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Cyclical patterns in number of cattle kept on farms, amount of beef produced, and price level of cattle have been described and explained in terms of "inventory cycles," "production cycles," and "price cycles." The factors which generate these "cycles" have been regarded as closely related to each other. In previous studies of these "cycles," however, the fundamental activities of the economic agents involved have seldom been identified explicitly. This is particularly evident with respect to the activities of beef breeders concerning their breeding cows.

The primary objective of this dissertation is to formulate a set of alternative hypotheses which are capable of explaining the fundamental activities of beef breeders concerning their breeding cows. The secondary objective is to extend the primary objective...
by constructing a multiple-relation model which explains the fundamental activities of beef breeders in conjunction with the determination of both calf and salvage cow price and the aggregate calf production process.

In order to pursue these objectives, the fundamental activities of a "representative" beef breeder concerning his breeding animals were systematically analyzed with respect to the optimum size and composition of his herd and the adjustment activities of his herd within one production period. The beef breeder's anticipation of a calf price was explicitly considered, since the beef breeder tends to expect a certain level of calf price at the beginning of a subsequent production period. An investment behavior relation, represented as a partially reduced form equation, was derived from the herd demand relation, mode of price anticipation, and herd adjustment relation.

For the primary objective, three refutable hypotheses were specified, as aggregate investment behavior relations, in the form of a partially reduced form equation. For the secondary objective, the investment behavior relation concerning the aggregate replacement heifers was developed. In addition, the calf price relation and the salvage cow price relation were derived from the corresponding aggregate supply and demand relations and equilibrium conditions. Finally, a calf production relation which
could describe and explain the calf production process was developed.

The empirical results obtained from applying multiple regression analysis to the available data suggest that the refutable hypotheses formulated for the investment activities of beef breeders explain, both qualitatively and quantitatively, almost all of the variation in the number of breeding cows adjusted and maintained at the beginning of each production period. In addition, the estimate of structural parameters of herd demand relation, the estimate of weight of price expectation, and the estimate of herd adjustment coefficient can be approximated. Furthermore, the investment behavior relation concerning the aggregate replacement heifers explains about 99 percent of the variation in the number of replacement heifers kept on farms at the beginning of each production period; the calf production relation explains almost all of the variation in the number of calves produced in each production period; the calf price relation explains about 88 percent of the variation in the price level of calves; and the salvage cow price relation explains about 86 percent of the variation in the salvage cow price.

From both theoretical and empirical analyses, it was concluded that the investment activities of beef breeders concerning their breeding cows can hardly be ignored if one is seriously
looking for a "cycle" (or "cycles") or for factors which generate a "cycle" (or "cycles") in the number of beef breeding cows kept on farms at the beginning of every production period.
An Analysis of Investment in the United States Beef Cattle Industry

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AN ANALYSIS OF INVESTMENT IN THE UNITED STATES BEEF CATTLE INDUSTRY

CHAPTER I

INTRODUCTION

The periodic or wavelike patterns in the number of beef and/or dairy cattle kept on farms, the amount of beef produced, and cattle prices have been generally regarded as "inventory cycles" (or "cattle cycle"), "production cycles" (or "beef cycle"), and "price cycles," respectively. The factors which generate these "cycles" have been regarded as closely related to each other. As these "cycles" require some kind of explanation, a number of attempts have been made to describe as well as explain the cyclical fluctuations in number of beef and/or dairy cattle kept on farms, number of beef cattle kept on feed, amount of beef produced, and cattle prices.

Various explanations concerning these cyclical fluctuations are available in terms of a few strategic, aggregate variables and economic data. The previous studies have been based upon either aggregate supply and demand relations or a self-generating mechanism rather than upon a systematically developed theoretical framework stemming from the fundamental activities of the economic agents involved. The empirical phenomena in number of beef and/or dairy cattle kept on farms, number of beef cattle kept on feed, amount of beef produced, and cattle prices may
be variously explained and interpreted within the framework of either supply and demand analysis or the self-generating mechanism. Nevertheless, it is desirable from the methodological point of view to consider whether the cyclical phenomena may better be explained by a set of hypotheses which are based on the activities of economic agents.

This dissertation presents an attempt to improve understanding of, and at least in part to explain, the cyclical fluctuations in number of beef breeding cows kept on farms in the United States. There are two objectives. The primary objective is to formulate a set of alternative hypotheses, with reference to existing hypotheses, which are capable of explaining the fundamental activities of beef breeders concerning their breeding cows and heifers, two years old and older. The primary objective extends to a secondary objective which formulates a multiple-relation model capable of explaining the investment activities of beef breeders concerning their breeding cows as well as replacement heifers in conjunction with the determination of calf prices, the determination of salvage cow prices, and the aggregate calf production processes.

In order to improve understanding of the cyclical phenomena associated with the number of beef breeding cows kept on farms, it would seem necessary to clarify the meaning of "cycles" as the
term has been used in previous studies. A review will also be made of hypotheses which have been submitted to explain the cyclical phenomena in number of beef and/or dairy cattle kept on farms, number of beef cattle kept on feed, amount of beef produced, and cattle prices. Through a brief review, the fundamental activities of a representative beef breeder will be identified as "investments." This preparation will provide insight for the study of "investment behavior" of breeding cattlemen concerning their breeding cows.

A. Definition of the Empirical Phenomena

The economic phenomena associated with the beef and/or dairy cattle sector of the livestock economy have often been described and discussed in terms of "cycles," without any explicit definition of "cycle." Describing the cyclical pattern in the number of beef and dairy cattle kept on farms, shown in Figure 1, Williams and Stout (93, p. 543) state:

"There have been only six major cattle cycles in the United States since 1896. The first two were rather uniform and regular. The third was shortened by depression and drought during the mid-thirties. The fourth may have been affected by World War II. The last complete cycle, beginning with a trough in 1949, probably was affected by the Korean Conflict and the exceptionally high prices incident thereto,
One reason for the high prices, however, is that the Korean Conflict caught the cattle industry at a low phase of the cycle with its marketing down. This cycle was relatively short, ending with a trough in 1958."

With reference to cyclical behavior in the number of beef cattle kept on farms and related prices, Ehrich (18, p. 3-4) states:

"Cycles in prices and numbers have not followed a perfectly consistent pattern in past years. Periods of from
6 to 8 years, while periods of decline varied from 3 to 10 years, and the rate of increase or decline has differed from cycle to cycle. Price cycles exhibited a similar lack of consistency in behavior. Consequently, it is desirable at the outset to examine graphically several alternative price and numbers series, applying rather subjective measures which are designed to identify historical behavior in some detail.

From the preceding paragraphs, it is apparent that "cattle cycle" (or "inventory cycle") has been used to designate the cyclical pattern in the time series data for the number of beef and/or dairy cattle kept on farms. Similarly, "production cycle" (or "beef cycle") and "price cycle" have been used to denote the cyclical pattern in the time series data for the amount of beef produced and the price of cattle, respectively. In other words, "cattle cycle," "production cycle," and "price cycle" are used to denote the ex post cyclical phenomena associated with the number of cattle kept on farms, the amount of beef produced, and the price of cattle, respectively. For convenience, the "cycle" of a given economic variable will be used in this dissertation to denote an ex post behavior pattern of time series data for the economic variable, with certain periodicity and amplitude.
B. A Review of the Literature Attempting to Explain the Cycles Associated with the Cattle Industry

The existence of cyclical phenomena in time series data for the strategic variables which characterize the beef and/or dairy cattle sector of livestock economy has raised a number of provocative questions. For example: Why are there cyclical fluctuations in the number of beef and/or dairy cattle kept on farms? Why are there cyclical fluctuations in the number of beef cattle kept on feed? Why are there cyclical fluctuations in the amount of beef produced? Why are there cyclical fluctuations in cattle prices? How did the cyclical fluctuations in such variables as the number of cattle kept on farms, the number of beef cattle kept on feed, the amount of beef produced, and cattle prices come about? How do the strategic variables influence one another? There have been a few explanations concerning these cyclical fluctuations. In some studies, the cyclical fluctuations in the variables which characterize the cattle sector of livestock economy have been analyzed with those which characterize the pork sector of livestock economy. However, it is not the intent of this dissertation to review the available literature dealing with various cycles associated with the cattle sector as well as other sectors of the livestock economy. Rather this dissertation will focus attention only on those earlier works which are germane to
the present study.

For convenience, explanations advanced in the literature, to explain the various cycles associated with the cattle industry, have been divided into three categories. First, there are those studies stipulating that the cycles associated with the cattle sector of the livestock industry are basically the result of the causal forces which are primarily exogenous to the cattle industry, e.g., the rise in consumers' demand for beef, wars, weather conditions, etc. The second category emphasizes such internal factors as the reaction of ranchers to economic data, technology of the growth process associated with beef cattle, and production lags. It also explains the cycles in terms of Cobweb Theorem or Servomechanism Control System. Third, there are hypotheses which suggest that the cyclical phenomena result from internal as well as external factors. These studies will be considered under the headings of exogenous hypotheses, and endogenous hypotheses, and exogenous-endogenous hypotheses.

1. **Exogenous Hypotheses**

The general theme of the exogenous hypotheses is to stress external phenomena as the major determinants generating periodic fluctuations in the strategic variables of cattle sector. Such factors as rainfall, feed supplies, and pasture conditions
are postulated as influencing the aggregate "demand" for cattle. Among those advancing such hypotheses are Burmeister (8) and Pearson (68).

In connection with the causes of changes in the aggregate demand for both beef and dairy cattle, Pearson (68, p. 4948) submits an exogenous hypothesis which was formulated by a group of economists as follows:

"One group says that the increasing demands are due to farmer demands and another to urban consumer demands for beef which, in turn, are related to national income, urban wages, employment, building [i.e., construction of buildings], business activity and so on, ad finitium. The argument is about as follows. When building is active, business is good and labor is fully employed. It is therefore concluded that consumers, with plenty of purchasing power, are willing to spend larger amounts of money on beef."

The test of the above hypothesis in the light of empirical evidence is carried out and interpreted by Pearson (68, p. 4948) who concluded:

"There was, however, no consistent tendency for the two series to move in the same or opposite directions nor was there much similarity between the amounts of change. There were seven periods when building activity and demand for cattle moved in opposite directions and seven when they moved in the same direction. If a 'knight of the elbow' tossed up [sic] a penny ten times and got five heads
and five tails, only a 'piker' would conclude that the penny was weighted."

Consequently, Pearson (68, p. 4950) disputes the above hypotheses as follows:

"The farmer demand for cattle was not consistently related to urban demand as measured by [construction of] building[s] which exhibited cyclical fluctuations. Nor was it related to urban demand as measured by the price level or business activity, neither of which exhibited regular cyclical fluctuations. Although not shown, the farmer demand for cattle was not related to urban employment, earnings of factory workers, national income and the like."

As an alternative to the above hypothesis, Pearson (68, p. 4951) submits his hypothesis as follows:

"Fluctuations in the farmer demand for cattle are due to forces found on the farms and ranges rather than in urban homes, restaurants and hotels. All cattlemen want to increase their herds as much as possible because they are optimists and because increasing volume decreases costs and increases profits. One of the dampers on this enthusiasm for expansion is the supply of feed."

On the basis of available data, Pearson (68, p. 4951) summarizes his study as follows:

"The best measure of the volume of roughages and pasture available is the Jennings's series. These data indicate that the volume of feed available for livestock has fluctuated from 1.7 to 2.5 tons per animal. The supply of roughages was positively
related to changes in the demand for cattle. When the feed supply increased, the demand for cattle increased."

2. **Endogenous Hypotheses**

One endogenous hypothesis is based on the self-generating mechanism, i.e., Cobweb Theorem. This theorem is advanced as the theoretical explanation of cyclical phenomena in the production and prices of some commodities. Ezekiel (22, p. 437-8) specified the conditions under which the Cobweb Theorem applies as follows:

"(1) where production is completely determined by the producers' response to price, under conditions of pure competition (where the producer bases plans for future production on the assumption that present prices will continue, and that his own production plans will not affect the market); (2) where the time needed for production requires at least one full period before production can be changed, once the plans are made; and (3) where the price is set by available supply."

On the basis of an explicit assumption that the Cobweb Theorem does in fact apply to the explanation of cyclical phenomena in the cattle sector of livestock economy, Williams and Stout (93, p. 541-2) explain the theoretical cobweb patterns as follows:

"An explanation of the self-generating aspects of the cycle can begin with any phase of it. Suppose, however, that production and marketings initially are stable and consistent with the equilibrium level of prices and that prices are in equilibrium. In Figure 22-8, it is assumed that an initial disturbance appears in the form of a rise in
the demand for livestock and meat products which triggers a price rise. Given disequilibrium, the cycle develops out of attempts by producers to bring production, marketings, and prices back into balance or equilibrium. Producers, acting rationally, withhold livestock from market to increase the size of their herds in the hope of capitalizing on future high prices. The reduction in marketings and slaughter has the effect of reinforcing the initial price rise. If the commodity under consideration is cattle, marketings and slaughter probably will continue to decline for two or three years after the initial disturbance--the time required to raise new animals to marketing age. But as marketings and slaughter begin to rise, prices drop. Producers continue to add to their herds, however, as long as prices are above the equilibrium level."

In regard to the movement in prices, viz., equilibrium price-cyclical bottom-equilibrium price, Williams and Stout (93, p. 542) explain the causal mechanism bringing about cyclical fluctuations as follows:

"When prices drop to equilibrium, inventories reach a cyclical peak. Marketings and slaughter rise further, prices drop below equilibrium, and herd liquidations begin to take place at a rapid pace. As the liquidation process continues, a point eventually is reached at which the productive capacity of herd no longer can sustain an upward trend in marketings. Marketings level off, reach a peak, and begin to fall. Prices begin to rise but inventories continue to fall. Inventory reductions are required until marketings have dropped to the level consistent with equilibrium prices."

Finally, the self-generating patterns associated with cattle sector are implied by Williams and Stout (93, p. 542) as follows:

"Although prices and marketings at this point have returned to the original equilibrium situation, inventories
are out of line with this situation. Marketings, as a result, continue to fall, prices continue to rise, and inventories again begin to build. The cycle regenerates itself and, according to theory, may continue endlessly."

A recent endogenous hypothesis has been formulated on the basis of another cycle-generating mechanism, i.e., Servomechanism Control System. This mechanism is closely related to the theory of inventory cycles in the general economy, as expounded by Lundberg (53), Metzler (58), and Nurkse (67), but with some interesting special considerations. The operation of Servomechanism Control System may be briefly explained as follows. The harmonic oscillations generated by Servomechanism Control Systems are but one example of a widely occurring phenomenon called "feedback." It occurs whenever a signal produces a response that acts after a delay to alter the signal, and has been detected and studied in relation to physiological processes, wildlife populations, electrical circuitry, and industrial plant operations. Particularly in relation to inventory control, it is a major topic in operations research, and by an obvious extension to the economy as a whole, it has been advanced as the mechanism of the inventory cycles of the general economy in recent years. The essential requirement for "feedback" is an unvarying response to a signal, which acts through a fixed delay to alter the signal in a predetermined manner.
Applying the operational scheme of Servomechanism Control System to the cattle sector of the livestock economy,
Ehrich (18, p. 25) explains the cyclical phenomena associated with beef cattle sector as follows:

"The characteristic behavior of production and prices suggests that harmonic motion is an appropriate model of cycle-generating forces in the United States' beef economy. Essentially, true harmonic motion involves stimulus, response, and 'feed-back,' which serves to alter the stimulus after a fixed delay. It appears that the physical characteristics of beef-cattle growth, uncertainty regarding future prices and producers' behavior in the face of uncertainty, combine to produce harmonic behavior in annual fluctuations in cattle prices and numbers. In particular, producers respond to prices (stimulus) by changing the rate of planned production (increasing or decreasing the breeding herd incrementally), the change in production is realized after a delay (physical growth limitations), and the price stimulus is altered by realized production (prices are unilaterally affected by pre-determined supplies)."

Following such an alternative endogenous hypothesis, Ehrich (18, p. 25) summarizes his statistical analyses as follows:

"Statistical estimates of the relationships among prices, inventory adjustments, and annual production of beef were consistent with the basic model. Of primary importance, the evidence supports the view that producers respond incrementally to deviations of price from equilibrium, and it serves to deny the existence of a conventional supply function for beef cattle. For, of course, the conventional concept of a supply function presupposes that producers adjust to a new level of planned output which is independent of the present level of output, in response to a change in price levels, rather than seeking to change the rate of planned output from current levels."
In addition, Ehrich points out the limitation of Cobweb Theorem as follows:

"Parenthetically, other evidence supports harmonic motion as a model of behavior for the hog economy as well as for the cattle economy. The Cobweb Theorem, which depends on the existence of a conventional supply curve, is then an inadequate model of dynamic economic behavior in either the hog or cattle economy."

3. **Exogenous and Endogenous Hypotheses**

A synthesis of the two diverse theoretical hypotheses considers both endogenous and exogenous factors as determining the course of the relevant variables which characterize the cattle sector of livestock economy. The general theme of this synthesis is expressed by Breimeyer (6, p. 16) as follows:

"Quite naturally, theories with respect to cycles in cattle, resembling those of the business cycle, are divided into those emphasizing outside factors and those favoring automatic self-generating properties. The idea of the automatic cycle is that over-expansion is followed by over-contraction and later by over-expansion once more, continuing in never-ending succession. The theory conforms to the 'cob-web theorem' of economic fluctuation developed by Mordecai Ezekiel."

In connection with the limitation of the endogenous hypotheses, Breimeyer (6, p. 16) indicates:

"Objections to the automatic interpretation are that it disregards outside factors such as demand, feed supply, and competitive position. It would be unfair and uncomplimentary to cattlemen to suggest that they are insensitive to such factors."
As far as the response of cattle producers is concerned, Breimeyer (6, p. 16) states:

"Cattle producers are, of course, motivated by price. No simple analysis of price response is possible. What price do cattlemen respond to: Past prices? Perhaps. Present prices? Perhaps these too. But mostly they respond to the expectations of future prices. My summary view is that cattle producers respond to all factors affecting them including current prices and expectations of future prices. They act within limitations imposed by the characteristics of the industry--a long life cycle, high investment, and few alternative enterprises to most producers. Because of these characteristics, responses are not quick, simple, or direct but take on the slow evolutions known as the cattle cycle."

Following the framework set out by Breimeyer, two econometric studies were carried out. First, an econometric model of the beef and pork sectors of the economy was formulated by Wallace and Judge (91). However, their study does not go beyond the estimation of structural parameters for the aggregate supply and demand relations for beef, pork, feeder cattle, and feed grain. Second, an econometric model which decomposes the beef and pork cycles was constructed by Maki (55) in order to facilitate the development of alternative market forecasting procedures. Maki does not offer any rigorous hypothesis about the causes of cycles associated with the cattle sector of the livestock economy since his primary interest deals with forecasting procedures. Nonetheless, Maki (55, p. 739) cites the crucial role of breeding cows within the framework of internal
mechanism of the beef production cycle as follows: "Changes in beef cow inventories perform an extremely critical role in accounting for both the period and the amplitude of the beef cycle."

A brief survey of available hypotheses concerning the cyclical phenomena associated with the cattle industry suggests that these hypotheses are not systematically developed from the fundamental activities of economic agents in the cattle industry. Furthermore, this survey suggests that a better understanding of the cyclical phenomena associated with the cattle industry is predicated on an understanding of the fundamental activities of economic agents in the cattle industry. Before this study considers the fundamental activities of beef breeders, notice will be taken of some qualifications of the previous endogenous hypotheses with reference to the explanation of fundamental activities of economic agents in the industry.

C. A Critique of Those Studies, Particularly the Endogenous Hypotheses

It has been accepted that cyclical patterns in time series data for those economic variables which characterize the cattle sector can be explained by a self-generating mechanism, i.e., either the Cobweb Theorem or the Servomechanism Control System. Perhaps the aggregate economic activities of such
economic agents as cattle ranchers, feed-lot operators, and 
slaughter-house operators may be adequately accounted for 
by the cycle-generating mechanism. However, if one is inter-
ested in explaining the causal mechanism of cyclical phenomena 
associated with the cattle sector, then one can hardly escape 
from some crucial questions concerning technology of the growth 
process associated with beef cattle and the fundamental economic 
activities of beef breeders, feed-lot operators, and slaughter-
house operators. For example: Why does a beef breeder (or 
beef breeders as a whole) maintain a herd of breeding animals-- 
cows, heifers, heifer calves, bulls, and bull calves? What are 
the roles of breeding females in the herd? What are the economic 
activities of beef breeders, feed-lot operators, and slaughter-
house operators, following the vertical chain of beef production 
process? In order to understand the limitations of the endogenous 
hypotheses, consideration will first be given to these three ques-
tions, followed by discussion of the qualifications of the Cobweb 
Theorem and Servomechanism Control Systems as alternative 
endogenous hypotheses.

There can be no doubt that a beef breeder (or beef breeders 
as a whole) maintain a herd of breeding animals over time in 
order to produce a series of calf crops which, in turn, yield a 
series of "economic returns."
Nor is it difficult to recognize the multiple role of breeding females in the herd: any given female at a given point in time can be viewed as (a) a finished good, (b) a good in process, or (c) a piece of fixed capital (or durable input). This is perhaps most dramatically apparent for a young heifer. If she has been well fed, she may be immediately marketable as medium or possibly better grade beef. Alternatively she may profitably be fed intensively for a short period with a consequent increase in weight and possibly in grade. A third alternative is to retain her in the breeding herd to produce calves.

The third question requires a more detailed response.

Beef breeders market the calves, except those for replacement, either to feed-lot operators or to slaughter-house operators, depending upon the class, grade, and other attributes of calves, while the feed-lot operators market the fed cattle to the slaughter-house operators who produce red meat for final consumers. In addition, the beef breeders market the breeding animals which are culled from their breeding herds either to feed-lot operators or to slaughter-house operators, depending upon the class, grade, and other attributes of the animals. Thus, according to the vertical chain of beef production process, the economic activities may be alternatively stated as follows: (a) The economic activities of slaughter-house operators depend on the aggregate demand
for red meat (of various classes and grades) and the aggregate supply of slaughter cattle (of various classes and grades); (b) the economic activities of feed-lot operators depend on the aggregate demand for fed cattle (by slaughter-house operators) and the aggregate supply of feeder cattle (of various classes and grades); finally, (c) the economic activities of beef breeders depend on the aggregate demand for feeder cattle and for slaughter cattle on one hand and the aggregate breeding herd maintained on the other hand.

Of first consideration are the limitations of the Cobweb Theorem as an endogenous hypothesis. Although some economists use this theorem, without qualification, to explain the cyclical phenomena in prices and quantities exchanged in the market, Ezekiel (22, p. 437-40) suggests several limitations. They may be briefly summarized as follows: First, the asymmetry in adjusting factors of production to the desired level of production is not adequately taken into account. For example, beef breeders as a whole cannot increase the number of breeding cows at once, but they can reduce the number of breeding cows immediately by culling them out. Second, there may not be any clearly "fixed" delay in supply responses of the producers. For instance, the beef breeders may not always have one-period supply responses. Third, the most serious limitation is that imposed by natural
conditions, e.g., weather conditions, range conditions, and feed supply, affecting the decision to maintain a herd of breeding animals. Finally, there is no commodity for which the third condition of the theorem—that the supply alone sets the price—is completely fulfilled. The last limitation implies that there are many farm products whose prices ordinarily show larger variations due to changes in supply than to all other influences combined; yet their prices are also influenced by changes in the supply of competing products, changes in the prosperity or income of consumers, and so on.

With respect to blanket application of the Cobweb Theorem to various commodity cycles, Ezekiel (22, p. 440-441) warns:

"Not all cyclical phenomena in individual industries are traceable to the 'cobweb reaction.' In durable or semi-durable goods, the average length of service of the equipment, and the bunching of replacements in recurring goods, may give rise to a separate cyclical phenomenon which has been called 'the replacement cycle.' In producer's goods, especially in producers' goods several steps removed from the final product, such as in machine tools or die making, the demand for the producers' goods may appear only when production of consumers' goods is increasing, and may disappear entirely when demand for the final product is stable. Similarly, demand for machinery to make the machines to make the final product may appear only when the demand for the final product is increasing at an increasing rate. The derived character of the demand

1/ See the third condition, which should be met for the application of the Cobweb Theorem, in section I, B, 2.
for producers' goods may thus give rise to cyclical phenomena in producers' goods' industries of a quite different character. Many recurring cycles in commodity prices may thus be found to be due to causes other than the cobweb reaction. The cobweb theorem as summarized here should be used as an hypothesis in studying the interactions of supply and demand only for those commodities whose conditions of pricing and production satisfy the special assumptions on which it is based, not as a blanket explanation of all industrial cycles."

The above limitations summarized by Ezekiel imply that the Cobweb Theorem cannot be blindly applied to the cyclical phenomena associated with the cattle sector of the livestock economy as long as the breeding animals, particularly breeding females, can be regarded as a collection of durable factors of production. It is therefore deemed desirable to formulate an alternative hypothesis (or a set of alternative hypotheses) which will explain the cyclical phenomena in number of cattle kept on farms, number of feeder cattle kept on feed, amount of beef produced, and cattle prices.

Also to be considered are the limitations of the other endogenous hypothesis, which is essentially founded on the Servomechanism Control System. Ehrich analyzes the beef cattle cycle by applying to it the model which was designed for study of the hog cycle and based upon operation of the Servomechanism Control System. There are several criticisms of the model formulated by Ehrich. First, his model is intended to
show the operation of cycle-producing mechanism without any systematically developed hypotheses. For example, Ehrich (18, p. 8-11) suggests:

"The observed behavior of cow numbers in relation to feeder-cattle prices, and the inverse relationship between beef production and prices, suggests that it is appropriate to view the underlying mechanism of the cattle cycle as harmonic motion. In essence, the presence of three conditions is essential to the validity of harmonic motion as a model of the cycle-producing mechanism. Larson (50) lists these as: 1) the existence of price as a signal effecting a response in terms of a change in the rate of planned production, 2) a change in the price signal due to the production response, and 3) a fixed delay."

In addition, Ehrich avers:

"The above elements appear to be present in the cattle economy. The price of feeder cattle induces a change in the rate of planned production, viz., the size of breeding herd is changed. Prices respond to realized production, while realized production lags the original price signal by from two to three years."

However, Ehrich does not elaborate on the conditions under which the price of feeder cattle induces a change in the rate of planned production.

Second, the strategic factors which seem to generate the cyclical phenomena in the number of beef cattle kept on farms, number of beef cattle kept on feed, amount of beef produced, and cattle prices are suggested without qualifications. In fact, Ehrich (18, p. 11) states:
"The key forces underlying the cycle appear to be three in number: (1) a fixed production lag, (2) the dependence of price on current production suggests the existence of a particular demand relation, and (3) the change in desired rate of production is proportional to the deviation of price from equilibrium levels."

Third, instead of developing an economic model, step by step, Ehrich (18, p. 11-12) lists several simplifying assumptions:

"Certain general assumptions are made at the outset: 1) competition is atomistic at all levels of the vertical production-marketing system, 2) there is uncertainty or lack of complete knowledge, 3) prices are determined by quantities and certain 'exogenous' variables, where 'exogenous' means that a variable affects variables within the beef economy, but is not in turn affected by them, 4) current supplies are determined by past decisions, 5) producers respond to price, given fixed production lags, 6) producers view current and past prices as the best estimate of future prices, in the face of uncertainty, and 7) the rate of change of production is proportional to the deviation of price from equilibrium."

These assumptions are not clearly incorporated into his model.

Fourth, economic agents in the cattle sector of the livestock economy and their economic activities are not clearly specified. It is pertinent to ask why and under what conditions any economic agent does respond to any given stimulus, e.g., price level. However, since Ehrich does not enumerate the fundamental activities of beef breeders, feed-lot operators, and slaughter-house operators, his model may be solely a collection of the statistical relationships among the strategic
variables which characterize the cattle industry.

Fifth, a crucial limitation of Ehrich's model is related to the identification of economic relations, i.e., "inventory relations," developed with a view toward quantifying the elements of changes in cattle production. Ehrich (18, p. 14) identified his model as follows:

"The relations may be summarized in words as follows: heifers on farms in year \( t \) depend on the lagged deviation of price from equilibrium and on calf numbers in \( t-1 \); 2) the annual change in cow numbers depends on the culling rate and on the replacement rate; 3) calves on farms on January 1 of year \( t \) depend on cow numbers on January 1 of the year \( t-1 \) and on the lagged price of feeder calves; and 4) number of all steers, heifers, and calves on farms on January 1 depend on the number of calves on farms the previous year and on the rate of marketing of calves, steers, and heifers during the two previous years. (Rate of marketing is taken to mean the rate of marketing for slaughter, whether from feedlots or directly from pasture.) The rate of marketings is affected by placements on feed and retention of breeding stock, which in turn are a function of lagged prices."

It should be noted, however, that since Ehrich does not take into account the role of breeding animals as a collection of durable factors and the fundamental activities of beef breeders, his "inventory relations" become those which relate the number of various classes of cattle kept on farms (or their changes over time) to arbitrary, strategic variables. Thus his "inventory relations" describe and/or explain the behavior of observable statistical variables rather than the behavior of beef breeders.
with regard to their breeding animals.

Finally, another crucial limitation of Ehrich's model is related to the insistence on the fixed production lag and the fixed response pattern of producers (i.e., beef breeders, feed-lot operators, and slaughter-house operators).

A brief critique of self-generating mechanisms as alternative endogenous hypotheses, together with a review of other hypotheses, implies that the explanation of any cyclical phenomena in the strategic variables characterizing the cattle industry should be based on a systematically developed set of hypotheses from the fundamental activities of economic agents rather than a blind application of any existing economic theorems to a set of time series data of these strategic variables.

D. Identification of the Fundamental Activities of a Beef Breeder as "Investment"

From the foregoing, it is clear that an explanation of the cyclical phenomena in time series data for the strategic variables characterizing the cattle industry entails a systematically developed hypothesis (or a set of hypotheses) which is based upon the fundamental activities of economic agents. For purposes of the present study a brief discussion of the fundamental activities of a beef breeder (or beef breeders as a group)
and the identification of his (or their) activities as "investment" is relevant.

Of primary consideration are the fundamental activities of a beef breeder within the framework of a beef production process. He maintains a herd of breeding animals (i.e., cows, heifers, heifer-calves, bulls, and bull-calves) as a collection of durable factors of production for every production period, the nature of the collection depending upon such factors as price level of feeder cattle, price of salvage cattle, available amount of feed resources, and biological properties of breeding animals. Following the technology of growth processes associated with beef cattle, the breeding cows are capable of producing calves for each production period. However, it should be recognized that the productivity of breeding animals changes as they grow older.2/ The beef breeder markets calves, except those for replacement, and breeding animals which are regarded as salvage cows. Consequently, the beef breeder gets some kind of "economic" returns" for those beef animals marketed.

The question arises: How can the fundamental activities of a beef breeder with respect to his herd of breeding animals be identified? One way is to regard the fundamental activities of a beef breeder as "investment" and the beef breeder as an "investor." If this terminology is adopted, the meaning of

2/ See Appendix I.
"investment" and "investor" should be qualified. In economic literature, many different meanings have been applied to "investment" and "investor." Often "investment" means simply buying something for money, whether it is a physical object, a security, or a promise of repayment. Investment may also be the transfer of a certain amount of wealth from one ownership, or employment, to another. Then nearly everybody would be an investor, to some extent. It is quite common to call the purchase of bonds or shares of stock an act of investment and to call those who engage in this transaction "investors."

This study will use the definitions of "investment" and "investor" stated by Haavelmo (32, p. 159):

"We define investment as acquisition of capital goods for productive purposes and investors as those who make decisions in this respect. That is, we think of the investor only as a person or decision unit who is responsible for the use of capital and for what happens to it. In fact, the investor will have to be a producer of some kind."

According to the narrow sense of words specified above, the beef breeder and his activities within the framework of cattle industry may be identified as the "investor" and "investment," respectively. Therefore, the fundamental activities of a beef breeder will be regarded henceforth as investment activity.
E. Statement of Problems

In order to improve understanding of the cyclical fluctuations in strategic, aggregate variables which characterize the cattle industry, it is now clear that the fundamental activities of beef breeders concerning their breeding cows can hardly be ignored. Thus, the primary questions to be probed in the present study are: How do beef breeders determine the size and composition of their breeding herds? What is the relation between these decisions and the empirical existence of cycles in the cattle industry? In order to broaden the understanding of the cyclical fluctuations in number of breeding cows kept on farms, number of replacement heifers kept on farms, calf price, salvage cow price, and size of calf crop, there are four secondary questions to be considered in the present study. They are: (a) How do beef breeders determine the optimum number of replacement heifers and adjust their actual number of replacement heifers to the desired level? (b) How is the calf price determined? (c) How is the salvage cow price determined? (d) What is the size of calf crop from a given size of aggregate herd of breeding cows?
F. Scope of This Study

Chapter II develops an investment behavior relation, together with other closely related behavior relations, which reflects both these characteristics which are shared by decisions to retain a breeding herd and the unique properties associated with the beef breeding cows. In making explicit the considerations which underlie the determination of a breeding herd of optimum size and composition, the model of the hypothetical breeding firm shows that a breeding herd of optimum size and composition depends not only on the optimum culling age but also on the available amount of feed resources. The optimum last (or culling) age is determined by the beef breeder; the available amount of feed resources is determined by weather conditions, range feed conditions, and so on. However, the retainment of a breeding herd of optimum size and composition differs from that of, say, nondurable inputs in that adjustment time and transaction costs for the breeding animals are somewhat greater than those for the nondurable inputs. In addition, if one or more of the economic data is not available to the beef breeder, he tends to estimate these data on the basis of his past experience and/or scientific experiment. Thus, the special properties of durable inputs and the mode of estimating some unknown economic data must be incorporated into a theory...
of investment behavior. This is done in the investment behavior relation set forth in this study.

Chapter III submits three alternative hypotheses and a multiple-relation model fulfilling the objectives of the present study. The theoretical constructs discussed for a representative breeding firm in Chapter II are utilized to formulate three alternative hypotheses which are capable of explaining the investment activities of beef breeders. By extending the hypotheses about the investment behavior of beef breeders, a multiple-relation model is formulated capable of explaining the investment behavior of beef breeders concerning their breeding cows as well as replacement heifers, the determination of calf price, the determination of salvage cow price, and the calf production process.

Chapter IV presents empirical evidence in support of the three alternative investment behavior hypotheses and those of the multiple-relation model. On the basis of the available data as well as statistical methods, the best approximation of the structural parameters and the explanatory power of the various hypotheses are obtained and summarized. In addition, the statistical tests employed are presented. However, in following the objective of the present study, the formulated hypotheses are evaluated in terms of explanatory power and the compatibility between the sign of expected coefficient and the sign of estimated
coefficient. Thus, many other difficulties in the estimation procedures and the statistical tests of validation associated with various hypotheses are purposely left out of consideration.

The last chapter recapitulates the hypotheses submitted in the present study as a set of refutable hypotheses. It also discusses the qualification of the refutable hypotheses and indicates the directions of future research which might improve understanding of the cyclical phenomena in the number of beef breeding cows kept on farms, the number of replacement heifers kept on farms, the number of beef cattle kept on feed, the number of slaughter cattle, and the associated price levels in terms of fundamental activities of economic agents involved.
CHAPTER II

AN ANALYSIS OF A BEEF BREEDING FIRM

In the preceding chapter one important implication, which deals with the objectives of the present study, is that the cyclical phenomena in time series data for those strategic variables characterizing the cattle industry may be best explained by means of analyzing the fundamental activities of economic agents--beef breeders, feed-lot operators, and slaughter-house operators--in the cattle industry.

Another implication, from the methodological point of view, is that in the past there have been very few attempts to formulate an aggregate dynamic model, based on a systematically developed theory of the firm. Thus it can be argued that the previous aggregate analyses, which are not explicitly based on the analysis of firm, are subject to grave deficiencies in at least two respects: (a) a set of economic relations thus formulated may not adequately describe and/or explain the economic activities in the cattle industry; and (b) an explicit interpretation of empirical results may be virtually impossible in the absence of a well developed theoretical structure.

Such implications prompt consideration of two theoretical
questions about the fundamental activities of a beef breeder concerning his herd. They are: (a) How would a "representative" beef breeder determine his "optimum" breeding herd? (b) How would he "adjust" his actual to his "optimum" breeding herd.

Upon such considerations, a model of investment behavior will be developed and stated as a refutable hypothesis.

The model of investment behavior derived in the present chapter is essentially based upon the investment activities of a beef breeder with respect to his breeding herd (which consists of breeding females of all ages) when he is faced with given prices, or price expectations, and wants to maximize his net "accumulated" discounted return from his investment in a breeding herd.

In developing the investment behavior model, the framework of the traditional theory of firm is modified and studied. The traditional, neoclassical theory of the firm is, however, subject to limitation in the area of investment in the stock of capital or durable input. That theory places emphasis on the flow of factor inputs and product outputs, and the only consideration given to the firm's stock position is buried in the production function. Therefore, the traditional theory may not be directly applied to the case of breeding firm in which a herd of breeding animals grows and reproduces its kind regularly over time.
according to the course of nature. Thus, a modified theory, in which emphasis is shifted from flows to stocks, is needed for studying the entrepreneur's activities.

