum proximity sounding since radiosondes were often released closer in time and space to the occurrence of hail-producing thunderstorms. This variation in degree of proximity is a serious problem since environmental conditions are thought to vary rapidly near the time and location of initiation of severe moist convection (Beebe, 1958; Ludlam, 1963). On the other hand, the use of a more representative sounding would be complicated by diurnal trends as well as the particularly strong horizontal variation of climatic conditions in the NHRE region, as indicated for example, by the analyses of Dodd (1965) and Fig. 1. Furthermore, the time and location of initiation of hailstorms are often quite ambiguous. Sometimes the most representative sounding for weaker classes of moist convection cannot be determined at all from available data. Even though the existence of clouds complicates the radiosonde sampling problem, days on which the weaker classes of convection occurred prior to sounding release are not discarded to avoid severe reduction of sample size.

The comparison between hail days and cumulus congestus days is given the most attention in this study since such a comparison is expected to produce the most definable physical differences without challenging the observational system and sample size difficulties. Of particular concern with the two intermediate classes of convection is that precipitating convection can dramatically modify the boundary layer and considerably less information was available on the time and location of nonhail-producing convection to estimate precipitation-related modification of the mixed layer. At the same time, significant differences between hail and cumulus congestus days are not completely anticipated *a priori* and differences between different classes of weaker convection seem to be considerably less than differences between weaker classes and hailstorms for many of the parameters considered (Fig. 5).

Caution must be emphasized in interpreting the statistics in this study because of a number of difficulties:

1) Enough cases are not available to test conclusions on an independent data set.

2) Although any given class of convective days represents less than 25% of the total days, independence between days is not guaranteed. Most statistics computed in this study were also computed after filtering the approximate seasonal trend from the data (Section 5).

3) While the statistics to be used are not considered to be particularly sensitive to small deviations from normality, such complications limit the quantitative nature of the interpretation of results. Several variables characterized by significant skewness and kurtosis (Table 1) are transformed according to

$$f_t = \ln(f+c),$$

where f and f_t are, respectively, the original and trans-

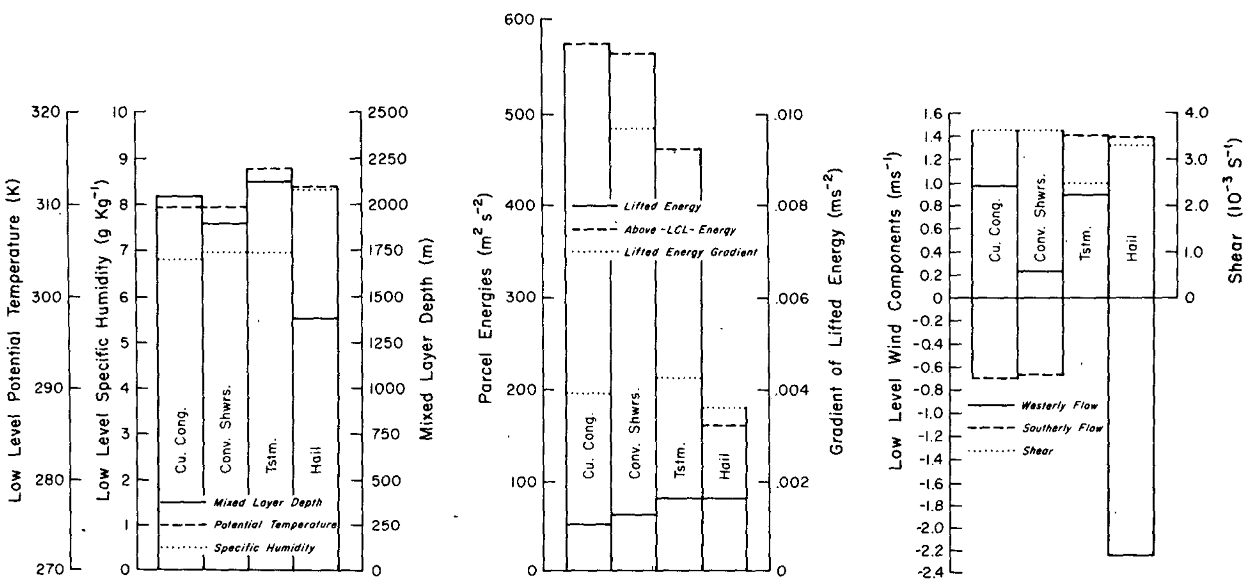

FIG. 5. Averages of variables for the four classes of moist convection based on 20 cumulus congestus days, 33 rain-shower days, 30 nonhail thunderstorm days and 38 hail days.

TABLE 1. Statistics for comparison between cumulus congestus and hail days. t-test values are unpaired, two-tailed, using pooled variance. Significance levels of 20, 10, 5, 2, 1 and 0.1% are shown.

