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Title A MATHEMATICAL METHOD OF EVALUATING THE RELIABILITY OF ELECTRONIC EQUIPMENT

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Reliability, the probability that a system will not fail but will perform correctly at least until some arbitrary time, is becoming an increasingly important concept in electronic system design. This probability is evaluated using probability density functions which are obtained analytically by forming mathematical expressions describing the failure distributions of the systems.

The most commonly used expression in reliability studies is the exponential function where the reciprocal of its mean corresponds to a failure rate. When several assumptions are made, the failure rate of the system is equal to the sum to the failure rates of the components which comprise the system. After the empirical failure rate values of the components have been adjusted to conform to thermal and electrical stress conditions, they may be summed to obtain the failure rate of the system. This method was used to calculate the failure rates for various military electronic equipment.

The calculated results were in close agreement with the experimental failure rates of this equipment.

On-off power cycling must also be considered in reliability studies since it has the effect of adding a substantial increment of value to the basic failure rate of a system. The effective failure rate and thus reliability of a system may be found by summing the basic failure rate and the increment of value due to power cycling. It was found, using multiple regression analysis and the results from an ARINC study of cycling failure rates of various equipment, that the increment of value added to the basic system failure rate might be predicted by considering only the tubes in a system and its power cycling rate.

A MATHEMATICAL METHOD OF EVALUATING THE RELIABILITY OF ELECTRONIC EQUIPMENT

by

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A MATHEMATICAL METHOD OF EVALUATING THE RELIABILITY OF ELECTRONIC EQUIPMENT

I. INTRODUCTION

The trend of today is towards automation but automation seems to require complexity. It is evident that for a complex system to operate correctly, most if not all of the parts which make up the system must operate correctly. If too many fail to do their assigned task, the system breaks down. The probability that a system will perform correctly for a given period of time is the system's reliability. Reliability is becoming an increasingly important concept, especially in the field of electronic system design, since it is so closely related to complexity. This paper is concerned with presenting a mathematical method which may be used to estimate the reliability of an electronic system. Although this is not a new problem, there have been few unified presentations which adequately develop this method. This thesis is presented in an attempt to remedy the situation.

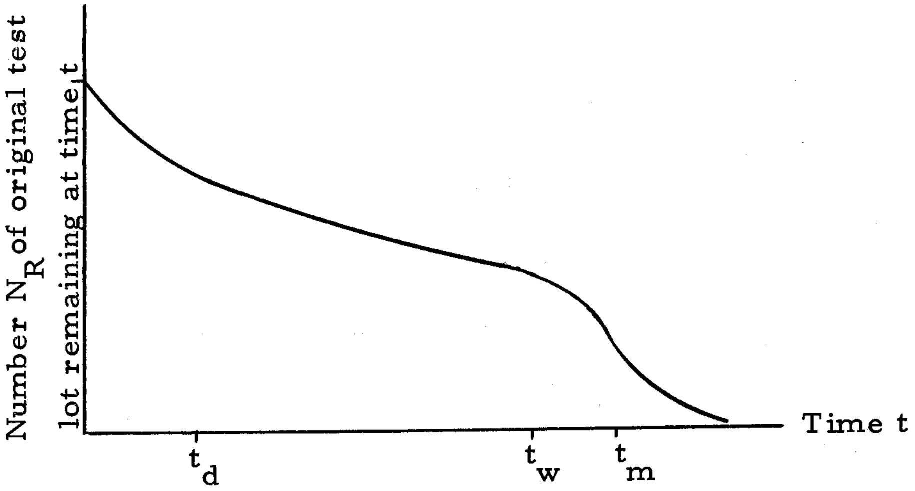
This investigation will begin by performing a hypothetical experiment. The basic types of failures that occur in this experiment shall be examined and statistical models to describe these failures shall be developed. It will be evident that the exponential failure density function describes the failures which occur in a system during its useful life. This distribution shall be used to calculate the reliability of a general system. Basic failure rates shall be derived and tabulated for various electronic components, e.g. capacitors, resistors, and relays, for use in the exponential distribution. It will be shown how these failure rates vary with voltage and temperature stresses. These basic failure rates shall be used to calculate the failure rate of various military electronic equipment. This investigation will be concluded by considering the effects of on-off cycling on the basic failure rate of electronic equipment.

II. STATISTICAL MODELS USED IN EVALUATING RELIABILITY

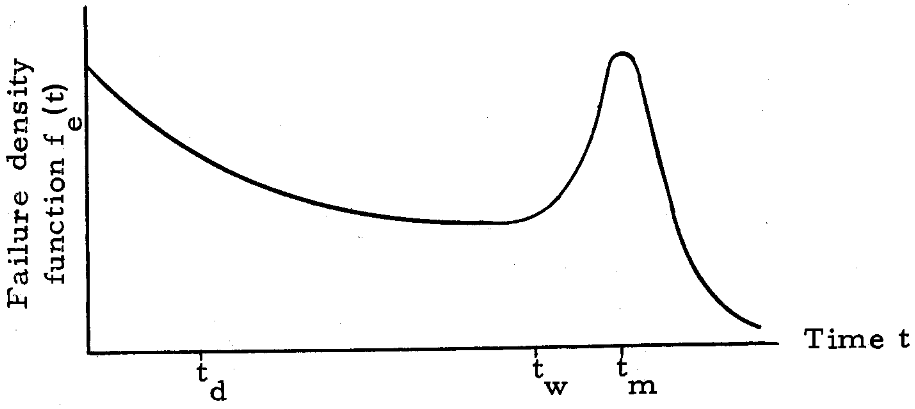
Failure Categories

In order to be able to evaluate the reliability of a system, we must first be able to describe the failures that may occur in the system. Thus, a hypothetical experiment shall be performed which will characterize the most commonly observed failures (12, p. 278). Then simple, but adequate, mathematical models shall be found to describe these failures.

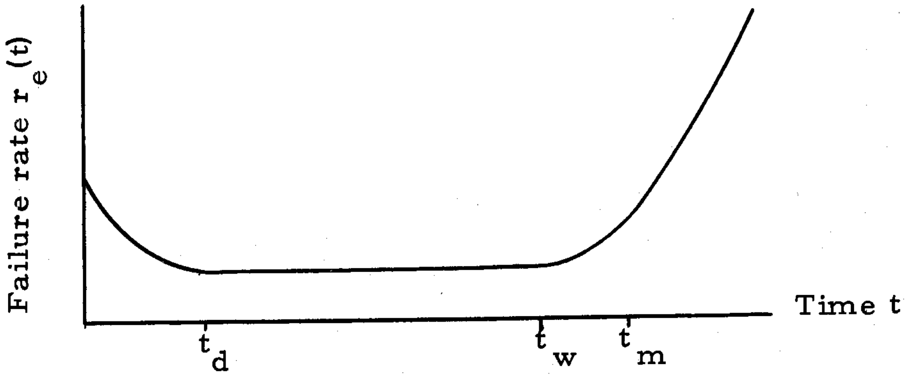
Suppose a very large number of similar independent systems are placed in some standard operating condition. If the time and number of failures that occur are carefully recorded, the number of remaining systems N_R still operating at time t might be characterized by Figure 1(a). Examination of this curve reveals that it contains three distinct regions. In the first region ($t < t_d$), a large number of failures occur in a short interval of time. In the second region ($t_d \leq t \leq t_w$), failures occur with less frequency than in the first. In the third region ($t > t_w$), failures occur rapidly until there are no remaining systems in the test lot. Failures in the first region are due primarily to faulty components and faulty workmanship. These failures become evident soon after the system is first operated. If these faults were corrected during the manufacturing process, the systems' failure curve obtained by the user would



(a) Number of original test lot remaining at time t.



(b) Failure density function for the test lot.



(c) Failure rate for the test lot.

Figure 1. Results of a hypothetical experiment.

contain only the second and third regions. Thus, the first region is often referred to as the "debugging" phase of operation or just the debugging region. The second region is called the chance failure region since there is no particular factor which produces failure. The third region is referred to as the wear-out region since failures are due primarily to systems deteriorating or "wearing out."

Let us define the failure density function $f_e(t)$ for the test lot as

$$f_e(t)=\left|dN_R/dt\right| \tag{1}$$

where dN_R/dt represents the slope of the curve of Figure 1(a) at any t. Examining Figure 1(b), it is seen that $f_e(t)$ decreases until $t=t_w$, rises to a maximum value, and then decreases towards zero.

The failure rate $r_e(t)$ for the test lot is defined to be

$$r_e(t) = \frac{dN_R}{dt}\Big/N_R \tag{2}$$

This quantity is shown in Figure 1(c). Evidently, $r_e(t)$ decreases to a constant value, remains constant until $t = t_w$, and then becomes progressively larger. As discussed in the following section, $r_e(t)$ is proportional to the conditional probability that the system will fail in the next increment of time dt, i. e. at time t + dt, given that it has survived until time t. Interpreting the second region of Figure 1(c) with this in mind, the conditional probability that a system which has survived until t_d will fail at t_d + dt is the same as the conditional probability that a system which has survived until t_w - dt will fail at t_w. Thus, the term chance is chosen to describe the failures in this region. In the debugging region, the conditional probability that a system will fail immediately after it is turned on (at dt) is greater than the conditional probability that a system which survives until a slightly later time t will fail immediately after this time, i. e. t + dt. However, in the wear-out region, the conditional probability that a system which has survived until some time will fail in the next increment of time becomes greater as the operating time t increases. These characteristics will be extremely useful in later work.

In this hypothetical experiment, the most commonly observed failure phenomena of systems have been presented. Now simple, but adequate, mathematical models must be developed which will describe this phenomena. To facilitate the development of these models, some basic concepts from probability theory need to be introduced.

Basic Concepts of Probability Theory

The time t at which a system fails is a numerical-valued random phenomenon. Since system operation begins at zero and may continue theoretically until infinity, t may extend from zero to infinity. It is convenient to define a probability density function f(t) such that the probability of the system failing when $t_1 \leq t \leq t_2$ is given by

$$P(t_1 \leq t \leq t_2) = \int_{t_1}^{t_2} f(t)\,dt \tag{3}$$

where f(t) must satisfy

$$\begin{aligned} f(t) &\geq 0 \quad \text{for} \quad t \geq 0 \\ &= 0 \quad \text{for} \quad t < 0 \end{aligned} \tag{4}$$

and (7, p. 166-174)

$$\int_0^\infty f(t)\,dt = 1 \tag{5}$$

Eq. 5 insures that the system will fail sometime after it is turned on and before a time equal to infinity is reached. The probability density function f(t) is analogous to the failure density function $f_e(t)$ of Figure 1(b). Since the probability density function describes a certain failure distribution, it might also be called a mortality function. To simplify the evaluation of Eq. 3, it is desirable to form the distribution function F(t) from the probability density function f(t). F(t) is defined as

$$F(t_1) = \int_0^{t_1} f(t)\,dt \tag{6}$$

Thus, we may write

$$f(t) = dF(t)/dt = F'(t) \tag{7}$$

Hence, the probability function may be reconstructed from the distribution function. The distribution function $F(t_1)$ of Eq. 6 gives the probability that the system will fail between zero and t_1. Therefore, Eq. 3 may be written as

$$P(t_1 \leq t \leq t_2) = F(t_2) - F(t_1) \tag{8}$$

To further aid in forming the mathematical models which describe the failures that occurred in the experiment, the notion of the force of mortality r(t) shall be used. The force of mortality r(t) is analogous to the failure rate $r_e(t)$ and is given by

$$r(t) = \frac{f(t)}{1-F(t)} = f(t) \Big/ \int_t^\infty f(x)\,dx \tag{9}$$

This quantity r(t) may be interpreted as a measure of the conditional probability that a system will fail in the next increment of time dt (at t + dt) given that it has survived until time t (8, p. 15). If failures occur independently in a system composed of n components, the force of mortality of the system r(t) is

$$r(t) = \sum_{i=1}^{n} r_i(t) \tag{10}$$

where $r_i(t)$ is the force of mortality of component i (9, p. 73). The systems tested in the experiment were assumed to be identical, so by Eq. 10

$$r_s(t) = nr(t) \tag{11}$$

where r(t) is the force of mortality of the system tested and $r_s(t)$ is the total force of mortality for the n systems. Since $r_s(t)$ is analogous to $r_e(t)$, $r_s(t)$ and r(t) differ from Figure 1(c) by only a constant multiple value. Let the force mortality of a system be written as

$$r(t) = r_1(t) + r_2(t) + r_3(t) \tag{12}$$

where $r_1(t)$, $r_2(t)$, and $r_3(t)$ have the following characteristics. Assign $r_2(t)$ to be equal to a constant value. To obtain a figure similar to Figure 1(c), $r_1(t)$ may assume some initial value $r(0)$ but it must approach zero as t approaches t_d. Conversely, $r_3(t)$ must be equal to zero until t becomes larger than t_w. Then $r_3(t)$ must become progressively larger.

Now we are in a position to determine the mathematical models which describe the failures that occurred. These models shall correspond to the various probability density functions which when substituted into Eq. 9 will yield force of mortality expressions which possess the characteristics just described, i. e. an $r(t)$ that decreases from a finite value to zero as t increases, an $r(t)$ that remains constant for all t, and an $r(t)$ that increases progressively from zero as t increases.

Infant Mortality

Failures which occur in the debugging region are said to be primarily of infant mortality type. This region is characterized by a decreasing force of mortality. Many infant mortality distributions may be found which have finite values of $r(0)$ which decrease continuously towards zero as time increases. Consider the function

$$\begin{aligned} f(t) &= c(1+at)^{-n} \quad \text{for } t \geq 0 \\ &= 0 \qquad\qquad\quad \text{for } t < 0 \end{aligned} \tag{13}$$

For this function to be a probability density function, Eqs. 4 and 5 must be satisfied. Evaluating Eqs. 4 and 5, it is evident that n must be greater than one,

$$c = a(n-1) \tag{14}$$

and a and c must be positive constants. Thus, we have taken an arbitrary function and found the conditions which when satisfied will yield a mortality function. To determine if this is an infant mortality function, the force of mortality of this f(t) is determined from Eq. 9 as

$$\begin{aligned} r(t) &= a(n-1)/(1+at) && \text{for } t \geq 0 \\ &= 0 && \text{for } t < 0 \end{aligned} \tag{15}$$

Since a is a positive constant and n is greater than one, r(t) has an initial value of a(n-1) which decreases towards zero as time t increases. Thus, this mortality function is of infant mortality type and might be considered as a possible mathematical model to describe part of the failure distribution in the debugging region. This function does not describe the entire failure distribution since failures which occur in this region are also partially due to chance. The chance mortality function will be discussed in the following section. However, since the failures due to chance occur throughout all time (since $r_2(t)$ is constant for all time), the distinguishing factor of region one is that infant mortality type failures also occur.