The investment behavior model of the firm presented below allows an additional perspective to be brought to the theory of the firm. The entrepreneurial motivation assumed in the present formulation is compatible with that in the traditional theory. In the traditional theory, profit is defined as the difference between current revenues and current costs specified in terms of units of output produced (and sold); or the net discounted (expected) return is formulated as the difference between the discounted (expected) return and the discounted (expected) cost in terms of units of durable input employed. However, in the present model, the net "accumulated" discounted (expected) return at the beginning of each production period is defined as the difference between the "accumulated" discounted (expected) return from retaining a herd of breeding animals at the beginning of each production period and the "accumulated" cost of retaining a herd of breeding animals at the beginning of each production period.3/ In addition, while optimization in the traditional

3/ The definition of net "accumulated" discounted return, "accumulated" discounted return, and "accumulated" cost will be given in section II, A, 1.
theory is constrained by the purely technological production function, that in the present formulation is constrained by the "accumulated" production function and the available amount of feed resources. 4/

An empirically testable investment behavior equation is immediately implied. Because the equation was derived in this manner, with the form of relation and its variables specified, the attendant assumptions necessary to the derivation are explicitly identified.

The change in orientation from the traditional theory to the proposed theoretical framework is a simple result of the change in the kind of question under consideration. Either of two basic questions is posed by the traditional theory: (a) What is the optimum rate of output, given product price, cost, and market structure? (b) What is the optimum rate of input, given product price and market structure? The answers to these questions are best obtained by using a flow approach and the formulation it employs. By redirecting attention to the firm's breeding herd of optimum size and composition, thus changing the underlying problems, the

4/ The "accumulated" production function is defined as that which relates the "accumulated" productivities of beef breeding cattle over age variable to the last age of breeding animals included.
implications of the model are of interest, and a modified view of the firm becomes pertinent.

This chapter is separated into four parts. In the first part, the optimum solutions of a hypothetical, stationary ranch operation will be analyzed under a set of simplifying assumptions. In the second part, the optimum solutions of a few alternative stationary processes of ranch operation will be exemplified. In the third section, a few numerical examples of the herd adjustment process over time will be discussed. In the last part, an economic model, which is designed primarily for explaining the investment activities of the representative rancher concerning his breeding cows, will be developed on the basis of theoretical discussions and some modifications of the initial simplifying assumptions. A short summary section concludes the chapter.

A. Optimum Solutions of Ranch Operation

The optimum solutions for a hypothetical ranch operation, as a set of choice criteria, will be determined and discussed in the present part. For simplicity, this part is divided into six sections. The first section presents the basic setting of the stationary ranch operation, a set of simplifying assumptions, and the definitions of the various quantities used in the development. The second section is devoted to the derivation of the
breeding herd of optimum size and composition. The third section deals with the determination of the number of calves produced from the optimum breeding herd. The fourth section presents the determination of the optimum number of calves sold. The fifth section is concerned with the determination of the optimum number of salvage cows. The effect, on the ranch operation of changes in economic data and available amount of feed resources will be discussed in the last section.

1. **The Problems, Assumptions, and Definitions**

Consider a beef breeding ranch where a "fixed" amount of feed resources is available to maintain a breeding herd of certain size and composition at the beginning of each production period. The rancher operates his ranch within the framework of a market economy in which such economic data as the price level of calves sold, the price level of salvage cows, the cost of care and maintenance, and the market rate of interest are known and stable over time. At the beginning of each production period, he replaces and/or retains and/or culls the breeding animals in the herd which is inherited from the preceding production period, depending upon the productivity of the breeding animal and the
optimum size and composition of the breeding herd.\(^5\) A herd of two-year old and older breeding animals, which are bred at the beginning of each production period, grows and produces calves at the end of the same production period (say, a calendar year). The breeder sells the calves, except those for replacement, produced in each production period in the calf market at the beginning of each subsequent production period (or at the end of same production period).

In considering such a ranch operation, the answers to four questions are appropriate. (a) What would be the breeding (and working) herd of optimum size and composition at the beginning of a given production period? (b) What would be the number of calves produced from the working herd of optimum size and composition at the end of a given production period? (c) What would be the number of calves sold at the beginning of a given production period? (d) What would be the number of breeding cows culled at the beginning of a given production period?

In order to answer the above questions, a large number of assumptions have to be made. Some of these assumptions undoubtedly are drastic simplifications; they will be modified later on.

\(^5\) The definition of breeding and working herd will be suggested immediately after a set of simplifying assumptions below.
(a) An initial assumption is that all breeding animals have equally long reproductive life—that is, each breeding animal is reproductive up to age M.

(b) All breeding animals are homogeneous with respect to productivity associated with age, flesh condition associated with age after calving, amount of feed fed per production period, amount of care and maintenance required per production period, and weight.

(c) The marginal productivity of each breeding animal rises up to a certain age, after which it declines with age. "Marginal" is used in the sense of "additional" as the animal becomes one year older than before. It implies that the accumulated (or total) productivity of a breeding animal rises at first at an increasing rate and then a decreasing rate as the animal becomes older.\(^6\)

(d) The amount of care and maintenance per production period is constant for each breeding animal, regardless of age. This implies that the accumulated (or total) amount of care and maintenance rises with age at a constant rate.

(3) The index of flesh condition of each breeding animal after calving declines with age. This implies that the accumulated

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\(^6\) This assumption is based upon the empirical study. See Appendix I and/or the detailed discussion of Burke (7).
(or total) index of flesh condition rises at a declining rate with respect to age. 7/

(f) The rancher follows the policy of maintaining an equal number of breeding animals in each age group of his breeding herd— that is, the age distribution of his breeding herd is uniform. 8/

(g) An assumption is made about the entrepreneur's optimization principle, that at the beginning of a given production period for the same production period he finds: (a) the "accumulated" discounted return from retaining a herd of breeding animals of all ages ranging from 0 to j inclusive; (b) the "accumulated" cost of care and maintenance incurred for breeding animals of all ages ranging from 0 to j inclusive; and (c) the "accumulated" salvage cost of breeding animals of all ages ranging from 0 to j inclusive. The assumption is made that he maximizes the net "accumulated" discounted return from a breeding herd with respect to the last age of breeding animals included in his

7/ This assumption is also based upon the discussion of Burke (7).

8/ Of course, the rancher may follow the policy of an unequal number of breeding animals in each age group of his breeding herd. For convenience, the present part and the following part will be predicated on an assumption of uniform age distribution. However, in the third and the last parts, this assumption will be modified.
breeding herd, or with respect to the period of investment on the breeding animals.

Together with these assumptions, the following definitions are made:

\[ b_t : \text{the breeding herd, which consists of breeding animals of all ages at the beginning of production period } t. \text{ It is measured in terms of the number of breeding animals according to age and specified in row vector as follows:} \]

\[ b_t : (n_t(i); \text{ where } i = 0, 1, \ldots, M) \]

\[ 2.1) \]

or

\[ 2.1a) \]

\[ b_t : (n_t(0), n_t(1), \ldots, n_t(M)), \]

where \( n_t(i) \) represents the number of \( i \)-year old breeding animals at the beginning production period \( t \);

\[ w_t : \text{the working herd, which consists of breeding animals two years old and older at the beginning of production period } t. \text{ It is similarly measured by the number of breeding animals according to age and expressed in row vector as follows:} \]

\[ w_t : (n_t(i); \text{ where } i = 2, 3, \ldots, M) \]

\[ 2.2) \]

or

\[ 2.2a) \]

\[ w_t : (n_t(2), n_t(3), \ldots, n_t(M)); \]

\[ f_t(i) : \text{the marginal productivity of an } i \text{-year old breeding animal, or of the } i \text{-th age group, at the beginning of production period } t. \text{ It is measured by the probability of calving by an } i \text{-year old breeding animal;} \]
$s_t(i)$: an index of flesh condition of an i-year old breeding animal at the beginning of production period $t$. It is measured by a quality index characterizing flesh condition after calving;

$p_{t+1}$: the price per calf sold for the production period $(t+1)$;

$p_{cm}$: the cost of care and maintenance incurred per breeding animal for the production period $t$;

$p_{co}$: the price level of salvage cows sold per animal for the production period $t$;

$r_t$: the market rate of interest for the period $t$;

$cc_t(i)$: the number of calves produced by the i-year old breeding animals at the end of production period $t$. It is approximated by the product of the number of i-year old breeding animals bred and their marginal productivity,

\[ cc_t(i) = n_t(i)f_t(i); \]

$v_{t+1}(i)$: the marginal return derived from retaining the i-year old breeding animals at the beginning of production period $(t+1)$, or at the end of production period $t$. It is measured by the product of $p_{ca}$ and $cc_t(i)$;

\[ v_{t+1}(i) = p_{ca} cc_t(i); \]

$c_t^p(i)$: the marginal cost of breeding animals when they become i-year old at the beginning of production period $t$, imputed by the alternative marginal cost (or value) of breeding animals as beef animals. This cost is
2.5) \[ c_t^P(i) = n_t(i) p_t^c o s_t(i); \]

\( c_t^s(i) \) : the marginal cost of care and maintenance incurred for the i-year old breeding animals during the production period \( t \). It can be represented as

\[ c_t^s(i) = n_t(i) p_t^{cm}; \]

\( v_t^d(i) \) : the marginal discounted return from retaining the i-year old breeding animals at the beginning of production period \( t \), that is,

\[ v_t^d(i) = v_{t+1}(i)/(1+r_t); \]

\( \Pi_t(i) \) : the net marginal discounted return from retaining the i-year old breeding animals for one production period at the beginning of production period \( t \), i.e.,

\[ \Pi_t(i) = v_t^d(i) - c_t^P(i) - c_t^s(i); \]

\( \Pi_t \) : the net accumulated discounted return from retaining breeding animals of ages ranging from 0 to \( j \) for one production period at the beginning of production period \( t \). It can be represented as

\[ \Pi_t = \sum_{i=0}^{j} \Pi_t(i) \]

where \( j \) is an explicit age variable which

\[ \text{2/ It should be noted that the net marginal discounted returns from the 0-year old and 1-year old breeding females are negative since these animals do not calve, whereas they need certain care and maintenance. The costs which are borne by the rancher for maintaining these animals may perhaps be regarded as "cost of waiting for calf crops" or "producer's time preference."} \]
denotes the last age of breeding animals included in the breeding (as well as working) herd at the beginning of production period t. 10/

2. The Breeding Herd of Optimum Size and Composition

From the simplifying assumptions stated in the preceding section, it is apparent that the optimum size of breeding herd is determined (or constrained) by the available amount of feed resources, whereas the optimum composition of the breeding herd is determined by the profitability of retaining breeding animals of various ages. Since it was assumed that the rancher follows a policy of maintaining an equal number of breeding animals in each age group, the crucial question is that of the optimum last age of breeding animals included in his breeding herd, or the optimum last age at which the breeding animals should be culled from the herd. The present section considers the optimum condition for maximizing the net accumulated discounted return from retaining a breeding herd and explains the optimum last:

10/ The "accumulated" cost of care and maintenance and the "accumulated" salvage cost are \( \sum_{i=0}^{j} c^s_t(i) \) and \( \sum_{i=0}^{j} c^p_t(i) \), respectively.
age of breeding animals included in the breeding herd in terms of the economic data. The determinants of the optimum size and composition can then be appropriately specified.

(1) **The optimum Last Age of Breeding Herd:** The net accumulated discounted return from retaining a breeding herd for one production period at the beginning of a given production period depends on the number of breeding animals included in each age group of the breeding herd, the productivity associated with age, the index of flesh condition associated with age, the last age of breeding animals included, the price level of calves sold, the price level of salvage cows sold, the cost of care and maintenance, and the market rate of interest. By a series of iterative substitutions, (i.e., (2.3) into (2.4); (2.4) into (2.7); (2.5), (2.6), and (2.7) into (2.8); and the (2.8) into (2.9)), \( \Pi_t \) as a function of the last age of breeding animals included, \( j \), can be expressed as

\[
\Pi_t = \sum_{i=0}^{j} \left\{ \left( \frac{p_{t+1}^a n_t(i) f_t(i)}{(1+r_t)} \right) - n_t(i) p_t^{cm} - n_t(i) p_t^{co} s_t(i) \right\} .
\]

Employing the assumptions (b), (c), (d), (e), and (f) and dropping the time subscripts give
\[ \Pi = n \left[ \sum_{i=0}^{j} p^{ca} f(i)/(1+r) \right. \\
- \sum_{i=0}^{j} p^{cm} \int_{0}^{j} f(i) \, di \\
- \sum_{i=0}^{j} p^{co} \int_{0}^{j} s(i) \, di \right], \]

where \( n \) represents an equal number of breeding animals in each age group. If the age variable is regarded as continuous, then

\[ \Pi = n \left[ \int_{0}^{j} p^{ca} f(i)/(1+r) \, di \\
- \int_{0}^{j} p^{cm} \, di - \int_{0}^{j} p^{co} \, s(i) \, di \right] \]

or

\[ \Pi = n \left[ p^{ca} / (1+r) \int_{0}^{j} f(i) \, di \\
- p^{cm} \int_{0}^{j} \, di - p^{co} \int_{0}^{j} s(i) \, di \right] \]

or

\[ \Pi = n \left[ \left( p^{ca} / [1+r] \right) (F(j) - F(0)) \\
- p^{cm} j - p^{co} (S(j) - S(0)) \right]. \]

Differentiating equation (2.12b) with respect to \( j \) under the assumption (g) and setting the result equal to zero lead to

\[ \frac{dF(j)}{dj} \]

2.13) \( p^{ca} / (1+r) \) \( \frac{dF(j)}{dj} \) \( = p^{cm} + p^{co} \frac{dS(j)}{dj} \).

11/ Functions \( F(j) \) and \( S(j) \) may best regarded as the "accumulated" production function and the "accumulated" salvage cattle production function, respectively.
There are two points to be noted in the necessary condition for maximizing the rancher's net accumulated discounted return. First, since he follows a policy of maintaining an equal number of breeding animals in each age group, \( n \) is independent of the necessary condition for maximizing his net accumulated discounted return with respect to the last age of breeding animals included. Second, the necessary condition requires that the net marginal return from the \( j \)-year old breeding animal(s) must be equated to the sum of marginal salvage cost and marginal cost of care and maintenance of the \( j \)-year old breeding animal(s).

The optimum last age of breeding animals included in the breeding herd, \( j^* \), can be then determined from the necessary condition. Solving equation (2.13) for \( j \) yields a general form of relation which relates the optimum last age of breeding animals included in the breeding herd and the economic data as follows:

\[
2.14) \quad j^* = G(p^{ca}, p^{cm}, p^{co}, r)
\]

where the economic data are regarded as the "parametric" variables. (A numerical example on the determination of the optimum last age of breeding animals included in the breeding herd will be shown in section II, B, 1.)

(2) The optimum Number of Breeding Animals in Each Age Group: The optimum number of breeding animals in each age group can now be determined by dividing the optimum total
number of breeding animals, which can be retained on the available amount of feed resources, by $j^* + 1$ (since the age 0 should be added). The optimum total number of breeding animals to be retained can be regarded as a monotonically increasing function of the available amount of feed resources. Then the optimum number of breeding animals in each age group, $n^*$, can be expressed as a function of the available amount of feed resources, $F$, as follows:

\[ 2.15) \quad n^* = \emptyset (F; j^*) \]

where $j^*$ is regarded as given.

(3) The Breeding Herd of Optimum Size and Composition:

Several factors must be considered which determine the breeding herd of optimum size and composition at the beginning of production period $t$. From what was advanced above, it is evident that the breeding herd of optimum size and composition depends upon the optimum last age of breeding animals included in the breeding herd and the optimum number of breeding animals in each age group. The breeding herd of optimum size and composition at the beginning of production period $t$, $b_t^*$, can then be regarded as a function of $j_t^*$ and $n_t^*$ and expressed as follows:

\[ 2.16) \quad b_t^* = H_0 (j_t^*, n_t^*) . \]
Substituting equations (2.14) and (2.15) into (2.16) yields

\[ b_t^* = H_1(p_{t+1}^{ca}, p_t^{cm}, p_t^{co}, r_t, F_t). \]

Thus the determinants of the breeding herd of optimum size and composition are specified in equation (2.17). Since the working herd is defined as a part of the breeding herd, the determinants of the working herd of optimum size and composition can be analogously expressed as follows:

\[ w_t^* = H_2(p_{t+1}^{ca}, p_t^{cm}, p_t^{co}, r_t, F_t). \]

3. The Number of Calves Produced

A supposition is that the rancher retains the working herd of optimum size and composition and breeds them at the beginning of production period \( t \). Then the total number of calves produced from the working herd of optimum size and composition at the end of production period \( t \), \( cc_t^* \), is the sum of the number of calves produced at the end of production period \( t \) by the breeding animals of ages ranged from 2 to \( j_t^* \), that is,

\[ cc_t^* = \sum_{i=2}^{j_t^*} cc_t^*(i). \]

In conjunction with equation (2.3), equation (2.19) becomes

---

12/ By substitution, equation (2.16) becomes as (2.16a)

\[ b_t^* = H_0(\emptyset (F; G(p_{t+1}^{ca}, p_t^{cm}, p_t^{co}, r_t)), \]

\[ G(p_{t+1}^{ca}, p_t^{cm}, p_t^{co}, r_t)). \] Since the primary interest is in the
Since the optimum number of breeding animals in each age group is constant, equation (2.20) can be alternatively expressed as follows:

\[
2.20a) \quad c^* = n^* \sum_{i=2}^{j^*} f(i).
\]

Thus the total number of calves produced from the working herd of optimum size and composition at the end of production period \(t\) is the product of the optimum number of breeding animals in an arbitrary age group for the production period \(t\) and the accumulated productivity of breeding animals of ages ranging from 2 to \(j^*_t\).

4. The Number of Calves Sold

The number of calves sold by the rancher at the beginning of a given production period within the framework of a certain stationary process is determined by the difference between the number of calves produced from the working herd at the end of the preceding production period and the number of replacement calves to be added to the breeding herd at the beginning of the same production period. Thus the optimum number of calves factors determining the optimum breeding herd, equation (2.16a) is simplified as (2.17).
sold at the beginning of production period \((t+1)\), \(c_{s_{t+1}}^*\), can be simply determined as follows:

\[
2.21) \quad c_{s_{t+1}}^* = c_{c_{t}}^* - n_{t+1}^*(0)
\]

where \(n_{t+1}^*(0)\) represents the optimum number of replacement calves, which are 0-year old at the beginning of production period \((t+1)\), to be added to the breeding herd at the beginning of production period \((t+1)\).

5. **The Number of Cows Culled**

The number of breeding cows culled at the beginning of production period \((t+1)\) depends upon the optimum size and composition of the breeding herd at the beginning of the same production period and the aging processes of breeding animals. Since the rancher operates his ranch under the case where the available amount of feed resources as well as the economic data are stable over time, his optimum size and composition of breeding herd remain the same. The optimum number of breeding cows culled at the beginning of production period \((t+1)\), \(s_{c_{t+1}}^*\), is then equal to the number of breeding animals in the last age group of the breeding (and working) herd of optimum size and composition during the production period \(t\), that is,
2.22) \[ s_{t+1}^c = n_t^* (j_t^*) \]

where \( n_t^* (j_t^*) \) designates the optimum number of breeding animals in the last age group during the production period \( t \).

(It should be noted that the number of cows culled, the number of replacement calves added, and the optimum number of breeding animals in the other age groups are equal as long as the optimum last age and the available amount of feed resources remain stable over time.)

6. The Effect of Changes in Economic Data and or Available Amount of Feed Resources on the Optimum Solutions for the Hypothetical Ranch Operation

The optimum solutions for the stationary process of ranch operation have been considered as a set of choice criteria. The present section will study the effect of once-and-for-all changes in economic data and/or available amount of feed resources on the optimum solutions for the stationary process of ranch operation. From what has been advanced above, it is apparent that if there is a once-and-for-all change in one, or more, of the economic data, then there will be a change in the optimum last age of breeding animals included in the breeding herd. Consequently, a once-and-for-all change in the optimum last age of breeding animals included in the breeding herd, with or without...
a change in the available amount of feed resources, brings about a change in the optimum size and composition of the working herd, the number of calves produced, the optimum number of calves sold, and the optimum number of breeding cows culled. Therefore, the effect of changes in economic data on the optimum last age of breeding animals included in the breeding herd and the effect of changes in the optimum last age and/or the available amount of feed resources on the ranch operation will be briefly studied.

Of first consideration is the effect of changes in economic data on the optimum last age of breeding herd. It is evident from equation (2.14) that changes in the optimum last age of the breeding herd depend on the changes in price level of calves sold, price level of salvage cows sold, cost of care and maintenance, and market rate of interest. In order to find out the effect of changes in economic data on the optimum last age, the economic data (as variables) are allowed to vary simultaneously. This is accomplished by total differentiation of equation (2.13),

\[
2.23) \quad \frac{dp}{(1+r)} \frac{ca}{dj} \frac{dF(i)}{dj} + \frac{p}{(1+r)} \frac{ca}{dj} \frac{d^2F(i)}{dj^2} dj \\
- \frac{p}{(1+r)^2} \frac{ca}{dj} \frac{dF(i)}{dj} dr = dp \frac{cm}{cm} + \frac{dS(i)}{dj} \frac{dp}{co} \\
+ \frac{p}{co} \frac{d^2S(i)}{dj^2} dj .
\]
Solving for \( dj \) and designating the change in the optimum last age by \( dj^* \) yield

\[
2.24) \quad dj^* = \frac{-\frac{dp^{ca}}{(1+r)} \frac{dF(j)}{dj} + \frac{pc}{(1+r)^2} \frac{dF(j)}{dj} dr + dp^{cm} + \frac{dS(j)}{dj} dp^{co}}{-\frac{pc}{(1+r)} \frac{d^2F(j)}{dj^2} - \frac{pc}{d^2S(j)} \frac{d^2S(j)}{dj^2}}
\]

where the denominator to the right in the above equation is expected to be negative because of earlier assumptions in section II, A, 1, i.e., \( d^2F(j)/dj^2 < 0 \) and \( d^2S(j)/dj^2 = 0 \).

The implications of equation (2.24) are obvious in regard to the variation in the optimum last age of breeding animals included in the breeding herd, \( dj^* \), depending upon the changes in \( p^{ca}, p^{cm}, p^{co}, \) and \( r \), viz., \( dp^{ca}, dp^{cm}, dp^{co}, \) and \( dr \), respectively. The variation in the optimum last age due to the changes in economic data can be briefly summarized: \( dp^{ca} \) brings about a positive effect on the optimum last age when \( dp^{cm} = dp^{co} = dr = 0 \) since the denominator to the right in equation (2.24) is negative as the sufficient condition for maximizing the net accumulated return. However, \( dp^{cm} \) and/or \( dp^{co} \)

13/ A numerical example will be given in section II, B, 2.
and/or dr bring about negative effects upon the optimum last age when \( dp_{ca} = 0 \). This generalization is, in fact, compatible with the qualifications of coefficients expected in the traditional, theoretical discussions, and it provides a priori qualifications on the coefficients of the economic variables in equation (2.14).

Next to be considered is the effect of changes in the optimum size and composition of breeding herd, the number of calves produced from the optimum size and composition of working herd, the number of calves sold, and the number of breeding cows culled. Three changes are to be considered: (a) the optimum last age changes because of changes in economic data; (b) the available amount of feed resources is changed; and (c) the optimum last age and the available amount of feed resources are simultaneously changed.

Under change (a), if the optimum last age is increased (decreased) as a result of changes in the economic data, then the number of age groups will be increased (decreased) and the optimum number of breeding animals in each age group will be decreased (increased). The number of calves produced from the optimum size and composition of the working herd will be increased (decreased) on account of increasing (decreasing) the optimum last age, as long as the number of breeding animals in each age group remains unchanged. Consequently, the number
of calves sold will be increased (decreased) and the number of breeding cows culled will be decreased (increased).

Under change (b), if the available amount of feed resources is increased (decreased) and if the optimum last age of breeding animals included in the breeding herd remains the same, then the optimum number of breeding animals in each age group will be increased (decreased), the number of calves produced from the optimum size and composition of the working herd will be increased (decreased), the number of calves sold will be increased (decreased), and the number of breeding cows culled will be increased (decreased).

Under change (c), if the available amount of feed resources is increased (decreased) and if the optimum last age of breeding animals included in the breeding herd is increased (decreased), then the optimum number of breeding animals in each age group will be increased (decreased) and the number of age groups in the breeding herd will be increased (decreased). Consequently, the number of calves produced, the number of calves sold, and the number of breeding cows culled will be increased (decreased).

B. Numerical Examples of Optimum Solutions

The present part will consider several numerical examples of the optimum solutions for a few alternative stationary
processes of ranch operation on the basis of the discussion presented in the preceding part. In the first section, the optimum solutions for a stationary process of ranch operation will be determined where the hypothetical values of economic data and the available amount of feed resources are assigned. In the second section, the effect of changes in economic data on the optimum last age will be reviewed in the light of a hypothetical diagram. In the third section, the optimum solutions for a few alternative stationary processes, which differ from the stationary process in section II, B, 1 as a result of change in the available amount of feed resources and/or the optimum last age, will be delineated. In the last section, a brief remark will be made as to the optimum solutions for the nonstationary processes of ranch operation.

1. **A Stationary Process of Ranch Operation**

A hypothetical case is that of a rancher who has feed resources to maintain 170 breeding animals every production period. The price level of calves sold is $20.00 per hundredweight, the price level of salvage cows sold is $15.00 per hundredweight, the cost of care and maintenance is zero, and the market rate of interest is five percent per annum. Furthermore, the marginal

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14/ On the basis of the simplifying assumption (b) in section II, A, 1, the price level of calves sold and salvage cows is conveniently measured per hundredweight rather than per animal.
productivity and the index of flesh condition associated with age are known to be those values in the second and third columns of Table 1, respectively. The net marginal discounted return from an i-year old breeding animal (or the i-th age group of breeding animals) and the marginal salvage cost of an i-year old breeding animal (or the i-th age group of breeding animals) are those values given in the fourth and fifth columns of Table 1, respectively. The net accumulated discounted return from the breeding herd and the accumulated salvage cost are presented in the six and seventh columns of Table 1, respectively.

On the basis of what was advanced in section II, A, the optimum size of breeding herd is 170 breeding animals for each production period. In order to determine the optimum composition of the breeding herd, it is necessary to find out the optimum last age of breeding animals included in the breeding herd. By comparing the net marginal discounted return with the marginal salvage cost in Table 1, 16 is found to be the optimum last age at which the net marginal discounted return barely covers the marginal salvage cost. Thus, the breeding herd of optimum size and composition at the beginning of production period \( t \), \( b_t^* \), is

\[
2.25) \quad b_t^* : (n_t^*(0), n_t^*(1), \ldots, n_t^*(16))
\]

where \( n_t^*(i) = 10 \) for \( i = 0, 1, \ldots, 16 \). The working herd of
Table 1. The hypothetical schedules for determining the optimum last age.

<table>
<thead>
<tr>
<th>i, j</th>
<th>f(i)</th>
<th>s(i)</th>
<th>( \frac{p^{\text{CAF}}(i)}{(1+r)} )</th>
<th>( p^{\text{CO}}(i) )</th>
<th>( \sum_{i=0}^{j} \frac{p^{\text{CAF}}(i)}{(1+r)} )</th>
<th>( \sum_{i=0}^{j} p^{\text{CO}}(i) )</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>$0.000$</td>
<td>$15.000$</td>
<td>$0.000$</td>
<td>$15.000$</td>
</tr>
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<td>1</td>
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<td>0.000</td>
<td>15.000</td>
<td>0.000</td>
<td>30.000</td>
</tr>
<tr>
<td>2</td>
<td>0.840</td>
<td>1.000</td>
<td>15.994</td>
<td>15.000</td>
<td>15.994</td>
<td>45.000</td>
</tr>
<tr>
<td>3</td>
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<td>0.951</td>
<td>16.374</td>
<td>14.268</td>
<td>32.368</td>
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</tr>
<tr>
<td>4</td>
<td>0.885</td>
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<td>49.218</td>
<td>82.840</td>
</tr>
<tr>
<td>5</td>
<td>0.900</td>
<td>0.861</td>
<td>17.136</td>
<td>12.911</td>
<td>66.354</td>
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<tr>
<td>6</td>
<td>0.910</td>
<td>0.819</td>
<td>17.326</td>
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<td>9</td>
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<td>10.571</td>
<td>134.517</td>
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<td>168.769</td>
</tr>
<tr>
<td>14</td>
<td>0.610</td>
<td>0.549</td>
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<td>8.232</td>
<td>204.288</td>
<td>177.001</td>
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<tr>
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<td>8.568</td>
<td>7.499</td>
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</tr>
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<td>0.472</td>
<td>6.474</td>
<td>7.086</td>
<td>229.421</td>
<td>199.366</td>
</tr>
</tbody>
</table>

(1) The marginal productivity of an i-year old breeding animal is approximated by Dr. John A. Edwards in the light of Burke's study (7). See Appendix I.

(2) The depreciation rate in flesh condition after calving is assumed to be five percent per annum.

(3) The marginal discounted return from an i-year old breeding animal, where the rate of interest is assumed to be five percent per annum and the price level of calves sold is $P.00 per hundred weight.

(4) The marginal salvage cost of an i-year old breeding animal, where the price level of salvage cows is $15.00 per hundred weight.

(5) The accumulated discounted return from a j-year old breeding animal.

(6) The accumulated salvage cost of a j-year old breeding animal.
optimum size and composition at the beginning of production period \( t \), \( w_t^* \), is

\[
2.25 \quad w_t^* : (n_t^* (2), n_t^* (3), \ldots, n_t^* (16)).
\]

The number of calves produced from the optimum working herd depends upon the marginal productivity associated with age and the optimum number of breeding animals included in each age group of the working herd. Since the optimum number of breeding animals in each age group is 10, the total number of calves produced according to equation (2.20a) is: \( cc_t^* = 117.00 \).

The optimum number of replacement calves and the number of breeding cows culled remain the same over time since there are no changes in economic data and the available amount of feed resources. That is, \( cs_{t+1}^* = cc_t^* - n_t^* (0) = 117.00 - 10 = 107.00 \) and \( sc_{t+1}^* = 10 \).

2. **Effect of Change in Economic Data on the Optimum Last Age**

The next task is to inquire into the effect of changes in the economic data on the optimum last age. While the direction and magnitude of changes in economic data on the optimum last age can be analyzed through a variety of numerical examples, this study limits itself to the effect of directional change in economic data on the optimum last age.
In connection with the study of the effect of change in economic data on the optimum last age, the reason can best be made clear by referring to Figure 2, which applies the initial optimum last age determined by the numerical values of economic data in the preceding section. The curve AA' in Figure 2 represents the accumulated discounted return from a breeding herd (of uniform age distribution) as a function of age. The curve a₀a₁ shows the marginal discounted return from an individual animal as a function of age. The curve BB' represents the accumulated cost of a breeding herd. The curve b₀b₁ shows the marginal cost of retaining an individual animal as a function of age. As it is apparent from Table 1 that the optimum last age is 16, the optimum last age can be shown as the age at which the marginal discounted return is at least equal to the marginal cost, viz., the age at which the curve a₀a₁ intersects b₀b₁. (Alternatively, the optimum last age can be shown as the age at which the change in the accumulated discounted return is equal to the change in the accumulated cost, since the accumulated magnitude at a given age is equal to the accumulated sum of marginal magnitudes up to the same age.)

15/ It should be recognized that the curve BB' presents the accumulated cost of a herd which consists of an equal number of breeding animals in each age group.
Figure 2. The determination of the optimum last age
Next to be considered is the effect of change in economic data on the optimum last age. Suppose that there is a fall in the price level of calves sold. Such a decrease in price will shift the curve $a_0a_1$ downward and reduce the optimum last age determined by the curves $a_0'a_1'$ and $b_0'b_1'$, ceteris paribus, and vice versa for a rise in the price level of calves sold. Alternatively, suppose that there is a rise in the salvage cow price (and/or the unit cost of care and maintenance). The rise in the salvage cow price will shorten the optimum last age since the curve $b_0'b_1'$ will be shifted upward to the curve $b_0'b_1'$, ceteris paribus; a fall in the salvage cow price will lengthen the optimum last age. A rise in the market rate of interest, since it will shift the curve $a_0a_1$ downward to the curve $a_0'a_1'$, will shorten the optimum last age, ceteris paribus, and vice versa for a fall in the market rate of interest. The results are consistent with the a priori qualifications on (2.14) indicated in section II, A, 6.

3. The Alternative Stationary Processes of Ranch Operation

Next to be studied is how the optimum solutions for the alternative stationary processes of ranch operation may result from those for an initial stationary process of ranch operation because of a once-and-for-all change in the optimum last age of breeding animals in the breeding herd and/or in the available
amount of feed resources.

For the initial stationary process of ranch operation, it is conveniently supposed that the optimum last age is nine and the available amount of feed resources is sufficient to maintain 100 breeding animals. Then the optimum number of breeding animals in each age group is ten, under the policy of maintaining an equal number of breeding animals in each age group. The number of breeding cows culled is ten since it is assumed that the economic data and the available amount of feed resources have remained stable over time. (See the optimum solutions for the initial stationary process of ranch operation in Table 2.)

The optimum stationary processes presented in Table 2 result from alternative once-and-for-all changes in the optimum last age and/or in the available amount of feed resources at the beginning of production period 1.

Suppose that there is an increase (a decrease) in the available amount of feed resources from 100 to 120 (from 100 to 80) whereas the optimum last age remains the same. Then the number of calves produced will be increased (decreased) from 70.65 to 84.78 (from 70.65 to 56.52). The number of calves sold will be increased (decreased) from 60.65 to 72.78 (from 60.65 to 48.52). The number of breeding cows culled will be increased (decreased) from 10 to 12 (from 10 to 8). (See cases 1
Table 2. The optimum solutions for the alternative stationary processes of ranch operation.

<table>
<thead>
<tr>
<th>t = 0</th>
<th>t = 1</th>
<th>Initial Stationary Process</th>
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<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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<td>(4)</td>
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<td>(5)</td>
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For the sake of simplicity, the optimum size of breeding herd for the fourth alternative stationary process is modified from 100.00 to 99.00.

(1) Case 1 represents the alternative stationary process in which the available amount of feed resources is once-and-for-all increased from the initial stationary process.

(2) Case 2 denotes the alternative stationary process in which the available amount of feed resources is once-and-for-all decreased from the initial stationary process.

(3) Case 3 designates the alternative stationary process in which the optimum last age is once-and-for-all decreased from the initial stationary process.

(4) Case 4 represents the alternative stationary process in which optimum last age is once-and-for-all increased from the initial stationary process.

(5) Case 5 denotes the alternative stationary process in which the available amount of feed resources and the optimum last age are increased from the initial stationary process.

(6) Case 6 designates the alternative stationary process in which the available amount of feed resources and the optimum last age are decreased from the initial stationary process.
Suppose that the optimum last age is once-and-for-all decreased (increased) from 9 to 7 (from 9 to 10) while the available amount of feed resources remains the same. The number of calves produced will be decreased (increased) from 70.65 to 66.25 (from 70.65 to 71.10). The number of calves sold is decreased (increased) from 60.65 to 53.75 (from 60.65 to 62.10). The number of breeding cows culled will be increased (decreased) from 10 to 12.5 (from 10 to 9). (See cases 3 and 4, respectively, in Table 2.)

Finally, suppose that the available amount of feed resources is once-and-for-all increased (decreased) so that the optimum number of breeding animals to be retained is 120 instead of 100 (90 instead of 100) and the optimum last age is once-and-for-all increased (decreased) from 9 to 11 (from 9 to 8). The number of calves produced will be increased (decreased) from 70.65 to 86.95 (from 70.65 to 61.95). The number of calves sold will be increased (decreased) from 60.65 to 76.95 (from 60.65 to 51.95). The number of breeding cows culled will remain the same. (See cases 5 and 6, respectively, in Table 2.)

From the numerical examples given above, it should be noted that if the optimum last age and the available amount of feed resources are stable over time, then the optimum solutions...
for the ranch operation are stable over time as well. That is, the breeding (and working) herd of optimum size and composition, the number of calves produced, the optimum number of calves sold, and the optimum number of breeding cows culled are stable over time as long as the economic data and the available amount of feed resources are stable over time. However, it should also be noted that if there is a once-and-for-all change in the optimum last age and/or the available amount of feed resources, then the optimum solutions for the ranch operation will be accordingly changed, as discussed above, from the level of the initial stationary ranch operation to the alternative level.