	Specific humidity q (g kg^{-1})	Potential temperature θ (K)	Mixed layer depth h (m)	Lifted energy E (m^{-2} s^{-2})	Lifted energy gradient ΔE (m s^{-2})	Required above-LCL-energy EC (m^2 s^{-2})	Westerly flow u (m s^{-1})	Southerly flow v (m s^{-1})	Troposphere shear $\lvert \mathbf{V}_z \rvert$ (s^{-1})
Mean (hail)	8.34	311.7	1316	79	0.04	162	−2.19	1.39	3.4×10^{-3}
Mean (cumulus congestus)	6.75	309.6	2034	49	0.04	568	0.98	−0.71	3.6×10^{-3}
Mean (total)	7.78	311.0	1564	69	0.04	302	−1.10	0.67	3.4×10^{-3}
Standard deviation	2.28	4.44	697.87	57.15	0.05	399.72	2.95	4.89	1.4×10^{-3}
Skewness	0.01	−0.42	0.69	0.84	0.23	0.47	0.38	0.46	0.46
Kurtosis	2.53	3.06	2.88	2.71	9.92	3.47	3.08	4.52	2.45
Two-tailed t test-	2.65	1.76	−4.25	1.98	−0.07	−4.17	−4.50	1.57	−0.63
Significance level	2%	10%	0.1%	10%		0.1%	0.1%	20%	
Discriminant function coefficient ($\times10^{-2}$)	8.6	−6.4	12.0	0.4	1.5	8.3	6.2	−2.3	0.4
Transformed variable			$\ln(h+500)$	$\ln(E+5)$	$\pm(\lvert\Delta E\rvert)^{\frac{1}{2}}$		$\ln(u+20)$	$\ln(v+45)$	
Two-tailed t-test			−4.14	2.53	−0.55		−4.37	1.66	
Significance level			0.1%	2%			1%	20%	
Discriminant function coefficient ($\times10^{-2}$)	8.4	−5.7	10.9	−1.6	1.5	9.6	5.7	−2.4	0.0
t-statistic in total regression model	1.61 20%	1.14	2.03 5%	0.31	0.47	1.91 10%	1.46 20%	0.74	0.02
t-statistic in 3-variable regression model	1.76 10%	1.76 10%	3.78 0.1%	1.72 10%	0.12	3.69 0.1%	1.66 20%	0.40	0.51
Discrimination function coefficient ($\times10^{-2}$), reduced model	7.9		12.5			15.0			

formed variables. The constant c is chosen iteratively by attempting to jointly minimize skewness and deviations of kurtosis from 3. General appearance of the distributions was also used to determine the necessity of transformations. The various statistical inferences, such as the Student t-test, were generally insensitive to the value of c. In fact, only statistics concerning the lifted energy are appreciably affected by the transformations and none of the principal conclusions of this study depend on whether or not transformations are used. The east-west flow component was unnecessarily transformed, which, however, has virtually no effect on the discussions in this study.

4) Various calculations assume that the population variances of the two classes are equal and use the pooled variance to estimate the population variance of the two groups.

5) Interpretation of statistical results is also complicated by the possibility that a given convective class may be comprised of distinctly different physical situations whose parameter means differ more than the means between convective classifications. The classification scheme is inexact.

6) Frequency distributions (Fig. 6) for various parameters are presented as a display of the available data and not as an estimate of the population frequency distribution since the sample sizes are small.

7) Statistical inferences estimate statistical relationships and not cause and effect. Whenever possible, care must be taken to distinguish between direct physical importance and indirect statistical importance due to correlation with other variables or correlations with dynamic forcing not explicitly included in this study.

8) The analyses in this study are primarily linear. The actual physical dependence on computed variables may in some cases be significantly nonlinear. This potential nonlinearity is difficult to isolate *a priori* since a given parameter may have a multiplicity of physical roles in the discrimination.

9) The data are subject to instrumentation errors and representation difficulties due to small scale horizontal inhomogeneity and turbulent fluctuations.

Items 1 and 7–9 are considered to represent the most serious difficulties of this study. Problems 1 and 9 can not be significantly alleviated while further attention is given to 7 and 8 in subsequent sections.

The statistical tools used here are the Student t-statistic for comparing class means, linear regression and correlation, and multivariate discriminant function analysis (Miller, 1962). Since the intended use and variable elimination procedures of the discriminant function analysis employed here are somewhat different than in the analysis of Miller, we briefly discuss the application of our methodology. The discriminant function is the linear combination of original variables which differs the most between classes; i.e., it is that linear combination which, except for a scaling factor, uniquely has the maximum Student t-statistic. Due to correlations between variables, the importance of the contribution of individual variables to the discriminant function may differ from expected importance based on individual Student t-statistics alone. That is, the multivariate technique estimates the statistical importance of variables in a simultaneous collective discrimination. Here we are more interested in the physical significance

