

A multi-species approach to management of European hake and blue whiting mixed fishery



Marcos Pérez-Pérez
M. Dolores Garza-Gil
Manuel M. Varela-Lafuente

Department of Applied Economics, University of Vigo

IIFET 2016 Scotland
Aberdeen, 11 - 15 July 2016

INTRODUCTION

European Union fishing fleet usually target to catch species in **mixed fisheries**.

Mixed fisheries show **increased ecological interdependence** between two or more target species. Catches of one species will also impact the natural growth of the others.

The possibilities of fishing these species must be **determined jointly**, using **multi-species models**.

Multi-species approaches in fisheries management are **increasingly recommended by scientists** because of the complex series of interactions between species that occurs in fisheries (Botsford et al., 1997; Mahe et al., 2007).

The modelling of mixed fisheries

The paradigm change in fisheries management

- Code of Conduct for Responsible Fisheries (FAO, 1995).
- Convention on Biological Diversity (Yakarta, 1995).
- Reykjavik Conference on Responsible Fisheries in the Marine Ecosystem (FAO, 2001).
- The ecosystem approach to fisheries. FAO Technical Guidelines for Responsible Fisheries (FAO, 2003).

Objective:

Development of a predator-prey model for two major species caught by the European Union fishing fleet on EU fishing grounds.

Species selection criteria:

- Commercial importance.
- Captured on EU fishing grounds.
- Significant trophic interaction.

➤ Predator: European hake *Merluccius merluccius*

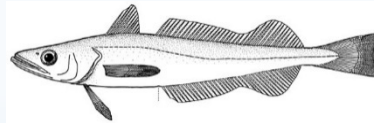
➤ Prey: Blue whiting *Micromesistius poutassou*

Background to the model

- **Lotka-Volterra**: Predator-prey population dynamics.
- **Gause** (1934): Assumes logistic growth.
- **Gordon-Schaefer** (1954): Classic bioeconomic model. Introduces capture rate and yield function.
- **Leslie and Gower** (1960): Logistic equations system to introduce L-V predator-prey dynamics in the G-S bioec. model.
- **Clark** (1985): Modifies predator-prey equations for exploited stocks, including the capture effect.
- **Brown *et al.*** (2005): Apply predator-prey model with capture to the populations of Nile perch and dagaa on Lake Victoria (África).