4. The Nonstationary Processes of Ranch Operation

A few numerical examples have been studied of the optimum solutions for an arbitrary stationary process of ranch operation and for alternative stationary processes, which differ from the arbitrary process because of a once-and-for-all change in the optimum last age and/or the available amount of feed resources. The optimum solutions for the ranch operation change over time, responding to a chain of changes in the optimum last age and/or the available amount of feed resources. The optimum solutions for a few alternative stationary processes described in Table 2 may be regarded as those for the ranch operation over
time, modifying the classification of alternative cases as the dimension of time. Thus it is evident that there is a chain of changes, instead of a once-and-for-all change, in the optimum last age and/or the available amount of feed resources, over time; that is, the optimum solutions fluctuate as the optimum last age and/or the available amount of feed resources change over time. (In fact, we can think of a large number of cases under which the optimum solutions fluctuate quite differently, depending upon the changes in the optimum last age and/or the available amount of feed resources over time.)

From the above numerical examples, which supplement discussions in section II, A, it is possible to conceive of a large number of both stationary and nonstationary processes of ranch operation as a set of choice criteria which depend on the optimum last age and the available amount of feed resources over time.

C. Adjustment of Breeding Herd

The optimum solutions for the stationary processes of ranch operation, as a set of choice criteria, were discussed in section II, A. The numerical examples of the optimum solutions for a few alternative stationary processes were presented in section II, B, in support of the discussion on a set of choice criteria. The next task is then to consider how the representative
beef breeder may operate his ranch over time, following the technology of growth process of breeding animals included in the breeding herd.

The ranch operation by a representative beef breeder for a series of production periods may perhaps best be regarded as the "herd adjustment" by the rancher in response to his (new) optimum breeding herd and the sale of calves and salvage cows over time. For this study of the ranch operation within such a framework, it is now deemed necessary to clarify what is meant by the "herd adjustment" or the "adjustment of breeding herd." The terms are synonymously used in the sense of any adjustments, i.e., replacing and culling breeding animals, in the actual breeding herd inherited from the preceding production period, in response to the rancher's (new) optimum breeding herd at the beginning of each production period under the policy of maintaining the "fixed" (or "variable") age distribution of breeding herd. 16/ In other words, the rancher who follows the policy of

16/ The "fixed" age distribution of breeding herd is here used as a particular age distribution (e.g., uniform age distribution) which is somehow preferred by the beef breeder over alternative age distributions. For example, if the representative rancher follows a uniform age distribution regardless of any change in the available amount of feed resources and/or the optimum last age, then the fixed age distribution is the uniform age distribution. The "variable" age distribution of breeding herd is used in the present study as an age distribution which is alternatively preferred by the rancher over the others, depending upon the age
a particular age distribution adds the replacement calves to and culls the breeding animals from his actual breeding herd inherited from the preceding production period, depending upon the available amount of feed resources and the optimum last age. As far as the sale of calves is concerned, the rancher sells the calves, except those for replacement, produced in each production period on the calf market at the beginning of each subsequent production period.

It should be noted, however, that the herd adjustment in a situation where the available amount of feed resources and the optimum last age are stable over time is different from one in which the available amount of feed resources and/or the optimum last age are not stable over time. As an illustration, consider the rancher who maintains a breeding herd of uniform age distribution at the beginning of a given production period. Suppose that the rancher breeds the breeding cows at the beginning of every production period. According to the technology of growth process associated with beef cattle, the breeding animals in the working herd produce calves at the end of every production period, and the breeding animals in the breeding herd become

distribution of breeding herd inherited from the preceding production period, the available amount of feed resources, and the optimum last age.
one year older at the beginning of every subsequent production period. In addition, suppose that the available amount of feed resources and the optimum last age are known and remain stable over time. The rancher would then adjust his actual to his optimum breeding herd of uniform age distribution by adding the replacement calves and culling the breeding animals, where the number of replacement calves added and the number of breeding animals culled are equal and stable over time. Also the rancher sells a constant number of calves at the beginning of every production period. Alternatively, suppose that the available amount of feed resources and the optimum last age are known but remain unstable over time. Then the rancher would adjust his actual to his optimum breeding herd of uniform, or alternative, age distribution by adding the replacement calves and culling the breeding animals where the number of replacement calves added and the number of breeding animals culled are not equal and are unstable over time. The number of calves sold at the beginning of each production period would, therefore, fluctuate over time.

The rancher attempts to adjust his actual to his (new) optimum breeding herd at the beginning of each production period under the principle of maximizing the net accumulated return. However, he may not, in fact, be able to do this for a
variety of reasons, e.g., technological rigidities associated with beef cattle or policy as to the composition of breeding herd. In the present part, three questions are studied: (a) Can the rancher adjust his actual to his (new) optimum, or desired, breeding herd at the beginning of every production period? (b) How will the herd adjustment influence (or generate) the time-path of the size of calf crop, the number of calves sold, and the number of breeding animals culled? (c) How may the herd adjustment within one production period be described and explained?

The first section will attempt to answer the first two questions by considering a few hypothetical herd adjustment processes of the rancher under the assumption that the beef breeder follows the policy of maintaining the uniform age distribution over time, as an example of "fixed" age distribution, even after any change in the optimum breeding herd. The second section will alternatively consider a few hypothetical herd adjustment processes of the rancher under the assumption that he follows the policy of maintaining the alternative age distribution over time, as an example of "variable" age distribution, after any change in the optimum breeding herd. The third question will be discussed in the last section.
1. Herd Adjustment Processes (1): Under the Policy of Uniform Age Distribution

A few hypothetical examples of the herd adjustment processes under the policy of uniform age distribution after any change in the optimum breeding herd will be presented and discussed in order to show how the rancher will adjust his actual breeding herd from a given level to an alternative level of uniform age distribution, or from one stationary process to an alternative stationary process. These examples are based upon the set of simplifying assumptions specified in section II, A, and the optimum solutions for a few alternative stationary processes of ranch operation exemplified in section II, B. In addition, it is assumed that the rancher is not able to buy any breeding animals from the market and that he culls the breeding animals (both under-aged and over-aged with respect to the optimum last age) by depending solely upon the policy of maintaining an equal

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17/ For the numerical examples of herd adjustment processes in the present section, i.e., section II, C, 1, it is assumed that the rancher who follows the policy of uniform age distribution does not cull the under-aged breeding animals regardless of their calving records. In order to differentiate the herd adjustment processes in the present section from those in the following section, i.e., section II, C, 2, the tables and figures in the present section are identified with the numeral 1 at the end of the description of tables and figures while the tables and figures in the following section are specified with the numeral 2 at the end of the description of tables and figures.
number of breeding animals in each age group.\(^{18/}\)

In order to consider the generation of the time-paths of herd adjustment, definition should be clarified of the actual breeding herd kept on the ranch at the beginning of production period \(t\), \((n_t(0), n_t(1), \ldots, n_t(j))\). The term is defined as the herd in the process of adjustment from one optimum to the (new) optimum breeding herd defined at the beginning of production period \(t\), \((n^*_t(0), n^*_t(1), \ldots, n^*_t(j^*))\), which is determined in terms of the optimum last age and the available amount of feed resources. In other words, \((n_t(0), n_t(1), \ldots, n_t(j))\) is thought of as the actual breeding herd maintained, and \((n^*_t(0), n^*_t(1), \ldots, n^*_t(j^*))\) as a desired, or optimum, breeding herd under the principle of optimization. This notation for the actual and optimum breeding herds can be shortened to \(b_t(n, j)\) and \(b^*(n, j^*)\), respectively. Furthermore, the actual and optimum breeding herds are designated as \(b_t(n)\) and \(b^*_t(n)\), respectively, if the composition of the breeding herd is disregarded, as denoted in Table 2.

As a starting point in the following time-paths of herd

\(^{18/}\) It should be noted that the present assumption will be modified in section II, C, 1.
adjustment, suppose that \( b^*_t (n^*, j^*) \) has been stable for a period long enough for the adjustment of \( b_t(n, j) \) to \( b^*_j (n^*, j^*) \) to be completed. The optimum solutions which have been stable up to and at the beginning of production period 0 can, for example, be identified as follows: (a) the optimum size of breeding herd, \( b^*_t (n) \), is 100, where \( t = \ldots, -2, -1, 0; \) (b) the optimum last age of the breeding herd, \( j^*_t \), is 9; (c) the optimum number of breeding animals in each age group, \( n^*_t \), is 10; (d) the optimum size of working herd, \( w^*_t (n) \), is 80; (3) the size of calf crop, \( cc^*_t \), is 70.65; (f) the optimum number of calves sold, \( cs^*_t \), is 60.65; and (b) the optimum number of cows culled, \( sc^*_t \), is 10. The actual solutions of the ranch operation for the production period 0 can then be specified as follows: (a) \( b_0^*(n) = b_0^*(n) = 100; \) (b) \( w_0^*(n) = w_0^*(n) = 80; \) (c) \( cc_0^* = cc_0^* = 70.65; \) (d) \( cs_0^* = cs_0^* = 60.65; \) and (c) \( sc_0^* = sc_0^* = 10. \)

Next to be considered are a few herd adjustment processes in response to a once-and-for-all change in the available amount of feed resources and/or the optimum last age. This will be followed by a brief discussion of the herd adjustment process in

\[19/\] It should be kept in mind that the size of the calf crop is measured at the end of each production period.
response to a chain of changes in the available amount of feed resources and/or the optimum last age.

(1) **Once-and-for-all Increase in the Available Amount of Feed Resources:** Table 3 presents a transitional process of herd adjustment carried out by the rancher from the initial stationary process of ranch operation to an alternative stationary process which results from a once-and-for-all increase in the available amount of feed resources while the optimum last age remains stable over time. It is supposed that the available amount of feed resources is once-and-for-all increased so that the rancher can maintain 120 breeding animals from the beginning of production period 1 on. The optimum solutions for an alternative stationary process of ranch operation for \( t = 1, 2, \ldots \), can be specified as follows: (a) \( b^*_t (n) = 120 \); (b) \( j^*_t = 9 \); (c) \( n^*_t = 12 \);

(d) \( w^*_t (n) = 96 \); (e) \( cc^*_t = 84.78 \); (f) \( cs^*_t = 72.78 \); and (g) \( sc^*_t = 12 \).

(See the optimum solutions for the initial and the first alternative stationary processes of ranch operation shown in Table 2.)

The upper part of Table 3 depicts the size and composition of the breeding (and working) herd and the movement of breeding animals in each age group over time according to the growth processes of beef cattle. It was assumed for the construction of Table 3 that the rancher began to replace 12 calves instead of 10
### Table 3. Herd adjustment: once-and-for-all increase in the available amount of feed resources (1).

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<sup>a</sup> In accordance with the definition of the actual size of calf crop, this solution is measured at the end of each production period. For example, the number of calves produced from the working herd of 80 breeding cows at the end of the production period 1 is 70.65.

<sup>b</sup> According to the definition of the actual number of calves sold, the present solution is measured at the beginning of each production period. The actual number of calves sold is defined as the difference between the number of calves produced in the production period (t-1) and the number of replacement calves added at the beginning of production period t. For example, the actual number of calves sold at the beginning of production period 3 is: 70.65 - 12.00 = 58.65.
calves from the beginning of production period 1 on, i.e.,
\[ n_t(0) = 12, \text{ where } t = 1, 2, \ldots \] First, the rancher takes 10 production periods to adjust his actual to his new optimum breeding herd so that an optimum breeding herd of uniform age distribution is again maintained. Second, the adjustment of breeding animals in the working herd commences from the beginning of production period 3. Third, the number of breeding animals culled in accordance with the technology of growth processes is 10 up to the beginning of production period 10, after which it becomes 12. Finally, under the present scheme of herd adjustment the available amount of feed resources is not fully utilized until the beginning of production period 10. 20/

The lower part reflects how the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled vary over time as a result of the herd adjustment.

The convergence of the actual with the optimum solution for the alternative stationary process of ranch operation throughout the transitional period may be made clear by reference to Figure 3. It should be noted in Figure 3 that the size of the actual

20/ A supposition is that the rancher markets the excess amount of feed resources in order to maintain an alternative age distribution.
Figure 3. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increase in the available amount of feed resources (1).
breeding (and working) herd is gradually adjusted to the new optimum size by increasing the number of replacement calves from 10 to 12, starting with the beginning of production period 1. Following the specified herd adjustment, the size of the calf crop remains the same for production periods 1 and 2, since the size of the working herd remains the same. From production period 3 onward, the size of the calf crop gradually rises toward its optimum level, reaching that level at the end of production period 10. As the size of the calf crop changes, the number of calves sold at first decreases, as a result of the increase in the number of replacement calves for production periods 1, 2, and 3, after which the number of calves sold rises until it reaches the optimum level at the end of production period 10. The number of breeding cows culled remains the same, since the optimum last age of the breeding herd is stable over time. However, as soon as the actual breeding herd is completely adjusted to the new optimum level, the number of breeding cows culled becomes 12 at the beginning of production period 11 and continues at this level as long as the optimum herd remains unchanged.

(2) Once-and-for-all Decrease in the Available Amount of Feed Resources: Table 4 describes the transitional process from the initial stationary process to an alternative stationary process resulting from a once-and-for-all decrease in the
Table 4. Herd adjustment: once-and-for-all decrease in the available amount of feed resources (1).

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\(a/\) See footnote a in Table 3.

\(b/\) See footnote b in Table 3.
available amount of feed resources while the optimum last age remains the same. Suppose that the available amount of feed resources is decreased in such a way that the rancher can maintain 80 breeding animals from the beginning of production period 1 on. The optimum solutions of an alternative stationary process for \( t = 1, 2, \ldots \) are specified as follows: (a) \( b_t^* (n) = 80 \); (b) \( j_t^* = 9 \); (c) \( n_t^* = 8 \); (d) \( w_t^* (n) = 64 \); (3) \( c_c^* = 56.52 \); (f) \( c_s^* = 48.52 \); and (g) \( sc_t^* = 8 \). (See the optimum solutions of the initial and second alternative stationary process of ranch operation shown in Table 2.)

The upper part of Table 4 presents the size and composition of the breeding (and working) herd and the movement of breeding animals in each age group over time in accordance with the technology of growth process. It is interesting to note two points. First, the number of breeding animals in each age group becomes 8 instead of 10 in accordance with the available amount of feed resources under the policy of uniform age distribution. Second, the breeding herd is completely adjusted to the optimum level at the beginning of production period 1. The lower section indicates that: (a) the size of the breeding herd is adjusted from 100 to 80 at the beginning of production period 1; (b) the size of the working herd is adjusted from 80 to 64 at the beginning of production.
Figure 4. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled; once-and-for-all decrease in the available amount of feed resources (1).
period 1; (c) the size of the calf crop is 56.52 at the end of production period 1; (d) the number of calves sold at the beginning of production period 1 rises to 62.65, and becomes stable at 48.52 from the beginning of production period 2; and (3) the number of breeding cows culled at the beginning of production period 1 rises to 28, and becomes stable at 8 from the beginning of production period 2. The time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled is plotted in Figure 4.

(3) Once-and-for-all Decreasing of the Optimum Last Age:

Table 5 shows a transitional process for the situation in which the optimum last age is once-and-for-all decreased at the beginning of production period 1 while the available amount of feed resources remains the same. The optimum solutions of an alternative stationary process for $t = 1, 2 \ldots$ under the present situation can be summarized as follows: (a) $b_t^*(n) = 100$; (b) $j_t^* = 7$; (c) $N_t^* = 12.50$; (d) $w_t^*(n) = 75$; (e) $cc_t^* = 66.25$; (f) $cs_t^* = 53.75$; and (g) $sc_t^* = 12.50$. (See the optimum solutions of the initial and the third alternative stationary process of ranch operation presented in Table 2.)

The upper part of Table 5 describes the size and
Table 5. Herd adjustment: once-and-for-all decreasing of the optimum last age (1).

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Note: See footnote a in Table 3.

Note: See footnote b in Table 3.
composition of the breeding (and working) herd and the movement of breeding animals in each age group over time, following the technology of the growth process. It is conveniently assumed that the rancher adjusts his breeding herd by increasing the number of replacement calves from 10 to 12.50, starting from the beginning of production period 1. It should first be noted that the rancher takes eight production periods to adjust his actual to his desired breeding herd of uniform age distribution. Second, the adjustment of breeding animals in the working herd commences from the beginning of production period 3, indicating a technological lag associated with the growth process. Third, the number of breeding animals culled at the beginning of production period 1 is 30, as a result of the change in the optimum last age, and becomes stable at 10 for the production periods from 2 to 8. However, at the beginning of production period 9, the number of breeding animals culled becomes 12.50. Finally, under the present scheme of herd adjustment the available amount of feed resources is not fully utilized until the beginning of production period 8. 21/ The lower part reflects how the size of breeding herd, the size of working herd, the size of calf crop, the number

21/ Again, a supposition is that the rancher markets the excess amount of feed resources in order to maintain an alternative age distribution.
of breeding animals culled would change as a result of the herd adjustment.

The dynamic changes over time are depicted in Figure 5. The size of the breeding herd is contracted as a result of shortening the optimum last age at the beginning of production period 1, then is gradually expanded to maintain the increased number of replacement calves, from the beginning of production period 1 on. Similarly, the size of the working herd is contracted as a result of shortening the optimum last age, and then is gradually adjusted to the new optimum level at the beginning of production period 8. Following the herd adjustment scheme specified above, the size of the calf crop is contracted for production periods 1 and 2, and then is finally raised to the new optimum level at the end of production period 8. The number of calves sold is decreased for production periods 2 and 3, and then is gradually raised to the new level at the beginning of production period 9. The number of breeding cows culled is increased at the beginning of production period 1 as a result of shortening the optimum last age. It remains at the initial optimum level for production periods 2 to 8, inclusive. The actual number of breeding cows culled is finally raised to the new optimum level at the beginning of production period 9.
Figure 5. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all decreasing of the optimum last age (1).
(4) Once-and-for-all Increasing of the Optimum Last Age:

Table 6 depicts the transitional process resulting from increasing the optimum last age at the beginning of production period 1 while the available amount of feed resources remains the same.

The optimum solutions of an alternative stationary process for \( t = 1, 2, \ldots \) can be recapitulated as follows: (a) \( b_t^* (n) = 99 \); (b) \( j_t^* = 10 \); (c) \( n_t^* = 9 \); (d) \( w_t^* (n) = 81 \); (e) \( cc_t^* = 62.10 \); and (g) \( sc_t^* = 9 \). (See the optimum solutions of the initial and the fourth alternative stationary process of ranch operation presented in Table 2, where the size of breeding herd is modified from 100 to 99 for the sake of simplicity.)

The upper part of Table 4 shows the size and composition of the breeding (and working) herd and the movement of breeding animals in each age group over time in accordance with the technology of the growth process. In addition, the breeding (and working) herd is completely adjusted to the desired level at the beginning of production period 1. The lower part indicates that:

(a) the size of the breeding herd is adjusted to the optimum size of the breeding herd at the beginning of production period 1;
(b) the size of the working herd is also adjusted to the optimum size of the working herd at the beginning of production period 1;
(c) the size of the calf crop is raised to the alternative optimum
Table 6. Herd adjustment: once-and-for-all increasing of the optimum last age (1). *

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\[ b_n \quad w_n \quad c_n \quad c_s \quad c_{sc} \]

\[ b_n \quad w_n \quad c_n \quad c_s \quad c_{sc} \]

* The size of breeding herd is modified from 100 to 99 in order to simplify the numerical example.

\[ a/\) See footnote a in Table 3.

\[ b/\) See footnote b in Table 3.
level from the beginning of production period 2; and (c) the number of breeding animals culled consists of one breeding animal in each age group, ranging from one to ten years old, at the beginning of production period 1. The time-paths of these variables can be seen in Figure 6.

(b) Once-and-for-all Increase in the Available Amount of Feed Resources and Increasing of the Optimum Last Age: Table 7 presents a transitional process of the breeding herd from the initial stationary process to an alternative stationary process which results from a once-and-for-all increase in the available amount of feed resources and a once-and-for-all increasing of the optimum last age at the beginning of production period 1. The optimum solutions of an alternative stationary process for $t = 1, 2, \ldots$ can be, for example, expressed as follows:

(a) $b_t^* (n) = 120$; (b) $j_t^* = 11$; (c) $n_t^* = 10$; (d) $w_t^* (n) = 100$; (e) $cc_t^* = 86.95$; (f) $cs_t^* = 76.95$; and (g) $sc_t^* = 10$. (See the optimum solutions for the initial and the fifth alternative stationary processes of ranch operation in Table 2.)

The upper part of Table 7 shows that the breeding herd is completely adjusted to the desired level at the beginning of production period 2 by extending the period of holding breeding animals. The optimum number of breeding animals in each age group remains the same. The lower part describes: (a) the size
Figure 6. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increasing the optimum last age (1).
Table 7. Herd adjustment: once-and-for-all increase in the available amount of feed resources and increasing of the optimum last age (1).

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\( w_t(n) \) 80.00 90.00 100.00 100.00 100.00
\( c_{se}^{a/} \) 70.65 79.00 86.95 86.95 86.95
\( s_{se}^{b/} \) 60.65 60.65 69.00 76.95 76.95
\( s_{sc}^{c/} \) 10.00 0.00 0.00 10.00 10.00

\(^a/\) See footnote a in Table 3.
\(^b/\) See footnote b in Table 3.
of the breeding herd is adjusted to the desired size at the beginning of production period 2; (b) the size of the working herd is also adjusted to the desired size at the beginning of production period 2; (c) the size of the calf crop is raised to the optimum level at the end of the production period 2; (d) the number of calves sold converges with the desired level at the beginning of production period 3; and (e) the number of breeding animals culled stays at zero for production periods 1 and 2 because of a once-and-for-all increasing of the optimum last age, then becomes stable at 10 from the beginning of production period 3. The time-paths of these variables can be seen in Figure 7.

(7) Once-and-for-all Decrease in the Available Amount of Feed Resources and Decreasing of the Optimum Last Age. Table 8 shows a transitional process of the breeding herd from the initial stationary process to an alternative stationary process which results from a once-and-for-all decrease in the available amount of feed resources and a once-and-for-all decreasing of the optimum last age at the beginning of production period 1. The optimum solutions of an alternative alternative process for $t = 1, 2, \ldots$ are, for example, specified as follows: (a) $b^*_t(n) = 90$; (b) $j^*_t = 8$; (c) $n^*_{t} = 10$; (d) $w^*_t(n) = 70$; (e) $cc^*_t = 61.95$; (f) $cs^*_t = 51.95$; and (g) $sc^*_t = 10$. (See the optimum solutions for the initial and the sixth alternative stationary processes of ranch operation
Figure 7. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increase in the available amount of feed resources and increasing of the optimum last age (1).
Table 8. Herd adjustment: once-and-for-all decreases in the available amount of feed resources and decreasing of the optimum last age (1).

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\( b_t \) See footnote a in Table 3.
\( c_t \) See footnote b in Table 3.
in Table 2.

The upper part of Table 8 shows how the actual breeding herd is adjusted to the desired breeding herd at the beginning of production period 1, following the technology of growth process. Responding to a once-and-for-all change in the optimum last age and the available amount of feed resources, the breeding herd is adjusted to the desired level at the beginning of production period 1 by adding 10 replacement calves and culling 20 over-aged cows. The number of calves sold at the beginning of production period 1 remains the same but it becomes stable at the desired level from the beginning of production period 2. The number of breeding animals (over-aged cows) culled becomes stable at 10 from the beginning of production period 2. The time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled can be alternatively seen from Figure 8.

(7) A Chain of Changes in the Available Amount of Feed Resources and/or the Optimum Last Age: This section considers the herd adjustment process in response to a chain of changes in the available amount of feed resources and/or the optimum last age. Instead of developing a large number of time-paths of herd adjustment, section II, C, 1, (7), confines itself to one hypothetical
Figure 8. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all decrease in the available amount of feed resources and decreasing of the optimum last age (1).
example and then discusses briefly the herd adjustment process responding to a chain of changes in the available amount of feed resources and/or the optimum last age.

Table 9 describes a hypothetical time-path of herd adjustment process responding to a chain of changes in the available amount of feed resources and the optimum last age over time. According to the assumed starting point of the time-path of herd adjustment, the rancher adjusted his actual to his desired breeding herd at the beginning of production period 0. In addition, it was assumed that the available amount of feed resources and the optimum last age at the beginning of each production period were those specified in the middle part of Table 9.

The upper part of Table 9 shows the movement of breeding animals in the breeding herd adjusted by the rancher over time. It was supposed, as an illustration, that at the beginning of production period 1 the available amount of feed resources was increased to maintain ten additional breeding animals and that the optimum last age was increased by one year. It was further assumed that the rancher would adjust his breeding (and working) herd by adding ten replacement calves as usual and culling zero breeding animals. Next it was supposed that at the beginning of production period 2 the available amount of feed resources and the optimum last age returned to their respective levels in
Table 9. Herd adjustment: a chain of changes in the available amount of reed resources and/or the optimum last age (1).

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production period 0. Then the breeding herd would be adjusted by adding 10 replacement calves and culling 20 breeding animals. The herd adjustment process for the following production periods can be interpreted in a manner similar to that of production periods 1 and 2.

The lower part of Table 9 reflects the time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled over time, and indicates the relationships among these solutions (see Figure 9). It is interesting to note a few points: (a) the difference in size between the breeding and working herds is 20, implying that the change in the size of the breeding herd equals the change in the size of the working herd; (b) the size of the calf crop varies with the change in the size of the working herd; (c) the time-paths of the number of calves sold and the number of breeding animals culled are different in pattern from those of the size of breeding herd, the size of working herd, the size of calf crop, indicating that the herd adjustment is carried out by changing the level of replacement calves added and the number of breeding animals culled.

The herd adjustment process described in Table 9 (and Figure 9) is based on the hypothetical chain of changes in the available amount of feed resources and the optimum last age. However, it
Figure 9. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: a chain of changes in the available amount of feed resources and/or the optimum last age (l).
should be noted that the chain of changes in the available amount of feed resources and the optimum last age were arranged in such a way that the optimum number of breeding animals in each age group remained the same and the herd adjustment could be easily carried out at the beginning of each production period. Alternatively, if the optimum number of breeding animals in each age group changes from period to period, then the rancher may not always adjust his actual to his optimum breeding herd of uniform age distribution at the beginning of each production period. For example, suppose that the optimum solutions of alternative stationary processes presented in Table 2 are regarded as those for the ranch operation period after period, modifying the classification of alternative stationary processes as that of production period in order (see Table 2). From the comparison of the optimum number of breeding animals in each age group between two adjacent periods, it is clear that the rancher cannot always adjust his actual to his optimum breeding herd of uniform age distribution at the beginning of each production period. Without any lengthy discussion on the alternative time-paths of herd adjustment, it is simply concluded that the rancher may not always adjust his actual to his optimum breeding herd of uniform age distribution because of the technological rigidities.
Before considering an alternative set of herd adjustment processes under the policy of "variable" age distribution, the implications of the above numerical examples can be summarized as follows: First, the rancher who follows the policy of maintaining a uniform age distribution may take longer than one production period to adjust his breeding herd in response to a once-and-for-all change, or a chain of changes, in the available amount of feed resources and/or the optimum last age. Second, there are asymmetries in the herd adjustment processes, implying that contraction in the size of breeding herd can, in general, be more quickly achieved than can an expansion in the size of breeding herd where the optimum last age remains the same. Third, the pattern and the "speed" of herd adjustment depends on the direction, magnitude, and type (i.e., once-and-for-all or a chain) of change in the available amount of feed resources and/or the optimum last age. Fourth, the optimum solutions—the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding cows—are causally related over time, following the technology of the growth process of breeding animals. Finally, in response to a well arranged chain of changes in the available amount of feed resources and/or the optimum last age, the "cyclical fluctuation" or "cycle" of each optimum solution may be generated by the
"stable" adjustment activities of the representative rancher concerning his breeding herd.

2. Herd Adjustment Processes (2): Under the Policy of Changing Age Distribution

This section considers how the rancher adjusts his actual to his (new) optimum breeding herd under the policy of changing (or "variable") age distribution. The analyses in the preceding section have been based on the assumption that the rancher follows the policy of maintaining uniform (or "fixed") age distribution after any change in his optimum breeding herd, together with other simplifying assumptions. However, it can be alternatively assumed that the rancher follows the policy of changing age distribution after any change in the available amount of feed resources and/or the optimum last age. From such a modification of one of the simplifying assumptions, the optimum solutions for the ranch operation become somewhat different from those defined and used in the preceding analyses. Therefore,

22/ For the numerical examples of herd adjustment processes in the present section, it is conveniently assumed that the rancher who follows the policy of changing age distribution does not cull the under-aged breeding animals, with respect to the optimum last age, regardless of their calving records.
the optimum solutions of the ranch operation under the modified framework will be discussed and an analysis made of how the rancher adjusts his breeding herd, following the growth process of breeding animals in the breeding herd.

Within the modified framework, an optimum breeding herd is defined as a breeding herd comprised of breeding animals of various ages (ranging from zero to the optimum last age), the number of calves added depending on the change in the available amount of feed resources. As an illustration, suppose that at the beginning of production period 0 the rancher has the amount of feed resources to maintain a herd of 100 breeding animals under the policy of uniform age distribution and that the optimum last age is 9. Now suppose that at the beginning of production period 1 the rancher has the amount of feed resources to maintain 120 breeding animals and the optimum last age remains the same. The optimum solutions are specified as follows: (a) the optimum breeding herd is \( n_1^* (0) = 30, n_1^* (1) = 10, ..., n_1^* (j^*) = 10 \); (b) the optimum working herd is \( n_1^* (2) = 10, n_1^* (3) = 10, ..., n_1^* (j^*) = 10 \); (c) the size of calf crop, \( c_{11}^* \), is 70.65; (d) the optimum number of calves sold, \( cs_{11}^* \) is: \( cc_0 - n_1(0) = 70.65 - 30 = 40.65 \); and (c) the optimum
number of breeding animals culled, $s_{c1}^*$, is 10.

In conjunction with the above modification in the optimum solutions, it is again assumed that the optimum solutions of the starting point of the following time-paths of herd adjustment are: (a) the size of breeding herd, $b_0^*(n) = 100$; (b) the optimum last age, $j_0^*$, is 9; (c) the optimum number of breeding animals in each age group, $n_0^*$, is 10; (d) the size of working herd, $w_0^*(n)$, is 80; (e) the size of calf crop, $c_{c0}^*$, is 70.65; (f) the optimum number of calves sold, $c_{s0}^*$, is 60.65; and (b) the optimum number of breeding animals culled, $s_{c0}^*$, is 10. In addition, it is assumed that the rancher is not able to purchase any breeding animals from the market and that he culls the over-aged breeding animals (with respect to the optimum last age). A few hypothetical herd adjustment processes will be studied in the following sections.

(1) Once-and-for-all Increase in the Available Amount of Feed Resources: Consider how a rancher adjusts his breeding herd in a situation where the available amount of feed resources is once-and-for-all increased while the optimum last age remains the same. Table 10 shows a herd adjustment process in response to a once-and-for-all increase in the available amount of feed resources at the beginning of production period 1. It is apparent from the upper part of Table 10 that there is a distortion in the
Table 10. Herd adjustment: once-and-for-all increase in the available amount of feed resources (2).

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See footnote a in Table 3.

See footnote b in Table 3.
age distribution at the beginning of production period 1. The "cohort" of 20 gradually passes through the age distribution, being 0-year old in production period 1, 1-year old in production period 2, and so on until it is finally culled from the breeding herd at the beginning of production period 11. In each production period from 2 to 10, ten breeding cows are culled every production period and, as the available amount of feed resources remains the same over time, ten replacement calves must be added. At the beginning of production period 11, however, 20 breeding animals are culled, so 20 replacement calves must be added. This new "cohort" clearly starts the processes all over again, and the "cycle" of the "cohort" will be indefinitely repeated, 20 being added every tenth production period and 10 in the intervening production periods.

From Figure 10, which reflects the lower part of Table 10, it should be observed how the size of working herd, the size of calf crop, and the number of calves sold change over time, resulting from the herd adjustment. The size of working herd changes along with the movement of the "cohort" of 20. It remains at 80 when the "cohort" of 20 is 0- and 1-year old, and becomes stable at 90 when the "cohort" of 20 is in the working herd. Thus the size of the working herd fluctuates between 80 and 90.
Figure 10. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increase in the available amount of feed resources (2).
The size of the calf crop over time follows a pattern of stable, rising, "relatively" stable, falling. It is due not only to the changing productivity of beef cattle associated with age but also to the growth process of the "cohort" of 20. Since the "cohort" of 20 does not beget any calves when it is 0- and 1-year old, the number of calves produced from the working herd is stationary in production periods 1 and 2. However, the additional number of calves produced from the working herd rises and falls along with the growth process of the "cohort" and the productivity associated with age. It is evident from the above example that the "cycle" of the number of calves produced will be indefinitely repeated, along with the rising, "relatively" stable, and falling.

On the basis of the number of calves produced from the working herd and the number of replacement calves added to the breeding herd, it is clear, as shown in the lower part of Table 10 as well as in Figure 10, that there is a somewhat different pattern of change in the number of calves marketed over time. It should be noted that the number of calves marketed at the beginning of production period 1 is decreased from 60.65 to 50.65 because of an increase in the number of replacement calves added. At the beginning of production periods 2 and 3, the number of calves marketed is 60.65, since the number of calves
produced and the number of replacement calves added remain the same. For production periods from 3 to 11, inclusive, the number of calves produced follows a pattern of rising, "relatively" stable, and falling, whereas the number of replacement calves added remains the same. Thus the number of calves marketed follows a pattern of rising, "relatively" stable, and falling for the period between production periods 2 and 12. The "cycle" in the number of calves marketed follows a pattern of falling, rising, "relatively" stable, and falling, along with the "cycle" in the number of calves produced.

(2) Once-and-for-all Decrease in the Available Amount of Feed Resources: Table 11 shows the herd adjustment process in a situation where the available amount of feed resources is once-and-for-all decreased at the beginning of production period 1 while the optimum last age remains the same. It should be observed from the upper part of Table 11 that no replacement calves are added at the beginning of production period 1 since the available amount of feed resources is decreased. The movement of the "cohort" of 0 brings about the distortion in the age distribution over time. From the lower part of Table 11, the size of the working herd stays at 80 when the "cohort" of 0 is 0- and 1-year old, and becomes stable at 70 when the "cohort" of 0 is in the working herd. In addition, the size of the calf crop
Table 11. Herd adjustment: once-and-for-all decrease in the available amount of feed resources (2).

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a/ See footnote a in Table 3.
b/ See footnote b in Table 3.
from the working herd changes with the movement of the "cohort" of 0. The number of calves marketed rises at the beginning of production period 1 since no replacement calves are added. However, it returns to the initial level for production periods 2 and 3, and follows the pattern of changes in the size of the calf crop for production periods 4 through 11. The time-path in the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled can be best observed from the lower part of Table 11 or from Figure 11.

(3) **Once-and-for-all Decreasing of the Optimum Last Age:**

Table 12 presents a pattern of herd adjustment process for a case in which the optimum last age is once-and-for-all decreased while the available amount of feed resources remains stable over time. The "cycle" of the "cohort" of 20 will be indefinitely repeated, 20 being added every ninth production period and 10 in the intervening production periods. In the present case, the size of the working herd decreases by 10 in production periods 1 and 2, and returns to the initial level for production periods 3 through 9. Therefore, the number of calves produced decreases from 70.65 to 61.95 for production periods 1 and 2, and it gradually rises and falls for production periods 3 through 9. The "cycle" of the number of calves produced follows a pattern of falling, rising,
Figure 11. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all decrease in the available amount of feed resources (2).
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Production Period

| b_t(n) | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| w_t(a) | 80.00 | 70.00 | 70.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 70.00 | 70.00 |
| c_t / b_t | 70.65 | 61.95 | 61.95 | 70.35 | 70.55 | 70.80 | 70.95 | 71.05 | 71.00 | 70.90 | 61.95 |
| c_t / c_t | 60.65 | 50.65 | 51.95 | 51.95 | 60.35 | 60.55 | 60.80 | 60.95 | 61.05 | 61.00 | 50.90 |
| sc_t | 10.00 | 20.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 20.00 | 10.00 |

Footnotes:

a/ See footnote a in Table 3.
b/ See footnote b in Table 3.
"relatively" stable, and falling. The number of calves sold, which is however decreased at the beginning of production period 1 from 60.65 to 50.65 on account of an increase in the number of calves marketed, follows a pattern of falling, rising, "relatively" stable, and falling. The lower part of Table 12, as well as Figure 12, describes the time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of animals culled.

(4) Once-and-for-all Increasing of the Optimum Last Age:

Table 13 shows a herd adjustment process in a situation in which the optimum last age is once-and-for-all increased while the available amount of feed resources remains the same. It can be easily seen from the upper part of Table 13 that the "cycle" of the "cohort" of 0 will be indefinitely repeated, with no replacement calves being added to the breeding herd every eleventh production period and 10 replacement calves in the intervening periods. Along with the movement of the "cohort" of 0, the size of the working herd rises by 10 for production periods 1 and 2, and returns to the initial level for production periods 3 through 11. From the lower part of Table 13, it is clear that the size of the calf crop follows a "cycle" of rising, falling, "relatively" stable, and rising, along with the change in the size of the working herd. In addition, the number of calves sold follows a
Figure 12. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all decreasing of the optimum last age (2).
Table 13. Herd adjustment: once-and-for-all increasing of the optimum last age (2).