Returning to infant mortality functions, a less obvious possibility is the Gamma density function,

$$f(t) = \frac{1}{a!b} \left(\frac{t}{b}\right)^{-a} \exp\left(-\frac{t}{b}\right) \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0 \qquad (16)$$

When b is greater than zero and a lies between minus one and zero, this function has a force of mortality which decreases towards 1/b (9, p. 79-82). Thus, this function is a possible model to describe the entire failure distribution of region one. Now let us consider the failures which occur in the second region.

Chance Mortality

Failures which occur in the second or chance region are said to be of chance mortality type. The characteristic of the chance mortality function is that it possesses a constant force of mortality. This was interpreted to mean that failures occur only because of chance in this second region.

Let us examine the exponential function

$$f(t) = \frac{1}{b} \exp(-t/b) \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0 \qquad (17)$$

which is equivalent to the Gamma function for $a = 0$. Making use of Eq. 6, the distribution function is

$$F(t) = 1 - \exp(-t/b) \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0 \qquad (18)$$

When b, the mean of the exponential function, is greater than zero, Eq. 4 is satisfied by Eq. 17. Since Eq. 18 satisfies the basic requirement of a mortality function given by Eq. 5, the exponential function may be used as a mortality function (7, p. 180). The force of mortality of the exponential function is

$$\begin{aligned} r(t) &= 1/b \quad \text{for } t \geq 0 \\ &= 0 \quad \text{for } t < 0 \end{aligned} \tag{19}$$

and thus, this mortality function may be considered to be of chance mortality type. The exponential function is the most commonly used distribution in reliability studies. It shall be discussed more fully in the following chapter. Let us now consider the model for failures which occur in the third region.

Wear-Out Mortality

The third region of Figure 1(c) is referred to as the wear-out region of the system. Here, the remaining systems in the test lot were found to fail comparatively quickly. It was also found that the force of mortality increases indefinitely from zero. Consider the normal density function given by

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}x^2\right) \quad \text{for } -\infty \leq x \leq \infty \tag{20}$$

where $x = (t-m)/\sigma$ is the standardized random variable, t is the random variable, and m and σ are the mean and standard deviation, respectively, of the normal density function. Since f(x) is always greater than or equal to zero, Eq. 4 is satisfied. The distribution function F(x), represented by $\Phi(x)$, is found by Eq. 6 to be

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{t-m}{\sigma}} \exp\left(-\tfrac{1}{2}x^2\right) dx = \Phi\left(\frac{t-m}{\sigma}\right) \tag{21}$$

Since Eq. 5 may be verified using Eq. 21, the normal function is a mortality function (7, p. 188-190). The force of mortality r(t) increases as t increases and asymptotically approaches the limit (9, p. 71).

$$\lim_{t \to \infty} r(t) = \frac{t-m}{\sigma^2} \tag{22}$$

Therefore, this function is of wear-out mortality type. Although other functions may be used to describe wear-out, e. g. the Gamma, Logarithmic normal, and Asymptotic functions, the normal distribution is the simplest to apply (1, p. 24-26 and 8, p. 13-18).

Failure Categories in Retrospect

These three failure categories just discussed represent the three basic failure modes of a system. The functions derived to represent these failures are only a few of many which could be found. The density functions we have given were derived on the basis of the force of mortality curve of Figure 1(c) which was obtained from a hypothetical experiment. In many cases, cost and time limitations do not allow actual experimental failure data to be compiled. In these cases, judgments must be made concerning which mathematical model should be used to describe the failures of a system. Hopefully, infant mortality type failures are lessened by the use of quality workmanship and materials. They should be found and corrected through use of operational testing before a system leaves the manufacturing plant. Assuming chance failures for much of the system's life seems to fit reality quite well. This shall be verified in Chapter V. This segment of a system's life may be extended using preventative maintenance techniques, i. e. replace certain elements before they begin to appreciably wear out.

If adequate experimental failure data could be accumulated, very involved statistical techniques could probably be used to obtain accurate mathematical models to represent the failures. The advantage of using this approach is, of course, that precise models may be found to describe the failures; and therefore, the reliability of the system may be much more accurately determined. Arbitrarily using the mortality functions previously developed will undoubtedly fail to describe the failure data exactly. Thus, the resulting reliability estimates may be somewhat in error, but in many cases, these estimates prove to be valuable aids in system design.

The infant, chance, and wear-out mortality discussions have been presented to show the general categories of system failures. To simplify the later work of evaluating system reliability, it will be assumed that all failures occur because of chance. As mentioned before, this seems to fit reality quite well as Chapter V will show. We shall now consider the exponential function (which characterizes chance mortality) in greater detail and show how it may be used to evaluate the reliability of electronic systems.

III. THE EXPONENTIAL DENSITY FUNCTION

System Failure Models

The exponential density function is given by Eq. 17 and its distribution function by Eq. 18. Suppose that this function represents the failure model for the components which make up the system. We may easily calculate the probability of failure for the system composed of n of these components provided we make several assumptions (12, p. 212). Firstly, each of the components must have a constant failure rate over the operating period of the system, i. e. component failures must occur randomly in time and therefore the exponential function describes this situation. Secondly, all system failures are due to random component failures (these are the only failures described by the exponential function). Thirdly, the components are assumed to fail independently of each other. Lastly, the failure of only one component results in failure of the system.

The probability of component i failing when $0 \leq t \leq T$ is given by Eqs. 8 and 18 as

$$P_i(0 \leq t \leq T) = F_i(T) - F_i(0) = 1 - \exp(-T/b_i) \qquad (23)$$

Thus, the probability that the component will survive at least until time T is by Eq. 5,

$$P_i(T \leq t \leq \infty) = 1 - P_i(0 \leq t \leq T) = \exp(-T/b_i) \qquad (24)$$

Therefore, the probability $P_s(\cdot)$ that the system will survive until at least time T is the product of the probabilities $P_i(\cdot)$ that each of the n independent components will survive until at least time T (9, p. 45), i. e.

$$\begin{aligned} P_s(t \geq T) &= P_1(t \geq T)\, P_2(t \geq T) \ldots P_n(t \geq T) \\ &= \exp(-T/b_1)\exp(-T/b_2) \ldots \exp(-T/b_n) \\ &= \exp(-T\,[\,(1/b_1)+(1/b_2)+\ldots+(1/b_n)\,]\,) \\ &= \exp(-T/b) \qquad (25) \end{aligned}$$

b, the mean of the exponential distribution for the system, is often referred to as the mean-time-between-failures or MTBF. Thus, the reciprocal of the MTBF of the system is equal to the sum of the reciprocals of the MTBF for each of the n components which make up the system, or

$$1/b = \sum_{i=1}^{n} (1/b_i) \qquad (26)$$

Define the reciprocal MTBF as the failure rate B (failures/some arbitrary time). Eq. 26 may then be written as

$$B = \sum_{i=1}^{n} B_i \qquad (27)$$

Before proceeding further, let us examine the various conditions which may effect this component failure rate B_i.

Factors to Consider When Making Reliability Studies

Examining Figure 2, it is evident that many factors must be considered when making reliability studies (11, p. 7). To begin this discussion, the operational, inherent, and use reliabilities of Figure 2 are defined as follows:

1. Inherent reliability: The probability that a system will perform properly when operated under the conditions for which it was designed.
2. Use reliability: The probability that each of the conditions shown will be performed or will affect the performance in the system as intended by the designer. These are the application factors which degrade the reliability of the system.
3. System operational reliability: The probability that the system will give the specified performance for a given period of time when used in the manner intended in any particular field condition. This probability is equal to the product of the inherent and the use reliabilities.

It is rather startling that so many seemingly unimportant factors should be considered in reliability studies. Some examples may help to emphasize the necessity of these considerations.

In the early 1950's, a group of engineers at Bell Telephone Laboratory examined 1135 components that had failed in various

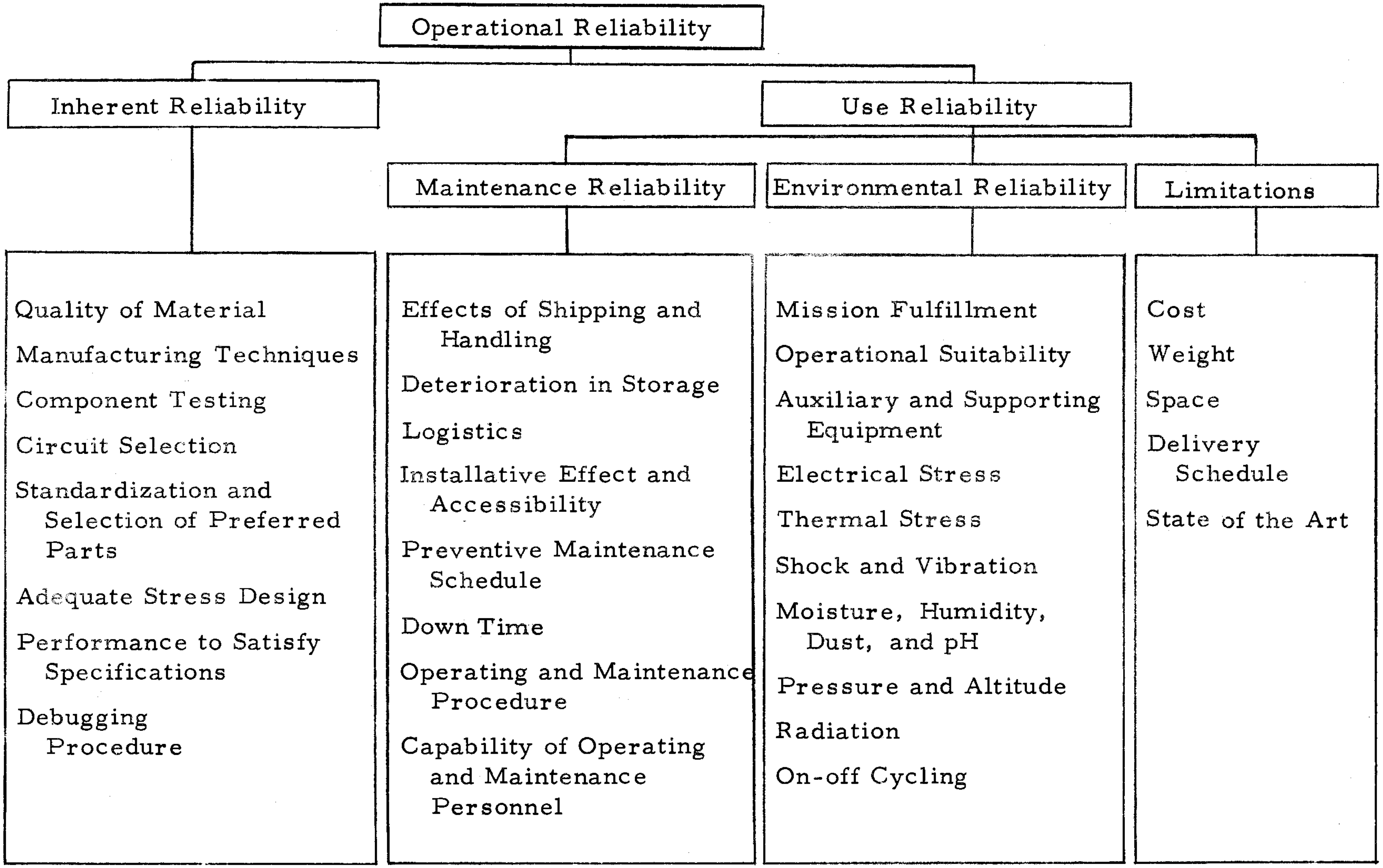


Figure 2. Factors to consider when making reliability studies.

electronic equipment and attempted to analyze the origin of the failure causes. They found that approximately 60% of the failures were due to faulty manufacturing, faulty components, and faulty engineering design (the failures associated with inherent reliability) while 40% were due to incorrect usage (the failures associated with use reliability) (3, p. 3).

E. Pieruschka has investigated the reliability growth of the German V-1 and V-2 missiles of World War II (9, p. 190). The initial operational reliability of the mass-produced missiles was zero at the beginning of 1944. By continual improvement in manufacturing techniques, this value was improved to 60% by August of the same year. An operational reliability of almost 85% was reached in the German experimental test missiles. Thus, these missiles had an inherent reliability that was greater than 85%.

At the present time, the high reliability requirements for the Minuteman ICBM have led to the initiation of a large scale reliability improvement program. Judging from the proposed failure rates of the components used in construction of the electronic equipment for the missile, this equipment should have an inherent reliability of almost 100% (12, p. 169-170). If similar standards could be achieved throughout production, it is possible this missile would have an inherent reliability of 95% or more.

These examples demonstrate the necessity of considering the factors of Figure 2 when making realistic reliability studies. Since most of these factors have not been quantatively described, different weighting coefficients have been developed in an attempt to account for them (5, p. 10). To simplify future considerations, it shall be assumed that the inherent and maintenance reliabilities are equal to one and the limitations are negligible. We shall now consider only the effects of electrical stress, thermal stress, and on-off cycling in determining the failure rate B of an electronic system.

Determination of System Failure Rate

Returning to Eq. 25, the operational reliability of the system may be found if the failure rate for each of the components which comprise the system may be determined. Assume that the component failure rate B_i can be represented as

$$B_i = B_{oi} + B_{ci}N_i \tag{28}$$

where B_{oi} represents a basic failure rate in failures/xy hours which is varied to conform to electrical and thermal stress information, B_{ci} the failures/x power on-off cycles, and N_i the number of power on-off cycles/y hours. Substituting Eq. 28 into Eq. 27, the system failure rate B becomes

$$\begin{aligned} B &= \sum_{i=1}^{n} (B_{oi} + B_{ci}N_i) = \sum_{i=1}^{n} B_{oi} + \sum_{i=1}^{n} B_{ci}N_i \\ &= B_o + B_c N \end{aligned} \tag{29}$$

since a power cycle applied to the system results in a power cycle applied to each component, i. e. $N_i = N$. For convenience, we will consider B_{oi} to be in failures/10^6 hours, B_{ci} in failures/10^3 cycles, and N_i in cycles/10^3 hours. Suppose an electronic device is composed of k types of components where there are n_i components of each type (assume that these n_i components of each type have the same MTBF to facilitate analysis). Then Eq. 28 becomes

$$B = \sum_{i=1}^{k} n_i(B_{oi} + B_{ci}N_i) = \sum_{i=1}^{k} n_iB_{oi} + \sum_{i=1}^{k} n_iB_{ci}N_i$$

$$= B_o + B_cN \tag{30}$$

Therefore

$$B_o = \sum_{i=1}^{k} n_iB_{oi} \tag{31}$$

$$B_c = \sum_{i=1}^{k} n_iB_{ci} \tag{32}$$

Examination of Eq. 30 reveals that the system failure rate B is composed of a base rate B_o and a term proportional to the power cycles applied to the system. Evidently, power cycling adds an increment of value to the nominal system rate B_o. Chapter VI shall be concerned with the cycling failure rate. The following chapter shall be devoted to methods of calculating base failure rates of electronic components.