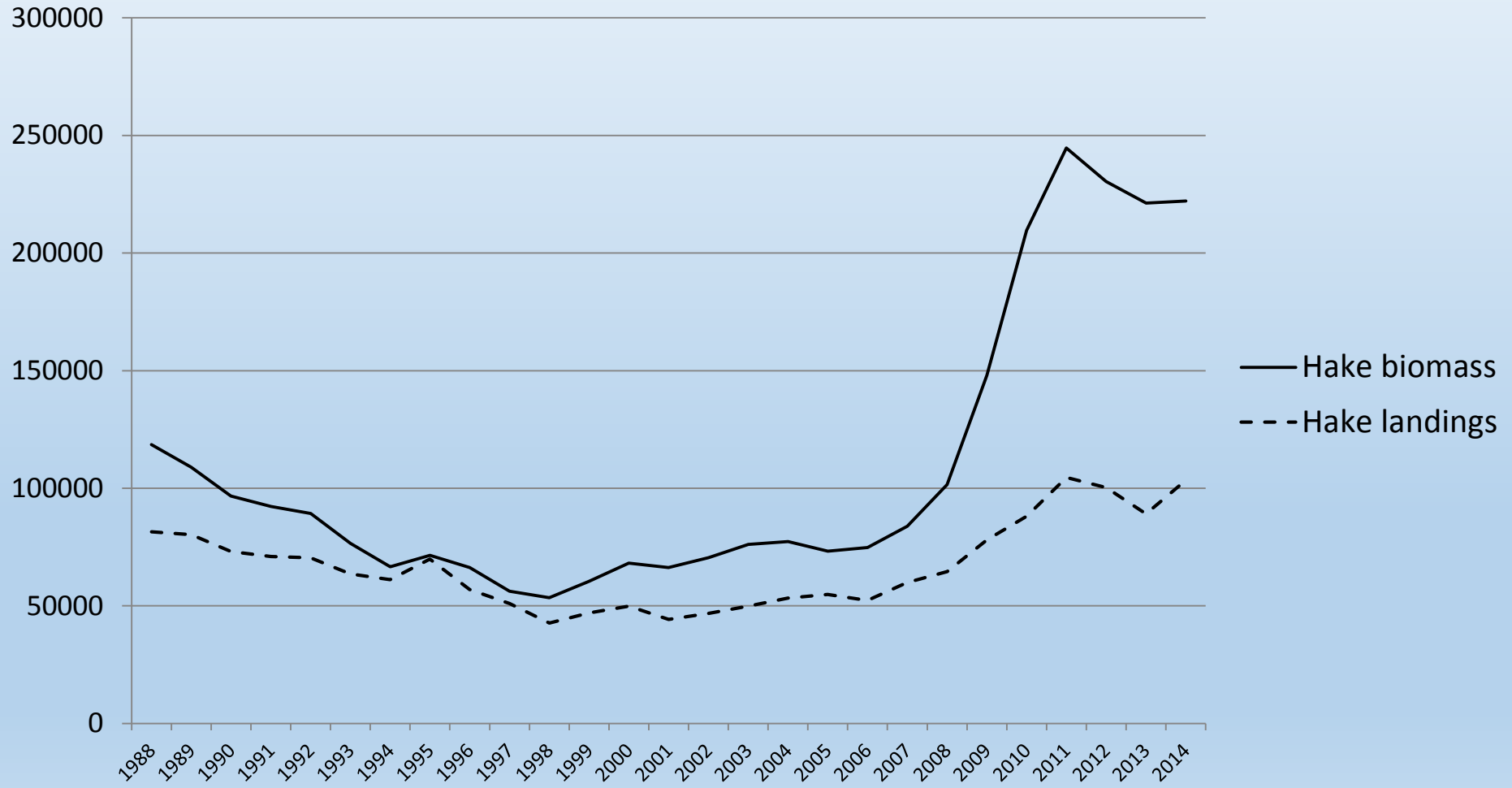
HAKE AND BLUE WHITING FISHERIES

European hake (*Merluccius merluccius*) (Linnaeus, 1758)

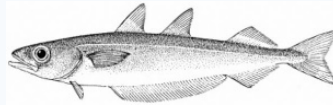


- Demersal benthopelagic species, widely distributed throughout the North-eastern Atlantic Ocean. Predator at the top of the Northeast Atlantic demersal trophic pyramid. Preys: anchovies, sardines, blue whiting, horse mackerel and mackerel.
- Hake is caught in mixed fisheries—along with blue whiting, megrim, monkfish, and Norway lobsters—by a multi-rigged fleet, using the following fishing methods: bottom trawling with doors, bottom pair trawling, bottom-set longline, and small fixed gillnets.
- **Stock status:**
 - **Northern:** SSB increasing, F has decreased significantly over the last decade (still above F_{MSY}), recruitment increasing.
 - **Southern:** SSB increasing (still close to B_{lim}), F well above the F_{MSY} , recruitment high in recent years.