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\( a/ \) See footnote a in Table 3.

\( b/ \) See footnote b in Table 3.
pattern of rising, falling, "relatively" stable, and rising since no replacement calves are added at the beginning of every eleventh production period. The time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled can be alternatively seen from Figure 13.

(5) Once-and-for-all Increase in the Available Amount of Feed Resources and Increasing of the Optimum Last Age: The herd adjustment process described in Table 14 shows how the rancher responds to a once-and-for-all increase in the available amount of feed resources and increasing of the optimum last age. From the upper part of Table 14, it should be noted that at the beginning of production period 1 the "cohort" of 20 is added and the productive life of breeding animals is extended by one year. In addition, the "cycle" of the "cohort" of 20 will be indefinitely repeated, 20 replacement calves being added to the breeding herd every eleventh production period and 10 replacement calves in the intervening production periods. Furthermore, at the beginning of every eleventh production period 20 breeding animals are culled out. From the lower part of Table 14, a few points can be noted. First of all, the size of the working herd follows a pattern of rising and falling. Second, along with the movement of the "cohort" of 20, the size of the calf crop follows a pattern
Figure 13. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increasing of the optimum last age (2).
Table 14. Herd adjustment: once-and-for-all increase in the available amount of feed resources and increasing of the optimum last age (2).

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| b(t) | 100.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 |
| w(t) | 80.00  | 90.00  | 90.00  | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 90.00  |
| c(t)  | 70.65  | 78.95  | 78.95  | 87.35  | 87.55  | 87.80  | 87.95  | 88.05  | 88.00  | 87.90  | 87.65  | 87.25  | 78.95  |
| c(t)  | 60.65  | 50.65  | 68.95  | 68.95  | 77.35  | 77.55  | 77.80  | 77.95  | 78.05  | 78.00  | 77.90  | 77.65  | 67.25  |
| s(t)  | 10.00  | 0      | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 10.00  | 20.00  |

See footnote a in Table 3.
See footnote b in Table 3.
of rising, "relatively" stable, and falling. Third, the number of calves sold follows the "cycle" of falling, rising, "relatively" stable, and falling. The time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled can be observed in Figure 14.

(6) Once-and-for-all Decrease in the Available Amount of Feed Resources and Decreasing of the Optimum Last Age:

Table 15 depicts the herd adjustment process for a situation in which the available amount of feed resources is once-and-for-all increased and the optimum last age is once-and-for-all decreased at the beginning of production period 1. The upper part of Table 15 describes the movement of breeding animals and the distortion in the age distribution over time. The lower part of Table 15 presents: First, the size of the working herd contracts to 70 for production periods 1 and 2 when the "cohort" of 0 is 0- and 1-year old, and remains stable at 60 for production periods 3 through 9 when the "cohort" of 0 stays in the working herd. Second, the size of the calf crop follows the "cycle" of falling, "relatively" stable, and rising. Third, the number of calves sold rises to 70.65 at the beginning of production period 1 since no replacement calves are added, and follows the pattern of rising, falling, "relatively" stable, and rising. The time-path
Figure 14. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all increase in the available amount of feed resources and increasing of the optimum last age (2).
Table 15. Herd adjustment: once-and-for-all decrease in the available amount of feed resources and decreasing of the optimum last age (2).

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<td>10.00</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( b_t(n) \) | 100.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 |

\( w_t(n) \) | 80.00 | 70.00 | 70.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 70.00 |

\( cc_t / \) | 70.65 | 61.95 | 61.95 | 53.25 | 53.25 | 53.10 | 52.75 | 52.85 | 52.90 | 53.00 | 61.95 | 61.95 |

\( cc_t / b_t \) | 60.65 | 70.76 | 51.95 | 51.95 | 43.55 | 43.35 | 43.10 | 42.95 | 42.85 | 42.90 | 53.00 | 51.95 |

\( sc_t \) | 10.00 | 20.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 10.00 | 0  |

\( a/ \) See footnote a in Table 3.

\( b/ \) See footnote b in Table 3.
of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled can be seen from Figure 15.

(7) *A Chain of Changes in the Available Amount of Feed Resources and/or the Optimum Last Age*: Table 16 presents a herd adjustment process in response to a chain of changes in the available amount of feed resources and/or the optimum last age. The available amount of feed resources to maintain a herd of breeding animals and the optimum last age for each production period are specified in the middle part of Table 16.

The upper part of Table 16 shows the movement of breeding animals and the changes in the age distribution. It is interesting to note two points. First, there is no unique "cycle" of the "cohort," as has been seen in the above examples, if there is a series of changes in the available amount of feed resources and/or the optimum last age. Second, the age distribution changes along with the growth process of breeding animals included in the breeding herd.

The lower part of Table 16 shows the time-path of the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled. A few points should be noted here. First, the size of the working herd does not follow the pattern of change in
Figure 15. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: once-and-for-all decrease in the available amount of feed resources and decreasing of the optimum last age (2).
Table 16. Herd adjustment: a chain of changes in the available amount of feed resources and/or the optimum last age (2).

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Production Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
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<td>2</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Working Herd</th>
<th>Breeding Herd</th>
</tr>
</thead>
<tbody>
<tr>
<td>b(_n)</td>
<td>b(_n)</td>
</tr>
<tr>
<td>w(_n)</td>
<td>w(_n)</td>
</tr>
<tr>
<td>c_{t,n}/b(_n)</td>
<td>c_{t,n}/b(_n)</td>
</tr>
<tr>
<td>c_{t,n}/c_{t,n}</td>
<td>c_{t,n}/c_{t,n}</td>
</tr>
<tr>
<td>t(_n)</td>
<td>t(_n)</td>
</tr>
</tbody>
</table>

*a* See footnote a in Table 3.

*b* See footnote b in Table 3.
Figure 16. Time-path of size of breeding herd, size of working herd, size of calf crop, number of calves sold, and number of breeding animals culled: a chain of changes in the available amount of feed resources and/or the optimum last age (2).
the size of the breeding herd because of the distortion in the age distribution as well as the change in the optimum last age.

Second, the size of the calf crop depends on not only size but also composition of the working herd. It should be observed, for example, that the size of the calf crop from a working herd of equal size for production periods 4, 7, and 8, i.e.,

\[ w_4(n) = w_7(n) = w_8(n) = 100, \]

would be 87.85, 88.60, and 88.50, respectively. Third, the number of calves sold changes along with the change in the size of the calf crop and the change in the number of replacement calves added. Finally, the number of breeding animals culled depends upon the number of breeding animals in each age group and the optimum last age.

Five implications are drawn from what has been seen under the policy of changing age distribution. First, the rancher adjusts his breeding herd at the beginning of every production period within the framework of the modified optimum solutions.

Second, there are also asymmetries in the herd adjustment process, except the cases described in sections II, C, 2, (1) and II, C, 2, (2). The herd adjustment process in response to a once-and-for-all increase in the available amount of feed resources and that in response to a once-and-for-all decrease in the available amount of feed resources are asymmetric with respect to the initial optimum solutions, if the optimum last age
remains stable and if the changes in the available amount of feed resources are equal. Third, the optimum solutions—the size of breeding herd, the size of working herd, the size of calf crop, the number of calves sold, and the number of breeding animals culled—are causally related, following the growth process of breeding animals in the herd. Fourth, the movement of "cohort" (or "cohorts") in the breeding herd over time distorts the age distribution of the breeding herd, and consequently generates the level of each solution over time. Finally, each solution may be generated by the adjustment activities of the rancher in response to the available amount of feed resources and the optimum last age over time.

3. Herd Adjustment Relation

In examining the tabular and graphic description of a few herd adjustment processes in sections II, C, 1, and II, C, 2, it should be noted that the time-paths of herd adjustment are asymmetric with respect to a given stationary herd adjustment. For example, the expansion in the size of the breeding (and working) herd and the contraction in the size of the breeding (and working) herd are not symmetric with reference to a given stationary herd adjustment. The herd adjustment processes exemplified above may be described and explained by a large
number of complicated relations pertaining to the direction
and magnitude of changes in the optimum last age and/or the
available amount of feed resources. However, as a first
approximation, a simple herd adjustment relation is considered
which is capable of explaining, at least in part, the sensitivity
of the rancher concerning his breeding herd in response to the
desired change in productivity of his breeding herd.

In order to formulate a herd adjustment relation, two
simplifying assumptions are recalled. First, the optimum last
age and the available amount of feed resources are known.
Second, at the beginning of each production period the rancher
adjusts his actual breeding herd inherited from the preceding
production period by means of adding (or replacing) and/or culling
the breeding animals, depending upon the breeding herd of optimum
size and composition as well as the policy of particular age dis-
tribution. As a first step in deriving a herd adjustment relation,
consideration is given to the technological growth process. On
the basis of the two assumptions recalled above and the discus-
sions given in sections II, C, 1, and II, C, 2, the growth process
between the before-herd adjustment at the beginning of production
period (t+1) and the after-herd adjustment at the beginning of
production period t may be expressed in matrix notation as
follows: 23/

\[ b_{t+1}^b (n, j) = b_t^a (n, j) [g_i, i=1], \]

where \( b_{t+1}^b (n, j) \) denotes the breeding herd before herd adjustment at the beginning of production period \((t+1)\), \( b_t^a (n, j) \) designates the breeding herd after herd adjustment at the beginning of production period \(t\), and \( g_i, i+1 \) for \( i = 0, 1, \ldots, M \) represents the proportion of breeding animals surviving from age \( i \) to \((i+1)\).

Alternatively, equation (2.27) can be expressed as follows:

\[
\begin{bmatrix}
  b_{t+1}^b (0) \\
  n_{t+1}^b (1) \\
  \vdots \\
  n_{t+1}^b (M+1) \\
\end{bmatrix}
= \begin{bmatrix}
  0 & 0 & \ldots & 0 \\
  g_{01} & 0 & \ldots & \ldots \\
  0 & g_{12} & \ldots & \ldots \\
  \vdots & \vdots & \ddots & \ddots \\
  0 & 0 & \ldots & g_{M-1,M} \\
\end{bmatrix}
\begin{bmatrix}
  n_t^a (0) \\
  n_t^a (1) \\
  \vdots \\
  n_t^a (M) \\
\end{bmatrix}
\]

where \( n_{t+1}^b (i) \) for \( i = a, 1, \ldots, M+1 \) denotes the number of

---

23/ Strictly speaking, the breeding herd was defined and represented by a row vector in section II, A, where the element of vector corresponds to the number of breeding animals in each age group. However, the breeding herd is alternatively represented by a column vector in the present part.
i-year old breeding animals in the breeding herd before herd adjustment at the beginning of production period (t+1),

\(n^a_t(i)\) for \(i = 0, 1, \ldots, M\) designates the number of i-year old breeding animals in the breeding herd after herd adjustment at the beginning of production period \(t\), and \(n^b_{t+1}(0) = 0\). 24/

Now applying the assumption (a) in section II, A, 1, i.e.,

\(g_i, i+1 = 1\) for \(i = 0, 1, \ldots, M\), equation (2.27a) becomes

\[
\begin{bmatrix}
  n^b_{t+1}(0) \\
  n^b_{t+1}(1) \\
  \vdots \\
  n^b_{t+1}(M) \\
  n^b_{t+1}(M+1)
\end{bmatrix}
= 
\begin{bmatrix}
  0 \\
  n^a_t(0) \\
  \vdots \\
  n^a_t(M-1) \\
  n^a_t(M)
\end{bmatrix}
\]

Next to be considered are herd adjustment activities. The rancher adjusts his actual breeding herd, inherited from the

\[
24/\text{For example, if the rancher follows the uniform age distribution, then } n^a_t(i) = \bar{n}^* \text{ for } i = 0, 1, \ldots, M.\]
preceding production period, to the optimum breeding herd at
the beginning of a given production period by adding (or replacing)
and/or culling the breeding animals. Perhaps the rancher's herd
adjustment activities may best be explained by relating the net
change (or adjustment) in the actual breeding herd at the begin-
nning of production period (t+1) to the difference between the opti-
mum breeding herd and the actual breeding herd before herd
adjustment at the beginning of production period (t+1) as follows:

\[
2.28) \quad \frac{a}{b} \left( b_{t+1}^a (n, j) - b_{t+1}^b (n, j) \right) = \left( b_{t+1}^*(n^*, j^*) - b_{t+1}^b (n, j) \right) [h_{ii}]
\]

where \(b_{t+1}^a (n, j)\) denotes the adjusted breeding herd at the be-
inning of production period (t+1), \(b_{t+1}^*(n^*, j^*)\) designates the
optimum breeding herd at the beginning of production period
(t+1), and \(h_{ii}\) for \(i = 0, 1, \ldots, M+1\) represents the "adjustment
coefficient" for the \(i\)-th age group. Alternatively, equation
(2.28) may be expressed as follows:

\[
2.28a) \quad \begin{bmatrix}
a_{t+1}^a (0) - b_{t+1}^b (0) \\
a_{t+1}^a (1) - b_{t+1}^b (1) \\
\vdots \\
a_{t+1}^a (M+1) - b_{t+1}^b (M+1)
\end{bmatrix}
\]
where \( n_{t+1}^a (i) - n_{t+1}^b (i) \) for \( i = 0, 1, \ldots, M+1 \) denotes the actual change in number of the \( i \)-year old breeding animals at the beginning of production period \((t+1)\), and \( n_{t+1}^* (i) - n_{t+1}^b (i) = 0\), \( i = 0, 1, \ldots, M+1 \) designates the desired change in number of the \( i \)-year old breeding animals at the beginning of production period \((t+1)\).

It is now possible to explain, at least in part, the rancher's herd adjustment activities concerning each individual age group.

The proportion of the actual change to the desired change in the number of breeding animals in the \( i \)-th age group at the beginning of production period \((t+1)\) may best be expressed as follows:

\[
2.29) \quad h_{ii} = \frac{\text{the actual change in the number of breeding animals in the } i\text{-th age group at the beginning of period } (t+1)}{\text{the desired change in the number of breeding animals in the } i\text{-th age group at the beginning of period } (t+1)}
\]
where \( i = 0, 1, \ldots, M+1 \) and it is provided that \( \dot{n}_{t+1}^* (i) - n_{t+1} (i) \neq 0 \). The desired change in the number of breeding animals in the 0-th age group at the beginning of production period \((t+1)\) is equal to the optimum number of breeding animals in the 0-th age group, since \( n_{t+1}^0 (0) = 0 \). The actual change in the number of breeding animals in the 0-th age group in response to the desired change could be made by the rancher at the beginning of production period \((t+1)\). (The herd adjustment can easily be accomplished by adding a certain portion of the calf crop from the preceding production period.) Thus, \( h_{00} = 1 \), implying that the herd adjustment for the 0-th age group is completed at the beginning of production period \((t+1)\). Next to be considered is the \((M+1)\)-th age group. Suppose that the optimum last age at the beginning of production period \((t+1)\) is \( M \). Then the desired change in the number of breeding animals in the \((M+1)\)-th age group becomes \( -n_{t+1}^b (M+1) \) since \( n_{t+1}^* (M+1) = 0 \), meaning that the number of breeding animals culled would be in \( n_{t+1}^b (M+1) \). The herd adjustment for this age group could be completed at the beginning of production period \((t+1)\). Thus, \( h_{M+1, M+1} = 1 \) since
\[(0 - \frac{n_{t+1}^b (M+1)}{n_{t+1} (M+1)}) / (0 - \frac{n_{t+1}^b (M+1)}{n_{t+1} (M+1)}) = - \frac{n_{t+1}^b (M+1)}{n_{t+1} (M+1)} - \frac{n_{t+1}^b (M+1)}{n_{t+1} (M+1)}\]

according to equation (2.29). Finally, consideration is given to other age groups between the one-year old age group and the optimum last age group, inclusive. For convenience, the speed of herd adjustment for those age groups under consideration may be generalized as follows. If the desired change in the number of breeding animals in the i-th age group at the beginning of production period \(t+1\), where \(i = 1, 2, \ldots, M\), is positive, then the actual change in the number of breeding animals in the i-th age group would be zero and consequently \(h_{ii} = 0\) for \(i = 1, 2, \ldots, M\). Alternatively, if the desired change in the number of breeding animals in the i-th age group at the beginning of production period \(t+1\) is negative, then the actual change in the number of breeding animals in the i-th age group could be completed in response to the desired change at the beginning of production period \(t+1\), i.e., \(h_{ii} = 1\) for \(i = 1, 2, \ldots, M\). For the above discussion, the adjustment coefficient for the individual age groups would take the numerical value between zero and unity, inclusive, i.e., \(0 \leq h_{ii} \leq 1\) for \(i = 0, 1, \ldots, M\).

\[25/\] This generalization is in fact based upon the assumption that the rancher is not able to purchase any breeding animals of various ages from the market.
What is needed, instead of each individual age group, to explain the rancher's adjustment activities with respect to his breeding herd is a relationship which will explain the change in his breeding herd within one production period. The next task then is to derive a relationship which is capable of explaining the rancher's adjustment activities with his breeding herd in response to the desired changes in his breeding herd. For such a task, an attempt could perhaps be made to relate the actual change in productivity of the breeding herd to the desired change in productivity of the breeding herd. The simplest hypothesis about the herd adjustment is: the change in actual breeding herd is proportional to the difference in productivity between the optimum and the actual breeding herd. This simple hypothesis may be written as follows:

\[ 2.30 \]

\[
\frac{M+1}{i=0} \sum f(i) \left( n_{t+1}^a (i) - n_{t+1}^b (i) \right)
\]

\[
= h \left[ \frac{M+1}{i=0} \sum f(i) \left( n_{t+1}^* (i) - n_{t+1}^b (i) \right) \right]
\]

26/ Now the crucial question is: how can the productivity of breeding herd be measured? In Chapters II and III, the productivity of breeding herd and working herd is measured in terms of capability of producing calves within one production period. However, in Chapter IV, the productivity of working herd will be approximated by the number of breeding cows and heifers two years old and older kept on farms on January 1.
where the term on the left-hand side of equation (2. 30) designates the actual change in productivity of breeding herd, \( h \) denotes the "adjustment coefficient of breeding herd" which reflects the rancher's adjustment activities concerning his breeding herd in response to the desired change in productivity of breeding herd, and the term on the right-hand side of equation (2. 30) represents the desired change in productivity of breeding herd. (It is interesting to note that if the adjustment coefficient for each individual group is assumed to be the adjustment coefficient of the breeding herd, and if equation (2. 28a) is premultiplied by a 1 by \( M+2 \) vector of productivities associated with age, the resultant equation is (2. 30).) Alternatively, equation (2. 30) can be expressed as follows:

\[
2. 30a) \quad h = M+1 \sum_{i=0}^{M+1} f(i) n_{t+1}^a (i) - \sum_{i=0}^{M+1} f(i) n_{t+1}^b (i) \\
= h \left[ M+1 \sum_{i=0}^{M+1} f(i) n_{t+1}^a (i) - \sum_{i=0}^{M+1} f(i) n_{t+1}^b (i) \right]
\]

For the discussion on the rancher's adjustment activities, equation (2. 30) can also be expressed as follows:

\[
2. 30b) \quad h = \frac{M+1}{M+1} \sum_{i=0}^{M+1} f(i) n_{t+1}^a (i) - \sum_{i=0}^{M+1} f(i) n_{t+1}^b (i)
\]
where the _a priori_ qualification on the value of $h$ may be specified such that $0 < h_0 \leq 1$ since $0 < h_{ii} \leq 1$ for $i = 0, 1, \ldots, M+1$. Now substituting equation (2.27b) for the productivity of actual breeding herd before herd adjustment in equation (2.30a) gives

$$2.30c) \quad \sum_{i=0}^{M+1} f(i) \frac{n^a_{t+1}(i)}{n^a_t(i)} - \sum_{i=0}^{M+1} f(i) n^a_t(i)$$

$$= h \left[ \sum_{i=0}^{M+1} f(i) n^{*}_{t+1}(i) - \sum_{i=0}^{M+1} f(i) n^a_t(i) \right].$$

The herd adjustment equation, i.e., (2.30c), deserves a few comments. First, the formulation of a herd adjustment relation is founded on a large number of simplifying assumptions, some of which may not be acceptable with reference to the world of reality. In particular, the assumption--$h_{ii} = h$ for $i = 0, 1, \ldots, M+1$--is a crucial one to support. Thus the herd adjustment equation submitted above should be regarded as an equation which approximates the adjustment activities of the rancher concerning his breeding herd within one production period. Second, the adjustment coefficient of the breeding herd for every production period can be measured (or described) by equation (2.30c) if the actual change and the desired change in productivity of the breeding herd are known. For situations in which the rancher follows the policy of "variable" age distribution outlined in section II, C, 2, the adjustment coefficient of the breeding herd
is one, since the actual change in productivity of the breeding herd can be adjusted to the desired change in productivity of the breeding herd within one production period. Alternatively, for situations in which the rancher follows the policy of "fixed" age distribution outlined in section II, C, 1, the adjustment coefficient of the breeding herd is one, if the desired change in productivity for each age group is negative while it is between zero and unity, inclusive, if the desired change in productivity for each age group is positive. Finally, equation (2.30c) may be used to approximate the change in productivity of the breeding herd at the beginning of production period (t+1), if the adjustment coefficient of the breeding herd and the desired change in productivity of the breeding herd are known at the beginning of production period (t+1).

D. **Economic Relations**

In the first part of this chapter, the determinants of the breeding (and working) herd of optimum size and composition under a set of simplifying assumptions were discussed. In addition, the size of the calf crop derived from the optimum working herd, the optimum number of calves sold, and the optimum number of breeding animals culled were discussed. In the second part, the numerical examples of optimum solutions for a few alternative stationary processes of ranch operation were
presented in support of the theoretical discussion given in section II, A. In the third part, it was seen that within the framework of hypothetical ranch operation the herd adjustment activities would generate the time-path of the size of calf crop, the number of calves sold, and the number of breeding animals culled. A herd adjustment relation was finally submitted which might describe and explain, at least in part, the herd adjustment activities of the rancher within one production period. In the present part, an attempt is made to formulate a model which is capable of explaining the ranch operation, with special reference to the investment activities of the rancher concerning his working herd and replacement heifers, the calf production process, and the marketing activities of the rancher in regard to the feeder calves and the breeding animals (which are not profitable in the breeding herd). 27/ This part is divided into five sections. The first section is devoted to development of an "investment behavior" relation by the representative rancher concerning his working herd. The second section specifies an "investment behavior" relation by the representative rancher concerning his

27/ Of course, the ranchers in the world of reality cull the breeding animals from their breeding herds, depending upon the calving records and other criteria. However, it is assumed that the "representative" beef breeder culls his breeding animals, depending only upon the optimum last age, the available amount of feed resources, and the policy of age distribution.
replacement heifers. The third section deals with the presentation of the calf production process. The fourth section submits the calf supply (or marketing) relation. In the last section, the cow supply (or marketing) relation is formulated.

1. "Investment Behavior" Relation Concerning Working Herd

On the basis of what was advanced above for exposing the fundamental activities of the rancher, the next task is to formulate an "investment behavior" relation which is capable of explaining not only the rancher's demand for the working herd but also his adjustment behavior within one production period. In the preceding discussion on the determinants of the optimum breeding (and working) herd, it was suggested that the optimum (or desired) working herd would be dependent upon the available amount of feed resources and the related economic data, as shown by equation (2.18). For convenience it was assumed that at the beginning of a given production period the rancher knew the price level of calves sold for the subsequent production period. In addition, it was assumed that the rancher would follow the policy of "fixed" (or "variable") age distribution after any changes in the available amount of feed resources and/or the economic data. Perhaps the rancher anticipates (or expects) the price of calves for the subsequent production period and then
approximates the optimum last age, since he does not know the price level for the subsequent production period. Perhaps the rancher may follow the policy of "normal" age distribution—the age distribution which maintains a stable relationship among the age groups with respect to the number of animals included in each age group. 28/ The rancher might follow the policy of "normal"

28/ The "normal" age distribution (or "long-run" age distribution) is referred as an age distribution which is determined by the rancher, depending upon the "expected" last age (i.e., the optimum last age anticipated), the available amount of feed resources, the breeding herd inherited from the preceding production period, and the mortality of the breeding animals. In order to exemplify the notion of a "normal" age distribution, suppose that a breeder maintains a stable herd of breeding animals with a stable amount of feed resources and the "expected" last age (which is same as the optimum last age). In addition, suppose that each breeding animal lives, at most, M years. The breeding herd may then be expressed as follows:

\[
\begin{align*}
\text{n}\,(0) &= g_{01}\,n\,(0) \\
\text{n}\,(1) &= g_{12}\,n\,(0) = g_{01}\,g_{12}\,n\,(0) \\
\text{n}\,(2) &= g_{23}\,n\,(1) = g_{01}\,g_{12}\,g_{23}\,n\,(0) \\
& \vdots \\
\text{n}\,(j^*) &= g_{j^*-1,j^*}\,n\,(j^*-1) = g_{01}\,\ldots\,g_{j^*-1,j^*}\,n\,(0),
\end{align*}
\]

where \( n\,(i) \) represents the number of \( i \)-year old breeding animals in a given production period for \( i = 0, 1, \ldots, j^* \) and \( g_{i,j} = 1 \) designates the proportion of breeding animals surviving from age \( i \) to \( i+1 \). Aggregating the number of breeding animals maintained by the rancher gives
age distribution and might adjust his actual to his "expected"
breeding (and working) herd, which is approximated by the
rancher in the light of "expected" last age and available amount of

\[
j^* \sum_{i=0}^{j^*} n(i) = n(0) \left( g_{01} + g_{01} g_{12} + \cdots + g_{01} g_{12} \cdots g_{j^*-1, j^*} \right),
\]

or \( n(0) = n(0) Z, \)

where \( Z \) denotes the terms inside of the parenthesis. By substi-
tuting for \( n(0) = \sum_{i=0}^{j^*} n(i)/Z \) into the above equations, the opti-
mum number of breeding animals in the age group, ranging from
0 to the optimum last age, may be expressed as follows:

\[
n^*(0) = (g_{01}/Z) \sum_{i=0}^{j^*} n(i)
\]

\[
n^*(1) = (g_{01} g_{12}/Z) \sum_{i=0}^{j^*} n(i)
\]

\[
\vdots
\]

\[
n^*(j^*) = (g_{01} g_{12} \cdots g_{j^*-1, j^*}/Z) \sum_{i=0}^{j^*} n(i)
\]

where \( n^*(i) \) represents the optimum number of \( i \)-year old breeding
animals. From the above equations, the optimum proportion of
breeding animals in each age group with respect to the optimum
size of breeding herd may be stated as follows:

\[
g_{01}/Z,
\]

\[
g_{01} g_{12}/Z,
\]

\[
g_{01} g_{12} g_{23}/Z,
\]

\[
\cdots \cdots \cdots ,
\]

\[
g_{01} g_{12} g_{23} \cdots g_{j^*-1, j^*}/Z.
\]
feed resources. In the present section, the rancher's demand for breeding cows at the beginning of a given production period and his adjustment activities within one production period when the price of calves is anticipated are considered. After such considerations, an "investment behavior" relation is submitted as a testable hypothesis which may explain the investment activities of the rancher concerning his working herd at the beginning of a given production period.

(1) The "Expected" Working Herd Demanded: The determinants of the optimum working herd under a set of simplifying assumptions were discussed in section II, A. However, two assumptions—(a) the rancher knows the price level of calves sold for the subsequent production period, and (b) the rancher follows the policy of "fixed" (or "variable") age distribution—are now subjected to modification. In the present section, the rancher's demand for "expected" working herd is therefore discussed.

In order to explain the rancher's demand for "expected" working herd, suppose that the rancher anticipates the subsequent

It should be noted that as long as the above proportions, the optimum size of breeding herd, and the optimum last age are known, the "normal" age distribution can be determined.
period's calf price in the light of past calf prices. In addition, suppose that the rancher follows his "normal" age distribution.  

Modifying the discussion in section II, A, 2, by incorporating the above suppositions, it is hypothesized that the rancher anticipates a certain price level of calves sold, estimates the optimum last age anticipated (i.e., "expected" last age), and consequently determines the optimum size and composition of his breeding (and working) herd, depending upon its "normal" age distribution. Then, the rancher's demand for the "expected" working herd at the beginning of production period (t+1) with such modifications may be explained by the relation as follows:  

\[ w_{t+1}^{**} (n^*, j^*) = H_3(p_{ca}^*, p_{co}^*, p_{cm}^*, r_t^*, f_t) \]

\[ \text{2.31)} \]

29/ For example, a normal age distribution of the breeding herd may be alternatively described by a probability density function as follows: 

\[ p(x) = A e^{-Bx} \quad \text{for} \quad 0 \leq x \leq j^* \]

where \( x \) denotes the age of breeding animals, \( j^* \) represents the optimum last age of the breeding herd, and \( A \) and \( B \) designate arbitrary positive constants.

30/ It should be noted here that the "expected" working herd demanded is measured by the optimum size and composition and explained by the economic data and the available amount of feed resources. However, it should be recognized that the productivity of the "expected" working herd demanded will be measured hereafter in terms of capability of producing calves within one production period.
where \( w_{t+1}^{**} (n^*, j^*) \) designates the "expected" (or "normal") working herd at the beginning of production period \((t+1)\), which is measured in terms of optimum size and optimum last age, and \( p_{t+1}^{ca^*} \) denotes the subsequent period's calf price anticipated at the beginning of production period \((t+1)\). It should be noted from equation (2.31) that the expected working herd at the beginning of production period \((t+1)\) depends upon the calf price level anticipated at the beginning of production period \((t+1)\) and upon other economic data of the preceding production period. On the basis of the discussion in section II, A, 6, it is reasonable to expect that the expected working herd is positively related to the anticipated price level of calves sold and the available amount of feed resources, and negatively related to other economic data specified in equation (2.31). The expected working herd in equation (2.31) may be alternatively measured in terms of productivity. Thus, the demand for the expected working herd may be alternatively explained by the relation as follows:

\[
2.31a) \quad \sum_{i=2}^{j^*} f(i) n_{t+1}^{**} (i) = H_3 (p_{t+1}^{ca^*}, p_t^{co}, p_t^{cm}, r_t, F_t),
\]

where \( \sum_{i=2}^{j^*} f(i) n_{t+1}^{**} (i) \) represents the productivity of expected working herd at the beginning of production period \((t+1)\).
(2) The "Expected" Price Level of Calves Sold: Next to be considered is how the rancher may expect (or anticipate) the future price level of calves sold. Perhaps, together with the determinants specified in equation (2, 31), he anticipates the future price level of calves sold according to the results derived from a statistical experiment and then determines his expected working herd. Alternatively, he may anticipate the future price level of calves sold in the light of his experience and/or guesswork and then determines his expected working herd. At any rate, it is reasonable to assume that the rancher follows some kind of scheme to derive an anticipated price level of calves sold. In order to explain the anticipation of the rancher as to the future price of calves, findings of previous studies are reviewed and a simple model of price expectation is submitted.

Within the last decade several studies of the accuracy of expectations in the industrial sector have appeared. These studies cover longer periods than do similar studies on the price expectations of farmers so that conclusions based on such studies are likely to be better substantiated than those based on survey of farmers. In a brief review of a number of recent studies of expectations based on interviews, Nerlove (65, p. 50) summarizes:

"the main results of the three studies [Theil (78), Modigliani and Sauerlender (53), and Eisner (20)]
examined indicate that there is widespread under-
estimation of actual changes and that forecasters
could generally do a better job at predicting the
levels of actual outcomes if they used some simple
mechanical device such as a projection of the cur-
rent value of the variable to be predicted."

In particular, with regard to the implication of Eisner's study,

Nerlove (65, p. 51) states:

"We may take Eisner's suggestion that expectations
may be based on a concept of the normal as a starting
point in our development of a model of expectation
formation. The discussion at this point may most
easily be couched in terms of prices and price expec-
tations. If more specific information is not available,
it seems reasonable to assume that the 'normal' price
expected for some future date depends in some way
on what prices have been in the past. Expectations of
'normal' price are, of course, shaped by a multitude
of influences, so that a representation of expected
price as a function of past prices may merely be a
convenient way to summarize the effects of these
many and diverse influences."

Then the question is: how may the rancher's price anti-
cipation be explained in terms of past prices? Of course, the
rancher determines his expected price level of calves sold
either on the basis of a mixture of experience, intuition, and
guesswork or on the basis of the results derived from some
statistical experiment. However, according to Eisner's
suggestion, a simple model relating the anticipated price level
of calves sold to the past observed prices may be expressed
as follows:
where $a_k$ is defined as the weight of the expectation attributed to the price level of calves sold in the $(t-k)$-th production period and $p_{t-k}^{ca}$ represents the price level of calves sold in the $(t-k)$-th production period.

(3) **The Herd Adjustment Equation:** What is needed to explain the rancher's adjustment activities—replacing and culling the breeding animals—associated with his working herd is a herd adjustment equation which relates the actual change to the desired change in the working herd. After considering how the rancher would adjust his working herd in response to the desired change in his expected working herd, which in turn depends upon the "expected" last age and the available amount of feed resources, a herd adjustment equation is submitted.

From discussions on the numerical examples of herd adjustment in section II, C, 1, it is apparent that the rancher who follows the policy of maintaining a uniform age distribution adjusts his actual to his optimum working herd at the beginning of each production period as long as there are no changes in the optimum last age and/or the available amount of feed resources.
That is, the rancher maintains a breeding (and working) herd of constant size and composition from period to period, by replacing and culling a constant number of breeding animals. (This example illustrates a stationary process of ranch operation.) It is also apparent that the rancher who follows the policy of maintaining a uniform age distribution may not adjust his actual to his optimum working herd at the beginning of every production period if there are any changes in the optimum last age and/or the available amount of feed resources.

From the numerical examples of herd adjustment processes in section II, C, 2, it is evident that the rancher who follows the policy of changing age distribution adjusts his actual to his optimum working herd at the beginning of every production period.

Now the question is: how can the adjustment activities of the rancher concerning his working herd be described and explained when he follows his "normal" age distribution? A very simple herd adjustment relation from the discussion in section II, C, 3, may best be modified for the breeding cows as follows:

$$2.33) \quad w_t^{a} (n, j) - w_t^{b} (n, j)$$

$$= (w_t^{**} (n^*, j^*) - w_t^{b} (n, j)) [h_{ij}],$$

where $w_t^{a} (n, j)$ denotes the adjusted working herd at the beginning of production period $(t+1)$, $w_t^{b} (n, j)$ designates the actual
working herd inherited from the preceding production period

prior to any herd adjustment, and $h_{ii}$ for $i = 2, 3, \ldots, M+1$
represents the adjustment coefficient of $i$-th age group. Alternatively, equation (2.33) can best be expressed as follows:

\[
\begin{bmatrix}
  n_{t+1}^a (2) & - & n_{t+1}^b (2) \\
  n_{t+1}^a (3) & - & n_{t+1}^b (3) \\
  \vdots & & \vdots \\
  n_{t+1}^a (M+1) & - & n_{t+1}^b (M+1)
\end{bmatrix}
\]

2.33a)

\[
\begin{bmatrix}
  n_{t+1}^{**} (2) & - & n_{t+1}^b (2) \\
  n_{t+1}^{**} (3) & - & n_{t+1}^b (3) \\
  \vdots & & \vdots \\
  n_{t+1}^{**} (M+1) & - & n_{t+1}^b (M+1)
\end{bmatrix}
\begin{bmatrix}
  h_{22} & 0 & 0 & 0 & 0 \\
  0 & h_{33} & \cdots & \cdots \\
  \vdots & \vdots & \ddots & \ddots \\
  0 & \cdots & \cdots & h_{M+1, M+1}
\end{bmatrix}
\]

where $(n_{t+1}^a (i) - n_{t+1}^b (i))$ denotes the actual change in number of breeding animals in the $i$-th age group and $(n_{t+1}^{**} (i) - n_{t+1}^b (i))$ designates the desired change in number of breeding animals in the $i$-th age group for $i = 2, 3, \ldots, M+1$. Assuming that $h_{ii} = h$ for $i = 2, 3, \ldots, M+1$ and premultiplying equation (2.33a) by a one by $M$ vector of productivities associated with each age
group, we have

\[
2.34) \sum_{i=2}^{M+1} \left[ f(i) \left( n_{t+1}^a(i) - n_{t+1}^b(i) \right) \right] = h \left[ \sum_{i=2}^{M+1} f(i) \left( n_{t+1}^{**}(i) - n_{t+1}^b(i) \right) \right],
\]

where the term on the left-hand side of equation (2.34) denotes the actual change in productivity of working herd, \( h \) designates the "adjustment coefficient of working herd" which measures the sensitivity of the rancher concerning his working herd in response to the desired change in productivity of optimum working herd, and the other term on the right-hand side of equation (2.34) denotes the desired change in productivity of the optimum working herd.