IV. EVALUATION OF COMPONENT FAILURE RATES

Introduction and Example

It has been shown that if the basic failure and cycling failure rates of the components making up a system may be found, the reliability of the system may be calculated using Eq. 30. This chapter is concerned only with presenting empirical values of the base failure rates of various components and showing how these may be adjusted to conform to certain electrical and thermal stress information. These values will be used in Chapter V to calculate the base failure rates of various military electronic systems.

Table 1 represents the empirical failure rates for various types of electrical components (5, p. 1-92 and 12, p. 5-190). These representative values have been determined through extensive experimentation by many different electronic research companies. In Table 1, column one represents the low, column two the average, and column three the high failure rates. These failure rate estimates may be used in Eq. 31 to obtain an average reliability and an estimate of the range of system reliability. To illustrate this fact, Table 1 and Eqs. 24 and 31 shall be used to calculate the failure rate of a typical AM radio whose components are listed in Table 2. Evidently, this particular AM radio contains 15 different types of electronic components. To apply Eq. 31, the number n_i of components of each

Table 1. Component Failure Rates (Failures/10^6 hours).

Component	Low	Average	High
Capacitors:			
General, fixed	0.30	0.50	0.90
General, variable	0.10	0.10	0.10
Fixed:			
Ceramic	0.01	0.04	0.20
Electrolytic	0.20	0.70	25.0
Glass	0.01	0.03	0.20
Mica	0.35	0.50	0.90
Paper	0.55	0.80	1.5
Tantalum	0.35	0.60	1.5
Connectors:			
1-2 pins		0.03	
3-6 pins		0.06	
7-9 pins		0.08	
10-17 pins		0.11	
17-20 pins		0.13	
Crystals, quartz	0.10	0.20	1.0
Diodes:			
Germanium	1.0	7.0	200.0
Silicon	1.0	3.5	50.0
Fuses		0.10	
Inductors	0.40	0.40	2.0
Lamps:			
Incandescent		1.0	
Neon		0.20	
Meters	0.30	0.50	0.70
Motors:			
Brush	20.5	20.5	20.5
Brushless	8.5	8.5	8.5
Relays:			
Magnetic	0.20	0.50	1.0
Thermal	0.30	1.0	2.0
Resistors:			
General, fixed	0.20	0.40	1.2
General, variable	0.60	0.80	2.3
Fixed:			
Composition	0.01	0.06	1.0
Film	0.40	0.65	1.0
Power film	1.7	2.0	2.5
Wirewound	1.0	1.4	2.5
Power wirewound	1.6	2.1	3.0

Table 1. Continued.

Variable:			
Composition	0.10	0.2	1.1
Power wirewound	1.7	2.2	3.1
Wirewound	1.1	1.5	2.6
Sockets:			
Transistor		0.06	
Tube		0.08	
Switches	0.05	0.10	0.15
Transformers	0.40	0.40	2.0
Transistors:			
Germanium	2.0	8.0	180.0
Silicon	2.0	4.0	40.0

Tubes:	Single Type	Double Type
Receiving:		
Subminiature:		
Diodes	2.0	3.0
Triodes	2.5	4.0
Pentodes	3.5	5.0
General	2.5	4.0
Miniature:		
Diodes	3.5	5.0
Triodes	5.0	8.0
Pentodes	8.0	13.0
General	6.0	9.0
Octal:		
Diodes	7.0	10.0
Triodes	9.0	15.0
Pentodes	15.0	25.0
General	10.0	15.0
Power:		
Diodes	10.0	15.0
Triodes	15.0	25.0
Pentodes	25.0	40.0
General	20.0	30.0
Transmitting:		
Power:		
Triode	50	
Pentode	100	
Beam pentode	100	
Microwave devices:		
Klystrons	50	
Magnetrons	100	
Microwave switching tubes	100	
Miscellaneous:		
Cathode ray	15 + 15x(number of guns)	
Voltage regulator	5	

Table 2. AM Radio Base Failure Rate B_o Calculation Sheet.

Component	Number	Low	Average	High
Capacitors:				
Fixed:				
Ceramic	3	0.03	0.12	0.60
Electrolytic	1	0.20	0.70	25.00
Mica	1	0.35	0.50	0.90
Paper	6	3.30	4.80	9.00
Variable	6	0.60	0.60	0.60
Connectors	2	0.06	0.06	0.60
Inductors	2	0.80	0.80	4.00
Lamps:				
Incandescent	1	1.00	1.00	1.00
Resistors:				
Fixed:				
Composition	10	0.10	0.60	10.00
Variable	1	0.60	0.80	2.30
Sockets:				
Tube	5	0.40	0.40	0.40
Switches	1	0.05	0.10	0.15
Transformers	4	1.60	1.60	8.00
Tubes:				
Miniature	4	24.00	24.00	24.00
Octal	1	10.00	10.00	10.00
B_o in failures/10^6 hours		43.09	46.08	96.55
B_o in failures/10^3 hours		0.043	0.046	0.096
If $P(t \geq T) = 0.95$, then T in hours is equal to		1200	1100	530
If $P(t \geq T) = 0.99$, then T in hours is equal to		235	215	105

type is multiplied by the appropriate failure rate of that component type found in Table 1. Since there are three failure rate estimates for each component type there are three columns of total failure rate values in Table 2. These values are used in Eq. 24 to compute the reliability of this AM radio. If the probability that the radio will operate continuously without failing until at least time T is equal to 95%, then by Eq. 24

$$P(t \geq T) = \exp(-TB_o) = 0.95 \tag{33}$$

Solving Eq. 33 for T,

$$T = 0.051/B_o \tag{34}$$

where B is in failures / 10^3 hours and T is in thousands of hours. If the probability is equal to 99%,

$$T = 0.01/B_o \tag{35}$$

The values of T listed in Table 2 were solved by using Eqs. 34 and 35 with the appropriate base failure rate B_o. The average values of T, i. e. 215 hours and 1100 hours, may be interpreted as signifying the approximate time T at which not more than a certain percentage, i. e. 1% and 5%, of identical radios in a large test lot have failed while being operated continuously. It is especially evident in the 95% case that there is a large discrepancy between the high and low failure rate values and thus the T values. Although these failure rates are useful for investigating the range of the reliability values, improved accuracy could be obtained by using more exact component failure rate values.

More exact failure rates may be found by taking into account the thermal and electrical stresses imposed upon each component. The remainder of this chapter explains the adjustment procedures that may be used for many types of electrical components. Each of the following sections is devoted to a particular component type. Each section begins with a discussion of the principal causes of component failures. Then the adjustment procedure is given as a series of steps. Each section is concluded by examining the characteristics of expressions which were derived. Once the adjusted values of the base failure rates are found for all the components in the system, these rates may be used in Eq. 31 to calculate the reliability of the system. This procedure is the same as was used to calculate the reliability of the AM radio. However, usually each component of a certain type may not have the same stresses imposed upon it. In this case, Eq. 29 would be more appropriate to use. We shall begin these adjustment procedures by considering capacitors.

Capacitors

Capacitors may fail because of current and voltage overloads, adverse frequency effects, high temperatures, pressure, humidity, shock, and vibration (12, p. 97-129). Current transients can produce dielectric failures and cause permanent changes in the capacitance value. Overrated voltage transients may produce internal corona, dielectric punctures, and reduced insulation resistance. Exceeding frequency ratings may result in poor operation, overheating, and dielectric punctures. Overheating decreases reliability, introduces capacitance drift, lowers the dielectric strength and insulation resistance, and shortens life. Pressure, shock, and vibrations may break hermetic seals, cause corrosion and fungus growths, reduce dielectric constants, and create higher leakage currents.

It has been mentioned before that to simplify the adjusted failure rate determinations, only effects of temperature and electrical stresses will be considered. The adjustment procedure is carried out as follows:

Step 1. Classify capacitor as to type, i. e. ceramic, electrolytic, glass, mica, paper, tantalum, or air.

Step 2. Determine the effective ambient temperature T in degrees Centigrade and the effective temperature coefficient K where

$$K=(T-40)/10 \tag{36}$$

Step 3. Determine the electrical stress ratio R_v where

$$R_v=V_p/V_R \tag{37}$$

V_p is the peak operating voltage, i. e. the sum of both the d. c. and a. c. applied voltages independent of the duty cycle, and V_R is the rated voltage.

Step 4. Choose the appropriate capacitor type from Table 1 and adjust its base failure rate using the appropriate equation (Eqs. 38 through 43) which follows.

Ceramic	$B=B_oR_v^{2.9}(1.4)^K$	(38)
Electrolytic	$B=B_oR_v^{5.0}(3.0)^K$	(39)
Glass	$B=B_oR_v^{3.8}(1.4)^K$	(40)
Mica	$B=B_oR_v^{3.6}(1.1)^K$	(41)
Paper	$B=B_oR_v^{2.6}(1.1)^K$	(42)
Tantalum	$B=B_o(1.2^K)\exp(-1.5(1-R_v))$	(43)

The base failure rate B_o is the failure rate in the second column of Table 1. The minimum failure rates are given in column one of Table 1. Appendix I gives an example to show how the effective failure rate expressions in this chapter have been derived from empirical curves. The reader who is interested in more fully appreciating these expressions should carefully review Appendix I.

In Eqs. 38 through 43 and most of the other effective failure rate equations in this chapter, B_o is corrected by a thermal and electrical stress term. It is enlightening to note the general trends of these equations.

If B_o is independent of electrical stress, the exponent of R_v must equal zero. If this exponent is greater than zero, the failure rate increases as the stress ratio R_v increases. If it is less than zero, the failure rate decreases as ratio R_v increases; but this case does not occur in real components. Thus we may conclude that those components with larger exponent values are more susceptible to electrical stresses than those with smaller values, e.g. electrolytic types compared with paper types. Hence, the order of their decreasing susceptibility (neglecting tantalum capacitors) to increases in electrical stresses is electrolytic, glass, mica, ceramic, and paper types.

If B_o is independent of thermal stress, then the base of the exponent K is equal to one. If the value of this base is greater than one, the failure rate increases as coefficient K increases. If the base is less than one, the failure rate decreases as T increases. Again this latter case does not occur in real components. Thus, components with larger values for this base are more susceptible than those with smaller values to thermal stresses, e.g. electrolytic types compared with paper or mica types. Therefore, the order of their decreasing susceptibility to changes in thermal stress is electrolytic, ceramic and glass, tantalum, and mica and paper types.

Diodes

Failure rates for semiconductor diodes seem to depend primarily on their junction temperatures (12, p. 41-68). These Centigrade temperatures may be expressed for lead mounted devices as

$$T_J = T + \theta_{JA} P_J \tag{44}$$

and for stud mounted devices as

$$T_J = T_C + \theta_{JC} P_J \tag{45}$$

Here T_J is the junction temperature, T the ambient temperature, T_C the case temperature, θ_{JA} the thermal resistance from junction to air, θ_{JC} the thermal resistance from junction to case, and P_J the average power dissipated at the junction. The value of thermal resistance is often given by the manufacturer. In the ideal heat sink, θ is equal to zero and the junction temperatures assume their limiting values. The failure of the semiconductor diode is assumed to be a function of its ambient temperature and its power dissipation. Their failure rate may be determined using the following steps:

Step 1. Determine the junction temperature T_J. If an adequate heat sink is being used, T_J is approximately equal to the ambient or case temperature by Eqs. 44 and 45.

Step 2. Determine the temperature coefficient K_d where for germanium diodes

$$\begin{aligned} K_d &= 0 && \text{for } T_J \leq 25^\circ C \\ &= (T_J - 25)/75 && \text{for } T_J > 25^\circ C \end{aligned} \tag{46}$$

and for silicon diodes

$$\begin{aligned} K_d &= 0 && \text{for } T_J \leq 25^\circ C \\ &= (T_J - 25)/125 && \text{for } T_J > 25^\circ C \end{aligned} \tag{47}$$

Step 3. Determine the peak rated power P_R of the diode from its specifications.

Step 4. Determine the average power P dissipated within the device as a function of its anticipated duty cycle.

Step 5. Calculate the stress ratio R_p where

$$R_p = P/P_R \tag{48}$$

P is the average power being dissipated by the diode and P_R is its rated power dissipation.

Step 6. Use the results of Steps 2 and 5 to solve for the effective failure rate using Eqs. 49 and 51.

The effective failure rate in failures/10^6 hours is given by

$$B = 1.0(500^{K_d})\exp(-7.7(1-R_p)) \tag{49}$$

When

$$K_d \leq 0.6(1-R_p) \tag{50}$$

the minimum failure rate in failures /10^6 hours is given by

$$B = 1.0 \exp(-2.5(1-R_p)) \tag{51}$$

It is apparent from Eq. 49 that the effective failure rate of germanium diodes is higher than the effective failure rate of silicon diodes at any arbitrary temperature when Eq. 50 is unsatisfied (they are equal when Eq. 50 is satisfied). It is also apparent that although both diode types are equally sensitive to power stresses, the effective failure rate increase for unit temperature increase is greater for germanium diodes than for silicon diodes.

Inductors and Transformers

The failure rates for inductors and transformers depend primarily upon the grades of insulation used in their construction, their ambient temperatures, and the electrical stresses imposed upon them (12, p. 131-141). Excessive voltage may lead to punctures and premature breakdown of the insulation. Corona may occur at points of high potential stress and may cause accelerated aging and weak spots in the insulation. Overrated currents cause overheating of the transformer and excessive core and coil expansions which may result in open or short circuits. Fluctuation of input frequency can be harmful. Below rated frequency, low reactance occurs and high currents may flow. Above rated frequency, excessive core losses may occur. In either case, the subsequent temperature rises may harm the insulation. The effective failure rates of these magnetic components may be found as follows:

Step 1. Determine the class of insulating materials:

(a) Inductors, Transformers, and Magnetic Amplifiers. Find their class by use of Table 3.