Hake biomass and landings (tons). 1988-2014

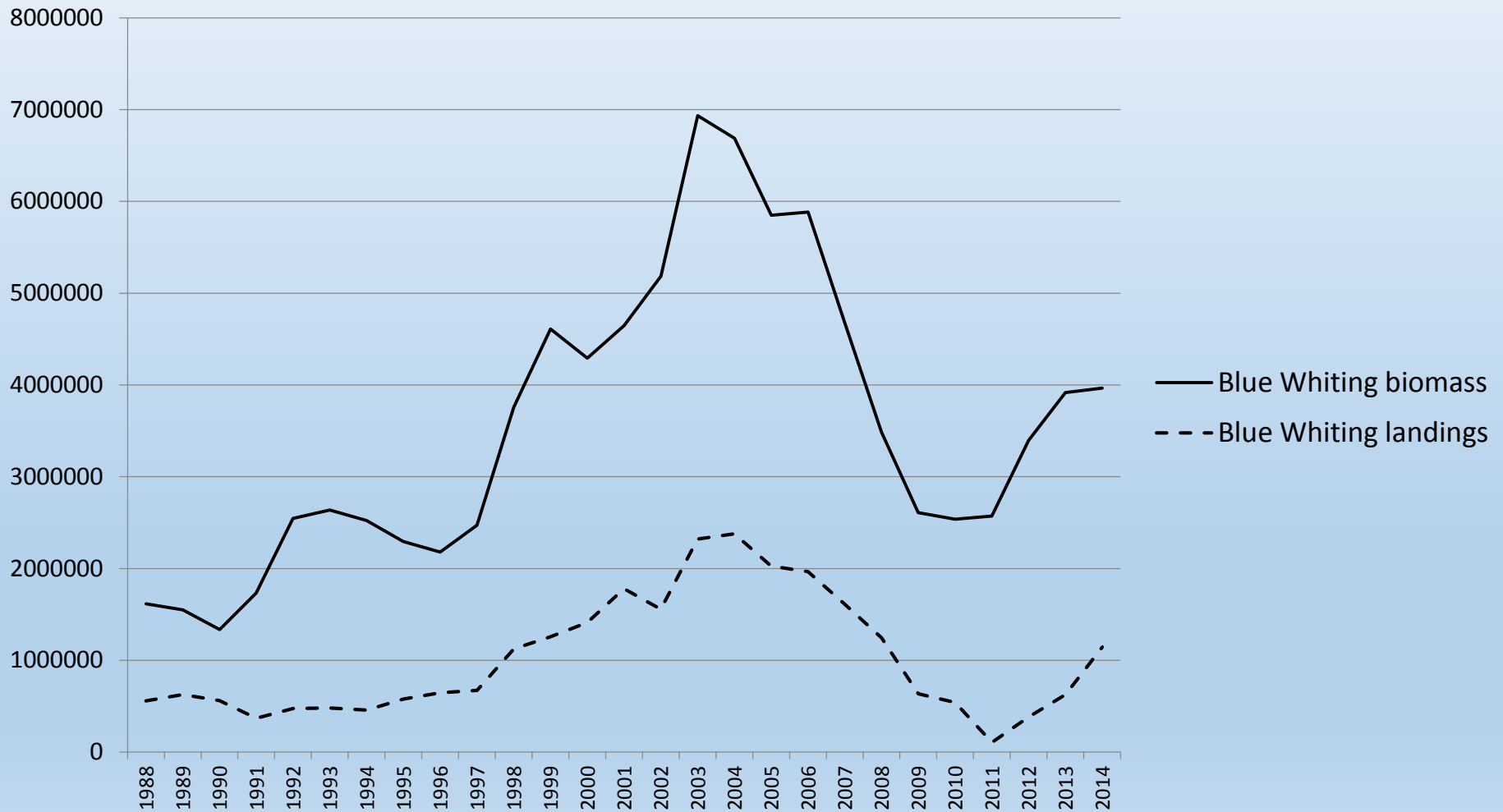


Blue Whiting (*Micromesistius poutassou*) (Risso, 1827)



- Demersal species of the gadus family, widely distributed throughout the North Atlantic Ocean and Mediterranean.
- Fishing methods: Bottom pair trawling (target species) and individual bottom trawling (by-catch).
- **Stock status:**
 SSB well above the B_{pa} , F increasing (management plan), recruitment increasing.

Blue Whiting biomass and landings (tons). 1988-2014



Predator-prey relationship between hake and blue whiting

- **Velasco and Olaso (1998)** study the European hake feeding (stomach contents) in the Cantabrian Sea (Div. VIIIc). They emphasize the **importance of the blue whiting as the main prey of hake at deep > 100 m**. Blue whiting becomes the **basis of the hake diet from the 40 cm long**, disappearing consumption of mackerel and other species.
- **Mahe et al. (2007)** shows that the blue whiting (28.9% by weight and 22% by number of ingested prey), is the **most important prey for hake in the southern Bay of Biscay** (Cantabrian Sea).
- **Cabral and Murta (2002)** study the stomach contents of hake in Portuguese waters (Div. IXa). They show that the blue **whiting is its most important prey** (numerical index of 23, rate of occurrence of 38, gravimetric index 59).

Application to hake and blue whiting fisheries

Theoretical model

Lotka-Volterra mathematic predator-prey model
(Volterra, 1926; Lotka, 1932)

Predator

$$\frac{dX}{dt} = r_m X \left[1 - \frac{X}{\bar{X}} \right] - h_m + \alpha XY \quad (3.1)$$

X = Hake biomass
 r_m = Hake intrinsic rate of growth
 $\bar{X}$ = Carrying capacity for the hake
 h_m = Hake catches
 α = Interaction coefficient H-BW

Prey

$$\frac{dY}{dt} = r_L Y \left[1 - \frac{Y}{\bar{Y}} \right] - h_L - \beta YX \quad (3.2)$$