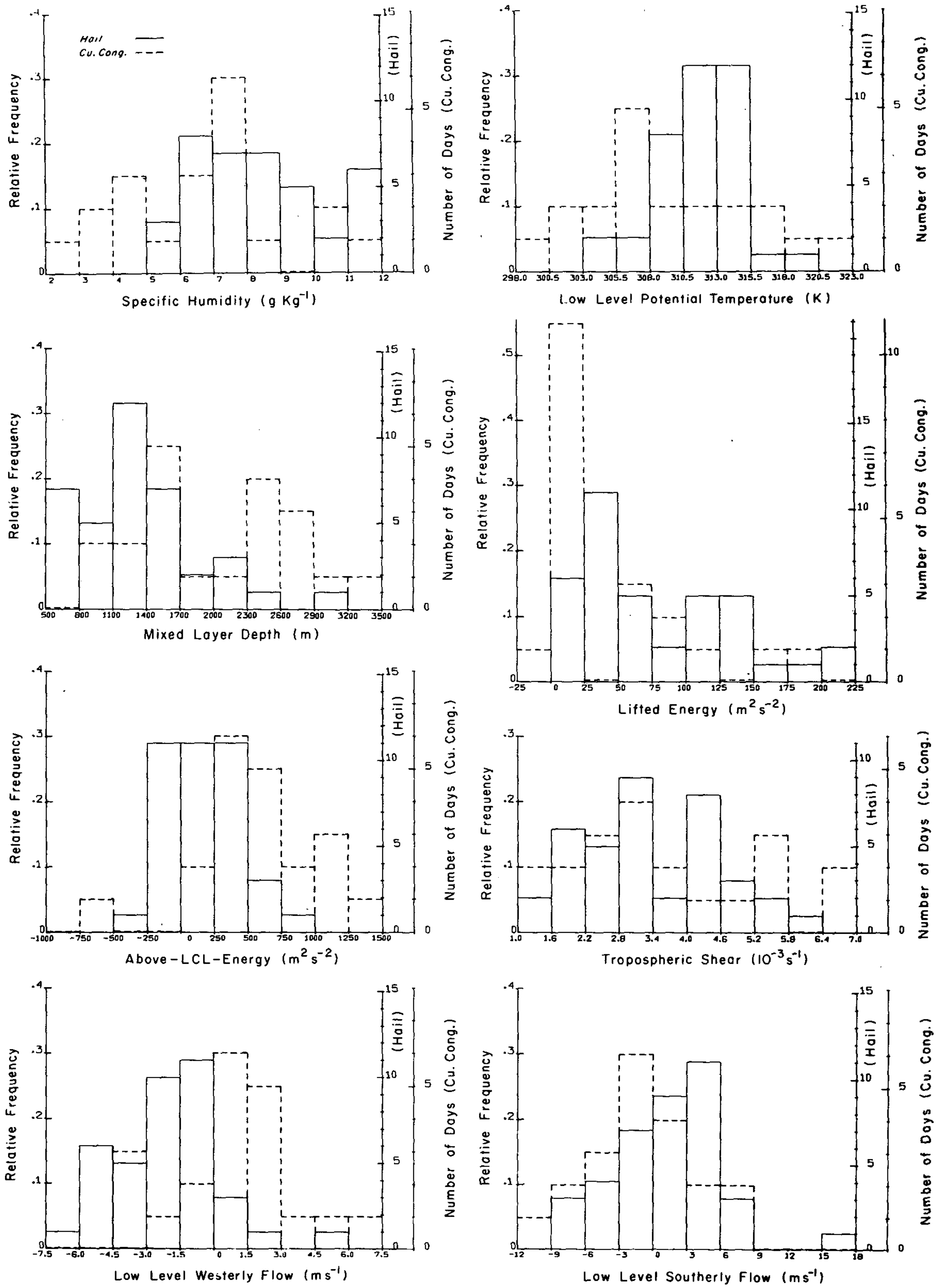

FIG. 6. Frequency distributions of 1400 L variables for hail days (solid lines) and cumulus congestus days (dashed lines).

of the relative importance of variables than the predictive power of the discrimination.

The discriminant function analysis can be applied simultaneously to all four classes of moist convection in which case more than one discriminant function might be required. However, for reasons discussed earlier in this section, we choose to discriminate only between cumulus congestus and hail days. The resulting simpler discriminant function is also more easily analyzed and presented from a physical point of view.

We represent the observational vector on the tth day of a given class as $\mathbf{x}_t$ for cumulus congestus days and $\mathbf{y}_t$ for hail days. These column vectors have nine elements representing the value of the nine variables (Table 1).

We scale the transformed variables by their standard deviations (before dividing the data into groups) so that the resulting discriminating vector more accurately indicates the relative importance of individual variables. We compute the pooled variance-covariance matrix

$$S_p=\frac{(n_1-1)S_1+(n_2-1)S_2}{n_1+n_2-2},$$

where S_1 and S_2 are covariance matrices for hail and cumulus congestus days, respectively, and are computed as follows:

$$S=\{s_{ij}\},$$

$$s_{ij}=\frac{1}{n-1}\sum_{t=1}^{n}\{(z_{it}-\bar{z}_i)(z_{jt}-\bar{z}_j)\},$$

$$\bar{z}_i=\frac{1}{n}\sum_{t=1}^{n}z_{it},\text{ etc.,}$$

where $\mathbf{z}$ is an observational vector, either $\mathbf{x}_t$ or $\mathbf{y}_t$, the subscript i or j identifies variables in the observational vector, and $n_1=38$ is the number of hail days and $n_2=20$ is the number of cumulus congestus days. We further define $\mathbf{d}$ as the vector difference of the means of the two groups which is a column vector of the form

$$\mathbf{d}=\bar{y}_i-\bar{x}_i.$$

The first discriminant function vector $\mathbf{a}$, which maximizes the discrimination between the two groups, is

$$\mathbf{a}\equiv S_p^{-1}\mathbf{d}$$

which is a solution of the eigenvalue problem (e.g., Timm, 1975)

$$(\mathbf{dd}'-\lambda S_p)\mathbf{a}=0. \quad (1)$$

The coefficients of the scaled discriminant function for the comparison between hail and cumulus congestus days are listed in Table 1. This discriminant function possesses a Hotelling's T^2 value of 49 (31 required for significance at the 1% level).