(b) R. F. and I. F. Transformers and Inductors. Find their class by use of Table 3, their grade by use of Table 4, and their total number of terminals n.

Step 2. Determine the effective temperature T_h of the magnetic component where

$$T_h = T + \Delta T \tag{52}$$

T is the ambient temperature surrounding the magnetic component while ΔT is the temperature rise within the magnetic component. Eq. 52 may be solved by use of the following equations (Eqs. 53 through 55):

(a) Transformers and Magnetic Amplifiers.

$$\Delta T = \Delta T_R(0.15 + 0.85(P_a/P_r)) \tag{53}$$

where ΔT is the temperature rise, ΔT_R the temperature rise for rated load operation, P_a the apparent power of the secondary, and P_r the rated apparent power of the secondary.

(b) Inductors.

$$\Delta T = \Delta T_R(I^2/I_R^2) \tag{54}$$

where I is the actual RMS current and I_R is the rated RMS current.

(c) R. F. and I. F. Transformers and Inductors. Let

$$T_h = T + 5^{\circ}C \tag{55}$$

Step 3. Determine the effective temperature coefficient K_h where

$$K_h = T_h/10 \tag{56}$$

Step 4. Use the results of Steps 1 and 3 and the appropriate equation that follows (Eqs. 57 through 61) to determine the basic failure rate of the magnetic component.

O and Q Classes $B = B_o(3.4)^{K_h - 4}$ (57)

A and R Classes $B = B_o(1.9)^{K_h - 4}$ (58)

B and S Classes $B = B_o(1.8)^{K_h - 6.5}$ (59)

H and T Classes $B = B_o(1.2)^{K_h - 8}$ (60)

C and U Classes $B = B_o(1.1)^{K_h - 11}$ (61)

If an R.F. or I.F. magnetic component has insulation of Class C, use Eq. 60. Again, B_o is equal to the failure rate in column two of Table 1. The minimum failure rate of a magnetic component is equal to B_o for values of K_h that make the exponent negative in Eqs. 57 through 61.

Step 5. Multiply the basic failure rate found in Step 4 by the appropriate degradation factor from Table 5 to determine the most realistic value of B.

There are several interesting characteristics exhibited by Eqs. 57 through 61. Firstly, the effective temperature includes the effects of power and current stresses. Thus, there is no electrical stress term included in these equations. Secondly, the equations are arranged with regard to their increasing values of maximum temperature ratings. Thirdly, the equations are arranged with regard to their decreasing values of failure rates at any arbitrary temperature. Lastly, they are likewise ordered with regard to their decreasing susceptibility to increases in temperature. It is rather striking to

Table 3. Insulating Material Classes Designated by AIEE Standard No. 1 and MIL-T-27A.

AIEE Designation With Maximum Temperature Rating	MIL-T-27A Designation With Maximum Temperature Rating	Description of Insulating Material
O, 90°C	Q, 85°C	Consists of cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in a liquid dielectric.
A, 105°C	R, 105°C	1. Cotton, silk, paper, and similar organic materials when either impregnated or immersed in a liquid dielectric. 2. Molded or laminated materials with cellulose filler, phenolic resins, and other resins of similar properties. 3. Films and sheets of cellulose acetate and other cellulose derivatives of similar properties. 4. Varnishes and enamels as applied to conductors.
B, 130°C	S, 130°C	Consists of mica, asbestos, fiberglass, and similar inorganic materials in buildup form with organic binding substances. A small proportion of Class A materials may be used for structural purposes only.

Table 3. Continued.

H, 180°C	T, 170°C	1. Mica, asbestos, fiberglass, and similar inorganic materials in buildup form with binding substances composed of silicone compounds or materials of similar properties. 2. Silicone compounds in rubbery or resinous forms, or materials with equivalent properties. A minute proportion of Class A materials must be used only when essential for structural purposes during manufacture.
C, no limit specified	U, > 170°C	Consists entirely of mica, porcelain, glass, quartz, and similar inorganic compounds.

Table 4. I. F. and R. F. Transformer and Inductor Grades.

Grade	Intended Use
1	Where maximum reliability, life, or operation under all climatic conditions is required. Resistant to immersion and moisture.
2	Flame resistant in addition to Grade 1.
3	Where little or no protection from climatic conditions is required. This assumes sealed assemblies.
4	Extreme resistance to shock and vibration in addition to Grade 1.
5	Where resistance to flame is required in addition to Grades 1 and 4.
6	Where little or no protection from climatic conditions is required but extreme resistance to shock and vibration is required.

Table 5. Degradation Factors for Magnetic Components.

Magnetic Component	Application	Multiplication Factors*	
		A	B
Transformers and Inductors	Plate and power transformers, filter chokes, etc.	0.5	1.0
	Filament, audio, interstage coupling, and low level pulse transformers.	0.3	0.6
	High level pulse transformers	0.8	1.6
Magnetic Amplifiers	$V_{out} \leq 50$ volts	0.3	0.6
	$50 \text{ volts} < V_{out} \leq 100$ volts	0.4	0.8
	$V_{out} > 100$ volts	0.5	1.0
R. F. and I. F. Inductors and Transformers	Grade 1 or 3	0.05n	
	Grade 2	0.1n	

*A: Hermetically sealed construction
B: Encapsulated construction

note that some of the failure rates remain constant until a relatively high temperature is reached, e. g. O, Q, A, and R Class failure rates remain constant until T = 40°C while C and U classes remain constant until T = 110°C.

Motors and Other Rotary Devices

The two major causes of failure in motors are of electrical and mechanical origin (12, p. 143-147). We may analyze these two failure types separately. The wire coil on the magnetic core is considered to compose an equivalent transformer. The information of the previous section may be used to calculate the failure rate for this electrical portion of the device. Mechanical failures may be attributed to bearings and brushes that fail due to frictional effects. Since friction is approximately proportional to speed, rotational speed shall be considered as the mechanical parameter. The failure rate may be determined as follows:

Step 1. Calculate the temperature rise ΔT using Eq. 54 and the effective temperature T_h using Eq. 52. Determine the class of insulation from Table 3.

Step 2. Use the appropriate failure rate equation (Eqs. 57 through 61) to calculate B for the electrical portion of the device.

Step 3. Adjust this failure rate to compensate for inherent differences between inductors and rotary devices by multiplying B by the appropriate factor in Table 6.

Step 4. Calculate the basic mechanical failure rate for brush devices by

$$B = 0.90S^2 \qquad (62)$$

and for brushless devices by

$$B = 0.40S^2 \qquad (63)$$

where S is in thousands of r.p.m. and B is in failures/10^6 hours.

Step 5. Add the failure rates found in Steps 3 and 4 to establish the effective failure rate for the rotary device.

Examining Eqs. 62 and 63, increasing the speed of a rotary device from 1000 r.p.m. to 2000 r.p.m. increases the failure rate by a factor of four. Increasing the speed to 3000 r.p.m. results in a failure rate that is nine times as large as the failure rate of the device at the original speed. Evidently, the speed of a device is extremely critical when estimating its failure rate.

Table 6. Adjustment Factors for Rotary Devices.

Device	Multiplication Factors
Motors, blowers, etc.	1.5
Generators, dynamotors, etc.	1.3
Synchros, resolvers, etc.	0.5

Relays and Switches

Relays and switches are electromechanical devices which are subject to both electrical and mechanical failures (12, p. 157-167). Some of these failures are due to poor contact alignment; open, contaminated, or pitted contacts; loss of resiliency in the springs, and open circuits in the coils. These devices have failure rates which depend substantially upon their cycling rate N_i. Although the failure rates depend substantially on the relay or switch type, realistic figures seem to be obtained by assuming their base failure rate B_{oi} of Eq. 28 to be numerically equal to their cycling failure rate B_{ci} (12, p. 163-165).

Resistors

It is convenient to consider resistors as being of composition, film, and wirewound types (12, p. 69-96). Composition resistors are inexpensive and come in a wide variety of standard values. Unfortunately, their resistance changes with humidity, temperature, and age. Film resistors are more expensive than the composition type, but their resistance is more constant over normal operating temperature ranges. The wirewound resistor is the most expensive but it has the most constant resistance for changes in temperature of the three types. To simplify the analysis, only the effects of thermal and electrical stresses will be considered. The failure rate is found as follows:

Step 1. Classify the resistor with respect to its type and power rating P_R.

Step 2. Determine the effective ambient temperature T for the resistor and calculate the temperature coefficient K using Eq. 36.

Step 3. Determine the electrical stress ratio R_p using Eq. 48.

Step 4. Determine the effective failure rate B using the results of Steps 1 through 3 and the appropriate equation that follows (Eqs. 64 through 68). Both fixed and variable resistors of the same type are described by the equation for that type.

Composition $\quad B = B_o R^{3.4}\, 2.0^K \quad (64)$

Film $\quad B = B_o R^{1.0}\, 1.2^K \quad (65)$

Power Film $\quad B = B_o R^{0.55}\, 1.0^K \quad (66)$

Power Wirewound $\quad B = B_o R^{2.0}\, 1.1^K \quad (67)$

Wirewound $\quad B = B_o R^{0.2}\, 1.1^K \quad (68)$

The minimum failure rate values are given in column one of Table 1.

As a means of comparison, these types of resistors are listed in order of their decreasing susceptibility to increases in electrical stresses, i. e. composition, power wirewound, film, power film, and wirewound types.

Their order of decreasing susceptibility to increases in thermal stresses is composition, film, wirewound and power wirewound, and power film types. This order confirms the introductory remarks of this section made concerning temperature stability. Evidently, composition resistors are the most susceptible to both electrical and thermal stress increases.

Transistors

The failure rates of transistors are considered to depend primarily on their collector junction temperature (12, p. 41-68). This temperature is assumed to be representative of the entire active transistor region. The approach and expressions derived in the diode section may be used (with a few appropriate changes) to calculate transistor failure rates.

To determine these failure rates, let T_J in Eqs. 44 through 47 be equal to the temperature of the collector junction. Follow Steps 1 through 5 developed in the diode section. For Step 6, use the appropriate equations that follow (Eqs. 69 through 71). The effective failure rate in failures/10^6 hours is given by

$$B = 2.0(666^{Kd}) \exp(-4.6(1-R_p)) \qquad (69)$$

When

$$K_d \leq 0.30(1-R_p) \qquad (70)$$

the minimum failure rate in failures/10^6 hours is given by

$$B = 2.0 \exp(-2.5(1-R_p)) \qquad (71)$$

The concluding remarks of the diode section are appropriate if the following exchanges in the discussion are made: Eq. 69 for Eq. 49, Eq. 70 for Eq. 50, Eq. 71 for Eq. 51, and the word transistor for diode.

Tubes

Electron tubes may fail due to burned filaments, overheating, loss of emission, short circuits and open circuits, microphonic effects, drift, insulation failures, and losses of bulb vacuum (6, p. 5). However, the most common failures are influenced by the heater voltage, the bulb temperature, and by the total power being dissipated in the tube (12, p. 13-40). The basic failure rates of electron tubes found in Table 1 reflect failures due to all types of failure causes. Adjustment factors will be applied to these values to determine effective failure rates. These factors will be functions of the significant stresses, i. e. the heater voltage and combined effect of bulb temperature and total power dissipation. The procedure is as follows:

Step 1. Classify the tube type and determine its basic failure type from Table 1.

Step 2. Determine the heater stress ratio R_H where

$$R_H = V_H / V_R \qquad (72)$$

V_H is the operating heater voltage and V_R is the rated voltage of the heater.

Step 3. Determine the actual power dissipation P in the electron tube where

$$P = P_H + P_P + P_G \tag{73}$$

P_H, P_P, and P_G are the actual dissipation of heater, plate, and grid of the tube, respectively.

Step 4. Determine the rated power dissipation P_R from the specifications and calculate the stress ratio R by use of Eq. 48.

Step 5. Determine the maximum bulb temperature T_B of the tube.

Step 6. Use the information from Steps 1, 2, 4, and 5 to determine the effective rate B when

$$B = B_o R^{0.5} R_H^{6.5} (0.90)^{(160-T_B)/10} \tag{74}$$

Examination of Eq. 74 reveals that the heater voltage has a rather spectacular effect on this effective failure rate B. Consider the situation when $R_H = 1$ and R and T_B are equal to constants. Increasing R_H by 10% results in a failure rate which is twice the normal value. Decreasing R_H by 10% results in a failure rate which is half the nominal value.

Concluding Remarks

In this chapter, representative failure rates for various electrical components have been given. It was shown how these rates could be used to calculate the failure rate and thus the reliability of an AM radio. If the stress relationships of the components making up the radio had been known, a more detailed failure rate analysis could have been performed. However, a problem would still have remained since there were no experimental figures with which to compare the calculated values.

Chapter V will present experimental failure rates of some military electronic equipment. The information in Chapter IV will be used to calculate their basic failure rates. Then these experimental and calculated values will be compared. It will be found that these values will be in close agreement.

V. CALCULATION OF BASE FAILURE RATES FOR VARIOUS MILITARY EQUIPMENT

ARINC Research Corporation has investigated many problems associated with determining the reliability of electronic parts and equipment. About 1957, they undertook a two-year study of the failures of 16 equipment types aboard the USS FORRESTAL (2). By using multiple regression techniques, they were able to ascertain values of the basic failure rate B_o and the cycling failure rate B_c for the various electronic equipment. The cycling failure rate B_c shall be investigated in Chapter VI. We will now concern ourselves with calculating the base failure rates B_o of this equipment.

It has been shown by Eq. 31 that if the number of each type of component in a system is known and the base failure rate of each of these components is known, the base failure rate of the system may be found. The number of components comprising the systems studied by ARINC are listed in Table 7. To obtain the base failure rates for the system, the base component failure rates of Table 1 must be used since the operational stress conditions of the components are not known (therefore the sophisticated adjustment procedures of their base failure rates presented in Chapter IV may not be used). The procedure to be used in estimating the B_o rates is identical to the technique used in forming the base failure rate estimates for the AM radio. This procedure,used in conjunction with the data of Table 7 and the failure rates of Table 1, yields the system base failure rates of Table 8. The experimental mean values B_{Eo} determined by ARINC and their standard deviations σ_{Eo} are given in the same table. Those readers interested in the details of how these results were obtained should refer to Appendix II. The experimental values of B_{Eo} and the average calculated values of B_{Co} are compared in Figure 3. The solid diagonal line represents the points at which the experimental and the calculated results agree. The dotted lines represent where the experimental value is equal to ± 50% of the calculated value. Evidently, 11 of the 16 systems had experimental failure rates which were calculated within 50% using the average failure rate values of Table 1. All of the experimental failure rates are within $0.3B_{Co}$ and $2B_{Co}$ limits. Considering the fact that the simplest possible approach has been used for calculating B_o, these results verify the usefulness of the method of analysis developed in the preceding chapters.