Y = Blue whiting biomass
 r_L = Blue whiting intrinsic rate of growth
 $\bar{Y}$ = Carrying capacity for blue whiting
 h_L = Blue whiting catches
 β = Interaction coefficient BW-H

Net profit function of the fishery at time t:

$$\pi(X, Y, h_m, h_L) = (P_m - C_m(X))h_m(t) + (P_L - C_L(Y))h_L(t) \quad (3.3)$$

P_m = Hake price

C_m = Hake capture cost

P_L = Blue whiting price

C_L = Blue whiting capture cost

Target function for the control problem :

$$J = \int_0^{\infty} e^{-\rho t} \pi[X(t), Y(t), h_m(t), h_L(t)] dt \quad (3.4)$$

Optimum controls: $h_m(t) = h_m^*$, $h_L(t) = h_L^*$ Maximize target function:

$$\max \int_0^{\infty} e^{-\rho t} [(P_m - C_m(X))h_m(t) + (P_L - C_L(Y))h_L(t)] dt \quad (3.5)$$

$$\begin{aligned} \frac{dX}{dt} &= F(X) - h_m + \alpha XY & 0 \leq h_m(t) \leq h_m \max & \quad 0 < X(t) \\ \frac{dY}{dt} &= G(Y) - h_L - \beta XY & 0 \leq h_L(t) \leq h_L \max & \quad 0 < Y(t) \end{aligned}$$

Hamiltonian function

$$\begin{aligned}
 H[X(t), Y(t), h_m(t), h_L(t), t; \lambda_1(t), \lambda_2(t)] = \\
 = e^{-\rho t} [(P_m - C_m(X))h_m(t) + (P_L - C_L(Y))h_L(t)] + \\
 + \lambda_1(t)[F(X) - h_m(t) + \alpha XY] + \lambda_2(t)[G(Y) - h_L(t) - \beta YX] \quad (3.6)
 \end{aligned}$$

λ_1 = Hake shadow price

$F(X)$ = Hake stock natural growth

λ_2 = Blue whiting shadow price

$G(Y)$ = Blue whiting stock natural growth

First order conditions:

$$\boxed{\frac{\partial H}{\partial h_m} = 0} \quad (3.7) \quad \boxed{\frac{\partial H}{\partial h_L} = 0} \quad (3.8)$$

$$\boxed{\frac{\partial \lambda_1}{\partial t} = \dot{\lambda}_1 = -\frac{\partial H}{\partial X}} \quad (3.9) \quad \boxed{\frac{\partial \lambda_2}{\partial t} = \dot{\lambda}_2 = -\frac{\partial H}{\partial Y}} \quad (3.10)$$

$$\boxed{\frac{\partial X}{\partial t} = \dot{X} = \frac{\partial H}{\partial \lambda_1}} \quad (3.11) \quad \boxed{\frac{\partial Y}{\partial t} = \dot{Y} = \frac{\partial H}{\partial \lambda_2}} \quad (3.12)$$

Commutation functions:

$$\sigma_1(t) = e^{-\rho t} (P_m - C_m(X)) - \lambda_1(t) \quad (3.16)$$

$$\sigma_2(t) = e^{-\rho t} (P_L - C_L(Y)) - \lambda_2(t) \quad (3.17)$$

Bang-bang controls:

$\sigma_1(t) < 0$	$\longrightarrow$	$h_m^* = 0$	Fishing marginal profit
$\sigma_1(t) > 0$	$\longrightarrow$	$h_m^* = h_m \max$	Investing in resource marginal profit
			vs
$\sigma_2(t) < 0$	$\longrightarrow$	$h_L^* = 0$	The optimal stationary solution is
$\sigma_2(t) > 0$	$\longrightarrow$	$h_L^* = h_L \max$	reached by the fastest way possible.

Singular case:

- Commutation functions = 0.
- Fishing marginal profit = Investing in resource marginal profit (shadow price).