Alternatively, equation (2.34) can be expressed as follows:

\[
2.34a) \sum_{i=2}^{M+1} f(i) n_{t+1}^a(i) - \sum_{i=2}^{M+1} b(i) n_{t+1}^b(i) = h \left[ \sum_{i=2}^{M+1} f(i) n_{t+1}^{**}(i) - \sum_{i=2}^{M+1} f(i) n_{t+1}^b(i) \right].
\]

It is a simple model, as a first approximation, which may not do justice to the complexity of the problem. It simply assumes that "response" is directly proportional to "stimulus."

Actually, the adjustment process is affected by a whole set of economic as well as biological considerations, and the relationship may not remain linear when any large changes in actual
working (and breeding) herd are called for. There may also be asymmetries in the herd adjustment activities as noted in sections II, C, 1 and II, C, 2. For example, in the phase of herd contraction while the optimum last age remains stable over time, it is possible to liquidate either a part or all of the breeding cows in the working herd. This provides a floor for the possible magnitude of downward adjustments. In the phase of herd expansion when the optimum last age remains stable over time a large increase in the size of the actual working herd may not be immediately attained in response to an increase in the size of the expected working herd. These complications are ignored in the present study. The most important reason for overlooking them is that it would require a rather complicated model to take them into account. And while equation (2, 34) may present an oversimplified picture of herd adjustment, it may perhaps explain, at least in part, adjustment activities of the rancher concerning his working herd within one production period.

(4) The "Investment Behavior" Relation: It is explicitly hypothesized under a large number of simplifying assumptions that at the beginning of each production period the rancher anticipates the price level of calves sold for the subsequent production period, approximates his expected working herd, and adjusts his actual working herd in response to his expected working herd.
The rancher's investment activities can be explained by a behavior equation which relates the actual working herd maintained at the beginning of a given production period to the determinants of the expected working herd and the actual working herd maintained at the beginning of the preceding production period. Thus is formulated an investment behavior relation as a "partially reduced-form equation" which is discussed by Hildreth and Jarret (38, p. 88 and 108) and Nerlove (66, p. 22-82).

As a simple device to explain the rancher's investment activities concerning his working herd, an economic model can be recapitulated as follows:

\[
2.31a) \quad \sum_{i=2}^{M+1} f(i) n_{t+1}^{**}(i) = H_3(p_{t+1}^{ca*}, p_t^{co}, p_t^{cm}, r_t, \bar{F}_t)
\]

\[
2.32) \quad p_{t+1}^{ca*} = \sum_{k=0}^{\lambda} a_k p_{t-k}^{ca} \quad \text{where } 0 \leq a_k \leq 1 \text{ and } \sum_{k=0}^{\lambda} a_k = 1
\]

\[
2.34a) \quad \sum_{i=2}^{M+1} f(i) n_{t+1}^{a}(i) - \sum_{i=2}^{M+1} f(i) n_{t+1}^{b}(i) = b \left[ \sum_{i=2}^{M+1} f(i) n_{t+1}^{**}(i) - \sum_{i=2}^{M+1} f(i) n_{t+1}^{b}(i) \right]
\]
putting equation (2.32) into (2.31a) and then equation (2.31a) into (2.34a) results in

\[ 2.35 \]
\[ \sum_{i=2}^{M+1} f(i) n_{i+1}^{a}(i) = \sum_{i=2}^{M+1} f(i) n_{i+1}^{b}(i) \]

\[ = h' \left[ \sum_{i=2}^{M+1} f(i) H_{3} \left( \sum_{k=0}^{\lambda} a_{k} p_{t-k}^{ca}, p_{t}^{co}, \right) \right. \]
\[ \left. - \sum_{i=2}^{M+1} f(i) n_{i+1}^{b}(i) \right] \]

Interest is directed toward the actual working herd adjusted at the beginning of production period (t+1) rather than the net change at the beginning of production period (t+1). Transforming equation (2.35), or adding the productivity of the actual working herd before herd adjustment at the beginning of production period (t+1) yields

\[ 2.36 \]
\[ \sum_{i=2}^{M+1} f(i) n_{i+1}^{a}(i) = (1 - h) \sum_{i=2}^{M+1} f(i) n_{i+1}^{b}(i) \]
\[ + h' \sum_{i=2}^{M+1} f(i) H_{3} \left( \sum_{k=0}^{\lambda} a_{k} p_{t-k}^{ca}, p_{t}^{co}, \right) \]
\[ p_{t}^{cm}, r_{t}, \bar{F}_{t} \).

Since it is assumed that there are no death losses in the number of breeding cows maintained, according to one of the simplifying assumptions, then equation (2.36) becomes
2.36a) \[
\sum_{i=2}^{M+1} f(i) n^a_{t+1}(i) = (1 - h') \sum_{i=2}^{M+1} f(i) n^a_{t}(i) \\
+ h \sum_{i=2}^{M+1} f(i) H_3(\lambda \sum_{k=0}^{\lambda} a_k p_{t-k}^c p_t^c, \rho_{cm}, r_t, F_t) \\
\]

since \[\sum_{i=2}^{M+1} f(i) n^b_{t+1}(i) = \sum_{i=2}^{M+1} f(i) n^a_{t}(i). \]
Equation (2.36), or (2.36a), may be called the "investment behavior" relation, although this type of equation has been referred to as a "short-run" demand equation for durable input [Griliches (26, p. 186)]. The a priori qualifications of equation (2.36a) may be specified as follows: the productivity of the working herd maintained at the beginning of production period (t+1) is positively related to the productivity of the working herd maintained at the beginning of production period t, the expected price level of calves sold, and the available amount of feed resources, and negatively related to the other economic data included in the equation.

2. "Investment Behavior" Relation Concerning Heifers

Following the procedure adopted for the derivation of investment behavior relation concerning working herd, an investment behavior relation which is capable of explaining the investment activities of the breeder concerning replacement heifers
may be derived from the three relations as follows:

\[ r_{t+1}^* = H_4 (p_{t+1}, p_t, p_t, r_t, F_t) \]

2.32)

\[ p_{t+1}^{ca^*} = \frac{\lambda}{k=0} a_k p_{t-k}^{ca} \]

where \( 0 < a_k < 1 \) and \( \sum_{k=0}^{\lambda} a_k = 1 \)

2.37b)

\[ r_{t+1} - r_t = h'' (r_{t+1}^* - r_t) \]

where \( r_{t+1}^* \) denotes the one-year old heifers desired (or expected) at the beginning of production period \( (t+1) \), \( r_{t+1} \) represents the one-year old heifers maintained at the beginning of production period \( (t+1) \), \( r_t \) designates the one-year old heifers maintained at the beginning of production period \( t \), \( h'' \) represents the adjustment coefficient associated with the one-year old heifers, and other variables are previously identified. Substituting (2.32) into (2.37a) and (2.37a) into (2.37b) and rearranging terms in (2.37b) give an investment behavior relation as follows:

\[ r_{t+1} = (1 - h'') r_t + h'' (r_{t+1}^* - r_t) \]

3. Calf Production Relation

Next to be considered is the technological relation which
explains the calf production process within a hypothetical ranch operation. On the basis of discussions in section II, A, the calf crop obtained at the end of each production period depends on the "primary" factor of production (i.e., working herd) and the "supplementary" factors of production (i.e., feed, care, maintenance, and other environmental and biological factors).

The size of the calf crop for each production period depends primarily upon the actual working herd of certain size and composition at the beginning of each production period. Since every breeding cow in the working herd is not certain to calve for various reasons (e.g., disease, weather conditions), it is reasonable to expect that the size of the calf crop is generally less than or equal to the size of working herd. However, it should be recalled from the numerical examples in section II, C, 2, (7), that the size of the calf crop derived from a given size of working herd would be different, depending upon the composition of the working herd.

The size of the calf crop also depends upon the supplementary factors of production for retaining a working herd of certain size and composition. Suppose that the rancher provides an "adequate" amount of feed, care, and maintenance for breeding cows during a given production period. If he improves the quality and quantity of feed and quality of care and maintenance beyond the
"adequate" level, then he may perhaps increase the size of his calf crop. Furthermore, if he introduces biological innovation, e.g., technological improvement for increasing the probability of obtaining twin calves from a prospective breeding cow, then it is possible for him to increase the size of his calf crop.

On the basis of the above considerations, a reasonable calf production relation which explains the calf production process, following the growth process of breeding animals in the working herd, may best be expressed as follows:

$$2.39) \quad c_{c t}^{M+1} = \sum_{i=2}^{M+1} f(i) n^a_t(i), Q_t$$

where $c_{c t}$ denotes the size of calf crop at the end of production period $t$, $\sum_{i=2}^{M+1} f(i) n^a_t(i)$ designates the productivity of the actual working herd at the beginning of production period $t$, and $Q_t$ represents the technological and environmental factors which influence the calf production process during the production period $t$. The a priori qualifications on the calf production relation imposed by the specification are: the size of calf crop is positively related to the size of working herd and the technological and environmental factors.
4. Supply of Beef Calves

At the beginning of every production period, the beef breeder supplies a large portion of the calf crop obtained in the preceding production period to the market and keeps the rest as replacements for the old breeding cows. In other words, he makes two decisions: (a) How many and what kind of calves should be marketed? (b) How many and what kind of calves should be kept? However, it is quite evident for a given size of calf crop that if he makes one decision, then the other must be automatically determined. Discussion will be confined to the former and to a calf supply relation, which will be used for constructing a calf price relation in the following chapter.

The sex composition of marketing calves is relatively stable over time. Neither all heifer calves nor all bull calves will be marketed if the rancher wants to replace old breeding cows and bulls. That is, the rancher attempts to market bull calves (e.g., steer calves) and nonprospective heifer calves. Since the relative number of old breeding cows to be replaced is in general greater than that of old breeding bulls to be replaced, it is reasonable to expect that the relative number of bull calves marketed with respect to the number of calves produced is greater than that of heifer calves marketed with respect
to the number of heifer calves produced.

The number of beef calves (both bull and heifer) marketed depends on the difference between the size of the calf crop and the number of replacement calves. By referring to the stationary ranch operation, it is evident that the number of replacement calves is stable as long as the optimum breeding herd remains stable over time. However, if the number of total replacement calves within the framework of a nonstationary ranch operation is under consideration, no hard and fast explanations in regard to the number of calves marketed can be made here. Nevertheless, it seems plausible hypothesis to state that the number of calves marketed may best be explained by the equation as follows:

\[ cs_{t+1} = H_6 (c_{t+1} n_{t+1}^{**} (0)) \]

where \( n_{t+1}^{**} (0) \) represents the optimum number of replacement calves at the beginning of production period \((t+1)\). The a priori qualification of the calf supply relation may be made as follows: the number of calves sold (or marketed) is related positively to the size of the calf crop and negatively to the optimum number of replacement calves maintained in the optimum breeding herd.
5. Supply of Salvage Cows

The rancher's marketing decision in regard to breeding cows is primarily based upon the profitability of retaining each breeding cow in the working herd. It was pointed out in the discussion associated with the stationary ranch operation that the number of breeding animals culled is stable over time as long as the optimum last age and the available amount of feed resources remain stable. However, when the ranch operation is considered in situations where the optimum last age and/or the available amount of feed resources are not stable, it is not unreasonable to explain the number of salvage cows supplied in terms of the expected working herd. In addition, it depends on the existing working herd. The supply relation of salvage cows may best be expressed as follows:

\[ sc_{t+1} = H \left[ \sum_{i=2}^{M+1} f(i) n_{t+1}^{**}(i), \sum_{i=2}^{M+1} f(i) n_{t}^{a}(i) \right] \]

The a priori qualification of the marketing relation of salvage cows may best be made as follows: the number of salvage cows marketed is negatively related to the expected working herd and positively related to the actual working herd inherited from the preceding production period.
E. Concluding Remarks

In this chapter a model of a cattle ranch was formulated which would provide an explanation of the economic activities of the beef breeder, depending upon the technology of growth process associated with beef cattle. In particular, the process whereby the rancher would determine his expected working herd was discussed. On the basis of this discussion, an investment relation concerning breeding cows and replacement heifers, a calf production relation, a calf supply relation, and a cow supply relation were formulated.

In developing such a model the assumptions deemed necessary were carefully identified. While these assumptions have seemed fairly restrictive at times, it is simply averred that they are reasonable and acceptable. Of course, the great advantage of such development lies precisely in the fact the assumptions are explicit and may be useful in profitably redirecting efforts if modifications should be desired at a later time.
CHAPTER III

INVESTMENT BEHAVIOR OF BEEF BREEDING CATTLEMEN 31/

As stated in Chapter I, the primary objective of the present study is to explain, qualitatively as well as quantitatively, the investment activities of beef breeders concerning their working herd. The secondary objective is to explain the investment activities of beef breeders concerning their working herd in conjunction with the determination of calf price, the determination of salvage cow price, the calf production process, and the investment activities of beef breeders concerning their replacement heifers. For such objectives, the present chapter is separated into two major parts. Part A deals with the formulation of three alternative hypotheses which may explain the investment activities of beef breeders concerning their working herd on the basis of what was advanced in the preceding chapter. In Part B a multiple-relation model is presented which is capable of explaining

31/ The optimum last age and the optimum working (and breeding) herd are defined and used for the ranch operation where the rancher knows all economic data, including calf price; the expected last age and the expected working (and breeding) herd are defined and used for the ranch operation where the rancher knows all economic data except calf price, which he anticipates. This difference should be recognized throughout the present thesis.
the investment activities of beef breeders concerning their working herd and replacement heifers, the determination of calf price, the determination of salvage cow price, and the calf production process. A short summary section concludes the chapter.

A. Investment Behavior Hypotheses

The present part is divided into five sections. The first section presents an initial model based on materials already advanced, plus three additional simplifying assumptions. In the second section, an initial model is reviewed with respect to the variables used in explaining the aggregate investment activities. The third section presents the revised model of aggregate investment behavior. In the fourth section, the identification of parameters associated with the revised model is discussed. In the last section, a few aggregate models are presented as testable hypotheses.

1. An Initial Model

Given the micro model, i.e., either (2.31a), (2.32), and (2.34a) or (2.36), an exact macro analogue can be obtained under a highly simplifying assumption of aggregation. For the sake of simplicity assume that every beef breeder behaves as if he were
the representative beef breeder who follows the policy of "normal" age distribution, as discussed in the preceding chapter. In addition, assume that equation (2.31a) is of linear form. Then the investment activities of the m-th beef breeder may be explained by a system of equation as follows:

\[ p_{ca}^{m, t+1} = a_0 p_{t}^{ca} + \ldots + a_\lambda p_{t-\lambda}^{ca} \]

\[ \sum_{i=2}^{M+1} f(i) n^{**}_{m, t+1} (i) = b_0 + b_1 p_{m, t+1}^{ca} + b_2 p_{t}^{co} + b_3 p_{t}^{cm} + b_4 r_t + b_5 F_{m, t} \]

3.1. a) \[ \sum_{i=2}^{M+1} f(i) n^{**}_{m, t+1} (i) = b_0 + b_1 p_{m, t+1}^{ca} + b_2 p_{t}^{co} + b_3 p_{t}^{cm} + b_4 r_t + b_5 F_{m, t} \]

3.1. b) \[ p_{ca}^{m, t+1} = a_0 p_{t}^{ca} + \ldots + a_\lambda p_{t-\lambda}^{ca} \]

3.1. c) \[ \sum_{i=2}^{M+1} f(i) n^{**}_{m, t+1} (i) = \sum_{i=2}^{M+1} f(i) n^{**}_{m, t} (i) - \sum_{i=2}^{M+1} f(i) n^{a}_{m, t} (i) \]

where \[ \sum_{i=2}^{M+1} f(i) n^{**}_{m, t+1} (i) \] represents the productivity of the working herd desired by the m-th beef breeder at the beginning of production period \( (t+1) \), \( p_{ca}^{m, t+1} \) denotes the future price level of calves anticipated by the m-th beef breeder at the beginning of production period \( (t+1) \), \( F_{m, t} \) designates the amount of feed resources available to maintain a herd of breeding animals for the

32/ See the discussion in footnote 28 for the "normal" age distribution.
production period \( t \), \( \sum_{i=2}^{M+1} f(i) n_{im}^a, t+1(i) \) represents the productivity of the actual working herd maintained by the \( m \)-th beef breeder at the beginning of production period \((t+1)\) after herd adjustment, and \( \sum_{i=2}^{M+1} f(i) n_{im}^a, t(i) \) denotes the productivity of the actual working herd maintained by the \( m \)-th beef breeder at the beginning of production period \( t \) after herd adjustment. Other variables have been defined in section II, A, 1, and \( b_0, b_1, b_2, b_3, b_4, b_5, a_0, \ldots, a_\lambda \), and \( h_m^t \) are arbitrary constants (or parameters) associated with the micro model. Now, suppose that all firms in the industry have identical parameters. Aggregating (3.1.a), (3.1.b), and (3.1.c) over \( s \) firms yields

\[
3.1.a') \quad \sum_{m=1}^{s} \sum_{i=2}^{M+1} f(i) n_{im}^{**}, t+1 = \sum_{m=1}^{s} (b_0 + b_1 p_{m, t+1}^{ca*} + \ldots + b_5 r_{m, t}^{ca} + b_4 r_{m, t} + b_5 b_{m, t})
\]

\[
3.1b') \quad \sum_{m=1}^{s} \sum_{i=2}^{M+1} f(i) n_{im}^{**}, t+1 = \sum_{m=1}^{s} (a_0 p_{t}^{ca} + \ldots + a_{\lambda} p_{t-\lambda}^{ca})
\]

\[
3.1c') \quad \sum_{m=1}^{s} \left[ \sum_{i=2}^{M+1} f(i) n_{im}^{**}, t+1(i) - \sum_{i=2}^{M+1} f(i) n_{im}^{a}, t(i) \right] = \left[ \sum_{m=1}^{s} (h_m^t)^{ca*} - \sum_{m=1}^{s} (h_m^t)^{ca} \right].
\]

On the basis of three additional assumptions, equations
(3.1.a'), (3.1.b'), and (3.1.c') become

\[ \begin{align*}
3.1.a') & \quad M+1 \sum_{i=2}^{s} f(i) n^{**}_{m, t+1(i)} = s b_{0} + s b_{1} p^{ca*}_{m, t+1} \\
& \quad + s b_{2} p^{co}_{t} + s b_{3} p^{cm}_{t} + s b_{4} r_{t} \\
& \quad + s b_{5} F_{m, t}
\end{align*} \]

\[ \begin{align*}
3.1.b') & \quad s p^{ca*}_{m, t+1} = s a_{0} p^{ca}_{t} + \ldots + s a_{\lambda} p^{ca}_{t-\lambda}
\end{align*} \]

\[ \begin{align*}
3.1.c') & \quad M+1 \sum_{i=2}^{s} f(i) n^{a}_{m, t+1(i)} = M+1 \sum_{i=2}^{s} f(i) n^{a}_{m, t(i)}
\end{align*} \]

Alternatively, equations (3.1.a''), (3.1.b''), and (3.1.c'') can be expressed as follows:

\[ \begin{align*}
3.2) & \quad \sum_{i=2}^{M+1} f(i) N^{**}_{t+1} (i) = b_{0} + b_{1} p^{ca*}_{t+1} + b_{2} p^{co}_{t} \\
& \quad + b_{3} p^{cm}_{t} + b_{4} r_{t} + b_{5} F_{t}
\end{align*} \]

\[ \begin{align*}
3.3) & \quad p^{ca*}_{t+1} = a_{0} p^{ca}_{t} + \ldots + a_{\lambda} p^{ca}_{t-\lambda}
\end{align*} \]

\[ \begin{align*}
3.4) & \quad \sum_{i=2}^{M+1} f(i) N^{a}_{t+1} (i) = \sum_{i=2}^{M+1} f(i) N^{a}_{t} (i)
\end{align*} \]
where $N^{**}_{t+1}(i)$ represents the number of i-year old breeding animals desired (or expected) by the beef breeders at the beginning of production period $(t+1)$, $\sum_{i=2}^{M+1} f(i) N^{**}_{t+1}(i)$ denotes the productivity of the aggregate working herd at the beginning of production period $(t+1)$, $N^{a}_{t+1}(i)$ and $N^{a}_{t}(i)$ designates the number of i-year old breeding animals adjusted and maintained by the beef breeders at the beginning of production period $(t+1)$ and $t$, respectively, $\sum_{i=2}^{M+1} f(i) N^{a}_{t+1}(i)$ and $\sum_{i=2}^{M+1} f(i) N^{a}_{t}(i)$ represent the productivity of the aggregate working herd adjusted and maintained by the beef breeders at the beginning of production period $(t+1)$ and $t$, respectively, $F_{t}$ represents the available amount of feed resources in any arbitrary breeding firm at the beginning of production period $t$ (i.e., $F_{t} = F_{m, t}$), $p^{ca*}_{m, t}$ denotes the future price level of calves anticipated by the representative breeder at the beginning of production period $(t+1)$ (i.e., $p^{ca*}_{t+1} = p^{ca*}_{m, t+1}$), and $b_0, b_1, b_2', b_3', b_4', b_5', a_0, \ldots, a_\lambda, \text{ and } h$ are arbitrary constants associated with the macro model. $^{33/}$

$^{33/}$ It should be noted that $a_0 = a_0', a_1 = a_1', \ldots, a_\lambda = a_\lambda$ and $b_1 = s b_1, b_2 = s b_2', \ldots, b_5 = s b_5$ since every beef breeder behaves as if he were the representative breeder. However, it should be recognized that the parameters associated with the micro model may be different from those associated with the macro model in numerical value if every breeder does not behave as if he were the representative breeder.
An aggregate behavior relation which explains, at least in part, the investment activities of beef breeders concerning their breeding cows can be formulated. Substituting (3.3) into (3.2) and then (3.2) into (3.4) yields

\[ 3.5) \quad \sum_{i=2}^{M+1} f(i) N^a_{t+1} (i) - \sum_{i=2}^{M+1} f(i) N^a_t (i) = \frac{h^i}{b_0} + b_1 (a_0 p^c_{t} + \ldots + a_\lambda p^c_{t-\lambda}) + b_2 p^c_{t} + b_3 p^c_{t} + b_4 r_t + b_5 F'_t \]

Adding \( \sum_{i=2}^{M+1} f(i) N^a_t (i) \) to both sides of (3.5) yields

\[ 3.6) \quad \sum_{i=2}^{M+1} f(i) N^a_{t+1} (i) = (1 - h^i) \sum_{i=2}^{M+1} f(i) N^a_t (i) + \frac{h^i b_0}{h'} + h^i b_1 a_0 p^c_{t} + \ldots + h^i b_1 a_\lambda p^c_{t-\lambda} + h^i b_2 p^c_{t} + h^i b_3 p^c_{t} + h^i b_4 r_t + h^i b_5 F'_t. \]

Thus, the linear investment behavior relation derived above suggests that the investment activities of beef breeders concerning their breeding cows at the beginning of a given production period can be explained by relating the aggregate working herd
maintained at the beginning of a given production period to the economic data in the preceding periods, the available amount of feed resources at the beginning of the preceding production period, and the aggregate working herd maintained at the beginning of the preceding production period.

2. **Criticisms of Initial Model**

Potentially useful extensions and refinements of the model specified by (3.2), (3.3), and (3.4) are easy to suggest. However, several modifications of the model, (3.6), will be made with respect not only to the determinants of the expected working herd in equation (3.2) but also to the assumption about the rancher's culling activities, incorporated into equation (3.4). It is then useful to discuss briefly criticisms of both the variables that will be taken into account in the reformulation and those that will not. A brief discussion for the rationale of modifying the assumption about the rancher's culling activities will be given, and the modified assumption will be explicitly made. The excluded variables from the model and the modifications submitted for the reformulation should be kept in mind as possible sources of specification error in the present study and as indicators of directions for future research.

34/ The modifications of equation (3.3) for the refutable hypotheses will be discussed in section III, A, 5.
(1) **Price Level of Calves Sold:** In order to explain the fundamental activities of the representative breeder concerning his breeding animals, the price of calves sold per animal has been conveniently used as an economic datum. However, it is evident that the price of calves sold per animal depends on the weight of calves marketed, the selling price of calves per unit of weight (hundredweight), and other attributes of calves marketed. Since the variation in the weights of calves sold and other attributes cannot be easily accounted for, the price of calves sold per hundredweight will be used as the appropriate measure of this theoretical variable.

(2) **Price Level of Salvage Cows:** For the sake of simplicity, the price of salvage cows sold per animal has been used as an economic datum. Of course, the price of salvage cows per animal depends on the weight of salvage cows, the marketing price of salvage cows per hundredweight, and other attributes of salvage cows. The price of salvage cows per hundredweight will be used as the economic variable, representing the price of salvage cows marketed per animal, since the variation in the weights of salvage cows marketed and other attributes of salvage cows marketed cannot easily be accounted.
(3) Available Amount of Feed Resources: One respect in which the model given by (2.31) is a crude approximation is in the treatment of the available amount of feed resources. It was supposed at the outset, in Chapter II, that a representative rancher attains a fixed amount of feed resources at the beginning of every production period. Alternatively, it was later supposed that he attains a variable amount of feed resources at the beginning of every production period. Both suppositions regard the available amount of feed resources as an important variable in determining (or constraining) the size of expected breeding (and working) herd. Nonetheless, a crucial question is: how can this conceptual variable, the available amount of feed resources, be modified as an empirical variable capable of explaining the investment activities of beef breeders as a whole? To answer this question, it would seem desirable to consider how the available amount of feed resources is generally obtained and what a proxy variable would be for the available amount of feed resources.

Each rancher obtains his feed resources from his range land and/or the feed market. In the world of reality, it seems reasonable to assume that each rancher relies heavily on the feed resources derived from his range land, since the available amount of feed resources in the market is fixed at a given point in time.
In turn, the feed resources derived from the range land depend on the conditions of range land—fertility of soil, weather, etc. In the present study, range feed condition is adopted as a proxy variable for the available amount of feed resources, although there may be a large number of alternative variables.

(4) **Unit Cost of Care and Maintenance:** In connection with the determination of an optimum as well as the expected working herd, it was shown in Chapter II that the optimum as well as the expected last age of breeding animals included in the breeding herd is partly influenced by the unit cost of care and maintenance. However, it is reasonable to expect that the cost of care and maintenance for a ranch firm is highly correlated with range condition, since the rancher keeps his herd on the range land. Furthermore, it is reasonable to expect that as range condition improves, unit cost of care and maintenance deteriorates and that as range condition deteriorates, unit cost of care and maintenance goes up. Such phenomena between the unit cost of care and maintenance and range condition tend to increase (or decrease) the size of working herd. Consequently, the coefficient of range condition (or of unit cost of care and maintenance) will tend to be biased if both range condition and unit cost of care and maintenance are included. \(^{35/}\) In addition, the present

\(^{35/}\) It is an example of multicollinearity problems. See the discussion in section IV, B, 1.
conceptual variable cannot be adequately approximated by any available empirical variable. Therefore, the unit cost of care and maintenance will not be included in the model as an explanatory variable.

(5) Market Rate of Interest\(^{36}\): The market rate of interest has also been regarded as an economic datum that influences the determination of the optimum (and expected) last age of breeding animals included in the breeding herd and consequently the optimum (and expected) working herd. The present economic datum was in fact introduced to yield a net accumulated discounted return from retaining a breeding herd for one production period. Since the rancher anticipates the price of calves sold and determines the expected last age as well as the expected working herd in terms of available economic data, he may be less sensitive to the market rate of interest than to the anticipated price of calves, the salvage cow price, and the range condition. (Perhaps the rancher may not consider the market rate of interest because he has been motivated by the anticipated price of calves.) On the basis of the above aversion, the present conceptual variable will be excluded from the model.

(6) The Assumption About the Rancher's Culling Activities:

In order to explain the investment activities of beef breeder concerning his breeding cows within the framework of hypothetical

\(^{36}\) Of course, the market rate of interest may be variously used and interpreted in the study of investment activities. It has been used here to determine the net accumulated discounted return from retaining a breeding herd for one production period.
ranch operation, it has been conveniently assumed that the representative breeder culls his breeding animals, depending only upon the optimum last age, the available amount of feed resources, and the policy of age distribution. Although the technological relationship between the productivity and the age of breeding cows in the aggregate is stable over time, some individual breeding animals may not follow such relationship. Consequently, these breeding animals may be culled from the working herd before they reach the optimum last age. In the world of reality, a breeder culls his breeding animals, depending upon such factors as the optimum last age, the available amount of feed resources, the policy of "normal" age distribution, and the policy of culling breeding animals. (Perhaps a beef breeder may follow a particular culling policy based on the calving record, e.g., culling a breeding cow whenever she does not calve for two consecutive production periods.) For the purpose of the present study, the assumption about the rancher's culling activity is, however, modified herefrom as follows: the representative beef breeder culls his breeding cows, depending upon the optimum last age, the available amount of feed resources, the "normal" age distribution, and the calving records.

37/ Recall the simplifying assumptions adopted for the numerical examples in the preceding chapter. It should be recognized that the herd adjustment was independent of the calving records of breeding animals.
3. **Revision of Initial Model**

In the first section of the present part, an initial model was presented which was formulated on the basis of a large number of simplifying assumptions. In the preceding section, some factors which influence the working herd expected were reviewed in order to modify the initial model. In the present section, an attempt is made to formulate a model which takes into account those modifications suggested in the preceding section and the stochastic errors.

The revised model can be specified as follows:

\[ M+1 \]
\[ \sum_{i=2}^{f(i)} N_{t+1}^{**} (i) = c_0 + c_1 P_{t+1}^{ca} + c_2 P_t^{co} \]
\[ + c_3 R_t \]

3.4) \[ M+1 \]
\[ \sum_{i=2}^{f(i)} N_{t+1}^{a} (i) - \sum_{i=2}^{f(i)} N_{t}^{a} (i) \]
\[ = \frac{1}{h} \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^{**} (i) - \sum_{i=2}^{M+1} f(i) N_{t}^{a} (i) \right], \]

where \( R_t \) denotes the range feed condition for the production period \( t \), and \( c_0 \), \( c_1 \), \( c_2 \), and \( c_3 \) are arbitrary constants.

Although the revised model, i.e., (3.3), (3.4), and (3.7),
capable of explaining the investment activities of cattle ranchers, it may be subject to stochastic errors. It is assumed that equation (3.3) is not subject to any stochastic errors. Initial discussion centers around the stochastic errors associated with the determination of expected working herd and the pattern of herd adjustment, after which equations (3.4) and (3.7) are revised.

The possible errors associated with the determination of expected working herd and consequently the pattern of herd adjustment may be due to the three reasons: (a) incomplete theory, (b) imperfect specification, and (c) incorrect process of aggregation. In connection with the first reason, the theoretical discussions presented in Chapter II are not necessarily complete, since an abstraction under a large number of simplifying assumptions can not explain everything. For example, the probability exists that significant variables (i.e., unit cost of care and maintenance and market rate of interest) were omitted from equation (3.2). As to the second reason, the form of relation may have been specified incorrectly. In other words, the expected working herd at the beginning of a subsequent production period depends nonlinearly upon those variables specified in equation (3.7). Finally, the process of aggregating over the heterogenous individual activities may be subject to error. Thus
it is quite evident from equation (3.4) that the three reasons suggested above for (3.7) might also bring about stochastic errors in specifying the pattern of herd adjustment within one production period.

The revised model, which takes into account the criticisms in the preceding section as well as the stochastic errors associated with the determination of expected working herd and the pattern of herd adjustment, can be written as follows:

\[
\sum_{i=2}^{M+1} f(i) N_{t+1}^{**} (i) = c_0 + c_1 p_{t+1}^{ca*} + c_2 p_t + c_3 R_t + U_{1t}
\]

3. 3)
\[
p_{t+1}^{ca*} = a_0 p_t^{ca} + \ldots + a_\lambda p_{t-\lambda}^{ca}
\]

3. 9)
\[
\sum_{i=2}^{M+1} f(i) N_{t+1}^a (i) - \sum_{i=2}^{M+1} f(i) N_t^a (i)
\]
\[
= h \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^{**} (i) - \sum_{i=2}^{M+1} f(i) N_t^a (i) \right] + U_{3t}
\]

where \( U_{1t} \) represents the stochastic errors associated with the determination of expected aggregate working herd and \( U_{3t} \) reflects the stochastic errors associated with the pattern of herd adjustment.
A revised aggregate investment behavior relation as a partially reduced form equation can be derived from equations (3.3), (3.8), and (3.9). Substituting (3.3) into (3.8) and then (3.8) into (3.9) yields

\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) - \sum_{i=2}^{M+1} f(i) N_t^a(i) \]

\[ = h \left( c_0 + c_1 a_0 p_t^{ca} + \ldots + a_\lambda p_{t-\lambda}^{ca} \right) \]

\[ + c_2 p_t^{co} + c_3 R_t + U_{1t} - \sum_{i=2}^{M+1} f(i) N_t^a(i) \]

\[ + U_{3t}^t. \]

Adding \( \sum_{i=2}^{M+1} f(i) N_t^a(i) \) to both sides of (3.10) and rearranging terms yield

\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) = (1 - h) \sum_{i=2}^{M+1} f(i) N_t^a(i) \]

\[ + h c_0 + h c_1 a_0 p_t^{ca} + \ldots \]

\[ + h c_1 a_\lambda p_{t-\lambda}^{ca} + h c_2 p_t^{co} + h c_3 R_t \]

\[ + h U_{1t} + U_{3t}. \]

4. **Identification**

The stochastic model, (3.11), in the preceding section is designed to explain, at least in part, the investment activities
of cattle ranchers, namely, the demand for the expected working herd, the anticipation of future price level of calves, and the pattern of herd adjustment. The present section deals with the identification of parameters specified either in (3.3), (3.8), and (3.9) or in (3.11) within the framework of multiple regression analysis.

According to the conventional multiple regression analysis, the multiple regression equation of (3.11) can be specified as follows:

\[
\sum_{i=2}^{M+1} f(i) N_{t+1}^{a}(i) = \sum_{i=2}^{M+1} f(i) N_{t}^{a}(i) + \sum_{i=2}^{M+1} f(i) N_{t}^{a}(i) + A_{0} p_{t}^{ca} + \ldots + A_{\lambda} p_{t-\lambda}^{ca} + B_{2} p_{t}^{co} + B_{3} R_{t} + V_{t}
\]

where

\[
B_{0} = h_{0} c_{0}
\]

\[
B_{1} = 1 - h_{1}
\]

\[
B_{2} = h_{2} c_{2}
\]

\[
B_{3} = h_{3} c_{3}
\]

\[
A_{0} = h_{0} a_{0} c_{1}
\]

\[
\ldots \ldots \ldots
\]

\[
\ldots \ldots \ldots
\]
\[
\begin{align*}
A_\lambda &= h' a_\lambda c_1 \\
V_t &= U_1 h' = U_3 t.
\end{align*}
\]

If the compound stochastic term, \( V_t \), satisfies the basic assumptions of the regression analysis, then the coefficients of the regression equation (3.12) can be derived and equated to the parameters of equations (3.3), (3.8), and (3.9) as follows:

\[
\begin{align*}
h' &= 1 - B'_1 \\
c_0 &= B'_0 / h' = B'_0 / (1 - B'_1) \\
c_1 &= A'_0 / h' a_0 = \ldots = A'_\lambda / h' a_\lambda \\
3.14) \\
&= A'_0 / (1 - B'_1) a_0 = \ldots = A'_\lambda / (1 - B'_1) a_\lambda \\
c_2 &= B'_2 / h' = B'_2 / (1 - B'_1) \\
c_3 &= B'_3 / h' = B'_3 / (1 - B'_1)
\end{align*}
\]

where \( A'_0, \ldots, A'_\lambda, B'_0, B'_2, \) and \( B'_3 \) are the estimated regression coefficients. It should be noted that the parameters of equations (3.3), (3.8), and (3.9) are identifiable.

5. Model of Investment Behavior of Beef Breeders Concerning Their Breeding Cows: Three Alternative Hypotheses 38/

38/ A comprehensive empirical study can be made by modifying the assumption about the mode of price anticipation. However, in the present study, it is supposed that the rancher anticipates the calf price on the basis of calf prices in one or two preceding production periods.
In the present section, what has been advanced for explaining the investment activities of beef breeders concerning their breeding cows is summarized and submitted as three competing hypotheses which are subjected to empirical tests (or verifications).

The general hypothesis of investment behavior formulated in the present part has been based on the three economic activities of beef breeders concerning their working herd, viz., anticipation of future price level of calves, determination of the expected working herd at the beginning of subsequent production periods, and adjustment of their actual to their expected working herd within one production period. Since the anticipated future price level and the expected working herd are not observable, a partially reduced form equation is used for explaining the investment behavior of beef breeders as well as for estimating the parameters.