Table 7. The number of various types of components which comprise some electronic systems.

Equipment	Capacitors	Connectors, Etc.	Quartz Crystals	Diodes	Inductors Transformers	Motors	Relays	Resistors	Tubes
Receivers:									
AN/GRC-27	203	140	38	0	76	2	5	143	30
AN/SRR-11	150	261	1	2	27	0	1	126	28
AN/SRR-12	174	284	1	2	41	0	1	123	29
AN/SRR-13A	200	122	2	1	43	0	1	123	29
AN/URR-27	103	44	0	1	46	1	0	95	23
AN/URR-35A	125	63	2	0	49	1	0	116	22
Transmitters:									
AN/GRC-27	222	121	38	2	76	8	10	216	42
AN/URT-2	861	527	1	18	338	9	77	771	128
AN/URT-7	86	54	4	3	29	1	4	84	18
TED-4	86	61	4	2	25	1	4	84	18
Radars:									
AN/SPA-4	139	138	2	0	29	10	1	431	67
AN/SPA-4A	139	138	0	4	29	10	1	431	67
AN/SPA-8A	265	258	0	0	39	13	12	867	136
AN/SPG-48	453	620	0	8	174	19	45	1222	253
MK-7	726	536	0	13	77	45	58	2103	293
VL-1	80	96	0	0	20	7	3	393	73

Table 8. Equipment Base Failure Rate B_o (Failures/10^3 hours).

	Experimental		Calculated B_o		
Equipment	B_{Eo}	σ_{Eo}	Low	Average	High
Receivers:					
AN/GRC-27	0.29	0.28	0.43	0.46	0.52
AN/SRR-11	0.34	0.21	0.19	0.25	0.35
AN/SRR-12	0.30	0.11	0.22	0.26	0.37
AN/SRR-13A	0.55	0.17	0.19	0.27	0.71
AN/URR-27	0.21	0.25	0.24	0.27	0.35
AN/URR-35A	0.15	0.14	0.27	0.30	0.38
Transmitters:					
AN/GRC-27	0.36	0.39	1.1	1.2	1.3
AN/URT-2	1.45	2.2	2.3	2.7	3.8
AN/URT-7	0.10	0.94	0.68	0.72	0.92
TED-4	0.64	0.05	0.60	0.62	0.80
Radars:					
AN/SPA-4	0.83	0.74	0.80	0.93	1.6
AN/SPA-4A	0.89	0.41	1.4	1.5	2.1
AN/SPA-8A	2.05	0.41	0.95	1.2	2.1
AN/SPG-48	4.39	3.4	4.5	4.7	5.2
MK-7	4.17	2.5	4.1	4.4	9.5
VL-1	0.47	0.22	1.1	1.2	1.5

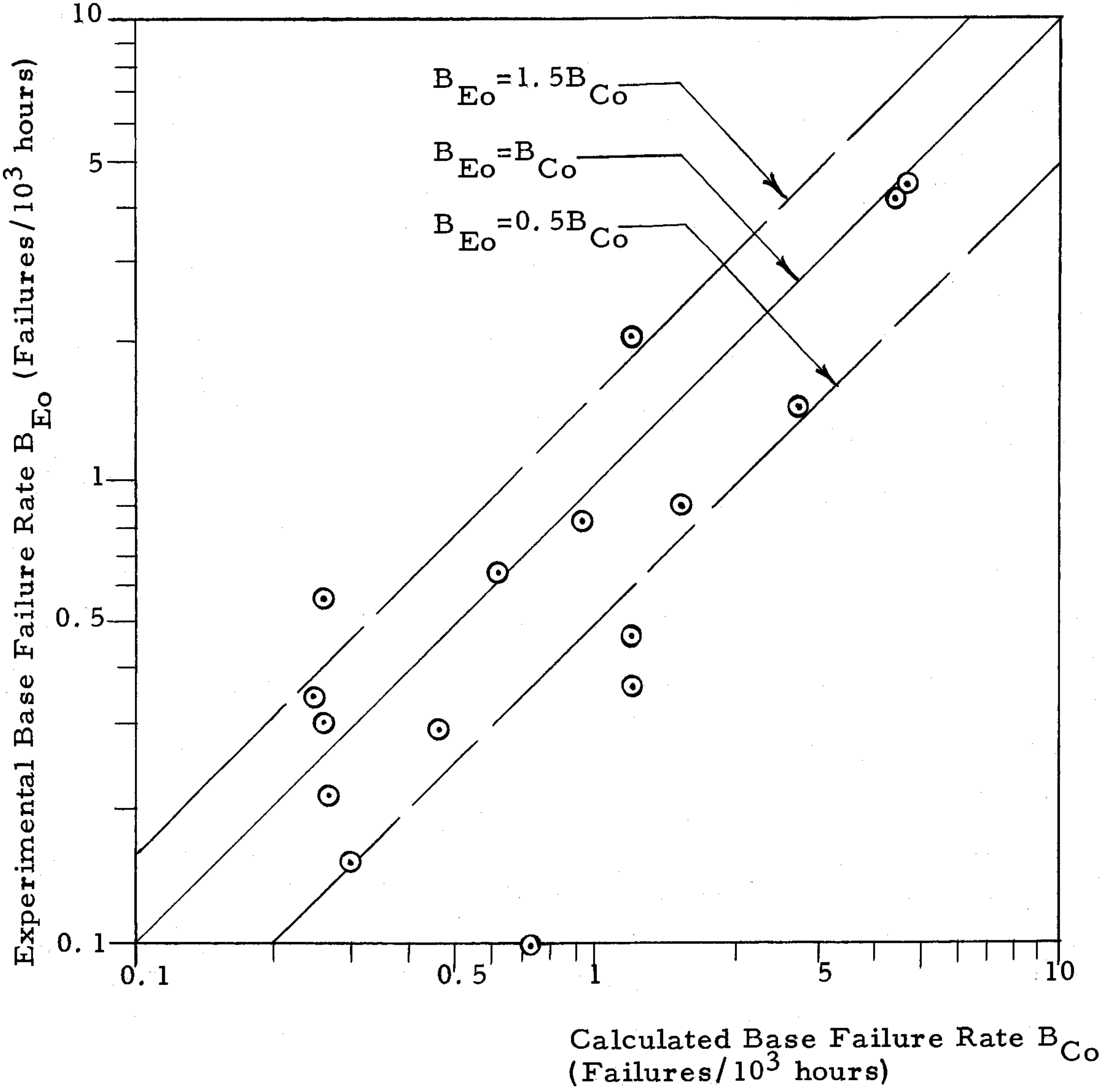


Figure 3. A comparison of the calculated average base failure rates B_{Co} and the experimental base failure rates B_{Eo} given in Table 8.

VI. INVESTIGATION OF THE CYCLING FAILURE RATE

Introduction

On-off power cycling was introduced in Figure 2 as an environmental condition which should be considered when making reliability studies. Mathematically, it was assumed that the effect of cycling on a component failure rate could be accounted for by Eq. 28. This equation indicated that power cycling added an increment of value to the component's base failure rate. The magnitude of this increment of value was proportional to both the system cycling failure rate and the power cycling rate applied to the system. It was indicated in Chapter V that ARINC had experimentally determined various system failure rates which had the form of Eq. 30. The base failure rate B_o for these systems was considered in Chapter V. Comparatively good agreement was obtained between the calculated and experimental system base failure rates. However, to the author's knowledge, no one has yet been able to quantatively describe and predict the effects of power cycling on system failure rates. This is a problem in the reliability field which has not been solved. If the cycling failure rate for various electrical components could be determined, then using Eq. 32, the susceptibility of a system to cycling failures could be predicted. This would form another valuable design tool. Thus, this chapter is devoted to determining the cycling failure rates for various components.

The experimental cycling failure rates (B_{Ec}) determined by ARINC for the various equipment are given in Table 9. The standard deviations σ_{Ec} of the experimental cycling failure rates were calculated using the method given in Appendix II. From the information of Tables 7 and 9, Eq. 32 might be used to calculate the component cycling failure rate B_{ci} provided n_i, k, and B_c are known and there are at least k independent equations. Let the general j^{th} cycling failure rate equation be

$$B_{jc} = \sum_{i=1}^{k} n_{ji} B_{ci} \tag{75}$$

where B_{jc} is the cycling failure rate value of the j^{th} equipment, n_{ji} is the number n_i of components of type i in the j^{th} equipment, B_{ci} is the component cycling failure rate of the i^{th} type component, and k is the number of component types in the equipment. Thus, using Eq. 75 and the data of Tables 7 and 9, the cycling failure rate expression for the AN/GRC-27 receiver is

$$\begin{aligned} 12 = {} & 203B_{c1}+140B_{c2}+38B_{c3}+0B_{c4}+76B_{c5}+2B_{c6} \\ & +5B_{c7}+143B_{c8}+30B_{c9} \end{aligned} \tag{76}$$

where B_{ci} is in failures/10^3 cycles for i = 1, 2, . . , 9. Evidently, from Tables 7 and 9, 15 additional cycling failure rate equations similar to Eq. 76 may be written. A system of J such equations may be expressed generally as

$$B_{jc} = \sum_{i=1}^{k} n_{ji} B_{ci} \quad \text{for } j = 1, 2, \ldots J \qquad (77)$$

Eq. 77 forms the basis for the analysis in this chapter.

Simultaneous Solutions

The first attempts at analysis involved the simultaneous solution of systems of equations in the form of Eq. 77 by the ALWAC-III computer. To facilitate notation and comparisons which follow, each step in the analysis is presented as a trial.

In Trial 1, the values of B_{Ec} in Table 9 were considered for the various receiver types. Since the B_{Ec} of the AN/GRC-27 was large compared to the other values (although it contained about the same number of components as the other receivers), it was decided not to use it in the analysis. There remained five cycling failure rates with which to form Eq. 77. This allowed solving for five component cycling failure rates. Thus, five element groups most often found in receivers were chosen which intuitively would have the highest cycling failure rates. Grouping similar components (assuming their cycling failure rates were about equal), these groups included capacitors, inductors and transformers, resistors, tubes, and connectors (including sockets and receptacles). Using the total number of elements in each group, five equations were formed from Eq. 77 (in this case of course $J = k = 5$). These five equations were solved simultaneously for the five component cycling failure rates. The Trial 1 entry in Table 10 gives the results which were found. It is disturbing that two negative component rates were obtained. Also, it is intuitively incorrect that tubes could have the lowest cycling failure rate.

In Trial 2, the Trial 1 procedure was repeated for the radar equipment case. The AN/SPA-4A was neglected because its B_{Ec} was small compared to the other rates (again it contained about the same number and types of components), and motors were considered in place of connectors. The Trial 2 entry in Table 1 gives the results which were found. Again negative results were obtained and again tubes had the lowest rate.

In Trial 3, the equipment types used in Trials 1 and 2 were combined. Arbitrarily eliminating the AN/URR-35A receiver from this group allowed the cycling failure rate for each of the component types of Table 7 to be determined. Although some negative rates were obtained as seen in Table 10, the order of the least susceptible to the most susceptible cycling components is intuitively almost correct, i. e. crystals, semiconductor diodes, inductors and transformers, resistors, connectors, capacitors, relays, tubes, and motors.

In Trial 4, it was decided to see the effect of repeating Trial 1 using different values of B_{Ec}. These adjusted values were obtained for each equipment by multiplying its B_{Ejc} value by the ratio of its average calculated B_{Cjo} value and the experimental mean value of B_{Ejo} found in Table 8. The results are shown in Trial 4 entry in Table 10. As would be expected, changing the B_{Ec} values in these equations has notably changed the simultaneous solutions.

In Trial 5, Trial 2 was repeated using adjusted B_{Ejc} values. Again this has a notable effect on altering the B_{ci} values.

For Trial 6, Trial 3 was repeated using these adjusted values. Comparing the results of these two trials, it is seen that the order of increasing B_{ci} components has almost been inverted. Evidently, a different approach is needed for this analysis since the simultaneous solutions depend so critically on the B_{Ec} values. Reflecting on the comparative largeness of the standard deviations σ_{Ec} for the cycling failure rates of Table 9, this method of simultaneous solution cannot be used successfully.

Multiple Regression Analysis

Multiple regression analysis is a standard statistical method which is used to solve more than k k-dimensional linear equations. It is used extensively in data analysis to obtain the best fit of a set of independent and dependent variables by an equation in the form of Eq. 75. This method is accomplished by first dividing each equation (in Eq. 77) by its standard deviation σ_{jc} (14, p. 44-58). Then

Eq. 77 becomes

$$(B_{jc}/\sigma_{jc}) = \sum_{i=1}^{k} (n_{ji}B_{ci}/\sigma_{jc}) \text{ for } j = 1, 2, \ldots J \qquad (78)$$

Then this system of J equations ($J > k$) is solved in the least squares sense. This consists of choosing the B_{ci} coefficients which will minimize Eq. 79, i. e.

$$\min \sum_{j=1}^{J} [(B_{jc}/\sigma_{jc}) - (\sum_{i=1}^{k} n_{ji}B_{ci}/\sigma_{jc})]^2 \qquad (79)$$

This solution gives the values of the component cycling failure rates which are the best least square approximations for the J equations (4, p. 56). The IBM 1620 computer was used to form these involved solutions.

The computer formed the solution through a reiterative process. Each step of this process yielded valuable statistical information. The procedure shall be described briefly since it has a bearing on the calculated results (10, p. 191-203). To simplify notation, Eq. 78 is written as

$$y_j = \sum_{i=1}^{k} a_i x_{ji} \quad \text{for } j = 1, 2, \ldots J \qquad (80)$$

The computer first calculated the k values of

$$\min \sum_{j=1}^{J} (y_j - a_i x_{ji})^2 \qquad (81)$$

for i=1, 2, . . , k by varying the value of a_i. It then selected the smallest value of Eq. 81 and printed out the value of the a_i coefficient which yielded this best least square approximation to the system of J equations. Say that the x_{j1} term yielded the closest approximation. The computer than calculated the k-1 values of

$$\min \sum_{j=1}^{J} (y_j - b_1 x_{j1} - b_i x_{ji})^2 \qquad (82)$$

for i = 2, 3, . . . , k by varying the values of b_1 and b_i. It again printed out the value of the b_1 and b_i coefficients which yielded the best least square approximation to the system of equations. The x_{ji} variable which was added was the one which made the greatest improvement in the approximation. Usually, a_1 would have its value changed to b_1 to make this improvement. This process was repeated until the coefficients of all the independent variables had been found. Thus, the last expression evaluated by the computer was

$$\min \sum_{j=1}^{J} (y_j - a_1 x_{j1} - a_2 x_{j2} - \dots - a_k x_{jk})^2 \qquad (83)$$

If $J \leq k$, the computer terminated the solution when J-1 coefficients had been calculated.