$$e^{-\rho t} (P_m - C_m(X)) = \lambda_1(t) \quad (3.18)$$

$$e^{-\rho t} (P_L - C_L(Y)) = \lambda_2(t) \quad (3.19)$$

Singular paths:

$$\dot{\lambda}_1 = -\rho e^{-\rho t} (P_m - C_m(X)) \quad (3.20)$$

$$\dot{\lambda}_2 = -\rho e^{-\rho t} (P_L - C_L(Y)) \quad (3.21)$$

From hamiltonian current value and first order conditions (3.9) y (3.10) we obtain the following equations:

$$\frac{\partial \lambda_1}{\partial t} = \dot{\lambda}_1 = -\frac{\partial H}{\partial X}$$

$$\dot{\lambda}_1 = e^{-\rho t} C_m'(X) h_m(t) - \lambda_1(t) [F'(X) + \alpha Y] + \lambda_2(t) \beta Y \quad (3.22)$$

$$\frac{\partial \lambda_2}{\partial t} = \dot{\lambda}_2 = -\frac{\partial H}{\partial Y}$$

$$\dot{\lambda}_2 = e^{-\rho t} C_L'(Y) h_L(t) - \lambda_1(t) \alpha X - \lambda_2(t) [G'(Y) - \beta X] \quad (3.23)$$

Sustituting (3.18), (3.19), (3.20) y (3.21), we obtain:

$$\begin{aligned} & -\rho e^{-\rho t} (P_m - C_m(X)) = \\ & = e^{-\rho t} C_m'(X) h_m(t) - e^{-\rho t} (P_m - C_m(X)) [F'(X) + \alpha Y] + e^{-\rho t} (P_L - C_L(Y)) \beta Y \end{aligned} \quad (3.24)$$

$$\begin{aligned} & -\rho e^{-\rho t} (P_L - C_L(Y)) = \\ & = e^{-\rho t} C_L'(Y) h_L(t) - e^{-\rho t} (P_m - C_m(X)) \alpha X - e^{-\rho t} (P_L - C_L(Y)) [G'(Y) - \beta X] \end{aligned} \quad (3.25)$$

From first order conditions
(3.11) y (3.12):

$$\frac{\partial H}{\partial \lambda_1} = \dot{X}$$

$$\dot{X} = F(X) - h_m(t) + \alpha XY \quad (3.26)$$

$$\frac{\partial H}{\partial \lambda_2} = \dot{Y}$$

$$\dot{Y} = G(Y) - h_L(t) - \beta YX \quad (3.27)$$

Steady state:

$$\dot{X}_u = \dot{Y}_u = 0$$

$$h_{mu} = F(X_u) + \alpha X_u Y_u \quad (3.28)$$

$$h_{Lu} = G(Y_u) - \beta Y_u X_u \quad (3.29)$$

Sustituting last equations in (3.24) and (3.25), we obtain:

$$\begin{aligned}
 & -\rho(P_m - C_m(X_u)) = \\
 & = C_m'(X_u)[F(X_u) + \alpha X_u Y_u] - (P_m - C_m(X_u))[F'(X_u) + \alpha Y_u] + [P_L - C_L(Y_u)]\beta Y_u; \\
 & [F'(X_u) + \alpha Y_u] - \frac{[P_L - C_L(Y_u)]\beta Y_u + C_m'(X_u)[F(X_u) + \alpha X_u Y_u]}{(P_m - C_m(X_u))} = \rho \quad (3.30)
 \end{aligned}$$

$$\begin{aligned}
 & -\rho(P_L - C_L(Y_u)) = \\
 & = C_L'(Y_u)[G(Y_u) - \beta Y_u X_u] - (P_m - C_m(X_u))\alpha X_u - (P_L - C_L(Y_u))[G'(Y_u) - \beta X_u]; \\
 & [G'(Y_u) - \beta X_u] + \frac{[P_m - C_m(X_u)]\alpha X_u - C_L'(Y_u)[G(Y_u) - \beta Y_u X_u]}{(P_L - C_L(Y_u))} = \rho \quad (3.31)
 \end{aligned}$$

Equations (3.30) and (3.31) constitute a system whose solution would give us the **steady state optimal biomass levels** for both species, $X_u = X^*$ and $Y_u = Y^*$.

Econometric annalysis

Functional form of the dynamics of each fish stock.

Quadratic form:

$$X_{t+1} = \alpha X_t - \beta X_t^2 + \gamma X_t Y_t - h_m$$
$$Y_{t+1} = \varphi Y_t - \mu Y_t^2 + \omega X_t Y_t - h_L$$

Exponential form:

$$X_{t+1} = \alpha e^{\beta X_t} + \gamma X_t Y_t - h_m$$
$$Y_{t+1} = \varphi e^{\mu Y_t} + \omega X_t Y_t - h_L$$

Potential form:

$$X_{t+1} = \alpha e^{\beta X_t + \gamma X_t Y_t} - h_m$$
$$Y_{t+1} = \varphi e^{\mu Y_t + \omega X_t Y_t} - h_L$$

The results indicate that **the quadratic form is the most appropriate for the growth functions of hake and blue whiting**, with a value of R^2 adjusted from 0,992661 and 0,984233, respectively (where this value is below 0.87 in all the other cases).