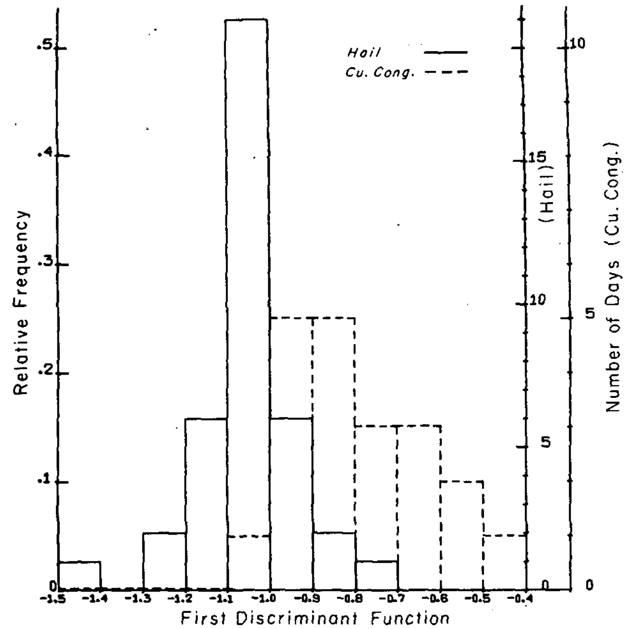

FIG. 7. Frequency distribution of the magnitude of the first discriminant function for hail and cumulus congestus days.

Fig. 7 shows that the frequency distribution for the first discriminant function indicates much more separation between hail and cumulus congestus days than is indicated by the frequency distributions of any individual variable (Fig. 6). Likewise, the Student t-test value for the discriminant function (square root of the Hotelling's T^2 test) is approximately 7, which is more than 60% greater than the maximum statistic for the individual variables. t-values are given here for qualitative perspective but cannot be interpreted in the same manner as ordinary t-statistics since the discriminant function is defined to maximize this statistic. In other words, due to some independence between variables, one can do a much better job of discriminating between hail and cumulus congestus days by evaluating the numerical value of the above first discriminant function compared to looking at any one variable (or arbitrary linear combination of variables). On the other hand, there is sufficient correlation between variables that some of the apparently important parameters (those with large coefficients in the scaled discriminant function), as well as the unimportant parameters, can be eliminated without seriously compromising the discrimination power. In fact, this elimination process is physically more illuminating than studying the behavior of the full nine-variable discriminant function.

To examine the relative importance of the variables in the total discrimination, we will follow a variable elimination procedure analogous to that given by Rao (1973, p. 551) and, in effect, similar to that suggested by Miller (1962, Chap. 2). The basic methodology is that of testing whether the discrimination power is significantly reduced by eliminating a given variable.

There are a number of mathematically equivalent ways of computing the basic test statistic for this problem. For the case of only two groups, here hail days and cumulus congestus days, it is possible to carry out the calculations by using ordinary multiple-regression methods. Let z be an index variable taking value $z=0$ for hail days and $z=1$ for cumulus congestus days, and consider regressing z on the p different predictor variables. The regression equation obtained is, up to a scale factor, the discriminant function (Snedecor and Cochran, 1967). Moreover, the usual partial F-test for assessing the joint significance of a subset of $p-s$ predictor variables is equivalent to Rao's test of whether the full discriminant function is significantly better than that based on just the remaining s variables (Pierce, 1977). The fact that z is not normally distributed is here irrelevant since the regression technique is mathematically equivalent to Rao's discriminant function method.

When $p-s=1$, the ordinary partial t-statistic of the potentially eliminated variable in the regression is the square root of the F-statistic. The variable with the smallest insignificant partial t-value in the regression model can be discarded without significantly influencing the discrimination. This process is applied sequentially to determine a simpler discriminant function.

Using this elimination algorithm, six of the nine variables can be eliminated with the explained variance decreasing from 46 to 40%. The t-statistics of the three remaining variables are listed in Table 1 as well as the t-statistic that each of the eliminated variables would have if restored to the model. The departing F value for the six discarded variables is only 1.09 (not even significant at the 50% level). Further elimination of variables greatly accelerates the rate of loss of information. We therefore choose to study the three-variable discriminant function (bottom line, Table 1). The Hotelling's T^2 value for the three-variable model is reduced to 37 compared to 49 for the full nine-variable model. However, the significance of the T^2 value is difficult to estimate since using the now reduced degrees of freedom does not take into account the fact that the reduced model has been formed by a statistical selection procedure (Miller, 1962; Pierce, 1977).

As an additional test of the significance of the three-variable discriminant function (in lieu of independent data), each observation is classified as a cumulus congestus or hail day based on the value of the three-variable discriminant function using the method given by Morrison (1976, Section 6.2). With this technique, the "threshold value" of the discriminant function is taken to be the mid-point between the respective means of the discriminant function for the two groups. The success of these classifications is summarized in Table 2. The misclassification probabilities are also estimated from the more reliable U method (jackknifing) of Lachenbruch and Mickey (see Morrison, 1976, Section 6.4). The U method consists of deleting an observation from the data set, computing a discriminant function from the remaining days, and then using this discriminant function to classify the deleted day. Repeating this procedure for each observation, we obtained (perhaps fortuitously) exactly the same misclassification rate for the three-variable model as with the preceding technique. Of more importance here is that the three-variable models perform nearly as well as the nine-variable model. Although the U classification scheme is not considered to be a sensitive variable elimination method by itself, the results in Table 2 lend comforting support to the variable eliminations suggested earlier in this section.

Table 2. Classification matrix for three-variable model. Parentheses contain results for the nine-variable model with and without the U-method, respectively. For example, the three-variable model predicted 18 of the days to be cumulus congestus days, of which 13 were actually cumulus congestus days, while five were unpredicted hail days.

Actual	Predicted: Cu cong.	Predicted: Hail
Cu cong.	13 (14, 12)	7 (6, 8)
Hail	5 (3, 5)	33 (35, 33)

Encouraged by the above tests, we will physically interpret (Section 5) only the behavior of the simpler three-variable discriminant function. But first we study between-class differences for each variable individually.