The first refutable hypothesis is: The beef breeders anticipate the future price level of calves only on the basis of two preceding price levels, determine the expected working herd, and adjust their actual to their expected working herd. This hypothesis can be explicitly specified by a model, as follows:

\[
3. \quad \sum_{i=2}^{M+1} f(i) N_{t+1}^{**}(i) = c_0 + c_1 p_{t+1}^{ca*} + c_t p_t + c_3 R_t + U_{1t}
\]
\[ p_{t+1}^{ca} = a_0 p_t^{ca} + a_1 p_{t-1}^{ca} + U_{2t} \]

\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) - \sum_{i=2}^{M+1} f(i) N_t^a(i) \]

\[ = h' \left( \sum_{i=2}^{M+1} f(i) N_{t+1}^{**}(i) - \sum_{i=2}^{M+1} f(i) N_t^a(i) \right) + U_{3t} \]

where \( U_{2t} \) designates the stochastic errors, i.e., the excluded past price levels associated with the anticipation of future price level of calves, \( a_0 + a_1 = 1 \), and \( 0 < h' < 1 \). The aggregate investment behavior relation from the above three equations becomes

\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) = B_0 + B_1 \sum_{i=2}^{M+1} f(i) N_t^a(i) + A_0 p_t^{ca} + A_1 p_{t-1}^{ca} + B_2 p_t^{co} \]

\[ + B_3 R_t + V_{1t} \]

where \( V_{1t} = h' U_{1t} + h' c_1 U_{2t} + U_{3t} \). It should be noted that the parameters of (3.16) are identifiable and that the expected signs of parameters of variables, except the price of salvage

\[ \frac{39}{39} \text{ The first refutable hypothesis supposes that } a_k = 0 \text{ where } k=2, 3, \ldots, \lambda. \text{ In other words, this hypothesis is a special model of equations (3.3), (3.8), and (3.9), where a restriction is imposed upon the mode of price anticipation a priori.} \]
cows, in (3.16) are positive.

The second hypothesis of investment behavior can be regarded as a special case of the first hypothesis, in which an a priori restriction is imposed on the pattern of herd adjustment. That is, the beef breeders adjust their actual to their expected working herd at the beginning of every production period. With such an a priori restriction, i.e., $h^1 = 1$,

$$M+1 \sum_{i=2}^{M+1} f(i) N_t^a (i) \quad \text{drops out of (3.16). Thus the investment behavior relation becomes}$$

$$3.17) \quad \sum_{i=2}^{M+1} f(i) N_{t+1}^a (i) = B_0 + A_0 \ p_{t}^{ca} + A_1 \ p_{t-1}^{ca}$$

$$+ B_2 \ p_{t}^{co} + B_3 \ R_t + V_{1t}^i .$$

The parameters of the structural relations are identifiable, and the expected signs of the parameters, except for the price of salvage cows, are positive.

The third hypothesis of investment behavior can be regarded as the alternative of the first hypothesis, in which the beef breeders do not take into account the price of calves in the preceding production period. That is, $a_1 = 0$. Under such an a priori restriction, $p_{t-1}^{ca}$ drops out of (3.16). Thus the investment behavior relation becomes
\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) = B_0 + B_1 \sum_{i=2}^{M+1} f(i) N_t^a(i) + A_0 p_{t}^{ca} + B_2 p_t^{co} + B_3 R_t + V_{1t} \]

It should also be noted that the parameters of this investment behavior relation are identifiable and that the expected signs of the parameters of variables, except that of the price of salvage cows, are positive.

**B. A Multiple-Relation Model**

The three alternative hypotheses submitted in the preceding part may explain the investment behavior of beef breeders as a whole concerning their working herds, where the price of calves and that of salvage cows are regarded as given data.\(^{40}\)

However, the calf price and the salvage cow price may hardly be regarded as given data within the framework of aggregate analysis since they influence the investment activities of beef breeders and are, in part, determined by the number of both beef calves and salvage cows marketed. The secondary objective of the present study, a multiple-relation model, which may explain the investment activities of beef breeders concerning their

\(^{40}\) The price levels of beef cattle, including the price level of calves sold and salvage cows, are treated in the present part as economic variables measured per 100 pounds.
working herd in conjunction with the determination of calf price, the determination of salvage cow price, the calf production process, and the investment activities of beef breeders concerning their heifers, will be constructed in the present part.

This part is composed of five sections. The first section deals with the derivation of a calf price relation. The second section is devoted to deriving a cow price relation. The third section presents the aggregate calf production relation. The fourth section deals with the investment activities of beef breeders concerning their heifers. A model consisting of an investment behavior relation concerning breeding cows, a calf price relation, a cow relation, a calf production relation, and an investment behavior relation concerning replacement heifers is submitted and discussed in the last section.

1. Calf Price Relation

It was assumed in the preceding chapter that the representative beef breeder views the price of calves sold as a given economic datum in order to maximize the net return from his investment in a herd of breeding animals. In the preceding section the price of calves sold was regarded as an explanatory variable in the investment behavior relation. However, the
influence of the marketing activities of beef breeders as a whole concerning their beef calves on the determination of calf price can hardly be ignored if one probes a provocative question, i.e., why are there fluctuations in the price of calves sold? In fact, the price of calves is a strategic variable which influences the investment activities of each beef breeder concerning his breeding (and working) herd and which is, in turn, influenced by the activities of the aggregate of beef breeders. In the present section, brief consideration is given to the determination of calf price under a set of simplifying assumptions, and a calf price relation which may explain the movement in the price of calves sold over time will be submitted as a refutable hypothesis.

To begin with, consider the aggregate demand relation. Feed-lot operators buy both beef and dairy calves in order to produce and sell slaughter steers and/or heifers to slaughter-house operators. The slaughter-house operators buy both beef and dairy calves in order to produce red meat for the final consumers. In fact, they are the intermediary producers of red meat for final consumption. According to the conventional theory of price, it may be supposed that the total number of both feeder and slaughter calves demanded by feed-lot and slaughter-house operators depends upon the price level of calves and per capita
disposable income. In addition, it is supposed that the number of calves demanded is negatively related to the price of calves and positively related to per capita disposable income. An aggregate demand for both feeder and slaughter calves may be expressed as follows:

\[ C_{t+1}^d = d_{10} + d_{11} p_{t+1}^{ca} + d_{12} Y_t \]

where \( C_{t+1}^d \) represents the total number of calves demanded by feed-lot and slaughter-house operators in the production period \((t+1)\), \( p_{t+1}^{ca} \) denotes the price level of calves sold for the production period \((t+1)\), and \( Y_t \) designates per capita disposable income for the production period \(t\). \( d_{10} \) is either positive or negative, \( d_{11} \) is expected to be negative, and \( d_{12} \) is expected to be positive.

Next consider the aggregate supply relation. It was observed in section II, D, 3, that the number of calves supplied by a representative beef breeder depends upon the size of the calf crop in the preceding production period and the optimum number of replacement calves at the beginning of every production period. In addition, note was taken that the optimum number of replacement calves depends upon the expected last age and the amount of feed resources at the beginning of every production period. The aggregate supply relation may be specified as one relation which explains the total number of calves supplied
by the beef breeders in terms of the size of calf crop, the expected last age, and the available amount of feed resources. However, it should be noted that both feeder and slaughter calves are marketed by dairy breeders as well as beef breeders and that dairy breeders may practice investment activities somewhat different from those of beef breeders. Then the question deals with the specification of an aggregate supply relation which explains the total number of calves supplied in terms of the strategic variables.

The number of calves supplied by cattlemen as a whole may be expressed on the basis of equation (2.40) as follows:

\[
CS_{t+1}^b = I_3 (CC_t^b, N_{t+1}^{**}(0))
\]

where \(CS_{t+1}^b\) represents the total number of both feeder and slaughter calves supplied by cattle ranchers at the beginning of production period \((t+1)\), \(CC_t^b\) designates the total number of calves produced by the breeding cows in production period \(t\), and \(N_{t+1}^{**}(0)\) denotes the optimum number of replacement calves at the beginning of production period \((t+1)\). Since the optimum number of replacement calves depends upon the expected last age and the available amount of feed resources, equation (3.20) may become
3.21) \[ CS^{b}_{t+1} = I_3 (CC^b_t, \text{ anticipated last age, available amount of feed resources}) \]

The number of dairy calves supplied by dairymen as a whole may be simply postulated as follows:

3.22) \[ CS^{d}_{t+1} = I_4 (CC^d_t, p^{ca}_t, \text{ prices of closely related commodities}) \]

where \( CS^{d}_{t+1} \) represents the total number of dairy calves supplied by the dairymen at the beginning of production period \((t+1)\) and \( CC^d_t \) denotes the total number of dairy calves produced in the production period \(t\). The aggregate supply of both beef and dairy calves may best be expressed as follows:

3.23) \[ C^s_{t+1} = CS^b_{t+1} + CS^d_{t+1} \]

\[ = I_3 (CC^b_t, \text{ anticipated last age, available amount of feed resources}) \]
\[ + I_4 (CC^d_t, p^{ca}_t, \text{ prices of closely related commodities}) \]

For the sake of simplicity and practicality, the aggregate supply relation is approximated as follows:

---

41/ The prices of closely related commodities may be: (a) price of feeder steers; (b) price of slaughter steers; and (c) price of slaughter heifers.
Since it is assumed that the total number of calves supplied depends linearly upon the variables specified in (3.24), the aggregate supply relation can be written as

\[ C_{t+1}^s = d_{20} + d_{21} CC_t^b + d_{22} CC_t^d + d_{23} p_t^{ca}, \]

where \( d_{20} \) is expected to be either positive or negative and \( d_{21}, d_{22}, \) and \( d_{23} \) are expected to be negative.

What is needed for deriving a calf price relation is an equilibrium condition. This condition asserts that for an equilibrium to exist in the market during a given period, the number of both beef and dairy calves demanded must be equal to the number of both beef and dairy calves supplied. In other words, no seller is left with marketable calves and no buyer with an unsatisfied demand. The equilibrium condition is then expressed as follows:

\[ C_{t+1}^d = C_{t+1}^s. \]

A calf price relation which explains the movement of the price of calves over time may now be derived from solving equations (3.19), (3.25), and (3.26) for \( p_{t+1}^{ca} \). Equating
equations (3.19) and (3.25) yields

\[ d_{10} + d_{11} p_{t+1}^{ca} + d_{12} Y_t = d_{20} + d_{21} CC_t^{b} + d_{22} CC_t^{d} + d_{23} p_t^{ca}. \]

Rearranging terms in (3.27), the relation is

\[ p_{t+1}^{ca} = \frac{1}{d_{11}} \left[ (d_{20} - d_{10}) - d_{12} Y_t \right] + d_{21} CC_t^{b} + d_{22} CC_t^{d} + d_{23} p_t^{ca}. \]

2. Cow Price Relation

In section III, A, the price of salvage cows was included in the investment behavior relation as an explanatory variable. Although the price of salvage cows was viewed as an economic datum by each individual beef breeder, it must be regarded as an economic variable which influences the investment activities of each individual beef breeder and which is, in turn, influenced by these activities in the aggregate. In the present section, the determination of salvage cow price is briefly considered, and a salvage cow price relation which explains the movement in the salvage cow price over time will be submitted.
Of first consideration is the aggregate demand for salvage cows. Feed-lot operators may buy old beef cows and dairy cows in order to produce slaughter cows. Slaughter-house operators buy slaughter cows from feed-lot operators, cattlemen, and dairymen. Since feed-lot and slaughter-house operators serve as intermediary producers of red meat for the final consumers, it may be postulated that the aggregate demand for both beef and dairy cows depends upon the price level of salvage cows and per capita disposable income. In addition, it is postulated that the number of beef and dairy cows demanded is negatively related to the price of salvage cows and positively to per capita disposable income. A linear aggregate demand function for both beef and dairy cows may then be expressed as follows:

\[ K_{t+1}^d = e_{10} + e_{11} p_{t+1}^{co} + e_{12} Y_t \]

where \( K_{t+1}^d \) represents the total number of both beef and dairy cows demanded by feed-lot and slaughter-house operators in period \( (t+1) \), \( p_{t+1}^{co} \) denotes the price of salvage cows in period \( (t+1) \), \( Y_t \) designates per capita disposable income in period \( t \), \( e_{10} \) is expected to be negative, and \( e_{12} \) is expected to be positive.

Next to be considered is the aggregate supply relation.
It was noted in section II, D, 4, that the number of cows culled by a representative beef breeder depends upon the actual working herd inherited from the preceding production period and the expected working herd at the beginning of every production period. Thus it would seem desirable to derive an aggregate relation which explains the total number of cows culled by both cattlemen and dairymen.

The total number of beef cows supplied by the beef breeders may be expressed on the basis of equation (2.41) as follows:

\[
S_{C_{b,t+1}} = \sum_{i=2}^{M+1} f(i) N_{t+1}^{b}(i), \quad \sum_{i=2}^{M+1} f(i) N_{t+1}^{**}(i)
\]

where \( S_{C_{b,t+1}} \) designates the total number of beef breeding cows culled at the beginning of production period \((t+1)\) before herd adjustment, and \( \sum_{i=2}^{M+1} f(i) N_{t+1}^{b}(i) \) represents the productivity of the aggregate working herd at the beginning of production period \((t+1)\) before herd adjustment, and \( \sum_{i=2}^{M+1} f(i) N_{t+1}^{**}(i) \) denotes the productivity of the aggregate working herd desired by the beef breeders at the beginning of production period \((t+1)\). Since the productivity of the expected working herd of an individual beef breeder depends on the anticipated last age and the amount of feed resources, equation (3.30) may be modified as follows:
The total number of dairy cows supplied may be simply postulated as follows:

\[ SC_{t+1}^d = I_7 \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^b (i), \text{ prices of closely related commodities} \right] \]

where \( SC_{t+1}^d \) denotes the total number of dairy cows supplied at the beginning of production period \((t+1)\) and \( \sum_{i=2}^{M+1} f(i) N_{t+1}^b (i) \) represents the productivity of the aggregate dairy working herd which is inherited from the preceding production period at the beginning of production period \(t\). The aggregate supply of both beef and dairy cows may be represented by the following equation:

\[ SC_{t+1}^b = I_6 \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^b (i), \text{ anticipated last age, available amount of feed resources} \right] \]

42/ See footnote 41 for some examples of the prices of closely related commodities.

43/ The productivity of dairy cattle associated with age may be different from that of beef cattle. However, it is supposed for convenience that they are identical. With such a supposition, the productivity of the aggregate dairy working herd is approximated.
3.33) \[ K_{t+1}^s = SC_{t+1}^b + SC_{t+1}^d \]

\[ = I_6 \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i), \text{anticipated last age, available amount of feed resources} \right] \]

\[ + I_7 \left[ \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i), \text{prices of closely related commodities} \right]. \]

However, for the sake of simplicity and practicality, this relation can be specified as follows:

3.34) \[ K_{t+1}^s = I_8 (p_t^{co}, \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i), \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i)). \]

When the total number of salvage cows supplied is assumed to depend linearly upon the variables specified in (3.34), the aggregate supply relation can be expressed as follows:

3.35) \[ K_{t+1}^s = e_{20} + e_{21} p_t^{co} + e_{22} \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i) \]

\[ + e_{23} \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i), \]

where \( e_{20} \) is either positive or negative and \( e_{21}, e_{22}, \) and \( e_{23} \) are expected to be positive.
In order to derive a cow price relation, assume that the salvage cow market is in equilibrium every production period. That is, the number of beef and dairy cows supplied is equal to the number of beef and dairy cows demanded every production period. The equilibrium condition is expressed as follows:

3.36) \[ K_{t+1}^d = K_{t+1}^s. \]

A cow price relation which explains the movement of the salvage cow price over time may be derived from solving (3.29), (3.35), and (3.36) for \( p_t^{co} \). Equating (3.29) and (3.35) yields

3.37) \[ e_{10} + e_{11} p_{t+1}^{co} + e_{12} Y_t = e_{20} + e_{21} p_t^{co} + e_{22} \sum_{i=2}^{M+1} f(i) N_{t+1}^b \]

\[ + e_{23} \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i). \]

Rearranging terms, a cow price relation is

3.38) \[ p_{t+1}^{co} = \left( \frac{1}{e_{11}} \right) \left[ (e_{20} - e_{10}) - e_{12} Y_t \right. \]

\[ + e_{21} p_t^{co} + e_{22} \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i) \]

\[ + e_{23} \sum_{i=2}^{M+1} f(i) N_{t+1}^b(i) \right]. \]
3. Calf Production Relation

The number of beef calves produced in the beef cattle industry during a given production period depends primarily upon the size and composition of the aggregate working herd, the amount of feed fed, care, and maintenance. On the basis of the calf production relation for the representative beef breeder, i.e., equation (2.39), the aggregate calf production relation for beef breeders as a whole may be represented as follows:

\[ CC_t^b = I_0 + \sum_{i=2}^{M+1} f(i) N_t^a(i), Q_t \]

where \( Q_t \) denotes the technological and environmental factors for the production period \( t \). If it is assumed that the aggregate calf crop depends linearly upon the aggregate working herd and the technological and environmental factors and it is further assumed that the range feed condition is regarded as a proxy variable for the technological and environmental factors, then equation (3.39) becomes

\[ CC_t^b = g_0 + g_1 \sum_{i=2}^{M+1} f(i) N_t^a(i) + g_2 R_t \]

where \( g_0 \) is either positive or negative, and \( g_1 \) and \( g_2 \) are expected to be positive.
4. Model of Investment Behavior of Beef Breeders Concerning Their Replacement Heifers: Three Alternative Hypotheses

In the present section, what has been advanced for explaining the investment activities of a beef breeder concerning his replacement heifers, i.e., equations (2.37a), (2.32), and (3.37b), is now modified for explaining the investment activities of beef breeders concerning their replacement heifers. By extending the three additional assumptions introduced in section III, A, 1, three competing hypotheses which are subject to empirical tests are developed below.

The first testable hypothesis is: The beef breeders anticipate the future price level of calves only on the basis of two preceding price levels, determine the desired number of replacement heifers, and adjust their actual to the desired level. This hypothesis can be explicitly specified by a model, as follows:

\[
H_{t+1}^{**} = c_0 + c_1 p_{t+1} + c_2 p_t + c_3 R_t + U_{1t}
\]
3.15) \[ p_{t+1}^{ca} = a_0 p_t^{ca} + a_1 p_{t-1}^{ca} + U_{2t} \]

3.42) \[ H_{t+1} - H_t = h'' (H_{t+1}^{**} - H_t) + U_{3t}^i \]

where \( H_{t+1}^{**} \) denotes the desired aggregate replacement heifers at the beginning of production period \((t+1)\), \( H_{t+1} \) represents the actual aggregate number of replacement heifers at the beginning of production period \((t+1)\), \( H_t \) designates the actual aggregate number of replacement heifers at the beginning of production period \(t\), \( U_{1t}^i \) and \( U_{3t}^i \) denote the stochastic errors, \( c_0^i \), \( c_1^i \), \( c_2^i \), \( c_3^i \), \( a_0 \), \( a_1 \), and \( h'' \) are arbitrary constants, \( a_0 + a_1 = 1 \), and \( 0 < h'' < 1 \). The aggregate investment behavior relation from the above three equations becomes

3.43) \[ H_{t+1} = B_0^i + B_1^i H_t + A_0^i p_t^{ca} + A_1^i p_{t-1}^{ca} \]
\[ + B_2^i p_t^{co} + B_3^i R_t + V_{1t}'' \]

where \( V_{1t}'' = h'' U_{1t}^i + h'' c_1^i U_{2t}^i + U_{3t}^i \), and \( B_0^i, B_1^i, A_0^i, A_1^i, B_2^i, \) and \( B_3^i \) are arbitrary constants. It should be noted that the parameters of (3.43) are identifiable and that the expected signs of parameters of variables, except the price of salvage cows, are positive.
The second hypothesis of investment behavior can be viewed as a special case of the first hypothesis, in which an a priori restriction is imposed on the pattern of herd adjustment. That is, the beef breeders adjust their actual to their expected number of replacement heifers at the beginning of every production period. With such an a priori restriction, \( H_t \) drops out of (3.43). The investment behavior relation becomes

\[
3.44) \quad H_{t+1} = B'_0 + A'_0 p^c_t + A'_1 p^c_{t-1} + B'_2 p^c_t \\
\qquad \quad + B'_3 R_t + V''_{1t}.
\]

The third hypothesis of investment behavior can be regarded as the alternative of the first hypothesis, in which the beef breeders do not take into account the price of calves in the preceding production period. That is, \( a_1 = 0 \). Under such an a priori restriction, \( p^c_{t-1} \) drops out of (3.43). Thus the investment behavior relation becomes

\[
3.45) \quad H_{t+1} = B'_0 + B'_1 H_t + A'_0 p^c_t \\
\qquad \quad + B'_2 p^c_t + B'_3 R_t + V''_{1t}.
\]
5. A Multiple-Relation Model

In the first part of the present chapter, three alternative hypotheses were submitted to explain the investment activities of beef breeders concerning their working herd. In the present section, the hypotheses about the determination of calf price and salvage cow price, the aggregate calf production process, and the investment activities of beef breeders concerning their replacement heifers are considered as an aspect of the study's secondary purpose. By integrating what has been advanced above, a model may be formulated which is capable of explaining (a) the investment activities of beef breeders concerning their working herd, (b) the determination of calf price, (c) the determination of salvage cow price, (d) the calf production process, and (e) the investment activities of beef breeders concerning their replacement heifers. In the present section, a brief discussion of the economic activities of beef breeders as well as closely related events is therefore given, and a model which is subject to empirical analyses is submitted.

The general hypothesis of investment behavior of beef breeders concerning their working herd was formulated to explain three economic activities, i.e., determination of expected working herd, anticipation of the future price level of calves, and adjustment of working herd. The investment behavior
relation was derived as a partially reduced form equation. It has been shown in the present section that the determination of calf price for a given production period may be explained by the calf price relation which is derived from the aggregate supply relation, the aggregate demand relation, and the equilibrium condition, with both beef breeders and dairymen supplying beef calves. In addition, it has been shown that the determination of salvage cow price may be explained by the cow price relation which is derived from the aggregate supply relation, the aggregate demand relation, and the equilibrium condition, where the beef breeders and dairymen supply salvage cows. Furthermore, it has been considered that the aggregate calf production process may be explained by a calf production relation, relating the number of calves produced to the aggregate working herd and range feed condition. Finally, it has been considered that the investment activities of beef breeders concerning their replacement heifers may be explained by the three competing investment relations which are derived as a partially reduced form equation.

The next task is to establish a causal chain of events, based upon the technology of the growth process associated with breeding animals kept on ranches. These events may be made
clear by referring to Figure 17, which summarizes the discussion. The upper part of Figure 17 depicts the herd adjustment activities at the beginning of each production period and the associated calf production process. $W_{t+\ell}^b$ and $H_{t+\ell}^b$ for $\ell = -1, 0, 1$ represent the aggregate working herd and the aggregate number of replacement heifers at the beginning of production period $(t+\ell)$ prior to any herd adjustment. $N_{t+\ell}^a(0)$, $H_{t+\ell}^a$, and $W_{t+\ell}^a$ for $\ell = -1, 0, 1$ denote the number of replacement calves maintained, the number of replacement heifers, and the aggregate working herd maintained at the beginning of production period $(t+\ell)$ after herd adjustment. $CC_{t+\ell}$ designates the size of calf crop obtained from the aggregate working herd at the end of production period $(t+\ell)$, where $\ell = -1, 0, 1$. The middle part reflects, in part, the determination of calf price; the lower part exposes, in part, the determination of salvage cow price.

Next to be considered are the relationships among the three aggregate economic events. First is the relationship between the investment activities of beef breeders concerning their breeding herd and the determination of price of calves. With respect to the investment behavior hypothesis, it can easily be seen from the upper and middle parts of Figure 17 that the aggregate working herd maintained at the beginning of production period $(t+1)$ depends upon the calf price in production periods $t$ and $(t-1)$. 
Figure 17. A hypothetical diagram showing the relationships among the investment activities of beef breeders, the determination of calf price, the determination of salvage cow price, and the calf production process.
In addition, it can be seen, with reference to the discussion on the determination of calf price, that the price of calves for the production period \( t \) is dependent upon the price of calves for the preceding production period and the size of calf crop in the preceding production period.

Next to be considered is the relationship between investment activities and the determination of salvage cow price. With reference to the investment behavior hypothesis, it can be seen from the upper and lower parts of Figure 17 that the investment activities at the beginning of production period \((t+1)\) depend upon the salvage cow price for production period \( t \).

With reference to the determination of salvage cow price, it can easily be seen that the salvage cow price for production period \( t \) depends upon the salvage cow price for production period \((t-1)\) and the aggregate working herd maintained at the beginning of production period \((t-1)\) prior to the herd adjustment.

Finally, to be considered is the relationship between investment activities and the determination of both calf price and salvage cow price. From the economic events presented in Figure 17, it is apparent that the investment activities of beef breeders concerning their working herd at the beginning of production period \((t+1)\) depend upon the calf price for production periods \( t \) and \((t-1)\) and the salvage cow price for production
period t. Nonetheless, it should be noted that the determination of calf price for production period \((t+1)\) depends on the number of calves marketed at the beginning of production period \((t+1)\), which is in turn dependent upon the number of beef calves produced at the end of production period \(t\) and the calf price for production period \(t\). In addition, it should be recognized that the determination of salvage cow price for production period \((t+1)\) depends upon the salvage cow price for production period \(t\) and the aggregate working herd maintained by the beef breeders at the beginning of production period \((t+1)\) prior to the herd adjustment.

These considerations suggest that a recursive model may be useful in explaining the investment activities of cattle ranchers (i.e., beef breeders) in conjunction with the determination of calf price, salvage cow price, and calf production. Since equations (3.28), (3.38), and (3.40) are formulated on the basis of a large number of simplifying assumptions, they are subjected to stochastic errors arising from (a) incompleteness of theory, (b) incorrect specification of relation, and (c) incorrect process of aggregation. For instance: (a) The price of calves for production period \((t+1)\) could depend upon the price of feeder steers for production period \(t\), which is left out of equation (3.28); (b) the price of salvage cows for production
period (t+1) may be dependent upon the price of slaughter steers for production period t, which is excluded from equation (3.88); and finally (c) the calf production relation may be a nonlinear relation between the total number of calves produced and both the aggregate working herd and range feed conditions. Taking into account the stochastic errors associated with each relation, the model may be specified as follows:

\[ M+1 \]

\[ \sum_{i=2}^{M+1} f(i) N_{t+1}^a (i) = B_0 + B_1 \sum_{i=2}^{M+1} f(i) N_t^a (i) + A_0 p_t^{ca} + A_1 p_{t-1}^{ca} + B_2 p_t^{co} + B_3 R_t + V'_{1t} + V_{2t} \]

\[ H_{t+1} = B_0' + B_1' H_t + A_0' p_t^{ca} + A_1' p_{t-1}^{ca} + B_2' p_t^{co} + B_3' R_t + V''_{1t} \]

\[ p_t^{ca} = D_0 + D_1 Y_t + D_2 CC_t^b + D_3 CC_t^c + D_4 p_t^{ca} + V''_{2t} \]

The first refutable hypothesis is submitted as the investment behavior relation concerning the aggregate working herd and the aggregate replacement heifers in the multiple-relation model. Certainly, the second or the third refutable hypothesis can be alternatively submitted as the investment behavior relation.
\[ 3.47 \]

\[
P_{t+1}^{co} = E_0 + E_1 Y_t + E_2 p_t^{co} + E_3 \sum_{i=2}^{M+1} f(i) N_t^a(i) \]
\[
+ E_4 \sum_{i=2}^{M+1} f(i) N_t^a(i) + V_{3t}^i
\]

\[ 3.48 \]

\[
CC_{t+1}^b = G_0 + G_1 \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) + G_2 R_{t+1} + V_{4t}^i
\]

where \( D_0, D_1, D_2, D_3, E_0, E_1, E_3, \) and \( E_4 \) are the coefficients of reduced form equations (3.46) and (3.47), \( G_0, G_1, \) and \( G_2 \) are the coefficients associated with calf production relation, \( V_{2t}^i, V_{3t}^i, \) and \( V_{4t}^i \) are stochastic errors, and others are previously designated.\(^{45}\)

The endogeneous variables are \( \sum_{i=2}^{M+1} f(i) N_{t+1}^a(i) \), \( H_{t+1}, p_{t+1}^{ca}, p_{t+1}^{co}, \) and \( CC_{t+1}^b \) while the predetermined variables are the other variables specified in the model.

\(^{45}\) It should be noted that the stochastic errors, \( V_{2t}^i \) and \( V_{3t}^i \) are composite stochastic errors. The former consists of the stochastic errors associated with the aggregate demand for calves and those associated with the aggregate supply of calves. The latter consists of the stochastic errors associated with the aggregate demand for salvage cows and those associated with the aggregate supply of salvage cows.
C. Concluding Remarks

In the first part of the present chapter, three alternative hypotheses were formulated with respect to the investment activities of beef breeders, i.e., the demand for expected working herd, the anticipation of future price level of calves, and the pattern of herd adjustment. In the second part, a multiple-relation model was submitted which explains the investment activities of beef breeders concerning their breeding cows as well as replacement heifers in conjunction with the determination of calf price, the determination of salvage cow price, and the calf production process. The models formulated in the present chapter have been stated in terms suitable for empirical verification.

In developing not only the three alternative hypotheses but also the multiple-relation model, a large number of simplifying assumptions have been explicitly specified. In particular, two crucial assumptions are made in the present chapter for the sake of simplicity and practicality: (a) every beef breeder in the beef cattle industry behaves as if he were the representative beef breeder who follows the policy of maintaining the "normal" age distribution, and (b) the models can be approximated in terms of linear equations. The plausibility of the simplifying assumptions...
may better be judged after a review of the empirical results in the following chapter.
CHAPTER IV

EMPIRICAL RESULTS

In this chapter, the investment hypothesis and multiple-relation model developed above are submitted to empirical test. The present chapter discusses the limitations and availability of the empirical data corresponding to the economic variables of interest and some of the methodological problems associated with the statistical analyses of economic models. The empirical results of the model will be presented and reviewed in the light of the theoretical discussions advanced above.

The present chapter is divided into four parts. Part A deals with the measurement of variables. Part B is devoted to a discussion of methodological problems. Part C presents the empirical results of the investment behavior hypotheses. Part D discusses the empirical results of the multiple-relation model formulated together with its variant. The last part summarizes the empirical results obtained in the present study.

A. Measurement of the Variables

In an attempt to validate a given model empirically, two key problems that must be solved are (a) the method of measuring
the variables that are required by theoretical considerations, and
(b) the selection of the time period from which to choose the data
to be used. Since these problems are important in dictating the
scope of the verifications made on the models of the present study,
it is appropriate to consider some of the issues which these
problems involve.

For the following statistical analyses, the sample will con-
sist of observations over time of all the variables in the models
presented in the preceding chapter. These variables have been
fairly well defined by their verbal descriptions and by the roles
that they play in the model. However, some of the variables may
not correspond very closely to any regularly compiled data, and
some error in these variables may be present in the data.
According to Haavelmo (30), it is often necessary to construct
from the available time series data measurements that will
correspond as closely as possible to the concepts employed in
the system. Thus the purpose here is to specify a sample period,
to describe the measurements of the economic variables used in
the preceding chapter, to indicate the sources of data, and to
present some rationalization of the choices of measurements. The
observations used in the present study are tabulated in
Appendices II and III.
1. Choice of Sample Period

The sample period for the present study extended from 1931 through 1964. The choice of 1931 as the initial observation and the choice of 1964 as the last observation were arbitrarily decided by the availability of time series data.

Another choice that had to be made was the time interval to which the observations should be referred. The data on beef breeding cows and heifers kept on farms are available as of January 1, whereas the data on prices are available for such marketing periods, as weeks, months, and years. Data on the calf crop (both beef and dairy calves), available for each calendar year, are not always identical with the calf production periods of ranchers throughout the country. The data on range feed conditions are available on the first day of each month for the 17 western states. After some examination of the available data, it was decided to use the calendar year as the basic time interval for the present study and to adjust the observations accordingly. Whatever basic time interval is used, there will always be some developments during the interval not fully reflected in the data representing the full interval. These factors contribute to the disturbances in the relations. For example, a year in which the range feed conditions were poor in the first or last six months
is not really similar to a year in which the range feed conditions were more uniformly good, even though the range feed index for the two years may be the same.

In principle, it may be possible to construct a model with different time periods for different variables. Complications in the construction and interpretation of such a model made it seem advisable to forego such an attempt in the present study and to adjust measurements to relate to a common time period, i.e., calendar year.

2. The Data and the Variables

(1) Available Data:

\[
\sum_{i=2}^{M+1} f(i) N_t^a (i) : \text{the productivity of the aggregate working herd adjusted and maintained by beef breeders at the beginning of production period } t. \text{ Although the aggregate working herd was discussed in terms of size and composition in the preceding chapter, the aggregate working herd is measured only in terms of size throughout the }
\]

46/ The stock variables, i.e., number of beef breeding cows and heifers two years old and older kept on farms, number of heifers kept on farms, number of beef calves produced, number of dairy cows and heifers two years old and older kept on farms, and number of dairy calves produced, are measured in units of 1,000 head in the present study.
statistical analysis since data on the size and composition of the working herd adjusted and maintained by each rancher are not available. This variable is, therefore, approximated by the number of breeding cows and heifers two years old and older kept on farms January 1, denoted by \( W_t(n) \). The data are from the U.S.D.A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330), Table 7.

\[ p_{ca} \]

\[ p_t \]

the price of calves sold per hundredweight.

The market price of calves is determined not only by the number of both beef and dairy calves demanded by feedlot operators and slaughter-house operators but also by the number supplied by beef breeders and dairy farmers. In fact, calf prices also vary according to grade and location of the market. For example, there are the choice grade veal calf price at Chicago and the choice grade steer calf price at Kansas City. Within the range of available data, the price of calves sold in the market for the present purpose is reasonably
compatible with the average calf price received by farmers. The data are from the U. S. D. A., *Livestock and Meat Statistics* (Statistical Bulletins Nos. 230 and 330), Table 40.

The salvage cow price per hundredweight. The data on cow prices for the exact counterpart of the theoretical discussion are not available. The time series data on cow prices, according to grade, are available only at Chicago. For the present purpose, the salvage cow price is approximated as the weighted annual average price per hundredweight for utility grade slaughter cows at Chicago. The data are from the U. S. D. A., *Livestock and Meat Statistics* (Statistical Bulletins Nos. 230 and 330), Table 162.

The number of replacement beef heifers maintained by beef breeders at the beginning of production period $t$. The present economic variable is best approximated by applying the data on the number of beef heifers kept on farms on January 1. The data are from the
The data are from the U. S. D. A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330), Table 7.

\[ M+1 \sum_{i=2}^{f(i)} N_t^a (i) \]

[\( f(i) N_t^a (i) \): the productivity of the aggregate working herd adjusted and maintained by dairymen at the beginning of production period \( t \). The observations on the size and composition of the aggregate dairy working herd cannot easily be derived since data on the composition of each individual working herd is not available. In addition, the productivities of the dairy working herd cannot be easily measured since each dairy cow produces dairy calves as well as milk. However, for the present purpose, this variable may be approximated by the number of dairy cows and heifers two years old and older kept on farms on January 1, denoted by \( W_t(n) \). The data are from the U. S. D. A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330).]
the price of feeder steers per hundredweight. Data on the feeder steers are not available for the United States as a whole, although time series data are available for regional markets such as South St. Paul, Chicago, and Kansas City. For the present purpose, this economic variable is approximated as the annual average stocker and feeder steer price per hundredweight for all weights at Kansas City. The data are from the U. S. D. A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330, Table 152.

the price of slaughter steers per hundredweight. The closely related beef animals in the salvage cow market may be regarded as slaughter steers (and/or heifers). The data on the price of slaughter steers (and/or heifers) for the United States as a whole are not available. After examination of available time series data, the present economic variable is approximated as the annual weighted average slaughter steer...
price per hundredweight for all grades at Chicago. The data are from the U. S. D. A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330), Table 156.

(2) Derived Data:

\[ CC^b_t : \]

the number of beef calves produced at the end of production period \( t \). The data on the number of beef calves born are not available although the number of beef and dairy calves combined is reported. The time series data on the number of beef calves born are derived according to the formula of proportion:

\[ CC^b_t = f_{1} CC_t, \]

where \( CC_t \) is the total number of both beef and dairy calves born during production period \( t \) and \( f_{1} \) is the proportion of the number of beef cows and heifers two years old and older to the total number of both beef and dairy breeding cows and heifers two years old and older kept on farms on January 1. The data on the total number of calves born are from the U. S. D. A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330), Table 24.
the range (feed) conditions in production period \( t \). This variable was used in the theoretical discussion as a proxy variable for the availability of feed resources and other supplementary factors of production. (In fact, the observations on feed, care, and maintenance are involved with various problems in measurement and are not available for all beef breeders.) The data on range feed conditions are available only for the 17 western states for the first day of each month. In accordance with the arbitrary choice of time period, the present economic variable is approximated by the annual average range feed condition for the 17 western states. The data on the range feed condition on the first day of each month are from the U.S. D.A., Livestock and Meat Statistics (Statistical Bulletins Nos. 230 and 330), Table 85.

the number of dairy calves produced. Since the total number of calves born in any given production period is fixed, the data on the number of dairy calves born are approximated by
subtracting the number of beef calves born $CC_t^b$, from the total number of both beef and dairy calves born in the production period $t$.