Only the quadratic forms have a **positive predator-prey interaction coefficient for hake while at the same time negative for blue whiting**, which would be typical of a trophic relationship.

Growth functions for hake and blue whiting:

Hake:
$$X_{t+1} = 1,98802 X_t - 0,0000029429 X_t^2 + 0,00000000183165 X_t Y_t - h_m$$

Blue whiting:
$$Y_{t+1} = 1,49868 Y_t - 0,0000000821632 Y_t^2 - 0,00000312827 X_t Y_t - h_L$$

Applied model

Starting from the expressions (3.30) and (3.31) from the theoretical model:

$$[F'(X_u) + \alpha Y_u] - \frac{[P_L - C_L(Y_u)]\beta Y_u + C_m'(X_u)[F(X_u) + \alpha X_u Y_u]}{(P_m - C_m(X_u))} = \rho \quad (3.30)$$

$$[G'(Y_u) - \beta X_u] + \frac{[P_m - C_m(X_u)]\alpha X_u - C_L'(Y_u)[G(Y_u) - \beta Y_u X_u]}{(P_L - C_L(Y_u))} = \rho \quad (3.31)$$

The functions are quadratic:

$$X_{t+1} = \alpha X_t - \beta X_t^2 + \gamma X_t Y_t - h_m$$

$$Y_{t+1} = \varphi Y_t - \mu Y_t^2 + \omega X_t Y_t - h_L$$

So the terms to substitute are:

$$F(X) = \alpha X_t - \beta X_t^2;$$

$$F'(X) = \alpha - 2\beta X_t$$

$$C_m(X_t) = a - bX_t;$$

$$C_m'(X_t) = -b$$

$$G(Y) = \varphi Y_t - \mu Y_t^2;$$

$$G'(Y) = \varphi - 2\mu Y_t$$

$$C_L(Y_t) = c - dY_t;$$

$$C_L'(Y_t) = -d$$

Expressions (3.30) and (3.31) are now:

$$[(\alpha - 2\beta X_u) + \gamma Y_u] - \frac{[P_L - (c - dY_u)]\omega Y_u + (-b)[(\alpha X_u - \beta X_u^2) + \gamma X_u Y_u]}{P_m - (a - bX_u)} = \rho \quad (3.30')$$

$$[(\varphi - 2\mu Y_u) - \omega X_u] + \frac{[P_m - (a - bX_u)]\gamma X_u - (-d)[(\varphi Y_u - \mu Y_u^2) - \omega Y_u X_u]}{P_L - (c - dY_u)} = \rho \quad (3.31')$$

Optimal catch expressions:

$$h_{mu} = \alpha X_u - \beta X_u^2 + \alpha X_u Y_u \quad (3.28')$$

$$h_{Lu} = \varphi Y_u - \mu Y_u^2 - \beta Y_u X_u \quad (3.29')$$

MSY biomass:

$$F(X) = \alpha X_t - \beta X_t^2 + \gamma X_t Y_t; F'(X) = \alpha - 2\beta X_t + \gamma Y_t$$

$$G(Y) = \varphi Y_t - \mu Y_t^2 - \omega X_t Y_t; G'(Y) = \varphi - 2\mu Y_t - \omega X_t$$

$$F'(X) = 0; Y_t = \frac{\alpha - 2\beta X_t}{\gamma}$$

$$G'(Y) = 0; Y_t = \frac{\varphi - \omega X_t}{2\mu}$$

$$\frac{\alpha - 2\beta X_t}{\gamma} = \frac{\varphi - \omega X_t}{2\mu}$$

$$X_{mrs} = \frac{\frac{\alpha}{\gamma} - \frac{\varphi}{2\mu}}{\frac{-\omega}{2\mu} + \frac{2\beta}{\gamma}} \quad (3.44)$$

$$Y_{mrs} = \frac{\alpha - 2\beta X_{mrs}}{\gamma} \quad (3.45)$$

Results

	Hake	Blue whiting
Optimal biomass	281.000 t.	4.871.500 t.
Optimal catch	328.766 t.	1.068.700 t.
MSY biomass	336.922 t.	2.706.178 t.
Shadow price	3.044,54 €/t.	171,40 €/t.
Benefits	1.052,26 mill. €	192,57 mill. €
Total benefits	1.244,83 mill. €	

The high value of the optimal hake catch corresponds to the end time of the year (cumulative figure).