4. Univariate comparisons

The univariate comparisons displayed in Figs. 5 and 6 and Table 1 indicate that the mixed layer tends to be thinner (significant at 0.1% level) and more moist (significant at 2% level) on hail days compared to cumulus congestus days. Only two of the 38 hail days occurred with a mixed layer depth greater than 2300 m (Fig. 6) while almost half (9 of 20) of the cumulus congestus days were associated with mixed layer depths greater than this value. Similarly, none of the hail days occurred with a 100 m specific humidity of less than 5 g kg^{-1} (a surface value of $\sim$6–6½ g kg^{-1}) while a little more than one-fourth of the cumulus congestus days occurred with conditions at least this dry (Fig. 6). However, the significance of such cutoff values cannot be reliably tested since the cutoffs are chosen *a posteriori*.

Cumulus congestus days also occur over a wider range of low-level temperatures than observed on hail days, although the difference of mean temperatures between the two classes is slight. Analyses not reported here indicate that cumulus congestus days occur under distinct and quite different types of environmental situations.

Fig. 5 also indicates that differences of mean moisture and mixed-layer depth between hail and nonhail-

TABLE 3. Linear correlation coefficients between variables for all days (upper right half), hail (lower left half) and cumulus congestus days (lower left half in parentheses). A correlation of 0.3 is required for significance at the 1% level.

	q	θ	h	E	ΔE	EC	u	v	$(\lvert \mathbf{V}_z \rvert)$
Specific humidity q		0.34	−0.52	−0.03	0.12	−0.71	−0.54	0.02	0.00
Potential temperature θ	−0.05 (0.57)		0.13	0.16	0.10	−0.47	−0.12	−0.11	−0.15
Mixed-layer depth h	−0.39 (−0.49)	0.54 (0.10)		−0.48	−0.15	0.34	0.61	−0.16	−0.09
Lifted energy E	−0.38 (0.29)	−0.07 (0.32)	−0.35 (−0.55)		0.42	0.10	−0.23	0.04	0.01
Lifted energy gradient ΔE	0.02 (0.37)	0.04 (0.25)	−0.06 (−0.46)	0.30 (0.82)		0.08	−0.09	0.01	0.06
Required above-LCL-energy EC	−0.63 (−0.70)	−0.35 (−0.46)	0.06 (0.19)	0.42 (0.09)	0.10 (0.07)		0.51	−0.18	0.11
Westerly flow u	−0.40 (−0.54)	0.23 (−0.20)	0.43 (0.53)	0.05 (−0.41)	−0.01 (−0.35)	0.34 (0.36)		−0.09	0.02
Southerly flow v	0.00 (−0.12)	−0.12 (−0.21)	−0.11 (−0.02)	0.01 (−0.04)	0.06 (−0.09)	−0.02 (−0.16)	0.06 (−0.05)		−0.29
Tropospheric shear $(\lvert \mathbf{V}_z \rvert)$	−0.01 (0.07)	−0.33 (0.01)	−0.20 (−0.09)	0.13 (−0.12)	0.11 (−0.03)	0.06 (0.11)	−0.14 (0.11)	−0.06 (−0.57)	

producing thunderstorms are much greater than differences between the three classes of less severe convection. The same can be said of above-LCL-energy and the low-level east-west flow component. The dramatic decrease of required above-LCL-energy reflects the expected destabilization preceding severe moist convection. On the other hand, the energy required to initiate moist convection increases slightly with increasing severity of moist convection. We can further speculate that this enhanced low-level stability on hail days is associated primarily with the inversion capping the mixed layer and implies the need for a forcing mechanism to initiate moist convection. Thus, partitioning the parcel energy into low-level and higher level contributions is indeed considerably more useful than computing one stability index.

Differences in shear between classes of convection severity is essentially limited to a slight increase in shear between nonhail and hail-producing thunderstorms. Shear is thought to be indicative of the relative low-level inflow into the storm as well as conducive to severe storm development through influences on the cloud-induced pressure field, updraft slope and downdraft strength (e.g., Newton and Newton, 1959; Browning and Ludlam, 1962; Newton, 1966). The average shear on hail days is 3.4×10^{-3} s^{-1}, which can be considered as typical of environments containing supercells (Marwitz, 1972a). However, all of the shear values on hail days (Fig. 6) are less than the 7.5–8×10^{-3} s^{-1} values characteristic of the two severely sheared cases reported by Marwitz (1972b), although shear values presented in this study are more representative of roughly the lower half of the troposphere instead of the cloud layer. In any case, the shear on days with weaker classes of moist convection is, in the mean, not significantly less than that on hail days.

The gradient of lifted energy associated with varying the level of parcel origin is found to be virtually unimportant in discriminating between hail and cumulus congestus days and where convenient, will be omitted from further consideration. The minimal change in moisture gradient and decreased mixed layer depths on hail days imply smaller moisture differences across the mixed layer on hail days. The smaller moisture difference together with thinner mixed layer depth on hail days may be of some significance since updrafts are presumably most vulnerable to entrainment in the mixed layer where the ambient turbulence is greatest.