To judge whether or not the approximation has improved with the addition of each independent variable, the standard deviation of y given x (denoted as $\sigma(y.x)$) is considered. This quantity is defined as

$$\sigma_n(y.x) = (Q_n/(N-p))^{\frac{1}{2}} \qquad (84)$$

where Q_n corresponds to the smallest minimum value found for the n^{th} least square approximation, N is the number of independent equations used, and p is the number of independent variables in the approximating equation, e. g. N=J and p=2 in Eq. 82. The difference between N and p is often called the degrees of freedom of the solution. Generally, $\sigma_n(y.x)$ is large for large values of N-p, it decreases to some minimum value, and then increases as N-p decreases towards zero. The best least square approximation for the system of J equations is obtained in the step at which $\sigma_n(y.x)$ assumes its minimum value. This arbitrary choice is generally used in statistical analysis because it seems to yield realistic results. Since the computer calculates and prints out this quantity and the values of the a_i's before every reiteration, it is a simple matter to judge when the best approximation is obtained. This was the criterion used to acquire the component failure rates in the following trials.

In Trials 7 and 8, the receiving equipment in Table 9 was considered. Since the results of multiple regression analysis improve as the degrees of freedom become larger, no equations were eliminated from the analysis. Although J=6 and p=9 in this case (there are more unknowns than equations), $\sigma_n(y.x)$ will hopefully reach a minimum and begin increasing before p=6. Examining the Trial 7 entry of Table 10, this was indeed the case since $\sigma_n(y.x)$ reached its smallest value when p=4. Although inductors and transformers have a small negative B_{ci} value, the total cycling rate B_c for a receiver should be positive because of the over-riding effects of the other comparatively large rates. The standard deviation of each of these values follow below the Trial 7 entry. The computer calculates these by an involved statistical method. Evidently, they are about equal to their respective rates. This is interpreted to mean that more reliable B_{ci} estimates cannot be made because of the relatively small degrees of freedom available in this analysis.

Trial 8 consisted of using the data of Trial 7 to determine a component rate which would facilitate more rapid (but unfortunately less accurate) cycling failure rate calculations for receivers. This was taken as the B_{ci} value which was found in the first step of the approximations and thus satisfied Eq. 81. Since the degree of freedom for this approximation was larger than in the previous trial, the standard deviation for this B_{ci} was much smaller.

Transmitting equipment was considered in Trials 9 and 10. In this case, only tubes, motors, and relays were considered since there were only four experimental transmitter cycling failure rate values available to use in the analysis. The Trial 9 entry in Table 10 gives the least square approximations for the B_{ci}'s which occurred when $p=2$. The standard deviations were equal to about half of their respective B_{ci} values.

Trial 10 consisted of determining the component failure rate which would be simpler to evaluate and which would give reasonable results. This was the B_{ci} value which satisfied Eq. 81 for transmitters.

In Trials 11 and 12, radar equipment was considered. Since there were the same number of radar types as there were receiver types, J=6 and p=9. Here, the best approximation involved only the three terms. In Trial 11, $\sigma_n(y.x)$ was minimized for p=3. Although a negative cycling failure rate was obtained for inductors and transformers, its effect in radar cycling rate calculations would undoubtedly be compensated for by the effects of the other coefficients. Evidently, the standard deviations are about half of their respective cycling rate values.

Trial 12 consisted of finding component failure rates from the data of Trial 11 which would simplify radar failure rate calculations. Here, p=2 was chosen as the approximation. The standard deviations were equal to about half of their respective B_{ci} values.

Although only combinations of diodes, inductors and transformers, motors, relays, and tubes are involved in these six trials, there are no general trends which are evident from the data. In an attempt to find more general failure rates, the component cycling failure rates were determined for all the components using all of the system failure rates (J=16, p=9). These overall rates are given in the Trial 13 entry. Their order of susceptibility to cycling (increasing failure rates) is: motors, inductors and transformers, resistors, capacitors, connectors, tubes, relays, crystals, and diodes. Since the standard deviations are twice as large as the respective failure rates in five out of the nine cases, this order is rather insignificant.

The best overall approximation given in the Trial 14 entry was obtained from the Trial 13 data where $p = 5$. Although the standard deviations are small compared to the respective B_{ci} values, it is unreasonable that diodes and especially quartz crystals could have a higher cycling failure rate than tubes.

The single component failure rate which satisfied Eq. 81 (again using the data of Trial 13) is given in the Trial 15 entry. Evidently, tubes give the best single least square approximation for the cycling failure rates for all the equipments.

Now that these various approximations have been found, all but the results of Trials 13 and 14 shall be used to calculate the cycling failure rates of this equipment. Then these calculated rates shall be compared with their respective experimental rates.

The best approximations of the system cycling failure rates are obtained by using the component cycling failure rates found in Trials 7, 9, and 11 in Eq. 75, where the appropriate value of n_{ji} is found in Table 7. These results are given in the B_{C1c} column of Table 9.

The component cycling failure rates found to simplify calculation procedures in Trials 8, 10, and 12, yield the system cycling failure rate B_{C2c} when substituted into Eq. 75.

The simplest and most general component cycling rate found in Trial 15 was used in Eq. 75 to determine the values of B_{C3c} for the system.

The best approximation (B_{C1c}) and the simplest and most general approximation (B_{C3c}) are compared with their experimental values (B_{Ec}) in Figs. 4 and 5, respectively. Evidently from Figure 4, only four of the 16 B_{Ec} values were less than $0.5B_{C1c}$ or greater than $2B_{C1c}$. All B_{Ec} values were within $0.25B_{C1c}$ and $4B_{C1c}$ values. However from Figure 5, eight of the 16 B_{Ec} values fell outside the $0.5B_{C3c}$ and $2.0B_{C3c}$ limits while three were outside $0.25B_{C3c}$ and $4B_{C1c}$ values. Clearly, better results were obtained by using the best approximation values; yet it is surprising that the single tube approximation gave such comparatively close results. In Figures 4 and 5, cycling failure rate estimates falling in the region above the $B_{Ec} = B_{Cc}$ line are optimistic while those in the region below this line are pessimistic. Thus, the calculated B_{Cc} estimates tend in these figures to be optimistic, i. e. the estimated cycling failure rate is generally smaller than the experimental cycling failure rate.

This analysis has been hampered by the relative lack of

Table 9. Equipment Cycling Failure Rate B_c (Failures/10^3 cycles).

Equipment	Experimental		Calculated B_c		
	B_{Ec}	σ_{Ec}	B_{C1c}	B_{C2c}	B_{C3c}
Receivers:					
AN/GRC-27	15.0	11.2	15.2	4.3	2.8
AN/SRR-11	2.9	4.4	2.5	1.5	2.6
AN/SRR-12	2.3	0.4	2.3	2.3	2.7
AN/SRR-13A	1.6	1.9	1.7	2.4	2.7
AN/URR-27	1.0	8.4	4.0	2.6	2.2
AN/URR-35A	3.9	3.4	3.4	2.7	2.1
Transmitters:					
AN/GRC-27	26.0	6.2	25.6	14.3	4.0
AN/URT-2	30.0	21.8	44.3	43.4	12.0
AN/URT-7	22.0	12.2	5.6	6.1	1.7
TED-4	5.7	0.7	5.6	6.1	1.7
Radars:					
AN/SPA-4	25.0	17.0	5.0	5.0	6.3
AN/SPA-4A	5.0	0.7	7.0	4.8	6.4
AN/SPA-8A	12.0	12.3	13.6	11.3	12.8
AN/SPG-48	24.0	1.4	23.7	23.5	23.8
MK-7	37.0	9.1	37.0	28.0	27.6
VL-1	15.0	10.6	6.6	5.5	6.9

Table 10. Component Cycling Failure Rates B_{ci} (Failures/10^3 Cycles).

Trial	Capacitors	Connectors, etc.	Crystals	Diodes	Inductors and Transformers	Motors	Relays	Resistors	Tubes
Trial 1	-0.002	0.004			0.01			0.11	-0.45
Trial 2	-0.87				2.0	1.08		0.61	-2.77
Trial 3	5.7	-0.4	-254.	-62.6	-11.8	43.6	9.4	-5.1	20.6
Trial 4	0.001	0.009			0.12			0.16	-0.90
Trial 5	-0.64				-0.07	7.6		-0.06	1.0
Trial 6	-33.3	2.4	1148.	363.	67.4	-221.	-61.4	28.5	-114.
Trial 7				0.56	-0.011	3.93	1.63		
σ_{ci}				0.48	0.025	1.27	0.67		
Trial 8					0.056				
σ_{ci}					0.005				
Trial 9						2.19			0.192
σ_{ci}						1.20			0.089

Table 10. Continued.

Trial	Capacitors	Connectors, etc.	Crystals	Diodes	Inductors and Transformers	Motors	Relays	Resistors	Tubes
Trial 10									0.336
σ_{ci}									0.054
Trial 11					-0.066		0.221		0.10
σ_{ci}					0.055		0.101		0.025
Trial 12							0.125		0.073
σ_{ci}							0.066		0.010
Trial 13	-0.006	0	0.515	1.306	-0.052	-0.133	0.283	-0.009	0.103
σ_{ci}	0.030	0.008	0.566	0.566	0.087	1.45	0.147	0.066	0.206
Trial 14	-0.008		0.484	1.044			0.295		0.021
σ_{ci}	0.003		0.101	0.033			0.064		0.020
Trial 15									0.094
σ_{ci}									0.008

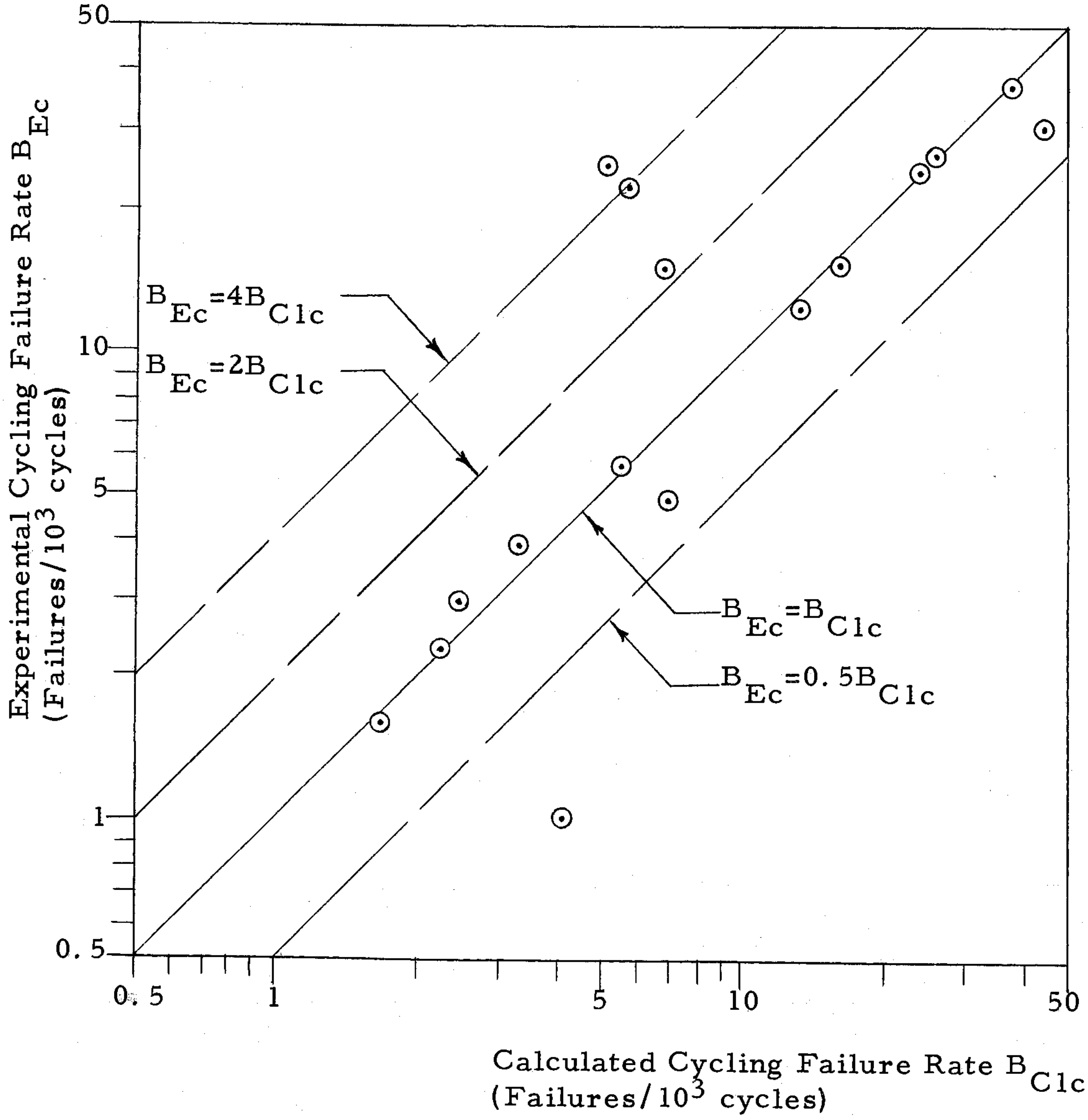


Figure 4. A comparison of the best calculated cycling failure rates B_{C1c} and the experimental failure rates B_{Ec} given in Table 9.