The optimal biomass is calculated based on annual estimates by ICES in indeterminate times of the year, and it may increase considerably towards the end of it due to the rapid growth of this species.

$$\alpha = 1,988$$

$$\beta = 0,00000294$$

$$\gamma = 0,00000000183$$

Net growth per ton:

$$1,988 + 0,009 - 0,82 = \mathbf{1,177 \text{ t.}}$$

The biomass at the end of the year is more than double than the existing at the beginning of the year (2,177 t. for each initial tonne).

Steady state:

Capture = Net growth (**1,177 t.** per ton).

Capture > Initial Biomass

Sensitivity analysis

Simulations:

- Variation of discount rate ρ .

ρ	=	-0,05	0	0,05	0,10	0,15
--------	---	-------	---	------	------	------

- Variation of prices.

P_m	=	4400	4500	4.619,23	4700	4800
-------	---	------	------	----------	------	------

PL	=	300	350	399,13	450	500
------	---	-----	-----	--------	-----	-----

Sensitivity of optimal biomass and catch results

Simulations		X_{it}	Y_{it}	h_{mit}	h_{lit}	h_m/X_{it}	h_l/Y_{it}
rho	-0,05	297.500	4.905.500	333.643	809.251	112,15 %	16,50 %
	0	290.500	4.882.300	331.765	921.642	114,20 %	18,88 %
	0,05	281.000	4.871.500	328.766	1.068.700	117,00 %	21,94 %
	0,1	275.000	4.812.750	326.572	1.169.377	118,75 %	24,30 %
	0,15	269.000	4.753.000	324.167	1.267.405	120,51 %	26,67 %
P_m	P_L						
4400	399,13	288.500	4.721.000	331.093	983.298	114,76 %	20,83 %
4500	399,13	287.500	4.755.900	330.810	991.807	115,06 %	20,85 %
4.619,23	399,13	281.000	4.871.500	328.766	1.068.700	117,00 %	21,94 %
4700	399,13	278.000	4.931.900	327.741	1.103.760	117,89 %	22,38 %
4800	399,13	277.000	4.969.300	327.396	1.112.406	118,19 %	22,39 %
P_L	P_m						
300	4.619,23	268.000	5.879.800	324.304	1.041.903	121,01 %	17,72 %
350	4.619,23	270.000	5.483.000	324.939	1.116.044	120,35 %	20,35 %
399,13	4.619,23	281.000	4.871.500	328.766	1.068.700	117,00 %	21,94 %
450	4.619,23	285.500	4.188.700	329.893	1.094.934	115,55 %	26,14 %
500	4.619,23	291.000	3.586.000	331.217	1.053.266	113,82 %	29,37 %
P_m	P_L						
4700	450	264.000	4.710.000	322.006	1.346.250	121,97 %	28,58 %
4500	350	310.000	5.110.100	336.374	557.278	108,51 %	10,91 %
4.619,23	399,13	281.000	4.871.500	328.766	1.068.700	117,00 %	21,94 %
4700	350	278.000	5.450.000	328.005	987.712	117,99 %	18,12 %
4500	450	290.000	4.100.000	331.205	1.043.912	114,21 %	25,46 %

Increment of discount rate

Increment of fishing effort

Increment of catch rate

Disminution of X_u, Y_u

In the case of the hake, the effective catch diminishes.



Greater efficiency is achieved with **low discount rates**.

Increment of hake price

Increases hake catch rate and decreases X_u .

The real catch decreases.