5. Interrelationships between variables

The variable reduction procedure invoked in Section 3 as well as certain significant correlations between variables (Table 3) indicate a redundancy of information. In this section, we study possible cause-and-effect relationships between variables and the influence of such relationships on the relative importance of individual variables in the total discrimination. For example, the various parcel energies depend crucially on mixed layer properties such as moisture and temperature, which are in turn related to mixed layer depth. All the variables are influenced by the east-west flow component through horizontal advections. We now discuss what we consider to be the most important relationships between variables.

a. Mixed-layer depth and low-level moisture

One of the direct influences of mixed-layer growth is dilution of low-level moisture. Daytime growth of the mixed layer over the high plains in summer is due primarily to entrainment of overlying free flow into the mixed layer. The influences of synoptic-scale vertical motions are normally secondary although not always negligible. Since the entrained free flow is typically much drier than mixed layer air (Fig. 2), entrainment-drying often exceeds effects due to surface evaporation and moisture advection (Mahrt, 1976). Sufficient mixed layer growth may lead to widespread cumulus development which in turn further enhances mass exchange between the mixed layer and drier free flow. This relationship between mixed layer depth and mixed layer moisture seems to be supported by the modest joint linear correlation of −0.52. (All correlations reported in the text, sometimes parenthetically, are for both classes combined, although correlations within each class are also reported in Table 3.)

To facilitate more quantitative examination of the relationship of moisture to other variables, we construct a multiple linear regression model for low-level moisture using only variables thought to be of direct influence. Appropriately transformed variables are used (Table 1). Backward elimination yields the following multiple regression model for the case where all retained variables are significant at the 1% level (with respect to rejection of the null hypothesis that the regression coefficient of an individual variable is zero):

$$q = \underset{(10\%)}{-14.1} \underset{(0.1\%)}{-0.58}\ln(h+500) \underset{(0.1\%)}{+0.44\theta} \left\{ \underset{(10\%)}{\ln(u+20)}, \quad \ln(v+45) \right\}.$$

Significance levels up to the 10% level are shown in parentheses. The brackets contain the discarded variables and the significance levels they would have if added to the model at this point. In addition to mixed-layer depth, moisture depends significantly on temperature, which might be explained through the dependence of saturation specific humidity on temperature. The low-level east-west flow component is eliminated in spite of a correlation of −0.54 with moisture. The influence of such flow on moisture is apparently already statistically represented through the dependence of mixed layer depth on easterly flow (correlation of 0.61). The latter dependence will now be explored in more detail.

b. *Dependence of mixed-layer depth on low-level flow*

Of all the variables, mixed-layer depth is most strongly correlated with the low-level westerly flow component. For an east-west gradient of mixed layer depth equal to 1.5×10^{-3} (Fig. 1) and an easterly flow component of 5 m s^{-1}, terrain-following flow would lead to a local reduction of mixed layer depth of about 150 m over a 6 h period. This effect is of marginal importance compared to diurnal effects which are an order of magnitude greater. However, the daytime growth of the mixed layer could be quite sensitive to changes in stratification associated with temperature advections. For example, increased low-level stratification on hail days (evident in Fig. 8) could effectively contribute to the observed slower growth rate of the mixed layer on hail days. The dependence of mixed layer growth on overlying free flow stratification is due to the fact that per unit growth (per unit entrained mass), more turbulent kinetic energy is converted to potential energy with increased free flow stratification; that is, for a given rate of generation of turbulence energy, the mixed layer grows more rapidly with weaker free flow stratification (e.g., Lilly, 1968 and Tennekes, 1973). Low-level easterly flow (upslope flow) favors a situation of Eulerian adiabatic cooling below the inversion since mean surface potential temperature tends to increase with surface elevation over the Great Plains. Weak temperature advection or warm air advection above the inversion would then act to additionally strengthen the low-level inversion. Estimates of inversion strength using several objective schemes (not reported here) do indeed indicate that the 1400 L low-level inversion is statistically stronger on hail days. Excessive inversion strengths of 7–8 K and particularly thin mixed-layer depths occurred on several hail days with significant easterly flow. The average 1400 L inversion strength for all types of days is only $\frac{1}{2}$–1 K, the exact value depending on computation technique.

A significant dependence of cloud cover (and thus heating and mixed layer growth) on the easterly flow component is not obvious from analyses of available but somewhat inadequate reports of hourly cloud cover at Sterling.

c. *Parcel energy*

The above-LCL-contribution to required parcel energy (parcel stability) is most highly correlated with low-level moisture (−0.71) reflecting the known important influence of latent heating on cloud buoyancy and its sensitivity to low level moisture in summer over the high plains. The importance of this dependence is also strongly supported by a backward step regression model for above-LCL-energy in which low-level moisture is by far the most important variable. A possible relationship between above-LCL-energy and east-west flow (−0.54) is expected from the dependence of tropospheric stratification on advections and synoptic situation.

d. *Independent importance of variables*

In the above subsections we have emphasized the relationships between variables. For example the low-level easterly flow component is highly correlated (>0.5) with each of the three variables which survive the elimination procedure invoked in Section 3, *viz.*, mixed-layer depth, low-level moisture and above-LCL-energy. In fact replacing any one of these surviving variables with the easterly flow component reduces the discrimination power much less than out-right elimination of one of them. On the other hand, the easterly flow component can be eliminated with little loss in discrimination power in spite of the fact that it has the highest individual t-statistic. That is, most of the discrimination ability of the easterly wind component involves relationships with the other three variables.