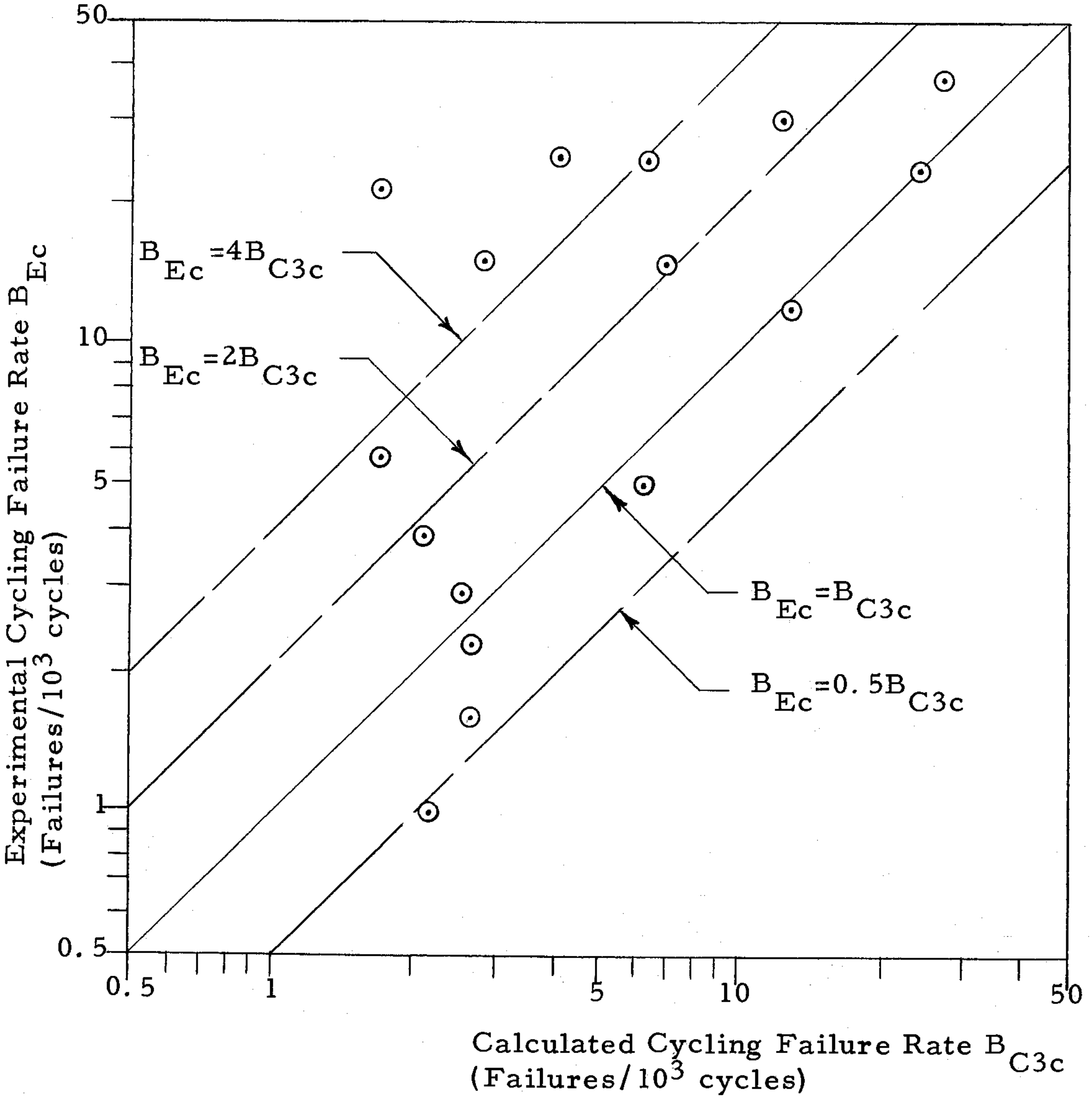


Figure 5. A comparison of the overall calculated cycling failure rates B_{C3c} and the experimental failure rates B_{Ec} given in Table 9.

representative information concerning the experimental failure rate B_{Ec}. As has already been discussed, these rates were obtained originally through use of multiple regression analysis. The data used for this analysis involved analyzing the failures which occurred in shipboard equipment. Since there were only about ten devices of each of the 16 types, and every device failed only about 15 times over the two-year test period, the B_{Ec}'s obtained are unlikely to be truly representative values for the equipment types. Since the results of this chapter were based upon these B_{Ec} values, these results may not be used with too great a confidence. However, this is a new problem and no other data has been systematically accumulated. The results which have been found are the best that may be derived from this data. Until new data becomes available, the component cycling failure rates which have been found cannot be verified. Although these values give usable cycling failure rates for the systems investigated, they may not necessarily be general enough to give usable results for analyzing other systems. However, keeping these limitations in mind, they might be used to give a mathematical means to estimate a system's cycling failure rate.

An Example

Consider again the AM radio of Chapter IV. The most representative rate determined in the regression analysis of Chapter VI was the overall rate for tubes where B_{ci}=0.094 failures/10^3 cycles. Since the AM radio contained five tubes (Table 2), its cycling failure rate B_c from Eq. 32 is

$$B_c=5(0.094) = 0.47 \text{ failures}/10^3 \text{ cycles} \tag{85}$$

The average base failure rate from Table 2 was

$$B_o= 46 \text{ failures}/10^6 \text{ hours} \tag{86}$$

Thus, the total failure rate B of the AM radio by Eq. 30 is

$$B = 46+0.47N = 46(1+0.01N) \text{ failures}/10^6 \text{ hours} \tag{87}$$

where N is in cycles/10^3 hours. It was seen in Table 2 that this radio would operate continuously for at least 1100 hours with a probability of 95%. Suppose that this AM radio is used five days a week for eight hours a day. It is turned on in the mornings, allowed to operate continuously for eight hours, and is then turned off. Neglecting the effects of cycling, this radio would operate for

$$1100 \text{ hours}/40 \text{ hours per week} = 27.5 \text{ weeks} \tag{88}$$

with a probability of 95%. Including cycling effects however, the radio would operate for only (by Eq. 34)

$$T = 0.051/(0.046(1+0.415)) = 775 \text{ hours} \tag{89}$$

or

$$775 \text{ hours}/40 \text{ hours per week} = 19.5 \text{ weeks} \tag{90}$$

since N = 41.5 cycles/10^3 hours (which corresponds to one power cycle/24 hours) with a probability of 95%. Apparently, cycling considerations have reduced T almost 30%. This example illustrates the need for including cycling effects in reliability models to allow more realistic reliability estimates of systems to be made.

VII. CONCLUSION

Reliability is a primary design goal in electronic systems. This paper has developed the notion of reliability, described it mathematically, and has then developed empirical values which could be used in conjunction with the mathematical theory to estimate the reliability of electronic equipment.

It was found that electronic system base failure rate estimates could be made by using empirically derived base failure rates for the components comprising the system. It was shown however, that to obtain more accurate failure rate estimates, the effects of on-off power cycling of the electronic equipment needed to be considered. From the data available, it was concluded that the cycling failure rate of the system could be found by considering only the tubes in the system. Using both the base failure rate and the cycling failure rate estimates enable realistic reliability estimates of systems to be made.

These concepts which have been formed and the empirical data which has been presented demonstrate the importance of designing reliability "into" an electronic system. A properly used but poorly designed system cannot operate reliably just as an improperly used but well designed system cannot operate reliably. Only by careful and meticulous design, conscientious manufacture, and proper use may an electronic system reach its optimum effectiveness.

BIBLIOGRAPHY

1. Aeronautical Radio, Inc. Electronic reliability in military applications. Washington, 1957. 287 p. (Aeronautical Radio publication no. 102)

2. Boteilho, R. J. Effects of cycling on reliability of electronic tubes and equipments. Washington, 1960. 2 vols. (ARINC Research Corporation. Publication no. 101-26-160)

3. Clement, Lewis M. Program on reliability of electronic equipment. Institute of Radio Engineers. Transactions of Professional Group on Quality Control PGQC-3:1-9. Feb. 1954. (Papers of Radio Fall Meeting, Toronto, Oct. 26-28, 1953)

4. Davis, Harry F. Fourier series and orthogonal functions. Boston, Allyn and Bacon, 1963. 403 p.

5. Earles, D. R. Reliability application and analysis guide. Denver, 1961. 134 p. (Martin Company publication no. MI-60-54)

6. Lusser, Robert. Unreliability of electronics-cause and cure. Huntsville, Ala., Redstone Arsenal, 1957. 13 p.

7. Parzen, Emanuel. Modern probability theory and its applications. New York, Wiley, 1960. 464 p.

8. Pieruschka, Erich. Mathematical foundation of reliability theory. Huntsville, Ala., Redstone Arsenal, 1958. 26 p.

9. Pieruschka, Erich. Principles of reliability. Englewood Cliffs, Prentice-Hall, 1963. 365 p.

10. Ralston, A. and Herbert S. Wilf. Mathematical methods for digital computers. New York, Wiley, 1960. 293 p.

11. U.S. Dept. of Defense. Reliability of military electronic equipment. Washington, 1958. 12 p. (Publication no. MIL-STD-441)

12. U.S. Dept. of Defense. Reliability stress and failure rate data for electronic equipment. Washington, 1962. 328 p. (Publication no. MIL-HDBK-217)

APPENDICES

APPENDIX I

The failure rates of electrical components are determined experimentally. To do this, a very large number of similar components are placed in similar electrical and thermal stress conditions. Then the number and time of the failures which occur are recorded. If the components are assumed to fail because of chance, a histogram of the failures which occurred per unit of time would be similar to Figure 6. If the population of the test lot was increased indefinitely and the experiment repeated, a frequency distribution curve of the failures could be obtained similar to the dotted curve of Figure 6. If the total number of units tested is divided into each of the ordinate values of Figure 6, the dotted distribution curve becomes a probability density function. Theoretically, this curve should be an exponential density function and may be used to calculate the average reliability of a component in the population tested. Since the mean of this mortality function may be easily found, the failure rate (the reciprocal of the mean of this distribution) of the component can be determined.

If this experiment is repeated using identical components for many different electrical and thermal stress conditions, a graph of the failure rate for the various conditions may be drawn. A graph similar to Figure 7 might well result. This graph forms an invaluable prediction aid to the electronic designer. If future components remain approximately identical to those tested, the designer may estimate their failure rates under virtually any stress condition he desires.

Now many companies have developed failure rate curves similar to Figure 7 (5 and 12). The curves which were used to derive the failure rate expressions in Chapter IV were obtained from Dept. of Defense publication MIL-HDBK-217 (12). These derived expressions were in close agreement with the results of derivations from Martin Company publication MI-60-54 (5). The procedure used in these derivations shall now be presented using the general purpose ceramic capacitor as the example (12, p. 117).

From Figure 7, the failure rate B is an exponential function of temperature T', or

$$B = B_o \exp(aT') \tag{91}$$

where B_o is a base failure rate and a is a constant. Denote any two sets of coordinates for a given R_v as (B_1, T_1') and (B_2, T_2'). Since

$$\ln B_1 - \ln B_2 = \ln(B_1/B_2) = a(T_1' - T_2') \tag{92}$$

if we take $B_1 = 10B_2$, then

$$a = \ln 10/(T_1' - T_2') = 2.3/(T_1' - T_2') \tag{93}$$

or

$$1/a = (T_1' - T_2')/2.3 \tag{93}$$

Now Eq. 91 may also be expressed as

$$B = B_o \exp(aT') = B_o C^{T'} \tag{95}$$

where C is a constant. Since the latter case is perhaps easier to visualize, solve Eq. 95 for C in terms of a.

Here

$$\exp(aT') = C^{T'} \tag{96}$$

and so

$$C = \exp(a) \tag{97}$$

Figure 8 may be constructed from Figure 7. Since the results are approximately straight lines on log-log paper, the general solution for the failure rate B in terms of voltage ratio R_v is

$$B = B_o R_v^{\,d} \tag{98}$$

where B_o is a base failure rate and d is a constant. Again, choosing any two sets of coordinates for a given T, say (B_1, R_{1v}) and (B_2, R_{2v}), d may be expressed as

$$d = \ln(B_1/B_2)/\ln(R_{1v}/R_{2v}) \tag{99}$$

Now the failure rate B may be expressed as a function of these two independent variables, i. e. the thermal and electrical stress, as

$$B=B_o(R_v^{\,d})(C^{T'}) \tag{100}$$

It is a simple matter to find the values of the constants from Figures 7 and 8. Let the base temperature and electrical stress ratio of a component be arbitrarily equal to 40°C and one, respectively. Let

$$T' = (T-40)/10 = K \tag{101}$$

If ($B_1 = 1.0$, $T_1 = 131^\circ C$) and ($B_2 = 0.1$, $T_2 = 63^\circ C$) when $R_v = 1$ from Figure 7, then by Eqs. 93 and 97,

$$\begin{aligned} a &= 2.3/(9.1-2.3) = 0.348 \\ C &= \exp(0.348) = 1.41 \end{aligned} \tag{102}$$

If ($B_1 = 0.040$, $R_{1v} = 1.0$) and ($B_2 = 0.014$, $R_{2v} = 0.7$) when $T = 40^\circ C$ from Figure 8, then by Eq. 99

$$d = \ln(0.040/0.014)/\ln(1.0/0.7) = 1.05/0.358 = 2.93 \tag{103}$$

Thus, the effective failure rate for this ceramic capacitor is given by Eq. 100 as

$$B = B_o R_v^{2.9} (1.4)^K \tag{104}$$

where B_o is the base failure rate of this capacitor found in column two of Table 1. The failure rate values in this column correspond to the component failure rates when $R_v = 1.0$ and $T = 40^\circ C$. Evidently from Figure 8, the minimum failure rate regardless of stresses is 0.01 failures/10^6 hours. The minimum failure rate values are given in column one of Table 1. Column three represents the failure rates where $R_v = 1.0$ and $T = 80^\circ C$.

Examination of Eq. 104 reveals that the failure rate of a component may be increased by increasing thermal and electrical stresses. This implies that the mean of the exponential distribution of Figure 6 is becoming smaller, i.e. failures are occurring more rapidly. Therefore, Eq. 104 is often called an accelerated life expression.

Accelerated life theory is extremely important in reliability studies. This may be easily seen by considering the ceramic capacitor. If it is assumed a piori that Eq. 104 describes the failure rate of this component, then large values of R_v and K may be used to produce a higher effective failure rate B, and thus, a smaller mean of the exponential mortality distribution. Then, knowing B, R_v, and K from an experiment, Eq. 104 may be used to calculate B_o. This method of evaluating B_o consumes either less time or fewer components and may therefore be more economical to carry out. Those interested in investigating this subject of accelerated life testing may find the following articles helpful:

Cary, Hall and Ralph E. Thomas. Accelerated testing as a problem of modeling. IRE Transactions. PGQC-6: 69-87.