Sensitivity of shadow prices results

Simulations		Lambda 1	Lambda 2	% variac. λ_1	% variac. λ_2
rho	-0,05	3.460,13	190,86	113,7%	111,4%
	0	3.252,88	180,62	106,8%	105,4%
	0,05	3.044,54	171,40	100,0%	100,0%
	0,1	2.866,19	160,92	94,1%	93,9%
	0,15	2.698,00	151,01	88,6%	88,1%
P _m	P _L				
4400	399,13	2.875,23	165,67	94,4%	67,4%
4500	399,13	2.965,12	167,00	97,4%	86,3%
4.619,23	399,13	3.044,54	171,40	100,0%	100,0%
4700	399,13	3.105,67	173,70	102,0%	113,1%
4800	399,13	3.195,56	175,21	105,0%	127,4%
P _L	P _m				
300	4.619,23	2.976,52	115,47	97,8%	96,7%
350	4.619,23	2.986,99	147,94	98,1%	97,4%
399,13	4.619,23	3.044,54	171,40	100,0%	100,0%
450	4.619,23	3.068,08	193,81	100,8%	101,3%
500	4.619,23	3.096,85	218,44	101,7%	102,2%
P _m	P _L				
4700	450	3.032,42	213,65	99,6%	124,6%
4500	350	3.082,84	133,75	101,3%	78,0%
4.619,23	399,13	3.044,54	171,40	100,0%	100,0%
4700	350	3.105,67	146,68	102,0%	85,6%
4500	450	2.978,20	190,44	97,8%	111,1%

Increment of discount rate

Decrease in both shadow prices.

This shows a growing appreciation of the current income generated by the capture of both species.

Increment of prices:

Shadow price increases. By increasing the resource price is increased fishing effort and catch rate, resource biomass decreases and it begins to be perceived as low, raising its shadow price.

Sensitivity of benefits results

Simulations		h_m	h_l	π_m	π_l	π_{total}	var. π_m	var. π_l	var. π_{tot}
rho	-0,05	333.643	809.251	1.098,15	146,92	1.245,07	104,4%	76,3%	100,0%
	0	331.765	921.642	1.079,19	166,47	1.245,66	102,6%	86,4%	100,1%
	0,05	328.766	1.068.700	1.052,26	192,57	1.244,83	100,0%	100,0%	100,0%
	0,1	326.572	1.169.377	1.034,46	207,96	1.242,42	98,3%	108,0%	99,8%
	0,15	324.167	1.267.405	1.016,14	222,36	1.238,51	96,6%	115,5%	99,5%
P_m	P_L								
4400	399,13	331.093	983.298	1.000,78	171,26	1.172,04	95,1%	88,9%	94,2%
4500	399,13	330.810	991.807	1.031,18	174,13	1.205,31	98,0%	90,4%	96,8%
4.619,23	399,13	328.766	1.068.700	1.052,26	192,57	1.244,83	100,0%	100,0%	100,0%
4700	399,13	327.741	1.103.760	1.070,04	201,55	1.271,60	101,7%	104,7%	102,2%
4800	399,13	327.396	1.112.406	1.099,85	204,79	1.304,65	104,5%	106,4%	104,8%
P_L	P_m								
300	4.619,23	324.304	1.041.903	1.014,79	126,48	1.141,27	96,4%	65,7%	91,7%
350	4.619,23	324.939	1.116.044	1.020,35	173,57	1.193,92	97,0%	90,1%	95,9%
399,13	4.619,23	328.766	1.068.700	1.052,26	192,57	1.244,83	100,0%	100,0%	100,0%
450	4.619,23	329.893	1.094.934	1.064.03	223,09	1.287,12	101,1%	115,9%	103,4%
500	4.619,23	331.217	1.053.266	1.078,32	241,87	1.320,19	102,5%	125,6%	106,1%
P_m	P_L								
4700	450	322.006	1.346.250	1.026,52	302,37	1.328,89	97,6%	157,0%	106,8%
4500	350	336.374	557.278	1.090,15	78,35	1.168,51	103,6%	40,7%	93,9%
4.619,23	399,13	328.766	1.068.700	1.052,26	192,57	1.244,83	100,0%	100,0%	100,0%
4700	350	328.005	987.712	1.070,90	152,30	1.223,21	101,8%	79,1%	98,3%

Increment of discount rate

Hake: the **greatest benefits occur with $\rho = -0.05$** , falling with increasing ρ , which is consistent with the results of the sensitivity analysis of catches (catch decreases with increasing ρ).

Mixed fishery: Maximum benefit with $\rho = 0$.



Discount rate = 0 recommended for managing this fishery.

Conclusions

This research work has developed the application of a multispecies bio-economic model of the predator-prey type to the mixed fishery of hake and blue whiting of the EU Atlantic waters. A better understanding of trophic relationships between fish stocks allows to develop multispecies fishery management models, which is the category under which most European fisheries fall.

The effective catch of hake is higher at low capture rates. The selection of low discount rates conduce to more efficient results.

The management of the mixed fishery of hake and blue whiting should base the objectives and technical measures in the use of discount rates close to zero. The maximum benefit is reached whit a low pressure over the resource.

THANK YOU