In contrast, each of the three remaining variables must retain a contribution to the total discrimination which is unique in terms of linear statistics. We now consider such contributions in more detail. The above-LCL-energy includes the influence of mid-tropospheric stratification which is, of course, independent of mixed-layer depth and low-level moisture. The importance of weakening stratification in the middle and upper troposphere to the development of deep moist convection is well documented. The mean profiles in Fig. 8 indicate weaker stratification on hail days in a layer roughly 2–4 km above the ground. On approximately one-fourth of the hail days, reduction in stratification takes the form of an elevated near-neutrally stratified layer, of a few kilometers thickness, beginning a few hundred meters above the mixed layer inversion. This second unstratified layer is also capped by an inversion and moisture drop, indicating that it could be associated with a deeper mixed layer from the previous day or a mixed layer which developed over much higher elevation to the west and overrode the surface-

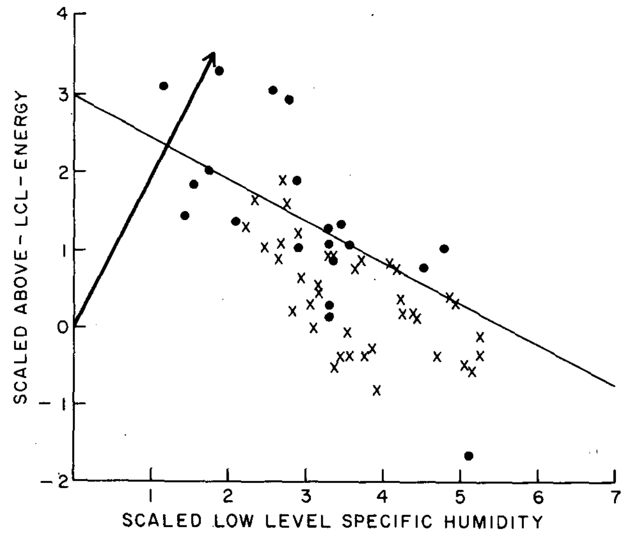

FIG. 9. Projection of data and direction of the three-variable first discriminant function for the subplane with coordinates corresponding to low level specific humidity and required above-LCL-energy for hail (crosses) and cumulus congestus (dots) days. The plotted vector indicates only direction since its position depends on the value of h defining the subplane. For reference, the thin solid line corresponds to a constant value of the discriminant function magnitude of −2.65.

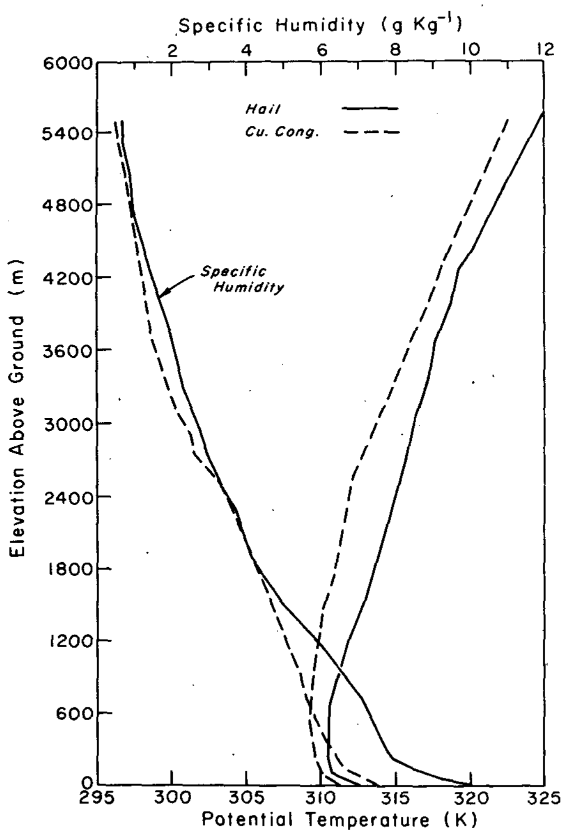

FIG. 8. Averaged 1400 L profiles of specific humidity and potential temperature for hail and cumulus congestus days.

based mixed layer. The existence of an elevated neutrally stratified layer preceding thunderstorm outbreaks has been frequently documented (Fawbush and Miller, 1953; Carlson and Ludlam, 1968; Danielsen, 1975; and others). An elevated unstratified layer also occurred on three of the 20 cumulus congestus days but in all cases was of thickness comparable to or thinner than 1500 m.

The importance of low-level moisture which is statistically independent of mixed-layer depth and above-LCL-energy could involve the influence of potential liquid water content of the updraft on the cloud physics processes governing hail formation (e.g., Danielsen *et al.*, 1972) and may also reflect influences due to liquid water drag. However, as discussed below, the role of moisture involves nonlinear relationships with mixed-layer depth and above-LCL-energy.

The fact that mixed layer depth retains an importance statistically independent of the other two variables likely involves the inverse relationship of cloud base temperature to mixed-layer depth. This inverse relationship can be argued from the observation that the afternoon condensation level is generally close to the mixed-layer top and hail days occur under a narrow range of surface temperatures. The cloud-base temperature is thought to be crucially related to cloud physics processes.

The independent importance of mixed layer depth may additionally reflect the difficulty of generating hail producing thunderstorms when the potential cloud thickness is restricted. That is, the mixed-layer top normally acts as a lower bound to the LCL for individual convective cloud elements. For example, on cumulus congestus days, the mixed-layer depth and therefore the condensation level, average more than 2 km above the surface which is already more than 1 km above sea level. Since the stratification of the tropopause and stratosphere acts to retard (although not terminate) convective cloud development and assuming the tropopause to be not significantly perturbed by surface elevation changes, the potential cloud thickness might be expected to be a few kilometers less over the high plains than 1000 km farther east where surface elevations are lower and average mixed layer depths are substantially less (Fig. 1).