B. Methodological Problems

Many important methodological problems arise when a hypothesis, or a set of hypotheses, is subject to empirical validation. Several are particularly relevant to the empirical validation of the hypotheses formulated in the preceding chapter. A brief discussion of four of these problems, together with their effects upon the empirical results, will suggest the limitations of the present study.

The methodological problems associated with the present study can easily be seen by reviewing the assumptions which should be met for the application of the least squares regression technique. In general, the least squares regression technique is based upon the following assumptions:

(a) the observations on the independent variables in the regression equation are nonstochastic constants; they are thus uncorrelated with each other and independent of the stochastic term;

(b) the stochastic term, e.g., $V_{1t}$ in (3.16), is distributed
independently with a finite variance and a zero mean; and

(c) the functional relationship is linear, as specified in Chapter III.

If the above assumptions are satisfied, then the parameter estimates obtained by regression are the best linear unbiased estimates. For the purpose of testing the statistical significance of the estimates obtained, it is also assumed that the distribution which the disturbances follow is normal. If one, or more, of the assumptions is not satisfied, then the empirical verification of the hypotheses may be subject to some limitations.

1. **Multicollinearity**

When two of the independent variables in a multiple regression equation are closely related to each other, assumption (a) is violated. In this case, the theoretically independent effects of these variables cannot be separated because they have moved jointly in the sample over which the test is being made. Although it is possible to estimate the joint effect of the two variables, it is impossible to identify the effects of each variable individually.

However, it is argued by Haavelmo (31, p. 260) that the estimate, $s^2$, say of $\sigma^2$, is not impaired by the fact that the independent variables are highly intercorrelated. His argument implies that the estimate of the standard error of the total equation, and thus
of the index of multiple determination, $R^2$, is unaffected.

High intercorrelations were observed, for example, in the fitted investment behavior equation, i.e., (4.1), which will be presented in the following part. They are: (a) between the price of calves for production period $t$, $p^c_a$, and that for production period $(t-1)$, $p^c_{t-1}$, and (b) between the price of calves for production period $t$, $p^c_a$, and that of salvage cows for production period $t$, $p^c_0$. The existence of collinearities in equation (4.1) suggests that the coefficients of these variables must be regarded with caution. Nonetheless, although these intercorrelations will hamper efforts to obtain precise estimates of the parameters associated with the investment behavior relations and other relations in the multiple-relation model, the existence of multicollinearity will not affect the interpretation of the explanatory power of the entire equation. This latter criterion is one of the more important by which the validity of the model can be judged in the present study.

2. Autocorrelated Disturbances

Assumption (b) requires that the $V'_{1t}$ in the investment behavior relation, for example, be distributed independently (i.e., $E(V'_{1t}, V'_{1t+s}) = 0$, $s \neq 0$). If the $V'_{1t}$ are autocorrelated, three consequences ensue, according to Johnston (41,
First, although the estimates of the regression coefficients remain unbiased, it is no longer true that the least squares technique assures a minimum sampling variance attached to these estimates. Second, it is likely that the usual formulae will underestimate this sampling variance, thereby exaggerating the statistical significance of the coefficients obtained. Third, predictions made from regression equations with autocorrelated disturbances will have sampling variances larger than those which can be obtained if condition (b) is fulfilled.

Tests for the presence of autocorrelated disturbances in the regression equations below were made with the Durbin-Watson "d" statistic whenever the test was appropriate. According to Durbin and Watson (16, 17), the d statistic is calculated in the following manner: let \( d_t \) be the unexplained residual from a least squares regression for observation \( t \) with \( n \) observations; then

\[
d = \frac{\sum_{t=2}^{n} (d_t - d_{t-1})^2}{\sum_{t=1}^{n} d_t^2};
\]

that is, the Durbin-Watson statistic equals the sum of the squares of the first difference of the residuals divided by the sum of squares of the residuals. If no serial correlation exists, the statistic tends to be two, while positive serial correlation in
the residuals tends to make the statistic less than two and negative serial correlation tends to make it greater than two. For a given level of significance Durbin and Watson have set upper and lower limits to the critical value of the statistic, below which significant negative serial correlation is indicated. For example, in the test for positive serial correlation, a value of the statistic above the upper limits indicates insignificant positive serial correlation, while a value below the lower limit indicates significant positive serial correlation. A value between the two limits means the test is inconclusive.

In connection with the limitations of the d statistic, Durbin and Watson (17) emphasized that the tests discussed by them do not apply to autoregressive models and other models in which lagged dependent variables appear as independent variables. Thus the test was applied only to the calf production relation.

3. Lagged Variables

Theoretical considerations have led to the formulation of a number of partially reduced form equations in which the lagged dependent variable appears on the right-hand side of the relations. A crucial question is: what are the consequences of using least squares regression when this assumption of independence between disturbance term and the independent variable is violated?
According to Johnston (41, p. 211-221), the answer is that the least squares estimate will be biased, although if the disturbance term follows a normal distribution, the estimate will tend to have the desirable asymptotic properties of consistency and efficiency.

For the problems at hand the danger of bias may be greatest for the estimate of the adjustment coefficient. The bias is likely to come about in the following way. It is not reasonable to suppose that the investment activities of beef breeders concerning their breeding cows are explained in terms of all of the "relevant" variables. Perhaps some "relevant" variables are left out. If both these omitted variables and the dependent variable are serially correlated, it is likely that the omitted variables will also be correlated with the lagged value of the dependent variable. This will lead to an upward bias in the estimate of the coefficient of the lagged dependent variable and a downward bias in the estimate of the adjustment coefficient. For this reason the estimated adjustment coefficients, which will be presented in the following section, must be treated with suspicion. Their low values may be the consequence of omitting variables that change slowly over time and/or the result of a slow rate of herd adjustment by the beef breeders to a difference between their actual working herd and the working herd they desire to maintain.
4. Errors of Measurement

When the observations of the variables in the regression equation contain errors of measurement, assumption (a) is violated, and the estimate of both the regression coefficient and the partial correlation coefficient for that variable will contain a downward bias. Thus these coefficients will appear, aside from sampling fluctuations, smaller than the true value.

C. Empirical Results of the Investment Behavior Hypotheses

The least squares regression estimates of the three investment behavior relations and their statistical tests, based on the available data, are presented in the present part. The first section deals with the empirical results of equation (3.16). The second section presents the empirical results of equation (3.17). The third section shows the empirical results of equation (3.18).

1. The Investment Behavior Relation

The empirical results of equation (3.16) are summarized as follows:

\[ W_{t+1}(n) = -6.168,4414 + 0.9513 W_t(n) + 66.4289 p_{ca}^t + 141.7421 p_{ca}^{t-1} \]

\[ (27.578) \quad (0.570) \quad (3.302) \]
The values of Student's "t" statistic (the absolute value of the ratio of the coefficient to its standard error) are shown in parentheses directly below the regression coefficients.

Several points deserve comment. First, the fit of the model to the data, as indicated by $R^2$, is evidence in support of the theoretical considerations advanced in the preceding chapters and suggests that further study of the properties of the model is indeed a worthwhile undertaking. Second, the sign of each estimated coefficient is compatible with what was expected a priori. Thus it is possible to find out the plausible estimates of structural parameters. Third, the individual significance tests of coefficients of (4.1), using the Student's t distribution, indicate that at the five percent level of significance, the tests for $W_t(n)$, $p_{t-1}^{ca}$, and $R_t$ are significant, since the table value is 2.05. Fourth, the joint significance test of the coefficients of (4.1), using the F distribution, indicates that at the one percent level of significance the test is highly significant, since $F_{28}^5 = 642.419$ against the table value of 3.76. Fifth, the test of autocorrelation in the disturbance terms cannot be carried out.

\[-151.2527 p_t^{co} + 78.6594 R_t \]

$(1.126)$ $(2.602)$

$R^2 = 0.996$. 

\[-151.2527 p_t^{co} + 78.6594 R_t \]
since the application of Durbin-Watson statistic is limited to those equations which do not include the lagged dependent variable as an explanatory variable. Sixth, the expected demand equation for the aggregate working herd in terms of size only can be derived from (4.1) as follows:

\[ W_{t+1}^{**} (n) = -12,666.2041 + 4,262.6354 p_{t+1}^{ca*} + 3,105,7990 p_t^{co} + 1,615.1830 R_t. \]

The estimated structural parameters of equation (4.2), for example, may be interpreted in terms of the ceteris paribus economic statement as follows: (a) a rise in the expected price level of calves of one dollar per hundredweight would bring about a rise in the demand for expected working herd of about 4,262,635 cows and heifers two years old and older, (b) a rise in the salvage cow price of one dollar per hundredweight would bring about a fall in the demand for expected working herd of about 3,105,799 cows and heifers two years old and older, and (c) an improvement in the range feed condition of one percent would bring about a rise in the demand for expected working herd of about 1,615,183 cows and heifers two years old and older. Seventh, the adjustment coefficient of the aggregate working herd derived from (4.1) is 0.0487, implying that about five percent of
desired change in the aggregate working herd is adjusted within one production period. Eighth, the weights of price expectation attributed to the past price level of calves can be derived from (4.1) as follows: (a) \( a_0 = 0.3191 \), and (b) \( a_1 = 0.6809 \). It is interesting to note that the beef breeders attribute more weight to the price level of calves in period \( (t-1) \), \( p_{t-1}^{ca} \), than in period \( t \), \( p_t^{ca} \), in order to anticipate the future price level of calves, \( p_{t+1}^{ca} \).

Finally, the predictive performance of equation (4.1) is summarized in Figure 18, where the calculated values correspond reasonably well to the actual values for the number of beef cows and heifers two years old and older kept on farms.

2. The Investment Behavior Relation Eliminating the Lagged Aggregate Working Herd

The empirical results of equation (3.17) may best be presented as follows:

\[
W_{t+1}(n) = 25,045.6445 + 1,637.8567 \ p_t^{ca} \\
+ 379.1829 \ p_{t-1}^{ca} - 1,784.0818 \ p_t^{co} \\
- 239.4665 \ R_t \\
(3.091) \\
(1.729) \\
(2.838) \\
(1.644)
\]

\( R^2 = 0.870 \), and \( d = 0.5515 \).
Figure 18. The number of beef cows and heifers two years old and older kept on farms on January 1.
The above investment behavior relation deserves several comments. First, the index of explanatory power, i.e., $R^2$, for equation (4.3) is respectable, but it is inferior to that for equation (4.1). Second, the sign of estimated coefficient for $R_t$ is not compatible with what was expected a priori, although others are compatible. Third, the individual significance tests of coefficients for (4.3), using the Student's $t$ distribution, indicate that at five percent level of significance the tests for the coefficient of $p_{t-1}^{ca}$ and $R_t$ are not significant. Fourth, the joint significance test for the coefficients of (4.3), using the $F$ distribution, shows that at the one percent level of significance the test is highly significant since the computed value, $F^4_{29} = 22.541$, is greater than the table value, $F^4_{29} = 4.04$. Fifth, the test of autocorrelation in the disturbance term for (4.3) indicates that there is positive correlation in the disturbance term on the basis of Durbin-Watson statistic at five percent level of significance. Sixth, equation (4.3) should be regarded as the investment behavior relation which explains the investment activities of beef breeders concerning their breeding cows, where the beef breeders adjust their working herd without any delay. Finally, no further statistical analyses were carried out since the sign of estimated coefficient for $R_t$ is not compatible with what was expected a priori.
3. The Investment Behavior Relation Eliminating the Price Level of Calves for Production Period (t-1)

The empirical results of investment behavior relation (3.18) may be summarized as follows:

4.4) \[ W_{t+1}(n) = -5,056,000 + 0.97416 W_t(n) \]
\[ + 192.0991 p_t^{ca} - 166.2778 p_t^{co} \]
\[ + 64.0983 R_t, \]
\[ (24.888) \]
\[ (1.506) \]
\[ (1.069) \]
\[ (1.851) \]
and \( R^2 = 0.993. \)

The above investment behavior relation deserves several comments. First, the index of explanatory power for equation (4.4) also supports the theoretical considerations in the preceding chapter. However, the index of (4.4) is slightly inferior to that of (4.1), although the index of (4.4) is superior to that of (4.3). Second, the signs of the estimated coefficients are compatible with the expected ones a priori. Third, the individual significance tests of the coefficients in (4.4), using the Student's t distribution, suggest that at the five percent level of significance, the tests for \( p_t^{ca}, p_t^{co}, \) and \( R_t \) are not significant. Fourth, the joint significance test for (4.4), using the F distribution, indicates that at the one percent significance level, the test is highly significant.
since the computed value, $F_{29}^4 = 596.546$, is greater than the table value, $F_{29}^4 = 4.04$. Fifth, the test of autocorrelation in the disturbance term cannot be carried out since regression equation (4.4) contains the lagged dependent variable as an explanatory variable. (Note the limitation associated with the application of Durbin-Watson statistic in section IV, B, 2.) Sixth, the expected demand equation for the aggregate working herd at the beginning of production period $(t+1)$ can be derived from equation (4.4) as follows:

4.5)  \[ W_{t+1}^{**}(n) = -195,968.9922 + 7,445.7000 p_{t+1}^{ca^*} \]
\[ - 6,444.8767 p_t^{co} + 2,484.4302 R_t. \]

The estimated structural parameters of (4.5) may best be interpreted in terms of the ceteris paribus economic statement as follows: (a) a rise in the expected price level of calves of one dollar per hundredweight would bring about a rise in the demand for expected working herd of about 7,445,700 cows and heifers two years old and older, (b) a rise in the salvage cow price of one dollar per hundredweight would bring about a fall in the demand for expected working herd of about 6,444,877 cows and heifers two years old and older, and (c) an improvement in the range feed condition of one percent would bring about a rise in the demand for expected working herd of about 2,484,430 cows.
and heifers two years old and older. Seventh, the coefficient of herd adjustment is 0.0258, implying that about three percent of the desired change in the aggregate working herd is adjusted within one production period. Eighth, it should be noted that the investment behavior relation under consideration is based upon the assumption that the beef breeders do not take into account \( p_{t-1}^{ca} \) in anticipating the future price level of calves. Finally, the predictive performance of equation (4.4) is shown in Figure 19, where the calculated values correspond reasonably well to the actual values for the number of beef cows and heifers two years old and older kept on farms over time.


The empirical results of the three alternative investment behavior hypotheses were presented and commented upon in the preceding sections. Now the fundamental questions which should be considered are: (a) What are the crucial tests to apply before accepting (or rejecting) the investment behavior hypotheses? (b) What are the implications to be drawn from the results of crucial tests on the investment behavior hypotheses? In the present section, the crucial tests for the primary purpose of this study are suggested, and the implications of the results from the crucial tests are summarized.
Figure 19. The number of beef cows and heifers two years old and older kept on farms on January 1.
Since the primary purpose of the present study is the explanation of the investment activities of beef breeders concerning their working herd, the coefficient of multiple determination (i.e., $R^2$) and the compatibility between the expected and actual signs of coefficients may best be regarded as the crucial tests. First, the coefficients of multiple determination for the regression equations, except equation (4.3), are relatively high so that the explanatory variables in any given regression equation account for about 99 percent of variations in the aggregate working herd at the beginning of each production period. Although the explanatory power of the second hypothesis is somewhat inferior to the other hypotheses, the three alternative hypotheses are acceptable with respect to the coefficient of multiple determination.

Second, the signs of the estimated coefficients in equations (4.1) and (4.4) are compatible with those expected a priori. However, the sign of the estimated coefficient for the range feed condition, $R_t$, in equation (4.3) is not compatible with the expectation. Thus the second hypothesis specified by equation (4.3) is not acceptable with respect to the sign test.

Three implications of the results from the crucial tests on the alternative investment behavior hypotheses are apparent. First, the initial investment behavior hypothesis, i.e., equation (3.16), is superior to the other hypotheses with respect to the
explanatory power and the compatibility between the expected and estimated signs. It should be recalled from the theoretical discussions of section III, A, that the second hypothesis was formulated as a special case of the initial hypothesis, by imposing a condition that the beef breeders adjust their working herd to their desired working herd at the beginning of every production period, i.e., \( h' = 1 \). With such an \textit{a priori} condition, the lagged working herd did not appear in equation (3.17) as an explanatory variable. In addition, it should be recalled that the third hypothesis was formulated as a special case of the first hypothesis, by imposing a condition that the beef breeders anticipate the future price level of calves to be at the current price level, i.e., \( a_0 = 1 \). With such an \textit{a priori} restriction, the price level of calves, i.e., \( p_{t-1}^{ca} \), did not appear in equation (3.18) as an explanatory variable. In effect, the empirical evidence suggests that the initial investment behavior hypothesis be regarded as the best hypothesis to explain the investment activities of beef breeders concerning their working herd at the beginning of each production period.

Second, the estimates of the structural parameters may best be approximated from the coefficients of the partially reduced form equation. Since the expected working herd and the future price level of calves were not directly observable
variables, the partially reduced form equation was formulated. On the basis of statistical results associated with equation (4.1), the investment activities of beef breeders with regard to their working herd may be explained by the three related activities, i.e., (a) the demand for expected working herd, (b) the anticipation of future price level of calves sold, and (c) the herd adjustment activity, where neither $h'$ nor $a_0$ is restricted a priori. As discussed in section IV, B, the estimates of (3.16) may be seriously biased, inconsistent, and inefficient because of the methodological problems associated with equation (3.16) as well as the least squares regression technique. Thus the estimates of the structural parameters should be regarded as the best approximation available for describing and explaining the investment activities of beef breeders concerning their breeding cows at the beginning of each production period.

Finally, the empirical results of equation (4.1) imply that the cyclical fluctuations associated with the size of the aggregate working herd over time can be generated by the model just formulated. In particular, the predictive performances of equation (4.1) suggest that the explanatory variables specified in equation (3.16) may be regarded as the factors which generate the cyclical fluctuations in number of breeding cows and heifers two years old and older maintained by the beef breeders.
D. Empirical Results of the Multiple-Relation Model

The empirical results of the multiple-relation model and its variant are presented and discussed briefly in the present part. Since those for the investment behavior relations concerning the aggregate working herd were considered in the preceding part, the empirical results for the investment behavior relation concerning the aggregate replacement heifers, the calf price relation, the cow price relation, and the calf production relation will be considered in the present part. The first section shows the empirical results of equation (3.43) and its variant. The second section presents the empirical results of equation (3.46) and its variant. The third section shows the empirical results of equation (3.47) and its variant. The fourth section presents the empirical results of equation (3.48). A brief summary concludes the present part.

A multiple-relation model may be statistically analyzed according to the simultaneous equation approach. For the purpose of this dissertation, the empirical results which are derived according to the simple equation approach are reported. For the direction of future research, the multiple-relation model formulated above may be statistically analyzed according to the simultaneous equation approach, with or without any modifications of the model.
The empirical results of equations (3.43), (3.44), and (3.45) are reported in the present section. The empirical results of equation (3.43) may be summarized as follows:

\[
H_{t+1} = -1,286.0586 + 0.8817 H_t \\
(16.073)
\]
\[
+ 8.7104 p_{t}^{ca} + 89.5625 p_{t-1}^{ca} \\
(0.524) \quad (2.295)
\]
\[
- 87.8306 p_{t}^{co} + 18.6204 R_t, \\
(1.903) \quad (1.801)
\]
\[
R^2 = 0.989
\]

Several points deserve comment. First, the fit of the model to the data supports the theoretical considerations in the preceding chapter. Second, the signs of the estimated coefficients are compatible with the expected ones a priori. Third, the individual significance tests of coefficients for equation (4.6), using the Student's t distribution, indicate that at the five percent level of significance only the coefficients of \(H_t\) and \(p_{t-1}^{ca}\) are significant. Fourth, the joint significance test for (4.6), using the F distribution, indicates that at the one percent significance level the test is highly significant, since \(F^5_{28} = 251.886\) against the table value of 3.76. Fifth, the test of autocorrelation in the
disturbance terms cannot be carried out since the application of Durbin-Watson statistic is limited to those equations which do not include the lagged dependent variable as an explanatory variable. Sixth, the expected demand equation for the replacement heifers can be derived from (4.6) as follows:

\[ H_{t+1}^{**} = -10,717.1549 + 819.2607 p_{t+1}^{ca} - 731.9215 p_t^{co} + 155.1696 R_t. \]

The estimated structural parameters of equation (4.7) may be interpreted in terms of the ceteris paribus economic statement as follows: (a) a rise in the expected calf price of one dollar per hundredweight would bring about a rise in the demand for expected replacement heifers about 819.261 head, (b) a rise in the salvage cow price of one dollar per hundredweight would bring about a fall in the demand for expected replacement heifers of about 731.922 head, and (c) an improvement in the range feed condition of one percent would bring about a rise in the demand for expected replacement heifers of 155.170 head. Seventh, the adjustment coefficient of the replacement heifers derived from (4.6) is 0.1163, implying that about 12 percent of desired changes in the replacement heifers is adjusted within one production period. Eighth, the weights of price expectation attributed to the past price level of calves can be derived from (4.6) as
follows: (a) $a_0 = 0.0886$, and (b) $a_1 = 0.9114$. It is interesting to note that the beef breeders attribute more weight to the price of calves in period $(t-1)$, $p_{t-1}^{ca}$, that in period $t$, $p_t^{ca}$, in order to anticipate the future price of calves, $p_{t+1}^{ca}$. Finally, the predictive performance of equation (4.6) is summarized in Figure 20, where the calculated values correspond reasonably well to the actual values for the number of replacement heifers kept on farms.

The empirical results of equation (3.44), which is regarded as a special form of equation (3.43), may be summarized as follows:

\[
H_{t+1} = 4,355.6563 + 91.5088 p_t^{ca} + 338.4334 p_{t-1}^{ca} - 359.5291 p_t^{co} - 26.2092 R_t' \tag{4.8}
\]

\[
R^2 = 0.874, \text{ and } d = 0.4490.
\]

The above investment behavior relation concerning replacement heifers deserves several comments. First, the index of explanatory power for equation (4.8) is respectable, but it is inferior to that for equation (4.6). Second, the sign of estimated coefficient for $R_t'$ is not compatible with what was expected a priori, although
Figure 20. The number of heifers kept on farms on January 1.
other signs are compatible. Third, the individual significance
tests of coefficients for (4.8), using the Student's t distribution,
indicate that at the five percent level of significance the tests
for the coefficient of \( p_t^{ca} \) and \( p_t^{co} \) are only significant. Fourth,
the joint significance test for the coefficients of (4.8), using the
F distribution, shows that at the one percent level of significance
the test is highly significant since the computed value, \( F_{29}^4 = 23.385 \),
is greater than the table value, \( F_{29}^4 = 4.04 \). Fifth, the test of
autocorrelation in the disturbance term in terms of Durbin-Watson
statistic indicates that there is positive correlation at the five
percent significance level. Sixth, equation (4.8) should be re-
garded as the investment behavior relation which explains the
investment activities of beef breeders concerning their replace-
ment heifers, where the beef breeders adjust their replacement
heifers without any delay. Finally, no further statistical analyses
were carried out since the estimated coefficient of \( R_t \) is contrary
to expectations.

The empirical results of equation (3.45), which is also re-
garded as a special form of (3.43), may be summarized as follows:
4.9) \[ H_{t+1} = -1,561.2485 + 0.9285 H_t \]
\[ + 19.8775 p_{ca}^c + 10.2428 p_{co}^c \]
\[ + 21.1791 R_t, \]
\[ R^2 = 0.987. \]

The above investment behavior relation deserves several comments. First, the index of explanatory power for equation (4.9) is respectable. It is slightly inferior to that for equation (4.6), but it is far superior to that for equation (4.8). Second, the sign of estimated coefficient for \( p_{co}^c \) is not compatible with what was expected a priori, although other signs are compatible. Third, the individual significance tests of coefficients for (4.9), using the Student's t distribution, indicates that at the five percent significance level only the test for the coefficient of \( H_t \) is significant. Fourth, the joint significance test for the coefficients of (4.9), using the F distribution, shows that at the one percent level of significance the test is highly significant since the computed value, \( F_{29}^4 = 273.336 \), is greater than the table value, \( F_{29}^4 = 4.04 \). Finally, no further statistical analyses were carried out since the estimated coefficient of \( p_{co}^c \) is not compatible with what was expected a priori.
2. **Calf Price Relation**

The regression equations for the calf price relation and its variant are reported in the present section.

The least squares estimates of equation (3.46) and other empirical results may best be summarized as follows:

\[
\begin{align*}
\text{ca}_{t+1} &= 7.1594 - 0.0001 \text{CC}_t^b + 0.0002 \text{CC}_t^d \\
&+ 0.7905 \text{ca}_t + 0.0053 \text{YT}_t \\
\end{align*}
\]

\[
\text{R}^2 = 0.8617.
\]

Several points deserve comment. First, the magnitude of \( \text{R}^2 \) suggests that the fit of the relation to the available data is reasonably good. Second, the sign of the coefficient for the size of dairy calf crop is contrary to what was expected a priori. Third, the individual significance tests of the coefficients for the above equation, using Student's \( t \), indicate that at the five percent level of significance only the coefficient of lagged calf price is significant. Fourth, the joint significance test of the coefficients for equation (4.10), using the \( F \) distribution, shows that at the one percent level of significance the test is highly significant since the computed value, \( F_{29}^4 = 45.168 \), is greater than the table value, \( F_{29}^4 = 4.04 \). Fifth, the application of the
Durbin-Watson statistic is not valid for the above regression equation since the lagged dependent variable is introduced as an explanatory variable. Finally, some modifications of equation (4.10) may be desirable to explain the determination of calf price.

An alternative hypothesis to that formulated in the preceding chapter is developed. Ranchers sell calves to feeders; feeders sell fat cattle to butchers. Perhaps the earlier formulation did not take into account the price of feeder steers, which may be regarded as a vertically related commodity in the production of red meat. Thus it is supposed that the aggregate demand for both beef and dairy calves depends upon the price of calves, the per capita disposable income, and the price of feeder steers. In addition, suppose that aggregate demand is positively related to the price of feeder steers.

The least squares estimates of the modified regression equation associated with the alternative hypothesis are as follows:

\[
4.11) \quad p_{t+1}^{ca} = 1.3654 - 0.0004 CC_t^b - 0.0003 CC_t^d - 0.5809 p_t^{ca} + 0.0089 Y_t + 1.3341 p_t^{fs} \\
(1.3440) \quad (0.5648) \quad (0.7674) \quad (1.7364) \quad (1.8357)
\]

\[
R^2 = 0.8766.
\]
Several points should be noted from the statistical results. First, the value of $R^2$ is slightly higher than that of equation (4.10), implying that the price of feeder steers accounts partly for the variation in the price of calves sold. Second, the most important point in including the price of feeder steers as an explanatory variable is that the sign of each individual coefficient is in accordance with what was expected a priori. Third, the individual significance tests show that at the five percent level of significance none of the coefficients is significant. Fourth, the joint significance test of all coefficients suggests that the test is highly significant at one percent level of significance, since the computed value, $F_{28}^5 = 39.76$, is greater than the table value, $F_{28}^5 = 3.76$. Fifth, the coefficients of equation (4.11) may best be interpreted in the form of the following ceteris paribus economic statements: (a) a rise in the number of calves produced of 10,000 head would bring about a fall in the price level of calves of about $4.20, (b) an increase in the number of dairy calves produced of 10,000 head would decrease the price of calves by about $2.50, (c) a rise in the price of calves in period $t$ of $1.00$ would bring about a fall in the price of calves in period $(t+1)$ of about $0.58, (d) an increase in the per capita disposable income of $100.00$ would increase the price of calves by $0.89,$ and (e) a rise in the price of feeder steers
of $1.00 would bring about a rise in the price of calves of about $1.33. Finally, the predictive performance of equation (4.11) is presented in Figure 21, where the calf prices calculated by using equation (4.11) do not correspond well to the actual calf prices.

3. Cow Price Relation

The least squares estimates of coefficients of equation (3.47) and other statistical results may be summarized as follows:

4.12) \[ p_{t+1}^{\text{co}} = -10.9428 + 0.7701 p_t^{\text{co}} + 0.0042 y_t \]

\[ (6.3491) \quad (1.2930) \]

\[ - 0.00003 W_t(n) + 0.0003 W_t(n), \]

\[ (0.1433) \quad (1.2062) \]

\[ R^2 = 0.8434. \]

The statistical results deserve some comments. First, the magnitude of \( R^2 \) is reasonably good. Second, the signs of coefficients for the lagged price of salvage cows, \( p_t^{\text{co}} \), and the size of dairy working herd, \( W_t(n) \), are not compatible with the signs expected \( \text{a priori} \). Third, the individual significance tests of coefficients suggest that at the five percent level of significance only the coefficient of \( p_t^{\text{co}} \) is significant.
Figure 21. The annual weighted average calf price received by farmer.
Fourth, the joint significant test of coefficients indicates that at the one percent level of significance the test is highly significant since the computed value, $F_{29}^4 = 39.08$ is far greater than the table value, $F_{29}^4 = 4.04$.

Although the above statistical results are consistent, at least in part, with the theoretical discussion, an alternative cow price relation within the framework of modified hypothesis may better explain the determination of salvage cow price. In order to submit an alternative cow price relation, suppose that the aggregate demand for salvage cows depends also upon the price of slaughter steers and that they are positively related. In addition, suppose that the aggregate supply of salvage cows depends also upon both the price of calves and the range feed condition and that these additional variables are negatively related to the number of salvage cows marketed. Then a reduced form equation for the price of salvage cows can be derived as an alternative hypothesis.

The least squares estimates and other statistical results associated with an alternative hypothesis may be summarized as follows:

\[
4.13) \quad p_{t+1}^{co} = -31.4821 + 1.1142 p_t^{ca} + 0.0721 p_t^{ss} \\
\quad (1.7629) \quad (0.2371)
\]
\[ -0.6680 p^c_t + 0.0535 R_t + 0.0034 Y_t \\
(0.8246) \quad (0.3742) \quad (0.7772) \]

\[ -0.000007 W_t(n) + 0.0009 \tilde{W}_t(n), \]

\[ (0.0247) \quad (2.1726) \]

\[ R^2 = 0.8639, \]

where \( p^{ss}_t \) denotes the price of slaughter steers for the production period \( t \). Several points deserve comment. First, the fit of the alternative reduced form equation to the available data is slightly better than that of the initial cow price relation. Second, the signs of coefficients for the variables, except for the size of dairy working herd, \( W_t(n) \), are compatible with the signs expected \textit{a priori}. Third, the individual significance tests of coefficients, using the Student's \( t \) distribution, show that at the five percent level of significance only the coefficient of \( W_t(n) \) is significant. Fourth, the joint significance test of coefficients, using the \( F \) distribution, suggests that at the one percent level of significance the test is highly significant since the computed value, \( F_{26}^7 = 23.37 \), is greater than the table value, \( F_{26}^7 = 3.42 \). Fifth, the coefficients of equation (4.13) may be interpreted in the form of \textit{ceteris paribus} economic statements as follows:

(a) a rise in the calf price of $1.00 would bring about a rise in the salvage cow price of about $1.11, (b) an increase in the
slaughter steer price of $1.00 would increase the salvage cow price of about $0.07, (c) a rise in the salvage cow price in period t of $1.00 would bring about a fall in the salvage cow price in period (t+1) of about $0.67, (d) an increase in the index of range feed condition of 10 percent would increase the salvage cow price by about $0.54, (3) a rise in the per capita disposable income of $100.00 would bring about a rise in the salvage cow price of about $0.34, (f) an increase in the size of beef working herd of 100,000 head would decrease the salvage cow price by about $0.70, and (g) a rise in the size of dairy working herd of 1,000 head would bring about a rise in the salvage cow price of about $0.90. Finally, the predictive performance of equation (4.13) is presented in Figure 22, which suggests that the calculated salvage cow prices do not correspond to the actual salvage cow prices.

4. Calf Production Relation

The least squares estimates of coefficients and other statistical results of equation (3.48) may now be presented as follows:

4.14) \[ \text{CC}_{t+1}^b = -3,712.8430 + 0.9094 W_{t+1}(n) + 32.1939 R_{t+1}, \]
\[ R^2 = 0.9965, \text{ and } d = 0.7641. \]
Figure 22. The annual average cow price of slaughter cows, utility, Chicago.
Several points deserve comment. First, the calf production equation fitted to the data explains quite well the variation in the number of calves produced. Second, the signs of coefficients are in accord with the signs anticipated. Third, the individual significance tests of coefficients, using the Student’s t distribution, show that the tests are significant at the five percent level. Fourth, the joint significance test of coefficients, using the F distribution, indicates that at the one percent level of significance the test is highly significant since the computed value, $F_{31}^2 = 4.412.91$ is far greater than the table value, $F_{31}^2 = 4.48$. Fifth, the test of autocorrelation in the disturbance term, using the Durbin-Watson statistic, suggests that at five percent level of significance there is a positive autocorrelation in the disturbance term. In connection with the sources of autocorrelation, Valvanis (90, p. 169) states: "Random terms are serially correlated when the time interval $t$ is too short, when overlapping observations are used, and when the data from which we estimate were constructed by interpolation." Since the data on the number of beef calves produced are derived from the number of both beef and dairy calves produced on the basis of proportion of number of beef breeding cows and heifers two years old and older to the total number of both beef and dairy cows and heifers two years old and older, the
possible source of autocorrelation in the disturbance may be due to the derivation of data by interpolation. (Note the consequences of autocorrelation in the disturbance term in section IV, B, 2.) Fifth, the coefficients of the calf production relation may best be interpreted in the form of ceteris paribus economic statements as follows: (a) a rise in the size of beef working herd of 10,000 head would bring about a rise in the number of calves produced of about 9,094 head, i.e., the "marginal rate of calving" is equal to 0.9094, and (b) an increase in the index of range feed condition of one percent would increase the number of calves produced by about 32,194 head. Finally, the predictive performance of equation (4.14) is summarized in Figure 23, which shows the good correspondence of the calculated size of calf crop to the actual size of calf crop.

4. Concluding Remarks on the Multiple-Relation Model

On the basis of the empirical results presented above, some relevant remarks with respect to the purpose of the present study are briefly recapitulated for the investment behavior relation concerning the aggregate replacement heifers, the calf price relation, the salvage cow price relation, and the calf production relation. First, the empirical results of the fitted investment behavior relation concerning the aggregate
Figure 23. The number of beef calves produced in each year.
replacement heifers and its variant support the hypotheses developed in Chapter III. It was noted that the explanatory power of equation (4.6) is superior to that of equations (4.8) and (4.9) and that the estimated coefficients of equation (4.6) are compatible with those expected \textit{a priori} while some estimated coefficients of equations (4.8) and (4.9) are not.

Second, the statistical results of the fitted calf price relation do not support the hypothesis submitted in Chapter III. Although the explanatory power of equation (4.10) was relatively high, the sign of the estimated coefficient for the size of dairy calf crop was not in accord with the expected sign. Therefore, it was alternatively assumed that the aggregate demand for both feeder and slaughter calves depends also upon the price of feeder steers. The empirical results of the modified calf price relation indicated that the coefficient of multiple determination was increased by one percent and that the signs of all of the estimated coefficients were compatible with those expected \textit{a priori}. The alternative hypothesis for the determination of calf price is then somewhat superior to the hypothesis formulated in Chapter III.

Third, the empirical results for the cow price relation which was formulated in Chapter III suggest that the estimated sign for the lagged cow price and the size of the dairy working
herd were not compatible with those expected a priori, although
the explanatory power of (4.12) was relatively high. Thus two
modifications in the simplifying assumptions were made: (a)
the aggregate demand for the salvage cows (i.e., both beef and
dairy cows) depends also upon the price of slaughter steers, and
(b) the aggregate supply of salvage cows depends also upon the
price level of calves and the range feed condition. The empirical
results of the modified cow price relation, i.e., (4.13), in-
dicated that the signs of all of the estimated coefficients, except
for the dairy working herd, were compatible with the expected
signs, and the coefficient of multiple determination was increased
by two percent. The alternative hypothesis about the determina-
tion of salvage cow price is still subject to further considerations.
That is, some efforts should be directed toward the construction
of better hypotheses about the determination of the salvage cow
price.

Finally, the empirical results support the hypothesis for-
mulated in Chapter III. The signs of the estimated coefficients of
(4.14) were compatible with the expected ones, and the explan-
atory power of (4.14) was quite satisfactory.
E. Summary

In the present chapter, three alternative investment behavior hypotheses, which were formulated in the foregoing chapter, were reviewed in the light of empirical evidence. In addition, other hypotheses for the investment activities of beef breeders concerning their replacement heifers, the determination of calf price, the determination of cow price, and the calf production process were also considered. It should be noted, however, that the empirical evidence and statistical tests presented in the present chapter might best be regarded as indicative and confirmative rather than conclusive, since there were some discrepancies between the theoretical constructs and the available data. For example, the aggregate working herd (i.e., the durable factor of calf production) was measured with respect only to size although the aggregate working herd was discussed in terms of size and composition in Chapter III. For the limited scope of the present study, it is now desirable to summarize the "tentative" conclusions which have been reached in the present study as follows:

(a) For the primary objective of the present study, three investment behavior hypotheses formulated in the present study explain the investment activities
of beef breeders concerning their working herd. On the basis of available data, the superiority of investment behavior relations with respect to $R^2$ can be described in descending order as follows: (3.16), (3.18), and (3.17).