Davis, D. J. An analysis of some failure data. Journal of the American Statistical Association 47:113-150. June 1952.

Earles, D. R. and M. F. Eddins. A theory of component part life expectancies. IRE Transactions. PGQC-8:252-267.

Epstein, Benjamin. Life test acceptance sampling plans when the underlying distribution of life is exponential. IRE transactions. PGQC-6:353-360.

Epstein, B. and M. Sobel. Life testing. Journal of the American Statistical Association 48:486-502. Sept. 1953.

Magistad, J. G. Some discrete distributions associated with life testing. IRE transactions. PGQC-7:1-11.

Nucci, E. J. Temperature effects on electronic reliability. IRE Transactions. PGQC-4:259-266.

Pettinato, Anthony D. Accelerated reliability testing. IRE Transactions. PGQC-7:241-252.

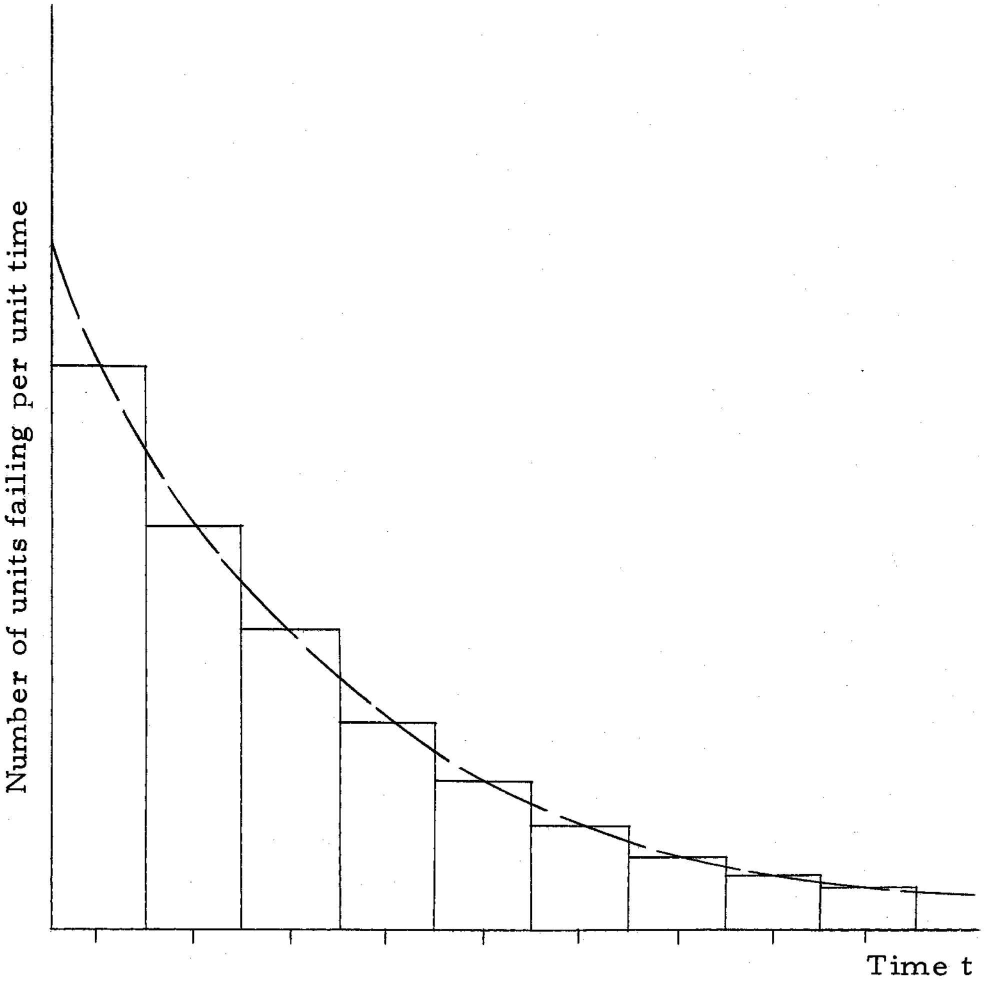


Figure 6. Histogram and frequency distribution of the failures occurring in a failure rate determination experiment.

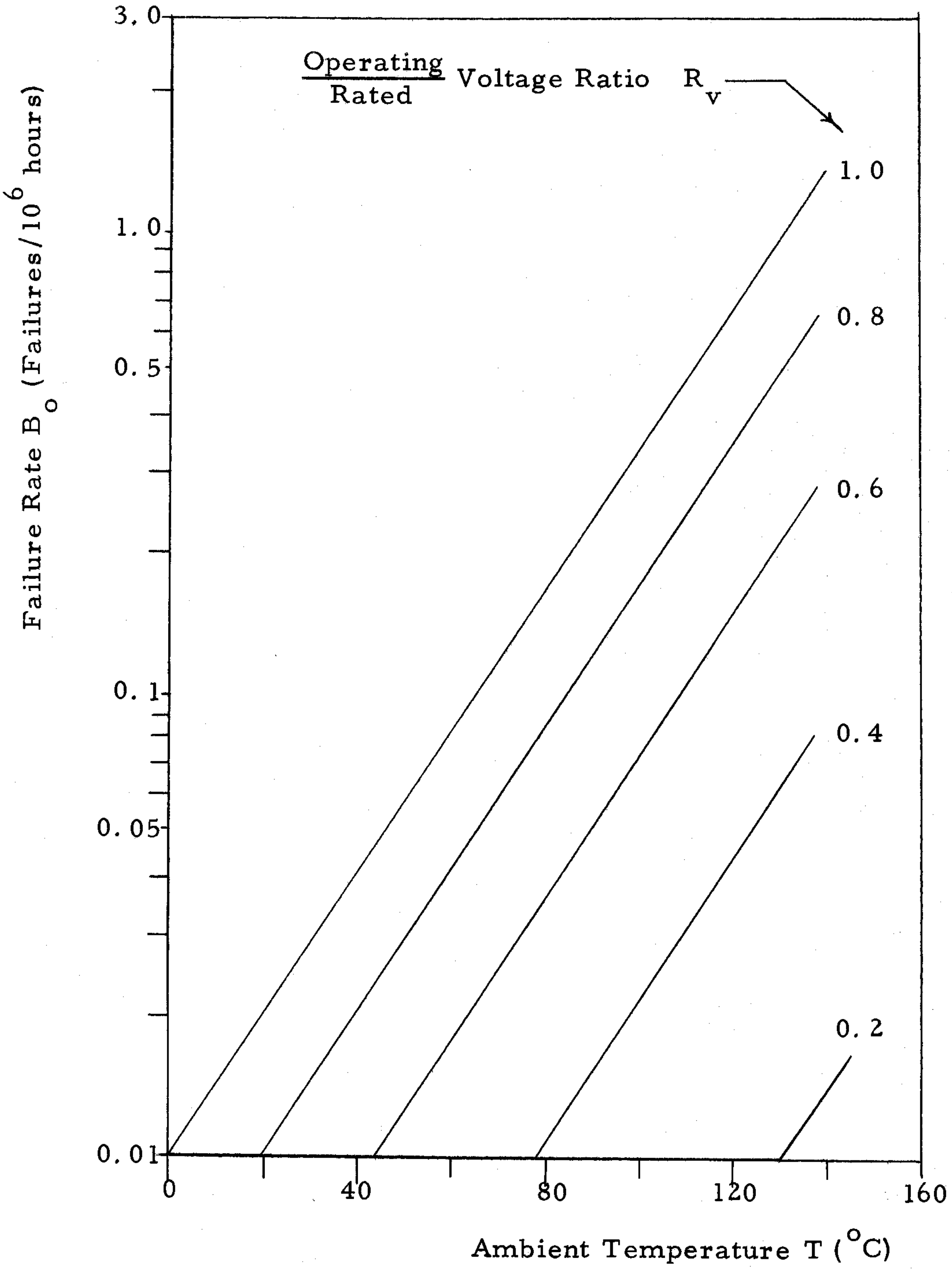


Figure 7. Failure rates of general purpose ceramic capacitors.

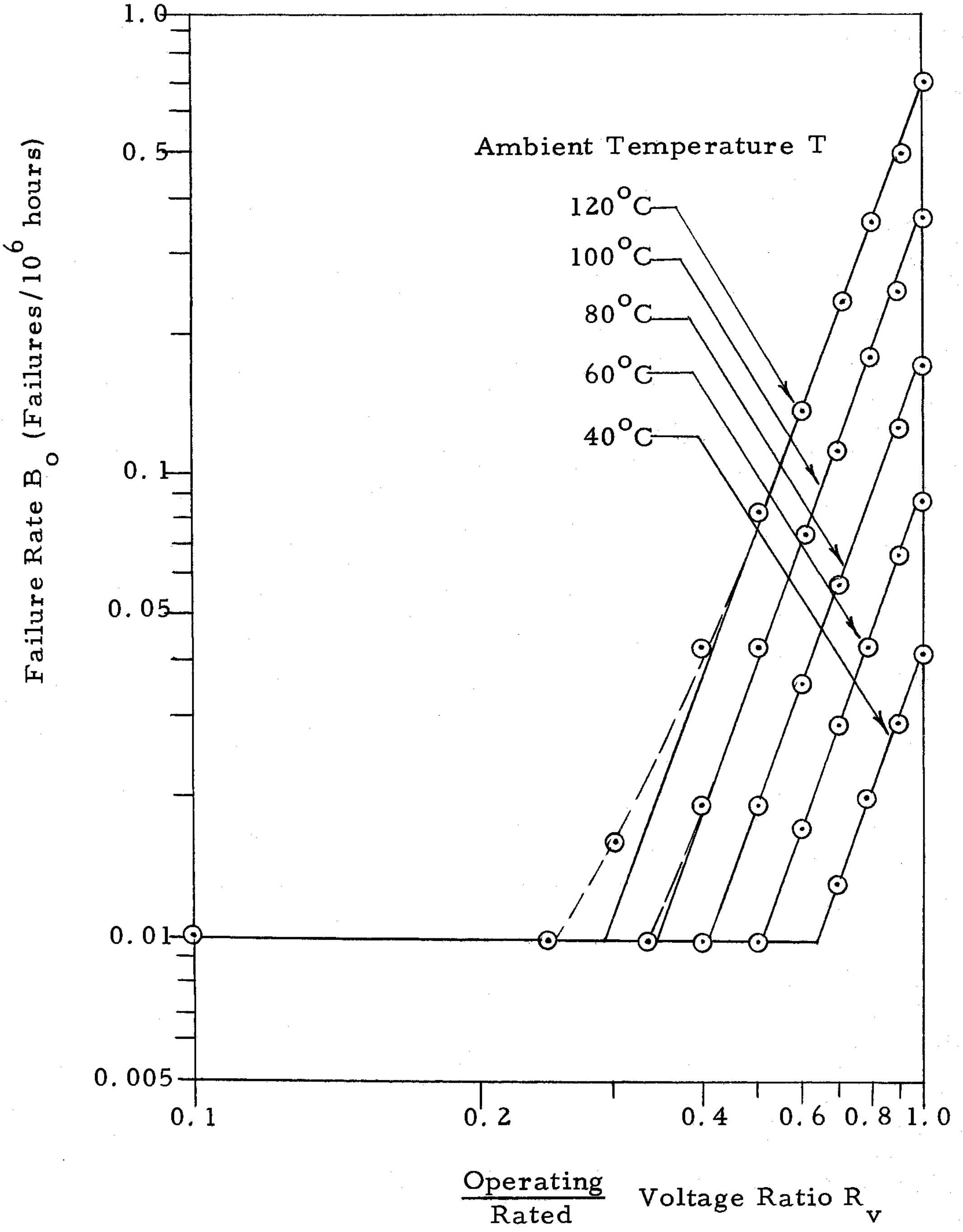


Figure 8. Failure rates of general purpose ceramic capacitors.

APPENDIX II

ARINC analyzed failures occurring in 158 pieces of military electronic equipment of 16 types during four periods of time, i. e. the first six months, the second six months, the second year, and the entire two years of operation. This was done so as to be able to investigate how the onset of wear-out in electronic tubes would effect the failure rate. For each of the equipment types, a base failure rate B_o and a cycling failure rate B_c was determined for each of the four periods of time. The base failure rate values shall be denoted as B_{1o}, B_{2o}, B_{3o}, and B_{4o} and the cycling failure rate values as B_{1c}, B_{2c}, B_{3c}, and B_{4c}. It was reasoned that the most realistic failure rate would result from the analysis performed over the longest length of time. Therefore, the mean of the four values was taken as

$$B_o = B_{4o}, \; B_c = B_{4c} \tag{105}$$

These are the mean values of B_o in Table 8 and B_c in Table 9. To obtain an estimate of the spread of these values, the standard derivations σ_o and σ_c were determined.

To serve as an example, consider the AN/SRR-13A receiver. Since σ_o and σ_c are found by exactly the same method, only σ_o shall be derived. If each of the three estimates B_{1o}, B_{2o}, and B_{3o} are equally likely to give the correct value of the equipment base failure rate B_o, then

$$P(B_o = B_{1o}) = P(B_o = B_{2o}) = P(B_o = B_{3o}) = 1/3 \tag{106}$$

The standard deviation σ_o of B_o is defined by (7, p. 203-210)

$$\sigma_o^2 = \sum_{j=1}^{3}[(B_{jo} - B_o)^2 \, P(B_{jo})] = (1/3) \sum_{j=1}^{3} (B_{jo} - B_o)^2 \tag{107}$$

The standard deviation σ_c of B_c is defined by Eq. 107 by interchanging σ_c for σ_o, B_{jc} for B_{jo}, and B_c for B_o. It was found for this receiver that $B_{1o} = 0.38$, $B_{2o} = 0.40$, $B_{3o} = 0.65$, and $B_{4o} = 0.55$ where B_{jo} is in failures/10^3 hours (2, vol. 2, p. 13). Thus, the mean of these values is by Eq. 104,

$$B_o = B_{4o} = 0.55 \tag{108}$$

Therefore, by Eq. 107,

$$\begin{aligned} \sigma_o^2 &= [(0.38-0.55)^2 + (0.40-0.55)^2 + (0.65-0.55)^2] \, (1/3) \\ &= (0.0288 + 0.0225 + 0.01) \, (1/3) = 0.0204 \end{aligned} \tag{109}$$

so

$$\sigma_o = (0.0204)^{\frac{1}{2}} = 0.14 \tag{110}$$

Although the standard deviation is not used directly in the analysis of Chapter V, it will prove valuable in the work of Chapter VI.