Visualization of the influence of these three variables on the total discrimination is facilitated by plotting the data vectors and direction vector of the three-variable discriminant function on subplanes defined by any two of the variables. The dot product of the data vector with the discriminant function vector ($\mathbf{z}_i \cdot \mathbf{a}$) yields the discriminant function magnitude for that observation. The discriminant function magnitude can therefore be quickly geometrically determined as the projection of the data vector on the discriminant function vector. For example, consider the plane defined by the low level specific humidity and required above-LCL-energy (Fig. 9). From only knowledge of individual t-statistics, one might expect the projection of the discriminant function vector onto this plane to be oriented along a line from the region of low moisture and large required energy (stability) toward the region with large moisture and small or negative required energy; in fact, hail days do tend to cluster in the latter region. However, the discriminant function is actually rotated one quadrant so that hail days, which yield low magnitudes of the discriminant function component, correspond to expected small required energy values but also small moisture values. Two complications contribute to this deviation from expectations based on univariate statistics. First, the two-dimensional component of the discriminant function vector is different from the two-dimensional discriminant function derived by using only two variables. The two-dimensional discriminant function computed from only moisture and required energy is considerably more aligned along the energy axis, indicating that some of the above reversed dependence on moisture is associated with the third variable, mixed-layer depth. Examination of the apparently anomalous cumulus congestus data points which have low required energy and high moisture indicates that such days exhibited mixed layers much deeper than those associated with neighboring hail day points. That is, in three-dimensional space where mixed layer depth is the third dimension, the apparently anomalous cumulus congestus points are quite separated from those hail day points which appear to be nearby in two-dimensional space. Second, some of the reversed moisture dependence is due to the fact that the statistical relationship between moisture and required energy is nonlinear and differs between hail and cumulus congestus days as is evident from higher order regression fits (not shown). For example, even though hail days are in general characterized by greater low-level moisture than cumulus congestus days, the moisture on hail days is less than would be predicted by the small required above-LCL energy and the modeled linear relationship between moisture and required energy. In other words, for a given required energy, hail days are drier than cumulus congestus days, but hail days, on the average, have much lower required energy than cumulus congestus days and on the whole have greater moisture.

These complications not only reveal the loss of information precipitated by eliminating any one of the three remaining important variables but also remind us of the limitation of linear analysis.

e. Seasonal adjustment

The possible seasonal dependence of the preceding results is estimated by repeating the multivariate analyses on seasonally adjusted data. The seasonally adjusted data (regardless of class) is the deviation from the least-squares, second-order regression fit with time (number of days after 15 May) using all the data from both classes. The seasonal variation in mixed layer depth and above-LCL-energy in the data period (15 May–10 August) is minimal, while seasonal variations in low-level temperatures and moisture are, of course, appreciable.

The influences of seasonal trend on the discrimination involves the higher relative frequency of hail day occurrence later in the season, and a seasonal variation in the relationship between variables. However, the bulk of the preceding conclusions are not significantly altered by seasonal adjustment. We only list the most noteworthy differences since the seasonal adjustment introduces an interdependence between data points beyond that already existing naturally.

1) The statistical importance of the mixed layer is enhanced substantially by seasonal adjustment.

2) The Hotelling's T^2 statistic of the discriminant function drops from 49 to 34 (remaining just significant at the 1% level) indicating that some of the differences between hailstorm and cumulus congestus environments are associated with seasonal variations.

3) Even though the seasonally adjusted low-level temperature differs only slightly between cumulus congestus and hailstorm environments, the low-level temperature becomes the second most important variable in the total discrimination. That is, hail days are unseasonably warm considering the thin mixed layer depth and normal increase of mixed layer depth with surface temperature. More extensive analyses of the influences of seasonal trends and the relationships between variables are discussed by Mahrt (1977).

6. Conclusions

1) The depth of the mixed layer over the high plains is a consistent discriminator between environments associated with hail-producing thunderstorms and environments leading to only cumulus congestus. Fig. 5 suggests that mixed layer depth is in general a useful discriminator between hailstorms and all classes of weaker moist convection. It is proposed in this study that the depth of the mixed-layer influences severe storm production in several direct ways as well as indirectly integrates several other important influences. As expected from previous studies, low-level moisture and parcel stability are also useful predictors.

2) Discrimination between environments is considerably more reliable if parcel energies are partitioned into contributions above and below the condensation level. The inversion capping the mixed layer is somewhat stronger on days with hailstorms while the stratification at higher levels is significantly decreased.

3) The tropospheric shear does not vary significantly between the classes of moist convection severity considered here. The possible subclassification of hail-producing thunderstorms, as has been considered on a case study basis in the literature, is currently being studied by the author and colleagues.

4) The energy required to lift a parcel to condensation often varies appreciably with elevation of parcel origin above ground, due to frequent significant decreases of moisture with height in the mixed layer. With such conditions, updrafts which originate lower in the mixed layer and entrain or consume less air in the upper part of the mixed layer lead to substantially greater buoyancy and total water content.

5) Although the goal of this investigation is to examine the importance of certain physical effects as opposed to constructing the best discrimination algorithm, it is worth noting that the between-class overlap of the multivariate discriminant function is minimal compared to the extensive between-class overlap of any one individual variable. Although the discriminant function could not be tested on independent data, the Hotelling's T^2 and U significance tests indicate the discriminant function to be statistically reliable.

In addition to subdividing hailstorm environments, the degree of proximity or representativeness of individual soundings warrants further attention. The inability of individual radiosonde releases to interrogate a highly fluctuating mixed layer occupied by plumes and other organized convections, is considered to also be a serious restriction although the problem is somewhat reduced by the statistical nature of this study.

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