(b) The signs of the estimated coefficients of (4.1) and (4.4) are compatible with those expected a priori while one of the coefficients of (4.3) is not. These empirical results imply that the investment behavior relations (3.16) and (3.18) are superior to (3.17) and that the lagged working herd can hardly be excluded from the investment behavior relation. (Alternatively, these results imply that the herd adjustment is not, on the average, completed within one production period.)

(c) The superiority of investment behavior relation (3.16) over others implies that beef breeders anticipate the future price level of calves sold, estimate their desired working herd, and adjust their actual working herd. The estimated coefficients of equation (4.1) provide the estimates of the adjustment coefficient, the weight of expectations, and the parameters of the expected demand relation, as presented in section IV, C.
In connection with the secondary objective of the present study, the explanatory performances of the investment behavior relation concerning the aggregate replacement heifers and the calf production relation, (4.6) and (4.14), respectively, are quite good while those of calf price relation and cow price relation, (4.10) and (4.12), respectively, are reasonably good. In regard to the compatibility between the estimated and expected signs, the investment behavior relation concerning the aggregate replacement heifers and the calf production relation are supported in the light of empirical evidence while the other relations are not. However, the modified calf price relation and cow price relation, (4.11) and 4.13), respectively, show that (i) the explanatory power of (4.11) is an improvement over that of (4.10), and the signs of the estimated coefficients are compatible with those expected a priori; and (ii) the explanatory power of (4.13) is an improvement over that of (4.12), and one of the estimated signs does not agree with what was expected a priori.
(e) The predictive performances of the investment behavior relation concerning the working herd, the investment behavior relation concerning the aggregate replacement heifers, and the calf production relation are respectable; those of the calf price relation and the cow price relation are not.
CHAPTER V

SYNOPSIS AND IMPLICATIONS

Three alternative hypotheses about the investment activities of beef breeders concerning their breeding cows as well as their replacement heifers, together with the hypotheses which explain the determination of calf price, the determination of salvage cow price, and the calf production process, reviewed in the light of empirical evidence are the direct outgrowth of the model of a representative beef breeding firm analyzed in Chapter II. As such, the hypotheses advanced in the present study differ markedly from previous hypotheses designed to explain the cyclical phenomena associated with the cattle industry. As indicated in Chapter I, the hypotheses formulated in previous cycle studies have often been directed toward selecting the strategic variables, e.g., price level of calves, price of choice steers, price of stocker and feeder steers, etc., which seem to be related to the number of beef breeding cows and heifers two years old and older and then attempting to establish statistical relationships between them. In particular, it has been hypothesized that the cyclical phenomena associated with the cattle industry (or cattle sector of livestock economy) are generated by a self-generating mechanism or the
interaction of the conventional supply and demand relations. The selection of variables in these earlier studies was made without benefit of any explicit theoretical model of the process being studied and verified. The self-generating mechanism was introduced without any explicit consideration of the fundamental activities of the economic agents involved. Two important weaknesses ensue from such procedure.

The first weakness concerns the specifics of the empirical work itself. With no underlying explicit theoretical foundation, it is impossible to avoid arbitrary specifications of the functional form validated and the form in which the various selected variables enter this model. Since no systematically developed theoretical basis has been developed for many of these models, the implications of changes in factors influencing the behavior of the economic agents involved cannot be usefully pursued unless they can be represented by changes in the variables of the empirical model.

The second weakness of previous empirical research concerns its larger role as a contribution to economic theory which advances the economist's view and understanding of the world. Since the assumptions employed are difficult to identify, lack of an appropriate theoretical basis severely hinders the interpretation which may be given the results. Yet the ultimate significance of economic research resides in its interpretation.
The empirical results of the present study are interesting because they support, and, at least in part, validate, the implications drawn from the hypothetical ranch operation described in Chapter II, from which the investment behavior relation concerning the working herd, the investment behavior relation concerning the replacement heifers, the calf price relation, the salvage cow price relation, and the calf production relation are explicitly derived and modified. These results have important theoretical consequences and implications, and it is to these that the present chapter is devoted.

This chapter is divided into four parts. In Part A, the reasoning which underlies the theoretical development of Chapters II and III is briefly recapitulated along with the evidence presented in support of the models. In Part B, the implications of the results for some of the theoretical questions raised in previous cycle studies are suggested. In Part C, an attempt is made to compare the present study with Ehrich's model. In Part D, some limitations of the model and several lines along which the present refutable hypotheses can be extended in future research are briefly noted.
A. The Models and Evidence: Brief Review

The motivational assumption upon which the refutable hypotheses for the primary objective of the present study are founded is that the representative beef breeder attempts to retain a working (and breeding) herd of optimum size and composition. It is in the search for the maximum net accumulated discounted return from the investment in the breeding animals that decisions are made to alter the rancher's actual working (and breeding) herd. Thus the optimum last age and the available amount of feed resources serve as the pivot variables of the ranch operation for a given stationary, biological process, and they are, consequently, the most important variables in determining the optimum working (and breeding) herd.

The "net investment"--the net change in the actual working (and breeding) herd maintained with respect to size and composition, over and above "normal" replacement and culling--is regarded as the result of decisions to adjust the actual to the optimum working herd. To understand the considerations underlying these decisions, attention must first be focused on the determination of a working herd of optimum size and composition. As specified in Chapter II, the net accumulated discounted return from a given breeding herd was defined as the difference between the accumulated
discounted return from the breeding animals of various ages at
the beginning of a given production period and the accumulated
cost of retaining them for one production period. Given the net
accumulated discounted return in terms of biological properties
of breeding animals and economic data, the decision problem of
a representative rancher is to determine his optimum working
herd and to adjust his actual to his optimum working herd.

The accumulated discounted return from the breeding animals
is taken as an increasing function of age (at first at an increasing
rate and then at a decreasing rate) based on the reproductivity
of beef breeding cows. The accumulated cost of retaining
breeding animals is also taken as an increasing function of age
(at a decreasing rate) based on the flesh condition after calving.
The constraint within which the herd adjustment decisions are
assumed to be made is that the optimum size of the working (and
breeding) herd is determined by the available amount of feed
resources. The economic data and the biological properties
associated with beef cattle determine the optimum last age of
breeding animals included in the working (and breeding) herd
under a set of policies in regard to the working (and breeding)
herd. Thus the available amount of feed resources imposes an
ultimate limit on the size of the working (and breeding) herd,
whereas the optimum last age determines the composition of the working (and breeding) herd under a particular policy of age distribution.

By solving the equations describing such a hypothetical ranch operation, the optimum working herd demanded could be expressed in terms of the price level of calves sold, the salvage cow price, the cost of care and maintenance, the market rate of interest, and the available amount of feed resources. In order to derive the demand relation for an optimum working herd, it was assumed that the beef breeder knows the price level of calves sold at the beginning of the subsequent production period. However, the rancher can, in reality, only anticipate the future price of calves. Therefore, a price expectation model was introduced as an approximation to the rancher's price anticipation. In connection with herd adjustment, it would seem that an instantaneous adjustment of the actual to the optimum (expected) working herd would be unlikely because the inevitable delays caused by such factors as the technological rigidities associated with beef cattle, the fixed number of breeding animals in the industry at a given point in time, and the rancher's rigidity associated with the policy of age distribution. In addition, it was assumed that the rancher follows the policy of maintaining his "normal" (or "long-run") age distribution rather than either "fixed" or
"variable" age distribution. By incorporating these assumptions into the theory of herd adjustment, the "net investment" was deemed to be proportional to the difference between the rancher's optimum and actual working herds. When the herd demand relation, the herd adjustment relation, and the mode of price anticipation were reduced and put into a form suitable for statistical analyses, a partially reduced form equation was derived. It explains the actual working herd adjusted and maintained at the beginning of a given production period in terms of (a) the expected calf price at the beginning of a given production period, (b) the available amount of feed resources at the beginning of a given production period, (c) the actual working herd adjusted and maintained at the beginning of the preceding production period, (d) the salvage cow price in the preceding production period, (e) the market rate of interest in the preceding production period, and (f) the cost of care and maintenance in the preceding production period. It is expected a priori that the dependent variable is related positively to the first three independent variables and negatively to the other independent variables.

For the primary objective of the present study, three investment behavior hypotheses were formulated in Chapter III. These hypotheses were essentially derived from the theoretical discussions advanced for the representative beef breeder. In
order to derive three alternative investment behavior hypotheses, it was assumed that every beef breeder in the industry behaves as if he were the representative beef breeder discussed in Chapter II and that the economic relations are in linear form. In addition, some modifications about the variables included were made. First, the price level of calves sold per hundredweight and the price level of salvage cows marketed per hundredweight were introduced in place of the price level of calves sold per animal and the price level of salvage cows per animal, respectively. Second, the available amount of feed resources was substituted by the proxy variable, the range feed condition. Finally, the market rate of interest and the cost of care and maintenance were excluded from the investment behavior relation. Furthermore, the assumption about the rancher's culling activities was modified so that the beef breeder would cull his breeding animals, depending upon the optimum last age, the available amount of feed resources, the "normal" age distribution, and the calving records. By incorporating the additional assumptions, the modifications of the variables included, and the modified assumption about the rancher's culling activities, three refutable hypotheses were submitted about the investment activities of the beef breeders concerning their breeding cows. The first hypothesis states: (a) the expected
working herd depends upon the anticipated price of calves sold, the salvage cow price, and the range feed condition; (b) the anticipated price of calves sold depends on the calf prices sold in basis of the calf price of the preceding production period, proportional to the difference between the expected and the actual working herd. The second hypothesis is the same as the first hypothesis, except for one modification in the herd adjustment relation: beef breeders as a whole adjust their actual working herd to the desired level without any delay. The third hypothesis is the same as the first hypothesis, except for one modification. The present hypothesis states that the beef breeders as a whole anticipate future price of calves sold on the basis of the calf price of the preceding production period.

For the secondary objective of the present study, a multiple-relation model was constructed in Chapter III which is capable of explaining the investment activities of beef breeders and the closely related economic and biological events. It attempted to explain the investment activities of beef breeders concerning their replacement heifers. In addition, it attempted to explain the determination of calf price, the determination of salvage cow price, and the calf production process, since the marketing activities of beef breeders seem to influence the determination of calf price and cow price.
The investment behavior relation concerning the aggregate replacement heifers is a partially reduced form equation which is constructed on the basis of the demand for replacement heifers desired, the anticipation of future price of calves, and the adjustment activities of beef breeders concerning their replacement heifers. Three alternative hypotheses were submitted. The first hypothesis states: (a) the expected (or optimum) number of replacement heifers depends upon the anticipated price level of calves sold, the salvage cow price, and the range feed condition; (b) the anticipated price of calves sold depends on the calf price of the past two periods; and (c) the "net investment" is proportional to the difference between the expected and the actual number of replacement heifers. The second hypothesis is the same as the first hypothesis, except for one modification. The second hypothesis states that beef breeders as a whole adjust their actual number of replacement heifers to the desired level without any delay. The third hypothesis is the same as the first hypothesis, except for one modification in the anticipation of future calf price: the beef breeders as a whole anticipate future price of calves sold on the basis of the price of the preceding production period.

The calf price relation is also a partially reduced form equation which is derived from the aggregate supply relation, the
aggregate demand relation, and the equilibrium condition. For convenience, it was assumed that the number of calves demanded by the feed-lot and slaughter-house operators depends on the price of calves sold and per capita disposable income and that the number of calves supplied by beef breeders and dairymen depend on the price of calves sold in the preceding production period, the size of the beef calf crop, and the size of the dairy calf crop. By assuming that the equilibrium condition is satisfied every production period, the calf price relation was derived as a partially reduced form equation.

The cow price relation was also a partially reduced form equation which is derived from the aggregate supply relation, the aggregate demand relation, and the equilibrium condition. For convenience, it was assumed that the number of salvage cows demanded by the feed-lot and slaughter-house operators depends on the salvage cow price and per capita disposable income and that the number of salvage cows supplied by beef breeders and dairymen depends on the beef working herd inherited, the dairy working herd inherited, and the salvage cow price in the preceding production period. By assuming that the equilibrium condition is satisfied every production period, the cow price relation was derived as a partially reduced form equation.
The aggregate calf production relation was submitted by relating the aggregate number of beef calves produced to the aggregate working herd and the range feed condition.

Because of difficulties in measuring the expected (or desired) working herd, expected (or desired) number of replacement heifers, expected calf price, and the actual working herd, the empirical results should be regarded as indicative and confirmative rather than conclusive evidence in support of the hypotheses. The working herd was measured with respect only to size, viz., the number of cows and heifers two years old and older kept on farms, on January 1, from 1931 to 1964, inclusive. The explanatory power of the three alternative hypotheses is rather good. All the coefficients, except two in the second hypothesis, were of the expected signs; some of the coefficients are not statistically significant. The predictive performances of the three hypotheses were respectable. However, the first hypothesis is better than the other hypotheses with respect to explanatory power and predictive performance. The empirical results of the first hypothesis yield the best available estimates of the parameters of demand for expected (or desired) working herd, the weight of price expectation, and the coefficient of herd adjustment. Thus, the empirical results and the attendant statistical tests support, at least in part, the foundation for the hypothesis. The statistical results
for the other relations in the multiple-relation model indicate that the investment behavior relation concerning the aggregate replacement heifers explained the investment activities of beef breeders concerning their replacement heifers and that its predictive performance was respectable. (It should be, however, noted that the investment activities of beef breeders concerning their replacement heifers were best explained by the partially reduced form equation, where neither $h''$ nor $a_0$ is restricted a priori.) In addition, the empirical results for the calf production relation explained the calf production process quite well, and its predictive performance was respectable. Furthermore, the statistical results of the calf price relation and cow price relation suggest that their explanatory powers are reasonably good but that the compatibilities between the expected signs and the estimated signs in both relations were not totally met. Therefore, the calf price and cow price relations were modified. By introducing the feeder steer price in the calf price relation, the explanatory power was somewhat improved and compatibility between the expected signs and the estimated signs was established. By introducing the slaughter steer price in the cow price relation, the explanatory power was somewhat improved, but the compatibility between the expected and the estimated signs was still subject to further examination. The predictive performances of
the revised calf price relation and the revised cow price relation were not respectable.

B. The Implications of the Present Study Toward Some Questions Raised by Previous Cycle Studies

The refutable hypotheses and the empirical evidence presented in this study provide an alternative perspective in understanding, not only the fundamental activities of beef breeders concerning their breeding animals, but also the cyclical behavior exhibited in the number of beef breeding animals kept on farms, the number of beef animals marketed, and the price level of beef cattle. The refutable hypotheses make explicit the rationale for retaining (or holding) breeding animals, with special reference to breeding cows and heifers two years old and older. Perhaps the breeding animals kept on farms may be regarded as "inventories" within a specific theoretical framework. However, it is apparent that the role of breeding animals as durable factors of production in the calf production process over time and the fundamental activities of beef breeders concerning their breeding animals can neither be ignored nor incorrectly identified if one is interested in looking for a cycle (or cycles).

The refutable hypotheses in the present study explain, qualitatively as well as quantitatively, the variation in the number
of cows and heifers two years old and older in terms of a few strategic variables which characterize the investment activities of the beef breeders concerning their working (and breeding) herd rather than in terms of a few strategic variables which are arbitrarily selected. The investment activities of beef breeders may be described and explained in terms of structural parameters derived from the estimated coefficients of an investment behavior relation, since the investment behavior relation is a partially reduced form equation from the herd demand relation, herd adjustment relation, and the mode of price anticipation.

The refutable hypotheses about the investment activities of beef breeders concerning their breeding animals explain, at least in part, the cyclical fluctuations in number of cows and heifers two years old and older kept on farms over time (as well as the number of replacement heifers kept on farms over time). In particular, the respectable predictive performance of the investment behavior relation suggests that the cyclical fluctuations in the size of working herd adjusted and maintained may be approximately generated by the investment behavior relation and that the explanatory variables in the investment behavior relation may be regarded as factors bringing about the cyclical fluctuations. Thus, the refutable hypotheses formulated in the present study may be considered as a hypothesis to be added to the set of
existing hypotheses designed to explain the cyclical phenomena associated with the beef cattle industry.

In light of the hypothetical ranch operation analyzed in Chapter II, it may be implied that the investment activities of beef breeders, as a whole, concerning their breeding animals over time influence the time-path of the number of calves produced, the number of calves sold, the number of salvage cows culled, and prices of beef cattle of various kinds.

C. Comparison with Ehrich's Model

Perhaps the model developed in the present study may be viewed as a modified version of Ehrich's model. Some fundamental differences between these models can easily be noted by considering the objectives of the study, the approach used for constructing the model, the specification of the stochastic model, and the empirical results. A brief comparison of the present model with Ehrich's will be made.

First, in connection with the objectives of the study, Ehrich (18, p. 3) specified four objectives of study as follows:

"(1) to describe the behavior of cattle prices and numbers since 1946 on an aggregate basis, viewing United States aggregates and annual average data; (2) to quantify those factors that affected prices and supplies of feeder cattle, and beef-cattle-inventory adjustment; (3) to determine the behavioral characteristics that underlie the cattle cycle; and (4) to provide estimates of national aggregate..."
demand, supply, and price relationships that will be useful background material for a regional analysis of these factors."

The primary objective of the present study is, however, to explain the investment activities of beef breeders concerning their breeding cows and heifers two years old and older. In addition, the secondary objective is to explain the investment activities of beef breeders concerning their breeding cows and heifers two years old and older as well as their replacement heifers, the determination of calf price, the determination of salvage cow price, and the calf production process. According to the specified objectives of the two studies, Ehrich's analysis deals with the behavior pattern of prices and numbers either in absolute magnitude or in their changes, while the present analysis deals with the behavior pattern of the economic agents involved, the determination of calf price, the determination of salvage cow price, and the calf production process. Thus the role of each model is properly differentiated.

Second, Ehrich submitted a set of simplifying assumptions at the outset for his economic model, while the present study introduced a large number of simplifying assumptions and drew implications from them for its economic model. Although Ehrich introduced a set of simplifying assumptions, he did not integrate their implications. Thus, for example, Ehrich was not clear as to why the beef breeder retains a herd of breeding animals.
However, the present study suggests the rationale for retaining a herd of breeding animals.

Third, with respect to the specification of the model, Ehrich's model consists of ten relations, while the present model consists of one partially reduced form equation for the primary purpose and four partially reduced form equations and one technological relation for the secondary purpose. Although some empirical variables may be commonly used in these models, the specified relations are different with respect to the variables included in the corresponding equations. For example, Ehrich identifies the "inventory" adjustment relation for beef cows by relating the annual change in beef cow numbers (in million head) to the number of cows slaughtered annually, lagged one year (in million head) and the derived number of heifers kept for breeding purposes, lagged one year (in million head). The present study identifies the investment behavior relation concerning the aggregate beef cows and heifers two years old and older by relating the number of beef cows and heifers two years old and older kept on farms at the beginning of a given production period to (a) the number of beef cows and heifers kept on farms at the beginning of the preceding period, (b) the calf prices in the two preceding periods, (c) the salvage cow price in the preceding period, and (d) the range feed condition in the preceding period. Thus it is
apparent that the structure of these models are different.

Fourth, the empirical results of Ehrich's model suggest that the heifer inventory adjustment relation (which relates the annual change in the number of heifers kept on farms to the average deviation from trend of annual average prices of stock and feeder steers at Kansas City during the current year and two years previous) explains about 44 percent of variation in the annual change in the number of heifers kept on farms. In addition, the empirical results show that the explanatory powers of other relations in Ehrich's model lie between 76 percent and 98 percent. The explanatory powers of the present model are respectable, as shown in Chapter IV. As far as the generation of the cyclical phenomena in numbers and prices is concerned, the investment behavior relation concerning the aggregate working herd, the investment behavior relation concerning the aggregate replacement heifers, and the calf production relation do generate the cyclical phenomena reasonably well while other relations do not. However, Ehrich's model was not designed to explain the generation of cyclical phenomena.

From the above comparisons, it is evident that the role of the present model is different from that of Ehrich's model and that the specification and the empirical results of the present model are different from those of Ehrich's model.
D. Qualifications and Possible Extensions

Before closing the present study, it is helpful to review its major shortcomings and to examine several further hypotheses. Four weaknesses exist in the present study.

First, the refutable hypotheses about the investment activities of beef breeders concerning their breeding animals were formulated on the basis of the fundamental activities of a representative beef breeder within the framework of a hypothetical ranch operation. For the sake of simplicity, the herd adjustment activities concerning the breeding herd were regarded as independent of each other. However, herd adjustment is not independent of the adjustments in replacement heifers and replacement calves. Perhaps the investment activities of beef breeders may be better explained by incorporating explicitly the investment activities of beef breeders concerning their cows and heifers two years old and older, replacement heifers, and replacement calves.

Second, the theoretical concepts of the working (and breeding) herd were discussed in terms of the number of animals and the age distribution of these animals. However, the working herd was measured empirically solely in terms of size, since the age distribution of breeding animals or any appropriate index to measure the composition of the working herd was not available.
Consequently, empirical verification of the measurement of the working herd was rather limited. Perhaps, by introducing an appropriate measurement of the age distribution of the working herd, the hypotheses of investment behavior of beef breeders concerning their working herd may be properly tested.

Third, the investment activities of beef breeders concerning their breeding cows were described and explained by the partially reduced form equation approach, because the expected price level of calves sold and the "normal" working herd were not available. Within the limited scope of the present study, no alternative estimation techniques which might measure and describe the investment activities of beef breeders concerning their working herd were considered. Perhaps some effort might profitably be directed toward formulating alternative estimation techniques by which the investment activities of beef breeders might better be described, explained, and predicted.

Fourth, for the secondary objective of the present study, the determination of calf price and the determination of cow price were explained by refutable hypotheses. However, it was observed in the preceding chapter that the relevant variables were not properly included in the relations when they were formulated in Chapter III. Perhaps the reformulation of the calf price relation and the cow price relation might better explain, and consequently
predict, the determination of both calf price and cow price.

Possibly one essential criticism can be directed to the aggregative nature of the verifications. It may well be that microeconomic studies will provide evidence that is not entirely consistent with the results summarized. However, as stated at the outset of the study, the primary concern is with aggregate activities. No effort has been directed toward formulating an accurate description of the process by which the investment activities of a given breeding ranch are carried out. Although it is certain that investment activities of individual beef breeders in the industry might on occasion be motivated by factors other than those which have been examined (or discussed), the investment activities and the parameter estimates for the present study are those which reflect the reactions of the aggregate economic community. The acceptability of the refutable hypotheses in this regard, as reported in Chapter IV, suggests that further work, especially along the lines indicated above, will be fruitful.

E. Concluding Remarks

This study was originally conceived as possessing a double objective—one primary, the other secondary. The primary objective was to formulate (or submit) a set of hypotheses capable of explaining the fundamental activities of beef breeders concerning
their breeding cows and heifers two years old and older. The secondary objective was to formulate a multiple-relation model to explain the investment activities of beef breeders concerning their breeding cows and heifers two years old and older in conjunction with other economic and technological relations capable of explaining the aggregate economic events associated with the investment activities.

The investment behavior relation of beef breeders, derived from the model of the representative breeding firm in Chapter II, was found in the preceding chapter to explain virtually all of the variations in the number of cows and heifers two years old and older kept on farms. The estimated coefficients of the investment behavior relation were consistent with the qualitative implications of the model; the parameters of expected demand relation for working herd, the weight of price expectation, and the coefficient of herd adjustment were approximated. The predictive performance was satisfactory. Thus, on the basis of the observed evidence, the approach adopted in the present study to the questions which were presented in Chapter I has been reasonably successful. In connection with the other relations in the multiple-relation model, the investment behavior relation concerning the aggregate replacement heifers and the aggregate calf production relation were implied by the investment behavior
relation concerning the rancher's replacement heifers and the calf production relation advanced in Chapter II, while the calf price relation and the salvage cow price relation were subject to the reformulation of hypotheses because these relations did not take into account all of the relevant variables. By incorporating some excluded variables into these relations, the hypothesis concerning the determination of calf price became compatible with the empirical evidence while the hypothesis concerning the determination of salvage cow price was not, implying that some further studies would be desirable for the determination of salvage cow price.

The model of the beef breeding firm may best be viewed as a part of the larger conceptual framework implied by the "general" theory of demand for durable input. In the present hypotheses, changes in the composition of a working (and breeding) herd maintained by a representative beef breeder depend not only upon changes in the optimum (or anticipated) last age but also upon the change in the policy as to the age distribution of the working (and breeding) herd. The view provided by the refutable hypotheses in the present study is substantially more fundamental than those previous hypotheses which were designed to explain "cattle cycles," "inventory cycles," "price cycles," and "production" cycle. The essential characteristics which distinguish
the present hypotheses from those previous hypotheses are the central role of the working herd in the ranch operation as a factor of production and the identification of the fundamental activities of beef breeders concerning their working herd.

An important conclusion which can be drawn from the present study, in relation to "cycle" studies, is that the rationale of retaining breeding animals as a collection of biological, durable inputs and the ranchers' adjustment activities concerning their breeding animals can hardly be disregarded if one is anxious to offer a satisfactory explanation of the processes which underlie fluctuations in the variables which characterize the beef cattle industry or the livestock economy.
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APPENDICES
APPENDIX I

THE TECHNICAL RELATIONSHIP BETWEEN
PRODUCTIVITY OF BEEF BREEDING COWS
AND AGE

The primary function of a breeding cow, or of any other breeding female, is to reproduce regularly throughout her life span. In general, the degree of reproductive efficiency of each breeding animal (or a herd of breeding females) determines a series of returns in physical terms, i.e., live calves, and consequently in money terms, i.e., some kind of "economic returns," from the breeding female (or a herd of breeding females) which is maintained by the beef breeder. The change in fertility associated with the age of beef breeding cows is one of the limiting factors in maintaining a breeding female (or a herd of breeding females). The rate of propagation in beef cattle is dependent upon the frequency of parturition as well as the length of the period for which a breeding cow is maintained as a durable factor of production (or a herd of breeding females is maintained as a collection of durable factors of production). As with other aspects of livestock production, fertility is affected by such factors as age, heridity, nutrition, and environment. In order to support the simplifying assumption (c) in section II, A, 1, the present
appendix recapitulates briefly some of the empirical results reported in Burke's study (7).

The primary consideration in his study was to single out the relationship between fertility of beef breeding cows under range conditions and age variable. In other words, his study dealt with the age variable in beef breeding cows in relation to fertility of beef breeding cows under range conditions, where other factors were assumed to be stable over time.

For such a partial analysis, the breeding and calving records of 4,470 pasture exposures of purebred Hereford females mated to 178 purebred Hereford bulls, were analyzed. The breeding animals under consideration were the property of the Wyoming Hereford Ranch, Cheyenne, Wyoming. They represented all of the animals that were pasture-bred in the period 1940 to 1952, inclusive.

The technological relationship between fertility and age was considered on the basis of breeding records of every female in the herd during the years surveyed, with respect only to pasture mating. As a matter of fact, three methods of breeding, (a) artificial, (b) hand, and (c) pasture, were employed by Wyoming Hereford Ranch. It was recognized by Burke that all females did not show a record of pasture breeding and that the females pasture bred in any one given year were not necessarily pasture
bred in the following year or years. In addition, it was realized that managerial practices had influenced the randomness of the data. However, it was conveniently supposed that the females were pasture bred to calve as three-year olds and that the first calf usually resulted from an artificial breeding or from hand breeding.

The criterion adopted in determining successful matings was that the cow give birth to a live calf. It was recognized that this criterion was not totally justifiable for those animals who had a successful conception but lost their calves prematurely for various reasons. Nevertheless, since it was impossible in many cases to distinguish between nonsuccessful conceptions and early abortions, a live calf criterion might be well justifiable.

The classification of the breeding animals was arbitrarily devised. The ages of females in this study went as high as 16 years at the time of breeding, while the ages of the bulls ran up to nine years of age. The number of two years old heifers was small in comparison with those in other age groups up to and including nine years of age. The number of animals in the age groups above ten was also small. Therefore, the females were arbitrarily grouped into five age groups: two's and three's, four's and five's, six's and seven's, eight's and nine's, and ten's and over. The bulls were conveniently divided into two groups,
a young age group and an old age group. This classification placed all those animals up to and including five years old in the young bull group and those bulls six and over in the old bull group.

Table 17 presents the fertility of purebred Hereford cows of all ages over the 12 calendar years. The year of peak conception failures, which occurred in 1948, was accounted for a severe and prolonged blizzard which occurred in the latter part of 1948. Thus Burke suggested that the 1948 records were not representative of the actual conception record of the cows. However, he did not account for the causes of rise in conception failures in 1950 and 1951. In any event, he computed and indicated that the average calf crop was 86 percent for pasture breeding for the 12-year period.

The data in Table 28 indicate the fertilities of purebred Hereford cows by age groups, without any consideration of the ages of bulls mated. Table 18 shows that the youngest group, which included the two and three year old females, comprised the smallest number of females. In addition, it shows that the four and five year old females were numerically the largest group. It should be noted that, on a percentage basis, the females in the six and seven year old age group were the most
Table 17. Fertility of Purebred Hereford Cows of All Ages Pasture Mated with Bulls of All Ages a/

<table>
<thead>
<tr>
<th>Year</th>
<th>Cows in herd each spring</th>
<th>Production results</th>
<th>Production calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>No.</td>
<td>No.</td>
</tr>
<tr>
<td>1940</td>
<td>251</td>
<td>237</td>
<td>14</td>
</tr>
<tr>
<td>1941</td>
<td>333</td>
<td>299</td>
<td>34</td>
</tr>
<tr>
<td>1942</td>
<td>277</td>
<td>257</td>
<td>20</td>
</tr>
<tr>
<td>1943</td>
<td>400</td>
<td>357</td>
<td>43</td>
</tr>
<tr>
<td>1944</td>
<td>452</td>
<td>387</td>
<td>65</td>
</tr>
<tr>
<td>1945</td>
<td>392</td>
<td>329</td>
<td>63</td>
</tr>
<tr>
<td>1946</td>
<td>459</td>
<td>407</td>
<td>52</td>
</tr>
<tr>
<td>1947</td>
<td>396</td>
<td>346</td>
<td>50</td>
</tr>
<tr>
<td>1948</td>
<td>457</td>
<td>337</td>
<td>120</td>
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<td>1949</td>
<td>362</td>
<td>319</td>
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</tr>
<tr>
<td>1950</td>
<td>349</td>
<td>282</td>
<td>67</td>
</tr>
<tr>
<td>1951</td>
<td>342</td>
<td>269</td>
<td>73</td>
</tr>
</tbody>
</table>

Totals and 4,470 3,826 644 86.10 13.90

avg. 447

a/ Reproduction of Burke's Table 1.

Table 18. Fertility of Hereford Cows by Age Groups Without Regard to Ages of Bulls or Year a/

<table>
<thead>
<tr>
<th>Age group at calving time</th>
<th>Production results</th>
<th>Production calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>No.</td>
</tr>
<tr>
<td></td>
<td>Years</td>
<td>No.</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td>701</td>
<td>599</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>1,165</td>
<td>1,016</td>
</tr>
<tr>
<td>6 &amp; 7</td>
<td>1,019</td>
<td>899</td>
</tr>
<tr>
<td>8 &amp; 9</td>
<td>797</td>
<td>688</td>
</tr>
<tr>
<td>10 &amp; over</td>
<td>788</td>
<td>624</td>
</tr>
</tbody>
</table>

Totals and 4,470 3,826 644 85.28 14.72

avg.

a/ Reproduction of Burke's Table 2.
Table 19. Fertility of Range Beef Cows by Age Groups When Mated to Young Bulls

<table>
<thead>
<tr>
<th>Age group at calving time</th>
<th>Production results</th>
<th>Production calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>No.</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td>608</td>
<td>521</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>812</td>
<td>704</td>
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<tr>
<td>6 &amp; 7</td>
<td>601</td>
<td>546</td>
</tr>
<tr>
<td>8 &amp; 9</td>
<td>485</td>
<td>428</td>
</tr>
<tr>
<td>10 &amp; over</td>
<td>521</td>
<td>417</td>
</tr>
<tr>
<td>Totals and avgs.</td>
<td>3,027</td>
<td>2,616</td>
</tr>
</tbody>
</table>

a/ Reproduction of Burke's Table 3.

Table 20. Fertility of Range Beef Cows by Age Groups When Mated to Old Bulls

<table>
<thead>
<tr>
<th>Age</th>
<th>No.</th>
<th>No.</th>
<th>No.</th>
<th>Percent</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 &amp; 3</td>
<td>93</td>
<td>78</td>
<td>15</td>
<td>83.87</td>
<td>16.13</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>353</td>
<td>312</td>
<td>41</td>
<td>88.38</td>
<td>11.62</td>
</tr>
<tr>
<td>6 &amp; 7</td>
<td>418</td>
<td>353</td>
<td>65</td>
<td>84.45</td>
<td>15.55</td>
</tr>
<tr>
<td>8 &amp; 9</td>
<td>312</td>
<td>260</td>
<td>52</td>
<td>83.33</td>
<td>16.67</td>
</tr>
<tr>
<td>10 and over</td>
<td>267</td>
<td>207</td>
<td>60</td>
<td>77.53</td>
<td>22.47</td>
</tr>
<tr>
<td>Totals and avgs.</td>
<td>1,443</td>
<td>1,210</td>
<td>233</td>
<td>83.51</td>
<td>16.49</td>
</tr>
</tbody>
</table>

a/ Reproduction of Burke's Table 4.
efficient when the bull age was not considered. In connection with the relationship between productivity and age, it was implied that there was a gradual increase in calf crop percentage to the six and seven year old age group, after which there followed a gradual decrease in calf crop percentage. (Alternatively, the relationship between fertility and age can be seen in Figure 24.)

The data in Table 19 show the productivities of the breeding females when they were mated to the young bulls. On a percentage basis, there were no significant differences in fertility levels between two and three, four and five, and eight and nine years old cows. However, the cows six and seven years old were significantly higher in fertility than all other age groups, whereas the cows over nine years old were significantly lower in fertility than all other age groups. It appeared that the peak efficiency was reached with the cows six and seven years old, after which there was general decline in the fertility of breeding females when they were mated to the young bulls. (The implied relationship between fertility and age when the breeding females were mated to the young bulls can be alternatively seen in Figure 24.)

The data in Table 20 demonstrate the productivities of the breeding females when they were mated to the old bulls. On a percentage basis, the results were essentially the same as those in Table 19, except that the four and five years old cows
Figure 24. Production results of cow age by bull age*

*Reproduction of Burke's Figure 1.
were the most fertile. The reason for this difference in fertility levels with different ages of bulls was not definitely ascertained. However, it is also noted that peak efficiency is reached with cows four and five years old, after which there is a steady decline. (The implied relationship between fertility and age when the breeding females were mated to the old bulls can be alternatively seen in Figure 24.)

On the basis of the empirical evidences recapitulated above, it was summarized by Burke (7, p. 8) that: "A peak in calf production was reached with six and seven year old cows. There was a gradual increase in calf crop percentage to this peak after which there followed a gradual decrease through the cows nine years of age. After cows became nine years of age or older, a sharp drop in calf production was noted."
### APPENDIX II

**AVAILABLE TIME SERIES DATA (Part A)**

<table>
<thead>
<tr>
<th>Year</th>
<th>(W_t(n))</th>
<th>(H_t)</th>
<th>(CC_t)</th>
<th>(W_t(n))</th>
</tr>
</thead>
<tbody>
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<td></td>
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<td>1,000 head</td>
<td>1,000 head</td>
<td>1,000 head</td>
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<tr>
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<td>8,997</td>
<td>2,704</td>
<td>24,355</td>
<td>22,440</td>
</tr>
<tr>
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<td>9,162</td>
<td>2,799</td>
<td>25,087</td>
<td>23,032</td>
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<tr>
<td>1931</td>
<td>9,809</td>
<td>3,015</td>
<td>26,056</td>
<td>23,820</td>
</tr>
<tr>
<td>1932</td>
<td>10,439</td>
<td>3,113</td>
<td>27,568</td>
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<tr>
<td>1933</td>
<td>11,346</td>
<td>3,414</td>
<td>28,935</td>
<td>25,936</td>
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<td>1934</td>
<td>12,678</td>
<td>3,656</td>
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<tr>
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<td>3,362</td>
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<td>11,048</td>
<td>3,493</td>
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<td>3,229</td>
<td>28,033</td>
<td>24,649</td>
</tr>
<tr>
<td>1938</td>
<td>10,132</td>
<td>3,136</td>
<td>27,787</td>
<td>24,466</td>
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*a/ Sources of the data are given in section IV, A.*
### APPENDIX II

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a/ Sources of the data are given in section IV, A.
APPENDIX III

DERIVED TIME SERIES DATA a/

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*a/ Sources of the data are given in section IV, A*.